CONQUERING

JUNIOR SECONDARY

MATHEMATICS`

STUDENT'S BOOK 2

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Preface

This books has been written with the aim of assisting learners who are facing some difficulties in mathematics. The book is syllabus oriented and includes almost all contents of the junior mathematics syllabus.

The language in this book is scaled to such an extent to suit leaner's mastery of language at this level. Examples are explained in a detailed and precise manner. Immediately after examples, an exercise is straight away given to consolidate what learners gained.   The concepts in this book are presented in such away that learners should find mathematics enjoyable like any other subject. The explanations are designed in a down to earth approach so that learners should not find any difficulties in mastering the subject matter.

Form 1 is the basic foundation of success in succeeding classes. It is for this reason that this book has been written in such away that learners should be equipped with all the concepts in a clear and simple way. It is my hope that teachers and learners will find this book helpful and indeed address some of the gaps that exists in the knowledge of mathematics in form one.

Marc Anthony Banda

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CHAPTER ONE

NUMBER PATTERNS

In this chapter we will learn about the mathematics of patterns

Activity: Investigation : Patterns

Can you spot any patterns in the following lists of numbers?

1. 2; 4; 6; 8; 10; ...
2. 1; 2; 4; 7; 11; ...
3. 1; 4; 9; 16; 25; ...
4. 5; 10; 20; 40; 80; ...

Common Number Patterns

Numbers can have interesting patterns. Here we list the most common patterns and how they are made.

Examples:

1. 1,4,7,10,13,16,19,22,25,...

This sequence has a difference of 3 between each number. The pattern is continued by adding 3 to the last number each time.

2. 3, 8 ,13, 18 ,23, 28, 33, 38,...

This sequence has a difference of 5 between each number. The pattern is continued by adding 5 to the last number each time.

3. 2 ,4, 8, 16, 32, 64, 128, 256,...

This sequence has a factor of 2 between each number. The pattern is continued by multiplying the last number by 2 each time.

4. 3 ,9, 27 ,81, 243, 729 ,2187,...

This sequence has a factor of 3 between each number. The pattern is continued by multiplying the last number by 3 each time.

Special Sequences

1. Triangular Numbers

1,3,6,10,15,21,28,36,45,...

This sequence is generated from a pattern of dots which form a triangle. By adding another row of dots and counting all the dots we can find the next number of the sequence.

2. Square Numbers

1,4,9,16,25,36,49,64,81,...

The next number is made by squaring where it is in the pattern. The second number is 2 squared

(22 $or\ 2 \times 2$) The seventh number is 7 squared ($7^2\ or\ 7 \times 7$) etc

3. Cube Numbers

1,8,27,64,125,216,343,512,729,...

The next number is made by cubing where it is in the pattern. The second number is 2 cubed

(23 $or\ 2 \times 2 \times 2$) The seventh number is 7 cubed ($7^3\ or\ 7 \times 7 \times 7$) etc

Fibonacci Numbers

0,1,1,2,3,5,8,13,21,34,...

The next number is found by adding the two numbers before it together. The 2 is found by adding the two numbers in front of it (1 + 1) The 21 is found by adding the two numbers in front of it (8 + 13) The next number in the sequence above would be 55 (21 + 34) Can you figure out the next few numbers?

Exercise 1

Write down each sequence and find the next two numbers.

1. 2, 6, 10, 14
2. 2, 9, 16, 23
3. 95, 87, 79, 71
4. 13, 8, 3, −2
5. 7, 9, 12, 16
6. 20, 17, 13, 8
7. 1, 2, 4, 7, 11
8. 1, 2, 4, 8
9. 55, 49, 42, 34
10. 10, 8, 5, 1
11. −18, −13, −9, −6
12. 120, 60, 30, 15
13. 27, 9, 3, 1
14. 162, 54, 18, 6
15. 2, 5, 11, 20
16. 1, 4, 20, 120
17. 2, 3, 1, 4, 0
18. 720, 120, 24, 6

The n^{th} term of a sequence (Notation)

A sequence does not have to follow a pattern but when it does we can often write down a formula to calculate the n^{th}-term, a_n. In the sequence

1; 4; 9; 16; 25;

where the sequence consists of the squares of integers, the formula for the n^{th}-term is

$$a_n = n^2$$

Therefore, we can generate a pattern, namely squares of integers.

Examples:

(a) For the sequence 4, 8, 12, 16, ...
The 10th term is $4 \times 10 = 40$.
The nth term is $4n$.

(b) For the sequence $(1 \times 2), (2 \times 3), (3 \times 4), (4 \times 5), \ldots$
The 10th term is 10×11.
The nth term is $n(n+1)$.

(c) For the sequence 5, 7, 9, 11, ...
The common difference between each term is 2 so the expression will contain $2n$.
The nth term is $2n + 3$, by inspection.

The General Formula

Example 1

Find the general formula for the following sequences

(a) 2,5,8,11,14...

(b) 0,4,8,12,16...

Solution

a. For the pattern 2,5,8,11,14,….

We can see that

The second term = 2 +3(2-1)

The 3^{rd} term = 2 +3(3-1)

For the 4^{th} term = 2+ 3(4-1) and so on

So, the general formula for the above sequence is 2 +3(n-1)

b. For the sequence 0, 4, 8, 12, 16

The second term = 0+4(2-1)

Third term = 0 +4(3-1)

4^{th} term = 0+4(4-1) and so on

Therefore the nth term = 4(n – 1)

Example 2

Find the nth term for

a. 0, 3, 8, 15, 24,-----
b. -1, 2, 7, 14, 23,-----

Solution

a. for the first term = 0^2-1
   2^{nd} term = 1^2-1
   3^{rd} term = 2^2-1
   4^{th} term = 2^3-1

   Therefore, the nth term = n^2-1
b. 1^{st} term = 1^2- 2
   2^{nd} term = 2^2-2
   3^{rd} term = 3^2-2
   4^{th} term = 4^2-2

   There fore, the nth term = n^2-2

Activity

The general term has been given for each sequence below. Work out the missing terms.

(b) 3;2;1;0;...;2 $-n + 4$
(c) 11;...;7;...;3 $-13 + 2n$

Exercise

1. Write down each sequence and select the correct formula for the nth term from the list given.

$\boxed{11n}$ $\boxed{10n}$ $\boxed{2n}$ $\boxed{n^2}$ $\boxed{10^n}$ $\boxed{3n}$ $\boxed{100n}$ $\boxed{n^3}$

(a) 2, 4, 6, 8, ...
(b) 10, 20, 30, 40, ...
(c) 3, 6, 9, 12, ...
(d) 11, 22, 33, 44, ...
(e) 100, 200, 300, 400, ...
(f) $1^2, 2^2, 3^2, 4^2, \ldots$
(g) 10, 100, 1000, 10 000, ...
(h) $1^3, 2^3, 3^3, 4^3, \ldots$

2. Look at the sequence: 5, 8, 13, 20, ...
Decide which of the following is the correct expression for the nth term of the sequence.

$\boxed{4n+1}$ $\boxed{3n+2}$ $\boxed{n^2+4}$

In questions **3** to **10** find a formula for the nth term.

3. 5, 10, 15, 20, ...
4. 2, 4, 8, 16, 32, ...
5. $(1 \times 3), (2 \times 4), (3 \times 5), \ldots$
6. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$
7. 7, 14, 21, 28, ...
8. 1, 4, 9, 16, 25, ...
9. $\frac{5}{1^2}, \frac{5}{2^2}, \frac{5}{3^2}, \frac{5}{4^2}, \ldots$
10. $\frac{3}{1}, \frac{4}{2}, \frac{5}{3}, \frac{6}{4}, \ldots$
11. 3, 7, 11, 15, ...
12. 5, 7, 9, 11
13. 7, 5, 3, 1
14. $-5, -1, 3, 7, \ldots$

Arithmetic sequence

The arithmetic sequence is given by the formula:

n^{th} term = a +(n – 1)d

where a is the first term , n is the number of terms and d is the common difference.

In an arithmetic sequence, the the difference between two succeeding terms is always the same.

For example , in the sequence 2,4,6,8,… the difference between 2 and 4 , 4 and 6 and so on is 2.

2 is called the common difference.

Example

Find the 17th term in the arithmetic sequence 12, 9, 6, … .

SOlUTION

In this case a = 12 and d = -3.

Using *nth term* $= a + (n - 1)d$, you obtain

17 th term $= 12 + (17 - 1) \times (- 3)$

$= 12 - 48$

$= -36.$

17

The 17th term is -36

Exercise

1 Are the following sequences arithmetic?

If so, state the common difference and the seventh term.

(i) 27, 29, 31, 33, … (ii) 1, 2, 3, 5, 8, … (iii) 2, 4, 8, 16, …

(iv) 3, 7, 11, 15, … (v) 8, 6, 4, 2, …

2 The first term of an arithmetic sequence is -8 and the common difference is 3.

(i) Find the seventh term of the sequence.

(ii) The last term of the sequence is 100.

How many terms are there in the sequence?

3 The first term of an arithmetic sequence is 12, the seventh term is 36 and the last term is 144.

(i) Find the common difference.

(ii) Find how many terms there are in the sequence.

CHAPTER TWO

QUADRATIC EXPRESSION AND EQUATIONS

A quadratic is any expression of the form $ax^2 + bx + c$, $a \neq 0$.

Factorization of quadratic expression

To factorize the expression, we look for two numbers such that their product is ac and their sum is b. a, b are the coefficient of x while c is the constant.

We can learn about how to factorise quadratics by looking at how two binomials are multiplied to get a quadratic. For example, $(x + 2)(x + 3)$ is multiplied out as:

$$(x + 2)(x + 3)$$

$$= x(x + 3) + 2(x + 3)$$

$$= (x)(x) + 3x + 2x + (2)(3)$$

$$= x^2 + 5x + 6.$$

Expand the following:

$(X+2)(X+2)$ ans: x^2+4x+4

$(X-3)(X-3)$ ans: x^2-6x+9

$(X+2)(X-2)$ ans: x^2-4

We see that the x^2 term in the quadratic is the product of the x-terms in each bracket. Similarly, the 6 in the quadratic is the product of the 2 and 3 the brackets. Finally, the middle term is the sum of two terms.

So, how do we use this information to factorise the quadratic?

Let us start with factorising x^2+5x+6 and see if we can decide upon some general rules. Firstly, write down two brackets with an x in each bracket and space for the remaining terms.

$$(\quad x \quad)(\quad x \quad)$$

Next decide upon the factors of 6. Since the 6 is positive, these are:

Factors of 6

1	6
2	3
-1	-6
-2	-3

Therefore, we have four possibilities:

Option 1	Option 2	Option 3	Option 4
$(x + 1)(x + 6)$	$(x - 1)(x - 6)$	$(x + 2)(x + 3)$	$(x - 2)(x - 3)$

Next we expand each set of brackets to see which option gives us the correct middle term.

Option 1	Option 2	Option 3	Option 4
$(x+1)(x+6)$	$(x-1)(x-6)$	$(x+2)(x+3)$	$(x-2)(x-3)$
x^2+7x+6	x^2-7x+6	$\underline{x^2+5x+6}$	x^2-5x+6

We see that Option 3 (x+2)(x+3) is the correct solution. As you have seen that the process of factorising a quadratic is mostly trial and error, however there is some information that can be used to simplify the process.

Method: Factorising a Quadratic expression

1. First factorise the entire equation by any common factor of the coefficients, so as to obtain an equation of the form $ax^2 + bx + c = 0$ where a, b and c have no common factors and a is positive.
2. Write down two brackets with an x in each bracket and space for the remaining terms.

   $(x \quad)(x \quad)$

3. Write down a set of factors for a and c.
4. Write down a set of options for the possible factors for the quadratic using the factors of a and c.
5. Expand all options to see which one gives you the correct answer.

There are some tips that you can keep in mind:

- If c is positive, then the factors of c must be either both positive and both negative. The factors are both negative if b is negative, and are both positive if b is positive. If c is negative, it means only one of the factors of c is negative, the other one being positive.
- Once you get an answer, multiply out your brackets again just to make sure it really works.

Example

Factorise x^2+4x+4

Look for two number such that their product is 1x4 = 4

Their sum is 4 where 4 is the coefficient of X

The numbers are 2 and 2

Rewrite 4x as 2x+2x thus,

$x^2+2x+2x+4$

$= x(x+2)+2(x+2)$

$= (x+2)(x+2)$

Example

Factorise : $8x^2+10x+3$

Solution

Look for two number such that their product is 8 x 3 = 24.

Their sum is 10 where 10 is the coefficient of x,

The number are 4 and 6,

Rewrite the term 10x as 4x + 6x, thus

$8x^2 +4x + 6x + 3$

Use the grouping method to factorize the expression

$= 4x (2x + 1) + 3 (2x + 1)$

$= (4x + 3) (2x + 1)$

Example

Factorize :$6x^2 – 13x +6$

Solution

Look for two number such that the product is 6 x 6 =36 and the sum is -13.

The numbers are -4 and – 9

Therefore,

$= 6x^2 – 4x – 9x +6$

$=2x (3x -2)-3(3x-2)$

$= (2x-3) (3x- 2)$

Example

Factorise $3x^2 + 13x + 4$.

(a) Find two numbers which multiply to give 12 and add up to 13.
In this case the numbers are 1 and 12.

(b) Split the '$13x$' term,
$3x^2 + x + 12x + 4$

(c) Factorise in pairs,
$x(3x + 1) + 4(3x + 1)$

(d) $(3x + 1)$ is common,
$(3x + 1)(x + 4)$

Exercise 1

1. Factorise the following:

(a) $x^2 + 8x + 15$ (b) $x^2 + 10x + 24$ (c) $x^2 + 9x + 8$ (d) $x^2 + 9x + 14$

(e) $x^2 + 15x + 36$ (f) $x^2 + 13x + 36$

2. Factorise the following:

(a) $x^2 - 2x - 15$
(b) $x^2 + 2x - 3$
(c) $x^2 + 2x - 8$
(d) $x^2 + x - 20$
(e) $x^2 - x - 20$

3. Find the factors for the following quadratic expressions:

(a) $2x^2 + 11x + 5$
(b) $3x^2 + 19x + 6$ (c) $6x^2 + 7x + 2$
(d) $12x^2 + 7x + 1$
(e) $8x^2 + 6x + 1$

4. Find the factors for the following trinomials:

(a) $3x^2 + 17x - 6$
(b) $7x^2 - 6x - 1$
(c) $8x^2 - 6x + 1$
(d) $2x^2 - 5x - 3$

5. factorise

(a) $x + x^2$
(b) $x^2 + 1 + 2x$
(c) $x^2 - 4x + 5$

(d) $16x^2 - 9$

(e) $4x^2 + 4x + 1$

Exercise 2

Factorise the following:

1. $x^2 + 7x + 10$
2. $x^2 + 7x + 12$
3. $x^2 + 8x + 15$
4. $x^2 + 10x + 21$
5. $x^2 + 8x + 12$
6. $y^2 + 12y + 35$
7. $y^2 - 11y + 24$
8. $y^2 + 10y + 25$
9. $y^2 + 15y + 36$
10. $a^2 - 3a - 10$
11. $a^2 - a - 12$
12. $z^2 + z - 6$
13. $x^2 - 2x - 35$
14. $x^2 - 5x - 24$
15. $x^2 - 6x + 8$
16. $y^2 - 5y + 6$
17. $x^2 - 8x + 15$
18. $a^2 - a - 6$
19. $a^2 + 14a + 45$
20. $b^2 - 4b - 21$
21. $x^2 - 8x + 16$
22. $y^2 + 2y + 1$
23. $y^2 - 3y - 28$
24. $x^2 - x - 20$
25. $x^2 - 8x - 240$
26. $x^2 - 26x + 165$
27. $y^2 + 3y - 108$
28. $x^2 - 49$
29. $x^2 - 9$
30. $x^2 - 16$

Exercise 3

Factorise the following:

1. $2x^2 + 5x + 3$
2. $2x^2 + 7x + 3$
3. $3x^2 + 7x + 2$
4. $2x^2 + 11x + 12$
5. $3x^2 + 8x + 4$
6. $2x^2 + 7x + 5$
7. $3x^2 - 5x - 2$
8. $2x^2 - x - 15$
9. $2x^2 + x - 21$
10. $3x^2 - 17x - 28$
11. $6x^2 + 7x + 2$
12. $12x^2 + 23x + 10$
13. $3x^2 - 11x + 6$
14. $3y^2 - 11y + 10$
15. $4y^2 - 23y + 15$
16. $6y^2 + 7y - 3$
17. $6x^2 - 27x + 30$
18. $10x^2 + 9x + 2$
19. $6x^2 - 19x + 3$
20. $8x^2 - 10x - 3$
21. $12x^2 + 4x - 5$
22. $16x^2 + 19x + 3$
23. $4a^2 - 4a + 1$
24. $12x^2 + 17x - 14$
25. $15x^2 + 44x - 3$
26. $48x^2 + 46x + 5$
27. $64y^2 + 4y - 3$
28. $120x^2 + 67x - 5$
29. $9x^2 - 1$
30. $4a^2 - 9$

Difference of two square method

The third simplest quadratic is made up of the difference of squares. We know that:

$(a + b)(a - b) = a^2 - b^2$.

This is true for any values of a and b, and more importantly since it is an equality, we can also write:

$a^2 - b^2 = (a + b)(a - b)$.

This means that if we ever come across a quadratic that is made up of a difference of squares, we can immediately write down what the factors are.

Example: Factorise $9x^2 - 25$

Step 1 : Examine the quadratic expression

We see that the quadratic is a difference of squares because:

$(3x)^2 = 9x^2$

and

$5^2 = 25.$

Step 2 : Write the quadratic as the difference of squares

$9x^2 - 25 = (3x)^2 - 5^2$

Step 3 : Write the factors

$(3x)^2 - 5^2 = (3x - 5)(3x + 5)$

Example

Factorise (a) $4a^2 - b^2$
(b) $3x^2 - 27y^2$

(a) $4a^2 - b^2 = (2a)^2 - b^2$
$= (2a - b)(2a + b)$

(b) $3x^2 - 27y^2 = 3(x^2 - 9y^2)$
$= 3[x^2 - (3y)^2]$
$= 3(x - 3y)(x + 3y)$

Exercise

Factorise the following:

1. $y^2 - a^2$ **2.** $m^2 - n^2$ **3.** $x^2 - t^2$ **4.** $y^2 - 1$
5. $x^2 - 9$ **6.** $a^2 - 25$ **7.** $x^2 - \frac{1}{4}$ **8.** $x^2 - \frac{1}{9}$

9. $4x^2 - y^2$ **10.** $a^2 - 4b^2$ **11.** $25x^2 - 4y^2$ **12.** $9x^2 - 16y^2$
13. $x^2 - \frac{y^2}{4}$ **14.** $9m^2 - \frac{4}{9}n^2$ **15.** $16t^2 - \frac{4}{25}s^2$ **16.** $4x^2 - \frac{z^2}{100}$
17. $x^3 - x$ **18.** $a^3 - ab^2$ **19.** $4x^3 - x$ **20.** $8x^3 - 2xy^2$
21. $12x^3 - 3xy^2$ **22.** $18m^3 - 8mn^2$ **23.** $5x^2 - 1\frac{1}{4}$ **24.** $50a^3 - 18ab^2$
25. $12x^2y - 3yz^2$ **26.** $36a^3b - 4ab^3$ **27.** $50a^5 - 8a^3b^2$ **28.** $36x^3y - 225xy^3$

Evaluate the following:

29. $81^2 - 80^2$ **30.** $102^2 - 100^2$ **31.** $225^2 - 215^2$ **32.** $1211^2 - 1210^2$
33. $723^2 - 720^2$ **34.** $3{\cdot}8^2 - 3{\cdot}7^2$ **35.** $5{\cdot}24^2 - 4{\cdot}76^2$ **36.** $1234^2 - 1235^2$
37. $3{\cdot}81^2 - 3{\cdot}8^2$ **38.** $540^2 - 550^2$ **39.** $7{\cdot}68^2 - 2{\cdot}32^2$ **40.** $0{\cdot}003^2 - 0{\cdot}002^2$

Factorisation by Grouping

One other method of factorisation involves the use of common factors. We know that the factors of $3x+ 3$ are 3 and $(x + 1)$. Similarly, the factors of $2x^2 + 2x$ are $2x$ and $(x + 1)$. Therefore, if we have an expression:

$2x^2 + 2x + 3x + 3$

then we can factorise as:

$2x(x + 1) + 3(x + 1)$.

You can see that there is another common factor: $x + 1$. Therefore, we can now write:

$(x + 1)(2x + 3)$.

We get this by taking out the $x + 1$ and see what is left over. We have a $+2x$ from the first term and a $+3$ from the second term. This is called factorisation by grouping.

Step 6 : Write the final answer

The factors of $7x+14y+bx+2by$ are $(7+b)$ and $(x+2y)$.

Exercise:

Factorisation by Grouping

1. Factorise by grouping: $6x + 9 + 2ax + 3$
2. Factorise by grouping: $x^2 - 6x + 5x - 30$
3. Factorise by grouping: $5x + 10y - ax - 2ay$
4. Factorise by grouping: $a^2 - 2a - ax + 2x$
5. Factorise by grouping: $5xy - 3y + 10x - 6$

Quadratic Equations

So far, we have met linear equations which have one solution only. Quadratic equations always have an x^2 term, and often an x term and a number term, and generally have two different solutions.

Solution by factors

Consider the equation $a \times b = 0$, where a and b are numbers. The product $a \times b$ can only be zero if either a or b (or both) is equal to zero. Can you think of other possible pairs of numbers which multiply together to give zero?

Example 1

Solve the equation $x^2 + x - 12 = 0$

Factorising, $(x-3)(x+4) = 0$

either $x - 3 = 0$ or $x + 4 = 0$

$x = 3$ $\quad$ $x = -4$

Example 2

Solve the equation $6x^2 + x - 2 = 0$

Factorising, $(2x-1)(3x+2) = 0$

either $2x - 1 = 0$ or $3x + 2 = 0$

$2x = 1$ $\quad$ $3x = -2$

$x = \frac{1}{2}$ $\quad$ $x = -\frac{2}{3}$

Example

Solve: $x^2 + 3x - 54 = 0$

Solution

Factorize the left hand side

$x^2 + 9x - 6x - 54 = 0$

$(x-6)(x+9) = 0$

$x - 6 = 0$, $x + 9 = 0$

Hence $x = 6$ or -9

Exercise

Solve the following equations:

1. $x^2 + 7x + 12 = 0$
2. $x^2 + 7x + 10 = 0$
3. $x^2 + 2x - 15 = 0$
4. $x^2 + x - 6 = 0$
5. $x^2 - 8x + 12 = 0$
6. $x^2 + 10x + 21 = 0$
7. $x^2 - 5x + 6 = 0$
8. $x^2 - 4x - 5 = 0$
9. $x^2 + 5x - 14 = 0$
10. $2x^2 - 3x - 2 = 0$
11. $3x^2 + 10x - 8 = 0$
12. $2x^2 + 7x - 15 = 0$
13. $6x^2 - 13x + 6 = 0$
14. $4x^2 - 29x + 7 = 0$
15. $10x^2 - x - 3 = 0$
16. $y^2 - 15y + 56 = 0$
17. $12y^2 - 16y + 5 = 0$
18. $y^2 + 2y - 63 = 0$
19. $x^2 + 2x + 1 = 0$
20. $x^2 - 6x + 9 = 0$
21. $x^2 + 10x + 25 = 0$
22. $x^2 - 14x + 49 = 0$
23. $6a^2 - a - 1 = 0$
24. $4a^2 - 3a - 10 = 0$
25. $z^2 - 8z - 65 = 0$
26. $6x^2 + 17x - 3 = 0$
27. $10k^2 + 19k - 2 = 0$
28. $y^2 - 2y + 1 = 0$
29. $36x^2 + x - 2 = 0$
30. $20x^2 - 7x - 3 = 0$

Solving quadratic equations with two terms

Example 1

Solve the equation $x^2 - 7x = 0$

Factorising, $x(x-7) = 0$

either $x = 0$ or $x - 7 = 0$

$x = 7$

The solutions are $x = 0$ and $x = 7$.

Example 2

Solve the equation $4x^2 - 9 = 0$

(a) Factorising, $(2x-3)(2x+3) = 0$

either $2x - 3 = 0$ or $2x + 3 = 0$

$2x = 3$ $2x = -3$

$x = \frac{3}{2}$ $x = -\frac{3}{2}$

(b) Alternative method

$$4x^2 - 9 = 0$$
$$4x^2 = 9$$
$$x^2 = \tfrac{9}{4}$$
$$x = +\tfrac{3}{2} \quad \text{or} \quad -\tfrac{3}{2}.$$

Exercise

Solve the following equations:

1. $x^2 - 3x = 0$
2. $x^2 + 7x = 0$
3. $2x^2 - 2x = 0$
4. $3x^2 - x = 0$
5. $x^2 - 16 = 0$
6. $x^2 - 49 = 0$
7. $4x^2 - 1 = 0$
8. $9x^2 - 4 = 0$
9. $6y^2 + 9y = 0$
10. $6a^2 - 9a = 0$
11. $10x^2 - 55x = 0$
12. $16x^2 - 1 = 0$
13. $y^2 - \frac{1}{4} = 0$
14. $56x^2 - 35x = 0$
15. $36x^2 - 3x = 0$
16. $x^2 = 6x$
17. $x^2 = 11x$
18. $2x^2 = 3x$
19. $x^2 = x$
20. $4x = x^2$
21. $3x - x^2 = 0$
22. $4x^2 = 1$
23. $9x^2 = 16$
24. $x^2 = 9$
25. $12x = 5x^2$
26. $1 - 9x^2 = 0$
27. $x^2 = \dfrac{x}{4}$
28. $2x^2 = \dfrac{x}{3}$
29. $4x^2 = \dfrac{1}{4}$
30. $\dfrac{x}{5} - x^2 = 0$

Word problems involving quadratic equations

Example

A rectangular room is 4 m longer than it is wide. If its area is 12 find its dimensions.

Solution

Let the width be x m .its length is then (x + 4) m.

The area of the room is x (x+4)

Therefore x (x + 4) = 12

- ✓ $x^2 + 4x = 12$
- ✓ $x^2+4x - 12=0$
- ✓ $x^2+6x-2x+2 = 0$
- ✓ $x(x+6) - 2(x+6)=0$
- ✓ $(x-2)(x+6)=0$
- ✓ $x=2$, $x = -6$

-6 is being ignored because length cannot be negative

The length of the room is x +4 = 2 +4

Exercise

1. When *x* is added to its square, *x* , the answer is 12. Find the two possible values of *x*.
2. When *x* is subtracted from its square, the answer is 20. Find the possible values of *x*.
3. Simone thinks of a number. She squares the number and adds twice the original number.
4. A rectangle has a length of (x + 5) cm and a width of *x* cm. Write down an expression for the area of the rectangle. , find the value of *x*.
5. The product of two consecutive odd numbers is 143. Find two positive numbers and two negative numbers that satisfy the condition.
6. The length of a picture is 10 cm more than the width. The area is 1200 cm. Find the dimensions of the picture.
7. Two integers differ by 6. The sum of the squares of these integers is 116. Find the two integers.

CHAPTER THREE

ALGEBRAIC EXPRESSION

Expanding the brackets

Brackets serve the same purpose as they do in arithmetic.

Example

Remove the brackets and simplify:

a.) 3 (a + b) – 2(a – b)
b.) 1/3a + 3 (5a + b – c)
c.) 2b + 3 (3 – 2 (a – 5)

Solution

a.) 3 (a + b) – 2(a – b) = 3a + 3b – 2a + 2b

= 3a - 2a + 3b + 2b

= a + 5b

b.) 1/3a + 3 (5a + b – c) = 1/3a + 15 a + 3b – 3c

$= \frac{1+45a^2}{3a}$+3b-3c

c.) 2b + a { 3 – 2 (a – 5)}

= 2b + a {3 – 2a + 10}

$=2b + 3a – 2 + 10a$

$= 2b + 3a + 10 a - 2$

$= 2b + 13a - 2$

The process of removing the brackets is called expansion while the reverse process of inserting the brackets is called factorization.

Exercise

Remove the brackets and collect like terms:

25. $3x + 2(x + 1)$ **26.** $5x + 7(x - 1)$ **27.** $7 + 3(x - 1)$
28. $9 - 2(3x - 1)$ **29.** $3x - 4(2x + 5)$ **30.** $5x - 2x(x - 1)$
31. $7x + 3x(x - 4)$ **32.** $4(x - 1) - 3x$ **33.** $5x(x + 2) + 4x$
34. $3x(x - 1) - 7x^2$ **35.** $3a + 2(a + 4)$ **36.** $4a - 3(a - 3)$
37. $3ab - 2a(b - 2)$ **38.** $3y - y(2 - y)$ **39.** $3x - (x + 2)$
40. $7x - (x - 3)$ **41.** $5x - 2(2x + 2)$ **42.** $3(x - y) + 4(x + 2y)$
43. $x(x - 2) + 3x(x - 3)$ **44.** $3x(x + 4) - x(x - 2)$ **45.** $y(3y - 1) - (3y - 1)$
46. $7(2x + 2) - (2x + 2)$ **47.** $7b(a + 2) - a(3b + 3)$ **48.** $3(x - 2) - (x - 2)$

Expanding two brackets

Example 1

$$\begin{aligned}(x + 5)(x + 3) &= x(x + 3) + 5(x + 3)\\ &= x^2 + 3x + 5x + 15\\ &= x^2 + 8x + 15\end{aligned}$$

Example 2

$$\begin{aligned}(2x - 3)(4y + 3) &= 2x(4y + 3) - 3(4y + 3)\\ &= 8xy + 6x - 12y - 9\end{aligned}$$

Example 3

$$\begin{aligned}3(x + 1)(x - 2) &= 3[x(x - 2) + 1(x - 2)]\\ &= 3[x^2 - 2x + x - 2]\\ &= 3x^2 - 3x - 6\end{aligned}$$

Exercise

Remove the brackets and simplify:

1. $(x + 1)(x + 3)$ **2.** $(x + 3)(x + 2)$ **3.** $(y + 4)(y + 5)$ **4.** $(x - 3)(x + 4)$ **5.** $(x + 5)(x - 2)$
6. $(x - 3)(x - 2)$ **7.** $(a - 7)(a + 5)$ **8.** $(z + 9)(z - 2)$ **9.** $(x - 3)(x + 3)$ **10.** $(k - 11)(k + 11)$

11. $(2x+1)(x-3)$
12. $(3x+4)(x-2)$
13. $(2y-3)(y+1)$
14. $(7y-1)(7y+1)$
15. $(3x-2)(3x+2)$
16. $(3a+b)(2a+b)$
17. $(3x+y)(x+2y)$
18. $(2b+c)(3b-c)$
19. $(5x-y)(3y-x)$
20. $(3b-a)(2a+5b)$
21. $2(x-1)(x+2)$
22. $3(x-1)(2x+3)$
23. $4(2y-1)(3y+2)$
24. $2(3x+1)(x-2)$
25. $4(a+2b)(a-2b)$
26. $x(x-1)(x-2)$
27. $2x(2x-1)(2x+1)$
28. $3y(y-2)(y+3)$
29. $x(x+y)(x+z)$
30. $3z(a+2m)(a-m)$

Be careful with an expression like $(x-3)^2$. It is not x^2-9 or even x^2+9.

$$\begin{aligned}(x-3)^2 &= (x-3)(x-3)\\ &= x(x-3)-3(x-3)\\ &= x^2-6x+9\end{aligned}$$

Another common mistake occurs with an expression like $4-(x-1)^2$.

$$\begin{aligned}4-(x-1)^2 &= 4-1(x-1)(x-1)\\ &= 4-1(x^2-2x+1)\\ &= 4-x^2+2x-1\\ &= 3+2x-x^2\end{aligned}$$

Remove the brackets and simplify:

1. $(x+4)^2$
2. $(x+2)^2$
3. $(x-2)^2$
4. $(2x+1)^2$
5. $(y-5)^2$
6. $(3y+1)^2$
7. $(x+y)^2$
8. $(2x+y)^2$
9. $(a-b)^2$
10. $(2a-3b)^2$
11. $3(x+2)^2$
12. $(3-x)^2$
13. $(3x+2)^2$
14. $(a-2b)^2$
15. $(x+1)^2+(x+2)^2$
16. $(x-2)^2+(x+3)^2$
17. $(x+2)^2+(2x+1)^2$
18. $(y-3)^2+(y-4)^2$
19. $(x+2)^2-(x-3)^2$
20. $(x-3)^2-(x+1)^2$
21. $(y-3)^2-(y+2)^2$
22. $(2x+1)^2-(x+3)^2$
23. $3(x+2)^2-(x+4)^2$
24. $2(x-3)^2-3(x+1)^2$

Factorization

Example

Factorize the following:

$3m + 3n = 3(m + n)$ (the common term is 3 so we put it outside the bracket)

Factorization by grouping

When the terms of an expression which do not have a common factor are taken pairwise, a common factor can be found. This method is known as factorization by grouping.

Example

Factorize:

a.) $3ab + 2b + 3ca + 2c$
b.) $ab + bx - a - x$

Solution

a.) $3ab + 2b + 3ca + 2c = b(3a + 2) + c(3a + 2)$

$= (3a + 2)(b + c)$

b.) $ab + bx - a - x = b(a + x) - 1(a + x)$

$= (a + x)(b - 1)$

Example

Factorise: (a) $x^2 + 7x$ (b) $3y^2 - 12y$ (c) $6a^2b - 10ab^2$

(a) x is common to x^2 and $7x$.

$\therefore \quad x^2 + 7x = x(x + 7)$

The factors are x and $(x + 7)$.

(b) $3y$ is common.

$\therefore \quad 3y^2 - 12y = 3y(y - 4)$

(c) $2ab$ is common.

$\therefore \quad 6a^2b - 10ab^2 = 2ab(3a - 5b)$

Exercise

Factorise

1. $x^2 + 5x$	**2.** $x^2 - 6x$	**3.** $7x - x^2$	**4.** $y^2 + 8y$
5. $2y^2 + 3y$	**6.** $6y^2 - 4y$	**7.** $3x^2 - 21x$	**8.** $16a - 2a^2$
9. $6c^2 - 21c$	**10.** $15x - 9x^2$	**11.** $56y - 21y^2$	**12.** $ax + bx + 2cx$

Example

Factorise $ah + ak + bh + bk$.

(a) Divide into pairs, $ah + ak \mid + bh + bk$.

(b) a is common to the first pair

b is common to the second pair

$a(h + k) + b(h + k)$

(c) $(h + k)$ is common to both terms.

Thus we have $(h + k)(a + b)$

Example

Factorise $6mx - 3nx + 2my - ny$.

$6mx - 3nx | + 2my - ny$

$= 3x(2m - n) + y(2m - n)$

$= (2m - n)(3x + y)$

Exercise 2

Factorise :

1. $ax + ay + bx + by$
2. $ay + az + by + bz$
3. $xb + xc + yb + yc$
4. $xh + xk + yh + yk$
5. $xm + xn + my + ny$
6. $ah - ak + bh - bk$
7. $ax - ay + bx - by$
8. $am - bm + an - bn$
9. $hs + ht + ks + kt$
10. $xs - xt + ys - yt$
11. $ax - ay - bx + by$
12. $xs - xt - ys + yt$
13. $as - ay - xs + xy$
14. $hx - hy - bx + by$
15. $am - bm - an + bn$
16. $xk - xm - kz + mz$
17. $2ax + 6ay + bx + 3by$
18. $2ax + 2ay + bx + by$
19. $2mh - 2mk + nh - nk$
20. $2mh + 3mk - 2nh - 3nk$
21. $6ax + 2bx + 3ay + by$
22. $2ax - 2ay - bx + by$
23. $x^2a + x^2b + ya + yb$
24. $ms + 2mt^2 - ns - 2nt^2$

Example

Simplify

$$\frac{16m^2 - 9n^2}{4m^2 - mn - 3n^2}$$

Solution

Num. (4m – 3n) (4m + 3n)

Den. $4m^2 - 4mn + 3mn - 3n^2$

(4m + 3n) (m – n)

(4m – 3n) (4m + 3n)

(4m + 3n) (m $\cancel{-}$n)

4m – 3n

m – n

Example

Simplify the expression.

$$\frac{18xy-18xr}{9xr-9xy}$$

Solution

Numerator

18x (y – r)

Denominator

9x (r – y)

Therefore

$=\frac{18x(y-r)}{9x(r-y)} =$

$$\frac{18x(r-y)(-1)}{9x(r-y)}$$

2 x(-1)

= - 2x

Example

Simplify

Solution

$$\frac{(3x-2a)(4x+3a)}{(3x+2a)(3x-2a)}$$

$$=\frac{4x+3a}{3x+2a}$$

Difference of Squares

We have seen that:

$$(ax + b)(ax - b) = a^2x^2 - b^2$$

Since 9.3 is an equation, both sides are always equal. This means that an expression of the form:

$a2x2 - b2$

can be factorised to

$(ax + b)(ax - b)$

Therefore,

$a^2x^2 - b^2 = (ax + b)(ax - b)$

For example, $x^2 - 16$ can be written as $(x^2 - 4^2)$ which is a difference of squares. Therefore the factors of $x^2 - 16$ are $(x - 4)$ and $(x + 4)$.

Example :

Factorise :

$81p^2 - 16 = (9p^2)^2 - 4^2$

$= (9p^2 + 4)(9p^2 - 4)$

$= (9p + 4)(9p - 4)$

Exercise

Simplify as far as possible:

1. $3x + 4y + 7y$
2. $4a + 7b - 2a + b$
3. $3x - 2y + 4y$
4. $2x + 3x + 5$
5. $7 - 3x + 2 + 4x$
6. $5 - 3y - 6y - 2$
7. $5x + 2y - 4y - x^2$
8. $2x^2 + 3x + 5$
9. $2x - 7y - 2x - 3y$
10. $4a + 3a^2 - 2a$
11. $7a - 7a^2 + 7$
12. $x^2 + 3x^2 - 4x^2 + 5x$
13. $\frac{3}{a} + b + \frac{7}{a} - 2b$
14. $\frac{4}{x} - \frac{7}{y} + \frac{1}{x} + \frac{2}{y}$
15. $\frac{m}{x} + \frac{2m}{x}$
16. $\frac{5}{x} - \frac{7}{x} + \frac{1}{2}$
17. $\frac{3}{a} + b + \frac{2}{a} + 2b$
18. $\frac{n}{4} - \frac{m}{3} - \frac{n}{2} + \frac{m}{3}$
19. $x^3 + 7x^2 - 2x^3$
20. $(2x)^2 - 2x^2$
21. $(3y)^2 + x^2 - (2y)^2$
22. $(2x)^2 - (2y)^2 - (4x)^2$
23. $5x - 7x^2 - (2x)^2$
24. $\frac{3}{x^2} + \frac{5}{x^2}$

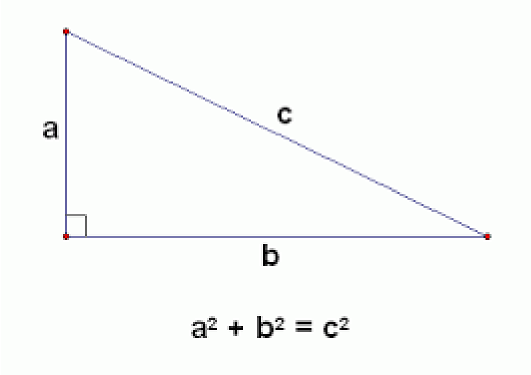
Remove the brackets and collect like terms:

25. $3x + 2(x + 1)$
26. $5x + 7(x - 1)$
27. $7 + 3(x - 1)$
28. $9 - 2(3x - 1)$
29. $3x - 4(2x + 5)$
30. $5x - 2x(x - 1)$
31. $7x + 3x(x - 4)$
32. $4(x - 1) - 3x$
33. $5x(x + 2) + 4x$
34. $3x(x - 1) - 7x^2$
35. $3a + 2(a + 4)$
36. $4a - 3(a - 3)$
37. $3ab - 2a(b - 2)$
38. $3y - y(2 - y)$
39. $3x - (x + 2)$
40. $7x - (x - 3)$
41. $5x - 2(2x + 2)$
42. $3(x - y) + 4(x + 2y)$
43. $x(x - 2) + 3x(x - 3)$
44. $3x(x + 4) - x(x - 2)$
45. $y(3y - 1) - (3y - 1)$
46. $7(2x + 2) - (2x + 2)$
47. $7b(a + 2) - a(3b + 3)$
48. $3(x - 2) - (x - 2)$

CHAPTER THREE

PYTHAGORAS THEOREM

Consider the triangle below:



Pythagoras theorem states that for a right-angled triangle, the square of the hypotenuse is equal to the sum of the square of the two shorter sides.

Example

In a right angle triangle, the two shorter sides are 6 cm and 8 cm. Find the length of the hypotenuse.

Solution

Using Pythagoras theorem

$c^2= a^2 +b^2$

where a= 6 , and b = 8

$\therefore c^2 = 6^2 +8^2$

$c^2= 36 +64$

$c^2=100$

$c = \sqrt{100}$

$\therefore c = 10cm$

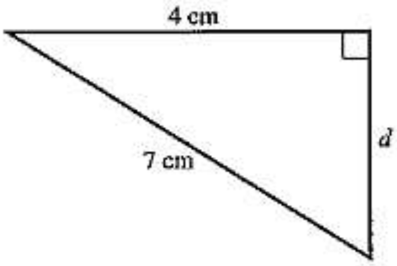
Example

Find the side marked *d*.

$d^2 + 4^2 = 7^2$

$d^2 = 49 - 16$

$d = \sqrt{33} = 5{\cdot}74$ cm (3 sig. fig.)

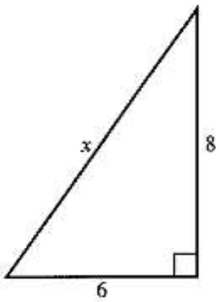


The *converse* is also true:
'If the square on one side of a triangle is equal to the sum of the squares on the other two sides, then the triangle is right-angled.'

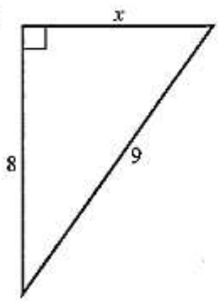
Exercise

In questions **1** to **10**, find x. All the lengths are in cm.

1.



2.



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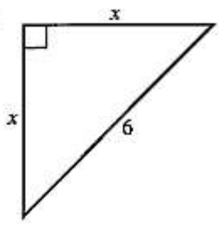

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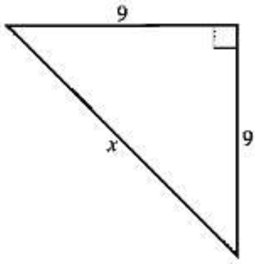

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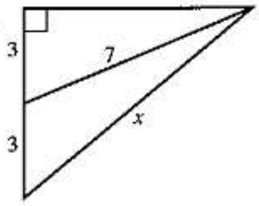

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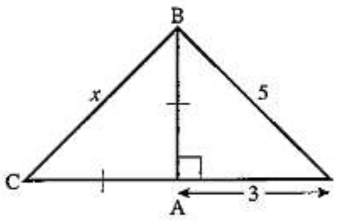

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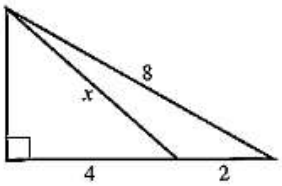

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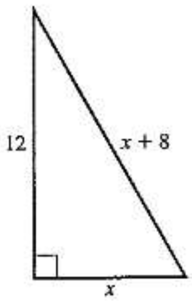
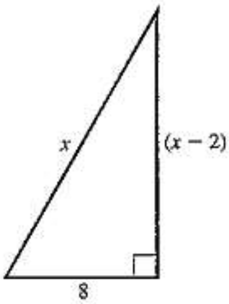
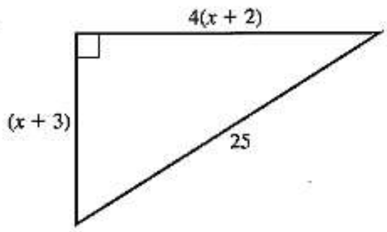


9.



10.



11. Find the length of a diagonal of a rectangle of length 9 cm and width 4 cm.

12. A square has diagonals of length 10 cm. Find the sides of the square.

13. A 4 m ladder rests against a vertical wall with its foot 2 m from the wall. How far up the wall does the ladder reach?

CHAPTER FOUR

LINEAR INEQUALITIES

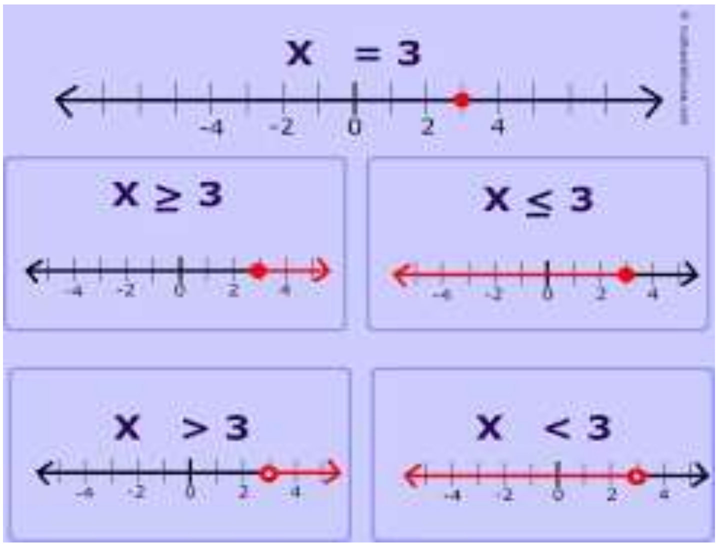
Inequality symbols

Statements connected by these symbols are called inequalities

<u>Representing linear inequalities on a number line</u>

Simple statements

Simple statements represents only one condition on a number line as follows

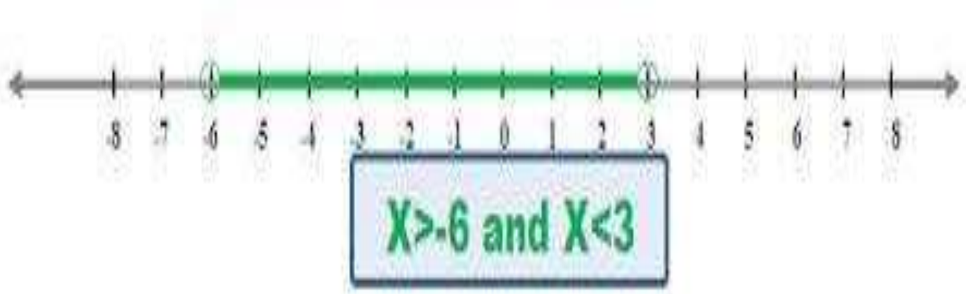


X = 3 represents specific point which is number 3, while x >3 does not it represents all numbers to the right of 3 meaning all the numbers greater than 3 as illustrated above. X< 3 represents all numbers to left of 3 meaning all the numbers less than 3.The empty circle means that 3 is not included in the list of numbers to greater or less than 3.

The expression means that means that 3 is included in the list and the circle is shaded to show that 3 is included.

<u>Compound statement</u>

A compound statement is a two simple inequalities joined by “and” or “or.” Here are two examples.



<u>Solution to simple inequalities</u>

<u>Example</u>

Solve the inequality

X – 1 > 2

Solution

Adding 1 to both sides gives ;

X – 1 + 1 > 2 + 1

Therefore, x > 3

Note;

In any inequality you may add or subtract the same number from both sides.

<u>Example</u>

Solve the inequality.

X + 3 < 8

Solution

Subtracting three from both sides gives

X + 3 – 3 < 8-3

X < 5

Example

Solve the inequality

2x +3 >2

Subtracting three from both sides gives

2 x + 3 – 3 >2 – 3

2x > -1

Divide both sides by 2 gives

X > $\frac{1}{2}$

Multiplication and Division by a Negative Number

Multiplying or dividing both sides of an inequality by positive number leaves the inequality sign unchanged

Multiplying or dividing both sides of an inequality by negative number reverses the sense of the inequality sign.

Example

Solve the inequality 1 -3x < 4

Solution

- 3x – 1 < 4 –

-3x < 3

∴ x > - 1 (divide by -3 to both sides)

Note that the sign is reversed X >-1

Example

Solve the following

3x -1 > -4

Solution

Solving the first inequality

3x – 1 > _ 4

3x > -3

X > -1

Example

Solve 4q+3<2(q+3) and represent the solution on a number line

Answer

Step 1 : Expand all brackets

$$4q + 3 < 2(q + 3)$$

$$4q + 3 < 2q + 6$$

Step 2 : Move all constants to the RHS and all unknowns to the LHS

$$4q + 3 < 2q + 6$$

$$4q - 2q < 6 - 3$$

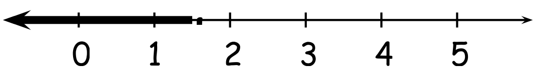
$$2q < 3$$

Step 3 : Solve inequality

$2q < 3$ Divide both sides by 2

$$q < 3/2$$

$$q < 1.5$$



Exercise: Linear Inequalities

1. Solve for x and represent the solution graphically:
   (a) $3x + 4 > 5x + 8$
   (b) $3(x - 1) - 2 \leq 6x + 4$
   (c) $-4(x - 1) < x + 2$
2. Solve the following inequalities. Illustrate your answer on a number line if x is a real number.
   (a) $-2 \leq x - 1 < 3$
   (b) $-5 < 2x - 3 \leq 7$

3. Solve for x: $7(3x + 2) - 5(2x - 3) > 7$. Illustrate this answer on a number line.

Exercise 2

Introduce one of the symbols $<$, $>$ or $=$ between each pair of numbers.

1. $-2, 1$
2. $(-2)^2, 1$
3. $\frac{1}{4}, \frac{1}{5}$
4. $0{\cdot}2, \frac{1}{5}$
5. $10^2, 2^{10}$
6. $\frac{1}{4}, 0{\cdot}4$
7. $40\%, 0{\cdot}4$
8. $(-1)^2, (-\frac{1}{2})^2$
9. $5^2, 2^5$
10. $3\frac{1}{3}, \sqrt{10}$
11. $\pi^2, 10$
12. $-\frac{1}{3}, -\frac{1}{2}$
13. $2^{-1}, 3^{-1}$
14. $50\%, \frac{1}{5}$
15. $1\%, 100^{-1}$

State whether the following are true or false:

16. $0{\cdot}7^2 > \frac{1}{2}$
17. $10^3 = 30$
18. $\frac{1}{8} > 12\%$
19. $(0{\cdot}1)^3 = 0{\cdot}0001$
20. $(-\frac{1}{5})^0 = -1$
21. $\frac{1}{5^2} > \frac{1}{2^5}$
22. $(0{\cdot}2)^3 < (0{\cdot}3)^2$
23. $\frac{6}{7} > \frac{7}{8}$
24. $0{\cdot}1^2 > 0{\cdot}1$

Solve the following inequalities:

25. $x - 3 > 10$
26. $x + 1 < 0$
27. $5 > x - 7$
28. $2x + 1 \leqslant 6$
29. $3x - 4 > 5$
30. $10 \leqslant 2x - 6$
31. $5x < x + 1$
32. $2x \geqslant x - 3$
33. $4 + x < -4$
34. $3x + 1 < 2x + 5$
35. $2(x + 1) > x - 7$
36. $7 < 15 - x$
37. $9 > 12 - x$
38. $4 - 2x \leqslant 2$
39. $3(x - 1) < 2(1 - x)$
40. $7 - 3x < 0$

Solve the following inequalities and show the results on a number line

1. $2x + 1 > 11$
2. $3x - 4 \leqslant 5$
3. $2 < x - 4$
4. $6 \geqslant 10 - x$
5. $8 < 9 - x$
6. $8x - 1 < 5x - 1($
7. $2x > 0$
8. $1 < 3x - 11$
9. $4 - x > 6 - 2x$
10. $\frac{x}{3} < -1$
11. $1 < x < 4$
12. $-2 \leqslant x \leqslant 5$
13. $1 \leqslant x < 6$
14. $0 \leqslant 2x < 10$
15. $-3 \leqslant 3x \leqslant 21$
16. $1 < 5x < 10$
17. $\frac{x}{4} > 20$
18. $3x - 1 > x + 19$

CHAPTER FIVE

INDICES AND LOGARITHMS

Index and base form

The power to which a number is raised is called index or indices in plural.

e.g. 10^2

2 is called the index and 10 is the base.

Laws of indices

For the laws to hold the base must be the same.

Rule 1

Any number, except zero whose index is 0 is always equal to 1

Example

$5^0=1$

Rule 2

To multiply an expression with the same base, copy the base and add the indices.

$x^b \times x^a = x^{(b+a)}$

Example

$2^3 \times 2^2 = 2^{(3+2)} = 2^5 = 32$

Rule 3

To divide an expression with the same base, copy the base and subtract the powers.

$x^b \div x^a = x^{(b-a)}$

Example

$2^6 \div 2^2 = 2^{(6-2)} = 2^4 = 16$

Rule 4

To raise an expression to the nth index, copy the base and multiply the indices

$(X^a)^b = x^{ab}$

Example

$(2^3)^2 = 2^{2\times 3}$

$= 32$

Rule 5

When dealing with a negative power, you simply change the power to positive by changing it into a fraction with 1 s the numerator.

$X^{-b} = \frac{1}{x^b}$

Example

$=2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

Fractional indices

Fractional indices are written in fraction form. In summary if. a is called the root of b written as .

Rule 6 : $x^{\frac{a}{b}} = \sqrt[b]{x^a}$

Example

$8^{\frac{2}{3}} = \sqrt[3]{8^2} = 2^2 = 4$

Example

Simplify:

(a) $x^7 \times x^{13}$ (b) $x^3 \div x^7$

(c) $(x^4)^3$ (d) $(3x^2)^3$

(e) $(2x^{-1})^2 \div x^{-5}$ (f) $3y^2 \times 4y^3$

(a) $x^7 \times x^{13} = x^{7+13} = x^{20}$

(b) $x^3 \div x^7 = x^{3-7} = x^{-4} = \frac{1}{x^4}$

(c) $(x^4)^3 = x^{12}$

(d) $(3x^2)^3 = 3^3 \times (x^2)^3 = 27x^6$

(e) $(2x^{-1})^2 \div x^{-5} = 4x^{-2} \div x^{-5}$
$= 4x^{(-2--5)}$
$= 4x^3$

(f) $3y^2 \times 4y^3 = 12y^5$

EXERCISE

Express in index form:

1. $3 \times 3 \times 3 \times 3$
2. $4 \times 4 \times 5 \times 5 \times 5$
3. $3 \times 7 \times 7 \times 7$
4. $2 \times 2 \times 2 \times 7$
5. $\frac{1}{10 \times 10 \times 10}$
6. $\frac{1}{2 \times 2 \times 3 \times 3 \times 3}$
7. $\sqrt{15}$
8. $\sqrt[3]{3}$
9. $\sqrt[5]{10}$
10. $(\sqrt{5})^3$

Simplify:

11. $x^3 \times x^4$
12. $y^6 \times y^7$
13. $z^7 \div z^3$
14. $z^{50} \times z^{50}$
15. $m^3 \div m^2$
16. $e^{-3} \times e^{-2}$
17. $y^{-2} \times y^4$
18. $w^4 \div w^{-2}$
19. $y^{\frac{1}{2}} \times y^{\frac{1}{2}}$
20. $(x^2)^5$
21. $x^{-2} \div x^{-2}$
22. $w^{-3} \times w^{-2}$
23. $w^{-7} \times w^2$
24. $x^3 \div x^{-4}$
25. $(a^2)^4$
26. $(k^{\frac{1}{2}})^6$
27. $e^{-4} \times e^4$
28. $x^{-1} \times x^{30}$
29. $(y^4)^{\frac{1}{2}}$
30. $(x^{-3})^{-2}$
31. $z^2 \div z^{-2}$
32. $t^{-3} \div t$
33. $(2x^3)^2$
34. $(4y^5)^2$
35. $2x^2 \times 3x^2$
36. $5y^3 \times 2y^2$
37. $5a^3 \times 3a$
38. $(2a)^3$
39. $3x^3 \div x^3$
40. $8y^3 \div 2y$
41. $10y^2 \div 4y$
42. $8a \times 4a^3$
43. $(2x)^2 \times (3x)^3$
44. $4z^4 \times z^{-7}$
45. $6x^{-2} \div 3x^2$
46. $5y^3 \div 2y^{-2}$
47. $(x^2)^{\frac{1}{2}} \div (x^{\frac{1}{3}})^3$
48. $7w^{-2} \times 3w^{-1}$
49. $(2n)^4 \div 8n^0$
50. $4x^{\frac{3}{2}} \div 2x^{\frac{1}{2}}$

Exercise 2

. Simplify as far as possible:

(a) 302^0 (b) 1^0 (c) $(xyz)^0$

(e) $(2x)^3$ (f) $(-2x)^3$ (g) $(2x)^4$

. Simplify, without using a calculator:

(a) $\frac{(-3)^{-3}.(-3)^{2}}{(-3)^{-4}}$.

(b) $(3^{-1} + 2^{-1})^{-1}$

(c) $\frac{9^{n-1}.27^{3-2n}}{81^{2-n}}$

Example

Evaluate:

(a) $9^{\frac{1}{2}}$ (b) 5^{-1}

(c) $4^{-\frac{1}{2}}$ (d) $25^{\frac{3}{2}}$

(e) $(5^{\frac{1}{2}})^3 \times 5^{\frac{1}{2}}$ (f) 7^0

(a) $9^{\frac{1}{2}} = \sqrt{9} = 3$

(b) $5^{-1} = \frac{1}{5}$

(c) $4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

(d) $25^{\frac{3}{2}} = (\sqrt{25})^3 = 5^3 = 125$

(e) $(5^{\frac{1}{2}})^3 \times 5^{\frac{1}{2}} = 5^{\frac{3}{2}} \times 5^{\frac{1}{2}} = 5^2$
$= 25$

(f) $7^0 = 1 \left[\text{consider} \frac{7^3}{7^3} = 7^{3-3} = 7^0 = 1\right]$

Remember: $a^0 = 1$ for any non-zero value of a.

EXERCISE

Evaluate the following:

1. $3^2 \times 3$	**2.** 100^0	**3.** 3^{-2}	**4.** $(5^{-1})^{-2}$
5. $4^{\frac{1}{2}}$	**6.** $16^{\frac{1}{2}}$	**7.** $81^{\frac{1}{2}}$	**8.** $8^{\frac{1}{3}}$
9. $9^{\frac{3}{2}}$	**10.** $27^{\frac{1}{3}}$	**11.** $9^{-\frac{1}{2}}$	**12.** $8^{-\frac{1}{3}}$
13. $1^{\frac{5}{2}}$	**14.** $25^{-\frac{1}{2}}$	**15.** $1000^{\frac{1}{3}}$	**16.** $2^{-2} \times 2^5$
17. $2^4 \div 2^{-1}$	**18.** $8^{\frac{2}{3}}$	**19.** $27^{-\frac{2}{3}}$	**20.** $4^{-\frac{3}{2}}$

21. $36^{\frac{1}{2}} \times 27^{\frac{1}{3}}$
22. $10\,000^{\frac{1}{4}}$
23. $100^{\frac{3}{2}}$
24. $(100^{\frac{1}{2}})^{-3}$
25. $(9^{\frac{1}{2}})^{-2}$
26. $(-16{\cdot}371)^{0}$
27. $81^{\frac{1}{4}} \div 16^{\frac{1}{4}}$
28. $(5^{-4})^{\frac{1}{2}}$
29. $1000^{-\frac{1}{3}}$
30. $(4^{-\frac{1}{2}})^{2}$
31. $8^{-\frac{2}{3}}$
32. $100^{\frac{5}{2}}$
33. $1^{\frac{4}{3}}$
34. 2^{-5}
35. $(0{\cdot}01)^{\frac{1}{2}}$
36. $(0{\cdot}04)^{\frac{1}{2}}$
37. $(2{\cdot}25)^{\frac{1}{2}}$
38. $(7{\cdot}63)^{0}$
39. $3^{5} \times 3^{-3}$
40. $(3\frac{3}{8})^{\frac{1}{3}}$
41. $(11\frac{1}{9})^{-\frac{1}{2}}$
42. $(\frac{1}{8})^{-2}$
43. $(\frac{1}{1000})^{\frac{2}{3}}$
44. $(\frac{9}{25})^{-\frac{1}{2}}$
45. $(10^{-6})^{\frac{1}{3}}$
46. $7^{2} \div (7^{\frac{1}{2}})^{4}$
47. $(0{\cdot}0001)^{-\frac{1}{2}}$
48. $\dfrac{9^{\frac{1}{2}}}{4^{-\frac{1}{2}}}$
49. $\dfrac{25^{\frac{3}{2}} \times 4^{\frac{1}{2}}}{9^{-\frac{1}{2}}}$
50. $(-\frac{1}{7})^{2} \div (-\frac{1}{7})^{3}$

Example

Simplify:

(a) $(2a)^3 \div (9a^2)^{\frac{1}{2}}$ (b) $(3ac^2)^3 \times 2a^{-2}$ (c) $(2x)^2 \div 2x^2$

(a) $(2a)^3 \div (9a^2)^{\frac{1}{2}} = 8a^3 \div 3a$
$= \frac{8}{3}a^2$

(b) $(3ac^2)^3 \times 2a^{-2} = 27a^3c^6 \times 2a^{-2}$
$= 54ac^6$

(c) $(2x)^2 \div 2x^2 = 4x^2 \div 2x^2$
$= 2$

Rewrite without brackets:

1. $(5x^2)^2$
2. $(7y^3)^2$
3. $(10ab)^2$
4. $(2xy^2)^2$
5. $(4x^2)^{\frac{1}{2}}$
6. $(9y)^{-1}$
7. $(x^{-2})^{-1}$
8. $(2x^{-2})^{-1}$
9. $(5x^2y)^0$
10. $(\frac{1}{2}x)^{-1}$
11. $(3x)^2 \times (2x)^2$
12. $(5y)^2 \div y$
13. $(2x^{\frac{1}{2}})^4$
14. $(3y^{\frac{1}{3}})^3$
15. $(5x^0)^2$
16. $[(5x)^0]^2$
17. $(7y^0)^2$
18. $[(7y)^0]^2$
19. $(2x^2y)^3$
20. $(10xy^3)^2$

Simplify the following:

21. $(3x^{-1})^2 \div 6x^{-3}$
22. $(4x)^{\frac{1}{2}} \div x^{\frac{3}{2}}$
23. $x^2y^2 \times xy^3$
24. $4xy \times 3x^2y$
25. $10x^{-1}y^3 \times xy$
26. $(3x)^2 \times (\frac{1}{9}x^2)^{\frac{1}{2}}$
27. $z^3yx \times x^2yz$
28. $(2x)^{-2} \times 4x^3$
29. $(3y)^{-1} \div (9y^2)^{-1}$
30. $(xy)^0 \times (9x)^{\frac{3}{2}}$
31. $(x^2y)(2xy)(5y^3)$
32. $(4x^{\frac{1}{2}}) \times (8x^{\frac{3}{2}})$
33. $5x^{-3} \div 2x^{-5}$
34. $[(3x^{-1})^{-2}]^{-1}$
35. $(2a)^{-2} \times 8a^4$
36. $(abc^2)^3$
37. Write in the form 2^p (e.g. $4 = 2^2$):
(a) 32 (b) 128 (c) 64 (d) 1
38. Write in the form 3^q:
(a) $\frac{1}{27}$ (b) $\frac{1}{81}$ (c) $\frac{1}{3}$ (d) $9 \times \frac{1}{81}$

STANDARD FORM

When dealing with either very large or very small numbers, it is not convenient to write them out in full in the normal way. It is better to use standard form. Most calculators represent large and small numbers in this way.

The number $a \times 10^n$ is in standard form when $1 \leqslant a < 10$ and n is a positive or negative integer.

Example

Write the following numbers in standard form:

(a) $2000 = 2 \times 1000 = 2 \times 10^3$

(b) $150 = 1{\cdot}5 \times 100 = 1{\cdot}5 \times 10^2$

(c) $0{\cdot}0004 = 4 \times \frac{1}{10\,000} = 4 \times 10^{-4}$

EXERCISE

Write the following numbers in standard form:

1. 4000 **2.** 500 **3.** 70 000 **4.** 60 **5.** 2400 **6.** 380
7. 46 000 **8.** 46 **9.** 900 000 **10.** 2560 **11.** 0·007 **12.** 0·0004
13. 0·0035 **14.** 0·421 **15.** 0·000 055 **16.** 0·01 **17.** 564 000 **18.** 19 million

In questions **1** to **12** give the answer in standard form.

1. 5000×3000 **2.** $60\,000 \times 5000$ **3.** $0{\cdot}000\,07 \times 400$ **4.** $0{\cdot}0007 \times 0{\cdot}000\,01$
5. $8000 \div 0{\cdot}004$ **6.** $(0{\cdot}002)^2$ **7.** $150 \times 0{\cdot}0006$ **8.** $0{\cdot}000\,033 \div 500$
9. $0{\cdot}007 \div 20\,000$ **10.** $(0{\cdot}0001)^4$ **11.** $(2000)^3$ **12.** $0{\cdot}005\,92 \div 8000$

13. If $a = 512 \times 10^2$ $b = 0{\cdot}478 \times 10^6$ $c = 0{\cdot}0049 \times 10^7$

LOGARITHM

Logarithmic Function Definition

The logarithmic function is defined as an inverse function to exponentiation. The logarithmic function is stated as follows

For x, a > 0, and a≠1,

$y = \log_a x$, if $x = a^y$

Then the logarithmic function is written as:

$f(x) = \log_a x$

The most 2 common bases used in logarithmic functions are base e and base 10. The log function with base 10 is called the common logarithmic function and it is denoted by $\log_{10}$ or simply log.

$f(x) = \log_{10}$

The log function to the base e is called the natural logarithmic function and it is denoted by $\log_e$.

$f(x) = \log_e x$

To find the logarithm of a number, we can use the logarithm table instead of using mere calculation. Before finding the logarithm of a number, we should know about the characteristics and mantissa part of a given number

- **Characteristic Part** – The whole part of a number is called the characteristic part. The characteristic of any number greater than one is positive, and if it is one less than the number of digits to the left of the decimal point in a given number. If the number is less than one, the characteristic is negative and is one more than the number of zeros to the right of the decimal point.
- **Mantissa Part** – The decimal part of the logarithm number is said to be the mantissa part and it should always be a positive value. If the mantissa part is in a negative value, then convert into the positive value.

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

How to Use the Log Table?

The procedure is given below to find the log value of a number using the log table. First, you have to know how to use the log table. The log table is given for the reference to find the values.

Step 1: Understand the concept of the logarithm. Each log table is only usable with a certain base. The most common type of logarithm table is used is log base 10.

Step 2: Identify the characteristics and mantissa part of the given number. For example, if you want to find the value of $\log_{10}(15.27)$, first separate the characteristic part and the mantissa part.

Characteristic Part = 15

Mantissa part = 27

Step 3: Use a common log table. Now, use row number 15 and check column number 2 and write the corresponding value. So the value obtained is 1818.

Step 4: Use the logarithm table with a mean difference. Slide your finger in the mean difference column number 7 and row number 15, and write down the corresponding value as 20.

15.27

N	0	1	2	3	4	5	6	7	8	9	Mean Difference							
											1	2	3	4	5	6	7	
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	
14	1431	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	7	
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	9	12	6	7	

Step 5: Add both the values obtained in step 3 and step 4. That is 1818+20= 1838. Therefore, the value 1838 is the mantissa part.

15.27

Row 15 + column 2 Row 15 + column 7

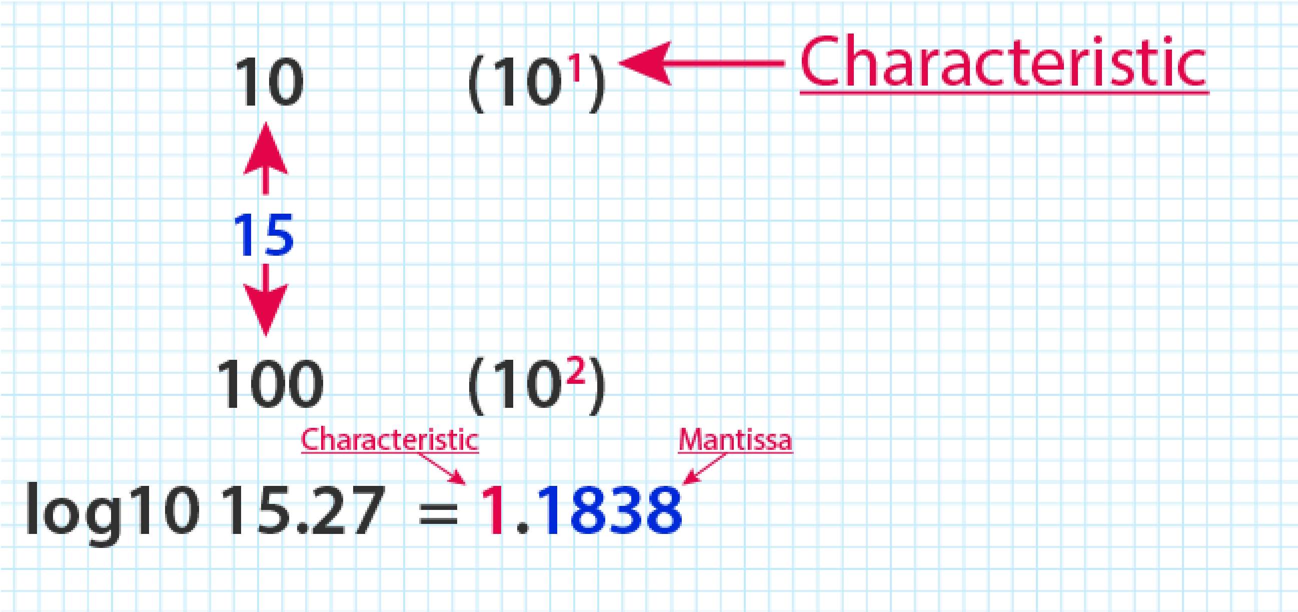
1818 + 20

= 1838

Mantissa

Step 6: Find the characteristic part. Since the number lies between 10 and 100, (10^1 and 10^2), the characteristic part should be 1.

Step 7: Finally combine both the characteristic part and the mantissa part, it becomes 1.1838.



Example

Here the sample example to find the value of the logarithmic function using the logarithm table is given.

Question:

Find the value of $\log_{10}$ 2.872

Solution:

Step 1: Characteristic Part= 2 and mantissa part= 872

Step 2: Check the row number 28 and column number 7. So the value obtained is 4579.

Step 3: Check the mean difference value for row number 28 and mean difference column 2. The value corresponding to the row and column is 3

Step 4: Add the values obtained in step 2 and 3, we get 4582. This is the mantissa part.

Step 5: Since the number of digits to the left side of the decimal part is 1, the characteristic part is less than 1. So the characteristic part is 0

Step 6: Finally combine the characteristic part and the mantissa part. So it becomes 0.4582.

Therefore the value of log 2.872 is 0.4582.

Example

Find the logarithm of:

379

Solution

379

$= 3.79 \times 10^2$

Log 3.79 = 0.5786

Therefore the logarithm of 379 is 2 + 0.5786= 2.5786

The whole number part of the logarithm is called the characteristic and the decimal part is the mantissa.

Logarithms of Positive Numbers less than 1

Example

Find Log to base 10 of 0.034

We proceed as follows:

Express 0.034 in standard form, i.e., $A X^n$.

Read the logarithm of A and add to n

Thus $0.034 = 3.4 \times 10^{-2}$

Log 3.4 from the tables is 0.5315

Hence $3.4 \times 10^{-2}=$

Using laws of indices add 0.5315 + -2 which is written as

$\bar{2}.5315$

It reads bar two point five three one five. The negative sign is written directly above two to show that it's only the characteristic is negative.

Example

Find the logarithm of:

0.00063

Solution

(Find the logarithm of 6.3)

= 0.7993

$=\bar{4}.7993$

Example

Find the logarithm of:

379

Solution

379 = 3.79 $x10^2$

Log 3.79 = 0.5786

ANTILOGARITHMS

Finding antilogarithm is the reverse of finding the logarithms of a number. For example the logarithm of 1000 to base 10 is 3. So the antilogarithm of 3 is 1000.In algebraic notation, if

Log x = y then antilog of y = x.

Example

Find the antilogarithm of 0.3031

Solution

Let the number be x

(Find the antilog, press shift and log then key in the number)

=-0.5184

Example

Use logarithm tables to evaluate: 456 ×398÷ 271

Number	Standard form	logarithm
456	4.56×10^2	2.6590
398	3.98×10^2	2.5999 add

2.6590+2.5999

= 5.2589

271	2.71×10^2	2.4330

subtract 5.2589 – 2.4330

= 2.8259

Next Find antilog of 2.8259

To find the exact number find the antilog of 2.8259 by letting the characteristic part to be the power of ten then finding the antilog of 0.8259. The mantisa (which is 2) in this case will determine where to place a decimal point. In this case the answer must be between 100 and 1000

= 669.7

- When numbers are multiplying, we add the logs
- When numbers are dividing , we subtract the logs
- When numbers has a power e.g. 345^6, we find the log of 345 = 2.5378 and then multiply by 6 =15.2269 and then find its antilog = 1.686×10^{15}
- When a number has a root e.g. $\sqrt[3]{234}$, we find log of 234 = 2.3692 and then we divide by 3 = 0.7897 and then find the antilog of 0.7897.

Example

Evaluate using log tables: $\frac{\sqrt[5]{3456}}{235^2}$

Number	Standard form	Logarithm
3456	3.456×10^3	$3.539\div 5 = 0.7078$
235	2.35×10^2	$2.3711\times 2 = 4.742$

Subtract 0.7078-4.742 = $\bar{4}.0342$

Antilog of 0.0342 = 1.0819= 0.00010819 9 the ($\bar{4}$ means shifting the decimal point to the left 4 times).

Exercise

Evaluate using log tables

a. $415.2\times 0.0761 \div 135$ b. log 0.02343 c. log 2567 d. $\frac{2343^3}{\sqrt[2]{3443}}$

e. $\frac{9879\times 123\div 2324}{\sqrt[3]{4567}\times 1111^2}$

CHAPTER SIX

QUADRILATERALS

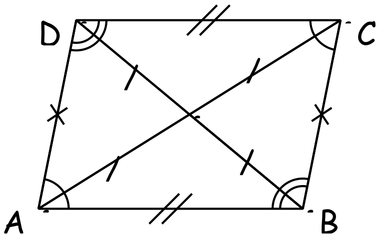
A quadrilateral is a four –sided plane figure. The interior angles of a quadrilateral add up to 360^0 .Quadrilaterals are also classified in terms of sides and angles.

PROPERTIES OF QUADRILATERALS

Properties of Parallelograms

In a parallelogram,

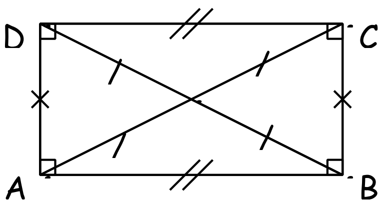
1. The parallel sides are parallel by definition.
2. The opposite sides are congruent.
3. The opposite angles are congruent.
4. The diagonals bisect each other.
5. Any pair of consecutive angles are supplementary.



Properties of Rectangles

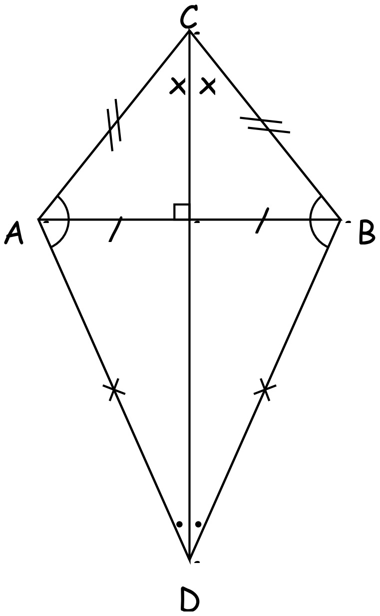
In a rectangle,

1. All the properties of a parallelogram apply by definition.
2. All angles are right angles.
3. The diagonals are congruent.



Properties of a kite

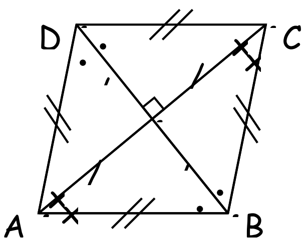
1. Two disjoint pairs of consecutive sides are congruent by definition.
2. The diagonals are perpendicular.
3. One diagonal is the perpendicular bisector of the other.
4. One of the diagonals bisects a pair of opposite angles.
5. One pair of opposite angles are congruent.



Properties of Rhombuses

In a rhombus,

1. Allthe properties of a parallelogram apply by definition.
2. Two consecutive sides are congruent by definition.
3. All sides are congruent.
4. The diagonals bisect the angles.
5. The diagonals are perpendicular bisectors of each other.
6. The diagonals divide the rhombus into four congruent right triangles.

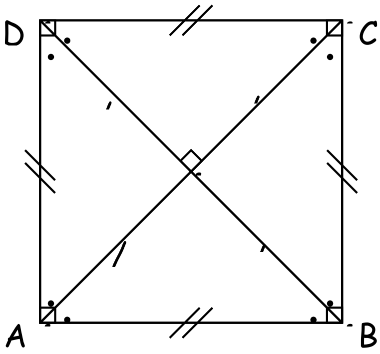


Properties of Squares

In a square,

1. All the properties of a rectangle apply by definition.

2. All the properties of a rhombus apply by definition.
3. The diagonals form four isosceles right triangles.

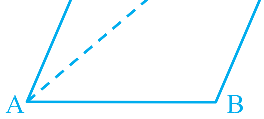


ACTIVITY

Cut out a parallelogram from a sheet of paper and cut it along a diagonal (see Fig. 8.7). You obtain two triangles. What can you say about these triangles?

Place one triangle over the other. Turn one around, if necessary. What do you observe?

Observe that the two triangles are congruent to each other.



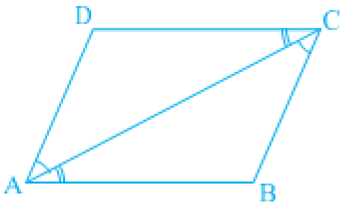
Repeat this activity with some more parallelograms. Each time you will observe that each diagonal divides the parallelogram into two congruent triangles.

Let us now prove this result.

<u>THEOREMS IN QUADRILATERALS</u>

Theorem 1 : *<u>A diagonal of a parallelogram divides it into two congruent triangles (two equal parts).</u>*

Proof : Let ABCD be a parallelogram and AC be a diagonal (see Fig. 8.8). Observe that the diagonal AC divides parallelogram ABCD into two triangles, namely, Δ ABC and Δ CDA. We need to prove that these triangles are congruent.



In Δ ABC and Δ CDA, note that BC || AD and AC is a transversal.

So, $\angle$ BCA = $\angle$ DAC (Pair of alternate angles)

Also, AB || DC and AC is a transversal.

So, $\angle$ BAC = $\angle$ DCA (Pair of alternate angles)

and AC = CA (Common)

So, Δ ABC $\equiv$ Δ CDA (ASA rule)

or, diagonal AC divides parallelogram ABCD into two congruent triangles ABC and CDA.

Now, measure the opposite sides of parallelogram ABCD. What do you observe? You will find that AB = DC and AD = BC.

This is another property of a parallelogram stated below:

Theorem 2: *<u>In a parallelogram, opposite sides are equal</u>*.

You have already proved that a diagonal divides the parallelogram into two congruent

triangles; so what can you say about the corresponding parts say, the corresponding sides? They are equal.

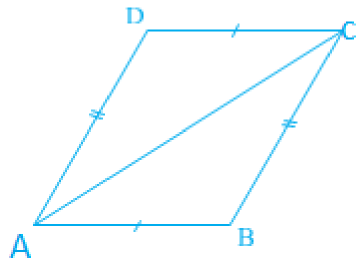
So, AB = DC and AD = BC

Now what is the converse of this result? You already know that whatever is given in a theorem, the same is to be proved in the converse and whatever is proved in the theorem it is given in the converse. Thus, Theorem can be stated thus :

If a quadrilateral is a parallelogram, then each pair of its opposite sides is equal. So its converse is :

Theorem 3: **<u>If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.</u>**

Can you reason out why?



Let sides AB and CD of the quadrilateral ABCD be equal and also AD = BC (see Fig. above). Draw diagonal AC.

Clearly, Δ ABC $\equiv$ Δ CDA (already proved)

So, $\angle$ BAC = $\angle$ DCA (alternate angles)

and $\angle$ BCA = $\angle$ DAC (alternate angles)

therefore we say that ABCD is a parallelogram

You have just seen that in a parallelogram each pair of opposite sides is equal and conversely if each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram. Can we conclude the same result for the pairs of opposite angles?

Draw a parallelogram and measure its angles. What do you observe? Each pair of opposite angles is equal.

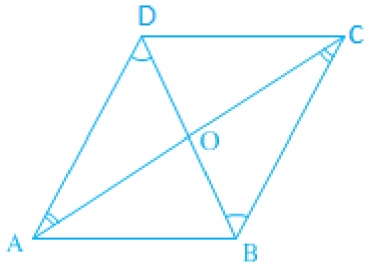
Repeat this with some more parallelograms. We arrive at yet another result as given below.

Theorem4 : In a parallelogram, opposite angles are equal.

Now, is the converse of this result also true? Yes. Using the angle sum property of a quadrilateral and the results of parallel lines intersected by a transversal, we can see that the converse is also true. So, we have the following theorem :

Theorem 5 : *If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.*

There is yet another property of a parallelogram. Let us study the same. Draw a parallelogram ABCD and draw both its diagonals intersecting at the point O (see Fig. below).



Measure the lengths of OA, OB, OC and OD. What do you observe? You will observe that

OA = OC and OB = OD. or, O is the mid-point of both the diagonals.

Repeat this activity with some more parallelograms.

Each time you will find that O is the mid-point of both the diagonals. So, we have the following theorem :

Theorem 6 : ***The diagonals of a parallelogram bisect each other*.**

Now, what would happen, if in a quadrilateral the diagonals bisect each other? Will it be a parallelogram? Indeed this is true.

This result is the converse of the result of

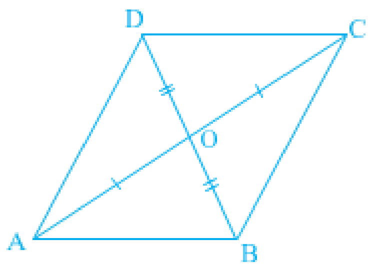
Theorem above. It is given below:

Theorem 7 : ***<u>If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.</u>***

You can reason out this result as follows:

Note that in Fig. below, it is given that OA = OC

and OB = OD.



So, Δ AOB $\equiv \Delta$ COD

Therefore, $\angle$ ABO = $\angle$ CDO (alternate angles)

From this, we get AB || CD

Similarly, BC || AD

Therefore ABCD is a parallelogram

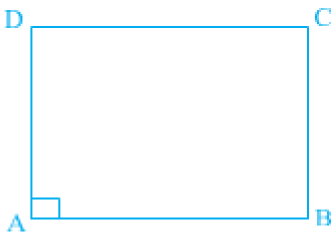
Example 1 : Show that each angle of a rectangle is a right angle.

Solution : Let us recall what a rectangle is.

A rectangle is a parallelogram in which one angle is a right angle.

Let ABCD be a rectangle in which $\angle$ A = 90°. We have to show that $\angle$ B = $\angle$ C = $\angle$ D = 90°

We have, AD || BC and AB is a transversal



So, $\angle A + \angle B = 180°$ (Interior angles on the same side of the transversal)

But, $\angle A = 90°$

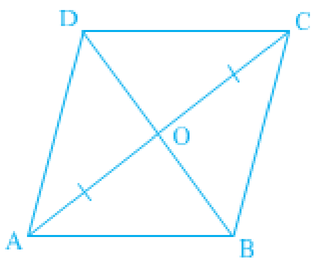
So, $\angle B = 180° - \angle A = 180° - 90° = 90°$

Now, $\angle C = \angle A$ and $\angle D = \angle B$

(Opposite angles of the parallellogram) So, $\angle C = 90°$ and $\angle D = 90°$.

Therefore, each of the angles of a rectangle is a right angle.

Example 2 : Show that the diagonals of a rhombus are perpendicular to each other.



Solution : Consider the rhombus ABCD (see Fig.above. You know that AB = BC = CD = DA (corresponding sides of congruent triangles)

Now, in Δ AOD and Δ COD,

OA = OC (Diagonals of a parallelogram bisect each other)

OD = OD (Common) AD = CD

Therefore, $\Delta AOD \equiv \Delta COD$

(SSS congruence rule)

This gives, $\angle$ AOD = $\angle$ COD

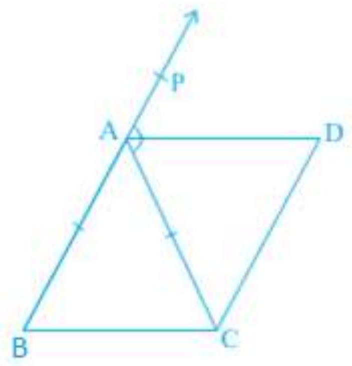
But, $\angle$ AOD + $\angle$ COD = 180° (Linear pair)

So, $2\angle$ AOD = 180°

or, $\angle$ AOD = 90°

So, the diagonals of a rhombus are perpendicular to each other.

Example 3 : ABC is an isosceles triangle in which AB = AC. AD bisects exterior angle PAC and CD || AB (see Fig. that follows).



Show that

(i) $\angle$ DAC = $\angle$ BCA and (ii) ABCD is a parallelogram.

Solution :
(i) Δ ABC is isosceles in which AB = AC (Given)
So, $\angle$ ABC = $\angle$ ACB (Angles opposite to equal sides)
Also, $\angle$ PAC = $\angle$ ABC + $\angle$ ACB
(Exterior angle of a triangle)

or, $\angle$ PAC = $2\angle$ ACB ------- (1)
Now, AD bisects $\angle$ PAC.---------- (2)
So, $\angle$ PAC = $2\angle$ DAC

Therefore,

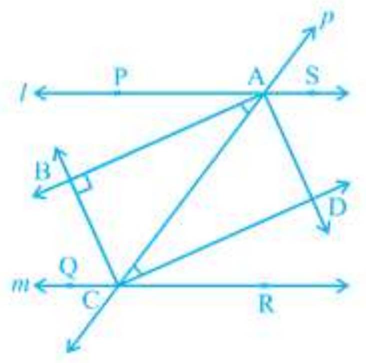
$2\angle$ DAC = $2\angle$ ACB [From (1) and (2)]
or, $\angle$ DAC = $\angle$ ACB
(ii) Now, these equal angles form a pair of alternate angles when line segments BC

and AD are intersected by a transversal AC. So, BC || AD

Also, BA || CD (Given)

Now, both pairs of opposite sides of quadrilateral ABCD are parallel. So, ABCD is a parallelogram.

Example 4 : Two parallel lines l and m are intersected by a transversal p (see Fig. 8.15). Show that the quadrilateral formed by the bisectors of interior angles is a rectangle.



Solution :

It is given that PS || QR and transversal p intersects them at points A and C respectively.

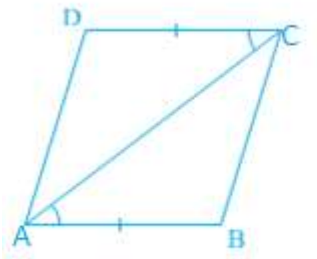
The bisectors of ∠ PAC and ∠ ACQ intersect at B and bisectors of ∠ ACR and ∠ SAC intersect at D.

We are to show that quadrilateral ABCD is a rectangle.

Now, ∠ PAC = ∠ ACR

(Alternate angles as $l \parallel m$ and p is a transversal)

Theorem 8: ***A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.***



Look at Fig 8.17 in which AB = CD and

AB || CD. Let us draw a diagonal AC. You can show that Δ ABC≡ Δ CDA by SAS congruence rule.

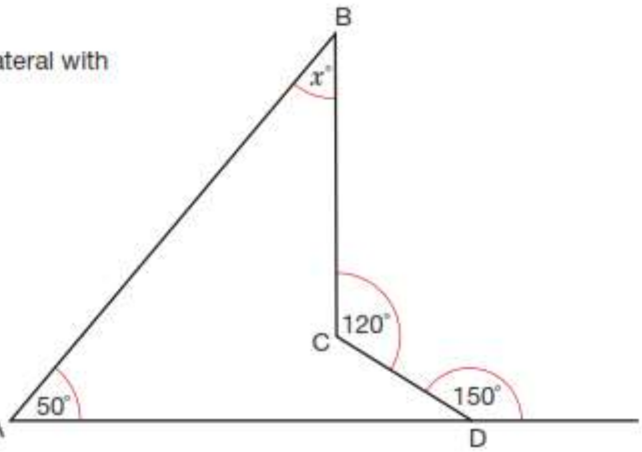
So, BC || AD (corresponding sides in a parallelogram)

And AB is also parallel to DC (same reason as above

Therefore ABCD is a parallelogram

Examples

ABCD is a quadrilateral with angles as shown.



a Find angle x.

b Show that BC is **perpendicular** to AD.

---

a $\angle ADC = 30°$ (angles on a straight line add up to 180°)

$\angle DCB = 240°$ (angles at a point **sum** to 360°)

$x = 360 - 50 - 30 - 240 = 40°$ (internal angles of a quadrilateral sum to 360°)

a $\angle ADC = 30°$ (angles on a straight line add up to 180°)

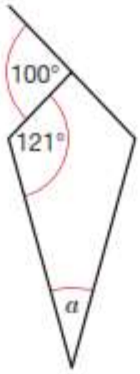
$\angle DCB = 240°$ (angles at a point **sum** to 360°)

$x = 360 - 50 - 30 - 240 = 40°$ (internal angles of a quadrilateral sum to 360°)

Exercise

1 ▶ Name a quadrilateral with no lines of symmetry and rotational symmetry of order two.

2 ▶ The diagram shows a kite. Find the size of angle a.

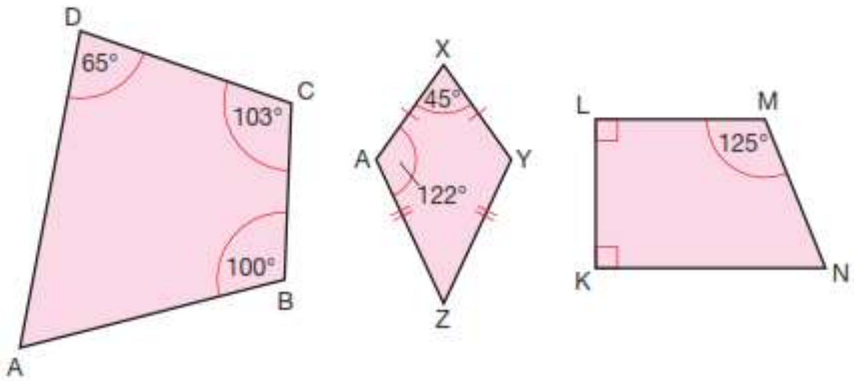


3 ▶ Work out the size of

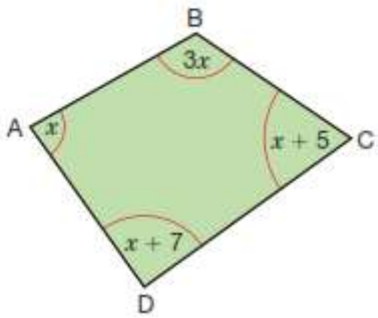
a angle DAB

b $\angle$AZY

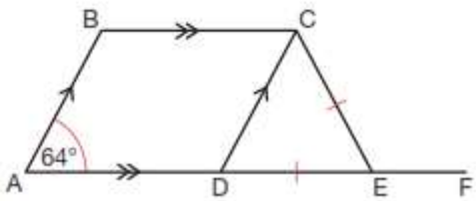
c M$\hat{N}$K



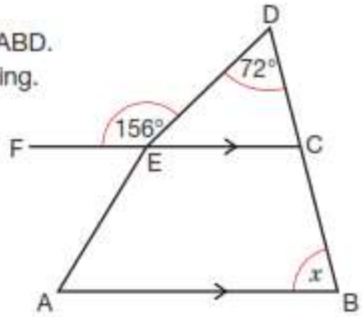
4 ▶ Work out the size of angle ABC. Give reasons for your working.



5 ▶ ABCD is a parallelogram.
CDE is an isosceles triangle.
ADEF is a straight line. Angle BAD = 64°.
Work out the size of angle CEF.
Give reasons for your working.



6 ▶ Work out the size of angle ABD.
Give reasons for your working.

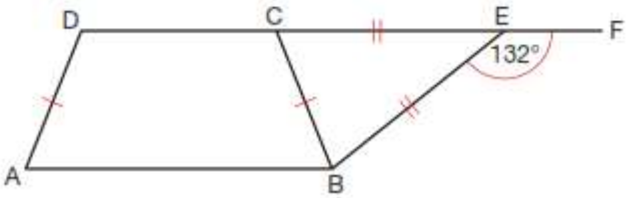


Exercise 2

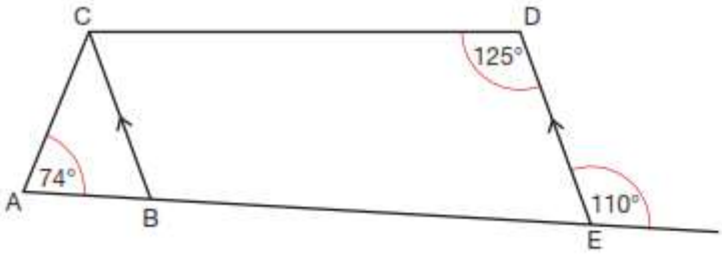
1 ▶ Name a quadrilateral with one line of symmetry and no rotational symmetry.

2 ▶ The four angles of a quadrilateral are 90°, $3x + 15°$, $x + 25°$ and $x + 55°$. Find x.

3 ▶ ABCD is an isosceles trapezium. BCE is an isosceles triangle.
DCEF is a straight line. Angle BEF = 132°.
Work out the size of angle DAB. Give reasons for your working.



4 ▶ Work out the size of angle ACB. Give reasons for your working.



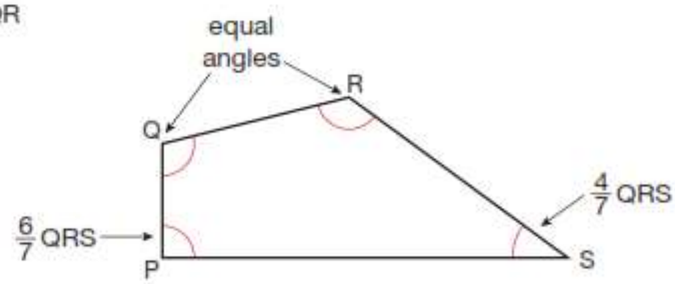
5 ▶ In this quadrilateral, angles PQR and QRS are equal.

Angle PSR is $\frac{4}{7}$ angle QRS.

Angle QPS is $\frac{6}{7}$ angle QRS.

a Find angle PSR.

b Show that angle QPS is a right angle.



CHAPTER SEVEN

CONGRUENT TRIANGLES

- ✓ Congruent triangles are those triangles that have corresponding sides and corresponding angle equal. The symbol for congruent trianles is $\equiv$ *or* $\cong$

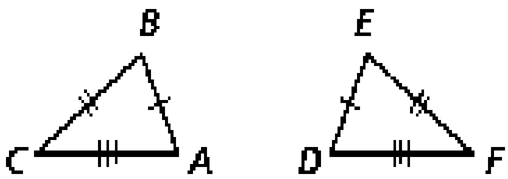
Proving congruency in triangles

A. <u>**side, side, side congruency theorem**</u>

Theorem

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$, then $\Delta ABC \cong \Delta DEF$.

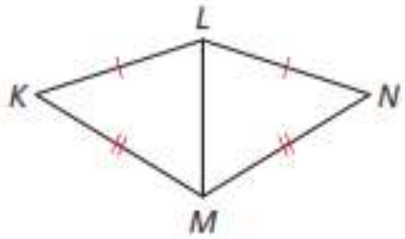


EXAMPLE 1 Using the SSS Congruence Theorem

Write a proof.

Given $\overline{KL} \cong \overline{NL}$, $\overline{KM} \cong \overline{NM}$

Prove $\triangle KLM \cong \triangle NLM$



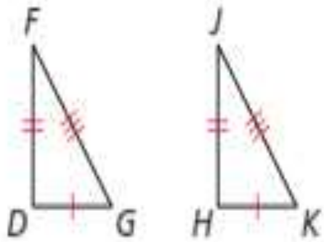
SOLUTION

STATEMENTS	REASONS
S **1.** $\overline{KL} \cong \overline{NL}$	**1.** Given
S **2.** $\overline{KM} \cong \overline{NM}$	**2.** Given
S **3.** $\overline{LM} \cong \overline{LM}$	**3.** Reflexive Property of Congruence (Thm. 2.1)
4. $\triangle KLM \cong \triangle NLM$	**4.** SSS Congruence Theorem

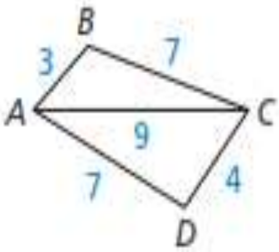
Exercise

Decide whether the congruence statement is true. Explain your reasoning.

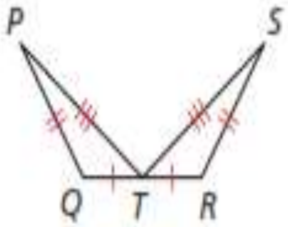
1. $\triangle DFG \cong \triangle HJK$



2. $\triangle ACB \cong \triangle CAD$



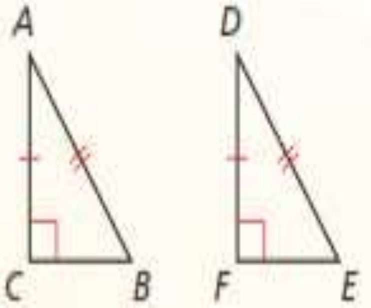
3. $\triangle QPT \cong \triangle RST$



B. RIGHT ANGLE HYPOTENUSE SIDE (RHS) THEOREM

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

If $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $m\angle C = m\angle F = 90^\circ$, then $\triangle ABC \cong \triangle DEF$.

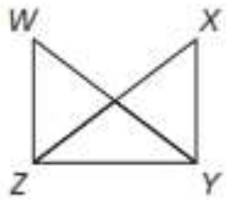


EXAMPLE

Write a proof.

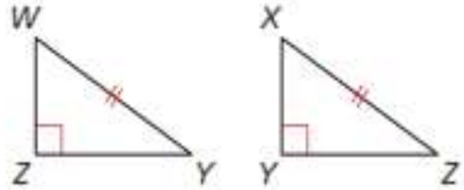
Given $\overline{WY} \cong \overline{XZ}$, $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$

Prove $\triangle WYZ \cong \triangle XZY$



SOLUTION

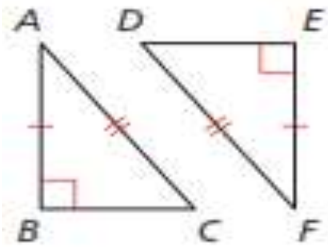
Redraw the triangles so they are side by side with corresponding parts in the same position. Mark the given information in the diagram.

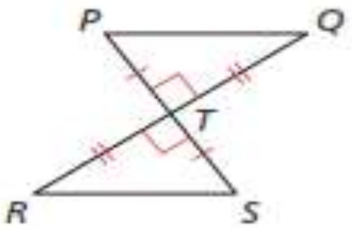


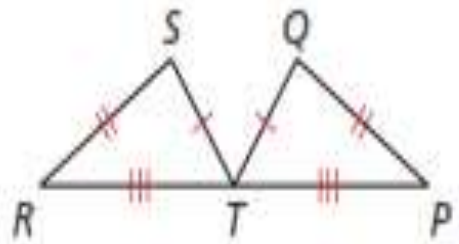
STATEMENTS	REASONS
H **1.** $\overline{WY} \cong \overline{XZ}$	**1.** Given
2. $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$	**2.** Given
3. $\angle Z$ and $\angle Y$ are right angles.	**3.** Definition of $\perp$ lines
4. $\triangle WYZ$ and $\triangle XZY$ are right triangles.	**4.** Definition of a right triangle
L **5.** $\overline{ZY} \cong \overline{YZ}$	**5.** Reflexive Property of Congruence (Thm. 2.1)
6. $\triangle WYZ \cong \triangle XZY$	**6.** HL Congruence Theorem

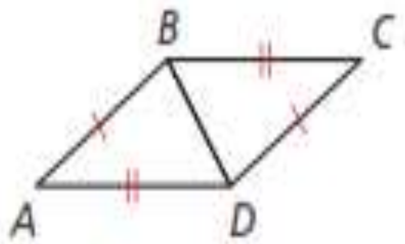
EXERCISE

Prove whether the figures below are congruent or not



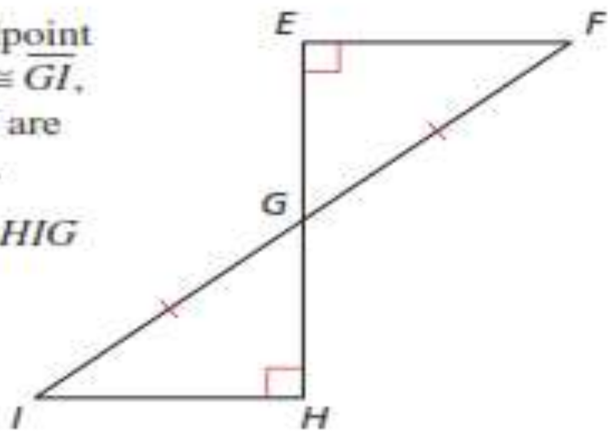






Given G is the midpoint of $\overline{EH}$, $\overline{FG} \cong \overline{GI}$, $\angle E$ and $\angle H$ are right angles.

Prove $\triangle EFG \cong \triangle HIG$



<u>c. Proving congruency by ASA and AAS</u>

Theorem **Angle-Side-Angle (ASA) Congruence Theorem**

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

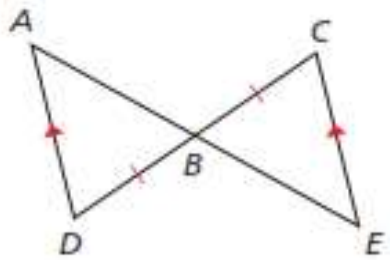
If $\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$, and $\angle C \cong \angle F$, then $\triangle ABC \cong \triangle DEF$.

EXAMPLE

Write a proof.

Given $\overline{AD} \parallel \overline{EC}$, $\overline{BD} \cong \overline{BC}$

Prove $\triangle ABD \cong \triangle EBC$



SOLUTION

STATEMENTS	REASONS
1. $\overline{AD} \parallel \overline{EC}$	1. Given
A 2. $\angle D \cong \angle C$	2. Alternate Interior Angles Theorem (Thm. 3.2)
S 3. $\overline{BD} \cong \overline{BC}$	3. Given
A 4. $\angle ABD \cong \angle EBC$	4. Vertical Angles Congruence Theorem (Thm 2.6)
5. $\triangle ABD \cong \triangle EBC$	5. ASA Congruence Theorem

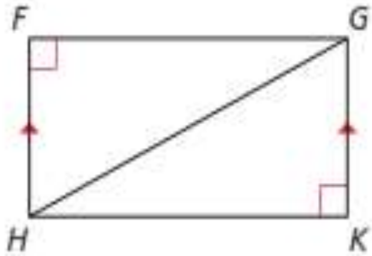
EXAMPLE

Show that the two figures are congruent

Write a proof.

Given $\overline{HF} \parallel \overline{GK}$, $\angle F$ and $\angle K$ are right angles.

Prove $\triangle HFG \cong \triangle GKH$



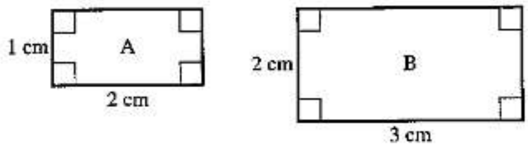
SOLUTION

STATEMENTS	REASONS
1. $\overline{HF} \parallel \overline{GK}$	1. Given
A 2. $\angle GHF \cong \angle HGK$	2. Alternate Interior Angles Theorem (Theorem 3.2)
3. $\angle F$ and $\angle K$ are right angles.	3. Given
A 4. $\angle F \cong \angle K$	4. Right Angles Congruence Theorem (Theorem 2.3)
S 5. $\overline{HG} \cong \overline{GH}$	5. Reflexive Property of Congruence (Theorem 2.1)
6. $\triangle HFG \cong \angle GKH$	6. AAS Congruence Theorem

CHAPTER EIGHT

SIMILARITY

Two triangles are similar if they have the same angles. For other shapes, not only must corresponding angles be equal, but also corresponding sides must be in the same proportion.
The two rectangles A and B are *not* similar even though they have the same angles.



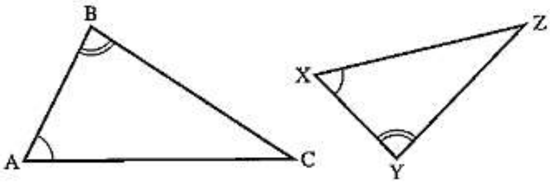
Example

In the triangles ABC and XYZ

$\hat{A} = \hat{X}$ and $\hat{B} = \hat{Y}$

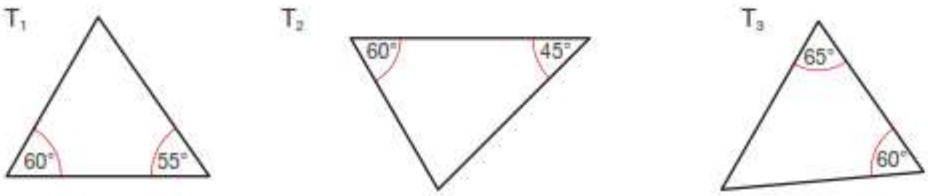
so the triangles are similar. ($\hat{C}$ must be equal to $\hat{Z}$.)

We have $\frac{BC}{YZ} = \frac{AC}{XZ} = \frac{AB}{XY}$



Example

which of these angles are similar to each other?



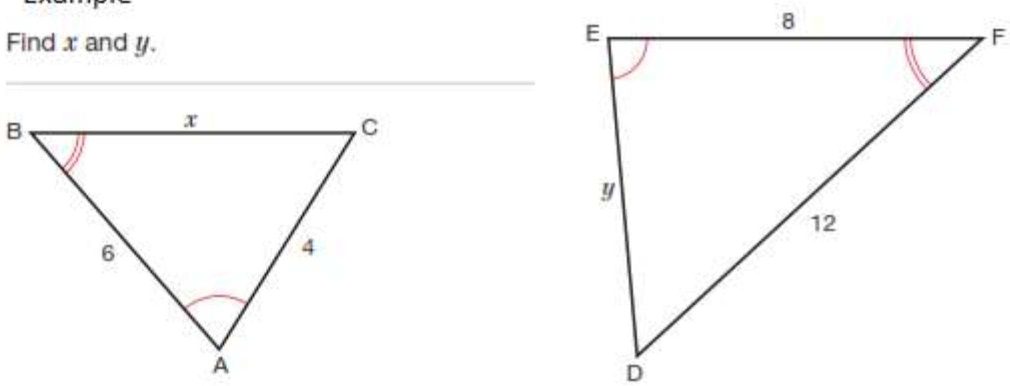
The angle sum of a triangle is 180°.

Therefore in T_1 the angles are 55°, 60° and 65°, in T_2 the angles are 45°, 60° and 75°, and in T_3 the angles are 55°, 60° and 65°.

Therefore the triangles T_1 and T_3 are similar.

Example

Find x and y.



$\hat{A} = \hat{E}$, $\hat{B} = \hat{F}$, $\hat{C} = \hat{D}$ Therefore $\triangle$ ABC is similar to $\triangle$ DEF.

$\Rightarrow \frac{AB}{EF} = \frac{AC}{DE} = \frac{BC}{DF} \Rightarrow \frac{6}{8} = \frac{4}{y} = \frac{x}{12}$ Corresponding sides are in the same ratio.

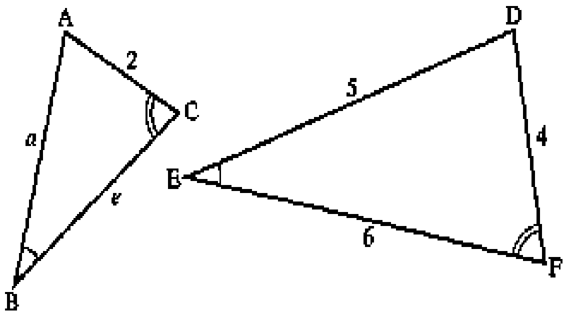
$\Rightarrow \frac{6}{8} = \frac{x}{12}$ $\Rightarrow 6 \times 12 = 8 \times x$ $\Rightarrow x = 9$

$\Rightarrow \frac{6}{8} = \frac{4}{y}$ $\Rightarrow 6 \times y = 8 \times 4$ $\Rightarrow y = 5\frac{1}{3}$

Exercise 1

Find the sides marked with letters in questions 1 to 11; all lengths are given in centimetres.

1.



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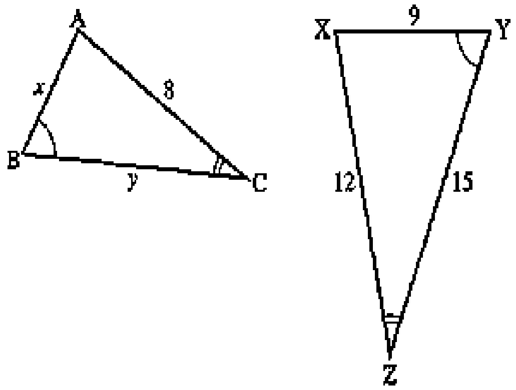

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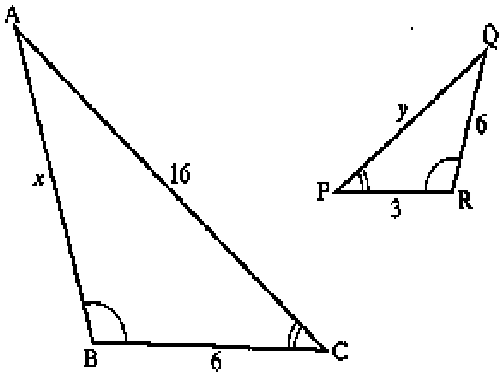

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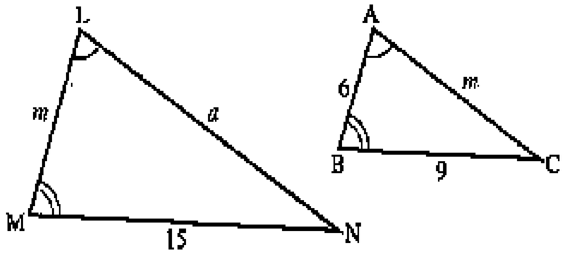

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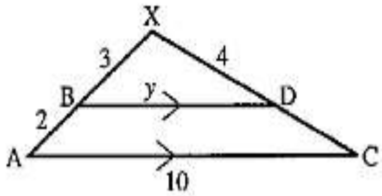

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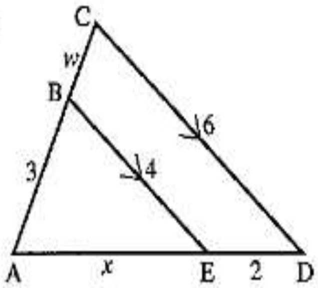

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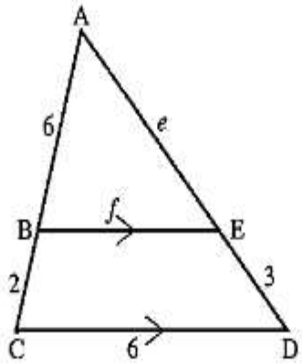

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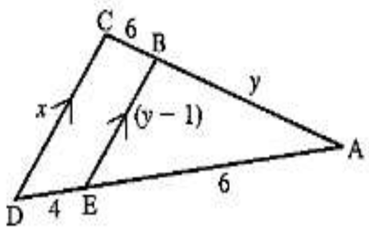
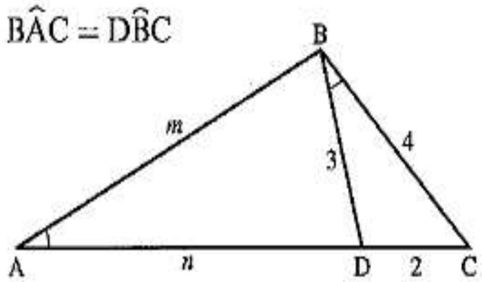
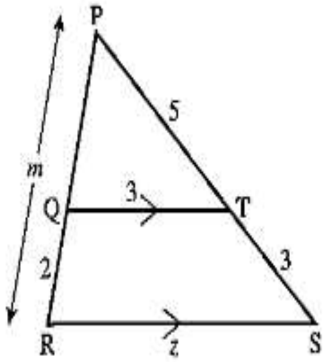


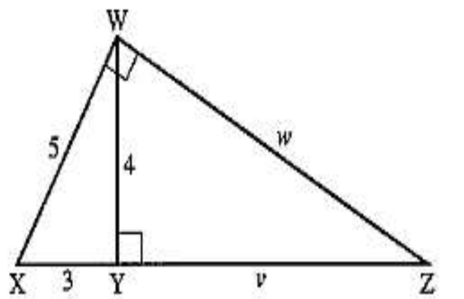
9. $B\hat{A}C = D\hat{B}C$



10.



11.



CHAPTER NINE

PROBABILITY

Definition

The likelihood of an occurrence of an event or the numerical measure of chance is called probability.

- If an event **cannot** happen the probability of it occurring is 0.
- If an event is **certain** to happen the probability of it occuring is 1.
- All probabilities lie between 0 and 1.

  You write probabilities using fractions or decimals.

Experimental probability (single event experiments)

This is where probability is determined by experience or experiment. What is done or observed is the experiment. Each toss is called a trial and the result of a trial is the outcome. The experimental probability of a result is given by:

$$\text{Probability} = \frac{\textit{the number of favourable outcomes}}{\textit{the total number of trials}}$$

A. Tossing a die

Example

A boy had a fair die with faces marked 1to6 .He threw this die once. The result of his experiment is shown below.

face	1	2	3	4	5	6

All possible outcomes are: 1,2,3,4,5,6

NOTE: Each number has a fair and equal chance

What is the experimental provability of getting?

a.)1 b.) 6 c) and old number d) an odd and an even number e) a prime number f) a multiple of 3

Solution

a.) P(1) = 1/6

b.) P (6)= 1/6

c) P (odd number) = 1/6 +1/6 + 1/6 = 3/6 = ½ (there are three odd numbers: 1,3 and 5)

d). P (odd +even number) = 6/6 =1 (3 odd numbers and 3 even numbers)

e) P(Prime number) = 3/6 = ½ (3 prime numbers)

f) P(multiple of 3) = 2/6 = 1/3 (2 multiples of 3)

TOSSING A COIN

A coin has two sides : head and tail. These two sides have fair chance when the coin is tossed.

Example

If a coin is tossed, find

i.) The probability of getting a head or a tail
ii.) Probability of getting a gead and a tail
iii.) Probability of getting a head

Solution

i. P (head)
½
ii. P(head) + P(tail)

= ½ + ½ = 1

Note;
If [OR] is used then we add and when and is used we multiply

iii. P (head and tail)
½ × ½ = ¼

Throwing cards

Example 1

The numbers 1 to 20 are each written on a card.
The 20 cards are mixed together.
One card is chosen at random from the pack.
Find the probability that the number on the card is:

(a) even (b) a factor of 24 (c) prime.

We will use '$p(x)$' to mean 'the probability of x'.

(a) $p(\text{even}) = \frac{10}{20} = \frac{1}{2}$

(b) $p(\text{factor of } 24) = p(1, 2, 3, 4, 6, 8, 12) = \frac{7}{20}$

(c) $p(\text{prime}) = p(2, 3, 5, 7, 11, 13, 17, 19) = \frac{8}{20} = \frac{2}{5}$

In each case, we have counted the number of ways in which a 'success' can occur and divided by the number of possible results of a 'trial'.

Theoretical probability

This can be calculated without necessarily using any past experience or doing any experiment. The probability of an event happening #number of favorable outcomes /total number of outcomes.

Example

A basket contains 5 red balls, 4green balls and 3 blue balls. If a ball is picked at random from the basket, find:

a.)The probability of picking a blue ball

b.) The probability of not picking a red ball

Solution

a.)Total number of balls is 12

The number of blue balls is 3

Solution

a.) therefore, P (a blue ball) =3/12

b.)The number of balls which are not red is 7.

Therefore P (not a red ball)= 7/12

Example

A bag contains 6 black balls and some brown ones. If a ball is picked at random the probability that it is black is 0.25.Find the number of brown balls.

Solution

Let the number of balls be x

Then the probability that a black ball is picked at random is 6/x

Therefore 6/x = 0.25

x = 24

The total number of bald is 24

Then the number of brown balls is 24 - 6 =18

EXERCISE

1. In a car park with 300 cars, there are 190 Opals. What is the probability that the first car to leave the car park is:
   A an Opal
   B not an Opal
2. Tamara has 18 loose socks in a drawer. Eight of these are orange and two are pink. Calculate the probability that the first sock taken out at random is:
   A Orange
   B not orangeC pink

D not pink

E orange or pink

F not orange or pink

3. A plate contains 9 shortbread cookies, 4 ginger biscuits, 11 chocolate chip cookies and 18 Jambos. If a biscuit is selected at random, what is the probability that:

A it is either a ginger biscuit OR a Jambo?

B it is NOT a shortbread cookie.

C

4. The children in a nursery school were classified by hair and eye colour. 44 had red hair and not brown eyes.

A How many children were in the school

B What is the probility that a child chosen at random has:

i. Brown eyes

ii. Red hair

5. A jar has purple, blue and black sweets in it. The probability that a sweet, chosen at random, will be purple is 1/7 and the probability that it will be black is 3/5.

A If I choose a sweet at random what is the probability that it will be:

i. purple or blue

ii. Black

iii. purple

B If there are 70 sweets in the jar how many purple ones are there?

C 1/4 if the purple sweets in b) have streaks on them and rest do not. How many purple sweets have streaks?

CHAPTER NINE

LINEAR SIMULTANEOUS EQUATIONS

Linear simultaneous Equations in Two Unknowns

Many problems involve finding values of two or more unknowns. These are often linked via a number of linear equations. For example, if I tell you that the sum of two numbers is 89 and their difference is 33, we can let the larger number be x and the smaller one y and write the given information as a pair of equations:

$x + y = 89$ (1)

$x - Y = 33.$ (2)

These are called simultaneous equations since we seek values of x and y that makes both equations true simultaneously. In this case, if we add the equations we obtain $2x = 122$, so $x = 61$. We can then substitute this value back into either equation, say the first, then $61 + y = 89$ giving $y = 28$.

Elimination method is used by making the coefficient of one of the variables in the two equations the same. Either add or subtract to form a single linear equation of only one unknown variable.

Example

Solve the simultaneous equations

$2x + 3y = 15,$

$-3y + 4x = 3.$

Solution

$2x + 3y = 15$ —— (1)

$-3y + 4x = 3$ —— (2)   (1) + (2):

$(2x + 3y) + (-3y + 4x) = 18$

$6x = 18$

$x = 3$

Substitute $x = 3$ into (1):

$2(3) + 3y = 15$

$3y = 15 – 6$

$y = 3$

$\therefore x = 3, y = 3$

Example 10

Using the method of elimination, solve the simultaneous equations

$5x + 2y = 10,$ $4x + 3y = 1.$

Solution

$5x + 2y = 10$ —— (1)

$4x + 3y = 1$ —— (2)

(1) × 3: $15x + 6y = 30$ —— (3)

(2) × 2: $8x + 6y = 2$ —— (4)

(3) – (4): $7x = 28$ $x = 4$

Substitute $x = 4$ into (2):

$4(4) + 3y = 1$

$16 + 3y = 1$

$3y = –15$

$y = –5$

$\therefore x = 4, y = –5$

Example

The cost of two skirts and three blouses is K600.If the cost of one skirt and two blouses of the same quality K 350, find the cost of each item.

Solution

Let the cost of one skirt be x kwachas and that of one blouse be y kwachas. The cost of two skirts and three blouses is 2x + 3y kwachas.

The cost of one skirt and two blouses is $x + 2y$ kwachas.

So, $2x + 3y = 600$……………….. (I)

$X + 2y = 350$ ……………………….. (II)

Multiplying equation (II) by 2 to get equation (III).

$2x + 4y = 700$ ……………… (III).

$2x + 3y = 600$…………………(I)

Subtracting equation (I) from (II), $y = 100$.

From equation (II),

$X + 2y = 350$ but $y = 100$

$X + 200 = 350$

$X = 150$

Thus the cost of one skirt is 150 kwachas and that of a blouse is 100 kwachas.

In solving the problem above, we reduced the equations from two unknowns to a single unknown in y by eliminating. This is the elimination method of solving simultaneous equations.

Example

$x + 2y = 8$ ...[1]
$2x + 3y = 14$ ...[2]

(a) Label the equations so that the working is made clear.
(b) Choose an unknown in one of the equations and multiply the equations by a factor or factors so that this unknown has the same coefficient in both equations.
(c) Eliminate this unknown from the two equations by subtracting them, then solve for the remaining unknown.
(d) Substitute in the first equation and solve for the eliminated unknown.

$x + 2y = 8$ ...[1]

[1] × 2 $\quad 2x + 4y = 16$ ...[3]
$2x + 3y = 14$ ...[2]

Subtract [2] from [3]
$y = 2$

Substituting in [1]
$x + 2 \times 2 = 8$
$x = 8 - 4$
$x = 4$

The solutions are $x = 4$, $y = 2$.

Example

$2x + 3y = 5$ ...[1]
$5x - 2y = -16$ ...[2]

[1] × 5 $\quad 10x + 15y = 25$ ...[3]
[2] × 2 $\quad 10x - 4y = -32$ ...[4]

[3] − 4 $\quad 15y - (-4y) = 25 - (-32)$
$19y = 57$
$y = 3$

Substitute in [1]
$2x + 3 \times 3 = 5$
$2x = 5 - 9 = -4$
$x = -2$

The solutions are $x = -2$, $y = 3$.

Exercise

1. $2x + 5y = 24$
$4x + 3y = 20$

2. $5x + 2y = 13$
$2x + 6y = 26$

3. $3x + y = 11$
$9x + 2y = 28$

4. $x + 2y = 17$
$8x + 3y = 45$

5. $3x + 2y = 19$
$x + 8y = 21$

6. $2a + 3b = 9$
$4a + b = 13$

7. $2x + 3y = 11$
$3x + 4y = 15$

8. $3x + 8y = 27$
$4x + 3y = 13$

9. $2x + 7y = 17$
$5x + 3y = -1$

10. $5x + 3y = 23$
$2x + 4y = 12$

11. $7x + 5y = 32$
$3x + 4y = 23$

12. $3x + 2y = 4$
$4x + 5y = 10$

13. $3x + 2y = 11$
$2x - y = -3$

14. $3x + 2y = 7$
$2x - 3y = -4$

15. $x + 2y = -4$
$3x - y = 9$

16. $5x - 7y = 27$
$3x - 4y = 16$

17. $3x - 2y = 7$
$4x + y = 13$

18. $x - y = -1$
$2x - y = 0$

19. $y - x = -1$
$3x - y = 5$

20. $x - 3y = -5$
$2y + 3x + 4 = 0$

21. $x + 3y - 7 = 0$
$2y - x - 3 = 0$

22. $3a - b = 9$
$2a + 2b = 14$

23. $3x - y = 9$
$4x - y = -14$

24. $x + 2y = 4$
$3x + y = 9\frac{1}{2}$

25. $2x - y = 5$
$\frac{x}{4} + \frac{y}{3} = 2$

26. $3x - y = 17$
$\frac{x}{5} + \frac{y}{2} = 0$

27. $3x - 2y = 5$
$\frac{2x}{3} + \frac{y}{2} = -\frac{7}{9}$

28. $2x = 11 - y$
$\frac{x}{5} - \frac{y}{4} = 1$

29. $4x - 0{\cdot}5y = 12{\cdot}5$
$3x + 0{\cdot}8y = 8{\cdot}2$

30. $0{\cdot}4x + 3y = 2{\cdot}6$
$x - 2y = 4{\cdot}6$

Solution by substitution

Substitution method is used when we make one variable the subject of an equation and then we substitute that into the other equation to solve for the other variable.

Example

Solve the simultaneous equations

$2x - 3y = -2$,

$y + 4x = 24$.

Solution

$2x - 3y = -2$ —— (1)

$y + 4x = 24$ —— (2)

From (1),

$-2 + 3y = 2x$

$x = -1 + \frac{3}{2}y$ —— (3)

Substitute (3) into (2):

$Y + 4(-1+\frac{3}{2}y) = 24$

$y - 4 + 6y = 24$

$7y = 28$

$y = 4$

Substitute $y = 4$ into (3):

$x = -1 + \frac{3}{2}y$

$= -1 + \frac{3}{2}(4)$

$= -1 + 6$

$= 5$

$\therefore x = 5, y = 4$

Example

Using the method of substitution, solve the simultaneous equations

$5x + 2y = 10, \qquad 4x + 3y = 1.$

Solution

$5x + 2y = 10$ —— (1)

$4x + 3y = 1$ —— (2)

From (1),

$2y = 10 - 5x$

$y = \frac{10-5x}{2}$ —— (3)

Substitute (3) into (2):

$4x + 3(\frac{10-5x}{2}) = 1$

$8x + 3(10 - 5x) = 2$

$8x + 30 - 15x = 2$

$7x = 28$

X= 4

Substitute $x = 4$ into (3):

$Y = \frac{10 - 5(4)}{2}$

$= -5$

$\therefore x = 4, y = -5$

Example : solve by substitution:

2x+3y =600 …………..1

X + 2y = 350-----------11

Taking equation (II) alone;

Subtracting 2y from both sides;

X = 350 – 2y

Substituting this value of x in equation 1;

2(350 – 2y) + 3y = 600

700 – 4y + 3y = 600

Y = 100

Substituting this value of y in equation 11;

X + 2(100) =350

X = 350 – 200

∴ X = 150

EXERCISE

Word problems

1. Find two numbers with a sum of 15 and a difference of 4.
2. Twice one number added to three times another gives 21. Find the numbers, if the difference between them is 3.
3. The average of two numbers is 7, and three times the difference between them is 18. Find the numbers.
4. In three years' time a pet mouse will be as old as his owner was four years ago. Their present ages total 13 years. Find the age of each now.
5. Find two numbers where three times the smaller number exceeds the larger by 5 and the sum of the numbers is 11.

Graphical solutions to simultaneous equations

Apart from substitution method and elimination method, we can also solve simultaneous equations by using graphs

Example

Solve the simultaneous equations
y = ½ x +2and *y* = 4 - *x* graphically.
First, make a table of values for each equation.

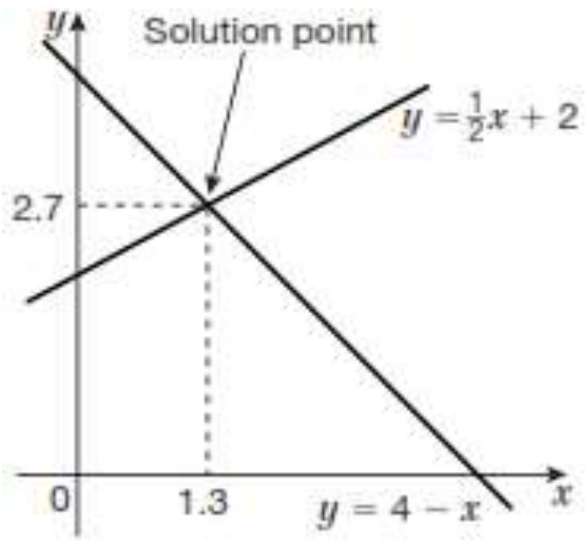
First, make a table of values for each equation.

x	0	2	4
$y=\frac{1}{2}x+2$	2	3	4

x	0	2	4
$y=4-x$	4	2	0

Next, draw accurate graphs for both equations on one set of axes.

The solution point is approximately $x = 1.3$, $y = 2.7$.



Exercise

1 ▶ Copy and complete these tables, then draw both graphs on one set of axes.

x	0	2	4
$y = x + 1$			

x	0	2	4
$y = 2x - 2$			

Solve the simultaneous equations $y = x + 1$, $y = 2x - 2$ using your graph.

2 ▶ On one set of axes, draw the graphs of $y = 3x - 1$ and $y = 2x + 1$ for $0 \leq x \leq 6$.
Then, solve the simultaneous equations $y = 3x - 1$ and $y = 2x + 1$ using your graph.

In Questions 3 and 4, solve the simultaneous equations graphically, using $0 \leq x \leq 6$.

3 ▶ $y = 2x + 2$
$y = 3x - 1$

4 ▶ $y = \frac{1}{2}x + 1$
$y = 4 - x$

CHAPTER TEN

RATIO AND PROPORTION

Ratio

The word 'ratio' is used to describe a fraction. If the *ratio* of a boy's height to his father's height is 4 : 5, then he is $\frac{4}{5}$ as tall as his father.

Example 1

Change the ratio 2 : 5 into the form

(a) $1:n$ (b) $m:1$

(a) $2:5=1:\frac{5}{2}$
$=1:2{\cdot}5$

(b) $2:5=\frac{2}{5}:1$
$=0{\cdot}4:1$

Example 2

Divide 200 kg in the ratio 1 : 3 : 4.
The parts are $\frac{1}{8}$, $\frac{3}{8}$ and $\frac{4}{8}$ (of 200 kg). i.e. 25 kg, 75 kg and 100 kg.

Exercise

In questions **1** to **8** express the ratios in the form $1:n$.

1. 2 : 6 **2.** 5 : 30 **3.** 2 : 100 **4.** 5 : 8
5. 4 : 3 **6.** 8 : 3 **7.** 22 : 550 **8.** 45 : 360

In questions **9** to **12** express the ratios in the form $n:1$.

9. 12 : 5 **10.** 5 : 2 **11.** 4 : 5 **12.** 2 : 100

Mixtures

Example

An alloy consists of magnesium, zinc and copper in the ratio 5:3:4. If an alloy compound weighs 720 g, how much of this would be magnesium, zinc and copper?

Solution

1. Add the total number of parts:

5 + 3 + 4 = 12 parts

2. Find out how much one part is:

720 g ÷ 12 = 60 g

3. Multiply each ratio by 60:

60 g × 5 = 300 g of magnesium

60 g × 3 = 180 g of zinc

60 g × 4 = 240 g of copper

Example

To make pink paint I mix red and white paint in the ratio 4:5. If I use 600 ml of red paint, how many litres of paint will I make?

1. Red paint = 4 parts = 600 ml
2. 600 ml ÷ 4 = 150 ml = One part
3. Total parts = 4 + 5 = 9
4. 150 ml × 9 = 1,350 ml of paint
5. Convert to litres by ÷ 1,000: 1,350 ÷ 1,000 = 1.35 litres of paint made

Proportion

The majority of problems where proportion is involved are usually solved by finding the value of a unit quantity.

Example :

Eight men can dig a trench in 4 hours. How long will it take five men to dig the same size trench?

8 men take 4 hours
1 man would take 32 hours
5 men would take $\frac{32}{5}$ hours = 6 hours 24 minutes.

Exercise

1. Three men build a wall in 10 days. How long would it take five men?
2. Nine milk bottles contain $4\frac{1}{2}$ litres of milk between them. How much do five bottles hold?
3. A car uses 10 litres of petrol in 75 km. How far will it go on 8 litres?
4. A wire 11 cm long has a mass of 187 g. What is the mass of 7 cm of this wire?
5. 80 machines can produce 4800 identical pens in 5 hours. At this rate
   (a) how many pens would one machine produce in one hour?
   (b) how many pens would 25 machines produce in 7 hours?
6. Three men can build a wall in 10 hours. How many men would be needed to build the wall in $7\frac{1}{2}$ hours?
7. If it takes 6 men 4 days to dig a hole 3 feet deep, how long will it take 10 men to dig a hole 7 feet deep?
8. Find the cost of 1 km of pipe at 7 cents for every 40 cm.
9. A wheel turns through 90 revolutions per minute. How many degrees does it turn through in 1 second?

DIRECT PROPORTION

There are several ways of expressing a relationship between two quantities x and y. Here are some examples.

x varies as y
x varies directly as y
x is proportional to y

These three all mean the same and they are written in symbols as follows.

$x \propto y$

The '$\propto$' sign can always be replaced by '$= k$' where k is a constant:

$x = ky$

Suppose $x = 3$ when $y = 12$;
then $3 = k \times 12$
and $k = \frac{1}{4}$
We can then write $x = \frac{1}{4}y$, and this allows us to find the value of x for any value of y and *vice versa*.

Example 1

y varies as z, and $y = 2$ when $z = 5$; find
(a) the value of y when $z = 6$
(b) the value of z when $y = 5$

Because $y \propto z$, then $y = kz$ where k is a constant.
$y = 2$ when $z = 5$
$2 = k \times 5$
$k = \frac{2}{5}$
So $y = \frac{2}{5}z$

(a) When $z = 6$, $y = \frac{2}{5} \times 6 = 2\frac{2}{5}$
(b) When $y = 5$, $5 = \frac{2}{5}z$
$z = \frac{25}{2} = 12\frac{1}{2}$

Example 2

The value V of a diamond is proportional to the square of its weight W.
If a diamond weighing 10 grams is worth K200, find:
(a) the value of a diamond weighing 30 grams
(b) the weight of a diamond worth S5000.

$V \propto W^2$

or $V = kW^2$ where k is a constant.

$V = 200$ when $W = 10$

$\therefore \quad 200 = k \times 10^2$

$k = 2$

So $V = 2W^2$

(a) When $W = 30$,

$V = 2 \times 30^2 = 2 \times 900$

$V = £1800$

So a diamond of weight 30 grams is worth \$1800.

(b) When $V = 5000$,

$5000 = 2 \times W^2$

$$W^2 = \frac{5000}{2} = 2500$$

$W = \sqrt{2500} = 50$

EXERCISE

1. Rewrite the statement connecting each pair of variables using a constant k instead of '$\propto$'.

(a) $S \propto e$ (b) $v \propto t$ (c) $x \propto z^2$

(d) $y \propto \sqrt{x}$ (e) $T \propto \sqrt{L}$ (f) $C \propto r$

(g) $A \propto r^2$ (h) $V \propto r^3$

2. y varies as t. If $y = 6$ when $t = 4$, calculate:

(a) the value of y, when $t = 6$

(b) the value of t, when $y = 4$.

3. z is proportional to m. If $z = 20$ when $m = 4$, calculate:

(a) the value of z, when $m = 7$

(b) the value of m, when $z = 55$.

4. A varies directly as r^2. If $A = 12$, when $r = 2$, calculate:

(a) the value of A, when $r = 5$

(b) the value of r, when $A = 48$.

5. Given that $z \propto x$, copy and complete the table.

x	1	3		$5\frac{1}{2}$
z	4		16	

6. Given that $V \propto r^3$, copy and complete the table.

r	1	2		$1\frac{1}{2}$
V	4		256	

7. Given that $w \propto \sqrt{h}$, copy and complete the table.

h	4	9		$2\frac{1}{4}$
w	6		15	

8. s is proportional to $(v - 1)^2$. If $s = 8$, when $v = 3$, calculate:

(a) the value of s, when $v = 4$

(b) the value of v, when $s = 2$.

Inverse variation

There are several ways of expressing an inverse relationship between two variables,

x varies inversely as y
x is inversely proportional to y.

We write $x \propto \frac{1}{y}$ for both statements and proceed using the method outlined in the previous section.

Example

z is inversely proportional to t^2 and $z = 4$ when $t = 1$. Calculate:
(a) z when $t = 2$
(b) t when $z = 16$.

We have $\quad z \propto \frac{1}{t^2}$

or $\quad z = k \times \frac{1}{t^2}$ (k is a constant)

$z = 4$ when $t = 1$,

$\therefore \quad 4 = k\left(\frac{1}{1^2}\right)$

so $\quad k = 4$

$\therefore \quad z = 4 \times \frac{1}{t^2}$

(a) when $t = 2$, $z = 4 \times \frac{1}{2^2} = 1$

(b) when $z = 16$, $16 = 4 \times \frac{1}{t^2}$

$$16t^2 = 4$$
$$t^2 = \tfrac{1}{4}$$
$$t = \pm\tfrac{1}{2}$$

1. Rewrite the statements connecting the variables using a constant of variation, k.

(a) $x \propto \frac{1}{y}$ (b) $s \propto \frac{1}{t^2}$ (c) $t \propto \frac{1}{\sqrt{q}}$

(d) m varies inversely as w

(e) z is inversely proportional to t^2.

2. b varies inversely as e. If $b = 6$ when $e = 2$, calculate:
(a) the value of b when $e = 12$
(b) the value of e when $b = 3$.

3. q varies inversely as r. If $q = 5$ when $r = 2$, calculate:
(a) the value of q when $r = 4$
(b) the value of r when $q = 20$.

4. x is inversely proportional to y^2. If $x = 4$ when $y = 3$, calculate:
(a) the value of x when $y = 1$
(b) the value of y when $x = 2\frac{1}{4}$.

5. R varies inversely as v^2. If $R = 120$ when $v = 1$, calculate:
(a) the value of R when $v = 10$
(b) the value of v when $R = 30$.

6. T is inversely proportional to x^2. If $T = 36$ when $x = 2$, calculate:
(a) the value of T when $x = 3$
(b) the value of x when $T = 1{\cdot}44$.

7. p is inversely proportional to $\sqrt{y}$. If $p = 1{\cdot}2$ when $y = 100$, calculate:
(a) the value of p when $y = 4$
(b) the value of y when $p = 3$.

8. y varies inversely as z. If $y = \frac{1}{8}$ when $z = 4$, calculate:
(a) the value of y when $z = 1$
(b) the value of z when $y = 10$.

9. Given that $z \propto \frac{1}{y}$, copy and complete the table:

y	2	4		$\frac{1}{4}$
z	8		16	

10. Given that $v \propto \frac{1}{t^2}$, copy and complete the table:

t	2	5		10
v	25		$\frac{1}{4}$	

11. Given that $r \propto \frac{1}{\sqrt{x}}$, copy and complete the table:

x	1	4		
r	12		$\frac{3}{4}$	2

12. e varies inversely as $(y - 2)$. If $e = 12$ when $y = 4$, find

(a) e when $y = 6$ (b) y when $e = \frac{1}{2}$.

13. M is inversely proportional to the square of l.

If $M = 9$ when $l = 2$, find:

(a) M when $l = 10$ (b) l when $M = 1$.

14. Given $z = \frac{k}{x^n}$, find k and n, then copy and complete the table.

x	1	2	4	
z	100	$12\frac{1}{2}$		$\frac{1}{10}$

CHAPTER ELEVEN

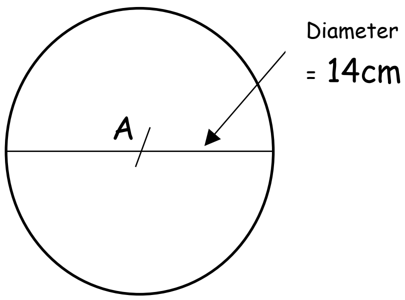
MENSURATION

The circle

The area of a circle = πr^2

Example

Find the area of the circle that follows



Solution

Area of the circle = πr^2

Radius = ½ of diameter = ½ $\times$ 14 = 7 cm

Therefore the area of the circle = $\frac{22}{7} \times 7 \times 7$= 154 cm^2

Circumference

Circumference (C) = $\pi \times D$, D is diameter

Example

Find the diameter of the circle if the circumference is 30 cm

Solution

C= $\pi \times D$

30 = $\frac{22}{7} \times D$

$\therefore$ D = (30$\times$ 7) $\div$ 22 = 9.55 (2 d.p)

Exercise

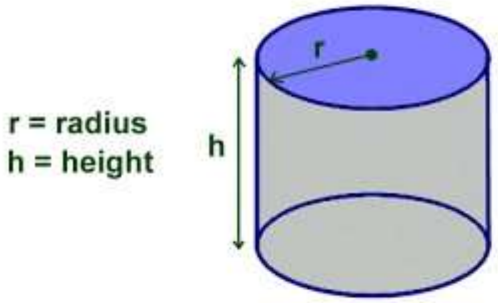
1. Find the area and diameter of the circle with the following diameters
   b. 15 cm
   c. 44 cm
   d. 18cm
   e. 14 cm

2. Find the radius and diameter of the circle given the following circumference

a. 60cm

b. 40 cm

Cylinder



Area of closed cylinder $= 2\pi r(r+h)$

Area of open cylinder (open one side) $= \pi r(r+h)$

Example

Find the area of the closed cylinder r =2.8 cm and h= 13 cm

Solution

$= 2\pi r(r+h)$

$2\times 3.14 \times 2.8(2.8 + 13)$

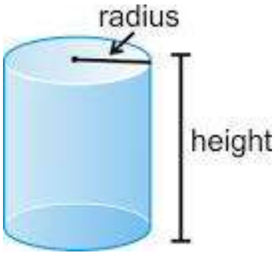
$17.584 \times 15.8 = 277.82872$

Note;

For open cylinder do not multiply by two, find the area of only one circle.

Cylinder

A prism with a circular cross-section is called a cylinder, see the figure below.



If you roll a piece of paper around the curved surface of a cylinder and open it out, you will get a rectangle whose breath is the circumference and length is the height of the cylinder. The ends are two circles. The surface area S of a cylinder with base and height h is therefore given by;

Example

Find the surface area of a cylinder whose radius is 7.7 cm and height 12 cm.

Solution

$2\pi r(r+h) = 2\pi r2 + 2\pi rh$

S =2 (7.7) x 12 + 2 (7.7) cm²

=15.4× 3.14 (19.7) cm²

=953.48 cm²

Volume of a cylinder

The volume of a cylinder is given by $\pi r^2 h$

Example

Find the volume of a cylinder with height 10cm and diameter 8 cm

Solution

Radius = ½ diameter = 4cm

= 3.14× 4 × 4 × 10 = 502.4 cm^2

The pyramid

Volume of a Pyramid The volume of a pyramid is found by:

$$V = \frac{1}{3}A \cdot h$$

where *A* is the area of the base and *h* is the height.

31.2

A cone is a pyramid, so the volume of a cone is given by

$$V = \frac{1}{3}\pi r^2 h.$$

A square pyramid has volume

$$V = \frac{1}{3}a^2 h$$

where *a* is the side length.

Worked Example

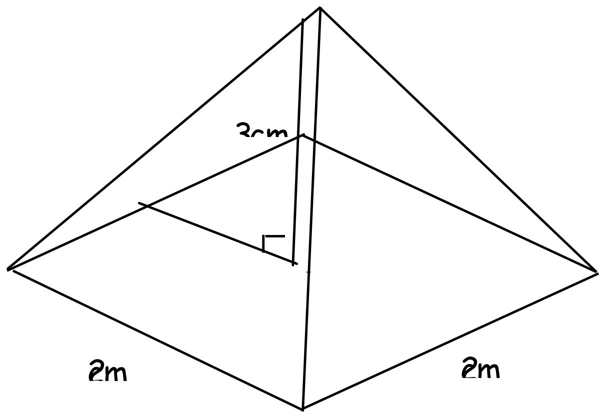
Question What is the volume of a square pyramid, 3cm high with a base Of 2cm

Step 1 : Determine the correct formula The volume of a pyramid is

$$V = \frac{1}{3}A \cdot h,$$

which for a square base means

$$V = \frac{1}{3}a \cdot a \cdot h.$$



Step 2 : Substitute the given values

$= 1/3 \times 2 \times 2 \times 3$

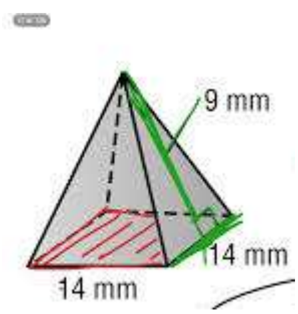
$= 4cm^3$

Surface area of a pyramid

The surface area of a pyramid is the sum of the area of the slanting faces and the area of the base.

Surface area = base area + area of the four triangular faces (take the slanting height marked green below)

Example ; find the surface area of the pyramid below



Solution

Surface area = base area + area of the four triangular faces

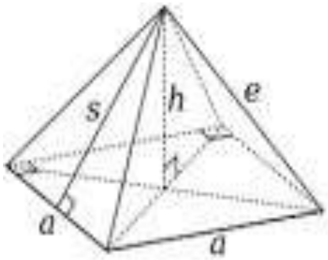
= (14 x 14) + (14 x 14)

= 196 + 252

= 448

Example

The figure below is a right pyramid with a square base of 4 cm and a slanting edge of 8 cm. Find the surface area of the pyramid.



a = 4 cm e = 8 cm

Surface area = base area + area of the four triangular bases

= (l x w) + 4 (½ base $\times$ $hieght$)

$4 \times 4 + 4\ (\ ½ \times 4 \times 8)$

=80

Exercise

1. Find the area of an open cylinder with height 15cm and radius 12 cm
2. Find the volume of a cylinder which has radius 14 cm and height 20cm
3. If the cylinder has volume 144cm^3 and diameter 16cm, find its height .
4. What is the volume of a square pyramid, 12cm high with a base 14 cm.
5. The right pyramid with a square base of 4 cm has a slanting edge of 8 cm. Find the surface area of the pyramid.

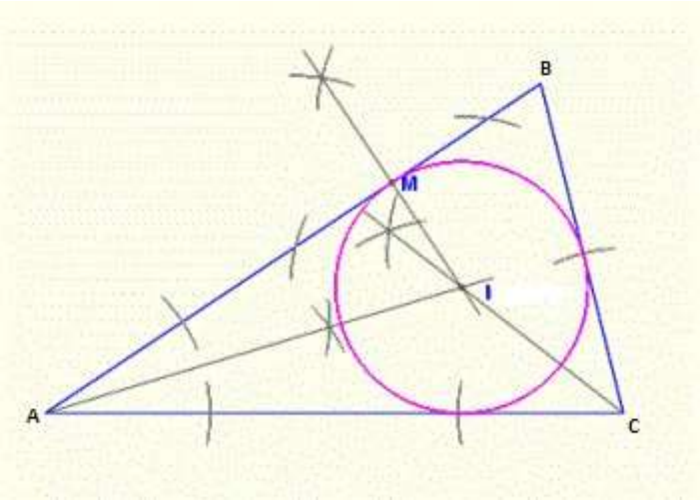
CHAPTER TWELVE

GEOMETRIC CONSTRUCTIONS

Circles and triangles

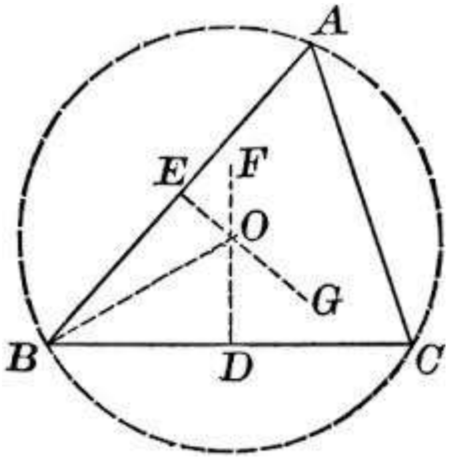
Inscribed circle

- Construct any triangle ABC.
- Construct the bisectors of the three angles
- The bisectors will meet at point I
- Construct a perpendicular from O to meet one of the sides at M
- With the centre I and radius IM draw a circle
- The circle will touch the three sides of the triangle ABC
- Such a circle is called an inscribed circle or in circle.
- The centre of an inscribed circle is called the incentre



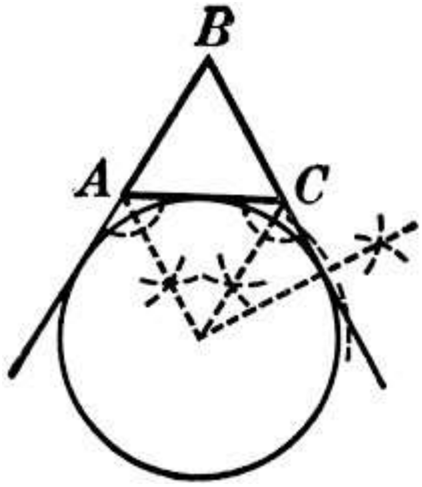
Circumscribed circle

- Construct any triangle ABC.
- Construct perpendicular bisectors of AB , BC, and AC to meet at point O.
- With O as the centre and using OB as radius, draw a circle
- The circle will pass through the vertices A , B and C as shown in the figure below



Escribed circle

- Construct any triangle ABC.
- Extend line BA and BC
- Construct the perpendicular bisectors of the two external angles produced
- Let the perpendicular bisectors meet at O
- With O as the centre draw the circle which will touch all the external sides of the triangle



Note;

Centre O is called the ex-centre

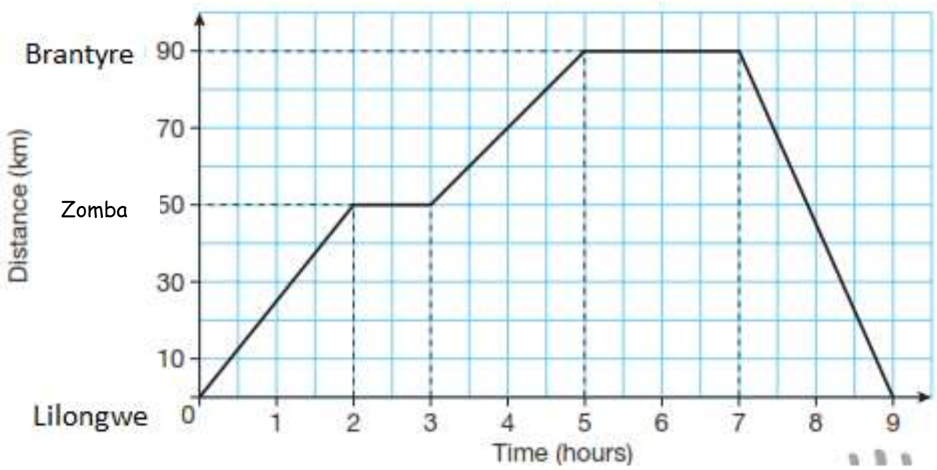
AO and CO are called external bisectors.

CHAPTER THIRTEEN

TRAVEL GRAPH

Travel graphs show motion. They make understanding how things move when compared to time much clearer by using diagrams.
A car goes from Lilongwe to Brantyre for a car show, and then returns to Lilongwe.
Here is a graph representing the distance in relation to the time of the journey.

Formula : Speed = $\frac{distance}{time\ taken}$



What is the speed of the car from Lilongwe to Zomba?
b The car breaks down at Zomba. For how long does the car break down?
c What is the speed of the car from Zomba to Brantyre?
d The car is transported by a recovery vehicle back to Lilongwe from Brantyre.
At what speed is the car transported?

a The speed from Lilongwe to Zomba is
50 km÷2 h
= 25 km/h.
b The car is at Zomba for 1 hour.
c The speed from Zomba to Brantyre is 40 km÷2 h= 20 km/h.

d The speed from Brantyre to Lilongwe is
90 km/2 h

= 45 km/h.

On a distance–time graph:

- The vertical axis represents the distance from the starting point.
- The horizontal axis represents the time taken.
- A horizontal line represents no movement.
- The gradient of the slope gives the speed (a straight line implies a constant speed).
- A positive gradient represents the outbound journey.
- A negative gradient represents the return journey.

Exercise 1

1. Ingar travels south on a motorway from Kasungu, while Franz travels north on the same road from Mzimba. This distance–time graph (showing the distance from Mzimba) shows the journeys of both travellers.

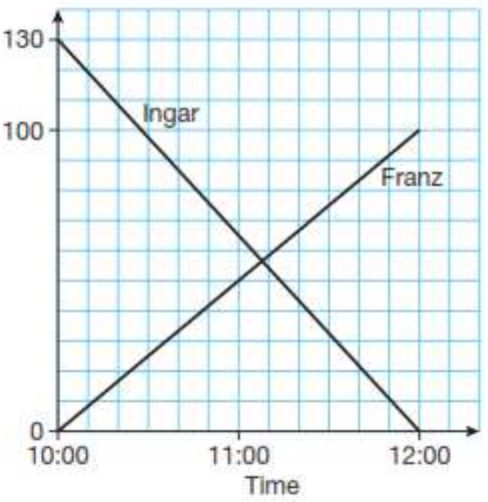
a What is Ingar's speed in kilometres per hour?

b What is Franz's speed in kilometres per hour?

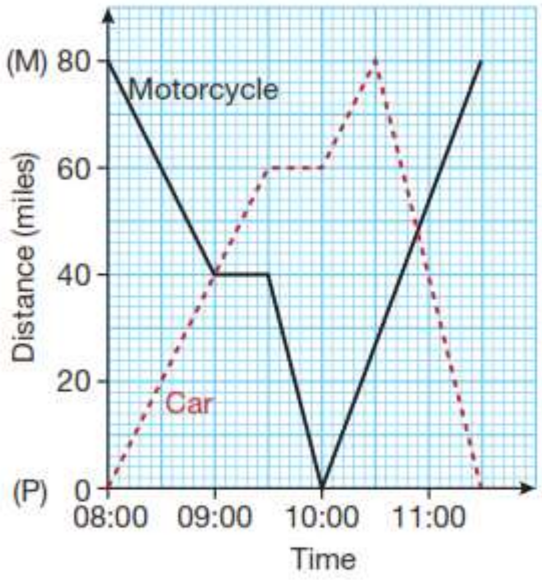
c At what time does Ingar reach Hannover?

d What is the distance between Ingar and Franz at 10:30?

e At what time do Ingar and Franz pass each other?



2. This distance–time graph shows the journeys of a car and a motorcycle between Mangochi (M) and Phalombe(P).



a When did the car stop, and for how long?
b When did the car and the motorcycle pass each other?
c What is the distance between the car and the motorcycle at 09:30?
d After the motorcycle's first stop, it increased its speed until it arrived in Phalombe. The speed limit on the road was 70 miles/hour
e Over the whole journey (excluding stops), what was the average speed of the car?

f What was the average speed of the motorcycle (excluding stops)?

Example

A van left Lilongwe for Brantyre at an average speed of 80 km/h. After half an hour, a car left Lilongwe for Brantyre at a speed of 100 km/h.

a.) Find the relative speed of the two vehicles.
b.) How far from Lilongwe did the car over take the van

Solution

Relative speed = difference between the speeds

= 100 – 80

= 20 km/h

Distance covered by the van in 30 minutes

Distance =

Time taken for car to overtake the van

Example

A truck left Mzimba at 7.00 am for Lilongwe at an average speed of 60 km/h. At 8.00 am a bus left Lilongwe for Mzimba at speed of 120 km/h .How far from Mzimba did the vehicles meet if Mzimba is 160 km from Lilongwe?

Solution

Distance covered by the lorry in 1 hour = 1 x 60

= 60 km

Distance between the two vehicle at 8.00 am = 160 – 100

= 100km

Relative speed = 60 km/h + 120 km/h

Time taken for the vehicle to meet =

=

Distance from Mzimba = 60 x   x 60

= 60 + 33.3

= 93.3 km

Exercise 2

1. Two Lorries A and B ferry goods between two towns which are 3120 km apart. Lorry A traveled at km/h faster than lorry B and B takes 4 hours more than lorry A to cover the distance.Calculate the speed of lorry B

2. A matatus left town A at 7 a.m. and travelled towards a town B at an average speed of 60 km/h. A second matatus left town B at 8 a.m. and travelled towards town A at 60 km/h. If the distance between the two towns is 400 km, find;

I.) The time at which the two matatus met

II.) The distance of the meeting point from town A

3. A bus started from rest and accelerated to a speed of 60km/h as it passed a billboard. A car moving in the same direction at a speed of 100km/h passed the billboard 45 minutes later. How far from the billboard did the car catch up with the bus?

4. Lilongwe and Mchinji are each 250km from Mzuzu. At 8.15am a lorry leaves Mzuzu for Lilongwe. At 9.30am a car leaves Mchinji for Lilongwe along the same route at 100km/h. Both vehicles arrive at Lilongwe at the same time.

(a) Calculate their time of arrival in Lilongwe

(b) Find the cars speed relative to that of the lorry.

(c) How far apart are the vehicles at 12.45pm.

5. A passenger notices that she had forgotten her bag in a bus 12 minutes after the bus had left. To catch up with the bus she immediately took a taxi which traveled at 95 km/hr. The bus maintained an average speed of 75 km/ hr. determine

(a) The distance covered by the bus in 12 minutes

(b) The distance covered by the taxi to catch up with the bus