Asset Pricing and Valuation

Lecture 4: Capital Asset Pricing Model

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Capital Asset Pricing Model

Assumptions (1)

- The Capital Asset Pricing Model (CAPM) is fundamental for modern finance (developed 50 years ago)
- Model which works when markets are in equilibrium
- The CAPM builds on the Markowitz Mean Variance Model

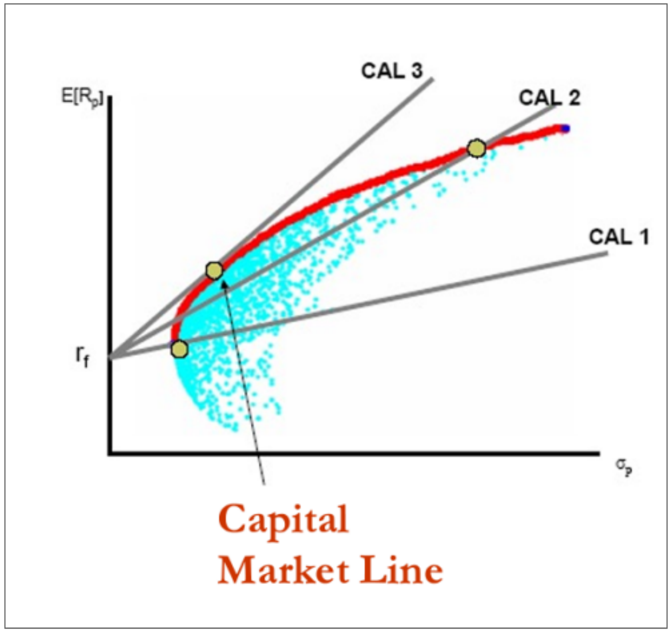
Assumptions (2)

- A static model, economic agents only focus on what happens one period ahead (1 month, 1 quarter, 1 year etc.)
- **Perfect competition** - There are many investors (each with different utility functions and initial wealth) - investors are price takers.
- **Homogeneous expectations** - All investors have the same expectations about the future (mean and variance of returns)
- **No transaction costs** - Investors can buy and sell assets without incurring any costs
- Investors optimize according to Markowitz theory (i.e. care about mean-variance trade-off)
- Share same information, therefore, their risk and return expectations for each asset are identical.

Markowitz Model

Which risk portfolio will the investor select

- An investor chooses a portfolio located on the efficient frontier
- The portfolio which provides the highest return adjusted to risk is Capital Allocation Line (CAL) 3.
- This always happens when the CAL is tangent to the Efficient Frontier.



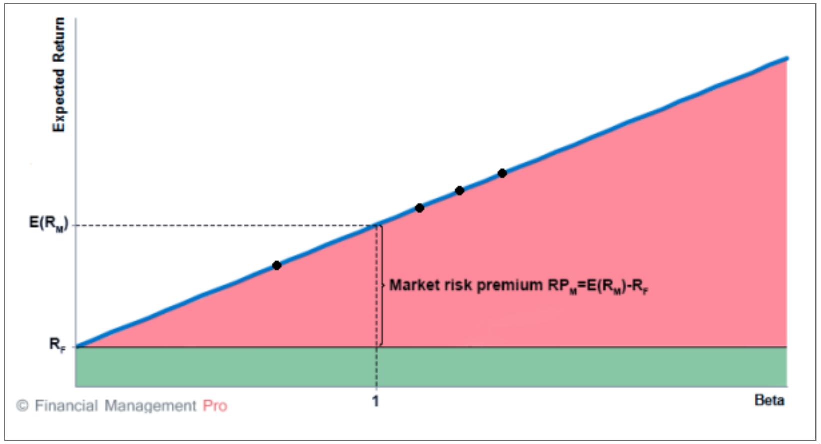
Security Market Line (SML)

- Agents hold diversified portfolios, therefore demand a risk premium which depends on the systemic risk of each asset (and not specific risk) - i.e. the risk inherent to the entire market (undiversifiable risk/market risk).
- Systemic risk depends on β, therefore investors demand a return that is a function of β.
- THe CAPM equation relates an asset's risk premium to;
  - The expected market risk premium
  - THe asset's systemic risk (its β)

CAPM:
$E[r_i] = r_f + \beta_i(r_m - r_f)$

Security Market Line (SML)

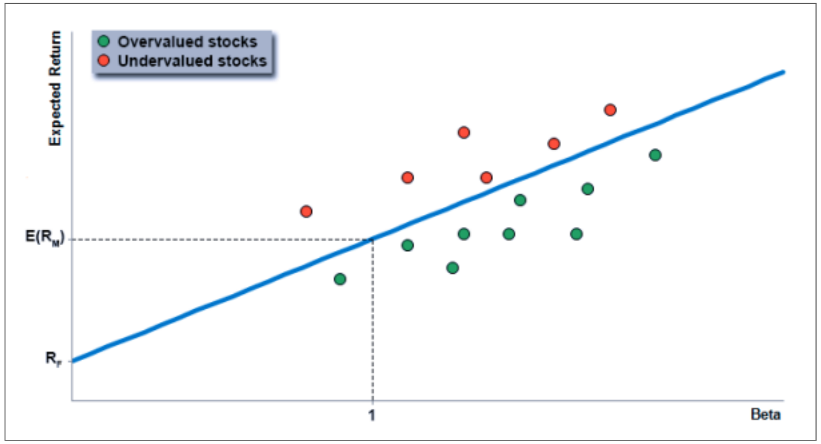
- The SML shows the relationship between expected return ($E(R_i)$) and its risk (β). I.e. it shows the expected return for any given beta/risk.
  - Zero β will have $E(R)$ equal to the risk free rate (r_f).
  - When β is 1, then we have the same risk as the market portfolio (thus expect the same return as the market) - higher than 1, we assume more risk than the market thus expect a higher return.
  - The slope of the SML is the market risk premium ($r_m - r_f$). The higher the market risk premium, the steeper the slope.
  - The SML can change slope over time and the y-axis intercept (R_f - depending on interest rates).
  - SML line can shift with changes in systematic risk changes - i.e. fundamental macroeconomic factors change (unexpected inflation, unemployment, GDP etc.).



r = r_f + beta(r_m - r_f)

- Yahoo Finance (show different betas)
- Each company has a different β and thus the expected returns will be different - each company sits at a different point on the SML (each black dot).

Overvalued / Uudervalued



- Some stocks might have a higher expected return than the SML and a lower expected return than the SML.
- If its above the SML - then the difference between the point and the SML some excess returns - i.e. the return is higher than what is predicted by the SML. i.e. eBay will have an α of say 15% (point above) - 13% (SML) = %2
- i.e. a return > predicted by the CAPM
- Conversely, if the point is below the SML then we are assuming greater risk than what the market dictates.
- **If the market portfolio is efficient** then all assets will lie on the SML and α will be zero.
  - If $\alpha > 0$ - improve on market portfolio by buying the stock (outperform the market)

- If $\alpha < 0$ - improve on market portfolio by selling the stock

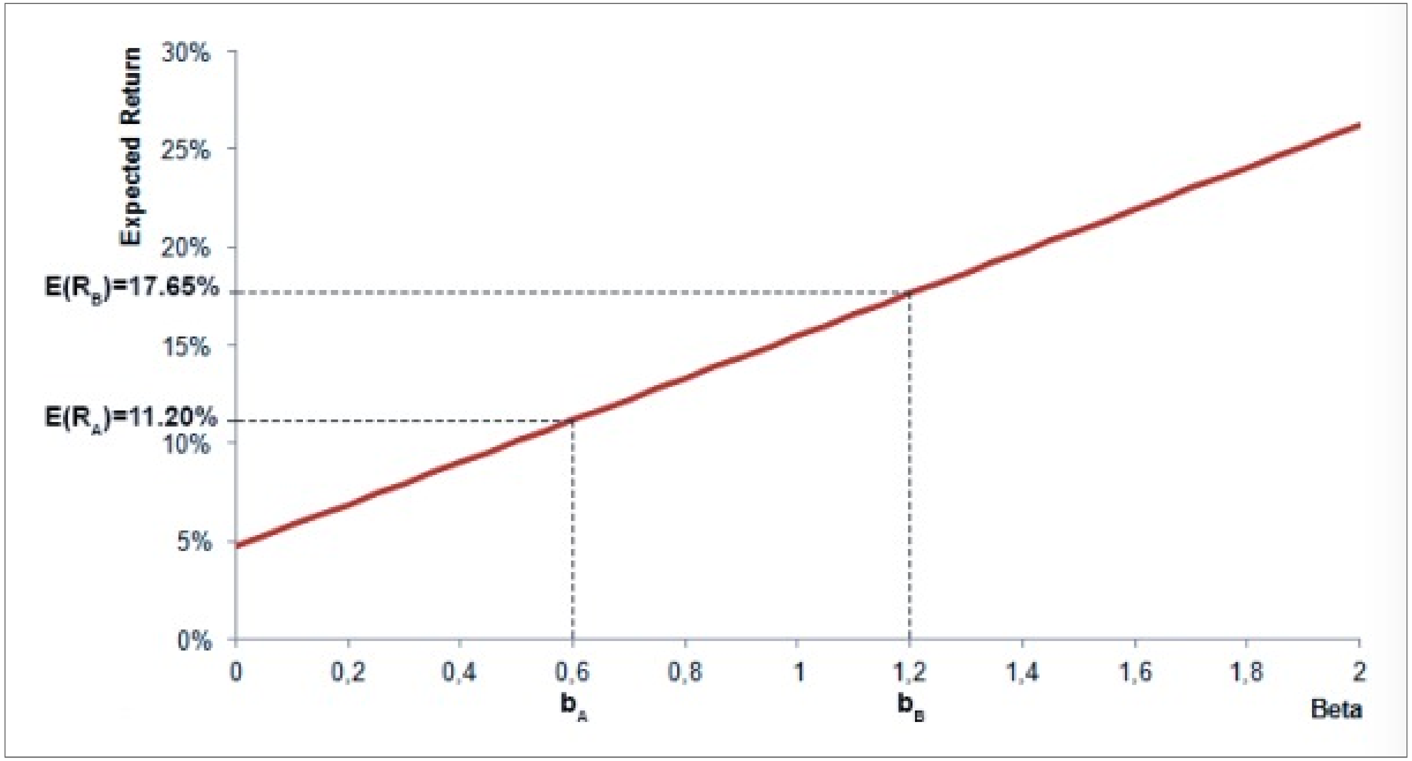
Security Market Line (SML)

Example:

- Assume R_f is 4.75% and expected market return is 15.50%. The SML equation looks like:

  $E(R_i) = 4.75\% + \beta_i(15.50\% - 4.75\%)$
  Suppose that; + **Security A** has a $\beta = 0.6$ + **security B** has a $\beta = 1.2$
- The expected returns of **A** is 11.20% and **B** is 17.65%.
- Therefore, a lower risk (lower β) means lower expected return.

Example


Expected Return
30%
25%
20%
E(R_B)=17.65%
15%
E(R_A)=11.20%
10%
5%
0%
0
0,2
0,4
0,6
0,8
1
1,2
1,4
1,6
1,8
2
b_A
b_B
Beta

Security Market Line (SML)

Example:

- Under the CAPM assumptions, determine AMADEUS's shares expected return taking into account that next year expected market return is 11.5%, one-year t-bills offer a return of 3.5% and AMADEU's β is 1.8.
  - Risk free rate: $r_f = 3.5\%$
  - Expected market return: $r_m = 11.5\%$
  - AMADEUS's β: $\beta_i = 1.8$
- Expected return
  $E[r_i] = r_f + \beta_i(r_m - r_f) = 3.5\% + 1.8(11.5\% - 3.5\%) = 17.9\%$

Calculating beta

How to calculate β

- β is the expected $\%$ change in a security's return, given a 1% change in the market return.
- i.e. we take the return of an individual stock R_i and the return of the market R_m and calculate the covariance between the two. Divide it by the variance of the market.

$$\beta = \frac{Cov(R_i, R_m)}{Var(R_m)}$$

How to calculate β example

Suppose we have;

Month	GOOG	SPY500	R_i_average	R_m_average	R_i_minus_R_i_average	R_i_minus_R_m_average	product_of_deviations	sum_procut_deviations	covarianc
January	0.11	0.08	0.07833333	0.05666667	0.03166667	0.02333333	0.0007388889	0.05466667	0.0109333
February	0.17	0.10	0.07833333	0.05666667	0.09166667	0.04333333	0.0039722222	0.05466667	0.0109333
March	0.21	0.13	0.07833333	0.05666667	0.13166667	0.07333333	0.0096555556	0.05466667	0.0109333
April	0.18	0.11	0.07833333	0.05666667	0.10166667	0.05333333	0.0054222222	0.05466667	0.0109333
May	-0.08	-0.03	0.07833333	0.05666667	-0.15833333	-0.08666667	0.0137222222	0.05466667	0.0109333
June	-0.12	-0.05	0.07833333	0.05666667	-0.19833333	-0.10666667	0.0211555556	0.05466667	0.0109333

- So the β is 1.85. This means that for every 1% change in the market, we expect a 1.85% change in the stock.

3 Factor Model

- **Objective**: Try to explain the expected returns of a security. ### Capital Asset Pricing Model (CAPM)
- Recall, CAPM is a single factor model - the market factor.
- CAPM:
  $r_i = r_f + \beta_i(r_m - r_f)$
- Takes only the risk free rate and the market premium into account.
- So, the expected returns of a security is just a function of one factor (systematic risk β) - how exposed the security is to the market.

(3 Factor Model)

- Fama and French - Common risk factors in the returns on stocks and bonds (1992)
  $r_i = r_f + \beta_{mkt}(r_m - r_f) + \beta_{SMB} + \beta_{HML}$
- SMB (small minus big) - takes market capitalization into account. Small cap stocks tended to have higher returns than large cap stocks. (size effect)
- HML (high minus low) - takes into account the book to market ratio. Stocks with high book to market ratios tended to have higher returns than stocks with low book to market ratios. (value effect)
- book-to market ratio = book value of equity / market value of equity
  - Recall, Book-value-of-equity = Total Assets - Total Liabilities
  - Market value of equity = Share price * Number of shares outstanding
- Idea:
  - SMB - Go long on portfolio of small cap stocks, short on portfolio of large cap stocks.
  - HML - Go long on portfolio of high book-to-market stocks, short on portfolio of low book-to-market stocks.
    - Low book-to-market are *growth* stocks - expected to grow
    - High book-to-market stocks are *value* stocks.

Extentions of CAPM and Fame French