Asset Pricing and Valuation

Lecture 6: Options

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Options

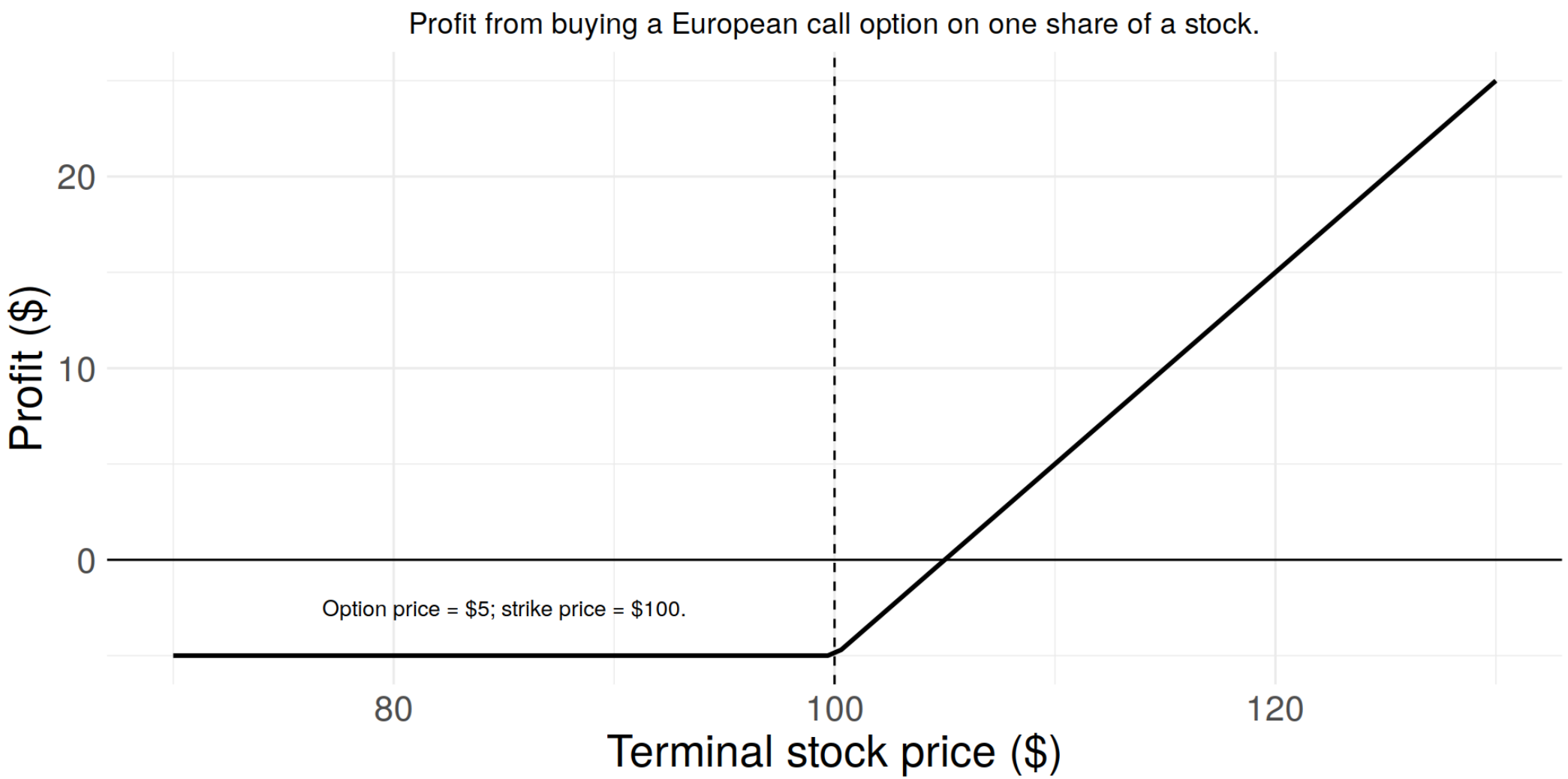
Options, Forwards and Futures

- Options - different from **forward** and **futures** contracts.
  - Options give the holder the **right** to do something but they do not have to exercise this right.
  - Forward and Futures, the two parties have committed themselves to some action.
  - Forwards and Futures cost the trader nothing
  - Options requires an up-front payment

Types of options

- A **call** options gives the holder of the option the **right** to **buy** an asset by a **certain date** for a **certain price**.
- A **put** option gives the holder the **right** to **sell** an asset by a **certain date** for a **certain price**.
  - **American** options can be exercised at any time up to the date of expiration
  - **European** options can be exercised only on the expiration date.

A call option



A call option

Example (hope stock price increases)

- Suppose current stock price is $98
- We buy a European call option with a strike of $100, expiry is 4 months
- The price of the option is $5
- Initial investment is $500

Stock price less than $100 on expiry

- If stock price is less than $100 at expiry, the investor will choose not to exercise
  - Suppose stock price is $90 on expiry
  - We do not exercise the contract since its cheaper to buy at $90 on the market.
  - No point buying a stock for $\$100$ when its trading at $90
  - We don’t exercise the contract - losing $500

Stock price more than $100 on expiry

- If stock price is $115 on expiry
  - We exercise the option
  - buy 100 shares for $100 each
  - Sell shares immediately, we make $\$15$ per share (minus $5 cost)

A put option


Profit from buying a European put option on one share of a stock.
30
20
10
0
Profit ($)
e = $7; strike price = $70.
40
60
80
100
Terminal stock price ($)

A put option

Example (hope stock price decreases)

- We buy a European put option with a strike of $70
- Suppose current stock price is $65, expiry in 3 months
- The price of the option is $7
- The initial investment is $700

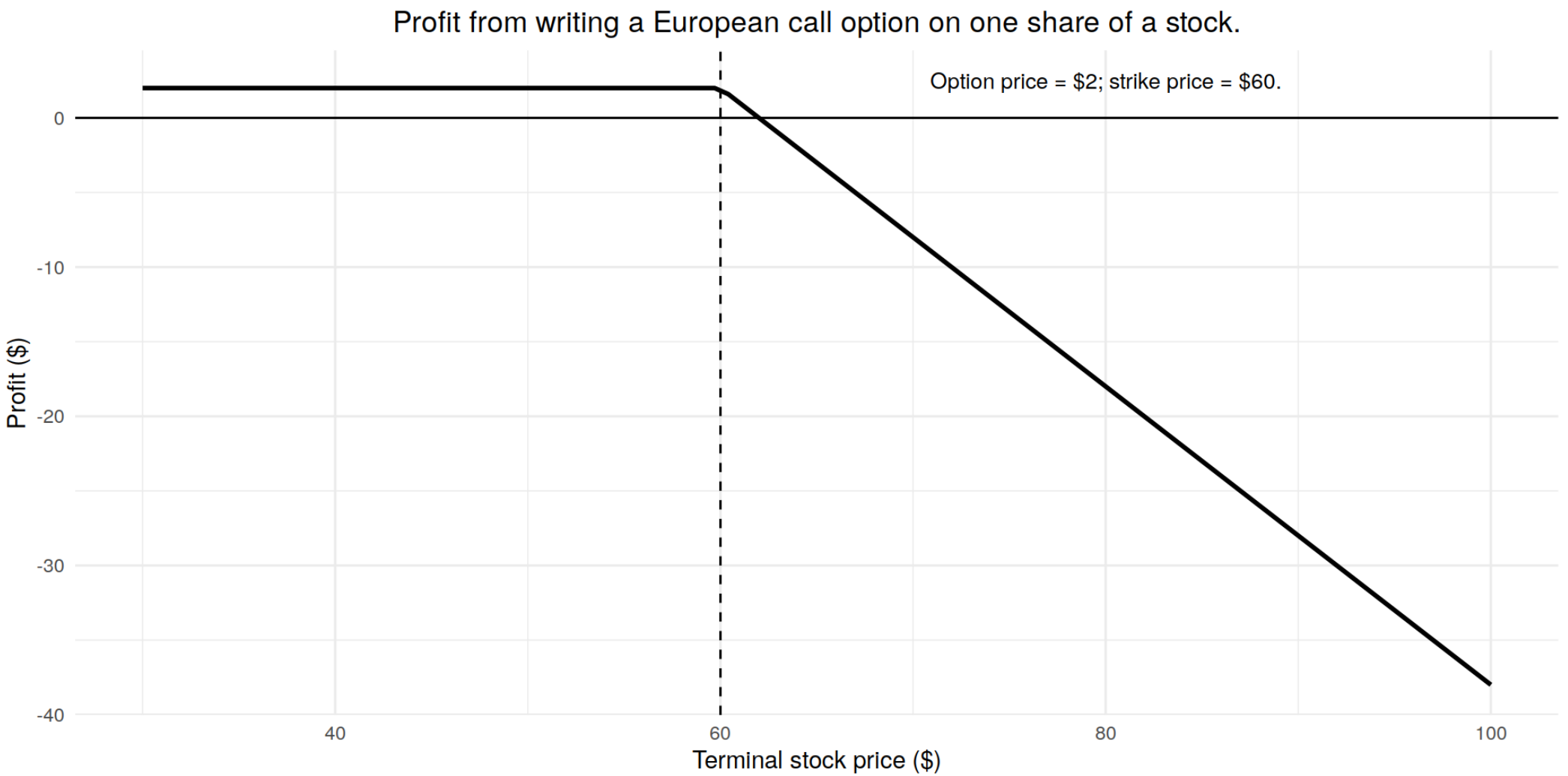
Stock price less than $70 on expiry

- Suppose stock price is $55 on expiry
  - The we exercise the contract
  - Why? because we can sell the stock for $\$70$ when its trading at $55
  - Our gain is $15 per share - $7 intial cost = $8 per share

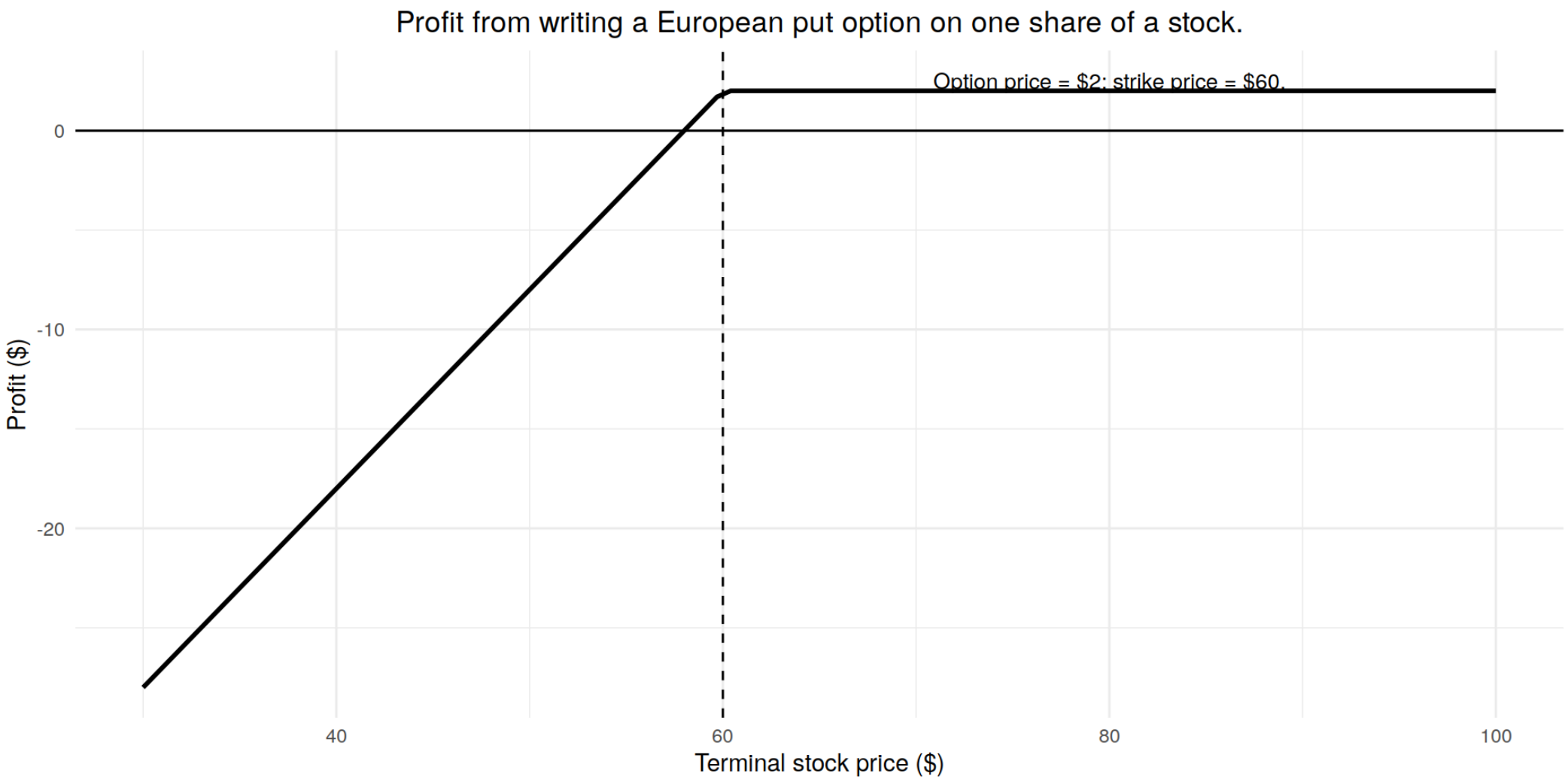
Stock price more than $70 on expiry

- The **put** contract is worthless (don't exercise)
- Lose the initial investment of $700

Writing a call



Writing a put



Option payoffs

- Its more *useful* to characterize a European option in terms of its payoff to the **purchaser** of the option.
- Then the initial cost of the option is not included in our calculations.
- Define K as the strike price and S_T as the final price of the **underlying asset** at expiry.
- The payoff in a **long call** option is $max(S_T - K, 0)$
  - Exercise if $S_T > K$
- The payoff to the **short call** holder is $-max(S_T - K, 0)$
- The payoff to the **long put** holder is $max(K - S_T, 0)$
- The payoff to the **short put** is $-max(K - S_T, 0)$

Option payoffs (Buyer Call vs Writter Call - Example (1) S(T) > K)

Option payoffs (Buyer Put vs Writter Put - Example (1) S(T) > K)

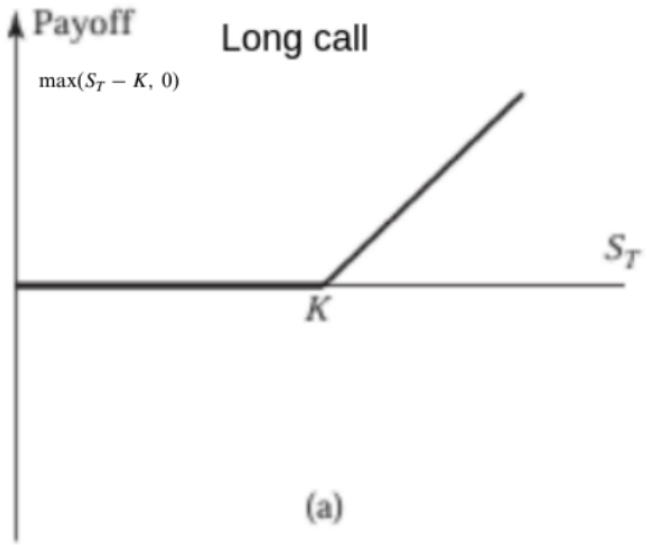
- Assume $S_T = 120$ and $K = 100$
- **Buyer of put** payoff:
  - Buy the right to sell a share at K to somebody (Writer of put)
  - $max(K - S_T, 0)$
  - $max(100 - 120, 0)$
  - $max(-20, 0)$
  - As the buyer of a **put** contract, we do not exercise the contract.
  - Why would we sell for $\$100$ something that is worth $\$120$ in the market at T
- **Writer of put** payoff:
  - Sell somebody (buyer of the put) the right to sell a share at K to us (writer of the put)
  - $-max(K - S_T, 0)$
  - $-max(100 - 120, 0)$
  - $-max(-20, 0)$
  - Recall, buyer of put did not exercise.
  - So we earn 0 payoff (in reality we would have earned the contract premium)

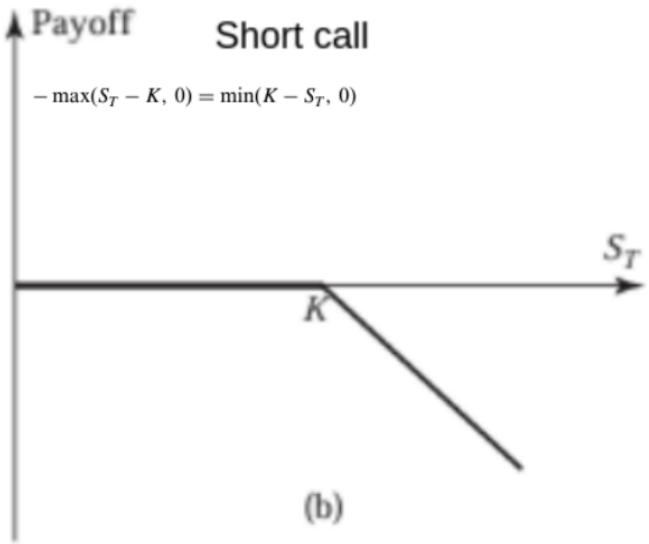
Option payoffs (Buyer Call vs Writter Call - Example (2) S(T) < K)

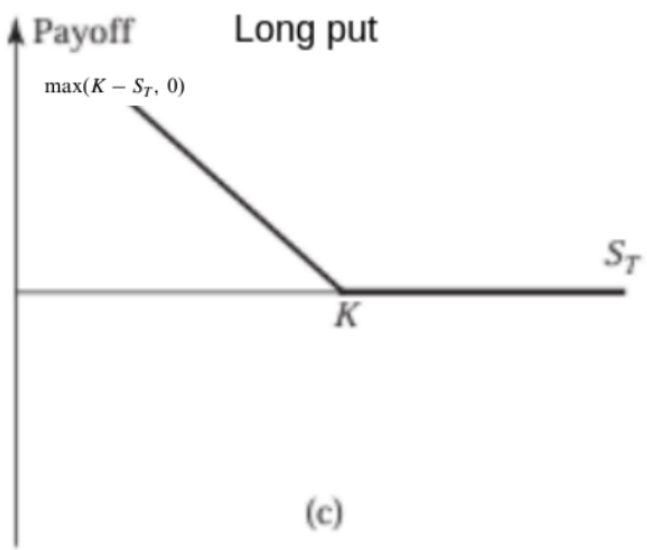
- Assume $S_T = 80$ and $K = 100$
- **Buyer of call** payoff:
  - **Buy** the right to **buy** a share at K from somebody (Writer of the call)
  - $max(S_T - K, 0)$
  - $max(80 - 100, 0)$
  - $max(-20, 0)$
  - Do not exercise contract (lose the premium we paid for the contract)
- **Writer of call** payoff:
  - Sell somebody (buyer of call) the right to buy a share at K from us (Writer of call)
  - $-max(S_T - K, 0)$
  - $-max(80 - 100, 0)$
  - $-max(-20, 0)$
  - Buyer never exercised, so we earn the options premium they paid to us

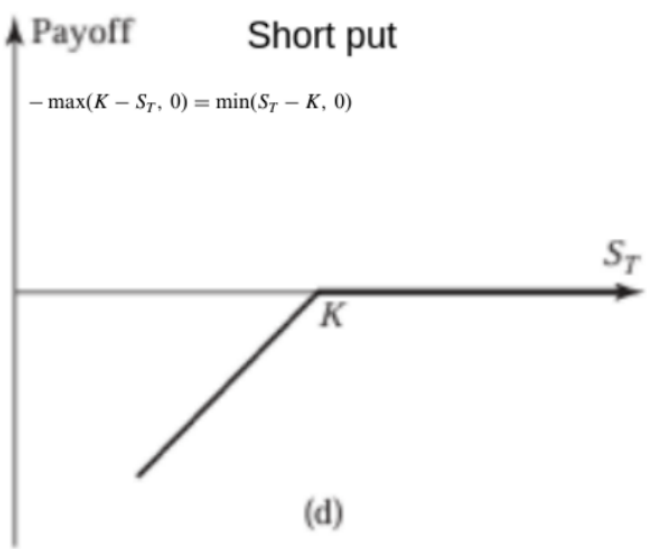
Option payoffs (Buyer Put vs Writter Put - Example (2) S(T) < K)

Option payoffs









Properties of stock options

Factors affecting option prices

- The current stock price, S_0
- The strike price, K
- The time to expiration, T
- The volatility of the underlying stock price, σ
- The risk-free interest rate, r
- The dividends that are expected to be paid

Factors affecting option prices

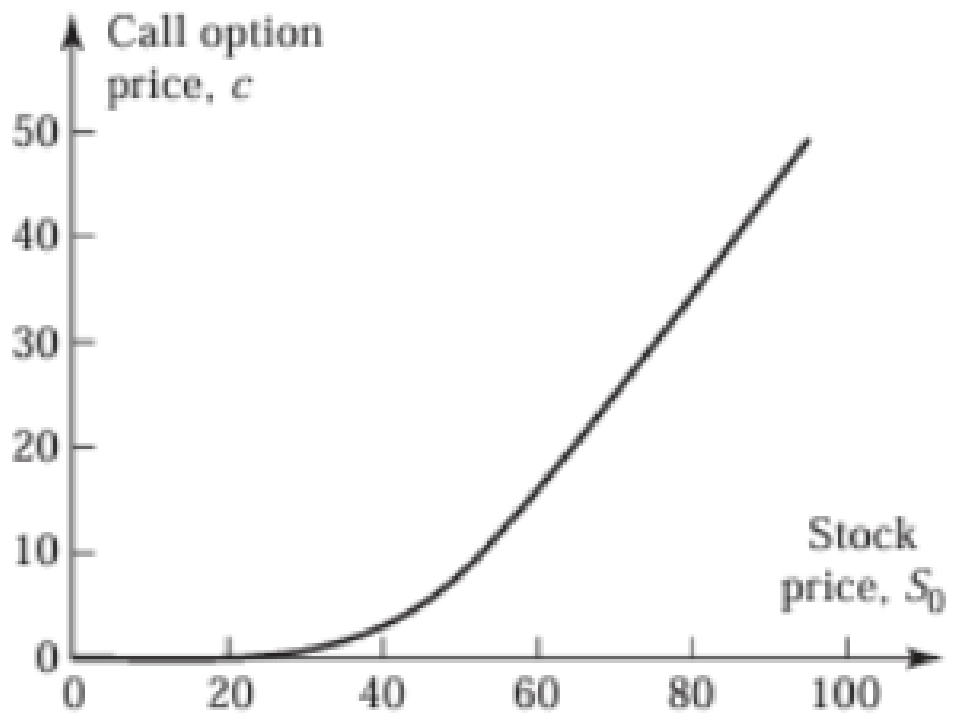
Variable	*European call*	*European put*	*American call*	*American put*
Current stock price	+	−	+	−
Strike price	−	+	−	+
Time to expiration	?	?	+	+
Volatility	+	+	+	+
Risk-free rate	+	−	+	−
Amount of future dividends	−	+	−	+

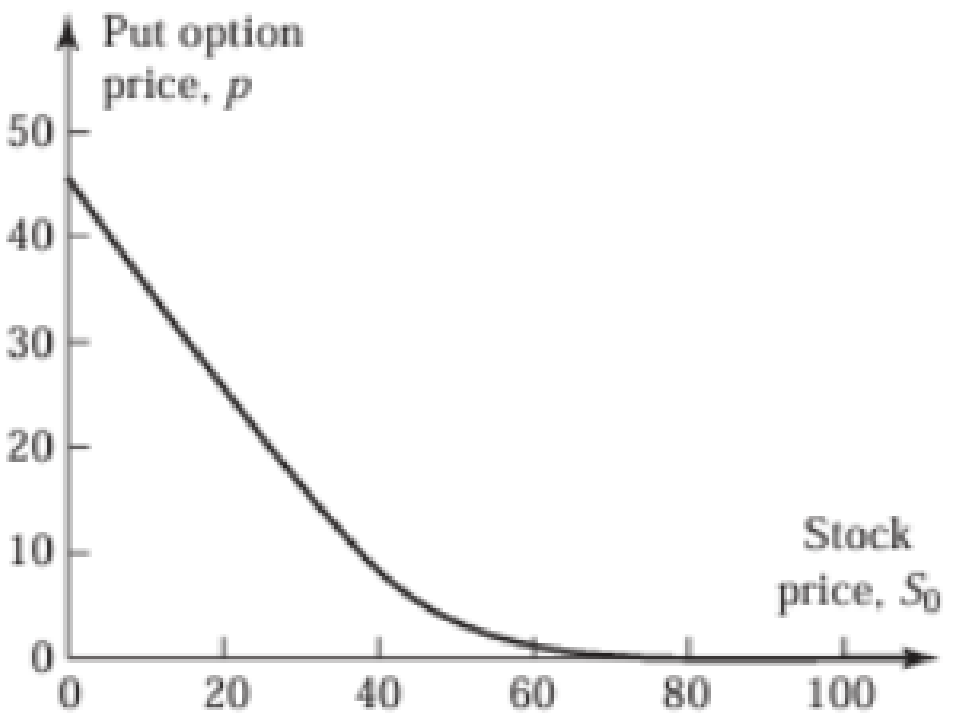
* + indicates that an increase in the variable causes the option price to increase;
− indicates that an increase in the variable causes the option price to decrease;
? indicates that the relationship is uncertain.

Effect on the price of an option when increasing one variable while keeping all others fixed.

Factors affecting option prices

- The effect of changes in **stock prices**
- Assuming $S_0 = 50, K = 50, T = 1, r = 5\%, \sigma = 30\%$
- Keeping (all variables) fixed, as S_t increases, the value of our option c increases (**call**)
- Keeping (all variables) fixed, as S_t increases, the value of our option c goes to zero (**put**)

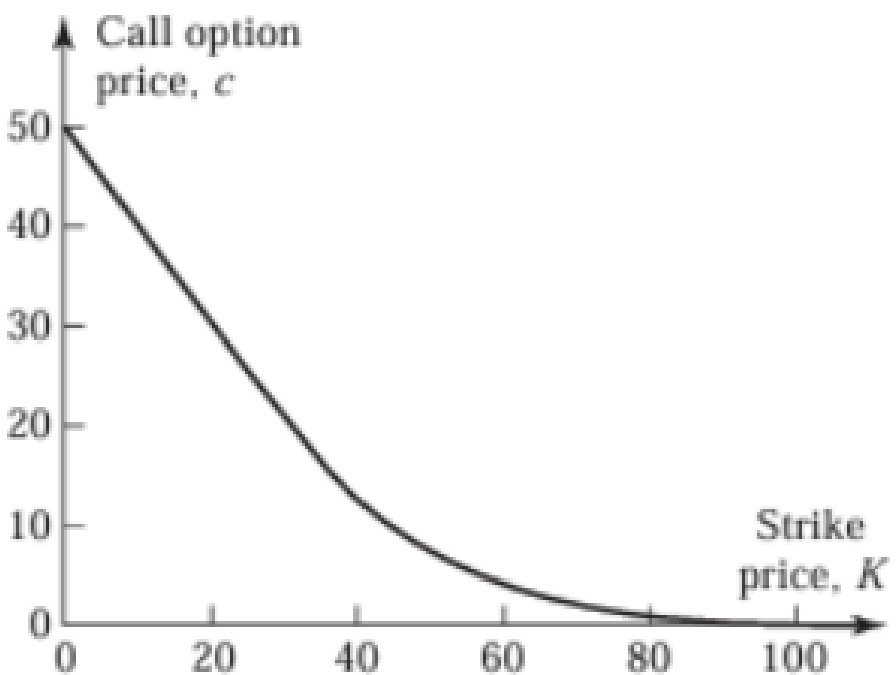


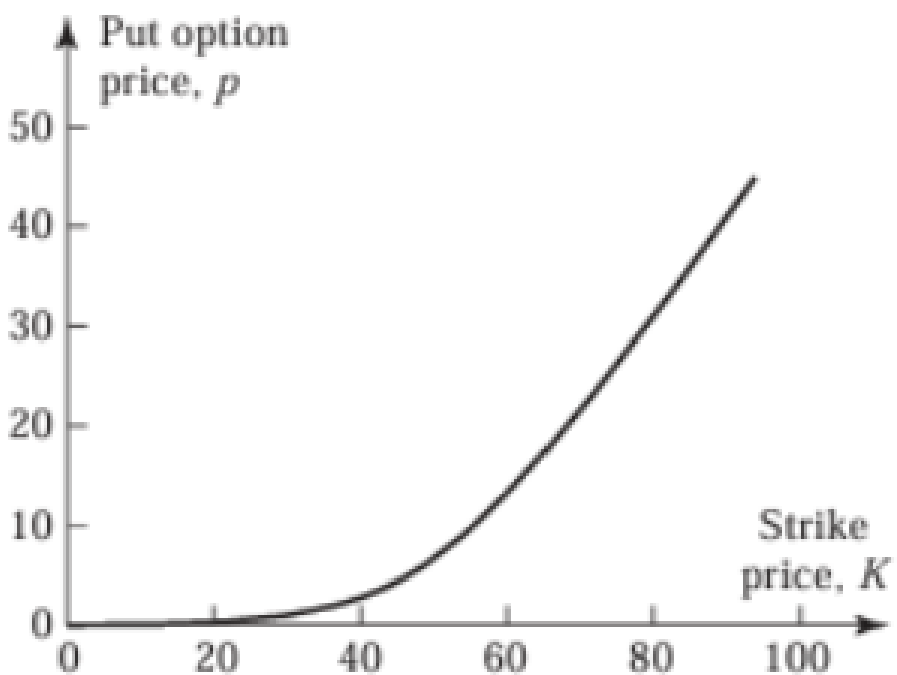


The effect of changes in **stock prices**

Factors affecting option prices

- The effect of changes in **strike prices**
- Assuming $S_0 = 50, K = 50, T = 1, r = 5\%, \sigma = 30\%$
- Keeping all variables fixed, as the value of K increases, the options price decreases (**call**)
  - We keep $S_0 = 50$ fixed, and we buy a **call** contract with $K = 1000$ - we are buying an out-of-the-money contract.
  - Further out-of-the-money we go, the cheaper the **call** contract.
- Keeping all variables fixed, as the value of K increases, the options price increase
  - We keep $S_0 = 50$ fixed, and we buy the right to sell at $K = 1000$, we are buying in-the-money
  - At time $T = 1$ when stock price is at, say 100 - we are still deep in the money

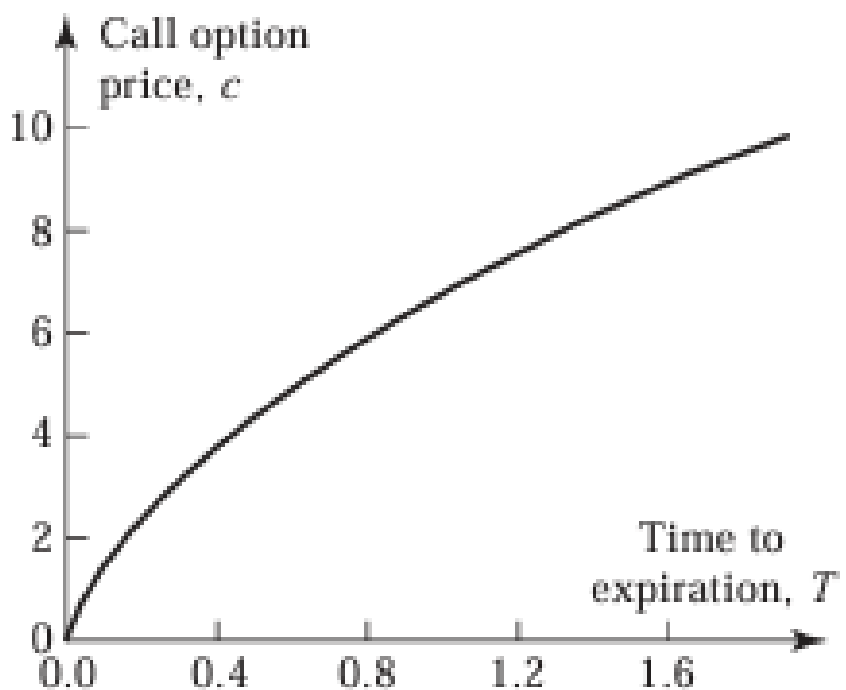


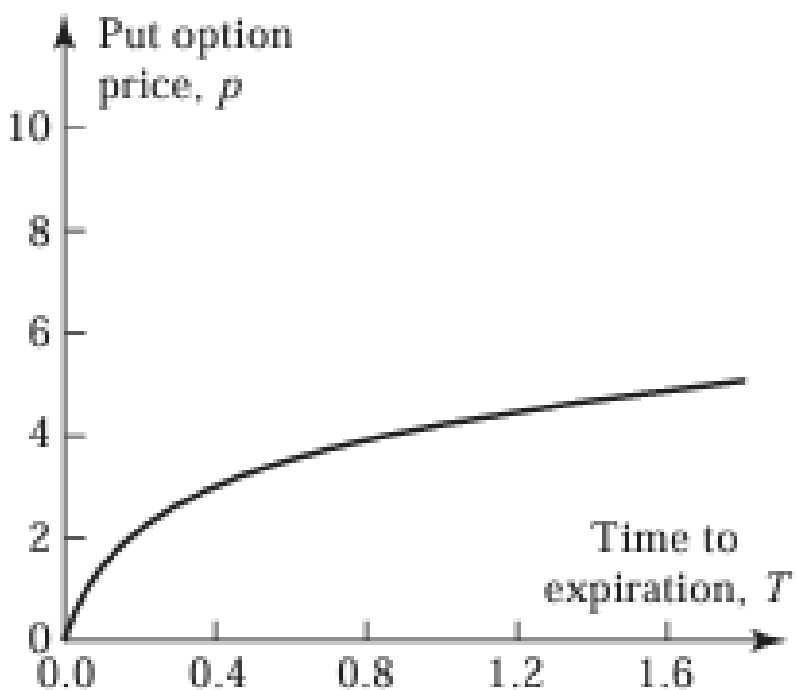


The effect of changes in **strike prices**

Factors affecting option prices

- The effect of changes in **expiration date**
- Assuming $S_0 = 50, K = 50, T = 1, r = 5\%, \sigma = 30\%$
- Keeping $S_0 = 50$ and $K = 50$ fixed, we are at-the-money
  - As we get *closer* to expiration T the options price increases in both **put** and **call**
  - Like a 50-50 coin toss.

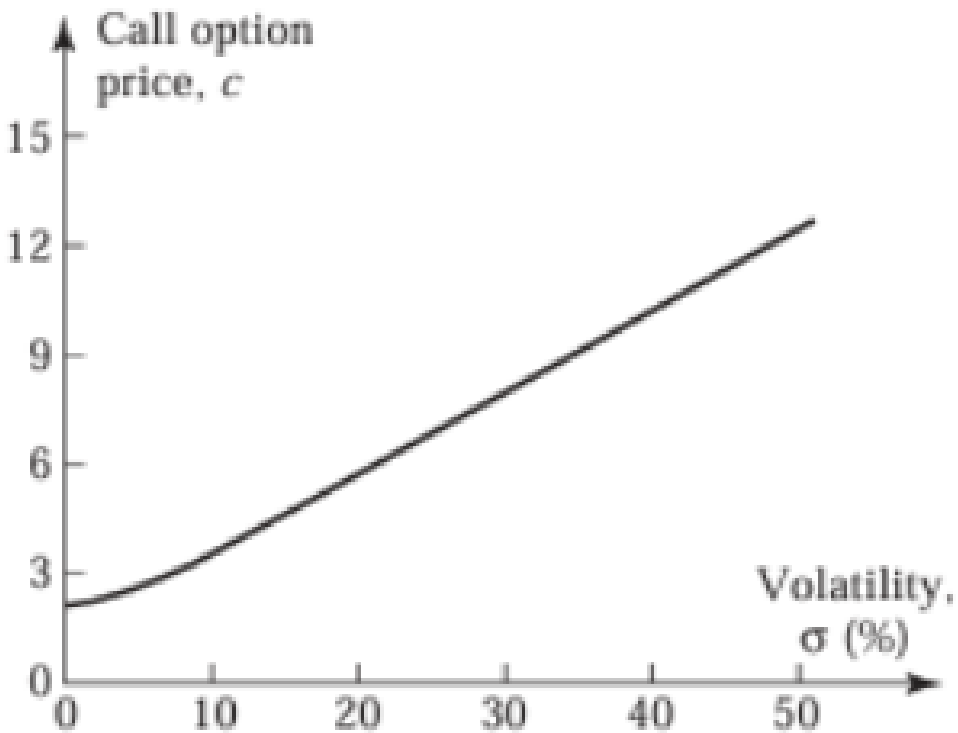


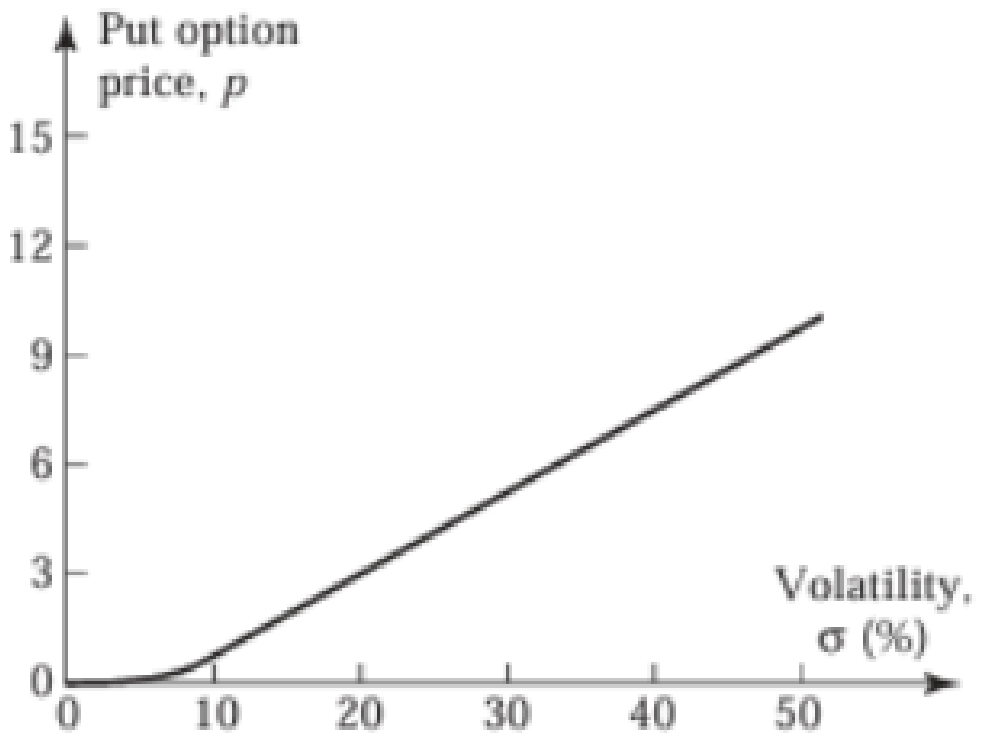


The effect of changes in **expiration date**

Factors affecting option prices

- The effect of changes in **volatility** σ
- Assuming $S_0 = 50, K = 50, T = 1, r = 5\%, \sigma = 30\%$
- High volatility increases likelihood that option ends in the money.





Factors affecting option prices

- **Call Options**
  - payoff $S_T > K$ - stock price higher than strike
  - **Stock price**
    - **call** options become **more** valuable as the **stock** price increases
  - **Strike Price**
    - **call** options become less valuable as the **strike** price increases
  - **Volatility** σ
    - **call** options become more valuable as the **volatility** increases
    - Limited downside risk when prices decrease since the most we can lose is the price of the option contract
- **Put Options**
  - payoff $K > S_T$ - strike price exceeds stock price
  - **Stock price**
    - **put** options become **less** valuable as the **stock** price increases **Strike Price**
    - **put** options become **more** valuable as the **strike** price increases **Volatility** σ
    - **put** - owners of **puts** benefit from price decrease and has limited downside risk in the event of price increases (value of the contract)

Options chains

What is an option chain?

- What is an option chain?
- It’s a table listing all the **calls**, **puts** and **strike prices** for a given option **expiration** for a single underlying asset
- We can quickly scan for **open interest**, **price changes** and **volume**.

Options chain example (TSLA trading at $212.14 a share - 19/01/2024)

Cadenas de opciones

FEB 16 '24 28 DÍAS | FEB 23 '24 35 DÍAS | MAR 01 '24 42 DÍAS | MAR 15 '24 56 DÍAS | MÁS

VISTA EN PESTAÑA | PUT/CALL | 10 STRIKES | SMART | TSLA | 100 | Mis cadenas

VI: 2.9%

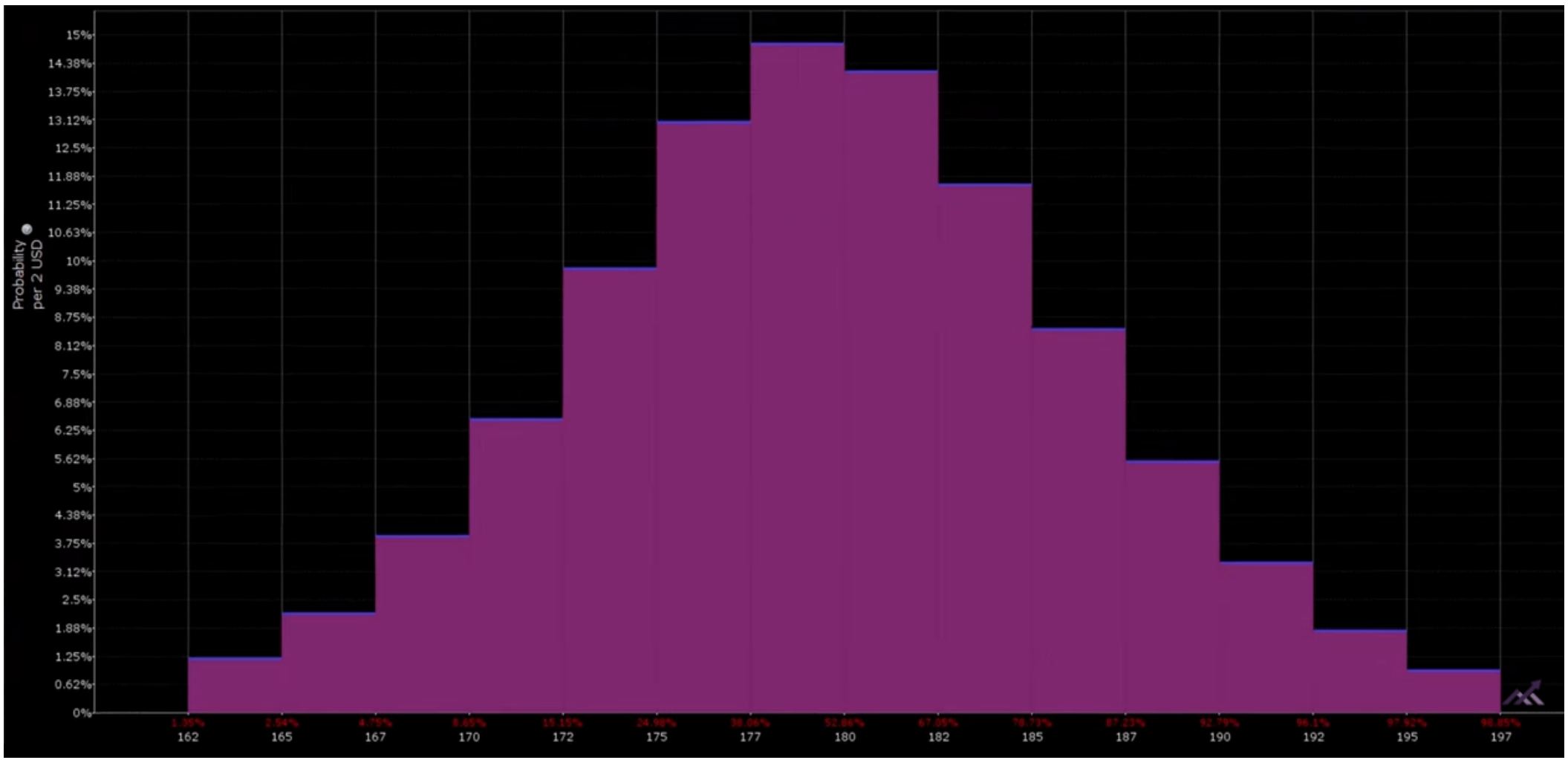
CALLS								EJERCICIO	PUTS						
BID x ASK	VOLUMEN	INTERÉS ABI...	DELTA	GAMMA	VEGA	THETA			BID x ASK	VOLUMEN	INTERÉS ABI...	DELTA	GAMMA	VEGA	THETA
27.50 x 28.35	51	36	0.787	0.008	0.206	-0.141		190	4.50 x 4.70	199	690	-0.215	0.008	0.207	-0.114
24.10 x 24.40	19	38	0.740	0.010	0.237	-0.152		195	5.75 x 5.95	325	472	-0.262	0.010	0.237	-0.125
20.70 x 21.00	78	155	0.688	0.011	0.261	-0.162		200	7.35 x 7.55	166	294	-0.315	0.011	0.262	-0.134
17.55 x 17.85	184	146	0.632	0.011	0.266	-0.169		205	9.20 x 9.40	84	130	-0.372	0.011	0.267	-0.140
14.75 x 15.05	673	310	0.573	0.012	0.282	-0.171		210	11.35 x 11.50	205	370	-0.431	0.012	0.282	-0.142
12.30 x 12.55	171	516	0.513	0.012	0.287	-0.171		215	13.85 x 14.15	405	502	-0.492	0.012	0.287	-0.141
10.10 x 10.35	306	341	0.453	0.012	0.281	-0.168		220	16.70 x 16.95	338	293	-0.554	0.012	0.280	-0.138
8.25 x 8.45	874	409	0.395	0.012	0.286	-0.162		225	19.55 x 20.35	228	429	-0.613	0.012	0.282	-0.132
6.65 x 6.90	959	732	0.340	0.011	0.268	-0.152		230	23.00 x 23.70	16	297	-0.669	0.012	0.264	-0.122
5.35 x 5.55	920	780	0.290	0.011	0.242	-0.141		235	26.70 x 27.40	18	71	-0.721	0.011	0.237	-0.111

Options chain example (TSLA trading at $212.14 a share - 19/01/2024)

- **Calls**
  - **Bid** price - buyers offer the price they are willing to pay for the option
  - **Ask** price - sellers ask for the price they want to sell the option
  - **Bid-Ask-spread** - difference between the **bid** and **ask** price
    - If **bid** price is **12.30** and ask is **12.55** then the mid-price is **12.425** (different for each option contract)
  - **Open interest** - number of contracts that have not been settled (more open interest the closer we are to expiry)
  - **Volume** - number of contract that have been bought and sold today
  - **Delta** - Measures the impact of a change in the price of the underlying
    - If the **delta** is 0.20 for a strike, then for ever $\$1$ move in the underlying our option contract would increase/decrease by $0.20 cents
    - e.g. a **delta** of 0.513 suggests that for every $\$1$ move in the underlying, the option will change by $\$0.513$
    - TSLA: https://finance.yahoo.com/quote/TSLA/
    - If TSLA went from 212 to 210 ($\$2$ decrease) then the **call** option would decrease by 0.513 * 2 = 1.026
      - If we paid the **mid-price**: **12.425 call** option then it would be worth **11.399**
    - If TSLA went from 212 to 150 ($\$62$ decrease) then the **call** option would decrease by 0.513 * 62 = 31.81
      - Basically worthless

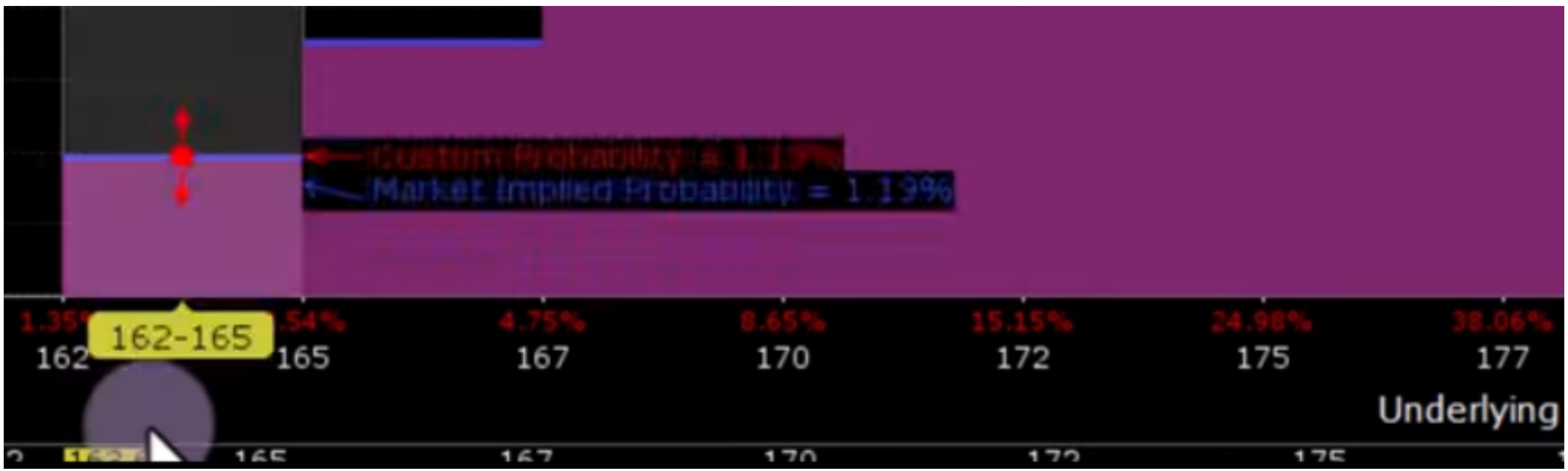
Probability distribution

- X-axis is the underlying price (or **strike prices**) on a Feb17 ’ 23 call option currently trading at 179 dollars
  - has probabilities



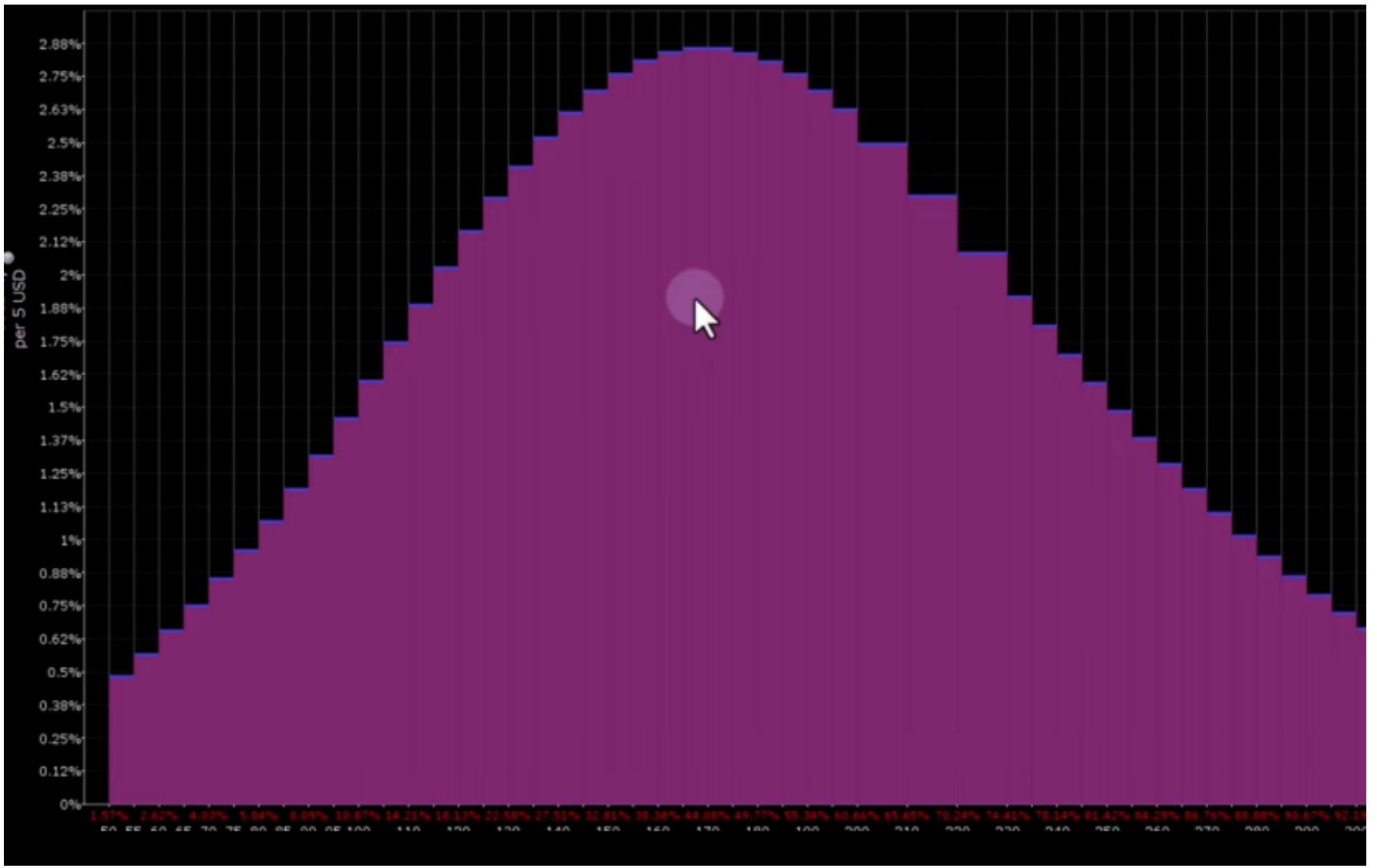
Probability distribution

- i.e. 1.35% probability that the stock will close on expiry at 162 a share.
- 2.54% chance it closes at 165
- 24.98% at close 175
- Therefore probability the stock closes **between** 162-165 is (2.54% - 1.35%) = 1.19%
- For the stock to close above 172 - the probability is (100-15.15) = 84.85
    - 85% chance the stock closes above 172 on expiry



Probability distribution

- A year long options chain - Jan 2025
- Has a wide distribution

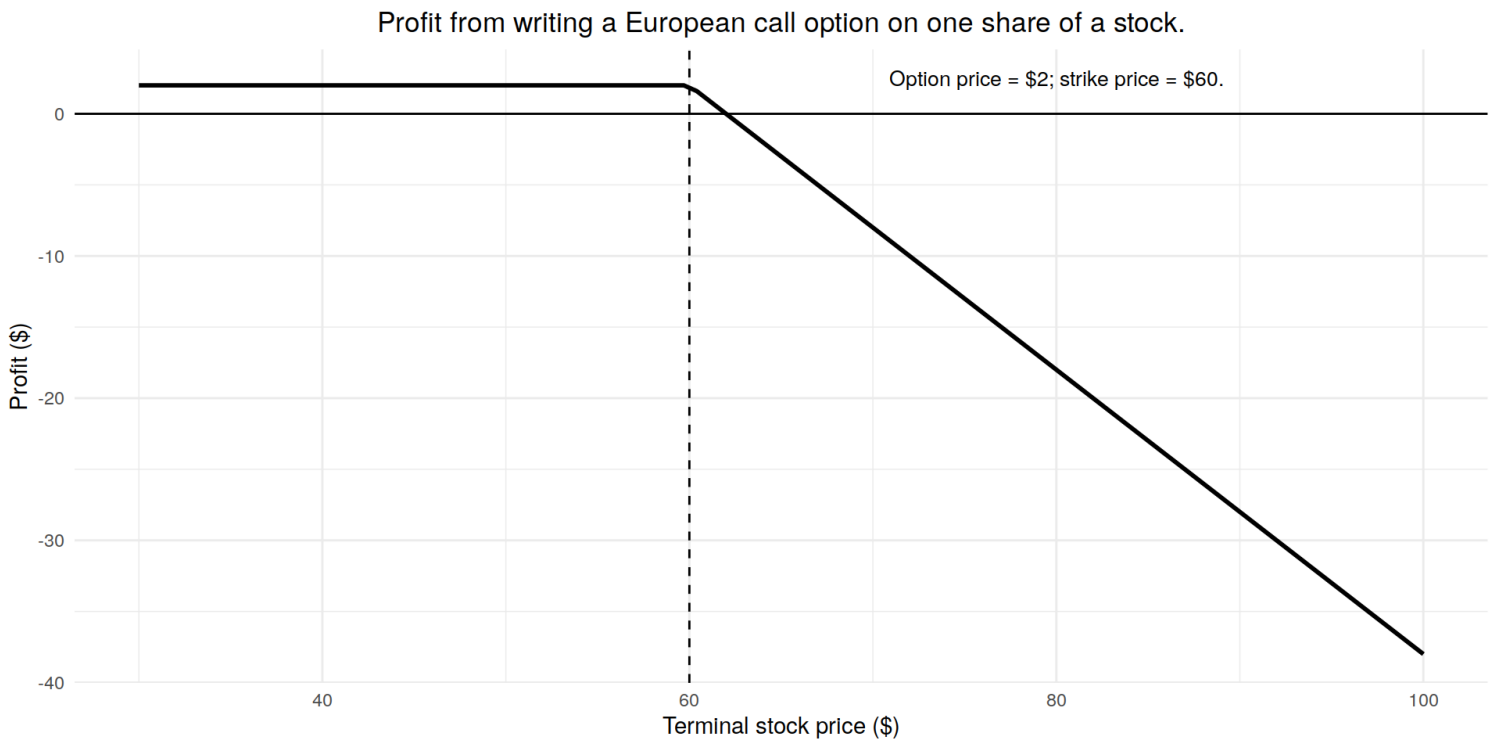


Trading strategies involving options

Strategy 1: Covered Calls

Covered call

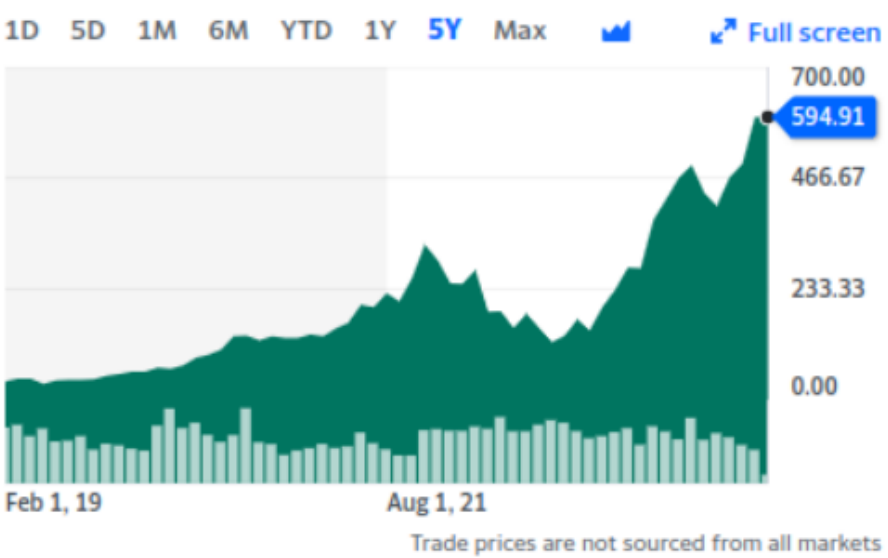
- **Objective** Sell a call option whilst owning at least 100 shares of the same stock.
- Writing just a call option is risky (unlimited loss potential)



- Combining with 100 shares of stock, we add no additional risk
- We can profit when stock price remains flat or declines
- Income generating strategy for equity owners (who believe stock will not go higher)
- Sell out-of-the-money calls which have a higher probability of expiring worthless

Covered call

- Suppose we bought 100 shares of NVDA in Sept 2022 at 121.39
- Its now trading at 594
- We can sell covered calls at $K = 650$ for $T = 3months$
- Collect premiums
- Hope $S_T > K$ so we can keep the premium and the stock
- Assume at expiry $S_T = 500$ - we keep the premium and stock
- Create a new covered call for $T = 3months$



Covered call

- When we **sell** a **call**, we are selling somebody the **right**, but not the obligation to **buy** a **stock** at a **strike** price on a specific **date**
- **Selling covered calls** - we get paid extra money as we hold a stock (we will be obligated to sell it if the contract is exercised)
- Generate money in the meantime whilst we hold the stock (best in flat / bearish markets)
- If we bought a stock at a good price and are willing to hold onto it for years, and if we think its overvalued, we can sell covered calls to generate income
  - of course, we risk losing the stock if the contract is exercised.
- We determine a price that our shares are overvalued (DCF, paying less dividends today than last year, PE ratio has fallen etc.)
  - Instead of waiting for the share price to become overvalued, we can plan the sale in advance
  - Determine a **fair value** and then **sell covered calls** at **strike** prices above these **fair values**
  - Generating extra income whilst we hold the stock

Covered call

- Consider the table for 4-month **call** options
- **strike** we would be obligated to **sell** the shares at, if options buyer decides to exercise their option
- **change** - shows recent changes in options pricing
- **Bid** - approximately what we will receive in option premiums per share if we sell the call
  - We get paid this amount by the buyer of the option
- **overview**
  - the **Strike** is the amount we agree to sell the shares for (if option exercised)
  - The big is roughly the premium we can expect to receive when we sell the option

Calls

Strike	Price	Change	Bid	Ask	Volume	Open Int
41.00	4.55	0.00	4.45	4.70	-	161
42.00	3.65	0.00	3.65	3.90	-	155
43.00	2.70	0.00	3.00	3.15	-	136
44.00	2.33	+0.32	2.35	2.41	13	607
45.00	1.72	+0.30	1.76	1.81	4	703
46.00	1.25	+0.26	1.25	1.30	28	1649
47.00	0.82	+0.12	0.85	0.89	1	1378
48.00	0.55	+0.08	0.54	0.58	2	286
49.00	0.33	+0.04	0.32	0.35	13	321

Example of covered call

- Suppose we bought 100 shares of a dividend paying stock (yield 3.33%) for $30 per share 5 years ago
  - We considered at $30 the stock to be undervalued
  - Today the stock is trading at $45 a share and paying dividends of 2.77% (we feel is now overvalued)
- Suppose we **sell** an option with a strike of $\$47$ - (above the stock price $S_0 = \$45$)
- We will receive a premium **bid** of 0.85 for writing the contract (0.85 * 100 = $85)
- So we are obligated to sell our shares (if exercised) for $\$47 * 100 = \$4700$

Example of covered call (possibility A)

- $S_T < \$47$ - stock price below strike 47
- Option buyer does not exercise the option (no need to sell our shares)
- We keep the premium of $\$85$ and the shares (plus dividends it pays out)
- We can sell another option
  - If share price increased to $\$46$ we can set a new strike of $\$48$ (maintain same margin)
  - Options premiums will be similar to what we received when setting the strike at $\$47$

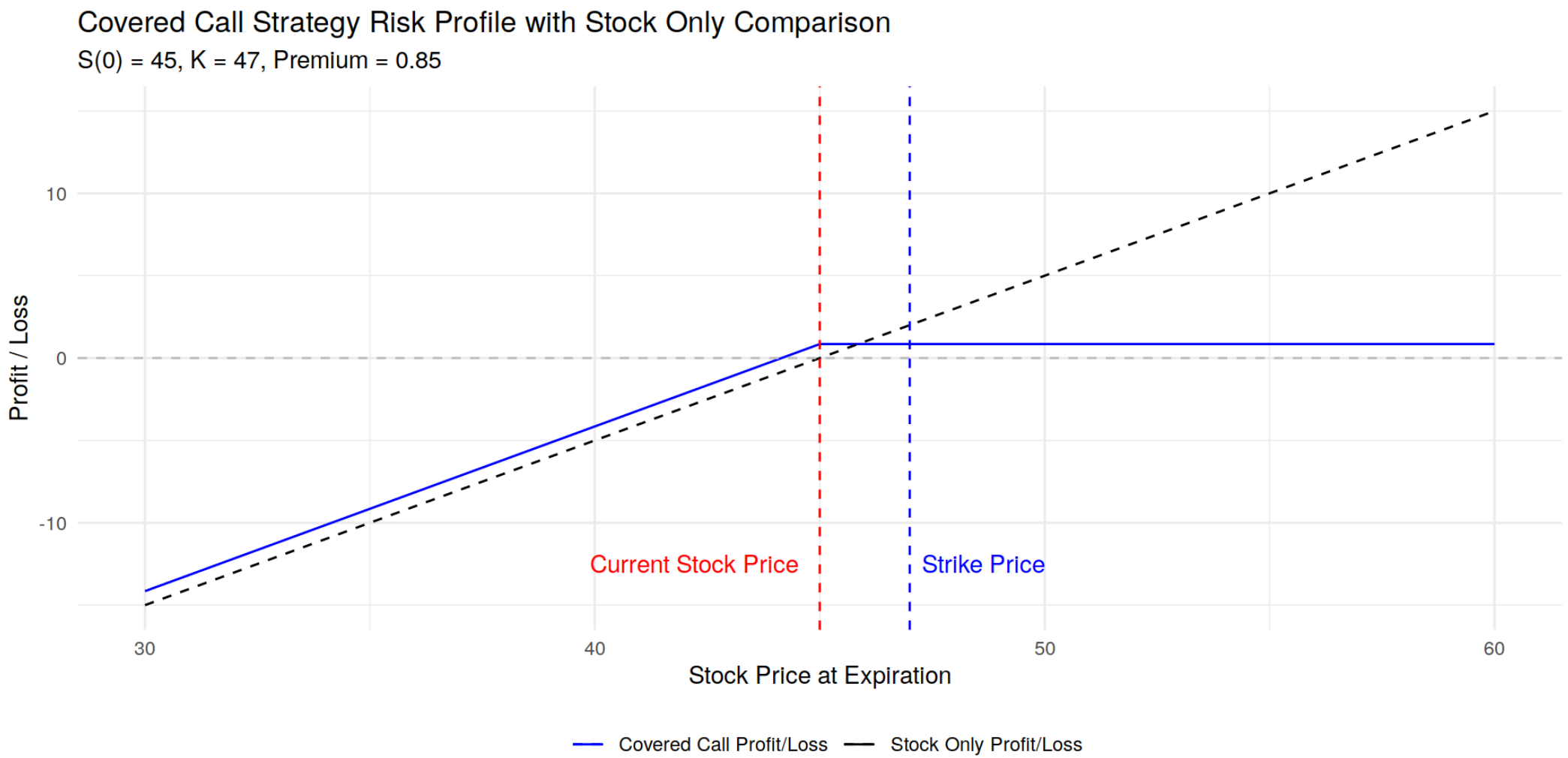
Example of covered call (possibility A)

- If dividends are $\$1.25$ a share ($\$0.3125$ per quarter) - over 4 months we receive 1 dividend payment or possibly 2 payments ($\$0.625$) or annualized $\$1.25$
- We earned $\$0.85$ in premiums - if we sold this 4 times in a year we would get $\$2.55$ in premiums (more than dividends)
- So annually we earn $\$1.25$ in dividends and $\$2.55$ in premiums (total $\$3.80$)
- The yields are $\$1.25/\$45 = 2.77\%$ and $ $2.55 / $45 = 5.66%$ - total return of 8.44%

Example of covered call (possibility B)

- $S_T > \$47$ - stock price above strike 47
- Buyer exercises and we have to sell 100 shares at $\$47$
- If stock is trading at $\$50$ - we still have to sell them for $\$47$
- We made $\$2$ in capital appreciation ($S_T = 47 - K = 45$), 0.85 in premiums plus dividends paid $\$0.625$
- Total gains per share $\$3.475$ for putting up a $\$45$ share principal (7% return, 22.5% annualized)
- We didn't sell for $\$50$ but we still made a good return (but we thought the stock was already overvalued)
- If the share price went to $\$60$ then we missed out on a lot of upside returns potential.

Example of covered call (payoff)



Strategy 2: Selling puts

Selling puts

- Selling put options, we give somebody the right, but not the obligation to force us to buy 100 shares of a company at a certain price K (strike price)
- They pay a premium to increase their flexibility and decrease our flexibility
- Get paid to do something we want to do?
  - Want to enter into an equity position
  - Say, we want to buy a company at a specific price but its trading higher than that price.

Selling puts

- Say, we want to buy shares of a company trading at $\$30.50$ per share
- Determine what a **fair value** for a stock is, using DCF etc.
- The **fair value** is less $\$30$
- We would wait for the stock to fall to buy it, invest in something else
- What is the whole market is overvalued and you can't find anything to invest in?
- **Sell a putt** option to have the obligation to buy the stock at a price on a specific date.
- We wanted to do this, but now we get paid to do it.

Selling puts (Example)

- **Strike** price we are obligated to buy the shares (if options buyer exercised)
- **Price** What the option has been trading at.
- **Bid** - Approximately what we get in premiums for selling the option
- **Stike** and **Bid** are the most important for option sellers

Puts

Strike	Price	Change	Bid	Ask	Volume	Open Int
26.00	0.34	-0.02	0.36	0.42	1	668
27.00	0.48	-0.05	0.53	0.57	1	151
28.00	0.70	-0.05	0.75	0.79	3	628
29.00	1.01	-0.05	1.04	1.10	1	490
30.00	1.46	+0.01	1.43	1.49	8	927
31.00	1.76	-0.12	1.93	2.00	8	3754
32.00	2.48	0.00	2.48	2.61	-	87
33.00	3.37	0.00	3.15	3.35	-	11
34.00	4.85	0.00	4.05	4.15	-	1

Selling puts (Example)

- Suppose we want to write a put option with a strike price of $\$30$ with current stock price trading at $\$30.50$ and the bid price is $\$1.43$ (premium for writing the contract)
  - We receive $\$143$ today - obligating ourselves to buy 100 shares if exercised at $\$30$ - $\$3000$
  - We need to have the cash available in our accounts in case of exercise.
  - If we did not have the cash - it would be a **naked put** - more risky.
  - Upfront cash needs $\$2,857$

Selling puts (Example) - Possibility A

- Stock price stays above $\$30$
- Option buyer does not exercise the contract (expires worthless)
- Why would the buyer force us to pay $\$30$ when the market is higher?
- Rate of Return = $\$143/\$2857 = 5\%$ over 4 months
- We can write another contract with $T = 4months$ and “roll the contract over”

Selling puts (Example) - Possibility B

- Stock price falls below $30 to $29.50
- We still keep the premium but we need to buy the stock for $30
- Essentially we paid $28.57 ($30 - $1.43) of $2857
- So we own the company (what we wanted) at a price less than the market price

Bull call spread

- Profit from a stock we feel *mildly* bullish about
- Combining a *buy call* and *sell call* simultaneously
- Net debit (sell call) less expensive than (buy call)
- Predefined risk/reward (know max gain / loss)

Bull call spread (example)

- Suppose $S_0 = 700$ and we think the stock is going to go up
- We don't want to spend 700 to buy the stock
  - We can buy a **call** option (cheaper) for $\$10$ with a strike K
    - Need to wait until the stock price goes up $\$10$ to start making money
  - We think the stock is going to go up but no more than $\$720$
  - So we want to buy the stock at 700 but would be happy selling at 720
    - We can sell a **call** option at 720 at $\$5$
  - We put a cap on the upside
    - Now our initial investment is less since we receive the premium $\$5$
    - putting a cap lowered our initial investment/risk (but limit profits)

Bull call spread (example)