COSC 458-647
Application Software Security

2’s Complement Arithmetic

(Adopted from projectLeadTheWay)

2’s Complement Arithmetic

This presentation will demonstrate

- That subtracting one number from another is the same as making one number negative and adding.
- How to create negative numbers in the binary number system.
- The 2’s Complement Process.
- How the 2’s complement process can be use to add (and subtract) binary numbers.

Negative Numbers?

- Digital electronics requires frequent addition and subtraction of numbers. You know how to design an adder, but what about a subtract-er?
- A subtract-er is not needed with the 2’s complement process. The 2’s complement process allows you to easily convert a positive number into its negative equivalent.
- Since subtracting one number from another is the same as making one number negative and adding, the need for a subtract-er circuit has been eliminated.

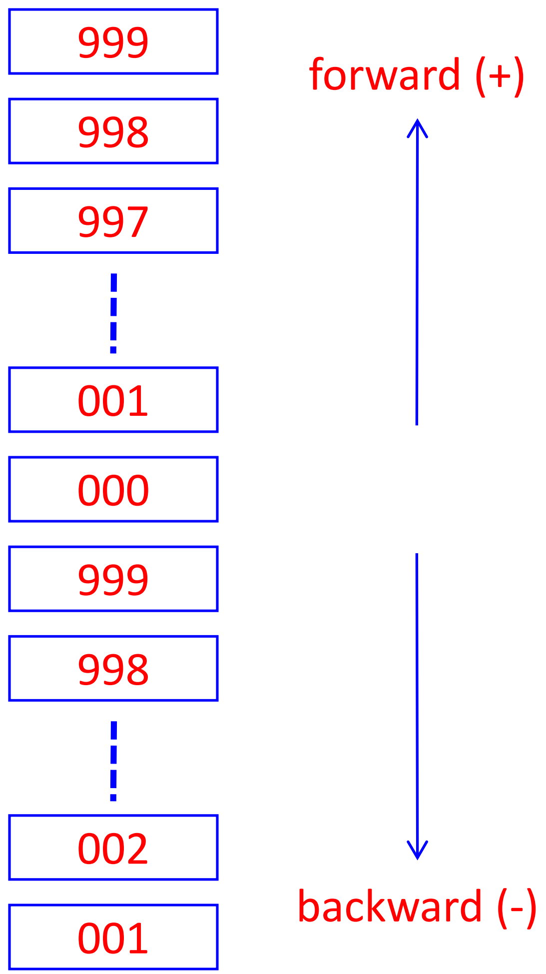
How To Create A Negative Number

- In digital electronics you cannot simply put a minus sign in front of a number to make it negative.
- You must represent a negative number in a *fixed-length* binary number system. All signed arithmetic must be performed in a *fixed-length* number system.
- A physical *fixed-length* device (usually memory) contains a fixed number of bits (usually 4-bits, 8-bits, 16-bits) to hold the number.

3-Digit Decimal Number System

A bicycle odometer with only three digits is an example of a fixed-length decimal number system.

The problem is that without a negative sign, you cannot tell a +998 from a -2 (also a 998). Did you ride forward for 998 miles or backward for 2 miles?



Negative Decimal

How do we represent negative numbers in this 3-digit decimal number system without using a sign?

→Cut the number system in half.

→Use 001 – 499 to indicate positive numbers.

→Use 500 – 999 to indicate negative numbers.

→Notice that 000 is not positive or negative.

+499	499
+498	498
+497	497
⋮	⋮
+001	001
000	000
-001	999
-002	998
⋮	⋮
-499	501
-500	500

pos(+)

neg(-)

"Odometer" Math Examples

$$\begin{array}{r} 3 \\ +\ \ 2 \\ \hline 5 \end{array} \qquad \begin{array}{r} 003 \\ +\ \ 002 \\ \hline 005 \end{array} \qquad\qquad \begin{array}{r} 6 \\ +\ (-3) \\ \hline 3 \end{array} \qquad \begin{array}{r} 006 \\ +\ \ 997 \\ \hline 1]003 \end{array}$$

Disregard Overflow

$$\begin{array}{r} (-5) \\ +\ \ 2 \\ \hline (-3) \end{array} \qquad \begin{array}{r} 995 \\ +\ \ 002 \\ \hline 997 \end{array} \qquad\qquad \begin{array}{r} (-2) \\ +\ (-3) \\ \hline (-5) \end{array} \qquad \begin{array}{r} 998 \\ +\ \ 997 \\ \hline 1]995 \end{array}$$

Disregard Overflow

It Works!

Complex Problems

- The previous examples demonstrate that this process works, but how do we *easily* convert a number into its negative equivalent?
- In the examples, converting the negative numbers into the 3-digit decimal number system was fairly easy. To convert the (-3), you simply counted backward from 1000 (i.e., 999, 998, 997).
- This process is not as easy for large numbers (e.g., -214 is 786). How did we determine this?
- To convert a large negative number, you can use the 10's Complement Process.

10’s Complement Process

The **10’s Complement** process uses base-10 (decimal) numbers. Later, when we’re working with base-2 (binary) numbers, you will see that the **2’s Complement** process works in the same way.

First, complement all of the digits in a number.

- A digit’s complement is the number you add to the digit to make it equal to the largest digit in the base (i.e., 9 for decimal). The complement of 0 is 9, 1 is 8, 2 is 7, etc.

Second, add 1.

- Without this step, our number system would have two zeroes (+0 & -0), which no number system has.

10’s Complement Examples

Example #1

```
-003
 ↓↓↓
 996
  +1
----
 997
```

Complement Digits

Add 1

Example #2

```
-214
 ↓↓↓
 785
  +1
----
 786
```

Complement Digits

Add 1

8-Bit Binary Number System

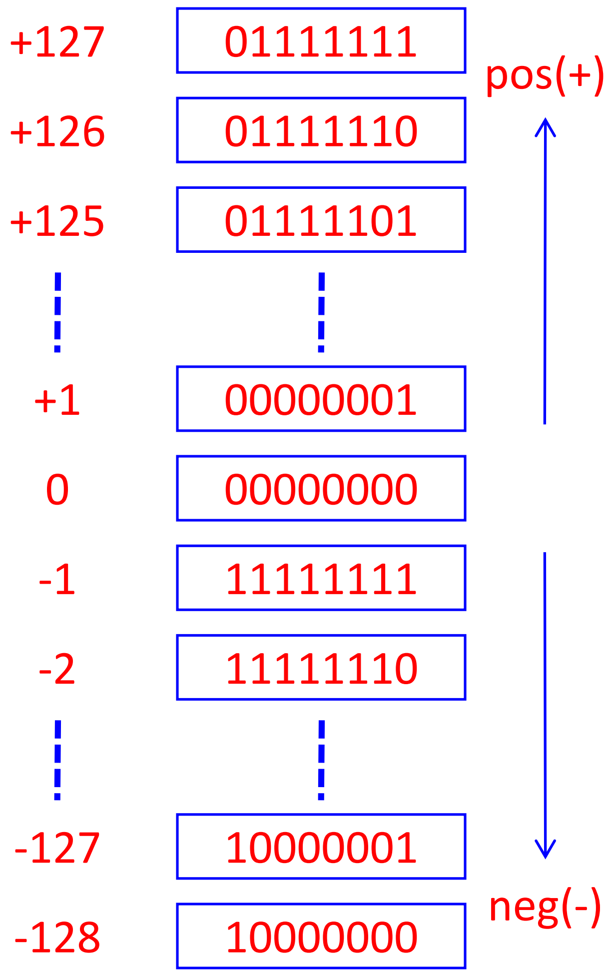
Apply what you have learned to the binary number systems. How do you represent negative numbers in this 8-bit binary system?

→Cut the number system in half.

→Use 00000001 – 01111111 to indicate positive numbers.

→Use 10000000 – 11111111 to indicate negative numbers.

→Notice that 00000000 is not positive or negative.



Sign Bit

- What did do you notice about the most significant bit of the binary numbers?
- The MSB is (0) for all positive numbers.
- The MSB is (1) for all negative numbers.
- The MSB is called the sign bit.
- In a signed number system, this allows you to instantly determine whether a number is positive or negative.

+127	01111111	pos(+)
+126	01111110	
+125	01111101	
⋮	⋮	
+1	00000001	
0	00000000	
-1	11111111	
-2	11111110	
⋮	⋮	
-127	10000001	
-128	10000000	neg(-)

2'S Complement Process

The steps in the **2's Complement** process are similar to the 10's Complement process. However, you will now use the base two.

First, complement all of the digits in a number.

- A digit's complement is the number you add to the digit to make it equal to the largest digit in the base (i.e., 1 for binary). In binary language, the complement of 0 is 1, and the complement of 1 is 0.

Second, add 1.

- Without this step, our number system would have two zeroes (+0 & -0), which no number system has.

2's Complement Examples

Example #1

```
  5 = 00000101
      ↓↓↓↓↓↓↓↓
      11111010
            +1
      --------
 -5 = 11111011
```

Complement Digits

Add 1

Example #2

```
-13 = 11110011
      ↓↓↓↓↓↓↓↓
      00001100
            +1
      --------
 13 = 00001101
```

Complement Digits

Add 1

Using The 2’s Compliment Process

Use the 2’s complement process to add together the following numbers.

$$\begin{array}{rr} & \text{POS} \\ + & \text{POS} \\ \hline & \text{POS} \end{array} \Rightarrow \begin{array}{rr} & 9 \\ + & 5 \\ \hline & 14 \end{array} \qquad \begin{array}{rr} & \text{NEG} \\ + & \text{POS} \\ \hline & \text{NEG} \end{array} \Rightarrow \begin{array}{rr} & (-9) \\ + & 5 \\ \hline & -4 \end{array}$$

$$\begin{array}{rr} & \text{POS} \\ + & \text{NEG} \\ \hline & \text{POS} \end{array} \Rightarrow \begin{array}{rr} & 9 \\ + & (-5) \\ \hline & 4 \end{array} \qquad \begin{array}{rr} & \text{NEG} \\ + & \text{NEG} \\ \hline & \text{NEG} \end{array} \Rightarrow \begin{array}{rr} & (-9) \\ + & (-5) \\ \hline & -14 \end{array}$$

POS + POS → POS Answer

If no 2's complement is needed, use regular binary addition.

$$
\begin{array}{rcr}
9 & \longrightarrow & 00001001 \\
+\ \ 5 & \longrightarrow & +\ 00000101 \\
\hline
14 & \longleftarrow & 00001110
\end{array}
$$

POS + NEG → POS Answer

Take the 2's complement of the negative number and use regular binary addition.

$$
\begin{array}{r} 9 \\ +\ (-5) \\ \hline 4 \end{array}
\quad
\begin{array}{c} \longrightarrow \\ \\ \longleftarrow \end{array}
\quad
\begin{array}{r} 00001001 \\ +\ 11111011 \\ \hline \cancel{1}]\,00000100 \end{array}
$$

8th Bit = 0: Answer is Positive
Disregard 9th Bit

$$
\left.\begin{array}{r} 00000101 \\ \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \\ 11111010 \\ +1 \\ \hline 11111011 \end{array}\right\}
$$

2's Complement Process

POS + NEG → NEG Answer

Take the 2's complement of the negative number and use regular binary addition.

$$
\begin{array}{r}
(-9) \\
+\quad 5 \\
\hline
-\boxed{4}
\end{array}
\qquad\qquad
\begin{array}{r}
11110111 \\
+\ 00000101 \\
\hline
11111100
\end{array}
$$

8th Bit = 1: Answer is Negative

To Check: Perform 2's Complement On Answer

$$
\begin{array}{r}
11111100 \\
\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \\
00000011 \\
+1 \\
\hline
00000100
\end{array}
\qquad\qquad
\begin{array}{r}
00001001 \\
\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \\
11110110 \\
+1 \\
\hline
11110111
\end{array}
$$

2's Complement Process

NEG + NEG → NEG Answer

Take the 2's complement of both negative numbers and use regular binary addition.

$$\begin{array}{rcr} (-9) & \longrightarrow & 11110111 \\ +\ (-5) & \longrightarrow & +\ 11111011 \\ \hline -\boxed{14} & \longleftarrow & \cancel{1}]\,11110010 \end{array}$$

2's Complement Numbers, See Conversion Process In Previous Slides

8^{th} Bit = 1: Answer is Negative
Disregard 9^{th} Bit

To Check: Perform 2's Complement On Answer

$$\begin{array}{r} 11110010 \\ \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \\ 00001101 \\ +1 \\ \hline 00001110 \end{array}$$