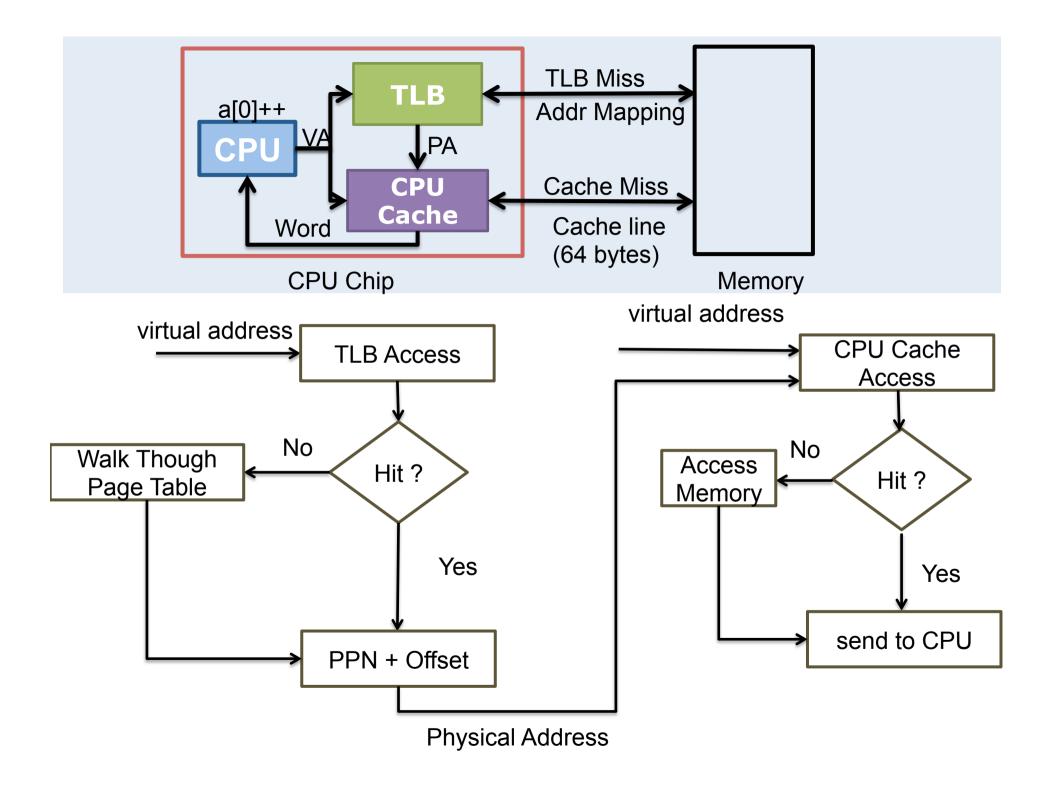
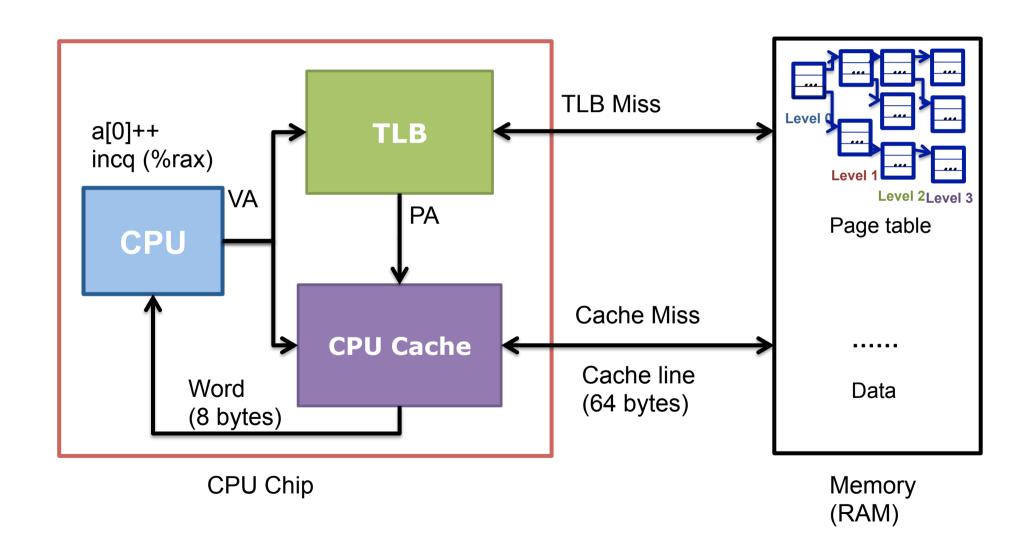
## **Cache-Friendly Code**

Jinyang Li

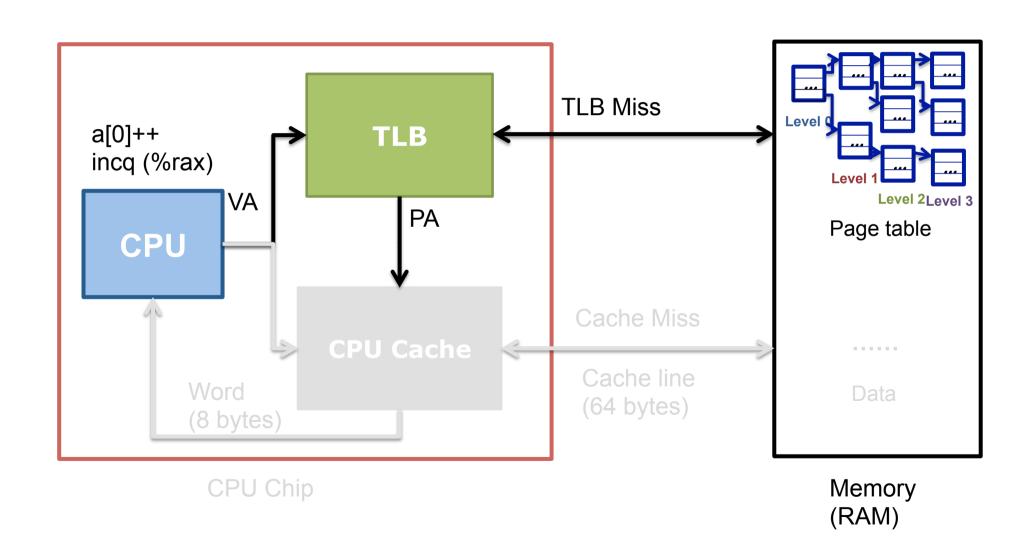
based on the slides of Tiger Wang

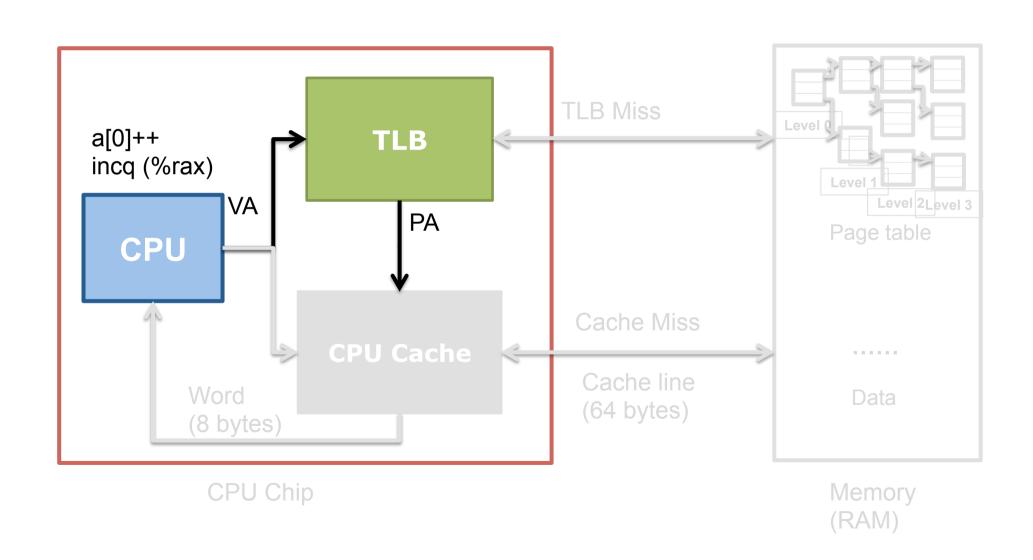


### **Step 1. Address Translation**

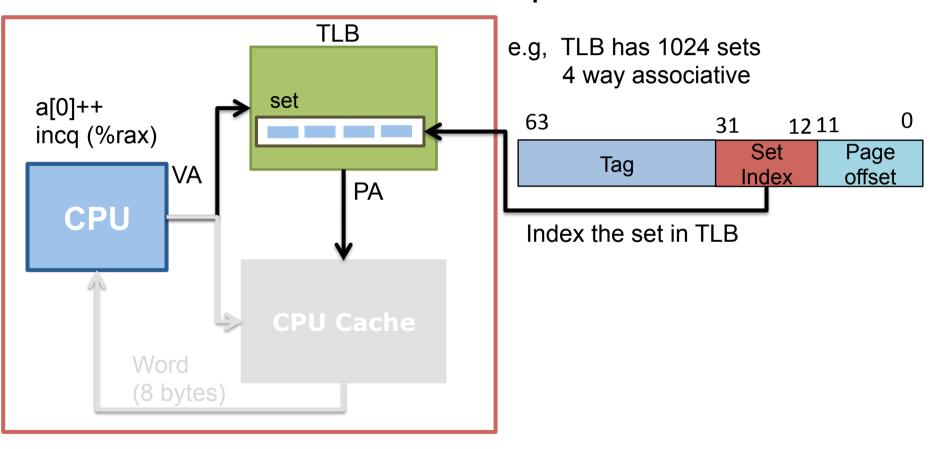


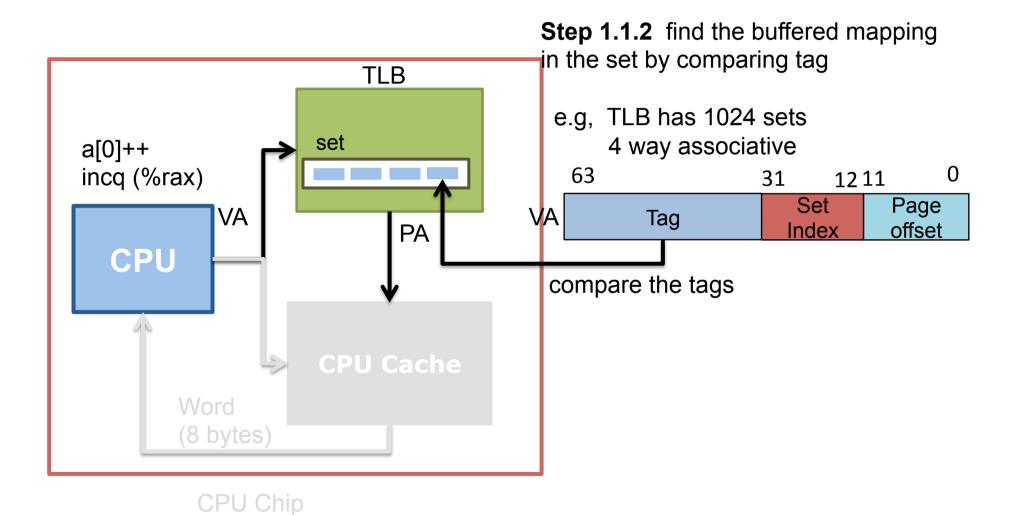
### **Step 1. Address Translation**

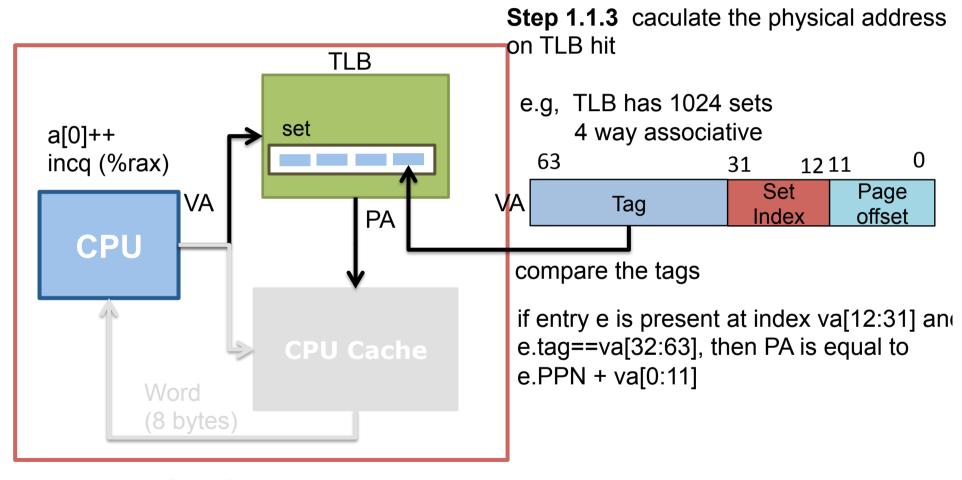




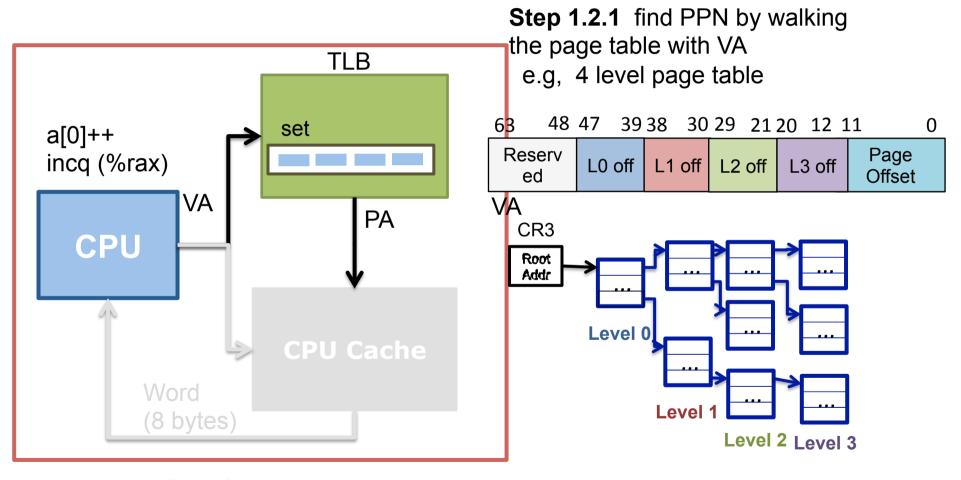
**Step 1.1.1** calculate the set index in TLB



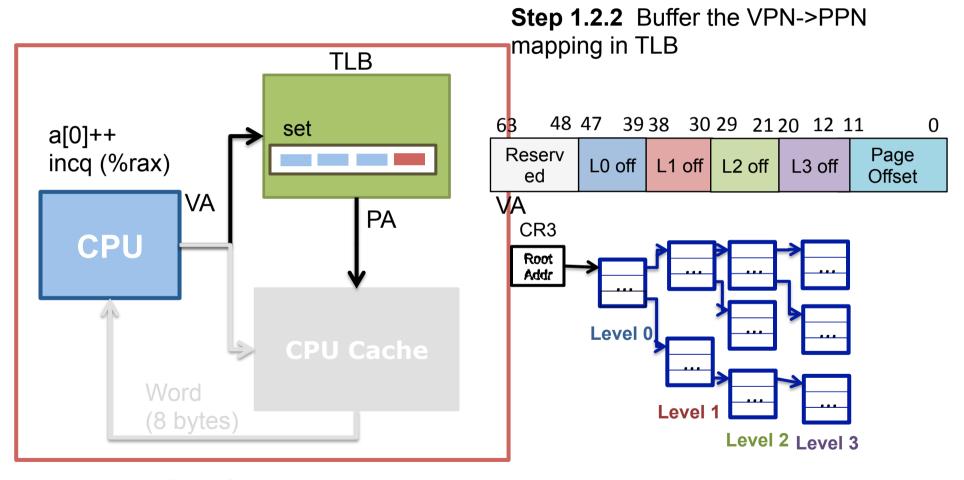




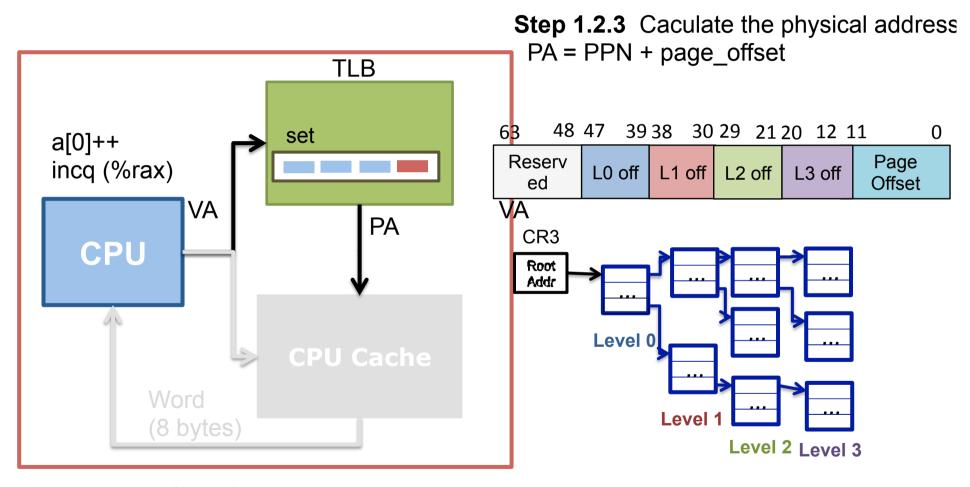
## Step 1.2 Walk Page Table on TLB Miss



## Step 1.2 Walk Through Page Table on TLB Miss

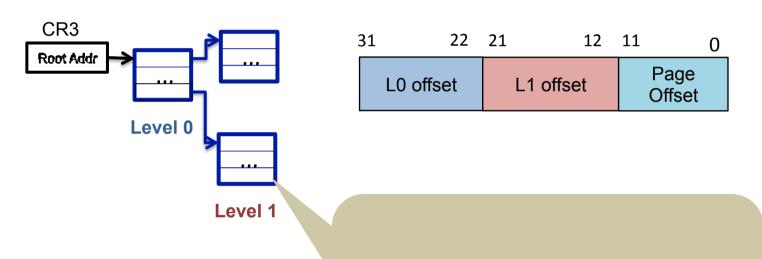


## Step 1.2 Walk Through Page Table on TLB Miss



#### **Exercise**

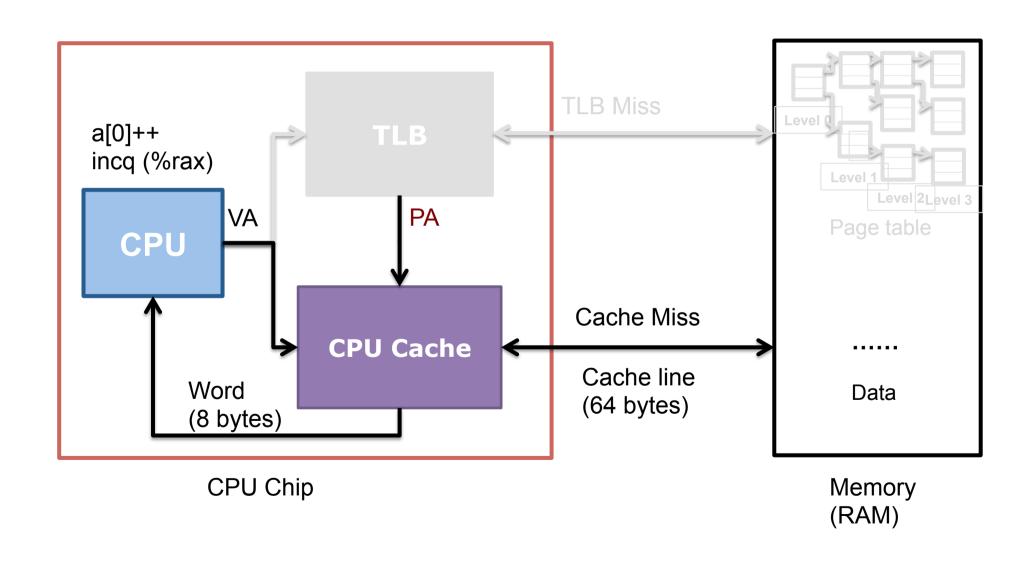
32 bit address, 2 level page table, 4K page size



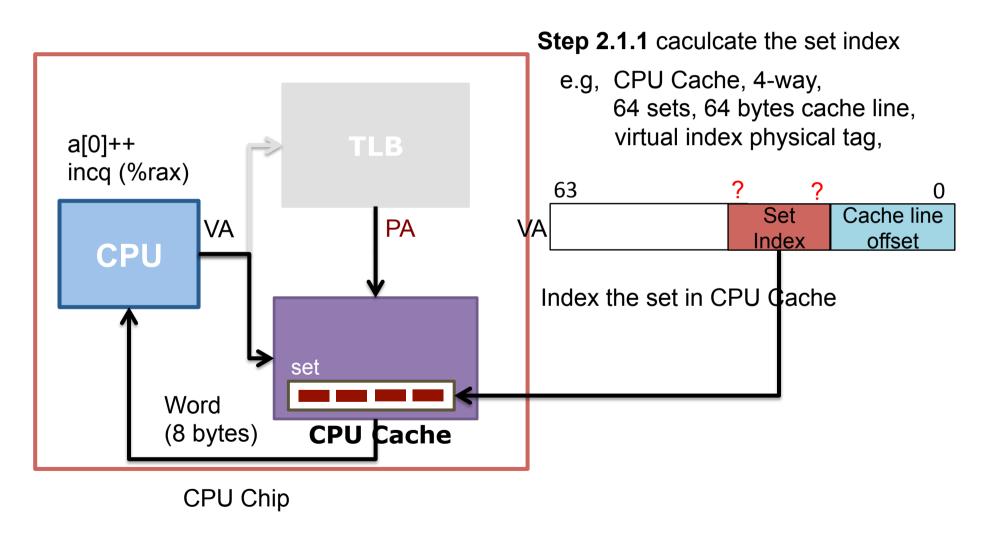
Given each page entry is 4-byte in size, how many entries per 4KB page?

 $4KB/4B = 2^10$  entries

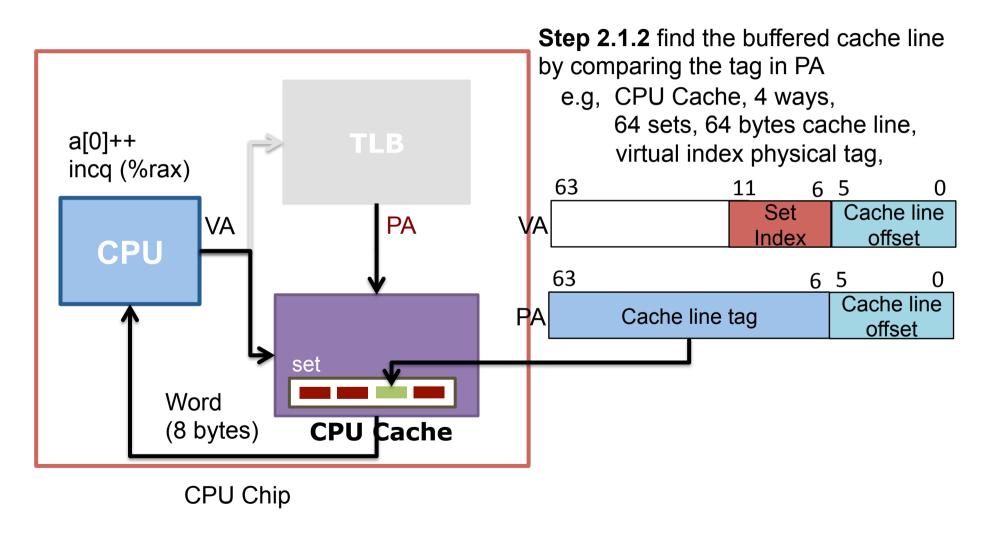
### Step 2. Fetch Data



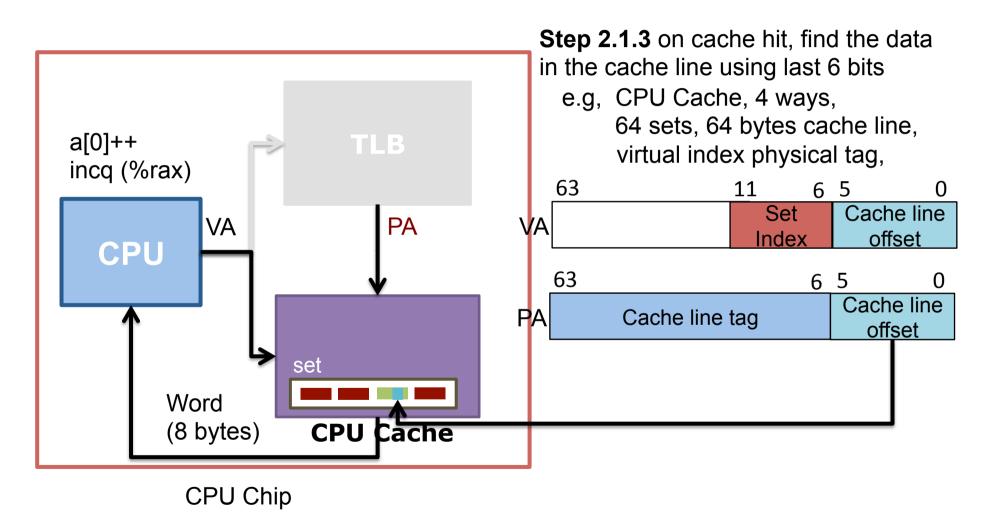
# **Step 2.1 Fetch Data** from CPU Cache



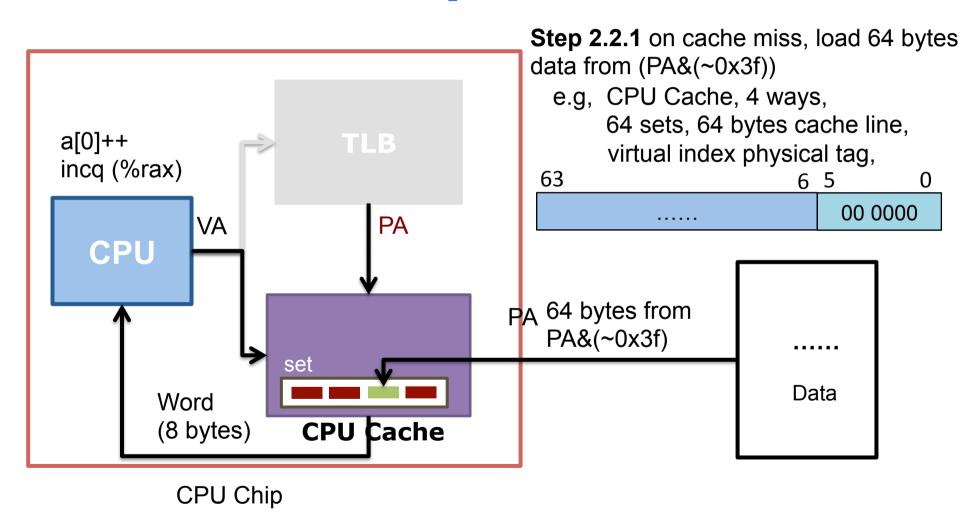
# **Step 2.1 Fetch Data** from CPU Cache



# **Step 2.1 Fetch Data** from CPU Cache



# **Step 2.2 Fetch Data from Memory on Cache Miss**



### **Writing Cache-Friendly Code**

#### Why?

- Programs with lower cache miss rates typically run faster
  - Miss rate: fraction of memory references not found in cache (misses/references)
  - Typical numbers: 3-10% for L1, can be quite small (<1%) for L2, depending on size

#### How to write cache friendly code?

Memory access pattern

Memory layout

#### Simple example: sum of 2D array

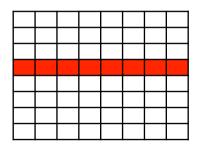
```
int64 sumarrayrows(int64** a, int r, int c)
{
   int i, j = 0;
   int64 sum = 0
   int i, j = 0;
   int i, j = 0;
   int64 sum = 0;

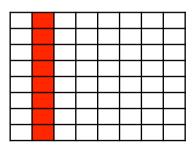
for (int i = 0; i < r; i++)
   for (int j = 0; j < c; j++)
      sum += a[i][j];
   return sum;
}</pre>
int64 sumarraycols(int64** a, int r, int c)

{
   int i, j = 0;
   int64 sum = 0;

for (int j = 0; j < c; j++)
      sum += a[i][j];
   return sum;
}
```

#### Which implementation is more cache friendly?





### Simple Example

```
int64 sumarrayrows(int64** a, int r, int c)
{
   int i, j = 0;
   int64 sum = 0
   int i, j = 0;
   int i, j = 0;
   int64 sum = 0;

for (int i = 0; i < r; i++)
   for (int j = 0; j < c; j++)
      sum += a[i][j];
   return sum;
}</pre>
int64 sumarraycols(int64** a, int r, int c)

{
   int i, j = 0;
   int64 sum = 0;

for (int j = 0; j < c; j++)
      sum += a[i][j];
   return sum;
}
```

#### How many cache misses?

#### Example:

- CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
- Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
- local variables: i, j, sum are stored in the registers

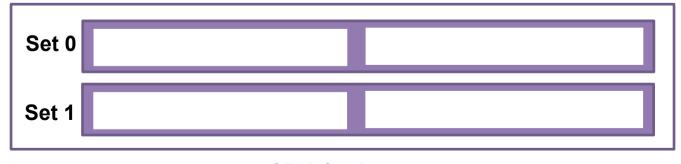
```
a[5][7]
a[5][1]
a[5][0]
a[4][7]
a[4][1]
a[4][0]
a[3][7]
a[3][1]
a[3][0]
a[2][7]
a[2][1]
a[2][0]
a[1][7]
a[1][1]
a[1][0]
a[0][7]
a[0][1]
a[0][0]
Memory
```

### Simple Example

#### Example:

- CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
- Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
- local variables: i, j, sum are stored in the registers

```
for (int j = 0; j < c; j++)
    for (int i = 0; i < r; i++)
        sum += a[i][j];</pre>
```

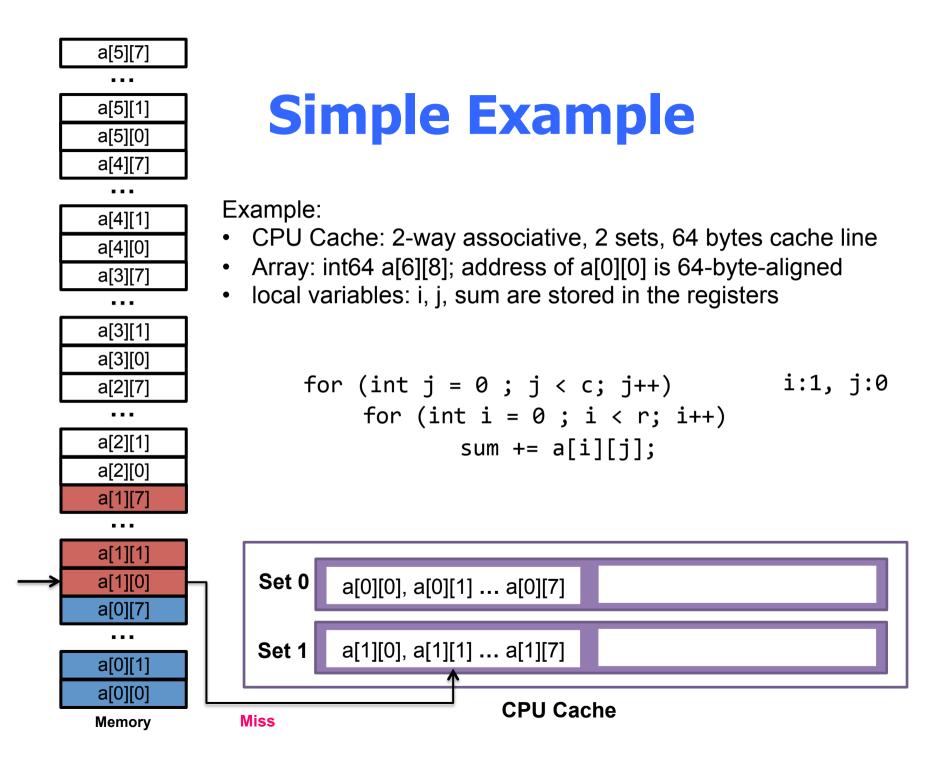


**CPU Cache** 

```
a[5][7]
                Simple Example
a[5][1]
a[5][0]
a[4][7]
             Example:
a[4][1]

    CPU Cache: 2-way associative, 2 sets, 64 bytes cache line

a[4][0]
             Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
a[3][7]
             • local variables: i, j, sum are stored in the registers
a[3][1]
a[3][0]
                   for (int j = 0; j < c; j++) i:0, j:0
a[2][7]
                         for (int i = 0; i < r; i++)
a[2][1]
                                  sum += a[i][j];
a[2][0]
                 Miss
a[1][7]
a[1][1]
               Set 0
a[1][0]
                       a[0][0], a[0][1] ... a[0][7]
a[0][7]
               Set 1
a[0][1]
a[0][0]
                                      CPU Cache
Memory
```



```
a[5][7]
                Simple Example
a[5][1]
a[5][0]
a[4][7]
            Example:
a[4][1]
              CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
a[4][0]
              Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
a[3][7]
              local variables: i, j, sum are stored in the registers
a[3][1]
a[3][0]
                    for (int j = 0; j < c; j++) i:2, j:0
a[2][7]
                          for (int i = 0; i < r; i++)
 . . .
a[2][1]
                                   sum += a[i][j];
a[2][0]
                      Miss
a[1][7]
a[1][1]
               Set 0
                        a[0][0], a[0][1] ... a[0][7]
a[1][0]
                                                    a[2][0], a[2][1] ... a[2][7]
a[0][7]
                       a[1][0], a[1][1] ... a[1][7]
               Set 1
a[0][1]
a[0][0]
                                       CPU Cache
Memory
```

```
a[5][7]
                 Simple Example
a[5][1]
a[5][0]
a[4][7]
            Example:
a[4][1]
               CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
a[4][0]
              Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
              local variables: i, j, sum are stored in the registers
a[3][7]
a[3][1]
a[3][0]
                                                                     i:3, j:0
                    for (int j = 0; j < c; j++)
a[2][7]
                           for (int i = 0; i < r; i++)
 . . .
a[2][1]
                                     sum += a[i][j];
                                                                                    Miss
a[2][0]
a[1][7]
a[1][1]
                Set 0
a[1][0]
                         a[0][0], a[0][1] ... a[0][7]
                                                     a[2][0], a[2][1] ... a[2][7]
a[0][7]
                        a[1][0], a[1][1] ... a[1][7]
                                                     a[3][0], a[3][1] ... a[3][7]
                Set 1
a[0][1]
a[0][0]
                                         CPU Cache
```

Memory

```
a[5][7]
                 Simple Example
a[5][1]
a[5][0]
a[4][7]
                Example:
a[4][1]
                  CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
a[4][0]
                  Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
a[3][7]
                  local variables: i, j, sum are stored in the registers
a[3][1]
a[3][0]
                    for (int j = 0; j < c; j++) i:4, j:0
a[2][7]
                          for (int i = 0; i < r; i++)
 . . .
a[2][1]
                                    sum += a[i][j];
               Miss
a[2][0]
a[1][7]
a[1][1]
                Set &
                        a[4][0], a[4][1] ... a[4][7]
a[1][0]
                                                    a[2][0], a[2][1] ... a[2][7]
a[0][7]
                Set 1
                        a[1][0], a[1][1] ... a[1][7]
                                                    a[3][0], a[3][1] ... a[3][7]
a[0][1]
a[0][0]
                                        CPU Cache
Memory
```

```
a[5][7]
a[5][1]
a[5][0]
a[4][7]
a[4][1]
a[4][0]
a[3][7]
a[3][1]
a[3][0]
a[2][7]
  . . .
a[2][1]
a[2][0]
a[1][7]
a[1][1]
a[1][0]
a[0][7]
a[0][1]
a[0][0]
```

Memory

### **Simple Example**

#### Example:

- CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
- Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
- local variables: i, j, sum are stored in the registers

```
for (int j = 0; j < c; j++) i:5, j:0
for (int i = 0; i < r; i++)
sum += a[i][j];
```

```
      Set 0
      a[4][0], a[4][1] ... a[4][7]
      a[2][0], a[2][1] ... a[2][7]

      Set 1
      a[5][0], a[5][1] ... a[5][7]
      a[3][0], a[3][1] ... a[3][7]
```

**CPU Cache** 

```
a[5][7]
                 Simple Example
a[5][1]
a[5][0]
a[4][7]
            Example:
a[4][1]

    CPU Cache: 2-way associative, 2 sets, 64 bytes cache line

a[4][0]
               Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
              local variables: i, j, sum are stored in the registers
a[3][7]
a[3][1]
a[3][0]
                    for (int j = 0; j < c; j++) i:0, j:1
a[2][7]
                          for (int i = 0; i < r; i++)
 . . .
a[2][1]
                                    sum += a[i][j];
a[2][0]
               Miss
a[1][7]
a[1][1]
                Set 0
a[1][0]
                        a[4][0], a[4][1] ... a[4][7]
                                                     a[0][0], a[0][1] ... a[0][7]
a[0][7]
                        a[5][0], a[5][1] ... a[5][7]
                                                     a[3][0], a[3][1] ... a[3][7]
                Set 1
a[0][1]
a[0][0]
                                        CPU Cache
Memory
```

a[5][7] Simple Example a[5][1] a[5][0] a[4][7] Example: a[4][1] CPU Cache: 2-way associative, 2 sets, 64 bytes cache line a[4][0] Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned a[3][7] • local variables: i, j, sum are stored in the registers a[3][1] a[3][0] for (int j = 0; j < c; j++) i:1, j:1 a[2][7] for (int i = 0; i < r; i++) . . . a[2][1] sum += a[i][j]; a[2][0] a[1][7] a[1][1] Set 0 a[1][0] a[4][0], a[4][1] ... a[4][7] a[0][0], a[0][1] ... a[0][7] a[0][7] Set 1 a[5][0], a[5][1] ... a[5][7] a[1][0], a[1][1] ... a[1][7] a[0][1] a[0][0] **CPU Cache** Miss Memory

```
a[5][7]
                 Simple Example
a[5][1]
a[5][0]
a[4][7]
             Example:
a[4][1]
                CPU Cache: 2-way associative, 2 sets, 64 bytes cache line

    Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned

a[4][0]
             • local variables: i, j, sum are stored in the registers
a[3][7]
a[3][1]
a[3][0]
                    for (int j = 0; j < c; j++) i:2, j:1
a[2][7]
                          for (int i = 0; i < r; i++)
 . . .
a[2][1]
                                    sum += a[i][j];
                       Miss
a[2][0]
a[1][7]
a[1][1]
                Set 0
a[1][0]
                        a[2][0], a[2][1] ... a[2][7]
                                                     a[0][0], a[0][1] ... a[0][7]
a[0][7]
                        a[5][0], a[5][1] ... a[5][7]
                                                     a[1][0], a[1][1] ... a[1][7]
                Set 1
a[0][1]
a[0][0]
                                        CPU Cache
Memory
```

a[5][7] Simple Example a[5][1] a[5][0] a[4][7] Example: CPU Cache: 2-way associative, 2 sets, 64 bytes cache line a[4][1] Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned a[4][0] • local variables: i, j, sum are stored in the registers a[3][7] a[3][1] a[3][0] for (int j = 0; j < c; j++) i:3, j:1 a[2][7] for (int i = 0; i < r; i++) . . . a[2][1] sum += a[i][j]; a[2][0] Miss a[1][7] a[1][1] Set 0 a[1][0] a[2][0], a[2][1] ... a[2][7] a[0][0], a[0][1] ... a[0][7] a[0][7] a[1][0], a[1][1] ... a[1][7] Set 1 a[3][0], a[3][1] ... a[3][7] a[0][1] a[0][0] **CPU Cache** Memory

```
a[5][7]
                 Simple Example
a[5][1]
a[5][0]
a[4][7]
              Example:
a[4][1]
              • CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
a[4][0]

    Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned

a[3][7]

    local variables: i, j, sum are stored in the registers

a[3][1]
a[3][0]
                     for (int j = 0; j < c; j++)
                                                                      i:4, j:1
a[2][7]
                           for (int i = 0; i < r; i++)
 . . .
a[2][1]
                                     sum += a[i][j];
a[2][0]
a[1][7]
                                                            Miss
a[1][1]
                Set 0
a[1][0]
                         a[2][0], a[2][1] ... a[2][7]
                                                      a[4][0], a[4][1] ... a[4][7]
a[0][7]
                         a[3][0], a[3][1] ... a[3][7]
                                                      a[1][0], a[1][1] ... a[1][7]
                Set 1
a[0][1]
a[0][0]
                                         CPU Cache
Memory
```

```
a[5][7]
a[5][1]
a[5][0]
a[4][7]
a[4][1]
a[4][0]
a[3][7]
a[3][1]
a[3][0]
a[2][7]
  . . .
a[2][1]
a[2][0]
a[1][7]
a[1][1]
a[1][0]
a[0][7]
a[0][1]
a[0][0]
Memory
```

## Simple Example

#### Example:

- CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
- Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
- local variables: i, j, sum are stored in the registers

```
for (int j = 0; j < c; j++) i:5, j:1
  for (int i = 0; i < r; i++)
      sum += a[i][j];</pre>
```

**Miss** 

```
      Set 0
      a[2][0], a[2][1] ... a[2][7]
      a[4][0], a[4][1] ... a[4][7]

      Set 1
      a[3][0], a[3][1] ... a[3][7]
      a[5][0], a[5][1] ... a[5][7]
```

**CPU Cache** 

a[5][7] Simple Example a[5][1] a[5][0] a[4][7] Example: a[4][1] CPU Cache: 2-way associative, 2 sets, 64 bytes cache line a[4][0] Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned a[3][7] • local variables: i, j, sum are stored in the registers a[3][1] a[3][0] for (int i = 0; i < r; i++) a[2][7] for (int j = 0; j < c; j++) a[2][1] sum += a[i][j]; a[2][0] a[1][7] a[1][1] Set 0 a[1][0] a[0][7] Set 1 a[0][1] a[0][0] **CPU Cache** Memory

```
a[5][7]
                Simple Example
a[5][1]
a[5][0]
a[4][7]
               Example:
a[4][1]
               CPU Cache – 2 ways, 2 sets, 64 bytes cache
a[4][0]
               line
a[3][7]
               Array – int64 a[6][8]
               The address of a[0][0] is cache line alignment
a[3][1]
a[3][0]
                   for (int i = 0; i < r; i++) i:0, j:0
a[2][7]
                         for (int j = 0; j < c; j++)
 . . .
a[2][1]
                                  sum += a[i][j];
a[2][0]
                 Miss
a[1][7]
a[1][1]
               Set 0
a[1][0]
                       a[0][0], a[0][1] ... a[0][7]
a[0][7]
               Set 1
a[0][1]
a[0][0]
                                      CPU Cache
Memory
```

#### a[5][7] Simple Example a[5][1] a[5][0] a[4][7] Example: CPU Cache: 2-way associative, 2 sets, 64 bytes cache line a[4][1] Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned a[4][0] • local variables: i, j, sum are stored in the registers a[3][7] a[3][1] a[3][0] for (int i = 0; i < r; i++) i:0, j:1 a[2][7] for (int j = 0; j < c; j++) a[2][1] sum += a[i][j]; a[2][0] Hit a[1][7] a[1][1] Set 0 a[1][0] a[0][0], a[0][1] ... a[0][7] a[0][7] Set 1 a[0][1] a[0][0] **CPU Cache** Memory

a[5][7] Simple Example a[5][1] a[5][0] a[4][7] Example: a[4][1] CPU Cache: 2-way associative, 2 sets, 64 bytes cache line a[4][0] Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned • local variables: i, j, sum are stored in the registers a[3][7] a[3][1] a[3][0] for (int i = 0; i < r; i++) i:0, j:7 a[2][7] for (int j = 0; j < c; j++) a[2][1] sum += a[i][j]; a[2][0] Hit a[1][7] a[1][1] Set 0 a[1][0] a[0][0], a[0][1] ... a[0][7] a[0][7] Set 1 a[0][1] a[0][0] **CPU Cache** Memory

a[5][7] Simple Example a[5][1] a[5][0] a[4][7] Example: a[4][1] CPU Cache – 2 ways, 2 sets, 64 bytes cache a[4][0] line a[3][7] Array – int64 a[6][8] The address of a[0][0] is cache line alignment a[3][1] a[3][0] for (int i = 0; i < r; i++) i:1, j:0 a[2][7] for (int j = 0; j < c; j++) a[2][1] sum += a[i][j]; a[2][0] a[1][7] a[1][1] Set 0 a[1][0] a[0][0], a[0][1] ... a[0][7] a[0][7] Set 1 a[1][0], a[1][1] ... a[1][7] a[0][1] a[0][0] **CPU Cache** Miss Memory

a[5][7] Simple Example a[5][1] a[5][0] a[4][7] Example: a[4][1] CPU Cache: 2-way associative, 2 sets, 64 bytes cache line a[4][0] Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned a[3][7] • local variables: i, j, sum are stored in the registers a[3][1] a[3][0] for (int i = 0; i < r; i++) i:1, j:1 a[2][7] for (int j = 0; j < c; j++) a[2][1] sum += a[i][j]; a[2][0] a[1][7] a[1][1] Set 0 a[1][0] a[0][0], a[0][1] ... a[0][7] a[0][7] Set 1 a[1][0], a[1][1] ... a[1][7] a[0][1] a[0][0] **CPU Cache** Hit Memory

a[5][7] Simple Example a[5][1] a[5][0] a[4][7] a[4][1] Example: CPU Cache: 2-way associative, 2 sets, 64 bytes cache line a[4][0] Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned a[3][7] local variables: i, j, sum are stored in the registers a[3][1] a[3][0] for (int i = 0; i < r; i++) i:1, j:7 a[2][7] for (int j = 0; j < c; j++) a[2][1] sum += a[i][j]; a[2][0] a[1][7] a[1][1] Set 0 a[1][0] a[0][0], a[0][1] ... a[0][7] a[0][7] Set 1 a[1][0], a[1][1] ... a[1][7] a[0][1] a[0][0] **CPU Cache** Hit Memory

```
a[5][7]
                 Simple Example
a[5][1]
a[5][0]
a[4][7]
             Example:
a[4][1]
                CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
a[4][0]

    Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned

a[3][7]
             • local variables: i, j, sum are stored in the registers
a[3][1]
a[3][0]
                    for (int i = 0; i < r; i++) i:2, j:0
a[2][7]
                          for (int j = 0; j < c; j++)
 . . .
a[2][1]
                                    sum += a[i][j];
a[2][0]
                       Miss
a[1][7]
a[1][1]
                Set 0
                        a[0][0], a[0][1] ... a[0][7]
a[1][0]
                                                    a[2][0], a[2][1] ... a[2][7]
a[0][7]
                        a[1][0], a[1][1] ... a[1][7]
                Set 1
a[0][1]
a[0][0]
                                        CPU Cache
Memory
```

```
a[5][7]
                Simple Example
a[5][1]
a[5][0]
a[4][7]
              Example:
a[4][1]
                CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
a[4][0]
                Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
              • local variables: i, j, sum are stored in the registers
a[3][7]
a[3][1]
a[3][0]
                    for (int i = 0; i < r; i++) i:2, j:1
a[2][7]
                          for (int j = 0; j < c; j++)
a[2][1]
                                   sum += a[i][j];
                     Hit
a[2][0]
a[1][7]
a[1][1]
               Set 0
                        a[0][0], a[0][1] ... a[0][7]
a[1][0]
                                                   a[2][0], a[2][1] ... a[2][7]
a[0][7]
                       a[1][0], a[1][1] ... a[1][7]
               Set 1
a[0][1]
a[0][0]
                                       CPU Cache
Memory
```

```
a[5][7]
                 Simple Example
a[5][1]
a[5][0]
a[4][7]
              Example:
                 CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
a[4][1]
                Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
a[4][0]
              • local variables: i, j, sum are stored in the registers
a[3][7]
a[3][1]
a[3][0]
                                                                      i:2, j:7
a[2][7]
              Hit
                           for (int j = 0; j < c; j++)
a[2][1]
                                     sum += a[i][j];
a[2][0]
a[1][7]
a[1][1]
                Set 0
                         a[0][0], a[0][1] ... a[0][7]
a[1][0]
                                                      a[2][0], a[2][1] ... a[2][7]
a[0][7]
                        a[1][0], a[1][1] ... a[1][7]
                Set 1
a[0][1]
a[0][0]
                                         CPU Cache
Memory
```

```
a[5][7]
                 Simple Example
a[5][1]
a[5][0]
a[4][7]
            Example:
a[4][1]
              CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
a[4][0]
              Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
a[3][7]
              local variables: i, j, sum are stored in the registers
a[3][1]
a[3][0]
                                                                     i:3, j:0
                    for (int i = 0; i < r; i++)
a[2][7]
                           for (int j = 0; j < c; j++)
 . . .
a[2][1]
                                     sum += a[i][j];
                                                                                    Miss
a[2][0]
a[1][7]
a[1][1]
                Set 0
                         a[0][0], a[0][1] ... a[0][7]
a[1][0]
                                                     a[2][0], a[2][1] ... a[2][7]
a[0][7]
                        a[1][0], a[1][1] ... a[1][7]
                                                     a[3][0], a[3][1] ... a[3][7]
                Set 1
a[0][1]
a[0][0]
                                         CPU Cache
Memory
```

a[5][7] Simple Example a[5][1] a[5][0] a[4][7] Example: CPU Cache: 2-way associative, 2 sets, 64 bytes cache line a[4][1] a[4][0] Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned local variables: i, j, sum are stored in the registers a[3][7] a[3][1] a[3][0] for (int i = 0; i < r; i++) i:3, j:1 a[2][7] for (int j = 0; j < c; j++) . . . a[2][1] sum += a[i][j]; Hit a[2][0] a[1][7] a[1][1] Set 0 a[1][0] a[0][0], a[0][1] ... a[0][7] a[2][0], a[2][1] ... a[2][7] a[0][7] a[3][0], a[3][1] ... a[3][7] a[1][0], a[1][1] ... a[1][7] Set 1 a[0][1] a[0][0] **CPU Cache** 

Memory

```
a[5][7]
                 Simple Example
a[5][1]
a[5][0]
a[4][7]
            Example:
a[4][1]
               CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
a[4][0]
               Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
a[3][7]
               local variables: i, j, sum are stored in the registers
a[3][1]
a[3][0]
                    for (int i = 0; i < r; i++) i:3, j:7
a[2][7]
                          for (int j = 0; j < c; j++)
 . . .
a[2][1]
                                    sum += a[i][j];
                                                                                 Hit
a[2][0]
a[1][7]
a[1][1]
                Set 0
                        a[0][0], a[0][1] ... a[0][7]
a[1][0]
                                                    a[2][0], a[2][1] ... a[2][7]
a[0][7]
                                                    a[3][0], a[3][1] ... a[3][7]
                        a[1][0], a[1][1] ... a[1][7]
                Set 1
a[0][1]
a[0][0]
                                        CPU Cache
Memory
```

```
a[5][7]
                 Simple Example
a[5][1]
a[5][0]
a[4][7]
              Example:
a[4][1]
                 CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
a[4][0]
                 Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
a[3][7]
                local variables: i, j, sum are stored in the registers
a[3][1]
a[3][0]
                    for (int i = 0; i < r; i++) i:4, j:0
a[2][7]
                          for (int j = 0; j < c; j++)
 . . .
a[2][1]
                                    sum += a[i][j];
               Miss
a[2][0]
a[1][7]
a[1][1]
                Set &
a[1][0]
                        a[4][0], a[4][1] ... a[4][7]
                                                    a[2][0], a[2][1] ... a[2][7]
a[0][7]
                Set 1
                        a[1][0], a[1][1] ... a[1][7]
                                                    a[3][0], a[3][1] ... a[3][7]
a[0][1]
a[0][0]
                                        CPU Cache
Memory
```

```
a[5][7]
               Simple Example
a[5][1]
a[5][0]
a[4][7]
             Example:
a[4][1]
               CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
a[4][0]
              Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
a[3][7]
              local variables: i, j, sum are stored in the registers
a[3][1]
a[3][0]
                  a[2][7]
                        for (int j = 0; j < c; j++)
a[2][1]
                                 sum += a[i][j];
            Hit
a[2][0]
a[1][7]
a[1][1]
              Set &
a[1][0]
                      a[4][0], a[4][1] ... a[4][7]
                                                a[2][0], a[2][1] ... a[2][7]
a[0][7]
              Set 1
                      a[1][0], a[1][1] ... a[1][7]
                                                a[3][0], a[3][1] ... a[3][7]
a[0][1]
a[0][0]
```

Memory

**CPU Cache** 

a[5][7] a[5][1] a[5][0] a[4][7] a[4][1] a[4][0] a[3][7] a[3][1] a[3][0] a[2][7] . . . a[2][1] a[2][0] a[1][7] a[1][1] a[1][0] a[0][7] a[0][1] a[0][0]

Memory

# **Simple Example**

- CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
- Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
- local variables: i, j, sum are stored in the registers

```
for (int i = 0; i < r; i++) i:4, j:7
for (int j = 0; j < c; j++)
sum += a[i][j];
```

```
      Set 0
      a[4][0], a[4][1] ... a[4][7]
      a[2][0], a[2][1] ... a[2][7]

      Set 1
      a[1][0], a[1][1] ... a[1][7]
      a[3][0], a[3][1] ... a[3][7]
```

**CPU Cache** 

```
a[5][7]
a[5][1]
a[5][0]
a[4][7]
a[4][1]
a[4][0]
a[3][7]
a[3][1]
a[3][0]
a[2][7]
  . . .
a[2][1]
a[2][0]
a[1][7]
a[1][1]
a[1][0]
a[0][7]
a[0][1]
a[0][0]
```

Memory

## **Simple Example**

- CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
- Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
- local variables: i, j, sum are stored in the registers

```
      Set 0
      a[4][0], a[4][1] ... a[4][7]
      a[2][0], a[2][1] ... a[2][7]

      Set 1
      a[5][0], a[5][1] ... a[5][7]
      a[3][0], a[3][1] ... a[3][7]
```

**CPU Cache** 

```
a[5][7]
a[5][1]
a[5][0]
a[4][7]
a[4][1]
a[4][0]
a[3][7]
a[3][1]
a[3][0]
a[2][7]
  . . .
a[2][1]
a[2][0]
a[1][7]
a[1][1]
a[1][0]
a[0][7]
a[0][1]
a[0][0]
```

Memory

# **Simple Example**

- CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
- Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
- local variables: i, j, sum are stored in the registers

```
for (int i = 0; i < r; i++) i:5, j:1

for (int j = 0; j < c; j++)

sum += a[i][j];
```

```
      Set 0
      a[4][0], a[4][1] ... a[4][7]
      a[2][0], a[2][1] ... a[2][7]

      Set 1
      a[5][0], a[5][1] ... a[5][7]
      a[3][0], a[3][1] ... a[3][7]
```

**CPU Cache** 

a[5][7] a[5][1] a[5][0] a[4][7] a[4][1] a[4][0] a[3][7] a[3][1] a[3][0] a[2][7] . . . a[2][1] a[2][0] a[1][7] a[1][1] a[1][0] a[0][7] a[0][1] a[0][0]

Memory

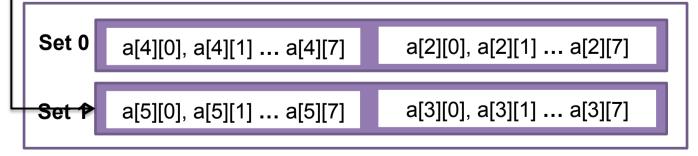
## **Simple Example**

- CPU Cache: 2-way associative, 2 sets, 64 bytes cache line
- Array: int64 a[6][8]; address of a[0][0] is 64-byte-aligned
- local variables: i, j, sum are stored in the registers

```
for (int i = 0; i < r; i++) i:5, j:7

for (int j = 0; j < c; j++)

sum += a[i][j];
```



**CPU Cache** 

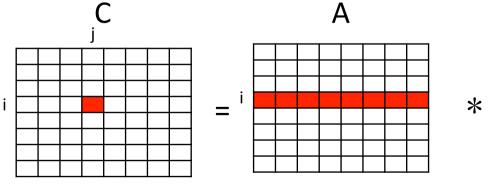
Cache: 2-way, 2 sets, 64B cacheline

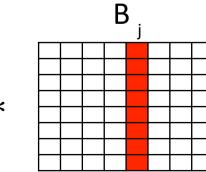
Matrix A, B, C: double[8][8]

1st elements of A,B,C, are 64B-aligned

```
C = A * B \rightarrow C[i,j] = A[i:] \cdot B[:j]
```

```
for (int i=0; i < N; i++) {
  for (int j=0; j < N; j++) {
   for (int k=0; k < N; k++)
        C[i][j] += A[i][k] * B[k][j];
  }
}</pre>
```





### **Matrix Multiplication**

Cache: 2-way, 2 sets, 64B cacheline Matrix A, B, C: double[8][8] 1st elements of A,B,C, are 64B-aligned

```
for (int i=0; i < N; i++) {
  for (int j=0; j < N; j++) {
   for (int k=0; k < N; k++)
        C[i][j] += A[i][k] * B[k][j];
  }
}
MM—ijk</pre>
```

Which one is cache friendly? Which one is worst?

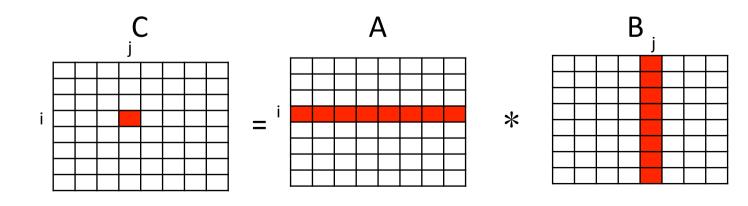
```
for (int i=0; i < N; i++) {
  for (int k=0; k < N; k++) {
    for (int j=0; j < N; j++)
     C[i][j] += A[i][k] * B[k][j];
          MM-iki
for (int k=0; k < N; k++) {
  for (int j=0; j < N; j++) {
    for (int i=0; i < N; i++)
     C[i][j] += A[i][k] * B[k][j];
          MM-kji
```

Cache: 2-way, 2 sets, 64B cacheline

Matrix A, B, C: double[8][8]

1st elements of A,B,C, are 64B-aligned

```
C = A * B → C[i,j] = A[i:] • B[:j]
for (int i=0; i < N; i++) {
  for (int j=0; j < N; j++) {
   for (int k=0; k < N; k++)
        C[i][j] += A[i][k] * B[k][j];
  }
}</pre>
```



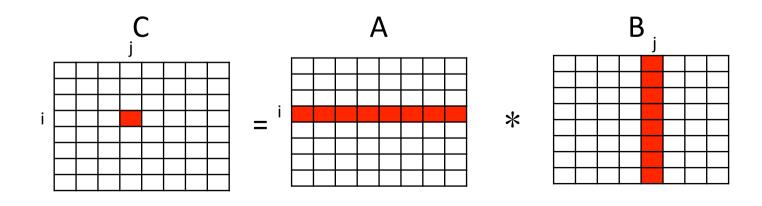
Cache: 2-way, 2 sets, 64B cacheline

Matrix A, B, C: double[8][8]

1st elements of A,B,C, are 64B-aligned

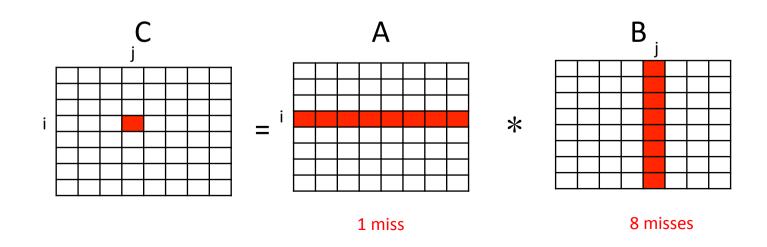
```
C = A * B \rightarrow C[i,j] = A[i:] • B[:j]

for (int i=0; i < N; i++) {
   for (int j=0; j < N; j++) {
     for (int k=0; k < N; k++)
        C[i][j] += A[i][k] * B[k][j];
   }
}</pre>
```



Cache: 2-way, 2 sets, 64B cacheline Matrix A, B, C: double[8][8] 1st elements of A,B,C, are 64B-aligned

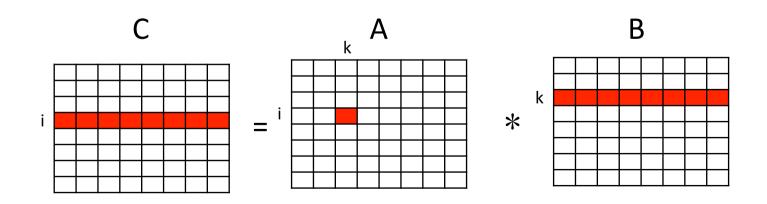
```
C = A * B → C[i,j] = A[i:] • B[:j]
for (int i=0; i < N; i++) {
  for (int j=0; j < N; j++) {
    for (int k=0; k < N; k++)
        C[i][j] += A[i][k] * B[k][j];
  }
}</pre>
```



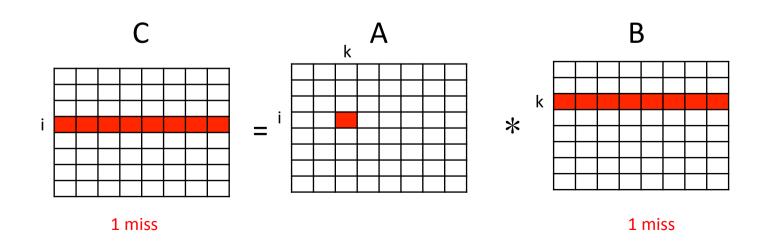
Cache: 2-way, 2 sets, 64B cacheline

Matrix A, B, C: double[8][8]

1st elements of A,B,C, are 64B-aligned



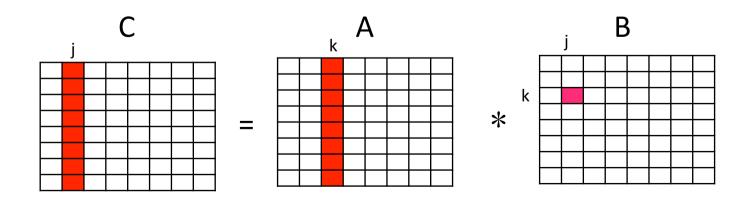
Cache: 2-way, 2 sets, 64B cacheline Matrix A, B, C: double[8][8] 1st elements of A,B,C, are 64B-aligned



Cache: 2-way, 2 sets, 64B cacheline

Matrix A, B, C: double[8][8]

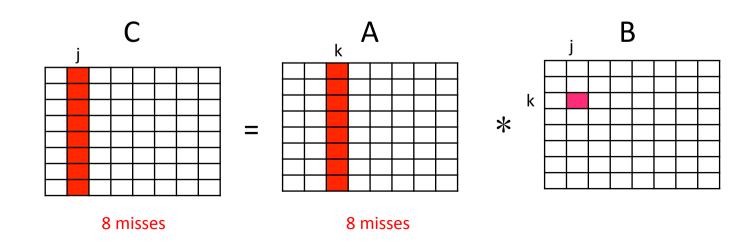
1<sup>st</sup> elements of A,B,C, are 64B-aligned



Cache: 2-way, 2 sets, 64B cacheline

Matrix A, B, C: double[8][8]

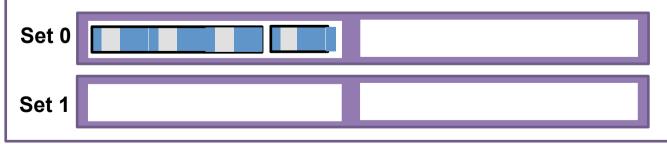
1<sup>st</sup> elements of A,B,C, are 64B-aligned



```
typedef struct {
typedef struct {
                         int64 t id;
  char g;
  int64_t id;
                         int64_t id1;
                                              for(int i = 0; i < N; i++) {
                                                     a[i].id = i;
                         int64 t id2;
  char g1;
                                                     a[i].id1 = i+1;
                         char g;
  int64 t id1;
                                                     a[i].id2 = i+2;
  char g2;
                         char g1;
                                                     a[i].g2 = 'y';
                                                     a[i].g1 = 'e';
                         char g2;
  int64 t id2;
                                                     a[i].g = 's';
                       } info;
} info;
```

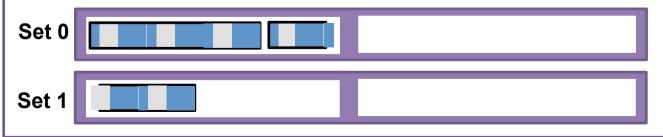
```
typedef struct {
                         typedef struct {
                           int64 t id;
  char g;
  int64_t id;
                           int64_t id1;
                                                 for(int i = 0; i < N; i++) {
                                                         a[i].id = i;
                           int64 t id2;
  char g1;
                                                         a[i].id1 = i+1;
                           char g;
  int64 t id1;
                                                         a[i].id2 = i+2;
  char g2;
                           char g1;
                                                         a[i].g2 = 'y';
                           char g2;
                                                         a[i].g1 = 'e';
  int64 t id2;
                                                         a[i].g = 's';
                         } info;
} info;
        16 bytes
                  32 bytes
                               g g1 g2
```

```
typedef struct {
                               for(int i = 0; i < N; i++) {
                                        a[i].id = i;
                                        a[i].id1 = i+1;
      char g;
                                        a[i].id2 = i+2;
      int64_t id;
                                        a[i].g2 = 'y';
      char g1;
                                        a[i].g1 = 'e';
                                        a[i].g = 's';
      int64 t id1;
      char g2;
      int64 t id2;
                              CPU Cache – 2 ways, 2 sets, 64 bytes cache line
                              Array – info a[2]
                              The address of a[0] is cache line alignment
    } info;
                          16 bytes
access a[0]
                                                                    id2
```



**CPU Cache** 

```
typedef struct {
                               for(int i = 0; i < N; i++) {
                                        a[i].id = i;
                                        a[i].id1 = i+1;
      char g;
                                        a[i].id2 = i+2;
      int64_t id;
                                        a[i].g2 = 'y';
      char g1;
                                        a[i].g1 = 'e';
                                       a[i].g = 's';
      int64 t id1;
      char g2;
      int64 t id2;
                              CPU Cache – 2 ways, 2 sets, 64 bytes cache line
                              Array – info a[2]
                              The address of a[0] is cache line alignment
    } info;
                          16 bytes
access a[1]
                                                                    id2
  Set 0
```



**CPU Cache** 

```
typedef struct {
                               for(int i = 0; i < N; i++) {
                                       a[i].id = i;
    int64 t id;
                                        a[i].id1 = i+1;
    int64 t id1;
                                        a[i].id2 = i+2;
    int64 t id2;
                                        a[i].g2 = 'y';
                                       a[i].g1 = 'e';
    char g;
                                       a[i].g = 's';
    char g1;
    char g2;
                              CPU Cache – 2 ways, 2 sets, 64 bytes cache line
                              Array – info a[2]
 } info;
                              The address of a[0] is cache line alignment
                                    32 bytes
access a[0]
  Set 0
  Set 1
```

**CPU Cache** 

```
typedef struct {
                               for(int i = 0; i < N; i++) {
                                       a[i].id = i;
    int64 t id;
                                        a[i].id1 = i+1;
    int64 t id1;
                                        a[i].id2 = i+2;
    int64 t id2;
                                        a[i].g2 = 'y';
                                       a[i].g1 = 'e';
    char g;
                                       a[i].g = 's';
    char g1;
    char g2;
                              CPU Cache – 2 ways, 2 sets, 64 bytes cache line
                              Array – info a[2]
 } info;
                              The address of a[0] is cache line alignment
                                    32 bytes
access a[1]
  Set 0
  Set 1
```