Asset Pricing and Valuation

Lecture 3: Markowitz Model

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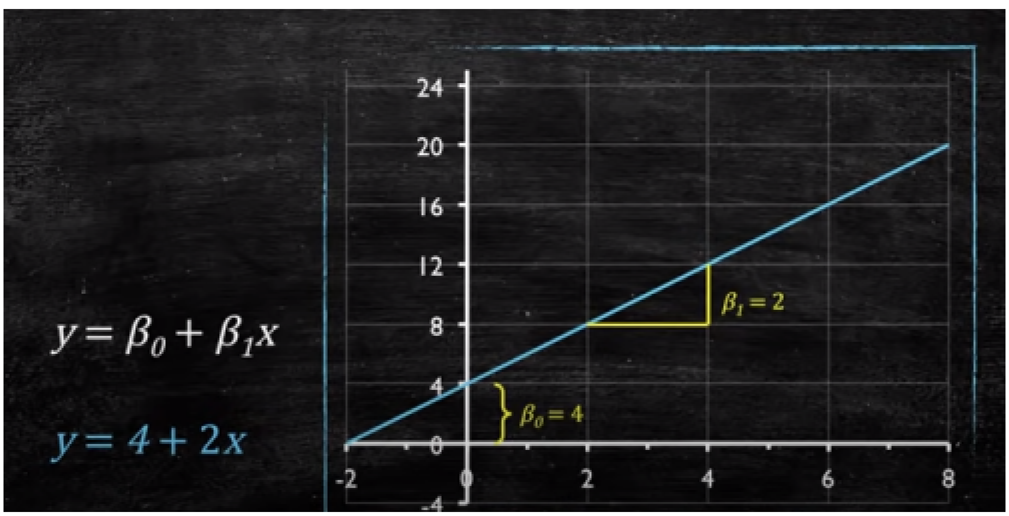
$$y = \alpha + \beta_1 \times x$$

- α is the intercept - β_1 is the slope

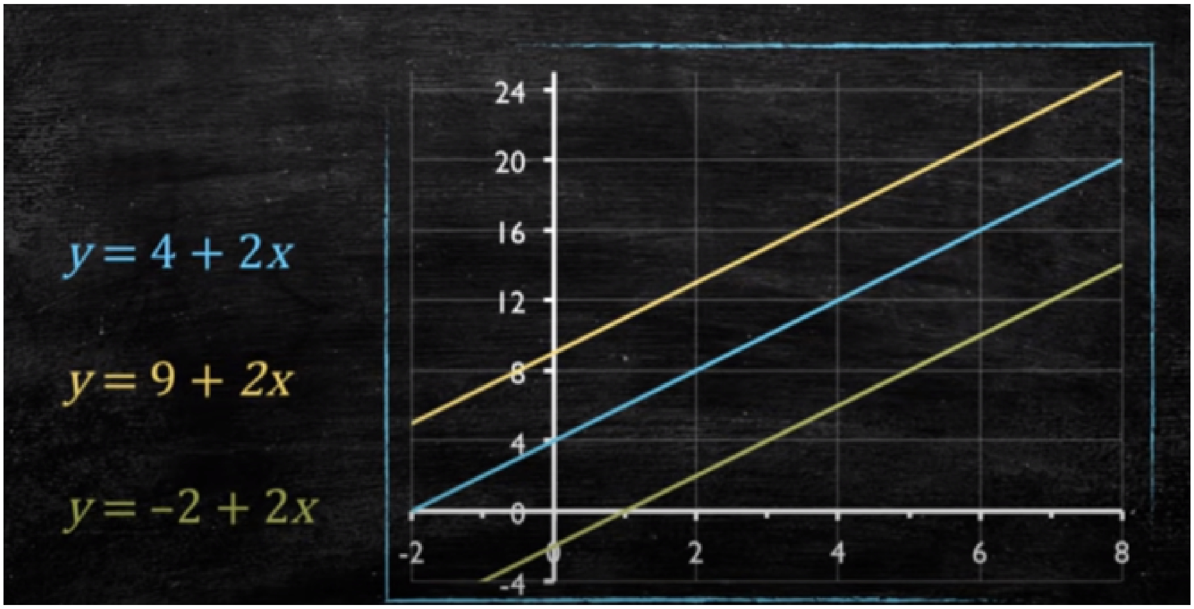
- If we have $\beta_0 = 4$ then the intercept is when we have the x-axis = 0 and the y-axis = 4
- If the slope is 2, then the slope tells us for every unit increase in the x-axis, the y-axis will increase twice as much.
- or relating to our risk/return - for a 2% increase risk, we get a 4% increase in return
- If $\beta = 2$ - then it

3 Regression introduction



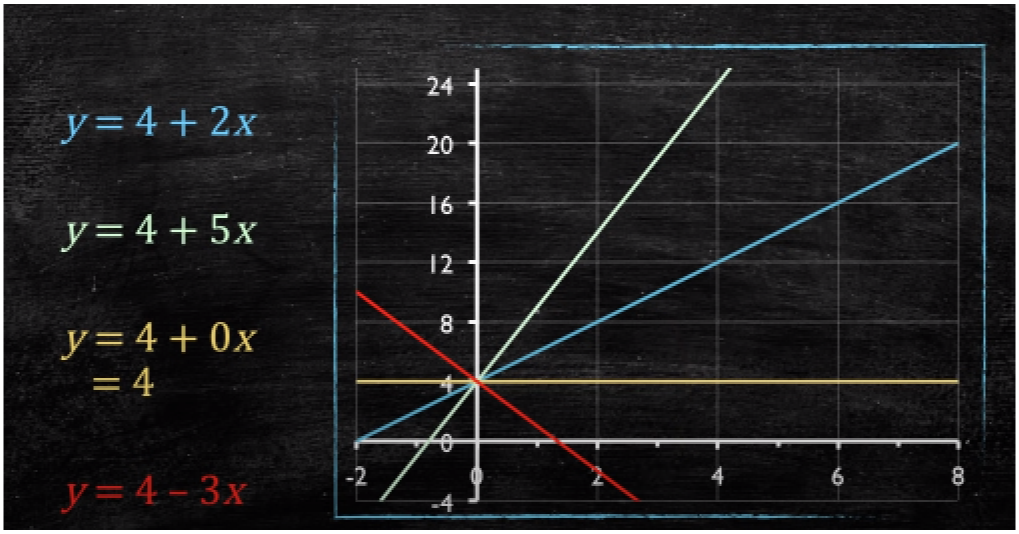
4 Changing intercepts

- Changing the intercept changes the y-axis
- The same as changing the risk free rate of return



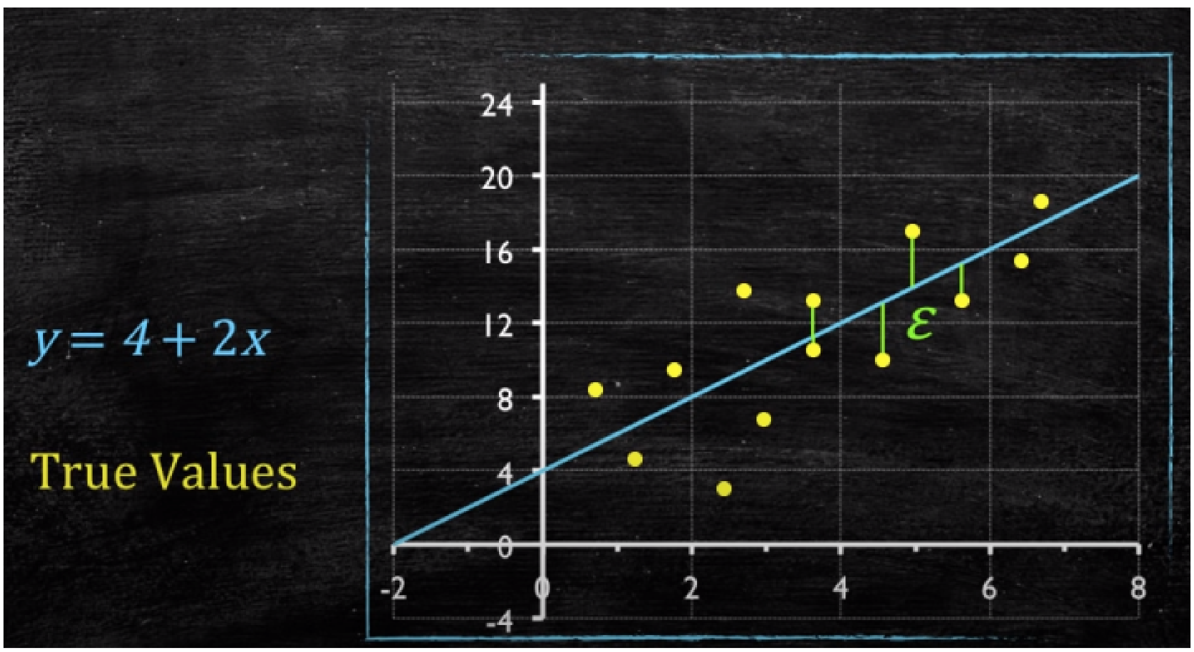
5 Changing the slope

- changing the sensitivity on y on values of x
- or we are changing the sensitivity of our beta value
- if we cange the slope from 2 to 5 then for every unit of x-axis we get 5 units of y-axis (steaper slope)
- y-axis is growing much faster than before
- Zero slope - means no relationship between x and y, y will always be 4
- If we have a negative slope, then for every unit increase in x, y will decrease by 2 units



6 beta in the real world

- Does not always sit on the same line



- Linear regression tries to minimize these errors
- but the linear model has errors so we must include these errors into our equation

$$y = \alpha + \beta x + \epsilon$$

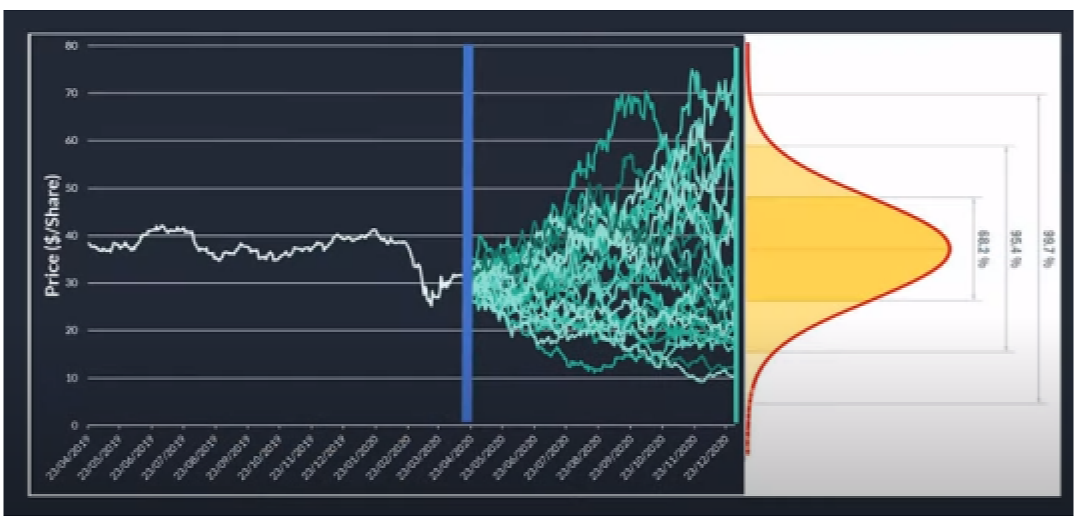
- y = dependent variable - x = independent variable - α = intercept - β = slope - ϵ = error term (we want to minimize)

6.1 Markowitz (1958) model

- Markowitz (1958) set the basic of Modern Finance and Modern Portfolio Theory
- Discusses how an investor could optimally decide to allocate their funds among risky assets
- Since then, extensive research has been done to improve the model
- The rational investors problem is to determine weights w to **optimally** construct a portfolio.
- In the mean-variance - we want to select one portfolio on the **efficient fronteir** subject to a constraint given by our risk-return preferences.

6.2 Mean-Variance

- Obtain risk estimations by looking at the distribution of past returns
- https://www.portfoliovisualizer.com/



6.3 Mean-Variance

- Choose our weights as to maximize the returns subject to volatility constraints - with the weights summing up to 1

-Maximize:

$$w^* = max_w w' \mu$$

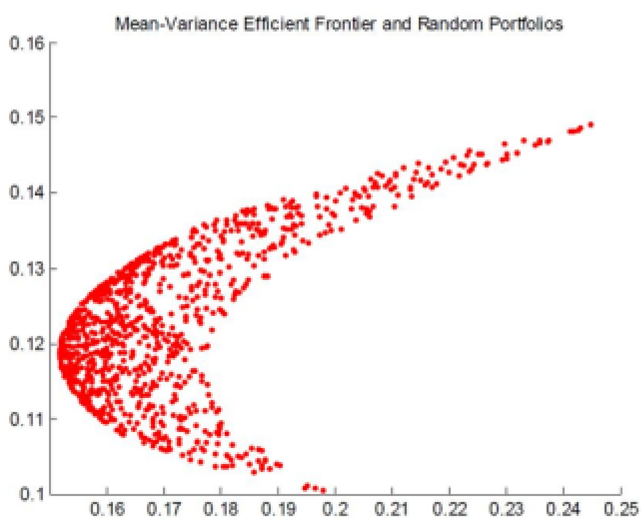
- Subject to:

$$\sqrt{w'\Omega w} = \sigma_p^* \sum_{i=1}^{K} w_i = 1$$

- Additionally,
- *No-short selling* - i.e. portfolio weights can't be negative $w_i >= 0$

6.4 Efficient Frontier

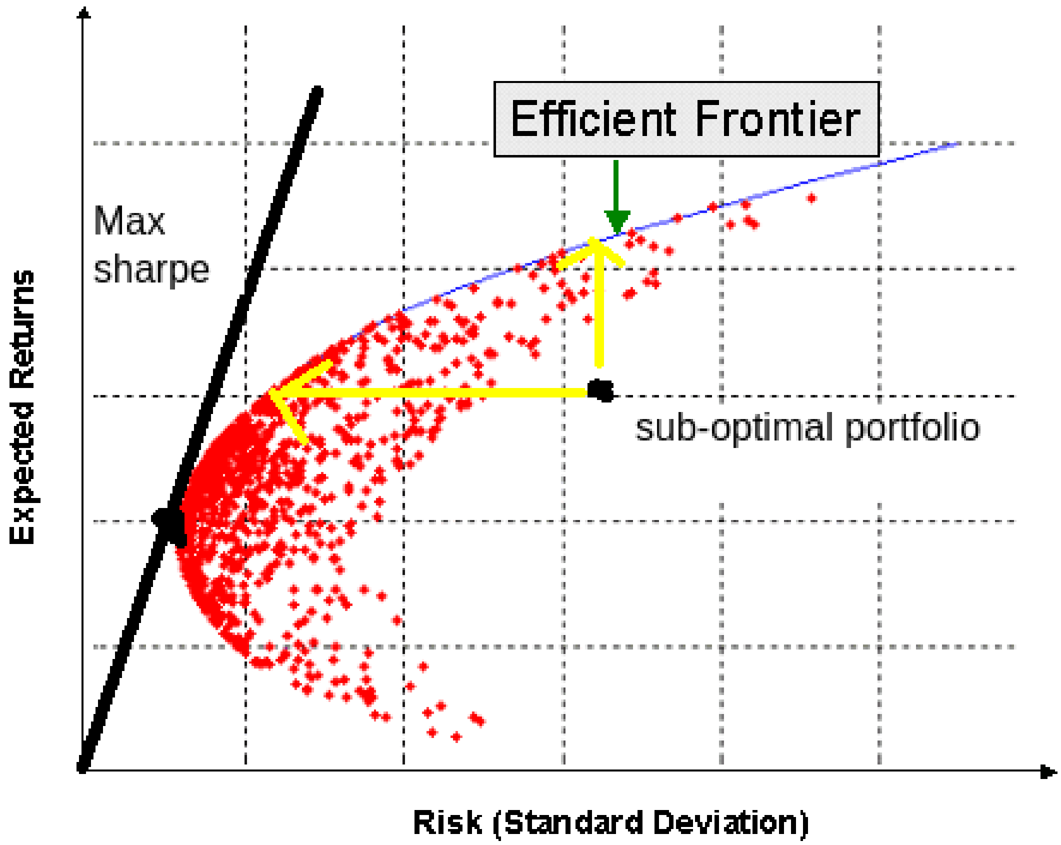
- Modern Portfolio Analysis - rational investors determine the optimal combination of assets of their portfolio on the basis of expected returns (means) of individual assets and risk (covariance matrix)
- Consider each point as an asset in the plot below
- For any particular return level (y axis) there is an optimal portfolio



6.5 Efficient frontier

- The portfolios which sit on the line reflect portfolios of all of the assets that provides the lowest risk for a given return
- These are the "most efficient" portfolio combinations
- We compute this for all potential returns (from minimum return stock to maximum)
- The maximum return portfolio - sits at the upper right hand side of the line

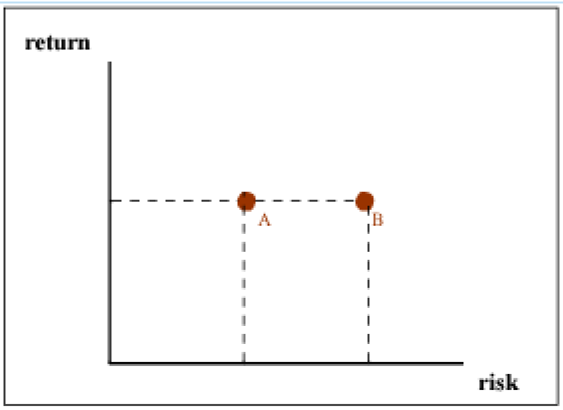
- As the expected returns fall we find other portfolios which are efficient

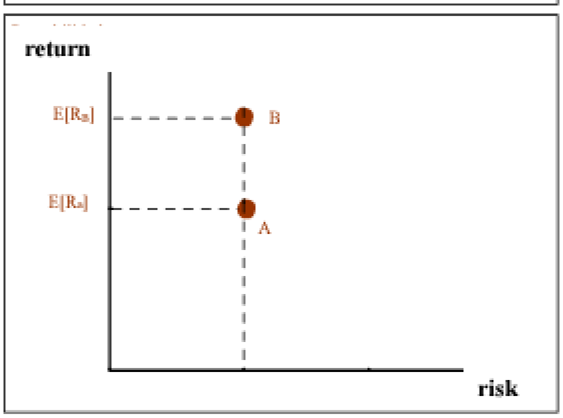


- Portfolios in red represent the *feasible set* of portfolios.
- The lower half of the returns - **we are reducing our returns but taking on more risk**
- Any portfolio below the **efficient frontier** is sub-optimal
  - Sub-optimal - i.e. inside the efficient frontier we assume the same risk but less return or we could obtain the same return but with less risk
- We can't obtain portfolios outside/above the efficient frontier
- If we draw a tangent line from the origin to the efficient frontier we obtain the *max sharpe ratio* portfolio.
- An investor always chooses a portfolio located on the efficient frontier.

6.6 Mean-Variance

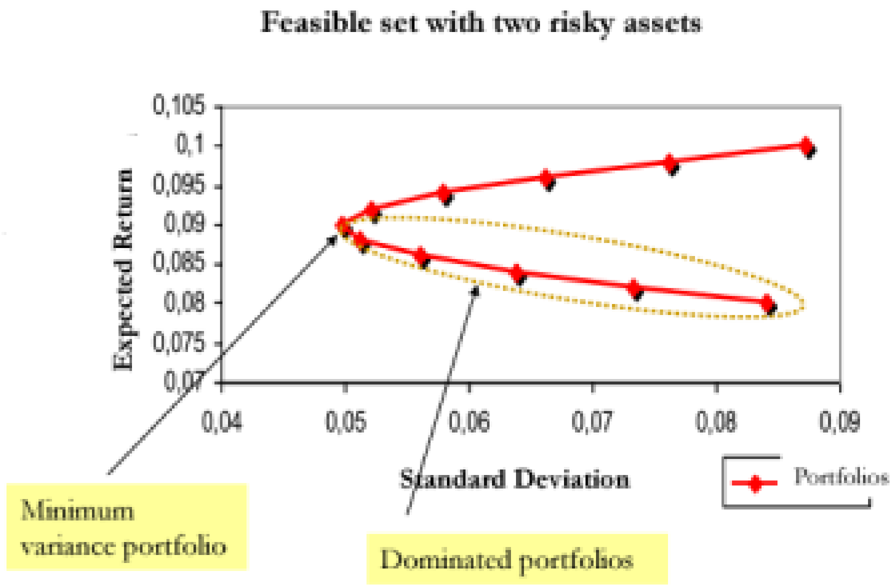
- Portfolio theory studies how to create portfolios that maximize an investors expected *utility*
- *utility* : The level of satisfaction associated with all possible outcomes, whether they result in shortfall or surplus
- It only looks at the means and variances
- The assumption of normal distribution must hold
- Investors are risk averse - i.e. minimize risk and max return
- Simulator: https://demonstrations.wolfram.com/TwoAssetMarkowitzFeasibleSet/





- In graph 1 (A is preferred) - In graph 2 (B is preferred)

6.7 Mean-Variance



6.8 Max-Sharpe Ratio

- Measures the risk to reward of a portfolio
- Sharpe ratio = return/volatility

$$Sharpe = \frac{Return - RiskFreeRate}{\sigma}$$

- we want a small denominator (σ) and a large numerator (return - risk free rate)
- The higher the Sharpe ratio is the better
- Sharpe greater than 1.0 is considered acceptable to good by investors
- A Sharpe higher than 2.0 is very good
- Sharpe greater than 3.0 is considered excellent.
- A ratio less than 1.0 is considered sub-optimal.

6.9 Sharpe Ratio Example

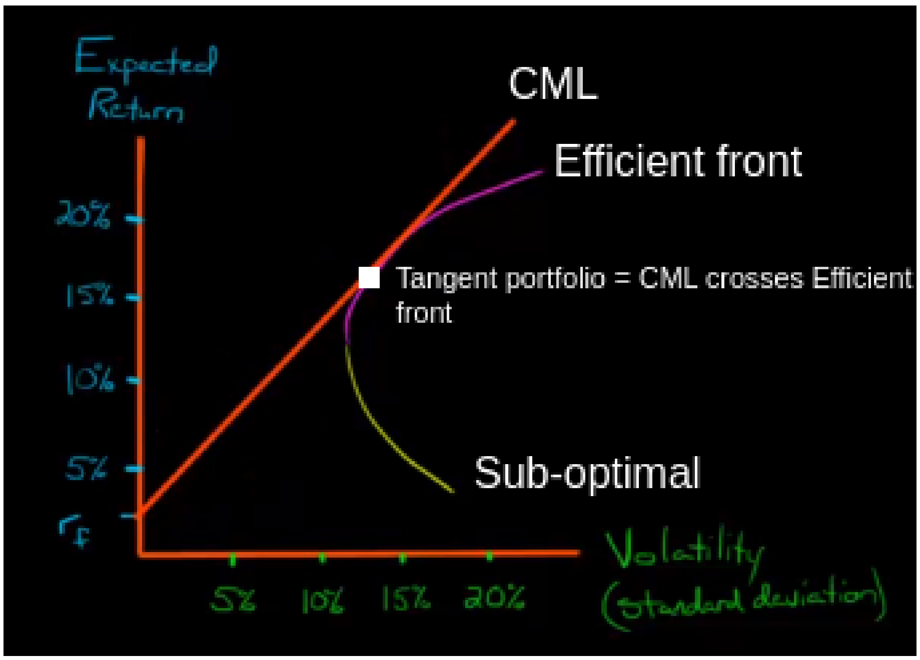
$$r_p = 14\% r_f = 4\% \sigma_{Rp} = 20\%$$

Then;

$$\text{Sharpe ratio} = \frac{14 - 4}{20} = 0.5$$

- What do we do with the Sharpe Ratio?
  - Its a unit of volatility to reward
  - we can rank and compare portfolios
  - Higher Sharpe ratio the better
  - i.e. if we have another portfolio with a Sharpe of 0.6 - then we get more reward per unit of risk from this portfolio than the previous portfolio

6.10 Sharpe Ratio (Example)

- 

- The capital market line (CML) represents portfolios that optimally combine risk and return.
- It is a theoretical concept that represents all the portfolios that optimally combine the risk-free rate of return and the market portfolio of risky assets.

- Under the capital asset pricing model (CAPM), all investors will choose a position on the capital market line
- Capital Market Line - a line from the R_f to the efficient frontier

$$\text{CML} = R_f + \frac{R_m - R_f}{\sigma_m} \times \sigma_p$$

- Equation tells us: Expected return on an *efficient portfolio* is equal to the risk-free rate plus a risk premium of the portfolio expressed as a percentage of the standard deviation of the market and multiplied by the standard deviation of the portfolio.
- The CML is the line that connects the risk-free rate to the efficient frontier
- It connects risk (measured by the standard deviation of the portfolio) to the *efficient* portfolios return
- Sharpe ratio is the portfolio which minimizes our unit of risk per return
- When analyzing a portfolio the Sharpe tells us how much reward are we getting per unit of risk for this portfolio

6.11 Sortino ratio

- The Sharpe ratio penalizes upside and downsize risk equally
- In stocks - upside volatility is a good thing (if we are long on a stock)
- The Sortino ratio only penalizes downside volatility (or volatility under some specified)
- Sharpe = (Average return) / Standard deviation
- Sortino = (Average return) / Downside deviation

$$\text{Sortino} = \frac{R_p - R_f}{\sigma_{NegativeRets}}$$

- Any positive return will not negatively affect the rating
- Look for a Sortino ratio of above 2.0

6.12 Treynor Ratio

- A percentage that measures the **reward-to-risk** of a portfolio
- Focuses on the **systematic risk** since it uses the portfolio beta
- Both the Sharpe and Treynor measure reward-to-risk (so we can still rank portfolios)
- Different from Sharpe since Sharpe focused on the portfolio total (systematic and unsystematic) risk σ whereas Treynor focuses on the systematic risk β

- Can only use the Treynor ratio on portfolios which are well diversified (where we have already diversified all of the non-systematic risk away)

$$T_P = \frac{\text{Excess return}}{\text{Beta of the portfolio}} = \frac{(r_p - r_f)}{\beta_p}$$

6.13 Treynor Ratio (Example)

$$r_p = 14\% r_f = 2\% \beta = 3$$

- So,

$$\text{Treynor}_p = \frac{r_p - r_f}{\beta_p} = \frac{14 - 2}{3} = 4\%$$

So the Treynor ratio of this portfolio is 4% - Can compare this to other portfolios - i.e. if a portfolio has a Treynor ratio of 3% then our original portfolio of 4% gives us more reward per unit of systematic risk.

7 About the CAPM model

- Developed in the 1960's by William Sharpe, Jack Treynor, John Lintner and Jan Mossin
- Extends the work of Harry Markowitz on portfolio theory
  - Deals with the valuation of assets held in a diversified portfolio - and less about the portfolio itself.
- Its based on the idea that not all risks are relevant if assets are hed in a diversified portfolio
- Previously we were interested in how our portfolio returns were related to the market returns

7.1 Type of risk

- Total risk of an **individual** stock is split into
  - Diversifiable risk
    * Also called unsystematic risk and its the firm specific risk - which can be diversified away as we increase the number of stocks in our portfolio - i.e. the risk is diversified away whent he stock is held in a portfolio. (e.g. labour strike, product recalls)

- Undiversifiable risk
  * Systemic risk - or - market risk, and relates to the risk which cannot be eliminated regardless of portfolio size. - oil price shocks, inflation, recessions, acts of nature, geo-political risk.

7.2 The concept of Beta

- According tot eh CAPM, **investors are rewarded only for market risk (systematic risk) - since non-systematic risk can be diversified away in an efficient portfolio**
- Beta is a measure of the systematic risk of a security
- Beta measures the sensitivity of a security's returns to the market returns
- Beta is obtained as the slope of the market model
  - $r_{it} = \alpha_i + \beta_i r_{mt} + \epsilon_{it}$
  - r_{it} is the return on security i at time t
  - r_mt is the return on the market at time t
  - β is the sensitivity of the security to the market (i.e. we are trying to measure this unverifiable risk through beta)

8 Jensen's Alpha

- The proportion of the excess return that is **not** explained by systemic risk (β)
- α is the difference between the actual return and the return predicted by CAPM
- i.e. if the CAPM provides us a ROI of 12% (expected) but the actual is 13% (so stock has a higher ROI than expected from CAPM)
- Then the α is 1% (13% - 12%)

$$r_i = r_f + \beta(r_m - r_f) + \alpha_i$$

Solving for α

$$\alpha_i = r_i - r_f - \beta_i(r_m - r_f)$$

8.1 Jensen's Alpha

Example:

$$r_f = 3\% r_m = 11\% \beta = 1.5 \text{actual return} = 17\%$$

- What is alpha?
- We might say that its 6% (17% - 11%) but this is not correct
- We need to use the CAPM model to calculate the expected return
  - We need to adjust for risk (β)
  - i.e. our beta is 1.5 - so when the market goes up we expect our portfolio to go up 1.5 times as much
  - So we expect it to out perform the market index
  - So α is a risk adjusted measure of return

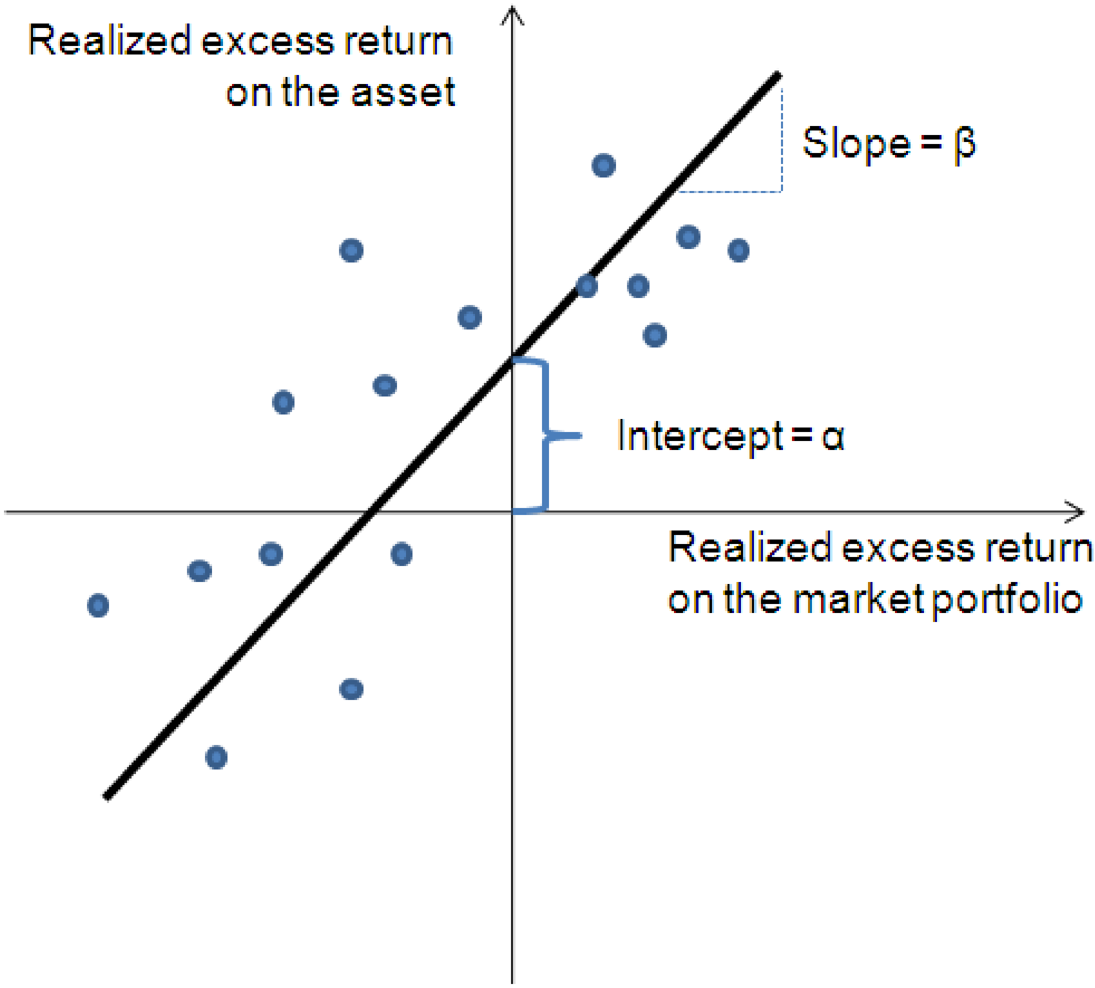
$$\alpha_i = 17 - 3 - 1.5 * (11 - 3)\alpha_i = 14 - 12 = 2\%$$

- Portfolio managers talk about alpha / seeking alpha

8.2 Jensen's Alpha

- Suppose we have;

$$r_i - r_f = \alpha_i + \beta_i(r_m - r_f)$$



8.3 Regression introduction

8.4 Linear regression

- Two main objectives of regression models
  - Establish a relationship between two variables
    - i.e. if one variables value increase and another decreases we aim to measure this relationship (through statistical significance)
    - Example - income and spending (positive relation ship .- people earn more, spend more (on average)), wage and gender (men earn higher than women on average (gender discrimination)), student height and exam scores (expect no relationship to exist)?

    * Forecast future values
    * If we know a companies sales grow over time, then we want to predict next quarter or next months sales
- **Deendent variable** - (Values depend on this variable - denoted a Y)
- **Independent variables** - (Variables helps to explain the dependent variable - denoted as X)