# Lecture 1

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## Table of contents

```
-- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
v dplyr     1.1.4     v readr     2.1.4
v forcats   1.0.0     v stringr   1.5.1
v ggplot2   3.5.1     v tibble    3.2.1
v lubridate 1.9.3     v tidyr     1.3.0
v purrr     1.0.2
-- Conflicts ------------------------------------------ tidyverse_conflicts() --
x dplyr::filter() masks stats::filter()
x dplyr::lag()    masks stats::lag()
i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to becom

Attaching package: 'janitor'


The following objects are masked from 'package:stats':

    chisq.test, fisher.test
```

# 1 Learning objectives

- Gain an intuition on investment valuation models.
- Describe an investment project according to its cash flows.
- Calculate and interpret the **Net Present Value** (NPV) of a project.
- To value a firm through **Discounted Cash Flow** (DCF) analysis.

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## 2 Introduction

- Valuation of a project is subjective.
- Each person will come up with their own valuation for a given project.
- Projects need funding (loans) and investors need to be compensated for the risk they take.

- What is the value of a cow?
- What is the value of a project?

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## 3 Value of a ticket

- What is the value of this ticket?


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## 4 Interest Rates

- Interest rates are the price the borrower of money is charged to borrow money.
- Suppose we take a loan for $X$ (principal) at an annual interest rate of $r$. We hold the loan for 1 year how much interest will be charged?

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### 4.1 Example

$$\text{Total interest (1 Year)} = rX \text{Outstanding Debt} = \text{Principal} + \text{Interest} = X + rX = (1+r)X$$

Suppose we borrow \$5000 at an annual interest rate of 7%. How much interest will we be charged in the first year? How much debt will we have if we make no payments?

$$\text{Total interest (1 Year)} = rX = 0.07 \times 5000 = \$350 \text{Outstanding Debt} = (1+r)X = (1+0.07) \times 5000 = \$5350$$

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## 5 Interest Rates

> **Note**
>
> - **Where do interest rates come from?**

- We go to a bank and ask for a loan, the bank tells us how much interest we need to pay back.
- Interest rates are the prices that are negotiated between the borrowers and lenders of money.
- Two types of markets for **borrowers** and **lenders**.

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## 6 Interest Rates

**Money Markets** - Wants to borrow money as cheaply as possible - Aims for lowest interest rate

**Lenders** - Wants to earn the highest possible rate (highest rates are riskier) - **Risk** - loss of their investment (not paid back if they go for high rates)

- Interest Rates are the prices negotiated on **Money Markets** and **Bond Markets** - but governments and central banks heavily influence these rates.

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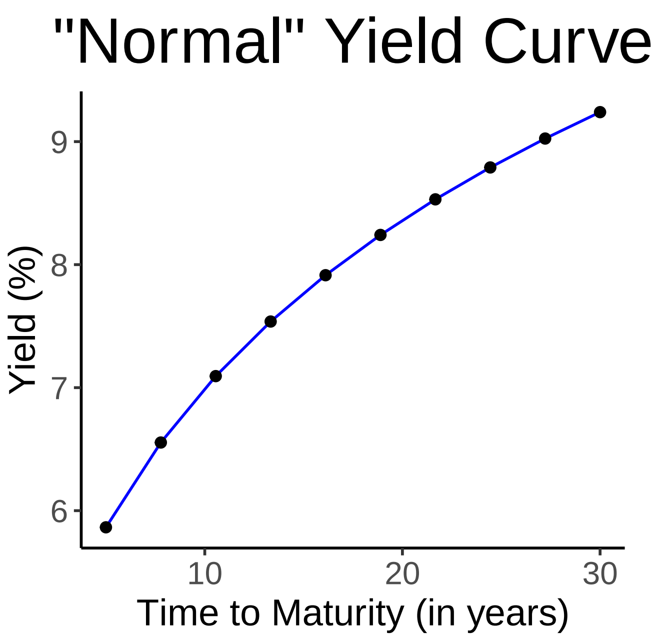
### 6.1 Term-Structure of Interest Rates

- Interest rates have a *term structure* that vary depending on the length of the loan.
- i.e. we will be charged different interest rates if we borrow money for 3 months than 5 years.




- i.e. the longer the loan term, the more time your money is tied up, thus the more risk you assume. With greater risk, you require a greater rewards, thus in the *Term-Structure of Interest Rates* we see higher yields for longer term loans.

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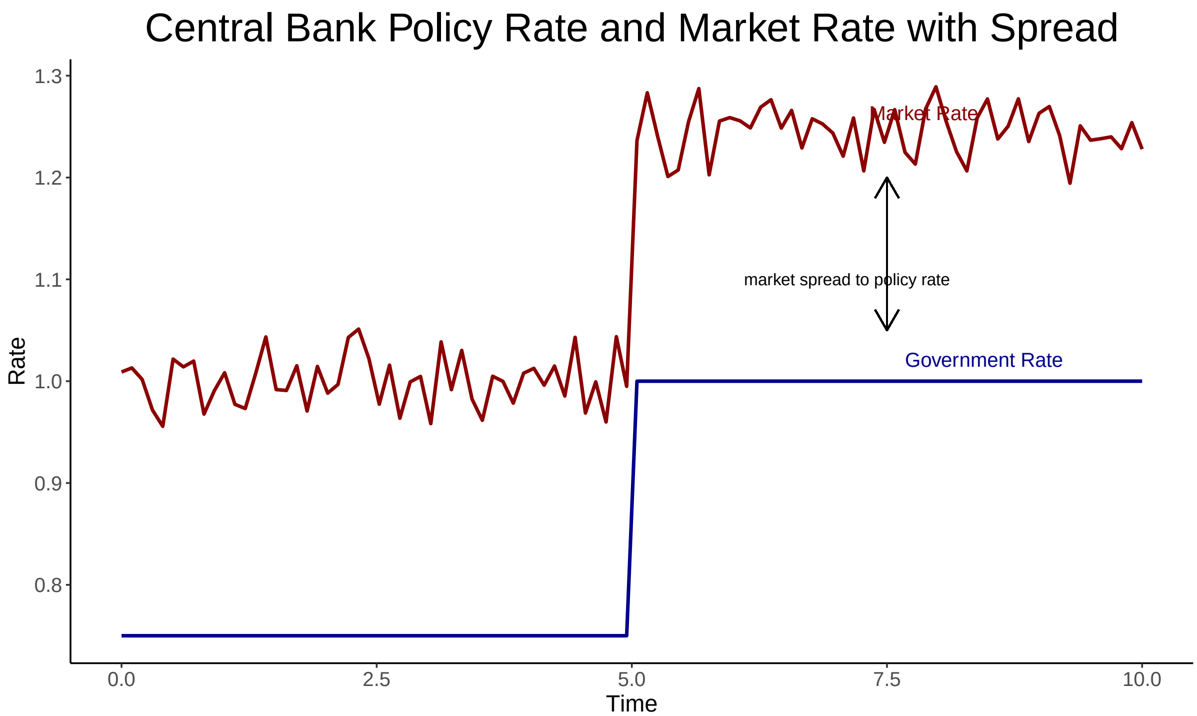
# 7 Interest Rates

- **Where do interest rates come from?**

- Central banks set an overall base rate for interest rates
- Once central banks set the base rate, the markets adjust (maintain a spread)




## 8 Interest Rates

- **Risk** is a force shaping interest rates
- Would you prefer \$100 today or \$100 in one years time?
- Would you prefer \$95 today or \$100 in one years time?
- Would you prefer \$90 today or \$100 in one years time?
- Choices like these help determine the interest rate.

$$\text{Interest Rate} = \frac{\text{Future Value} - \text{Present Value}}{\text{Present Value}} \text{Interest Rate} = \frac{\$100 - \$95}{\$95} = 5.26\% \quad | \quad 11.11\% \quad | \quad 0\%$$

- Minimum interest required in order to prefer \$95 today over \$100 in a year.

## 9 Time Value of Money

- Most people would prefer \$100 today over \$100 in a year which shows that money has a time value. Money today is worth more than the same amount in the future.
- A dollar received in 1 year is worth less than a dollar received today (Inflation, Opportunity Cost, Risk).
- Put \$1 today in the bank, it earns interest, in one years time we have 1 dollar plus the interest $\$1(1 + r)$.

### 9.1 Future Value

- What is the Future Value?
- If we invest a sum of money at a fixed interest rate for a fixed period of time, the **future value** is the amount of money we will have at the end of the period.
  - The value of an asset at a future date based on an assumed growth rate.
  - How much an investment today is worth in the future.
  - How much our money today will be worth after a certain period.

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### 9.2 Present Value

- What is the Present Value?
  - Its the current value of a future sum of money (of streams of cash flows).
  - We discount the Future Value by our estimated Rate of Return.
  - i.e. A sum of money today is likely worth more than the same sum in the future.

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### 9.3 Time lines

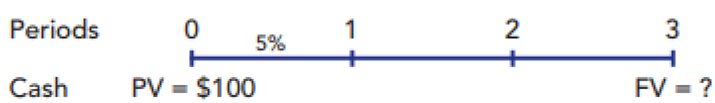


Figure 1: Time Line

- Want to find the unknown cash flow FV=? at t=3
- Single cash outflow invested at t=0

### 9.4 Time lines

> Question (5 minutes)
>
> Set up a time line to illustrate the following: You currently have $2,000 in a three year certificate of deposit (CD) that pays a guaranteed 4% annually.

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## 9.5 Future Value

- Would you prefer $1 today or $1 in 1 years time?
- A dollar in hand today is worth more than a dollar to be received in the future
  - We can invest it
- Process going from FV to PV is called **compounding**

> Question (10 minutes)
>
> We plan to deposit $100 in a bank that pays a guaranteed 5% interest each year. How much would you have at the end of Year 3?

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### 9.5.1 Definitions

- PV = Present value, or beginning amount = $100.
- FV = Future value, or ending amount, after N periods.
- CF = Cash flow. Cash flows can be positive or negative.
- CF = PV = the cash flow at Time 0
- CF would be the cash flow at the end of Period 3.
- Interest earned per year and is based on the balance at the beginning of each year, and we assume that it is paid at the end of the year.

> Tip
>
> Find the FV of $100 compounded for 3 years at 5%

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## 9.6 Step by step approach

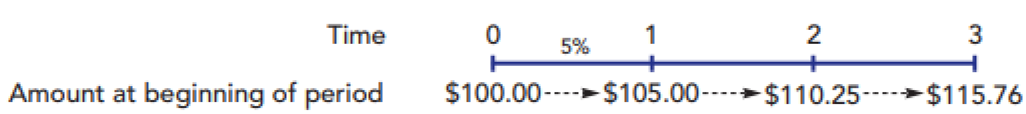


Figure 2: Time Line

- **Year 1**:

  - Amount at the end of Year 1: ( $100 + 5 = 105$ )
- **Year 2**:
  - Amount at the end of Year 2: ( $105 + 5.25 = 110.25$ )
- **Year 3**:
  - Final balance at the end of the process: ( 115.76 )

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## 9.7


Figure 3: Time Line

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Question 5 minutes

What is the difference between simple interest and compound interest? What would the FV of \$100 be after 5 years at 10% **compound** interest and **simple** interest.

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## 9.8 Solution

- Simple interest is calculated by multiplying loan principal by the interest rate and then by the term of the loan
- Compound interest calculates on the initial principal and all of the previously accumulated interest. Generates "interest on interest".
- Solutions:
  - Compounded \$161.05
  - Simple \$150.00

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## 9.9 Present Values

- Finding PV is the reverse of finding FV.
- Future value: $FV_N = PV(1+I)^N$
- Present Value: $PV = \frac{FV_N}{(1+I)^N}$

Question 10 minutes

1) A broker offers to sell you a Treasury bond that 3 years from now will pay \$115.76.
2) A bank is offering a guaranteed 5% interest on three year certificates of deposits (CDs).
3) **Whats the most we should pay for the bond?**

- Treasury bond: An interest-bearing bond issued by the US Treasury.
- Certificate of deposit, or CD, is a type of savings account offered by banks and credit unions

4) If we don't buy the bond, we will buy the CD.
5) The 5% rate paid on CDs is our **opportunity cost** or **rate of return** we could earn on an *alternative investment of similar risk*.

## 9.10 Present Values - solution step by step

- Present Value: $PV = \frac{FV_N}{(1+I)^N}$

1) We know in t=3 we will get \$115.76 from T-bond
2) $t_2 = 115/1.05 = 110.25$
3) $t_1 = 110.25/1.05 = 105$
4) $t_0 = 105/1.05 = 100$
5) Alternatively; $115.76/(1.05)^3 = 100$

- Recall, from FV example. If we invested \$100 at 5% it would grow to 115.76 in 3 years.
- We would also have \$115.76 after 3 years if we bought the T-bond
- Question: Whats the most we should pay for the bond?
  - The most we should pay is \$100 - its "fair price"
  - If we can buy cheaper, we should buy it.
  - If its more than \$100 we buy the CD.
  - If bond price = \$100 then we are indifferent.
- \$100 is the present value of \$115.76 due in 3 years at a 5 percent rate
- Finding PVs is called discounting (reverse of compounding)

### 9.11 Present Values discount rates

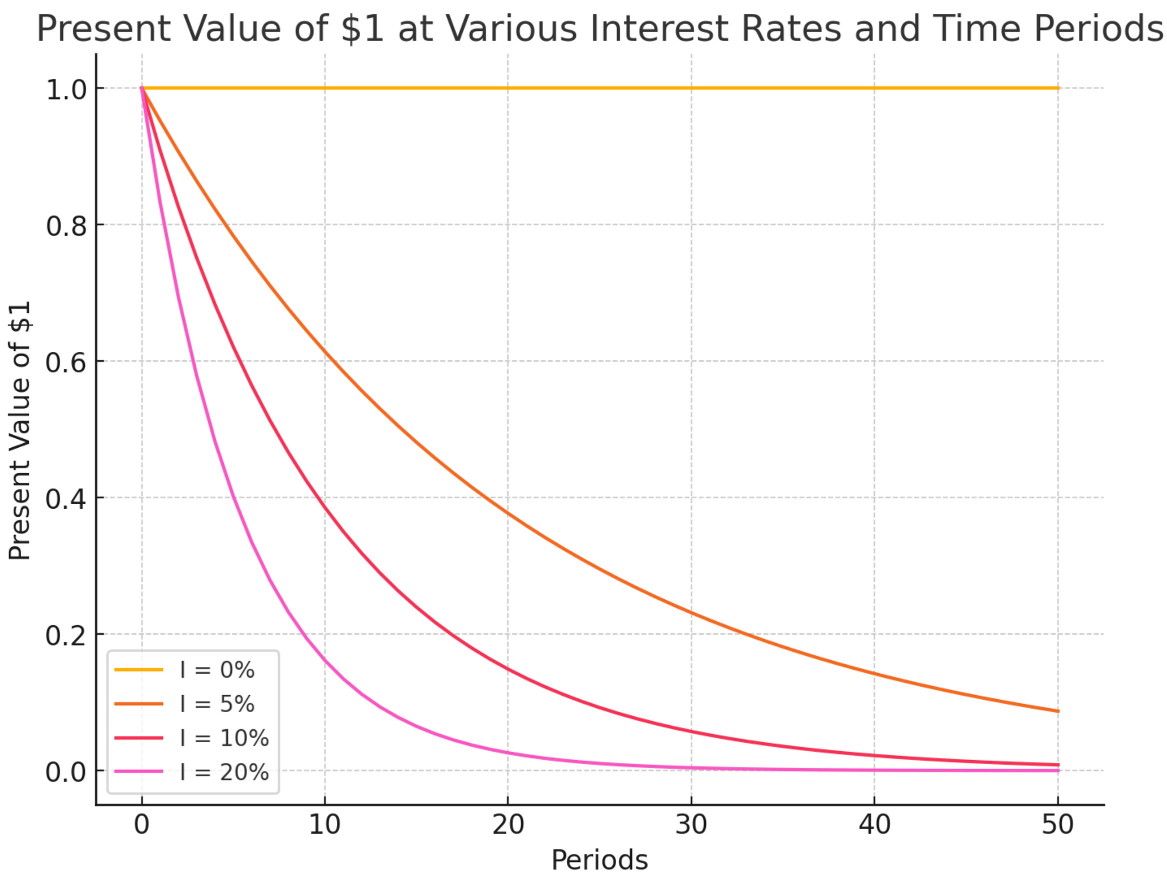


Figure 4: Time Line

- PV of a sum to be received in the future decreases and approaches zero as the payment date is extended.
- PV for faster for higher interest rates.
  - E.g. at a 20% discount rate, $1 million due in 100 years would be worth only $0.0121 today
    - Why? because $0.0121 would grow to $1 million in 100 years when **compounded** at 20%.

## 9.12 Present Values discount rates

Question 10 minutes

What is discounting and how is it related to compounding? How is the FV equation related to the PV equation?

- Discounting is the process of determining the **present value** of a **future** amount of money.
- Compounding is the process of determining the **future value** of a **present** amount of money.
- Discounting is based on the idea that money today is worth more than money in the future (due to earnings potential)
- Compounding involves the growth of money over time as interest is earning on both the principal and interests that accumulate "interest on interest"

1) Compounding moves forward in time, it takes the present value of money and projects its value into the future.
2) Discounting moves backwards in time, it takes a future amount of money and determines its equivalent value today.

- The **present value** is the **future value discounted** by $(1+r)^n$
- The **future value** is the **present value compounded** by $(1+r)^n$
- Discounting is essentially reverse compounding

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## 9.13 Present Values discount rates

Question 10 minutes

Suppose a U.S. government bond promises to pay \$2249.73 in 3 years at an interest rate of 4%. How much is the bond worth today? How would it change if the bond matured in 5 years? What if the interest rate on the 5 year bond were 6%?

Question 10 minutes

How much would \$1,000,000 due in 100 years be worth today if the discount rate was 5% and then 20%?

- Solution on paper

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### 9.14 Finding the interest rate

Question 10 minutes

- Suppose we know a bond costs \$100 and will return \$150 after 10 years (we know PV, FV and N)
- We want to find the **rate of return** on the bond.

- Solution on paper

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### 9.15 Finding the interest rate

1. Start with the future value formula:

$FV = PV \times (1+i)^n$

2. Divide both sides by $PV$:

$\frac{FV}{PV} = (1+i)^n$

3. Take the $n$-th root of both sides:

$\left(\frac{FV}{PV}\right)^{\frac{1}{n}} = 1 + i$

4. Subtract 1 from both sides to isolate $i$:

$i = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$

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### 9.16

> Question 10 minutes
>
> U.S. Treasury offers to sell a bond for $585.43. No payments will be made until the bond matures in 10 years from now, at which time it will be redeemed for $1000. What interest rate would you earn if you bought the bond? What rate of return would you earn if you could buy the bond for %550 or for $600?

- We know the FV
- We know the PV
- We know N

- Solution on paper

---

### 9.17

> Question 10 minutes
>
> Microsoft earned $0.12 per share in 1994. 10 Years later, in 2004, it earned $0.14. What was the growth rate in Microsoft's earnings per share (EPS) over 10 years?

- Solution on paper

- Compound Annual Growth Rate (CAGR) is conceptually similar to an interest rate.
- Both describe the rate at which an investment or value grows over time.

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### 9.18 Finding the number of years

- We might want to know how long it will take to accumulate a given sum of money
- i.e. We know what we have and how much we want and we know various rates of returns
- How long will it take us to get to our objective amount of money?

Question 10 minutes

We believe that we could retire comfortably if we had $1 million, our current savings is $500,000 and we found an investment product which gives us 4.5% We want to know our retirement date!

- Solution on paper

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## 9.19 Finding the number of years

$FV = PV \times (1+i)^n$

$\frac{FV}{PV} = (1+i)^n$

$\ln\left(\frac{FV}{PV}\right) = \ln(1+i)^n$

$\ln\left(\frac{FV}{PV}\right) = n \times \ln(1+i)$

$n = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+i)}$

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Question 10 minutes

How long would it take $1,000 to double if we invested in a savings account that pays 6% annually? How long at 10%?

- Solution on paper

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Question 10 minutes

Microsofts EPS were $1.04 in 2004 and its CAGR in the prior 10 years was 24.1% per year! If that growth rate was maintained, how long would it take for the EPS to double?

- Solution on paper

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## 9.20 Annuities

- So far, we dealt with "lump sums" or single payments.
- Many assets provide a series of "cash inflows" over time.
  - i.e. automobile and mortgage payments require a series of payments. (fixed intervals, called **annuity** - paid at the end of the period/year.)
- E.g. Receive \$100 in payments at the end of each of the next 3 years is a 3-year annuity.

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## 9.21 FV of an ordinary annuity

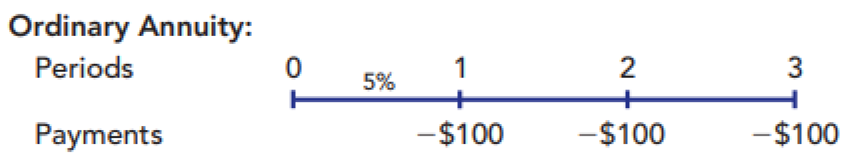


Figure 5: Time Line

- How much will we have at the end of the 3rd year?
  - First payment earns interest for 2 periods
  - Second payment earns interest for 1 period
  - Third payment earns no interest
  - $100(1.05)^2 + 100(1.05)^1 + 100(1.05)^0 = 315.25$
  - $FV_{annuity} = PMT(1+i)^{N-1} + PMT(1+i)^{N-2} + PMT(1+i)^{N-3} + ... + PMT(1+i)^0$
  - $FVA = PMT \times \left(\frac{(1+i)^n - 1}{i}\right)$

---

## 9.22 FV of an ordinary annuity

> Question 10 minutes
>
> Suppose we plan to buy a house in 5 years time, we can save \$2500 per year. We can deposit the money in a bank which pays 4% interest. We make the first deposit at the end of the year. How much will we have after 5 years? What if the interest increased to 6% or lowered to 3%

- Solution on paper

---

## 9.23 PV of an ordinary annuity

- To find the FV of an annuity we **compounded** the deposits
- To find the PV of an annuity we **discount** them.
    - i.e. find the PV of future streams of payments.

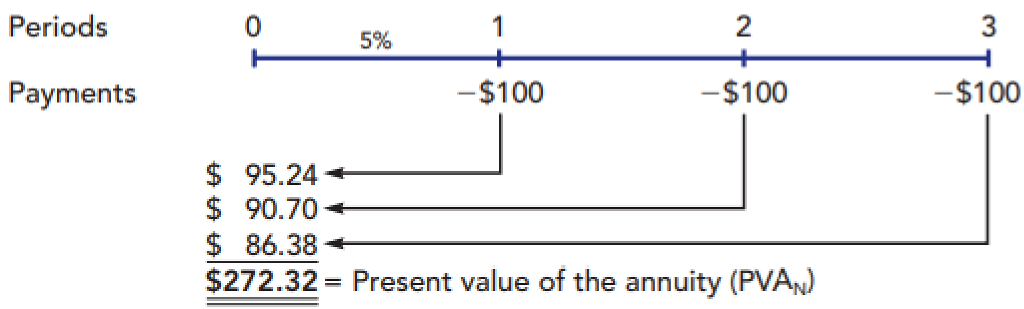


Figure 6: Time Line

- $\frac{100}{(1+0.05)^1} + \frac{100}{(1+0.05)^2} + \frac{100}{(1+0.05)^3}$
- $PVA = PMT \times \left(\frac{1-\frac{1}{(1+i)^n}}{i}\right)$
- $PVA = 100 \times \left(\frac{1-\frac{1}{(1+0.05)^3}}{0.05}\right)$

## 9.24 PV of an ordinary annuity

Question 10 minutes

What is the present value of an ordinary annuity with 10 payments of $100 with interest 10%? What if the rate is 4%? what about 0%?

- Solution on paper

### 9.25 Finding annuity payments

Question 10 minutes

Suppose we need to accumulate $10,000 and have it available to us in 5 years time. Our saving is currently 0 but we can earn 6%. How large must our deposits be to accumulate $10,000.

- FV = 10,000
- PV = 0
- N = 5
- i = 6%
- Solution on paper

### 9.26 Finding number of periods N

Question 10 minutes

Suppose we make end-of-year deposits of $1,200 per year, earn 6% on our savings, how long would it take us to reach $10,000?

- Solution on paper

### 9.27 Finding number of periods N

To solve for $n$ in the future value of an annuity formula:

$FV_{annuity} = PMT \times \left(\frac{(1+i)^n-1}{i}\right)$

#### 9.27.1 Step-by-Step Solution:

1. Isolate the fraction involving $n$:

$\frac{FV_{annuity}}{PMT} = \frac{(1+i)^n-1}{i}$

2. Multiply both sides by $i$:

$i \times \frac{FV_{annuity}}{PMT} = (1+i)^n - 1$

3. Add 1 to both sides to isolate $(1+i)^n$:

$(1+i)^n = 1 + i \times \frac{FV_{annuity}}{PMT}$

4. Take the natural logarithm (ln) of both sides:

$\ln((1+i)^n) = \ln\left(1 + i \times \frac{FV_{annuity}}{PMT}\right)$

5. Apply the logarithmic property $\ln(a^b) = b \cdot \ln(a)$:

$n \cdot \ln(1+i) = \ln\left(1 + i \times \frac{FV_{annuity}}{PMT}\right)$

6. Solve for $n$:

$n = \frac{\ln\left(1+i\times\frac{FV_{annuity}}{PMT}\right)}{\ln(1+i)}$

# 10 Uneven cash flows

- Many financial payments involve uneven cash flows..
  - i.e. Dividends on stocks typically increase over time.
  - We denote **Cash Flows** for uneven cash flows
  - **PMT** for equal payments

Cash flows can be: 1) Streams of payments + additional final lump sum (bonds) 2) all othe runeven streams (stocks and capital investments)

## 11 Uneven cash flows

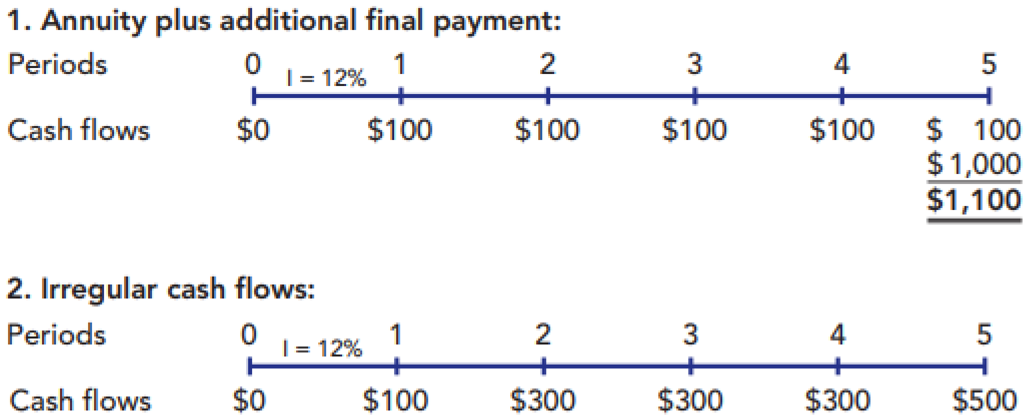


Figure 7: Time Line

$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \cdots + \frac{C_N}{(1+r)^N}$

$PV = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t}$

> Question 10 minutes
>
> Find the present values of the following 2 streams of cash flows

## 12 Uneven cash flows

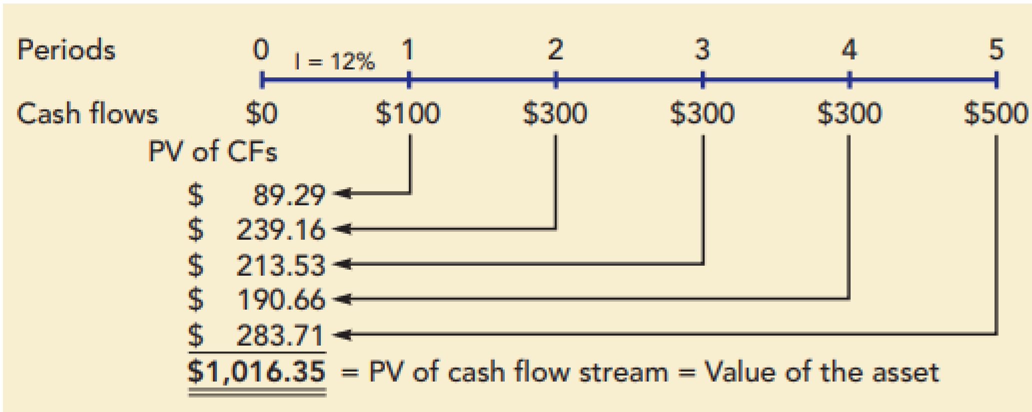


Figure 8: Time Line

Question 10 minutes

What's the present value of a 5 year annuity of \$100 + an additional \$500 in year 5? Interest is 6%. What would the PV be if \$100 payments occured in year 1 to 10 and \$500 at the end of year 10?

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## 13 Uneven cash flows

Question 10 minutes

Whats the present value of the following uneven cash flow streams? \$0 at Time 0, \$100 in Year 1 (or at Time 1), \$200 in Year 2, \$0 in Year 3, and \$400 in Year 4 if the interest rate is 8 percent?

## 14 Net Present Value

Question 10 minutes

Our manager has come to us and asked us to evaluate two potential projects that the company could invest in. We are asked to evaluate which project is the best one. The **discount rate** is 7% for both projects. Each project lasts 5 years, the initial investment for each project is 15000 and \$20000 respectively. The cash flows for each of the projects are the following:

Cash Flow Streams
Comparing Project A and Project B

| Year | Project A | Project B |
|---|---|---|
| 0 | $-\$15,000.00$ | $-\$20,000.00$ |
| 1 | $\$3,000.00$ | $\$2,000.00$ |
| 2 | $\$3,000.00$ | $\$4,000.00$ |
| 3 | $\$5,000.00$ | $\$6,000.00$ |
| 4 | $\$5,000.00$ | $\$8,000.00$ |
| 5 | $\$5,000.00$ | $\$10,000.00$ |

## 15 NPV Solution

Let's assume your company has two potential projects it can start. How can we decide which project is the better option?

Your company's weighted average cost of capital is 7%, so 7% will be the discount rate for both projects. Each project lasts five years. The initial investment and cash flows for the two projects are:

$$NPV_A = -15000 + \frac{3000}{(1+0.07)} + \frac{3000}{(1+0.07)^2} + \frac{5000}{(1+0.07)^3} + \frac{5000}{(1+0.07)^4} + \frac{5000}{(1+0.07)^5}$$

$$NPV_B = -20000 + \frac{2000}{(1+0.07)} + \frac{4000}{(1+0.07)^2} + \frac{6000}{(1+0.07)^3} + \frac{8000}{(1+0.07)^4} + \frac{10000}{(1+0.07)^5}$$

### 15.1 NPV Solution

Discounted Cash Flow Analysis
Comparing Project A and Project B

| Year[1] | Project A ($)[1] | Project B ($)[1] |
|---|---|---|
| Year 1 | $2,803.74 | $1,869.16 |
| Year 2 | $2,620.32 | $3,493.75 |
| Year 3 | $4,081.49 | $4,897.79 |
| Year 4 | $3,814.48 | $6,103.16 |
| Year 5 | $3,564.93 | $7,129.86 |
| NPV | $1,884.95 | $3,493.72 |

[1]All values are presented in present value terms, discounted at 7%

- Both projects profitable.
- Project B > Project A.
- Also need to consider other factors.
  - Is the NPV of Project B high enough to warrant a bigger initial investment?
  - What about intangible benefits, such as strategic positioning and brand equity?
  - Higher riskier investments might not be captured by NPV.
  - Market conditions might change and affect the cash flow streams.
  - Environmental and social impact - is project B more sustainable?

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## 16 NPV Interpretation

- How do we interpret NPV=0?
  - The Present Value (PV) of the future cash flows is equal to the initial costs.
  - The expected return on the project compensates for its level of risk.
- How do we interpret NPV > 0?
  - The PV of the future cash flows is higher than the initial costs.
  - The expected return on the project is above the minimum required for the level of risk it generates.
  - In colloquial terms, we can say that this investment makes us "richer" or increases the value of the company.
- How do we interpret NPV < 0?
  - The PV of the future cash flows is lower than the initial costs.

- The expected return on the project is below the minimum required for assuming such a risk.
- We can say that this investment reduces our wealth.

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# 17 NPV Interpretation

The main task of a finance director is to identify projects with positive NPV (Net Present Value).

- The idea is to accept (reject) projects with NPV >= 0 (< 0).
  - When the NPV is positive, we increase our wealth as long as we invest.
    * The money needed to start a project is less than the PV (Present Value) of the future cash flows that the project generates.
- What can we do if several projects have positive NPVs?
  - NPVs can be added, therefore increasing the number of positive NPV projects increases our wealth even further.
- What can we do if all available projects have a negative NPV?
  - We should not invest in projects with negative NPVs.

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## 17.1 Net Present Value

Recall;

$$\text{NPV} = -I_0 + \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2}, \ldots, \frac{C_5}{(1+r)^5} + \frac{C_6}{(1+r)^6}$$

$$\text{NPV} = -I_0 + \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}$$

Where;$C_t$ = Cash flow in period t$r$ = Discount rate$I_0$ = Initial investment$T$ = Number of periods

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### 17.2 Discount Rate

- Changing discount rates

$$-200 = -1000 + \sum \frac{200}{1+0}^{1} + \frac{200}{1+0}^{2} + \frac{200}{1+0}^{3} + \frac{200}{1+0}^{4} + \frac{200}{1+0}^{5} + \frac{200}{1+0}^{6}$$

$$83.44 = -1000 + \sum \frac{200}{1+0.03}^{1} + \frac{200}{1+0.03}^{2} + \frac{200}{1+0.03}^{3} + \frac{200}{1+0.03}^{4} + \frac{200}{1+0.03}^{5} + \frac{200}{1+0.03}^{6}$$

$$871.05 = -1000 + \sum \frac{200}{1+0.10}^{1} + \frac{200}{1+0.10}^{2} + \frac{200}{1+0.10}^{3} + \frac{200}{1+0.10}^{4} + \frac{200}{1+0.10}^{5} + \frac{200}{1+0.10}^{6}$$

```r
r = 0.10
res = 200/((1+r)^1) + 200/(1+r)^2 + 200/(1+r)^3 + 200/(1+r)^4 + 200/(1+r)^5 + 200/(1+r)^6
I_0 = 1000
print(I_0 - res)
```

```
[1] 128.9479
```

---

### 17.3 Discount Rate

- How do we determine the discount rate?
- The discount rate is the return we expect to get from **alternative** investments with similar risk.
  - **Risk-Free Rate** - government bonds) - *no risk*.
  - **Risk Premium** - Added to the *risk-free* rate to account for the extra risk of the investment.
  - **Company's Cost of Capital**: For businesses, it often reflects the Weighted Average Cost of Capital (WACC), considering both debt and equity financing costs.

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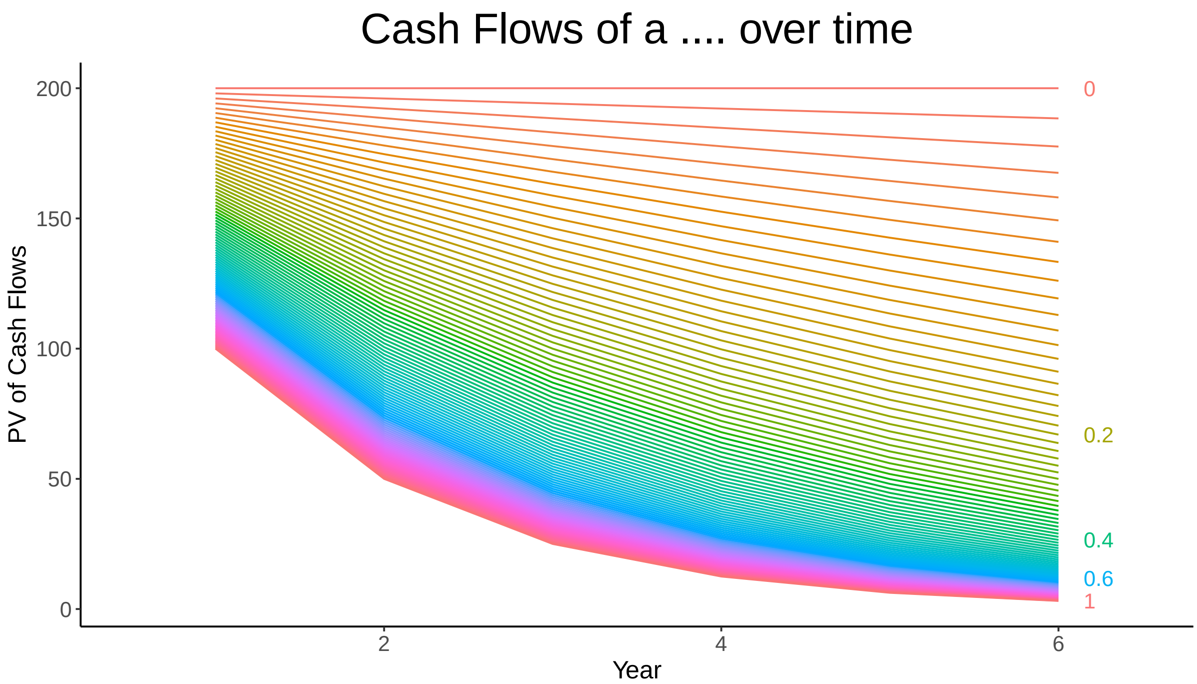
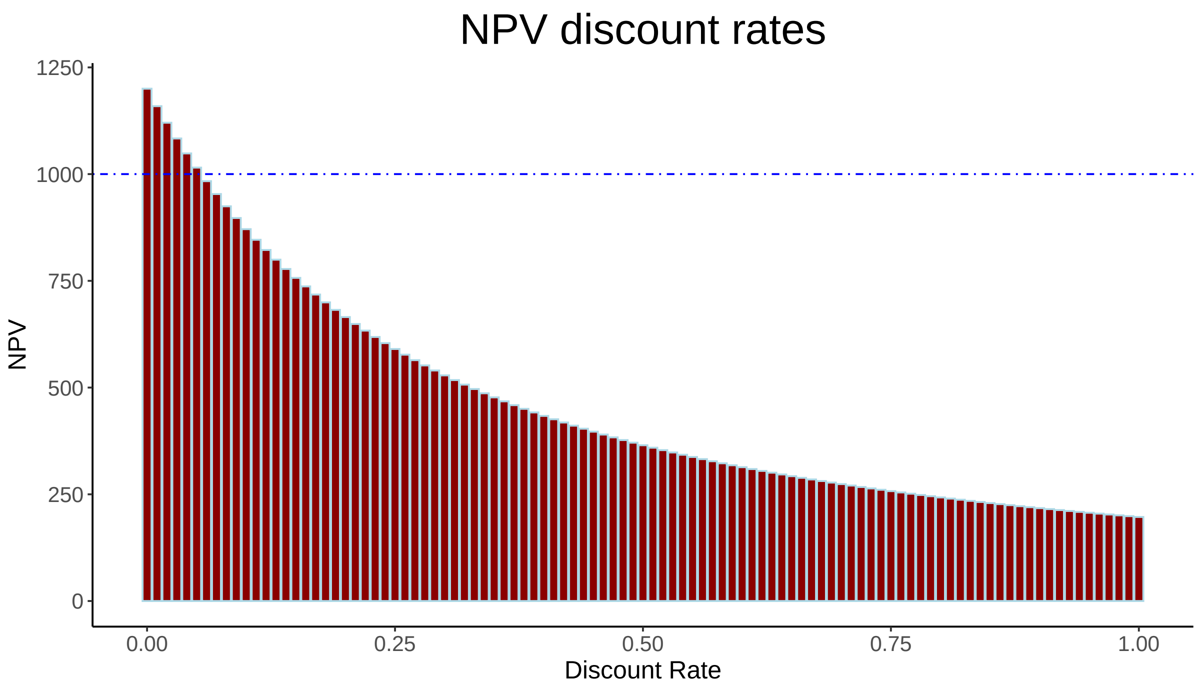
## 17.4 Cash Flows




- As the discount rate increases the **Cash Flows** are worth less today.
- The sum of the **cash flows**, the **NPV** decreases as well.

### 17.5 NPV discount rates




### 17.6 Importance of the discount rate

- **Reflects risk** - Higher discount rates are used for riskier investments. If the cow's milk production is uncertain, or milk prices are volatile, you might use a higher discount rate to account for that risk.
- **Affects NPV** - A higher discount rate makes future money worth less today, decreasing the NPV. A lower discount rate makes future money worth more today, increasing the NPV.
- **Decision-Making** - By adjusting the discount rate, you can make more informed decisions. If even at a high discount rate the NPV remains positive, it signifies a robust investment opportunity.
- **Limitations**
  - NPV is sensitive to the discount rate
  - NPV is sensitive to the cash flow estimates (we assume we know the future cash-flows)

## 17.7 NPV Example

> Question 10 minutes
>
> Your company produces fertilizer and currently traded at NYSE with latest stock price of \$27.3 and 1,000 common shares outstanding. You are thinking to invest in a new product,an organic fertilizer with an initial required investment of $\$30,000$. You believe that the new organic fertilizer will produce an annual cash revenue of $\$20,000$ and cash costs (including taxes) will be $\$14,000$ per year. We will wind down the business in 8 years and the plant, property and equipment will be worth $\$2,000$ at that time. With a 15% opportunity cost of capital, should you take the investment?

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## 17.8 NPV Solution

Net Present Value
Example

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Initial.Cost | −\$30 | — | — | — | — | — | — | — | — |
| Inflows | — | \$20 | \$20 | \$20 | \$20 | \$20 | \$20 | \$20 | \$20 |
| Outflows | — | −\$14 | −\$14 | −\$14 | −\$14 | −\$14 | −\$14 | −\$14 | −\$14 |
| Net.Inflows | — | \$6 | \$6 | \$6 | \$6 | \$6 | \$6 | \$6 | \$6 |
| Salvage | — | — | — | — | — | — | — | — | \$2 |
| Net.Cash.Flow | −\$30 | \$6 | \$6 | \$6 | \$6 | \$6 | \$6 | \$6 | \$8 |

## 17.9 NPV Solution

The NPV of the new investment is:

$$\text{NPV} = -30,000 + 6,000\frac{(1.15)^8 - 1}{0.15(1.15)^8} + \frac{2,000}{1.15^8} = -2,422$$

- The new investment will lose value of $-\$2,422$. Per share this translates to a loss of $-\$2,422/1,000 = -\$2.42$ that will drop the stock price from their current \$27.3 to \$24.9 ($-9\%$ negative return). - Would you accept this investment?

NPV

An investment should be accepted if the NPV is positive and rejected if it is negative.

---

### 17.10 Property Valuation

Question 10 minutes

How would you value a property?

- If we are valuing a property, we could value it from the ground up - brick and mortar (net asset approach to valuing a property/company)
- Could look at similar houses and see if ours is worth something similar. (multiples based approach)
- Value a property but looking at the rental income it generates (DCF approach)

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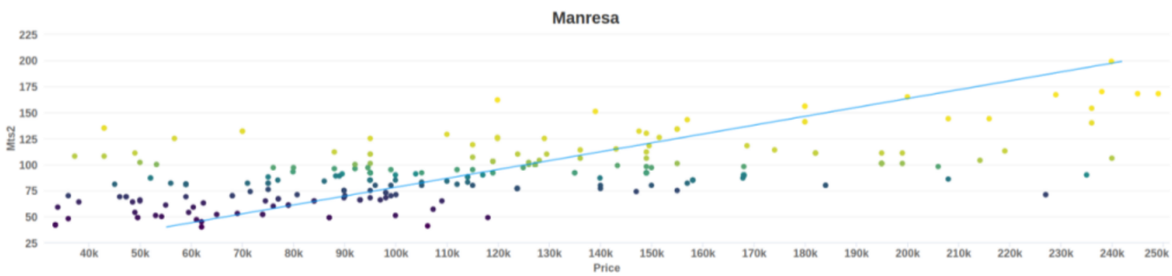
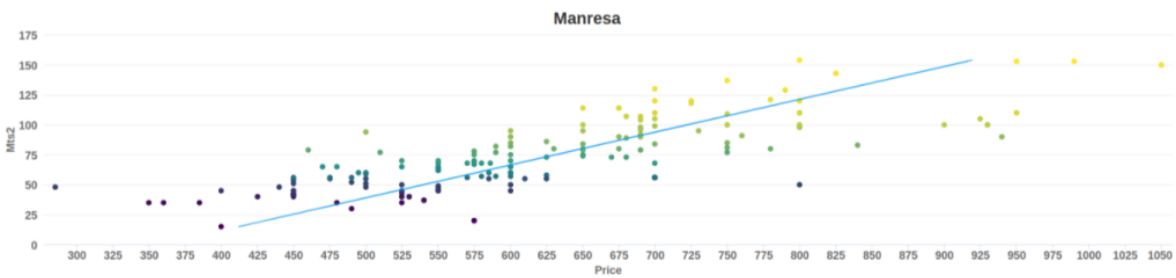
## 17.11 Property Valuation




Listing Prices




Rental Prices

## 17.12 Property Valuation

**DCF Assumptions**

| | |
|---|---|
| Vacancy | 8% |
| General Expenses | €10.00 |
| Growth Rate Income | 5% |
| Growth Rate Expenses | 3% |
| Cap Rate | 8% |

Note: A simple example.

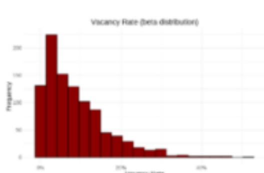
**Discounted Cash Flow Analysis**

All figures are presented in €

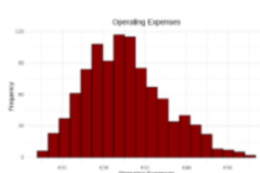
| Process | Projection (€) | | | | | |
|---|---|---|---|---|---|---|
| | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 | Year 6 |
| Potential Gross Revenues | € 6,900 | € 7,245 | € 7,601 | € 7,987 | € 8,386 | € 8,806 |
| General Vacancy | € 550 | € 580 | € 610 | € 640 | € 670 | € 705 |
| Effective Gross Revenues | € 6,350 | € 6,665 | € 6,990 | € 7,347 | € 7,715 | € 8,100 |
| Operating Expenses | € 880 | € 905 | € 930 | € 960 | € 990 | € 1,020 |
| Net Operating Income | € 5,470 | € 5,760 | € 6,060 | € 6,387 | € 6,725 | € 7,080 |
| Valuation at Sale | | | | | | € 88,500 |
| IRR | | | | | | 11 % |

The probability that this property will be profitable: 0.91

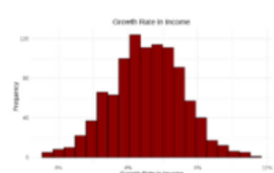

Figure 1: Property ID: 948



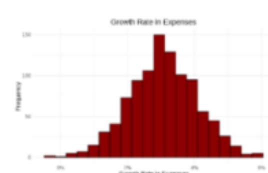
Vancancy Rates



Operating Expenses



Growth Rate in Income



Growth Rate in Expenses

## 18 Dividend growth model

- Shares are valued as any other asset:
  - Calculate the **Present Value** of the future cash flows
- Shareholders expect to receive two types of cash flows:
  - **Dividends** - cash payments made by the company to its shareholders
  - **Capital gains** - the difference between the price at which the shares are bought and sold
- The return per share of firm $i$ obtained between period 0 and 1 (one period) is:

$$R_i = \frac{DIV_1 + P_1 - P_0}{P_0}$$

---

## 19 Value a stock

- A shares PV is given by;

$$P_0 = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \cdots + \frac{D_N}{(1+r)^N} = \sum_{t=1}^{t=N} \frac{D_t + P_N}{(1+r)^t}$$

- $D$ is the expected dividend for period $t$, $P_N$ is the expected share price in period $N$. - $r$ is the expected return offered by *shares in an equivalent risk class* (return offered by alternative investment opportunities with the same risk).
- Dividends per share (DPS) = Earnings per share * pay out ratio
- The general formula to value stocks is:

$$P_0 = \sum_{t=1}^{t=\infty} \frac{D_t}{(1+r)^t}$$

- Use infinity since shares do not have a "maturity date" (company can still go bankrupt, acquired)

---

# 20 Example

Calculate the price of one share of GOOGL considering that expected dividends per share for the next two years are 2€ and 3€ respectively and the estimated selling price in two years is €11.5. The expected return offered by shares with a similar level of risk is 8.8%.

```
[1] 14.08751
```

$$P_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \frac{P_2}{(1+r)^2} = \frac{2}{(1+0.088)^1} + \frac{3}{(1+0.088)^2} + \frac{11.5}{(1+0.088)^2} = 14.09$$

## 20.1 Gordon Growth Model

- Used to determine the intrinsic value of a stock based on a future series of dividends that grow at a constant rate.
- Used to calculate a fair value for the stock taking into consideration dividend payments.
- Assume dividends grow at a constant rate.
- Used for companies with stable growth rates in dividends per share.

Dividend Growth

| Company | Years |
|---|---|
| IBM | 28 |
| Caterpillar | 29 |
| Chevron | 36 |
| Exxon Mobil | 41 |
| McDonald's | 47 |
| Pepsi Co | 51 |

## 20.2 Gordon Growth Model

- Assumptions:
  - Company exists forever
  - Dividends grow at a constant rate $g$ forever

$$P = \frac{D_1}{(r-g)}$$

Where;

$P$ = Current stock price$g$ = Expected growth rate of dividends$r$ = Required rate of return$D_1$ = Value of nex

### 20.3 Gordon Growth Model

Question 10 minutes

Consider a company trading at \$110 per share. The company requires 8% minimum rate of return $r$ and will pay \$3 dividends per share next year ($D_1$), the growth is expected to be 5% annually ($g$).

### 20.4 Gordon Growth Model

$$P = \frac{D_1}{(r-g)} = \frac{3}{(0.08-0.05)} = \$100$$

According to the Gordon growth model, the shares are \$10 overvalued.

### 20.5 Gordon Growth Model

- Limitations
  - Establishes a value for a company without accounting for market conditions
  - Assumes dividends grow at a constant rate forever
  - Ignores non-dividend factors such as brand loyalty, customer retention and intangible assets (which all add to a companies value)
  - Assumes dividend growth rate is stable
  - Can't be used for valuing stocks which *do not* pay a dividend (growth stocks usually do not pay).

### 20.6 Example

Suppose a company historically has been paying \$3 in dividends per year for 20 years. The expected return is 8.8% and the dividends are expected to grow at 3%. Calculate the price of the stock using the Gordon Growth Model.

```
[1] 86.2069
```

$$P_0 = \frac{D_1}{(r-g)} = \frac{3}{(0.088-0.03)} = 86.21$$

## 20.7 Example

Question 10 minutes

INCARSA Corporation is in the business of selling and distributing computers. The firm has announced that it will pay a 2€ dividend per share at the beginning of next year, a 3€ dividend per share the year after and a constant dividend per share equal to 1.8€ growing at 2% there after. The required rate of return for firms in this sector is 10%.

## 20.8 Solution

- Compute the price at which one share of INCARSA Corp. is expected to trade in the secondary market.

$$P_0 = \frac{2}{1.1} + \frac{3}{(1.1)^2} + \frac{1.8}{(0.1-0.02)\times(1.1)^2} = 22.89$$

- The firm has 2million shares outstanding.

$$\text{Equity Value}_0 = 22.89 \times 2M = 45.7851M$$

- Compute the value of the firm's equity. Earnings for this year are equal to 6,000,000€. Compute the payout ratio and the EPS.

$$\text{EPS} = \frac{6M}{2M} = 3\text{Pay-out}_1 = \frac{2}{3}$$