CHAPTER 1 :MATRIX

A matrix is a simple mathematical structure that holds numerical information in rectangular or tabular form.
A matrix can consist of any number of complete rows and columns. The value at the intersection of a row and column is referred to as a cell, element or data item.

ADDITION AND SUBRACTION OF MATRICES

Two matrices can be added or one matrix subtracted from another only if they have identical sizes.
Addition is performed by adding together corresponding elements. Similarly for subtraction.

Examples

Let A = $\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$, B = $\begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix}$ and C = $\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 1 \end{bmatrix}$

Required

Find the following where possible and give a reason where not possible:

a. 3B – 2A

3B – 2A = 3$\begin{bmatrix} -2 & -1 \\ 3 & 2 \end{bmatrix}$ -- 2 $\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$

= $\begin{bmatrix} -6 & -3 \\ 9 & 6 \end{bmatrix}$ - $\begin{bmatrix} 4 & -2 \\ 2 & 0 \end{bmatrix}$

= $\begin{bmatrix} -6-4 & 3--2 \\ 9-2 & 6-0 \end{bmatrix}$

= $\begin{bmatrix} -10 & 5 \\ 7 & 6 \end{bmatrix}$

Let A = $\begin{bmatrix} 4 & -3 \\ 8 & 7 \end{bmatrix}$, B = $\begin{bmatrix} 3 & 4 \\ 2 & 0 \end{bmatrix}$, identify the following matrices

a. X = 3A + 2B = 3$\begin{bmatrix} 4 & -3 \\ 8 & 7 \end{bmatrix}$ + 2 $\begin{bmatrix} 3 & 4 \\ 2 & 0 \end{bmatrix}$

= $\begin{bmatrix} 12 & -9 \\ 24 & 21 \end{bmatrix}$ + $\begin{bmatrix} 6 & 8 \\ 4 & 0 \end{bmatrix}$

= $\begin{bmatrix} 12+6 & -9+8 \\ 24+4 & 21+0 \end{bmatrix}$

=$\begin{bmatrix} 18 & -1 \\ 28 & 21 \end{bmatrix}$

b. E = 3A - 2B = 3$\begin{bmatrix} 4 & -3 \\ 8 & 7 \end{bmatrix}$ + 2 $\begin{bmatrix} 3 & 4 \\ 2 & 0 \end{bmatrix}$

= $\begin{bmatrix} 12 & -9 \\ 24 & 21 \end{bmatrix}$ - $\begin{bmatrix} 6 & 8 \\ 4 & 0 \end{bmatrix}$

= $\begin{bmatrix} 12-6 & -9-8 \\ 24-4 & 21-0 \end{bmatrix}$

=$\begin{bmatrix} 6 & -17 \\ 20 & 21 \end{bmatrix}$

EXAMPLE – ADDITION OF MATRICES

Betty and Dorothy went shopping. Betty bought 3kg of meat, 2kg of sugar and 6kg of beans. Dorothy bought 4 kg of meat, 1kg of sugar and 6 kg of beans. The following week, they also went shopping again. Calculate how much items did each of them buy.

Formulation of matrices

$$\begin{array}{cc} & \begin{array}{ccc} Meat(kg) & Sugar\ (kg) & Bean(kg) \end{array} \\ \begin{array}{c} Betty \\ Dorothy \end{array} & \begin{pmatrix} 3 & 2 & 6 \\ 4 & 1 & 6 \end{pmatrix} \end{array}$$

Second trip

$$\begin{array}{cc} & \begin{array}{ccc} Meat(kg) & Sugar\ (kg) & Bean(kg) \end{array} \\ \begin{array}{c} Betty \\ Dorothy \end{array} & \begin{pmatrix} 6 & 6 & 6 \\ 0 & 5 & 3 \end{pmatrix} \end{array}$$

Addition

This can be written more briefly as below:

$$= \begin{array}{c} Betty \\ Dorothy \end{array} \begin{pmatrix} 3 & 2 & 6 \\ 4 & 1 & 6 \end{pmatrix} + \begin{pmatrix} 6 & 6 & 6 \\ 0 & 5 & 3 \end{pmatrix}$$

$$= \begin{array}{c} Betty \\ Dorothy \end{array} \begin{pmatrix} 3+6 & 2+6 & 6+6 \\ 4+0 & 1+5 & 6+3 \end{pmatrix}$$

$$= \begin{array}{c} Betty \\ Dorothy \end{array} \begin{pmatrix} 9 & 8 & 12 \\ 4 & 6 & 9 \end{pmatrix}$$

Betty bought 9kg of meat, 8 kg of sugar and 12 kg of beans while Dorothy bought 4kg of meat, 6 kg of sugar and 9 kg of beans

EXAMPLE- SUBTRACTING MATRICES

At Sacred Heart Community Day Secondary School, Form 4A and Form 4B are organizing a party. They are collecting soft drinks and packets of biscuits from the members of the classes. For the party they have collected soft drinks shown in the form of matrix below:

$$\begin{array}{cc} & \begin{array}{cc} Drinks & Biscuits \end{array} \\ \begin{array}{c} Form\ 4A \\ fORM\ 4B \end{array} & \begin{pmatrix} 14 & 7 \\ 21 & 11 \end{pmatrix} \end{array}$$

The number they need is given by the matrix below:

$$\begin{array}{cc} & \begin{array}{cc} Drinks & Biscuits \end{array} \\ \begin{array}{c} Form\ 4A \\ fORM\ 4B \end{array} & \begin{pmatrix} 24 & 16 \\ 42 & 28 \end{pmatrix} \end{array}$$

Find the number of soft drinks and biscuit that are still needed.

Using matrix,

$$\begin{array}{c} Form\ 4A \\ fORM\ 4B \end{array} = \begin{array}{c} \begin{array}{cc} Drinks & Biscuits \end{array} \\ \begin{pmatrix} 24 & 16 \\ 42 & 28 \end{pmatrix} \end{array} - \begin{array}{c} \begin{array}{cc} Drinks & Biscuits \end{array} \\ \begin{pmatrix} 14 & 7 \\ 21 & 11 \end{pmatrix} \end{array}$$

$$=\begin{pmatrix} Drinks & Biscuits \\ 24-14 & 16-7 \\ 42-21 & 28-11 \end{pmatrix}$$

$$=\begin{pmatrix} Drinks & Biscuits \\ 10 & 9 \\ 21 & 17 \end{pmatrix}$$

Form 4A still need to collect 10 soft drinks and 9 biscuits while Form 4B still need to collect 21 soft drinks and 17 biscuits.

CONDITION AND PROCEDURE FOR MATRIX MULTIPLICATION

- Two matrices can be multiplied from left to right on the condition that the number of columns in the left- hand matrix equals the number of rows in the right-hand matrix.
- Matrix multiplication is performed by each row of the left-hand matrix in turn multiplying each column of the right-hand matrix.
- When we do multiplication

a. The number of columns of the first matrix must be equal to the number of rows of the second matrix.
b. And the result will have the same number of rows as the first matrix and the same number of columns as the second matrix.

MULTIPLICATION OF A MATRIX BY A NUMBER

When we multiply a matrix by a number we multiply every element of the matrix by that number.

For example, if the weekly order is A and the double order B , the B = 2A. A matrix can be multiplied by any number, including decimals and fractions

If A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and k is a number.

Then kA = $k\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$

EXAMPLE

Every week, three customers of a Shoprite store make the same order. The storekeeper records their order in a matrix.

$$\begin{array}{c} Harry \\ Fanwell \\ Jeoffrey \end{array}\begin{pmatrix} Sugar(kg) & Salt\ (kg) & Flour\ (kg) \\ 75 & 35 & 25 \\ 50 & 10 & 0 \\ 15 & 50 & 90 \end{pmatrix}$$

One week the store is going to be closed, so the customers each ask for a double order the week before. How will the storekeeper write down this double order?

$$=2\begin{pmatrix} Sugar(kg) & Salt\ (kg) & Flour\ (kg) \\ 75 & 35 & 25 \\ 50 & 10 & 0 \\ 15 & 50 & 90 \end{pmatrix}$$

$$=\begin{pmatrix} Sugar(kg) & Salt\ (kg) & Flour\ (kg) \\ 2\ x\ 75 & 2\ x\ 35 & 2\ x\ 25 \\ 2\ x\ 50 & 2\ x\ 10 & 2\ x\ 0 \\ 2\ x\ 15 & 2\ x\ 50 & 2\ x\ 90 \end{pmatrix}$$

=

$$= \begin{array}{c} Harry \\ Fanwell \\ Jeoffrey \end{array} \begin{pmatrix} Sugar(kg) & Salt\ (kg) & Flour\ (kg) \\ 150 & 70 & 50 \\ 100 & 20 & 0 \\ 30 & 100 & 180 \end{pmatrix}$$

MATRIX X COLUMN MATRIX

Example

A shop was selling skirts for K3, 000 each and dresses for K5,000. Angela and Kelvin bought some clothes as listed in the table.

	Skirt	Dresses
Angela	3	2
Chifuniro	4	5

1. Use matrix multiplication to find the number of clothes each of them bought.

$$\begin{pmatrix} Angela \\ Chifuniro \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 3{,}000 \\ 5{,}000 \end{pmatrix} = \begin{pmatrix} 3\ x\ 3{,}000 + & 2\ x\ 5{,}000 \\ 4\ x\ 3{,}000 + & 5\ x\ 5{,}000 \end{pmatrix}$$

$$= \begin{matrix} Angela \\ Chifuniro \end{matrix} = \begin{pmatrix} K19{,}000 \\ K37{,}000 \end{pmatrix}$$

2. Given that a = $\begin{pmatrix} 3 & 0 \\ -4 & 4 \end{pmatrix}$ and b = $\begin{pmatrix} 2 & -1 \\ -1 & 0 \end{pmatrix}$. Calculate ab.

Required to find

$$\textbf{ab}= \begin{pmatrix} \mathbf{3} & \mathbf{0} \\ \mathbf{-4} & \mathbf{4} \end{pmatrix} \begin{pmatrix} \mathbf{2} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}.$$

$$=\begin{pmatrix} \mathbf{3x2 + 0x - 1} & \mathbf{3\ x - 1 + 0x0} \\ \mathbf{-4x2 + 4x - 1} & \mathbf{-4x - 1 + 4x0} \end{pmatrix}$$

$$=\begin{pmatrix} \mathbf{6 + 0} & \mathbf{-3 + 0} \\ \mathbf{-8 + -4} & \mathbf{4 + 0} \end{pmatrix}$$

$$=\begin{pmatrix} \mathbf{6} & \mathbf{-3} \\ \mathbf{-12} & \mathbf{4} \end{pmatrix}$$

3. A point T has the coordinates $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$. The matrix which transforms T into T^- is $\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$. Calculate the coordinates of $\boldsymbol{T^-}$**.**

The matrix that transform T into T^-= $\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$.

$T^{-}= \begin{pmatrix}2 & 0\\1 & 1\end{pmatrix}\begin{pmatrix}3\\2\end{pmatrix}$

$= \begin{pmatrix}2\ x\ 3+0\ x\ 2\\1\ x\ 3+1\ x\ 2\end{pmatrix}$

$=\begin{pmatrix}6+0\\3+2\end{pmatrix}$

$= \begin{pmatrix}6\\5\end{pmatrix}$

The coordinates of $T^{-}= (6,5)$

4. A and B are two matrices. If A = $\begin{pmatrix}1 & 2\\4 & 3\end{pmatrix}$, find B given that A^2= A + B.

A^2= A + B

$= \begin{pmatrix}1 & 2\\4 & 3\end{pmatrix}^2=\begin{pmatrix}1 & 2\\4 & 3\end{pmatrix}+B$

$= \begin{pmatrix}1 & 2\\4 & 3\end{pmatrix}\begin{pmatrix}1 & 2\\4 & 3\end{pmatrix} = \begin{pmatrix}1 & 2\\4 & 3\end{pmatrix}+B$

$=\begin{pmatrix}1\ x\ 1+2\ \ x\ 4 & 1\ x\ 2+2\ x\ 3\\4\ x\ 1+3\ x\ 4 & 4\ x\ 3+3\ x\ 3\end{pmatrix} = \begin{pmatrix}1 & 2\\4 & 3\end{pmatrix}+B$

$= \begin{pmatrix}1+8 & 2+6\\4+12 & 8+9\end{pmatrix}=\begin{pmatrix}1 & 2\\4 & 3\end{pmatrix}+B$

$= \begin{pmatrix}9 & 8\\16 & 17\end{pmatrix}-\begin{pmatrix}1 & 2\\4 & 3\end{pmatrix}$ = B.

$=\begin{pmatrix}9-1 & 8-2\\16-4 & 17-3\end{pmatrix}=B$

B = $\begin{pmatrix}8 & 6\\12 & 14\end{pmatrix}$

5. Given that $\begin{pmatrix}3c & c\\5 & 1\end{pmatrix}\begin{pmatrix}4\\2\end{pmatrix}=\begin{pmatrix}28\\22\end{pmatrix}$, find the value of c.

$\begin{pmatrix}3c & c\\5 & 1\end{pmatrix}\begin{pmatrix}4\\2\end{pmatrix}=\begin{pmatrix}28\\22\end{pmatrix}$

$\begin{pmatrix}3c\ x\ 4+c\ x\ 2\\5\ x\ 4+1\ x\ 2\end{pmatrix}=\begin{pmatrix}28\\22\end{pmatrix}$

$\begin{pmatrix}12c\ +2c\\20+2\end{pmatrix}=\begin{pmatrix}28\\22\end{pmatrix}$

$\begin{pmatrix}14c\\22\end{pmatrix}=\begin{pmatrix}28\\22\end{pmatrix}$

14c = 28

c = 2

6. Given that A = $\begin{pmatrix}0 & 1\\5 & 1\end{pmatrix}$ and B =$\begin{pmatrix}2 & 3\\-1 & 4\end{pmatrix}$, find A + 2B.

Required to find A+2B= $\begin{pmatrix}\mathbf{0} & \mathbf{1}\\\mathbf{5} & \mathbf{6}\end{pmatrix}$ **+ 2**$\begin{pmatrix}\mathbf{2} & \mathbf{3}\\\mathbf{-1} & \mathbf{4}\end{pmatrix}$

$$= \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{5} & \mathbf{6} \end{pmatrix} + \begin{pmatrix} \mathbf{4} & \mathbf{6} \\ \mathbf{-2} & \mathbf{8} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{0+4} & \mathbf{1+6} \\ \mathbf{5\pm 2} & \mathbf{6+8} \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 7 \\ 3 & 14 \end{pmatrix}$$

7. Calculate the values of a and b if $\begin{pmatrix} 3a & 18 \\ 4 & 6 \end{pmatrix} - \begin{pmatrix} 2a & 2 \\ 2 & -6 \end{pmatrix} = \begin{pmatrix} 4 & 16 \\ 2 & b \end{pmatrix}$

$$\begin{pmatrix} \boldsymbol{3a} & \mathbf{18} \\ \mathbf{4} & \mathbf{6} \end{pmatrix} - \begin{pmatrix} \boldsymbol{2a} & \mathbf{2} \\ \mathbf{2} & \mathbf{-6} \end{pmatrix} = \begin{pmatrix} \mathbf{4} & \mathbf{16} \\ \mathbf{2} & \boldsymbol{b} \end{pmatrix}$$

$$\begin{pmatrix} \boldsymbol{3a-2a} & \mathbf{18-2} \\ \mathbf{4-2} & \mathbf{6--6} \end{pmatrix} = \begin{pmatrix} \mathbf{4} & \mathbf{16} \\ \mathbf{2} & \boldsymbol{b} \end{pmatrix}$$

$$\begin{pmatrix} \boldsymbol{a} & \mathbf{16} \\ \mathbf{2} & \mathbf{12} \end{pmatrix} = \begin{pmatrix} \mathbf{4} & \mathbf{16} \\ \mathbf{2} & \boldsymbol{b} \end{pmatrix}$$

a. = 4

b. =12

8. Find the values of x and y in the following matrix equation.

$$\begin{pmatrix} 6 & 3 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ -4 \end{pmatrix}$$

$$\begin{aligned} 6xx + 3xy &= 12 \\ 2xx \pm 3xy &= -4 \end{aligned}$$

6x+3y=12 (1)

2x -3y =-4 (2) (Add both sides)

8x = 8 (divide both sides by 4)

x=1

Substitute 4 for x in equation (1)

6(1) + 3y = 12

6 +3y = 12

3y = 6 (divide both sides by 3)

Y = 2

The values of x = 1 and the value of y = 2

9. Given that A $= \begin{pmatrix} 2 & 6 \\ 1 & -4 \end{pmatrix}$, B $= \begin{pmatrix} -3 & 2 \\ 0 & -5 \end{pmatrix}$, and C $= \begin{pmatrix} -1 & 8 \\ -5 & 7 \end{pmatrix}$, simplify $\frac{1}{4}(A - B + C)$.

$$\frac{1}{4}(A - B + C) = \frac{\mathbf{1}}{\mathbf{4}}\left(\begin{pmatrix} \mathbf{2} & \mathbf{6} \\ \mathbf{1} & \mathbf{-4} \end{pmatrix} - \begin{pmatrix} \mathbf{-3} & \mathbf{2} \\ \mathbf{0} & \mathbf{-5} \end{pmatrix} + \begin{bmatrix} \mathbf{-1} & \mathbf{8} \\ \mathbf{-5} & \mathbf{7} \end{bmatrix}\right)$$

$$=\frac{1}{4}\begin{pmatrix} 2--3 & 6-2 \\ 1-0 & -4--5 \end{pmatrix}+\begin{pmatrix} -1 & 8 \\ -5 & 7 \end{pmatrix}$$
$$=\frac{1}{4}\begin{pmatrix} 5 & 4 \\ 1 & 1 \end{pmatrix}+\begin{pmatrix} -1 & 8 \\ -5 & 7 \end{pmatrix}$$
$$=\frac{1}{4}\begin{pmatrix} 5\pm 1 & 4+8 \\ 1\pm 5 & 1+7 \end{pmatrix}$$
$$=\frac{1}{4}\begin{pmatrix} 4 & 12 \\ -4 & 8 \end{pmatrix}$$
$$=\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$$

10. Show that $\begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 5 & -1 \end{pmatrix}+\begin{pmatrix} 8 & -12 \\ -8 & -1 \end{pmatrix}$ is a zero matrix.

$$=\begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 5 & -1 \end{pmatrix}+\begin{pmatrix} 8 & -12 \\ -8 & -1 \end{pmatrix}$$
$$=\begin{pmatrix} 4x-2+0x5 & 4x3+0x-2 \\ 1x-2+2x5 & 1x3+2x-1 \end{pmatrix}+\begin{pmatrix} 8 & -12 \\ -8 & -1 \end{pmatrix}$$
$$=\begin{pmatrix} -8+0 & 12+0 \\ -2+10 & 3\pm 2 \end{pmatrix}+\begin{pmatrix} 8 & -12 \\ -8 & -1 \end{pmatrix}$$
$$=\begin{pmatrix} -8 & 12 \\ 8 & 1 \end{pmatrix}+\begin{pmatrix} 8 & -12 \\ -8 & -1 \end{pmatrix}$$
$$=\begin{pmatrix} -8+8 & 12-12 \\ 8-8 & 1-1 \end{pmatrix}$$
$$=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Indeed $\begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 5 & -1 \end{pmatrix}+\begin{pmatrix} 8 & -12 \\ -8 & -1 \end{pmatrix}$ is zero matrix

EXERCISE

1. A and B are two matrices. If A = $\begin{pmatrix} 0 & 1 \\ 3 & 2 \end{pmatrix}$, find B given that A^2 = A + B.
2. Given that A =$\begin{pmatrix} 3 & -1 \\ 1 & 0 \end{pmatrix}$ and B =$\begin{pmatrix} 0 & 1 \\ 2 & -5 \end{pmatrix}$, find 2A$-B^2$.
3. Given that matrix P =$\begin{pmatrix} 7 & 5 \\ 2 & 4 \end{pmatrix}$ and Q =$\begin{pmatrix} 3 & 10 \\ 5 & 1 \end{pmatrix}$, find PQ.
4. Given that matrix P = $\begin{pmatrix} 3 & 4 \\ 1 & -2 \end{pmatrix}$, Q = $\begin{pmatrix} 1 & 3 \\ 6 & 1 \end{pmatrix}$ and R = $\begin{pmatrix} 2 & 0 \\ -1 & 4 \end{pmatrix}$, find 3$(Q-PR)$.
5. T and R are two matrices. Given that T =$\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$and R = $\begin{pmatrix} 0 & 3 \\ -1 & 1 \end{pmatrix}$, find 3R$-T^2$.
6. Given that A = $\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$ and B = $\begin{pmatrix} 0 & -10 \\ 8 & 14 \end{pmatrix}$. Calculate a + $^1/_2\,B$.
7. Given that M = $\begin{pmatrix} 3 & 0 \\ 1 & -2 \end{pmatrix}$, N = $\begin{pmatrix} -1 & 2 \\ 0 & k \end{pmatrix}$ and MN = $\begin{pmatrix} -3 & 6 \\ -1 & -1 \end{pmatrix}$, find the value of k.
8. Given that M = $\begin{pmatrix} a & d \\ 1 & 2 \end{pmatrix}$ and N = $\begin{pmatrix} 5 & 2 \\ 1 & 3 \end{pmatrix}$, find MN.
9. Blouses and skirts for school uniform were selling at K500 and K250 respectively. Phiri family bought two blouses and four shirts, while Mwale family bought three blouses and six shirts for their family.

Present this information in two matrices. Using matrix multiplication, calculate the amount of money each family spent on the clothes.

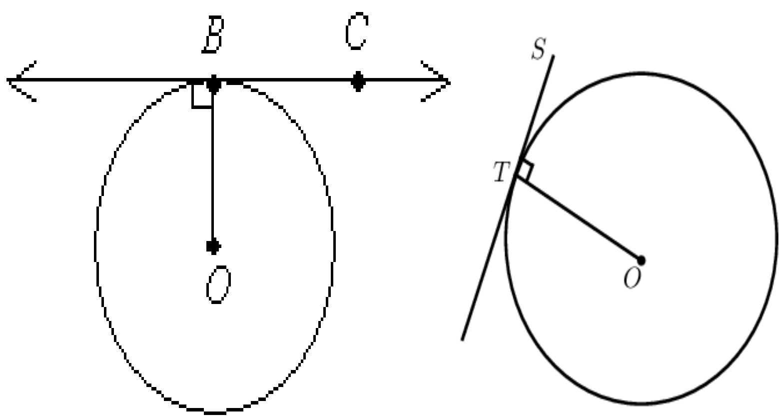
10. Given that $\begin{pmatrix} 6 & 8 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} w & -4 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find the value of w.

11. Given that the matrix V = $\begin{pmatrix} 3 & 5 \\ -1 & 1 \end{pmatrix}$ and W = $\begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}$. Find VW-V.

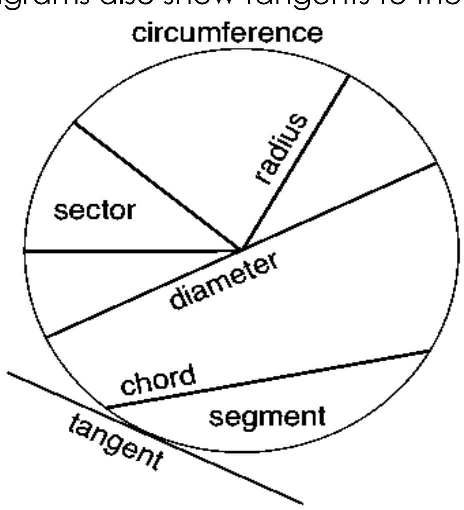
CHAPTER 2: CIRCLE: TANGENT PROPERTIES

TANGENT PROPERTIES

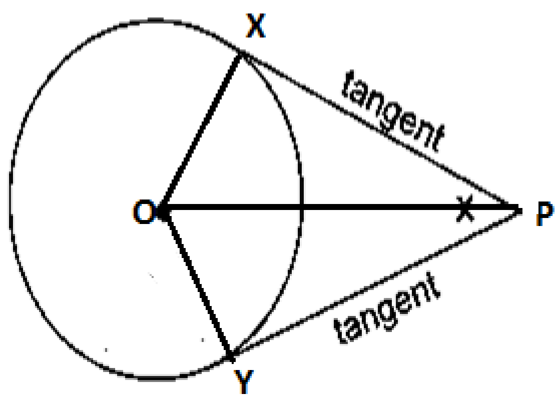
- A tangent is a straight line that touches the circumference at only one point.
- In other words a tangent is a line that touches a circle ason only one point. A tangent is perpendicular to the radius drawn from from the point of the contact to the centre of the circle.
- The point of tangency is where a tangent line touches the circle.



- In the above diagram, the line containing the points B and C is a tangent to the circle.Itouches the circle at B and is perpendicular to the ridus OB. Point B is called the point of tangency.
- The following diagrams also show tangents to the circle.



THE TWO TANGENTS FROM A POINT TO A CIRCLE ARE EQUAL IN LENGTH.



We can prove this by showing triangles PXO and PYO in the diagram above are congruent.

In the diagram, OX = OY (radii)

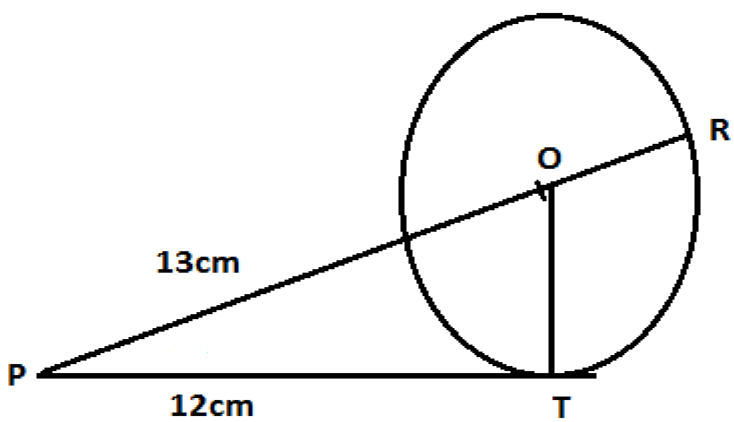
Angle PXO = angle PYO (radii perpendicular to tangents)

PO is common to both triangles

So triangles PXO and PYO are congruent (SAS), and PX = PY.

Example

P is 13 cm from the centre O of the circle. PT is a tangent to the circle and is 12cm long.Angle OPT = 30°.



Find

a. OT
b. Angle POT
c. Angle OTR

a. Triangle POT is right-angled at T, so
$OT^2 = OP^2 - PT^2$ **(Pythagoras'theorem)**

= $13^2 - 12^2$

= 169-144

= 25

OT = 5CM

b. **Angle POT = $90^\circ - 30^o$ = 60° (3rd angle in triangle)**

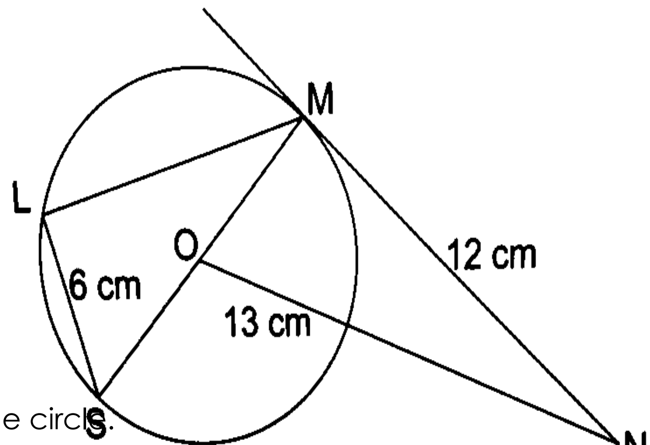
c. **Angle OTR = Angle ORT (angles of isosceles triangle)**

= $\frac{1}{2}$ x 60°

= **30° (base angles of isosceles)**

EXAMPLE

In figure below, MN is a tagent at M to the circle, centre O. ON = 13cm, NM =12cm and LS =6cm.



Calculate

a. The diameter of the circle.

b. The length of chord LM.

a. OM is perpendicular to MN (Tangent is perpendicular to radius)

Therefore, angle OMN = = 90°

Therefore, triangle OMN is a right –angled triangle.

In a triangle OMN

OM^2= MN^2 + OM^2 (Pythagoras theorem)

13^2= 12^2 + OM^2

OM^2 = 13^2- 12^2

OM^2 = 25

OM = $\sqrt{25}$

= 5cm (radius)

Therefore, diameter SM = 2 x OM

= 2 x 5cm

= 10cm

b. Angle SLM = 90° (Angle in a semi-circle)

Therefore, triangle LMS is a right-angled triangle.

In triangle LMS

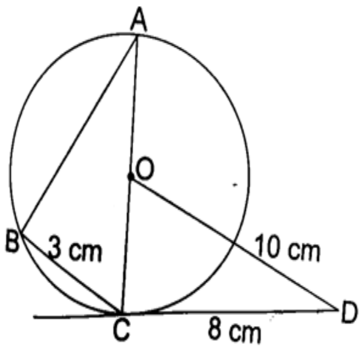
SM^2= LM^2 + LS^2

10^2= LM^2 + 6^2

LM^2 = 10^2- 6^2

LM^2 =64
LM = $\sqrt{64}$
LM = 8cm

In figure 4 below, DC is a tangent at C to the circle ABC, centre O. OD=10cm, DC=8cm and BC=3cm.

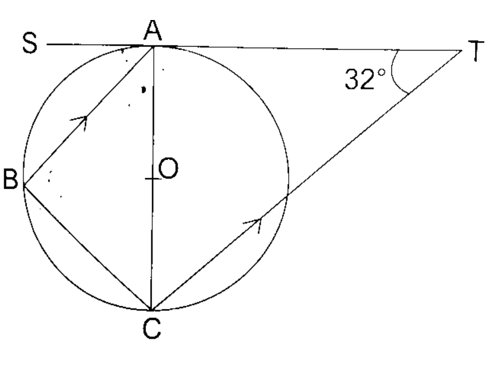


Find the length of the chord AB.

Angle OCD = 90^o (Radius perpendicular to tangent)
In triangle OCD
$OC^2 = 10^2 - 8^2$ **(Pythagoras)**
$OC^2 = 100^2 - 64^2$
$OC^2 = 36$
OC = $\sqrt{36}$
Angle ABC = 90^o**(Angle in semi-circle)**
$AB^2 = AC^2 - BC^2$ **(Pythagoras)**
$AC = 2\ x\ 6cm$
=12cm (Diameter)
$AB^2 = 12^2 - 3^2$ **(Pythagoras)**
$AB^2 = 144^2 - 9^2$ **(Pythagoras)**
$AB^2 = 135$
AB = $\sqrt{135}$
AB = 11.62cm.

Example

Figure below shows a tangent to a circle ABC with centre O. Line CT is parallel to BA and angle ATC = 32^o.



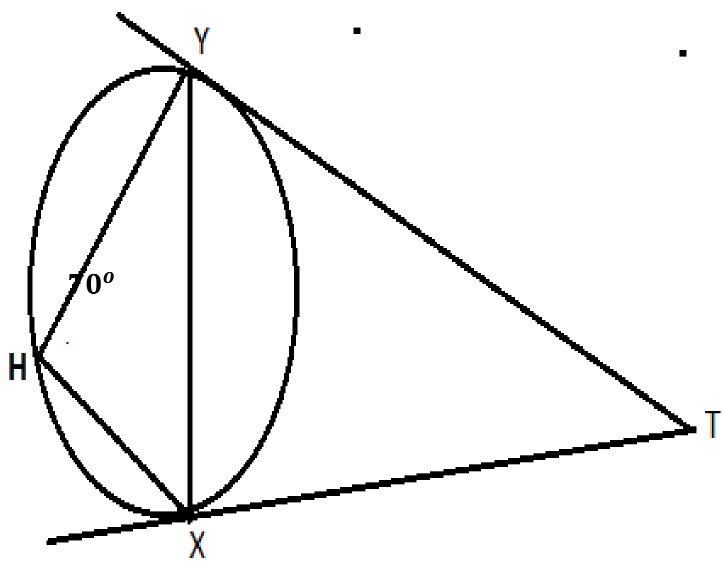
Calculate angle ACB.

SOLUTION

Since TA is a tangent to a circle centre O, OA (radius) is perpendicular to AT at a point of contact
Angle CAT = 90^o (OA is perpendiclar to TA)
Angle ACT = $180^o - (90^o + 32^o)$ (Angle sum of triangle)
= 54^o
Angle TCA = CAB = 54^o (Alternate angles)
Angle ABC = 90^o (angle in the semi-circle or angls in the alternate segment)
∴ Angle ACB = = $180^o - (90^o + 54^o)$ (Angle sum of triangle)
= $\mathbf{36^o}$

EXAMPLE

In figure below, TX and TY are tangents to the circle XHY at X and Y.



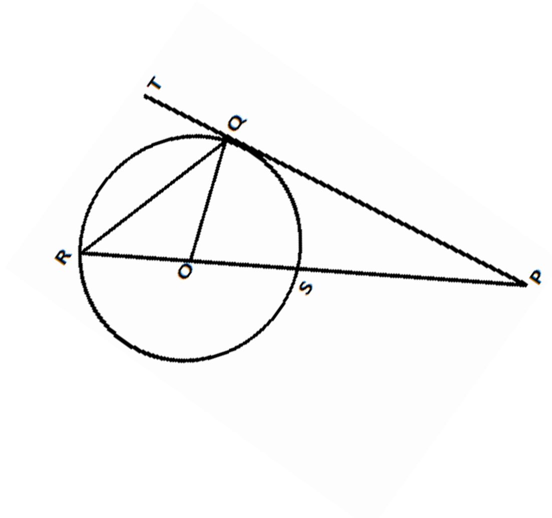
If angle XHY = 70^o, calculate angle XTY.

SOLUTION

TY = TY (Tangents from the same exterior point)
Therefore, triangle TXY is isosceles
Angle TXY = 70^o (Angles in the alternate segment are equal).
AngleTYX = TXY = 70^o (base angles of isosceles triangle)
Therefore, angle XTY = 180^o-$(70^o + 70^o)$ (Angle sum of triangle)
= 40^o

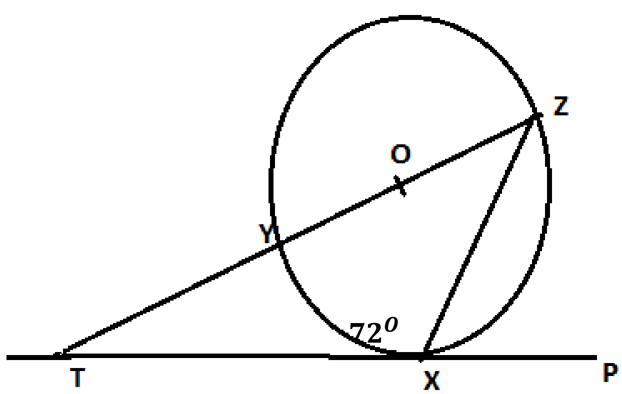
Example

Figure below shows a circle **QRS** with centre **O**. **PQT** is a tangent to the circle at **Q** and **PSOR** is a straight line.



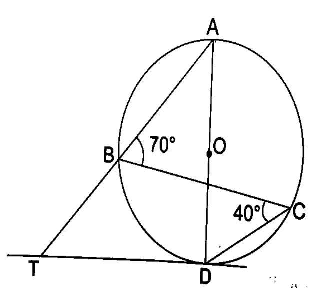
If angle RQO = 28^o, calculate angle RPQ.

In figure below, XYZ is a circle with centre O. TXP is a tangent to the circle at X . The diameter ZY produced meets the tangenbt at T.



If angle ZXP = 72^0, calculate the value of the angle XTY.

Figure is a circle ABDC centre O. TD is a tangent, TBA is a straight line and AD is a diameter.



If angle ABC= = 70^O, and angle BCD = = 40^O, CALCULATE

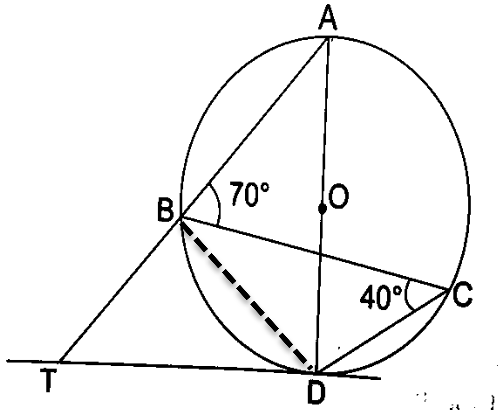
a. Angle DBC
b. Angle ATD.

Given : Circle ABDC with centre O, tangent TD, diameter AD and straight line TBA.

To calculate

a. Angle DBC

Join :BD



Angle ADC = Angle ABC (Angle in the same segment)

Therefore, angle ADC = 70^O

Angle ACD = 90^O **(Angle in the semi-circle)**

Angle ACB + angle BCD = 90 (complementary angles)

Angle ACB + 40^O= 90^O

Angle ACB = $90^O - 40^O$

= 50^O

But angle DBA = Angle ACB (angles in te same segment)

Therefore, angle DBA =50^O

Angle DBC = Angle DBA + Angle ADC

Angle DBC = 50^O+ 70^O

Angle DBC = 120^O

b. Angle ATD

Angle TAD = Angle BCD (angles in the same segment)

Angle TAD = 40^O

Angle TDA = 90^O **(AD is perpendicular to tangent TD)**

In triangle ATD:

Angle ATD + angle TDA + angle TAD = 180^o

Angle ATD + $90^o + 40^o = 180^o$

Angle ATD = $180^o - 90^o - 40^o$

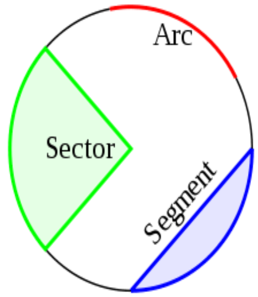
Therefore,angle ATD = 50^o

ALTENATE SEGMENT THEREOM

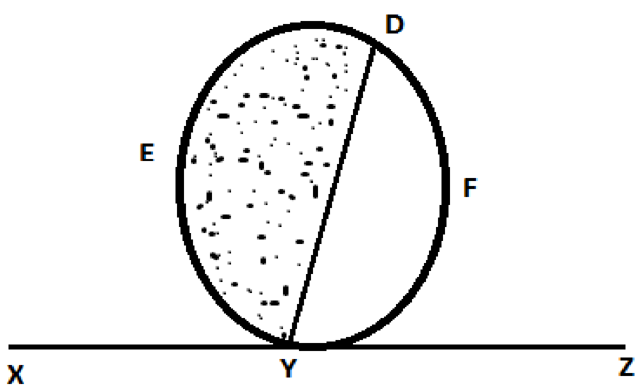
Angles in the alternate segments are equal.

SEGEMENT

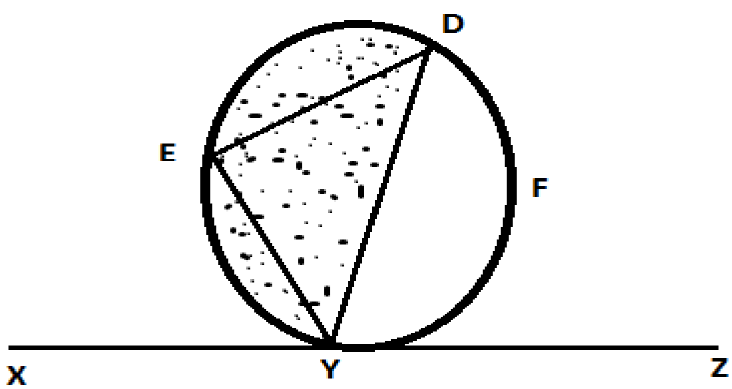
- It the space between a chord and the circumference.



In the figure segement DEY is on the opposite side of the chord DY to angle DYZ. So seggment DEY is alternate to segment DFY.

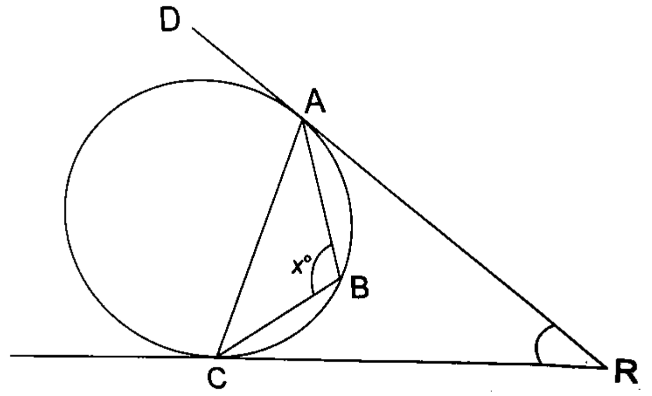


By joining ED and EY , angle DEY is said to be in the alternate segment to angle DYZ. Similarly angle DEY is in alternate segment to angle EYX.



EXAMPLE

In figure below, AR ,CR are tangents at A and C respectively to the circle ABC, angle ARC = 62° and angle ABC= x°.



Calculate the size of angle x.

AR = CR (Tangents from exterior point are equal)

Therefore, angle RAC = Angle RCA

Angle RAC = $\frac{1}{2}(180^\circ - 62^\circ)$

In triangle ABC, angle BAC = $180^\circ - 62^\circ$

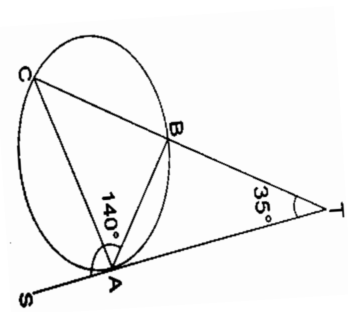
$= 59^\circ$

Angle DAC = $180^\circ - 59^\circ$ (Aadjacent angles on the straight line)

$= 121^\circ$

Angle x = Angle DAC =121° (Angles in the alternate segment)

In figure below, SAT is the tangent at A to the circle ABC and CBT is a straight line. Angle BAS =140° and angle ATC = 35°



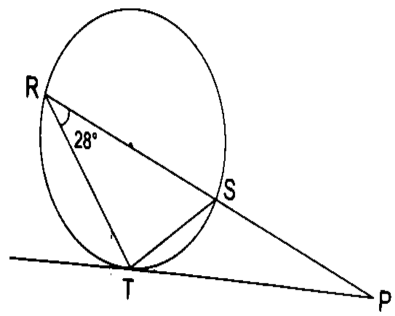
Calculate

a. Angle ABC.
b. Angle ACB.

a. Angle TAB = $180^\circ - 140^\circ$ = 40° (Adjacent angles on the straight line)
Angle ABC = 40°+ 35°(Exterior angles of TriangleTBA)
b. Angle ACB = Angle TAB = 40° (Angles in the alternate segment are equal)

EXAMPLE

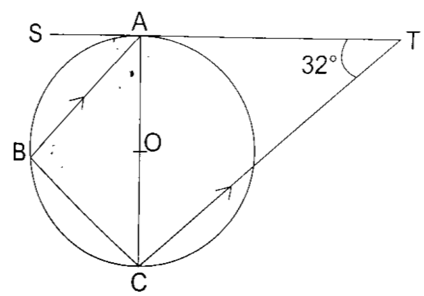
In figure below, RS is a diameter of the circle. PT is a tangent to the circle. Angle TRS = 28°.



Calculate the angles of triangle STP.

Angle RTS = 90° (Angle in the semi-circle)
Angle TRS = Angle STP = 28° (Angles in the alternate segment)
Angle TSP = $28^{\circ} + 90^{\circ} = 118^{\circ}$ (Exterior angles of Triangle TSP
Angle SPT = $180^{\circ} - (118 + 28) = 34^{\circ}$

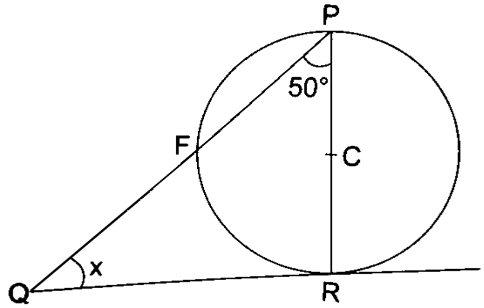
Figure shows a tangent to a circle ABC with centre O. Line CT is parallel to BA angle ATC = 32^{O}



Calculate angle ACB.

Given : Circle ABC centre O with TA tangent at A, CT//BA, angle ATC =32^{O}
To calculate angle ACB.
Angle SAB = 32^{O} **(Corrresponding angles; CT//BA)**
Angle ACB = Angle SAB (angles in the alternate segment)
Therefore, angle ACB = 32^{O}

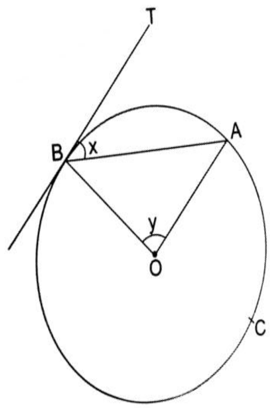
In figure 2, PR is a diameter and QR is a tangent to the circle RPF AT r. Angle P = $\mathbf{50^{O}}$.



Find the angle marked x.

Angle PRQ = 90^o (Radius perpendicular to tangent)
$50^o + 90^o + x = 180^o$ (Angle sum of angles in Triangle PQR)
x =$180^o - 50^o - 90^o$
x =40^o

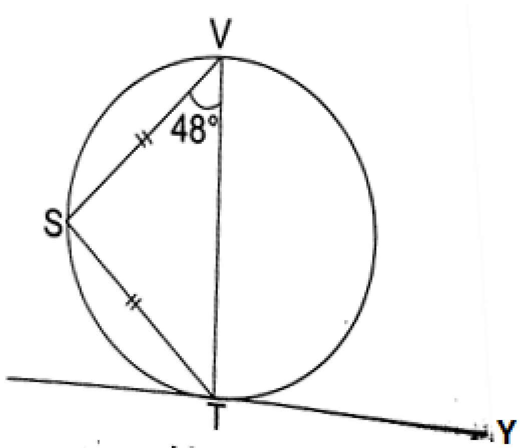
In figure 5, BT is a tangent at B to a circle ABC centre O.



Prove that angle AOB = 2 (angle ABT).

Join BC AND AC
Angle ABT =Angle ACB (angles in the alternate segment)
Y = 2 angle ACB (angle at centre is equal 2 times angle at crcumference)
Therefore Y = 2 angle ABT
Therefore
ANGLE aob = 2 times angle ABT.

In figure 1, TY is atangent to the circle TVS.

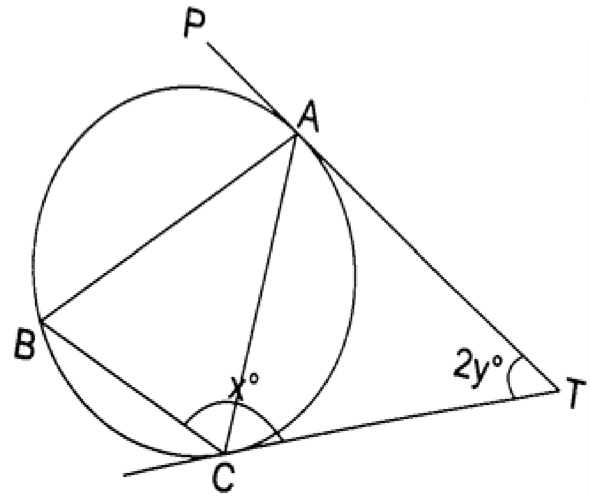


If angle SVT = $48^o$and VS = ST, calculate angle VTY.

Angle VTS = 48^o (base angles of isosceles triangle VTS)

Angle VST = $180^o - 48^o - 48^o$ (angle sum of angles in a Triangle)
Angle VST = 84^o

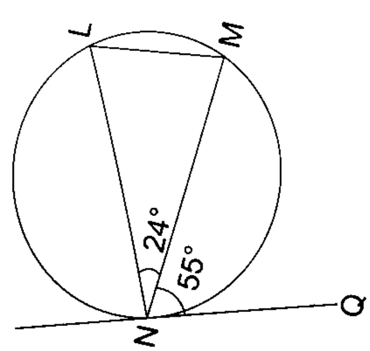
In figure 2, PAT and CT are tangents to the circle ACB.



Given that angle BCT = x^o and angle CTA = $2y^o$, express angle PAB in terms of x^o and y^o

Angle ACT = Angle CAT = $\frac{180^o - 2y^2}{2}$ (Tangents from exterior point)
= $(90 - y)^o$
Angle PAB = Angle ACB (Angles in alternate segments)
= $x^o - (90 - y)^o$
= $x^o + y^o - 90^o$

In figure 2QN is a tangent to the circle LMN at N.



If angle QNM = 55^o and angle LMN = 24^o, calculate angle NML.

Given :QN tangent to circle LMN at N, angle QNM = 55^o and angle LMN = 24^o
Required to find Angle NML.

Now, Angle NLM = Angle QNM = 55^o (ANGLES IN THE ALTERNATE SEGMENT)

Angle NLM = 55^o

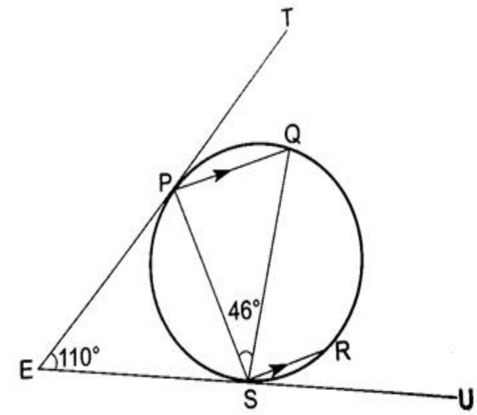
Angle NML = $180^o - (55^o + 24^o)$ (Angle sum of angles in Triangle)

= 101^o

The value of angle NML is 101^o

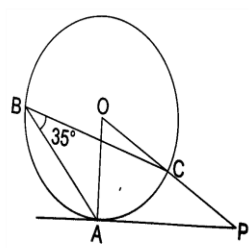
EXERCISE

1. In figure 3, ET and EU are tangents to the circle PQRS at P and S respectively. Angle TEU **=** 110^o, angle PSQ = 46^o and PQ is parallel to SR.



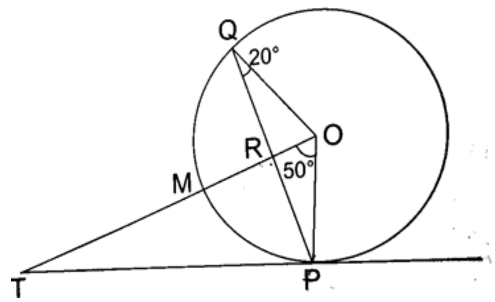
Calculate the value of angle RSU.

2. Figure 2 shows a circle ABC centre O . OCT is a straight line and AP is a tangent to the circle at A.



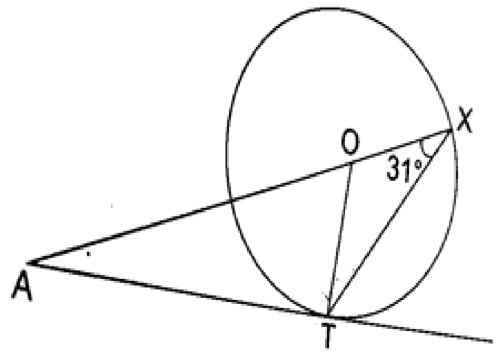
If angle ABC = 35^o, calculate the value of angle APO.

3. Figure 3 shows a circle MPQ centre O. TP is a tangent to the circle at P and TMRO is a straight line.



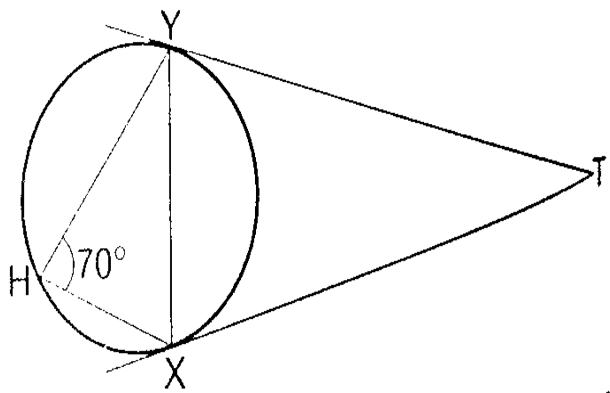
If angle RQO = 35^o and angle TOP = 50^o, show that TP =TR.

4. In figure 4, AT is a tangent to the circle centre O. Angle OXT = 31^o and AOX is a straight line.



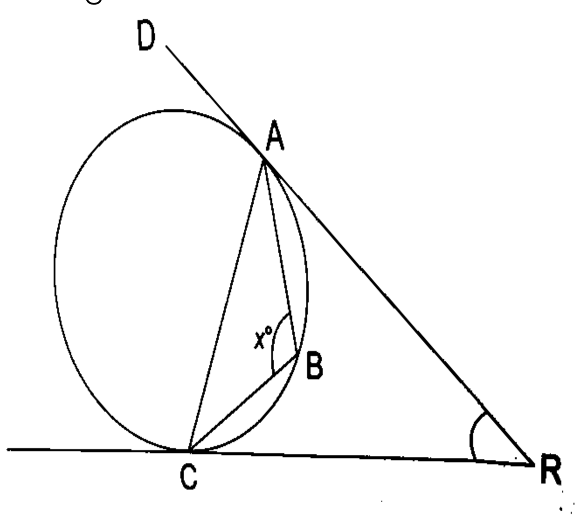
Calculate angle OAT.

5. In figure 5, TX and TY are tangents to the circle XHY at X and Y.



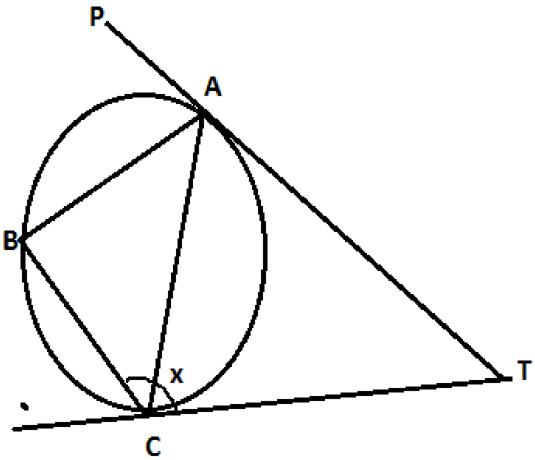
If angle XHY = 70^o, calculate angle XTY.

6. In figure 6 , AR , CR are tangents at A and C respectively to the circle ABC. Angle ARC = 62^o and angle ABC = x^o.



Calculate the size of angle x^o.

In figure below, PAT and CT are tangents to the circle ACB.



Given that angle BCT = x^o,and angle CTA= $2y^o$, express angle PAB in terms of $x^o$and y^o.

CHAPTER 3: STATISTICS

MEAN

What is the mean?

- The mean is the average of a set of numbers.
- It is found by adding up the set of numbers and then dividing the total by the number of data points in the set.

HOW TO FIND THE MEAN

The "Mean" is computed by adding all of the numbers in the data together and dividing by the number elements contained in the data set.

Mean = $\frac{\sum X}{N}$

Example:

Data Set = 2, 5, 9, 3, 5, 4, 7

Number of Elements in Data Set = 7

Mean = (2 + 5 + 9 + 7 + 5 + 4 + 3) / 7 = 5

EXAMPLES

Find the mean of 5, 7, 8 and 4.

Step1: Add up the numbers to give a total of 8 + 5+ 7 + 13

Total = 5+7+8+4

= 24

Mean = $\frac{24}{4}$ = 6

Median

The “Median” is the middle value of a set of ordered numbers. The "Median" of a data set is dependent on whether the number of elements in the data set is odd or even. First reorder the data set from the smallest to the largest then if the number of elements are odd, then the Median is the element in the middle of the data set.

If the number of elements is even, then the Median is the average of the two middle terms.

EXAMPLES: ODD NUMBER OF ELEMENTS

Data Set = 2, 5, 9, 3, 5, 4, 7

Reordered = 2, 3, 4, 5, 5, 7, 9

Median = 5

EXAMPLE: EVEN NUMBER OF ELEMENTS

Data Set = 2, 5, 9, 3, 5, 4

Reordered = 2, 3, 4, 5, 5, 9 - the middle terms are 4 and 5

Median = (4 + 5) / 2 = 4.5 - the median is the average of the two middle terms

Mode

The "Mode" for a set of data is the value that occurs most often. The "Mode" for a data set is the element that occurs the most often.

It is not uncommon for a data set to have more than one mode.

This happens when two or more elements occur with equal frequency in the data set. A data set with two modes is called bimodal.

A data set with three modes is called trimodal.

EXAMPLES: SINGLE MODE

Data Set = 2, 5, 9, 3, 5, 4, 7

Mode = 5

EXAMPLES: BIMODAL

Data Set = 2, 5, 2, 3, 5, 4, 7

Modes = 2 and 5

EXAMPLES: TRIMODAL

Data Set = 2, 5, 2, 7, 5, 4, 7

Modes = 2, 5, and 7

Range

The "Range" is the difference between the largest value and smallest value in a set of data.

First reorder the data set from smallest to largest then subtract the first element from the last element.

Example:

Data Set = 2, 5, 9, 7, 5, 4, 3

Reordered = 2, 3, 4, 5

Range = (9 - 2) = 7

Worked Example 2

Five people play golf and at one hole their scores are 3, 4, 4, 5, and 7
For these scores, find
(a) The mean
(b) The median
(c) The mode
(d) The range.

a. The numbers are already in order and the middle number is 4. So median = 4
b. The score 4 occurs most often, so,
mode = 4
c. The range is the difference between the smallest and largest numbers, in this case 3 and 7, so
range = 7 – 3 = 4

Example

There are 10 girls and 20 boys in a class. The average age of the pupils in the class is 13 years 8 months. If the average age of the girls is 13 years, find the average age of the boys.

Sum of ages of all pupils = 30 x (13 years 8Months)
$=30 \times 13\frac{2}{3}$ yearrs
= 410 years
Sum of the ages of the girls = 10 x 13 years
= 130 years
Sum of the ages of the boys= 410-130
=280 years
Average of the boys = $\frac{280}{20}$ = 14 years.

EXERCISE

1. **Find the mean, mode and median of the following set of numbers.**
   3,1,2,1,7,10,1,0,2
2. **Find the mean, median and mode of the following set of numbers.**
   9,10,14,13,13,12,11,11,10,9,7,8,8,9,9
3. **Find the median of the following set of numbers**
   4,1,7,9,1,3,8,1,7,5
4. **A bag is found to contain nails of the following sizes**

Length in cm	4	5	6	7	8	9
Number of nails	3	6	10	9	8	4

Calculate

a. The modal length
b. The mean length of the nails

5. In January 1998, maize was sold at different prices in different parts of the country. The following were some of the prices per kg.
   K5.16, K6.52, K4.44, K3.64, K6.06, K4.21, K6.08, K7.18, K5.77 and K8.33.
   Find
   a. The range of the prices
   b. The median
   c. The mean price.

STANDARD DEVIATION

To calculate standard deviation, we first calculate the mean and then find out the amount by which each value differs from this mean. This is called the deviation from the mean.

Example

Find the standard deviation of the following marks

4 6 7 10 13

Solution

Mean = $\frac{\sum x}{n}$

Mean = $\frac{4+6+7+10+13}{5}$ = $\frac{40}{4}$ =5

Marks	Deviation From Mean	Square Of Deviation
4	-4	16
6	-2	4
7	-1	1
10	2	4
13	5	25
Total	0	50

Variance = $\frac{50}{5}$

= 10

Standard deviation = $\sqrt{\frac{(x-\bar{x})^2}{n}}$

= $\sqrt{\frac{50}{5}}$

= $\sqrt{10}$

=

EXAMPLE 2

In 10 games over a season, a netball team scored the following:

17, 41,28,21,38,45,63,8,15,34

Find the mean and standard deviation of the number of goals

Mean, $\bar{x}= \frac{\sum x}{n} = \frac{17+41+28+21+38+45+63+8+15+34}{10}$

$= \frac{310}{10}$

=31

Score	Deviation from mean (31)	Square of deviation
17	-14	196
41	10	100
	-3	9
28	-10	100
21	7	49
38	14	196
45	32	1024
63	-23	529
81	-16	256
34	3	9
Total	0	2468

Total squared deviation = $\sum(x-\bar{x})^2$ **=2468**

Standard deviation = $\sqrt{\frac{\sum(x-\bar{x})^2}{n}}$

$= \sqrt{\frac{2468}{10}}$

=15.7 to 1 decimal place.

Example

The table below shows the ages of the students in a class as follows

14	15	14	16	14
14	15	17	15	18
14	15	15	16	15
16	15	14	13	15

Required

a. Construct the frequency table for the data
b. Find mean and standard deviation for the data.

solution

a. Frequency **table**

Ages(x)	Tally	Frequency	fx
13	/	1	13
14	~~////~~//	6	84
15	~~////~~////	8	120
16	///	3	48
17	/	1	17
18	1	1	18
Total		20	300

b. Mean = $\frac{\sum fx}{n}$

$= \frac{300}{20}$

c. Standard deviation

Ages(x)	Deviation	Square of deviation	Frequency	Total deviation	Total squared deviation
x	$(x-\bar{x})$		f		
13	-2	4	1	-2	4
14	-1	1	6	-6	6
15	0	0	8	0	0
16	1	1	3	3	3
17	2	4	1	2	4
18	3	9	1	3	9
Total			20	0	26

Standard deviation $= \sqrt{\frac{\sum f(x-\bar{x})^2}{n}} = \frac{26}{20}$

= 1.14 years to 2 decimal places

The table below shows ages of 5 pupils with the mean age of 12.6 years.

Age (years)	Deviation from mean	Square of deviation
10	-2.6	6.76

11	-1.6	2.56
13	0.4	0.16
14		
15	2.4	5.76
Total	0	

Copy and complete the table to calculate the variance of the ages.

SOLUTION

Age (years)	Deviation from mean	Square of deviation
10	-2.6	6.76
11	-1.6	2.56
13	0.4	0.16
14	A=1.4	A^2=1.96
15	2.4	5.76
Total	0	C=17.2

Mean age =12.6

A = 14-12.6 =1.4

$A^2 = 1.4^2 = 1.96$

C +6.76+2.56+0.16+1.96+5.76 = 17.2

Variance = $\frac{C}{Number\ of\ ages} = \frac{17.2}{7}$ = **3.44**

The table below shows the deviation(d) from the mean of marks and the frequencies(f) of the marks pupils scored in a test.

Mark	f	D	d^2	fd^2
20	1	-11	121	121
23	1	-8	64	64
26	2	-5	25	50
27	1	-4	16	16
30	3	-1	1	3
34	2	3	9	18
35	3	4	16	48
40	2	9	81	162

Using information from the table, calculate

a. Total number of pupils
b. Mean
c. Standard deviation to 3 significant figures

SOLUTION

a. $\sum f$= 1+1+2+1+3+2+3+2 =15

b. Mean = $\frac{Sum\ of\ marks}{Number\ of\ pupils}$

Mark	f	
20	1	20
23	1	23
26	2	52
27	1	27
30	3	90
34	2	68
35	3	105
40	2	80
	15	465

Mean = $\frac{465}{15}$ =31

The mean of the marks is 31

c. Standard deviation

f	fd^2
1	121
1	64
2	50
1	16
3	3
2	18
3	48
2	162
15	482

Variance = $\frac{482}{15}$ =32.1

Standard deviation =$\sqrt{\frac{\sum fd^2}{\sum f}}$ = $\frac{486}{15}$

= 5.67

CHAPTER 4: SOLVING QUADRATIC EQUATIONS

1. Given that the roots of a quadratic equation y =$x^2 + ax + b$ are -3 and 2, find the values of a and b.

If x =-3, then x +3 and if x=2, then x-2
(x+3)(x-2) $\equiv x^2 + ax + b$
$x^2 - 2x + 3x - 6 \equiv x^2 + ax + b$
$x^2 + x - 6 \equiv x^2 + ax + b$
Since $x^2 + x - 6$ and $x^2 + ax + b$ are identical. A =1 and b-6

2. Given that the roots of a quadratic equation in y are -5 or 3. Formulate the equation in the form $ay^2 + by + c = 0$

Then it is y=-5 and y=3
The factors are (y+3) and(y-3)
The quadratic equation will be
(y+5) (y-3) =a$y^2 + by + c =$
$y^2 - 3y + 5y - 15 = ay^2 + by + c$
$y^2 + 2y - 15 = ay^2 + by + c$
The quadratic equation is $y^2 + 2y - 15.$

3. **Solve the equation $2^{2a} - 5(2^a) + 4$=0**

Let 2^a =x
$x^2 - 5x + 4 = 0$
(x-4)(x-1)
Either x-4 =0 or x-1 =0
$\therefore$ **x=4 or x =1**
2^a**=4 or** $2^a = 1$
2^a**=** 2^2 **or** 2^a **=**2^o
a= 2 or a =0

Solve for x if $(2^x)^2 - 9(2^x) + 8 = 0.$

Let $2^x = m$
Substitute m for 2^x in the original equation:
$m^2 - 9m + 8 = 0$
$m^2 - m - 8m + 8$ **=0**
$m(m - 1) - 8(m - 1) =$
(m-8)(m-1) =0
$(2^x - 1)(2^x - 8) = 0$ **(substituting 2^x for m)**
Either $(2^x - 1) = 0$, 2^x=2^0, x =0
Or $(2^x - 8) = 0$, 2^x=2^3, x=3
$\therefore$ **x =0 or x =3**

EQUATIONS OF QUADRATIC FUNCTIONS

- The **general form of a quadratic function** presents the function in the form

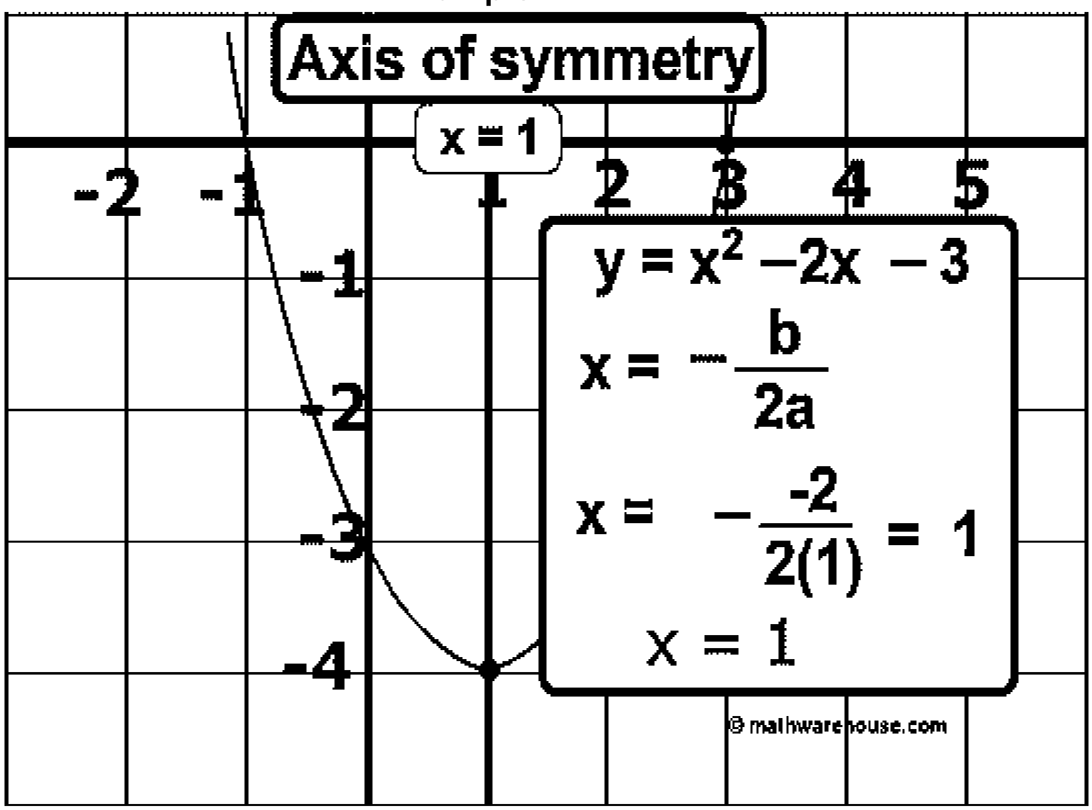
  F(x) = $ax^2 + bx + c$

  Where a, b, and c are real numbers and a $\neq 0$.
- If a > 0, the parabola opens upward.
- If a<0, the parabola opens downward.
- We can use the general form of a parabola to find the equation for the axis of symmetry.

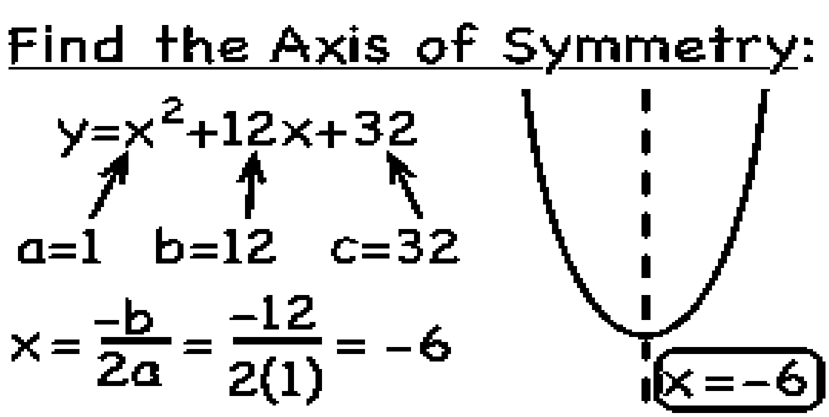
- The axis of symmetry is defined by x = $= \frac{-b}{2a}$.

  If we use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, to solve $ax^2 + bx + c = 0$ for x intercepts, or zeros, we find the value of x halfway between them is always x $= \frac{-b}{2a}$, the equation for the axis of symmetry.

Example

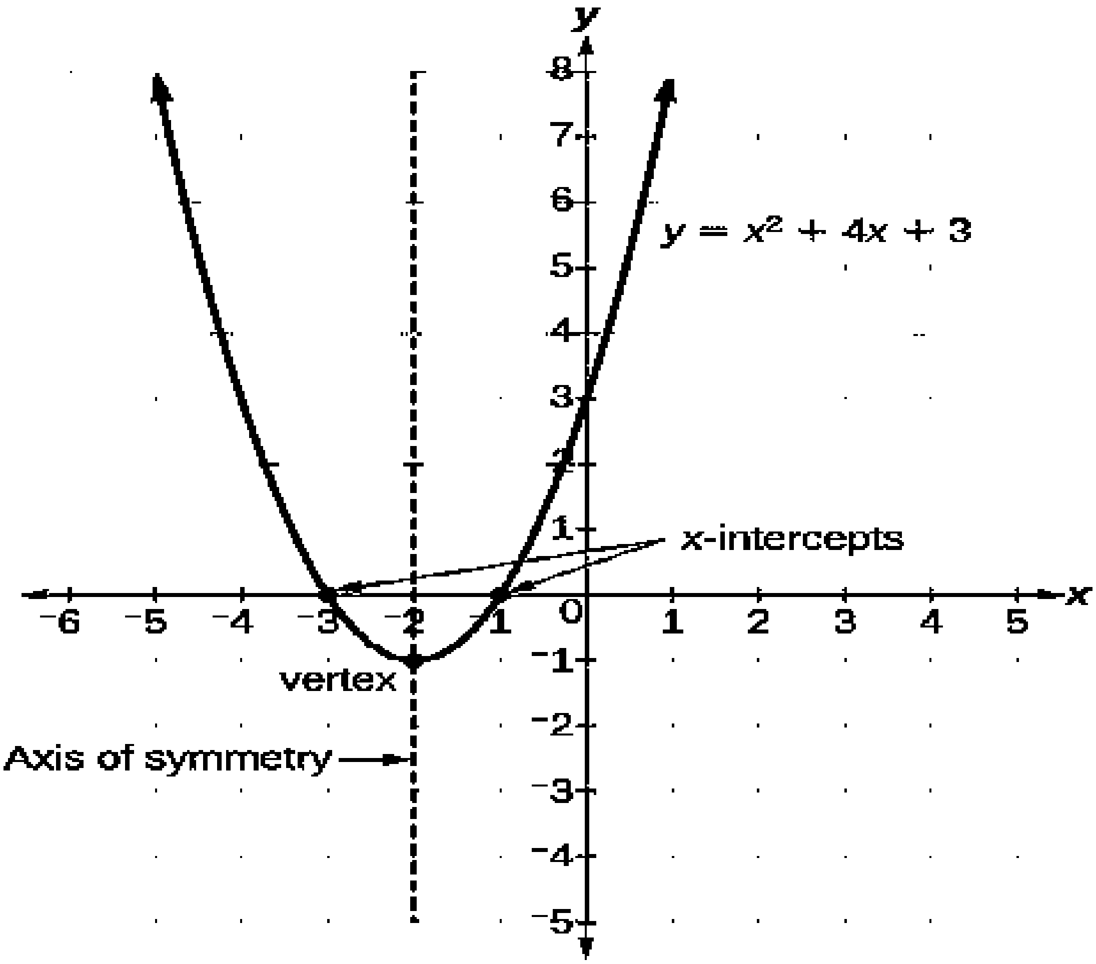


Example



Example

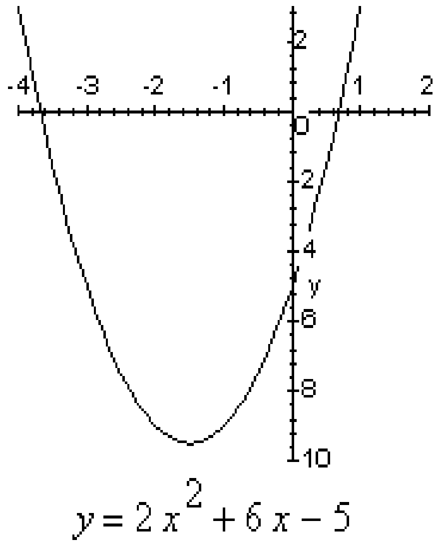
- The figure below shows the graph of the quadratic function written in general form as $y = x^2 + 4x + 3$.

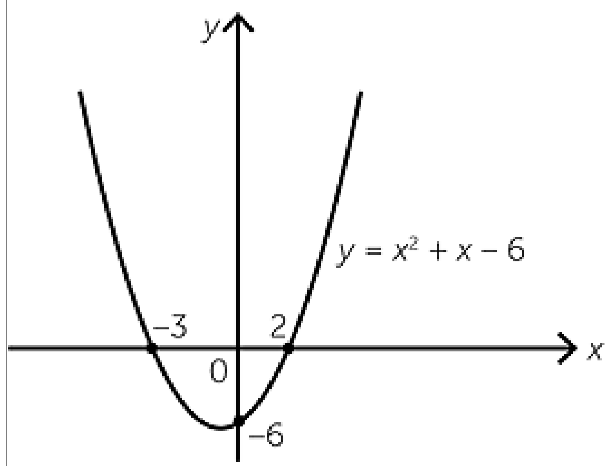


- **In this form, a= 1, b= 4 and c = 3. Because a > 0, the parabola opens upward. The axis of symmetry is x =$\frac{-4}{2(1)}$,= -2 divides the graph in half. The vertex always occurs along the axis of symmetry. For the parabola that opens upwards, the vertex occurs at the lowest point on the graph, in this instance, (-2,-1). The x-intercepts, those points are found where the parabola crosses the x-axis, occur at (-3,0) and (-1,0).**

Exercise

1. Determine the roots, y-intercept and line of symmetry for the quadratic functions by the graphs below.





Example

- Find the axis of symmetry of the graph of y = $x^2 - 6x + 5$ using the formula.

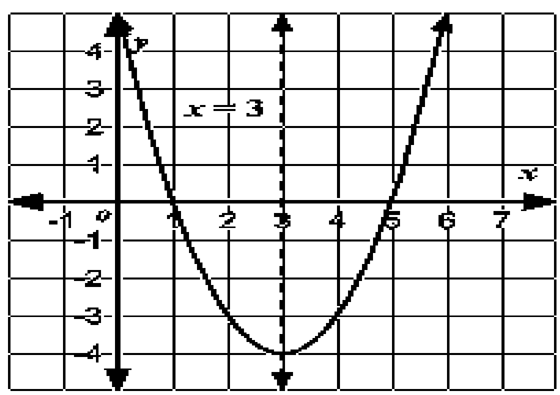
Solution

For a quadratic function in standard form$y = ax^2 + bx + c$, the axis of symmetry is a vertical line x = =$\frac{-b}{2a}$.

Here, a = 1, b= -6 and c= 5

Substitute

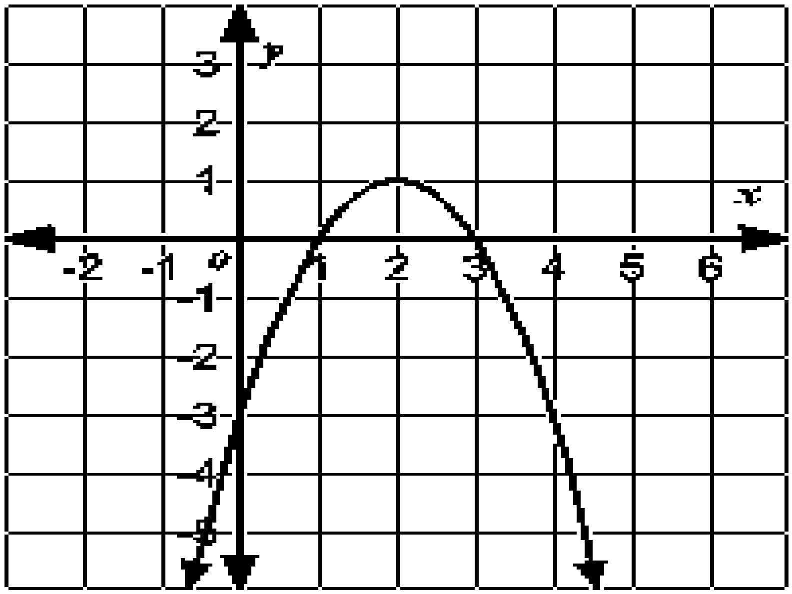
x= =$\frac{-6}{2(1)}$,= 3



Therefore, the line of symmetry is x = 3.

Example

Find the axis of symmetry of the parabola shown below.



The x-coordinate of the vertex is the equation of the axis of symmetry of the parabola. The vertex of the parabola is (2, 1). So, the axis of symmetry is the line x = 2.

SOLVING QUADRATIC EQUATIONS USING QUADRATIC FORMULA

If $\mathbf{a}\boldsymbol{x^2} + \boldsymbol{bx} + \boldsymbol{c} = 0$, then x = $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Where **a** is the coefficient of x^2

b is the coefficient of x

c is the coefficient of x^o (constant)

Quadratic formula can be used to solve any quadratic equation that has a solution. It is used when the quadratic expression cannot be easily factorized. Usually, it is used when you are told to correct your answer to decimal places.

Example

Solve the equation $x^2 - 5x - 2 = 0$, giving your answer correct to two decimal places.

a=1,b=-5,c=-2

$$\mathbf{x} = \frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-2)}}{2(1)}$$

$$= \frac{5\pm\sqrt{25+8}}{2}$$

$= \frac{5 \pm \sqrt{33}}{2}$

$\sqrt{33} = 5.745$

$\therefore \frac{5+5.745}{2}$ or $\frac{5-5.745}{2}$

Either x = $\frac{10.745}{2}$ or $\frac{-0.745}{2}$

Either x =5.3725 or -0.3725

Either x = 5.37or -0.37

Exercise

4. Solve the equation $2t^2 - t - 5 = 0$, giving your answer correct to two decimal places.
5. Solve the equation$(x-2)^2$=2 giving your answer correct to two decimal places.
6. Solve the equation $3x^2 + 8x + 1 = 0$, giving your answer correct to two decimal places.
7. Solve the equation $2a^2 + 4a = 3$, giving your answer correct to three significant figures.
8. Solve the equation$x^2 + 5x - 1 = 0$, giving your answer correct to two decimal places.

CHAPTER 5: ARITHMETIC PROGRESSIONS AND GEOMETRIC PROGRESSION

ARITHMETIC PROGRESSION

It is the sequence of numbers called terms in which any term after the first can be obtained from its immediate predecessor by adding a fixed number called the common difference (d).

The first term of an arithmetic progression is usually denoted by a.

Thus, the arithmetic progression takes the general form: a, a +d, a + 2d, a + 3d, …. etc.

FORMULAE FOR AN ARITHMETIC PROGRESSION

1. **nth term of AP = a + $(n-1)$d where a= first term, d= common difference, n= number of terms**
2. **the sum of Arithmetic progression of the first n terms**

   Sum of Arithmetic progression = $\frac{n}{2}(2a + (n-1)d)$ when first term, number of terms and common difference are known.
3. **Sum of Arithmetic progression = $\frac{n}{2}(2a + l)$ when first term, number of**

terms and last term are known

Find the 12th term of the following arithmetic progression series: 6 + 11 + 16…….

nth term of AP = a + $(n-1)$**d**

12th term of AP = 6 $+(12-1)$**5**

= 6 + (11x 5)

= 6 + 55

= 61

Find the sum of the first 12 terms of AP series: 6 + 11 + 16…….

Sum of 12 terms of Arithmetic progression = $\frac{n}{2}(2a+\ (n-1)d)$

= $\frac{12}{2}(2x\,6+\ (12-1)5)$

= 6 (12 + (11x 5))

= 6 (12 + 55)

= 6 x 67

= 402

Find the sum of the following terms of arithmetic progression series:

4 + 8 + 12 + 16 +…………………………96.

Nth term of AP = a + $(n-1)$**d**

96 = 4 + $(n-1)$**4**

96 – 4 = $(n-1)$**4**

92 = $(n-1)$**4**

$\frac{92}{4}=\frac{4}{4}(n-1)$

23 = n-1

24 = n

Sum of 24 terms of Arithmetic progression = $\frac{n}{2}(2a+\ (n-1)d)$

= $\frac{24}{2}(2x\,4+$

$(24-1)4)$

= 12 (8 + 92)

= 12 x 100

= 1200

The 10th term and 4th term of AP series are 40 and 16 respectively. Find

a. The common difference and the first tem of the AP series.
b. The 20th term of AP series.
c. The sum of 10 terms of AP series.

Nth term of AP = a + $(n-1)$d

40 = a + (10-1)d

40 = a + 9d

16 = a + (4-1)d

16 = a + 3d

a + 9d = 40

Subtract (a + 3d = 16)

6d = 24

d = 4 (the common difference)

a + 9(4) = 40

a = 40 -36

= 4 (the first term)

d. The 20th term of AP series = a + $(n-1)$d

= 4 + (20 – 1)4

= 4 + (19 x 4)

= 4 + 76

= 80

The sum of 10 terms of AP series =$\frac{n}{2}(2a + (n-1)d)$

= $\frac{10}{2}(2\,x\,4 + (10-1)4)$

= 5 (8 + 36)

= 5 x 44

= 220

Geoffrey begins a saving scheme which follows an arithmetic progression pattern. She finds out that after making 20 savings she had K105, 000 in her account and after 40 savings she had accumulated K410, 000.

Required:

a. Calculate her initial saving and periodic saving.
b. Calculate the amount of money that will be in her account after her 100th saving.

The sum of 10 terms of AP series =$\frac{n}{2}(2a + (n-1)d)$

105,000 = $\frac{20}{2}(2a + (20-1)d)$

105,000 = 10 (2a +19d)

10, 500 = 2a + 19d

410,000 = $\frac{40}{2}(2a + (40-1)d)$

410, 000 = 20 (2a + 39d)

20,500 = 2a + 39d

10,500 = 2a + 19d

20d =10,000

a = 500

10,500 - 2(500) = 19d

10500 - 1000 = 19d

9500 = 19d

d = 500

Her initial saving was K500 and periodic saving was K500.

Her 100th saving = a + (n-1)d

= 500 + (100-1)500

= K50,000

The product of the 2nd term and 5th term of the arithmetic progression is 90 and its common difference is 3. Find the first term of arithmetic progression

2nd term = a + (2-1)3

= a + 3

5th term = a + (5-1)3

= a + 12

(a + 3)(a + 12) =90

a^2 **+12a + 3a + 36 = 90**

a^2 **+ 15a -54 = 0**

$(a-3)(a+18)$ **= 0**

$a = 3 \text{ or } a = -18$

A clerk employed in a certain company, starts with a salary of K120, 000 per annum. At the beginning of each year he gets an increment of K20, 000

Required:

Find:

a. The salary that he gets in the third, fourth, fifth and sixth years of his employment.
b. The total sum of money that he gets for the first five years,

a. nth term = a + (n-1)d

3rd = 120,000 +(3-1)20,000

= K160,000

4^{th} = K160,000 + 20,000

= K180,000

5^{th} = K180,000 + 20,000

= K200,000

6^{th} = K200,000 + 20,000

= K220,000

b. The sum of first 5 terms of AP series =$\frac{n}{2}(2 + (n-1)d)$

$$= \frac{5}{2}(2(120,000) + (5-1)20,000)$$

$$= \frac{5}{2}(240,000 + 80,000)$$

$$= \frac{5}{2}\ x\ 320,000$$

= K800,000

The 2nd term of an arithmetic progression exceeds the 5th term by 18.
Required:
Find the common difference of the progression

Nth term = a + (n-1)d

5^{th} term = a + (5-1)d

= a + 4d

2^{th} term = a + (2-1)d

= a + d

a + d – (a + 4d) = 18

a + d –a -4d =

-3d = 18

d = -6

The common difference of the progression = - 6.

The salary of a teacher grows by K3, 000 each year. If the starting salary in his grade is K18, 000, find the teacher's salary in the 5^{th} year.

2^{th} term = + (2-1)d

5^{th} term = 18,000+ (5-1)3,000

= 18,000 + (4 x 3,000)

= 18,000 + 12,000

= 30,000

The teacher's salary in the 5^{th} year is K30,000

An employee, who received fixed annual increments, had a final salary of K90, 000 after 10 years. If her total salary was K650, 000 over the 10 years, what was her initial salary**?**

Sum of Arithmetic progression = $\frac{n}{2}(2a+\ l)$

650, 000 = $\frac{10}{2}(2a+\ 90{,}000)$

650,000 = 5$(2a+\ 90{,}000)$

$\frac{650{,}000}{5} = \frac{5(2a+\ 90{,}000)}{5}$

130,000 = 2a + 90,000

130,000 – 90,000 = 2a

40,000 = 2a

a = 20,000

Her initial salary was K20,000

Insert 5 numbers between 8 and 26 such that the resulting series is an AP.

nth term = a + (n-1)d

There are 7 numbers in total from 5 to 26.

26 = 8 + (7-1)d

26 = 8 + 6d

6d = 18

D = 3

2^{nd} term = 8 +3 = 11

3^{rd} term = 11 + 3 = 14

4^{th} term = 14 + 3 = 17

5^{th} term = 17 + 3 = 20

6^{th} term = 20 + 3 = 23

The AP series is 8 + 11 + 14 + 17 + 20 + 23 + 26 + ………

An arithmetic progression has 14 terms. The sum of the first 8 terms is 96. The sum of the next 6 terms is 156. Calculate

a. The common difference.
b. The first term
c. The 10^{th} term.

a. **For an AP , sum of the term =** $\frac{n}{2}(2a+(n-1)d)$ **a +(n-1)d, where a is the first term and d is common difference.**

S = $\frac{N}{2}(2a+(n-1)d)$ **where S is the sum and n is the number of terms.**

Sum of 8 terms is 96

96 =$\frac{8}{2}(2a+(8-1)d)$ **(divide both sides by 4**

96 =4$(2a+7d)$

24 = 2a +7d (1)

Sum of 14 terms = 96 + 156 = 252

Therefore 252 =$\frac{14}{2}(2a+(14-1)d)$

252 =$7(2a+(14-1)d)$ **(divide both sides by 7)**

36 = 2a +13d (2)

Solve equation (1) and equation (2) simultaneously

(2)-(1) yields:

36-24 = 13d-7d

12 =6d

2 = d

Therefore, the common difference ,d is =2

b. **Finding the first term, a**

Substitute d= 2 in equation (1)

2a = 24-7d

2a = 24-7 x2

2a = 24-14

2a = 10

A = 5

Therefore, the first term is 5.

c. **Finding the 10**th **term = a (n-1)d**

= 5 + (10-1)2

= 5 + (9x2)

= 5 + 18

= 23

The tenth term is 23.

The first term of an arithmetic progression, AP is 3 and its first, fourth and thirteenth terms form a geometric progression, GP.

a. if the common difference of the AP is d, write down expressions for the fourth term and thirteenth term.
b. Find d.
c. Find the tenth term of the AP.

a. For an AP, nth term = a+(n-1)d where a is the first term. Here is 3.

4^{th} term = 3 +(4-1)d

= 3 + 3d

= 3(1+d)

13^{th} term =3(13-1)d

=3+12d

= 3(1+4d)

b. AP is 3 + 3(1+d) + 3(1+4d)

To find the common difference, the second term should be divided by the first term and the third term should be divided by the second term since they form GP.

$\frac{3(1+d)}{3}=\frac{3(1+4d)}{3(1+d)}$ **(cross multiply)**

$(1+d)(1+d) = 1+4d)$ **remove brackets**

$$1+2d+d^2 = 1+4d$$

$$d^2+2d-4d+1-1=0$$

$$d^2-2d=0$$

d(d-2)=0

d= 0 or d = 2

Therefore, the value of d is 2 since 0 does not give an AP.

c. 10^{th} term = 3 +(10-1)2

= 3 + 9 x 2

= 3 +18

= 21

The first term of an arithmetic progression is 16 and the sixth term is 86. Calculate

a. The common difference.

b. The sum of the first 21 terms

The first term of the AP is 16 and the common difference is d.

a. The 6^{th} term = a + (n-1)d, where a = first term and d= common difference

= a + (6-1)d

=a + 5d if a = 16 gives

6^{th} term = 16 + 5d = 86

5d = 86-16

5d = 70

Therefore, d =14

b. The sum of the first 21 terms is

Sum = $\frac{n}{2}(2a+(n-1)d)$

Sum = $\frac{21}{2}(2\,x16+(21-1)14)$

= 3276

Given the arithmetic progression 21, 17, 13,…

Find

a. The 41st term

b. The sum of the first 41 terms.

a. nth term = $a+(n-1)d$

Since first term, a = 21 and the common difference, 17-21= -4

41st term = 21 +(41-1)-4

= 21 +(40x-4)

= 21 -160

= -139

b. The sum of the first 41

Sum = $\frac{n}{2}(2a+(n-1)d)$

= $\frac{41}{2}(2x\,17+(41-1)-4)$

= -2419

The first three terms of an AP are 19,16, 13, …

a. Which term of the progression is -41?

b. Find n if the sum of the first n terms is 44.

a. Let the number of terms be n. If the first term is 19 and the second term is 16, the common difference is 16-19= -3

nth term = $a+(n-1)d$

-41 = $a+(n-1)d$

- 41 = 19 + (n-1)-3

-41 = 19 + (-3n +3)

-41 = 19+3 -3n

-41 = 22 -3n

-41-22 = -3n

-3n = -63

n = 21

The term which has the value -41 is the 21st term.

b. Sum of n terms = $\frac{n}{2}(2a+(n-1)d)$

44 = $\frac{n}{2}(2x\,19+(n-1)-3)$

88 = n(38-3n+3)

88 = n(41-3n)

88= 41n$-3n^2$

$3n^2-41n+88=0$

Factorise

(3n-8)(n-11) =0

Either 3n-8= 0 or n-11 = 0, but n must be a whole number

Therefore, n= 11

In an Arithmetic Progression, the ratio of the seventh term to the eleventh term is 19:31 and the sum of these two terms is 25. Find

a. The first term.
b. The common difference.
c. The sum of the first twenty terms.

a. nth term = $a+(n-1)d$

The 7th term of the AP is a+6d and the 11th term is a + 10d

$\frac{a+6d}{a+10d}$ = $\frac{19}{31}$ **(cross multiply)**

31(a=6d) =19(a+10d)

31a +18d =19a +190d

12a-4d =0 (Equation (1)

The sum of the two terms is 25

(a+6d) + (a+10d) =25

2a +16d =25(Equation 2)

Solve equation (1) and (2) simultaneously

12a -4d =0

2a +16d =25

Multiply equation (1) by 4

48a -16d = 0 (Equation 3)

2A +16d =25 (Equation (4)

Add equations (3) and (4)

50 a = 25

a = $\frac{1}{2}$

Substitute a = $\frac{1}{2}$ in equation (1) to find d.

12 ($\frac{1}{2}$) -4d = 0

6-4d = 0

d = $1\frac{1}{2}$

The first term is $\frac{1}{2}$ and the common difference = $1\frac{1}{2}$

b. Sum of the first 20 terms

Sum = $\frac{n}{2}(2x\ 19 + (n-1) - 3)$

Sum $= \frac{20}{2}\left(2x\frac{1}{2} + (20-1)1\frac{1}{2}\right)$

= 295

Given the Arithmetic progression -2, 1, 4…., calculate the 40^{th} term.

The first term, a = -2, the common difference, d = 1- -2 = 3

nth term = $a + (n-1)d$

40^{th} term = -2 + (40-1)3

= -2-3 + 120

= 115

The nth term of a given series is 5n-13. Calculate the first 3 terms.

1^{ST} term = 5(1)-13

= 5-13

=-8

2^{nd} term = 5(2)-13

= 10-13

= -3

3^{rd} term = 5(3)-13

= 15-13

= 2

The first three terms of an AP are -8,-3 and 2

Find the number of terms in the progression 1200, 1280, 1360… 2080.

nth term = $a + (n-1)d$

First ,a = 1,200

Common difference,d = 1,280-1,200 =80

nth term =2,080

2,080 = 1,200 + (n-1)80

2,080-1,200 = 80n-80

980+80 =80n

80n= 960 (divide both sides by 80)

n = 12

There are 12 terms in the progression.

The series 4, 9, 14,19….. are in arithmetic progression. Calculate the 21st term.

The first term, a = 4, and the common difference of AP, d = 9-4 = 5

The 21st term of AP = a + (n-1)d

= 4 + (21-1)5

= 4 + (20 x 5)

= 4 + 100

= 104

The nth term of Arithmetic progression is (3n-2).Calculate the sum of the first four terms.

First term, a = 3(1)-2

= 3-2

= 1

Second term, a + d = 3(2)-2

= 6-2

= 4

Third term , a + d + d = 3(3)-2

= 9-2

= 7

The sum of the first 4 terms = $\frac{n}{2}(2a + (n-1)d)$

Common difference, d = (a +d) –a

= 4-1

= 3

The sum of the first 4 terms = $\frac{4}{2}(2x\,1 + (4-1)3)$

= 2 (2 +(3 x3))

= 2 (2 + 9)

= 2 x11

= 22

The sum of the first 4 terms is 22.

An orchard is in the shape of a trapezium. There are 13 fruit trees in the first row, 15 in the second, 17 in the next and so on. If there are 47 trees in the last row, how many trees are there?

Nth	Number of fruit tree
1	13
2	15
3	17
n	47

The number of fruit trees in the rows are increasing arithmetically. We therefore have an arithmetic Progression as follows:

13, 15, 17,…..47

Number of terms

nth term = a +(n-1)2

Where a is the first term, n is the number of terms and d is the common difference.

From the AP, a =13, d =15-13 = and n = ?

nth term = 13 + (n-1)2

47 = 13 + 9n-1)2

47-13 +2 =2n

36 = 2n

n= 18

The AP has 18 terms

Sum of 18 terms of the AP= $\frac{n}{2}(2a+(n-1)d)$

= $\frac{18}{2}(2x\ 13+(18-1)2)$

= 9 (26 +(17)2)

= 9(26 +34)

= 9 x 60

=540

Or AP= $\frac{n}{2}(a+l)$ **where a= first term and l = last term**

$= \frac{18}{2}(13+47)$

= 9 x 60

= 540

Given that 2x, x, x+3,….. are terms in the Arithmetic Progression. Calculate the value of x.

d = x-2x or d = (x + 3)-x
x-2x = (x+3)-x
-x = 3 (multiply by -1 both sides
x =-3

EXERCISE

1. The sum of n terms of an arithmetic progression is $\frac{5n^2+3n}{2}$. Calculate the first two terms of the arithmetic progression.
2. The nth term of an Arithmetic Progression is 5n-3. Calculate the sum of the first 6 terms of AP.
3. The ratio of the 2^{nd} term to the 7^{th} term of an arithmetic progression is 1:3and their sum is 20. Calculate the sum of the first 10 terms of the progression.
4. Find the sum of the first 20 terms of the arithmetic progression 4,2,0,…
5. The 9^{th} term of an arithmetic progression y, y+4,y+8,….. is 37. Find the value of y.
6. The first term of an arithmetic progression is 5 and the last term is 43. If the sum of the terms is 480, calculate the number of terms.
7. Find the firs of an arithmetic progression given that the 20^{th} term = 100 and the 22^{nd} term = 108.
8. The fifth term of an arithmetic progression is 26 and twelfth is 75. Find the eighth term.
9. The 8^{th} term and 3^{rd} term of arithmetic progression are 28 and 46 respectively. Find

a. the first term and the common difference of arithmetic progression
b. The sum of its first 10 terms of the series.

10. The 10^{th} tem of arithmetic progression is 51 and the sum of its first 12 terms is 402. Calculate

a. The first term and the common difference
b. 20^{th} term of this arithmetic progression.

GEOMETRIC PROGRESSION

A geometric progression is a sequence of numbers called <u>terms</u>, in which any term after the first can be obtained from its immediate predecessor by multiplying by a fixed number, called the <u>common ration (r)</u> .
The n^{th} term of geometric progression = ar^{n-1}
The sum of the first n terms of GP =$\frac{a(r^n-1)}{(r-1)}$

The sum of the first n terms of GP = $\frac{a}{(1-r)}$ when r < 1.

EXAMPLES

Find the 10th term of the following geometric progression series 3 + 12 + 48 + …..

The n^{th} term of geometric progression = ar^{n-1}

The 10^{th} term of geometric progression = 3 x 3^{10-1}

= 3 x 3^9

= 2,187

The 6th term of the geometric progression is 128, and the third tem is 16. Find the first term and the common ratio of the geometric progression.

The n^{th} term of geometric progression = ar^{n-1}

The 6^{th} term of geometric progression = ar^{6-1}

128 = ar^5

The 3^{th} term of geometric progression = ar^{3-1}

16 = ar^2

$\frac{ar^5}{ar^2} = \frac{128}{16}$

r^3 = 8

$\sqrt[3]{r^3} = \sqrt[3]{8}$

r = 2

16 = ax 2^2

4a = 16

a = 4

The first term is 4 and the common ratio is 2

Find the 11th term and the sum of the first 20 terms of the geometric progression 4, 8, 16, 32, 64 ……….

The n^{th} term of GP = ar^{n-1}

The 11^{th} term of GP = 4 x 2^{11-1}

= 4 x 2^{10}

= 4096

The sum of the first 20 terms of GP = $\frac{a(r^n-1)}{(r-1)}$

= $\frac{4(2^{20}-1)}{(2-1)}$

= 4,194,300

Find the sum of the following geometric progression 16,4,1,0.25 ……….

The sum of the first n terms of GP = $\frac{a}{(1-r)}$ when r < 1.

a = 16 and r = $\frac{1}{4} = 0.5$

The sum of the first n terms of GP = $\frac{a}{(1-r)}$

= $\frac{16}{(1-0.5)}$

= 8

Find the number of terms in the following terms of geometric progression 4, 8, 16 ………..4096.

The n^{th} **term of geometric progression = ar^{n-1}**

4096 = 4 x r^{n-1}

$\frac{4096}{4}$ = $\frac{4 \times 2^{n-1}}{4}$

1024 =2^{n-1}

$2^{n-1} = 2^{10}$

$(n-1)$ = **10**

n = 11

There are 11 terms in the AP.

During a tree planting week, a student planted 4 trees on the first day, 12 trees on the second, 36 trees on the third and so on. By the end of the period, he had planted 4,372 trees.

Required:

How many days did it take the student to plant the 4,372 trees?

The sum of the first n terms of GP = $\frac{a(r^n-1)}{(r-1)}$

4,372 = $\frac{4 \times (3^n-1)}{(3-1)}$

$\frac{2 \times 4{,}372}{4}$ = $\frac{4 \times (3^n-1)}{2 \times 4}$ **x 2**

2,186 = 3^n– 1

3^n = 2,187

3^n = 3^7

n = 7

In a geometric progression, the ratio of the fifth term to the second term is 27:8. Calculate the common ratio.

Let the first be a and the common ratio be r. Then

2nd term = ar

5th term = ar^4 and

The common ratio = $\frac{ar^4}{ar}$

$= r^3$

$r^3 = \frac{27}{8}$

r = $\sqrt[3]{\frac{27}{8}}$

r = $\frac{3}{2}$

The common ratio is $\frac{3}{2}$

In a geometric progression, the third term is 24, and the sixth term is -192. Find

a. The common ratio

b. The first term

a. Let the first term be a and the common ratio be r. Then

ar^2**= 24 and** ar^5**= -192**

By division, $\frac{ar^5}{ar^2} = \frac{-192}{24} = -8$

$r^3 = -8$

$\sqrt[3]{r^3} = \sqrt[3]{-8}$

R = -2

b. Substitute r = -2 into ar^2**= 24**

$a(-2^2)$**= 24**

4a = 24

a = 6

The first term is 6

The sixth term of a G.P is 48 and the fourth term is 12. If the common ratio is negative, find the second term.

Since the 6th term of the GP is 48 , then

ar^5 **= 48, where a is the first term and r is the common ratio.**

12 is the fourth term

ar^3 **= 12**

$\frac{ar^5}{ar^3} = \frac{48}{12}$

r^2**= 4**

$\sqrt{r^2} = \sqrt{4}$

r = ± 2

Since the common ratio is negative, r = -2

From ar^3 **= 12, we get a**

$a(-2)^3$ = **12**

a = $\frac{12}{-8}$

a = $\frac{3}{-2}$

The second term of GP = ar

= $\frac{3}{-2}x - 2$

= 3

The second of the GP is 3.

The sum of the first three positive numbers which are in a GP is 52. The square of the second number is equal to four times the third number. Find the second term of the progression.

Nth term of a GP = ar^{n-1}

First term = ar^{1-1}

= ar^0

= a x 1

= a

Second term of GP = ar^{2-1}

= = ar

3rd term of a GP =a r^2

From the statement:

a+ ar $+$ **a** r^2 **= 52 (1)**

$(ar)^2$**= 4**(ar^2)

a^2r^2**= 4**ar^2

$\frac{a^2r^2}{ar^2}$ = $\frac{4ar^2}{ar^2}$ **(divide equation (2) by** ar^2

a= 4

Substitute 4 for a in equation 1

4+ $4r$ $+$ **4** r^2 **= 52 (divide both sides by 4). This gives:**

1 +r +r^2 **= 13**

$r^2 + r - 13 + 1) = 0$

$r^2 + r - 12 = 0$ **(Factorise the left hand side of the equation)**

(r + 4)(r-3) = 0

Therefore, r = -4 or r = 3

Since all the 3 terms of a GP are positive, then, r = 3

Therefore, the second term of the progression is (ar)= 4 x 3

= 12.

The first three of a GP are x+1, $x^2 - 1$, and $(x^2 - 1)(2x - 4)$. Calculate the value of x.

Given 3 terms of a GP

Common ratio = $\frac{x^2-1}{x+1}$ = x-1

Or the common ration = $\frac{(x^2-1)(2x-4)}{x^2-1}$= 2x-4

Since the common ratio is constant

x -1 = 2x-4

x-2x = -4 + 1

-x = -3 (divide both sides by -1

x = 3

Therefore, the value of x = 3

The new types of bricks are laid down on tables. On the first table there are 2 bricks, on the second table there are 4 bricks, on the third table there are 9 bricks and so on. If on the nth table there are 1024 bricks, calculate the value of n.

nth table	Number of bricks
1	2
2	4
3	8
n	1024

The number of bricks increasing geometrically. We therefore have a geometric progression (GP) as follow.

2, 4, 8…

Common ratio = $\frac{4}{2}$ = 2

The nth term = ar^{n-1} where a is the first term r. is the common ratio and n is the number of terms.

1024 = 2 x 2^{n-1} (divide both sides 2)

512 = 2^{n-1}

$2^9 = 2^{n-1}$

Since the bases are equal, the powers must also be equal

9 = n-1

n = 9 +1

n = 10

The value of n is 10

1. The sum of the first two terms of a geometric progression is 4. If the first term is 3, find the common ratio.

2. Find the sum of the first 12 terms of the following GP: $\frac{1}{2187}, \frac{1}{729}, \frac{1}{243},$, …..Give your answer correct to 2 decimal places.
3. The sum of the first n terms of a geometric progression, GP is $2^{(n+1)} - 1$. Calculate
   a. The first term of the GP.
   b. The common ratio of the GP.
4. The first term of a geometric progression is 81, and the common ratio is $\frac{1}{3}$, calculate the fourth term.
5. The second term of a geometric progression is -6 and the fourth term is -54. Calculate the common ratio, given that it is negative.
6. If the 4th and 7th terms of a geometric progression are 4 and 32 respectively, calculate the first term.

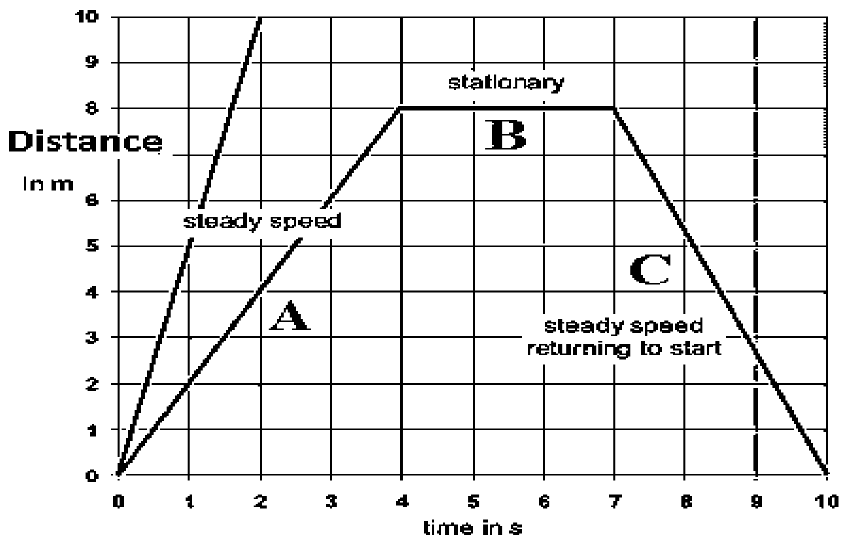
CHAPTER 6: DISTANCE-TIME GRAPHS

A distance-time graph is a useful way to represent the motion of an object. It shows how the distance moved from a starting point changes over time.

DISTANCE-TIME GRAPH

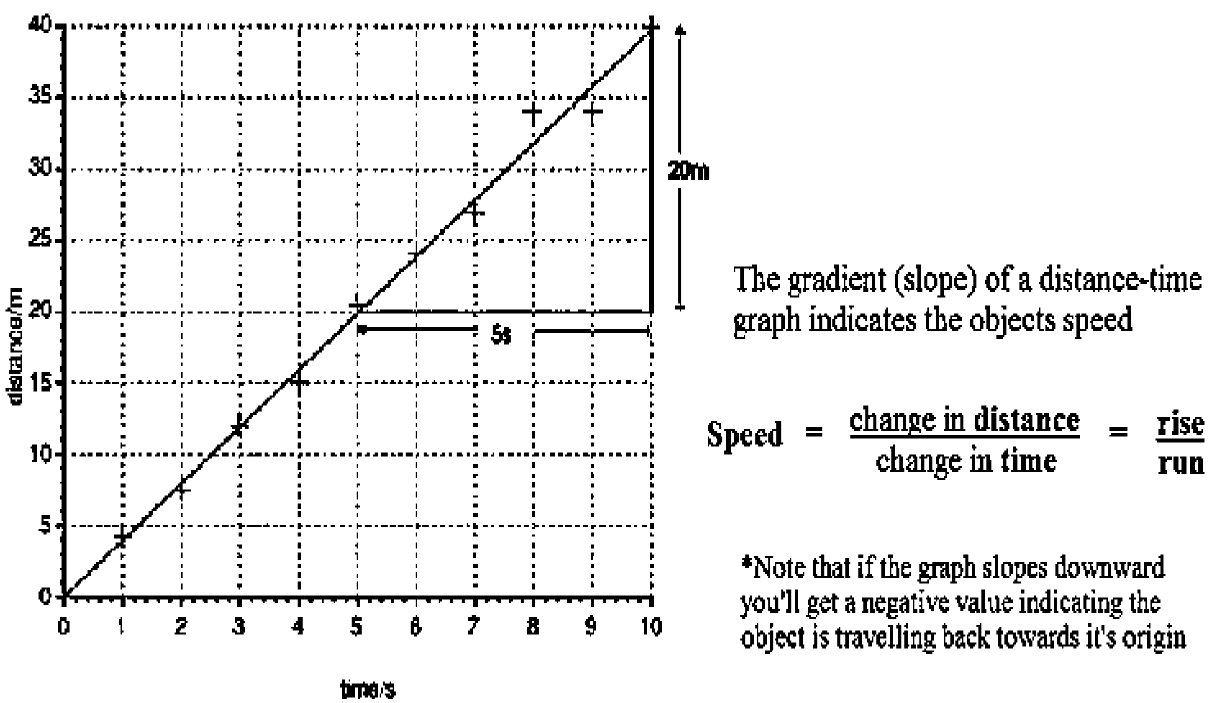
- Distance is the total length travelled by an object. The standard unit is the **'metre'**
- Distance -time graph shows how far an object has travelled in a given time.
- Distance is plotted on the Y-axis (LEFT) and Time is plotted on the X -axis (BOTTOM)

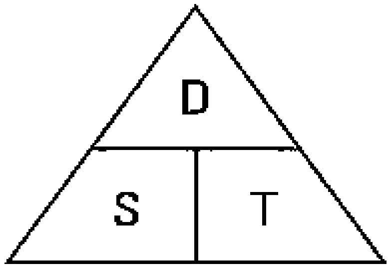
DESCRIBING THE MOTION OF AN OBJECT



During **Part A** of the journey, the object travels 8 m in 4 second sat the average speed of 2m/second. It is travelling at a constant velocity/speed.
During **Part B** of the journey, the object travels 0m in 3 seconds. It is stationary for 3 seconds.
During **Part C** of the journey, the object travels -8m in 3 seconds. It is travelling at a constant velocity of -2./second back to its starting point, our reference point 0.Negative means the object is moving backwards while positive means the object is moving forward.

CALCULATING SPEED FROM A DISTANCE-TIME GRAPH



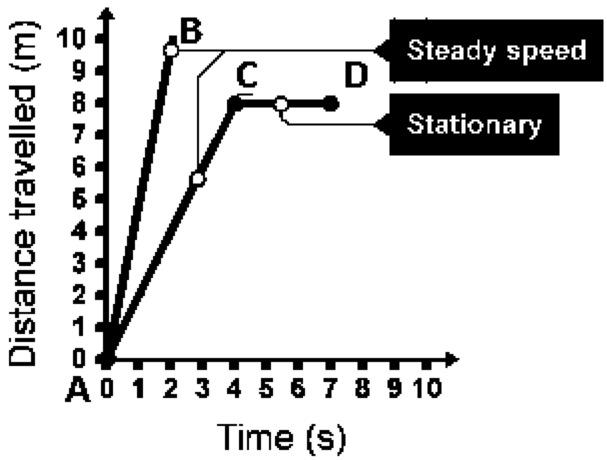


$D = S \times T$

$S = D \div T$

$T = D \div S$

- In a distance-Time graph, the gradient of the line is equal to the speed of the object. The greater/steeper the gradient the faster the object is moving.



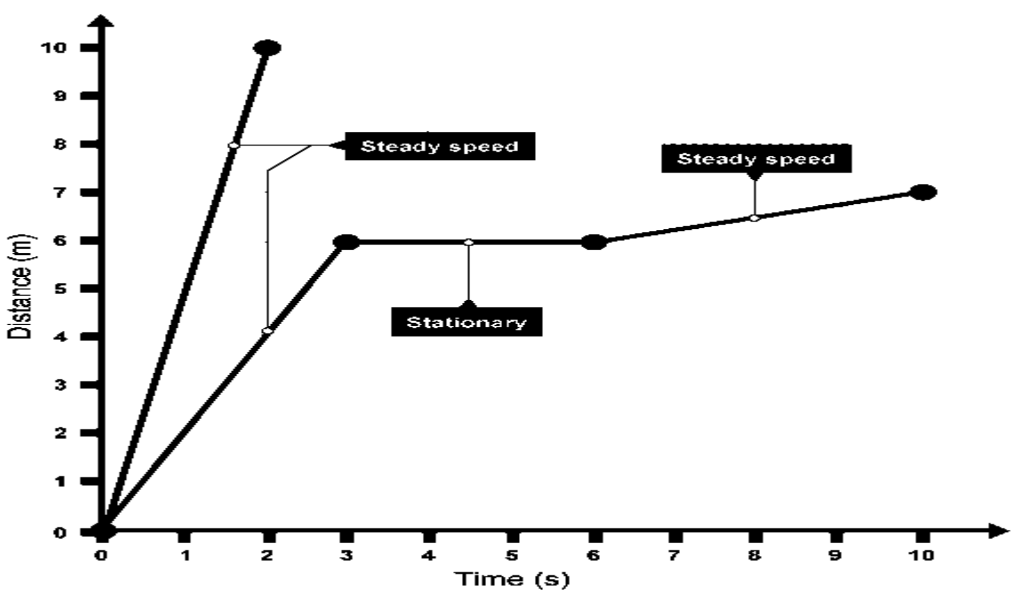
Calculate the speed of the object represented by line AB in the graph from 0 to 4 seconds.

Speed = Distance/Time

= 8m/4seconds

= 2m/s

In a distance-time graph



- **Distance travelled is plotted on the vertical (y) axis**
- **Time taken is plotted on the horizontal (x) axis**
- **The gradient of the line is equal to the speed. This means that the line is:**
  a. **Horizontal for a stationary object (because the distance stays the same)**
  b. **A straight diagonal for an object moving at a constant speed**
  c. **The steeper the line, the greater the gradient and the greater the speed.**
- **The vertical axis of a distance -time graph is the distance travelled from the start and the horizontal axis is the time-taken from the start.**

EXAMPLE

A bus driver drives at a constant speed which is indicated by speedometer and the driver measures the time taken by the bus for every kilometer. The driver notices that the bus travels 1km in every 2 minutes.

DISTANCE (IN KM)	1	2	3	4	5	6	7
TIME (IN MIN)	2	4	6	8	10	12	14

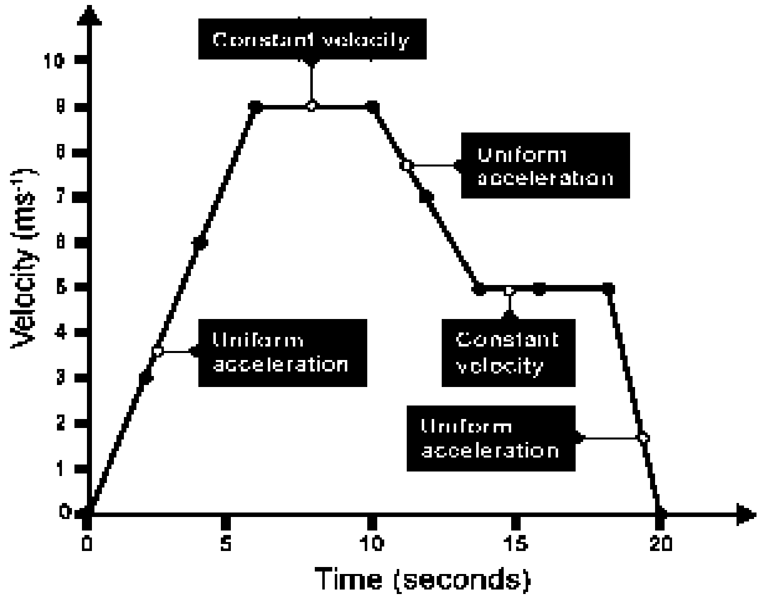
Calculate at speed was he travelling.

Speed= (Final position-Initial)/Time

= (7-1)/(14-2)= 0.2km/h

SPEED-TIME GRAPH/VELOCITY-TIME GRAPHS

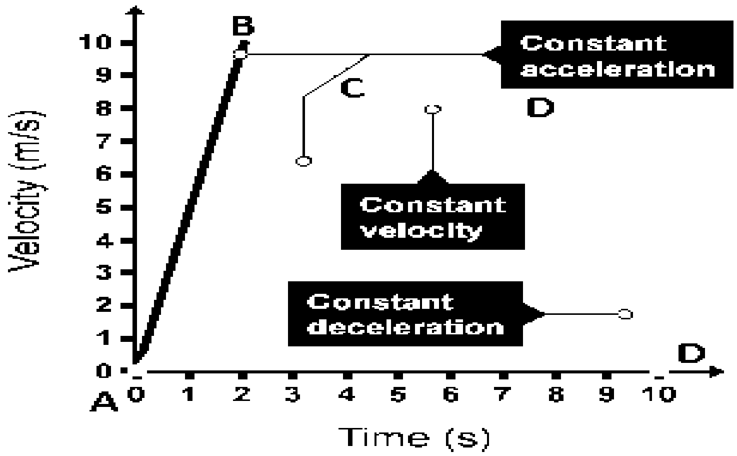
- A velocity -time graph shows the speed and direction on objects travels over a specific period of time.
- A Velocity-time graph is also called speed-time graph.
- Speed-time graphs are very useful when describing the movement of an object. We can use them to determine whether or not the object is moving at any point in time. We can also use them to see what speed the object is travelling at that point in time.
- Using the data from the graph, we can calculate any acceleration, the change in speed and the change in time.
- We can also use graphs to calculate distance travelled. The area under a speed-time graph represents the distance travelled.
- The area under a **speed-time graph** represents the distance travelled. This is a **velocity-time graph** of an object moving in a straight line due North.



- In the graph above, the object starts from rest, accelerates for 6 seconds. It stays at a constant velocity of 9 m per second for 4 seconds then slows down to 5m/s in a further 4 seconds. It travels at 5m/s for another 4 seconds then rapidly slows down and comes to rest.
- A horizontal line on a speed-time graph represents a constant speed.
- A sloping line on a speed-time graph represents acceleration. The sloping line shows that the speed of the object is changing. The object is either speeding up or slowing down.
- The vertical axis of a velocity -time graph is the velocity of the object.
- The horizontal axis is the time from the start.
- The steeper the slope of the line the greater the acceleration. If the line slopes downwards from left to right on the graph, this means that the object is slowing down. This motion is sometimes referred to as 'deceleration'. Deceleration is acceleration where the speed of the object is decreasing with time.

FEATURES OF THE VELOCITY-TIME GRAPHS

1. When an object is moving with a constant velocity/speed, the line on the graph is horizontal.
2. When the horizontal line is at zero velocity, the object is at rest.
3. When an object is undergoing constant **acceleration,** the line on the graph is straight but sloped.
4. Curved lines on velocity-time graphs also show changes in velocity, but not with a constant **acceleration or deceleration**.
5. The diagram shows some typical lines on a velocity -time graphs.



- The steeper the line, the greater the acceleration of the object
- Notice that a line sloping downwards with a negative gradient represents an object with a constant deceleration- it is slowing down.
- The acceleration can be calculated by dividing the change in velocity measured in metres per second by the time taken for the change in seconds.
- The units of acceleration are m/s/s or m/s^2

Constant acceleration, AB = Change in velocity (m/s)÷ Time taken (s)

= 10m/s÷ 2s

=5 m/s^2

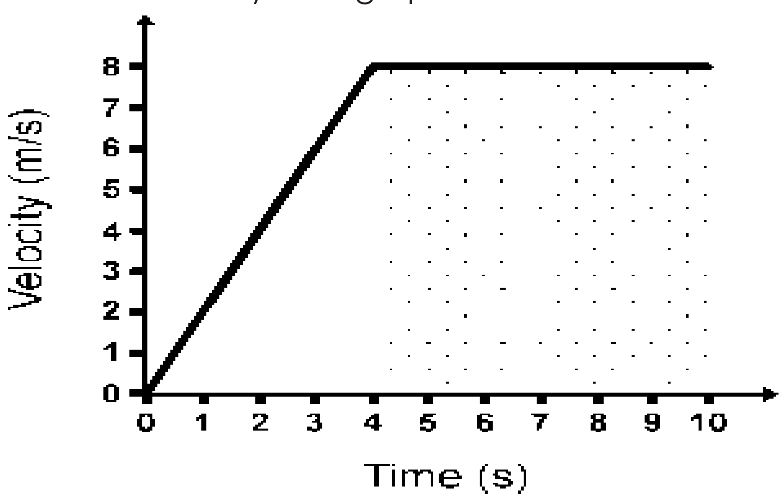
Constant acceleration, AC = Change in velocity (m/s)÷ Time taken (s)

= 8m/s÷ 2s

= 2m/s^2

FINDING DISTANCE OF SPEED-TIME GRAPH

Here is a velocity-time graph.



- The area under the line in a velocity-time graph represents the distance travelled.

- To find the distance travelled in the graph above, you need to find the area of the light--blue triangle and the dark-blue rectangle.

1. **Area of Light-Blue Triangle**

   The width of the triangle is 4 seconds and the height is 8 metres per second. To find the area, you use the equation

   Area of triangle = ½ x base x height

   = ½ x 8 x 4 m

   = 16m

2. **Area of Dark-Blue Rectangle**

   The width of the rectangle is 6 seconds and the height is 8 metres per second, so the area = Width x Height

   = 8 m/s x 6seconds

   = 48m

3. **Area under the Whole Graph**

   The area of triangle plus the area of the rectangle gives the area of the whole graph

   Total distance/area = 16 + 48 m

   = 64m

   A train travelling at 96km per hour takes 50 minutes for a journey. At what speed must it travel to reduce the time to 40minutes?

Total distance = time x speed

= 96km/h x 50/60

=80km

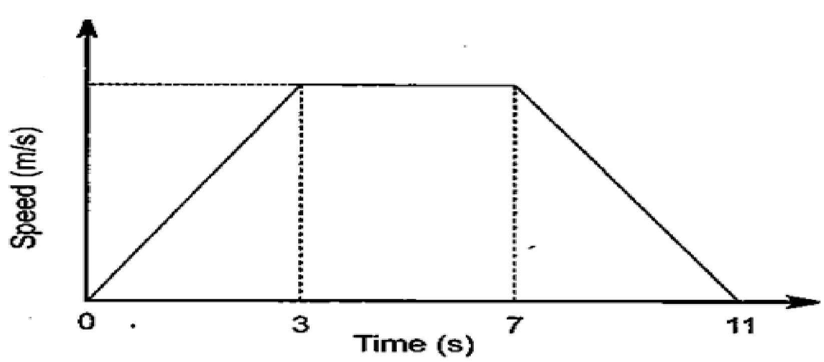
Speed = Distance ÷Time

= 80km 40/60hrs

=53.3km/hr

EXAMPLES

1. Figure below shows the speed-time graph of a moving object.



If the total distance travelled by the object was 90m, calculate the speed V of the object.

Total distance = Area under the graph

Area $= \frac{1}{2}$ **(Sum of parallel sides) x h**

$= \frac{1}{2}(4+11)x\,h$

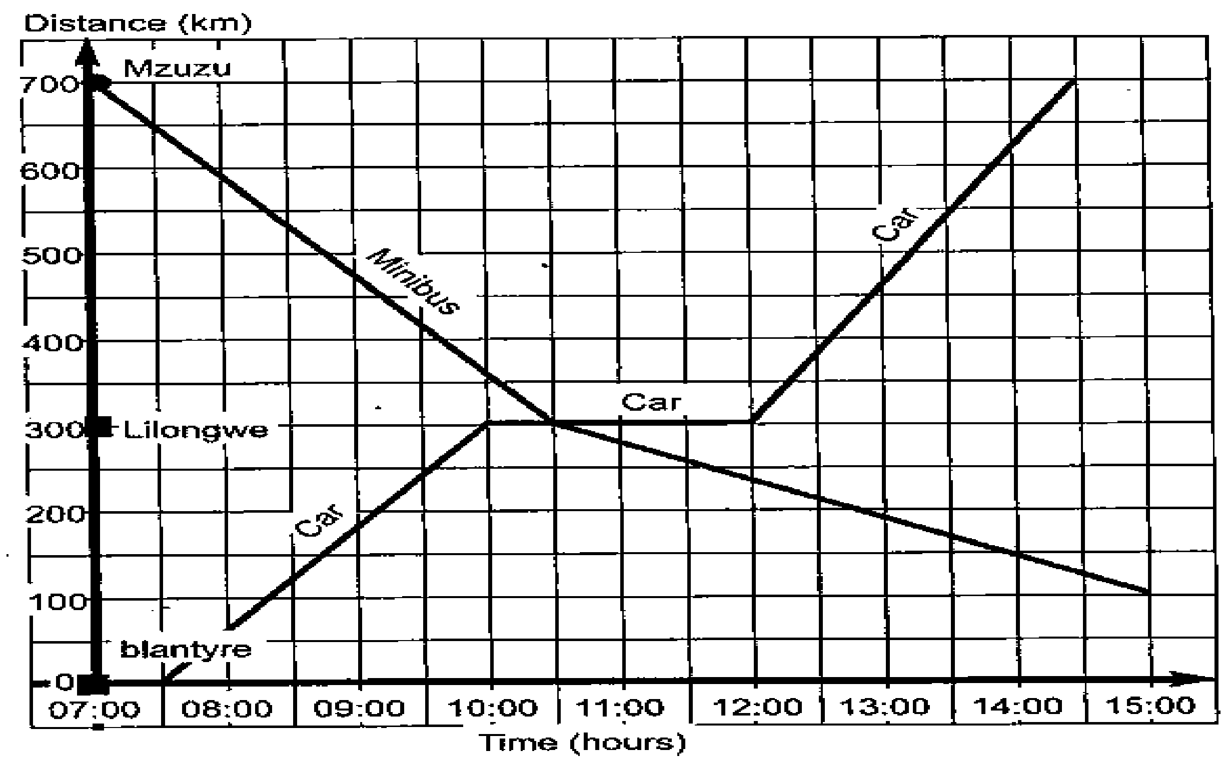
$= \frac{1}{2}(15)x\,h$

But $\frac{1}{2}(15)x\,h$ **= 90**

h = 12m

Speed V = 12m/s

2. Figure shows travel graph of a minibus that leaves Mzuzu at 07:00 hours and arrives in Blantyre at 15: 00 hours and a car that leaves Blantyre at 07:30 arrives in Mzuzu at 14:30 hours. From Mzuzu, the minibus travels at a constant speed and arrives in Lilongwe at 10:30 hours and immediately proceeds to Blantyre at another constant speed. From Blantyre the car travels at a constant speed and arrives in Lilongwe at 10:00 hours. At noon the car leaves for Mzuzu at a constant speed.



Calculate the average speed of the minibus during the time when the car stopped in Lilongwe.

Time at which the car stopped in Lilongwe = t_1= 10:00

Distance of the Minibus from Blantyre at 10:00 = d_1= 360km

Departure time of the car from Lilongwe =t_2= 10:00

Distance of the Minibus from Blantyre at 12:00 = d_1= 230km.

Distance travelled by the Minibus when the car stopped in Lilongwe

=d_1-d_2

= 360km-230km

= 130km

Time taken = t_1-t_2

= 12:oo-10:00

= 2hours

The Minbus travelled a distance of 130km in hours when the car stopped in Lilongwe.

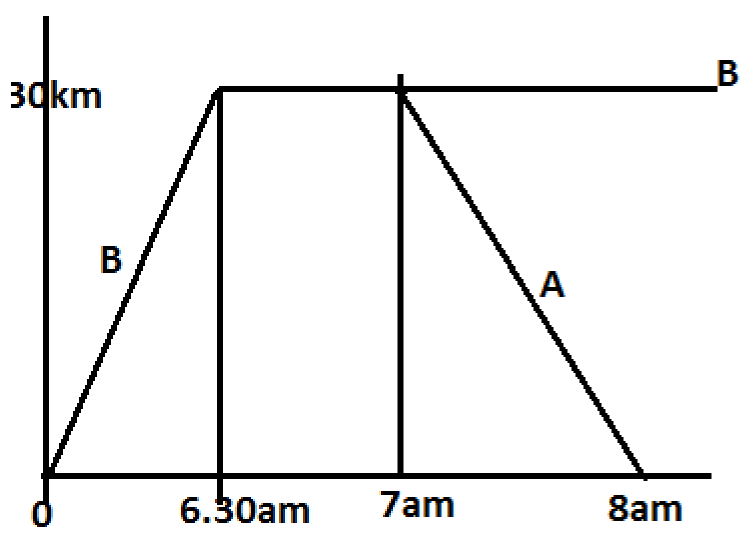
∴ **Average speed = $\frac{Distance}{Time\ taken}$**

= $\frac{130km}{2hours}$

= 65km/hour

∴ **Average speed of the Minibus when the car stopped in Lilongwe is 65km/hour**

3. Figure below shows travel graphs of two vehicles.



Find

a. the speed of B between 6am and 6.30am.

b. the speeds of A and B when they met at 7am.

a. Distance covered by B between 6am and 6:30am = 30km

Time taken = 30min= 1.5hr

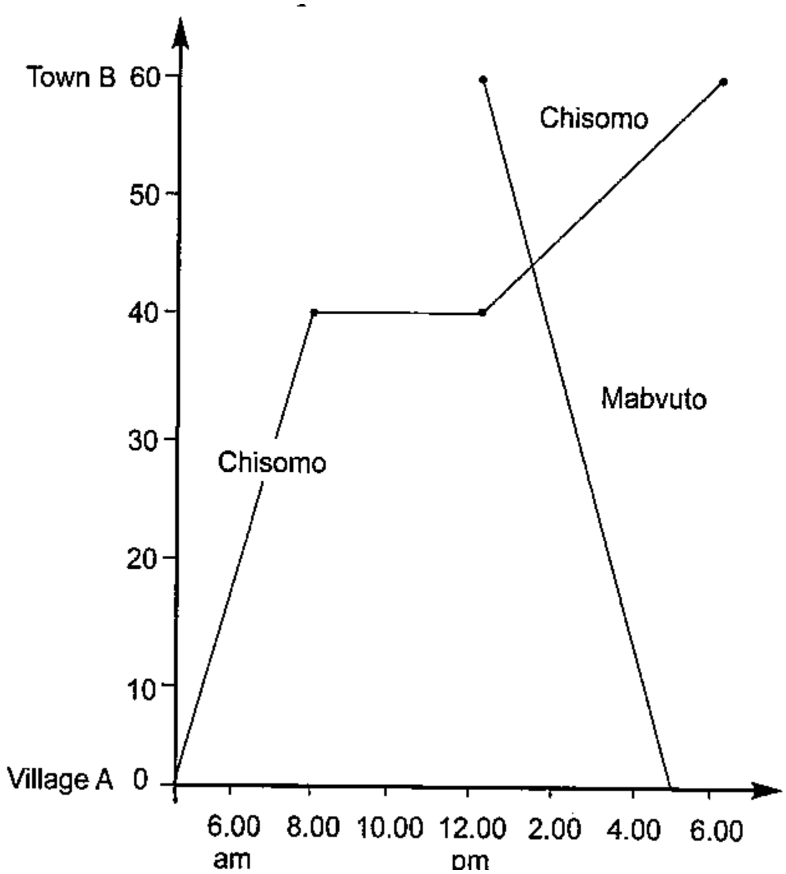
Speed = Distance÷ $Time$

=30km ÷ 0.5hrs

=6km/hour

b. The speed of A was 30km/hour and the speed of B was 0km/hour when they met at 7am.

4. The graph in figure below shows Chisome's journey from village A to town B and Mavuto's journey from town B to village A. Use the graph to answer the questions that follow.



a. At what time did Chisomo reach town B?
b. How far away from village A did Chisomo meet Mavuto?
c. How far away was Chisomo from Mavuto at 2pm?
d. For how long was Chisomo's speeds 0km/hour?
e. What was Mavuto's average speed?
f. What was Chisomo's average speed for the whole journey?

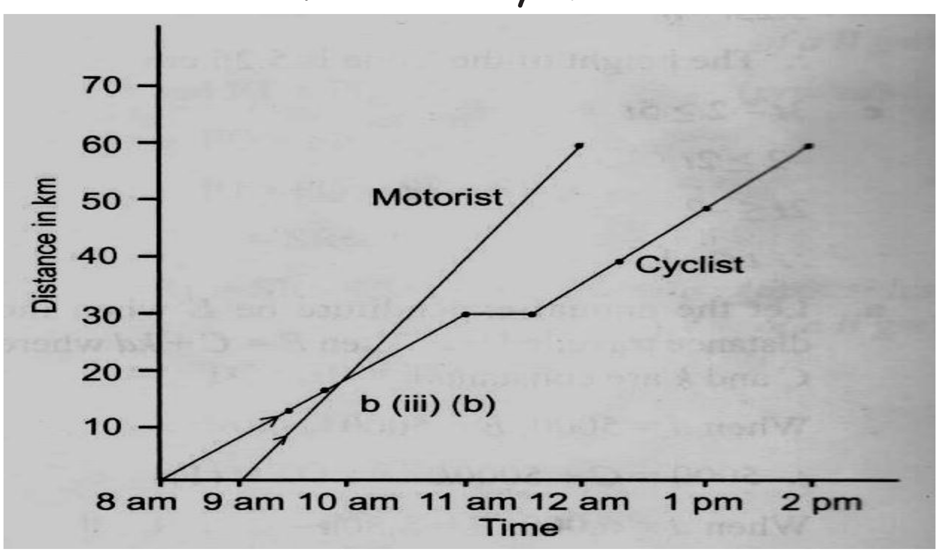
a. Chisomo reached town B at 6.00pm

b. Chisomo met Mavuto 44.5km away from village A.
c. At 2pm, Chisomo was 11km away from Mavuto.
d. Chisomo's speed was 0km/hour for 3hours.
e. Mavuto's average speed was 12km/hour.
f. **Chisomo's average speed for the whole journey was 60km ÷ 12hours = 5km/hour.**

5. A motorist leaves Zomba at 8.00am to cycle to Blantyre 60km away. For the first 3hours he travels at a steady speed of 12km per. He travels at a steady speed of 12km per hour. He then stops for 30 minutes and afterwards finishes his journey at a steady speed of 8km per hour.

a. Using a scale of 2cm to represent 1hour on the horizontal axis and 2cm to represent 10km on the vertical axis, draw the travel graph of the cyclist.
b. On the same scale and axis, draw the graph of a motorist travelling at a steady speed, who leaves Zomba at 9.00am for Blantyre and overtakes the cyclist at 10.30am.
c. From your graph find
   i. The time the cyclist arrives in Blantyre.
   ii. The speed of the motorist.

a. Plotting the graphs that illustrate the journeys of a cyclist and a motorist between Zomba and Blantyre.

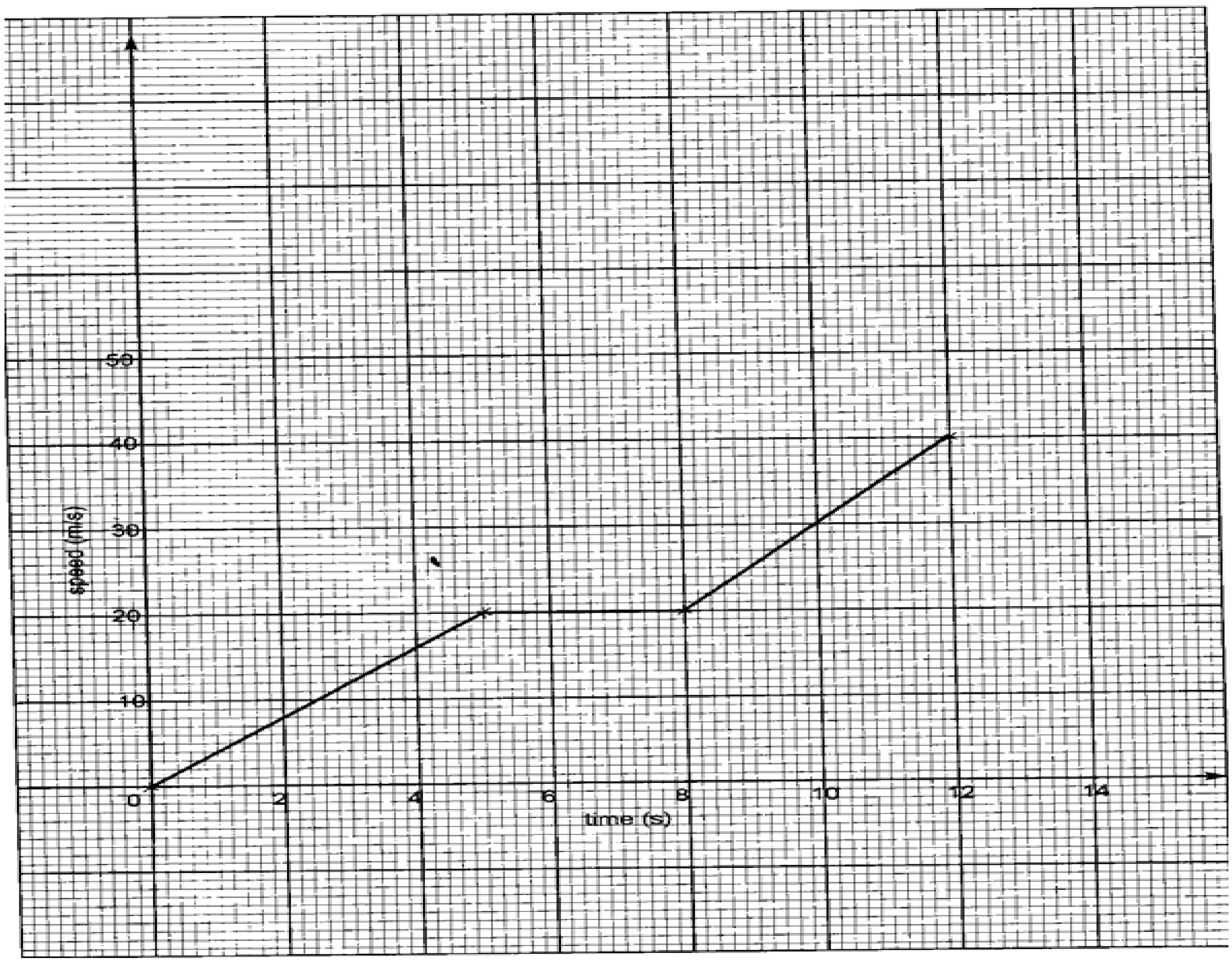


b. The cyclist arrives in Blantyre at 2.30pm.
c. The speed of the motorist is 20km/hour.

6. A train accelerates uniformly from rest and reaches a speed of 20m/s in 5 seconds. It then continues moving at the same speed for 3 seconds after which it accelerates constantly for 4 seconds and reaches a speed of 40m/s.

Using a scale of 2cm to represent 10m/s on the vertical axis and 2cm to represent 2 seconds on the horizontal axis, draw a speed-time graph to represent the motion of the train.

Plotting the graph

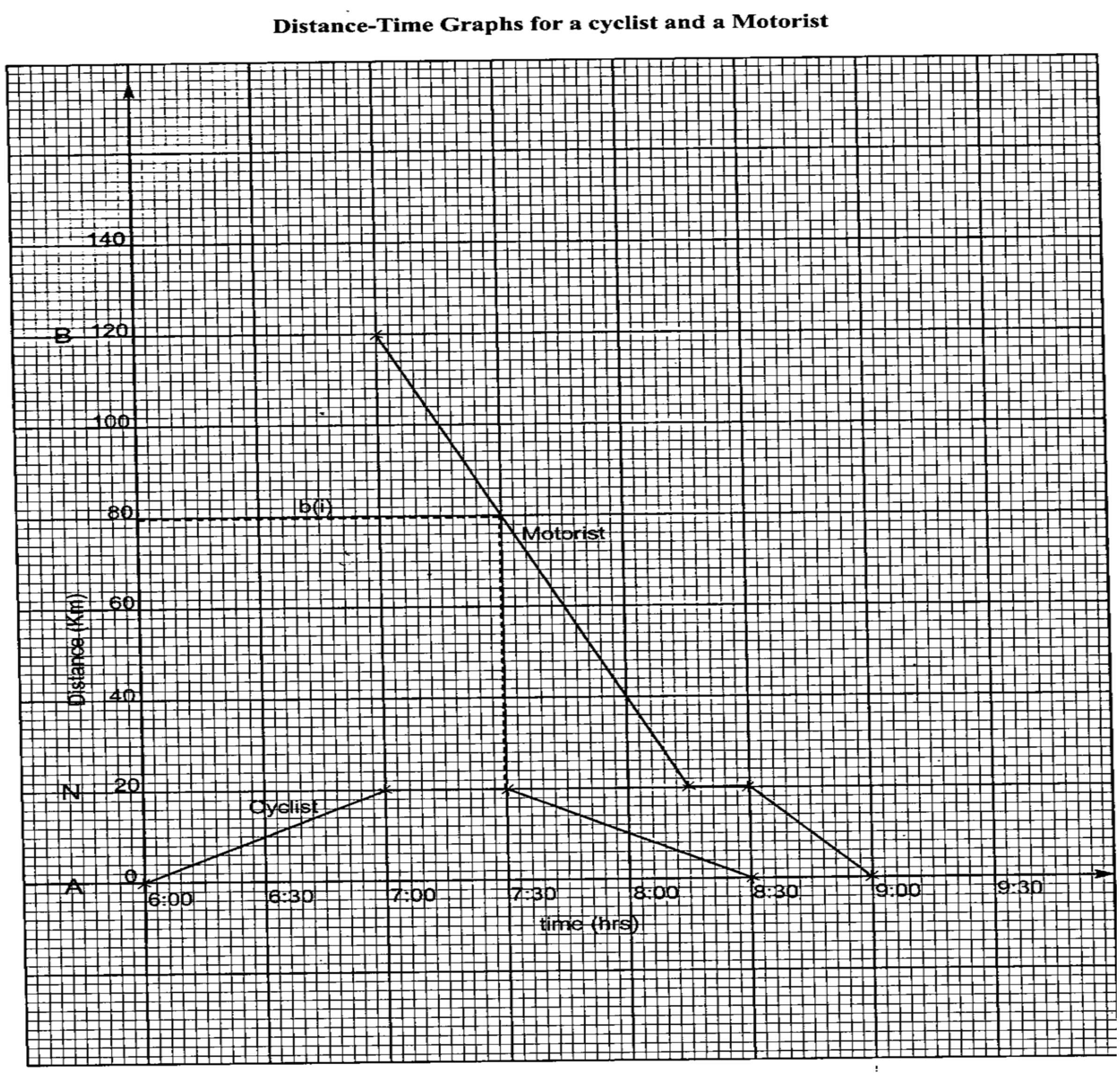


7. Villages X and Y are 120km apart. A cyclist leaves village X at 6.00am for Bwemba market 20km away towards Y travelling at an average speed of 20km/h. He spends 30 minutes at the market before returning to his home at X. He reaches home at 9.30am. A motorist leaves village Y for village X at 7.00am travelling at an average speed of 80km/h. She stops at Bwemba market for 15 minutes before proceeding to village X. She arrives at X at 9.00am.

a. Taking 2cm to represent 20km on the vertical axis and 2cm to represent $\frac{1}{2}$ h on the horizontal axis, and using the same axes, draw travel graphs of the cyclist and the motorist.
b. Using the graphs to answer the following questions
   (i) At what time was the motorist 80km away from the village?
   (ii) How far from the market was the motorist when the cyclist was leaving Bwemba market?
   (iii) When did the motorist overtake the cyclist?

SOLUTION

a. The graph



a. **The motorist was 80km away from X at 7:30am.**
b. **The cyclist left the market at 7:30 am , at this time, the motorist was 80km away from X. Hence the motorist was 80km-20km = 60km away from the market.**
c. **To find the time that the motorist overtook the cyclist, locate the intersection of the two which occurs at 8:48am.**
d. **Distance from Bwemba market to village X is 20km, and the time it takes for the motorist to travel from the market to X is $\frac{1}{2}$ h .**

Average speed = $\frac{Distance}{Time}$

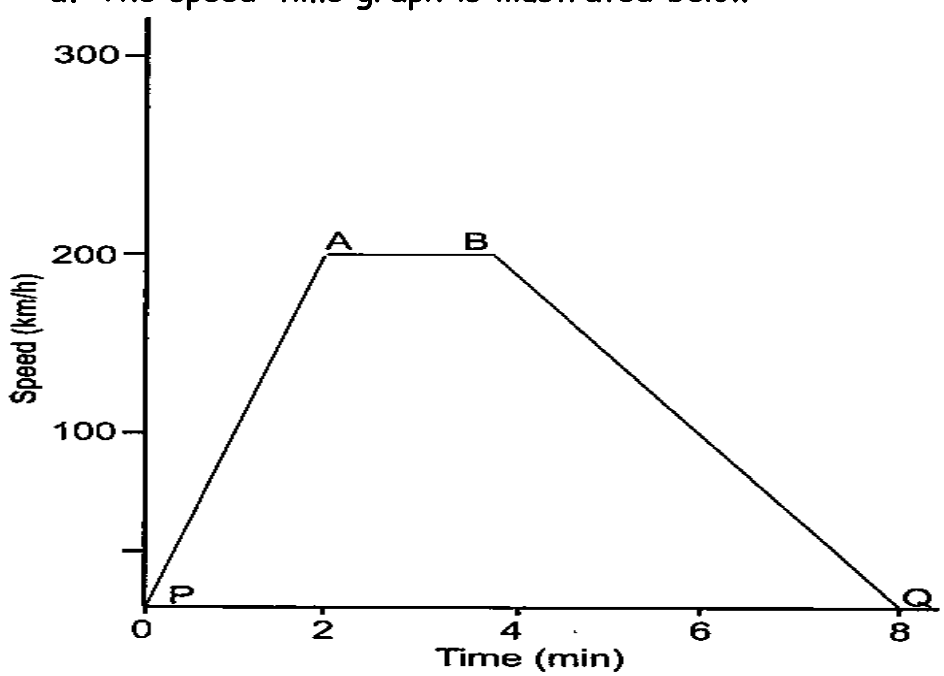
$=\frac{20km}{0.5h}$

= 40km/h

8. A vehicle travels from **P** to **Q** in 8 hours. It starts from rest at P increasing its speed steadily to 200km/hour in 2hours. It then travels at that speed for 1 hour. Finally the vehicle reduces its speed steadily until it stops, 5 hours later.

a. Sketch a speed-time graph of the vehicle.
b. Under the graph in a , calculate the distance the vehicle has travelled from **P** to **Q.**

a. **The speed-time graph is illustrated below**



b. On a speed-time graph, the distance is the area under the graph.

Distance = Area of trapezium PABQ.

$= \frac{1}{2}(AB + PQ)h$

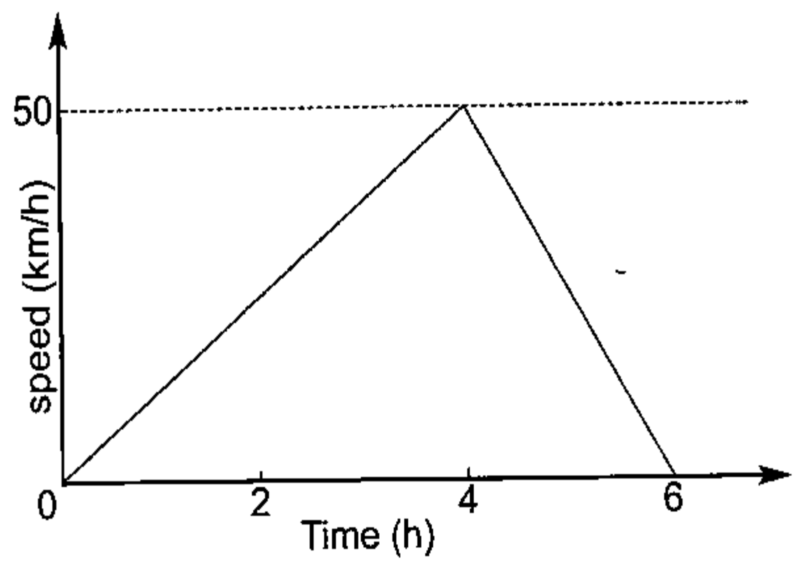
$= \frac{1}{2}(1 + 8)X\ 200$

$= \frac{1}{2}\ x\ 9x\ 200$

=900

∴ The distance travelled from P to Q is 900km

9. Figure shows a speed –time graph for a car during the first 6 hours.



Calculate the total distance travelled during 6 hours.

On a speed –time graph the total distance is the under the graph.
Since the graph is triangular in shape, then
Area = $\frac{1}{2}$ X Base X Height (Area of Triangle)
∴ Distance = $\frac{1}{2}$ x 6h x 50km/h
= 150km
∴ The distance travelled during the 6 hours is 150km.

10. Table A shows the times taken and the distances travelled by an express bus on its journey from Lilongwe to Mzuzu.

Distance travelled	Station	Times
0	Lilongwee	6.00am Dep

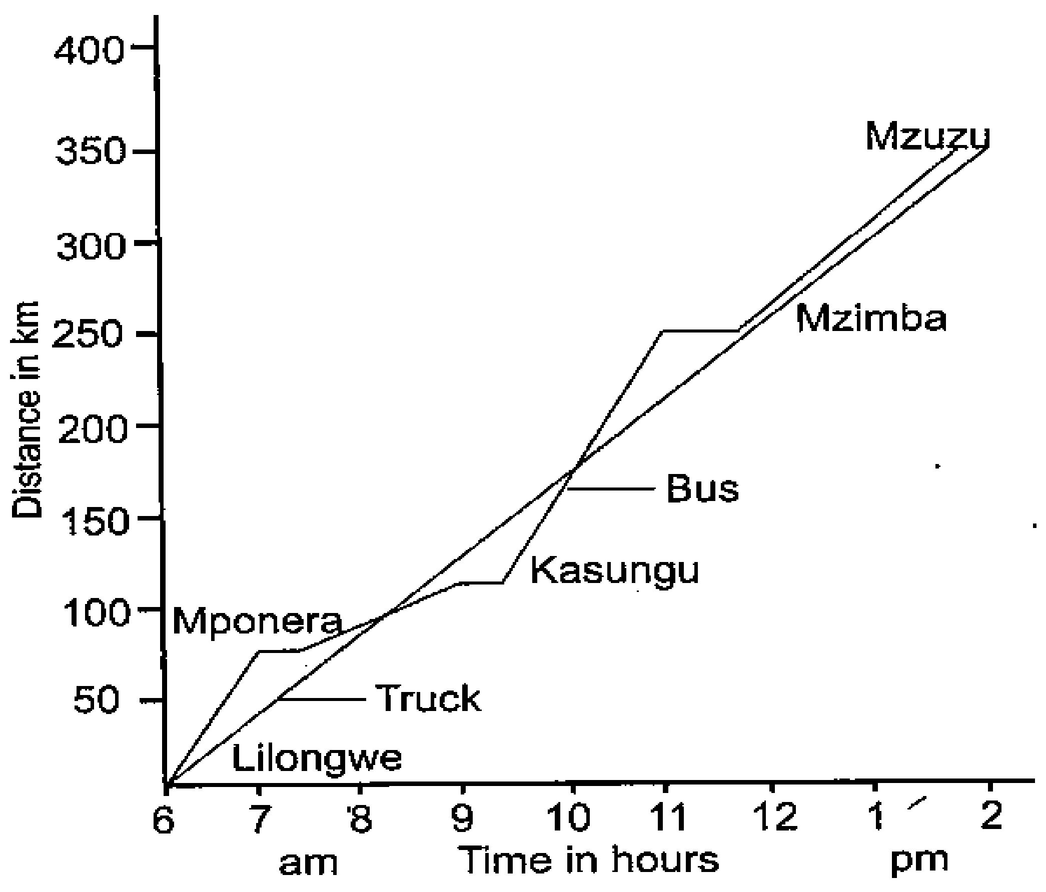
70	Mponela	7.00am Arriv 7.12 am
120	Kasungu	9.00am Arrive 9.21 am Dep
270	Mzimba	11.1am Arriv
360	Mzuzu	1.30 Aerrive

On the same day, trick driver left Lilongwe at the same time as the bus, and drove to Mzuzu at an average speed of 45km per hour.

Using a scale of 2cm to represent 1hour on the horizontal axis, and 2 cm to represent 50km on the vertical axis , draws graphs of the journey on the same set of axes.

b. Using your graphs to find when the bus overtook the truck.
c. Using your graphs to average speed of the bus between Lilongwe and Kasungu.

a. Travel graphs of a bus and a truck



b. The bus overtook the truck at 10.15 am, 90km away from Lilongwe.

c. Average speed of the bus between Lilongwe and Kasungu

Average speed $= \frac{\textbf{Total distance travelled}}{\textbf{Total time taken}}$

$= \frac{120km}{3h}$

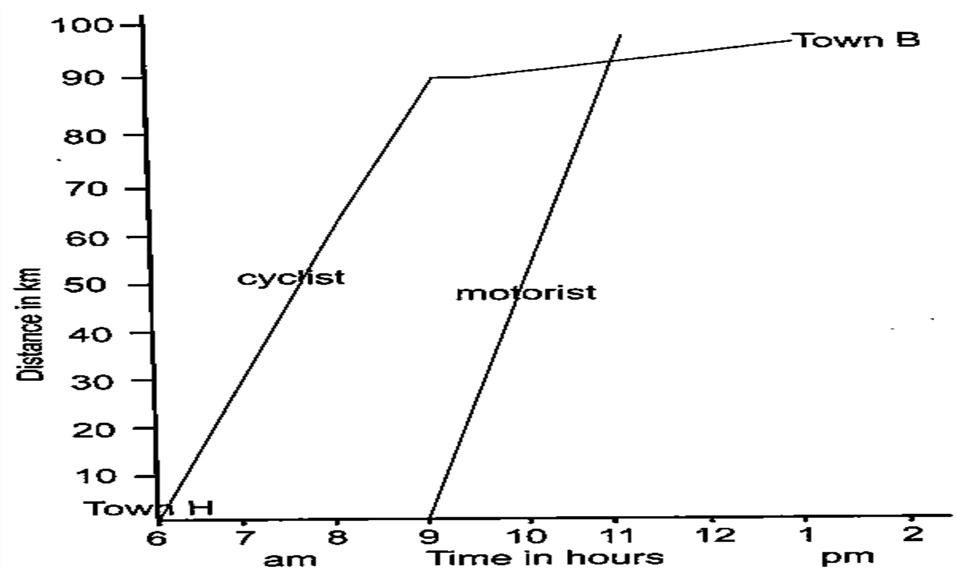
= 40km/hour

11. Town **H** and town **B** are 100kilometres apart. A cyclist leaves town H for town B at 6 a.m. at average speed of 30km/hour. After travelling for 3 hours the bicycle has a puncture, and for 30 minutes he tries to mend it but fails. He decides to walk the rest of the journey arriving at town **B** at 12.00 noon. A motorist leaves town **H** for town **B** at 9.00 a.m and travels at an average speed of 50km/hour.

a. Using a scale of 2cm to represent 10km on the vertical axis and 2cm to represent 1 hour on the horizontal axis, draw on the same axes the travel graphs of the cyclist and the motorist.

b. Use your graphs to answer the following questions
   (i) What was the walking speed of the cyclist after failing to mend the puncture?
   (ii) At what time did the motorist pass the cyclist?
   (iii) At what time did the motorist reach town **B**?
   (iv) What was the difference in the arrival times between the cyclist and the motorist?

c. Travel graphs showing the journeys of a cyclist and a motorist between towns H and B.



d. Using the graphs

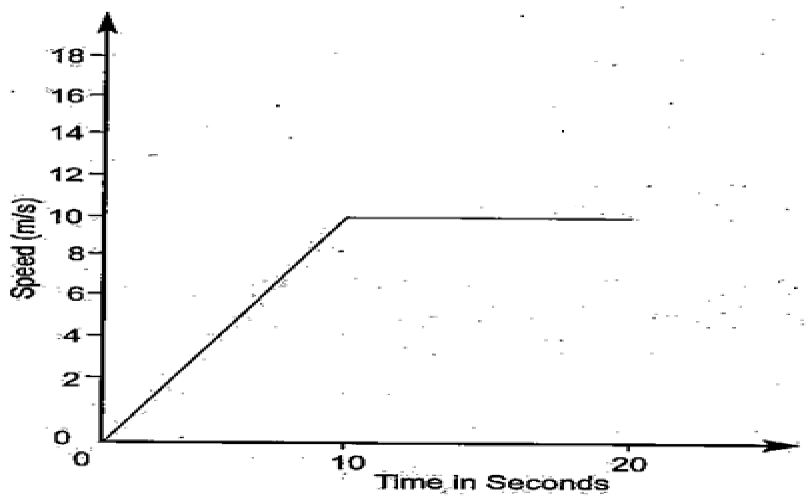
(i) It was 4km/hour

(ii) The motorist passed the cyclist at 10.54 a.m.

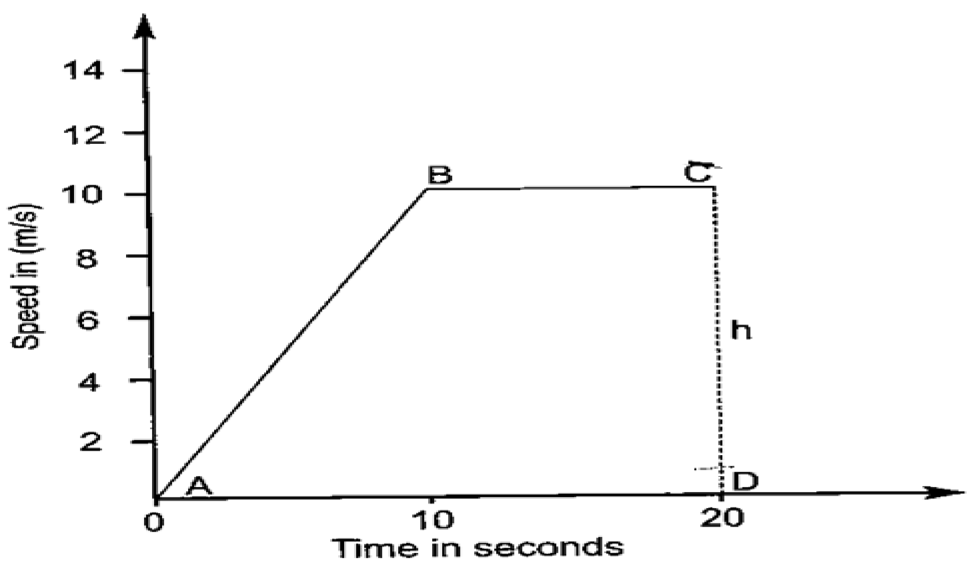
(iii) The motorist reached town B at 11.a.m.

(iv) The difference in the arrival times between the cyclist and the motorist was 1 hour.

12. Figure shows a speed-time graph for a particle during the first 20seconds of its motion.



Calculate the particle's average speed during the 20seconds.



Average speed = = $\frac{Total\ distance\ travelled}{Total\ time\ taken}$

Now distance = Area of trapezium ABCD.

= $\frac{1}{2}$ (BC + AD)h

= $\frac{1}{2}$ (10 +200) x 10

= 150m

Time taken = 20seconds

∴ Average speed = $\frac{150m}{20s}$

= 7 $\frac{1}{2}$m/s

The particles average speed during the 29 seconds is 7 $\frac{1}{2}$m/s

Note: On a speed-time graph, distance is the area under the graph

13. X and Y are two towns 200km apart. A lorry leaves X at noon and travels at 50km per hour for one hour. It then stops for 30 minutes; and then continues to Y at 60km per hour. A second lorry leaves Y at 12.30 pm and travels for one hour at 40km per hour. It then travels at a certain constant speed and reaches X at 4.30pm.

a. Using a scale of 2cm to represent 20km on the vertical axis and 4cm to represent 1 hour on the horizontal axis, draw graphs to illustrate the two journeys on the same axes.

b. Use your graph to find

(i) How far away from X the Lorries met. Give your answer to the nearest kilometer.

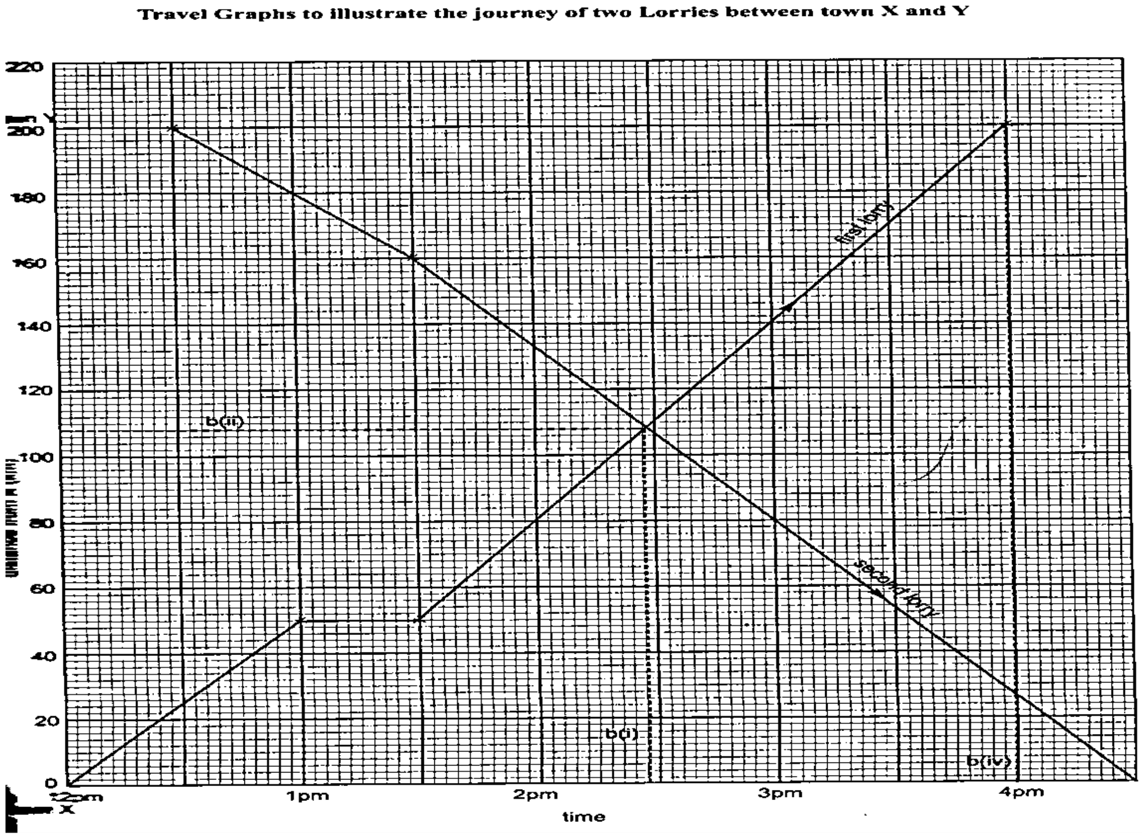
(ii) The time at which the Lorries met.

(iii) The speed at which the second lorry was travelling when the two Lorries met. Give your answer to the nearest kilometer per hour.

(iv) The time the first lorry arrived at Y.

SOLUTION

a. Graph



b. Using the graphs

(i) The lorries met 108 km away from X (to the nearest km)

(ii) The lorries met at 2.28pm (to the nearest minutes)

(iii) Speed of second lorry $= \frac{Distance}{Time}$

$= \frac{160km}{3hours}$

$= 53\ \frac{1}{3}$ **km/hour**

∴ Speed of the second lorry is 53km/h (to the nearest km/hr)

(iv) The first lorry arrived at Y at 4pm.

EXAMPLE

14. A motorist travelling at an average speed of 150km/h covers a section of road in 24 minutes. Another motorist takes 6 minutes longer for the same second of the road; calculate his average speed in kilometres per hour.

Distance travelled by 1st motorist = speed x time

= 150km x $\frac{24}{60}$**hours**

= 60km

Second motorist travels the same distance in a time= 24 minutes + 6minutes

=30 minutes or $\frac{1}{2}$ h

∴ Average speed of second motorist = $\frac{Distance}{time}$

= 60km ÷ $\frac{1}{2}$ h

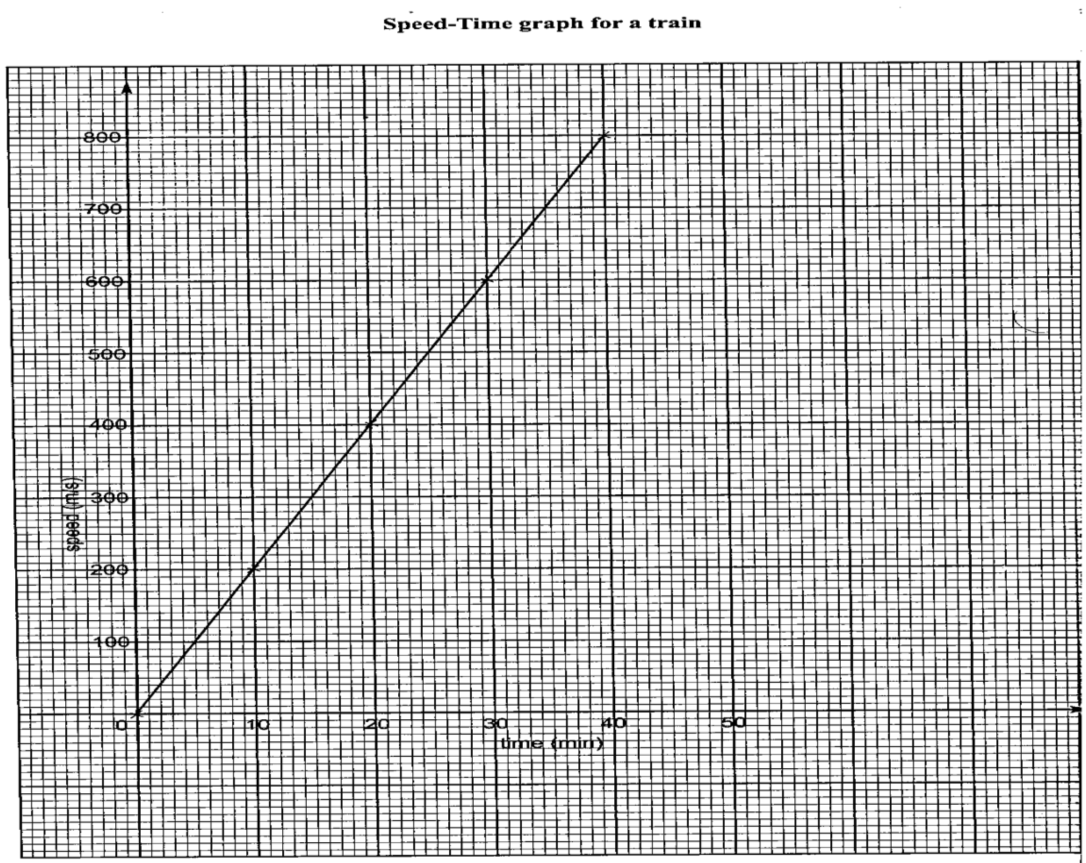
=120km/h

15. Table below shows the speed of a train recovered every 10 seconds.

Speed(m/s)	200	400	600	800
Time(s)	10	20	30	40

Using a scale of 2cm to represent 100m/s on the vertical axis and 2cm to represent 10 seconds on the horizontal axis, draw a speed-time graph and use it to calculate the acceleration of the train.

Plotting the graph



Acceleration = $\frac{100m/s - 0m/s}{40s - 0s}$

$$=\frac{800m/s}{40s}$$

= **200m/s/s**

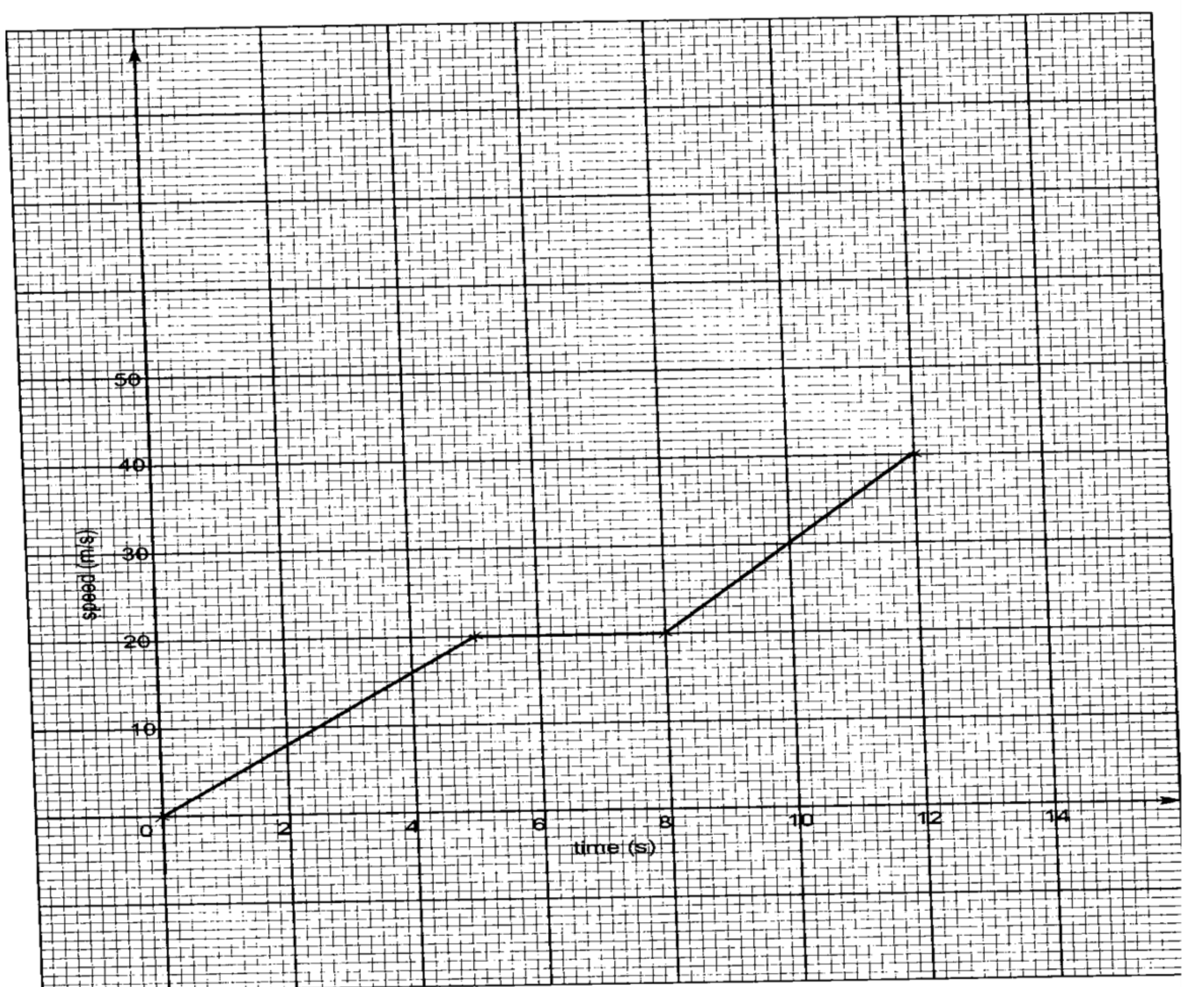
An object starts from rest and accelerates uniformly at $5,/s^2$

16. A train accelerates uniformly from rest and reaches a speed of 20m/s in 5 seconds. It then continues moving at the same speed for 3 seconds after which it accelerates constantly for 4 seconds and reaches a speed of 40m/s.

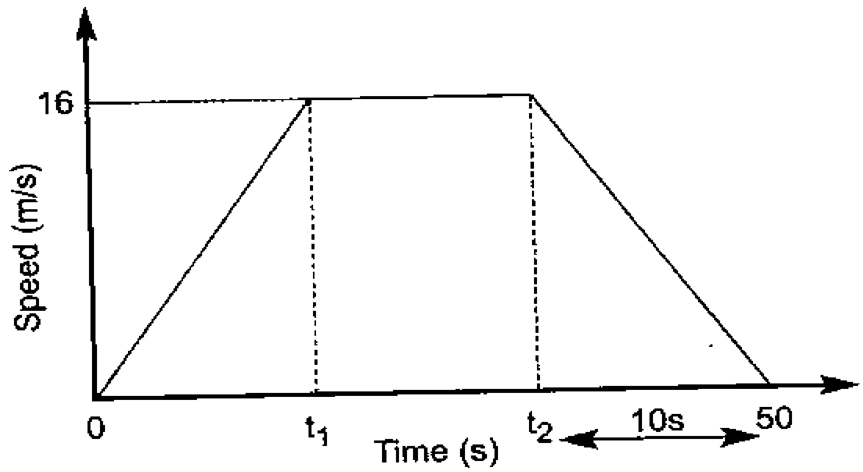
Using a scale of 2cm to represents 10m/s on the vertical axis and 2cm to represent 2 seconds on the horizontal axis draw a speed-time graph to represent the motion of the train.

SOLUTION

Speed-Time graph for a Train



17. Figure below is the speed time graph of a car. The car starts from rest and accelerates at 2m/s^2 for t_1 seconds its speed reaching 16m/s. It then maintains this speed for sometime after which it decelerates uniformly for 10seconds and stops.



Calculate the distance that the car travelled from time t_1 seconds to the t_2 when it started decelerating.

Distance travelled = Area under graph

Required to find time between t_1 and t_2

Time from O to t_1 = $\frac{16m/s}{2m/s}$

= 8seconds

Time between t_1 and t_2 = 50-(8+10)

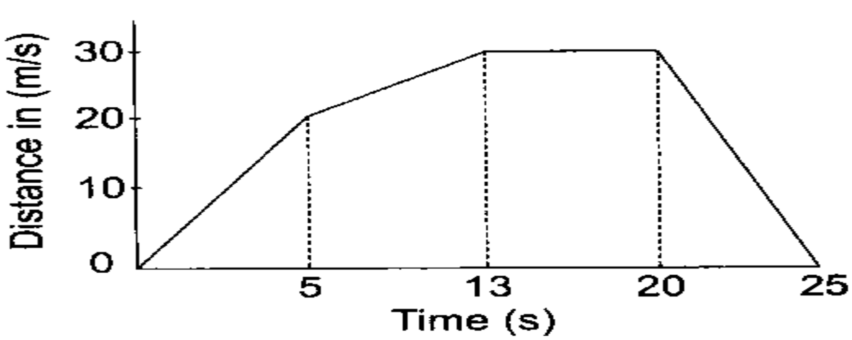
= 32 seconds

∴Distance between t_1 and t_2 = $\frac{16m}{s}\ x\ 32s$

= 512m

Distance travelled from time t_1 and t_2 = 512m

18. **Figure 3 shows a distance –time graph of an object.**



Calculate the speed of the object between t=5 and t = 13.

On a Distance/time graph, speed is the gradient

Speed = gradient = $\frac{Change\ in\ distance}{Chane\ in\ time}$

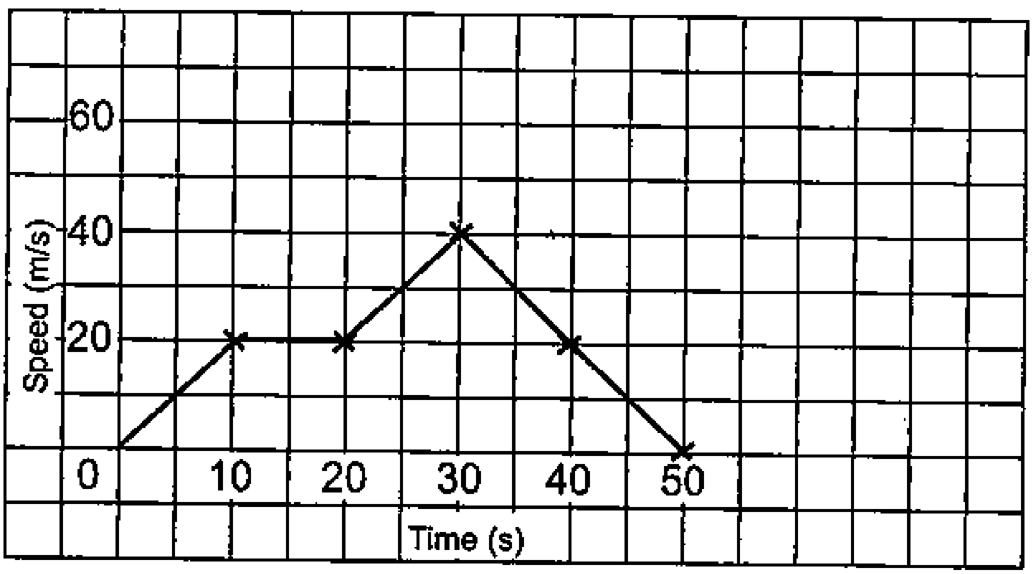
Speed = $\frac{30m-20m}{13s-5s}$

$= \frac{10m}{8s}$

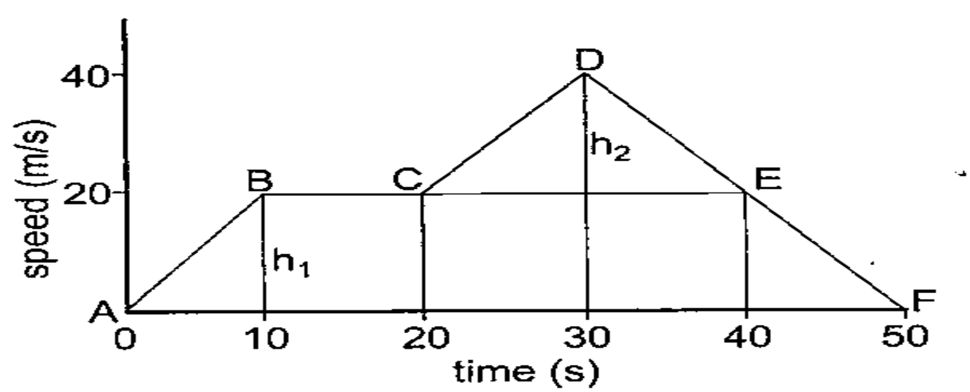
= **1.25m/s**

The speed of the object between t=5 and t= 13 is 1.25m/s

19. Figure 2 shows a speed-time graph of an object.



SOLUTION



Distance travelled = Area of the figure ABCDEF

Area of trapezium ABEF + Area of triangle DCE

Area of a trapezium ABEF = $\frac{1}{2}(BE + AF)h_1$

E = 40-10 =30

AF =50-0

h_1**= height = 20-0**

Area of trapezium ABEF = $\frac{1}{2}(30 + 50)X20$

= 80 X 10

= 800

Area of triangle DCE = $\frac{1}{2}$ **X CE X** h_2

CE = 40-20 = 20

h_2**= height of triangle DCE**

40-20 = 20

Area of triangle DCE = $\frac{1}{2}$ **X 20 X 20**

= 200

Area of the figure ABCDEF = 800+ 200

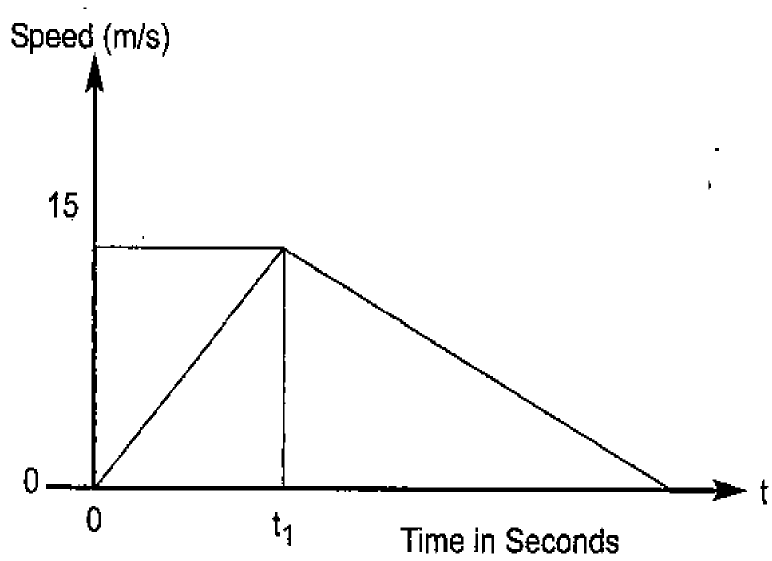
= 1,000

Distance travelled = 1,000m

= 1km

∴ The distance covered by the object is 1km

20. Figure 2 is a speed –time graph of a moving object.



Given that the acceleration for the first t_1 is 0.3ms^2, calculate the value of t_1.

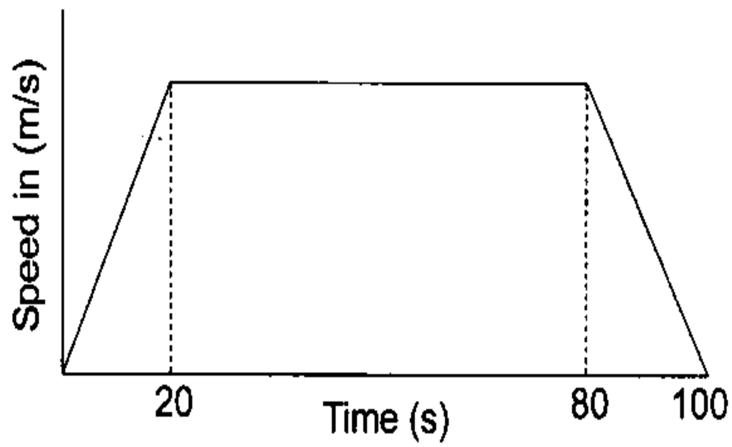
Acceleration = $\frac{\textit{Change in speed}}{\textit{Change in time}}$

0.3m/s/s = $\frac{\frac{15m}{s} - 0}{t_1}$

= $\frac{15m/s}{0.\frac{3m}{s}/s}$

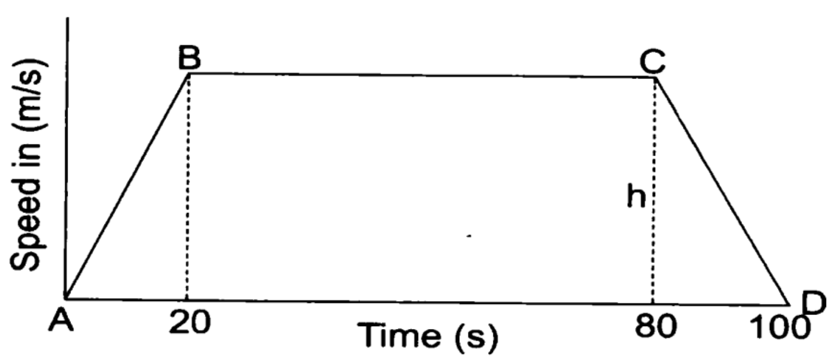
= **50s**

21. Figure shows a speed-time graph of a car.



Given that the total distance travelled is 3,200m., calculate the deceleration during the last 20 seconds.

SOLUTION



Distance travelled = area under the speed-time graph.

= Area of the trapezium ABCD

= $\frac{1}{2}\ (BC + AD)h$

3,200m = $\frac{1}{2}$ $(80 - 20) + (100 - 0)h$

3,200m = $\frac{1}{2}$ $(60 + 100)h$

3200m = $\frac{1}{2}$ $(160)h$

80h = 3200m

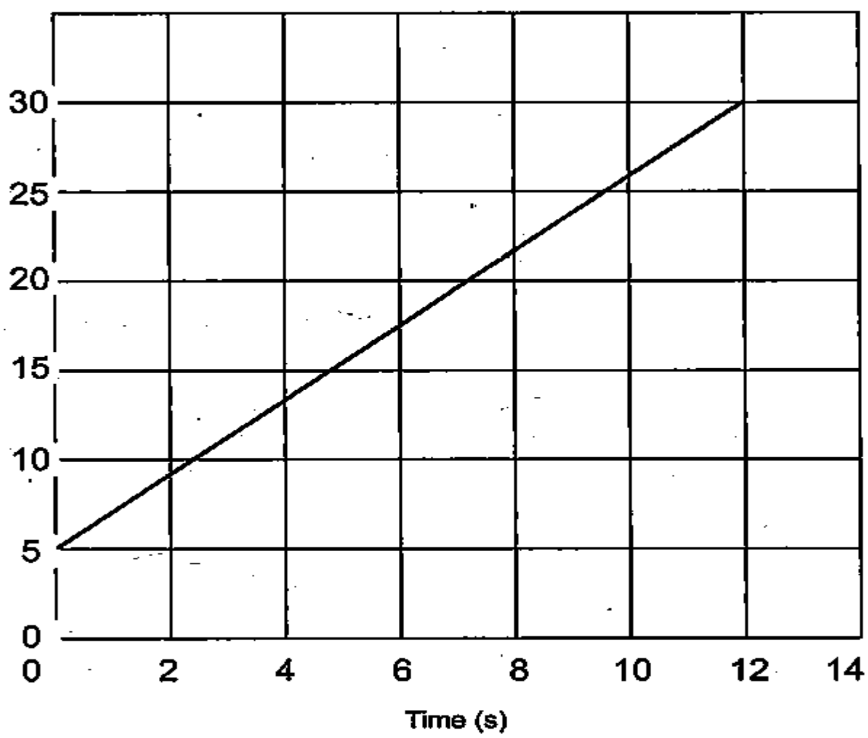
H=40m/s

Deceleration = gradient of CD

= $\frac{0-40m/s}{100s-80s}$

=-2m/s^2

22. Figure below shows the speed-time graph of a moving object.



Use the graph to find the acceleration of the object.

Acceleration = Chane in velocity ÷ Change in time

By reading of values from the graph

Acceleration = $\frac{30m/s-5m/s}{12-0s}$

= 25m/s ÷ 12s

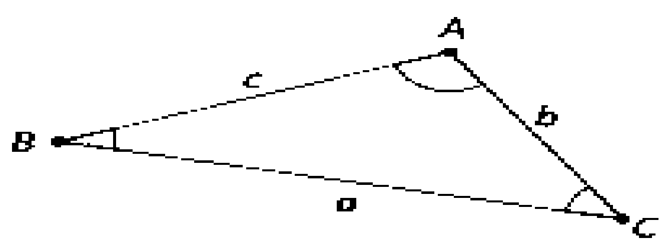
= 2.8m/s/s

CHAPTER 7: TRIGONOMETRY - SINE RULE

WHEN DO YOU DECIDE TO USE SINE RULE

1. The Sine Rule is important because it is used to solve problems about triangles which are not right- angled triangles.
2. If we know two angles of a triangle and one of the sides, then use the sine rule to find the other side.
3. If we know two sides of a triangle and an angle which is not between them, then use the sine rule to find another angle. There may be two possible answers.

Sine Rule or Law of Sines



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example

In triangle ABC, BC = 7cm, angle ABC = 40^O and angle ACB = 55^O.

Required

Find AC.

Angle A = $180^O - 40^O - 55^O$

= 85^O

a = 7cm

Using Sine Rule, $\frac{b}{\sin 40^O} = \frac{7}{85^O}$

b = $\frac{7sin40^O}{\sin 85^O}$

= 4.52cm to 3 s.f.

EXAMPLE

In triangle PQR, PQ= 8cm, QR = 11cm and angle P =55^o. Find angle R.

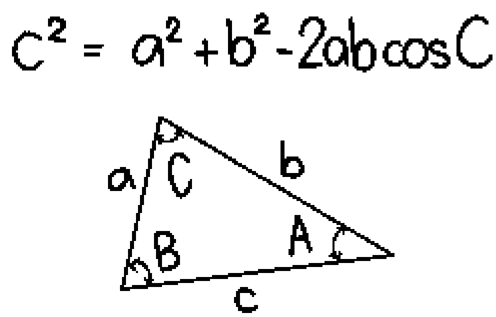
Using Sine Rule, $\frac{8}{\sin R} = \frac{11}{55^O}$

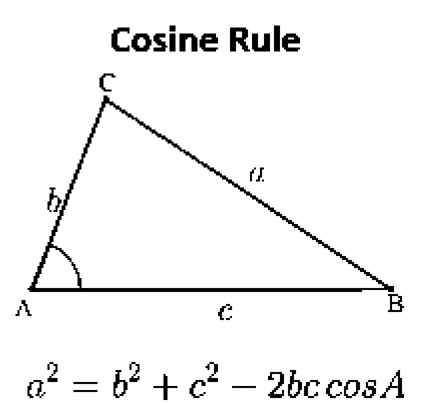
Sin R = $\frac{8sin55^O}{11}$

= 0.5958

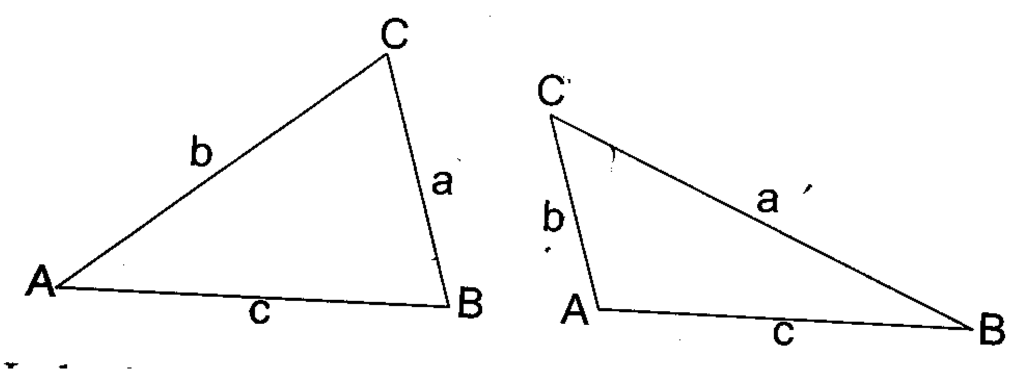
= 36.6° (By using calculator)

COSINE RULE





FINDING SIDE BC=a FROM THE FIGURE BELOW



Cos A = $\frac{b^2 + c^2 - a^2}{2bc}$

Cos B = $\frac{a^2 + c^2 - b^2}{2ac}$

Cos C = $\frac{a^2+b^2-c^2}{2ab}$

When do you decide to use cosine rule

1. The Cosine Rule is important because it is used to solve problems about triangles which are not right- angled triangles
2. Ife we know the three sides of a triangle then use the cosine rule to find the angles **- SSS**
3. If we know two sides of a triangle and the angle between them then use the cosine rule to find the third side- **SAS.**

Example

Three sides of a triangle are 7cm, 8cm and 9cm long. Find the angle opposite the longest side.

Cos A = $\frac{b^2+c^2-a^2}{2bc}$

Cos A = $\frac{7^2+8^2-9^2}{2x7x8}$

=0.2857

Cos A = 73.4^o

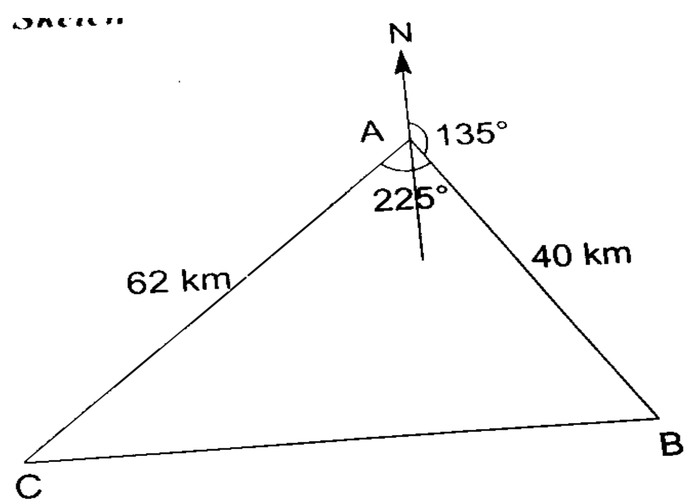
BEARINGS

Village B is on bearing of 135^o and a distance of 40km from village A. Village C is on bearing of 225^o and a distance of 62km from the village A.

a. Show that A, B and C form a right angled triangle.

b. Calculate angle ACB to the nearest degree.

Sketch



i Need to show that one of the angles o
ΔABC is a right angle.

Reflex $\angle NAC = 225°$

Obtuse $\angle NAB = 135°$

$\angle BAC$ = Reflex $\angle NAC$ – Obtuse $\angle NAB$

$= 225° - 135°$

$= 90°$, which is a right angle.

A, B, C form a right-angled triangle.

ii Calculating $\angle ACB$:

Now, $\tan \angle ACB = \frac{AB}{AC}$

$\tan \angle ACB = \frac{40 \text{ km}}{62 \text{ km}}$

$\tan \angle ACB = \frac{40}{62}$

$\angle ACB = \tan^{-1}) \left(\frac{40}{62}\right)$

$= 33°$ (*to the nearest degree*)

$\therefore$ The value of angle ACB is $33°$.

CHAPTER 8: POLYNOMIALS-REMAINDER THEOREM

The remainder theorem states that when f(x) is divided by (x + a) is f(-a)

1. **Use the remainder theorem to find the remainder when f(x) = $x^3 - 3x^2 + 6x + 5$ is divided by (x-2).**

The remainder is f(2)

If x = 2, $x^3 - 3x^2 + 6x + 5 = 2^3 - 3 \times 2^2 + 6 \times 2 + 5$

= 8-12 +12 + 5

=13

2. **Find the remainder when f(x) = $x^3 + 2x^2 - 5x - 6$ is divided by (x +2).**

If x + 3 = 0, x = -3

F(-3) = $(-3)^3 + 2(-3)^2 - 5(-3) - 6$

= -27 +18 +15 -6

=0

The remainder is 0.

FACTOR THEOREM

Factor theorem states that when polynomial f(x) is divided by (x+a), the remainder is f(-a). This is because f(x) = (x+a)Q(X) +R.

If f(-a) = 0, the remainder is 0, and f(x) = (x + a)Q(x) and (x+a) is a factor of f(x).

1. **Use the factor theorem to find a factor of the expression F(x) = $4x^3 - 20x^2 19x + 15$ and hence factorise it.**

We considers the factors of 15 which include $\pm 1, \pm 3, \pm 5$ *and* ± 15

F(3) = $4(3)^3 - 20(3)^2 19(3) + 15$

= 108 -180 +57 +15

= 0

So (x-3) is a factor of f(x).

$$
\begin{array}{r}
4x^2 - 8x - 5 \\
x-3\sqrt{4x^3 - 20x^2 19x + 15} \\
\underline{4x^3 - 12x^2} \\
-8x^2 + 19x \\
\underline{-8x^2 + 24x} \\
-5x + 15 \\
-\underline{5x + 15} \\
0
\end{array}
$$

$4x^3 - 20x^2 19x + 15 = (x-3)(4x^2 - 8x - 5)$

= (x-3)(2x +1)(2x-5)

2. One of the factors of $x^3 + -qx - 20$ is x + 4. Find
a. the value of q.
b. The other two linear factors.

$x^3 + -qx - 20$= 0 when x =-4 if x+4 is a factor, then

$(-4)^3 + -q(-4) - 20$= 0

-64-4q-20 =0

4q=-84

q = -21

$$x^2 \quad - 4x \;\; \text{-5}$$

$$\text{X+4}\sqrt{x^3 \qquad - 21x - 20}$$

$$- \underline{(x^3 + 4x^2)}$$

$$- 4x^2 - 21x$$

$$-\underline{(-4x^2 - 16x)}$$

$$-5x \;\; +15$$

$$-\underline{(5x \; +15)}$$

$$0$$

The other linear factors $x^2 - 4x$ **-5 = (**$x^2 + x - 5x - 5)$

= x(x+1)-5(x+1)

= (x+5)(x+1)

The other two factors are x +5 and (x+1).

3. Use the Remainder Theorem to factorise $x^3 - 3x^2 - 6x + 8$, and hence solve the equation $x^3 - 3x^2 - 6x + 8 = 0$.

Factors of 8 are$\pm 1, \pm 2, \pm 4$ *and* ± 8

Substitute x = 1 in the f(x)

F(1) = $(1)^3 - 3(1)^2 - 6(1) + 8$

= 1-3 -6 +8

=9-9

= 0

Therefore, x-1 is the factor of the polynomial.

To find other linear factors using division method

$$x^2 - 2x - 8$$

$$\text{x-1}\sqrt{x^3 - 3x^2 - 6x + 8}$$

$$\underline{x^3 - x^2}$$

$$-2x^2 - 6x$$

$$\underline{-2x^2 + 2x}$$

$$-8x \;\; +8$$

$$-\underline{8x + 8}$$

$$0$$

The other two linear factors are $x^2 - 2x - 8$**=**$x^2 + 2x - 4x - 8$

=x($x^2 + 2x) - 4(4x - 8)$

= x(x+2)-4(x+2)

=(x-4)(x+2)

∴(x-1) =(x-4)(x+2) =0

x=1 or x=4 or x=-2

4. Show that x + 1 is a factor of $x^3 - x^2 - 10x - 8$ and hence solve the equation $x^3 - x^2 - 10x - 8 = 0$.

Substitute x=-1 into the f(x)= $\mathbf{x^3 - x^2 - 10x - 8}$

F(-1)= $(-1)^3 - (-1)^2 - 10(-1) - 8$

= -1-1+10-8

= 10-10

=0

Therefore x+1 is the factor of $\mathbf{x^3 - x^2 - 10x - 8}$

To solve the equation $\mathbf{x^3 - x^2 - 10x - 8 = 0}$**, hence the need to find other linear factors of** $\mathbf{x^3 - x^2 - 10x - 8}$

$$\begin{array}{r} x^2 - 2x - 8 \\ x+1\overline{\big)\, x^3 - x^2 - 10x - 8} \\ \underline{x^3 + x^2} \\ -2x^2 - 10x \\ \underline{-2x^2 - 2x} \\ -8x \;\; -8 \\ -\underline{8x \; -8} \\ 0 \end{array}$$

The other factors linear factors : $\boldsymbol{x^2 - 2x - 8}$ **=** $\boldsymbol{x^2 + 2x - 4x - 8}$

= x($\boldsymbol{x^2 + 2x) - 4(-4x - 8)}$

= x(x+2)-4(x+2)

= (x-4)(x+2)

(x+1)(x-4)(x+2) =0

X =-1, or x=4 or x =-2

5. Given that 3x-1 is a factor of $6x^3 + ax^2 + x - 2$,
   a. Find the value of a, and
   b. Use the value of a to solve the equation $6x^3 + ax^2 + x - 2 = 0$

Substitute x =$\frac{1}{3}$ **into the f(x)=** $\mathbf{6x^3 + ax^2 + x - 2}$,

$\mathbf{6\,(\tfrac{1}{3})^3 + a(\tfrac{1}{3})^2 + \tfrac{1}{3} - 2}$,

$\frac{2}{9} + \frac{a}{9} + \frac{1}{3} - 2 = 0$ **(multiply all terms by 9**

2 +a+3-18=0

a-13 =0

a=13

By inspection, $6x^3 + 13x^2 + x - 2$, **=(3x-1)($2x^2 + 5x + 2$)**

0 = (3x-1)(2x+1)(x+2)

3x-1 =0, x =$\frac{1}{3}$

2x+1 = 0, x =$-\frac{1}{2}$

X =-2

6. Show that 2x-1 is a factor of $8x^3 + 20x^2 - 2x - 5$. Hence find the other two linear factors of the expression. What are the values of x for which $8x^3 + 20x^2 - 2x - 5 = 0$?

a. Substitute x =$\frac{1}{2}$ into a f(x)

$$8(\frac{1}{2})^3 + 20(\frac{1}{2})^2 - 2(\frac{1}{2}) - 5 = 0$$

1+5-1-5 =0

6-6 =0

Therefore, 2x-1 is the factor of expression $8x^3 + 20x^2 - 2x - 5$

b. $8x^3 + 20x^2 - 2x - 5$ **= (2x-1)($4x^2 + 12x + 5$) by inspection method**

= (2x-1)(2x+5)(2x+1)

The other two factors are 2x+5 and 2x+1

c. $8x^3 + 20x^2 - 2x - 5 = 0$

(2x-1)(2x+5)(2x+1) =0

∴ 2x-1=0, x =$\frac{i}{2}$, 2x =-5,x =-2.5,

$2x + 1 = 0$, x =-0.5

7. If the expression $2x^3 + px^2 - 8x - 3$ is exactly divisible by 2x+1, use the Remainder Theorem to find the value of p. If p has this value, factorise completely.

Since 2x+1 is a factor, then x =$-\frac{1}{2}$,

$$2\left(\frac{-1}{2}\right)^3 + p\left(\frac{-1}{2}\right)^2 - 8\left(\frac{-1}{2}\right) - 3 = 0$$

$\frac{-1}{4} + \frac{p}{4} + 1 = 0$ **(Multiply all terms by 4p**

-1 +p+4 =0

p+3 =0

P=-3

The expressions now becomes

$\mathbf{2x^3 - 3x^2 - 8x - 3}$

By inspection

$\mathbf{2x^3 - 3x^2 - 8x - 3}$ **=(2x+1)**$(x^2 - 2x - 3)$

= (2x+1)(x-3)(x+1)

8. If x-2 is a factor of $2x^3 - 7x^2 + 10x + K$, find the value of K.

Since x-2 is a factor, then x =2

$\mathbf{2(2)^3 - 7(2)^2 + 10(2) + K}$**=0**

16-28+20+K =0

36-28 +K=0

K =-8

9. Find the values of k and b in the expression $x^3 - kx^2 - bx + 2$ given that x-1 and x + 1 are factors of the expression.

Since x-1 is a factor, then x =1

$\mathbf{(1)^3 - k(1)^2 - b(1) + 2}$ **=0**

1-k-b+2 =0

k+b=3(1)

Since x+1 is a factor, when x =-1

$\mathbf{(-1)^3 - k(-1)^2 - b(-1) + 2}$ **=0**

-1-k+b+2 =0

k-b =1 (2)

Solve equations (1) and (2) simultaneously

k+b =3(1)

k-b =1 (2)

Add(1) and (2)

2k =4

K =2

Substitute k=2 into (1) to find b.

2+b =3

b=1

10. If 5a +2 is a factor of $5a^3 - 8a^2 - 19a - 6$, find the other factors.

$$\begin{array}{r l} & \boldsymbol{a^2 - 2a - 3} \\ \mathbf{5a+2} & \overline{)\ \mathbf{5a^3 - 8a^2 - 19a - 6}} \\ & -(\ \underline{5a^3 + 2a^2}) \end{array}$$

-$10a^2 - 19a$

-($\underline{-10a^2 - 4a}$)

-15a-6

(-$\underline{15a\ -6}$)

0

The other two linear factors: $a^2 - 2a - 3$ **=**$a^2 + a - 3a - 3$

= a(a+1)-3(a+1)

=(a-3)(a+1)

The others are a-3 and a+1.

11. When a polynomial, $x^3 + px^2 - 3x + 4$.is divided by x + 2, the remainder is 14. Calculate the value of P.

If x =-2; $(-2)^3 + p(-2)^2 - 3(-2) + 4 = 14$

-8+4p+6+4=14

4p+2=14

4p=14-2

4p=12 (divide both sides by 4)

P = 3

12. The remainder when x(x+b)(x-2b) is divided by x-b is -16. Find the value of b.

If x**=b; b(b+b)(b-2b)=-16**

2b^2x-b =-16

-2b^3= -16 (divide both sides by -2)

b^3= 8

b =$\sqrt[3]{8}$

b =2

13. When a polynomial $x^3 + kx^2 + x - k$ is divided by (x-k) the remainder is 2. Calculate the value of k.

Given that x-k is a factor, x =k

Substitute k for x in the f(k)

$\mathbf{k^3 + k\,(k^2) + k - k}$ **= 2**

$\mathbf{k^3 + k^3 + k - k}$ **= 2**

2 $\mathbf{k^3}$ =2 (divide both sides by 2

k^3 =1

K = $\sqrt[3]{1}$

K =1

The value of k is 1

14. Find the remainder when $2x^3 - 13x^2 - 8x + 12$ is divided by 2x-1.

Given the polynomial $2x^3 - 13x^2 - 8x + 12$ using the remainder theorem,

Let 2x-1 = 0, x = $\frac{1}{2}$

$$2(\frac{1}{2})^3 - 13(\frac{1}{2})^2 - 8(\frac{1}{2}) + 12 = 0$$

$\frac{1}{4}$ - $\frac{13}{4}$ + -4 +12 = 0

$\frac{1}{4}$ - $\frac{13}{4}$ + 8 = 0

$\frac{1-13+32}{4}$ =0

$\frac{-12+32}{4}$ =0

$\frac{20}{4}$ =0

5

The remainder is 5

15. $m^2 - m - 2$ is a factor of $m^3 - 2m^2 - pm + c$.When the polynomial is divided by m + 2 the remainder is -12. Find the values of p and c.

Given $m^2 - m - 2 = 0$

$m^2 + m - 2m - 2$ = 0

m(m+1)-2(m+1) =

(m-2)(m+1) =0

Either m-2 =0 or m+1 =0

M=2 or m=-1

Substitute -1 for m in $m^3 - 2m^2 - pm + c$

$-1^3 - 2(-1)^2 - p(-1) + c$ =

-1-2+p+c =0

P+c =3 (1)

Substitute 2 for m in $m^3 - 2m^2 - pm + c$ = 0

$(2)^3 - 2(2)^2 - p(2) + c$ =0

8-8-2p +c =0

2p-c=0 (2)

Solving the two equations for p and c

P +c =3

+ (2p-c =0)

3p =3 (divide both sides by 3)

P=1

Substitute for p in (1)

P+c =3

1+c =3

c=3-1

c=2

The value of p is 1 and the value of c is 2.

16. Use the remainder theorem to prove that (x-y) is a factor of the polynomial x^2(y-2) +y^2(2-x).

x=y, substitute in f(y)= x^2(y-2) +y^2(2-x).

$(y)^2$**(y-2) +y^2(2-y)=0**

$y^3 - 2y^2$ + $2y^2$-y^3=0

$y^3 - y^3 + 2y^2$ - $2y^2$ =0

0 = 0

Therefore x-y is the factor of the expression x^2(y-2) +y^2(2-x).

17. If (x+1) is a factor of $x^3 - 5x^2 + 2x + 8$, find the other two factors hence find the roots of the equation $x^3 - 5x^2 + 2x + 8 = 0$.

Finding other factors of $x^3 - 5x^2 + 2x + 8$ given x+1 is a factor.

$\boldsymbol{x^2 - 6x + 8}$

(x+1)$\sqrt{x^3 - 5x^2 + 2x + 8}$

-($\underline{x^3 + x^2}$)

-6$x^2 + 2$

-($\underline{6x^2 - 6x}$)

8x+8

(8x +8)

0

The other two linear factors:

$x^2-6x+8 = x^2-2x-4x+8$

= x(x-2)-4(x-2)

=x-4)(x-2)

The other two factors are x-4 and x-2

Finding the roots of the equation x^3-5x^2+2x+8 =0

(x+1)(x-4)(x-2) =0

X=-1,x=4 or x=2

18. Given that $(4x^2-9)(Bx+C)$ is identical to $16x^3$ $+24x^2-36x-54$, calculate the values of B and C.

Since $(4x^2-9)(Bx+C) \equiv 16x^3$ $+24x^2-36x-54$,

$4x^3B+4x^2C$-9Bx-9C$) \equiv 16x^3$ $+24x^2-36x-54$,

$4x^3B$=$16x^3$(Divide both sides by $4x^3$

B =4

$4x^2C$=$24x^2$ (divide both sides by$4x^2$

C=6

9Bx =36x (Divide both sides by 9x)

B=4

-9C =-54 (divide both sides by -9)

C = 6

Therefore, the value of B is 4 and the value of C is 6.

19. When the polynomial ax^2+bx+c is divided by (x +1) and (x+3) it gives a remainder of 2 in each case. Find the polynomial.

Given the polynomial ax^2+bx+c

Since the polynomial leaves a remainder of 2 when divided by (x+1)and (x+3) in each case then

(x+1)(x+3)+remainder = ax^2+bx+c

$x^2+3x+x+3+2$= ax^2+bx+c

x^2+4x+5= ax^2+bx+c

Equating coefficients

a$x^2=ax^2$, a=1

4x=bx, b=4

5=c, the c=5

20. Solve the equation $2x^3 - 5x^2 + x + 2 = 0$

The factors of 2 are ± 1 and ± 2

Let x =1

Substituting 1 for x in the polynomial gives

$$2(1)^3 - 5(1)^2 + (1) + 2 = 0$$

2-5+1+2 =0

5-5=0

0=0

Therefore, x=1 is a solution of the equation and hence taking 1 to the left hand side gives(x-1)as a factor of the expression $2x^3 - 5x^2 + x + 2$

Γ8Dividing the expression by the factor

$$2x^2 - 3x - 2$$

x-1 **(x+1)**$\sqrt{\mathbf{2x^3 - 5x^2 + 2x + 8}}$

-$\underline{(\ \mathbf{2x^3 + 2x^2})}$

-3$\mathbf{x^2 + 2}$

-($\underline{\mathbf{-3x^2 + 3x}}$ **)**

-2x+2

-(-2x +2)

0

$\mathbf{2x^3 - 5x^2 + 2x + 8} = (\mathbf{x - 1})\ 2x^2 - 3x - 2$

$=(\mathbf{x - 1})\ 2x^2 + x - 4x - 2$

=(x-1)x(2x+1)-2(2x+1)

= (x-1)(x-2)(2x+1)=0

Either (x-1)=0,x=1

(x-2) =0,x=2

(2x+1)=0, x=-$\frac{1}{2}$

21. When $x^3 + 5x^2 + Kx + 3$ is divided by (x+2), the remainder is 1. Use the Remainder Theorem, find the value of K.

Let x+2 =0, x=-2

Substituting -2 for x in the polynomial

$(-2)^3 + 5(-2)^2 + \mathbf{K}(-2) + 3$**=1**

-8+20-2K+3=1

15-2K =1

-2k=1-15

-2k =-14 (divide both sides by -2

K =7

∴ The value of K is 7

22. If (y+2) and (y-3) are factors of $2y^3 + by^2 + cy - 6$, find the values of b and c.

Since (y+2) and (y-3) are factors, then using the factor theorem,

Let y+2 =0, y=-2

$\mathbf{2(-2)^3 + b(-2)^2 + c(-2) - 6}$ **=0**

-16 +4b-2c-6=0

4b-2c-22 =0

4b-2c=22

Let(y-3)=0,y=3

$\mathbf{2(3)^3 + b(3)^2 + c(3) - 6}$

54+9b+3c-6=0

9b+3c+48=0

9b+3c=-48

We have two equations as follows

4b-2c=22 (1)

9b+3c=-48 (2)

(1)x9 and(2)x4

36b-18c=198 (3)

36b+12c=-192 (4)

(3)-(4)

-30c=390

C=-13(divide both sides by -30)

Substitute -13 for c in (1)

4b-2(-13) =22

4b+26 =22

4b=-4 (divide both sides by 4)

B=-1

The value of b is -1 and the value of c is -13

23. Given that (x+3)$(x+1)^2 \equiv Ax^3 + Bx^2 + Cx + D$, find the value of C.

(x+3)$(x+1)^2 \equiv Ax^3+Bx^2+Cx+D$

(x+3)$(x^2+2x+1) \equiv Ax^3+Bx^2+Cx+D$

x(x^2+2x+1)+3$(x^2+2x+1) \equiv Ax^3+Bx^2+Cx+D$

$x^3+2x^2+x+3x^2+6x+3 \equiv Ax^3+Bx^2+Cx+D$

$x^3+5x^2+7x+3 \equiv Ax^3+Bx^2+Cx+D$

Since the two sides are identical, then,

7x =Cx (divide both sides by x)

C= 7

∴The value of C is 7.

EXERCISE

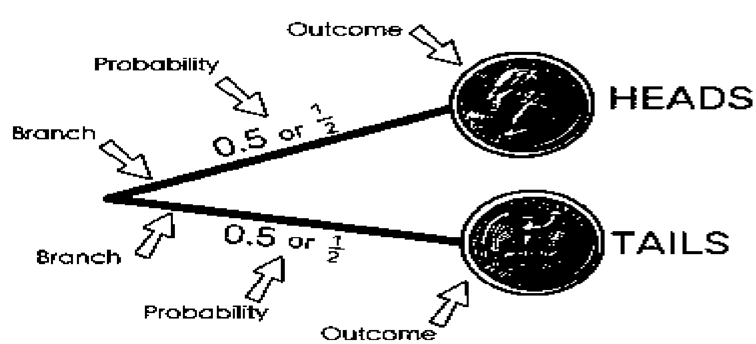
1. Show that k+3 is a factor of $k^3+3k^2-4k-12$.
2. Given that (x+1) and (x-3) are two factors of the polynomial a$x^3+bx-6)$, calculate the values of a and b.
3. Given that (x-2) is a factor of $x^3-6x^2+11x+p$, find p.
4. Given that x+2 is a factor of $2x^3-3x^2-11x+6$, solve the equation: $2x^3-3x^2-11x+6$=0
5. Given that $(3x^2+x-6)(x+1)$ *and* $3x^3+kx^2+hx-6$ are identical, find the value of k and h.
6. Solve the following equations
   a. $x^3-x^2-x+1=0$
   b. $2x^3+11x^2+10x-8=0$

CHAPTER 9: PROBABILITY

PROBABILITY OF TOSSING A COIN ONCE

- **Let's start with a common probability event:**
- **Flipping a coin that has heads on one side and tails on the other.**

TOSSING A COIN

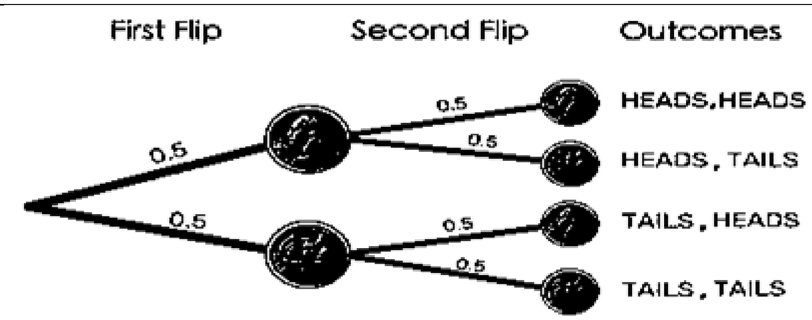


- ✓ **This simple probability has two branches**
- ✓ **One for each possible outcome heads or tails.**
- ✓ **Notice that the outcome is located at the end-point of a branch- This is where a tree diagram ends.**
- ✓ **Also notice that the probability of each outcome occurring is written as a decimal or a fraction. In this case, the probability for either outcome (flipping a coin and getting heads or tails) is fifty-fifty which is 0.5 or $\frac{1}{2}$.**

TOSSING A COIN TWICE

A probability tree diagram for flipping a coin twice

TOSSING A COIN TWICE



Notice the tree diagram, you can see that there are four possible outcomes when flipping a coin twice

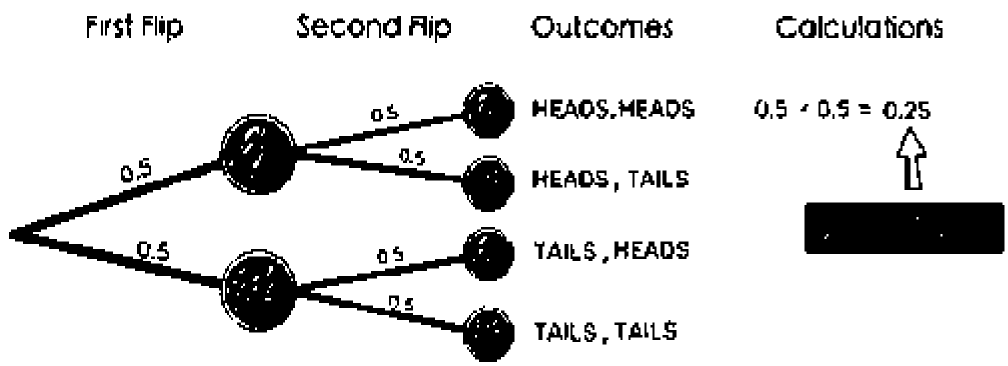
a. **Heads/Heads**
b. **Heads/Tails**
c. **Tails/heads**
d. **Tails/Tails**

And since there are four possible outcomes, there is a 0.25 or $\frac{1}{4}$ probability of each outcome occurring. So, for example, there is a 0.25 probability of getting heads twice in a row.

HOW TO FIND PROBABILITY

The rule for finding the probability of a particular event in a probability

tree diagram occurring is to multiply the probabilities of the corresponding branches.



For example, to prove that there is 0.25 probability of getting two heads in a row, you would multiply 0.5x0.5 (Since the probability of getting a heads on the first flipping is 0.5 and the probability of getting Heads on the second is also 0.5)

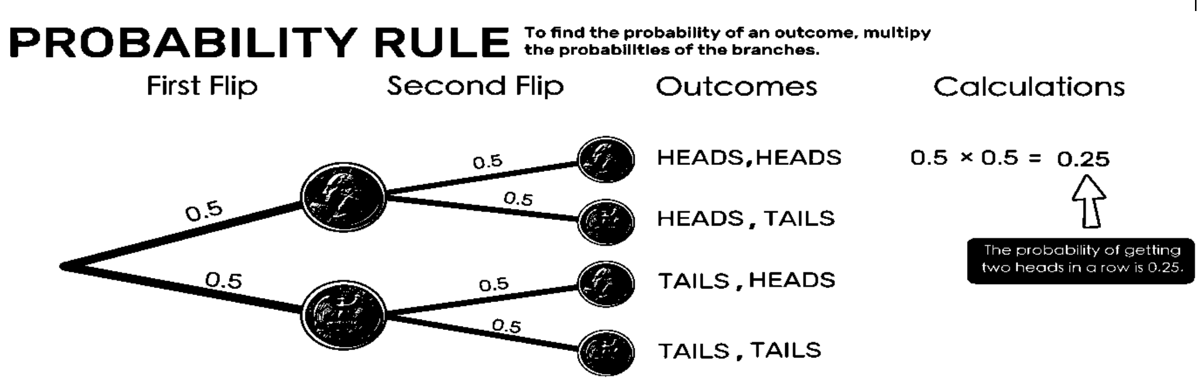
(2 Heads) = HXH

= 0.5X.5

= 0.25

TOSSING A COINT THREE TIMES

Repeat this process on the other three outcomes as follows, and add allof the outcome probabilities together as follows



Note that the sum of the probability of all of the outcomes should always equal.The probability tree diagram can be used to draw several conclusions such as

a. **The probability of getting heads first and tails second is**

   (Heads/Tails) = HXT

   = 0.5X.5

   = 0.25

b. **The probability of getting at least one tails from two consecting flips is 0.25+0.25+0.25 = 0.75**

c. **The probability of getting both a heads and a tails is 0.25 +0.25 = 0.5**

INDEPENDENT EVENTS AND DEPENDENT EVENTS

What is an independent Events?

In the toss tree diagram example, the outcome of each coin is independent of the outcome of the previous toss. That means that the outcome of the first toss had no effect on the probability of the outcome of the second toss. This situation is known as an independent event.

WHAT IS A DEPENDENT EVENT

Unlike an independent event, a dependent event is an outcome that depends on the event that happened before it. These kinds of situations are a bit trickier when it comes to calculating probability, but you can still use a probability tree diagram to help you.

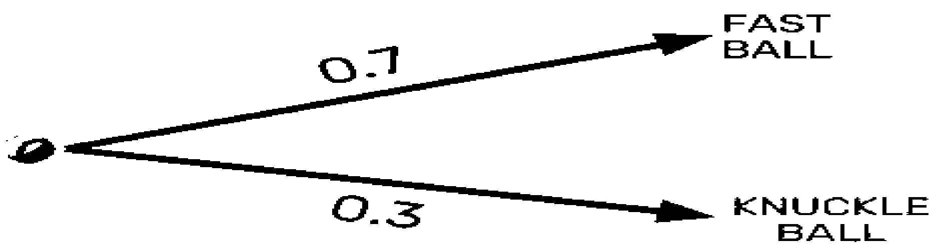
HOW TO MAKE A TREE DIAGHRAM

Example

Harry is a basket pitcher who throws two kinds of pitches, a fastball, and a knuckleball. The probability of throwing a strike is different for each pitch. The probability of throwing a fastball for a strike is 0.6 while the probability of throwing a knuckleball for a strike is 0.2. Harry throws fastballs more frequently that he throws knuckleballs. On average, for every 10 pitches he throws, 7 of them are fastballs (0.7 probability) and 3 of them are knuckleballs (0.3 probability). So what is the probability that the pitcher will throw a strike on any given pitch?

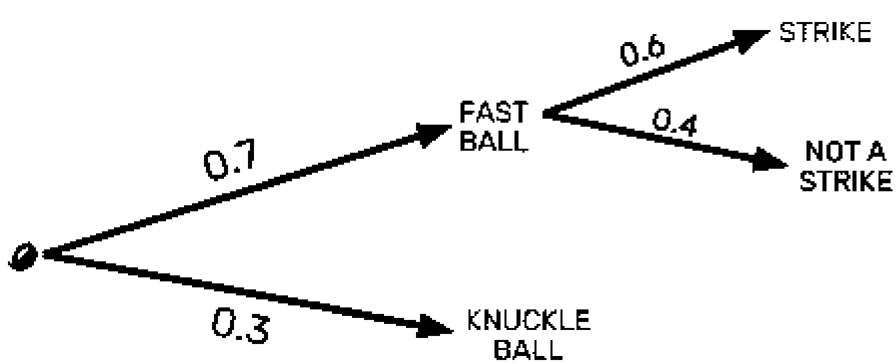
Solution

To find the probability that Harry will throw a strike, start by drawing a tree diagram that shows the probability he will throw a fastball or a knuckleball.



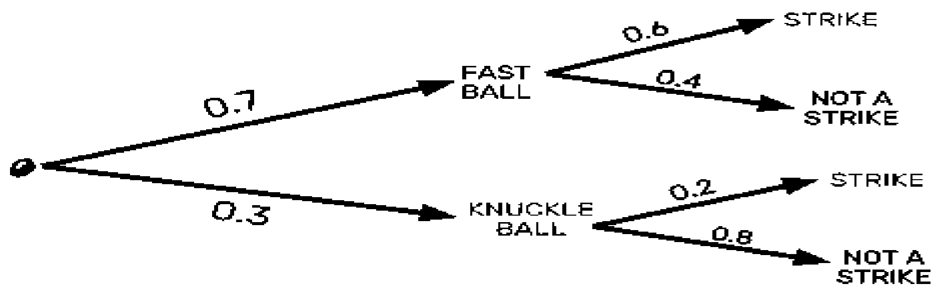
The probability of Harry throwing a fastball is 0.7 and the probability of him throwing a knuckle is 0.3. Notice that the sum of the probabilities is 1 because 0.7+0.3 is 1.00

Next, add branches for each pitch to show the probability for each pitch being a strike, starting with the fastball:



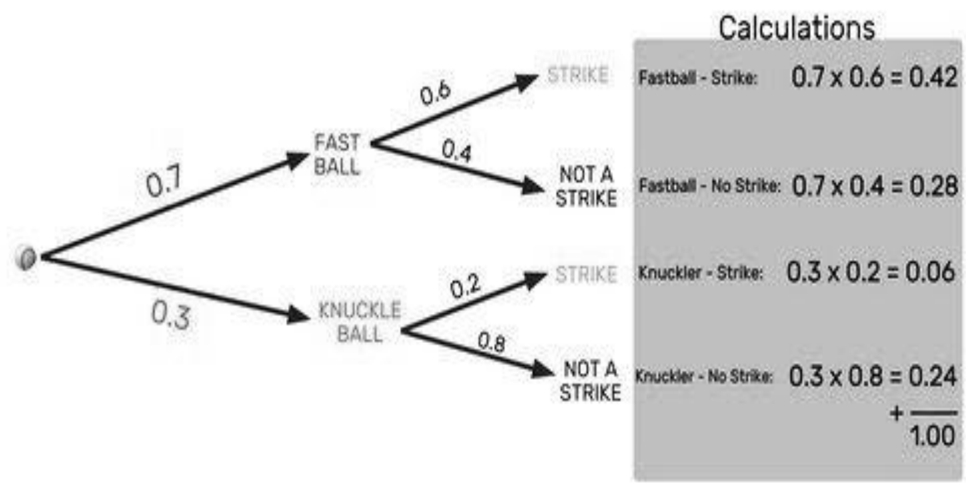
Remember that the probability of Harry throwing a fastball for a strike is 0.6, so the probability of him not throwing it for a strike is o.4 (Since 0.6+0.4)

Repeat this process for the knuckleball:

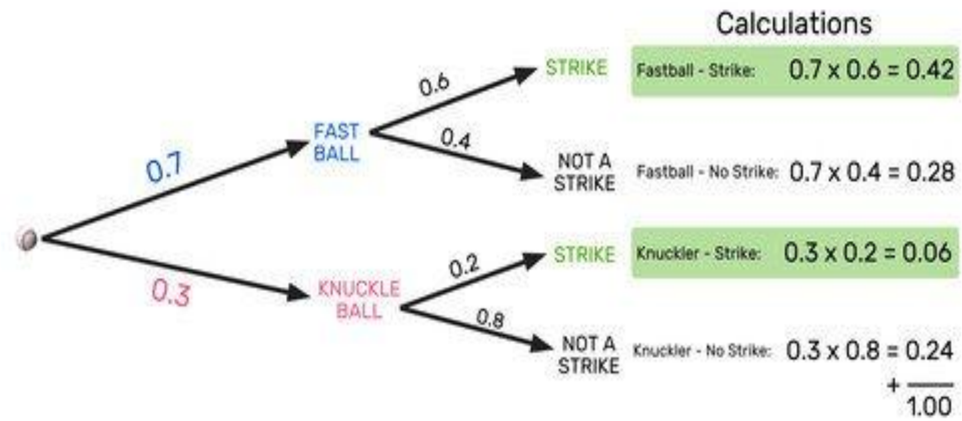


Remember that the probability of Harry throwing a knuckleball for a strike is 0.2, so the probability of him not throwing it for a strike is 0.8 (since 0.2 + 0.8) = 1.00

Now that the probability tree diagram has been completed, you can perform your outcome calculations. Remember that the sum of the probability outcomes has to equal one:



Since you are trying to figure out the probability that Harry will throw a strike on any given pitch, you have to focus on the outcomes that result in him throwing a strike: fastball for a strike or knuckleball for a strike:



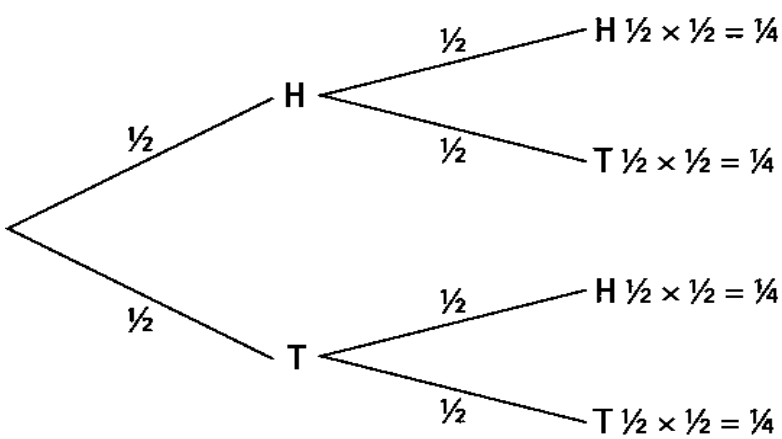
The last step is to add the strike outcome probabilities together:

0.42+0.06=0.48

The probability of Harry throwing a strike is 0.48 or 48%

FURTHER EXAMPLE

What is the probability of throwing a head and a tail if you toss two coins?



At the first event- The first coin toss, you have a 1.2 chance of the coin coming up heads and a $\frac{1}{2}$ chance of the coin coming up tails. On the left of the tree, the line splits into two parts

a. One with H for heads at the end labeled $\frac{1}{2}$

b. Another with T for tails at the end, so labeled $\frac{1}{2}$.

At the second event-the second coin each of the branches splits again. The outcomes and probabilities are the same as before for each, so you get a branch with H at the end labeled $\frac{1}{2}$ and a branch with T at the end also labeled $\frac{1}{2}$.

You can use the tree to work out the probability of any combination of events. For example, to find the probability of the first coin being a head and the second a tail, you follow the branch from the first split to H , and then the branch from there to T, and then times the labels-the probabilities together. You get a a half of a half = 1/2x1/2 =1/4

PROBABILITY TREE DIAGRAMS

Probability can be represented using tree diagrams. Each branch of the tree represents an outcome. A probability tree has two main parts

a. the branches

b. and the ends

The probability is generally written on the branches while the outcome is written on the ends of the branches.

HOW TO USE A PROBABILITY TREE

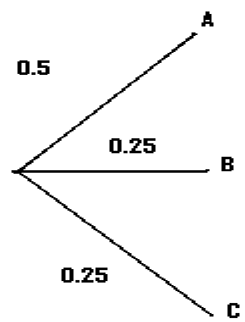
Example

An airplane manufacturer has three factories A, B and C which produce 50%, 25% and 25% respectively, of the particular airplane. Seventy percent of the airplanes produced in factory A are passenger airplanes, 25% of those produced in factory B are passenger airplanes, and 25% of the airplanes produced in factory C are passenger airplanes. If an airplane produced by the manufacturer is selected at random, calculate the probability the airplane will be the passenger plane.

Step 1

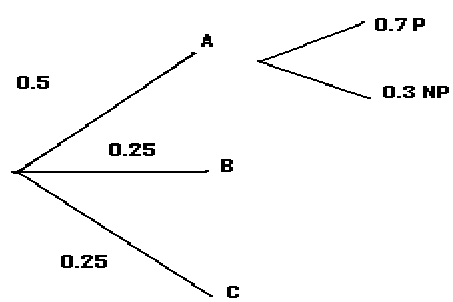
Draw lines to represent the first set of options in the Question. Label them

9our question list A, B and C so that is what we will use here).



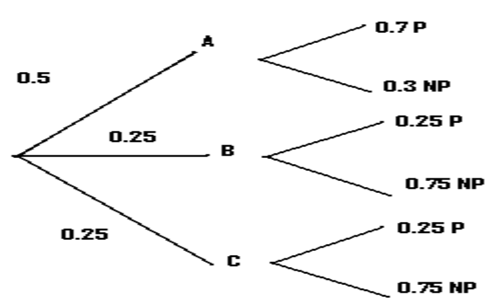
Step 2

Draw the next set of branches, in our case, we were told that 70% of factory A's output was passenger. Converting to decimals, we have 0.7P ("P" is just my own shorthand here for Passenger) AND 0.3np (Not Passenger)



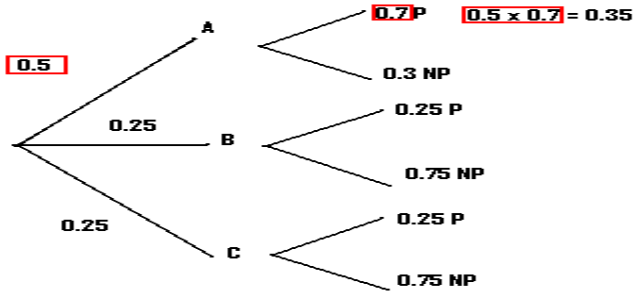
Step 3

Repeat step 3 as branches as you are given



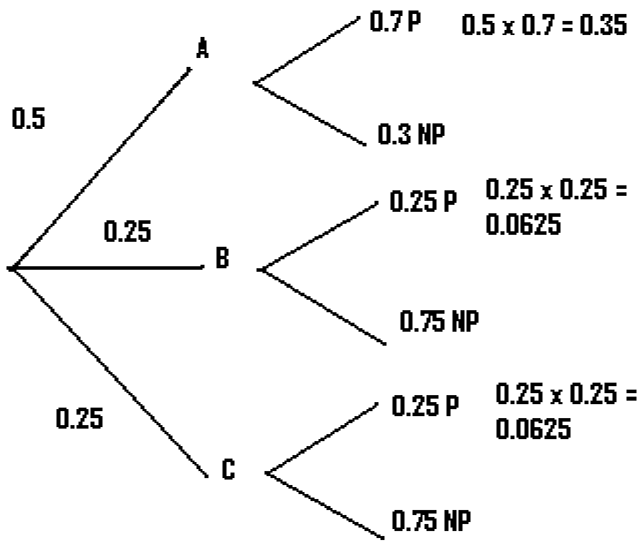
Step 4

Multiply the probabilities of the first branch that produces the desired result together. In our case, we want to know about the production of passenger planes, so we choose the first branch leads to P.



Step 5

Multiply the remaining branches that produce the desired result. In our example, there are two more branches that can lead to P.



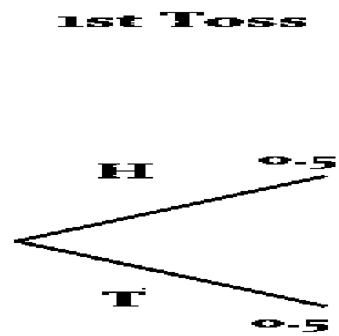
Step 6

Add up all of the probabilities you calculated in steps 4 and 5.In our example, we had:

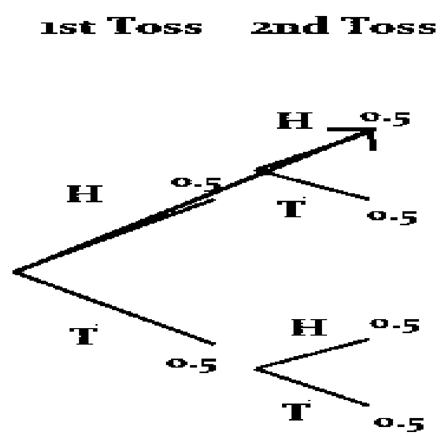
$0.35 + 0.625 + 0.625 = 0.475$ Answer

EXAMPLE

If you toss a coin three times, what is the probability of getting 3 heads?



The second step is to figure out your probability of getting a heads by tossing the coin once. The probability is 0.5 (you have a 50% probability of tossing a heads and 50% probability of tossing a tails). Those probabilities are represented at the ends of each branch.



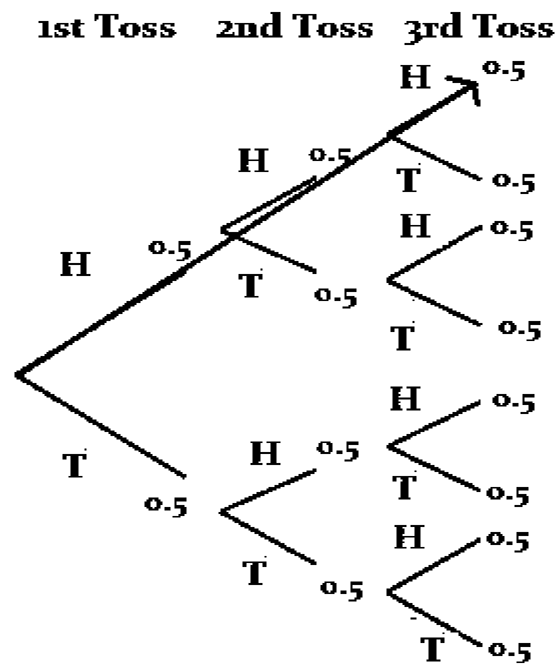
Then you add two branches to each branch to represent the second coin toss.

The probability of getting two heads (HH) = HXH

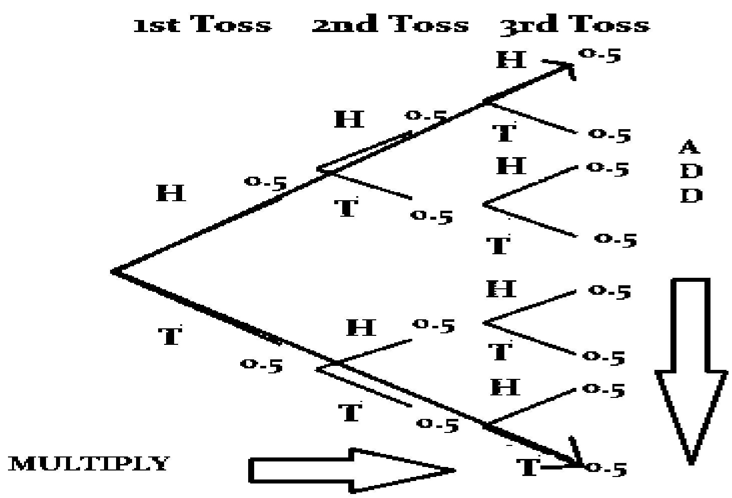
= 0.5X0.5

=0.25

This makes sense because your possible results for one head and one tail are HH, HT, TT or TH (each combination has a 25% probability).



Finally, add a third row because we were trying to find the probability of throwing 3 heads. Multiplying across the branches

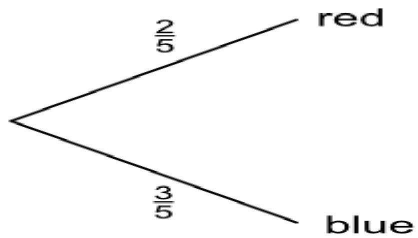


For HHH we get = 0.5x0.5x0.5 = 0.125 or 12.5%

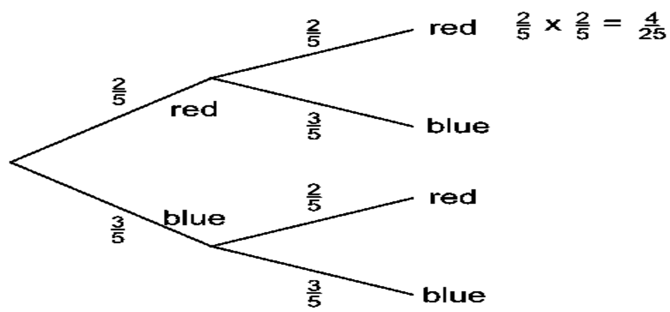
DRAWING TREE DIAGRAMS

All outcomes must be shown from each node. For example, a bag of balls contains 4 red balls and 6 blue balls

P(RED) = 2/5 and P(BLUE) =3/5)

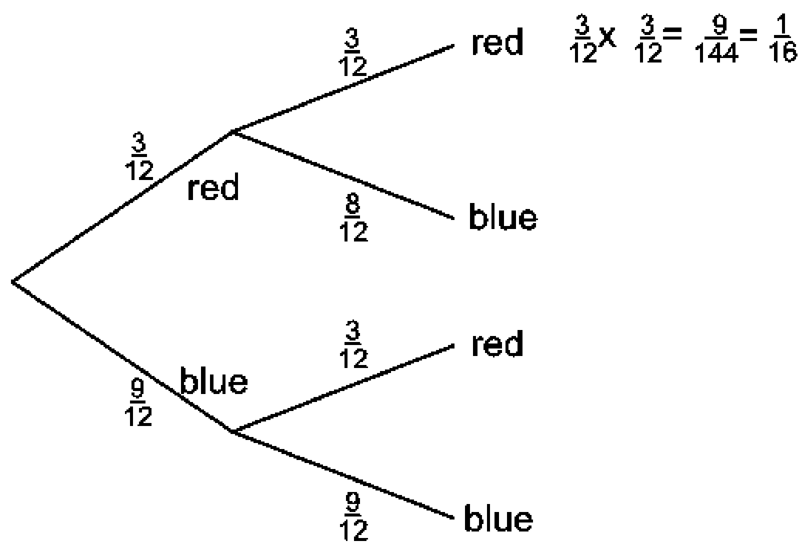


If a ball is drawn, replaced and the second ball is drawn



Multiply the probabilities along branches to calculate the probability of two consecutive events. The probability of drawing two red balls is

P(RED) X p(red) = 2/5x2/5 = 4/25



Exercise

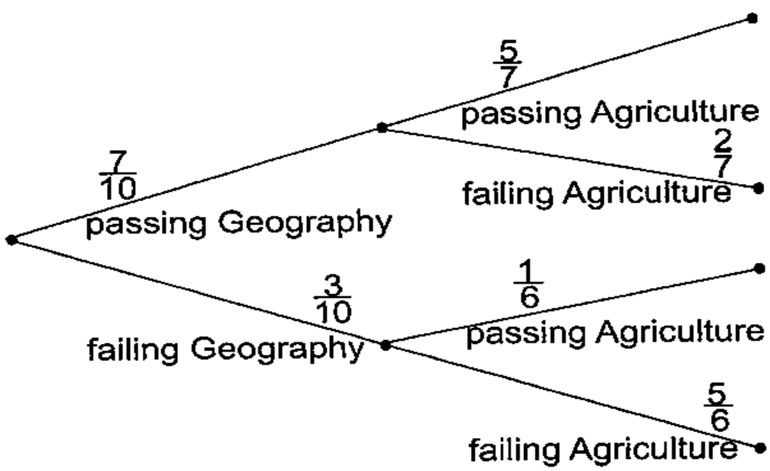
A standard 1 pupil has 3 coca cola bottle tops and 7 Fanta bottle tops in the same pocket. What is the probability of picking 1 Fanta bottle top from the pocket?

There are 10 bottle tops in the pocket.

P(Fanta top) = 7/10

EXAMPLE

Figure below is a tree diagram illustrating the probability of a student passing Agriculture and Geography in an examination. The probability of passing Geography in the examination is $\frac{5}{7}$. The probability of passing Agriculture after one failed Geography is $\frac{1}{6}$.



a. Calculate the probability of a student passing Agriculture.

$$P(\text{Passing Agriculture}) = \frac{7}{10} \times \frac{5}{7} + \frac{3}{10} \times \frac{1}{6}$$

$$= \frac{35}{70} + \frac{3}{60}$$

$$= \frac{210}{420} + \frac{21}{420}$$

$$= \frac{231}{420}$$

$$= \frac{11}{20}$$

b. **Calculate the probability of a student failing Agriculture**.

P*(Failing Agriculture)* $=\frac{7}{10} \, x \, \frac{2}{7} + \frac{3}{10} \, x \, \frac{5}{6}$

$=\frac{14}{70} + \frac{15}{60}$

$=\frac{84}{420} + \frac{105}{420}$

$=\frac{210}{420} \quad =\frac{1}{2}$

EXAMPLE

1. The Probability that it rains on a Monday is $\frac{1}{4}$, the probability that the teacher will be present on that day when it rains is $\frac{1}{6}$ and the probability that the teacher will be present when it does not rain is $\frac{1}{10.}$ Draw a tree diagram and label all the probabilities

P(it rains) = $\frac{1}{3}$ (given)

P(No rains) = 1-$\frac{1}{3}$ = $\frac{2}{3}$

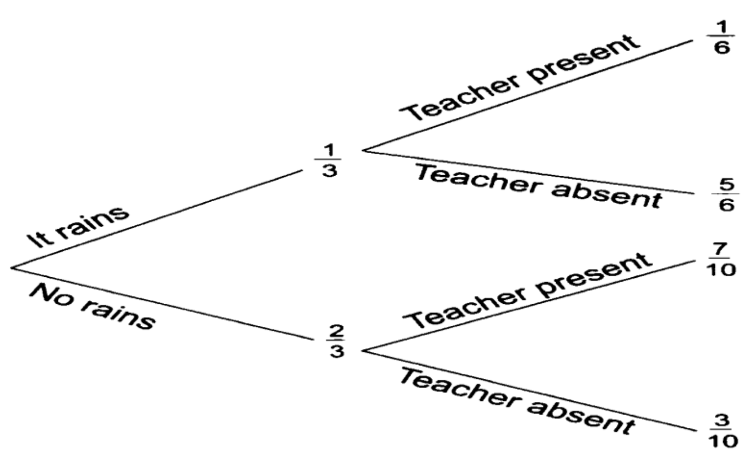
P(Teacher present when it rains) = $\frac{1}{6}$

P(Teacher absent when it rains) =1- $\frac{1}{6}$ = $\frac{5}{6}$

P(Teacher present when no rains) = $\frac{7}{10}$

P(Teacher absent when no rains) = 1-$\frac{7}{10}$ = $\frac{3}{10}$

The tree diagram will look as follows



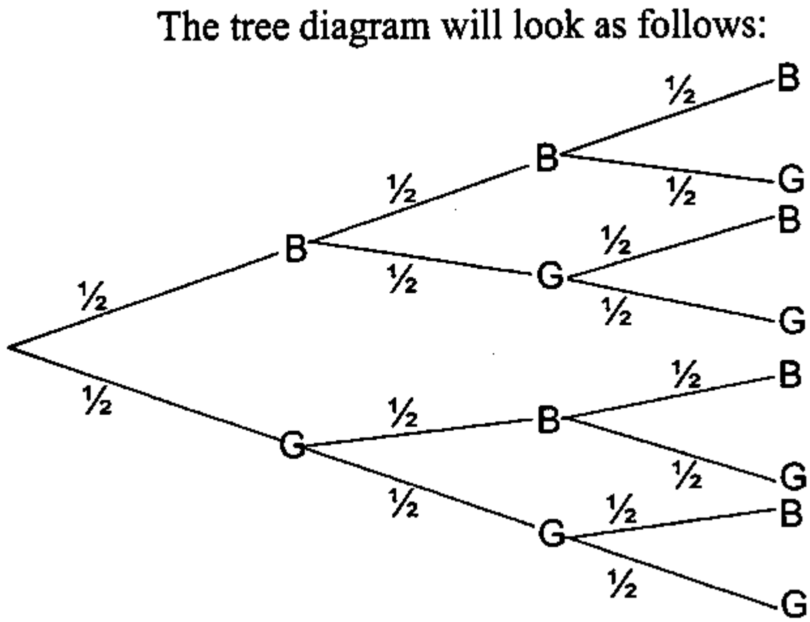
2. A family has 3 children born at different times. assuming it is equally likely to have a baby boy or a baby girl,

a. Draw a tree diagram to show the possibility of having a boy or a girl on each of the three births.

b. Calculate the probability that the family has 2 boys and 1 girl in any order of births.

a. Let B be all event having a baby boy and G an event of having a baby girl. Since it is equally likely to have a baby boy or a bay girl, then P(B) = $\frac{1}{2}$, P(G)= $\frac{1}{2}$.

The tree diagram will look as follows:



b. P(2B and 1G) = P(BBG) + P(BGB) +P(GBB)

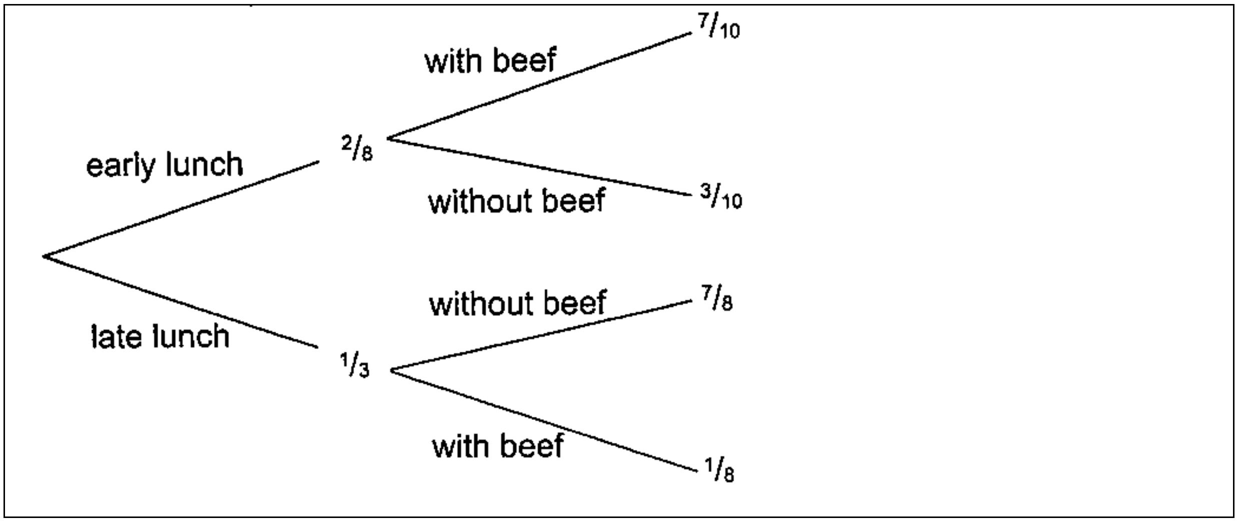
$$= \left(\frac{1}{2}\ x \frac{1}{2}\ x\ \frac{1}{2}\right) + \left(\frac{1}{2}\ x \frac{1}{2}\ x\ \frac{1}{2}\right) + \left(\frac{1}{2}\ x \frac{1}{2}\ x\ \frac{1}{2}\right)$$

$$= \left(\frac{1}{8}\ x \frac{1}{8}\ x\ \frac{1}{8}\right)$$

$$= \frac{3}{8}$$

∴The probability that the family ha 2 boys and 1 girl in any order is $\frac{3}{8}$

3. The probability of having early lunch at a boarding school is $\frac{2}{3}$. When the lunch is early, the probability of having beef is $\frac{7}{10}$ and when late, the probability of having beef is $\frac{1}{8}$. Draw a tree diagram to represent this information completeling al branches.



4. Two fair dice, each with six faces numbered 0,1,2,3,4, 5 are rolled at the same time. Table 1 shows the possible sums of the numbers on the dice.

	Die	0	1	2	3	4	5
Die	0	0	1	2	3	4	5
	1	1	2	3	4	5	6
	2	2	3	4	5	6	7
	3	3	4	5	6	7	8
	4	4	5	6	7	8	9
	5	5	6	7	8	9	10

Calculate the probability of getting

a. A sum less than 4.

b. A sum which is a multiple of 5.

a. Let the number of all possible outcomes be N.

Let n be the number of events, E_1 such that E_1 is the sum less than 4.

Counting from the table, N =36

E_1 **constitutes the following :0,1,2,3,1,2,3,2,3,3**

Counting the elements of E_1**, then n = 10**

$(E_1) = \frac{n}{N}$

$P(E_1) = \frac{10}{36}$

$= \frac{5}{18}$

∴**The probability of getting a sum less than 4 is** $\frac{5}{18}$

b. Let n be the number of events, E_2 **such that** E_2 **is a sum which is multiple of 5.**

E_1 **consists of the following : 5,5,5,5,5,5,10**

Counting the elements of E_2**, then n =7**

Using N as the total number of all possible events from a. above, then

$P(E_2) = \frac{n}{N}$

$P(E_2) = \frac{7}{36}$

∴**The probability of getting a sum which is a multiple of is** $\frac{7}{36}$

5. Table 6 shows the distribution of ages of learners in a form 2 class.

Age	14	15	16	17	18	19
Number of learners	2	10	8	4	9	3

What is the probability of picking at random a learner of 18 years of age?

Let N be the total number of learners and n the number of learners with 18 years of age.

From the table, N = 2+10+8 +4 + 9 + 3

= 36

n =9

Let E be an event of picking a learner of 18 years of age

∴ P(E) = $\frac{n}{N}$

= 9/35

= $\frac{1}{4}$

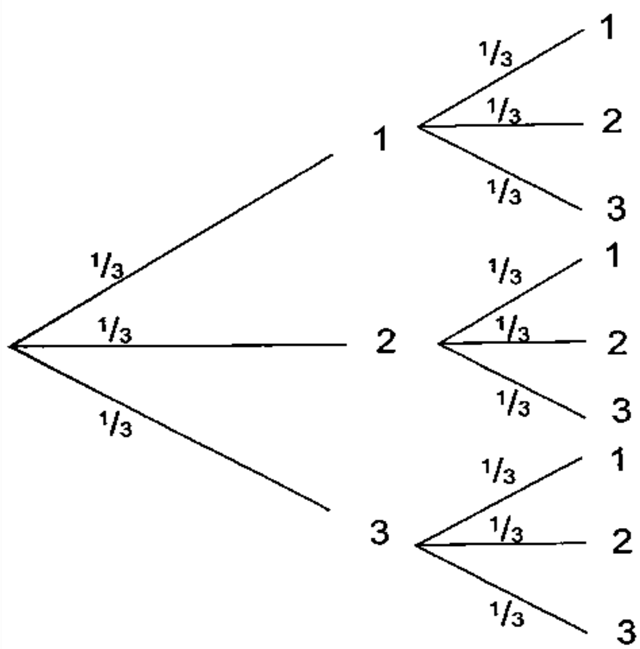
The probability of picking at random a learner of 18 years of age is 1/4

6. A set of three cards numbered 1,2 and 3 respectively, are placed in a bag. Draw a tree diagram and use it to find

a. How many two-digit numbers can be formed by selecting with replacement two cards with different digits in any order?

b. The probability that any two-digit number formed with different digits is even.

Tree diagram



From the tree diagram, the two digits number with different digits in any order are :12,13,21,23,31,32

∴There are 6 numbers.

Let N be total number of outcomes and n be the number of events required.

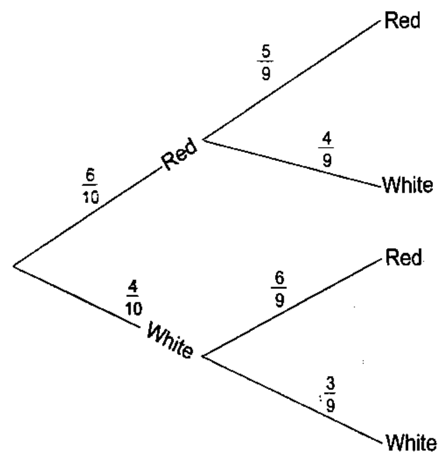
From the tree diagram, the required events, E are 12,32

n = 2, N = 9

P(E)= $\frac{n}{N}$ = 2/9

The probability that any two digits number formed with different digits is 2/9.

7. Figure 1 is a tree diagram which shows the probability of picking two balls one at a time without replacement from a bag containing 4 red and 4 white balls.



Use the tree diagram to calculate the probability of picking two balls of different colours, leaving your answer in its simplest form.

Solution

Probability of picking two balls of different colours:

The balls picked can be either order Red, White , or WHITE, Red

P(RW) = $\frac{6}{10}\ X\ \frac{4}{9} = \frac{4}{15}$

P(RW) = $\frac{6}{9}\ X\ \frac{4}{10} = \frac{4}{15}$

∴ P(RW) or P(WR0 = $\frac{4}{15} + \frac{4}{15}$

= $\frac{8}{15}$

8. A bag contains beans, groundnuts and maize seeds. The probability of getting at random a bean seed is 1/5, a groundnut seed is x/15 and a maize seed is 1/3. Find the value represented by x.

P(bean) = 1/5

P(g/nut) = x/15

P(maize) =1/3

Total probability = 1

1/5 +x/15 + 1/3 = 1

$$\frac{3+x+15}{15} = 1$$

8 + x =15

x = 15-8

x = 7

9. In Ziweto village, families are allowed to keep three types of animals: chickens, goats and pigs. The probability that a family will keep a chicken, a goat and a pig is ½,1/5 and 1/10 respectively.

   a. Draw a tree diagram to illustrate this information.

   b. Calculate the probability that a family will keep only one of animal.

P(Keeping chickens) = ½

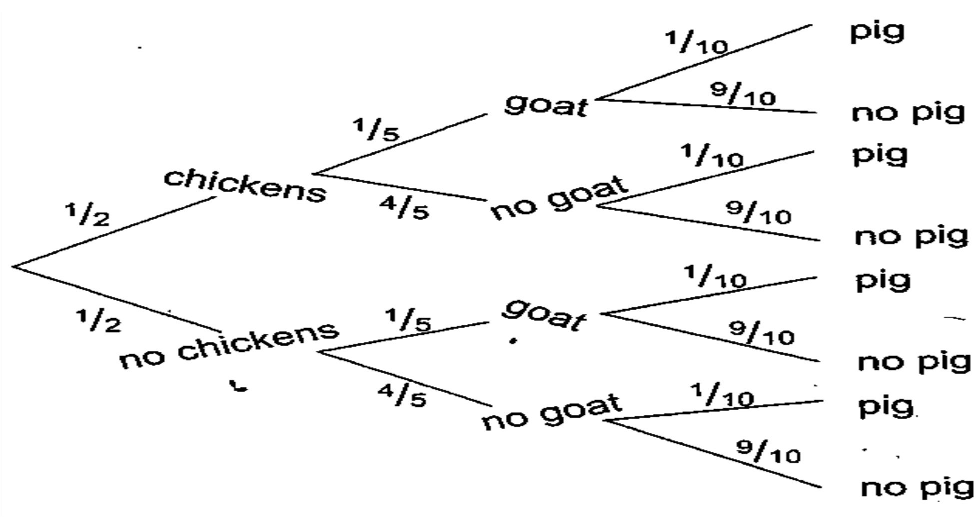
P(Keeping chickens) =1/2

P(Keeping goat) = 1/5

P(Not keeping goat) = 4/5

P(Keeping pig) = 1/10

P(Not keeping pig) =9/10



Probability of one type of animal P(Chicken) or P(pig) or P(goat)

P(chickens only) =1/2 X 4/5 X 9/10) = 36/100

P(pig only) = ½ X 4/5 X 1/10 = 4/100

P(goat only) = ½ x 1/5 x 9/10 = 9/100

P(chickens or pig or goat only) = 36/100+ 4/100+ 9/00)

= 49/100

The probability that a family will keep only one type of animal is 49/100.

10. A box has 5 cards marked 1,2,3,4 and 5. Another box has 2 cards marked A and B. If a card is picked randomly from each box at the same time, calculate the probability of picking a card marked 3 from one box and a card marked B from the other box.

Probability space for the first box = $(\mathbf{1,2,3,4,5})$

Probability of picking 3, P(3) = $\frac{1}{5}$

Probability space for 2nd box = $(\boldsymbol{A,B})$

Probability of picking B, P(B) = $\frac{1}{2}$

Probability of picking a card marked 3 and a B

P(3 and B) = P(3) X P(B)

= 1/5 X ½

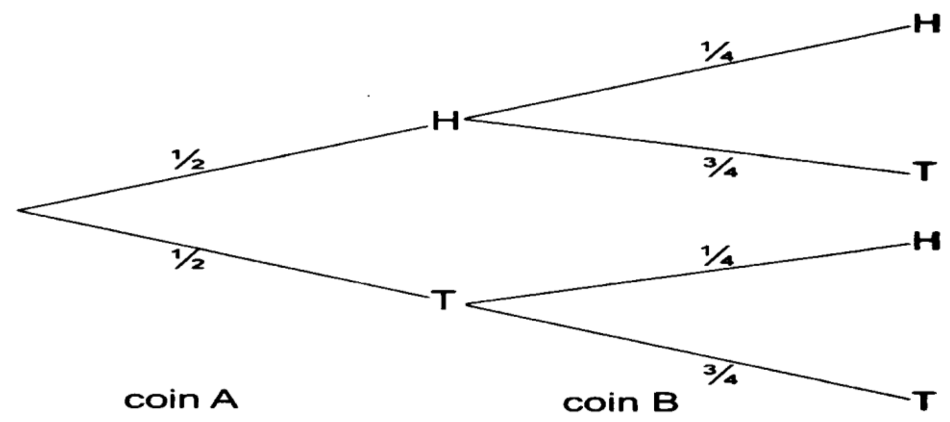
= 1/10

The probability of picking a card marked 3 from one box and a card marked B from the other box = 1/10

Exercise

1. The probability of a bus arriving early at the depot is $\frac{1}{10}$ and arriving late is $\frac{3}{5}$. If 400 buses are expected at the depot during the day, calculate the number of buses that are likely to arrive at the depot on time.

2. Coin A is tossed followed by coin B. The probability that coin A shows head is $\frac{1}{2}$ while the probability that coin B shows head is $\frac{1}{4}$. Using a tree diagram, calculate the probability that both coins A and B shows tails.



P(TT) $=\frac{1}{2}\ x\ \frac{1}{2}$

$= \frac{1}{4}$

Therefore the probability that both cons A and B shows tails is $\frac{1}{4}$

EXERCISE

1. A coin is tossed three times. Calculate the probability that the first and the third tosses give heads as outcomes, regardless of what the outcome of the second toss is.
2. Complete the table below showing all the possible total outcomes of throwing two dice at the same time.

	1	2	3	4	5	6
1	2	3	4	5	6	
2	3	4	5	6	7	
3				7		
4						
5						
6						12

From the table, write down the probability of getting a total of

a. 2
b. 7

3. A bag contains 3 blue beads, 5 white beads and 6 red beads.
a. If one bead is picked, what is the probability of picking a red bead?
b. If a red bead is picked, what is the probability of picking a second red bead?
4. A standard 1 pupil has 3 coca cola bottle tops and 7 Fanta bottle tops in the same pocket. What is the probability of picking 1 Fanta bottle top from the pocket?
5. A box contains 5 red, 9 white and 7 blue pieces of chalk. If one piece of chalk is picked at random,
   a. Find the probability that it is blue.
   b. Suppose a white piece of chalk was picked and not replaced. What is the probability that the second chalk picked is red?
6. The letters **M, A, L, A, W, I** are written on separate identical cards. The cards are placed upside down. If one card is picked at random, what is the probability that it has the letter A?
7. A fair die is rolled once. What is the probability of obtaining
   a. A prime number?
   b. Perfect square?
8. A bookshelf contains 10 Mathematics books, 9 History books and 11 books on sports. All books have the same dimensions. If Chimwemwe selects a book at random, what is the probability that is is a book on sports?
9. A box contains 5 red balls, 8 white balls and 7 black balls. If one ball is selected at random, calculate the probability that it is white or black.
10. A fair die and an unbiased coin are rolled and tossed respectively.
    a. Draw a probability space table for possible outcomes of the die and the coin.
    b. Use the probability space table to find the probability of obtaining an even number on the die and a tail on the coin.

CHAPTER 10 : GRAPHS OF CUBIC FUNCTIONS

Y= x^3 An equation such as $4x^3 + 12x^2 = 3$, which has a term in x^3 is called a cubic equation. It takes the form of y = $\boldsymbol{4x^3 + Bx^2 + Cx}$

Cubic equations usually have three roots (x-values). It crosses x- axis at three points.

CHARACTERISTICS OF GRAPHS OF CUBIC FUNCTIONS

Copy and complete the following table which shows a set of values of x and y that satisfy the equation Y = $x^3 - 5x^2 - 4x + 20$

x	-3	-2	-1	0	1	2	3	4	5	6
y	-40	0		20	12		-10	-12	0	32

Taking a scale of 2cm to represent 1 unit on the horizontal axis and 2cm to represent 10 units on the vertical axis, draw the graph of y Y = $x^3 - 5x^2 - 4x + 20$

a. Use your graph to solve the following equations

i. **Y** = $\boldsymbol{x^3 - 5x^2 - 4x + 20 = 0}$

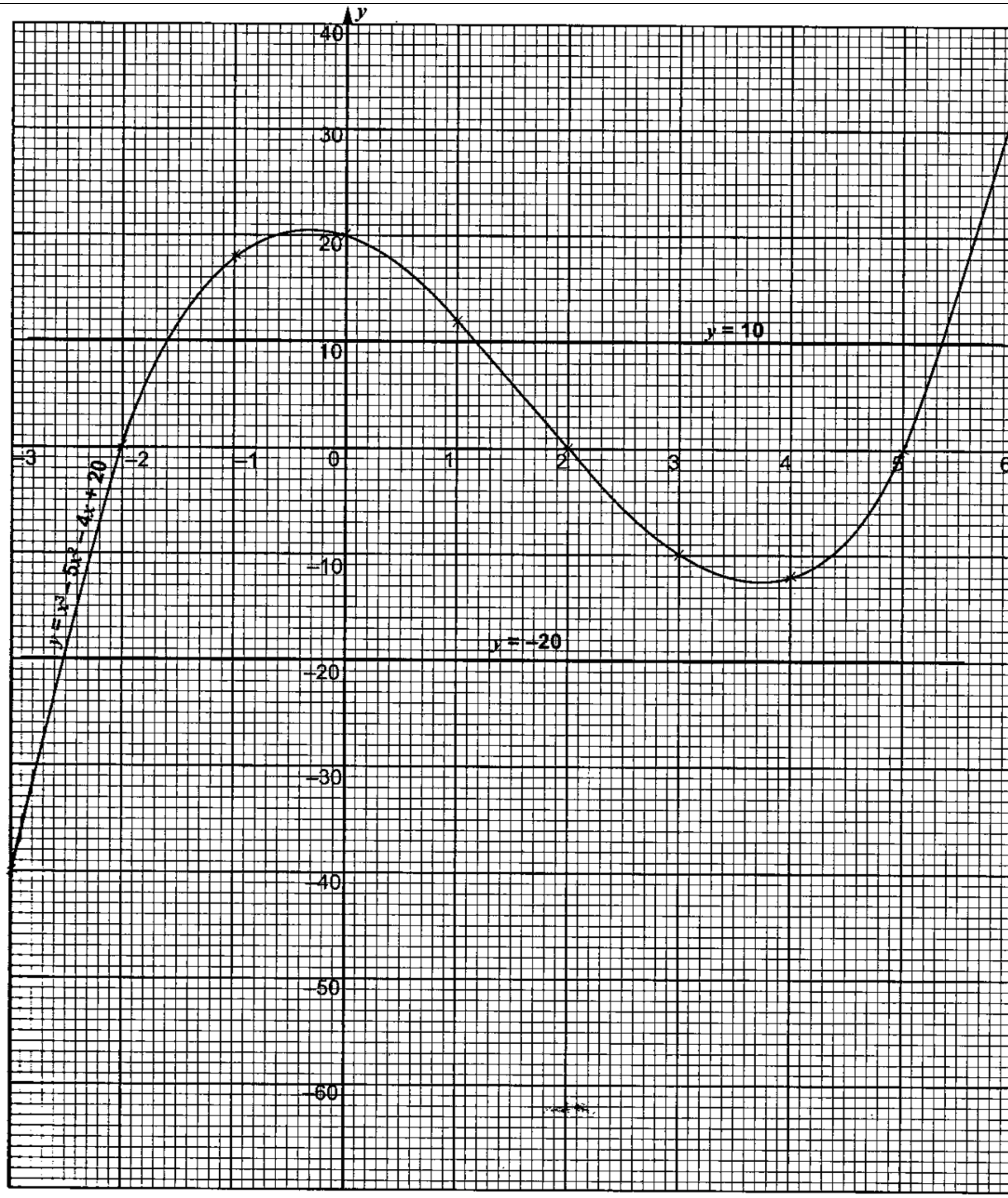
ii. **Y** = $\boldsymbol{x^3 - 5x^2 - 4x + 20 = 10}$

iii. **Y** = $\boldsymbol{x^3 - 5x^2 - 4x = -40}$

The completed table is as follows (underlined numbers are the ones added in)

Y = $\boldsymbol{x^3 - 5x^2 - 4x + 20}$

x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
y	**-40**	**0**	**18**	**20**	**12**	**0**	**-10**	**-12**	**0**	**32**



b. Solving the equations using the graphs

(i) $x^3 - 5x^2 - 4x + 20$

Y = 0

Either x = -2, 2, or 5

(ii) $x^3 - 5x^2 - 4x + 20$ **= 10**

Y = 10

Either x = -1.6, x = 1.15 or x = 5.35

(iii) $x^3 - 5x^2 - 4 = -40$

$x^3 - 5x^2 + 40 = 0$

$x^3 - 5x^2 + -4x + 20 = 0 - 40 + 20$

$x^3 - 5x^2 - 4x + 20 = -20$

Y = -20

Complete the following table of values for **Y =** $x^3 - 3x^2 + 2$

x	-2	-1	0	1	2	3	4
y	-18				-2		18

Taking 2cm to represent 1 unit on the horizontal axis and 1 cm to represent 2 units on the vertical axis, draw the graph of Y = $x^3 - 3x^2 + 2$ for values of x from -2 to 4.

a. Use your graph to solve the following equations

(i) $x^3 - 3x^2 + 2 = 0$

(ii) $x^3 - 3x^2 - x + 2$

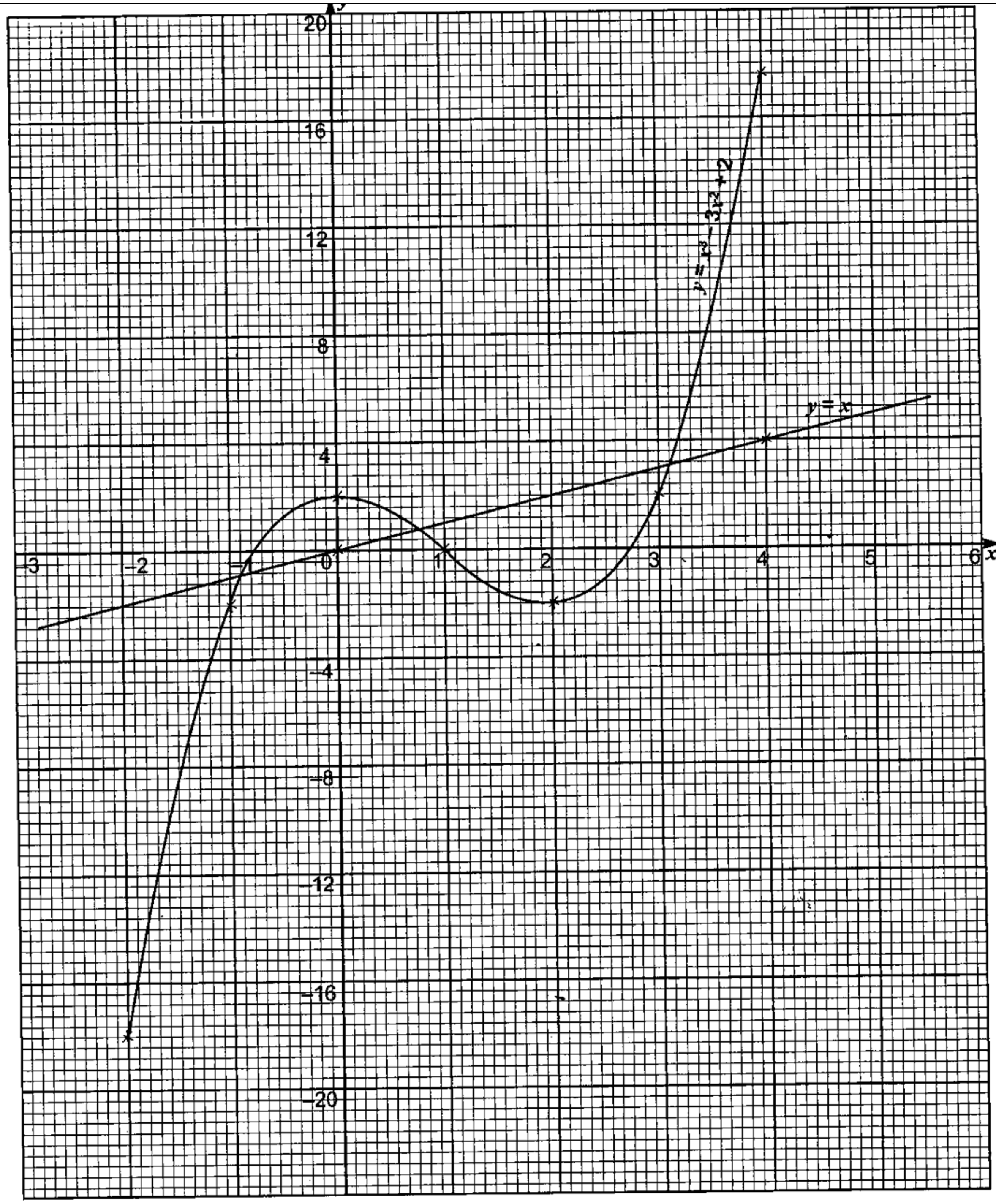
b. For what values of x is $x^3 - 3x^2 + 2 \geq 0$?

The completed table is as follows (underlined numbers are the ones added in)

Y = $x^3 - 3x^2 + 2$

x	-2	-1	0	1	2	3	4
y	-18	<u>-2</u>	<u>2</u>	<u>0</u>	-2	<u>3</u>	18

The graph is plotted below



a. Solving the equations using the graph

(ii) $x^3 - 3x^2 + 2 = 0$

Y = 0

Either x = -0.7, x = 1 or x = 2.7

(iii) $x^3 - 3x^2 - x + 2$

$x^3 - 3x^2 - x + x + 2$ = 0+ x

$x^3 - 3x^2 + 2$ = 0+ x

Y = x

Draw the line y = x on the graph and the required solution is at the intersection of the graphs

x	-2	-1	0	1	2	3	4
y	-2	- 1	0	1	2	3	4

Either x = -0.8, x = 0.7 or x = 3.

$b. x^3 - 3x^2 + 2 \geq 0$

The range of values for which y ≥ 0 is -0.7 $< x < 1$ and x $>$2.7

Copy and complete the table of values for the equation y =x $(x^2 - x - 6)$

x	-3	-2	-1	0	1	2	3	4
y	-18	0		0	-6		0	24

Using a scale of 2cm to represent 1 unit on the x- axis and 2cm to represent 5 units on the y-axis, draw the graph of y =x $(x^2 - x - 6)$

Use your graph to solve the equation x $(x^2 - x - 6) + 5$

Completing the table of values of $(x^2 - x - 6)$

x	-3	-2	-1	0	1	2	3	4
y	-18	0	4	0	-6	-8	0	24

Given : y = x $(x^2 - x - 6)$

If x = -1, then y = -1$(-1^2 - (-1) - 6)$

y= -1(1+1-6)

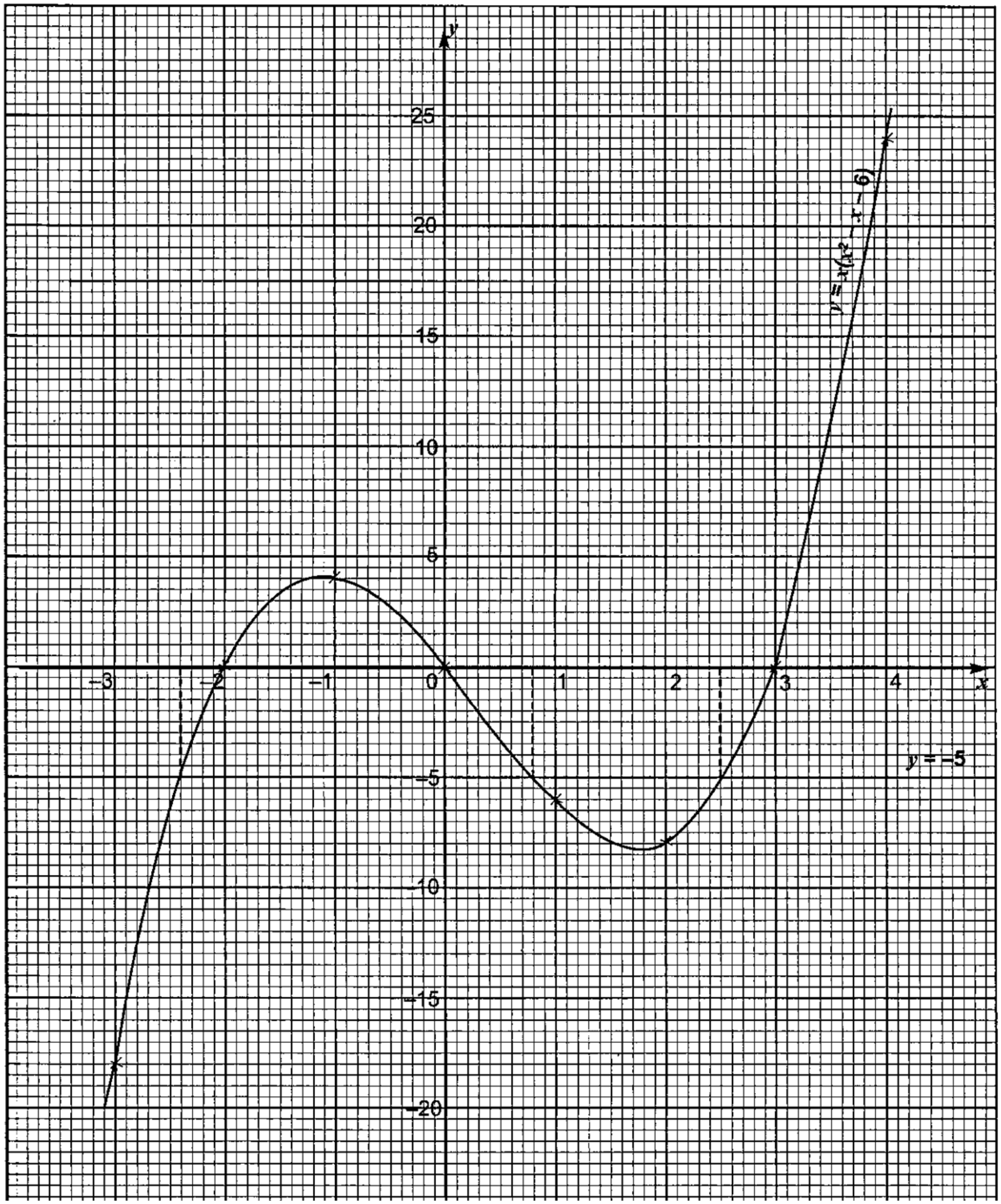
y =-1x-4

if x = 2, then y = 2$(2^2 - 2 - 6)$

y =2 x -4

y = -8

a. Graph



Solving the equation x $(x^2 - x - 6) + 5 = 0$

Y = -5

Draw the graph of y =-5 on the same pair of axes where the graph of y =x $(x^2 - x - 6) + 5$ is drawn

The solutions lies where the graphs of the two equations intersect

From the graphs, x = -2.4 or x = 0.8 or x = 2.5

Copy and complete the table of values for the equation y = $(x+1)(x^2+x-6)$

x	-4	-3	-2	-1	0	1	2	3
y	-18	0	4	0	-6	-8	24	

Completing the table below y = $(x+1)(x^2+x-6)$

x	-4	-3	-2	-1	0	1	2	3
y	-18	0	4	-0	-6	-8	24	0

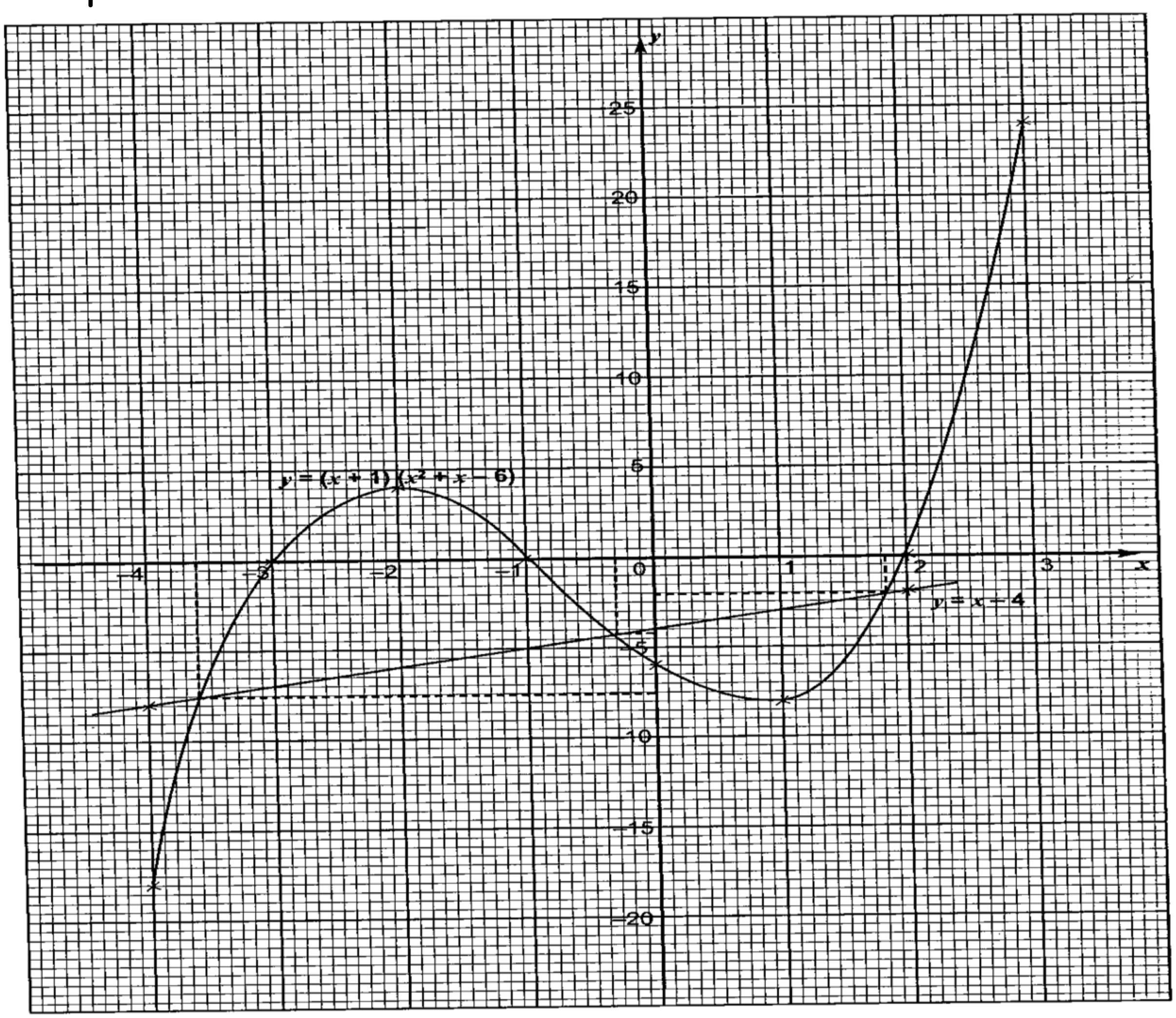
If x =-1, then

Y = $(-1+1)((-1)^2+(-1)-6))$

= 0(1-1-6)

= 0

b. Graph



c. Y = x -4

y = $x^3 + 2x^2 - 5x - 6$

From the equation y = (x+1)$(x^2 + x - 6)$

= x)$(x^2 + x - 6))(x^2 + x - 6)$

=$x^3 + 2x^2$-5x-6

= (x +1)$(x^2 + x - 6) = x^3 + 2x^2 - 5x - 6$

Draw the graph of y=x-4 on the same pair of axes where the graph of Y =$x^3 + 2x^2 - 5x - 6$ has been drawn

Table of values of y =x-4 is given by

X	-4	-1	3
y	-8	-5	-1

The solutions of the simultaneous equations lie where the graphs of the two equations intersect.

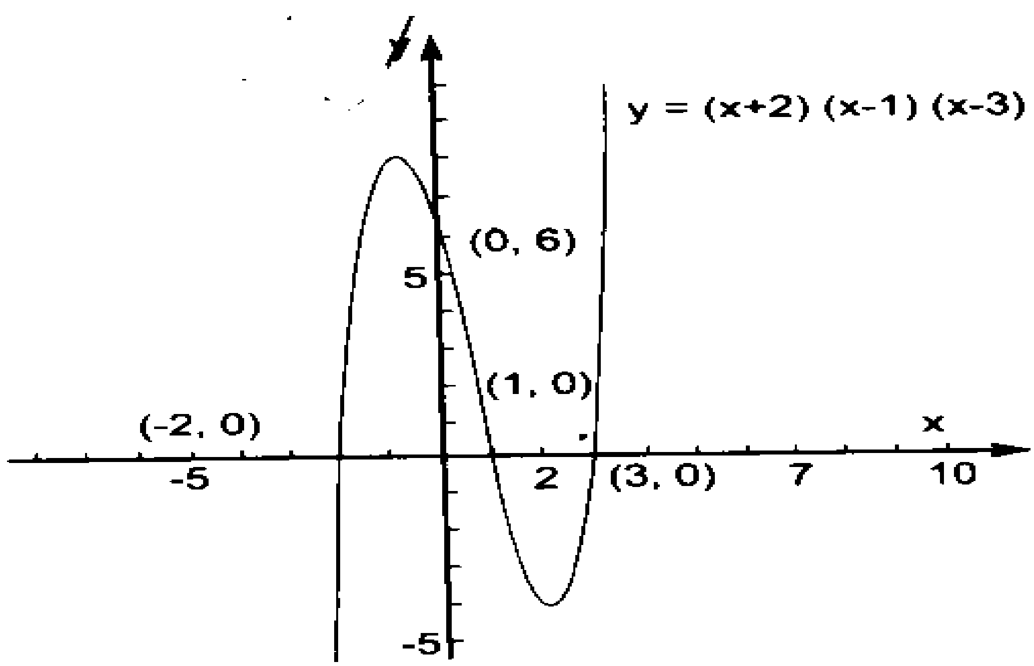
According to the graphs

X = 1.83; y=-2;

x = -0.4; y =-4.4

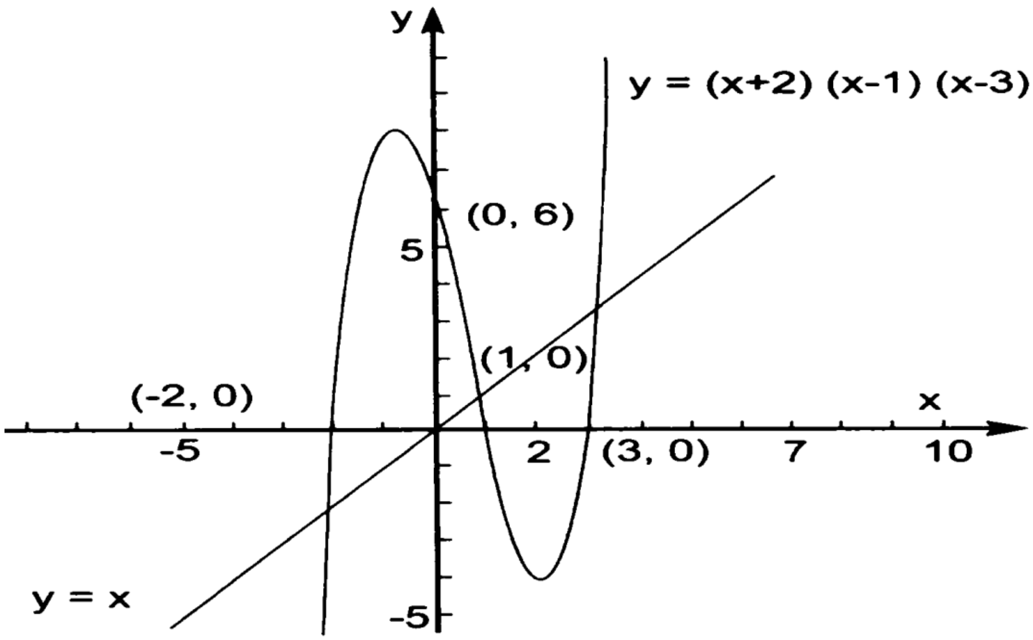
x =-3.6; y =-7.5

Figure below shows the graph of y = x+2)(x-1)(x-3) .



Use the graph to find the solutions to the following equation $x^3 - 2x^2 - 6 + 6 = 0$

The solutions lies where the graph of y= (x+2)(x-1)(x-3) and y = x intersect.

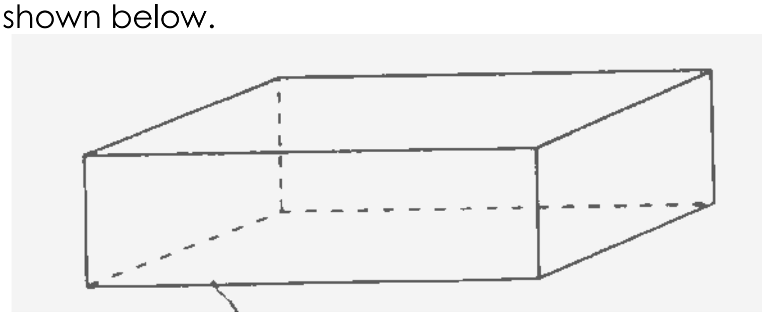


From the graph, x =3.2 or x =0.8 or x =-2

CHAPTER 11: THRE-DIMENSIONAL FIGURES

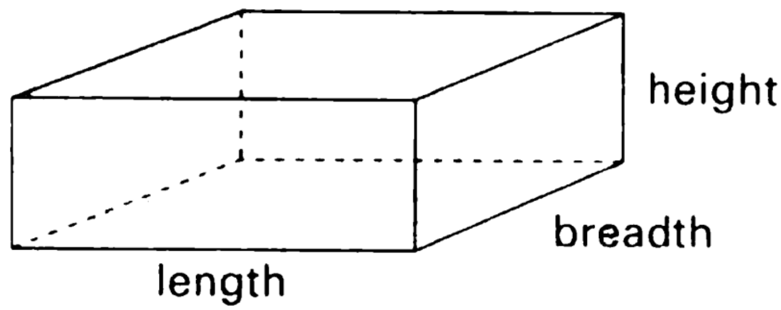
PRISMS

Prisms are solids which have the same cross-section throughout their length.
A cuboid is a prism. The cross-section is a rectangle. A cuboid is a rectangular prism as shown below.

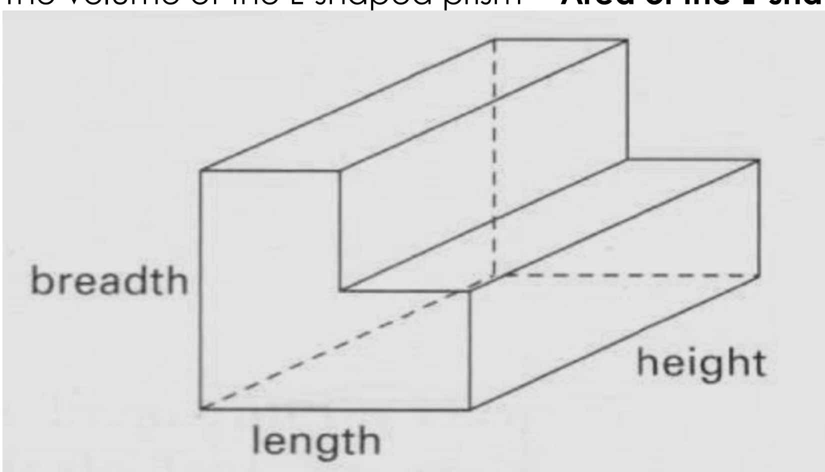


VOLUME OF A PRISM

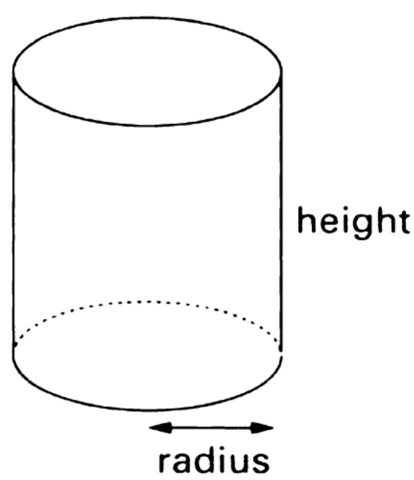
The volume of a cuboid = **Length x breadth x height**



Volume of a triangular prism = **Area of triangle x height**
The volume of the L-shaped prism = **Area of the L-shaped end x height**



VOULUME OF THE CYLINDER



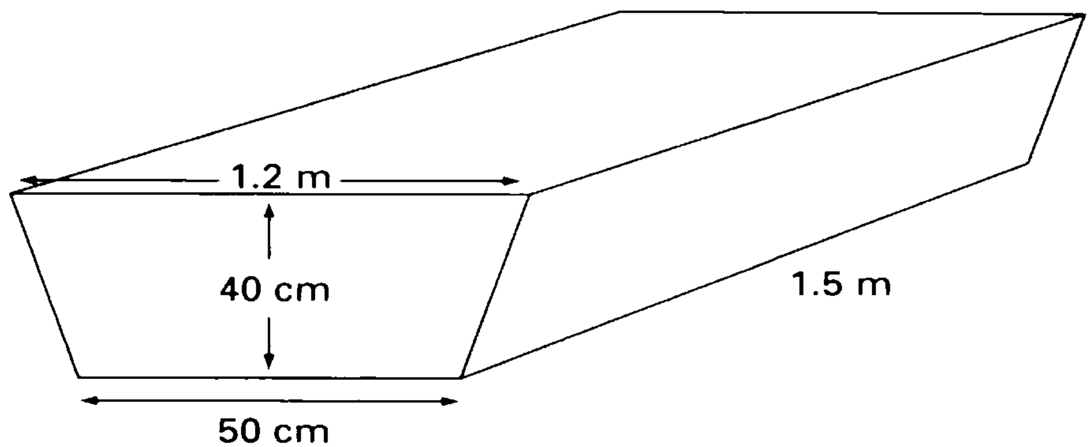
If the circular cross-section has a radius, r and the height of the cylinder is

h, then the volume= area of circle x height.

= $\pi r^2 h$

VOLUME OF THE OF A TROUGH WITH TRAPEZOIDAL END

Find the volume of a trough with length 1.5m and trapezoidal ends as shown in the diagram below.



The end is a trapezium with parallel sides of 1.2m and 50cm (=0.5m) which are 40cm (=0.4m) apart, area of trapezium = $\frac{1}{2}$ (0.5 + 1. 2) x 0.4m^2

= 0.34m^2

Volume = Area of cross-section x length

= 0.34 x 1.5

= 0.51m^3

The cross-section of a lead pipe is a rectangle. The internal dimensions are 10cm by 12 cm and the walls of the pipe are 5mm thick. The pipe is 100cm long. Calculate

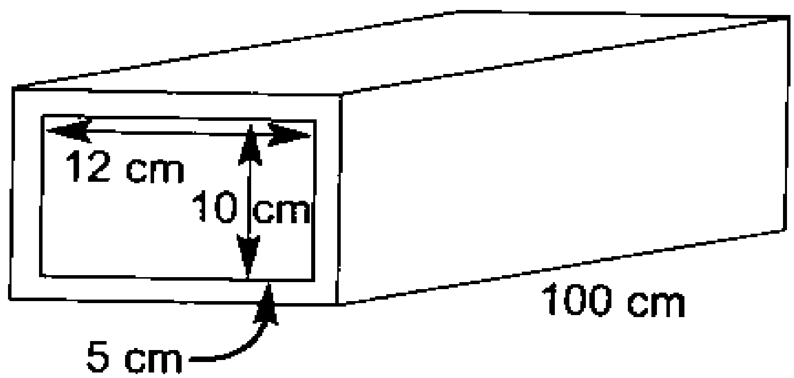
a. The volume of lead used in making the pipe.
b. The mass of the pipe in kg if the density of lead is 11.4g/cm^3.Give your answer to 3 significant figures.

a. Area of cross-section of pipe

= (12 + 0.5 +0.5) x (10 +0.5 + 0.5)-12 x 10

= 13 x 11 -12 x 10 = 143 -120

=23cm^2



Volume = area of cross-section x length

= 23 x 100cm^3

b. Mass = density x volume

= 11.4 x 2300g

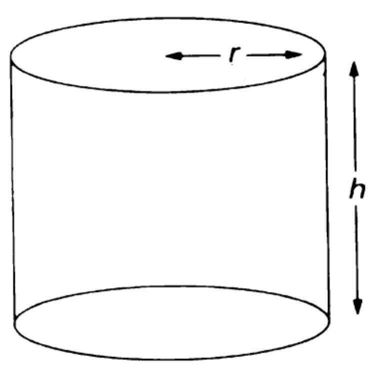
= 26220 g

= $\frac{26220}{1000}$ kg

$=$ 26.22kg

$\therefore$ Mass is 26.2kg to 3 s.f.

SURFACE AREA OF THE CURVED SURFACE OF A CYLINDER



- **The area of the curved surface of the cylinder = $2\pi rh$**
- **The area of the circular base of the cylinder = 2 πr^2**
- **The surface area of a cylinder with a base (open at one end)**

= $2\pi rh$ +2 πr^2

=2πr (h + r)

Example

A cylindrical steel pipe, open on both ends, has external radius of 21cm and internal radius of 14cm. It is 2m long. Take $\pi = \frac{22}{7}$. Calculate

a. The internal surface area
b. The volume of steel used to make the pipe.

a. Internal surface area = $2\pi rh$

= 2 x $\frac{22}{7}$ x 14cm x 200cm

= 17600cm^2

b. To find the volume of steel used to make the pipe = $\pi r^2 h$

volume of steel used to make the pipe =Volume of pipe – Volume of empty space

Volume of steel = $\frac{22}{7}$x 21cm x 21cm x 200cm -$\frac{22}{7}$x 14cm x 14cm x 200cm

= 277200cm^3 $-$123200cm^3

Volume of steel used = 154000cm^3

A metal bar of length 231mm and diameter 56 mm is melted and cast into washers. Each washer is 2mm thick with an internal diameter of 14mm and external diameter of 28mm. Calculate the number of washers obtained assuming no loss of metal.

The metal bar and the washer are in cylindrical shape

∴ Volume of metal bar = $\pi r^2 h$

Radius = diameter $\div 2$

= 56$\div 2$ = 28mm

Volume of metal bar = $\pi\, x\, (28mm)^2\, x\, 231mm$

Volume of metal bar needed per washer = $\pi r^2 h\ (ext)$- $\pi r^2 h$(int)

$$r^2 ext = external\ diameter\ \div 2$$

$$= \mathbf{28mm} \div 2 = 14mm$$

$$r^2 ext = internal\ diameter\ \div 2$$

$$= \mathbf{14mm} \div 2 = 7mm$$

Volume of metal bar needed per washer =$\pi x\ 2mm(14mm^2 - 7mm^2)$

$$=\pi x\ 2mm(14mm + 7mm)(14mm - 7mm)$$

$$= \pi x\ 2mm\ x\ 21mm\ x\ 7mm$$

$$= (\pi x\ 2\ x\ 21\ x\ 7)mm$$

Number of washers = $\frac{\pi\ x 28\ x 28\ x\ 231mm^2}{\pi\ x\ 2\ x 21\ x\ 7mm^2}$

= 14 x 4 x 11

= 616

Therefore 6166 washers will be obtained from the metal bar.

Calculate the total surface area of a solid circular cylinder whose diameter and height are 2.58cm and 4.27cm respectively. (Take π =3.14)

Diameter, d = = 2.58cm, height, h = 4.27cm

Radius, r = $\frac{Diameter}{2}$ = 2.58$\div 2$ = 1.29

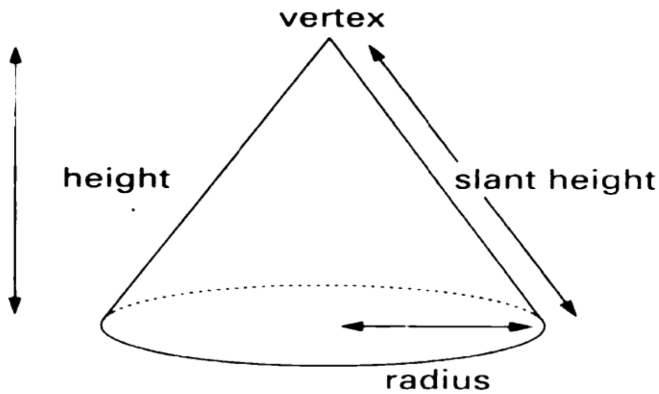
Surface area = 2$\pi r^2 + 2\pi rh$

= 2 x 3.14 x 1.29 x 1.29 x 4.27 + 2 x 3.14 x 1.29 x 4.27cm^2

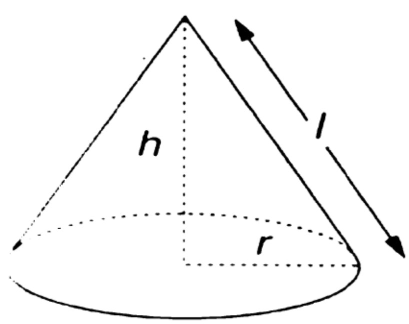
= 45.04cm^2

CURVED SURFACE OF A CONE

The figure below shows a shape of the cone.

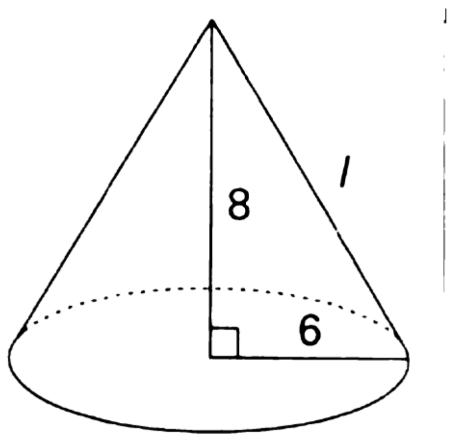


Area of curved surface of a cone **=** $\pi r l$ where r is the radius of the base and l is the slant height.



Example

Find the curved surface area of a cone which is 8cm high and a base radius of 6cm which is shown below.(Take $\pi = 3.14$)



By Pythagoras, $l^2 = 6^2 + 8^2$

= 36 + 64

= $100cm^2$

L = 10cm

Curved surface area = πrl

= 3.14 x 6 x 10

= $188.4cm^2$

Area of base = πr^2

= $3.14 \times 6^2 cm^2$

= $113.04cm^2$

The total area = $188.4 + 113.04cm^2$

= $301.44cm^2$

Find the height of a cone with a curved surface area of $440cm^2$ and a slant height of 20cm. (Take = $\frac{22}{7}$)

Curved surface area of cone = $\pi rl = 440cm^2$

$$\frac{22}{7}\ x\ r\ x\ 20 = 440$$

R = $\frac{440\ x\ 7}{22\ x\ 20}$

By Pythagoras, $h^2 + r^2 = l^2$

$$h^2 + 7^2 = 20^2$$

$$h^2 = 20^2 - 7^2$$

$$h^2 = 400 - 49$$

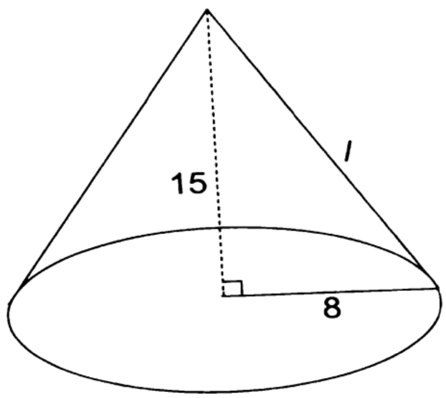
$$h^2 = 351$$

h = $\sqrt{351}$

h = 18.7cm to 3 s.f

A cone with a radius 8cm is 15cm high. Calculate the total surface area of the cone. Take $\pi = 3.14$

curved surface area of cone = $\pi r l$



$l^2 = 15^2\ x + 8^2$

$l^2 = \sqrt{289}$

$= 3.14 \times 8$

Area of the cone= $\pi r l$

$= 3.14 \times 8 \times 17$

$= 427.04 cm^2$

VOLUME OF A CONE

Volume of a cone **=** $\frac{1}{3}\pi r^2 h$

The volume of cone is $\frac{1}{3}$ of the volume of the cylinder with the same radius and height.

Examples

Find the volume of the cylinder and the cone with radius of 3cm and height 7cm. (**Take** $\pi = \frac{22}{7}$)

Volume of cylinder = $\frac{22}{7}\ x\ 3\ X\ 3\ x\ 7\ cm^3$

= 198 cm^3

Volume of cone	= $\frac{1}{3}\ x \frac{22}{7}\ x\ 3\ x\ 3\ x\ 7cm^3$
	= $66cm^3$

The volume of a cone is 2112cm^3. If the height of the cone is 14c, find its radius (Take$\pi = \frac{22}{7}$)

Volume of the cone = $\frac{1}{3}\ \pi r^2 h$

$r^2 = \frac{1}{3}\ \pi r^2 h$

$r^2 = \frac{2112\ x\ 3\ x\ 7}{22\ x\ 14}$

= **144**

R = $\sqrt{144}$

Radius, r = 12cm.

The volume of a right circular cone of height 2.97 cm is 29.4cm^3. Find the base radius of the cone, correct to 2 s.f. Take π = 3.142. Volume of the cone

Vo**lume of the cone** = $\frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi r^2 h$ = **29.4**

$\frac{1}{3}\ x\ 3.142\ x\ r^2\ x\ 2.97$ = **29.4**

$r^2 = \frac{29.4\ x\ 3}{3.142\ x\ 2.97}$

r^2 =**9.452**

r = $\sqrt{9.452}$

= 3.07

Therefore, the base radius of the cone is 3.07

The volume of a cone is $66cm^3$. If the height is 7cm, calculate its radius. (Take $\pi = \frac{22}{7}$ and Volume of cone = $\frac{1}{3}\pi r^2 h$

Volume of cone = $\boldsymbol{\frac{1}{3}\pi r^2 h}$

$$\mathbf{66} = \frac{1}{3} \, x \, \frac{22}{7} \, x \, r^2 \, x \, 7$$

$$r^2 = 66 \times \frac{3}{22}$$

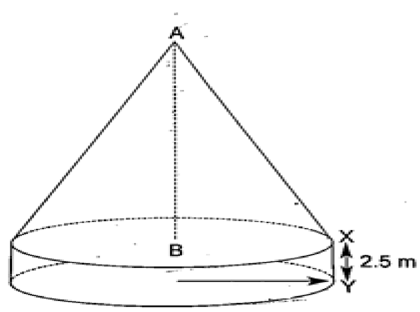
$$R = 3$$

Radius of the cone is 3cm

Exercise

The volume of a cone is $\mathbf{462cm^3}$. If its height is **7cm**, calculate its radius. (Take $\pi = \frac{22}{7}$ and Volume of cone = $\frac{1}{3}\pi r^2 h$

Figure shows a large tent which is composed of a cylinder below a conical top. The base has a radius of 12m. The height of the cone; AB is 9m and the vertical side XY is 2.5 m.



Calculate, giving your answers to three significant figures.

a. The length of the slanting side AX
b. The area of ground covered by the base.
c. The area of material needed to make the tent (conical top and cylindrical side).

a. In ΔABX, $AX^2 = AB^2 + BX^2$ **(Pythagoras)**

$$= 9^2 + 12^2$$

$$= 81 + 144$$

$$= 225$$

= $\sqrt{225}$

= 15m

b. Area of ground covered = Area of circle radius 12m

= $\pi 12^2$

= 3.14 x 144

= $452.16m^2$

= $452m^2$ to 3 s.f.

c. Curved surface area of conical top = π x 12 x 15

= 3.14 x 180

= $565.2m^2$

Surface area of cylindrical side = 2π x 12 x 2.5

= 3.14 x 60

= $188.4m^2$

Area of material required = 565.2 + $188.4m^2$

= $753.6m^2$

=$754m^2$ to 3 s.f.

A solid cylinder of radius 1.5cm and height 7cm is recast into a solid cone radius 3cm. find the height of the cone. (Volume of the cone = $\frac{1}{3}\pi r^2 h$).

Volume of a cylinder = $\pi r^2 h$

Volume of the cone = $\frac{1}{3}\pi r^2 h$

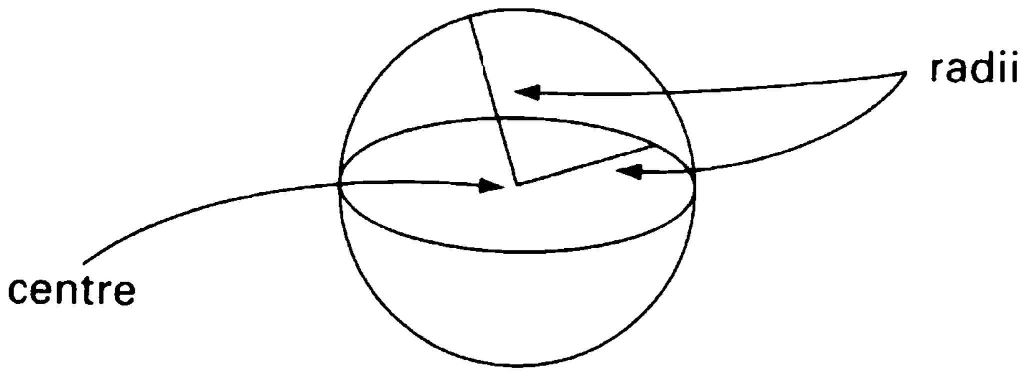
Volume of the cone = $\frac{1}{3}\pi\, x\, .5\, x\, 1.5\, x7$ = $\pi r^2 h$ (Divide both sides by 3π)

h = 0.75 x 7

h = 5.25

∴ The height of the cone is 5.25cm

VOLUME OF THE SPHERE



The volume of a sphere is $\frac{4}{3}\pi r^3$

Example

Find the volume of a spherical tennis ball with a radius of 3.18. (**Take** $\pi = \frac{22}{7}$)

The volume of the tennis ball is $\frac{4}{3}\pi r^3$ = $\frac{4}{3}\, x \frac{22}{7}\, x\, 3.18\, x\, 3.18\, cm^3$

= $134.6 cm^3$

SURFACE AREA OF A SPHERE

The surface area of a sphere is $4\pi r^2$

Example

Find the surface area of a spherical gas balloon with a diameter of 10 metres.

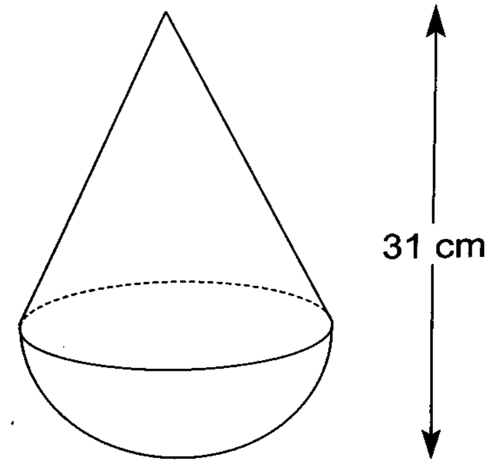
The radius of the balloon is 10 $\div 2 = 5m$

The surface area of a spherical gas balloon = $4\pi r^2$

= 4 × 3.14 × 5^2

= 314 m^2

Figure 2 shows a solid in the form of a cone and a hemisphere. The diameter of the hemisphere is 14cm and the height of the solid is 31cm. (Take π *to be* $^{22}/_{7}$).



Calculate

a. The total surface area of the solid.

b. The volume of the solid. Give your answer to one decimal place.

a. Radius of hemisphere = radius of cone = $\frac{14}{2}$ = 7cm

Height of cone = 31cm-7cm

= 24cm

Slant height = $\sqrt{24^2 + 7^2}$ cm (Pythagoras)

=$\sqrt{576 + 49}$

= $\sqrt{625}$

= 25cm

Surface area of cone = πrl

= $\frac{22}{7}$ x 7 x 25cm^2

= 550cm^2

Surface area of hemisphere = $\frac{1}{2}$ x 4π x 7^2

= 308cm^2

∴ Total surface area = 550 + 308cm^2

= 858cm^2

b. Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} x \frac{22}{7} x\, 7^2 x\, 24$

$= 1232cm^3$

Volume of hemisphere = $\frac{1}{2} \times \frac{4}{3} x (\frac{22}{7}) \times 7^2$

$= 718.667cm^3$

Total volume = $1232 + 718.667cm^3$

$= 1950.67cm^3$ **to 2 d.p**

c. Volume of sphere with radius, r = $\frac{4}{3}\pi r^3$

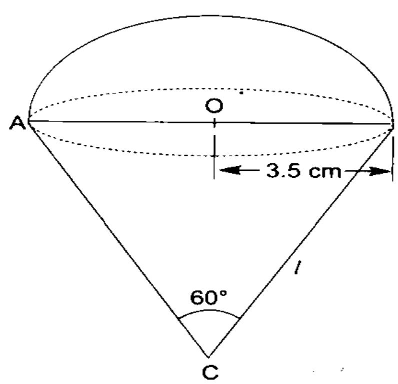
$\therefore \frac{4}{3}\pi r^3 = 1950.67$

$r^2 = 1950.67 \times \frac{3}{4} \times \frac{7}{22}$

R = 7.750

= 7.8cm

Figure below represents a solid block made from a right cone and a hemispherical top. The radius of the hemisphere OB = 3.5 and angle ACB = 60°



Calculate the surface area of the block. (Curved surface area of a cone = πrl, surface area of a sphere = volume of sphere = $\frac{4}{3}\pi r^2$). Take $\pi = \frac{22}{7}$.

To prove: Surface area of the block.

Construction: Join OC

Now surface area of the block = curved surface area of a cone + surface area of a hemisphere.

Curved surface area of a cone = πrl

Since we have a right cone, then

AC=BC and OC $\perp$AB

OC bisect $< ACB$

Angle OCB = $\frac{<ACB}{2}$ = $\frac{60^\circ}{2}$ = 30°

Sin $<$OCB = $\frac{OB}{CB}$

Sin 30^o =$\frac{3.5cm}{l}$

L = $\frac{3.5cm}{sin\ 30^o}$

L = 3.5 $\div 0.5$

L = 7cm

$\therefore$ Curved surface area of a cone = $\frac{22\ x\ 3.5cm\ x\ 7cm}{7}$

= $77cm^2$

Surface area of a hemisphere = $\frac{4\pi r^2}{2}$

Surface area of sphere = $\frac{2\ x\ 22\ x\ 3.5^2}{7}\ cm^2$

= $77cm^2$

$\therefore$Surface area of the block = $77cm^2$ +$77cm^2$

= 154cm^2

The diameter of some spherical lead balls is 0.2cm. Given that the density of lead is 11.4g/cm^3. Calculate

a. The mass of each ball.
b. The number of balls which can be made from 10kg of lead. (Take π = 3.142; Volume of sphere = $\frac{4}{3}\pi r^{2.}$).

a. Mass of ball = density x volume

Volume of ball = $\frac{4}{3}$ x 4.142 x $0.2^3 cm^3$

= 0.032

Mass of ball = $\frac{1}{3}$ x 3.142 x 0.032 x 11.4

= 0.382g

Mass of each ball is 0.38 g to 2 s.f.

b. Number of balls = $\frac{mass\ of\ lead}{mass\ of\ one\ ball}$

=$\frac{10kg}{0.382g}$

= $\frac{10{,}000}{0.382}$

= 26178

Number of balls that can be made is 26, 000 to 2 s.f.

A metal sphere of radius 4 cm is recast into solid cylinder of base radius 8cm. assuming that there is no loss of metal, calculate the height of the cylinder. (Volume of the sphere is $\frac{4}{3}\pi r^3$)

Since there is no loss of metal, the volume of the sphere must be equal to the volume of the cylinder.

Volume of the cylinder = $\pi r^2 h$

$$\frac{4}{3}\pi r^3 = \pi r^2 h$$

$$\frac{4}{3}\ x\ 4\ x\ 4\ x\ 4 = 8\ x\ 8\ x\ h$$

h = $\frac{4\ x\ 4\ x\ 4\ x\ 4}{3\ x\ 8\ x\ 8}$ cm

h = 1 $\frac{1}{3}$ cm

Calculate the total surface area of a solid hemisphere of radius 21cm. (Area of a sphere = $4\pi r^2$; Take$\pi = \frac{22}{7}$)

Area of a sphere = $4\pi r^2$

Area of hemisphere = $4\pi r^2 \div 2$

= $2\pi r^2$

= 2 x $\frac{22}{7}$ x 21 x $21cm^2$

= $2772cm^2$

Therefore, the total surface area of a solid hemisphere is $2772cm^2$

A cylindrical metal bar whose volume is $594cm^3$ is melted down and cast into a sphere. Calculate the radius of the sphere, leaving your answer correct to 2 decimal places. (Volume of sphere$\frac{4}{3}\pi r^3$, Take $\pi = 3.14$)

Volume of cylinder = Volume of the sphere

594 = $\frac{4}{3}\pi r^3$

594 = $\frac{4}{3}$ $x\ 3.14\ xr^3$

594 x 3 = 12.568r^3

r^3= $\frac{1782}{12.568}$

r = $\sqrt[3]{\frac{1782}{12.568}}$

= 5.21cm

Therefore, the radius of the sphere is 5.21cm.

A pond 12m in diameter has a shape of a hemisphere and is full of water. The pond is emptied and all the water poured into a cylindrical tank of radius 5m. Assuming there is no loss of water; calculate the height of water in the tank. (Volume of sphere = $\frac{4}{3}\pi r^3$)

Volume of hemisphere = $\frac{4\ x\frac{1}{2}}{3}\pi r^3$

= $\frac{4}{3}x\frac{1}{2}$ $x\ 3.14\ x\ x\ 6\ x\ 6\ x\ 6m^3$

= 452.16

Volume of the cylinder = $\pi r^2 h$

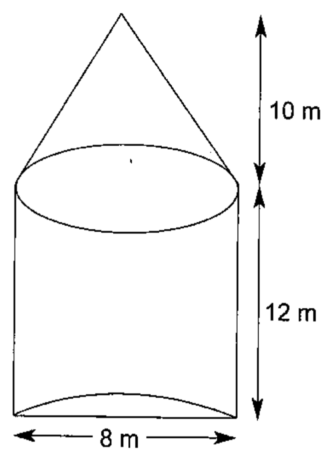
$\pi r^2 h = 452.16$

$3.14\ x\ 5\ x\ 5h = 452.16$

H = $\frac{452.16}{3.14\ x\ 5\ x\ 5}$

= 5.76

Figure shows a cone placed on top of a cylinder. The height of the cone is 10 m and that of a cylinder is 12m. The diameter of both cone and cylinder is 8m.



Calculate the total volume of the shape to 2 decimal places.(Volume of a cone = $\frac{1}{3}\pi r^2$, take $\pi = 3.14$)

Volume of the shape = Volume of the cylinder + Volume of the cone

Volume of the cylinder = $\pi r^2 h$

= $3.14\ x\ \left(\frac{8m}{2}\right)^2\ x\ 12m$

= $602.88m^3$

Volume of the cone = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3}\ x\ 3.14\ x\ \left(\frac{8m}{2}\right)^2\ x\ 10m$

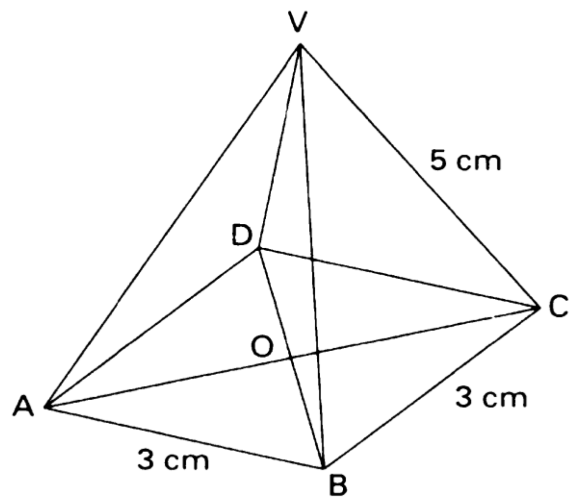
= $167.47m^3$

Volume of the shape = 602.88$m^3 + 167.47m^3$

= 770.35m^3

VOLUME OF PYRAMID

The figure below shows a right Pyramid which has a square base of side 3cm and slant edges 5cm long. Find its volume.

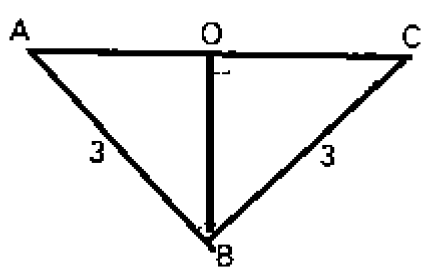


Volume of Pyramid = $\frac{1}{3}$ $x\ area\ of\ base\ x\ height$

We know the area of the base; it is 3 x 3 = 9cm^2

We need to find OV, the height of the Pyramid First, we have to find OB.

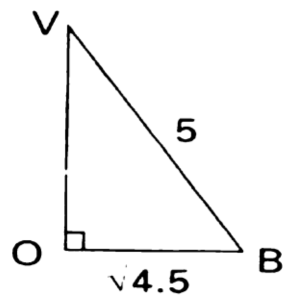
Let us look at the triangle ABC.



By Pythagoras, $OB^2 + OC^2 = 3^2$

OC = OB (half diagonals of a square)

So 2 x OB^2 and OB = $\sqrt{4.5}$



Now we look at triangle BOV.

$VO^2 = 5^2 - (\sqrt{4.5})^2$

Vo = $\sqrt{20.5}$ **cm**

The volume of the Pyramid = $\frac{1}{3}$ **x area of base x height**

$= \frac{1}{3} \times 9 \times \sqrt{20.5}$

$13.6cm^3$

The volume of the pyramid is $60cm^3$ and its base is $20cm^2$. Calculate the height of the pyramid.

Volume of the pyramid = $\frac{1}{3}$ **x base area x height**

$60cm^3 = \frac{1}{3} \times 20 \times$ **height (Multiply both sides by 3)**

$180cm^3 = 20cm^2 \times$ **height**

Height = $\frac{180m^3}{20cm^2}$

Height = 9cm

Therefore, the height of the pyramid is 9cm.

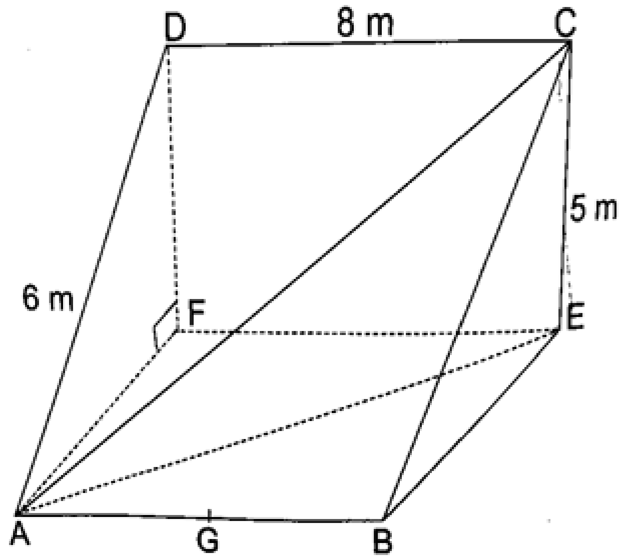
GEOMETRY IN THREE DIMENSIONS

ANGLES BETWEEN LINES AND PLANES

The angle between a line and a plane is defined as the angle between the line and its projection on the plane.

ExampleS

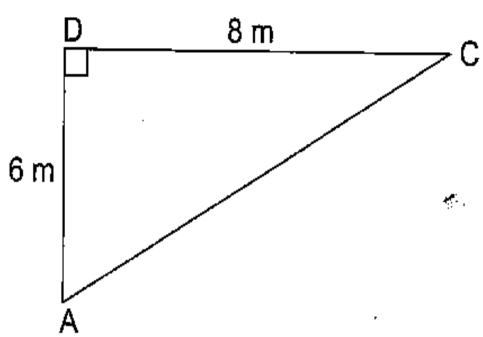
Figure below shows a prism with a horizontal rectangular base ABEF. The faces ABCD and CDFE are rectangles. The planes CDFE, ABEF are perpendicular to each other. The point G is the midpoint of AB . The length DC = 8m, AD = 6m and CE = 5m.



Calculate

a. The length of AC and AE.
b. The angle that AC makes with the plane ABEF
c. The angle ACG.

a. In Triangle ACD



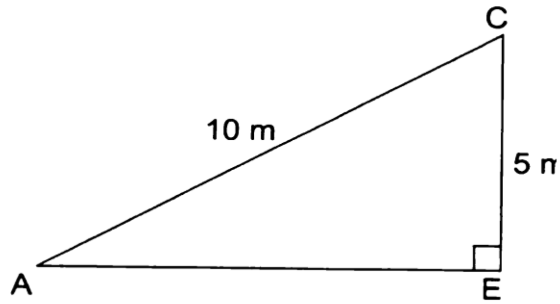
$AC^2 = 6^2 + 8^2$

= 100

AC = $\sqrt{100}$

=10

In Triangle AEC,



10^2 = AE^2 + 5^2

AE^2 =100-25

AE = $\sqrt{75}$

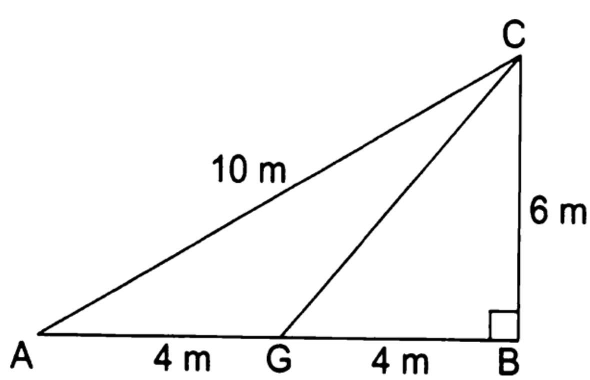
AE = 8.660M

b. The required angle is angle CAE in Triangle ACE.

Sin Angle CAE = $\frac{5}{10}$

= 30°

c. In Triangle ACB,



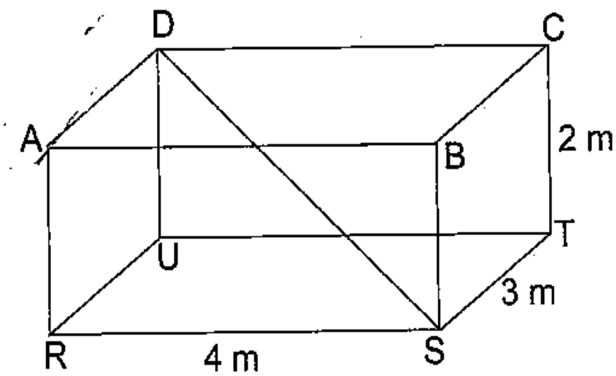
Tan Angle ACB = $\frac{8}{6}$ = 1.3333

Angle ACB = 53.129°

Angle ACG = Angle ACB – Angle GCB

∴acg = 19.4^{o}

Figure below shows a solid cuboid of length 4m, width 3m and height 2m, calculate



Calculate

a. The length of CS, correct to three decimal places.

b. The angle DSC to the nearest degrees.

In triangle BCS, $BC^2 + BS^2 = CS^2$ **(Pythagoras)**

$3^2 + 2^2 = CS^2$

$13 = CS^2$

CS = $\sqrt{13}$

CS = 3.62 to 3 s.f.

In triangle, CDS

$DC^2 + CS^2 = DS^2$ **(Pythagoras)**

$62 + 3.61^2 = DS^2$

DS^2 **= 49.03**

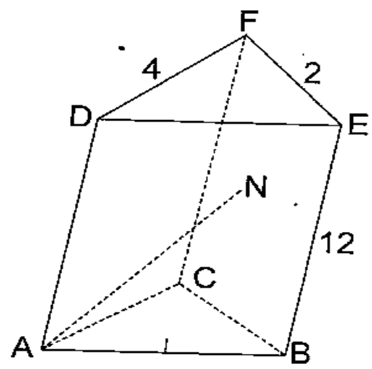
DS = $\sqrt{49.03}$

DS = 7.003

Sin angle DSC = $\frac{DC}{DS}$ = $\frac{6}{7}$

Angle DSC = 59^0

Figure below is a triangular prism with DE = 4cm, EF= 2cm, DF = 4cm and BE = 12cm.



If N is the point where diagonals of the face FEBC meet, find the angle which NA makes with the plane.

Draw line from N to BC bisecting BC at X

NX = $\frac{1}{2}$EB = 6

Join AX

Angle AXB =90°

$AX^2 = AB^2 - BX^2$

$AX^2 = 4^2 - 1^2$

AX = $\sqrt{15}$

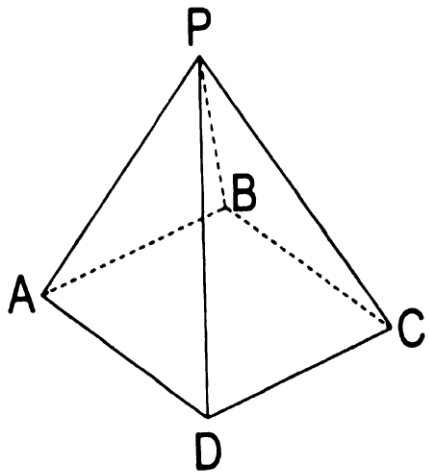
AX = 3.87

Tan ϑ =$\frac{6}{3.87}$

= .55

Tan ϑ =57°

Figure below shows a right pyramid with vertex P and PA = PB = PC = PD = 13CM. ABCD is a rectangular base with AB =8cm and AD = 6CM



Calculate

a. The height of the pyramid

b. The angle between the base and an edge.

a. Join AC and BD intersecting at O

The following can be extracted from the diagram

Since ABCD is a rectangle, then $< ADC = 90^{\circ}$

CD =AB = 8CM

$AC^2 = AD^2 + CD^2$ **(Pythagoras theorem)**

$=(6^2 + 8^2)cm^2$

$= (36 + 64)cm^2$

$= 100cm^2$

AC =$\sqrt{100}$

= 10

Since diagonals of a rectangle bisect each other, then

AO =OC = (10$\div 2$**)cm**

= 5cm

In triangle, APO, angle POA = 90^o

$\therefore PO^2 = PA^2 - AO^2$

= $(13^2 - 5^2)cm^2$

= $169\text{-}25cm^2$

= $144cm^2$

PO = $\sqrt{144}$

= 12CM

The height of the pyramid is 12cm.

b. **We need to find angle PAO**

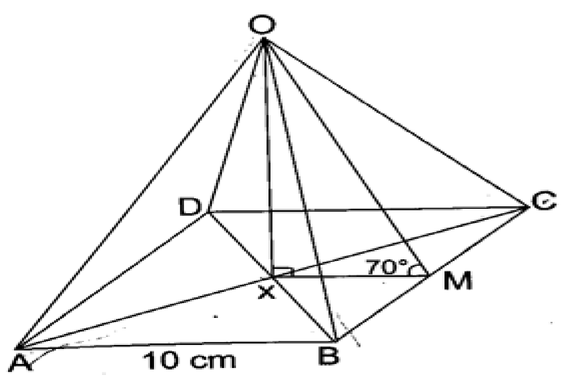
c. **Sin** $< PAO = \frac{12cm}{13cm}$

Sin^{-1} **=67°**

The angle between the base and an edge is 67°

EXERCISE

Figure below is a right pyramid with a square base ABCD of side 10cm. Each of the four triangular faces is inclined at 70° to the base.



Calculate

a. The perpendicular height OX of the pyramid.
b. The length of the slant edge OA.
c. The angle between the edge OA and the base plane ABCD.
d. The area of the face OAB.

CHAPTER 12 :LINEAR PROGRAMMING

Linear programming is a mathematical modeling technique in which a linear function is maximized or minimized when subjected to various constraints. This technique has been useful for guiding quantitative decisions in business planning, in industrial engineering and to a lesser extent in the social and physical sciences.

TERMINOLOGIES OF LINEAR PROGRAMMING

1. **Decision variables**
   - These are variables which will be used as a function of the objective function.
   - These variables decide your output. The decision maker can control the value of an objective function using the decision variable.
   - When you solve any linear programming problem, you first need to identify the decision variables.
2. **Constraints**
   - These are set of restrictions or situational conditions.
   - Constraints can be in equalities or inequalities form.
   - Constraints restrict
3. The objective function
4. Feasible solution
5. The infeasible region
6. The feasible region
7. Non-negativity constraints

CORNER POINT METHOD

The optimal solution to a LPP, if it exists, occurs at the corners of the feasible region.

The method includes the following steps

Step 1: Find the feasible region of the LLP.

Step 2: Find the co-ordinates of each vertex of the feasible region.

These co-ordinates can be obtained from the graph or by solving the equation of the lines.

Step 3: At each vertex (corner point) compute the value of the objective function.

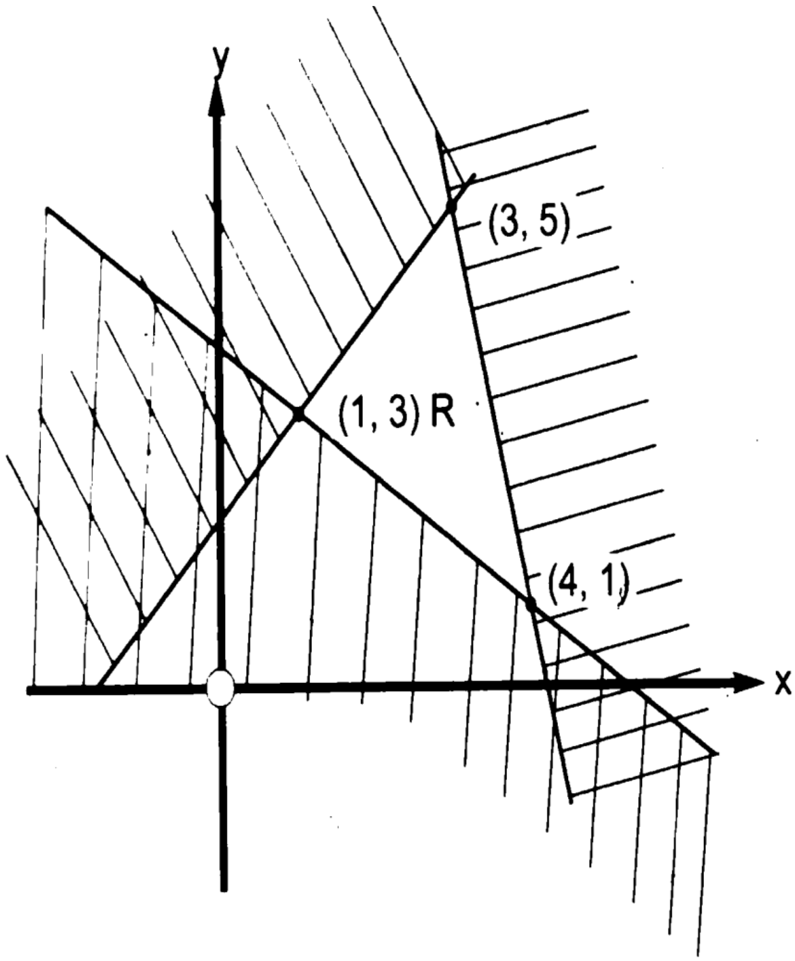
Step 4: Identify the corner point at which the value of the objective function is maximum

(or minimum depending on the LP)

The co-ordinates of this vertex is the optimal solution and the value of Z is the optimal value

EXAMPLES

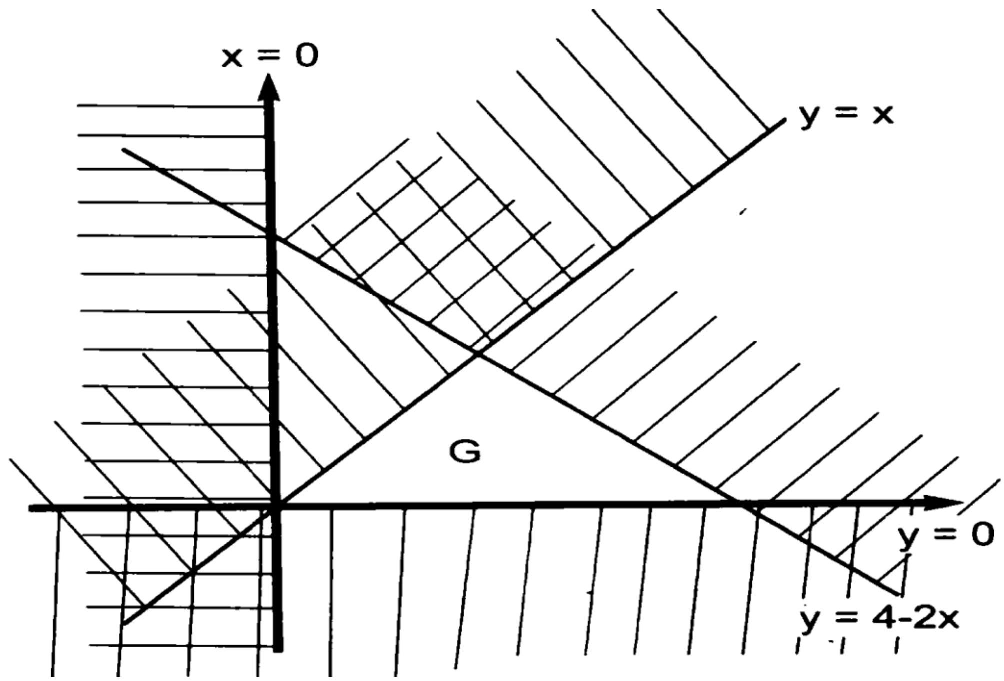
Figure below shows the region R bounded by three inequalities



Calculate the maximum value of 5x-4y + 8 in this region.

To find the maximum value of:
Corner points: (3,5), (1,3), (4,1)
Substitute the corner points (3,5), (1,3), (4,1) in the function 5x -4y + 8
(3,5) : 5(3)-495) +8 =3
(1,3) : 5(1)-4(3) +8 =1
(4,1) : 5(4)-4(1) + 8 =24
The maximum value is 24.

Figure below shows the region G bounded by inequalities.



Write the three inequalities that describe the region G.

For y =4-2x
Y +2x = 4 9subtract -2x both sides)
According to the shaded region, then y +2x $\leq$ 4
For y = x
y-x = 0 (According to the shaded region, then y-x $\geq$ 0
For y = 0 (According to the shaded region, then y $\geq$ 0
Ignoring the inequality x $\geq$ 0 does not change the indicated region
$\therefore$ The three inequalities are:

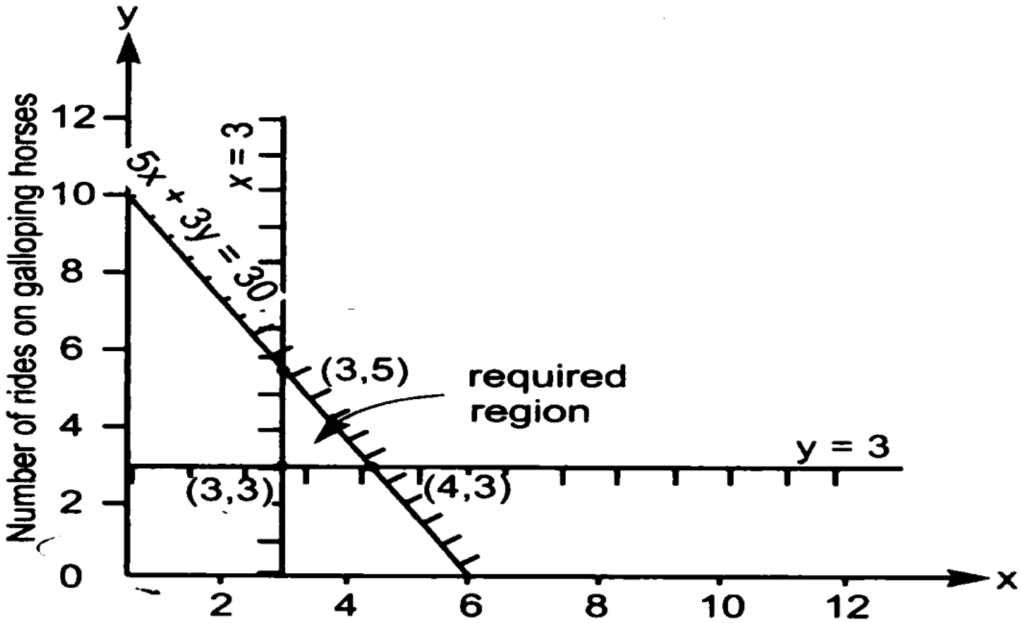
a. **y +2x $\leq$ 4**
b. **y$\leq$ x**
c. **y $\geq$ 0**

At a fair, a ride on bumper-cars costs K500 and on galloping horses K300. A boy has K3, 000 to spend and wants to have at least three rides on each. How can he do if he decides to spend as much as possible of his K3, 000? There are in fact only two ways of doing it. Does either of these give him any change out of his money?

Let x represent number of rides on bumper cars and let y represent number of rides on galloping horses.
$\therefore$ x $\geq$ 3
y $\geq$ 3 and
500x + 300y $\leq$ 3,000

$5x + 3y \leq 30$

The graph representing the above three inequalities.



Evaluate the cost of the rides at each of the coordinates of the corner points.

(3,3) Cost = 3 x K500 + 3 x K300
= k2,400

(3,5), Cost = 3 x K500 + 5 x K300
=K3,000

(4,3), Cost = 4 xK500 + 3 xK300
= K2,900

∴ He spends much of his K3,000 when he has 3 rides on bumper cars and 5 rides on galloping horses.

A shopkeeper buys x metres of material at K10 per metre and y metres of material at K20 per metre.

a. If she has to buy more 250 m of material, write an inequality connecting x and y.
b. If she has K4,000 available, write another inequality connecting x and y.
c. Represent these inequalities on a graph and shade out the regions not required.
d. The shopkeeper makes a profit of 50t per metre on the cheaper material and 75t per metre on the expensive material. Use your graph to find to the

nearest the number of metres of each material which must be bought to give the maximum profit. State the profit.

e. $x + y > 250$

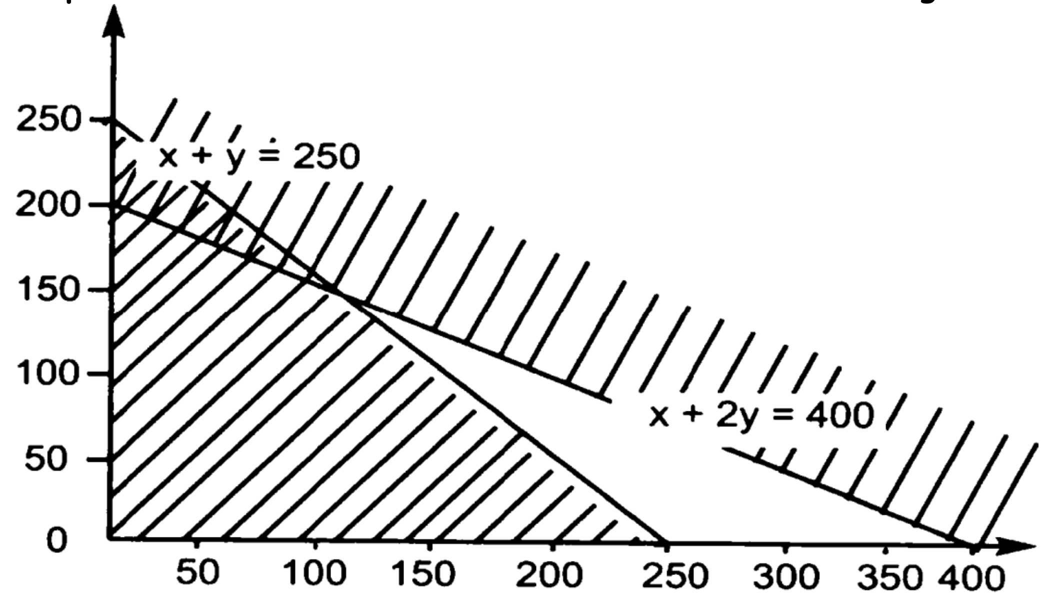
f. **Cost of x meters at K20/m = K10x**

Cost of y meters at K20/m = K20Y

$\therefore$ **10X + 20Y $\leq$ 4000**

The point (395,5) gives the maximum profit.

g. **Graphs to show constraints on the amount of material bought.**



395m of the cheaper material and 5 m of expensive material must be bought to give the maximum profit. The profit is K201.25.

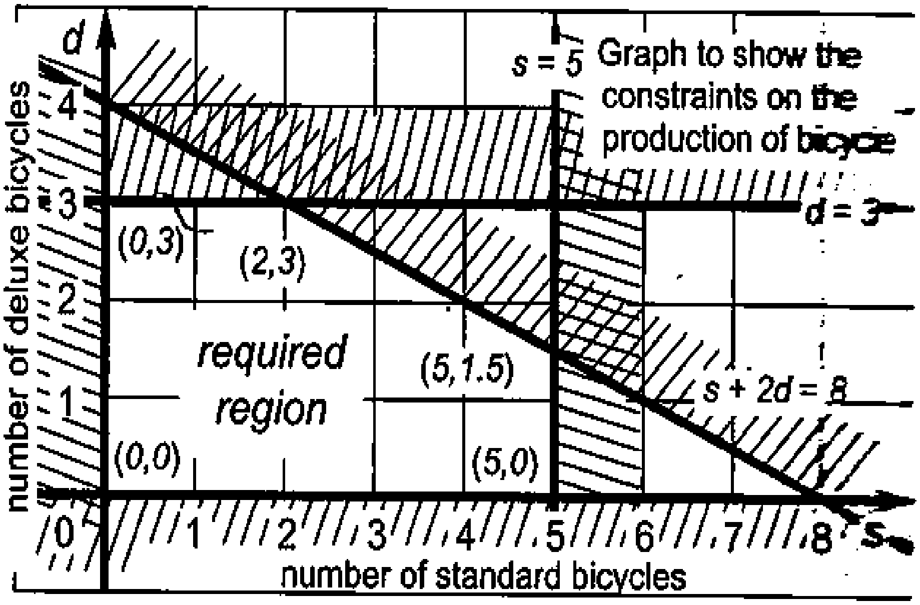
A small bicycle assembly plant makes two models, Standard and Deluxe. No more than 5 Standard and 3 Deluxe bicycles can be made daily. There are 8 workers at the plant and it takes 1man to make a Standard model and 2 men to make a Deluxe model daily.

a. Write down the constrain or limitations and present them graphically.
b. If the profit on a standard model is K100 and the profit on a Deluxe is K300, how many of each type must be made each day to make the maximum profit?

Let s represent the number of Standard bicycles and d represent the number of Deluxe bicycles.

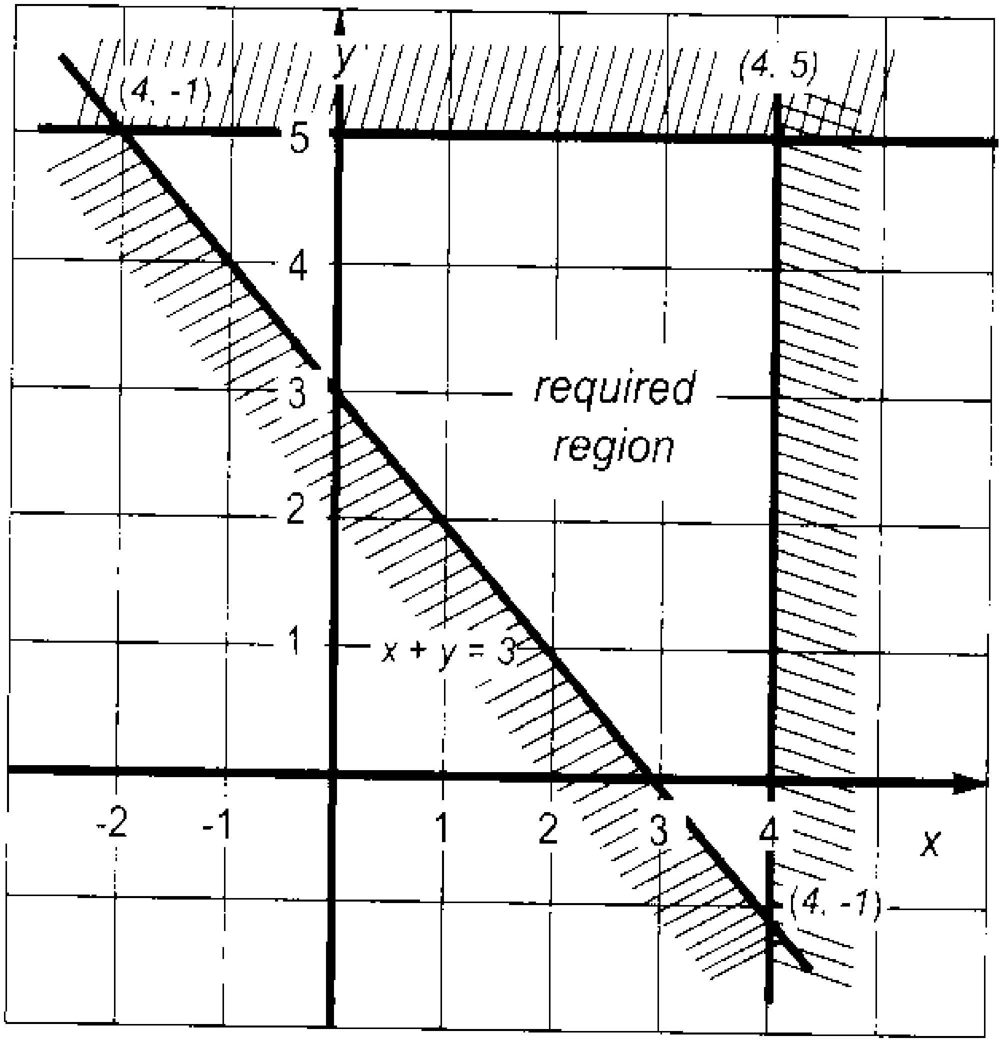
a. From the second sentence

S ≤ 5
S ≤ 3
Form third sentence
S + 2d ≤ 8
Also s ≥0,



b. Objective function is P=100+300d.
Evaluate P at each each of the vertices of the region satisfied by the inequalities.
(0,0), P = 0
(5,0), P 100x5+0 =500
(5,3/2), P =100x5+300x3/2)
950
(2,3), P =100x2 +300x3
=1100
(2,3),P=0+300 x3=900
∴ Maximum profit occurs when 2 Standard and 3 Deluxe models are made per day.

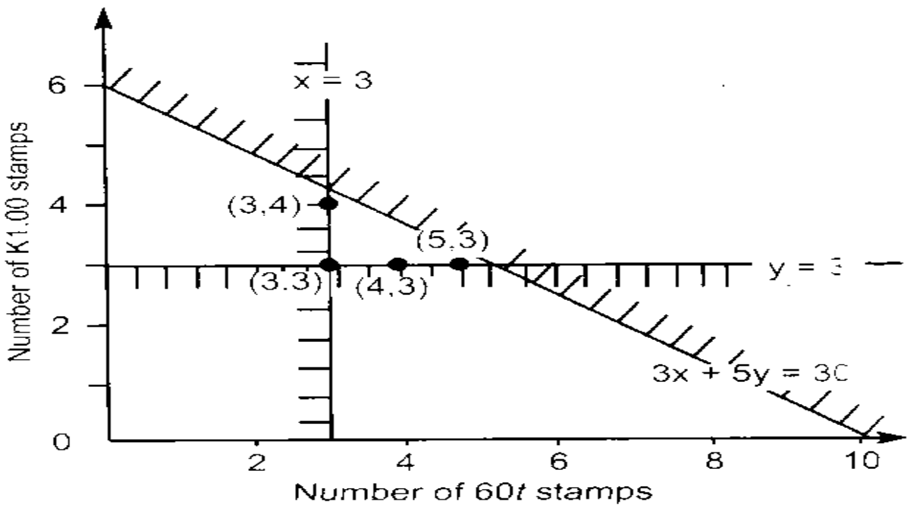
Sketch the area defined by the following inequalities giving coordinates of vertices.
x≤4
y ≤ 5 and
x +y ≥ 3



Ulemu has K6.00 for buying stamps. He has to buy stamps at 60t each and at K1.00 each. He has to buy at least three of each type.

a. Write down three inequalities from this information.
b. Use a graph to illustrate the region defined by the inequalities.
c. Which combination of stamps would give the largest number of stamps?

Lt x represent 60t stamps and y represents K1.00 stamps.
From the first sentence
$60x + 100y \leq 600$
$3x+5y \leq 30$
From the last sentence,
$x \leq 3$, and
$y \leq 3$



The objective function is N= x+y , where N is the number of stamps bought. Evaluate N at each of the vertices of the region satisfied by each inequalities.
(3,3), N = 3+3 = 6
(4,3), N= 4+3 = 7
(5,3), N =5+3 = 8
93,4), N = 3+4 = 7
5+60t stamps and 3(K1.00) stamps gives the largest number of stamps.

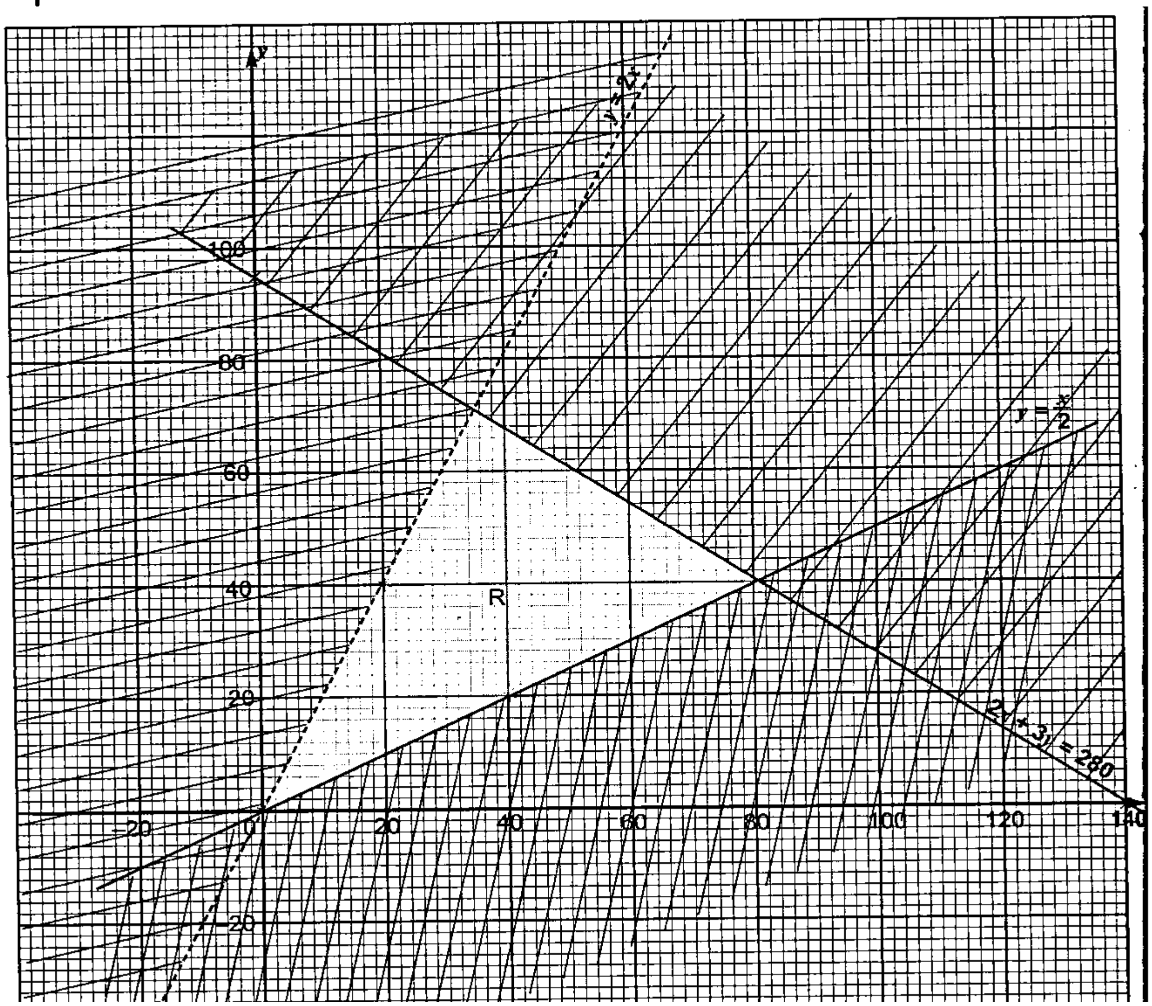
A city assembly decides to construct a $1400m^2$ car park for Lorries and minibuses. A minibus will fit on a $10m^2$ space while a lorry requires $15m^2$ of space. The number of Lorries has to be greater or equal to half the number of minibuses. The number of Lorries has to be less than twice the number of minibuses.

a. If x represent the number of minibuses and y represent the number of lorries, write down one inequality involving x and y in addition to y ≥ $\frac{x}{2}$ and y <2x
b. Using a scale of 2cm to represent 20 units on both axes, draw the region R bounded by the three inequalities.
c. Use your graph to find the maximum number of vehicles (minibuses and lorries) that can be parked on a car park of such a size.

a. If 1 lorry requires $15m^2$space, then y lorries require $15ym^2$ space
If 1 minibus requires $10m^2$ space, then x minibuses require $10xm^2$.
With only $1400m^2$ space available, then
10x +15y ≤1400
2x + 3y ≤ 280 (divide each term by 5)

∴The inequality is $2x + 3y \leq 280$

b. Graph



c. Need to find the maximum of x + y coordinate bounding the feasible region

(0,0)= 0+0 =0

(80,40) = 80+40 =120

(36,70) =36 +70 = 106

Therefore, 80 minibuses and 40 lorries are required to maximize the objective function.

A farmer gets a loan of K50, 000 to buy sheep and goats only. Sheep costs K5, 000 each while goats cost K2, 000 each. She would like to spend at least K10, 000 more on sheep than on goats.

a. If x represents the number of sheep and y the number of goats, write down two inequalities in x and y in addition to $x \geq 0$ and $y \geq 0$.

b. Using a scale of 2cm to represent 2 units on the horizontal axis and 2cm to represent 5 units on the vertical axis, draw the region which represents the four inequalities.
c. How many sheep and goats can the farmer buy to have the maximum number of animals with the loan.

a. Each sheep costs K5,000 and each goat cost K2,000

The total amount of money to be spent on buying sheep = K5,000x
Total amount of money to be spent on buying goats = K2, 000y.

Since the farmer has an amount of K5,0000

$5000x + 2{,}000y \leq 5{,}000$

$5x + 2y \leq 50$

The farmer intends to spend at least K10,000 more on sheep than on goats, then

$5000x - 2000y \geq 1{,}000$

$5x - 2y \geq 10$ **(divide each term by 1,000)**

$\therefore$ **The two inequalities are as follows**

(i) $5x + 2y \leq 50$

(ii) $5x - 2y \geq 10$

b. x= 0 is the y-axis

y = 0 is the x-axis

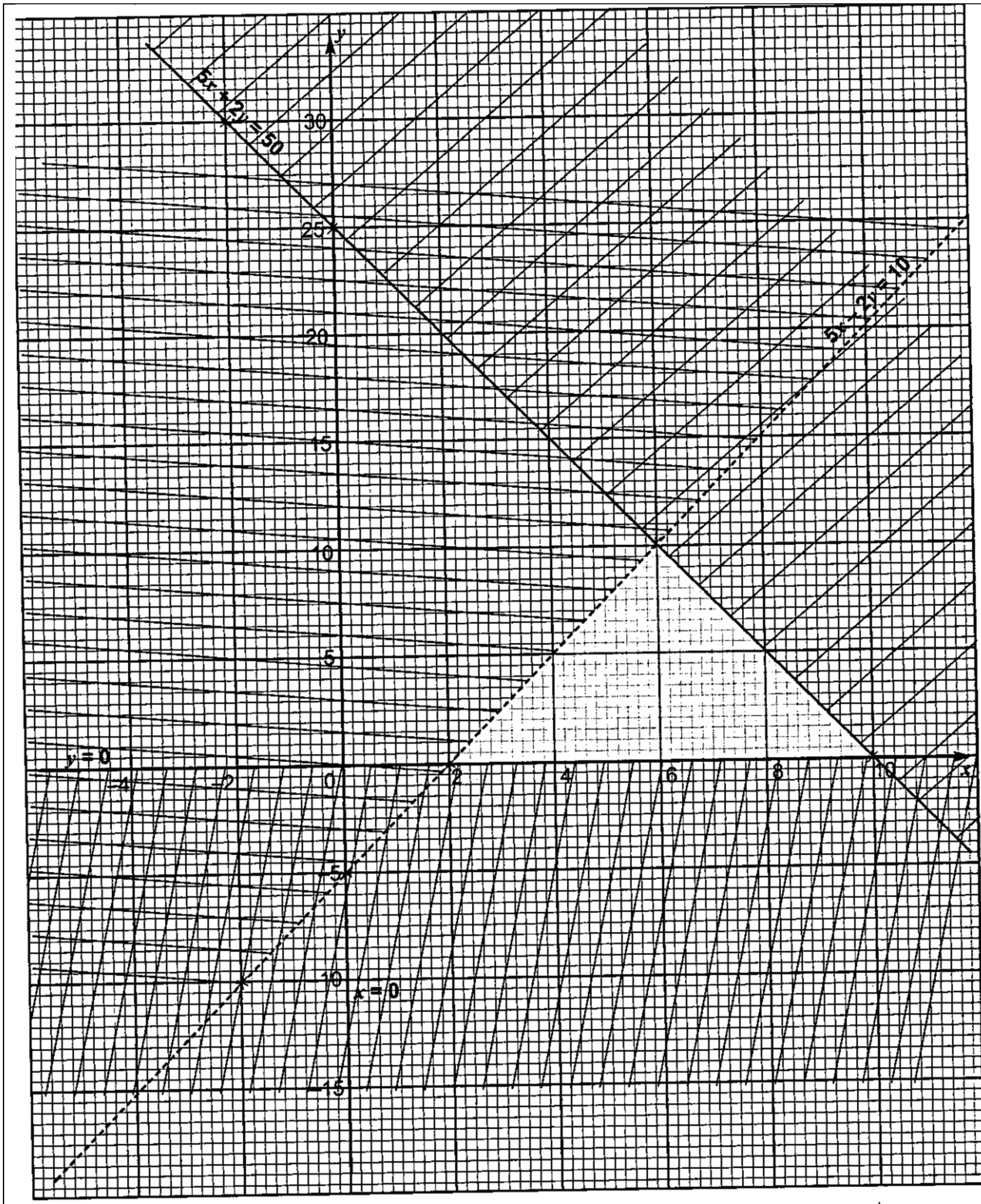
(iii) $5x + 2y \leq 50$

x	0	6	10
y	25	10	0

(iv) $5x - 2y \geq 10$

x	0	2	4
y	-5	0	5

Graph



c. Objective function = x + y

Need to maximize the objective function

$x \geq 0$; $y \leq 0$; $5x + 2y \leq 50$; $5x - 2y \geq 10$

Coordinates bonding the wanted region

(2,0), (10,0)(6,10)

Substitute the coordinates in the objective function

(x,y)	x + y
(2,0)	2+0 =2
(10,0)	10+0 =10
(6,10)	6+10 =16

Since 16 is the maximum, the farmer should therefore buy 6 sheep and 10 goats.