Asset Pricing and Valuation

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```
-- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
v dplyr     1.1.4     v readr     2.1.5
v forcats   1.0.0     v stringr   1.5.1
v ggplot2   3.5.1     v tibble    3.2.1
v lubridate 1.9.3     v tidyr     1.3.1
v purrr     1.0.2
-- Conflicts ------------------------------------------ tidyverse_conflicts() --
x dplyr::filter() masks stats::filter()
x dplyr::lag()    masks stats::lag()
i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to becom

Adjuntando el paquete: 'janitor'


The following objects are masked from 'package:stats':

    chisq.test, fisher.test
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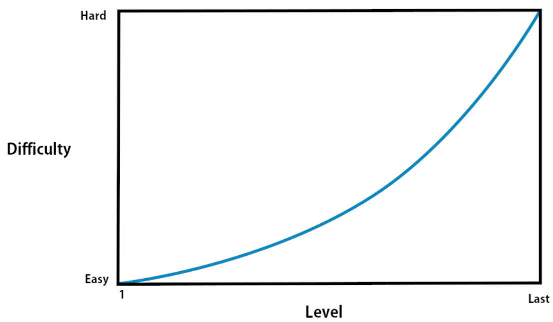
1 Course Overview

- **Valuation**: Key concepts of asset pricing and valuation.
- **Risk and Return of a Portfolio**: Understanding portfolio performance.
- **Markowitz Portfolio Optimization**: Efficient frontier and asset allocation.
- **Capital Asset Pricing Model**: The relationship between risk and expected return.
- **Fama and French 3 & 5 Factor Model**: Multi-factor models and their application.
- **Seeking Alpha (Lazy Prices)**: Exploring alpha and market inefficiencies.
- **(If time) ML for Portfolio Optimization**: Machine learning techniques for optimization.

- **Derivatives**: Understanding financial derivatives and their pricing.
- **Options**: Introduction to options and strategies for pricing and hedging.

2 Course Overview



3 Today's Learning Objectives

- **Understand Investment Valuation Models**: Gain an intuition for various models used in investment analysis.
- **Investment Project Analysis**: Learn how to describe an investment project based on its cash flows.
- **Time Value of Money**
- **Net Present Value (NPV)**: Learn to calculate and interpret the NPV of a project.
- **Discounted Cash Flow (DCF) Valuation**: Understand how to value a firm using DCF analysis.

4 Why Valuation Matters

- **Informs Sound Investment Decisions**: Helps investors identify profitable and safe opportunities by assessing risk and return.
- **Compares and Ranks Projects/Opportunities**: Provides a basis to compare the potential of different investment opportunities objectively.
- **Essential in Corporate Finance & Banking**: A critical tool in corporate finance, investment banking, and portfolio management for assessing value and making strategic decisions.

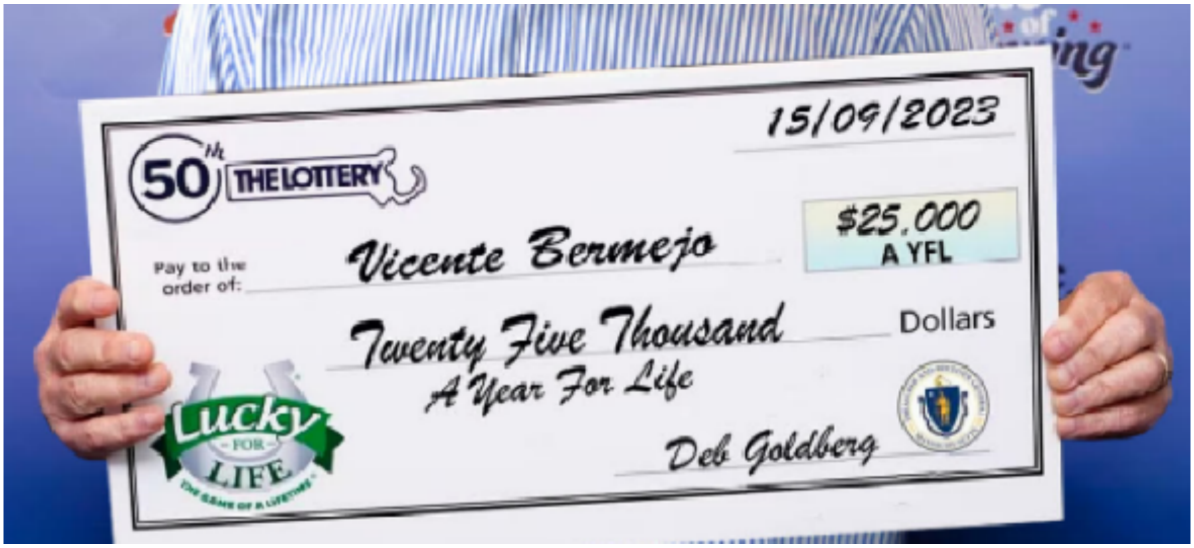
5 Misconceptions About Valuation

- Myth 1: A Valuation is an Objective Search for "True" Value – **Truth 1.1**: All valuations are biased. The only questions are how much and in which direction. – **Truth 1.2**: The direction and magnitude of bias are directly proportional to who pays you and how much you are paid.
- Myth 2: A Good Valuation Provides a Precise Estimate of Value – **Truth 2.1**: There are no precise valuations. – **Truth 2.2**: The payoff to valuation is greatest when the valuation is least precise.
- Myth 3: The More Quantitative a Model, the Better the Valuation (we should use complex models?) – **Truth 3.1**: Let inputs to the model are usually better. – **Truth 3.2**: Simpler valuation models perform better than complex ones.

6 Basis for all valuation approaches

- Decisions on which assets are undervalued and which are overvalued are based on: – Perceptions that markets are inefficient (i.e. mistakes in pricing assets) – Assumption about how and when these inefficiencies will get corrected.
- In efficient markets, the market price is the best estimate of value.
- The purpose of any valuation model is then the justification of this value.

7 Value of a winning lottery ticket



8 Value of a project

- Valuation of a project is subjective.
- Each person will come up with their own valuation for a given project.
- Projects need funding (loans) and investors need to be compensated for the risk they take.
- What is the value of a cow?
- What is the value of a project?

9 Interest Rates

- Interest rates are the price the borrower of money is charged to borrow money.
- Suppose we take a loan for X (principal) at an annual interest rate of r. We hold the loan for 1 year how much interest will be charged?

9.1 Interest Rates

$$\text{Total interest (1 Year)} = rX \text{Outstanding Debt} = \text{Principal} + \text{Interest} = X + rX = (1+r)X$$

10 Interest Rates

Question:
Suppose we borrow \$5000 at an annual interest rate of 7%. How much interest will we be charged in the first year? How much debt will we have if we make no payments? What about year 2?

11 Year 1

Answer:
The total interest and outstanding debt can be calculated as follows:

$$\text{Total interest (1 Year)} = rX = 0.07 \times 5000 = \$350 \text{Outstanding Debt} = (1+r)X = (1+0.07) \times 5000 = \$5350$$

12 Year 2

Answer:
For the second year, we calculate the interest on the new outstanding debt (\$5350):

$$\text{Total interest (2nd Year)} = r \times \text{Outstanding Debt (End of Year 1)} = 0.07 \times 5350 = \$374.50 \text{Outstanding Debt}$$

13 Interest Rates

> Note
>
> - **Where do interest rates come from?**

- We go to a bank and ask for a loan, the bank tells us how much interest we need to pay back. (Where does this number come from?)
- Interest rates are the prices that are negotiated between the borrowers and lenders of money.
- Two types of markets for **borrowers** and **lenders**.

14 Interest Rates

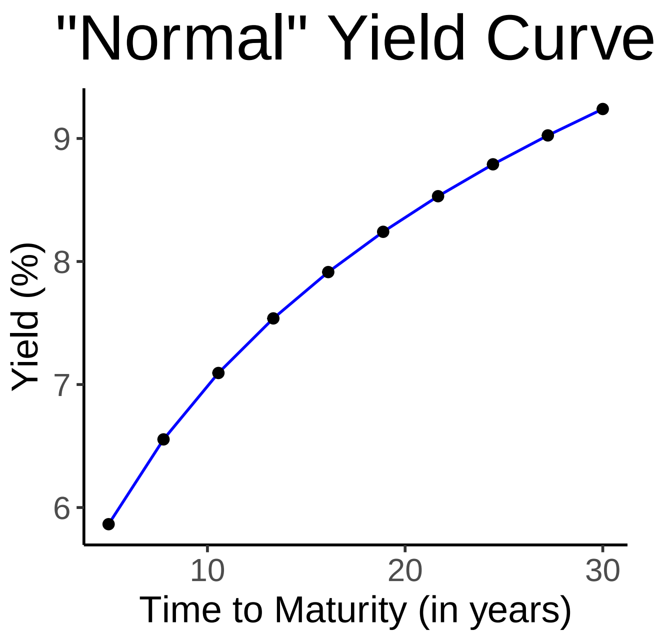
Money Markets - Wants to borrow money as cheaply as possible - Aims for lowest interest rate

Lenders - Wants to earn the highest possible rate (highest rates are riskier) - **Risk** - loss of their investment (not paid back if they go for high rates)

- Interest Rates are the prices negotiated on **Money Markets** and **Bond Markets** - but governments and central banks heavily influence these rates.

15 Term-Structure of Interest Rates

- Interest rates have a *term structure* they vary depending on the length of the loan.
- i.e. we will be charged different interest rates if we borrow money for 3 months than 5 years.
- i.e. the longer the loan term, the more time your money is tied up, thus the more risk you assume. With greater risk, you require a greater rewards, thus in the *Term-Structure of Interest Rates* we see higher yields for longer term loans.



16 Interest Rates

> **Note**
>
> - **Where do interest rates come from?**

- Central banks set an overall base rate for interest rates
- Once central banks set the base rate, the markets adjust (maintain a spread)

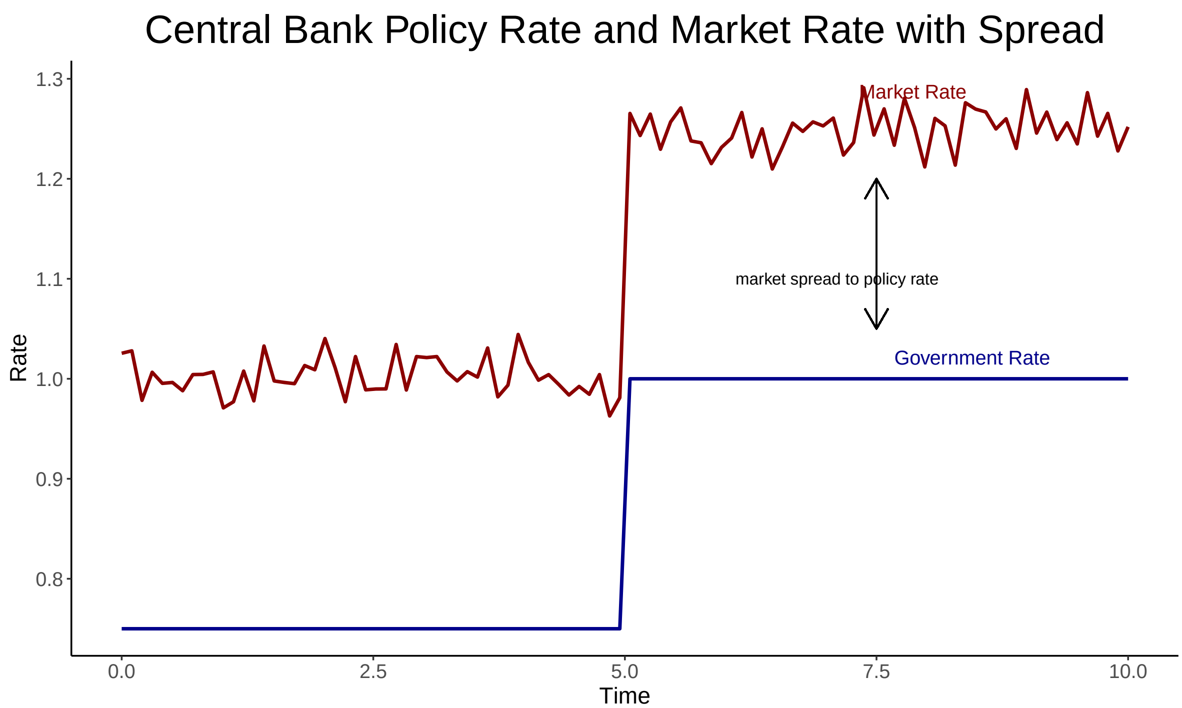
17 Role of central banks

- **Monetary policy:** Central banks set interest rates, control the money supply, and influence short-term interest rates.
- **Currency regulation:** Central banks issue notes that are legal tender.
- **Lender of last resort:** Central banks lend money to other banks during times of crisis to prevent them from collapsing.

- **Banking system oversight:** Central banks oversee the banking system and provide banking services to commercial banks, depository institutions, and the federal government.
- **Cash reserves:** Central banks keep cash reserves for commercial banks. Commercial banks are required to keep a portion of their deposits in cash reserves with the central bank.
- **Financial system stability:** Central banks promote financial system stability, produce economic research/reports.

18 Central Banks

19 Interest Rates



20 Interest Rates

- **Risk** is a force shaping interest rates
- Would you prefer \$100 today or \$100 in one years time?
- Would you prefer \$95 today or \$100 in one years time?

- Would you prefer \$90 today or \$100 in one years time?
- Choices like these help determine the interest rate.

$$\text{Interest Rate} = \frac{\text{Future Value} - \text{Present Value}}{\text{Present Value}} \text{Interest Rate} = \frac{\$100 - \$95}{\$95} = 5.26\% \quad | \quad 11.11\% \quad | \quad 0\%$$

- Minimum interest required in order to prefer \$95 today over \$100 in a year.

21 Time Value of Money

- Most people would prefer \$100 today over \$100 in a year which shows that money has a time value. Money today is worth more than the same amount in the future.
- A dollar received in 1 year is worth less than a dollar received today (Inflation, Opportunity Cost, Risk).
- Put \$1 today in the bank, it earns interest, in one years time we have 1 dollar plus the interest $\$1(1 + r)$.
- So a dollar received in the future is worth less than a dollar today.
- **Question** - So how much is that dollar actually worth today?

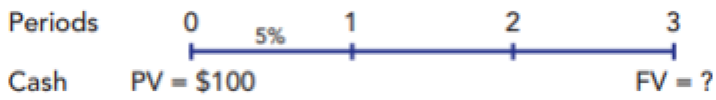
22 Future Value

- What is the Future Value?
- If we invest a sum of money at a fixed interest rate for a fixed period of time, the **future value** is the amount of money we will have at the end of the period.
  - The value of an asset at a future date based on an assumed growth rate.
  - How much an investment today is worth in the future.
  - How much our money today will be worth after a certain period.

23 Present Value

- What is the Present Value? – Its the current value of a future sum of money (of streams of cash flows). – We discount the Future Value by our estimated Rate of Return. – i.e. A sum of money today is likely worth more than the same sum in the future.

24 Time lines



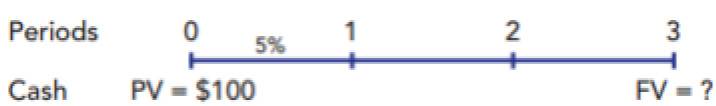
- Want to find the unknown cash flow FV=? at t=3
- Single cash outflow invested at t=0
- Process going from PV to FV is called **compounding**
  - Growing money forward in time
- Process going from FV to PV is called **discounting**
  - Bringing future money back to its value today

25 Definitions

- PV = Present value, or beginning amount = $100.
- FV = Future value, or ending amount, after N periods.
- CF = Cash flow. Cash flows can be positive or negative.
- Interest earned per year and is based on the balance at the beginning of each year, and we assume that it is paid at the end of the year

> **Question (10 minutes)**
>
> We plan to deposit $100 in a bank that pays a guaranteed 5% interest each year.
> How much would you have at the end of Year 3?

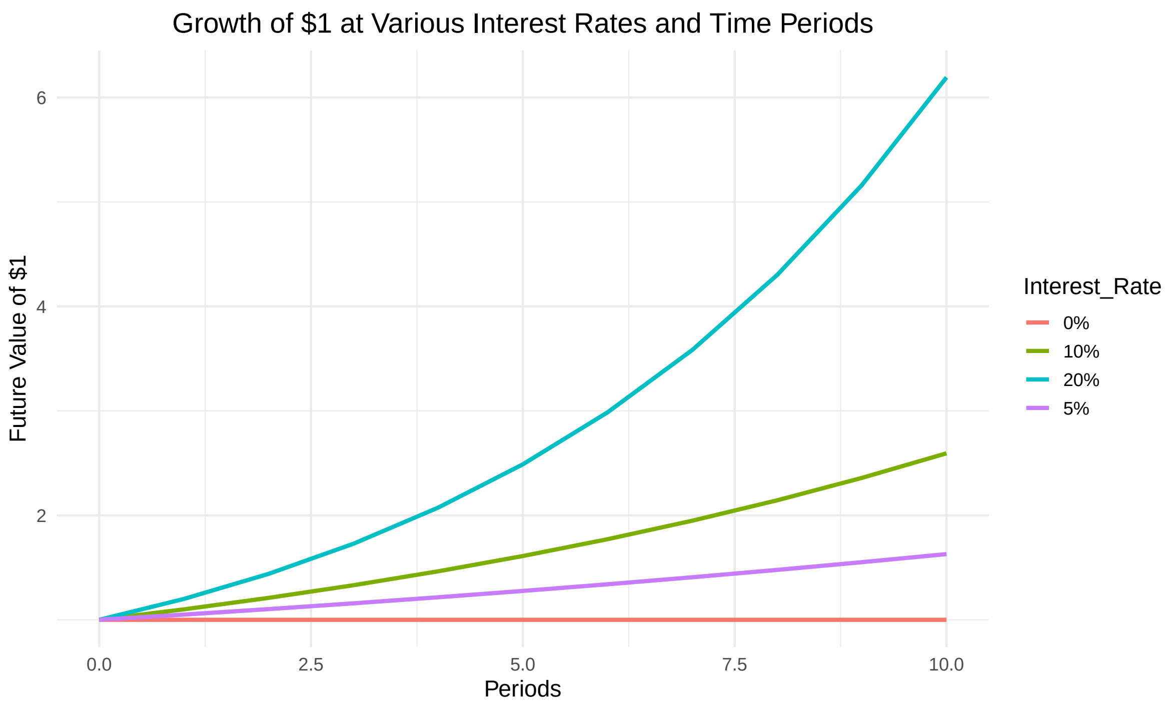
26 Answer



- **Year 1**
  - Amount at the end of Year 1: $(100 + 5) = 105$
- **Year 2**
  - Amount at the end of Year 2: $(100 + 5.25) = 110.25$

- **Year 3**
  - Final balance at the end of process: (115.76)

27 Answer



28 Present Value

- **Finding Present Value (PV)** is the reverse of finding **Future Value (FV)**.
- **Future Value**: $FV_N = PV \times (1 + I)^N$
- **Present Value**: $PV = \frac{FV_N}{(1+I)^N}$

Question (10 minutes)

1) A broker offers to sell you a Treasury bond that 3 years from now will pay $115.76
2) A bank is offering a guaranteed 5% interest on three year certificates of deposits (CDs).
3) **Whats the most we should pay for the bond?**

- **Definitions**

  A) Treasury bond: An interest-bearing bond issued by the US Treasury
  B) Certificate of deposit, or CD, is a type of savings account offered by banks and credit unions.

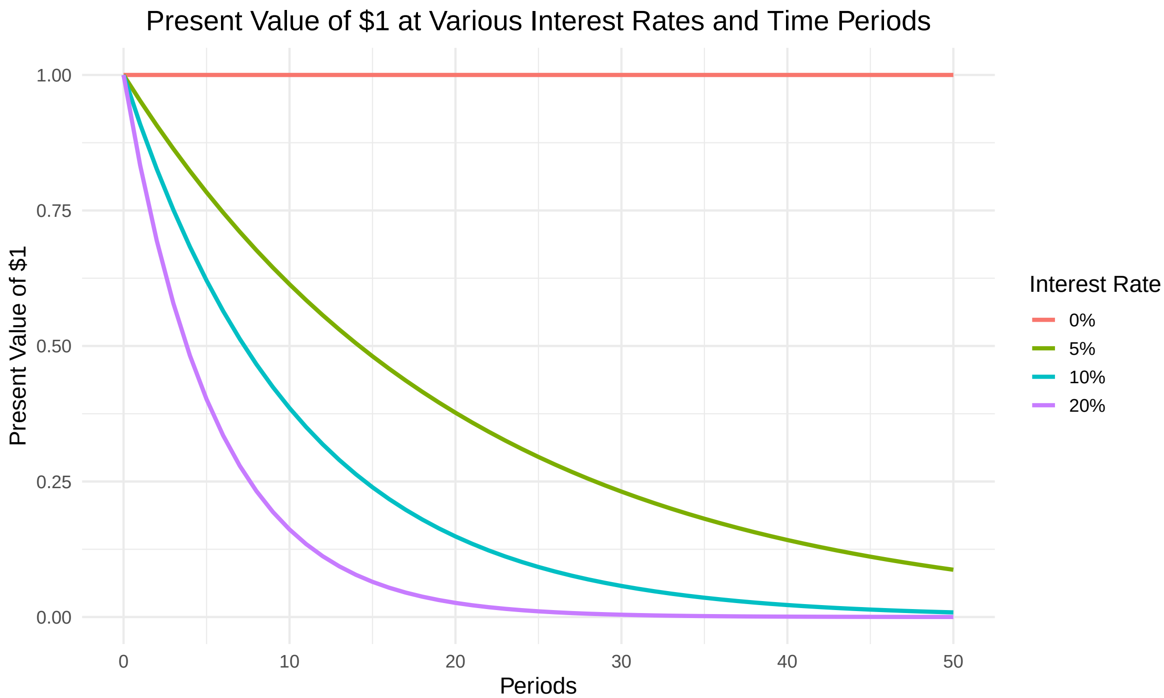
4) If we don't buy the bond, we will buy the CD.
5) The 5% rate paid on CDs is our **opportunity cost** or **rate of return** we could earn on an *alternative investment of similar risk*.

29 Present Value

1) We know in t=3 we will get \$115.76 from T-bond
2) $t_2 = 115/1.05 = 110.25$
3) $t_1 = 110.25/1.05 = 105$
4) $t_0 = 105/1.05 = 100$
5) Alternatively; $115.76/(1.05)^3 = 100$

- Recall, from FV example. If we invested \$100 at 5% it would grow to 115.76 in 3 years.
- We would also have \$115.76 after 3 years if we bought the T-bond
- Question: Whats the most we should pay for the bond?
  - The most we should pay is \$100 - its "fair price"
  - If we can buy cheaper, we should buy it.
  - If its more than \$100 we buy the CD.
  - If bond price = \$100 then we are indifferent.
  - 100 is the present value of \$115.76 due in 3 years at a 5 percent rate

30 Present Values discount rates



- PV of a sum to be received in the future decreases and approaches zero as the payment date is extended.
- A 20% discount rate, $1 million due in 100 years would be worth only $0.0121 today
  - Why? because $0.0121 would grow to $1 million in 100 years when **compounded** at 20%.

31 Present Values discount rates

Question (10 minutes)

What is discounting and how is it related to compounding? How is the FV equation related to the PV equation?

- Discounting is the process of determining the **present value** of a **future** amount of money.
- Compounding is the process of determining the **future value** of a **present** amount of money.

- Discounting is based on the idea that money today is worth more than money in the future (due to earnings potential)
- Compounding involves the growth of money over time as interest is earning on both the principal and interests that accumulate "interest on interest"

1) Compounding moves forward in time, it takes the present value of money and projects its value into the future.
2) Discounting moves backwards in time, it takes a future amount of money and determines its equivalent value today.

- The present value is the future value discounted by $(1 + r)^n$
- The future value is the present value compounded by $(1 + r)^n$

32 Present Value discount rates

Question (10 minutes)

Suppose a U.S. government bond promises to pay \$2249.73 in 3 years at an interest rate of 4%. How much is the bond worth today? How would it change if the bond matured in 5 years? What if the interest rate on the 5 year bond were 6%?

```
# Given values
FV = 2249.73

# Case 1: Bond matures in 3 years at 4% interest
r1 = 0.04
N1 = 3
PV1 = FV / (1 + r1) ** N1

# Case 2: Bond matures in 5 years at 4% interest
N2 = 5
PV2 = FV / (1 + r1) ** N2

# Case 3: Bond matures in 5 years at 6% interest
r2 = 0.06
PV3 = FV / (1 + r2) ** N2

print("
      Bond matures in 3 years at 4% interest: The bond is worth approximately $2000.00 today
      Bond matures in 5 years at 4% interest: The bond is worth approximately $1849.11 today
      Bond matures in 5 years at 6% interest: The bond is worth approximately $1681.13 today
```

```
      As you can see, the present value decreases as the maturity lengthens or the interest i
      ")
```

```
[1] "\n      Bond matures in 3 years at 4% interest: The bond is worth approximately $2000.0
```

33 Present Value discount rates

Question (10 minutes)

How much would $1,000,000 due in 100 years be worth today if the discount rate was 5% and then 20%?

```
# Given values
FV = 1000000
N = 100

# Case 1: Discount rate of 5%
r1 = 0.05
PV1 = FV / (1 + r1) ** N

# Case 2: Discount rate of 20%
r2 = 0.20
PV2 = FV / (1 + r2) ** N

print("
      With a 5% discount rate: The present value is approximately $7,604.49.
      With a 20% discount rate: The present value is approximately $0.01.
      ")
```

```
[1] "\n      With a 5% discount rate: The present value is approximately $7,604.49.\n      Wi
```

34 Finding the interest rate

Question (10 minutes)

- Suppose we know a bond costs \$100 and will return \$150 after 10 years (we know PV, FV and N)
- We want to find the rate of return on the bond.

35 Finding the Interest Rate

1) Start with the future value formula:

$$FV = PV \times (1+i)^n$$

2) Divide both sides by PV:

$$\frac{FV}{PV} = (1+i)^n$$

3) Take the n-th root of both sides:

$$\left(\frac{FV}{PV}\right)^{\frac{1}{n}} = 1+i$$

4) Subtract 1 from both sides to isolate i:

$$i = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$$

4) Plug in the values

$$i = \left(\frac{150}{100}\right)^{\frac{1}{10}} - 1i = 4.14$$

36 Finding the interest rate

Question (10 minutes)

U.S. Treasury offers to sell a bond for \$585.43. No payments will be made until the bond matures in 10 years from now, at which time it will be redeemed for \$1000. What interest rate would you earn if you bought the bond? What rate of return would you earn if you could buy the bond for %550 or for \$600? - We know FV - We know PV - We know N

```
# Given values
FV = 1000
N = 10

# Case 1: Buying the bond for $585.43
PV1 = 585.43
r1 = (FV / PV1) ** (1 / N) - 1

# Case 2: Buying the bond for $550
PV2 = 550
r2 = (FV / PV2) ** (1 / N) - 1

# Case 3: Buying the bond for $600
PV3 = 600
r3 = (FV / PV3) ** (1 / N) - 1

print("
      If you buy the bond for $585.43, the rate of return is approximately 5.50% per year.
      If you buy the bond for $550, the rate of return is approximately 6.16% per year.
      If you buy the bond for $600, the rate of return is approximately 5.24% per year.
      ")
```

```
[1] "\n      If you buy the bond for $585.43, the rate of return is approximately 5.50% per y
```

37 Finding the interest rate

Question (10 minutes)

Microsoft earned $0.12 per share in 1994. 10 Years later, in 2004, it earned $0.14. What was the growth rate in Microsoft's earnings per share (EPS) over 10 years?

```
# Given values
EPS_initial = 0.12
EPS_final = 0.14
N = 10

# Calculate CAGR (Compound Annual Growth Rate)
CAGR = (EPS_final / EPS_initial) ** (1 / N) - 1
CAGR
```

```
[1] 0.01553449
```

```
print("
      The compound annual growth rate (CAGR) of Microsoft's earnings per share (EPS) over the
      ")
```

```
[1] "\n      The compound annual growth rate (CAGR) of Microsoft's earnings per share (EPS) o
```

- Compound Annual Growth Rate (CAGR) is conceptually similar to an interest rate.
- Both describe the rate at which an investment or value grows over time.

38 Finding the number of years

- We might want to know how long it will take to accumulate a given sum of money
- i.e. We know what we have and how much we want and we know various rates of returns
- How long will it take us to get to our objective amount of money?

Question (10 minutes)

We believe that we could retire comfortably if we had $1 million, our current savings is $500,000 and we found an investment product which gives us 4.5% We want to know our retirement date!

```
# Given values
PV <- 500000
FV <- 1000000
r <- 0.045

# Calculate the number of years (N) to reach $1 million
N <- log(FV / PV) / log(1 + r)

# Print the result
cat("It will take approximately", round(N, 2), "years for your savings of $500,000 to grow to
```

```
It will take approximately 15.75 years for your savings of $500,000 to grow to $1,000,000 at
```

39 Finding the number of years

$$FV = PV \times (1 + i)^n$$

Rearrange to solve for (n):

$$\frac{FV}{PV} = (1 + i)^n$$

Taking the natural logarithm of both sides:

$$\ln\left(\frac{FV}{PV}\right) = \ln(1 + i)^n$$

Which simplifies to:

$$\ln\left(\frac{FV}{PV}\right) = n \times \ln(1 + i)$$

Finally, solving for (n):

$$n = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1 + i)}$$

40 Finding the number of years

> **Question (10 minutes)**
>
> How long would it take $1,000 to double if we invested in a savings account that pays 6% annually? How long at 10%?

```
# Given values
PV <- 1000
FV <- 2 * PV  # Doubling the initial investment

# Interest rates
r1 <- 0.06  # 6% interest
r2 <- 0.10  # 10% interest

# Calculate the number of years (N) to double the investment at 6% and 10%
N1 <- log(FV / PV) / log(1 + r1)
N2 <- log(FV / PV) / log(1 + r2)

# Print the results
cat("At 6% interest, it will take approximately", round(N1, 2), "years to double your investm
```

```
At 6% interest, it will take approximately 11.9 years to double your investment.
```

```
cat("At 10% interest, it will take approximately", round(N2, 2), "years to double your invest
```

```
At 10% interest, it will take approximately 7.27 years to double your investment.
```

41 Finding the number of years

> **Question (10 minutes)**
>
> Microsoft's EPS were $1.04 in 2004 and its CAGR in the prior 10 years was 24.1% per year! If that growth rate was maintained, how long would it take for the EPS to double?

```r
# Given values
r <- 0.241  # CAGR of 24.1%

# Calculate the number of years (N) to double the EPS
N <- log(2) / log(1 + r)

# Print the result
cat("At a CAGR of 24.1%, it will take approximately", round(N, 2), "years for the EPS to doul
```

```
At a CAGR of 24.1%, it will take approximately 3.21 years for the EPS to double.
```

42 Uneven cash flows

- Many financial payments involve uneven cash flows. – i.e. Dividends on stocks typically increase over time. – We denote Cash Flows for uneven cash flows
  - Cash flows can be:
    1) Streams of payments + additional final lump sum (bonds)
    2) all other uneven streams (stocks and capital investments)

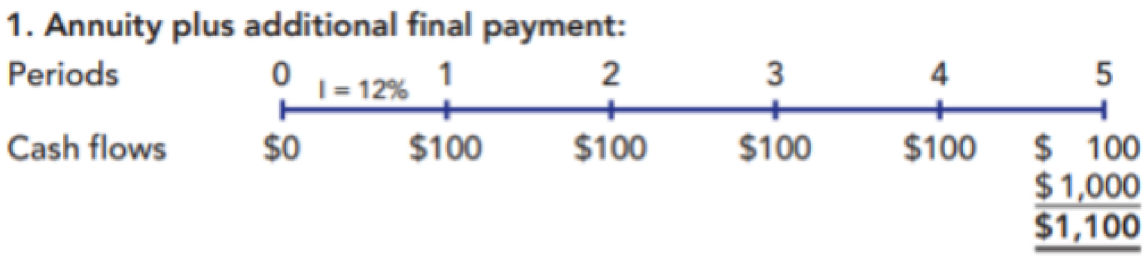
43 Uneven cash flows

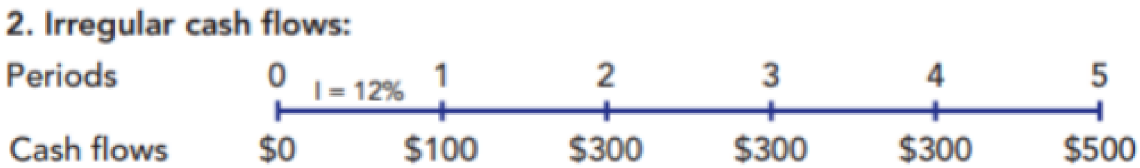
Question (10 minutes)

Find the present values of the following 2 streams of cash flows.

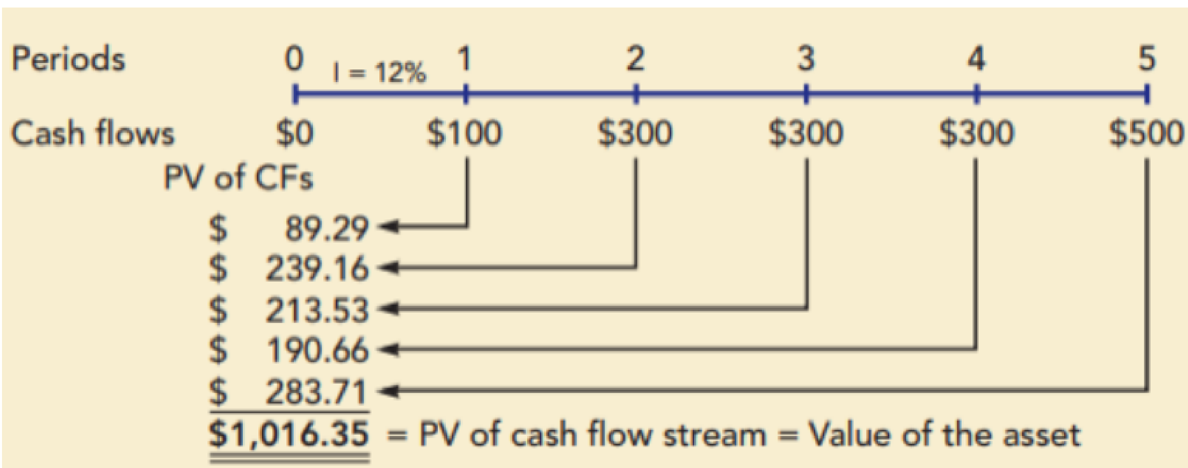
$$PV = \frac{CF_1}{(1+i_1)} + \frac{CF_2}{(1+i_2)^2} + \frac{CF_3}{(1+i_3)^3} + \cdots + \frac{CF_n}{(1+i_N)^N}$$

$$PV = \sum_{t=1}^{n} \frac{CF_t}{(1+i_t)^t}$$





44 Uneven cash flows



45 Uneven cash flows

Question (10 minutes)

Whats the present value of the following uneven cash flow streams? $0 at Time 0, $100 in Year 1 (or at Time 1), $200 in Year 2, $0 in Year 3, and $400 in Year 4 if the interest rate is 8 percent?

```r
CF <- c(0, 100, 200, 0, 400)  # Cash flows
i <- 0.08  # Interest rate

# Calculate Present Value (PV)
PV <- sum(CF / (1 + i)^(0:4))  # 0:4 represents the time periods 0 to 4

# Print the result
cat("The present value of the cash flows is:", round(PV, 2), "\n")
```

```
The present value of the cash flows is: 558.07
```

- $$PV = \frac{0}{(1+0.08)^0} + \frac{100}{(1+0.08)^1} + \frac{200}{(1+0.08)^2} + \frac{0}{(1+0.08)^3} + \frac{400}{(1+0.08)^4}$$
- $$PV = \sum_{t=1}^{4} \frac{CF_t}{(1+0.08)^t}$$

46 Risk Adjusted Value: Two Basic Propositions

- The value of an asset is the risk-adjusted **present value** of the cash flows
- $$\text{Value of asset} = \frac{E(CF_1)}{(1+r)} + \frac{E(CF_2)}{(1+r)^2} + \frac{E(CF_3)}{(1+r)^3} + \cdots + \frac{E(CF_n)}{(1+r)^n}$$
  - If **IT** does not affect the expected cash flows or the riskiness of the cash flows, **IT** cannot affect value.
  - For an asset to have value, the expected cash flows have to be positive some time over the life of the asset.
  - Assets that generate cash flows early in their life will be worth **more** than assets that generate cash flows later. (Time value of money)

47 Net Present Value (Question)

Our manager has come to us and asked us to evaluate two potential projects that the company could invest in. We are asked to evaluate which project is the best one. The **discount rate** is 7% for both projects. Each project lasts 5 years, the initial investment for each project is 15000 and \$20000 respectively. The cash flows for each of the projects are the following:

Cash Flow Streams

Comparing Project A and Project B

Year	Project A	Project B
0	-$15,000.00	-$20,000.00
1	$3,000.00	$2,000.00
2	$3,000.00	$4,000.00
3	$5,000.00	$6,000.00
4	$5,000.00	$8,000.00
5	$5,000.00	$10,000.00

48 Net Present Value (Solution)

Question (10 minutes)

Our manager has come to us and asked us to evaluate two potential projects that the company could invest in. We are asked to evaluate which project is the best one. The discount rate is 7% for both projects. Each project lasts 5 years, the initial investment for each project is 15000 and $20000 respectively. The cash flows for each of the projects are the following:

$$NPV_A = -15000 + \frac{3000}{(1+0.07)} + \frac{3000}{(1+0.07)^2} + \frac{5000}{(1+0.07)^3} + \frac{5000}{(1+0.07)^4} + \frac{5000}{(1+0.07)^5}$$

$$NPV_B = -20000 + \frac{2000}{(1+0.07)} + \frac{4000}{(1+0.07)^2} + \frac{6000}{(1+0.07)^3} + \frac{8000}{(1+0.07)^4} + \frac{10000}{(1+0.07)^5}$$

49 Net Present Value (Solution)

- Both projects profitable.
- Project B > Project A.
- Also need to consider other factors.
  - Is the NPV of Project B high enough to warrant a bigger initial investment?
  - What about intangible benefits, such as strategic positioning and brand equity?
  - Higher riskier investments might not be captured by NPV.
  - Market conditions might change and affect the cash flow streams.
  - Environmental and social impact - is project B more sustainable?

Discounted Cash Flow Analysis
Comparing Project A and Project B

Year[1]	Project A ($)[1]	Project B ($)[1]
Year 1	$2,803.74	$1,869.16
Year 2	$2,620.32	$3,493.75
Year 3	$4,081.49	$4,897.79
Year 4	$3,814.48	$6,103.16
Year 5	$3,564.93	$7,129.86
NPV	$1,884.95	$3,493.72

[1] All values are presented in present value terms, discounted at 7%

50 NPV Interpretation

- How do we interpret NPV=0?
  - The Present Value (PV) of the future cash flows is equal to the initial costs.
  - The expected return on the project compensates for its level of risk.
- How do we interpret NPV $>$ 0?
  - The PV of the future cash flows is higher than the initial costs.
  - The expected return on the project is above the minimum required for the level of risk it generates.
  - In colloquial terms, we can say that this investment makes us “richer” or increases the value of the company.
- How do we interpret NPV $<$ 0?
  - The PV of the future cash flows is lower than the initial costs.
  - The expected return on the project is below the minimum required for assuming such a risk.
  - We can say that this investment reduces our wealth.

51 NPV Interpretation

The main task of a finance director is to identify projects with positive NPV (Net Present Value).

- The idea is to accept (reject) projects with NPV $>=$ 0 ($<$ 0).
  - When the NPV is positive, we increase our wealth as long as we invest.

      * The money needed to start a project is less than the PV (Present Value) of the future cash flows that the project generates.

- What can we do if several projects have positive NPVs?
  - NPVs can be added, therefore increasing the number of positive NPV projects increases our wealth even further.
- What can we do if all available projects have a negative NPV?
  - We should not invest in projects with negative NPVs.

52 Net Present Value

Recall;

$$\text{NPV} = I_0 - \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2}, ..., \frac{C_5}{(1+r)^5} + \frac{C_6}{(1+r)^6}$$

$$\text{NPV} = I_0 - \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}$$

Where;C_t = Cash flow in period tr = Discount rateI_0 = Initial investment$T$ = Number of periods

53 Discount Rates

$$200 = 1000 - \sum \frac{200}{1+0}^1 + \frac{200}{1+0}^2 + \frac{200}{1+0}^3 + \frac{200}{1+0}^4 + \frac{200}{1+0}^5 + \frac{200}{1+0}^6$$

$$1083.44 = 1000 - \sum \frac{200}{1+0.03}^1 + \frac{200}{1+0.03}^2 + \frac{200}{1+0.03}^3 + \frac{200}{1+0.03}^4 + \frac{200}{1+0.03}^5 + \frac{200}{1+0.03}^6$$

$$871.05 = 1000 - \sum \frac{200}{1+0.10}^1 + \frac{200}{1+0.10}^2 + \frac{200}{1+0.10}^3 + \frac{200}{1+0.10}^4 + \frac{200}{1+0.10}^5 + \frac{200}{1+0.10}^6$$

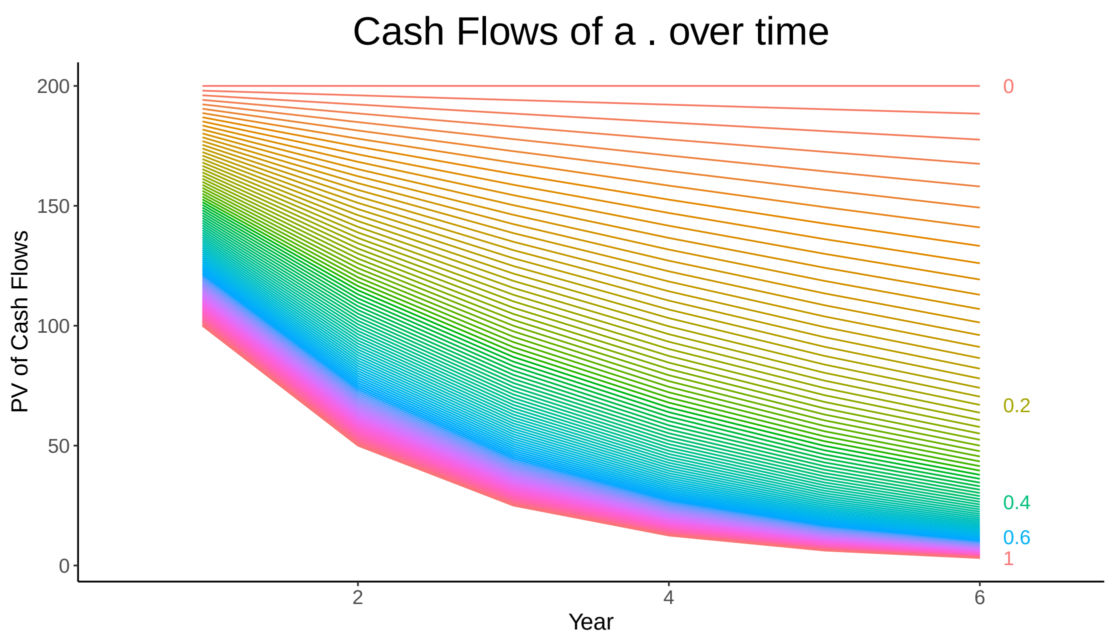
```
r = 0.10
res = 200/((1+r)^1) + 200/(1+r)^2 + 200/(1+r)^3 + 200/(1+r)^4 + 200/(1+r)^5 + 200/(1+r)^6
I_0 = 1000
print(I_0 - res)
```

```
[1] 128.9479
```

54 Discount Rate

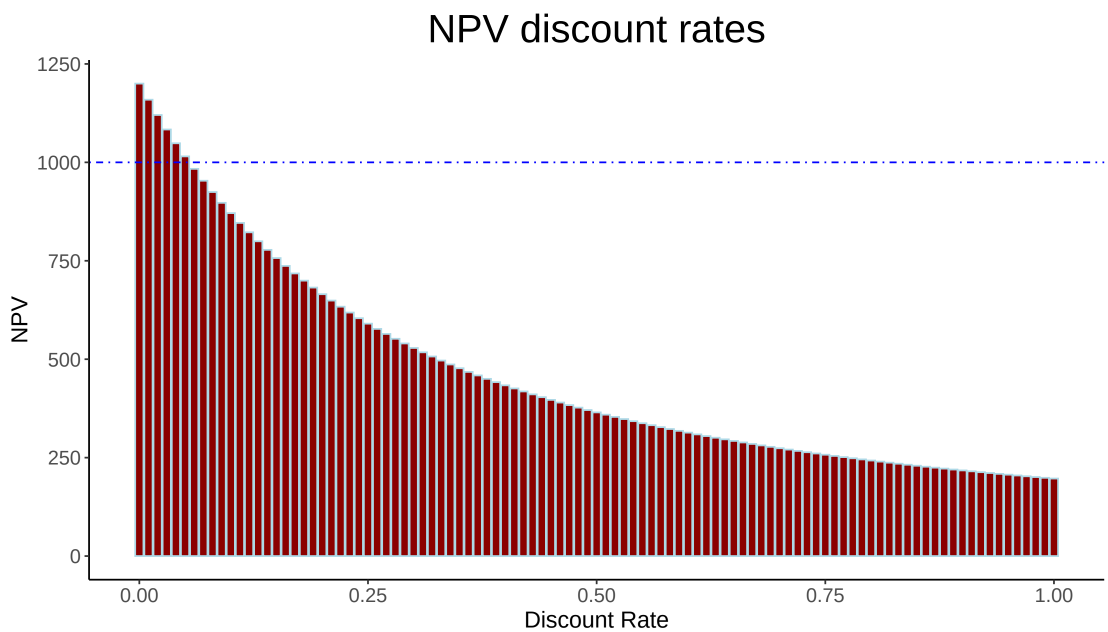
- How do we determine the discount rate?
- The discount rate is the return we expect to get from **alternative** investments with similar risk.
  - **Risk-Free Rate** - government bonds - *no risk*.
  - **Risk Premium** - Added to the *risk-free* rate to account for the extra risk of the investment.
  - **Company's Cost of Capital**: For businesses, it often reflects the Weighted Average Cost of Capital (WACC), considering both debt and equity financing costs.

55 Cash Flows / Discount Rate



- As the discount rate increases the **Cash Flows** are worth less today.
- The sum of the **cash flows**, the **NPV** decreases as well.

56 NPV discount rates



57 Importance of the discount rate

- **Reflects risk** - Higher discount rates are used for riskier investments. If the cow's milk production is uncertain, or milk prices are volatile, you might use a higher discount rate to account for that risk.
- **Affects NPV** - A higher discount rate makes future money worth less today, decreasing the NPV. A lower discount rate makes future money worth more today, increasing the NPV.
- **Decision-Making** - By adjusting the discount rate, you can make more informed decisions. If even at a high discount rate the NPV remains positive, it signifies a robust investment opportunity.
- **Limitations**
  - NPV is sensitive to the discount rate
  - NPV is sensitive to the cash flow estimates (we assume we know the future cashflows)

Net Present Value
Example

	0	1	2	3	4	5	6	7	8
Initial.Cost	-$30	—	—	—	—	—	—	—	—
Inflows	—	$20	$20	$20	$20	$20	$20	$20	$20
Outflows	—	-$14	-$14	-$14	-$14	-$14	-$14	-$14	-$14
Net.Inflows	—	$6	$6	$6	$6	$6	$6	$6	$6
Salvage	—	—	—	—	—	—	—	—	$2
Net.Cash.Flow	-$30	$6	$6	$6	$6	$6	$6	$6	$8

58 NPV Question

Question (10 minutes)

Your company produces fertilizer and currently traded at NYSE with latest stock price of \$27.3 and 1,000 common shares outstanding. You are thinking to invest in a new product,an organic fertilizer with an initial required investment of \$30, 000. You believe that the new organic fertilizer will produce an annual cash revenue of \$20, 000 and cash costs (including taxes) will be \$14, 000 per year. We will wind down the business in 8 years and the plant, property and equipment will be worth \$2, 000 at that time. With a 15% opportunity cost of capital, should you take the investment?

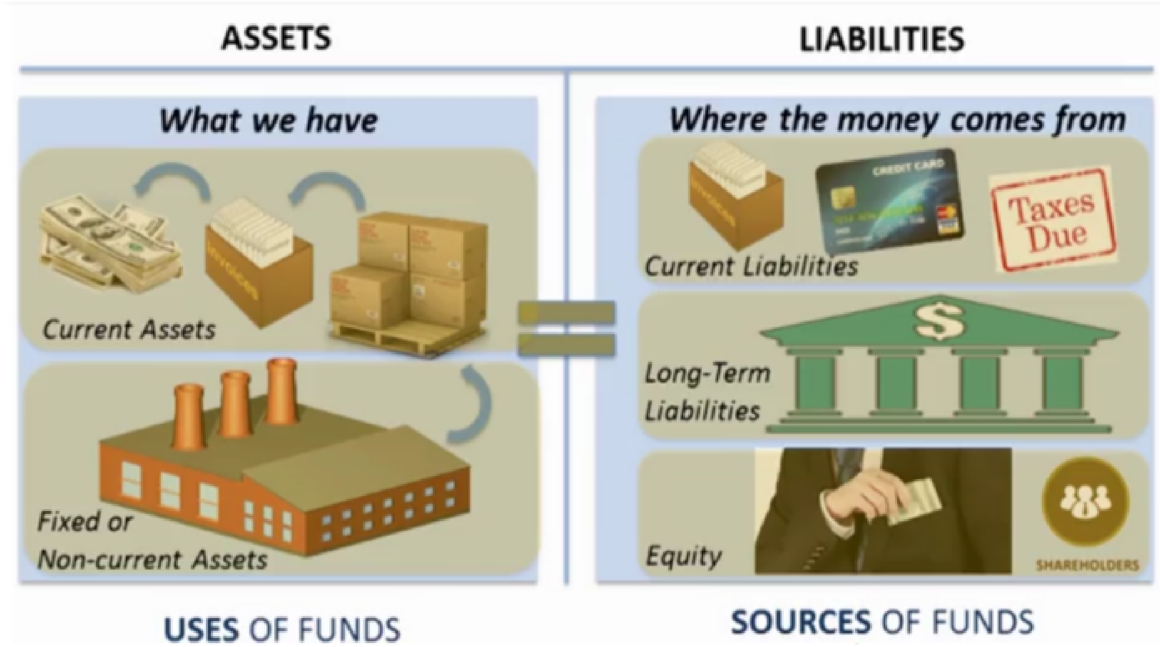
59 NPV Solution

The NPV of the new investment is:

$$\text{NPV} = -30,000 + 6,000\frac{(1.15)^8 - 1}{0.15(1.15)^8} + \frac{2,000}{1.15^8} = -2,422$$

- The new investment will lose value of $-\$2,422$. Per share this translates to a loss of $-\$2,422/1,000 = -\$2.42$ that will drop the stock price from their current \$27.3 to \$24.9 (-9% negative return). - Would you accept this investment?

60 Financial Ratios



61 Financial Ratios

- **Price to Book value**
  - Total assets of \$3 billion
  - Total liabilities of \$2 billion
  - 100 million outstanding shares
  - Current share price of \$15
  - Book value = \$3 - \$2 = \$1 billion
  - Per share book value = \$1 billion / 100 million shares = \$10 per share
  - Price to book value = \$15 / \$10 = 1.5

62 Financial Ratios

- **Price to sales**
  - Lower P/S ratio - more attractive the investment
  - Sales is 500 million
  - 100 million outstanding shares
  - Current share price of \$15

  - Per share sales = \$500 million / 100 million shares = \$5 per share
  - Price to sales = \$15 / \$5 = 3

63 Financial Ratios

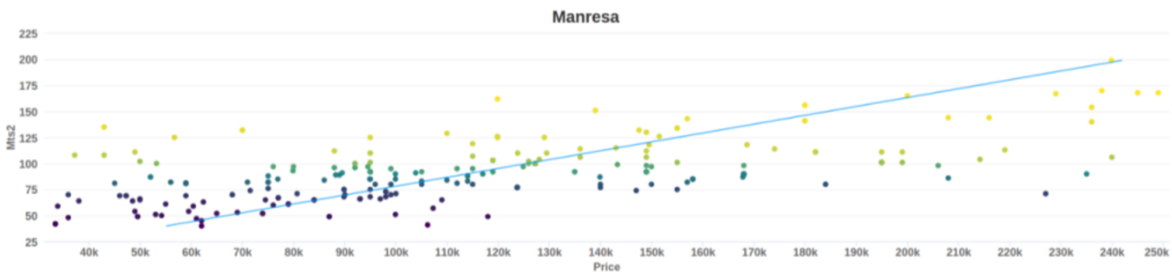
- **Price to earnings**
  - Earnings per share = Net Income / shares outstanding
  - Net income is 200 million
  - 100 million outstanding shares
  - Current share price of \$15
  - Earnings per share = \$200 million / 100 million shares = \$2 per share
  - Price to earnings ratio = market value per share / earnings per share
  - Price to earnings = \$15 / \$2 = 7.5

64 Property valuation

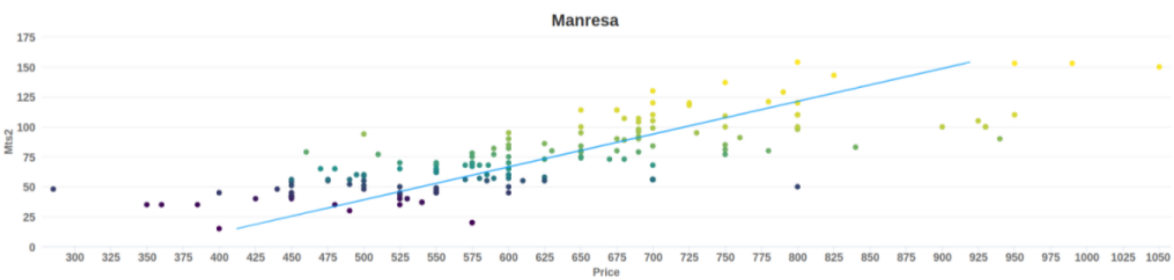
> **Question (10 minutes)**
>
> How would you value a property? In BCN/Surrounding area

- If we are valuing a property, we could value it from the ground up - brick and mortar (net asset approach to valuing a property/company)
- Could look at similar houses and see if ours is worth something similar. (multiples based approach)
- Value a property but looking at the rental income it generates (DCF approach)

65 Property Evaluation



Listing Prices



Rental Prices

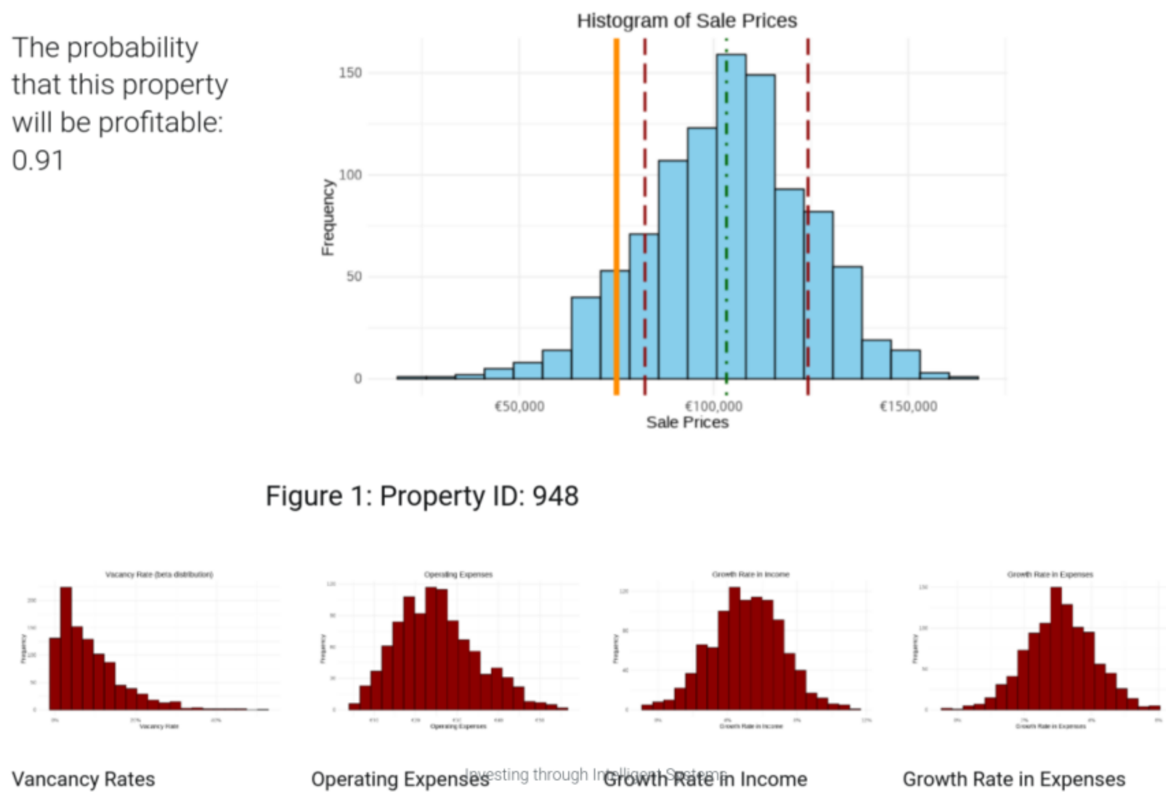
66 Property Evauation

DCF Assumptions	
Vacancy	8%
General Expenses	€10.00
Growth Rate Income	5%
Growth Rate Expenses	3%
Cap Rate	8%
Note: A simple example.	

Discounted Cash Flow Analysis

All figures are presented in €

	Projection (€)					
Process	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**
Potential Gross Revenues	€ 6,900	€ 7,245	€ 7,601	€ 7,987	€ 8,386	€ 8,806
General Vacancy	€ 550	€ 580	€ 610	€ 640	€ 670	€ 705
Effective Gross Revenues	€ 6,350	€ 6,665	€ 6,990	€ 7,347	€ 7,715	€ 8,100
Operating Expenses	€ 880	€ 905	€ 930	€ 960	€ 990	€ 1,020
Net Operating Income	€ 5,470	€ 5,760	€ 6,060	€ 6,387	€ 6,725	€ 7,080
Valuation at Sale						€ 88,500
IRR						11 %



67 Discounted Cash Flow (DCF)

- Fair Value
  - The fair value of a stock is the present value of all future cash flows.
  - A company earns *cash flows* and we expect them to earn the same in the future.
  - We discount the future cash flows back to today
  - We can obtain a fair value per share for a company.

68 Discounted Cash Flow (Example)

- Forecast that a company is going to generate \$100 million every year for 5 years.
- What do we think the company is worth? 500 million?
- Time value of money - is \$100 million in 5 years worth the same as \$100 million today?
- Inflation erodes the value of money over time.

- We need to *discount the future values back to the present value (today)

$$PV = \frac{FV}{(1+r)^n}$$

- So 100m in 1 years time is roughly worth 90m today, 100m in 5 years time is roughly equal to 62m today etc.
- The higher the interest rates, the lower the value of the cash flows and therefore lower the value of the firm overall.
- If company issues 100m shares then each share is worth about $3.79 a share (when IR are 10%)

```
  time cash_flow InterestRate       PV      NPV
1    1       100          0.1 90.90909 379.0787
2    2       100          0.1 82.64463 379.0787
3    3       100          0.1 75.13148 379.0787
4    4       100          0.1 68.30135 379.0787
5    5       100          0.1 62.09213 379.0787
```

69 Dividend Growth Model

- Shares are valued as any other asset:
  - Calculate the **present value** of the future cash flows
- Shareholders expect to receive two types of future cash flows
  - **Dividends** - cash payments made by the company to its shareholders
  - **Capital gains** - the difference between the price at which the shares are bought and sold
- The return per share of firm **i** obtained between period 0 and 1 (one period) is:

$$R_i = dfracDIV_i + P_1 - P_0 P_0$$

70 Value a stock

A share's PV is given by;

$$P_0 = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \cdots + \frac{D_N}{(1+r)^N}$$