MATH3202 Assignment 1

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Note: Data from Matthew Smith will be used throughout this report.

Introduction

The aim of this report is to produce a model optimising the usage of four natural gas generators, and power distribution within the network of transmission lines and substation nodes operated by ELECTRIGRID. The formulation of the model, and the development of the result for each further communication with ELECTRIGRID is documented throughout the report.

Section A - Mathematical Formulation

Sets

Ultimately, the mathematical formulation for this problem directly addresses the final communication, however, it is acknowledged that each communication builds upon the previous ones. Thus, very similar mathematical formulations to the one developed below can be used for every communication, with any irrelevant data and constraints removed in order to suit the problem. Nonetheless, the sets for this problem can be defined as follows.

$$\begin{aligned} T &: \text{Periods} \\ N &: \text{Nodes} \\ A &: \text{Arcs} \\ G &: \text{Generators} \\ L &: \text{Arcs with no power limit} \end{aligned}$$

Data

The data used to solve this problem has been defined below.

$$\begin{aligned} D_{nt} &: \text{demand of node } n \in N \text{ for the period } t \in T \text{ (in MW)} \\ X_n &: \text{x coordinate of node } n \in N \text{ (in km)} \\ Y_n &: \text{y coordinate of node } n \in N \text{ (in km)} \\ Q_g &: \text{maximum capacity of generator } g \in G \text{ (in MW)} \\ C_g &: \text{cost for each generator } g \in G \text{ (in \$/MWh)} \\ loss &: \text{power loss over the arcs (= 0.1 \%/km)} \\ ArcLimit &: \text{power limit for each arc } a \in A \text{ such that } a \notin L \text{ (= 129 MW)} \\ \Delta P_{max} &: \text{maximum change in power for a generator (= 216 MW)} \\ S_a &: \text{start node (node 1) for arc } a \in A \\ E_a &: \text{end node (node 2) for arc } a \in A \\ d_{ij} &: \text{distance from node } i \in N \text{ to node } j \in N \text{ (in km)} \end{aligned}$$

Note that the distance can be defined as follows.

$$d_{ij} = \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2}$$

Variables

The variables within this problem include the power distributed by each generator and the power flowing from one node to another via an arc/transmission line. These are defined as follows.

F_{ijt} : Flow from node $i \in S_a$ to node $j \in E_a$ for the arc $a \in A$ and for the period $t \in T$ (in MW)
P_{gt} : Power output from generator $g \in G$ for the period $t \in T$ (in MW)

Objective Function

The ultimate objective of this problem was to minimise the cost spent to power the substation nodes within the ELECTRIGRID network. Therefore, it was determined that the objective should be formulated such that the sum of the each generator output multiplied by 4 times the cost of that generator (to model the 4 hours the generators operate for over a period) was minimised over the 6 periods throughout the day. This can be described mathematically, as shown below.

$$\text{Minimise Cost} \Rightarrow \text{Cost} = \sum_{t \in T} \sum_{g \in G} 4 P_{gt} C_g$$

Constraints

For the final communication, the following 6 constraints had to be set according to the information given from the company.

Constraint 1 - Power Flow Through Generators
The first constraint accounts for the fact that the net power output in each generator (power flowing out subtract power flowing in) must be equal to the power required for each generator. Ultimately, the losses in communication 2 must be applied to the flow into the generator that is coming from another nearby node. Mathematically, this can be described as follows.

$$\sum \text{Flow out} - \sum \text{Flow in} = P_{gt}$$
$$\Rightarrow \sum_{\substack{j \in E_a \\ \text{s.t. } g \in S_a}} F_{gjt} - \sum_{\substack{i \in S_a \\ \text{s.t. } g \in E_a}} F_{igt}(1 - loss \times d_{ig}) = P_{gt} \ , \forall g \in G, \forall t \in T$$

Constraint 2 - Power Flow Through Nodes
For all other substation nodes that have a unique demand, the net flow into the node (power flowing in subtract power flowing out) must meet the specific demand of that node. Again, the losses in communication 2 must be applied to the flow into the node that is coming from another nearby node. As such,

$$\sum \text{Flow in} - \sum \text{Flow out} = D_{nt}$$
$$\Rightarrow \sum_{\substack{i \in S_a \\ \text{s.t. } n \in E_a}} F_{int}(1 - loss \times d_{in}) - \sum_{\substack{j \in E_a \\ \text{s.t. } n \in S_a}} F_{njt} = D_{nt} \ , \forall n \in N \setminus G, \forall t \in T$$

Constraint 3 - Generator Capacity
As specified in the first communication, each generator has a capacity that cannot be exceeded during any period in the day, as described through the following constraint.

$$P_{gt} \leq Q_g \Rightarrow \forall g \in G, \forall t \in T$$

Constraint 4 - Arc Capacity
In communication 3, it was specified that the flow going through the transmission lines also had capacities. The constraint described below accounts for this change.

$$F_{ijt} \leq ArcLimit \Rightarrow \forall i \in S_a,\ \forall j \in E_a,\ \forall a \in A,\ \forall t \in T$$

Constraint 5 - Change in Generator Power
Ultimately, communication 5 states that the change in power output from a generator cannot change by more than 216MW for one time period to another. For periods 1 through to 5, this can be described as follows.

$$|P_{gt} - P_{g(t-1)}| \leq \Delta P_{max} \Rightarrow \forall g \in G,\ \forall t \in (1, 2, 3, 4, 5)$$

For period 0, it was assumed that the company is looking to operate the substation network for more than one day, which consequently means that the generator outputs for the first period of the day are constrained by the outputs from the last period from the previous day. As such,

$$|P_{g(t=0)} - P_{g(t=5)}| \leq \Delta P_{max} \Rightarrow \forall g \in G$$

Constraint 6 - Non-Zero Powers
Lastly, it is important to state that constraints must be placed on the variables, namely the generator power outputs and power flows through the lines, so they have non-zero values.

$$F_{ijt} \geq 0 \Rightarrow \forall i \in S_a,\ \forall j \in E_a,\ \forall a \in A,\ \forall t \in T$$

$$P_{gt} \geq 0 \Rightarrow \forall g \in G,\ \forall t \in T$$

Section B - Client Information

Abstract

Communication 1

Generator	Power (MW)	Cost ($)
Generator 1 (Node 18)	362.0	582,096
Generator 2 (Node 19)	569.0	969,576
Generator 3 (Node 28)	786.0	1,433,664
Generator 4 (Node 33)	186.0	374,976
Totals	1903.0	3,360,312

Communication 1 represented the most simplistic scenario. Results of the model could be verified by simply adding up the total demand and subtracting the cheapest generators' capacity first, leaving the most expensive generator to top off the remaining demand. Ultimately, if the capacities of the cheaper geenrators are increased, or the total demand is decreased, then the minimal cost could be decreased further.

Communication 2

Generator	Power (MW)	Cost ($)
Generator 1 (Node 18)	362.0	582,096
Generator 2 (Node 19)	569.0	969,576
Generator 3 (Node 28)	786.0	1,433,664
Generator 4 (Node 33)	312.26	629,524
Totals	2029.26	3,614,860

When considering loss in transmission wires, it is apparent that the total power requirement, and hence, the total cost is increased. To come to the results shown above it was assumed that power loss could be represented by $P_{out} = P_{in} \times (1 - Loss \times Distance)$, rather than a more realistic scenario where $P_{out} = P_{in} \times (1 - Loss)^{Distance}$. Across the entire network, a total of 126.26MW was lost through the transmission lines in the optimal solution which resulted in an increase of $254548 to the total cost.

Communication 3

Generator	Power (MW)	Cost ($)
Generator 1 (Node 18)	362.0	582,096
Generator 2 (Node 19)	563.21	959,708
Generator 3 (Node 28)	786.0	1,433,664
Generator 4 (Node 33)	318.84	642,783
Totals	2030.05	3,618,250

Having maximum power limits on transmission wires is another realistic constraint. This added information had the effect of increasing the minimum cost slightly. This price increase correlates to extra losses incurred by power being redirected to the substation nodes in a longer path when the power limit was previously being exceeded, therefore, the losses in transmission wires and hence the total overall power generated are increased. Ultimately, it appears that the arcs surrounding generator 2 may be exceeding their capacities as slightly less power is being produced from here whereas more power is being distributed from generator 4 (the more expensive one) as opposed to communication 2.

Communication 4

Variable	Value
Maximum Power Required	2121.39MW
Minimum Operation Cost	$2,600,777

	Power Outputs (MW)					
Generator	12am-4am	4am-8am	8am-12pm	12pm-4pm	4pm-8pm	8pm-12am
Generator 1 (Node 18)	362.0	362.0	362.0	362.0	362.0	362.0
Generator 2 (Node 19)	307.65	453.25	519.66	536.27	569.0	554.96
Generator 3 (Node 28)	92.68	227.82	485.39	688.89	786.0	708.56
Generator 4 (Node 33)	0	71.54	101.32	127.89	404.39	112.54
Totals	762.33	1114.6	1468.37	1715.05	2121.39	1738.05

	Costs ($)					
Generator	12am-4am	4am-8am	8am-12pm	12pm-4pm	4pm-8pm	8pm-12am
Generator 1 (Node 18)	97,016	97,016	97,016	97,016	97,016	97,016
Generator 2 (Node 19)	87,374	128,722	147,584	152,301	161,596	157,608
Generator 3 (Node 28)	28,174	69,257	147,558	209,423	238,944	215,401
Generator 4 (Node 33)	0	24,037	34,045	42,972	135,876	37,814
Totals	212,564	319,032	426,203	501,711	633,432	507,839

In the real world, power distribution varies with time - power consumption at midnight (when everyone but STEM students are sleeping) is intuitively lower than 6pm when everyone is at home with all the lights on watching TV, cooking dinner, or catching up on lectures. Given data for demand varying with time, it is seen that the minimum cost for running the network across the day is noticeably lower. This is due to a lower average demand; however, this does not mean that the peak demand is not higher, bringing higher costs due to increased usage of the more expensive generators - but for a decreased duration. The calculations assumed that demand is constant across the 4 hour time periods, but if more data is provided the model could more precisely model the cost over a day.

Communication 5

Variable	Value
Maximum Power Required	2121.39MW
Minimum Operation Cost	$2,641,021

	Power Outputs (MW)					
Generator	12am-4am	4am-8am	8am-12pm	12pm-4pm	4pm-8pm	8pm-12am
Generator 1 (Node 18)	**146.0**	362.0	362.0	362.0	362.0	362.0
Generator 2 (Node 19)	338.96	442.07	519.66	536.27	569.0	554.96
Generator 3 (Node 28)	**381.14**	238.45	454.45	623.31	786.0	597.14
Generator 4 (Node 33)	0	71.54	129.93	188.39	404.39	**216.0**
Totals	**866.09**	1114.06	1466.05	1709.97	2121.39	1730.09

	Costs ($)					
Generator	12am-4am	4am-8am	8am-12pm	12pm-4pm	4pm-8pm	8pm-12am
Generator 1 (Node 18)	39,128	97,016	97,016	97,016	97,016	97,016
Generator 2 (Node 19)	96,264	125,547	147,584	152,301	161,596	157,608
Generator 3 (Node 28)	115,866	72,490	138,154	189,487	238,944	181,530
Generator 4 (Node 33)	0	24,037	43,658	63,300	135,876	72576
Totals	251,258	319,090	426,412	502,102	633,432	508,730

Communication 5 introduced the constraint that the generators had an upper limit on how much change in production could occur in-between the 4 hour periods. This constraint negatively effected the minimum cost by approximately $41,000. The primary reason behind the cost increase is the need to keep the most expensive generators (Nodes 28 and 33) operating at higher productions to allow the capacity at the demand spike from 4pm - 8pm.

The four underlined values from the Power Outputs table will be investigated in more detail. Unlike in Communication 4, the Generator 1 12am-4am power output is not it's maximum output - it is instead 216MW less that the previous, and next time period. The reason for this correlates to the productions of Generators 3, and 4 in particular. Although for previous constraints, production in these two most expensive generators was minimised across all time periods, the production necessary to meet demand in peak hours requires the adjacent time periods to 'ramp up' and 'ramp down' production. The 8pm-12am time period has the second highest average demand across the daily cycle; however, it is followed by the lowest demand of the day (12am-4am). It can be seen that all generators drop in production by the 216MW limit; however, the total production in the 12am-4am time period still exceeds the similar value for Communication 4 by approximately 104MW. This extra production is not required by the nodes, so instead the extra power is dissipated in the transmission lines as losses.

Since the Communication 4 constraints achieved a lower daily operation cost, if minimising the result from this extra constraint is desired there are several recommendations put forward. The first would be making some improvements to the generator design allowing for greater changes in production between time periods. The greatest change in production from Communication 4 is Generator 3 at approximately 620MW. The closer the change can be to this value, the closer the minimum operational cost will be to the Communication 4 solution. In addition, adding energy storage such as batteries will allow the cheaper generators to use some of their excess capacity in off-peak times, and this power can be used in peak times to reduce the required generation of Generators 3, and 4. This would result in less required usage at off-peak times for Generators 3, and 4, while still acknowledging the production change limits.