Lecture 2

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0.1 Introduction

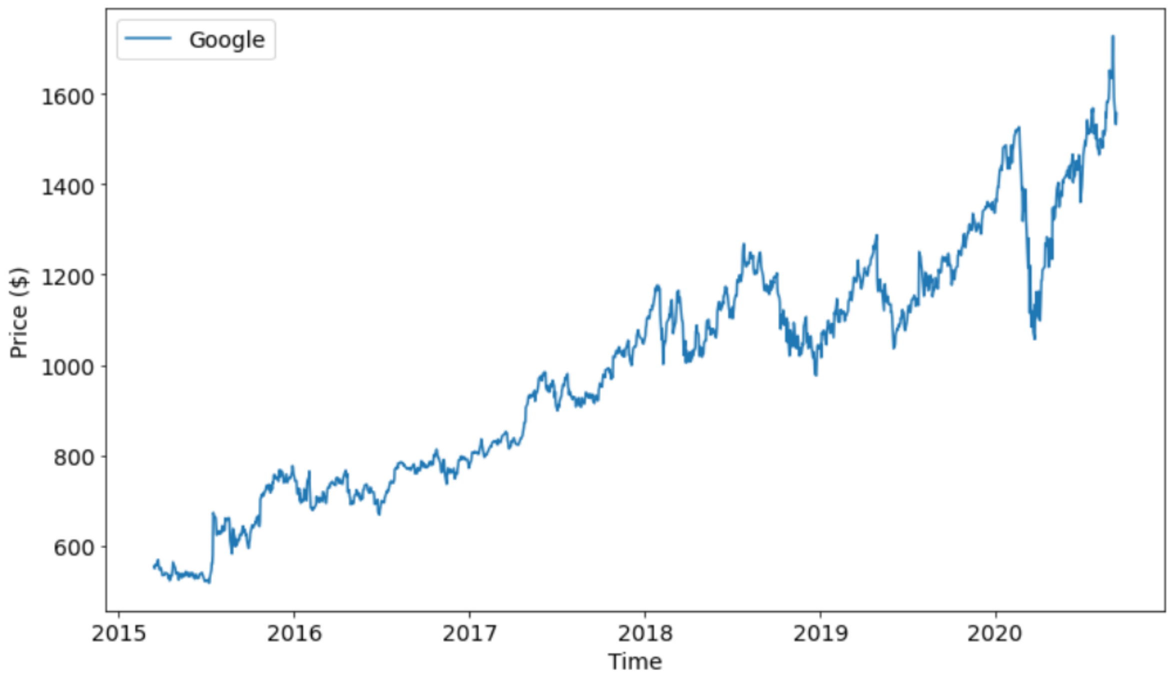
0.1.0.1 Stocks

- Represent ownership in a company (shareholder)
  - You own a percentage of the chairs, vehicles, buildings - and percentage of its profits.
  - Receive dividends (not on all stocks)

0.1.0.2 Bonds

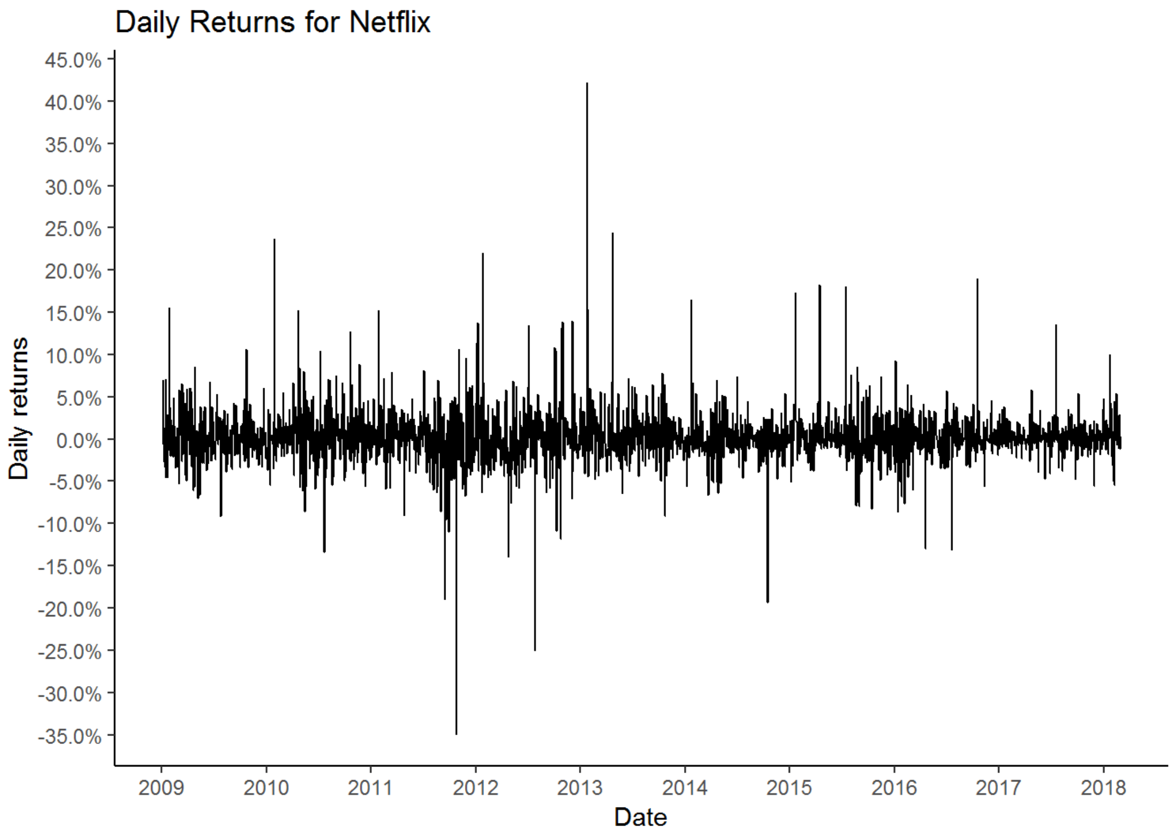
- Lend a company money
- Fixed income investment

0.2 Stock Prices, Returns, Cumulative Returns



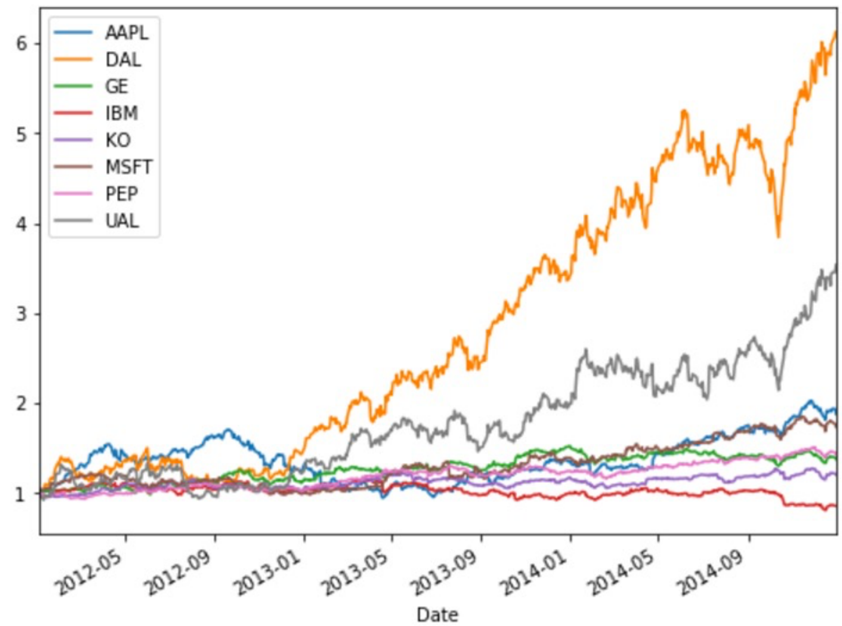
- Can I compare Google to Coca Cola share price?

0.3 Stock Prices, Returns, Cumulative Returns

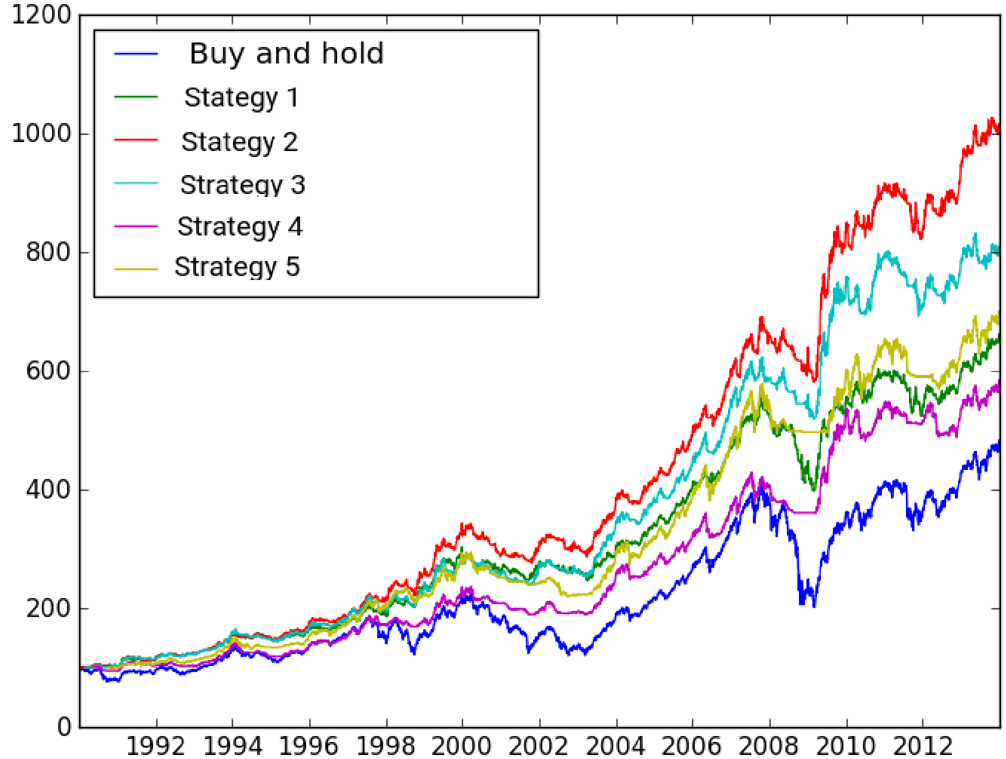


0.4 Stock Prices, Returns, Cumulative Returns

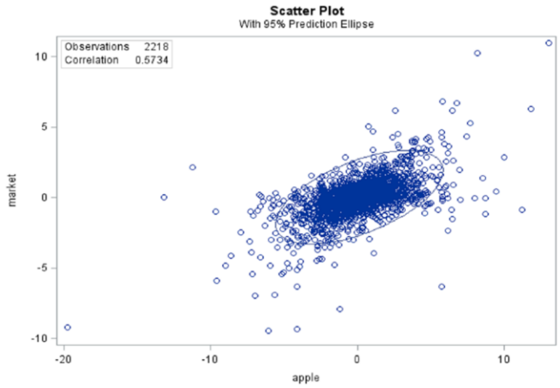
In [19]:
```
# plot all the cumulative returns to get an idea
# of the relative performance of all the stocks
daily_cr.plot(figsize=(8,6))
plt.legend(loc=2);
```



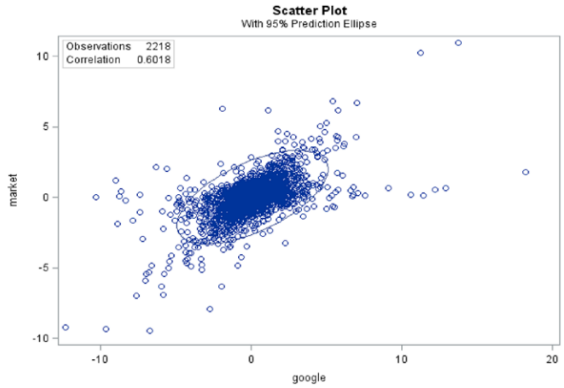
0.5 Stock Prices, Returns, Cumulative Returns



0.6 Stock Prices, Returns, Cumulative Returns



Apple



Google

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1 Diversification

> 💡 Question
>
> What is diversification? Why do people diversify?

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2 Diversification

- Select variety of stocks in different industries.
- **Optimal** to own few stocks across many industries.
  - How many stocks are optimal?
- Why do people diversify?
  - Investors try to stabilize their portfolio.
  - More stocks we own, more chance of finding NVDA
  - Limit our **market exposure** - if stock falls X%, portfolio not affected much.

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3 Diversification

> 💡 Question
>
> Is the following a diversified portfolio?

Bank	Percentage
Bank of America	30%
Chase Bank	20%
Wells Fargo	20%
Citigroup	30%

- Yes, but focused on banking
- Should diversify across different assets (stocks, bonds, real estate, gold etc.)

4 Portfolio

> Question
>
> What is a portfolio? How do we select a portfolio? If 2 assets have the same returns, which do we prefer?

5 Portfolio

- Collection of assets (shares, bonds, derivatives, real estate, etc)
- Each asset represents a % of the total portfolio value. The weight of each asset is represented by W_i with $\sum_{i=1}^{n} W_i = 1$
- Selection: Want to maximize our return and minimize risk.
- Given 2 assets with same returns, prefer asset with the lower risk.
- Risk taking investors demand an extra **premium** for taking on risk - AKA **risk premium**

6 Portfolio

> Question
>
> Which investment vehicle do you prefer and why?

	Small Caps	Blue Chips	Long Term Bonds	Medium Term Bonds	Treasury Bills
Avg Annual Return	18.29%	12.49%	5.53%	5.30%	3.85%
Annualized Standard Deviation	39.28%	20.35%	8.18%	6.33%	2.25%

7 Expected Returns

$$R_i = \frac{P_1 - P_0}{P_0}$$

$$E[R_i] = \frac{E[CF_{i,1}] + E[P_1] - P_0}{P_0}$$

Where $E[CF_{i,1}]$ = expected cash flow $E[P_1]$is the expected price

8 Optimal Portfolio

Question (10 minutes)

How many companies do you want in your portfolio? What is the optimum?

9 Optimal Portfolio

Question (10 minutes)

What do we see here?

Year	Securities	Avg. Return	Excess	Year	Securities	Avg. Return	Excess
1997	207	33%	6%	2007	224	5%	2%
1998	145	29%	15%	2008	243	-37%	1%
1999	158	21%	8%	2009	292	26%	-17%
2000	313	-9%	-19%	2010	282	15%	-6%
2001	340	-12%	-13%	2011	239	2%	2%
2002	303	-22%	-5%	2012	234	16%	-1%
2003	274	29%	-13%	2013	260	32%	-4%
2004	288	11%	-6%	2014	259	14%	-1%
2005	255	5%	-3%	2015	230	1%	3%
2006	246	16%	0%	2016	262	12%	-2%

- 20 years of data,
  - 8 of the years produced an excess return over the S&P500 index.
  - 12 years resulted in the S&P500 outperforming the simulated portfolios.

– The number of assets in the portfolio ranged from 145 to 340.

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10 Optimal Portfolio

Question (10 minutes)

What do we see here?

Year	Avg. Return	Year	Avg. Return
1997	27%	2007	3%
1998	14%	2008	-38%
1999	13%	2009	44%
2000	10%	2010	21%
2001	1%	2011	1%
2002	-18%	2012	17%
2003	42%	2013	37%
2004	17%	2014	15%
2005	8%	2015	-2%
2006	16%	2016	14%

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11 Optimal Portfolio

Year	Securities	Avg. Return	Year	Avg. Return SPY	Excess
1997	207	33%	1997	27%	6%
1998	145	29%	1998	14%	15%
1999	158	21%	1999	13%	8%
2000	313	-9%	2000	10%	-19%
2001	340	-12%	2001	1%	-13%
2002	303	-22%	2002	-18%	-5%
2003	274	29%	2003	42%	-13%
2004	288	11%	2004	17%	-6%
2005	255	5%	2005	8%	-3%
2006	246	16%	2006	16%	0%

Year	Securities	Avg. Return	Year	Avg. Return SPY	Excess

Year	Securities	Avg. Return	Year	Avg. Return SPY	Excess
2007	224	5%	2007	3%	2%
2008	243	-37%	2008	-38%	1%
2009	292	26%	2009	44%	-17%
2010	282	15%	2010	21%	-6%
2011	239	2%	2011	1%	2%
2012	234	16%	2012	17%	-1%
2013	260	32%	2013	37%	-4%
2014	259	14%	2014	15%	-1%
2015	230	1%	2015	-2%	3%
2016	262	12%	2016	14%	-2%

Question (10 minutes)

What do we see here?

Year	10 Stock	25 Stock	50 Stock	100 Stock	150 Stock	Year	10 Stock	25 Stock	50 Stock	100 Stock	150 Stock
1997	26.99%	27.59%	27.42%	27.41%	27.40%	2007	2.04%	3.09%	4.01%	4.01%	2.99%
1998	14.06%	13.33%	14.25%	13.83%	13.79%	2008	-39.70%	-38.91%	-38.98%	-38.38%	-38.73%
1999	11.66%	11.70%	13.74%	13.12%	13.32%	2009	44.80%	44.00%	43.64%	43.48%	43.97%
2000	9.61%	9.96%	9.18%	9.18%	9.43%	2010	21.36%	21.22%	21.00%	21.01%	21.21%
2001	1.68%	1.61%	0.57%	0.43%	0.57%	2011	0.94%	0.81%	1.14%	1.23%	1.14%
2002	-16.48%	-16.49%	-17.40%	-17.55%	-17.50%	2012	15.99%	15.86%	16.06%	16.14%	16.08%
2003	41.06%	41.26%	41.20%	41.38%	41.43%	2013	37.20%	36.87%	37.24%	36.42%	36.95%
2004	17.60%	17.70%	16.94%	16.92%	17.20%	2014	15.08%	14.97%	13.66%	13.86%	14.32%
2005	8.47%	8.68%	6.30%	5.35%	6.04%	2015	-1.61%	-1.83%	-1.96%	-2.46%	-1.93%
2006	15.33%	15.72%	15.32%	15.59%	15.91%	2016	13.85%	13.25%	13.53%	13.65%	13.71%

- From the S&P500, 100 individual random portfolios holding 10, 25, 50, 100 and 150 stocks were generated.

- The returns of the each of the different portfolios are very similar across each year

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12 Optimal Portfolio

Year	10 Stock	25 Stock	50 Stock	100 Stock	150 Stock	Year	10 Stock	25 Stock	50 Stock	100 Stock	150 Stock
1997	10.82%	6.66%	4.36%	3.28%	2.69%	2007	11.09%	6.83%	4.65%	2.45%	2.07%
1998	12.06%	8.37%	5.41%	3.70%	3.06%	2008	7.80%	4.66%	3.36%	2.13%	1.55%
1999	14.97%	9.48%	7.77%	5.10%	3.83%	2009	18.20%	10.55%	7.65%	4.64%	3.05%
2000	15.15%	8.81%	5.69%	4.01%	2.41%	2010	8.88%	4.70%	3.46%	2.08%	1.79%
2001	10.41%	6.73%	4.80%	3.33%	2.27%	2011	6.59%	5.03%	3.26%	2.36%	1.84%
2002	8.62%	5.46%	3.53%	2.57%	1.99%	2012	7.13%	4.73%	3.23%	2.19%	1.58%
2003	13.53%	9.21%	5.96%	3.62%	2.73%	2013	10.87%	6.51%	4.63%	2.79%	2.04%
2004	9.90%	5.75%	3.66%	2.55%	1.99%	2014	5.87%	3.89%	3.49%	2.34%	1.63%
2005	7.95%	5.19%	3.36%	2.52%	1.94%	2015	8.12%	4.34%	3.16%	2.34%	1.55%
2006	6.93%	4.08%	3.11%	1.98%	1.59%	2016	8.29%	4.33%	3.01%	2.18%	1.55%

- Standard deviations
- Significant reduction in SDs from 10 stock portfolio to 25 stock portfolio. Then it tails off.

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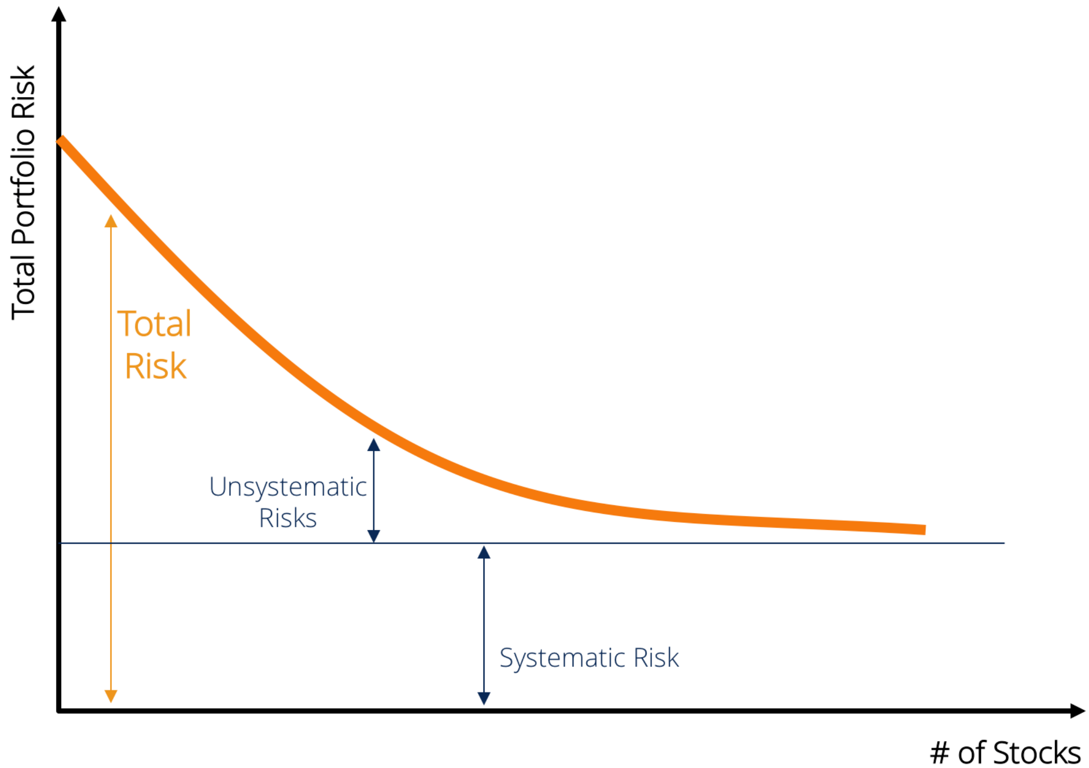
13 Optimal Portfolio

14 Optimal portfolio



Total Risk = Systematic Risk + Unsystematic Risk

15 Risk and Return

> Question (10 minutes)
>
> A financial analyst wants to calculate the expected return of investing in a portfolio of shares whose return is 1.5% with a 15% probability, 5% return with a 25% probability and 4% otherwise.

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16 Risk and Return

- $$E[R_i] = p_1 E[R_1] + p_2 E[R_2] + p_3 E[R_3] =$$
- $$0.15(0.015) + 0.25(0.05) + 0.6(0.04) = 0.0387 = 3.87\%$$

---

17 Risk and Return

- We do not always know the probabilities of expected returns of stocks.
- We don't know what the future probabilities look like.
- What do we do?
  - Take historical returns Yfinance

---

18 Risk and Return

- For a single asset:
  - $$E[R_i] = \mu_i = \frac{1}{T}\sum_{t=1}^{T} R_t$$
  - Where T is the number of historic data points used and R_t is the return in period t.

Question

How do we calculate the return for a portfolio?

---

19 Risk and Return

- For a portfolio with N assets, the portfolio expected return is the weighted average of the expected return of the individual assets.
- $E(R_p) = w_1 E(R_1) + w_2 E(R_2) + \cdots + w_N E(R_N) = \sum_{i=1}^{N} w_i E(R_i)$
- Where p stands for portfolio.

20 Compute a portfolios return

Question

The following table shows the expected return from investing in assets A and B under two possible scenarios. (S=1 and S=2) along with their probabilities (1/3) and (2/3).

1) Compute the expected return of a portfolio with 50% invested in each asset - the risk free rate of return is 2%. 2)Compute the expected return of a portfolio with 40% invested in asset A and 60% invested in asset B. 3)Compute the expected return of a portfolio 30% invested in asset A and 40% invested in asset B and the remainder invested in the risk-free asset.

	S = 1 (Prob = 1/3)	S = 2 (Prob = 2/3)
Asset A	20%	-3%
Asset B	40%	0%

---

21 Compute a portfolios return

$$E[R_A] = \frac{1}{3}20\% + \frac{2}{3}(-3\%) = 4.67\% E[R_B] = \frac{1}{3}40\% + \frac{2}{3}(0\%) = 13.33\% E[R_P] = \frac{4.67 + 13.33}{2} = 9\%$$

$$E[R_P] = 0.4 * 4.67\% + 0.6 * 13.33\% = 9.85\%$$

$$E[R_P] = 0.03 * 4.67\% + 0.4 * 13.33\% + 0.03 * 2\% = 7.8\%$$

---

22 Risk and Return

- Risk is related to the dispersion of returns relative to its expected return
- Previously, we estimated **returns** by taking the historical mean returns of an asset
- Now, we do the same but for *risk*
  - We take the historical variance of an asset's returns

$$\sigma_i^2 = \frac{1}{T-1}\sum_{t=1}^{T}(R_{i,t} - \mu_i)^2 \mu \text{ is the average return of asset i}$$

  - The standard deviation is the square root of the variance
  -

$$\text{volatility} = \sigma_i = \sqrt{\sigma_i^2}$$

---

23 Risk and Return

Question

Determine the historical **average** return and the historical **risk** of investing in shares of the following company

Year	End of Year Price
2010	12.5
2011	13.2
2012	14.6
2013	14.2
2014	13.9
2015	14.5

Year	End of Year Price
2016	14.9
2017	15.8
2018	15.6

---

24 Risk and Return

- 1) Compute the returns
  - $$R = \frac{V_f - V_i}{V_i} \text{ where } V_f \text{ is the final value and } V_i \text{ is the initial value}$$
- 2) Compute the historical average returns
  - $$\mu_i = \frac{1}{T}\sum_{t=1}^{T} R_{i,t}$$
- 3) Compute the historical standard deviation
  - $$\sigma_i = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(R_{i,t} - \mu_i)^2}$$

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25 Risk and Return

- $\mu_i = \frac{1}{8}\sum_{t=2010}^{2018} R_t = 2.90\%$
- $\sigma_i = \sqrt{\frac{1}{8}\sum_{t=2010}^{2018}(R_t - 2.90\%)^2} = 4.7\%$

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26 Risk and Return

```
-- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
v dplyr     1.1.4     v readr     2.1.4
v forcats   1.0.0     v stringr   1.5.1
v ggplot2   3.5.1     v tibble    3.2.1
v lubridate 1.9.3     v tidyr     1.3.0
v purrr     1.0.2
-- Conflicts ------------------------------------------ tidyverse_conflicts() --
x dplyr::filter() masks stats::filter()
x dplyr::lag()    masks stats::lag()
i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to becom
Loading required package: xts

Loading required package: zoo


Attaching package: 'zoo'


The following objects are masked from 'package:base':

    as.Date, as.Date.numeric



######################### Warning from 'xts' package ##########################
#                                                                              #
# The dplyr lag() function breaks how base R's lag() function is supposed to  #
# work, which breaks lag(my_xts). Calls to lag(my_xts) that you type or       #
# source() into this session won't work correctly.                            #
#                                                                              #
# Use stats::lag() to make sure you're not using dplyr::lag(), or you can add #
# conflictRules('dplyr', exclude = 'lag') to your .Rprofile to stop           #
# dplyr from breaking base R's lag() function.                                #
#                                                                              #
# Code in packages is not affected. It's protected by R's namespace mechanism #
# Set `options(xts.warn_dplyr_breaks_lag = FALSE)` to suppress this warning.  #
#                                                                              #
################################################################################
```

```
Attaching package: 'xts'


The following objects are masked from 'package:dplyr':

    first, last


Loading required package: TTR

Registered S3 method overwritten by 'quantmod':
  method            from
  as.zoo.data.frame zoo
```

year	end_of_year_price	returns	historical_returns	historical_volatility
2010	12.5	NA	0.02900409	0.04672851
2011	13.2	0.05600000	0.02900409	0.04672851
2012	14.6	0.10606061	0.02900409	0.04672851
2013	14.2	-0.02739726	0.02900409	0.04672851
2014	13.9	-0.02112676	0.02900409	0.04672851
2015	14.5	0.04316547	0.02900409	0.04672851
2016	14.9	0.02758621	0.02900409	0.04672851
2017	15.8	0.06040268	0.02900409	0.04672851
2018	15.6	-0.01265823	0.02900409	0.04672851

---

27 Portfolio of Three Assets

Expected Return: - The expected return of a portfolio with three assets is the weighted average of the expected returns of each asset: - $E(R_p) = w_1 E(R_1) + w_2 E(R_2) + w_3 E(R_3)$

Where:

- $E(R_p)$ = Expected return of the portfolio
- $E(R_1), E(R_2), E(R_3)$ = Expected returns of assets 1, 2, and 3
- w_1, w_2, w_3 = Portfolio weights for assets 1, 2, and 3, respectively (where $w_1 + w_2 + w_3 = 1$)

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28 Variance

- The variance of a three-asset portfolio takes into account the individual variances and the covariances between the assets:
- $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\text{Cov}(R_1, R_2) + 2w_1w_3\text{Cov}(R_1, R_3) + 2w_2w_3\text{Cov}(R_2, R_3)$

Where:

- σ_p^2 = Variance of the portfolio
- $\sigma_1^2, \sigma_2^2, \sigma_3^2$ = Variances of assets 1, 2, and 3, respectively

29 Covariances

- To compute the covariances:
- Covariance between Asset 1 and Asset 2:
  - $\text{Cov}(R_1, R_2) = (R_1 - R_1^\mu)(R_2 - R_2^\mu)$
- Covariance between Asset 1 and Asset 3:
  - $\text{Cov}(R_1, R_3) = (R_1 - R_1^\mu)(R_3 - R_3^\mu)$
- Covariance between Asset 2 and Asset 3:
  - $\text{Cov}(R_2, R_3) = (R_2 - R_2^\mu)(R_3 - R_3^\mu)$

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30 Standard Deviation

30.0.0.1 Standard Deviation:

- The standard deviation of the portfolio is:
- $\sigma_p = \sqrt{\sigma_p^2}$
- These are the fundamental formulas for calculating the expected return and variance (risk) of a portfolio consisting of two and three assets.

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31 Diversification

- 3M company (Minnesota Mining and Manufacturing) - created some of the most well known brands (Scotch tape, Post-it notes, etc.)
- But also is heavily diversified in Health Care, Security and Protection Services, Communications etc.
- Nestle also diversified: https://en.wikipedia.org/wiki/List_of_Nestl%C3%A9_brands
- Samsung - Financial Services, IT, Ship building, chemicals
- Why did they all diversify?
  - To reduce risk
  - Enter new markets
  - Synergies

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32 Over-diversification

- Investing in too many assets is not optimal
- As we continue to diversify - we reach the systematic market risk (the risk inherent to the entire market or market segment)

- Go to case Portfolio Return-Risk