

**CHAPTER 1****QUADRATIC EQUATIONS**

An equation is an expression used to find unknown e. g  $x - a = 6$

Quadratic equations are equations whose highest power (exponent or degree) is 2.

e. g  $2x^2 - 2x - 3 = 0$ ,  $ax^2 + bx + c$  are quadratic

**FACTORISING QUADRATIC EQUATION**

Factors are numbers which can be multiplied to give a product.

For example  $12 = 4 \times 3$ , in this case 4 and 3 are factors while 12 is a product

**Example 1; factorize  $x^2 + 7x + 6$**

**Solution**

In the expression, 6 is constant

We need **two** numbers whose **sum** is 7 and **product** is 6

The numbers we want are 1 and 6

So  $x^2 + 7x + 6 = x^2 + x + 6x + 6$

$$= x(x+1) + 6(x+1)$$

$$= (x+1)(x+6)$$

**Example 2, factorize  $3x^2 - 8x - 16$**

**Solution**

**$3x^2 - 8x - 16$**

$3 \times 16 = 48$ , so we need to find two factors that their summation should be **-8** and their product should be **48**.

Factors are -12 and 4,

$$3x^2 - 12x + 4x - 16$$

$$3x(x - 4) + 4(x - 4)$$

$$(3x + 4)(x - 4)$$

**Example 3, factorize  $6x^2 + x - 22$**

**Solution**

$$6x^2 + x - 22$$

$$6x^2 - 11x - 12x - 22$$

$$x(6x - 11) - 2(6x - 11)$$

$$(x - 2)(6x - 11)$$

**EXERCISE**

Factorize the following

1.  $x^2 + x - 12$
2.  $2x^2 - 3x + 1$
3.  $2x^2 + 17x + 30$
4.  $x^2 - xy + 6y^2$

**FACTOSING USING DIFFERENT OF TWO SQUARES**

A square is a product found when you multiply the number by itself once.

$$\text{e. g } 25 = 5 \times 5$$

$$x^2 = x \times x$$

A general formula for factorizing using different of two squares.

$$(A^2 - B^2) = (A + B)(A - B)$$

**EXAMPLES**

1. Factorize;  $a^2 - 4$

**Solution**

$$a^2 - 2^2$$

$$A = a, B = 2$$

$$(a + 2)(a - 2)$$

2.  $4a^2 - 9$

**Solution**

$$(2a)^2 - 3^2$$

$$A = 2a, B = 3$$

$$4a^2 - 9 = (2a+3)(2a-3)$$

$$3. \quad 2x^2 - 18y$$

**Solution**

$$2x^2 - 18y^2 = 2x^2 - 18y^2$$

$$= 2(x^2 - (3y)^2)$$

$$A = x, B = 3y$$

$$2x^2 - 18y^2 = 2(x+3y)(x-3y)$$

**NB: Different of two cubes and sum of two cubes are applied sometimes. The general formula;**

$$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$$

$$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$$

### **EXAMPLE**

$$1. \quad \text{Factorize; } r^3 - 8 = r^3 - 2^3$$

$$A = r, B = 2$$

$$r^3 - 8 = (r-2)(r^2 + 2r + 2^2)$$

$$= (r-2)(r^2 + 2r + 4)$$

$$2. \quad \text{Factorize; } 1 - 27y^3 = 1^3 - (3y)^3$$

$$A = 1, B = 3y$$

$$1 - 27y^3 = (1-3y)(1^2 + 3y + (3y)^2)$$

$$= (1-3y)(1 + 3y + 9y^2)$$

### **EXERCISE**

$$1. \quad 16 - y^2$$

$$2. \quad 49x^2 - 16$$

$$3. \quad (x^2 - x - 8)^2 - 4(2x-1)^2$$

$$4. \quad x^3 + 1$$

### **SOLVING QUADRATIC EQUATIONS BY FACTORIZATION**

To solve quadratic equations, we need to factorize first then equate each factor to zero.

#### **EXAMPLES**

$$1. \quad \text{Solve } x^2 + 3x = 0$$

**Solution**

$$x^2 + 3x = 0$$

$$x(x+3) = 0$$

$$x=0 \text{ or } x+3=0$$

$$x = -3$$

$$2. \quad \text{Solve } x^2 - 5x + 6 = 0$$

**Solution**

$$x^2 - 5x + 6 = 0$$

$$x^2 - 2x - 3x + 6 = 0$$

$$x(x-2) - 3(x-2) = 0$$

$$(x-2)(x-3) = 0$$

$$(x-2)=0 \text{ or } (x-3)=0$$

$$x = 2 \quad x = 3$$

$$3. \quad \text{Solve } 3x^2 - 2x - 8 = 0$$

**Solution**

$$3x^2 - 2x - 8 = 0$$

$$3x^2 - 6x + 4x - 8 = 0$$

$$3x(x-2) + 4(x-2) = 0$$

$$(x-2)(3x+4) = 0$$

$$x-2=0 \text{ or } 3x+4=0$$

$$x = 2 \quad 3x = -4$$

$$x = \frac{-4}{3}$$

$$4. \quad \text{Solve } x^2 - 4 = 0$$

**Solution**

$$x^2 - 4 = 0 = x^2 - 2^2$$

$$= (x+2)(x-2) = 0$$

$$= x+2=0 \text{ or } x-2=0$$

$$x = -2 \quad x = 2$$

$$5. \quad \text{Solve } 2r^2 - 18 = 0$$

**Solution**

$$2r^2 - 18 = 0$$

Divide by 2

$$r^2 - 9 = 0$$

$$r^2 - 3^2 = 0$$

$$(r-3)(r+3) = 0$$

$$r = 3 \text{ or } r = -3$$

**EXERCISE**

Solve the following equations

1.  $x^2 - 3x - 10 = 0$
2.  $a^2 + 5a - 15 = 0$
3.  $x(x + 4) = 0$
4.  $4(x - 1)^2 = 9$

**COMPLETING THE SQUARE**

A square is a product we get after multiplying a number by itself once e.g

$$3 \times 3 = 9 \text{ or } 6 \times 6 = 36$$

Completing the square refers to changing a quadratic equation to the form  $a(x + b)^2 + c$ .

When completing the square, multiply the coefficient of  $x$  by half, then add and subtract the result while being squared.

Brackets are introduced to prevent subtracting numbers.

**EXAMPLE**

Factorize the following by completing the square;

1.  $x^2 - 4x + 12$
2.  $3x^2 + 6x - 5$

**Solutions****1.  $x^2 - 4x + 12$** 

Divide -4 by half which is -2

$$\{x^2 - 4x + (-2)^2\} - (-2)^2 + 12$$

$$(x - 2)^2 - 4 + 12$$

$$(x - 2)^2 + 8$$

**2.  $3x^2 + 6x - 5$** 

$3\left(x^2 + 2x - \frac{5}{3}\right)$  takeout coefficient  $x^2$  3)

$$3\{(x^2 + 2x + (1)^2) - (1)^2 - \frac{5}{3}\}$$

$$3(x + 1)^2 - 1 - \frac{5}{3}$$

$$3(x + 1)^2 - \frac{3-5}{3}$$

$$3\{(x + 1)^2 - \frac{8}{3}\}$$

$$3(x + 1)^2 - \frac{8}{3} \times 3$$

(To remove brackets multiply it with 3)

$$3(x + 1)^2 - 8$$

**SOLVING BY COMPLETING THE SQUARE.**

Solving by completing the square we factorize by completing the square first.

**EXAMPLE**

Solve the following by completing the square

1.  $x^2 + 4x - 5 = 0$
2.  $x^2 - 7x + 2 = 0$

**Solutions****1.  $x^2 + 4x - 5 = 0$** 

$$\{(x^2 + 4x + (2)^2) - (2)^2 - 5 = 0$$

$$(x + 2)^2 - 4 - 5 = 0$$

$$(x + 2)^2 - 9 = 0$$

$$(x + 2)^2 = 9$$

$$x + 2 = \pm\sqrt{9}$$

$$x = -2 + 3 \text{ or } x = -2 - 3$$

$$x = 1 \quad x = -5$$

**2.  $x^2 - 7x + 2 = 0$** 

$$x^2 - 7x + \left(\frac{-7}{2}\right)^2 - \left(\frac{-7}{2}\right)^2 + 2 = 0$$

$$\left(x - \frac{7}{2}\right)^2 - \frac{49}{4} + 2 = 0$$

$$\left(x - \frac{7}{2}\right)^2 - \frac{41}{4} = 0$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{41}{4}$$

$$x - \frac{7}{2} = \pm\sqrt{\frac{41}{4}}$$

$$x = \frac{7}{2} + \frac{\sqrt{41}}{2} \quad \text{or} \quad x = \frac{7}{2} - \frac{\sqrt{41}}{2}$$

$$= \frac{7 + \sqrt{41}}{2}$$

$$= 6.70$$

$$\frac{7 - \sqrt{41}}{2}$$

$$0.30$$

**EXERCISE**

- Factorize the following by completing the square
  - $2x^2 + 8x + 3$
  - $3x^2 + 5x + 11$
  - $2y^2 - 7x + 4$
- The following by completing the square
  - $x^2 + 6x - 1 = 0$
  - $4x^2 + 5x - 3 = 0$
  - $3a^2 - 7a + 7 = 0$
  - $x^2 + 36x - 36 = 0$

**SOLVING QUADRATIC EQUATIONS USING FORMULA**

The general formula for solving quadratic equations is  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**EXAMPLE**

- $2x^2 + 3x - 5 = 0$ , solve using the formula leaving your answer to 2 decimal place.

**Solution**

$$2x^2 + 3x - 6 = 0$$

$$a=2 \quad b=3 \quad c=-6$$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-6)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{9+48}}{4}$$

$$= \frac{-3 \pm \sqrt{57}}{4}$$

$$x = \frac{-3+7.55}{4} \quad \text{or} \quad \frac{-3-7.55}{4}$$

$$= \frac{4.45}{4} \quad = \frac{-10.55}{4}$$

$$= 1.14 \quad = -2.64$$

- Solve  $3y^2 - 4y - 8 = 0$  using the formula, leaving the answer to 2 decimal place.

**Solution**

$$3y^2 - 4y - 8 = 0$$

$$A=3 \quad b=-4 \quad c=-8$$

$$y = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-8)}}{2(3)}$$

$$y = \frac{4 \pm \sqrt{16+96}}{6}$$

$$y = \frac{4 \pm \sqrt{112}}{6}$$

$$y = \frac{4+10.56}{6} \quad \text{or} \quad \frac{4-10.58}{6}$$

$$= \frac{14.56}{6} \quad = \frac{-6.58}{6}$$

$$y = 2.43 \quad \text{or} \quad y = -1.10$$

**EXERCISE**

Solve the following using the formula leaving your answer to 2 Dec. place

- $x^2 + 7x - 3 = 0$
- $2x^2 - 7x - 3 = 0$
- $5x^2 + 8x - 2 = 0$
- $3x^2 - 12x + 10 = 0$

**Forming equations given its roots****EXAMPLES**

- The roots of quadratic equation are 4 and -3, formulate appropriate equation

**Solution**

The roots are 4 and -3

$$x = 4 \quad \text{or} \quad x = -3$$

$$x - 4 = 0 \quad \text{or} \quad x + 3 = 0$$

$$(x - 4)(x + 3) = 0$$

$$x^2 - x - 12 = 0$$

3. 5 is one root of the equation  $x^2 + mx - 15$ . find  $m$  and the other root

**Solution**

$$x^2 + mx - 15 = 0$$

$$(5)^2 + m(5) - 15 = 0$$

$$25 + 5m - 15 = 0$$

$$\frac{10}{-5} = -\frac{5m}{5}$$

$$m = -2$$

So the equation is  $x^2 - 2x - 15 = 0$

$$x^2 - 5x + 3x - 15 = 0$$

$$x(x-5) + 3(x-5) = 0$$

$$(x-5)(x+3) = 0$$

$$x = 5 \text{ or } x = -3$$

The second root is -3

**EXERCISE**

- Find the simplest version of the Equation whose roots are  $\frac{5}{7}$  and -1
- If 3 is one root of equation  $x^2 - mx + 16 = 0$ . Find  $m$  and the other root.
- Find the value of  $c$  if one root of  $3x^2 - 8x + c = 0$  is three times the other.
- The equation  $2x^2 - x + m = 0$  has one root of  $-\frac{3}{2}$ . Find  $m$  and the other root.

**WORD PROBLEMS**

Translating the English into mathematical equation word problems is often quite challenging.

Here is the list of some of the phrases and their mathematical translations.

| ENGLISH                                      | MATHEMATICS                        |
|--|------------------------------------|
| Sum of                                       | +                                  |
| The difference between                       | -                                  |
| The product of                               | $\times$                           |
| The quotient of                              | $\div$                             |
| The number increased by 3                    | $X + 3$                            |
| The number decreased by 3                    | $X - 3$                            |
| Reciprocal of a number                       | $\frac{1}{x}$                      |
| An even number                               | $2x$                               |
| Odd number                                   | $2x + 1$                           |
| Two consecutive even or odd numbers          | $X$ and $x + 2$                    |
| Two consecutive numbers (They differ by one) | $X$ and $x + 1$<br>$X$ and $x - 1$ |

NB:

- Even numbers** are numbers that can be exactly divided by two e.g 2, 4, 6 e.t.c
- Odd numbers** are numbers that cannot be divided exactly by 2 e.g 1, 3, 5, 7, 9 e.t.c
- Prime numbers** are numbers that can be divided only by itself and 1 e.g 2, 3, 5, 7, 11 e.t.c

**EXAMPLES**

- Bertha's father** is six times as old as Bertha's age. The product of their ages is 150 years. What are their respective ages.

**Solution**

Let **Bertha's** age be  $x$

$\therefore$  **Her father's** age is  $6x$

$$x \times 6x = 150$$

$$6x^2 = 150$$

$$x^2 = \frac{150}{6}$$

$$x = 25$$

$$x = \pm\sqrt{25}$$

$$x = 5 \quad \text{or} \quad -5$$

$\therefore$  **Bertha's** age is 5 and his father's age is  $6 \times 5 = 30$

2. The product of two consecutive even numbers is 168. Find the numbers

### Solution

Let the smaller number be  $x$

$\therefore$  The other number is  $x + 2$

$$x(x + 2) = 168$$

$$x^2 + 2x = 168$$

$$x^2 + 2x - 168 = 0$$

$$x^2 + 14x - 12x - 168 = 0$$

$$x(x + 14) - 12(x + 14) = 0$$

$$(x - 12)(x + 14) = 0$$

$$x - 12 = 0 \quad \text{or} \quad x + 14 = 0$$

$$x = 12 \quad \quad \quad x = -14$$

$\therefore$  The numbers are 12 and 14 or -12 and -14.

3. Eugenio's taxi is three years older than Geoffrey's taxi. In two years' time, the product of their ages of two taxis will be 54. How old is each taxi at present.

### Solution

Let Geoffrey's taxi age be  $y$

$\therefore$  Eugenio's age taxi will be  $y + 3$

In two years' time each taxi age will increase by 2

Geoffrey's tax age in two years' time =  $(y+2)$  yrs

Eugenio's taxi age in two years =  $(y+5)$

$$(y+2)(y+5) = 54$$

$$y^2 + 7y + 10 = 54$$

$$y^2 + 7y - 44 = 0$$

$$y^2 + 11y - 4y - 44 = 0$$

$$y(y + 11) - 4(y + 11) = 0$$

$$(y - 4)(y + 11) = 0$$

$$y = 4 \quad \text{or} \quad y = -11$$

$\therefore$  Geoffrey's taxi is 4 years old and Eugenio's taxi is  $4 + 3 = 7$  years old.

### **EXERCISE**

1. Lawrence is four times older than his brother Geoffrey. If the product of their ages is 96. What are their respective ages.
2. If  $2n$  ( $n + 1$ ) and ( $n - 1$ ) ( $3n - 1$ ) are consecutive numbers. What numbers are they?
3. Find the consecutive old numbers such that the square of the smaller one added to the twice the square of greater is 211.
4. The sum of the square of two consecutive positive old numbers is 650. Find the numbers.

### SOLVING SIMULTANEOUS EQUATION

Example

1. Solve  $x^2 - y^2 = 16$   
 $x + y = 8$

### Solution

$$x^2 - y^2 = 16 \dots\dots\dots \text{i)}$$

$$x + y = 8 \dots\dots\dots \text{ii)}$$

$$x = 8 - y \dots\dots\dots \text{iii)}$$

Substitute  $x = 8 - y$  in i)

$$(8 - y)^2 - y^2 = 16$$

$$64 - 16y + y^2 - y^2 = 16$$

$$64 - 16 = 16y$$

$$48 = 16y$$

$$y = 3$$

Substitute  $y = 3$  in iii)

$$x = 8 - 3$$

$$= 5$$

$$\begin{aligned} 2. \text{ solve } x + y &= 2 \\ x^2 - y^2 &= 52 \end{aligned}$$

### Solution

$$x + y = 2 \dots\dots\dots \text{i)}$$

$$x^2 - y^2 = 52 \dots\dots\dots \text{ii)}$$

$$x = 2 - y \dots\dots\dots \text{iii)}$$

Substitute  $x = 2 - y$  in ii)

$$(2 - y)^2 - y^2 = 52$$

$$4 - 4y + y^2 - y^2 = 52$$

$$-4y = 48$$

$$y = -12$$

Substitute  $y = -12$  in iii)

$$x = 2 - (-12)$$

$$= 2 + 12$$

$$= 14$$

### **EXERCISE**

1. Solve  $a - b = 5$ ,  $ab = 6$
2. Solve  $3x + 4y = 5$   
 $x^2 + xy = y + 1$
3. Solve  $y = 2(x + y - 1) = -2(x + y + 3)$
4. Solve  $r + s = 2$   
 $r^2 - s^2 - rs - s = 0$

## **CHAPTER 2**

### **SURDS**

A surd is any expression that involve a root.

#### MULTIPLICATION OF SURDS

When two or more surds are to be multiplied, they should be simplified first if possible, then whole numbers should be taken with whole numbers and surds with surds.

#### EXAMPLE

1.  $\sqrt{5} \times \sqrt{10}$
2.  $\sqrt{12} \times \sqrt{3}$
3.  $(4\sqrt{3})^2$

$$\text{NB; } \sqrt{a} \times \sqrt{b} = \sqrt{ab}, \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\sqrt{a} + \sqrt{b} \pm \sqrt{ab}$$

#### Solutions

1.  $\sqrt{5} \times \sqrt{10}$   
 $\sqrt{5} \times \sqrt{5} \times \sqrt{2}$   
 $\sqrt{25} \times \sqrt{2}$   
 $5\sqrt{2}$
2.  $\sqrt{12} \times \sqrt{3}$   
 $\sqrt{4} \times \sqrt{3} \times \sqrt{3}$   
 $2 \times \sqrt{9}$   
 $2 \times 3$   
 $6$
3.  $(4\sqrt{3})^2$

$$4^2 \times (\sqrt{3})^2$$

$$16 \times \sqrt{9}$$

$$16 \times 3$$

$$48$$

**DIVISION OF SURDS**

If a fraction has a surd in the denominator, it is usually best to rationalize the denominator. To rationalize the denominator means to make the denominator into a rational number. This is done by multiplying the numerator and the denominator of the fraction by a surd of the denominator.

**EXAMPLES**

1. Simplify  $\frac{\sqrt{18}}{\sqrt{2}}$

**Solution**

$$= \frac{\sqrt{18}}{\sqrt{2}} = \frac{\sqrt{18}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{36}}{4}$$

$$= \frac{6}{2}$$

$$= 3$$

2. Simplify  $\frac{1}{\sqrt{2}}$

**Solution**

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{2}}{2}$$

3. Simplify  $\frac{2\sqrt{3}}{\sqrt{6}}$

**Solution**

$$= \frac{2\sqrt{3}}{\sqrt{6}} = \frac{2\sqrt{3}}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{2\sqrt{18}}{\sqrt{6}}$$

$$= \frac{2\sqrt{2} \times \sqrt{9}}{\sqrt{36}}$$

$$= \frac{2\sqrt{2} \times 3}{6}$$

$$= \sqrt{2}$$

**ADDITION AND SUBTRACTION OF SURDS**

Usually we add or subtract like surds and only whole numbers

E. g  $2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$

$$2\sqrt{2} - 3\sqrt{2} = -\sqrt{2}$$

**EXAMPLES**

1. Simplify  $\sqrt{12} + \sqrt{3}$

**Solution**

$$= \sqrt{4} \times \sqrt{3} + \sqrt{3}$$

$$= 2\sqrt{3} + \sqrt{3}$$

$$= 3\sqrt{3}$$

2. Simplify  $2\sqrt{150} - \sqrt{96} - \sqrt{216}$

**Solution**

$$= 2\sqrt{25} \times \sqrt{6} - \sqrt{16} - \sqrt{16} \times \sqrt{6} - \sqrt{36} \times \sqrt{6}$$

$$= 2 \times 5\sqrt{6} - 4\sqrt{6} - 6\sqrt{6}$$

$$= 10\sqrt{6} - 10\sqrt{6}$$

$$= 0$$

3. Simplify  $\frac{\sqrt{3}}{1} + \frac{3}{\sqrt{3}} - \frac{1}{\sqrt{27}}$

**Solution**

$$= \frac{\sqrt{3}}{1} + \frac{3}{\sqrt{3}} - \frac{1}{\sqrt{27}}$$

$$= \frac{\sqrt{3} \times \sqrt{27} + \sqrt{9} - 1}{\sqrt{27}}$$

$$= \frac{\sqrt{81} + 3 - 1}{\sqrt{27}}$$

$$= \frac{9 + 2}{\sqrt{27}}$$

$$= \frac{11}{\sqrt{9} \times \sqrt{3}}$$



$$\begin{aligned}
 &= \frac{11}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
 &= \frac{11\sqrt{3}}{3 \times \sqrt{9}} \\
 &= \frac{11\sqrt{3}}{9}
 \end{aligned}$$

**CONJUGATE SURDS**

A pair of expressions in a surd form are said to be conjugate surds. E . g

$2\sqrt{2} - 3\sqrt{2}$ ,  $3 + \sqrt{2}$  and  $3 - \sqrt{2}$  are examples of conjugate surds.

**EXAMPLE**

1. Expand  $(\sqrt{2} + \sqrt{5})(\sqrt{7} + \sqrt{2})$

**Solution**

$$\begin{aligned}
 &= \sqrt{14} + \sqrt{4} + \sqrt{35} + \sqrt{10} \\
 &= \sqrt{14} + 2 + \sqrt{35} + \sqrt{10} \\
 &= \sqrt{14} + \sqrt{35} + \sqrt{10} + 2
 \end{aligned}$$

2. Expand  $(\sqrt{6} + \sqrt{3})(\sqrt{6} + \sqrt{3})$

**Solution**

$$\begin{aligned}
 &= \sqrt{36} + \sqrt{18} + \sqrt{18} + \sqrt{9} \\
 &= 6 + 2\sqrt{18} + 3 \\
 &= 9 + 2 \times \sqrt{9} \times \sqrt{2} \\
 &= 9 + 2 \times 3\sqrt{2} \\
 &= 9 + 6\sqrt{2} \\
 &= 3(3 + 2\sqrt{2})
 \end{aligned}$$

**RATIONALIZING CONJUGATE SURDS**

When the denominator is a conjugate surd, multiply both the numerator and the denominator with the conjugate surds.

**EXAMPLE**

1. Rationalize  $\frac{1}{4+2\sqrt{7}}$

**Solution**

$$\begin{aligned}
 &= \frac{1}{4+2\sqrt{7}} \times \frac{4-2\sqrt{7}}{4-2\sqrt{7}} \\
 &= \frac{4-2\sqrt{7}}{16-4 \times 7} \\
 &= \frac{4-2\sqrt{7}}{16-28} \\
 &= \frac{2(2-\sqrt{7})}{-12} \\
 &= \frac{(2-\sqrt{7})}{-6} \\
 &= \frac{(\sqrt{7}-2)}{6}
 \end{aligned}$$

**EXERCISE**

1. Simplify

- $(\sqrt{2})^2$
- $\sqrt{5} \times \sqrt{15}$
- $2\sqrt{54} - \sqrt{24} - \sqrt{216}$
- $(2\sqrt{3} - 4\sqrt{2})(\sqrt{3} + 2\sqrt{2})$
- $\sqrt{75} + \sqrt{48} - 2\sqrt{27}$
- $\frac{11+\sqrt{5}}{3-\sqrt{2}}$
- $\frac{2+\sqrt{2}}{(2-\sqrt{2})}$
- $\frac{\sqrt{2}+2\sqrt{5}}{(\sqrt{5}-\sqrt{2})}$

**CHAPTER 3****CIRCLE GEOMETRY****PARTS OF A CIRCLE****1. Circumference**

It is the distance around the circle

**2. Diameter**

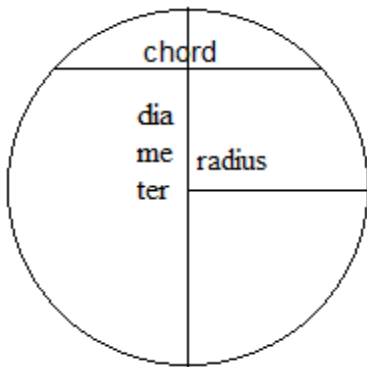
It is a line that divide a circle into two equal parts.

**3. Radius**

It is the half of the diameter

**4. Chord**

Is a line which joins two parts of the circle.

**5. Arc**

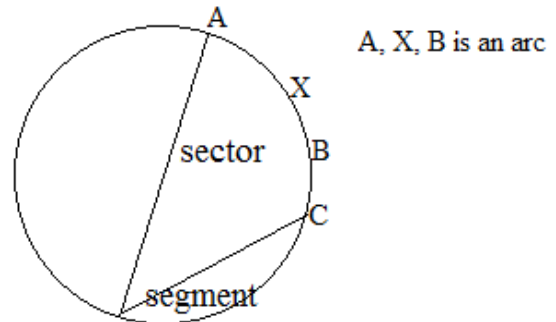
Part of circumference or fractional of the circumference.

**6. Segment**

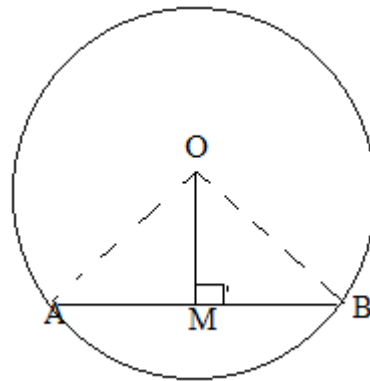
Is an area between an arc and two radii.

**7. A sector**

Is area between an arc and two radii

**CIRCLE PROPERTIES (PART ONE)****Theorems;**

1. If a line is drawn from the center of the circle to the mid-point of the chord, it bisects the chord.  
(Reason, perpendicular bisector)



**Given;** circle Centre O, chord  $AB \perp OM$

**To prove;**  $AM = BM$

**Construction:** join A and B to O respectively

**Proof;** In  $\Delta s$  AMO and BMO

$AO = BO$ , radii

MO is common

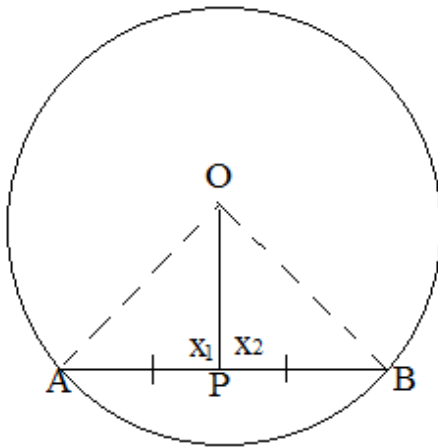
$\angle AMO = \angle BMO$ , given right  $\angle s$

$\therefore \Delta AMO \equiv \Delta BMO$ , RHS

$\therefore AM = BM$

**Theorem 2 converse**

If a line is drawn from the circle to the midpoint of the chord, then that line is perpendicular to the chord

**PROVE**

**Given;** circle Centre O, AP=AB, mid-point P.

**To prove;**  $OP \perp AB$

**Construction** = join O to A and B

**Proof;** in  $\triangle OPA$  and  $\triangle OPB$

AO= BO, radii

OP is common

AP=BP, given

$\therefore \triangle OPA \equiv \triangle OPB$

$\therefore x_1 = x_2$

$x_1 + x_2 = 180^\circ$ , adj  $\angle$ s on a str. line

$2x = 180^\circ$

$x = 90^\circ$

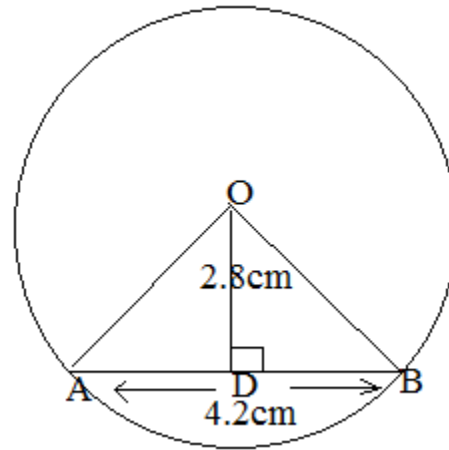
$\therefore x_1 = x_2 = 90^\circ$

$\therefore OP \perp AB$

**EXAMPLE**

1. A chord 4.2cm long is 2.8cm from the Centre of the circle. Calculate radius of the circle.

**Solution**



$AD = BD$ ,  $OD \perp$  bisector

$$\therefore BD = \frac{4.2}{2}$$

$$= 2.1\text{cm}$$

$BO^2 = OD^2 + DB^2$ , Pythagoras theorem

$$BO^2 = 2.8^2 + 2.1^2$$

$$BO^2 = 7.84 + 4.41$$

$$BO = \sqrt{12.25}$$

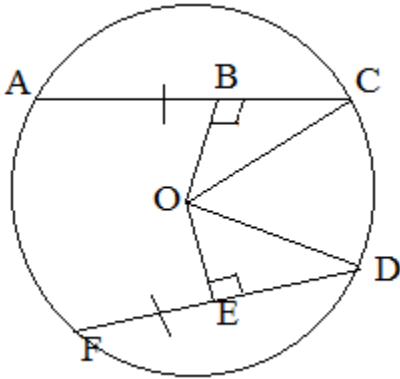
$$= 3.5\text{cm}$$

$\therefore$  The radius is 3.5cm

**Theorem 3**

Equal chords are equidistant from the Centre of the circle.

**PROVE**



**Given;** Centre O, chords  $AC=FD$ ,  $OB$  and  $OE$  are  $\perp$ s

**To prove:**  $OB=OE$

**Construction;** join  $OC$  and  $OD$

**Proof;**  $AC=FD$ , given

$$AB=BC, OB \perp \text{bisector}$$

$$FE=DE, OE \perp \text{bisector}$$

$$\therefore BC=ED$$

In  $\triangle OBC$  and  $\triangle OED$

$$OC=OD, \text{ given}$$

$$BC=ED, \text{ proved}$$

$$\hat{B} = \hat{E}, \text{ given right angles}$$

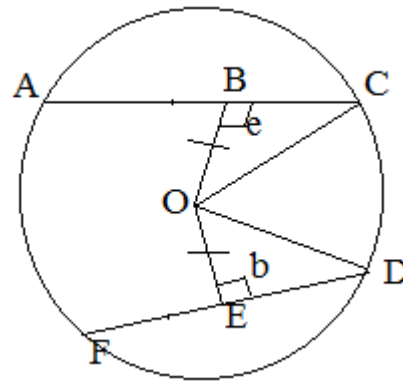
$$\therefore \triangle OBC \equiv \triangle OED, (\text{RHS})$$

$$\therefore OB=OE$$

**Theorem 4**

Chords which are equidistant from the Centre of the circle are equal to each other.

**PROVE**



**Given;** circle Centre O, equal chords,  $\perp$ s  $OB$  and  $OE$ , chords  $AC$  and  $FD$ .

**To prove;**  $AC=FD$

**Construction;** join  $OC$  and  $OD$

**Proof:**  $AB=BC$ ,  $OB \perp$  bisector

$$FE=DE, OE \perp \text{bisector}$$

In  $\triangle OBC$  and  $\triangle OED$

$$OB=OE, \text{ given}$$

$$OC=OD, \text{ radii}$$

$$e=d, \text{ given right angles}$$

$$\therefore \triangle OBC \equiv \triangle OED, (\text{RHS})$$

$$\therefore BC=ED$$

$$\therefore AB=BC=FE=DE$$

$$AB+BC=AC$$

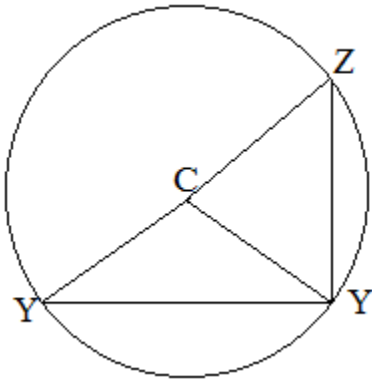
$$FE+DE=FD$$

$$\therefore AC=FD$$

$$AC=FD$$

**EXAMPLE**

In a circle Centre C, if the chords XY, YZ are equal. Prove that XY bisects CY and  $\widehat{XYZ}$ .

**Solution**

**Given;** Centre C,  $XY=YZ$

**To prove;**  $\widehat{XYC} = \widehat{ZYC}$  or CY bisects  $\widehat{XYZ}$

**Construction:** join C to Z and X

**Proof:** in  $\triangle CXY$  and  $\triangle CZY$

$$XY=ZY, \text{ given}$$

$$CX= CZ, \text{ radii}$$

CY is common

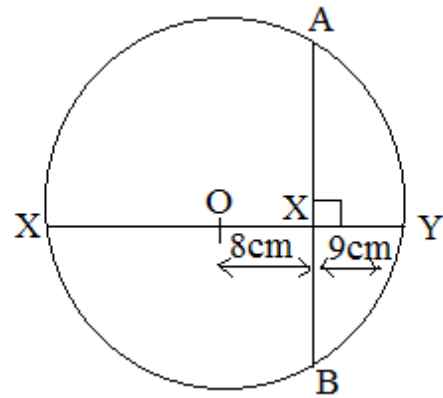
$$\therefore \triangle CXY \equiv \triangle CZY \text{ (SSS)}$$

$$\therefore \widehat{XYC} = \widehat{ZYC}$$

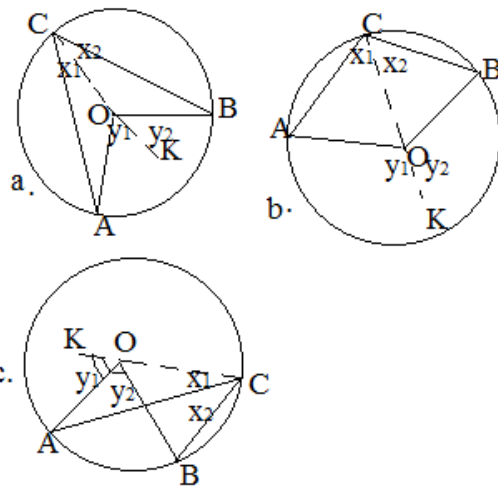
$$\therefore \text{CY bisects } \widehat{XYZ}$$

**EXERCISE**

1. A chord 7cm long is drawn in a circle of radius 3.7cm long. Calculate distance of a chord from the Centre of the circle.
2. The figure below shows a circle AYBX and Centre O. XY cuts AB at C such that angle  $\widehat{ACY}=90^\circ$

**PART TWO****ANGLE PROPERTIES****Theorem 5**

Angle at the center is twice angle formed at the circumference.



**Given;** center O, arc AB forming  $\widehat{AOB}$  at the Centre,  $\widehat{ACB}$  at the circumference

**To prove:**  $\widehat{AOB} = \widehat{ACB}$

**Construction:** draw CO and produce it to K

**Proof:** in figures above

$$y_1 = x_1 + \widehat{A}, \text{ ext. } \angle \text{ of } \triangle = \text{sum of int. opp } < \text{ s in a } \triangle$$

$$\text{But } x = \widehat{A},$$

$$\therefore y_1 = x_1 + x_2$$

$$= 2x_2$$

In the figures **a** and **b**

$$\therefore y_1 + y_2 = 2x_1 + 2x_2$$

$$y_1 + y_2 = 2(x_1 + x_2)$$

$$\therefore \widehat{AOB} = 2\widehat{ACB}$$

In figure **c**

$$Y_2 - y_1 = 2(x_2 - x_1) = 2(x_2 - x_1)$$

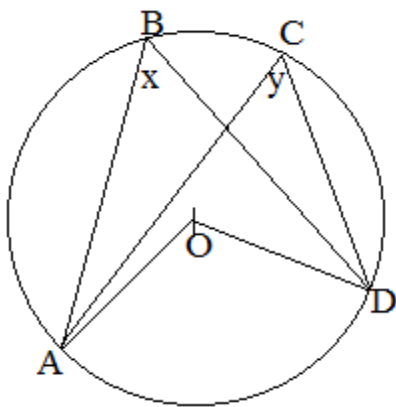
$$\therefore \widehat{AOB} = 2\widehat{ACB}$$

### Theorem 6

Angles in the same segment of a circle are equal.

Reason,  $\angle$  s in same seg.

**PROVE**



**Given;** circle ABCD,  $\widehat{ABD}$  and  $\widehat{ACD}$

**To prove;**  $\widehat{ABD} = \widehat{ACD}$

**Construction;** join O to A and B respectively

**Proof;**  $\widehat{AOD} = 2x$ ,  $\angle$  at the Centre =  $2 \times \angle$  @ Circumference

$\widehat{AOD} = 2y$ ,  $\angle$  At the Centre =  $2 \times \angle$  @ Circumference

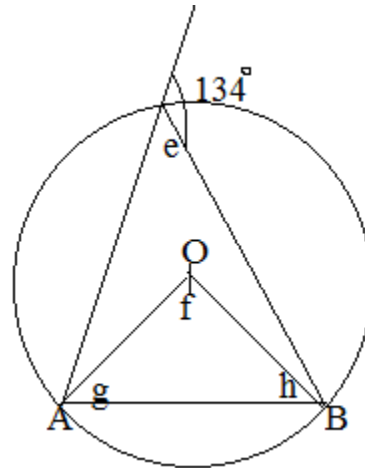
$$\therefore 2x = 2y$$

$$\therefore x = y$$

$$\therefore \widehat{ABD} = \widehat{ACD}$$

### EXAMPLES

- Find the  $e$ ,  $f$ ,  $g$ , and  $h$  if O is the Centre of the circle.



#### Solution

$$e + 134^\circ = 180^\circ, \text{sum } \angle \text{ s in a } \Delta$$

$$e = 180^\circ - 134^\circ$$

$$e = 46^\circ$$

$$f = 2 \times e, \angle \text{ @ the Centre} = 2 \times \angle \text{ @}$$

Circumference

$$f = 2 \times 46^\circ$$

$$= 92^\circ$$

$$AO = BO, \text{radii}$$

$$\therefore \Delta AOB \text{ is isosceles}$$

$$\therefore g = h$$

$$g + h + f = 180^\circ, \text{sum } \angle \text{ s in } \Delta$$

$$2h + 96^\circ = 180^\circ$$

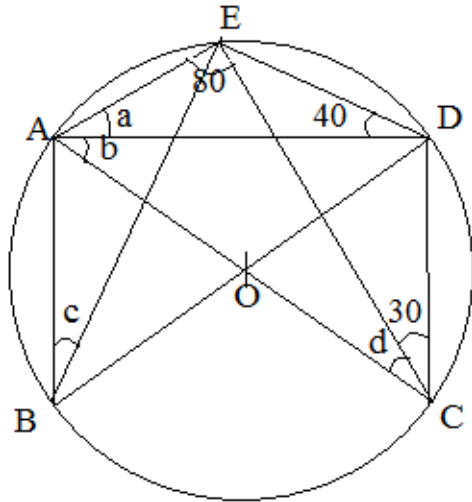
$$2h = 180^\circ - 96^\circ$$

$$h = \frac{88^\circ}{2}$$

$$\therefore g = 44^\circ$$

$$y = 44^\circ$$

2. Find all letters in the figure below



**Solution**

$a = 30^\circ$ ,  $\angle$  in the same seg

$c = 40^\circ$ ,  $\angle$  in the same seg

$c = d$ ,  $\angle$  in the same seg

$$\therefore d = 40^\circ$$

In  $\triangle ACE$ ,

$$\hat{A} + \hat{C} + \hat{E} = 180^\circ, \text{ sum } \angle \text{ in a } \triangle$$

$$(a + b) + d + 80^\circ = 180^\circ$$

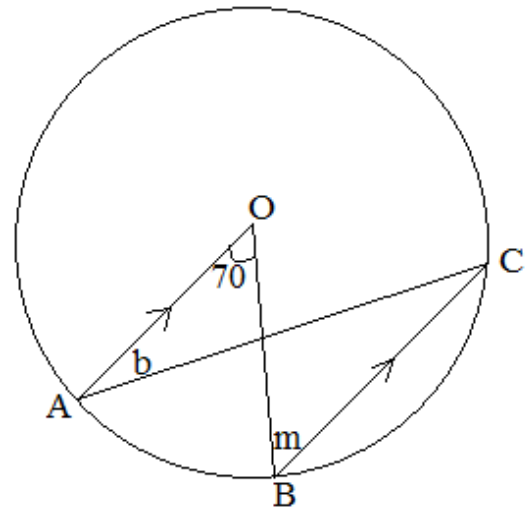
$$30^\circ + b + 40^\circ + 80^\circ = 180^\circ$$

$$b = 180^\circ - 150^\circ$$

$$b = 30^\circ$$

**EXERCISE**

1. Find marked angles in the circles below

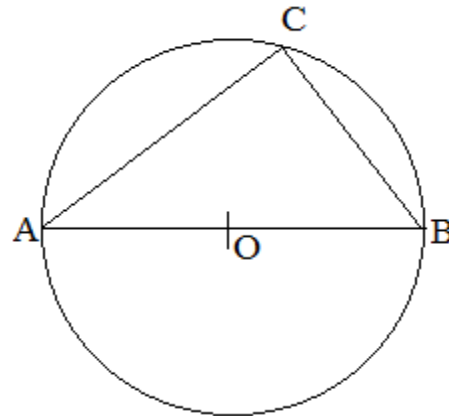


**Theorem 7**

Angle in a semicircle is a right angle.

Reason =  $\angle$  in a semicircle

**PROVE**



**Given;** circle Centre O, diameter AB and  $\angle ACB$

**To prove:**  $\angle ACB = 90^\circ$

**Proof:**  $\angle AOB = 2\angle ACB$ ,  $\angle$  At the Centre = 2  $\times$   $\angle$

At circumference

But  $\angle AOB = 180^\circ$ ,  $\angle$  on a str line

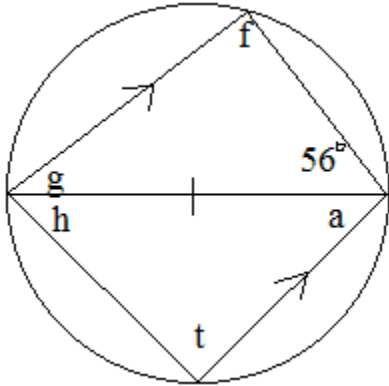
$$\therefore 180^\circ = 2\angle ACB$$

$$\frac{180^\circ}{2} = \angle ACB$$

$$\therefore \widehat{ACB} = 90^\circ$$

**EXAMPLE**

1. Find the marked angles in the figure below.

**Solution**

$f = 90^\circ$ ,  $\angle$ s in a semicircle

$f + g + 56^\circ = 180^\circ$ , sum  $\angle$ s in a  $\Delta$

$$90^\circ + g + 56^\circ = 180^\circ$$

$$g = 180^\circ - 146^\circ$$

$$g = 34^\circ$$

$$a = g, \text{ alt } \angle \text{s}$$

$$\therefore a = 34^\circ$$

$$t = 90^\circ < \text{s in a semicircle}$$

$$h + a + 90^\circ = 180^\circ, \text{ sum } \angle \text{s in a } \Delta$$

$$h + 34^\circ + 90^\circ = 180^\circ$$

$$h = 180^\circ - 126^\circ$$

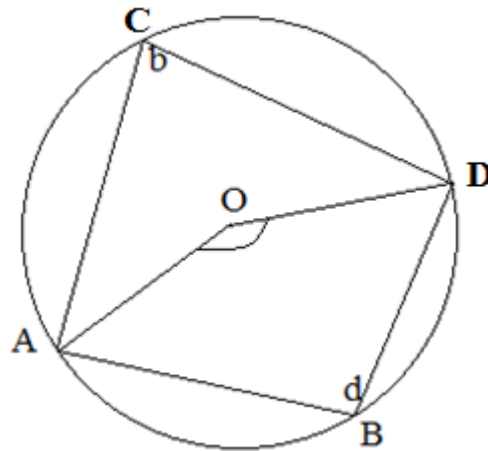
$$= 56^\circ$$

**CYCLIC QUADRILATERAL**

A cyclic quadrilateral is the quadrilateral which has a circle passing through its vertices.

**Theorem 8**

The sum of interior opposite angles of a cyclic quadrilateral is  $180^\circ$  Or the interior opposite  $\angle$ s of a cyclic quadrilateral are supplementary.



**Given;** Centre O, cyclic quad ABCD

**To prove;**  $\widehat{ABD} + \widehat{ACD} = 180^\circ$

**Construction;** join O to A and C respectively

**Proof;** reflex  $\widehat{AOD} = 2b$ ,  $\angle$  @ a centre =  $2 \times \angle$  @ circum

**Obtuse  $\widehat{AOD} = 2d$ ,  $\angle$  @ a centre =  $2 \times \angle$  @ circum**

But  $2b + 2d = 360^\circ$ ,  $\angle$ s @ a point

$$2(b + d) = 360^\circ$$

$$b + d = \frac{360^\circ}{2}$$

$$b + d = 180^\circ$$

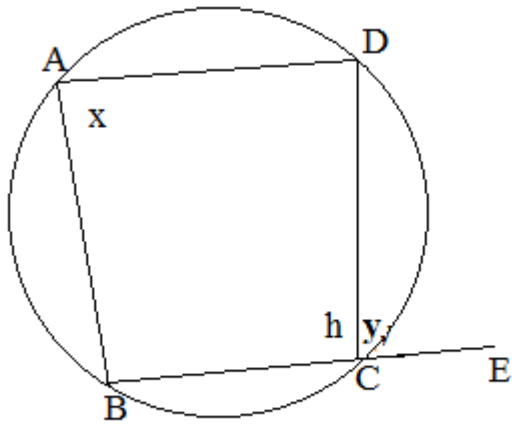
$$\therefore \widehat{ABC} + \widehat{ADC} = 180^\circ$$

**Theorem 9**

The interior opposite angles of a cyclic quadrilateral is equal to the exterior angle.

**PROVE**





**Given;** cyclic quad,  $\widehat{BAD}$  and  $\widehat{DCE}$

**To prove:**  $\widehat{BAD} = \widehat{DCE}$

**Proof;**  $x + h = 180^\circ$ , sum int. opp.  $\angle$ s in

Cyclic quad

$$\therefore x = 180^\circ - h$$

$$y + h = 180^\circ, \angle \text{ s on a str line}$$

$$\therefore y = 180^\circ - h$$

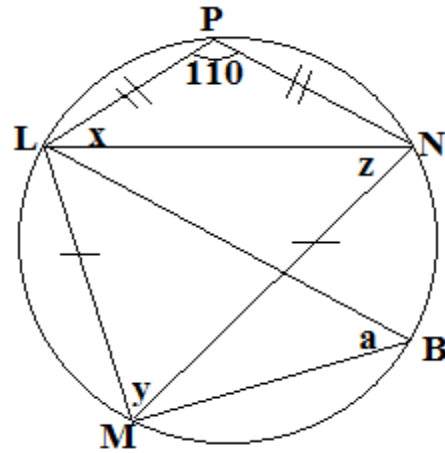
$$\therefore x = y$$

$$\therefore \widehat{BAD} = \widehat{DCE}$$

**Reason,** ext.  $\angle$  = inter.  $\angle$  in cyclic quad

### EXAMPLE

Find the lettered angles.



### Solution

In  $\Delta LPN$ ,  $LP = NP$ , given

$\therefore \Delta LPN$  is an isoscles

$\therefore \widehat{N} = x$ , base angles

$$x + \widehat{N} + 110^\circ = 180^\circ, \text{ sum } \angle \text{ s in } \Delta$$

$$x + x + 110^\circ = 180^\circ$$

$$2x = 180^\circ - 110^\circ$$

$$2x = 70^\circ$$

$$\therefore x = 35^\circ$$

$$110^\circ + y = 180^\circ, \text{ opp. int. } \angle \text{ s in cyclic q.}$$

$$y = 180^\circ - 110^\circ$$

$$y = 70^\circ$$

In  $\Delta LMN$ ,  $LM = NP$ , given

$\therefore \Delta LMN$  is an isoscles

$\therefore \widehat{L} = z$  base angles

$$z + \widehat{L} + y = 180^\circ, \text{ sum } \angle \text{ s in } \Delta$$

$$z + z + 70^\circ = 180^\circ$$

$$2z = 180^\circ - 70^\circ$$

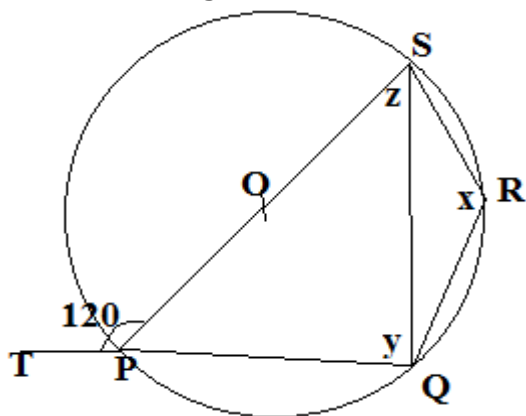
$$2z = 110^\circ$$

$$\therefore z = 55^\circ$$

$$z = a, \angle \text{ s in same seg}$$

$$\therefore a = 55^\circ$$

2. Find lettered angles



**Solution**

$$x = 120^\circ, \text{ ext. } \angle \text{ of a cyclic quad}$$

$$y = 90^\circ, \angle \text{ s in a semicircle}$$

$$z + y = 120^\circ, \text{ ext. } \angle \text{ in } \Delta = \text{sum int. opp. } \angle \text{ s}$$

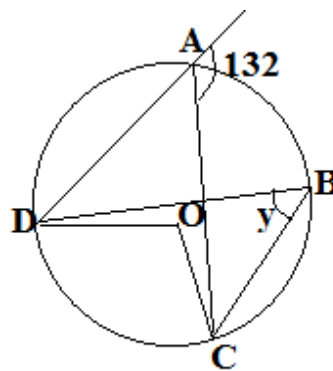
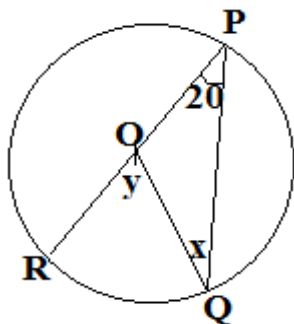
$$z + 90^\circ = 120^\circ$$

$$z = 120^\circ - 90^\circ$$

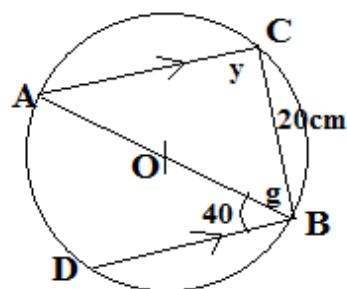
$$z = 30^\circ$$

**EXERCISE**

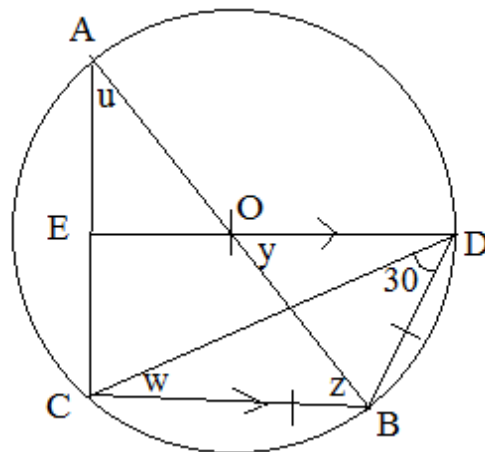
1. Find all lettered angles in the figures below



2. AOB is a diameter. Find the value of  $g$  and  $y$



3. AB is a diameter. AC and ED are straight lines. Find all letters

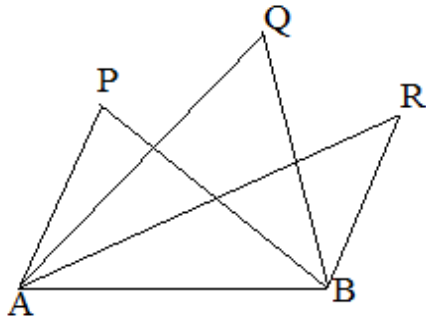


**CONCYCLIC POINTS**

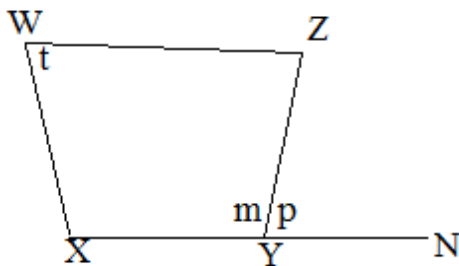
Concyclic points are points which lie on a circle.

Points are concyclic;

- I. If the angles subtended by the same line such as APB, AQB, ARB.



- II. If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is a cyclic



If  $t + m = 180^\circ$ , then **WXYZ** is cyclic Quad.

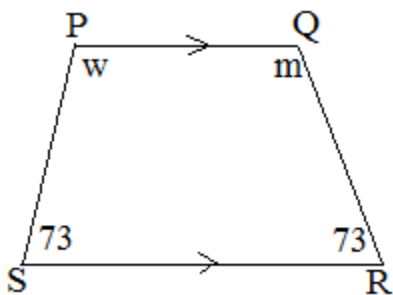
If  $t = p$ , then **WXYZ** is cyclic quad

#### EXAMPLE

**PQRS** is a trapezium having **PQ** parallel to **SR** and  $\widehat{PSR} = \widehat{QRS} = 73^\circ$ . Prove that **PQRS** is a cyclic quadrilateral.

#### Solution

NB; A trapezium is a quadrilateral with two opposite sides parallel



Given; trapezium **PQRS**,  $\widehat{PSR} = \widehat{QRS} = 73^\circ$ .

To prove; **PQRS** is a cyclic quad

Proof:  $w + \widehat{PSR} = 180^\circ$ , allied  $\angle$ s

$$W + 73^\circ = 180^\circ$$

$$W = 180^\circ - 73^\circ$$

$$w = 107^\circ$$

$m + \widehat{PSR} = 180^\circ$ , allied  $\angle$ s

$$m + 73^\circ = 180^\circ$$

$$m = 180^\circ - 73^\circ$$

$$m = 107^\circ$$

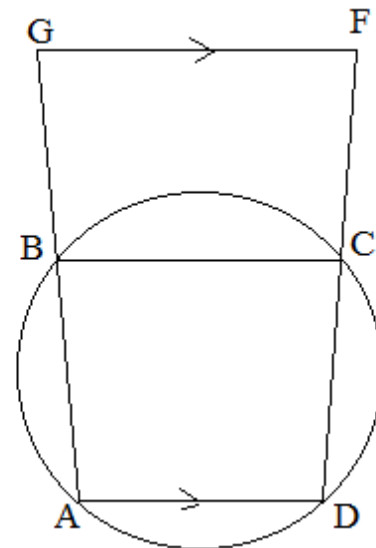
$$\therefore m = w$$

$\therefore w + \widehat{QRS} = 180^\circ$ ,  
but this is the sum of 2 int opp  $\angle$ s of a cyclic quad

$\therefore$  **PQRS** is cyclic quad.

#### EXERCISE

- Figure below shows a circle **ABCD** in which **AB** and **DC** are produced to **G** and respectively such that **GF** is parallel to **AD**. (**MANEB 2006**)



Prove that quadrilateral **GBCF** is cyclic

#### **CHAPTER 4**

## ALGEBRAIC FRACTIONS

This topic will deal with;

- Adding algebraic fraction
  - subtracting algebraic fraction
  - dividing algebraic fraction
  - multiplying algebraic fraction
- A. ADDING ALGEBRAIC FRACTION**

### EXAMPLE

Simplify the following

$$1. \frac{y+2}{2} + \frac{x-2}{3}$$

### Solution

$$\frac{y+2}{2} + \frac{x-2}{3}$$

$$\frac{3(y+2)+2(x-2)}{6}$$

$$\frac{3y+6+2x-4}{6}$$

$$\frac{3y+2x+2}{6}$$

$$2. \frac{n}{3} + \frac{m-1}{3} + \frac{m}{6}$$

### Solution

$$\frac{n}{3} + \frac{m-1}{3} + \frac{m}{6}$$

$$\frac{2 \times n + 2(m-1) + m}{6}$$

$$\frac{2n+2m-2n+m}{6}$$

$$\frac{3m}{6}$$

$$\frac{m}{2}$$

$$3. \text{ Simplify } \frac{3}{2x-4} + \frac{2}{6-3x}$$

### Solution

$$\frac{2}{2(x-2)} + \frac{2}{3(2-x)}$$

$$\frac{3}{2(x-2)} + \frac{2}{-3(x-2)}$$

NB;  $a - b = -(b - a)$

Thus  $2 - x = -(2 - x)$

$$\frac{3}{2(x-2)} - \frac{2}{3(x-2)}$$

$$\frac{9-4}{6(x-2)}$$

$$\frac{5}{6(x-2)}$$

### **EXERCISE**

Simplify the following

$$1. \frac{2p+r}{r} + \frac{4p}{4r}$$

$$2. \frac{3mn}{2m+2n} + \frac{5mn}{3m+3n}$$

$$3. \frac{2r-3}{4} + \frac{2-r}{3}$$

$$4. 3 + \frac{2b}{a-b}$$

### **EXAMPLES**

$$1. \text{ Simplify } \frac{m+3}{m^2-3n+2} + \frac{7m+5}{m^2+m-2}$$

### Solution

$$\frac{m+3}{(m-2)(m-1)} + \frac{7m+5}{(m+2)(m-1)}$$

$$\frac{(m+3)(m+2) + (7m+5)(m-2)}{(m-2)(m-1)(m+2)}$$

$$\frac{m^2+3m+2m+6+7m^2-14m+5m-10}{(m-2)(m-1)(m+2)}$$

$$\frac{8m^2 - 4m - 4}{(m-2)(m-1)(m+2)}$$

$$\frac{4(2m^2 - m - 1)}{(m-2)(m-1)(m+2)}$$

$$\frac{4(2m+1)(m-1)}{(m-2)(m-1)(m+2)}$$

$$\frac{4(2m+1)}{(m-2)(m+2)}$$

2. Simplify  $\frac{a+b}{4} + \frac{(b-2a)}{5}$

**Solution**

$$\frac{a+b}{4} + \frac{(b-2a)}{5}$$

$$\frac{5(a+b) - 4(b-2a)}{20}$$

$$\frac{5a + 5b - 4b + 8a}{20}$$

$$\frac{13a + b}{20}$$

3. Simplify  $\frac{c(3-c)}{c^2 + 3c - 10} + \frac{c-1}{c+5}$

**Solution**

$$\frac{c(3-c)}{c^2 + 3c - 10} + \frac{c-1}{c+5}$$

$$\frac{c(3-c)}{(c+5)(c-2)} + \frac{c-1}{c+5}$$

$$\frac{c(3-c) + (c-1)(c-2)}{(c+5)(c-2)}$$

$$\frac{3c - c^2 + c^2 - 3c + 2}{(c+5)(c-2)}$$

$$\frac{2}{(c+5)(c-2)}$$

4. Simplify  $\frac{1}{2n-3} + \frac{1}{2n+1} - \frac{1}{n-1}$

**Solution**

$$\frac{(2n+1)(n-1) + (2n-3)(n-1) - (2n-3)(2n+1)}{(2n-3)(2n+1)(n-1)}$$

$$\frac{2n^2 - n - 1 + 2n^2 - 5n + 3 - (4n^2 - 4n - 3)}{(2n-3)(2n+1)(n-1)}$$

$$\frac{4n^2 - 6n + 2 - 4n^2 + 4n + 3}{(2n-3)(2n+1)(n-1)}$$

$$\frac{5-2n}{(2n-3)(2n+1)(n-1)}$$

## EXERCISE

Simplify the following

1.  $\frac{5}{d^2-16} + \frac{2}{(4d-d)^2}$

2.  $\frac{5}{d^2-2d-8} + \frac{2}{d^2-6d+8}$

3.  $\frac{3}{2(x+y)} - \frac{1}{3(x+y)}$

4.  $\frac{2}{m-n} + \frac{3}{a+2}$

5.  $\frac{2}{2t^2+3t-2} + \frac{1}{t+2} - \frac{1}{2t-1}$

## DIVISION OF ALGEBRAIC EXPRESSIONS

### EXAMPLE

1. Simplify  $\frac{x^2-5x+6}{x} \div (x-3)^2$

**Solution**

$$\frac{\frac{x^2-5x+6}{x}}{(x-2)(x-3)} \times \frac{1}{(x-3)(x-3)}$$

$$\frac{x-2}{x(x-3)}$$

2. Simplify  $\left(\frac{1}{x} - \frac{1}{y}\right) \div \frac{x^2-y^2}{x^2y^2}$

**Solution**

$$\frac{(y-x)}{xy} \times \frac{x^2y^2}{x^2-y^2}$$

$$\frac{(y-x)}{xy} \times \frac{x^2 y^2}{(x+y)(x-y)}$$

$$\frac{-(x-y)}{xy} \times \frac{x^2 y^2}{(x+y)(x-y)}$$

$$\frac{-xy}{x+y}$$

$$\frac{xy}{y-x}$$

## MULTIPLICATION OF ALGEBRAIC FRACTIONS

### EXAMPLES

1. Simplify  $\frac{x+1}{x^2-4} \times \frac{x+2}{5}$

#### Solution

$$\frac{x+1}{x^2-4} \times \frac{x+2}{5}$$

$$\frac{x+1}{x^2-2^2} \times \frac{x+2}{5}$$

$$\frac{x+1}{(x+2)(x-2)} \times \frac{x+2}{5}$$

$$\frac{x+1}{5(x-2)}$$

2. Simplify  $\frac{n^2-9}{n^2-n} \times \frac{n^2-3n+2}{n^2+n-6}$

#### Solution

$$\frac{n^2-9}{n^2-n} \times \frac{n^2-3n+2}{n^2+n-6}$$

$$\frac{n^2-3^2}{n(n-1)} \times \frac{(n-1)(n-2)}{(n-2)(n+3)}$$

$$\frac{(n+3)(n-3)}{n(n-1)} \times \frac{(n-1)(n-2)}{(n-2)(n+3)}$$

$$\frac{n-3}{n}$$

## SOLVING ALGEBRAIC FRACTIONS

### EXAMPLE

1. Solve  $\frac{4}{x^2-x-2} + \frac{3}{x^2-4} = \frac{2}{x^2+3x+2}$

#### Solution

$$\frac{4}{(x-2)(x+1)} + \frac{3}{(x-2)(x+2)} = \frac{2}{(x+2)(x+1)}$$

$$\frac{4(x+2)+3(x+1)=2(x-2)}{(x-2)(x+1)(x+2)}$$

$$4(x+2) + 3(x+1) = 2(x-2)$$

$$4x + 8 + 3x + 3 = 2x - 4$$

$$7x + 11 = 2x - 4$$

$$5x = -15$$

$$x = -3$$

NB: When solving algebraic fractions, if the denominator has been used in both sides (the equal to side and other side), then we disregard the denominator

2. Solve  $\frac{3}{2b+5} = \frac{2}{b+2}$

#### Solution

$$\frac{3}{2b+5} = \frac{2}{b+2}$$

$$3(b+2) = 2(2b+5)$$

$$3b+6 = 4b+10$$

$$-b = 4 \quad \therefore b = -4$$

## WORD PROBLEMS

### EXAMPLE

Mada Kalima bought some fish for k4000. If the fish had each been sold for k20 less, then he would have bought 10 more. How much was one fish sold.

#### Solution

Total money = k4000

Let the current price be k x

$$\therefore \text{The number of fish at } x \text{ kwacha} = \frac{4000}{x}$$

If the price had been each sold for k20 less, the current = (x- 20)

$$\therefore \text{Number of fish at } (x-20) \text{ kwacha} = \frac{4000}{x-20}$$

Number of fish bought due to decrease in price is 10 more than that of the current price.

$$\therefore \frac{4000}{x-20} - \frac{4000}{x} = 10$$

$$\frac{4000x - 4000(x-20) = 10x(x-20)}{x(x-20)}$$

$$4000x - 4000(x - 20) = 10x(x - 20)$$

$$4000x - 4000x + 8000 = 10x^2 - 200x$$

$$8000 = 10x^2 - 200x$$

$$10x^2 - 200x - 8000 = 0$$

Divide by 10

$$x^2 - 20x - 800 = 0$$

$$x^2 - 100x + 80x - 800 = 0$$

$$x(x - 100) + 80(x - 100) = 0$$

$$(x - 100)(x + 80) = 0$$

$$(x - 100) = 0 \text{ or } (x + 80) = 0$$

$$x = 100 \quad x = -80$$

$\therefore$  The cost of one fish was k100 each.

### EXERCISE

1. Simplify  $(xy - \frac{x^2y}{x+y}) \div \frac{x^2y}{x+y}$

2.  $\frac{3d^2-12}{9d^2} \times \frac{6d^2}{4d+8}$

3. solve  $\frac{b+1}{2b-3} = 2 - \frac{b+3}{2b+3}$

4. **Beauty** drove a distance of 40km on a return journey, she drove 20km/hour

faster and took 2hrs less. What was her speed on outward journey

5. A taxi driver takes k36 in fare per trip with a full load of passengers, if he increases his fare by k1 per passenger she can make the same amount when carrying three passengers less. How many passengers did he originally carry?

**CHAPTER 5****SET THEORY**

A set is a collection of objects or items

**Types of sets**

- i. equal sets
- ii. Empty sets
- iii. Infinite sets
- iv. Finite sets

**EQUAL SETS**

Sets with same elements and equal elements e. g

$$A = \{a, b, c\}$$

$$B = \{a, b, c\}$$

$$\therefore A = B$$

**EMPTY SET**

Is a set with no elements e. g  $C = \{\}$

$\emptyset$  a symbol for an empty set.

**INFINITE SET**

A set which has uncountable number of elements e. g  $A = \{\text{negative numbers}\}$

**FINITE SETS**

Has countable number of elements e. g

$$A = \{\text{whole numbers from 1 to 10}\}$$

**SET SYMBOLS**

1.  $\in$  = An element of

Example,  $8 \in A$  means 8 is an element of A

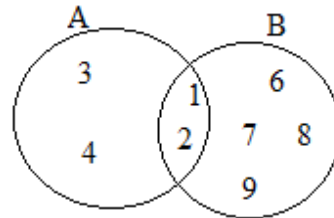
2.  $\notin$  = is not an element of
3.  $\mu$  or  $\forall$  = universal set
4.  $\cup$  = union
5.  $\cap$  = intersection
6.  $\subset$  = subset of

**UNION OF SETS**

The union of two sets is the set of all elements that are members of either set

**EXAMPLE**

$$A = \{1, 2, 3, 4\} \quad B = \{1, 2, 6, 7, 8\}, \text{ find } A \cup B.$$

**Solution**

$$\therefore A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

**INTERSECTION SETS**

The intersection of two sets is the set of elements that are in both sets. E. g in the above example

$$A \cap B = \{1, 2\}$$

An intersection of sets have elements that are common to given sets

**SET BUILDER NOTATION****Real numbers (R)**

Are all categories of numbers e. g negative, fraction, positive numbers e. t. c

**Natural numbers (N)**

Are positive whole numbers excluding 0

**Whole numbers (W)**

Are all natural numbers including 0

**Prime numbers (P)**

Are numbers which can be divided by one and it self e. g  $P = \{2, 3, 5, 7, 11, 13, \dots\}$



**EXAMPLE**

1.  $A = \{x : x \in \mathbb{R}, x^2 = 0\}$

Read as A is a set of values of x such that x is an element of a real number and x square is equal to zero.

**Solution**

$$A = \{x : x \in \mathbb{R}, x^2 = 0\}$$

$$x^2 = 0$$

$$x = \sqrt{0}$$

$$x = 0$$

$$\therefore A = \{0\}$$

2.  $B = \{x : x \in \mathbb{R}, x^2 - 3 = 0\}$

**Solution**

$$x^2 - 3 = 0$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

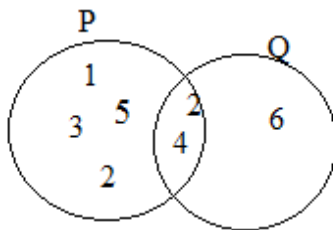
$$\therefore B = \{\pm\sqrt{3}\}$$

$$B = \{+\sqrt{3}, -\sqrt{3}\}$$

NB:  $\cap$  (A) means number of elements in A

3.  $P = \{1, 2, 3, 4, 5\}$  and  $Q = \{2, 4, 6\}$

Find a.  $n(P \cap Q)$ , b. all subsets of Q

**Solution**

a.

$$(P \cap Q) = \{2, 4\}$$

$$n(P \cap Q) = 2$$

NB: a set with n elements has  $2^n$  subsets. Every set has got an empty set

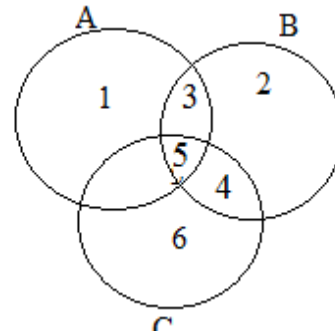
b.  $Q = \{2, 4, 6\}$

$$n(Q) = 3$$

$$\text{Number of subsets} = 2^3 = 8$$

4. Given that  $A = \{1, 3, 5\}$ ,  $B = \{2, 3, 4, 5\}$  and  $C = \{2, 4, 5, 6\}$

Find a.  $(A \cup B) \cap C$ , b.  $A \cup B \cup C$  d.  $A \cap C$

**Solution**

a.  $(A \cup B) = \{1, 2, 3, 4, 5\}$

$$C = \{3, 4, 5, 6\}$$

$$\therefore (A \cup B) \cap C = \{3, 5\}$$

b.  $A \cup B \cup C = \{1, 2, 3, 4, 5, 6\}$

c.  $A \cap C = \{ \}$

**UNIVERSAL SET (U)**

It is a collection of all sets

**COMPLEMENT OF A SET**

These are elements that are absent in a set but are present in a universal set.

$A'$  = means complements of A

**EXAMPLES**

1. Given that  $\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

a. If  $A = \{1, 3, 5, 7, 9\}$  list  $A'$

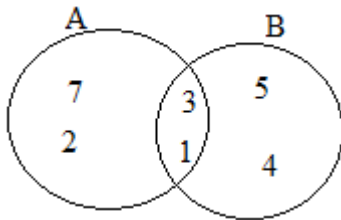
b. If  $P = \{2, 3, 5, 7\}$  list  $P'$

**Solution**

a.  $A' = \{2, 4, 6, 8, 10\}$

b.  $P' = \{1, 4, 6, 8, 9, 10\}$

### NUMBER OF ELEMENTS IN A TWO SET PROBLEMS



$$n(A \cup B) = 6$$

$$n(A \cap B) = 2$$

$$n(A) = 4$$

$$n(B) = 4$$

$$\begin{aligned} \text{In general, } n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 4 + 4 - 2 \\ &= 6 \end{aligned}$$

#### EXAMPLE

1. Given that  $n(A \cup B) = 14$ ,  $n(A) = 10$ ,  $n(B) = 9$ , find  $n(A \cap B)$ .

**Solution**

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$14 = 10 + 9 - n(A \cap B)$$

$$14 = 19 - n(A \cap B)$$

$$\begin{aligned} n(A \cap B) &= 19 - 14 \\ &= 5 \end{aligned}$$

2. Given that  $n(A) = \{2, 4, 6, 8, 10\}$ ,  $n(B) = \{4, 8, 12, 16\}$ . Show that,  
 $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

**Solution**

$$n(A \cup B) = 7$$

$$n(A) = 5$$

$$n(B) = 4$$

$$n(A \cap B) = 2$$

By substituting;

$$7 = 5 + 4 - 2$$

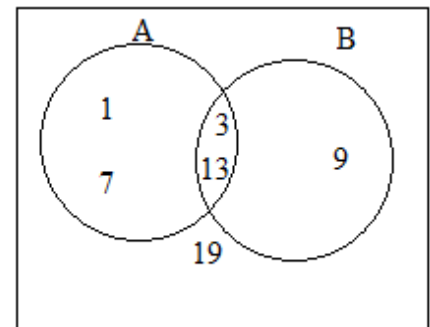
$$7 = 9 - 2$$

$$7 = 7$$

$$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

#### EXERCISE

1. The diagram below shows sets A, B and C the universal set.



3. List  $A \cap B'$  and  $B \cap A'$

ii. What are  $n(A \cap B)$  and  $n(A \cup B)$

2. Given  $\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$   
 $A = \{2, 4, 6, 8\}$

Find  $A'$  and  $(A')'$

3. Given that  $A = B = \{1, 2, 3, 4\}$ , show that  
 $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

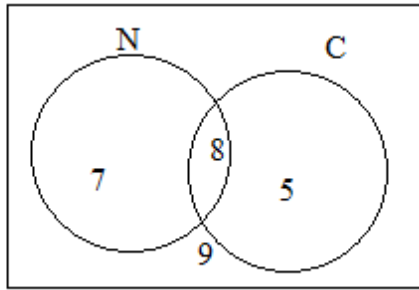
### WORD PROBLEMS

#### EXAMPLE

1. In a class, 15 pupils like nature studies, 13 like craft and 8 like both subjects. Use the Venn diagram to find how many pupils are there in the class

**Solution**

NB: always start with intersection



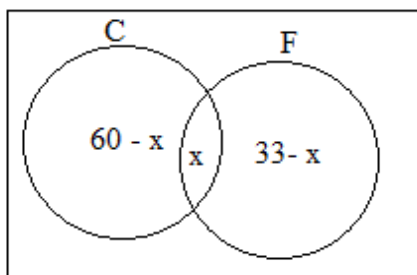
$$\begin{aligned}\text{Number of pupils} &= 7 + 8 + 5 + 9 \\ &= 29\end{aligned}$$

NB; Make sure the elements in N add up to 15 and in C add up to 13.

8 is subtracted from 15 and 13 because 8 pupils like both subjects.

2. In a class, 60 pupils study Chichewa and 33 study French. All pupils study at least one of the languages. If there are 70 pupils in the class, how many study both languages?

**Solution**



$$60 - x + x + 33 - x = 70$$

$$93 - x = 70$$

$$93 - 70 = x$$

$$x = 23$$

∴ 23 Pupils study both languages

3. In a group of 43 pupils, 29 are girls and 26 like dancing, 7 do not like dancing. How many boys do not like dancing?

**Solution**

$$\text{Number of pupils} = 43$$

$$\text{Number of girls} = 29$$

$$\therefore \text{Number of boys} = 43 - 29$$

$$= 14$$

$$\text{Number of people who do not like dancing} = 7$$

$$\text{Number of girls who like dancing} = 29 - 7$$

$$= 22$$

$$\text{No. of boys who like dancing} = 14 - 22$$

$$= -8$$

**But** there are 14 boys

$$\therefore \text{No. of boys who do not like dancing} = 14 - 4$$

$$= 10$$

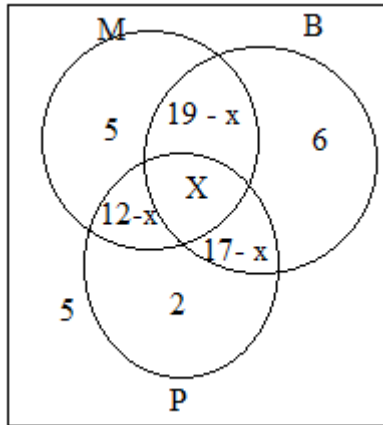
### **THREE SET PROBLEMS**

#### **EXAMPLE**

- Class of 50 students wrote test in mathematics, biology and physical science. The results of the tests were as follows
  - 12 passed maths and physical
  - 19 passed maths and biology
  - 17 passed biology and physical
  - 2 passed physical only
  - 5 passed maths only
  - 6 passed biology only

If 5 students failed all the three subjects and  $x$  passed all subjects use Venn diagram to calculate the value of  $x$ . (MANEB 2013)

**Solution**



$$x + 5 + 19 - x + 12 - x + 17 - x + 6 + 2 + 5 = 50$$

$$66 - 2x = 50$$

$$66 - 50 = 2x$$

$$16 = 2x$$

$$x = 8$$

### EXERCISE

- In a class of 50 students, each of the students ate at least one of the following types of fruits; banana, mango and orange. It was found that
  - $(x + 1)$  ate all the three types of fruits
  - 9 students ate mangoes and oranges only
  - 8 students ate bananas and mangoes only
  - 5 students ate bananas and oranges only
  - X ate bananas only
  - $(x - 1)$  students ate mangoes only

Represent the data on a Venn diagram hence find the value of  $x$

## CHAPTER 6

## MAPPING AND FUNCTIONS

Mapping and functions looks at relationships that exist among or between elements

### RELATIONS

**Mary** "is a friend of" **mercy**

**Mary**.....first element

**Is the friend of**.....defining condition

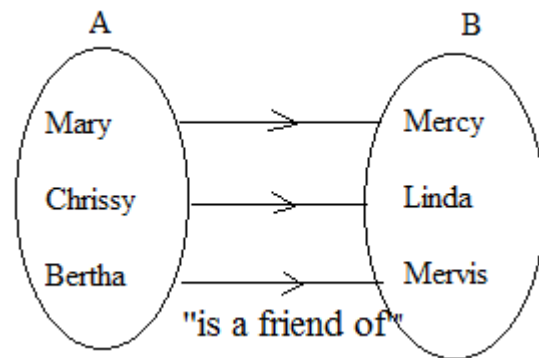
**Mercy**.....second element

(Mary, Mercy).....ordered pair

### TYPES OF RELATIONS

Arrow diagrams are used to map relations

#### 1). ONE TO ONE RELATIONS



The above arrow diagram is **example of one to one relation**

In this relation, every member of **A** has one image in **B**

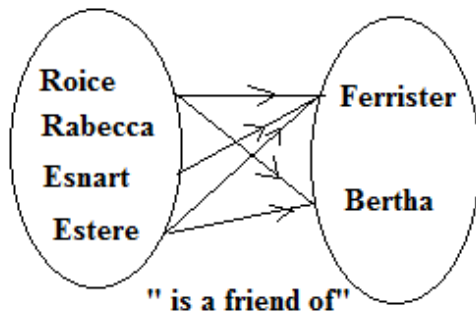
Every member of **B** is the image of exactly one member of **A**;

Mercy is the image of Mary

Linda is the image of Chrissy

Mervis is the image of Bertha

## 2). MANY TO ONE RELATION



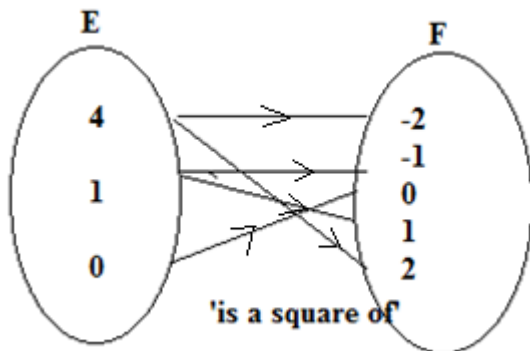
In the above diagram, many elements of C have the same elements in D

No elements of C have more than one elements in D

## 3). ONE TO MANY RELATION

If  $E = (0,1,4)$   $F = (-2,-1,0,1,2)$

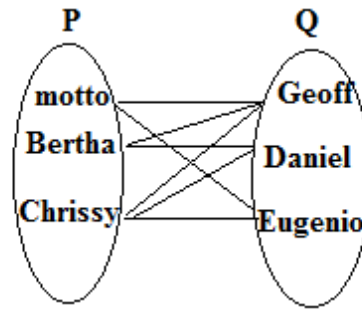
And the relation is a square of



In the above diagram, some elements of E have more than one image in F

No image of F is the image of more than one element of E

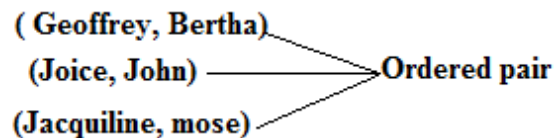
## 4). MANY TO MANY RELATIONS



In many to many to many relation, some elements of P have more than one image of Q.

Some elements of Q are more images of many elements of P.

## DOMAIN AND RANGE



**Domain** is a set of the first elements of all ordered pair.

$\therefore D = \{\text{Geoffrey, Joyce, Jacqueline}\}$

**Range** is the set of all the second elements of all ordered pair.

$\therefore R = \{\text{Bertha, John, Mose}\}$

## EXAMPLES

Given the domain and range of the following relation from A to B where  $A = \{1,2,3\}$  and  $B = \{1,2,3,4,5\}$

a.  $P = \{(x, y) : x \in A, y \in B \text{ and } y = 2x\}$

$\therefore P = \{(1,2), (2,4)\}$

$D(P) = \{1, 2\}$

$R(P) = \{2, 4\}$

b.  $R = \{(x, y) : x \in A, y \in B \text{ and } x = 2\}$

$\therefore R = \{(2, 2)\}$

$\therefore D(R) = \{2\}$

$$R(S) = (2)$$

$$c. S = \{(x, y) : x \in A, y \in B \text{ and } x = 2y\}$$

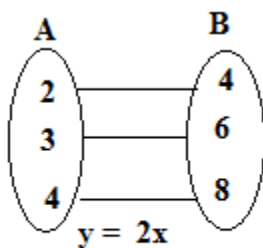
$$\therefore S = \{(2, 1)\}$$

$$\therefore D(S) = (2)$$

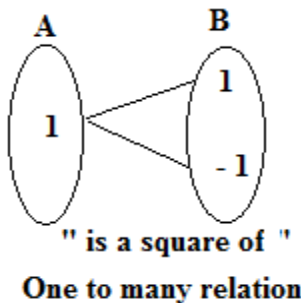
$$R(S) = (1)$$

## FUNCTIONS

A function is a special kind of relation. It is a rule which assigns to each element  $x$  of a set  $A$  just one element of a set  $B$ .



one to one relation



One to many relation

## FUNCTION NOTATION

We can write a function in different ways or notations e.  $y = x^2$ , can be written as  $f(x) = x^2$ , or  $f : x \rightarrow x^2$ .

Where  $y / f(x) / f : x \rightarrow$  is a domain and  $x^2$  is the range.

## EXAMPLES

1. A function  $f$  from  $P$  to  $Q$  is difficult by the formulae  $f(t) = t + 3$ , where  $t$  is an

element of  $P$  and  $Q$  consists of the elements of the form  $t + 3$ .

### Solution

$$f : p \rightarrow Q$$

$$f(p) = Q$$

$$f(t) = t + 3$$

$$P = \{1, 2, 3, 4, 5\}$$

$$f(1) = 1 + 3 = 4$$

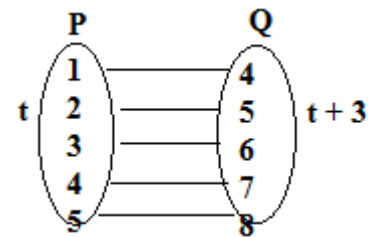
$$f(2) = 2 + 3 = 5$$

$$f(3) = 3 + 3 = 6$$

$$f(4) = 4 + 3 = 7$$

$$f(5) = 5 + 3 = 8$$

$$\therefore Q = \{4, 5, 6, 7, 8\}$$



2. If  $f(x) = 2x + 1$  and  $g(x) = x^2$ , find

$$a. fg(3) \quad b. gf(3) \quad c. gf(-2)$$

### Solution

$$a. g(3) = 3^2$$

$$= 9$$

$$\therefore fg(3) = 2(g(3)) + 1$$

$$= 2(9) + 1$$

$$= 19$$

$$b. f(3) = 2(3) + 1$$

$$= 7$$

$$\therefore gf(3) = 7^2$$

$$= 49$$

$$c. f(-2) = 2(-2) + 1$$

$$= -4 + 1$$

$$= -3$$

$$\therefore gf(x) = (-3)^2$$

$$= (-3)^2$$

$$= 9$$

**MANEB QUESTION**

1. Given that  $g(x) = \frac{3x}{x+1}$  calculate the value of  $x$  if  $g(x) = 2$

**Solution**

$$g(x) = \frac{3x}{x+1}$$

$$2 = \frac{3x}{x+1}$$

$$2x + 2 = 3x$$

$$2 = 3x - 2x$$

$$x = 2$$

2. Given that  $g(x) = dx - 5$ , and that  $g(2) = 1$ . Find the value of  $x$ .

**Solution**

$$g(2) = d(2) - 5$$

$$\text{But } g(2) = 1$$

$$\therefore 1 = 2d - 5$$

$$1 + 5 = 2d$$

$$6 = 2d$$

$$\therefore 3 = d$$

3. Given that  $f(x) = x^2 + 1$  and  $g(x) = kx + n$  where  $n$  and  $k$  are constants.  $f(0) = g(0)$  and  $g(2) = 15$ . Find  $k$  and  $n$ .

**Solution**

$$f(x) = x^2 + 1$$

$$f(0) = (0)^2 + 1$$

$$= 1$$

$$g(0) = k(0) + n$$

$$= 0 + n$$

$$= n$$

$$\text{But } g(0) = f(0)$$

$$\therefore 1 = n$$

$$g(2) = k(2) + 1$$

$$15 = 2k + 1$$

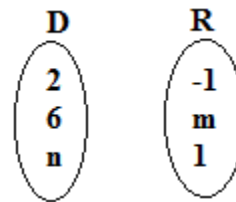
$$15 - 1 = 2k$$

$$14 = 2k$$

$$\therefore k = 7$$

**EXERCISE**

1. Figure below shows an arrow diagram for the function  $y = 2x - 5$ . Find the value of  $m$  and  $n$



2. Given that  $f(x) = \frac{2x^3}{3} + 1$ . Find  $g(-1)$  in its simplest form.
3. Given that  $f(x) = 2x + 3$  and  $g(x) = \frac{5}{x+2}$ . find  $fg(x)$
4. The function  $y = 2 + x$  has range  $\{3, 6\}$ , find its domain.

**CHAPTER 7****CHANGING SUBJECT OF THE FORMULA**

To make unknown variable subject, it must have coefficient and power 1 and all the other terms and numbers must go to the other side of the equation

In the equation  $x = 2y - 3$ ,  $x$  is the subject while  $y$  is not.

**STEPS TO FOLLOW**

- Begin by clearing fractions, brackets and root signs
- Rearrange the formula so that all the terms which contain the new subject are on one side of the equals sign and the rest on the other
- If more than one term contain the subject, take it outside the bracket
- Divide both sides by the bracket and simplify as far as possible

**EXAMPLES**

Make  $x$  subject

$$1. \quad 4xy = 2 - 2x$$

$$4xy + 2x = 2$$

$$x(4y + 2) = 2$$

$$x = \frac{2}{4y+2}$$

$$\therefore x = \frac{2}{2(2y+1)}$$

$$x = \frac{1}{2y+1}$$

$$2. \quad \frac{a}{x} + b = c$$

$$\frac{a}{x} = c - b$$

$$\frac{a}{x} = \frac{c-b}{1}$$

$$a = x(c - b)$$

$$\frac{a}{c-b} = x$$

$$3. \quad a = \frac{2b+3x}{3b-2x}$$

$$a(3b - 2x) = 2b + 3x$$

$$3ab - 2ax = 2b + 3x$$

$$3ab - 2b = 3x + 2ax$$

$$3ab - 2b = x(3 + 2a)$$

$$\frac{3ab-2b}{3+2a} = x$$

$$\therefore x = \frac{b(3a-2)}{3+2a}$$

**EXERCISE**

Make the letter in the bracket subject

$$1. \quad \frac{x}{2a} + \frac{x}{3a} = b \dots \dots \dots (x)$$

$$2. \quad a\sqrt{x} - 1 = b \dots \dots \dots (x)$$

$$3. \quad x(a - b) = b(c - x) \dots \dots (x)$$

$$4. \quad \sqrt[3]{\frac{x}{a}} = b \dots \dots \dots (x)$$

**EXAMPLES**

Make  $x$  subject of the formula

$$1. \quad \frac{a}{a-x} = \frac{b}{b+x}$$

**Solution**

$$\frac{a}{a-x} = \frac{b}{b+x}$$

$$a(b + x) = b(a - x)$$

$$ab + ax = ab - bx$$



$$ax + bx = ab - abx(a + b) = 0$$

$$x = \frac{0}{a+b}$$

$$x = 0$$

2. Make  $x$  subject

$$a(a^2 - x) = b(b^2 - x)$$

$$a^3 - ax = b^3 - bx$$

$$bx - ax = b^3 - a^3$$

$$x(b - a) = (b - a)(b^2 + ab + a^2)$$

$$x = \frac{(b - a)(b^2 + ab + a^2)}{(b - a)}$$

$$x = (b^2 + ab + a^2)$$

3. Make  $a$  subject of the formula  $c = \frac{ab\sqrt{2g}}{\sqrt{a^2 - b^2}}$

**Solution**

$$c = \frac{ab\sqrt{2g}}{\sqrt{a^2 - b^2}}$$

$$c\sqrt{a^2 - b^2} = ab\sqrt{2g}$$

$$a^2 - b^2 = \left(\frac{ab\sqrt{2g}}{c}\right)^2$$

$$a^2 - b^2 = \left(\frac{a^2b^2(2g)}{c^2}\right)$$

$$c^2(a^2 - b^2) = 2a^2b^2g$$

$$c^2a^2 - c^2b^2 = 2a^2b^2g$$

$$c^2a^2 - 2a^2b^2g = a^2b^2$$

$$a^2(c^2 - 2b^2g) = c^2b^2$$

$$a^2 = \frac{c^2b^2}{c^2 - 2b^2g}$$

$$a = \sqrt{\frac{c^2b^2}{c^2 - 2b^2g}}$$

$$a = \frac{cb}{\sqrt{c^2 - 2b^2g}}$$

## EXERCISE

Make the latter in the brackets subject of the formula

$$1. H = \frac{w^2}{2g} (R^2 - r^2) \quad (r)$$

$$2. T = \frac{mu^2}{k} - 2mg \quad (k)$$

$$3. y^2 = 2a(x - 4a) \quad (x)$$

$$4. H = \frac{A}{a - 4b} - R^2 \quad (A)$$

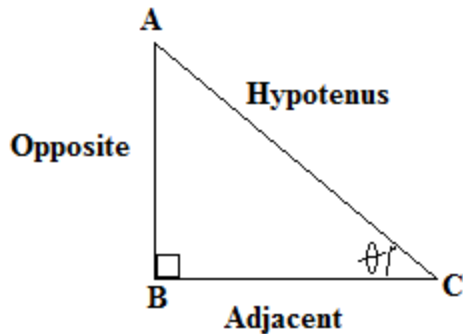
$$5. D = \sqrt{\frac{2v^2d}{g} + \frac{d^2}{4}} \cdot \frac{d}{2} \quad (d)$$

$$6. T = 2\pi \sqrt{\frac{h^2 + k^2}{2gh}} \quad (k)$$

**CHAPTER 8****TRIGONOMETRY**

**Trigono** means triangle and **metron** means measures. Trigonometry is a Greek word

It deals with size and angles of triangles

**TRIGONOMETRIC RATIOS**

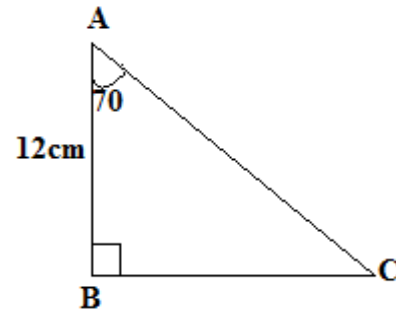
$$1. \text{ Sine } \theta = \frac{\text{opposite side}}{\text{hypotenuse side}} = \frac{AB}{AC}$$

$$2. \text{ Cos } \theta = \frac{\text{adjacent}}{\text{hypotenuse side}} = \frac{BC}{AC}$$

$$3. \text{ Tan } \theta = \frac{\text{opposite side}}{\text{Adjacent}} = \frac{AB}{BC}$$

**EXAMPLE**

1. ABC is a triangle in which **AB**= 12cm, angle **B**= 90° and angle **A**= 70°. Find the length of **BC** leaving your answer to 2 significant figures.

**Solution**

$$\text{Tan } A = \frac{BC}{AB}$$

$$\text{Tan } 70^\circ = \frac{BC}{12\text{cm}}$$

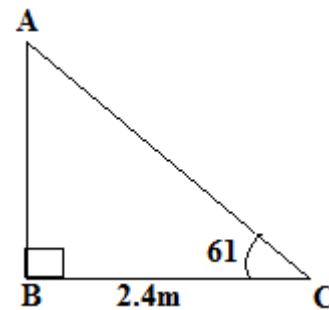
$$2.7475 = \frac{BC}{12\text{cm}}$$

$$2.7475 \times 12\text{cm} = BC$$

$$\therefore BC = 32.9700$$

$$= 33\text{cm}$$

2. A ladder makes 61° with the ground. The base of the ladder is 2.4m from a vertical wall. How far does the ladder reach up the wall? To 3 significant figures

**Solution**

$$\text{Tan } 61^\circ = \frac{AB}{2.4\text{m}}$$

$$1.8040 = \frac{AB}{2.4\text{m}}$$

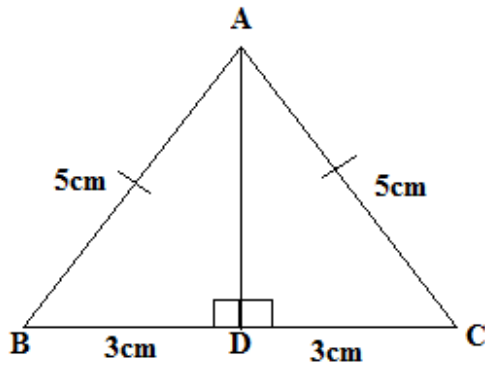
$$1.8040 \times 2.4\text{cm} = AB$$

$$\therefore AB = 4.329$$

$$= 4.33\text{m}$$

3. A triangle of sides 5cm, 5cm and 6cm. what are its angles?

**Solution**



**Construction:** join a perpendicular bisector AD

In  $\Delta ABC$

$$\cos B = \frac{3\text{cm}}{5\text{cm}}$$

$$= 0.6000$$

$$\cos^{-1} 0.6000 = 53.2^\circ$$

$$\therefore \angle B = 53.2^\circ$$

But angle B = angle C, (base angles)

$$\therefore \angle C = 53.2^\circ$$

In  $\Delta ABC$

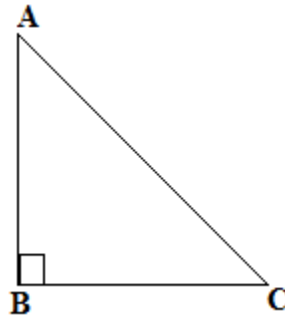
$$\angle A + \angle B + \angle C = 180^\circ, \text{ Sum of } \angle\text{s in a } \Delta$$

$$\angle A + 53.2^\circ + 53.2^\circ = 180^\circ$$

$$\angle A = 180^\circ - 106.4^\circ$$

$$= 73.6^\circ$$

### RELATIONSHIP BETWEEN ANGLES OR RATIOS



$$\sin A = \frac{BC}{AC}$$

$$\cos C = \frac{BC}{AC}$$

$$\therefore \sin A = \cos C$$

$$\text{But } \angle A + \angle C = 90^\circ$$

$$\angle A = 90^\circ - \angle C$$

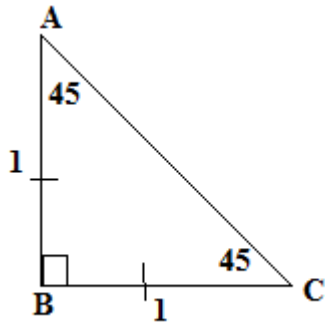
$$\text{But } \sin A = \cos C$$

$$\therefore \sin A = \cos (90^\circ - A)$$

$$\cos A = \sin (90^\circ - A)$$

**In general**

1.  $\sin \theta = \cos (90^\circ - \theta)$
2.  $\cos \theta = \sin (90^\circ - \theta)$
3.  $\tan \theta = \frac{\sin \theta}{\cos \theta}$
4.  $\sin \theta = \sin (180^\circ - \theta)$
5.  $\cos \theta = -\cos (180^\circ - \theta)$
6.  $\sin^2 \theta + \cos^2 \theta = 1$

**SPECIAL ANGLES****0°, 30°, 45°, 60°, 90°**

$$AC^2 = 1^2 + 1^2, \text{ Pythagoras}$$

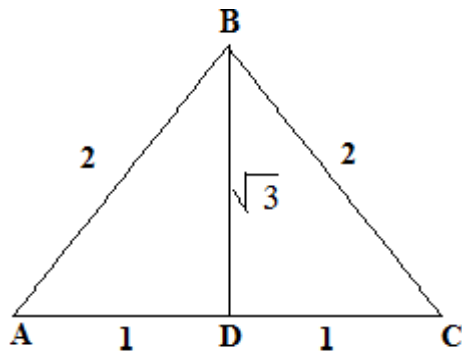
$$\therefore AC = \sqrt{2}$$

In  $\triangle ABC$ 

$$1. \sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$2. \cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$3. \tan 45^\circ = \frac{1}{1} = 1$$

**30° and 60°** uses equilateral triangleIn  $\triangle ABD$ 

$$AB^2 = BD^2 + AD^2, \text{ Pythagoras}$$

$$2^2 = BD^2 + 1^2$$

$$BD^2 = 4 - 1$$

$$AD = \sqrt{3}$$

1.  $\sin 30^\circ = \frac{1}{2}$
2.  $\cos 30^\circ = \frac{\sqrt{3}}{2}$
3.  $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
4.  $\sin 60^\circ = \frac{\sqrt{3}}{2}$
5.  $\cos 60^\circ = \frac{1}{2}$
6.  $\tan 60^\circ = \sqrt{3}$

| Angle | 0° | 30°                  | 45°                  | 60°                  | 90°      |
|-------|----|----------------------|----------------------|----------------------|----------|
| Sin   | 0° | $\frac{1}{2}$        | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1        |
| Cos   | 1° | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$        | 0        |
| Tan   | 0° | $\frac{\sqrt{3}}{3}$ | 1                    | $\sqrt{3}$           | $\infty$ |

**EXAMPLE**

1. Without using a calculator or mathematical table, show that

$$\sin 30^\circ + \cos 45^\circ + \tan 60^\circ = \frac{1 + \sqrt{2} + 2\sqrt{2}}{2}$$

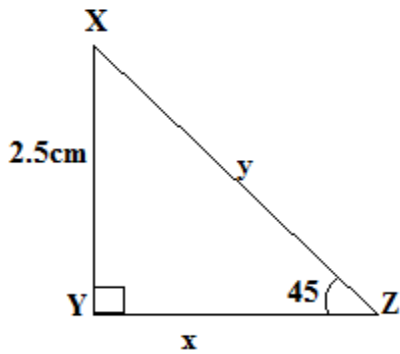
**Solution**

$$\sin 30^\circ + \cos 45^\circ + \tan 60^\circ = \frac{1}{2} + \frac{\sqrt{2}}{2} + \sqrt{3}$$

$$= \frac{1 + \sqrt{2} + 2\sqrt{2}}{2}$$

$$\therefore \sin 30^\circ + \cos 45^\circ + \tan 60^\circ = \frac{1 + \sqrt{2} + 2\sqrt{2}}{2}$$

2. Find the value of x and y in the figure below without using a calculator or a four figure table.

**Solution**

$$\tan 45^\circ = \frac{2.5\text{cm}}{x}$$

$$1 = \frac{2.5\text{cm}}{x}$$

$$x = 2.5\text{cm}$$

$$\sin 45^\circ = \frac{2.5\text{cm}}{y}$$

$$\frac{\sqrt{2}}{2} = \frac{2.5\text{cm}}{y}$$

$$y\sqrt{2} = 2.5\text{cm} \times 2$$

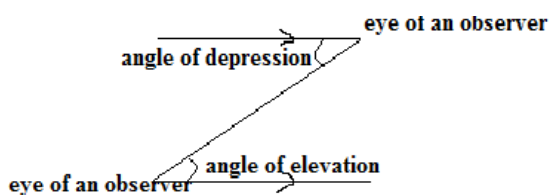
$$y = \frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$y = \frac{5\sqrt{2}}{2}$$

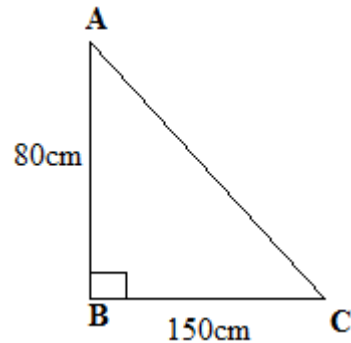
$$y = 2.5\sqrt{2}$$

**ANGLES OF ELEVATION AND DEPRESSION**

- I. Angles of depression and elevation are measured from eye level of an observer.
- II. Angle of depression is always equal to angle of elevation.

**EXAMPLE**

1. A church tower is 80m high. Find the angle of elevation of its top from a point on the ground 150m away on the level ground.

**Solution**

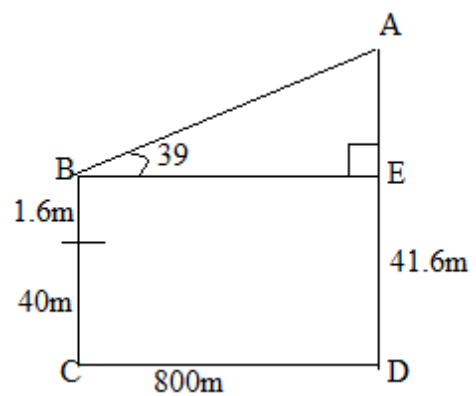
$$\tan C = \frac{80}{150}$$

$$= 0.5333$$

$$\tan^{-1} 0.5333 = 28.1^\circ$$

$$\therefore C = 28.1^\circ$$

2. A boy is standing on top of a building 40m tall watching an airplane in the sky. He estimates that the airplane is vertically above a point on the ground 800m from the base of the building. If the angle of elevation is  $39^\circ$ , and the boy is 1.6 tall. How high is the airplane above the ground?

**Solution**

In  $\Delta ABC$ ,  $BE = CD = 800\text{m}$

$$\tan = \frac{AE}{BE}$$

$$\tan 39^\circ = \frac{AE}{800\text{m}}$$

$$0.8098 = \frac{AE}{800\text{m}}$$

$$AE = 0.8098 \times 800\text{m}$$

$$= 647.84\text{m}$$

$$AD = AE + ED$$

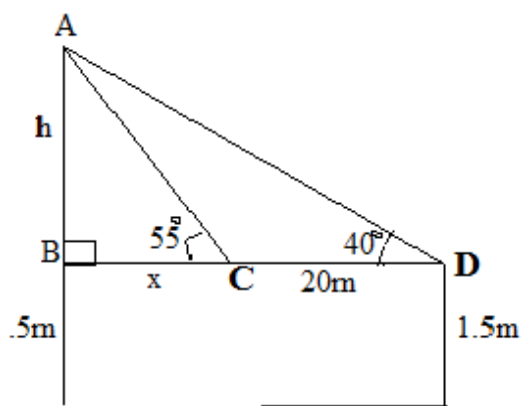
$$AD = 647.84 + 41.6$$

$$= 689.44\text{m}$$

$\therefore$  The airplane is 689.44m above the ground

3. A girl is looking up at the top of the building. She measures the angle of elevation of the building as  $40^\circ$ . She walks 20m towards the building and finds that the angle of elevation is now  $55^\circ$ , the girl is 1.5m tall. How far was she from the building when she started? How tall is the building?

**Solution**



$$\text{In } \Delta ABC; \tan 55^\circ = \frac{h}{x}$$

$$h = x \tan 55^\circ \dots\dots\dots \text{i)}$$

$$\text{In } \Delta ABD; \tan 40^\circ = \frac{h}{x+20}$$

$$h = (x + 20) \tan 40^\circ \dots\dots\dots \text{ii)}$$

$$\text{In i) put } h = x \tan 40^\circ$$

$$x \tan 55^\circ = (x + 20) \tan 40^\circ$$

$$1.4281x = 0.8391(x + 20)$$

$$1.4281x = 0.8391x + 16.7820$$

$$1.4281x - 0.8391x = 16.7820$$

$$0.5890x = 16.7820$$

$$x = \frac{16.7820}{0.5890}$$

$$= 28.492$$

$$= 28.5\text{m}$$

$$BD = 20\text{m} + 28.5\text{m} = 48.5\text{m}$$

$$\text{In i) put } x = 28.5\text{m}$$

$$h = 1.4281 \times 28.5\text{m}$$

$$= 40.7\text{m}$$

$\therefore$  The girl was 48.5m when she started

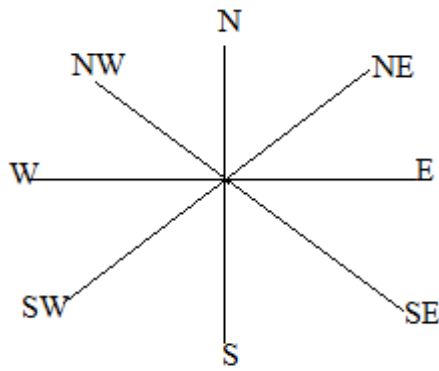
$$\text{The height of the building} = 40.7\text{m} + 1.5\text{m}$$

$$= 42.2\text{m}$$

### **BEARING**

Bearing is the direction of something as compared to the other.

It uses cardinal points e. g North, South, East, West e. t. c



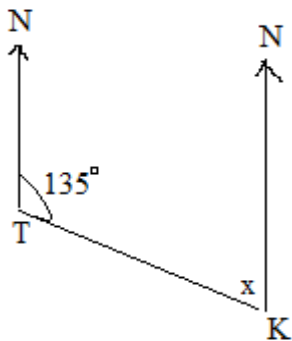
## TWO TYPES OF BEARING

- I. Three figure bearing
- II. Campus bearing

### THREE FIGURE BEARING

1. Measured in degrees
2. Uses three digits
3. If it is an acute angle, a zero may be added
4. It is measured from north clockwise

#### **EXAMPLE**



Find the bearing of

- a. K from T
- b. T from k

#### **Solution**

NB: we draw always north arrow at the point we need to find bearing.

∴ The bearing of K from T is  $135^\circ$

- b. Bearing of T from K

$$x + 135^\circ = 180^\circ \text{ allied angles}$$

$$x = 180 - 135^\circ$$

$$x = 45^\circ$$

$x + \angle K = 360^\circ$  angles at a point

$$\angle K = 360^\circ - 45^\circ$$

$$= 315^\circ$$

The bearing of T from K  $315^\circ$

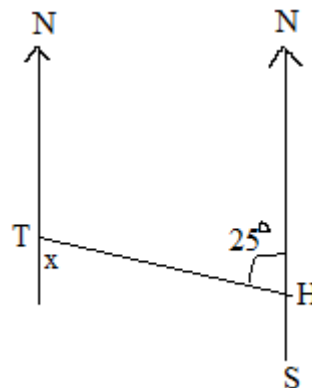
### CAMPUS BEARING

1. It is based on cardinal points
2. The angle is measured from north or south (whichever is nearer) turning east or west.
3. An acute angle is always taken

#### **EXAMPLE**

Find the campus bearing in the diagram below

- a. H from T
- b. T from H



#### **Solution**

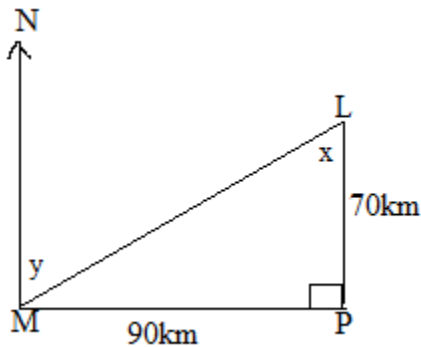
- a. The bearing of T from H is  $N25^\circ W$
- b.  $x = 25^\circ$ , alt angles

∴ The bearing of H from T is  $S25^\circ E$

#### **EXAMPLE**

1. A ship sails 70km from Likoma Island and then 90km west. What is the bearing of Likoma Island from its new position?

**Solution**



NB: we always draw a north arrow at point of bearing.

$$\tan L = \frac{90\text{km}}{70\text{km}}$$

$$= 1.2857$$

$$\angle L = 52.1^\circ$$

$$\therefore x = 52.1^\circ$$

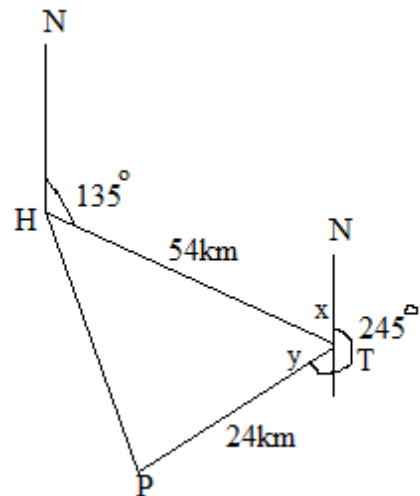
$$x = y, \text{ alt angles}$$

$$\therefore y = 52.1^\circ$$

$\therefore$  The bearing of Likoma Island from its new position is **052°** or **N52°E**

2. My mother leaves her home and travels on a bearing of 135° at a speed of 18km/h, after 3hrs she changes direction and continues on a bearing of 245° for another 24km. how far from home is she?

**Solution**



$$x + 155^\circ = 180, \text{ Allied angles}$$

$$x = 180^\circ - 155$$

$$x = 25^\circ$$

$$x + y + 245^\circ = 360, \angle \text{ s at a point}$$

$$25^\circ + 245^\circ + y = 360^\circ$$

$$y = 360^\circ - 270^\circ$$

$$y = 90^\circ$$

$\therefore \triangle HTP$  is a right angled triangle

$$\therefore HP^2 = HT^2 + TP^2$$

$$HP^2 = 24^2 + 54^2$$

$$HP^2 = 576 + 2916$$

$$HP = \sqrt{3492}$$

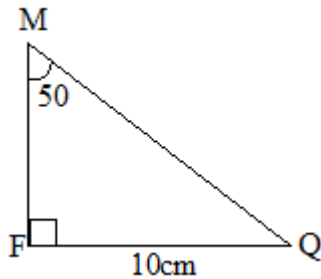
$$= 59.09\text{km}$$

$\therefore$  She is 59.1km away from home

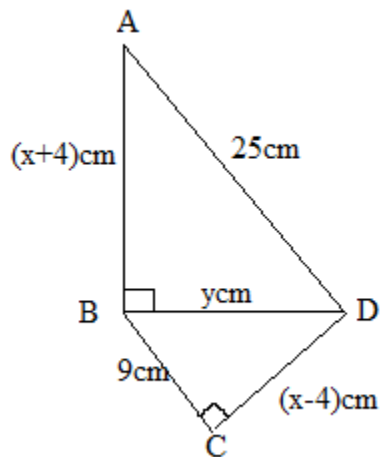
### EXERCISE

1. In the figure, find **MF** and area of triangle **MFO**.



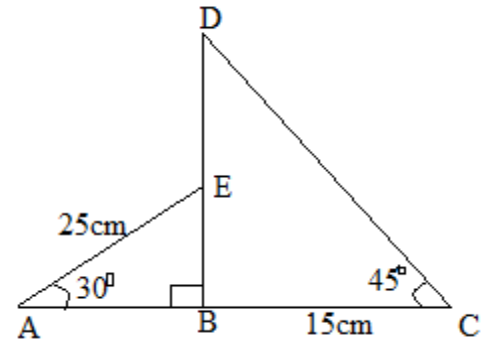


2. Town A is 56km from town B on a bearing of  $205^\circ$ 
  - a. Write down the bearing of B from A
  - b. Town C is 34km from town A on a bearing of  $115^\circ$ 
    - I. Find the bearing of C from B
    - II. Find the distance of C from B
3. The figure below ABD and BCD are right angled at B and C respectively.  $AB = (x+4)\text{cm}$ ,  $AD = 25\text{cm}$ ,  $BD = y\text{cm}$ ,  $BC = 9\text{cm}$  and  $CD = (x-4)$ .



Find the value of  $x$  and  $y$  and angle CBD

4. In the figure below, DB is perpendicular to the line ABC,  $AE = 25\text{cm}$ ,  $BC = 15\text{cm}$ , angle  $EAB = 30^\circ$ , angle  $BCD = 45^\circ$



Without using calculator, find the length of DE.

5. Mrs. Motto who is 1.6 tall stands 30m from a vertical tree and finds that the angle of elevation of the top of the tree is  $40^\circ$ . Find the height of the tree assuming the tree is perpendicular to the ground
6. Village B is on the bearing  $135^\circ$  and the distance of 40km from the village C is on a bearing of  $225^\circ$  and a distance of 62km from the village A.
  - a. Show that A, B and C form a right angled triangle
  - b. Calculate angle ACB to the nearest degree

**CHAPTER 9****EXPONENTIAL AND  
LOGARITHM FUNCTIONS**

$y = x^2$  It's an exponential function of the power 2

2 Is a power

$x$  Is a base

**LAWS OF INDICES**

- Any number to power 0 is equal to  $1a^0 = 1$  e. g  $5^0 = 1$
- Any number to power 1 is the same as that number  $a^1 = a$  e. g  $5^1 = 5$
- When multiplying numbers of the same bases, add powers e. g  
 $m^a \times m^b = m^{a+b}$
- When dividing numbers of the same base, subtract powers e. g  
 $m^a \div m^b = m^{a-b}$
- Any number with a negative power is the equal to the reciprocal of that number e. g

$$m^{-a} = \frac{1}{m^a}$$

$$6. m^{\frac{a}{b}} = (\sqrt[b]{m})^a$$

**SOLVING EXPONENTIAL FUNCTIONS**

When solving exponential functions, it is necessary to identify an appropriate base from a given number and raise it to a certain power.

**EXAMPLES**

- $2^x = 64$
- $7^n = \sqrt{7}$
- $8^x = 0.25$
- $2^{3x} = 4^{-1}$

**Solutions**

$$1. 2^x = 64$$

$$2^x = 2^6$$

$$\therefore x = 6$$

$$2. 7^n = \sqrt{7}$$

$$7^n = 7^{\frac{1}{2}}$$

$$\therefore n = \frac{1}{2}$$

$$3. 8^x = 0.25$$

$$2^{3x} = \frac{25}{100}$$

$$2^{3x} = \frac{1}{4}$$

$$2^{3x} = 4^{-1}$$

$$2^{3x} = 2^{-2}$$

$$3x = -2$$

$$x = \frac{-2}{3}$$

$$4. 2^{3x} = 4^{-1}$$

$$2^{3x} = 2^{-2}$$

$$3x = -2$$

$$x = \frac{-2}{3}$$

**MORE EXAMPLES**

$$1. \text{ Solve, } 2^{2x} + 5 \times 2^x + 4 = 0$$

**Solution**

$$2^{2x} - 5 \times 2^x + 4 = 0$$

$$\text{Let } 2^x = y$$

$$\therefore y^2 - 5y + 4 = 0$$

$$(y - 4)(y - 1) = 0$$

$$y = 4 \text{ or } y = 1$$

$$\text{But } 2^x = y$$

$$\therefore 2^x = 4 \quad \text{or} \quad 2^x = 1$$

$$2^x = 2^2 \quad 2^x = 2^0$$

$$\therefore x = 2 \quad x = 0$$

$$2. \text{ Solve, } 2^{2b+1} - 17(2^b) + 8 = 0$$

**Solution**

$$2^{2b+1} - 17(2^b) + 8 = 0$$

$$2^{2b} 2^1 - 17(2^b) + 8 = 0$$

$$\text{Let } 2^b = x$$

$$2x^2 - 17x + 8 = 0$$

$$(2x - 1)(x - 8) = 0$$

$$2x = 1 \text{ or } x = 8$$

$$x = \frac{1}{2} \text{ or } x = 8$$

$$\text{But } 2^b = x$$

$$\therefore 2^b = \frac{1}{2} \text{ Or } 2^b = 8$$

$$2^b = 2^{-1} \quad 2^b = 2^3$$

$$b = -1 \quad b = 3$$

### EXERCISE

$$1. \text{ Solve for } m \text{ and } n \quad 2^m \times 7^n = 392$$

$$2. \text{ Solve, } 3^{2x} - 6 \times 2^x + 9 = 0$$

$$3. (2^x)^2 - 9 \times (2^x) + 8 = 0$$

### LOGARITHM

$\log_a b = x$  It's a logarithm function

$\log_a b = x$  Is the same as  $b = a^x$

$b = a^x$  It's an exponential function

### RULES OF LOGARITHMS

$$1. \log_a a = 1$$

$$2. \log_x mn = \log_x m + \log_x n$$

$$3. \log_m 1 = 0$$

$$4. \log_x m^n = n \log_x m$$

$$5. \log_x \frac{m}{n} = \log_x m - \log_x n$$

### EXAMPLES

Express the following as a log of a single numbers

$$1. \log_3 8 + \log_3 5$$

Solution

$$\log_3 8 + \log_3 5 = \log_3 8 \times 5$$

$$= \log_3 40$$

$$2. \log_{10} 3 + \log_{10} 4 - \log_{10} 6$$

Solution

$$\log_{10} 3 + 2 \log_{10} 4 - \log_{10} 6$$

$$\log_{10} 3 + \log_{10} 4^2 - \log_{10} 6$$

$$\log_{10} \frac{3 \times 16}{6}$$

$$\log_{10} \frac{48}{6}$$

$$\log_{10} 8$$

$$3. \text{ Simplify } \frac{\log \sqrt{5}}{\log 25}$$

Solution

$$\frac{\log \sqrt{5}}{\log 2} = \frac{\log 5^{\frac{1}{2}}}{\log 5^2}$$

$$= \frac{\frac{1}{2} \log 5}{2 \log 5}$$

$$= \frac{1/2}{2}$$

$$= \frac{1}{2} \div 2$$

$$= \frac{1}{4}$$

### EVALUATING LOGARITHM

#### EXAMPLES

$$1. \text{ If } \log_{10} 2 = 0.301 \text{ and } \log_{10} 3 = 0.477, \text{ find the value of}$$

$$\text{a. } \log_{10} 6$$

Solution

$$\log_{10} 6 = \log_{10} 3 \times 2$$

$$\begin{aligned}
 &= \log_{10} 3 + \log_{10} 2 \\
 &= 0.477 + 0.301 \\
 &= 0.778
 \end{aligned}$$

b.  $\log_2 5$

**Solution**

$$\begin{aligned}
 \log 5 &= \log_{10} \frac{10}{2} \\
 &= \log_{10} 10 - \log_{10} 2 \\
 &= 1 - 0.301 \\
 &= 0.699
 \end{aligned}$$

2. Given that  $\log_{10} 2 = 0.301$  and  $\log_{10} 3 = 0.477$ . without using calculator; find  $\log_{10}(\tan 60^\circ)$ .

**Solution**

In special angles,  $\tan 60^\circ = \sqrt{3}$

$$\begin{aligned}
 \therefore \log_{10}(\tan 60^\circ) &= \log_{10} \sqrt{3} \\
 &= \log_{10} 3^{\frac{1}{2}} \\
 &= \frac{1}{2} \log_{10} 3 \\
 &= \frac{1}{2} \times 0.477 \\
 &= 0.2385
 \end{aligned}$$

3. If  $\log_5 x = a$  and  $\log_5 y = b$ , express the following in terms of a and b.  $\log_5 xy^3$

**Solution**

$$\begin{aligned}
 \log_5 xy^3 &= \log_5 x + \log_5 y^3 \\
 &= \log_5 x + 3\log_5 y \\
 &= a + 3b
 \end{aligned}$$

4. Evaluate without using a calculator or four figure table.  $\log_3 81$

**Solution**

$$\begin{aligned}
 \log_3 81 &= y \\
 \therefore 81 &= 3^y \\
 3^4 &= 3^y \\
 y &= 4
 \end{aligned}$$

## EXERCISE

Given that  $\log_{10} 2 = 0.301$  and  $\log_{10} 3 = 0.477$ . evaluate

- $\log_{10} 108$
- $\log_{10} 0.5$
- $\log_{10} 6$
- Evaluate  $\frac{1}{2} \log_{10} 25 - 2 \log_{10} 3 + \log_{10} 180$ . Without using a calculator.
- $\log_8 \sqrt{2}$

## SOLVING LOGARITHMS

### EXAMPLE

Given that  $\log_x 8 + \log_x 4 = 5$ , find the value of x

**Solution**

$$\begin{aligned}
 \log_x 8 + \log_x 4 &= 5 \\
 \log_x 8 \times 4 &= 5 \\
 \log_x 32 &= 5 \\
 32 &= x^5 \\
 2^5 &= x^5 \\
 \therefore x &= 2
 \end{aligned}$$

Solve the equation;

$$\log_{10}(2m + 6) = 1 + \log_{10}(m - 1)$$

**Solution**

$$\begin{aligned}
 \log_{10}(2m + 6) &= 1 + \log_{10}(m - 1) \\
 \log_{10} 2m + 6 &= \log_{10} 10 + \log_{10} m - 1 \\
 \log_{10} 2m + 6 &= \log_{10} 10(m - 1) \\
 2m + 6 &= 10m - 10 \\
 6 + 10 &= 10m - 2m \\
 16 &= 8m \\
 \therefore m &= 2
 \end{aligned}$$

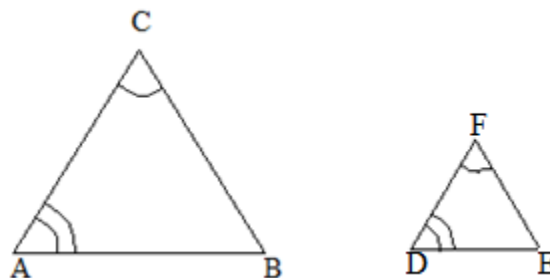
**EXERCISE**

Solve the following

1.  $\log_y 864 - \log_y 6 = 2$
2.  $\log_7 343 = 2x - 5$
3.  $\log_9 27^k = k + 1$
4.  $\log_a 3x - \log_a x - 5 = \log_a 8$
5. Given that  $\log p - \log q = 2 \log r$ . Find  $p$  in terms of  $q$  and  $r$
6. Solve,  $\log_3 w + \frac{3}{\log_3 w} = 4$

**CHAPTER 10****SIMILAR FIGURES****THEOREMS**

1. If two triangles are **equiangular**, their corresponding sides **are proportional**.



**Given:**  $\triangle ABC$  and  $\triangle DEF$ ,  $\angle A = \angle D$ ,  
 $\angle C = \angle F$

**To prove:**  $\frac{AC}{DF} = \frac{BC}{EF} = \frac{AB}{DE}$

**Proof:** in  $\triangle ABC$  and  $\triangle DEF$

$\angle A = \angle D$ , given

$\angle C = \angle F$ , given

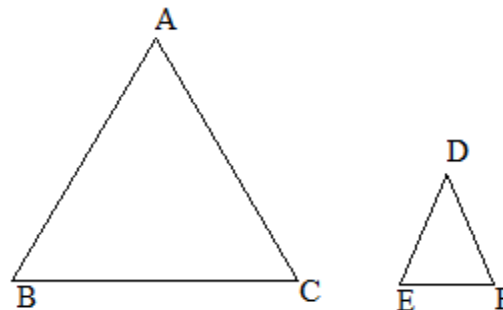
$\therefore \angle B = \angle E$ , sum  $\angle$  in a  $\triangle$

$\therefore \triangle ABC \sim \triangle DEF$  (a. a. a)

$$\therefore \frac{AC}{DF} = \frac{BC}{EF} = \frac{AB}{DE}$$

2. **Converse theorem**

If the corresponding sides of two triangles **are proportional**, then the triangles **are equiangular**.



**Given:**  $\frac{AC}{DF} = \frac{BC}{EF} = \frac{AB}{DE}$

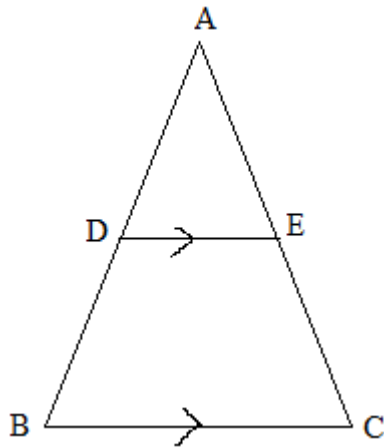
**To prove**  $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

**Proof:**  $\frac{AC}{DF} = \frac{BC}{EF} = \frac{AE}{DE}$ , given

$\therefore \triangle ABC \sim \triangle DEF$

$\therefore \angle A = \angle D, \angle B = \angle E, \text{ and } \angle C = \angle F$

3. For any given triangle, if a parallel line is drawn from one source of the triangle to the other side, then the two triangles formed are similar.



**Given:**  $\triangle ADE$  and  $\triangle ABC$ ,  $DE \parallel BC$

**To prove:**  $\triangle ADE \sim \triangle ABC$

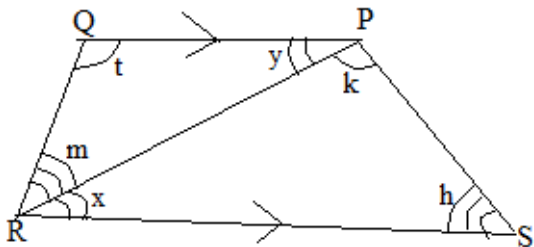
**Proof:** in  $\triangle ADE$  and  $\triangle ABC$

$\therefore DE \parallel BC$

$\therefore \triangle ADE \sim \triangle ABC$  (a. a. a)

### EXAMPLES

1. Figure below PQRS is a trapezium with  $PQ \parallel SR$  and  $\angle PQR = \angle SPR$ . Prove that  $PQ \times QR = PQ \times RS$



### Solution

**Given:**  $\triangle PQR$  and  $\triangle RPS$ ,  $PQ \parallel SR$ , trapezium PQRS,  $\angle PQR = \angle SPR$

**To prove:**  $PQ \times QR = PQ \times RS$

**Proof:** in  $\triangle PQR$  and  $\triangle RPS$

$x = y$ , alt angles

$t = k$ , given

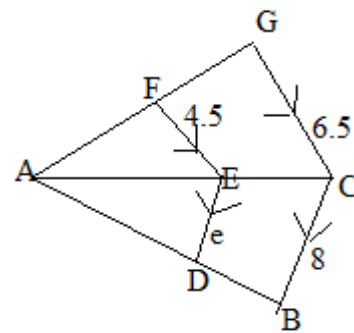
$\therefore m = h$ , sum angles in a  $\triangle$

$\therefore \triangle PQR \sim \triangle RPS$ , (a. a. a)

$$\therefore \frac{QR}{PS} = \frac{PQ}{RP}$$

$\therefore PQ \times QR = PQ \times RS$

2. Find the value of  $e$  in the figure below



### Solution

In  $\triangle AGC$  and  $\triangle AFE$ ,

$GC \parallel FE$ , given

$\therefore \triangle AGC \sim \triangle AFE$

$$\therefore \frac{AG}{AF} = \frac{GC}{FE} = \frac{AC}{AE}$$

$$\frac{6.5}{4.5} = \frac{AC}{AE}$$

In  $\triangle ABC$  and  $\triangle ADE$ ,

$BC \parallel DE$ , given,

$\therefore \triangle ABC \sim \triangle ADE$

$$\therefore \frac{AC}{AE} = \frac{8}{e}$$

$$\frac{6.5}{4.5} = \frac{8}{e}$$

$$6.5 \times e = 36$$

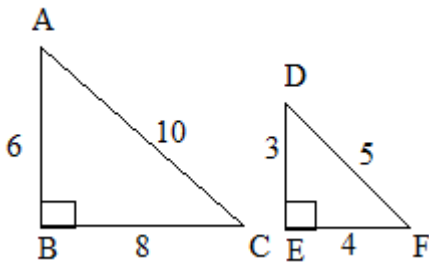
$$e = \frac{36}{6.5}$$

$$e = 5.5\text{cm}$$

### AREAS OF SIMILAR FIGURES

#### 1. Scale factor

Consider similar triangles below,



$$\frac{AC}{DF} = AC:BF$$

$$\frac{10}{5} = 10:5$$

2:1 is a scale factor

In general the ratio of the corresponding sides of two similar figures will give the scale factor.

#### 2. Ratio of the areas of similar figures

The ratio of the areas of two similar figures is equal to the ratio of the squares of their corresponding sides.

For example in the triangles ABC and DEF above;

$$\text{Area of } \triangle ABC = \frac{1}{2} \times 8 \times 6$$

$$= 24\text{cm}^2$$

$$\text{Area in } \triangle DEF = \frac{1}{2} \times 4 \times 3$$

$$= 6\text{cm}^2$$

$$\text{Area of } \triangle ABC : \text{area of } \triangle DEF = 24 : 6$$

$$= 4 : 1$$

$$\text{And } AB^2 : DE^2 = 6^2 : 3^2$$

$$= 36 : 9$$

$$= 4 : 1$$

$$\text{THUS } \frac{\text{area ABC}}{\text{Area DEF}} = \left(\frac{AB}{DE}\right)^2$$

$$\therefore \text{Area factor} = 4 : 1 \text{ (2}^2, 1)$$

In general if the lengths of corresponding sides of two similar figures are in the ratio **k: 1**, **k<sup>2</sup>: 1** is called area factor and **k: 1** is called scale factor.

#### EXAMPLE

1. Triangles **ABC** and **XYZ** are similar, the size of triangle **ABC** are **6cm, 7cm, and 8cm**. the shortest side of triangle **XYZ** is **2cm**. given that the area of triangle **XYZ** is **4.5cm<sup>2</sup>**. Calculate the area of triangle **ABC**

#### Solution

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle XYZ} = \left(\frac{6}{2}\right)^2$$

$$\frac{\text{Area of } \triangle ABC}{4.5\text{cm}^2} = \frac{36}{4}$$

$$\text{Area of } \triangle ABC = \frac{4.5\text{cm}^2 \times 36}{4}$$

$$= 40.5\text{cm}^2$$

2. The area of two similar triangles **ABC** and **XYZ** are in the ratio 1:16. If the height of the smaller triangle is **2cm**, calculate the height of the bigger triangle

#### Solution

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle XYZ} = \left(\frac{h_1}{h_2}\right)^2$$

$$\frac{1}{16} = \left(\frac{2}{h^2}\right)^2$$

$$\frac{1}{16} = \frac{4}{h^2}$$

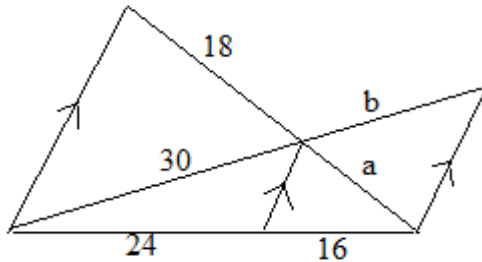
$$h^2 = 64$$

$$h = \sqrt{64}$$

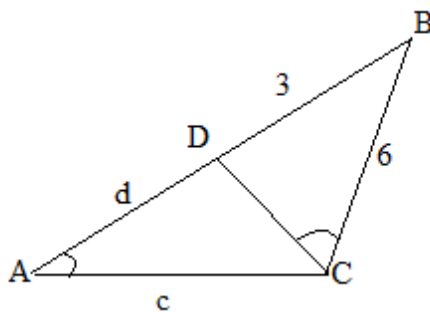
$$= 8\text{cm}$$

**EXERCISE**

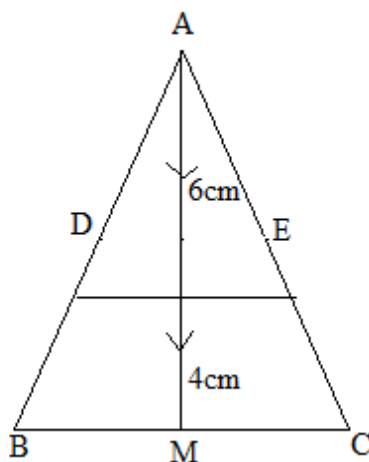
1. Find the value of the letters



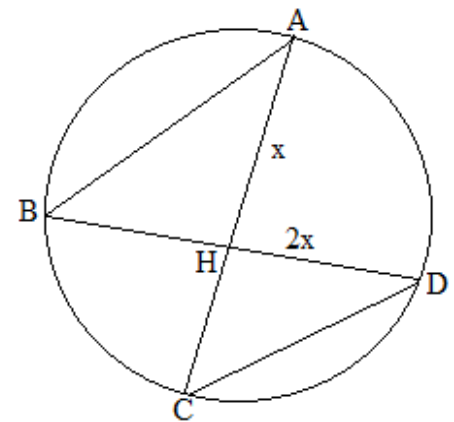
2.



3. Figure below shows two similar triangles ADE=12cm in which  $DE \parallel BC$ . The area of triangle ADE=12cm<sup>2</sup>. If the height of triangle ADE and trapezium DECB are 6cm and 4cm respectively. Calculate the length of BC



4. In the figure below, circle ABCD has chord AC and BD intersecting at H. length of line HD is twice that of HA.



Find the ratio of areas of triangle ABH to triangle CDH.

5. The areas of two similar triangles are **ABC** and **HKL** are 100cm<sup>2</sup> and 256cm<sup>2</sup> respectively. If the length of AB is 5cm, calculate the length of **HK** (MANEB).



**CHAPTER 11****TRANSFORMATION**

Transformation simply means complete change

**TYPES OF TRANSFORMATIONS**

1. Translation
2. Reflection
3. Rotation (out of syllabus)
4. Enlargement

**1. TRANSLATION**

To move from one place to the other.

$\vec{AB} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ , this is a translation vector or displacement vector or column vector.

- 2 means move two spaces to the right (x)
- -4 means move 4 spaces downwards (y)



In general:

**Coordinates + translation = coordinates**

**Of object      of vector      of image**

**EXAMPLE**

1. A (0, 2) is a point on the Cartesian graph, find the coordinates of its image under translation vector  $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$ .

**Solution**

C. Object + C. vector = C. image

$$\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ -2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$\therefore$  The coordinates of image are (-3, 0)

2. A point P (-2, 4) is translated to point P'. If P is 5 units down and three units to the right. Find the coordinates of P'.

**Solution**

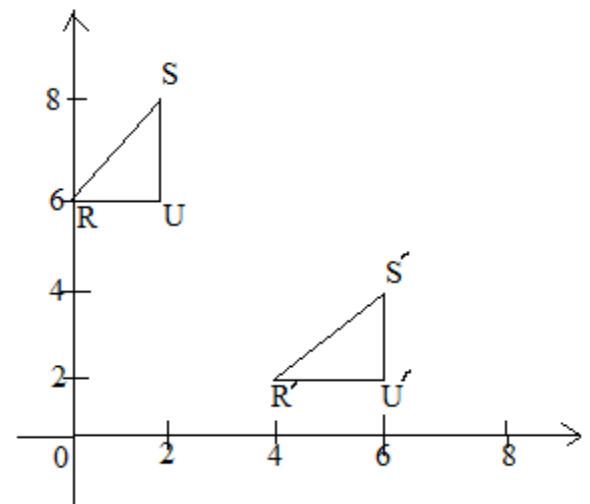
Object + Vector = Image

$$\begin{pmatrix} -2 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\therefore p' = (1, -1)$$

3. Triangle RSU has vertices R (0, 6), (2, 6), and (2, 8). Draw the graph on the paper provided, triangle RSU and its image under translation vector  $\begin{pmatrix} 4 \\ -4 \end{pmatrix}$

**Solution**

$$\therefore R' = (4, 2), \quad S' = (6, 4) \quad U' = (6, 2)$$

**EXERCISE**

1. Point A (3, 4) is translated to A' = (7, 9). Calculate the translation vector
2. P (1, 2) has the image Q (-2, -3) under translation. Find the translation vector.

**REFLECTION**

Reflection means to turn over

To have reflection, you need to have a mirror (line of symmetry)

The distance from the object to the mirror must be equal to the distance from the image to the mirror.

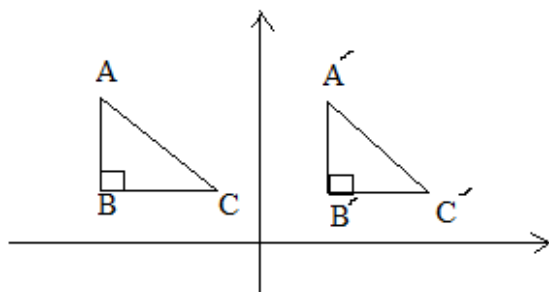
The line joining the object point and the image point must always be perpendicular to the mirror of symmetry

Reflection changes direction but not shape.

The line of symmetry is a line that divides an object into two equal parts such that the corresponding sides match each other when bent.

### EXAMPLE

1. In the figure below, reflect triangle ABC in the y-axis



### ENLARGEMENT

Means making something small or large

To enlarge a figure the following are used

- Centre of enlargement
- Scale factor

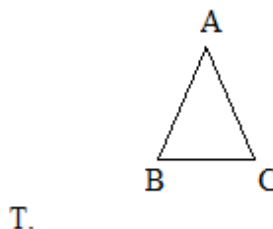
#### PROCEDURES

- Join Centre to each object's points
- Measure each length from the Centre to the object point
- Multiply each length by the scale factor to get the length from the Centre to image point
- When the scale factor is a fraction, the image will be smaller

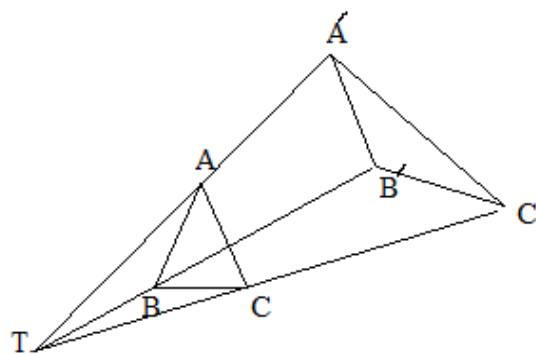
- When the scale factor is negative, the image becomes upside down
- When scale factor is a whole number, the image will be larger than the object

### EXAMPLES

Enlarge the figure below using a scale factor of 2.  
T is the center of enlargement.



#### Solution

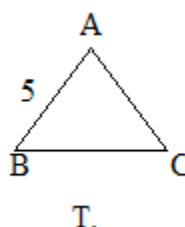


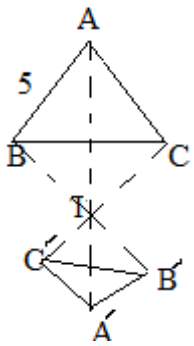
$$TA=3 \quad \therefore TA' = 3 \times 2 = 6$$

$$TB=2 \quad \therefore TB' = 2 \times 2 = 4$$

$$TC=4 \quad \therefore TC' = 4 \times 2 = 8$$

2. Enlarge triangle ABC using scale factor  $-\frac{1}{2}$  and Centre T



**Solution**

NB: When the scale factor is negative

- The Centre is between image and object
- The image is upside down

**HOW TO FIND SCALE FACTOR**

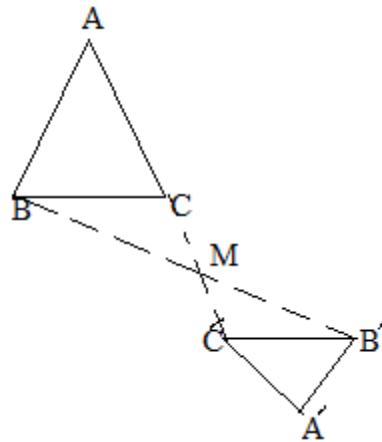
$$\text{Scale factor} = \frac{\text{length of image}}{\text{length of object}}$$

$$\text{Or} = \frac{\text{line from image to the centre}}{\text{line from object to the centre}}$$

NB: when finding scale factor consider the size of and shape (is it upside down or not?)

**FINDING CENTRE**

Corresponding points of the object and image, where the lines meet that is the Centre of enlargement.

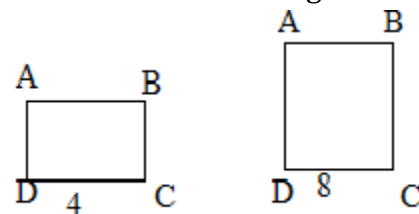
**EXAMPLE**

M is the Centre of enlargement and scale

$$\text{factor} = \frac{B'C'}{BC}$$

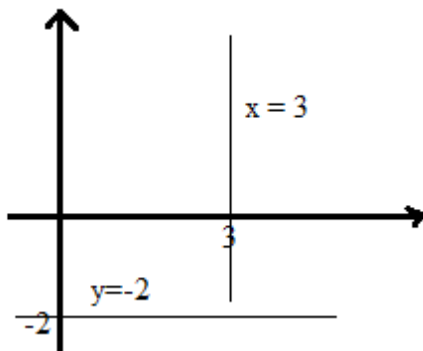
**EXERCISE**

1. A triangle has vertices D(0, 2), E(4,6) and F(6, 2).
  - a. Using the scale of 2cm to represent 2 units on both axes, draw triangle DEF on the graph paper provided
  - b. On paper provided enlarge DEF using O (0,2) as a Centre
2. Find the Centre of the enlargement and the scale factor for the figure below.



**CHAPTER 12****COORDINATE GEOMETRY**

- $x$  – coordinates are those coordinates along  $x$ -axis
- $y$  – coordinates are those coordinates along  $y$  –axis
- All equations in terms of  $x$  –coordinates gives vertical lines.
- All equations in terms of  $y$  –coordinates gives horizontal lines

**SKETCHING A STRAIGHT LINE GRAPH**

When sketching a straight line graph you need to know coordinates of two points.

**EXAMPLE**

$y = 2x + 6$  (6 is a constant or  $y$ -intercept)

Sketching

- You need to know where graph cuts  $y$ -axis and  $x$ -axis
- The graph cuts the  $y$  axis where  $x=0$
- The graph cuts the  $x$  axis where  $y=0$

So,  $y = 2x + 6$

When  $x=0$

$$y = 2(0) + 6$$

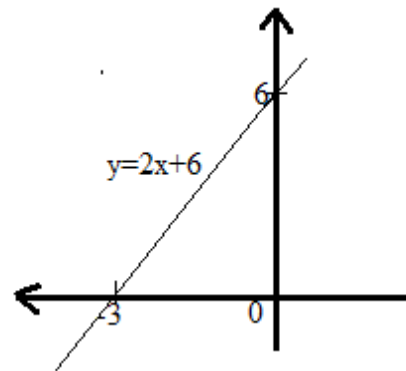
$$y = 6$$

When  $y=0$

$$0 = 2x + 6$$

$$-6 = 2x$$

$$x = -3$$



$\therefore -3$  is  $x$ - intercept

**GRADIENT (M)**

Gradient is a slope of a line.

You can find gradient;

- From a given equation
- Using two coordinates
- From a given graph

**A. FROM A GIVEN EQUATION**

We can find gradient from a given equation by making  $y$  subject of the formula. The coefficient of  $x$  is the gradient.

**EXAMPLE**

$$1. \quad 2y - 3x = 0$$

$$2y = 3x$$

$$y = \frac{3}{2}x$$

$$\therefore \text{Gradient} = \frac{3}{2}$$

$$2. \quad y = 2 - x$$

$$\therefore \text{Gradient} = -1$$

**B. GRADIENT FROM COORDINATES OF TWO POINTS**

$$\text{Gradient (m)} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

**EXAMPLE**

- Find the gradient of a line passing through  $(0, -2)$  and  $(2, 4)$

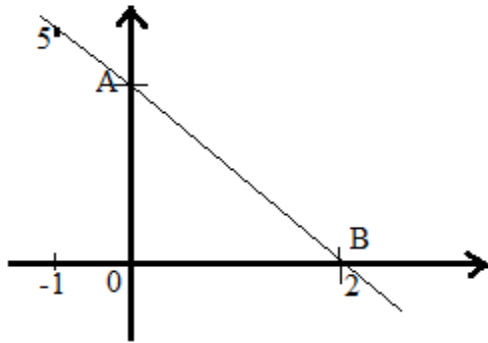
**Solution**

$$(0, -2) \quad (2, 4)$$

$$\begin{aligned}\text{Gradient} &= \frac{4 - (-2)}{2 - 0} \\ &= \frac{6}{2}\end{aligned}$$

**C. GRADIENT FROM A GIVEN GRAPH****EXAMPLE**

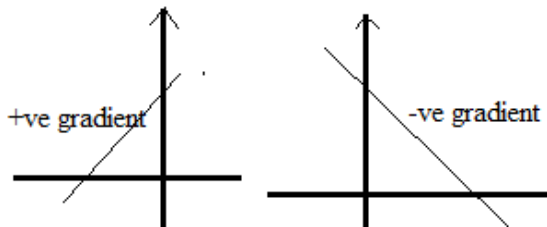
Find the gradient of the line AB below



$$\begin{aligned}&(2, 0) \quad (-1, 5) \\ \therefore m &= \frac{5 - 0}{-1 - 2} \\ &= \frac{5}{-3}\end{aligned}$$

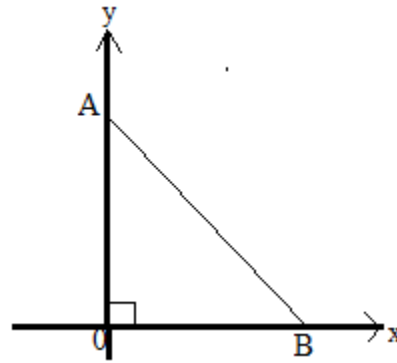
NB: when line slopes upwards from left to right, then the gradient is positive.

: Gradient will be negative if the line slopes from left to right.



The gradient of straight line is zero

The gradient of a vertical line is undefined

**TANGENT OF AN ANGLE AND GRADIENT**

$$\text{Gradient AB} = \frac{AO}{BO}$$

$$\tan B = \frac{AO}{BO}$$

$\therefore$  Gradient of AB = Tan of AB

In general,  $\tan \theta = \text{Gradient}$

**FINDING EQUATION OF A STRAIGHT LINE**

The equation can be found when given the following

- Gradient and y-intercept
- Gradient and coordinate of a point
- Coordinates of two points

In general, equation for all linear equation is  $y = mx + c$

Where,  $m = \text{gradient}$

$c = \text{y-intercept}$

**A. Given gradient and y-intercept****EXAMPLE**

Find the equation of a line where gradient is -2 and y-intercept is 10.

**Solution**

$$y = mx + c$$

$$m = -2, \quad c = 10$$

$$y = -2x + 10$$

**B. Given a gradient and coordinates of a point****EXAMPLE**

Find the equation of a straight line passing through (3, -4), where gradient is 2.

**Solution**

$$y = mx + c$$

$$m = 2, y = -4, x = 3$$

$$\therefore -4 = 2(3) + c$$

$$-4 = 6 + c$$

$$c = -10$$

$$\text{But } m = 2$$

$$\therefore y = 2x - 10$$

**C. Given coordinates of two points****EXAMPLE**

A straight line passes through points (3, 1) and (4, 2). Find the equation of the line

**Solution**

$$(3, 1) (4, 2)$$

$$m = \frac{2-1}{4-3}$$

$$= 1$$

$$m = 1 \quad y = 1 \quad x = 3$$

$$y = mx + c$$

$$1 = 1(3) + c$$

$$1 = 3 + c$$

$$1 - 3 = c$$

$$c = -2$$

$$\therefore y = x - 2$$

**EXERCISE**

1. A straight line passes (3, 1) and (b, 2). Find the value of b if the gradient of the line is 1.
2. Find the gradient of a straight line whose equation is  $\frac{y+2x}{4} = \frac{x}{3}$
3. A gradient of a straight line passing through (-2, 5) is  $\frac{1}{2}$ . Find the equation of the straight line.
4. Given that A (1, -1) and B (7, -9). Find the equation of the line AB

**GRADIENT OF PARALLEL LINES**

All parallel lines have got equal gradient.

**EXAMPLE**

A straight line  $3y = x$  is parallel to a line which passes through a point (4, -5). Find the other equation.

**Solution**

$$3y = x$$

$$y = \frac{x}{3} = \frac{1}{3}x$$

$$\therefore m = \frac{1}{3} \quad y = -5 \quad x = 4$$

$$\therefore y = mx + c$$

$$-5 = \frac{1(4)}{3} + c$$

$$-5 = \frac{4}{3} + c$$

$$c = \frac{-19}{3}$$

$$\therefore y = \frac{1}{3}x - 19$$

2. A straight line  $y + (b - 2)x + 10 = 0$  is parallel to  $y = 2bx - 3$ . find the value of b.

**Solution**

$$\begin{aligned}
 y + (b - 2)x + 10 &= 0 \\
 y &= -(b - 2)x - 10 \\
 \therefore 2b &= -(b - 2) \\
 2b &= -b + 2 \\
 2b + b &= 2 \\
 3b &= 2 \\
 b &= \frac{2}{3}
 \end{aligned}$$

### GRADIENT OF A PERPENDICULAR LINE

The gradient of perpendicular line is upside down negative of the other gradient.

Thus,  $m_1 \times m_2 = -1$  or the product of gradient of perpendicular lines is -1

### **EXAMPLE**

A straight line passing through a point (7, 2) is perpendicular to line  $4y = 3 - x$ . Find the equation of this line, hence find the coordinates of the point of intersection of the two lines.

### Solution

$$4y = 3 - x$$

$$y = \frac{3}{4} - \frac{x}{4}$$

$$m = -\frac{1}{4}$$

$$m \times \frac{-1}{4} = -1$$

$$m = \frac{-1 \times 4}{-1}$$

$$m = 4$$

$$y = mx + c$$

Taking (7, 2)

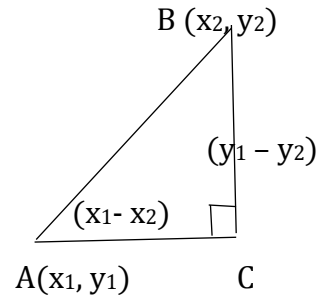
$$m = 4 \quad x = 7 \quad y = 2$$

$$2 = 4(7) + c$$

$$c = -26$$

$$\therefore y = 4x - 26$$

### DISTANCE BETWEEN TWO POINTS



$$AB^2 = AC^2 + BC^2, \text{ Pythagoras theorem}$$

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

### **EXAMPLE**

1. A straight line passes A (4, -2) and B(3, -1). Find the distance of the line AB.

### Solution

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$A (4, -2) \quad B (3, -1)$$

$$AB = \sqrt{(3 - 4)^2 + (-1 - (-2))^2}$$

$$= \sqrt{1 + (-1 + 2)^2}$$

$$= \sqrt{1 + 1}$$

$$= \sqrt{2}$$

### **EXERCISE**

1. A straight line passes through point A (b, -2) and B (3, -1). Find the value of b if the distance of line AB is  $\sqrt{2}$ .
2. Find the equation of a straight line which passes through (1, -2) perpendicular to  $2x + 3y - 3 = 0$
3. Find the equation of the line

- a. Through (3, 4) parallel to  $y=2x+5$
- b. Through (-2, 3) parallel to  $6y + 2x - 5=0$
4. The distance between M (3, 5) and N (8, a) is 13. Find the value of a.
5. Calculate the distance between x and y intercepts of the graph  $y = 12 - 2.4x$
6. The straight line  $2x + 3y + 5 = 0$  and  $3x = -2y$  intersect at point M. without drawing graphs find the coordinates of M.

A statement containing a  $<$  or  $\leq$  is called an inequality.

### Symbol and their meanings

$<$  (less than)

$>$  (greater than)

$\leq$  (less than or equal to)

$\geq$  (greater than or equal to)

### LINES TO USE

( $<$ ,  $>$ ) Dotted line (-----)

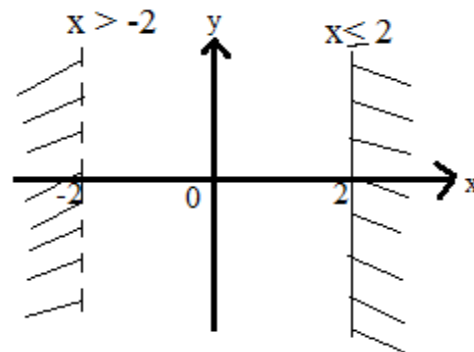
( $\leq$ ,  $\geq$ ) Solid line (—————)

When shading inequalities on the graph, shade the unwanted region.

### EXAMPLE

Show the inequality  $x > -2$  and  $x \leq 2$  on the graph and shade on the unwanted region.

### Solution



2. Sketch  $0 \leq x < 8$

### Solution

$$0 \leq x < 8$$

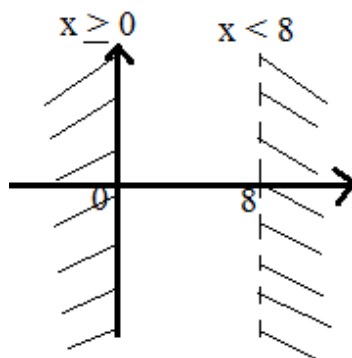
$$x < 8, \quad x \geq 0$$

## CHAPTER 13

## LINEAR INEQUALITIES

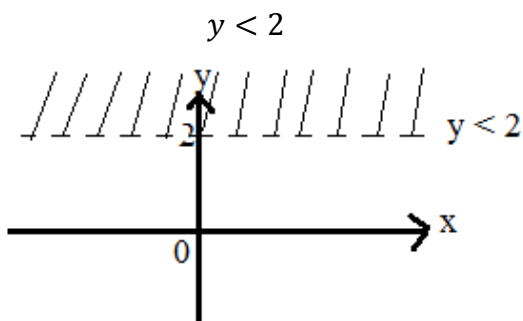
A statement containing ( $=$ ) sign is called an equation.





3. Sketch  $y < 2$

**Solution**



## INEQUALITIES IN TWO VARIABLES

### **EXAMPLE**

1. Sketch  $y \leq 2x + 3$  in the graph

**Solution**

$$y = 2x + 3$$

When  $x = 0$

$$y = 2(0) + 3$$

$$y = 3 \quad (0, 3)$$

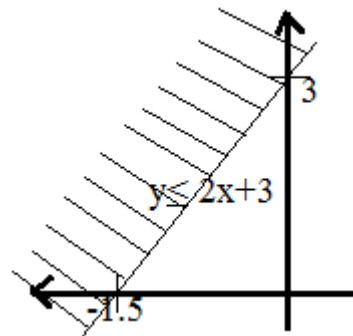
When  $y = 0$

$$0 = 2x + 3$$

$$-3 = 2x$$

$$x = -\frac{3}{2}$$

$$= -1.5 \quad (-1.5, 0)$$



## TWO OR MORE INEQUALITIES

### Example

1. Sketch the region

$$Q \{(x, y): 3 < x + 3y \leq 6\}$$

**Solution**

$$3 < x + 3y$$

$$x + 3y \leq 6$$

**a.  $3 < x + 3y$**

When  $x = 0$

$$3 = (0) + 3y$$

$$y = 1 \quad (0, 1)$$

When  $y = 0$

$$3 = x + 3(0)$$

$$x = 3 \quad (3, 0)$$

**b.  $x + 3y = 6$**

When  $x = 0$

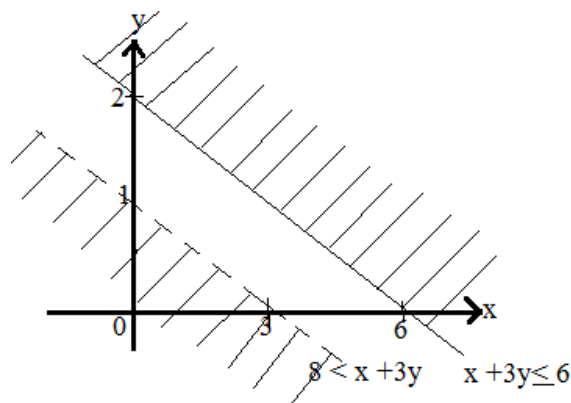
$$(0) + 3y = 6$$

$$y = 2 \quad (0, 2)$$

When  $y = 0$

$$x + 3(0) = 6$$

$$x = 6 \quad (6, 0)$$



2. Sketch defined by the following inequalities giving coordinates and vertices

- $x \leq 4$
- $y \leq 5$
- $x + y \geq 3$

**Solution**

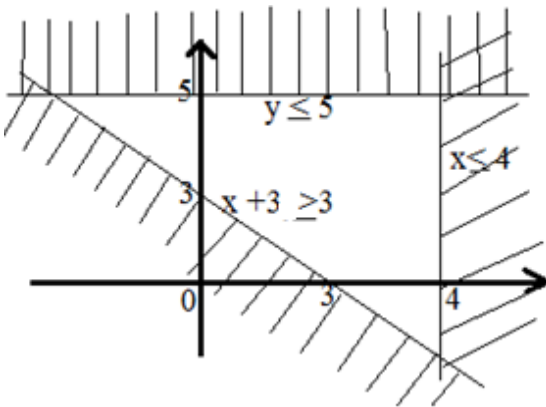
$$x + y = 3$$

$$x = 0, (0) + y = 3$$

$$y = 3 \quad (0, 3)$$

$$y = 0, \quad x + (0) = 3$$

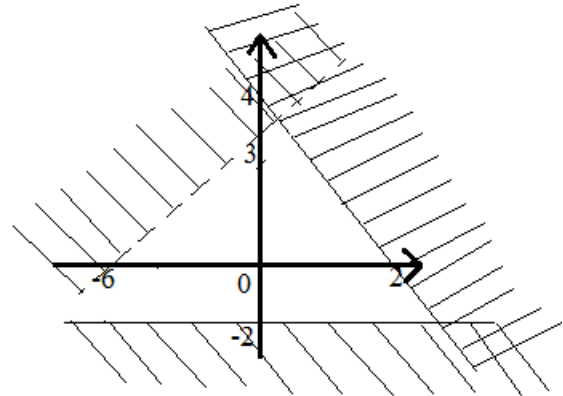
$$x = 3 \quad (3, 0)$$



**FINDING EQUATIONS FROM A GIVEN GRAPH**

**EXAMPLE**

Find three inequalities which describes the graph below.



**Solution**

$$1. \quad y \geq -2$$

$$2. \quad (2, 0) \quad (0, 4)$$

$$m = \frac{4-0}{0-2}$$

$$m = \frac{4}{-2} = -2$$

Taking  $(0, 4)$

$$y = mx + c$$

$$4 = -2(0) + c$$

$$c = 4$$

$$y = -2x + 4$$

$$\therefore y \geq -2x + 4$$

$$3. \quad (-6, 0) \quad (0, 3)$$

$$m = \frac{3-0}{0-(-6)}$$

$$m = \frac{3}{6} = \frac{1}{2}$$

Taking  $(-6, 0)$

$$y = mx + c$$

$$0 = \frac{1}{2}(-6) + c$$

$$c = 3$$

$$y = \frac{1}{2}x + 3$$

$$\therefore y < \frac{1}{2}x + 3$$

**EXERCISE**

1. M is the set of points (x, y) which satisfies the following inequalities

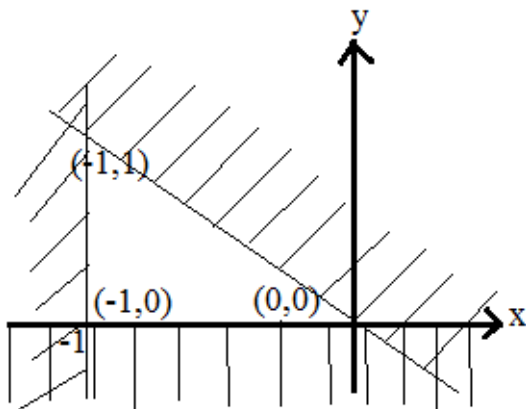
- i.  $y - x \leq 1$
- ii.  $2x \leq 5$
- iii.  $5y > -4x$
- iv.  $y \leq 2$

Show on the graph the region represented by M

2. On the same axes, sketch the graphs of the regions described by the following inequalities.

- i.  $x \geq 0$
- ii.  $y \geq 0$
- iii.  $y \leq 3x + 2$
- iv.  $y + 4x < 8$

3. The fig below shows unshaded region bounded by inequalities. Write down three inequalities from the graph.



## CHAPTER 14

## VARIATIONS

To vary means to change.

### TYPES OF VARIATIONS

- 1. Direct variations
- 2. Inverse variations(inverse)
- 3. Joint variations
- 4. Partial variations

### DIRECT VARIATION

Variation in which increase in one variable results in the other variable, e. g

$$D \propto T$$

Distance varies with time.

$$D = kt, \text{ where } k \text{ is constant.}$$

### EXAMPLES

- i. If P is proportional to q and p=30 when q=6, find the constant proportional and find the value of p when q=24.

#### Solution

$$P \propto q$$

$$p = kq$$

$$p=30, q=6$$

$$\therefore 30 = k \times 6$$

$$\frac{30}{6} = k$$

$$k=5$$

$$\therefore p=5q, \text{ when } q=24$$

$$P=5 \times 24$$

$$=120$$

- ii. If P varies directly as the cube of q, and p=6 when q=2, find p when q=8

#### Solution

$$P \propto q^3$$

$$p=kq^3$$

$$p=6, q=2$$

$$6=k(2)^3$$

$$6=8k$$

$$\frac{6}{8} = k$$

$$k = \frac{3}{4}$$

$$\therefore p = \frac{3}{4}q^3$$

$$q = 8$$

$$P = \frac{3}{4} \times 8^3$$

$$P = \frac{3}{4} \times 8 \times 8 \times 8$$

$$P = 38.4$$

### INVERSE VARIATIONS

This is the variation in which one quantity increases while the other quantity decreases.

### EXAMPLES

1. If P varies inversely as q and p=4, when q=6, find p when q=8

#### Solution

$$p \propto \frac{1}{q}$$

$$p = \frac{k}{q}$$

$$p=4, q=6$$

$$4 = \frac{k}{6}$$

$$\therefore k=24$$

$$\therefore p = \frac{k}{q}$$

$$q=8$$

$$\therefore p = \frac{24}{8} = 3$$

2. The number of beats per minutes of a pendulum varies inversely as the square root of its length. An **81cm** long pendulum makes **24** beats per minutes. Calculate the number of beats per minutes of a **25cm** pendulum.

#### Solution

$$n \propto \frac{1}{\sqrt{l}}$$

$$n = \frac{k}{\sqrt{l}}$$

$$L=81\text{cm}, n=24\text{beats}$$

$$24 = \frac{k}{\sqrt{81}}$$

$$9 \times 24 = k$$

$$\therefore k=216$$

$$\therefore n = \frac{216}{\sqrt{l}}$$

$$l=25, k=216$$

$$\therefore n = \frac{216}{\sqrt{25}}$$

$$= \frac{216}{5}$$

$$= 43.2\text{beats}$$

### JOINT VARIATIONS

This is the combination of variations

### EXAMPLES

1. If  $x \propto zy$  and  $x = 8$  when  $y = 2$  and  $z = 1$ . find  $x$  when  $y = 3$  and  $z = 2$

#### Solution

$$x \propto zy$$

$$x = kzy$$

$$x=8, z=1, y=2$$

$$\therefore 8 = k \times 1 \times 2$$

$$8 = 2k$$

$$K=4$$

$$\therefore x = 4zy$$

$$y = 3, z = 2$$

$$\therefore x = 4 \times 3 \times 2$$

$$= 24$$

$$x = 24$$

2. Suppose that  $w$  varies directly as  $z^2$  and inversely as  $xy$ , and that  $w=10$  when  $x=15, y=2$  and  $z=5$ . Find  $z$  when  $w=2, x=8$  and  $y=27$

**Solution**

$$w \propto \frac{z^2}{xy}$$

$$w = \frac{kz^2}{xy}$$

$$w = 10, x = 15, y = 2, z = 5$$

$$10 = \frac{k5^2}{15 \times 2}$$

$$10 = \frac{25k}{32}$$

$$300 = 25k$$

$$k = 12$$

$$w = \frac{12z^2}{xy}$$

$$w = 2, x = 8 \text{ and } y = 27$$

$$\therefore 2 = \frac{12 \times z^2}{8 \times 27}$$

$$2 = \frac{12z^2}{216}$$

$$432 = 12z^2$$

$$z^2 = 36$$

$$z = \pm \sqrt{36}$$

$$\therefore z = 6 \text{ or } -6$$

### **PARTIAL VARIATION**

Involves three set of numbers.

The first set gives first equations.

The second set gives second equations.

The third set gives answer to the equation.

### **EXAMPLES**

1.  $a$  is partly constant and partly varies as  $b$  when  $b=4, a=60$  and when  $b=12, a=100$
- a) Find the relationship between  $a$  and  $b$
- b) Find  $a$  when  $b=3$

**Solution**

a).  $a \propto c + b$

$$a = c + kb$$

$$a = 60, b = 4$$

$$\therefore 60 = c + 4k \dots \dots \dots (1)$$

$$b = 12, a = 100$$

$$\therefore 100 = c + 12k \dots \dots \dots (2)$$

$$100 = c + 12k$$

$$- 60 = c + 4k$$

$$40 = 8k$$

$$K = 5$$

In equation 2 substitute with  $k=5$

$$60 = c + 4(5)$$

$$60 = c + 20$$

$$60 - 20 = c$$

$$40 = c$$

b). when  $b = 3$

$$\therefore a = 40 + 5(3)$$

$$a = 40 + 15$$

$$a = 55$$

### **EXERCISE**

1. The distance travelled by a lorry varies directly as the time taken for the journey. Find how far the lorry travels in 5 hours if it can travel 86 km in 1:15 hour.
2.  $A$  varies jointly with  $b$  and with  $c$ , and  $a=12$  when  $b=3$  and  $c=2$
- a) Find an equation which expresses  $a$  in terms of  $b$  and  $c$ .

c. Find the value of a when  $b = \frac{1}{4}$  and  $c = \frac{1}{2}$

3. A quantity b varies jointly with r and t, and  $b=108$  when  $r=3$  and  $t=6$ . Find an equation which expresses b in terms of r and t.
5. Given that  $x \propto \frac{y}{z^2}$  and  $x=12$  when  $y=2$  and  $z=1$ , Find y when  $z=2$  and  $x=15$ .
6. The cost of running a private secondary school per is partly constant and partly varies as the square of the number of students enrolled in the school. If the school has 100 students, the cost per day is k4000 and if there are 160 students the cost per day is k7900. Calculate the cost of running the school if it has 240 students.

## CHAPTER 15

## STATISTICS

Statistics refers to a set of data such as average, mode, median standard deviation e. t. c

Data is the unprocessed information.

### FREQUENCY TABLE

Construct a frequency table from the ages of form four students

20, 17, 15, 18, 19, 17, 17, 18, 20, 17, 19, 19, 18, 19, 17, 18, 17, 18, 18, 20, 17, 21, 20, 19, 17, 17, 18, 17, 16, 16, 16, 16, 20, 18, 17, 19, 20, 19, 19, 18, 18, 19, 21, 18, 17, 21, 18, 18, 19, 18, 18

### Solution

| Score | Tally                          | Frequency       |
|-------|--------------------------------|-----------------|
| 15    | /                              | 1               |
| 16    | ///                            | 3               |
| 17    | //// //// /                    | 11              |
| 18    | //// <del>////</del><br>/// // | 17              |
| 19    | //// //// /                    | 11              |
| 20    | //// //                        | 7               |
| 21    | //                             | 2               |
|       |                                | $\Sigma f = 52$ |

Ways of showing data from the frequency table

1. Histogram
2. Frequency polygon

### HISTOGRAM

- ✓ It uses rectangles/ bars.
- ✓ The area of a triangle is proportional to frequency
- ✓ The width of rectangles are equal in each bar.

- ✓ It must have a title and the axes must be labeled

### PIE CHART

A pie chart is a circular way of showing data in relation to the whole data.

The circle represents the total, each slice represents the named parts of the data.

### **EXAMPLE**

- At a certain political meeting, a researcher observed that out of 90 people, 30 people had black shirts, 20 had blue shirts, 10 had white shirts, 5 had no shirts. Show the information on a pie chart.

### **Solution**

90 people represent  $360^\circ$

Black shirts = 30 people

$$= \frac{30 \times 360^\circ}{90}$$

$$= 120^\circ$$

Yellow shirts = 20 people

$$= \frac{20 \times 360^\circ}{90}$$

$$= 80^\circ$$

Blue shirts = 25 people

$$= \frac{25 \times 360^\circ}{90}$$

$$= 100^\circ$$

White shirts = 10 people

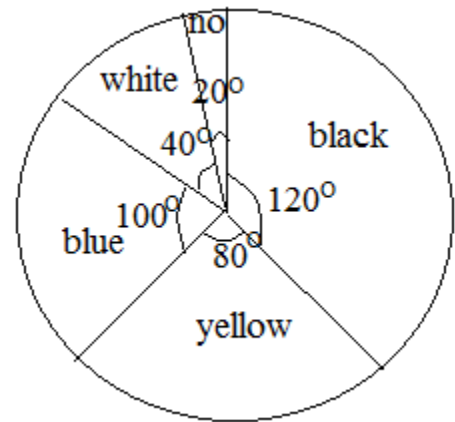
$$= \frac{10 \times 360^\circ}{90}$$

$$= 20^\circ$$

No shirt = 5 people

$$= \frac{5 \times 360^\circ}{90^\circ}$$

$$= 20^\circ$$



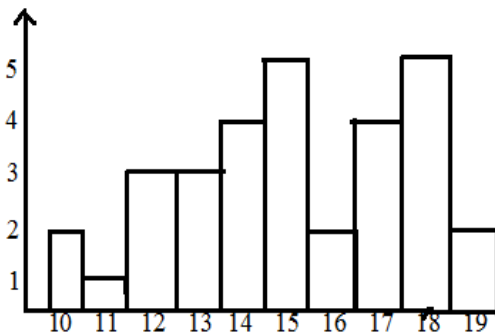
- A teacher was concerned that her pupils were taking too long to answer examination questions. She asked them to record how long it took them to answer a certain questions. These are the times in minutes that they told her.  
10, 15, 18, 17, 15, 13, 18, 19, 12, 15, 17, 18, 13, 14, 12, 14, 14, 18, 19, 17, 15, 17, 15, 13, 18, 19, 12, 11, 10, 14, 16, 16
  - Draw up a frequency table for this information
  - Draw a histogram to show this information
  - Examination question is worth 10% of the 2 hours paper.
    - How long should each student be spending on the questions
    - What percentage of the students are spending on the questions

### **Solution**

a.

| Score (x) | tally | Frequency       |
|-----------|-------|-----------------|
| 10        | //    | 2               |
| 11        | /     | 1               |
| 12        | ///   | 3               |
| 13        | ///   | 3               |
| 14        | ////  | 4               |
| 15        | ##### | 5               |
| 16        | //    | 2               |
| 17        | ////  | 4               |
| 18        | ##### | 5               |
| 19        | ///   | 3               |
|           |       | $\Sigma f = 32$ |

a. TIME TAKEN FOR A STUDENT TO ANSWER A QUESTION



b. 1. 2 hour = 120 minutes

$$\therefore 10\% = \frac{10}{100} \times 120$$

= 12 minutes

$\therefore$  Each student should spend 12 minutes for a question

12 minutes = 10%

2. Greater than 12 min = more

People who solve in greater than 12 minutes

$$= 32 - 6 = 26$$

$$= \frac{26}{32} \times 100$$

$$= 81.25\%$$

### GROUPED DATA

When we have a large amount of data, it is useful to group the data into classes

This makes data display and calculations simple.

### Things to be known

- I. Class (group)
- II. Class limits
- III. Class boundaries
- IV. Class mid points
- V. Class width

### EXAMPLE

The following are weights of 30 pupils (in kg)

45, 62, 35, 54, 48, 55, 48, 59, 52, 40, 54, 46, 59, 51, 32, 37, 49, 42, 53, 38, 37, 35, 53, 46, 48, 44, 33, 52, 54, 44

- a. Using classes of 30- 34, 35- 39, 40- 44 and so on, construct a frequency table for this data.
- b. For the second class (35- 39), write down the lower and upper class boundaries and hence find class width.
- c. What percentage of pupils had their weight at least 52 kg?

### Solution

30- 34 or 35- 39 are class limits

30 or 35 are lower class limits

34 or 39 are upper class limits

NB; To find class boundaries, subtract 0.5 from lower class limits and add 0.5 to upper class limits. E. g for 30- 39, 29.5 is a lower class boundary and 34.5 is the upper class boundary

$$\text{i.e. } 30 - 0.5 = 29.5$$

$$34 + 0.5 = 34.5$$



**Class width**

Is the difference between the upper and lower class boundary.

I.e. for 30- 34

$$\text{Class width} = 34.5 - 29.5$$

$$= 5$$

**Class mid points**

$$\text{Mid-point} = \frac{\text{upper class b} + \text{lower class b}}{2}$$

$$\text{i.e. } \frac{34.5 + 29.5}{2}$$

$$= \frac{64.0}{2}$$

$$= 32$$

**a. Frequency table**

| Score (x) | Tally       | Frequency(f)    | Mid-point |
|-----------|-------------|-----------------|-----------|
| 30- 34    | //          | 2               | 32        |
| 35- 39    | ////<br>/   | 6               | 37        |
| 40- 44    | ////        | 4               | 42        |
| 45- 49    | ////<br>//  | 7               | 47        |
| 50- 54    | ////<br>/// | 8               | 52        |
| 55- 59    | //          | 2               | 57        |
| 60- 64    | /           | 1               | 62        |
|           |             | $\Sigma f = 30$ |           |

a. For 35- 39

$$\text{Upper class boundary} = 39.5$$

$$\text{Lower class boundary} = 34.5$$

$$\therefore \text{Class width} = 39.5 - 34.5$$

$$= 5$$

b. At least means  $\geq$

11 pupils had  $\geq 52$  kg

$$\therefore \text{percentage} = \frac{11}{30} \times 100\%$$

$$= 36.7\%$$

**MEAN (AVERAGE), MODE AND MEDIAN****1. MEDIAN**

A number in the middle of a given scores.

**Odd number of scores**

To find the median scores first arrange the number of scores in either ascending or descending order.

Formula for finding position for given odd numbers is  $\left(\frac{n+1}{2}\right)^{\text{th}}$

**EXAMPLE**

Find the median for 5, 6, 7, 7, 8

**Solution**

5, 6, 7, 7, 8

$n = 5$  (Odd number)

$$\text{Median} = \frac{5+1}{2} = 3^{\text{rd. position}}$$

$$\therefore \text{Median} = 7$$

**Even number of scores**

Arrange scores in ascending or descending order.

Formula for finding **position** for a given even

number of scores is  $\frac{\left(\frac{n}{2}\right)^{\text{th}} + \left(\frac{n}{2} + 1\right)^{\text{th}}}{2}$

**EXAMPLE**

Find the median given the following ages

5, 5, 6, 7, 7, 8

**Solution**

5, 5, 6, 7, 7, 8

$n = 6$  (Even number)

$$\text{Median} = \frac{6}{2} = 3^{\text{rd. position}}$$

$$= 6$$

$$\text{Median} = \frac{6}{2} + 1 = 4^{\text{th. Position}}$$

$$= 7$$

$$\therefore \text{Median} = \frac{6+7}{2}$$

$$= 6.5$$

## 2. MODE

Score with highest frequency

EXAMPLE

For the following set of data find the mode

2, 2, 5, 8, 6, 2, 3, 4, 4

**Solution**

Mode = 2

## 3. MEAN (average)

The symbol for mean is  $\bar{x}$

EXAMPLE

1. Find mean of 2, 7, 8, 3, 10

**Solution**

$$\text{Mean} = \frac{2+7+10+8+3}{5}$$

$$= \frac{30}{5}$$

$$= 6$$

## MEAN FOR GROUPED DATA

The formula for finding mean of grouped data.

$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}$$

## EXAMPLE

- The following marks were obtained by class at Nguludi Secondary school in a mathematics examination marked out of 30.  
30, 26, 29, 23, 25, 26, 24, 26, 27, 26, 22, 24, 27, 28, 25, 27, 26, 28, 26, 26, 28, 25, 24, 25, 24, 28, 26, 25, 27, 26

  - Work out mean, median and mode for this data.
  - Students which obtained more than 87% in the test received a small prize. What percentage of students received a prize?

**Solution**

First draw the frequency table

| Score (x) | Tally   | Freq. (f)     | fx              |
|-----------|---------|---------------|-----------------|
| 22        | /       | 1             | 22              |
| 23        | /       | 1             | 23              |
| 24        | ////    | 4             | 96              |
| 25        | ////    | 5             | 125             |
| 26        | //// // | 8             | 208             |
| 27        | ////    | 4             | 108             |
| 28        | ////    | 5             | 140             |
| 29        | /       | 1             | 29              |
| 30        | /       | 1             | 30              |
|           |         | $\sum f = 30$ | $\sum fx = 781$ |

$$\text{a. Mean } (\bar{x}) = \frac{\sum fx}{\sum f}$$

$$= \frac{781}{30}$$

$$= 26.3$$

$$\text{Median} = \frac{26+26}{2}$$

$$= 26$$

$$\text{Mode} = 26$$

$$\text{b. } 30 \text{ marks} = 100\%$$

$$y = 87\%$$

$$y = \frac{30 \times 87\%}{100\%}$$

$$= 26.1$$

∴ Students who get more than 87% are 11

(Those above 26.1 marks)

$$\begin{aligned} \therefore \text{Percentage} &= \frac{11}{30} \times 100\% \\ &= 36.7\% \end{aligned}$$

### EXERCISE

1. Height of students at Nguludi secondary school in form four are;

| Height (cm) | Number of students |
|-------------|--------------------|
| 60- 65      | 7                  |
| 65- 70      | 11                 |
| 70- 75      | 17                 |
| 75- 80      | 20                 |
| 80- 85      | 16                 |
| 85- 90      | 9                  |

Find the mean height for the class

2. Find the mean from the following scores

| Score     | 1  | 2  | 3  | 4  | 5  | 6  |
|-----------|----|----|----|----|----|----|
| Frequency | 26 | 16 | 21 | 15 | 24 | 18 |

3. Find mean, mode and median for  
2, 2, 1, 2, 7, 10, 20, 11.

## CHAPTER 16

### QUADRATIC GRAPHS

Examples of quadratic graph expressions are;

$$y = ax^2, \quad y = 6x^2 - 5x - 2$$

#### WHAT DO WE NEED TO KNOW

- How to copy and complete the tables of the quadratic expressions
- How to draw graph of quadratic expressions
- The minimum and maximum values of quadratic equations on the graph
- How to find solutions of quadratic equations using the graph
- How to solve simultaneous Equations using the graph
- How to find the turning points of the quadratic graph
- Line of symmetry
- How to come up with the quadratic equations from the given graph

In general if the coefficient of quadratic expression is negative the parabola curves downwards (∩-shaped) if it is positive, it curves upwards (U-shaped)

(∩-shaped) Parabola gives maximum values

(U-shaped) parabola gives minimum values

- The line of symmetry pass at the turning point.

#### EXAMPLE

Table below shows some values of the equation,  
 $y = x^2 + x - 2$

|     |    |    |    |    |   |   |
|-----|----|----|----|----|---|---|
| $x$ | -4 | -3 | -2 | -1 | 0 | 1 |
| $Y$ | 10 |    | 0  | -2 |   | 0 |

- Copy and complete the table
- Using the scale of 2cm to represent 2 units on the  $y$ -axis and 2cm to represent 1 unit on the  $x$ -axis, draw the graph  
 $y = x^2 + x - 2$
- Use your graph to solve  $x^2 + 2x - 3 = 0$

**Solutions**

$$\begin{aligned}
 y &= x^2 + x - 2 \\
 &= (-3)^2 + (-3) - 2 = 4 \\
 y &= x^2 + x - 2 \\
 &= (0)^2 + (0) - 2 = -2
 \end{aligned}$$

$$y = x^2 + x - 2$$

|     |    |    |    |    |    |   |
|-----|----|----|----|----|----|---|
| $x$ | -4 | -3 | -2 | -1 | 0  | 1 |
| $Y$ | 10 | 4  | 0  | -2 | -2 | 0 |

**TURNING POINT**

The turning point is found by using

- Completing the square method
- Calculus

In the above equation,

$$\begin{aligned}
 &x^2 + x - 2 \\
 &\left(x^2 + x + \left(\frac{1}{2}\right)^2\right) - \left(\frac{1}{2}\right)^2 - 2 \\
 &\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - 2 \\
 &\left(x + \frac{1}{2}\right)^2 - \frac{1 - 8}{4} \\
 &\left(x + \frac{1}{2}\right)^2 - \frac{9}{4} \\
 \therefore \text{Turning point} &= x = -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 &= -0.5 \\
 &-\frac{9}{4} = -2.25 \\
 &(-0.5, -2.25)
 \end{aligned}$$

Using calculus,

NB: If  $y = x^n$  then  $\frac{dy}{dx} = nx^{n-1}$

$$\frac{dy}{dx} = 2x + 1 = 0$$

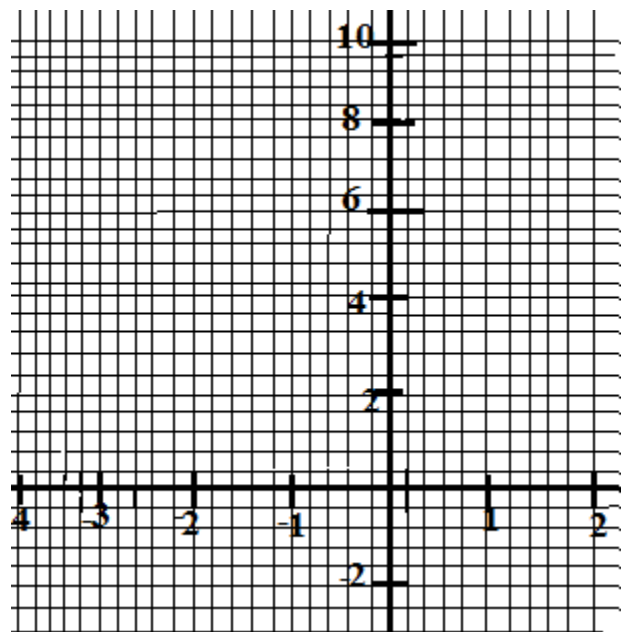
$$2x = -1$$

$$x = -0.5$$

Put  $x = -0.5$  in the equation

$$\begin{aligned}
 y &= (-0.5)^2 - 0.5 - 2 \\
 &= -2.25
 \end{aligned}$$

$$(-0.5, -2.25)$$



- From the graph,  
 $x = -2$  or  $x = 1$
- We subtract the new equation from the original equation one, then we plot the new expression after finding the  $y$ -intercept and  $x$ -intercept. Where the line intersect the cubic line on the graph gives the solution.

$$\begin{aligned}
 y &= (x^2 + 2x - 2) - (x^2 + x - 3) \\
 &= x^2 + 2x - 2 - x^2 - x + 3 \\
 &= x + 1
 \end{aligned}$$

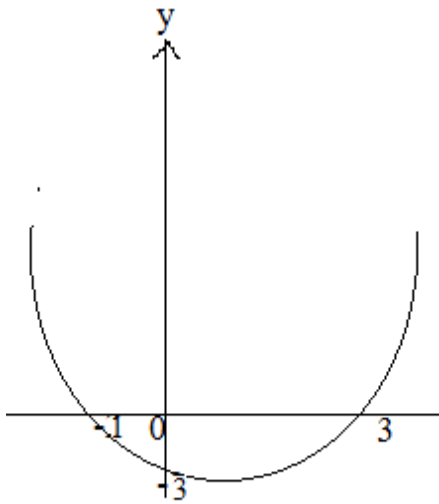
When  $y=0$ ,  $x + 1 = 0$   
 $x = -1$   $(-1, 0)$

When  $x = 0$ ,  $y = 0 + 1$   
 $y = 1$   $(0, 1)$   
 $\therefore$  from the graph,  $x =$        $y =$

### HOW TO FIND EQUATION OF THE GRAPH

#### **EXAMPLE**

- i. Find the equation of the graph below in terms of  $y = ax^2 + bx + c$



#### **Solution**

$$x = -1 \text{ or } x = 3$$

$$x + 1 = 0, \quad x - 3 = 0$$

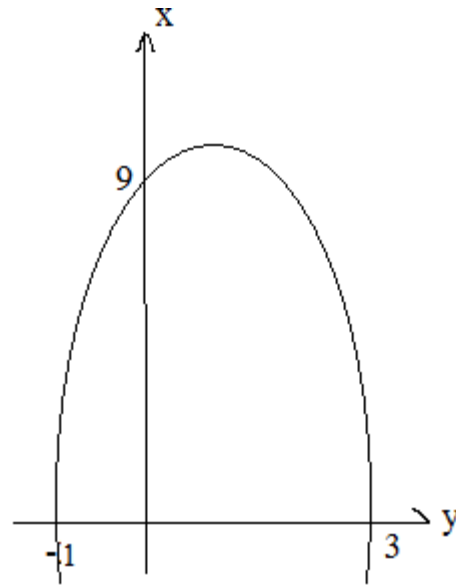
$$(x + 1)(x - 3) = 0$$

$$x^2 - x - 3 = 0$$

$$\therefore y = x^2 - x - 3$$

#### **EXAMPLE 2.**

The figure below shows the graph of equation,  $y = ax^2 + bx + c$



Find the equation of the graph

#### **Solution**

$$x = -1 \text{ or } x = 3$$

$$x + 1 = 0 \quad x - 3 = 0$$

$$(x + 1)(x - 3) = y$$

$$x^2 - 3x + x - 3 = y$$

$$x^2 - 2x - 3 = y$$

Divide  $-3$  by  $y$  – intercept  $= \frac{9}{-3}$

$= -3$  and the answer multiply the equation

$$-3(x^2 - 2x - 3) = y$$

$$-3x^2 + 6x + 9 = y$$

$$9 + 6x - 3x^2 = y$$

# BOOK

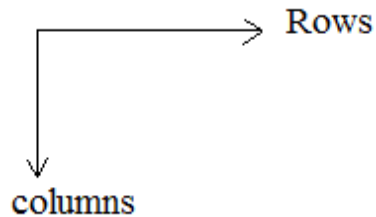
# 4

**MATHEMATICS IS A SYSTEMATIC WAY OF  
WASTING OUR TIME**

## CHAPTER 1

## MATRIX

Matrix is an array of numbers i. e rows and columns of numbers.



e. 5 packets of sugar at k10 each

6 under wears at k30 each

8 sweets at k5 each

This data can be represented in matrices form as

$$\begin{bmatrix} 5 & 6 & 8 \end{bmatrix} \begin{bmatrix} 10 \\ 30 \\ 5 \end{bmatrix}$$

5, 6, 8, 10, 30, 5 are components or elements

### NAMING MATRICES

When naming matrices, we start with counting number of rows then number of columns.

#### 1. ROW MATRIX

$\begin{bmatrix} 2 & 4 & 7 \end{bmatrix}$ , this is a  $1 \times 3$  matrix

#### 2. COLUMN MATRIX

$\begin{bmatrix} 10 \\ 7 \\ 0 \end{bmatrix}$ , this is a  $3 \times 1$  matrix

#### 3. ZERO MATRIX

A zero matrix is a matrix which has zeros as its elements.

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

#### 4. SQUARE MATRIX

It's a matrix which has equal number of rows and columns

e. g  $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

#### ADDITION OF MATRICES

When adding matrices together, add corresponding elements together.

Only those matrices of the same order can be added together.

#### EXAMPLE

1. Given that  $A = \begin{bmatrix} 4 & -9 \\ 6 & 7 \end{bmatrix}$   $B = \begin{bmatrix} 8 & 22 \\ -5 & 0 \end{bmatrix}$

Find  $A + B$ .

#### Solution

$$A + B = \begin{bmatrix} 4 & -9 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 8 & 22 \\ -5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 8 & -9 + 22 \\ 6 + (-5) & 7 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 13 \\ 1 & 7 \end{bmatrix}$$

#### SUBTRACTION OF MATRICES

Corresponding elements must be subtracted together.

Only those matrices of the same order can be subtracted together.

#### EXAMPLE

1. Given that

$$A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & -2 & 4 \end{bmatrix} B = \begin{bmatrix} -1 & 0 & -2 \\ 3 & -2 & -1 \end{bmatrix}$$

Find  $A - B$

#### Solution

$$A - B = \begin{bmatrix} 3 & 2 & 5 \\ 1 & -2 & 4 \end{bmatrix} - \begin{bmatrix} -1 & 0 & -2 \\ 3 & -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 - (-1) & 2 - 0 & 5 - (-2) \\ 1 - 3 & -2 - (-2) & 4 - (-1) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 7 \\ -2 & 0 & 5 \end{bmatrix}$$

2. Given that

$$A = \begin{bmatrix} 1 & 2 & c \\ 2 & 4 & 5 \\ 3 & d & 8 \end{bmatrix} B = \begin{bmatrix} 4 & 3 & -1 \\ 3 & 2 & b \\ 2 & 2 & -4 \end{bmatrix} C = \begin{bmatrix} 5 & a & 4 \\ 6 & 6 & 1 \\ 5 & 0 & e \end{bmatrix}$$

Find  $a, b, c, d$  and so that  $A + B = C$

#### Solution

$$\begin{bmatrix} 1 & 2 & c \\ 2 & 4 & 5 \\ 3 & d & 8 \end{bmatrix} + \begin{bmatrix} 4 & 3 & -1 \\ 3 & 2 & b \\ 2 & 2 & -4 \end{bmatrix} = \begin{bmatrix} 5 & a & 4 \\ 6 & 6 & 1 \\ 5 & 0 & e \end{bmatrix}$$

$$\begin{bmatrix} 5 & 5 & c - 1 \\ 6 & 6 & 5 + b \\ 5 & 2 + d & 4 \end{bmatrix} = \begin{bmatrix} 5 & a & 4 \\ 6 & 6 & 1 \\ 5 & 0 & e \end{bmatrix}$$

$$\therefore a = 5$$

$$5 + b = 1$$

$$b = 1 - 5$$

$$\therefore b = -4$$

$$c - 1 = 4$$

$$c = 4 + 1$$

$$\therefore c = 5$$

$$2 + d = 0$$

$$\therefore d = -2$$

$$\therefore e = 4$$

#### MULTIPLICATION OF MATRIX BY A NUMBER (SCALAR)

If  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  and  $k$  is the number then,

$$KA = k \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

#### EXAMPLE

1. Given the following matrices,

$A = \begin{bmatrix} 3 & 1 & 0 \\ 2 & 5 & 6 \end{bmatrix}$   $B = \begin{bmatrix} 2 & -3 & 2 \\ 1 & 2 & -3 \end{bmatrix}$ . Calculate matrix  $5A - 3B$

**Solution**

$$\begin{aligned} 5A - 3B &= 5 \begin{bmatrix} 3 & 1 & 0 \\ 2 & 5 & 6 \end{bmatrix} - 3 \begin{bmatrix} 2 & -3 & 2 \\ 1 & 2 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 15 & 5 & 0 \\ 10 & 25 & 30 \end{bmatrix} - \begin{bmatrix} 6 & -9 & 6 \\ 3 & 6 & -9 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 14 & -6 \\ 7 & 19 & 39 \end{bmatrix} \end{aligned}$$

### **MULTIPLYING MATRICES TOGETHER**

To multiply matrices together, we multiply the first number in the row matrix by the first number in the column matrix.

A second number in the row matrix must multiply by the second number in the column matrix and so on.

Add the products together.

NB: we can only multiply matrices together if the number of columns in the first matrix is equal to the number of rows in the second matrix.

### **EXAMPLE**

Given that  $V = \begin{bmatrix} 3 & 5 \\ -1 & 1 \end{bmatrix}$  and  $W = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$ . Find  $VY$ .

**Solution**

$$\begin{aligned} VY &= \begin{bmatrix} 3 & 5 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 3 \times (-1) + 5 \times 0 & 3 \times 1 + 5 \times 2 \\ -1 \times (-1) + 1 \times 0 & -1 \times 1 + 1 \times 0 \end{bmatrix} \\ &= \begin{bmatrix} -3 \times 0 & 3 + 10 \\ 1 + 0 & -1 + 2 \end{bmatrix} \\ &= \begin{bmatrix} -3 & 13 \\ 1 & 1 \end{bmatrix} \end{aligned}$$

### **WORD PROBLEMS**

#### **EXAMPLE**

1. A shop was selling skirts for K3000 each and dresses at K5000 each.

**Bertha** and **Taonga** bought some clothes listed in the table.

|        |   |   |
|--------|---|---|
| Bertha | 3 | 2 |
| Taonga | 4 | 5 |

Find how much each person spent on the clothes

**Solution**

$$\begin{aligned} &= \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 3000 \\ 5000 \end{bmatrix} \\ &= \begin{bmatrix} 9,000 + 10,000 \\ 12,000 + 25,000 \end{bmatrix} \\ &= \begin{bmatrix} 19,000 \\ 37,000 \end{bmatrix} \end{aligned}$$

$\therefore$  Bertha spent K19,000

Taonga spent K 37,000

2. Mphatso wants to buy 3 pencils, 2 rulers and 4 ball pens. Bali wants to buy 1 pencil, 1 ruler and 8 ball pens. In Zomba pencils and rulers costs K60 each and ball pens cost K20 each. In Mzuzu pencils and rulers costs K80 each and ball pens cost K30 each.

Use matrix multiplication to find how much more these items will cost Mphatso and Bali in Mzuzu

**Solution**

|         | Pencil ruler pen   |   |   | Zomba   | Mzuzu |
|---------|--|---|---|---|-------|
| Mphatso | 3  | 2 | 4 | 60  | 80    |
| Bali    | 1  | 1 | 8 | 60  | 80    |
|         |  |   |   | 20  | 30    |
|         | $= \begin{bmatrix} 3 & 2 & 4 \\ 1 & 1 & 8 \end{bmatrix}$ |   |   | $\begin{bmatrix} 60 & 80 \\ 60 & 80 \\ 20 & 30 \end{bmatrix}$ |       |



Zomba                      Mzuzu

$$= \begin{bmatrix} 180 + 120 + 80 & 240 + 160 + 120 \\ 60 + 60 + 160 & 80 + 80 + 240 \end{bmatrix}$$

$$= \begin{bmatrix} 380 & 520 \\ 280 & 400 \end{bmatrix}$$

∴ Mphatso will pay **K520 – K380 = K140**,

More in Mzuzu

∴ Bali will pay **K400 – K280 = K120**,

More in Mzuzu

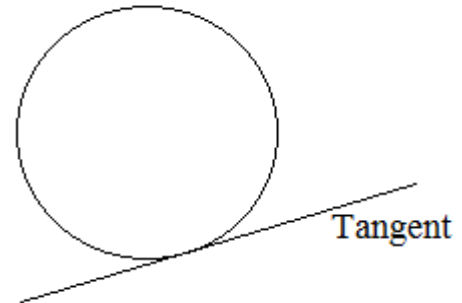
### EXERCISE (MANEB papers)

- Given that  $P = \begin{bmatrix} 7 & 5 \\ 2 & 4 \end{bmatrix}$  and  $Q = \begin{bmatrix} 3 & 10 \\ 5 & 1 \end{bmatrix}$ ,  
Find PQ
- Given that Matrix  $P = \begin{bmatrix} 3 & 4 \\ 1 & -2 \end{bmatrix}$   
 $Q = \begin{bmatrix} 1 & 3 \\ 6 & 1 \end{bmatrix}$  and  $R = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$ . Find  $3(Q - PR)$ .
- T and R are two matrices. Given the  
 $T = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$   $R = \begin{bmatrix} 0 & 3 \\ -1 & 1 \end{bmatrix}$ . Find  $3R - T^2$ .
- Find **x** and **y** in the matrix below  
 $\begin{bmatrix} 6 & 3 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \end{bmatrix}$
- Show that  
 $\begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 5 & -1 \end{bmatrix} + \begin{bmatrix} 8 & -12 \\ -8 & -1 \end{bmatrix}$  is a zero matrix
- Given that  $M = \begin{bmatrix} 3 & 0 \\ 1 & -2 \end{bmatrix}$ ,  $N = \begin{bmatrix} -3 & 2 \\ -1 & k \end{bmatrix}$ ,  
and  $MN = \begin{bmatrix} -3 & 6 \\ -1 & -1 \end{bmatrix}$ . Find the value of **k**

## CHAPTER 2

### TANGENT PROPERTIES

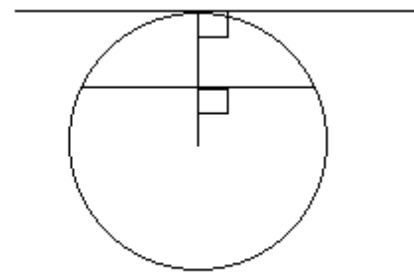
Tangent is a straight line which touches the circumference of a circle at only one point even after producing it.



#### Theorem 1

A tangent to a circle is perpendicular to the radius from point of contact to the Centre of the circle.

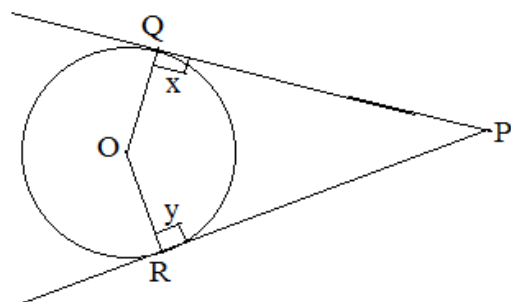
**Reason,  $tan \perp$  to radius**



#### Theorem 2

The two tangents from an external point to a circle are equal in length.

**Reason,  $tan$  from ext. pt.**



**Given;** two tangents PQ and PR

**To prove;**  $PQ = PR$

**Construction;** draw  $\perp$ s QO and RO and line

PO

**Proof;** in  $\Delta$ s PQO and PRO

Q0= R0, radii

$x = y \tan \perp \text{ to radius}$

P0 is common

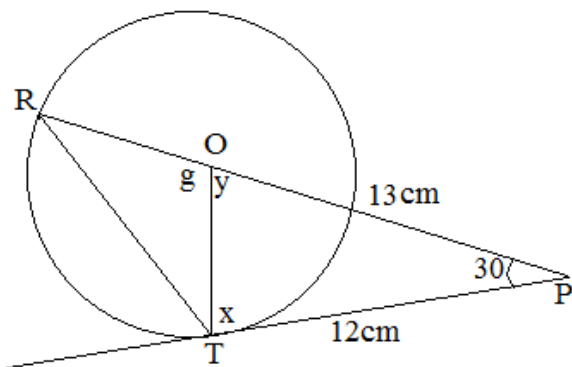
$$\therefore \triangle PQO \equiv \triangle PRO \text{ (RHS)}$$

$$\therefore PQ=PR$$

### EXAMPLE

1. P is 13cm from the Centre O of the circle, PT is a tangent to a circle and is 12cm long. Angle OPT=30°. find
- OT
  - $\widehat{POT}$
  - $\widehat{OTR}$

**Solution**



$x = 90^\circ$ ,  $\tan \perp$  to radius

- a. In  $\Delta PTO$   
 $PO^2 = PT^2 + OT^2$ , Pythagoras  
 $13^2 = 12^2 + OT^2$   
 $169 = 144 + OT^2$   
 $169 - 144 = OT^2$   
 $25 = OT^2$   
 $OT = \sqrt{25}$   
 $\therefore OT = 5$

- b. In  $\Delta\mathbf{OPT}$

$$y + 30^\circ + x = 180^\circ, \text{sum} < s \text{ in } \Delta$$

$$y + 30^\circ + 90^\circ = 180^\circ$$

$$y = 180^\circ - 120$$

$$y = 60^\circ$$

$$\therefore \angle P\hat{O}T = 60^\circ$$

- c.  $y + g = 180^\circ$ , adj.  $\angle$  on str. line

$$60^\circ + g = 180^\circ$$

$$g = 120^\circ$$

$$O_R = O_T, \text{ radii}$$

$\therefore \Delta OTR$  is isosceles

$$M=n, \text{ base} < s$$
$$M + n + g = 180^\circ, \text{ sum } < s \text{ in a } \Delta$$

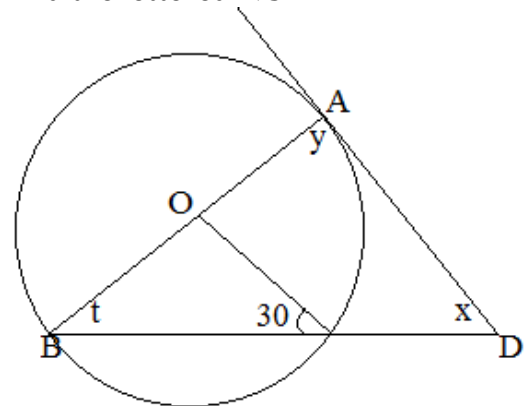
$$2n = 180^\circ - 120^\circ$$

$$2n=60^\circ$$

$$N = 30^\circ$$

$$\therefore \theta_{TR} = 30^\circ$$

2. Find the lettered  $< s$ .



### Solution

$y = 90^\circ$ ,  $\tan \perp$  to radius

In  $\triangle OBC$

OB=OC, radii

$\therefore \triangle OBC$  is an isosceles

$$\therefore t = 30^\circ, \text{base} < s$$

in  $\triangle ABD$

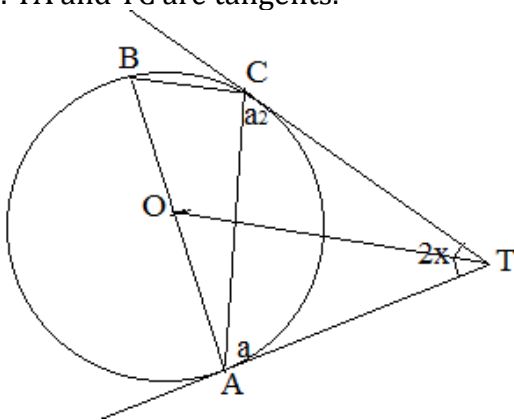
$$x + y + x = 180^\circ, \text{sum} < s \text{ in a } \Delta$$

$$30^\circ + 90^\circ + x = 180^\circ$$

$$X = 180^\circ - 120^\circ$$

$$x = 60^\circ$$

3. AB is a diameter of a circle ABCD Centre O. TA and TC are tangents.



If  $\widehat{ATC} = 2x$ , show that  $\widehat{BAC} = x$

**Solution**

In  $\triangle ATC$ ,

$AT = CT$ , Tan from ext point

$\therefore \triangle ACT$ , is an isosceles

$\therefore a = a_2$ , base  $\angle$ s

$a + a_2 + 2x = 180^\circ$ , sum  $\angle$ s in  $\triangle$

$$2x + 2a = 180^\circ$$

$$2(x + a) = 180^\circ,$$

$$x + a = \frac{180^\circ}{2}$$

$$x + a = 90^\circ$$

$$\therefore x = 90^\circ - a$$

$\widehat{BAT} = 90^\circ$ , Tan  $\perp$  to radius

$$\widehat{BAC} = \widehat{BAT} - a,$$

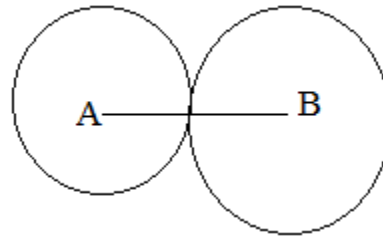
$$\therefore \widehat{BAC} = 90^\circ - a$$

$$\text{But } 90^\circ - a = x$$

$$\therefore \widehat{BAC} = x$$

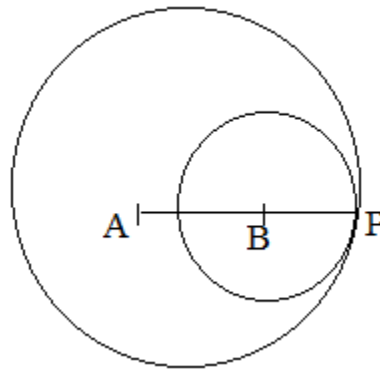
### **CONTACT OF CIRCLES**

Circles can touch internally or externally. Where the distance between their centers is the sum of their radii.



$$AB = AT + TB$$

When they touch internally the distance between their centers is the difference between radii.



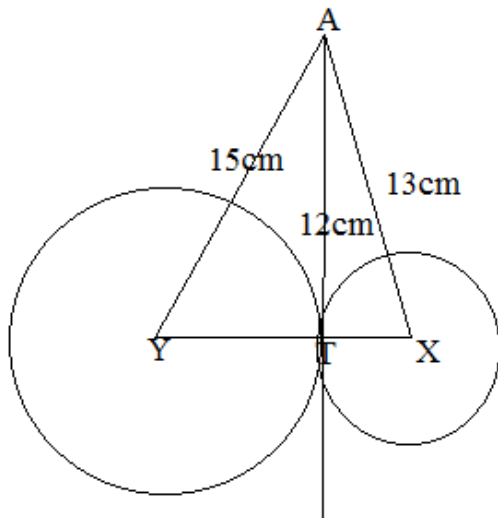
$$AB = AP - BP$$

### **EXAMPLE**

Two circle centers X and Y touch externally at T. A is a point on their common tangent such that  $AT = 12\text{cm}$ ,  $AX = 13\text{cm}$  and  $AY = 15\text{cm}$ .

- I. Calculate XY
- II. If the circles touch internally instead of externally what is XY?

**Solution**



$$\widehat{ATY} = 90^\circ, \text{ Tan } \perp \text{ to radius}$$

$$\widehat{ATX} = 90^\circ, \text{ Tan } \perp \text{ to radius}$$

$\therefore \Delta ATY$  and  $\Delta ATX$  are right  $\triangle$ s

In  $\Delta ATY$

$$AY^2 = YT^2 + AT^2, \text{ pythagorus theorem}$$

$$15^2 = YT^2 + 12^2$$

$$225 - 144 = YT^2$$

$$81 = YT^2$$

$$YT = \sqrt{81}$$

$$YT = 9\text{cm}$$

In  $\Delta ATX$ ,

$$AX^2 = AT^2 + XT^2, \text{ Pythagoras theorem}$$

$$13^2 = 12^2 + XT^2$$

$$169 - 144 = XT^2$$

$$25 = XT^2$$

$$XT = \sqrt{25}$$

$$= 5\text{cm}$$

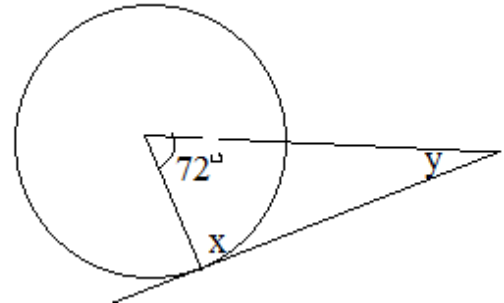
$$\begin{aligned} \text{I. } XY &= YT + TX \\ &= 9\text{cm} + 5\text{cm} \\ &= 14\text{cm} \end{aligned}$$

$$\begin{aligned} \text{II. } XY &= YT - TX \\ &= 9\text{cm} - 5\text{cm} \end{aligned}$$

$$= 4\text{cm}$$

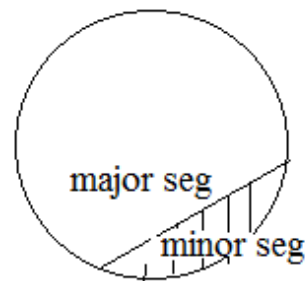
### EXERCISE

- AD is a diameter of a circle. AB is a chord and AT is a tangent.
  - State the size of  $\angle ABT$
  - If  $\angle BAT$  is an acute of  $x^\circ$ , find the value of  $\angle DAB$  in terms of  $x$ .
- Calculate the marked angles



### Theorem 3

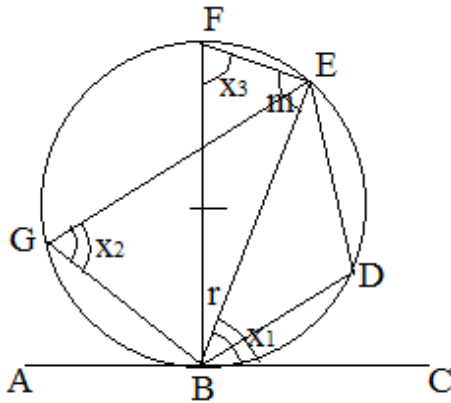
#### ALTERNATE SEGMENTS



If a straight line touches a circle and from the point of contact a chord is drawn, the angles which chord makes with the tangent are equal to the angles in the alternate segments

**Reason,**  $\angle$ s in alt seg. are equal.

**PROVE**



**Given;** a circle, str. line ABC, chord BC,  $\widehat{CBE}$  and  $\widehat{BGE}$ ,  $\widehat{ABE}$  and  $\widehat{BDE}$

**To prove;  $\widehat{CBE} = \widehat{BGE}$  and  $\widehat{ABE} = \widehat{BDE}$**

**Construction:** draw diameter BE, join E to F

**Proof:**  $r + x = 90^\circ$ ,  $\tan \perp$  to radius

$$x = 90^\circ - r$$

$m = 90^\circ$ ,  $\angle s$  in a sem circle

In  $\triangle BEF$

$$r + x_3 + m = 180^\circ, \text{ sum } < s \text{ in a triangle}$$

$$r + x_3 + 90^\circ = 180^\circ$$

$$r + x_3 = 180 - 90^\circ$$

$$r + x_3 = 90^\circ$$

$$x_3 = 90^\circ - r$$

$$\therefore X_1 = X_3$$

But  $x_2 = x_3$ , < s in same seg.

$$\therefore X_2 = X_3$$

$$\therefore \widehat{CE} = \widehat{BE}$$

$$X_2 + \widehat{BDE} = 180^\circ, \text{ sum intr. Opp. } \angle\text{s in cyclic}$$

Quad.

$$\therefore \angle BDE = 180^\circ - x_2$$

$$\hat{A}\hat{B}E + x = 180^\circ, \text{ } \angle s \text{ on a str line}$$

$$\angle A\hat{B}E = 180^\circ - x$$

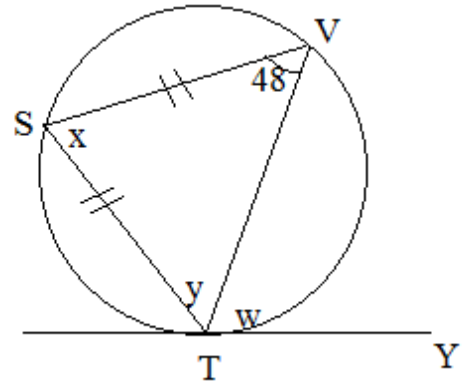
$$\therefore \hat{A}BE = 180^\circ - x_2 (x_1 = x_2)$$

$$A\hat{B}E = B\hat{D}E$$

### EXAMPLE

1. TY is a tangent to the circle TVS. If  $\angle STV = 48^\circ$  and  $VS = ST$ . what is  $\angle VTY$ ?

### Solution



SV= ST, given

$\therefore \Delta STV$ , is isosceles

$$\therefore y = 48^\circ$$

$$x + y + 48^\circ = 180^\circ, \text{sum} < s \text{ in } \Delta$$

$$x + 48^\circ + 48^\circ = 180^\circ$$

$$x = 180^\circ - 96^\circ$$

$$x = 84^\circ$$

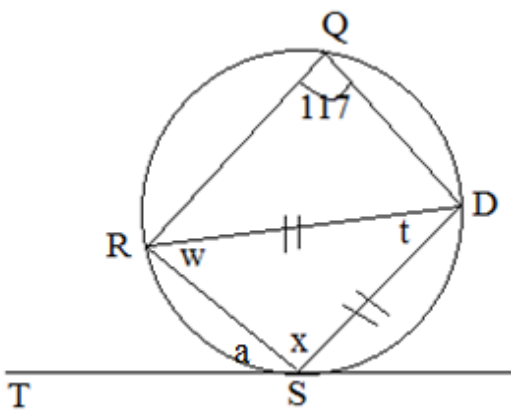
$w = x, < s$  in alt seg

$$w = 84^\circ$$

$$\therefore \hat{V\hat{T}Y} = 84^\circ$$

2. TS is a tangent to a circle PQDS. If  $\mathbf{DR=PS}$  and angle  $\mathbf{DQR=117^\circ}$ , calculate angle RST

**Solution**



$$\begin{aligned} x + 117^\circ &= 180^\circ, \text{sum int. opp} \\ &< s \text{ in cyclic quad} \\ x &= 180^\circ - 117^\circ \\ &= 63^\circ \end{aligned}$$

In  $\Delta$  **DRS**

RD= DS, given

$\therefore \Delta DRS$ , is an isosceles

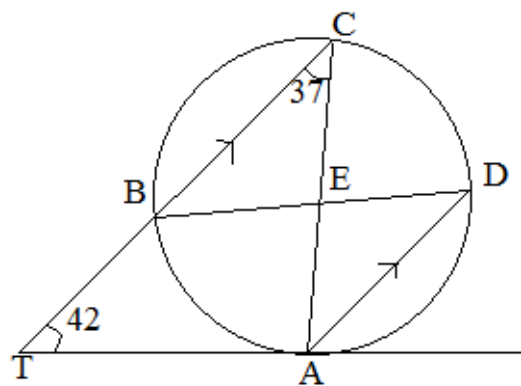
$$\therefore x = w, \text{base} < s$$
$$\therefore w = 63^\circ$$
$$x + w + t = 180^\circ, \text{sum} < s \text{ in a } \Delta$$
$$63^\circ + 63^\circ + t = 180^\circ$$
$$t = 180^\circ - 126$$
$$t = 54^\circ$$

$a = t = 54^\circ, < s$  in alt seg

$$a = 54^\circ$$
$$\therefore \text{D}\hat{\text{S}}\text{T} = 54^\circ$$

## EXERCISE

1. AT is a tangent to the circle ABCD, angle  $\text{BAC} = 64^\circ$  and angle  $\text{CAT} = 72^\circ$ . Calculate  $\widehat{\text{BCA}}$  and  $\widehat{\text{CDA}}$
2. If angle  $\text{ACB} = 37^\circ$  and angle  $\text{ATB} = 42^\circ$ , Calculate  $\widehat{\text{ABT}}$  and  $\widehat{\text{AEB}}$

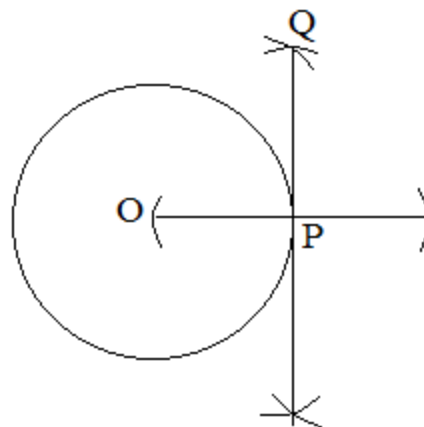


## CONSTRUCTION

### Construction of a tangent to a circle at a point on the circumference.

## PROCEDURES

1. Draw a circle Centre O and mark P on its circumference.
2. Join O to P and produce it to K
3. Through P, construct line PQ perpendicular to OP. then PQ is the required tangent.

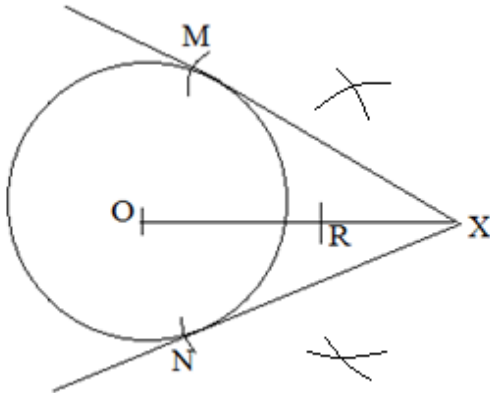


### Construction of a tangent from an external point.

## PROCEDURES

- i. Draw a circle Centre O and mark a point X outside the circle.
- ii. Join O to X
- iii. Bisect OX and let the mid-point of OX be R.
- iv. With radius from R, draw arcs cutting the circle at M and N. join M to X and N

to X. then XM and XN are the required tangents.



## CHAPTER 3

## STATISTICS

### RANGE

It's the difference between the highest and the lowest score. E. g. 10, 2, 7, 100, 20

$$\begin{aligned}\text{Range} &= 100 - 2 \\ &= 98\end{aligned}$$

### EXAMPLE

- At the end of term, the lowest in the mathematics test was 34%. The range is 52%. How many marks did the top student score?

#### Solution

$$\text{Range} = \text{highest} - \text{lowest}$$

$$52\% = \text{highest} - 34\%$$

$$52\% + 34\% = \text{highest}$$

$$\therefore \text{Highest} = 86\%$$

### INTERQUATILE RANGE

It is the difference between lower quartile and upper quartile.

The score at the middle of the lower half of score is called lower quartile range (it is a  $\frac{1}{4}$  way along the distribution)

$$QL = \left(\frac{n+1}{4}\right)^{\text{th}} \text{ score.}$$

The score of the middle of the upper half of score is called upper quartile range (QU).

Because it is a  $\frac{3}{4}$  way along the distribution.

$$QU = \frac{3}{4}(n + 1)$$

The median is also called second quartile.

### EXAMPLE

- Find the lower, second and upper quartiles of the following data.  
10, 7, 2, 11, 15, 17, 12, hence find the interquartile and semi quartile range.

**Solution**

10, 7, 2, 11, 15, 17, 12

Lower quartile (QL) =  $\frac{(n+1)}{4}$ th score

$$= \frac{(7+1)}{4} \text{th score}$$

$$= 2^{\text{nd}} \text{ score}$$

$$\underline{= 7}$$

Upper quartile =  $\frac{3}{4}(n+1)$ th score

$$= \frac{3}{4}(7+1) \text{th score}$$

$$= \left(\frac{24}{4}\right) \text{th score}$$

$$= 6^{\text{th}} \text{ score}$$

$$\underline{= 17}$$

$$\text{Median} \underline{= 11}$$

Interquartile range = 17 - 7

$$\underline{= 10}$$

$$\text{Semi-quartile} = \frac{1}{2} \times 10$$

$$\underline{= 5}$$

**EXERCISE**

1. For the following set of numbers state,
    - i. The lowest quartile
    - ii. The median
    - iii. The interquartile range
    - iv. The upper quartile
    - v. The semi-interquartile range
- 8, 8, 9, 11, 14, 17, 21, 25, 29, 35, 37, 43.

**STANDARD DEVIATION****VARIANCE (S<sup>2</sup>)**

It is the average of deviations from mean.

Standard deviation is the square root of variance(s)

Deviation is the value we get after subtracting mean ( $\bar{x}$ ) from given score(x).**EXAMPLE**

1. Find the standard deviation for 5, 6, 2, 9, 13, 25, 18, 10

**Solution**

$$\text{Mean}(\bar{x}) = \frac{2+5+6+10+9+13+25+18}{8}$$

$$= \frac{88}{8}$$

$$= 11$$

| Score (x) | Deviation from mean(x- $\bar{x}$ ) | Squared deviation             |
|-----------|------------------------------------|-------------------------------|
| 2         | -9                                 | 81                            |
| 5         | -6                                 | 36                            |
| 6         | -5                                 | 25                            |
| 9         | -2                                 | 4                             |
| 10        | -1                                 | 1                             |
| 13        | 2                                  | 4                             |
| 18        | 7                                  | 49                            |
| 25        | 14                                 | 196                           |
| Total     | 0                                  | $\Sigma(x - \bar{x})^2 = 396$ |

$$\text{Variance (S}^2\text{)} = \frac{\Sigma(x - \bar{x})^2}{n}$$

$$= \frac{396}{8}$$

$$= 49.5$$

$$\text{Standard deviation(S)} = \sqrt{49.5}$$

$$= 7.03$$

**GROUPED DATA****EXAMPLE**

1. Find correct to 1 decimal place, the mean standard deviation of the following; 26, 29, 29, 30, 31, 33, 35, 33, 35



**Solution**

| Score (x) | Tally | Freq.(f)       | $fx$              |
|-----------|-------|----------------|-------------------|
| 26        | /     | 1              | 26                |
| 29        | //    | 2              | 58                |
| 30        | /     | 1              | 30                |
| 31        | /     | 1              | 31                |
| 33        | ///   | 3              | 99                |
| 35        | /     | 1              | 35                |
| Total     |       | $\Sigma f = 9$ | $\Sigma fx = 279$ |

$$\begin{aligned}\text{Mean}(\bar{x}) &= \frac{\Sigma fx}{\Sigma f} \\ &= \frac{279}{9} \\ &= 31\end{aligned}$$

$$\therefore \text{Mean} = 31$$

| X     | $(x - \bar{x})$ | f              | $f(x - \bar{x})$ | $(x - \bar{x})^2$ | $f(x - \bar{x})^2$             |
|-------|-----------------|----------------|------------------|-------------------|--------------------------------|
| 26    | -5              | 1              | -5               | 25                | 25                             |
| 29    | -2              | 2              | -4               | 4                 | 8                              |
| 30    | -1              | 1              | -1               | 1                 | 1                              |
| 31    | 0               | 1              | 0                | 0                 | 0                              |
| 33    | 2               | 3              | 6                | 4                 | 12                             |
| 35    | 4               | 1              | 4                | 16                | 16                             |
| Total |                 | $\Sigma f = 9$ |                  |                   | $\Sigma f(x - \bar{x})^2 = 62$ |

$$\begin{aligned}S^2 &= \frac{\Sigma f(x - \bar{x})^2}{\Sigma f} \\ S &= \sqrt{\frac{\Sigma f(x - \bar{x})^2}{\Sigma f}} \\ &= \sqrt{\frac{62}{9}} \\ &= \sqrt{6.886} \\ &= 2.6\end{aligned}$$

2. The ages of the students in a class were as follows; 14, 15, 14, 16, 14, 14, 15, 17, 15, 18, 14, 15, 15, 16, 15, 14, 13, 15, 15, 16. Find

- Mean
- Standard deviation.

**Solution**

| Score (x) | Tally       | Freq.(f)        | $fx$              |
|-----------|-------------|-----------------|-------------------|
| 13        | /           | 1               | 13                |
| 14        | ////<br>/   | 6               | 84                |
| 15        | ////<br>/// | 8               | 120               |
| 16        | ///         | 3               | 48                |
| 17        | /           | 1               | 17                |
| 18        | /           | 1               | 18                |
|           |             | $\Sigma f = 20$ | $\Sigma fx = 300$ |

$$\begin{aligned}\text{Mean}(\bar{x}) &= \frac{\Sigma fx}{\Sigma f} \\ &= \frac{300}{20} \\ &= 15\end{aligned}$$

| Score x | $(x - \bar{x})$ | F | $f(x - \bar{x})$ | $(x - \bar{x})^2$ | $F(x - \bar{x})^2$ |
|---------|-----------------|---|------------------|-------------------|--------------------|
| 13      | -2              | 1 | -2               | 4                 | 4                  |
| 14      | -1              | 6 | -6               | 1                 | 6                  |
| 15      | 0               | 8 | 0                | 0                 | 0                  |
| 16      | 1               | 3 | 3                | 1                 | 3                  |
| 17      | 2               | 1 | 2                | 4                 | 4                  |
| 18      | 3               | 1 | 3                | 9                 | 9                  |
|         | $= 20$          |   | 0                |                   | $= 26$             |

$$\begin{aligned}S^2 &= \frac{\Sigma (x - \bar{x})^2}{\Sigma f} \\ &= \frac{26}{20} \\ &= 1.3 \\ S &= \sqrt{1.3} \\ &= 1.14 \text{ years}\end{aligned}$$

**EXERCISE**

1. If the variance of 2 and 2a is 9, find the value of a
2. Find mean and standard deviation for; 7, 8, 10, 12, 14, 16, 17, 20.
3. Given the following information, find the standard deviation.

| Order size (x) | Number of orders (f) |
|----------------|----------------------|
| 150            | 10                   |
| 250            | 28                   |
| 350            | 42                   |
| 500            | 50                   |
| 300            | 20                   |

4. Find the standard deviation for the data below, using grouped data.

| Ages      | Frequency |
|-----------|-----------|
| 100- 200  | 10        |
| 200-300   | 28        |
| 300- 400  | 42        |
| 400- 600  | 50        |
| 600- 1000 | 20        |

**CHAPTER 4****PROGRESSIONS AND SERIES**

- Arithmetic progression
- Sequence
- Term, first term, common sequence
- Rule
- Infinite sequence
- $n^{\text{th}}$  term
- sum of arithmetic progression

**ARITHMETIC PROGRESSION**

A sequence whereby the terms increase or decrease by the same amount is called **arithmetic progression**.

E. g 1, 3, 5, 7 ..... This sequence is increasing by adding 2 to the previous term.

8, 6, 4 ..... This sequence is decreasing by adding -2 to the previous sequence.

**SEQUENCE**

It is an arrangement of numbers arranged according to their definite rule.

Two conditions are needed to come up with a sequence

- i. first term
- ii. rule

**Rule;** to previous term ***add 2*** to get the next term.

2, 4, 6, 8, 10 ..... sequence

**INFINITE SEQUENCE**

It is a sequence in which last or first term may not be known.

E. g 1, 3, 5 .....

**FINITE SEQUENCE**

A sequence whereby last and first term are known.

E. g 1, 3, 5 ..... 13

**n<sup>th</sup> TERM OF AN AP**

1, 3, 5, 7.....

1 is a first term (a)

$$3-1=2$$

$$5-3=2$$

$$7-5=2$$

∴ 2 is a common difference (d)

**1, 3, 5, 7.....**1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>th</sup>, 4<sup>th</sup> ..... n<sup>th</sup>

$$1, 1+2, 1+2+2, 1+2+2+2.....n^{\text{th}}$$

Substitute a=1 and d=2

$$a, a+d, a+d+d, a+d+d+d..... n^{\text{th}}$$

$$a^{\text{st}}, a+d^{2\text{nd}}, a+2d^{3\text{rd}}, a+3d^{4\text{th}}..... n^{\text{th}}$$

$$a, a+(2-1)d, a+(3-1)d, a+(4-1)d..... n^{\text{th}}$$

let n = position

$$\therefore a + (n-1) d$$

n = number of terms

**EXAMPLE**

- For 1, 3, 5..., find the 7<sup>th</sup> term.

**Solution**

$$\begin{aligned} 7^{\text{th}} \text{ term} &= 1 + (7-1)2 \\ &= 1+12 \\ &= 13 \end{aligned}$$

- How many are there in the AP 5, 12, 19.....82.

**Solution**

$$a=5, d=7 \text{ } n=? \text{ } n^{\text{th}} \text{ term} = 82$$

$$n^{\text{th}} \text{ term} = a + (n-1)d$$

$$82 = 5 + (n-1)7$$

$$82 = 7n - 2$$

$$82 + 2 = 7n$$

$$84 = 7n$$

$$n = 12$$

- Find the formula for the n<sup>th</sup> term of the sequence of odd numbers 1, 3, 5, 7..... And find thousand<sup>th</sup> odd number.

**Solution**

1, 3, 5, 7.....

$$A=1, d=2$$

$$n^{\text{th}} \text{ term} = 1 + (n-1)2$$

$$= 1 + 2n - 2$$

$$= 2n - 1$$

$$\text{Thousand}^{\text{th}} \text{ odd number} = 2(1000) - 1$$

$$= 2000 - 1$$

$$= 1999$$

**EXERCISE**

- Find the tenth term of the AP 3, 8, 15.
- How many terms are there in an AP 45, 41, 37.....1
- Find the formula for the n<sup>th</sup> term of even numbers 2, 4, 6, 8 hence find
- The thousand<sup>th</sup> even number.

**EXAMPLES**

- The twenty fifth term of an AP is 43 and the fifty ninth term is 26. Find the first term and the common difference and the thirtieth term.

**Solution**

$$25^{\text{th}} \text{ term} = a + (25-1) d$$

$$43 = a + 24d \text{..... (i)}$$

$$59^{\text{th}} \text{ term} = a + (59-1) d$$

$$26 = a + 58d \text{..... (ii)}$$

$$43 = a + 24d$$

$$(26 = a + 58d)$$

$$17 = -34d$$

$$d = -\frac{34}{17}$$

$$= -\frac{1}{2}$$

$$\text{In i. put } d = -\frac{1}{2}$$

$$43 = a + 24\left(-\frac{1}{2}\right)$$

$$43 = a - 12$$

$$43 + 12 = a$$

$$a = 55$$

$$30^{\text{th}} = 55 + (30-1) \cdot -\frac{1}{2}$$

$$= 55 + 29 \times -\frac{1}{2}$$

$$= 55 - 14.5$$

$$= 40.5$$

- An AP has fourth term 8 and seventh term 17. Find the first term and the common

difference. Find an expression for the  $n^{\text{th}}$  term.

**Solution**

$$4^{\text{th}} \text{ term} = a + (4-1)d$$

$$8 = a + 3d \dots\dots\dots (i)$$

$$7^{\text{th}} \text{ term} = a + (7-1)d$$

$$17 = a + 6d \dots\dots\dots (ii)$$

ii) - i) gives  $17 = a + 6d$

$$(8 = a + 3d)$$

$$9 = 3d$$

$$d = \frac{9}{3}$$

$$d = 3$$

In i.) Put  $d = 3$

$$8 = a + 3(3)$$

$$8 - 9 = a$$

$$-1 = a$$

$$a = -1$$

$$n^{\text{th}} \text{ term} = a + (n-1)d$$

$$= -1 + (n-1)3$$

$$= -1 + 3n - 3$$

$$= 3n - 4$$

**SUM OF TERMS IN AN AP**

The formula for finding sum of  $n^{\text{th}}$  term of an AP is;

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

**EXAMPLE**

1. An arithmetic progression has first term 3 and common difference 2. Find the sum of the first 50 terms.

**Solution**

$$a = 3, d = 2, n = 50$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{50} = \frac{50}{2}(2(3) + (50-1)2)$$

$$= 25(6+98)$$

$$= 25 \times 104$$

$$S_{50} = 2600$$

2. The second term of an AP is 15 and the fifth term is 21. Find the common difference, the first term and the sum of the first ten terms.

**Solution**

$$2^{\text{nd}} \text{ term} = a + d(2-1)$$

$$15 = a + d \dots\dots\dots (i)$$

$$5^{\text{th}} \text{ term} = a + d(5-1)$$

$$21 = a + 4d \dots\dots\dots (ii)$$

ii) - i); gives  $21 = a + 4d$

$$(15 = a + d)$$

$$6 = 3d$$

$$d = \frac{6}{3} = 2$$

in i) put  $d = 2$

$$15 = a + 2$$

$$15 - 2 = a$$

$$a = 13$$

$$S_{10} = \frac{10}{2}(2(13) + (10-1)2)$$

$$= 5(26+18)$$

$$= 220$$

3. How many terms of an AP 1, 3, 5.....are required to make a sum of 1525?

**Solution**

$$a = 1, d = 2$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$1521 = \frac{n}{2}(1) + (n-1)2$$

$$1521 = \frac{n}{2}(2 + 2n - 2)$$

$$1521 = \frac{n}{2} \times 2n$$

$$1521 = n^2$$

$$n = \sqrt{1521}$$

$$= 39$$

4. The sixth term of an AP is twice the third term, and the first term is 3. Find the common difference and the 10<sup>th</sup> term.

**Solution**

$$6^{\text{th}} \text{ term} = 3 + d(6-1)$$

$$= 3 + 5d$$

$$\begin{aligned}
 3^{\text{rd}} \text{ term} &= 3 + d(3-1) \\
 &= 3 + 2d \\
 \text{But } 6^{\text{th}} \text{ term} &= 2 \times 3^{\text{rd}} \text{ term} \\
 \therefore 3 + 5d &= 2(3 + 2d) \\
 3 + 5d &= 6 + 4d \\
 5d - 4d &= 6 - 3 \\
 d &= 3 \\
 10^{\text{th}} \text{ term} &= 3 + 3(10 - 1) \\
 &= 3 + 3(9) \\
 &= 30
 \end{aligned}$$

**EXERCISE**

1. The twentieth term of an AP is 55 and the eightieth term is 255. Find the first term, the common difference and the thirtieth term
2. How many terms are there in the following series  
200, 192, 184.....120
3. The first term of an AP is 2 and the last term is 59. If the sum of their terms is 610, find the common difference.
4. The sum of n terms of an AP for all values of n is  $5n^2 - 2n$ . find the first term and the common difference
5. Find the value of n if the value of the first n terms of the two APs 2, 5, 8..... and 47, 45, 43 are equal

**GEOMETRIC PROGRESSION (GP)**

$1^{\text{st}}, 2^{\text{nd}}, 4^{\text{th}}, 8^{\text{th}} \dots n^{\text{th}} \text{ term}$

$1, 1 \times 2, 1 \times 2 \times 2, 1 \times 2 \times 2 \times 2 \dots n^{\text{th}} \text{ term}$

$a \times 2^0, a \times 2^1, a \times 2^2, a \times 2^3 \dots n^{\text{th}} \text{ term}$

(Position and power differs by 1)

$$\frac{a \times 2}{a} = \frac{a \times 2 \times 2}{a \times 2} = \frac{a \times 2 \times 2 \times 2}{a \times 2 \times 2} = 2$$

2 is a common ratio

$a, ar^1, ar^2, ar^3 \dots ar^{n-1}$

$\therefore n^{\text{th}} \text{ term of a GP} = ar^{n-1}$

**EXAMPLE**

1. Write down the sixth term and the  $n^{\text{th}}$  term of the GP. 2, 1,  $\frac{1}{2}$  ...

**Solution**

$$2, 1, \frac{1}{2}, \dots$$

$$a = 2, r = \frac{1}{2}, n = 6$$

$$6^{\text{th}} \text{ term} = 2 \times \left(\frac{1}{2}\right)^{6-1}$$

$$= 2 \times \left(\frac{1}{2}\right)^5$$

$$= \frac{1}{16}$$

2. The sixth term of GP is 16 and the third term 2, find the first term and the common ratio

**Solution**

$$6^{\text{th}} \text{ term} = ar^{6-1}$$

$$16 = ar^5 \dots \dots \dots \text{i)}$$

$$3^{\text{rd}} \text{ term} = ar^{3-1}$$

$$2 = ar^2 \dots \dots \dots \text{ii)}$$

$$\text{i)} \quad \div \quad \text{ii)} \text{ gives } \frac{16}{2} = \frac{ar^5}{ar^2}$$

$$8 = r^3$$

$$2^3 = r^3$$

$$r = 2$$

in ii) put  $r = 2$

$$2 = a \times 2^2$$

$$2 = a \times 4$$

$$2 = 4a$$

$$a = \frac{1}{2}$$

**SUM OF TERMS IN A GP**

The formula for finding sum of terms in a GP where  $r > 1$  is;

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Where  $r < 1$  the formula is;

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

**EXAMPLE**

1. A GP has first term 3 and common ratio 2. Find the sum of the first ten term.

**Solution**

$$a = 3, r = 2, n = 10$$

$$\begin{aligned} S_{10} &= \frac{3(2^{10} - 1)}{2 - 1} \\ &= \frac{3(1024 - 1)}{1} \\ &= 3 \times 1023 \\ &= 3069 \end{aligned}$$

2. What is the largest number of terms of GP,  $8 + 24 + 74 + \dots$  that will give less than 3,000,000

**Solution**

$$A = 8, r = 3, s_n = 3,000,000$$

$$S_n = \frac{8(3^n - 1)}{3 - 1}$$

$$\frac{8(3^n - 1)}{3 - 1} < 3,000,000$$

$$4(3^n - 1) < 3,000,000$$

$$3^n - 1 < \frac{3,000,000}{4}$$

$$3^n < 750,000 + 1$$

$$3^n < 750,001$$

$$\log 3^n < \log 750,001$$

$$n \log 3 < \log 750,001$$

$$n < \frac{\log 750,001}{\log 3}$$

$$n < \frac{5.8751}{0.4771}$$

$$n < 12.31$$

$$n < 12$$

$\therefore$  The largest number is 12

**EXERCISE**

1. Write down the sixth term and the  $n^{\text{th}}$  term of
  - i. 2, 4, 8 .....
  - ii. 3, -6, 12 .....
2. The first term of a GP is 5 and the common ratio is 2. Find the common ratio
3. The third term of a GP is 6 and the eighth term is 192. Find the first term of the series and the common ratio.
4. Mr. Motto saves from his profit as follows, K3 the first week, K6 the second week, K12 the third week, K24 the fourth week and so on. If he manages to keep on saving under this system,
  - i. How much would he save in the tenth week
  - ii. What would be the total amount of his savings in the first ten weeks
5. The sum of  $n$  terms of a GP is 14. The first term is 2, find the possible values of the sum of first five terms.
6. The first and fourth terms of a GP are  $\frac{x}{y^2}$  and  $\frac{y}{x^5}$ . Find the second and third terms.
7. In a GP a product of the first and 7<sup>th</sup> terms is equal to the 4<sup>th</sup> term. Given that the sum of the first and 4<sup>th</sup> term is 9. Find the 1<sup>st</sup> term and common ratio.

**CHAPTER 5****TRAVEL GRAPHS**

We shall look at

1. Velocity- time graph
2. Acceleration – time graph
3. Deceleration – time graph

To calculate the distance covered, find the areas under the given graph.

The formula for finding the area depends on the shape of the graph.

$$\text{Triangle} = \frac{1}{2} \text{ base} \times \text{height}$$

$$\text{Rectangle} = \text{length} \times \text{breadth}$$

$$\text{Trapezium} = \frac{1}{2}(\text{sum of sides}) h$$

**VELOCITY TIME GRAPH**

$$\text{Velocity} = \frac{\text{distance}}{\text{time}}$$

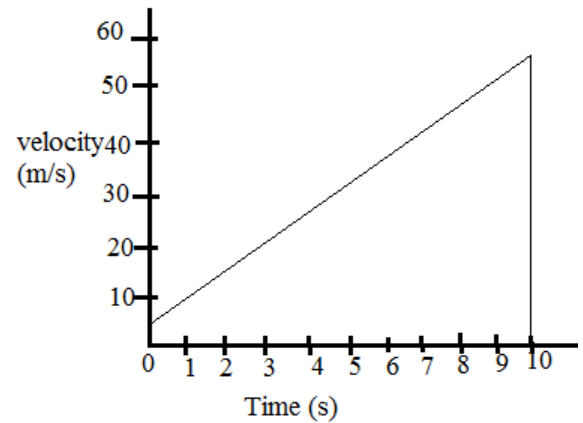
**Velocity** is the rate of change of distance for given time

**EXAMPLE**

1. Draw a velocity time graph for a body which starts with initial velocity of 4m/s and continues to move for a further 10s with acceleration of 5m/s<sup>2</sup>. Use your graph to find the distance covered by the body in this time.

**Solution**

|           |   |   |    |    |    |    |    |    |    |    |    |
|-----------|---|---|----|----|----|----|----|----|----|----|----|
| Time S    | 0 | 1 | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| Velo city | 4 | 9 | 14 | 19 | 24 | 29 | 34 | 39 | 44 | 49 | 54 |



To find distance covered, calculate the area under the graph.

$$\text{Area of trapezium} = \frac{1}{2}(\text{Sum of sides})h$$

$$= \frac{1}{2} \left( \frac{4m}{s} + \frac{54m}{s} \right) \times 10s$$

$$= \frac{1}{2} \times \frac{58m}{s} \times 10s$$

$$= 290m$$

$$\therefore \text{Distance traveled} = 290m$$

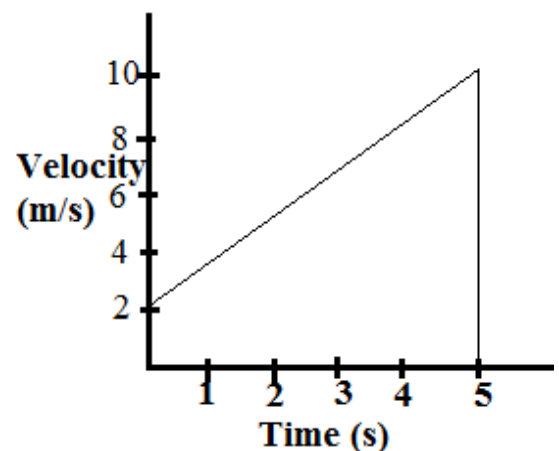
**ACCELERATION**

$$\text{Acceleration} = \frac{\text{velocity}}{\text{time}} = \frac{m/s}{s}$$

$$= m/s^2$$

**EXAMPLE**

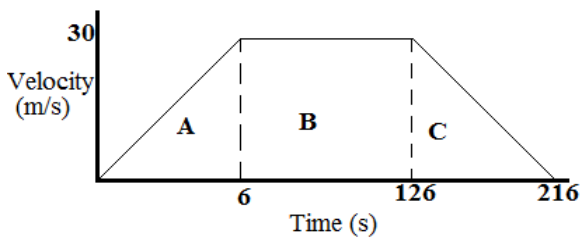
1. Find the acceleration of the graph below



**Solution**

$$\begin{aligned}
 \text{Acceleration} &= \frac{\text{velocity}}{\text{time}} \\
 &= \frac{(10-2)\text{m/s}}{5\text{s}} \\
 &= \frac{8\text{m/s}}{5\text{s}} \\
 &= 1.6\text{m/s}^2
 \end{aligned}$$

2. A car starts from rest and is uniformly accelerated at  $5\text{m/s}^2$  until it attains a velocity of  $30\text{m/s}$ . it maintains this velocity for  $120\text{s}$  after which it is brought to the rest in another  $90\text{s}$ . Sketch the velocity time graph for this motion and use it to find the distance covered by the car. Find the deceleration for the last part.

**Solution**

$$\begin{aligned}
 \text{Area A} &= \frac{1}{2} \times 6\text{s} \times \frac{30\text{m}}{\text{s}} \\
 &= 90\text{m}
 \end{aligned}$$

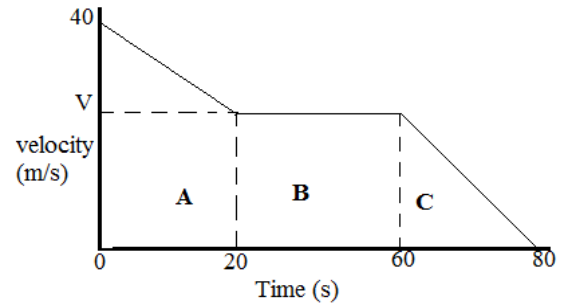
$$\begin{aligned}
 \text{Area B} &= \frac{30\text{m}}{\text{s}} \times 120\text{s} \\
 &= 3600\text{m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Area C} &= \frac{1}{2} \times 90\text{s} \times \frac{30\text{m}}{\text{s}} \\
 &= 1350\text{m}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Distance} &= 90\text{m} + 3600\text{m} + 1350\text{m} \\
 &= 5040\text{m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Deceleration} &= \frac{0-30\text{m/s}}{90\text{s}} \\
 &= \frac{-30\text{m/s}}{90\text{s}} \\
 &= -0.3\text{m/s}^2
 \end{aligned}$$

3. Figure below is a speed time graph of a velocity. The train decelerates  $40\text{m/s}$  to  $V\text{m/s}$  in  $20\text{s}$ . It then maintain this speed for the next  $40\text{seconds}$  after which it decelerates uniformly for  $20\text{seconds}$  and stops.



Given that the total distance travelled is  $1600\text{m}$ . Calculate

- Value of  $V$
- Deceleration during the first  $20\text{seconds}$

**Solution**

$$\begin{aligned}
 \text{Area of trapezium A} &= \frac{1}{2}(\text{sum of sides}) \times h \\
 &= \frac{1}{2} \left( \frac{V+40\text{m}}{\text{s}} \times 20\text{s} \right) \\
 &= 10(V+40)\text{m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Area B} &= 40\text{s} \times \frac{Vm}{\text{s}} \\
 &= 40Vm
 \end{aligned}$$

$$\begin{aligned}
 \text{Area C} &= \frac{1}{2} \times 20\text{s} \times \frac{Vm}{\text{s}} \\
 &= 10Vm
 \end{aligned}$$

$$A+B+C=1600\text{m}$$

$$\therefore 10(V+40) + 40V + 10V = 1600\text{m}$$

$$10V+400+50V=1600\text{m}$$

$$60V=1600\text{m}-400\text{m}$$

$$60V=1200\text{m}$$

$$V=20\text{m/s}$$



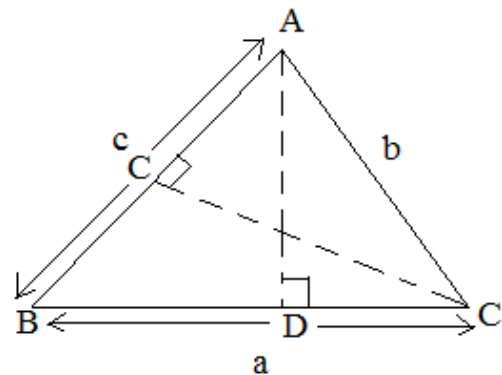
$$\begin{aligned}
 \text{Deceleration} &= \frac{\text{change in speed}}{\text{change in time}} \\
 &= \frac{(20-40)\text{m/s}}{20\text{s}} \\
 &= \frac{-20\text{m/s}}{20\text{s}} \\
 &= -1 \text{ m/s}^2
 \end{aligned}$$

**EXERCISE**

1. A car starts from the rest and is uniformly accelerated at  $5\text{m/s}^2$  until it attains a velocity of  $30\text{m/s}$ . It maintains this velocity for  $120\text{s}$  after which it is brought to the rest in another  $90\text{s}$ . Sketch the velocity time graph for this motion and use it to find the distance covered.

**CHAPTER 6****TRIGONOMETRY****SINE AND COSINE RULES**

Sine and cosine rules are used to find unknown sides and angles of triangles which are not right angled.

**SINE RULE**

In  $\triangle ABD$

$$\sin B = \frac{AD}{AB}$$

$$\sin B = \frac{AD}{c}$$

$$c \sin B = AD$$

In  $\triangle ADC$

$$\sin C = \frac{AD}{AC}$$

$$\sin C = \frac{AD}{b}$$

$$b \sin C = AD$$

$$\therefore c \sin B = b \sin C$$

Divide both sides by  $c$  and  $b$

$$\therefore \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\text{In general; } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

**EXAMPLE**

1. Find the unknown side a

**Solution**

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 50^\circ}{a} = \frac{\sin 60^\circ}{12}$$

$$\frac{0.7660}{a} = \frac{0.8660}{12}$$

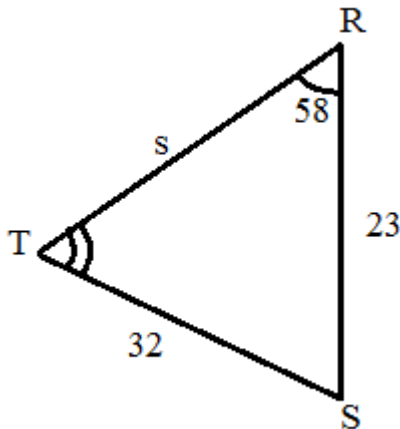
$$12 \times 0.7660 = a \times 0.8660$$

$$\frac{9.192}{0.866} = a$$

$$a = 10.6 \text{ cm}$$

2. In triangle **RST**, **RS**=23cm, **ST**=32cm, angle **R**=58. Find angle T and RT.

**Solution**



$$\frac{\sin T}{23} = \frac{\sin 58^\circ}{32}$$

$$\sin T = \frac{0.8480 \times 23}{32}$$

$$= 0.6095$$

$$\sin^{-1} 0.6095 = 37.6^\circ$$

$$\frac{\sin S}{S} = \frac{\sin R}{r}$$

$$\hat{S} + 58^\circ + 37.6^\circ = 180^\circ, \text{ Sum } <s \text{ in a } \Delta$$

$$\hat{S} = 180^\circ - 95.6^\circ$$

$$= 84.4^\circ$$

$$\frac{\sin 84.4^\circ}{S} = \frac{\sin 58^\circ}{32}$$

$$32 \sin 84.4^\circ = S \sin 58^\circ$$

$$\frac{32 \sin 84.4^\circ}{\sin 58^\circ} = S$$

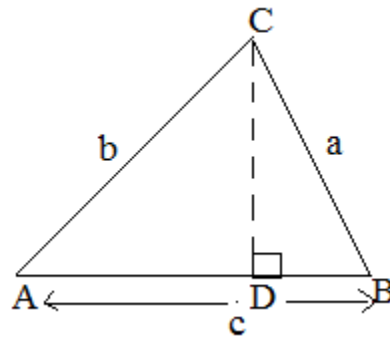
$$S = \frac{32 \sin 0.9952}{\sin 58^\circ}$$

$$= \frac{31.8482}{0.8480}$$

$$= 37.7 \text{ cm}$$

$$\therefore RT = 37.7 \text{ cm}$$

**COSINE RULE**



In  $\Delta ABC$ ,

$$BC^2 = DC^2 + DB^2, \text{ Pythagoras}$$

$$\text{But } DB = AB - AD$$

$$\therefore DB = c - AD$$

$$\therefore BC^2 = DC^2 + (c - AD)^2$$

$$BC^2 = DC^2 + c^2 - 2c \cdot AD + AD^2$$

**Rearranging**

$$BC^2 = DC^2 + AD^2 + c^2 - 2c \cdot AD$$

$$\text{But } DC^2 + AD^2 = b^2$$

$$\therefore BC^2 = b^2 + c^2 - 2c \cdot AD$$

In  $\Delta ADC$ ,

$$\cos A = \frac{AD}{b}$$

$$b \cos A = AD$$

$$\therefore BC^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

**In general;**

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

**To find angles make Cosine subject**

$$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

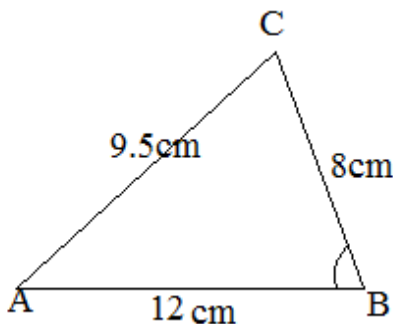
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

### EXAMPLE

- Find the stated angle that in a triangle ABC given that  $a=8\text{cm}$ ,  $b=9.5\text{cm}$ ,  $c=12\text{cm}$ . find angle B

**Solution**



**Solution**

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{8^2 + 12^2 - 9.5^2}{2 \times 8 \times 12}$$

$$= \frac{177.75}{192}$$

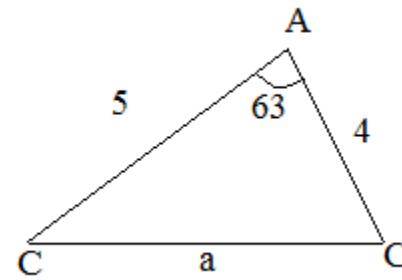
$$= 0.6133$$

$$\cos^{-1} 0.6133 = 52.2^\circ$$

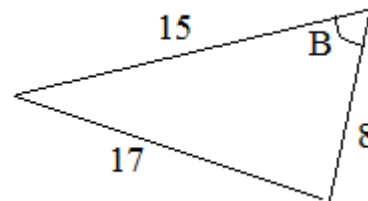
$$\therefore \hat{B} = 52.2^\circ$$

### EXERCISE

- Find the unknown side in the triangle below



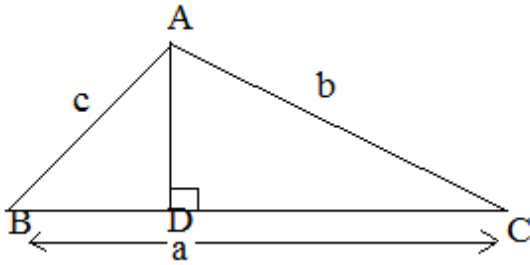
- In a triangle ABC,  $AB=8\text{cm}$ ,  $AC=12\text{cm}$  and  $\hat{A}=36^\circ$ . Find BC
- In triangle XYZ,  $x=5\text{cm}$ ,  $y=6\text{cm}$  and  $\hat{Z}=60^\circ$ . Show that  $z=\sqrt{31}$
- Find the labeled angles in these triangle



### MIXED PROBLEMS

Here are tips to help you to know which rule to use (between cosine and sine rule).

- SSS:** if the three sides triangle are known then use **cosine rule** to find angles
- SAS:** if two sides of a triangle are known and the angle between them is known, then use **cosine rule** to find the other side
- ASA:** if one side and two angles are known then use **sine rule** to find the other side
- SSA:** if the two sides of a triangle are known and the angle which is not between them, then use **sine rule** to find the other angle (there may be two possible outcomes)

**AREA OF A TRIANGLE**

$$\text{Area } \Delta = \frac{1}{2} \text{ base} \times \text{height}$$

$$\text{Area } \Delta ABC = \frac{1}{2} a \times AD$$

In a  $\Delta ABD$

$$\sin B = \frac{AD}{c}$$

$$c \sin B = AD$$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2} ac \sin B$$

$$\text{Area of } \Delta ABC = \frac{1}{2} ab \sin C$$

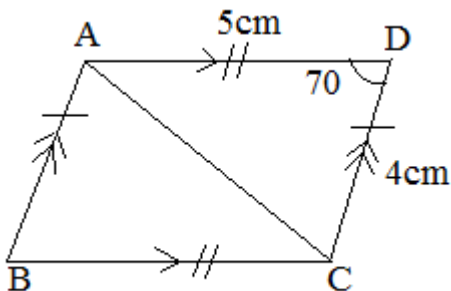
$$\text{Area of } \Delta ABC = \frac{1}{2} bc \sin A$$

NB: These formula expresses the area in terms of two sides and their included angle.

**EXAMPLE**

1. A parallelogram has sides of 4cm and 5cm and an angle of  $70^\circ$ . What is its area

**Solution**



NB: diagonal bisects the //gram

$$\Delta ADC \equiv \Delta CBA$$

$$\text{Area of } \Delta ADC = \frac{1}{2} ac \sin D$$

$$= \frac{1}{2} \times 5\text{cm} \times 4\text{cm} \times \sin 70^\circ$$

$$= 10\text{cm} \times 0.9370\text{cm}$$

$$= 9.4\text{cm}^2$$

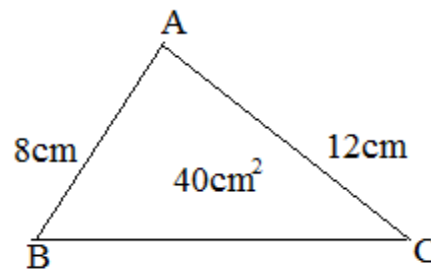
$$\therefore \text{Area of a //gram} = 2 \times 9.4\text{cm}^2$$

$$= 18.8\text{cm}^2$$

**Finding an angle**

A triangle has an area of  $40\text{cm}^2$ . The two adjacent sides of a triangle are 8cm and 12cm long. What is the angle between them?

**Solution**



$$\text{Area of } \Delta ABC = \frac{1}{2} bc \sin A$$

$$40 = \frac{1}{2} \times 8 \times 12 \times \sin A$$

$$40 = 4 \times 12 \times \sin A$$

$$\frac{40}{48} = \sin A$$

$$\sin A = 0.8333$$

$$A = 56.4^\circ$$

$$\text{Or } A = 123.6^\circ$$

$$(\sin 56.4^\circ) = \sin (180^\circ - 56.4^\circ)$$

**BEARING**

Many problems involving bearings can be solved using the sine and cosine rules.

**EXAMPLE**

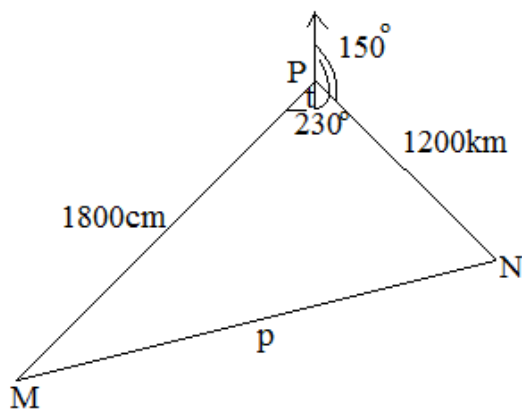
1. Two planes leaves an airport at the same time. One flies on a bearing of  $150^\circ$  at  $400\text{km/h}$  of  $230^\circ$  at  $600\text{km/h}$ . how far apart are the two planes after three hours

**Solution**

Distance = time  $\times$  speed

$$\begin{aligned}\therefore \text{Distance (MP)} &= 400\text{km/h} \times 3\text{hrs} \\ &= 1200\text{km}\end{aligned}$$

$$\begin{aligned}\text{Distance (NP)} &= 600\text{km/h} \times 3\text{hrs} \\ &= 1800\text{km}\end{aligned}$$



$$150^\circ + t = 230^\circ$$

$$t = 230^\circ - 150^\circ$$

$$= 80^\circ$$

$$p^2 = m^2 + n^2 - 2mn \cos P$$

$$p^2 = 1200^2 + 1800^2 - (1200)(1800) \cos 80^\circ$$

$$= 1440000 + 3240000 - 2320000 \times 0.1736$$

$$= 4680000 - 749952$$

$$p^2 = 3930048$$

$$p = \sqrt{3930048}$$

$$= 1982.43$$

$$\therefore p = 1980 \text{ (to 3 sig. fig)}$$

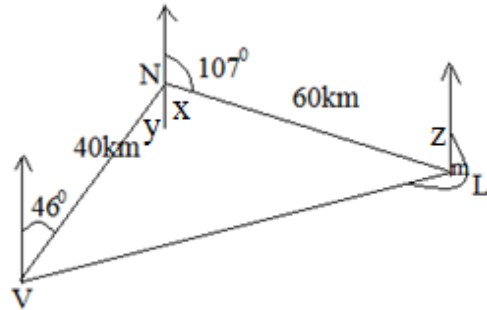
$$\therefore \text{They are 1980km apart}$$

2. A man drives from his village to town, a distance of  $40\text{km}$  on a bearing of  $046^\circ$ , he

then drives from the town to a lake  $60\text{km}$  away on a bearing of  $107^\circ$ .

- How far away is the lake from his house
- What is the bearing of his house from the lake?

**Solution**



$$y = 46^\circ, \text{ alt } < s$$

$$x + 107^\circ = 180^\circ, < s \text{ on str. Line}$$

$$x = 180^\circ - 107^\circ$$

$$= 73^\circ$$

$$\therefore \widehat{VOL} = x + y$$

$$= 46^\circ + 107^\circ$$

$$= 119^\circ$$

$$n^2 = l^2 + v^2 - 2lv \cos N$$

$$n^2 = 40^2 + 60^2 - 2(40)(60) \cos 119^\circ$$

$$n^2 = 1600 + 3600 - 4800 \times -0.4848$$

$$n^2 = 5200 + 2327.04$$

$$n^2 = 7527.04$$

$$n = \sqrt{7527.04}$$

$$= 86.8\text{km}$$

$$\text{a. } \therefore \text{They are 86.8km apart}$$

$$\begin{aligned}\cos L &= \frac{v^2 + n^2 - l^2}{2vn} \\ &= \frac{60^2 + 86.759^2 - 40^2}{2 \times 60 \times 86.759} \\ &= \frac{9527.04}{10411.080}\end{aligned}$$

$$\begin{aligned}\cos L &= 0.9151 \\ &= 23.8^\circ\end{aligned}$$

Z = x, alt angles

$$\therefore z = 73^\circ$$

$$23.8^\circ + 73^\circ + m = 360^\circ, < s @ a point.$$

$$\begin{aligned}m &= 360^\circ - 96.8^\circ \\ &= 263.2^\circ\end{aligned}$$

$\therefore$  The bearing of home from the lake is  
263.2°

### EXERCISE

- Given that A=(0,0), B=(4, 3) and C=(4, -3).
  - Find the area of triangle ABC
  - Calculate angle BAC
- What is the area of an equilateral triangle with sides 10cm long?
- A triangle has an area of 60cm<sup>2</sup>. Two adjacent sides of the triangle are 8cm and 12cm long. What is the angle between them?
- An aero plane leaves airport A on a bearing of N23°E and flies for 340km to another airport B and flies on a bearing of N60°W to another airport C. If the airport A and C are 680km apart. Calculate the bearing of airport A from airport C.
- A boat sails 40km from point A to B east of A on a lake. Lodge P is south of point B. the bearing of Q from point A is S52°E and the bearing of lodge P from B is S39°W. if P is 60km from Q. calculate the bearing of lodge Q from lodge P.

## CHAPTER 7

## POLYNOMIALS

Poly means many.

$3x^2$  monomial

$3x^2 + 2x$  Polynomial

### DIVISION OF POLYNOMIAL

#### EXAMPLE

- Find the quotient and the remainder of the following;

$$x^2 + 2x + 1 \text{ Divided by } x + 1$$

We start with dividing the highest power of x by (x+1), dividing  $x^2$  by x, give x so put x on top line, multiply x by x(x+1) and subtract the result from the first two terms of the polynomial.

$$\begin{array}{r} \text{I.} \quad \quad \quad x + 1 \\ x + 1 \overline{) x^2 + 2x + 1} \\ \underline{-(x^2 + x)} \phantom{+ 1} \\ x + 1 \\ \underline{-(x + 1)} \\ 0 \end{array}$$

$\therefore$  The quotient is  $x + 1$  and the remainder is 0

NB; in terms of quotient, divisor and the remainder, the polynomial can be expressed as

$$\text{Polynomial} = \text{divisor} \times \text{quotient} + \text{Remainder}$$

- Find the quotient and the remainder when  $x^3 - 3x + 4x + 1$  is divided by  $x - 2$

**Solution**

$$\begin{array}{r}
 x^2 - x + 2 \\
 x - 2 \overline{) x^3 - 3x^2 + 4x + 1} \\
 \underline{-(x^3 - 2x^2)} \phantom{+ 1} \\
 x^2 - 4x \phantom{+ 1} \\
 \underline{-(-x^2 + 2x)} \phantom{+ 1} \\
 2x + 1 \phantom{+ 1} \\
 \underline{-(2x - 4)} \\
 5
 \end{array}$$

$\therefore$  The quotient is  $x^2 - x + 2$  and the remainder is 5.

**EXERCISE**

Find the quotient and the remainder of the following,

1.  $3x^3 + 5x^2 + 7x - 14$  divided by  $x - 1$
2.  $x^3 - 4x^2 + x + 2$ , divided by  $x^2 - 3$
3.  $8x^3 + 6x^2 - 3x - 1$  divided by  $2x + 1$

**REMAINDER THEOREM**

The remainder when  $f(x)$  is divided by  $x + a$  is  $f(-a)$

**EXAMPLE**

Use remainder theorem to find the remainder when  $5x^3 + 4x^2 + 7x - 12$  is divided by  $x - 1$

**Solution**

$$x - 1 = 0$$

$$x = 1$$

$$f(1) = 3(1)^3 + 4(1)^2 + 7(1) - 12$$

$$= 3 + 4 + 7 - 12$$

$$= 2$$

$\therefore$  2 is the remainder

**EXAMPLE 2**

When  $x^3 + 3x^2 + ax + b$  is divided by  $(x + 1)$  the remainder is 5 and when it is divided by  $x - 2$ , the remainder is 8. Find  $a$  and  $b$

**Solution**

$$x + 1 = 0$$

$$x = -1$$

$$f(-1) = (-1)^3 + 3(-1)^2 + a(-1) + b$$

$$5 = -1 + 3 - a + b$$

$$5 - 2 = b - a$$

$$3 = b - a \dots\dots\dots(i)$$

$$x - 2 = 0$$

$$x = 2$$

$$f(2) = (2)^3 + 3(2)^2 + a(2) + b$$

$$8 = 8 + 12 + 2a + b$$

$$8 - 20 = 2a + b$$

$$-12 = 2a + b \dots\dots\dots(ii)$$

In (ii) put  $b = a + 3$

$$\therefore -12 = 2a + (a + 3)$$

$$-12 = 2a + a + 3$$

$$-12 - 3 = 3a$$

$$-15 = 3a$$

$$a = -5$$

In (i) put  $a = -5$

$$\therefore 3 = b - (-5)$$

$$b = -2$$

**EXERCISE**

1. Use the remainder theorem to find the following expressions
  - i.  $x^3 - 8x^2 - 4x + 5$  divided by  $(x - 2)$
  - ii.  $7x^3 - 7x - 42$  divided by  $(x - 2)$
  - iii.  $4x^3 - 5x + 4$  divided by  $(2x - 1)$

2. When  $x - 3$  is divided into  $x^3 - 4x^2 + 5x + k$  find  $k$ .
3. When  $ax^3 + bx^2 + 3x - 4$  is divided by  $(x - 1)$  the remainder is 3, and when it is divided by  $(x + 2)$ , the remainder is 6. Find  $a$  and  $b$
4.  $(x - 1)$  and  $(x + 1)$  are factors of the expression  $x^3 + ax^2 + bx + c$ . the expressions leaves the remainder of 12 when it is divided by  $(x - 2)$ . What are the values of  $a, b$  and  $c$

### FACTOR THEOREM

If  $f(a) = 0$ , then  $(x + a)$  is a factor of  $f(x)$

#### **EXAMPLES**

Factorize,  $t^3 - 6t^2 + 11t - 6$

There are two methods of factorizing polynomials

- i. Using factors of the constant
- ii. By inspection

#### Solution

- i. By using factors of the constant.  
We find the factors of the constant, in this case

The factors of 6 are;  $\pm 1, \pm 2, \pm 3, \pm 6$

Let  $t = 1$

$$\begin{aligned} f(1) &= (1)^3 - 6(1)^2 + 11(1) - 6 \\ &= 1 - 6 + 11 - 6 \\ &= 0 \end{aligned}$$

$\therefore (t - 1)$  is a factor

$$\begin{array}{r} t-1 \overline{) t^3 - 6t^2 + 11t - 6} \\ \underline{-(t^3 - t^2)} \phantom{- 6} \\ -5t^2 + 11t \phantom{- 6} \\ \underline{-(-5t^2 + 5t)} \phantom{- 6} \\ 6t - 6 \\ \underline{-(6t - 6)} \\ 0 \end{array}$$

$$\begin{aligned} \therefore t^3 - 6t^2 + 11t - 6 &= \\ &= (t - 1)(t^2 - 5t + 6) \\ &= (t - 1)(t - 2)(t - 3) \end{aligned}$$

- ii. By inspection

Used when one of the factors is known

#### **EXAMPLE**

Factorize  $2x^3 - x^2 - 2x + 1$

If one factor is  $(x - 1)$

#### Solution

$$\begin{aligned} (x - 1)(ax^2 + bx + c) &= 2x^3 - x^2 - 2x + 1 \\ &= ax^3 + bx^2 + cx - ax^2 - bx - c \\ &= ax^3 + bx^2 - ax^2 + cx - bx - c \\ &= ax^3 + (b - a)x^2 + (c - b)x - c \end{aligned}$$

#### **(Factorizing)**

$$\therefore a = 1, b - a = -1, -c = 1$$

$$b - 2 = -1, c = -1$$

$$b = 1$$

$$\therefore (x - 1)(2x^2 + x - 1) \text{ We substitute}$$

$$(x - 1)(x + 1)(2x - 1)$$

#### **EXERCISE**

1. Factorize the following polynomials by factor method
  - i.  $x^3 + 3x^2 - 6x - 8$
  - ii.  $6y^3 + 5y^2 - 21y + 10$
  - iii.  $2k^3 + k^2 + k + 2$
  - iv.  $6x^3 - 13x^2 + x + 2$
2. Factorize the following polynomials given one factor
  - i.  $x^3 - 2x^2 - 5x + 6, (x - 1)$
  - ii.  $2x^3 + x^2 - 8x - 4, (x - 2)$
  - iii.  $x^3 - 4x^2 + x + 6, (x + 1)$
3. Show that  $12x^3 + 16x^2 - 5x - 3$  is divisible by  $(2x - 1)$  and hence find the other factors of the expression



**IDENTICAL POLYNOMIALS**

These are polynomials which when expanded, gives the same expression

**EXAMPLE**

Given that  $2x^3 + kx^2 + mx + 1$  is identical to  $(2x - 1)(x^2 + x + 1)$  find  $k$  and  $m$ .

**Solution**

$$\begin{aligned} 2x^3 + kx^2 + mx + 1 &= (2x + 1)(x^2 + x + 1) \\ &= 2x^3 + 3x^2 + 3x + 1 \\ \therefore k &= 3, m = 3 \end{aligned}$$

**EXERCISE**

1. Given that  $By^3 - Cy^2 - y + 3$  is identical to  $(y - 1)(4y^2 - 1)$ . Find the value of B and C
2. Given that  $(3x^2 + x - 6)(x + 1)$  and  $3x^2 + kx^2 + hx - 6$  are identical, find the value of k and h.

**SOLVING POLYNOMIAL**

Factor theorem is used when solving polynomial.

**EXAMPLE**

$$x^3 - x^2 - x + 1 = 0$$

**Solution**

Factors of 1 =  $\pm 1$

Let  $x = 1$

$$\begin{aligned} f(1) &= (1)^3 - (1)^2 - (1) + 1 \\ &= 0 \end{aligned}$$

$$f(1) = 0$$

$\therefore x - 1$  is a factor

$$\begin{array}{r} x^2 + 0 - 1 \\ x - 1 \overline{) x^3 - x^2 - x + 1} \\ \underline{-(x^3 - x^2)} \phantom{+ 1} \\ 0 - x \phantom{+ 1} \\ \underline{-(0 - 0)} \phantom{+ 1} \\ -x + 1 \\ \underline{-(-x + 1)} \\ 0 \end{array}$$

$$(x + 1)(x^2 - 1) = 0$$

$$(x + 1)(x - 1)(x + 1) = 0$$

$$(x + 1)(x + 1)(x - 1) = 0$$

$$\therefore x = -1 \text{ or } x = 1$$

**EXERCISE**

Solve the following

1.  $x^3 + 3x^2 - x - 3 = 0$
2.  $t^3 - 13t - 12 = 0$
3.  $4y^3 - 12y^2 - y + 3 = 0$
4.  $x^3 - 3x^2 + 3x - 1 = 0$
5.  $x^3 - 9x + 10 = 0$

**CHAPTER 8****PROBABILITY**

Means a chance

$$\text{Probability} = \frac{\text{no. of ways an event can occur}}{\text{total no. of possible outcomes}}$$

$$P(\text{certainty}) = 1$$

$$P(\text{impossible event}) = 0$$

**Generally**, probability is between 0 and 1 inclusive

$$P \geq 0$$

$$P \leq 1$$

$$\text{Thus } 0 \leq p \leq 1$$

$$p(\text{success}) + p(\text{not success}) = 1$$

**EXAMPLE**

Given that the probability of having girls at a school is  $\frac{7}{10}$ , what is the probability of not having girls.

**Solution**

$$p(\text{not having}) + p(\text{having}) = 1$$

$$p(\text{not having}) = 1 - \frac{7}{10}$$

$$p(\text{not having}) = \frac{3}{10}$$

**EXAMPLE 2**

What is the probability of getting an even number when a die is rolled?

**Solution**

The numbers are, 1, 2, 3, 4, 5, 6

Even numbers are; 2, 4 and 6

$$\begin{aligned} \therefore p(\text{even numbers}) &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

**EXAMPLE**

A bag contains 3 red and 7 white balls. What is the probability that when I pick one ball out, it is a red ball?

**Solution**

There are 3 red ball

$$\therefore p(\text{red ball}) = \frac{3}{10}$$

**PARK OF CARDS**

In total there are 54 cards but in mathematics we only use 52 cards

| Clubs | Hearts | spades | Diamond |
|-------|--------|--------|---------|
| Ace   | Ace    | Ace    | Ace     |
| King  | King   | King   | king    |
| Queen | Queen  | queen  | Queen   |
| Pack  | Pack   | Pack   | Pack    |
| 10    | 10     | 10     | 10      |
| 9     | 9      | 9      | 9       |
| 8     | 8      | 8      | 8       |
| 7     | 7      | 7      | 7       |
| 6     | 6      | 6      | 6       |
| 5     | 5      | 5      | 5       |
| 4     | 4      | 4      | 4       |
| 3     | 3      | 3      | 3       |
| 2     | 2      | 2      | 2       |

**EXAMPLE**

A card is picked from a park of cards. What is the following probabilities?

- P( picking an ace)
- P(picking an ace of diamond)
- P(picking a red card)
- P(picking a red heart)
- P(picking two red heart)
- P(a card with a prime number on it)

**Solution**

$$\text{i. } P(\text{ace}) = \frac{4}{52} = \frac{1}{13}$$

$$\text{ii. } P(\text{ace of diamond}) = \frac{1}{52}$$

$$\text{iii. } p(\text{red card}) = \frac{26}{52} = \frac{1}{2}$$

$$\text{iv. } p(\text{heart}) = \frac{13}{52} = \frac{1}{4}$$

$$\text{v. } p(2 \text{ red heart}) = \frac{1}{52}$$

$$\begin{aligned} \text{vi. } p(\text{card with prime no.}) &= \frac{16}{52} \\ &= \frac{4}{13} \end{aligned}$$

### EXERCISE

1. A die is rolled once, calculate the probability of getting the following
  - a. Even number
  - b. Prime or odd number
  - c. Score of 6
  - d. Score of 10
2. A bag contains 6 red balls and 8 blue balls. A ball is removed, what is the probability that this ball is blue
3. A card is picked at random from pack of cards. What are these probabilities
  - a.  $p(\text{king})$
  - b.  $P(\text{diamond})$
  - c.  $p(18 \text{ of hearts})$

### EXPERIMENTAL PROBABILITIES

$$\text{Exp. Prob} = \frac{\text{no. of favourable outcomes}}{\text{total number of trials}}$$

### EXAMPLE

One weekend, the number of goals scored by the teams in a football league matches were as follows.

| No. of goals | 0 | 1  | 2  | 3 | 4 | 5 |
|--------------|---|----|----|---|---|---|
| No. of teams | 8 | 10 | 12 | 2 | 3 | 5 |

What is the probability that a team picked at random had the following score?

- a. No goals

- b. More than one goal
- c. Less than mean number of goals per football match.

### Solutions

$$\text{a. } P(\text{no goals}) = \frac{8}{40} = \frac{1}{5}$$

$$\begin{aligned} \text{b. } P(>1 \text{ goal}) &= \frac{12+2+3+5}{40} \\ &= \frac{22}{40} \\ &= \frac{11}{20} \end{aligned}$$

$$\text{c. Mean} = \frac{0(8)+1(10)+2(12)+3(2)+4(3)+5(5)}{40}$$

$$= \frac{77}{40}$$

$$= 1.9$$

$$\approx 2$$

$$\begin{aligned} \therefore P(< \text{mean no. of goals}) &= \frac{8+10}{40} \\ &= \frac{9}{20} \end{aligned}$$

### EXERCISE

In a traffic survey, the number of people (including the driver) in each car passing a school was recorded and the results were as follows

| No. of people | 1  | 2  | 3  | 4  | 5  |
|---------------|----|----|----|----|----|
| No. of cars   | 17 | 12 | 14 | 21 | 16 |

What is the probability that a car picked at random has;

- i. Exactly 4 passengers
- ii. Less than 2 passengers
- iii. Less than the average number of passengers

In a survey, the heights of pupils in form 3 were recorded as follows

|                   |       |       |       |       |       |
|-------------------|-------|-------|-------|-------|-------|
| Heights of pupils | 140-9 | 150-9 | 160-9 | 170-9 | 180-9 |
| No. of pupils     | 2     | 12    | 14    | 21    | 16    |

What is the probability that a pupil picked at random is

- More than 169cm tall
- Less than 160cm tall

### POSSIBILITY/ PROBABILITY /SAMPLE SPACE

Possibility space show all the possible outcomes by listing them down

It is a complete set of possible results or outcomes of an experiment

#### EXAMPLE

Three coins are tossed. State the possibility space and use it to find the following

- P(three heads)
- P(two heads)
- P(one head)
- P(0 head)
- P(not head)

Solutions

The possible outcomes are;

HHH, HHT, HTH, THH,

HTT, THT, TTH, TTT

- $P(3\text{heads})=p(\text{HHH})=\frac{1}{8}$
- $P(2\text{heads})=p(\text{HHT}) \text{ or } P(\text{HTH}) \text{ or } p(\text{THH})$   
 $=\frac{3}{8}$
- $P(1\text{head})=p(\text{THT}) \text{ or } P(\text{TTH}) \text{ or } p(\text{HTT})$   
 $=\frac{3}{8}$
- $P(0 \text{ head})= p(\text{TTT})$   
 $=\frac{1}{8}$

$$\begin{aligned} \text{v. } P(\text{no 0 head}) &= 1 - p(0 \text{ head}) \\ &= 1 - \frac{1}{8} \\ &= \frac{7}{8} \end{aligned}$$

#### EXAMPLE 2

Use the possibility space for tossing two dice to calculate these probabilities

- P(a score of 10 or more)
- P(an even score)
- P(score of less than 12)
- P(a score of 13)

#### Solution

1<sup>st</sup> die

|                       |   |   |   |    |    |    |
|-----------------------|---|---|---|----|----|----|
|                       | 1 | 2 | 3 | 4  | 5  | 6  |
| 1 <sup>st</sup> die 1 | 2 | 3 | 4 | 5  | 6  | 7  |
| 2 <sup>nd</sup> die 2 | 3 | 4 | 5 | 6  | 7  | 8  |
| 3                     | 4 | 5 | 6 | 7  | 8  | 9  |
| 4                     | 5 | 6 | 7 | 8  | 9  | 10 |
| 5                     | 6 | 7 | 8 | 9  | 10 | 11 |
| 6                     | 7 | 8 | 9 | 10 | 11 | 12 |

NB; for score add number of die.

- $P(\text{score of 10 or more})$   
 $=\frac{6}{36}$   
 $=\frac{1}{6}$
- $P(\text{even score})$   
 $=\frac{18}{36}$   
 $=\frac{1}{2}$
- $P(\text{score} < 12) = \frac{35}{36}$
- $P(\text{score of 13}) = 0$

#### EXERCISE

1. Three digits 2, 1 and 3 are written in random order to make a two digit number.
  - a. list the possibility space
  - b. what is the probability that a number formed is even
  - c. What is the probability that the number formed is prime
2. Construct the possibility space for 4 coins and use it to calculate these probabilities
  - i. P(4 heads)
  - ii. P(less than 4 heads)
  - iii. P(2 heads)
3. A number is selected from the set  $S=\{1,2,3...18\}$ . Find the following
  - i. P(prime number)
  - ii. P(even number)
  - iii. P(multiple of 6)
4. A coin is tossed and a die is rolled. Illustrate the possible outcomes on the possibility space. Find the probability that the outcomes will be;
  - i. A head and a 1
  - ii. A head and a 3
  - iii. A tail and a 5

### TREE DIAGRAMS

Use branches.

The probability is found using the branches.

It is clear and more flexible illustrations of outcomes and their probabilities other than possibility space.

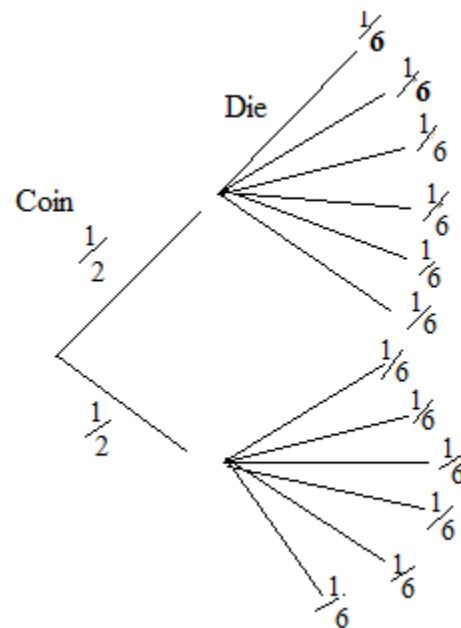
The probabilities of the individual outcomes are shown on these branches.

NB; at any intermediate stage of at final count, the sum of the branches must be equal to one

### EXAMPLES

I Toss a coin and roll a die, draw the tree diagram to show all the possible outcomes. Use it to find P (having a head and an even number)

### Solution



$$P(\text{H and even \#}) = P(\text{H,2}) + P(\text{H,4}) + P(\text{H,6})$$

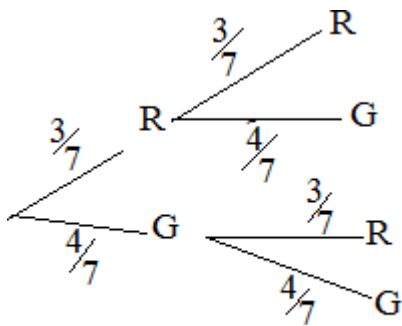
$$= \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{6}$$

$$= \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$$

$$= \frac{1}{4}$$

2. A bag contains 3 red beads and 4 green beads. A bead is taken at random, color noted and a bead replaced. If two successive draws are made, find the probability that the result will be;
  - i. 2 red balls
  - ii. 1 red and 1 green

### Solution

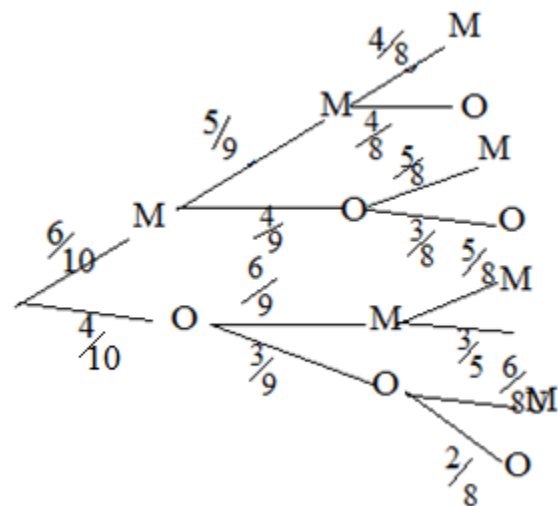


i. 
$$P(RR) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$$

ii. 
$$P(\text{red and green}) = \frac{6}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{3}{9} = \frac{24}{90} + \frac{12}{90} = \frac{36}{90} = \frac{2}{5}$$

3. A basket contains 6 mangoes and 4 oranges. Three fruits are removed from it without replacement. Use a tree diagram to work out the following probabilities.
- P (three mangoes are removed)
  - P(a mango and two oranges)
  - P(not picking 3 mangoes)

### Solution



a. 
$$P(3 \text{ mangoes}) = P(MMM) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{120}{720} = \frac{1}{6}$$

$$= \frac{1}{6}$$

b. 
$$P(MOO \text{ or } OMO \text{ or } OOM) = \left( \frac{6}{10} \times \frac{4}{9} \times \frac{3}{8} \right) + \left( \frac{4}{10} \times \frac{6}{9} \times \frac{3}{8} \right) + \left( \frac{4}{10} \times \frac{3}{9} \times \frac{6}{8} \right) = \frac{72}{720} + \frac{72}{720} + \frac{72}{720} = \frac{216}{720} = \frac{3}{10}$$

c. 
$$P(\text{not } 3) = 1 - P(MMM) = 1 - \frac{1}{6} = \frac{5}{6}$$

4. A bag contains beans, g nuts and maize seed. The probability of getting at random a bean seed is  $\frac{1}{5}$ , a g nut seed is  $\frac{x}{15}$  maize is  $\frac{1}{3}$ . Find the value of x

### Solution

$$\frac{x}{15} + \frac{1}{3} + \frac{1}{5} = 1$$

(sum of prob = 1)

$$x + 5 + 3 = 15$$

$$x + 8 = 15$$

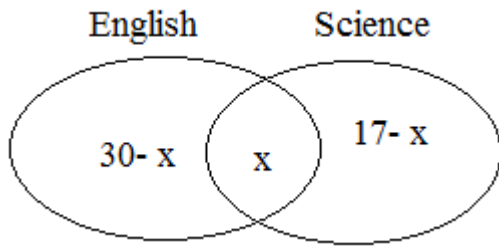
$$x = 15 - 8 = 7$$

### USING VENN DIAGRAM TO FIND PROBABILITY

#### EXAMPLE

In a class of 30 pupils like English and 17 like science. All 46 students say they like at least one of these subjects. What is the probability that a pupil chosen at random likes exactly one subject?

### Solution



Let those taking both subjects be  $x$

$$\therefore 30 - x + x + 17 - x = 46$$

$$47 - 46 = x$$

$$x = 1$$

$$\therefore P(\text{pupil like exactly one sub}) = \frac{45}{46}$$

### EXERCISE

1. In a plastic bag, there are  $x$  blue pens, 6 black pens and 4 red pens. If the probability of picking a red pen is  $\frac{1}{5}$ . Calculate number of blue pens
2. A coin A is tossed followed by coin B. The probability that a coin A shows head is  $\frac{1}{2}$ , while the probability that coin B shows head is  $\frac{1}{4}$ . Using the tree diagram, calculate the probability that both coins A and B shows tails
3. The probability of a bus arriving early at a depot is  $\frac{1}{10}$  and arriving late is  $\frac{3}{10}$ . If 400 buses are expected at the depot during the day, calculate the number of buses that are likely to arrive at the depot in time.
4. The probability of having early lunch at a boarding is  $\frac{2}{3}$ , when lunch is early, the probability of having a beef is  $\frac{7}{10}$  and when late the probability of having a beef is  $\frac{1}{8}$ .

Draw a tree diagram to represent this information completing all branches.

## VECTORS

Any translation vector can be written as  $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

Coordinates + translation = coordinates of object + vector of image

Thus  $O + V = I$

### EXAMPLE

- Find the coordinates of the image of (-3, 4) under translation vector  $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ .

**Solution**

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} -6 \\ 6 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$\therefore$  The coordinates of image are (-6, 6)

- $\underline{p} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}, \underline{q} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ 
  - Find the column vector which is parallel to  $\underline{p}$  and six times as long.
  - Find column vector which is four times the length of  $\underline{q}$  but in opposite direction.

**NB:** If  $k$  is a positive // then  $k\overline{AB}$ , is  $k$  times the length of  $\overline{AB}$ , and in the same direction  $\overline{AB}$ .

In other words  $k\overline{AB}$ , is parallel to  $\overline{AB}$ , and has length  $k|\overline{AB}|$  or  $k\underline{a}$  is parallel to  $\underline{a}$  and of length  $k|\underline{a}|$ .

### VECTORS ARE PARALLEL IF

- There is a common vector
- They have the same direction

**Solution**

$$a. \underline{p} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$

$$\therefore 6\underline{p} = 6\begin{pmatrix} -2 \\ -3 \end{pmatrix} = \begin{pmatrix} -12 \\ -18 \end{pmatrix}$$

$$b. \underline{q} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$-4\underline{q} = -4\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 12 \\ -4 \end{pmatrix}$$

- $P = (5, -3)$ 
  - If  $\overline{PQ} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ . Find the coordinates of Q
  - $\overline{PR} = 3\overline{PQ}$ , find the coordinates of R

*solution*

$$a. P + \overline{PQ} = Q$$

$$\begin{pmatrix} 5 \\ -3 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ -4 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\therefore Q = (7, -4)$$

$$b. \overline{PR} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\therefore 3\overline{PR} = 3\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$$

$$P + \overline{PR} = R$$

$$\begin{pmatrix} 5 \\ -3 \end{pmatrix} + \begin{pmatrix} 6 \\ -3 \end{pmatrix} = R$$

$$\begin{pmatrix} 11 \\ -6 \end{pmatrix} = R$$

$\therefore$  The coordinates of R = (11, -6)

### EXERCISE

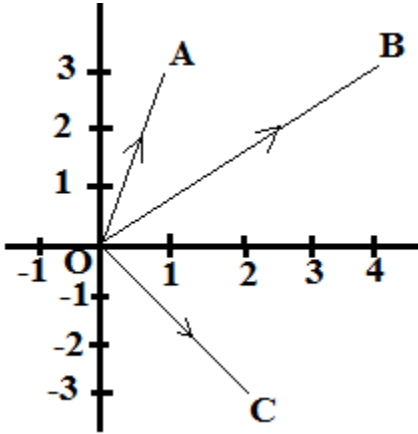
- $A = (2, 1)$ , is mapped on to point (3, -1) under a translation. What would be the image of  $C = (-2, -3)$  under the same translation vector.
- Vector  $\underline{g} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ , vector  $\underline{h} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$ .
  - Find the image of  $A = (0, -1)$  under a translation  $(2\underline{g} - 3\underline{h})$
  - Find the image of  $B = (-2, -1)$  under the translation  $3(\underline{g} - \underline{h})$

### POSITION VECTORS



Positional vectors can be used to specify the position of a point with respect to a fixed point.

On a Cartesian coordinates, the fixed point is usually the origin O



In positional vectors, the coordinates of an object is (0, 0)

$$\begin{aligned}\vec{OA} &= \begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ 3 \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\vec{OB} &= \begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 4 \\ 2 \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\vec{OC} &= \begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ -3 \end{pmatrix}\end{aligned}$$

In general, in position vectors, translation vector is equal to coordinates of image

#### EXAMPLE

If  $k = (2, 3)$  and  $M = (5, 6)$ . Find the following

- The positional vectors of M and K
- $\vec{KM}$
- $\vec{MK}$

#### Solution

$$\begin{aligned}\text{a. } \vec{OK} &= \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \\ \vec{OM} &= \begin{pmatrix} 5 \\ 6 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\text{b. } \vec{KM} &= M - K \\ &= \begin{pmatrix} 5 \\ 6 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ 3 \end{pmatrix}\end{aligned}$$

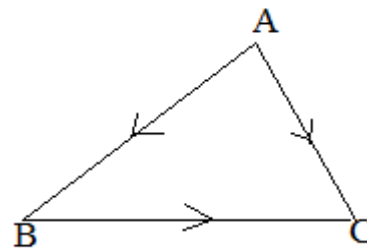
$$\begin{aligned}\text{c. } \vec{MK} &= -\vec{KM} \\ &= \begin{pmatrix} -3 \\ -3 \end{pmatrix}\end{aligned}$$

#### EXERCISE

- If  $T = (8, 0)$ ,  $W = (2, 0)$  and  $Z = (5, 0)$ . Find the following.
  - The positional vector of T and Z
  - $\vec{WZ}$  and  $\vec{ZT}$ .
- $\underline{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$  and  $\underline{b} = \begin{pmatrix} -6 \\ -12 \end{pmatrix}$ . Work out  $\frac{1}{2}\underline{a} + \frac{2}{3}\underline{b}$ .

### ADDITION OF VECTORS

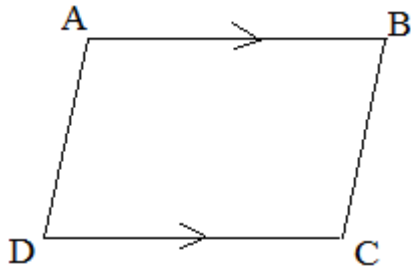
#### TRIANGLE LAW OF ADDITION OF VECTORS



$$\vec{AB} + \vec{BC} = \vec{AC}$$

#### PARALLELOGRAM

A quadrilateral is a //gram when two opposite sides are equal and parallel.



When opposite  $\angle$ s are equal,

$$\overrightarrow{AD} // \overrightarrow{BC}$$

$$\overrightarrow{AD} = \overrightarrow{BC}$$

$\therefore$  ABCD is a //gram

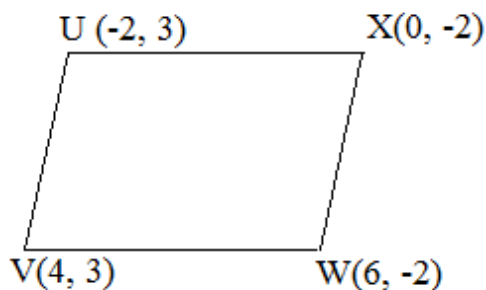
**A quadrilateral is //gram when**

- One pair of the sides is equal and // to each other.
- Opposite angles are equal
- Two opposite sides are equal

### EXAMPLE

1. Show that quadrilateral UVWX which has  $U(-2, 3)$ ,  $V(4, 3)$ ,  $W(6, -2)$  and  $X(0, -2)$  is a //gram.

**Solution**



$$\overrightarrow{UX} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

$$\overrightarrow{VW} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

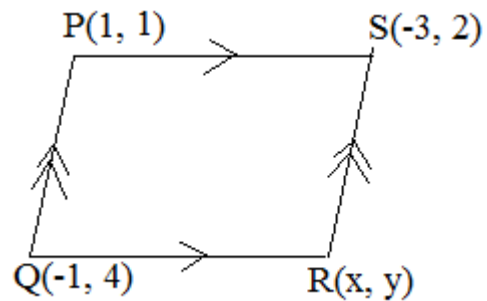
$$= \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

$$\therefore \overrightarrow{UX} = \overrightarrow{VW} \text{ and } \overrightarrow{UX} // \overrightarrow{VW}$$

$\therefore$  UVWX is a //gram

2. In a parallelogram PQRS,  $P = (1, 1)$ ,  $Q = (-1, 4)$ ,  $S = (-3, 2)$ . Find the coordinates of R.

**Solution**



$$\overrightarrow{PS} = \overrightarrow{QR}, \text{ opp. Sides of a //gram}$$

$$\therefore \overrightarrow{QR} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$$

$$Q + \overrightarrow{QR} = R$$

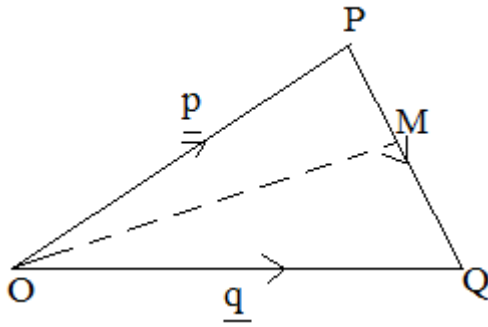
$$\begin{pmatrix} -1 \\ 4 \end{pmatrix} + \begin{pmatrix} -4 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} -5 \\ 5 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\therefore R = (-5, 5)$$

### **VECTOR ALGEBRA**

We can apply triangle law of addition of vectors to position vectors.



$$\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$$

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$= \underline{q} - \underline{p}$$

### MID POINT OF A VECTOR

If M is a mid-point in the figure above.

$$\overrightarrow{PM} = \frac{1}{2} \overrightarrow{PQ} = \frac{1}{2} (\underline{q} - \underline{p})$$

$$\overrightarrow{OM} = \overrightarrow{OP} + \overrightarrow{PM}$$

$$= \underline{p} + \frac{1}{2} (\underline{q} - \underline{p})$$

$$= \frac{1}{2} (\underline{p} + \underline{q})$$

### EXAMPLE

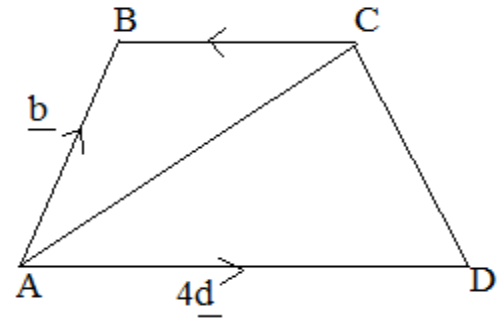
1. ABCD is a quadrilateral in which BC // AD.  $\overrightarrow{AD} = 4\underline{d}$ ,  $\overrightarrow{AB} = \underline{b}$ ,  $\overrightarrow{BC} = \frac{3}{4} \overrightarrow{AD}$ . Find

a.  $\overrightarrow{BC}$

b.  $\overrightarrow{AC}$

c.  $\overrightarrow{CD}$  (in terms of  $\underline{b}$  and  $\underline{d}$ )

**solution**

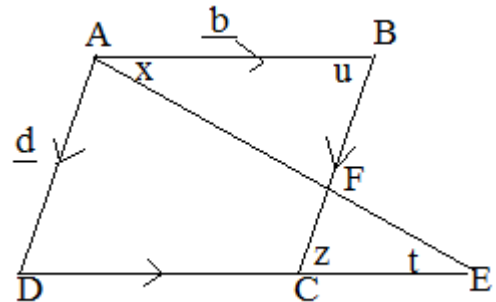


a.  $\overrightarrow{BC} = \frac{3}{4} \overrightarrow{AD}$   
 $= \frac{3}{4} \times 4\underline{d}$   
 $= 3\underline{d}$

b.  $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$   
 $= \underline{b} + 3\underline{d}$

c.  $\overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AD}$   
 $\underline{b} + 3\underline{d} + \overrightarrow{CD} = 4\underline{d}$   
 $\overrightarrow{CD} = 4\underline{d} - \underline{b} - 3\underline{d}$

2. In the figure, ABCD is a //gram  $\overrightarrow{DC} = 4\underline{d}$ ,  $\overrightarrow{AB} = \underline{b}$  and  $\overrightarrow{AD} = \underline{d}$



- a. Show that  $\Delta s$  ABC and ECF are similar  
 b. Show that  $\overrightarrow{BF} = 4\overrightarrow{FC}$   
 c. Show that  $\overrightarrow{AF} = \frac{1}{5} ((5\underline{b} + 4\underline{d}))$   
 d. Show that  $\overrightarrow{AF} = \frac{1}{4} (5\underline{b} + 4\underline{d})$

### **Solution**

- a.  $\overline{AB} // \overline{DC}$ , opp. Sides of //gram  
 In  $\Delta s$  ABF and ECF

$$x = t, \text{ alt } \angle s$$

$$u = z, \text{ alt } \angle s$$

$$\angle CFE = \angle AFB, \text{ vert. opp. } \angle s$$

$$\therefore \Delta ABC \sim \Delta ECF \text{ (a. a. a)}$$

$$\therefore \frac{AB}{EC} = \frac{BF}{CF} = \frac{AF}{EF}$$

$$\text{b. } \therefore \frac{\overrightarrow{AB}}{\overrightarrow{EC}} = \frac{\overrightarrow{BF}}{\overrightarrow{CF}}$$

But  $\overrightarrow{AB} = \overrightarrow{DC}$ , opp. Sides of a //gram

$$\overrightarrow{DC} = 4\overrightarrow{CE}$$

$$\frac{\overrightarrow{DC}}{4} = \overrightarrow{CE}$$

$$\therefore \overrightarrow{EC} = \frac{-b}{4}$$

$$\therefore \frac{\overrightarrow{AB}}{\overrightarrow{EC}} = \frac{b}{-b/4}$$

$$= -4$$

$$-4 = \frac{\overrightarrow{BF}}{\overrightarrow{CF}}$$

$$-4\overrightarrow{CF} = \overrightarrow{BF}$$

$$\overrightarrow{CF} = -\overrightarrow{FC}$$

$$\therefore \overrightarrow{BF} = -4(-\overrightarrow{FC})$$

$$\overrightarrow{BF} = 4\overrightarrow{FC}$$

$$\text{c. } FC = \frac{d}{5}$$

$$\overrightarrow{AF} = \overrightarrow{AB} + \overrightarrow{BF}$$

$$= \underline{b} + 4\left(\frac{\underline{d}}{5}\right)$$

$$= \frac{5\underline{b} + 4\underline{d}}{5}$$

$$= \frac{1}{5}(5\underline{b} + 4\underline{d})$$

$$\text{d. } \overrightarrow{AD} + \overrightarrow{DE} = \overrightarrow{AE}$$

$$\underline{d} + \frac{5\underline{b}}{4} = \overrightarrow{AE}$$

$$\frac{4\underline{d} + 5\underline{b}}{4} = \overrightarrow{AE}$$

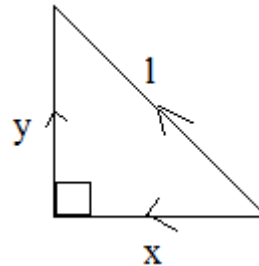
$$\overrightarrow{AE} = \frac{1}{4}(4\underline{d} + 5\underline{b})$$

$$= \frac{1}{4}(5\underline{b} + 4\underline{d})$$

## LENGTH OF A VECTOR

$$\overrightarrow{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Length of vector  $AB = |AB|$



$l^2 = x^2 + y^2$ , Pythagoras theorem

$$l = \sqrt{x^2 + y^2}$$

## EXAMPLE

- Find the length of vector  $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$

**Solution**

$$\left| \begin{pmatrix} -3 \\ 4 \end{pmatrix} \right| = \sqrt{(-3)^2 + (4)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$= 5 \text{ units}$$

- If  $a = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$  and  $b = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ , find t in the following equation,  
 $|2a - b| = t|a - b|$

**Solution**

$$2a - b = 2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$\therefore |2a - b| = \sqrt{(-1)^2 + (3)^2}$$

$$= \sqrt{1 + 9}$$

$$\begin{aligned}
 &= \sqrt{10} \\
 a - b &= \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} \\
 &= \begin{pmatrix} -2 \\ 1 \end{pmatrix} \\
 \therefore |a - b| &= \sqrt{(-2)^2 + (1)^2} \\
 &= \sqrt{5} \\
 \therefore \sqrt{10} &= t\sqrt{5} \\
 t &= \frac{\sqrt{10}}{\sqrt{5}} \\
 t &= \sqrt{2}
 \end{aligned}$$

### PARALLEL LINES

Parallel lines have common vector and same direction.

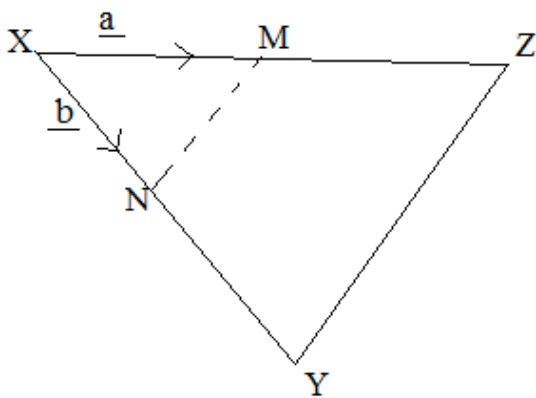
### EXAMPLE

1. Show that  $\begin{pmatrix} 4 \\ -8 \end{pmatrix}$  and  $\begin{pmatrix} 3 \\ -6 \end{pmatrix}$  are parallel.

**Solution**

$$\begin{aligned}
 &\begin{pmatrix} 4 \\ -8 \end{pmatrix} \text{ And } \begin{pmatrix} 3 \\ -6 \end{pmatrix} \\
 &4\begin{pmatrix} 1 \\ -2 \end{pmatrix} \text{ and } 3\begin{pmatrix} 1 \\ -2 \end{pmatrix} \\
 &\therefore \text{ They are } //
 \end{aligned}$$

2. In figure M and N are the mid points of  $\overrightarrow{XZ}$  and  $\overrightarrow{XY}$  respectively.



Show that  $\overrightarrow{MN}$  and  $\overrightarrow{ZY}$  are parallel.

**Solution**

$XM = MZ$ , M is a mid-point

$XN = NY$ , M is a mid-point

$$\overrightarrow{XM} + \overrightarrow{MN} = \overrightarrow{XN}$$

$$\underline{a} + \overrightarrow{MN} = \underline{b}$$

$$\overrightarrow{MN} = \underline{b} - \underline{a}$$

$$\overrightarrow{XZ} = 2\underline{a} \text{ And } \overrightarrow{XY} = 2\underline{b}$$

$$\overrightarrow{XZ} + \overrightarrow{ZY} = \overrightarrow{XY}$$

$$2\underline{a} + \overrightarrow{ZY} = 2\underline{b}$$

$$\overrightarrow{ZY} = 2\underline{b} - 2\underline{a}$$

$$= 2(\underline{b} - \underline{a})$$

Since  $(\underline{b} - \underline{a})$  is common in both  $\overrightarrow{MN}$  and  $\overrightarrow{ZY}$

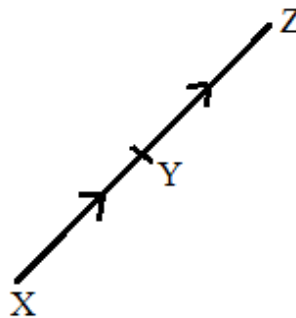
$$\therefore \overrightarrow{MN} // \overrightarrow{ZY}$$

### COLLINEAR POINTS

Points are collinear if they lie in a straight line.

If two vectors are equal, they are parallel and have the common point.

i.e.



$$XY // YZ$$

And Y is common

∴ The points X, Y and Z are collinear (they lie in straight line)

### EXAMPLE

1. Show that X(-4, 4), Y(2, 7) and Z (6,9) are collinear

#### Solution

$$\overrightarrow{XY} = \begin{pmatrix} 2 \\ 7 \end{pmatrix} - \begin{pmatrix} -4 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 3 \end{pmatrix} = 3 \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\overrightarrow{YZ} = \begin{pmatrix} 6 \\ 9 \end{pmatrix} - \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

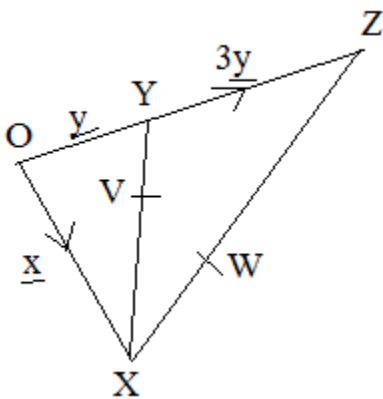
$$= \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

∴  $\overrightarrow{XY} // \overrightarrow{YZ}$  and Y is common

∴ X, Y and Z are collinear

### EXAMPLE 2(MENEB 2016)

Figure below shows positional vectors  $\overrightarrow{OX} = \underline{x}$ .  $\overrightarrow{OY}$  is produced to a point Z where  $OY:YZ=1:3$ . V is a point on  $\overrightarrow{YX}$  such that  $\overrightarrow{YV}:\overrightarrow{VX} = 1:2$  and W is a point on  $\overrightarrow{XZ}$  such that  $\overrightarrow{XW}:\overrightarrow{WZ} = 1:2$



Show that, O, V and W are collinear

#### Solution

In  $\Delta OZX$ ,  $\overrightarrow{OZ} + \overrightarrow{ZX} = \overrightarrow{OX}$

$$4\underline{y} + \overrightarrow{ZX} = \underline{x}$$

$$\overrightarrow{ZX} = \underline{x} - 4\underline{y}$$

$$\overrightarrow{XZ} = -(\underline{x} - 4\underline{y})$$

$$= 4\underline{y} - \underline{x}$$

$$\overrightarrow{XW} + \overrightarrow{WZ} = \overrightarrow{XZ}$$

$$\text{But } 2\overrightarrow{XW} = \overrightarrow{WZ}$$

$$3\overrightarrow{XW} = 4\underline{y} - \underline{x}$$

$$= \frac{4\underline{y} - \underline{x}}{3}$$

$$\overrightarrow{OY} + \overrightarrow{YX} = \overrightarrow{OX}$$

$$\overrightarrow{YX} = \underline{x} - \underline{y}$$

$$\text{But } 2\overrightarrow{YV} = \overrightarrow{VX}$$

$$\therefore \overrightarrow{YV} + 2\overrightarrow{YV} = \overrightarrow{YX}$$

$$3\overrightarrow{YV} = \underline{x} - \underline{y}$$

$$= \frac{\underline{x} - \underline{y}}{3}$$

$$\therefore \overrightarrow{VX} = \frac{2(\underline{x} - \underline{y})}{3}$$

$$= \frac{2\underline{x} - 2\underline{y}}{3}$$

$$\overrightarrow{VX} + \overrightarrow{XW} = \overrightarrow{VW}$$

$$\frac{2\underline{x} - 2\underline{y}}{3} + \frac{4\underline{y} - \underline{x}}{3} = \overrightarrow{VW}$$

$$\overrightarrow{VW} = \frac{2\underline{y} + \underline{x}}{3}$$

$$\overrightarrow{OY} + \overrightarrow{YV} = \overrightarrow{OV}$$

$$\underline{y} + \frac{x-y}{3} = \overrightarrow{OV}$$

$$\overrightarrow{OV} = \frac{2y+x}{3}$$

$\therefore \overrightarrow{OV} // \overrightarrow{VW}$  and V is common

$\therefore O, V$  and  $W$  are collinear

## CHAPTER 10

### LINEAR PROGRAMMING

In inequalities, the unshaded region on the graph is the region which satisfies all the conditions

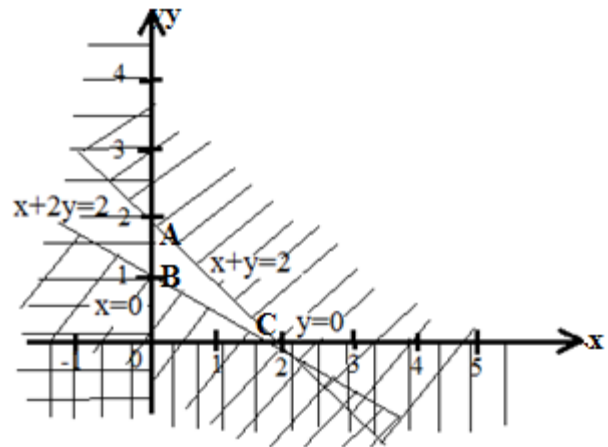
The unshaded region is called feasible or accessible region

#### EXAMPLE

Show the feasible region representing the following inequalities;

- I.  $x + 2y \geq 2, x + y \leq 2, x \geq 0, y \geq 0$
- II. Find the minimum and maximum value of  $2x + 5y$

**Solution**



To find minimum and maximum values of an expression within the region, we only consider the values at the vertices in the feasible region and substitute in the given expression.

$$\text{At } A(0,2)=2(0)+5(2)=10$$

$$\text{At } B(0,1)=2(0)+5(1)=5$$

$$\text{At } C(2,0)=2(2)+5(0)=4$$

$\therefore$  The minimum value of  $2x + 5y$  is 4 at C

The maximum value is 10

### EXERCISE

1. Show the region representing the following inequalities;
  - i.  $x + y \leq 8, x + 3y > 12, x > 0, y > 0$
  - ii. Find the minimum value of  $x - 4y$  in the unshaded region.
2. i. show the feasible region representing the following inequalities;  
 $x + y > 0, 3x + 2y \leq 9, x \geq 0, y \geq 0$ 
  - iii. find the maximum and minimum value of  $7x + 2y$  in the feasible region

### LINEAR PROGRAMMING AND ITS APPLICATIONS

In linear programming questions, we must find the maximum or minimum value of the objective function while satisfying a set of constraints function.

Points to follow when solving linear programming questions;

- i. summarize the information in a table
- ii. write down the objective function and all the necessary constraints
- iii. graph the feasible region
- iv. identify all corner points
- v. find the value of the objective function at each corner points
- vi. for the bounded region the solution is given by the corner point giving the optimal value (minimum and maximum) of the objective function
- vii. For the unbounded region, check that the solutions actually exist. If it does it will occur at the corner point

### EXAMPLES

A shopkeeper wants to buy up to 500 necklaces for her shop. She has K9000 to spend. There are two types of necklaces available, one type costing K30 each and the other costing K10 each. She wants to buy at least 100 of each type.

- i. If the shopkeeper buys  $x$  of the expensive type and  $y$  of the cheaper type of necklace, show graphically the region in which points  $(x, y)$  lie, using a scale of 1cm to represent 100 units on each axis
- ii. The profit is K10 on the expensive necklaces and K5 on the cheaper necklaces. How many of each type of necklaces should she buy to make the most profit

### Solution

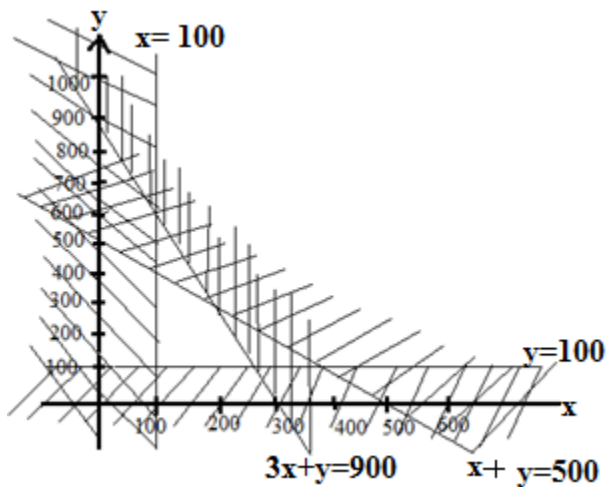
| Types  | Number | Cost  | Profit |
|--------|--------|-------|--------|
| Type A | $x$    | $30x$ | $10x$  |
| Type B | $y$    | $10y$ | $5y$   |
| Total  | 500    | 9000  | P      |

Using the table, we can have the following inequalities

$$\begin{aligned}
 x + y &\leq 500 \dots\dots\dots (i) \\
 30x + 10y &\leq 9000, \\
 3x + y &\leq 900 \dots\dots\dots (ii) \\
 x &\geq 100 \dots\dots\dots (ii) \\
 y &\geq 100 \dots\dots\dots (iii)
 \end{aligned}$$

NB. At least means greater than or equal to ( $\geq$ )  
 At most means less than or equal to ( $\leq$ )





$$P=10x + 5y$$

$$\text{At } A(100,400)=10(100)+5(400) \\ =K3000$$

$$B(200,300)=10(200)+5(300) \\ =K3500$$

$$C(250,100)=10(250)+5(100) \\ =K3000$$

$$D(100,100)=10(100)+5(100) \\ =K1500$$

The maximum profit is K3500 at  
B(200,300)

∴ She should buy 200 of the expensive necklaces and 300 of the cheaper necklaces

### EXAMPLE 2

A transport company has three 30-passenger buses and nine 15-passenger buses. The company contracts to transport more than 120 passengers a day to the national parks. It cost k3000 per day to run each 30-passenger bus and k1000 per day to run each 15-passenger bus and the company must spend less than k12000 per day in order to meet the costs, if  $x$  and  $y$  are the numbers of 30-passenger and 15 passenger buses each day;

- i. Show that  $2x + y > 8$

- ii. Write down the other inequalities involving  $x$  and  $y$
- iii. Illustrate the solution set of the four inequalities on the graph paper provided and shade the unwanted region

### Solution

| Types   | Number | cost  | profit  |
|---------|--------|-------|---------|
| 30-pBus | $x$    | $30x$ | $3000x$ |
| 15-pBus | $y$    | $15y$ | $1000y$ |
| Total   |        | 120   | 12000   |

$$30x + 15y > 120$$

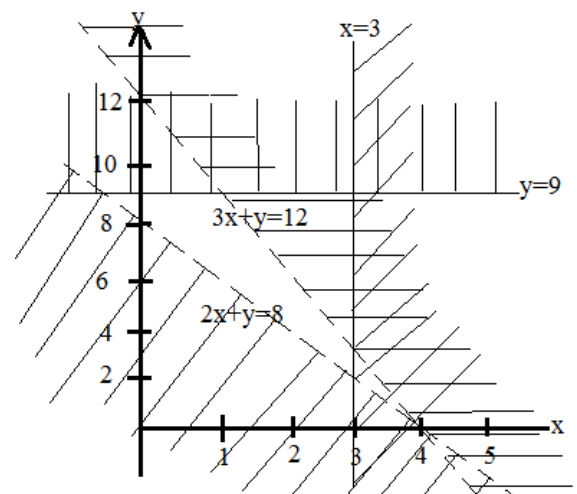
$$\therefore 2x + y > 8 \dots\dots\dots \text{i}$$

$$x \leq 3 \dots\dots\dots \text{ii}$$

$$y \leq 9 \dots\dots\dots \text{iii}$$

$$3000x + 1000y < 12000$$

$$3x + y < 12 \dots\dots\dots \text{iv}$$



### EXERCISE

1. Vendor decides to spend up to k600 to buy sweets and biscuits. She does not want to buy more than 10 packets of each. A packet of sweet costs k40 and that of biscuits costs k80. The profit on each packet of sweet is k10 and on its packet of biscuits is k15.

- i. By taking  $x$  to represent the number of packets of sweet and  $y$  to represent the number of packets of biscuits, write down four inequalities that satisfies the above information
  - ii. Using the scale of 2cm to represent 2 units on axes, draw graphs to show the region represented by the inequalities
  - iii. Use your graph to find the number of packets of sweets and biscuits respectively for the vendor to maximize profit
2. A business man wants to buy  $x$  meters of low quality cloth at k200 per meter and  $y$  meters of high quality cloth at k400 per meter. He decided to
- . Buy at most a total of 1000 meters of cloth
  - . Spend at least a total sum of k80, 000
- a. Write down two inequalities in addition to  $x \geq 0, y \geq 0$
  - b. Using a scale of 2cm to represent 200 units on both axes, draw on the graph paper the regions represented by the four inequalities
  - c. A business man will make a profit of k10 per meter on the cheaper cloth and k20 per meter on the expensive cloth. How many meters of cloth of each type of cloth must he buy to get the maximum profit?

## CHAPTER 11:

## THREE DIMENSIONS FIGURES

A PRISMS; A prism is any solid object which have uniform cross-section

Examples of prisms;

- i. Triangular prism
  - has cross-section of a triangle
- ii. Hexagonal prism
  - Its cross-section is a hexagon
- iii. Cuboid
  - Its cross-section is a rectangle
- iv. L-shaped prism
  - Its cross-section has an L-shape

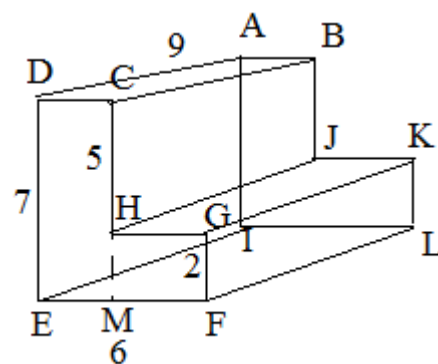
### VOLUME OF A PRISM

For any prism;

$$V = \text{cross-sectional area} \times \text{height (length)}$$

### EXAMPLES

Calculate the volume of the prism below



### Solution

$$\text{Area of rect. DCME} = 7 \times 2 = 14\text{cm}^2$$

$$\text{Area of rect. HGFM} = 4 \times 2 = 8\text{cm}^2$$

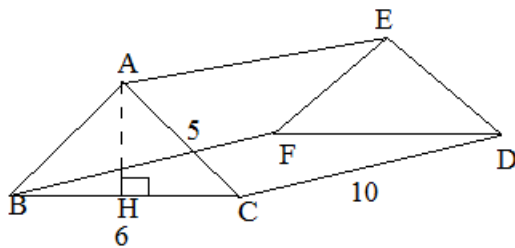
$$\therefore \text{Area of L-shape} = (14+8) \text{ cm}^2 = 22\text{cm}^2$$

$$\therefore \text{Volume} = 22\text{cm}^2 \times 9\text{cm} = 198\text{cm}^3$$

**EXAMPLE 2**

A cross-section of a prism is a triangle with two sides of 5cm and the other side 6cm long. The length of the prism is 10cm. calculate,

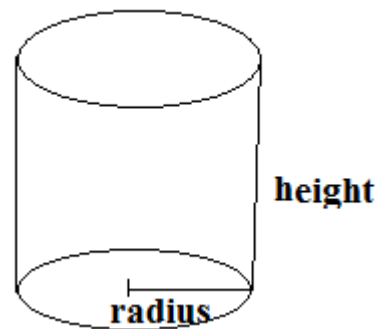
- The height of the triangle
- The area of the triangle
- The volume of the prism.

**Solution**

- In  $\triangle ABH$ ,  $BH = \frac{6}{2}$ ,  $AH \perp$  bisector  
 $= 3\text{cm}$   
 $AB^2 = AH^2 + BH^2$ , Pythagoras  
 $25 = AH^2 + 9$   
 $25 - 9 = AH^2$   
 $16 = AH^2$   
 $AH = \sqrt{16}$   
 $= 4\text{cm}$   
 $\therefore \text{Height} = 4\text{cm}$
- Area  $\triangle ABH = \frac{1}{2} \times 6 \times 4$   
 $= 12\text{cm}^2$
- Volume  $= 12\text{cm}^2 \times 10\text{cm}$   
 $= 120\text{cm}^3$

**EXERCISE**

- Calculate the volume of the prism below. All the lengths are in cm (drawing)
- The cross-section of prism 20cm long has an area of  $25\text{cm}^2$ , what is the volume of the prism
- The cross-section of a prism is a triangle ABC with angle  $ABC = 90^\circ$ ,  $AB = 10\text{cm}$  and  $BC = 6\text{cm}$ . if the prism is 30cm high, what is its volume?

**VOLUME OF THE CYLINDER**

Area of the circle  $= \pi r^2$  (cross-sectional area)

$$\therefore \text{Volume} = \text{area circle} \times \text{height}$$

$$V = \pi r^2 h$$

**EXAMPLES**

- Figure shows the cross-section of a cylindrical water main pipe which is 10m long. The pipe has an inner radius of 30cm and outer radius 37cm. find the area of the material needed to make the pipe in litres

NB: it is necessary for the units to be the same, so 10m should be in cm too here is the table to assist in converting units,

Km hm dam m dm cm mm

Kg hg dag g dg cg mg

1 0 0 0 0 0 0

1 0 0 0 0 0

1 0 0 0 0

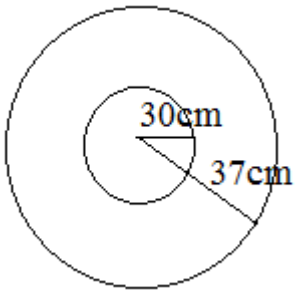
1 0 0 0

1 0 0

1 0

1

### Solution



$$1\text{m} = 100\text{cm}$$

$$10\text{m} = \text{more}$$

$$= 100\text{cm} \times 10$$

$$= 1000\text{cm}$$

$$V \text{ of inner cylinder} = \frac{22}{7} \times 30 \times 30 \times 1000$$

$$= \frac{19800000\text{cm}^3}{7}$$

$$= 2828571. \text{ cm}^3$$

$$V \text{ of outer cylinder} = \frac{22}{7} \times 37 \times 37 \times 1000$$

$$= 4302571.4\text{cm}^3$$

$$\therefore V \text{ of the material} = 2828571.4 - 4302571.4$$

Needed

$$= 1474000\text{cm}^3$$

$$\text{In litres the volume} = \frac{1474000}{1000}$$

$$= 1474\text{l}$$

$$\text{In general the vol.} = \pi R^2 h - \pi r^2 h$$

$$= \pi h(R^2 - r^2)$$

$$= \pi h(R - r)(R + r)$$

### EXAMPLE

A lead has internal radius 2.3cm and external radius 3.7cm. if  $1\text{cm}^3$  of lead weighs 11.4g, what is the weight of a 14cm length of wire

### Solution

$$14\text{m} = 14 \times 100\text{cm}$$

$$= 1400\text{cm}$$

$$\text{Volume} = \pi h(R - r)(R + r)$$

$$= \frac{22}{7} \times 1400(3.7 + 2.3)(3.7 - 2.3)$$

$$= 4400(6)(1.4)$$

$$= 36960\text{cm}^3$$

$$1\text{cm}^3 = 11.4\text{g}$$

$$36960\text{cm}^3 = \text{more}$$

$$= \frac{36960\text{cm}^3 \times 11.4\text{g}}{1\text{cm}^3} = 421344\text{g}$$

$$\therefore 14\text{m length wire weigh } 421344\text{g}$$

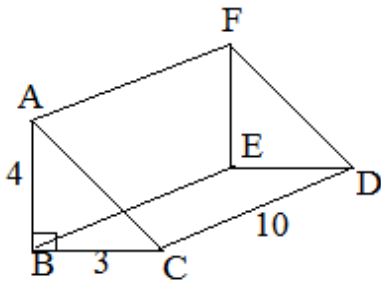
**AREA OF A PRISM**

The surface area of any prism is the sum of areas of all its faces.

To find surface area of the prism we work out the areas of all faces and add them up.

**EXAMPLE**

Find the surface area of the prism below lengths are in cm



$$\text{Area of } \triangle ABC = \frac{1}{2} \times 4 \times 3 = 6\text{cm}^2$$

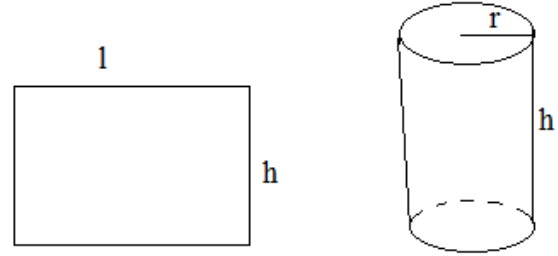
$$\therefore \triangle FED = 6\text{cm}^2$$

$$\text{Area of rect. BCDE} = 10 \times 3 = 30\text{cm}^2$$

$$\text{Area of a rect. ABEF} = 4 \times 10 = 40\text{cm}^2$$

$$\text{Area of rect. ACDF} = 5 \times 10 = 50\text{cm}^2$$

$$\begin{aligned} \therefore \text{Surface area} \\ &= 6\text{cm}^2 + 6\text{cm}^2 + 30\text{cm}^2 + 40\text{cm}^2 + 50\text{cm}^2 \\ &= 132\text{cm}^2 \end{aligned}$$

**SURFACE AREA OF A CYLINDER**

$$\text{Length of rect.} = \text{length of circumf.} = 2\pi r$$

$$\text{Area of a rect.} = \text{length} \times \text{height}$$

$$\therefore \text{Area of a cylinder} = 2\pi r$$

$$\text{But the area of a circle} = \pi r^2$$

$$\therefore \text{Surface area of a cylinder (open at one end)} \\ = 2\pi r h + \pi r^2$$

$$\text{Surface area of a cylinder closed at both ends} = 2\pi r h + 2\pi r^2$$

$$= 2\pi r(h + r)$$

**EXAMPLE**

1. The cartridge of a bullet is cylinder closed at one end. The cartridge is 70mm long and has a diameter of 14mm. find the surface area of the cartridge ( $\frac{22}{7}$ ).

**Solution**

$$S. A = 2\pi r h + \pi r^2$$

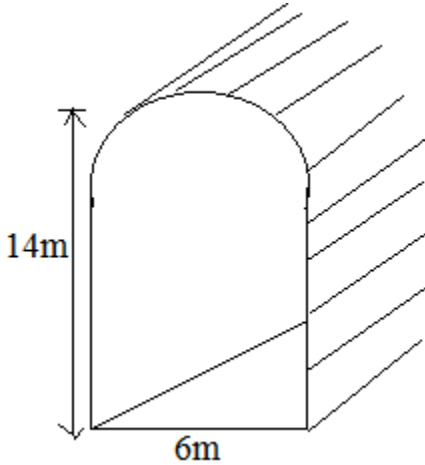
$$h = 70\text{mm}, r = 7\text{mm}$$

$$\therefore S. A = 2 \times \frac{22}{7} \times 7 \times 70 + \frac{22}{7} \times 7 \times 7$$

$$= 3080 + 154$$

$$= 3234\text{mm}^2$$

2. The figure below shows a tunnel 200m long and 14m high. The roof of the tunnel is semicircular with a diameter of 6m.
- It is proposed to paint inside of the tunnel (but not the floor). What is the area to be painted?
  - Painting cost K2 per square meter. What is the total cost of painting the tunnel?

**Solution**

$$\text{Radius} = \frac{6m}{2} = 3m$$

a.  $\therefore AB = 14m - 3m = 11m$

$$\begin{aligned} \therefore \text{Area of a rect. (outside)} &= 11m \times 200m \\ &= 2200m^2 \end{aligned}$$

$$\therefore \text{area of two rectangles} = 2 \times 2200 = 4400m^2$$

$$\text{Area of the roof} = \frac{1}{2} \times 2\pi rh \text{ (semicircular)}$$

$$= \pi rh$$

$$\therefore \text{Area} = \frac{22}{7} \times 3m \times 200m$$

$$= 1885.7m^2$$

$$\therefore \text{The area to be painted} = 1885.7 + 4400$$

$$= 6285.7m^2$$

b.  $1m^2 = 2k$   
 $6285.7m^2 = x$

$$\frac{1m^2}{6285.7m^2} = \frac{2k}{x}$$

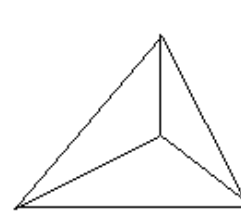
$$X = K12571.4$$

$$\therefore \text{Total cost} = K 12571.40$$

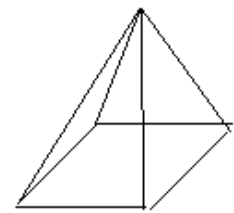
**PYRAMIDS**

A pyramid is a solid whose base is a plane figure and whose side faces meet in a point or vertex.

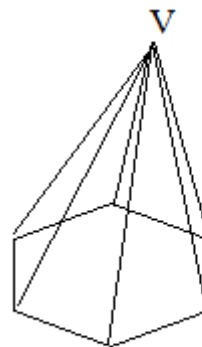
If the vertex is directly above the Centre of a regular base, it is called a right pyramid. If the base is a circle, then the pyramid is called a Cone.

**Examples of pyramids**

Triangle-based  
pyramid (tetrahedron)



square based  
pyramid



Hexagonal based  
pyramid



Cone

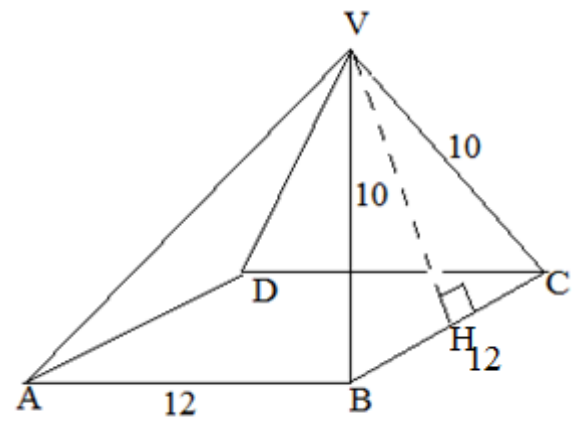
**SURFACE AREA OF A PYRAMID**

We work out the area of all faces and add up them.

**EXAMPLE**

A square based pyramid has a base side of 12cm. The other faces are isosceles triangles with equal sides of length 10cm. what is the surface area of the pyramid.

**Solution**



In  $\Delta VHC$ ,  $HC = \frac{12}{2}$ ,  $VH \perp$  bisector

$$= 6\text{cm}$$

$$VC^2 = VH^2 + HC^2$$

$$100 - 36 = VH^2$$

$$VH^2 = 64$$

$$VH = \sqrt{64}$$

$$= 8\text{cm}$$

$$\text{Area of } \Delta VHC = \frac{1}{2} \times 12\text{cm} \times 8\text{cm}$$

$$= 48\text{cm}^2$$

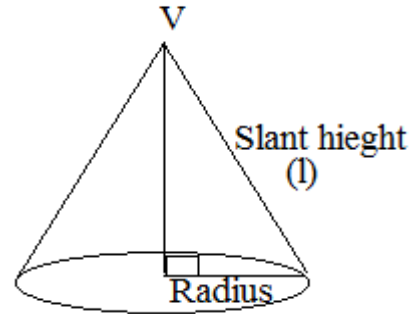
$$\text{Area of rect. ABCD} = 12\text{cm} \times 12\text{cm}$$

$$= 144\text{cm}^2$$

$$\therefore \text{Total surface area} = 4 \times 48\text{cm}^2 + 144\text{cm}^2$$

$$= 336\text{cm}^2$$

**SRFACE AREA OF A CONE**

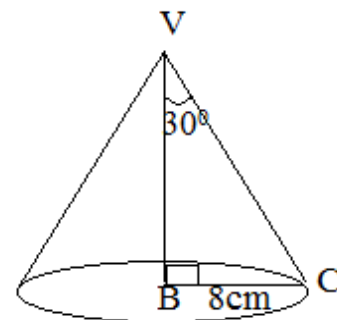


Surface area of a cone (open)  $= \pi r l$

S. A of a cone when closed  $= \pi r l + \pi r^2$

**EXAMPLE**

1. Find the total surface area of a cone shown in the figure below



**Solution**

$$\sin 30^\circ = \frac{8\text{cm}}{VC}$$

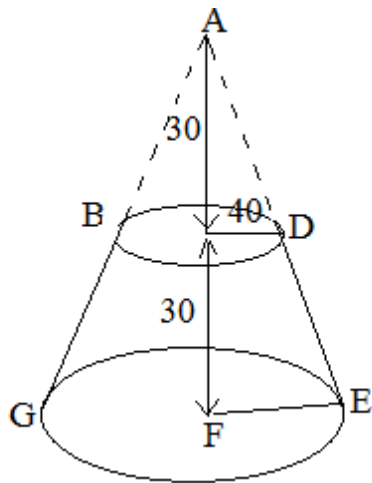
$$0.5 \times 8\text{cm} = VC$$

$$VC = 4\text{cm}$$

$$\therefore \text{Surface area} = \frac{22}{7} \times 8\text{cm} \times 4\text{cm} + \frac{22}{7} \times 8^2$$

$$= 301.43\text{cm}^2$$

2. Fig below shows a lump-shade made out of the base of a cone, with the top part removed. Find area of material to make the lampshade

**Solution**

$$\text{In } \triangle ACD, AD^2 = 40^2 + 30^2$$

$$AD = \sqrt{25,000}$$

$$= 50\text{cm}$$

$$\text{Area of a cone ABD} = 3.14 \times 40\text{cm} \times 50\text{cm}$$

$$= 6280\text{cm}^2$$

$$\text{In } \triangle ACD \text{ and } AFE, \angle F = \angle C, \text{ right } \angle$$

A is common

$$\therefore \triangle ACD \sim \triangle AEF$$

$$\therefore \frac{AF}{AC} = \frac{AE}{AD}$$

$$\frac{60}{30} = \frac{AE}{50}$$

$$AE = 100$$

$$AE^2 = EF^2 + AF^2$$

$$1000 - 3600 = AF^2$$

$$AF = \sqrt{6400}$$

$$AF = 80\text{cm}$$

$$\therefore \text{Area of a cone AGF} = 3.14 \times 80 \times 100$$

$$= 25120\text{cm}^2$$

$$\text{Area of material used to make lampshade} = 25120\text{cm}^2 - 6280\text{cm}^2$$

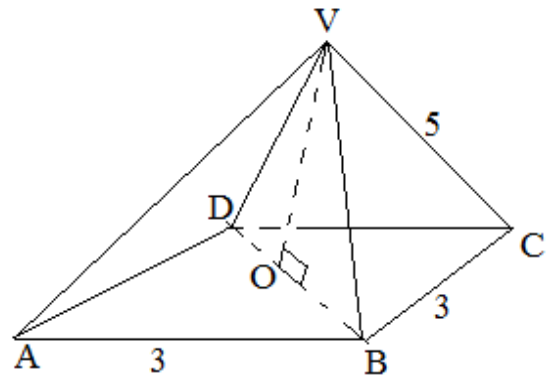
$$= 18840\text{cm}^2$$

**VOLUME OF A PYRAMID**

$$V \text{ of a pyramid} = \frac{1}{3} \text{ base area} \times \text{height}$$

**EXAMPLE**

Find the volume of a right pyramid which has a square base of side 3cm and slant edges 5cm long

**Solution**

$$\text{In } \triangle ABD, DB^2 = 3^2 + 3^2, \text{ Pythagoras}$$

$$= \sqrt{18}$$

$$DB = 3\sqrt{2}$$

$$\therefore OB = \frac{3\sqrt{2}}{2}$$

$$\text{In } \triangle VOB, VO^2 + OB^2 = VB^2, \text{ Pythagoras}$$

$$VO^2 = 5^2 - \left(\frac{3\sqrt{2}}{2}\right)^2$$



$$= 25 - \frac{9 \times 2}{4}$$

$$= \sqrt{20.5}$$

$$= 4.53 \text{ cm}$$

Area of a square ABCD =  $3 \text{ cm} \times 3 \text{ m}$

$$= 9 \text{ cm}^2$$

$$\therefore \text{Volume} = \frac{1}{3} \times 9 \text{ cm}^2 \times 4.53 \text{ cm}$$

$$= 13.6 \text{ cm}^2$$

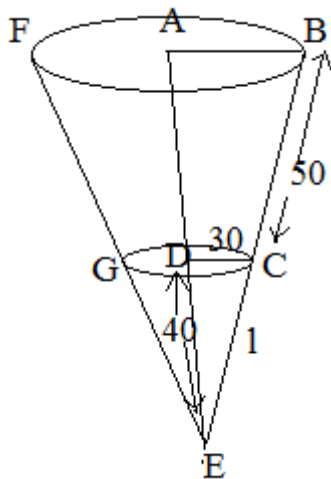
### VOLUME A CONE

Since the base of the cone is a circle,

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

### EXAMPLE

Find the volume of the basket in the diagram below,



### Solution

$$l^2 = 40^2 + 30^2, \text{pythagorus}$$

$$= \sqrt{2500}$$

$$l = 50 \text{ cm}$$

In  $\Delta ABE$  and  $DCE$

$\angle A = \angle D$ , Right  $\angle$ s

$\angle E$  is common

$\therefore \Delta ABE \sim \Delta DCE$

$$\therefore \frac{AB}{DC} = \frac{AE}{DE}$$

$$AB = \frac{100}{50} \times 30$$

$$= 60 \text{ cm}$$

$$\frac{AB}{DC} = \frac{AE}{DE}$$

$$\frac{60}{30} = \frac{AE}{40}$$

$$AE = 80 \text{ cm}$$

$$\therefore \text{Vol. of cone FBE} = \frac{1}{3} \times \frac{22}{7} \times 60 \times 60 \times 80$$

$$= 301,714.29 \text{ cm}^2$$

$$\text{Volume of GCE} = \frac{1}{3} \times \frac{22}{7} \times 30 \times 30 \times 40$$

$$= 37,714.29 \text{ cm}^3$$

$$\therefore \text{Volume of a bucket} = 264000 \text{ cm}^3$$

### VOLUME OF A SPHERE

$$\text{Surface area of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

### EXAMPLE

- Find the volume of a spherical ball with a radius of 3.14. Take  $\pi = 3.14$

### Solution

$$\text{The vol. of a tennis ball} = \frac{4}{3} \times 3.14 \times 3.18$$

$$= 134.6\text{cm}^3$$

2. A metal cylinder with length of 32cm and radius 3cm is melted down to form a sphere. What is the radius of the sphere?

**Solution**

$$\text{Volume of a cylinder} = \frac{22}{7} \times 3 \times 3 \times 32$$

$$= 905.14\text{cm}^3$$

$$\therefore \text{Volume of a sphere} = 905.14\text{cm}^3$$

$$\frac{4}{3}\pi r^3 = 905.14\text{cm}^3$$

$$\frac{4}{3} \times \frac{22}{7} \times r^3 = 905.14$$

$$r^3 = 216$$

$$r = \sqrt[3]{216} = 6\text{cm}$$

## **GEOMETRY IN THREE DIMENSIONS**

### **A PLANE**

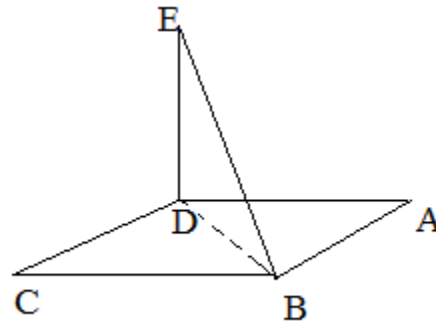
A plane is any flat surface such as a floor. The walls of the box are the planes

### **LINES AND PLANES**

A straight line can be put in a plane. When two lines meet their line of intersection is always a straight line.

### **ANGLES BETWEEN A LINE AND A PLANE**

It is the angle between a line and its projection on a plane.

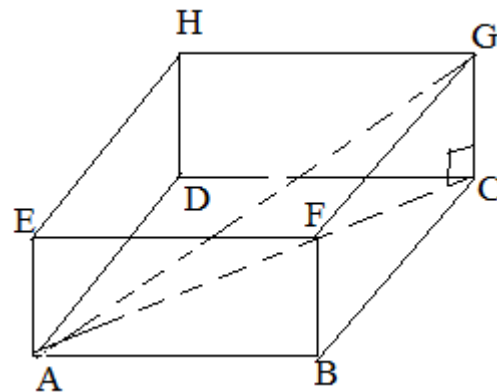


ABCD is a plane while BE is the line and BD is a line of projection

The angle between the line AB and the plane ABCD is angle EBD

### **EXAMPLE**

Figure below show the water tank with rectangular faces. AB=8m, BC=5m, and CG=3m



- Calculate the length AG leaving your answer in a simplified surd form
- The angle AG makes with the plane ABCD

**Solutions**

$$\begin{aligned} \text{In } \triangle ABC, AC^2 &= AB^2 + BC^2, & \text{Pythagoras} \\ &= 8^2 + 5^2 \\ &= \sqrt{89} \end{aligned}$$

In  $\triangle AGC$ ,

$$AG^2 = AC^2 + GC^2, \text{ Pythagoras}$$

$$= \sqrt{89}^2 + 3^2$$

$$=89+9$$

$$= \sqrt{98}$$

$$=7\sqrt{2}$$

In general the longest diagonal can be found by the following formulae

$$AC^2 = l^2 + w^2$$

$$\text{And } AG^2 = AC^2 + GC^2(h)$$

$$\therefore AG^2 = l^2 + w^2 + h^2$$

$$= \sqrt{l^2 + w^2 + h^2}$$

$$\text{In } \triangle GAC, \sin A = \frac{3}{7\sqrt{2}} = 0.3030$$

$$\angle A = 18^\circ \text{ (To the nearest degree)}$$

### ANGLES BETWEEN TWO PLANES

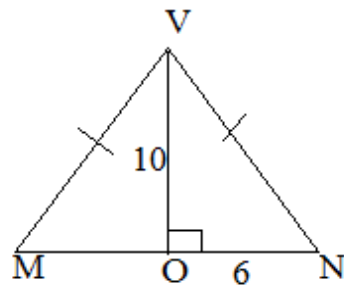
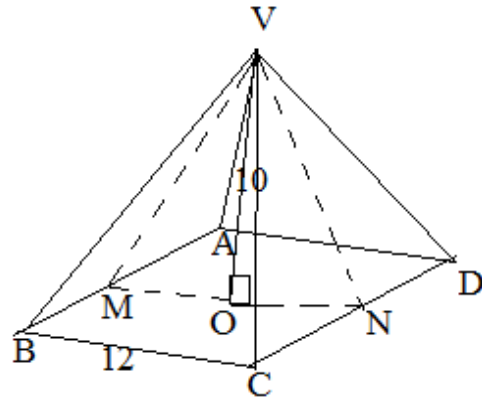
Is the angle between two lines one on the first plane and the other on the second plane that both meet at the line of intersection of the planes at right angle

#### **EXAMPLE**

VABCD is a pyramid on a square base ABCD of side 12cm. the height of the pyramid is 10cm. calculate the angle between,

- i. ABV and VCD
- ii. VCD and ABCD

#### **Solution**



- i. In  $\triangle VON$ ,  $\tan V = \frac{6}{10} = 0.6$   
 $\therefore \angle VON = 30.96^\circ = 31^\circ$   
 $\therefore \angle MVN = 2 \times \angle VON = 62^\circ$

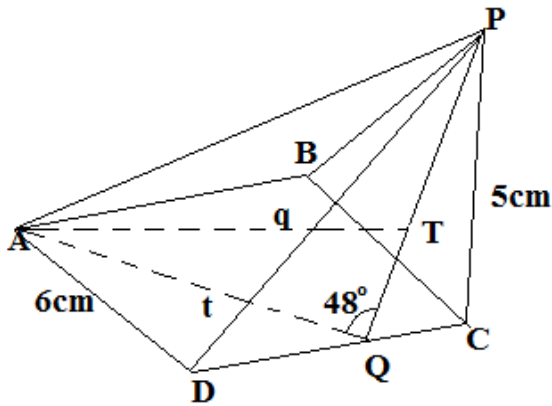
$\therefore$  The angle between ABV and ABCD is  $62^\circ$

- ii.  $\tan N = \frac{10}{6} = 1.6667$   
 $\angle N = 59^\circ$

$\therefore$  The angle between VCD and ABCD =  $59^\circ$

**EXAMPLE(2016)**

Figure below is a square based pyramid PABCD. T is the midpoint of the slant height OQ.



If  $\angle AQP = 48^\circ$ ,  $PC = 5\text{m}$ , and  $AD = 6\text{m}$ , calculate the length of AT giving your answer to 2 decimal place

**Solution**

In  $\triangle PDC$ ,  $QC = DQ$ ,

$$\therefore QC = 3\text{m}$$

In  $\triangle PQC$ ,  $PQ^2 + QC^2 = PC^2$ , Pythagoras

$$PQ^2 = 5^2 - 3^2$$

$$= 25 - 9$$

$$= \sqrt{16}$$

$$= 4\text{m}$$

$\therefore TQ = 2\text{m}$ , T midpoint

In  $\triangle ADQ$ ,  $AQ^2 = AD^2 + DQ^2$ , Pythagoras

$$AQ^2 = 6^2 + 3^2$$

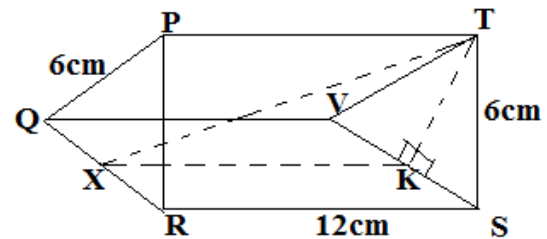
$$= \sqrt{45}$$

$$= 6.7\text{m}$$

$$\begin{aligned} \text{In } \triangle ATQ, \text{ using cosine rule,} \\ q^2 &= a^2 + t^2 - 2at \cos Q \\ &= 2^2 + \sqrt{45}^2 - 2 \times 2 \times \sqrt{45} \times \cos 48^\circ \\ &= \sqrt{31.0453} \\ &= 5.57\text{m} \\ \therefore AT &= 5.57\text{m} \end{aligned}$$

**TRIANGULAR PRISM****EXAMPLE**

Figure below is a prism with a cross-section of equilateral triangles PQR. PRST, PQST, AND QRSV are rectangles, such that  $RS = 12\text{cm}$ ,  $PQ = 6\text{cm}$  and X is the midpoint of QR.



Calculate

- Length of TX giving your answer correct to 2 decimal place
- The angle TX Makes with the base QRSV

**SOLUTION**

I. In  $\triangle VTS$ ,  $VK = KS = 3\text{cm}$ ,  $TK \perp$  bisector  
In  $\triangle TKS$

$$\begin{aligned} TK^2 &= 6^2 - 3^2, \text{ Pythagoras} \\ &= 36 - 9 \\ &= \sqrt{27} \end{aligned}$$

$$\begin{aligned} \text{In } \triangle TXK, TX^2 &= XK^2 + TK^2 \\ &= 12^2 + \sqrt{27}^2 \\ &= 144 + 27 \\ &= \sqrt{171} \\ &= 13.08\text{cm} \end{aligned}$$

$$\text{II. } \cos x = \frac{XK}{TX}$$

$$\begin{aligned}
 &= \frac{12}{13.08} \\
 &= 0.9174 \\
 X &= 23.4^0
 \end{aligned}$$

## CHAPTER 12

### GRAPHS OF CUBIC FUNCTIONS

Cubic graphs are graphs of cubic equation or cubic expression.

A cubic equation is the one in which the highest power of  $x$  is 3. E. g  $x^3$ ,  $2x^3 + 3$ .

Just as in quadratic graphs, a table of values need to be drawn when drawing cubic graphs so that corresponding points are found.

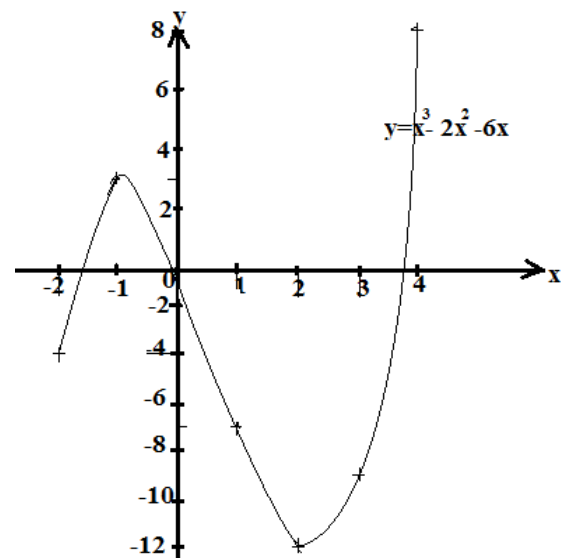
#### EXAMPLE

- Complete the table below and draw the graph  $y = 2x^3 - 2x^2 + 6x$

|     |    |    |   |   |   |   |   |
|-----|----|----|---|---|---|---|---|
| $x$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| $y$ |    |    |   |   |   |   |   |

#### Solution

|     |    |    |   |    |     |    |   |
|-----|----|----|---|----|-----|----|---|
| $x$ | -2 | -1 | 0 | 1  | 2   | 3  | 4 |
| $y$ | -4 | 3  | 0 | -7 | -12 | -9 | 8 |



#### SOLVING SILMUTANEOUS EQUATION GRAPHICALLY

#### EXAMPLE

Complete the table below and draw the graph  $y = 2x^3 - 2x^2 - 6x$  and use it to solve  $y = x - 2$ .

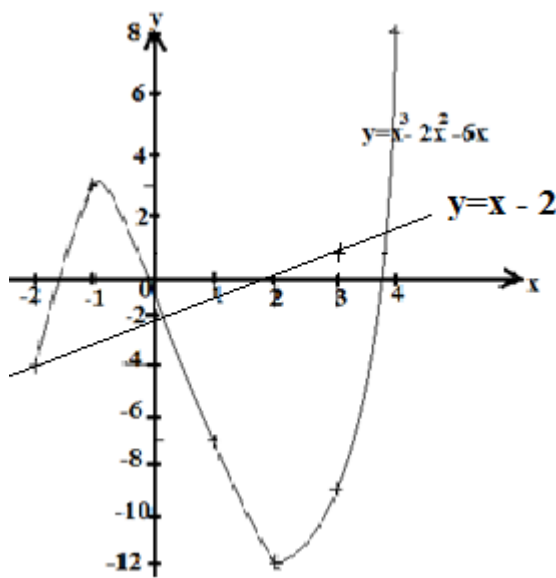
|     |    |    |   |   |   |   |   |
|-----|----|----|---|---|---|---|---|
| $x$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| $y$ |    |    |   |   |   |   |   |

**Solution**

|     |    |    |   |    |     |    |   |
|-----|----|----|---|----|-----|----|---|
| $x$ | -2 | -1 | 0 | 1  | 2   | 3  | 4 |
| $y$ | -4 | 3  | 0 | -7 | -12 | -9 | 8 |

To draw  $y = x - 2$  the graph  $y = x - 2$  we need a table like this.

|     |    |    |   |
|-----|----|----|---|
| $x$ | -2 | 0  | 3 |
| $y$ | -4 | -2 | 1 |



$$\therefore x = -2, 0.25, 3.8$$

## SUPPLEMENTARY EXERCISE

1. Expand

$$\left(x^{\frac{1}{2}} + 1 + x^{-\frac{1}{2}}\right)\left(x^{\frac{1}{2}} + 1 + x^{-\frac{1}{2}}\right)$$

2. In a GP, the product of the 1<sup>st</sup> and 7<sup>th</sup> term is equal to the fourth term. Given that the sum of the 1<sup>st</sup> and 4<sup>th</sup> terms is 9. Find

- The first term
- Common ratio

3. Then 1<sup>st</sup> term of GP exceeds the second term by 4 and the sum of the 2<sup>nd</sup> and 3<sup>rd</sup> term is  $2\frac{1}{3}$ . Find the first three terms

4. If  $\log_a x$ ,  $\log_a x + 3$  and  $\log_a x + 12$  are three consecutive terms of an AP. Find the value of  $x$

5. Find the angle that is supplementary to the complement of the complementary of base angle of any isosceles triangle?

6. Factorize  
 $(2a^2 + 3ab - 5b^2)(2a^2 - 5ab - 3b^2)$

7. At Mr. Chitopa's party, there were 240 people. There were 20 more men than women and there were 20 more adults than children. Find how many women were there.

8. Droben is 20 years older than her sister Bertha. In ten years, Bertha's age will be half of Droben. Find how old is Bertha now.

9. 8 percent of 36 is 72% of what number?

10. A committee is composed of  $w$  women and  $m$  men. 3 women and 2 men are added to the committee and if one person is selected at random from the enlarged committee, what is the probability that a person selected is a woman?

11. Given that the area of a cube is  $25\text{cm}^2$ . Calculate its volume.

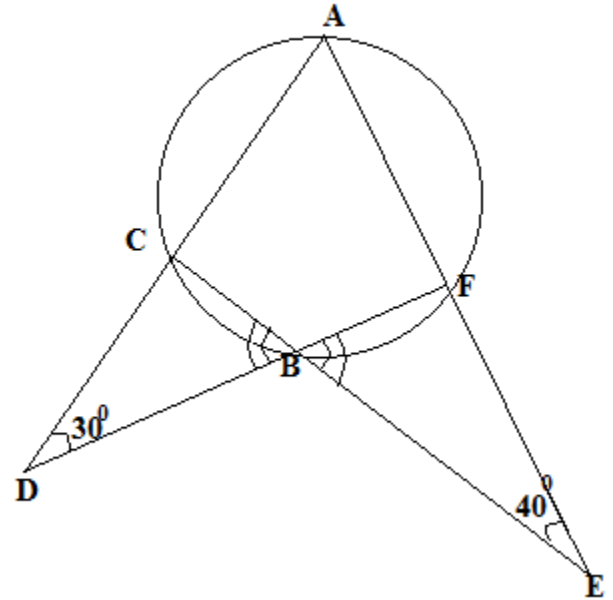
12. Solve for  $w$

$$\log_3 w + \frac{3}{\log_3 w} = 4$$

13. Given the function  $f(x) = 2x - 1$ , find  $f^{-1}(x)$
14. Function  $f$  and  $g$  are defined by  $f(x) = 2x + 3$  and  $g(x) = \frac{5}{x-2}, x \neq 2$ . find the value of  $x$  for which  $ff(x) = gf(2)$
15. Given that the function  $f(x) = 3^x$  and  $g(x) = 5x - 2$ . What is the value of  $x$  if  $gf(x) = 403$
16. There was a tea party at Mr. Motto's house. All people at the party brought their Cuts, in all they were 22 heads and 72 feet. Find how many people and cuts were there.
17. Simplify  $\sqrt[4]{16t} + 3\sqrt[4]{t} - \sqrt[4]{81t}$
18. The sides of two square fields are in the ratio 4:5. The area of the larger field is 1296cm<sup>2</sup> greater than the smaller field. Find the area of the larger field.
19. Given that  $n(A) = 7x + 13, n(B) = 6x + 17, n(C) = 8x + 20, n(A \cap B \cap C) = x, n(A \cap B \cap C^1) = 2x, n(C \cap A \cap B^1) = 4x, n(A^1 \cap B \cap C) = 3x, n(A \cup B \cup C) = 150$ .
- Draw a Venn diagram to show this information
  - What is  $n(A \cap B \cap C^1)$
20. The 1<sup>st</sup> and 4<sup>th</sup> terms of a GP are  $\frac{x}{y^2}$  and  $\frac{y}{x^5}$ . Find the second and third terms
21. Geoffrey conducted a research on a sample about the relationship between profit and the age of a company and the following are sums he calculated from the sample.
- $$\sum x = 1,310$$
- $$\sum x^2 = 28,000$$
- $$\sum y = 5,700$$
- $$\sum y^2 = 927,000$$
- $$\sum xy = 65,500$$

Where  $y$  represents annual profit and  $x$  represents the age of each firm in years. Calculate **mean** and **standard deviation** of  $x$  and  $y$ .

22. Figure below shows a circle **ABCF**,  $\widehat{CBD} = \widehat{FBE}$ ,  $\widehat{AEB} = 40^\circ$  and  $\widehat{ADB} = 30^\circ$ . Find the angle **BAC**



23. The first two terms of an AP are  $\frac{1}{a+2}, \frac{1}{a+1}$ ; find the  $n^{\text{th}}$  term.
24. If 4 is a solution of the equation  $x^2 - 3x + k = 10$ , where  $k$  is a constant, what will be the other solution
25. Simplify  $\left(a^{-\frac{1}{2}}b^{\frac{3}{2}}\right)^{-2}$
26. In a multiple-choice exam paper, there are 5 questions and 4 choices per question. Find the probability of
- Guessing all right
  - A pass if pass mark = 80%
27. In a factory production line, it is found that on average, 1 bulb in 10 is faulty. Find the probabilities that in the next 5 bulbs produced;

- a. None is faulty
- b. 1 is faulty
- c. Less than 3 are faulty

28. A stone is dropped into the lake from a height of 5m. Acceleration in air is  $10\text{m/s}^2$ . How long will the stone to reach the bottom of the lake if it is 10m deep.

29. Factorize  $8x^2 - x^5$

30. In a multiple choice exam paper, each question has 3 answers to choose from. What are the probabilities of
- a. Guessing a right answer
  - b. A wrong answer

If the paper consists of 4 questions, what are the probabilities of guessing correctly,

- a. None right
- b. One right
- c. Two right
- d. Three right
- e. Four right

31. The vertices of a quadrilateral are  $A(1,7)$ ,  $B(4,2)$ ,  $C(-3,-2)$ ,  $O(-5,4)$ .  $P, Q, R, S$  are mid-points of  $AB, BC, CD$ , and  $DA$  respectively.

- a. Find the position vectors  $\mathbf{p}, \mathbf{q}, \mathbf{r}, \mathbf{s}$  and hence find the vectors  $\widehat{SP}$  and  $\widehat{RQ}$ .
- b. Show that  $\widehat{SP}$  is equal and parallel to  $\widehat{RQ}$  and that  $PQRS$  is a parallelogram.

32. A Fair die is tossed 5times find the probability of obtaining

- a. 5 heads
- b. 2 heads and 3 tails
- c. Greater than 2 heads
- d. Greater than or equal to 2 heads

33. The probability that **Geoffrey** will beat **Lawrence** in a mathematics exam is  $\frac{8}{10}$ , of beating **Eugenio** is  $\frac{4}{5}$ , of beating **Gumzy** is  $\frac{1}{4}$ . Find the probability that

- a. Geoffrey will beat all.
- b. Geoffrey will beat Eugenio, Gumzy but not Lawrence

- c. The probability that no one will beat Geoffrey

# THE END

# GRORY BE TO GOD