Asset Pricing and Valuation

Lecture 2: Risk and Return

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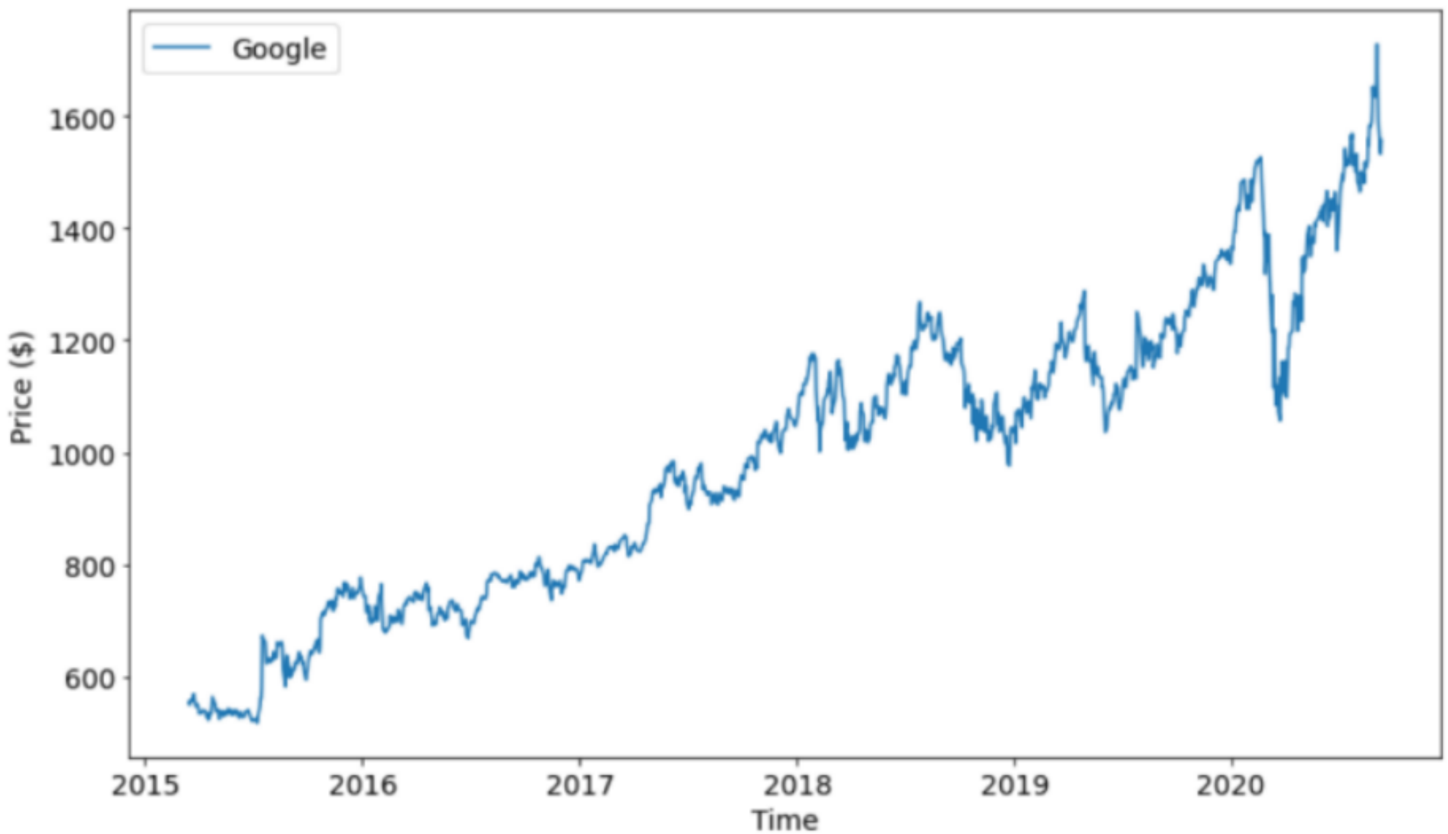
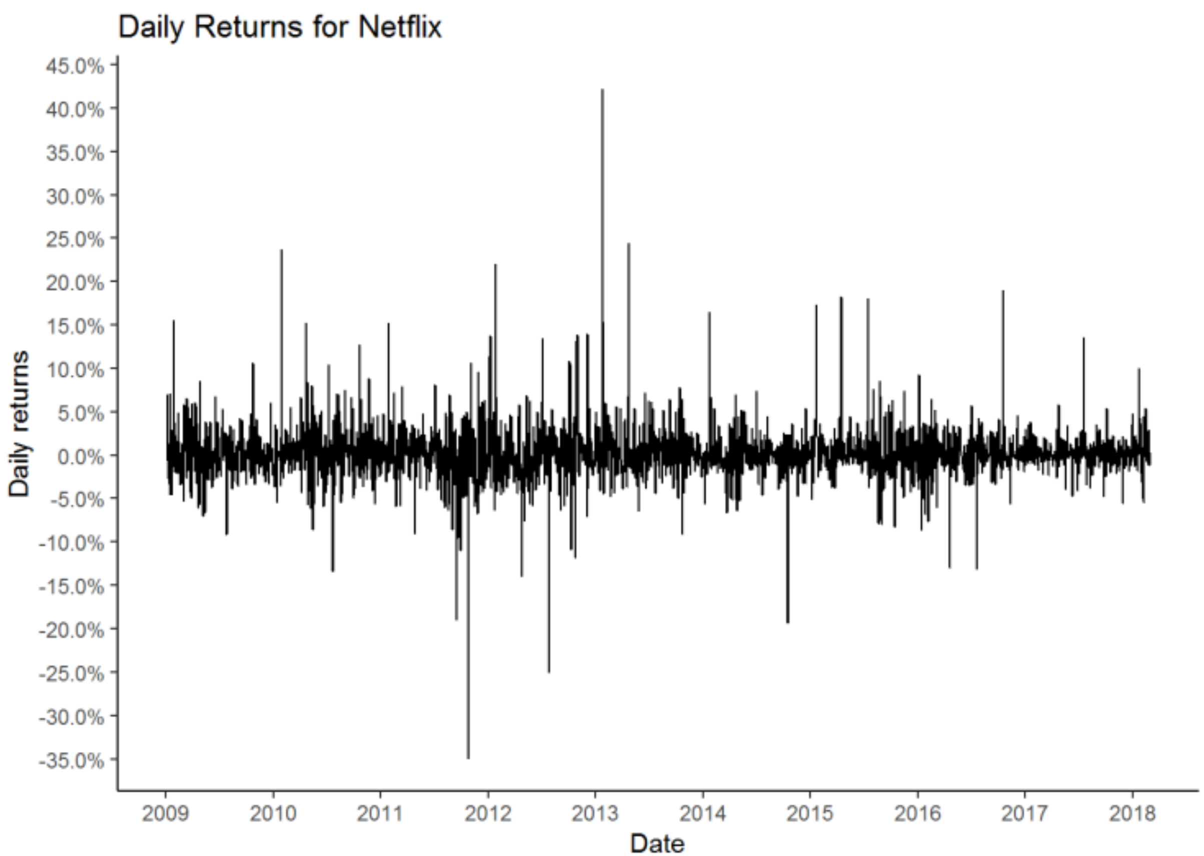
Learning Objectives

- Definitions for value of a stock
- Diversification

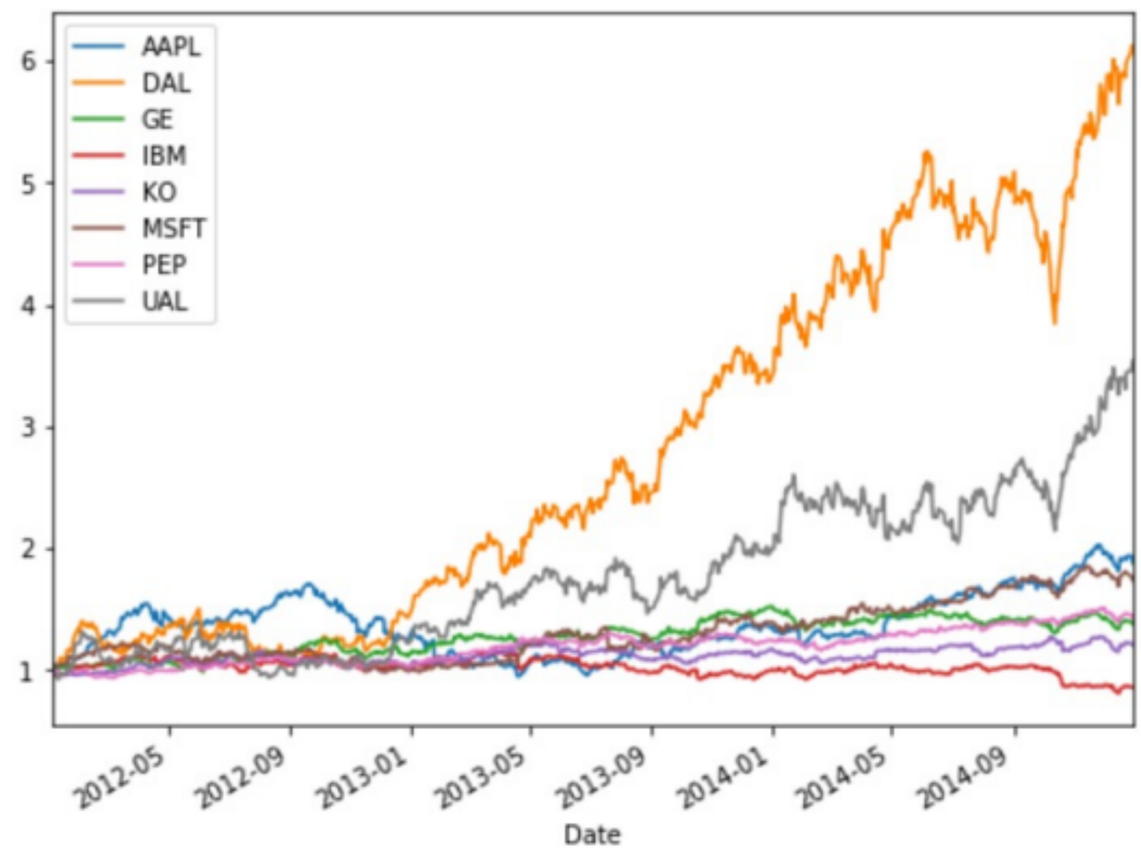
Stock Prices, Returns, Cumulative Returns



Stock Prices, Returns, Cumulative Returns

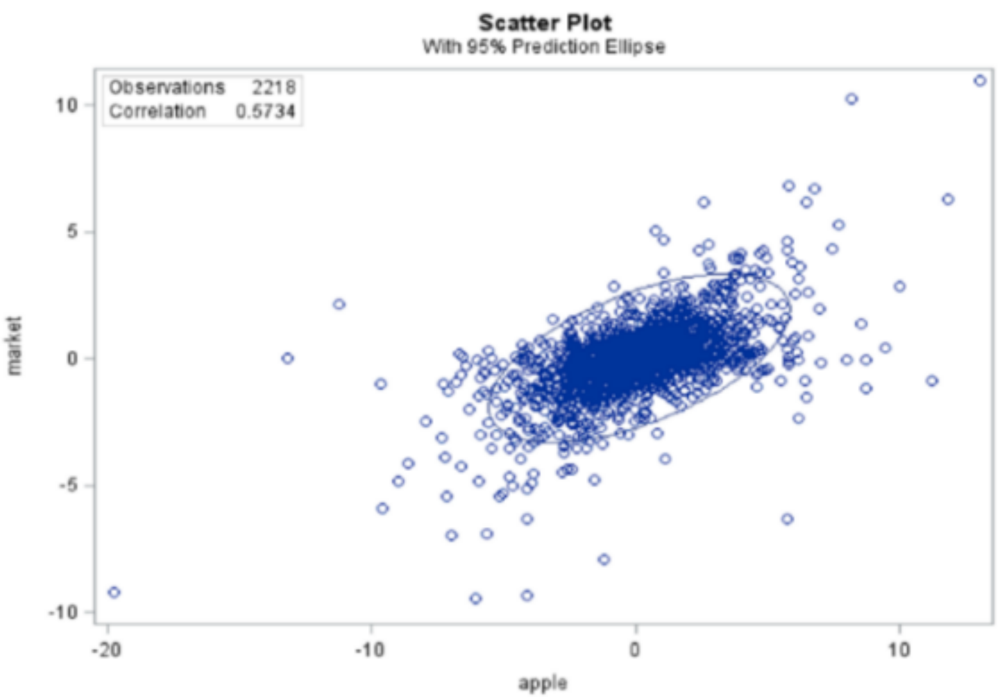


Stock Prices, Returns, Cumulative Returns



Stock Prices, Returns, Cumulative Returns

Stock Prices, Returns, Cumulative Returns



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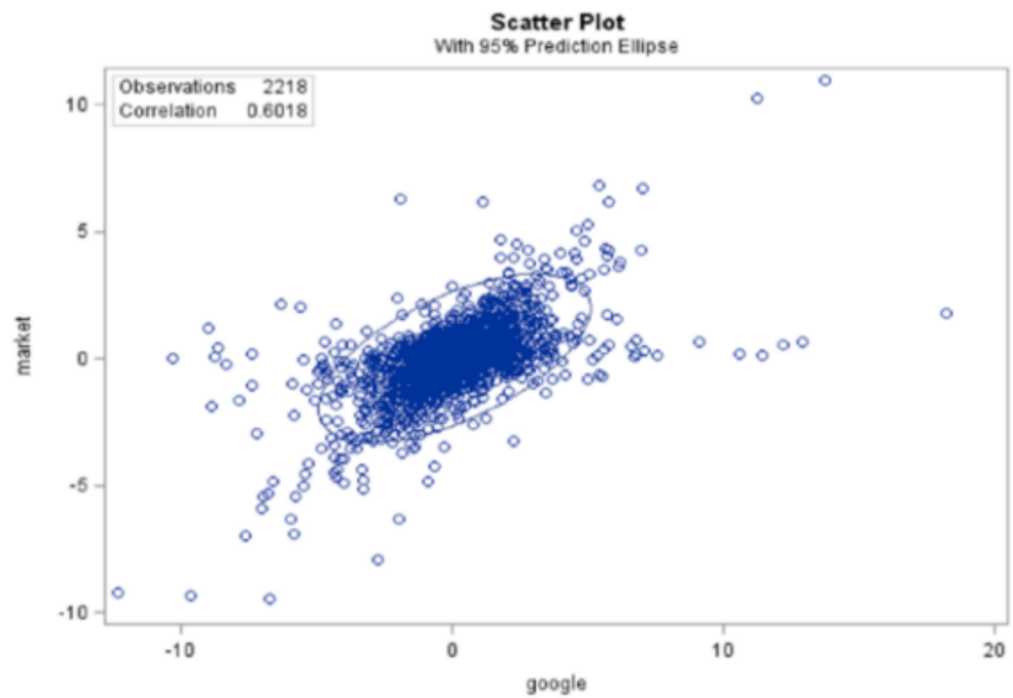

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Diversification

> **Tip**
>
> - **What is diversification? Why do people diversify?**

- Select variety of stocks in different industries.
- **Optimal** to own few stocks across many industries. – How many stocks are optimal?
- Why do people diversify?
  - Investors try to stabilize their portfolio. – More stocks we own, more chance of finding “NVDA” – Limit our market exposure - if stock falls X%, portfolio not affected much.

Diversification

Question

Is the following a diversified portfolio?

Percentage	
Bank	Percentage
Bank of America	100%

Diversification

Question

Is the following a diversified portfolio?

Percentage	
Bank	Percentage
Bank of America	30%
Chase Bank	20%
Wells Fargo	20%
Citigroup	30%

- Yes, but focused on banking

Diversification

Question

Is the following a diversified portfolio?

Percentage	
Bank	Percentage
Bank of America	30%
Nvidia	20%
McDonald's	20%
Caterpillar	30%

- Yes, but focused only on stocks

Diversification

Question

Is the following a diversified portfolio?

Percentage	
Stock	Percentage
SPY500	30%
U.S. Bonds	20%
U.S. Cash	20%
U.S. Real Estate	30%

- Yes, but focused only in the U.S.

Diversification

> **Question**
>
> Is the following a diversified portfolio?

Percentage	
Stock	Percentage
MSCI	30%
U.K. Bonds	20%
Crypto	20%
U.S. Real Estate	30%

- Yes, but contains only 4 asset classes - add more assets.

Portfolio

- What is a portfolio?
  - A collection of assets (shares, bonds, derivatives, real estate, etc) held by an institution or individual.
  - Each asset represents a $\%$ of the total portfolio value. The weight of each asset is represented by W_i with $\sum_{i=1}^{n} W_i = 1$
- How do we select our portfolios?
- **We want to maximise our return and minimise risk**
- Given two assets with the same returns, the one with lowest risk is preferred.
- We prefer **certain** cash-flows than a **risky* cash flow with the** same** expected value. (useful for forecasting, allocation of resources and planning)
- Risk-taking investors demand an extra premium for taking on more risk - known as the **risk premium**

Portfolio

Portfolio Metrics					
Metric	Small Caps	Blue Chips	Long Term Bonds	Medium Term Bonds	Treasury Bills
Avg Annual Return	18.29%	12.49%	5.53%	5.30%	3.85%
Annualized Standard Deviation	39.28%	20.35%	8.18%	6.33%	2.25%

- Blue-chip stocks are from companies that are large, well-established, and financially sound.
- E.G. Anheuser-Busch InBev, Pfizer, Nestle, Nike, Sanofi

Expected returns

$$E[R_i] = \frac{E[CF_{i,1}] + E[P_1] - P_0}{P_0}$$

where

$$E[CF_{i,1}] = \text{expected cash flow}$$
$$E[P_1]\text{is the expected price}$$

Question

How many companies do you want in your portfolio? What is the optimium?

Optimal Portfolio

Question

What do we see here?

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1997–2006				
	Securities	Stocks Avg. Return	SPY Avg. Return	Excess
1997	207	33%	27%	6%
1998	145	29%	14%	15%
1999	158	21%	13%	8%
2000	313	-9%	10%	-19%
2001	340	-12%	1%	-13%
2002	303	-22%	-18%	-5%
2003	274	29%	42%	-13%
2004	288	11%	17%	-6%
2005	255	5%	8%	-3%
2006	246	16%	16%	0%

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2007–2016				
	Securities	Stocks Avg. Return	SPY Avg. Return	Excess
2007	224	5%	3%	2%
2008	243	-37%	-38%	1%
2009	292	26%	44%	-17%
2010	282	15%	21%	-6%
2011	239	2%	1%	2%
2012	234	16%	17%	-1%
2013	260	32%	37%	-4%
2014	259	14%	15%	-1%
2015	230	1%	-2%	3%
2016	262	12%	14%	-2%

- 20 years of data, – 8 of the years produced an excess return over the S&P500 index. – 12 years resulted in the S&P500 outperforming the simulated portfolios.

Optimal Portfolio

Question

What do we see here?

Annual Stock Returns (Part 1)

Year	Stock_10	Stock_25	Stock_50	Stock_100	Stock_150
1997	26.99%	27.59%	27.42%	27.41%	27.40%
1998	14.06%	13.33%	14.25%	13.83%	13.79%
1999	11.66%	11.70%	13.74%	13.12%	13.32%
2000	9.61%	9.96%	9.18%	9.18%	9.43%
2001	1.68%	1.61%	0.57%	0.43%	0.57%
2002	-16.48%	-16.49%	-17.40%	-17.55%	-17.50%
2003	41.06%	41.26%	41.20%	41.38%	41.43%
2004	17.60%	17.70%	16.94%	16.92%	17.20%
2005	8.47%	8.68%	6.30%	5.35%	6.04%
2006	15.33%	15.72%	15.32%	15.59%	15.91%

Annual Stock Returns (Part 2)

Year	Stock_10	Stock_25	Stock_50	Stock_100	Stock_150
2007	2.04%	3.09%	4.01%	4.01%	2.99%
2008	-39.70%	-38.91%	-38.98%	-38.38%	-38.73%
2009	44.80%	44.00%	43.64%	43.48%	43.97%
2010	21.36%	21.22%	21.00%	21.01%	21.21%
2011	0.94%	0.81%	1.14%	1.23%	1.14%
2012	15.99%	15.86%	16.06%	16.14%	16.08%
2013	37.20%	36.87%	37.24%	36.42%	36.95%
2014	15.08%	14.97%	13.66%	13.86%	14.32%
2015	-1.61%	-1.83%	-1.96%	-2.46%	-1.93%
2016	13.85%	13.25%	13.53%	13.65%	13.71%

- From the S&P500, 100 individual random portfolios holding 10, 25, 50, 100 and 150 stocks were generated. The returns of the each of the different portfolios are very similar across each year

Optimal Portfolio

Question

What do we see here?

Annual Stock Returns (1997–2006)

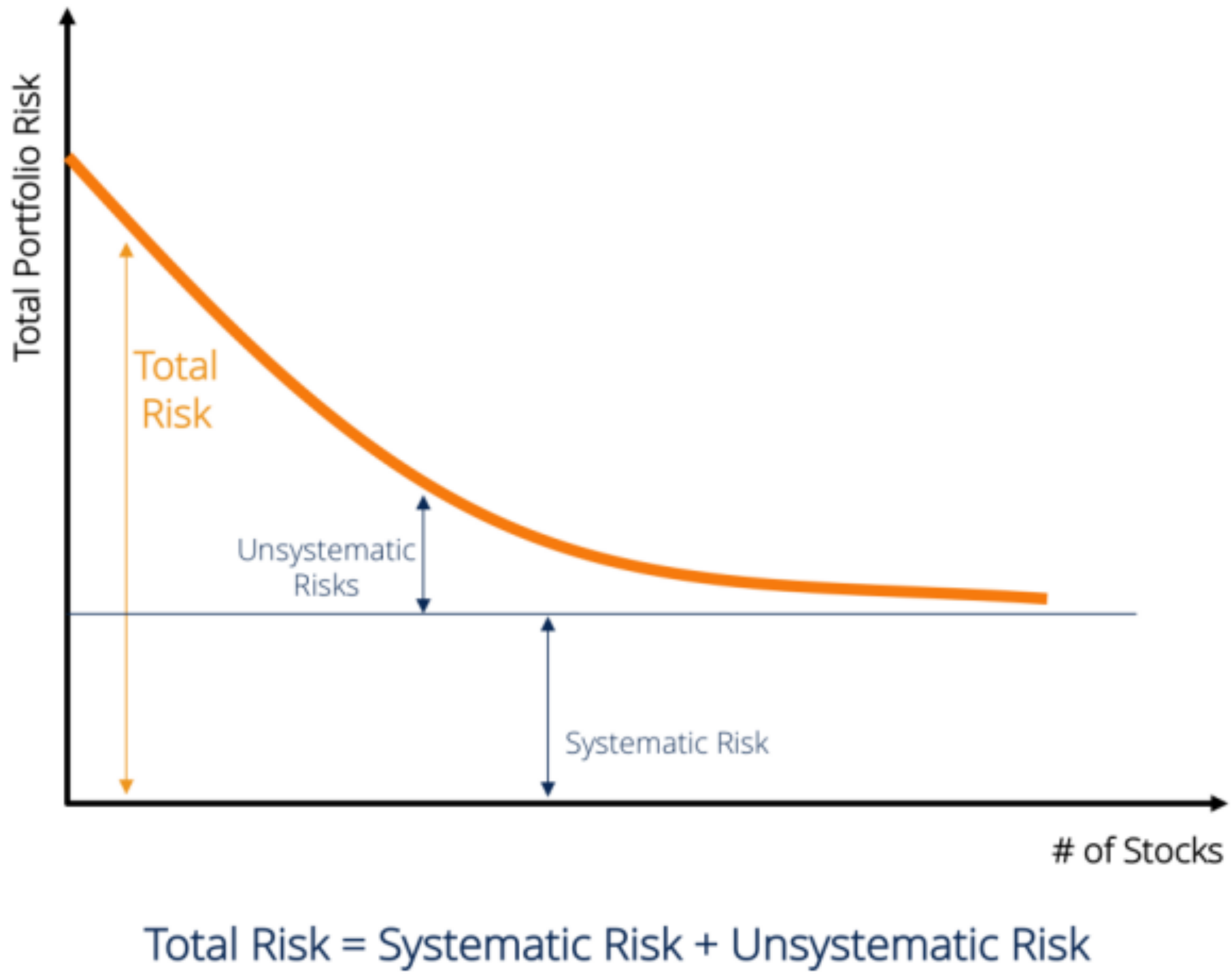
Year	Stock_10	Stock_25	Stock_50	Stock_100	Stock_150
1997	10.82%	6.66%	4.36%	3.28%	2.69%
1998	12.06%	8.37%	5.41%	3.70%	3.06%
1999	14.97%	9.48%	7.77%	5.10%	3.83%
2000	15.15%	8.81%	5.69%	4.01%	2.41%
2001	10.41%	6.73%	4.80%	3.33%	2.27%
2002	8.62%	5.46%	3.53%	2.57%	1.99%
2003	13.53%	9.21%	5.96%	3.62%	2.73%
2004	9.90%	5.75%	3.66%	2.55%	1.99%
2005	7.95%	5.19%	3.36%	2.52%	1.94%
2006	6.93%	4.08%	3.11%	1.98%	1.59%

Annual Stock Returns (2007–2016)

Year	Stock_10	Stock_25	Stock_50	Stock_100	Stock_150
2007	11.09%	6.83%	4.65%	2.45%	2.07%
2008	7.80%	4.66%	3.36%	2.13%	1.55%
2009	18.20%	10.55%	7.65%	4.64%	3.05%
2010	8.88%	4.70%	3.46%	2.08%	1.79%
2011	6.59%	5.03%	3.26%	2.36%	1.84%
2012	7.13%	4.73%	3.23%	2.19%	1.58%
2013	10.87%	6.51%	4.63%	2.79%	2.04%
2014	5.87%	3.89%	3.49%	2.34%	1.63%
2015	8.12%	4.34%	3.16%	2.34%	1.55%
2016	8.29%	4.33%	3.01%	2.18%	1.55%

- Standard deviations. Significant reduction in SDs from 10 stock portfolio to 25 stock portfolio. Then it tails off.

Optimal portfolio



Optimal portfolio

Question

A financial analyst wants to calculate the expected return of investing in a portfolio of shares whose return is 1.5% with a 15% probability, 5% return with a 25% probability and 4% otherwise.

- $$E[R_i] = p_1 E[R_1] + p_2 E[R_2] + p_3 E[R_3] = 0.15(0.015) + 0.25(0.05) + 0.6(0.04) = 0.03$$
- However, we do not always know the probability distribution of the expected returns of a specific asset - so know what future probabilities may look like.
- We can use the **historic** returns in order to obtain as estimate for the probability distributions of the expecte
- Use the historic average return for the expected return
- $$E[R_i] = \mu_i = \frac{1}{T} \sum_{t=1}^{T} R_t$$

  Where T is the number of historic data points used and R_t is the return in period t.

Compute a portfolios return

- For a portfolio with N assets, the portfolio expected return is the weighted average of the expected return of the individual assets.

-

$$E(R_p) = w_1 E(R_1) + w_2 E(R_2) + \cdots + w_N E(R_N) = \sum_{i=1}^{N} w_i$$

Where p stands for portfolio.

Compite a portfolios return

Question

The following table shows the expected return from investing in assets A and B under two possible scenarios. (S=1 and S=2) along with their probabilities (1/3) and (2/3).

1. Compute the expected return of a portfolio with 50% invested in each asset - the risk free rate of return is 2%.
2. Compute the expected return of a portfolio with 40% invested in asset A and 60% invested in asset B. 3)Compute the expected return of a portfolio 30% invested in asset A and 40% invested in asset B and the remainder invested in the risk-free asset.

S.1..Prob.1.3.	S.2..Prob.2.3.
0.2	-0.03
0.4	0.00

Compute a portfolios return

- A. compute the expected return of a portfolio with 50% invested in each asset - the risk free rate of return is 2%.

  - $$E[R_A] = \frac{1}{3}20\% + \frac{2}{3}(-3\%) = 4.67\%$$
    $$E[R_B] = \frac{1}{3}40\% + \frac{2}{3}(0\%) = 13.33\%$$
    $$E[R_P] = \frac{4.67 + 13.33}{2} = 9\%$$

- B. Compute the expected return of a portfolio with 40% invested in asset A and 60% invested in asset B.

  - $$E[R_P] = 0.4 * 4.67\% + 0.6 * 13.33\% = 9.85\%$$

- C. Compute the expected return of a portfolio 30% invested in asset A and 40% invested in asset B

Risk and Return

- Risk is related to the dispersion of returns relative to its expected return
- Previously, we estimated **returns** by taking the historical mean returns of an asset
- Now, we do the same but for *risk*
  - We take the historical variance of an asset's returns

$$\sigma_i^2 = \frac{1}{T-1}\sum_{t=1}^{T}(R_{i,t} - \mu_i)^2$$

$$\mu \text{ is the average return of asset i}$$

  - The standard deviation is the square root of the variance
  -

$$\text{volatility} = \sigma_i = \sqrt{\sigma_i^2}$$

Risk and Return

Question

Determine the historical **average** return and the historical **risk** of investing in shares of the following company:

year	end_of_year_price
2010	12.5
2011	13.2
2012	14.6
2013	14.2
2014	13.9
2015	14.5
2016	14.9
2017	15.8
2018	15.6

Risk and Return

- 1. Compute the returns
  - $$R = \frac{V_f - V_i}{V_i}$$
    $$\text{where } V_f \text{ is the final value}$$
    $$\text{and } V_i \text{ is the initial value}$$
- 2. Compute the historical average returns
  - $$\mu_i = \frac{1}{T}\sum_{t=1}^{T} R_{i,t}$$
    $$\mu_i = \frac{1}{8}\sum_{t=2010}^{2018} R_t = 2.90\%$$
- 3. Compute the historical standard deviation
  - $$\sigma_i = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(R_{i,t} - \mu_i)^2}$$

Risk and Return

▸ Show the code

year	end_of_year_price	returns	historical_returns	historical_volatility
2010	12.5	NA	0.02900409	0.04672851
2011	13.2	0.05600000	0.02900409	0.04672851
2012	14.6	0.10606061	0.02900409	0.04672851
2013	14.2	-0.02739726	0.02900409	0.04672851
2014	13.9	-0.02112676	0.02900409	0.04672851
2015	14.5	0.04316547	0.02900409	0.04672851
2016	14.9	0.02758621	0.02900409	0.04672851
2017	15.8	0.06040268	0.02900409	0.04672851
2018	15.6	-0.01265823	0.02900409	0.04672851

Portfolio of Three Assets

Expected Return: The expected return of a portfolio with three assets is the weighted average of the expected returns of each asset:

$$E(R_p) = w_1 E(R_1) + w_2 E(R_2) + w_3 E(R_3)$$

Where: -

$$E(R_p) = \text{Expected return of the portfolio}$$

-

$$E(R_1), E(R_2), E(R_3) = \text{Expected returns of assets 1, 2, and 3}$$

-

$$w_1, w_2, w_3 = \text{Portfolio weights for assets 1, 2, and 3}$$

- where

$$w_1 + w_2 + w_3 = 1$$

Portfolio of Three Assets

The variance of a three-asset portfolio takes into account the individual variances and the covariances between the assets:

$$\sigma_p^2 = w_1^2\,\sigma_1^2 + w_2^2\,\sigma_2^2 + w_3^2\,\sigma_3^2 + 2\,w_1\,w_2\,\mathrm{Cov}(R_1, R_2) + 2\,w_1\,w_3\,\mathrm{Cov}(R_1, R_3) + 2\,w_2\,w_3\,\mathrm{Co}$$

Where: -

$$\sigma_p^2 = \text{Variance of the portfolio}$$

-

$$\sigma_1^2, \sigma_2^2, \sigma_3^2 = \text{Variances of assets 1, 2, and 3, respectively}$$

Computing Covariances

To compute the covariances:

- Covariance between Asset 1 and Asset 2:

$$\mathrm{Cov}(R_1, R_2) = \left(R_1 - R_{\mu_1}\right)\left(R_2 - R_{\mu_2}\right)$$

- Covariance between Asset 1 and Asset 3:

$$\mathrm{Cov}(R_1, R_3) = \left(R_1 - R_{\mu_1}\right)\left(R_3 - R_{\mu_3}\right)$$

- Covariance between Asset 2 and Asset 3:

$$\mathrm{Cov}(R_2, R_3) = \left(R_2 - R_{\mu_2}\right)\left(R_3 - R_{\mu_3}\right)$$

Compute Standard Deviation

- The standard deviation of the portfolio is:

$$\sigma_p = \sqrt{\sigma_p^2}$$

- These are the fundamental formulas for calculating the expected return and variance (risk) of a portfolio consisting of two and three assets.