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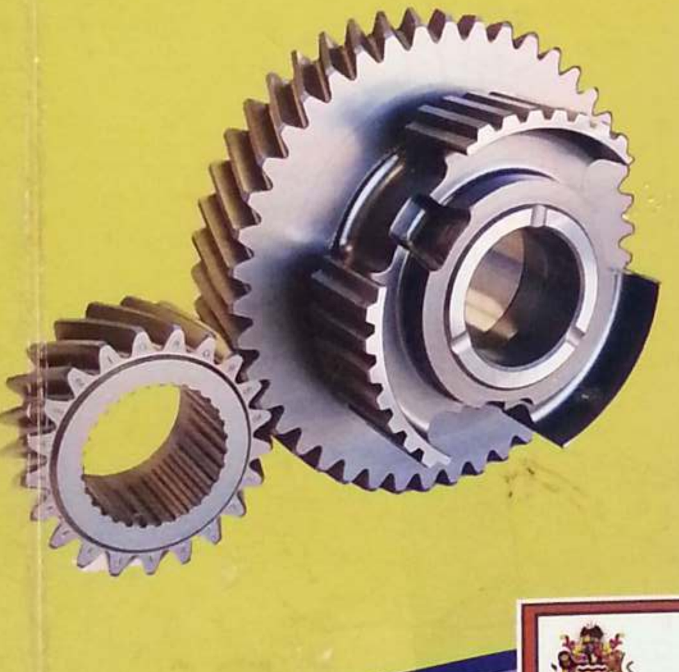
Senior Secondary Physics Student's Book

Form 3



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PHYSICS

Student's Book
Form 3

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TABLE OF CONTENTS

PROPERTIES OF MATTER

MECHANICS

OSCILLATIONS AND WAVES

Scientific Investigation and Skills

Outcome

The students will be able to use physics laws, principles, theories, and relations to explain and creatively exploit phenomena to generate and test theories as well as critically analyze and evaluate scientific data from observations and experimentations

UNIT

1 Measurements II

Success Criteria

By the end of this unit, you must be able to:

- Use suitable instruments and units for various measurements.

Introduction

In Form 1, we were introduced to quantities and their measurements. We differentiated between fundamental and derived quantities. We looked at their SI units, symbols and prefixes for the units smaller and greater than the SI units. When learning how to measure the quantities, we mainly focused on how to measure them in relatively large quantities.

In this unit, we will first review what we learnt then discuss how to measure very small quantities of length, volume, mass and time.

1.1 Fundamental and derived quantities of measurements

Fundamental quantities are those quantities which are not defined in terms of other quantities. In physics, there are 7 fundamental quantities of measurements, namely: *length, mass, time, thermodynamic temperature, electric current luminous intensity and amount of substance.*

Table 1.1 Shows basic fundamental physical quantities, their SI units and symbols.

Table 1.1

Fundamental quantity	SI Unit	Symbol
1. Length	metre	m
2. Mass	kilogram	kg
3. Time	second	s
4. Thermodynamic temperature	kelvin	K
5. Electric current	ampere	A
6. Luminous intensity	candela	cd
7. Amount of substance	mole	mol

Derived quantities

Quantities which are expressed in terms of the fundamental quantities are referred to as the *derived quantities*.

For example, *speed* is a derived quantity. It's derived from distance and time, since it's the rate of distance travelled i.e.

Speed = $\frac{\text{distance (m)}}{\text{time (s)}}$. The SI unit for speed is *metres per second (m/s)*.

Some derived units have been given special names. For example, *work* is measured in newton metre (Nm) and has been given a unit called *joule (J)*.

Table 1.2 shows some of the derived quantities and names of their derived unit.

Table 1.2

Derived quantity	Name of (SI) unit	Symbol of (SI) unit	Base units
Moment	newton.metre	N.m	kg m^2/s^2
Work	joule	J	kg m^2/s^2
Energy	joule	J	kg m^2/s^2
Power	watt	W	kg m^2/s^3
Pressure	pascal	Pa	kg/ms^2
Charge	coulomb	C	As

1.2 Standard notation (Scientific notation)

Consider a molecule whose diameter is about 0.000 000 001 m, and the radius of the earth which is about 6 370 000 m. It is cumbersome to state these values due to their many decimal places. We can express these values in standard notation.

A number N is said to be expressed in standard notation (scientific notation) when written in the form $A \times 10^n$, where A is greater than or equal to 1 and less than 10 i.e. $1 \leq A < 10$ and n = 1, 2, 3, ...n, is an integer representing the number of steps moved to the left or right on A to get N. A number with negative n is less than one and that with positive n is greater than one.

Example 1.1

Express the following values in standard form:

(a) The radius of the sun is 695 800 km.

(b) The cross section area of a thread is 0.000 000 171m^2.

Solution

(a) 695 800 km = $6.\underbrace{95800} \times 10^5$ km = 6.958×10^5 km

(b) 0.000 000 171m^2 = $\underbrace{0000000 1}.71m^2 \times 10^{-7} = 1.71 \times 10^{-7}\ m^2$

1.3 Prefixes for SI units

Consider an Ebola virus 0.000 014m long. Let us also consider the memory capacity of a flash disk which is 8 000 000 000 bytes. We see that the length of Ebola virus is too small while memory of the flash disk is too large and is a challenge to read, state or write easily without making a mistake. Some words have been used as short-cuts to writing values of such magnitude. The words are associated with certain magnitude. For example a word like micro stands for $\frac{1}{1\ 000\ 000}$ or $\times 10^{-6}$, giga for 1 000 000 000 or $\times 10^{9}$. Since these words are used or fixed before the SI units they are called *prefixes*.

Thus, we say that the length of the Ebola virus is 14 micro metres written as 14 μm, and the memory capacity of the flash disk 8 Giga bytes written as 8 Gb.

> ! By the way, one way to prevent contacting the Ebola virus is avoiding contact with body fluid of the infected person including blood, mucus, saliva, feases and urine. Take care !!!

Table 1.3 shows some common multiples and submultiple prefixes, their symbols and magnitude.

Table 1.3

Prefix	Symbol	Magnitude
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
mill	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
deca	da	10^{1}
hecto	h	10^{2}
kilo	k	10^{3}
Mega	M	10^{6}
Giga	G	10^{9}
Tera	T	10^{12}
Peta	P	10^{15}
Exa	E	10^{18}

Exercise 1.1

1. Define the term *fundamental quantity*. Give one example.
2. Name three derived quantities and their base units.
3. Give the SI units of the following:
   (a) Power
   (b) Voltage
   (c) Number of particles in a container
   (d) Speed of a moving body
4. Express the following in scientific notation.
   (a) 0.134 kg (b) 7 017 m^3
   (c) 161 256 m^2 (d) 8.000 136 g
5. Write the following numbers in standard notation.
   (a) 0.006 14 mm (b) $\frac{1}{100\ 000}$ km (c) 511 500 g
   (d) $\frac{5}{90\ 000}\ \Omega$ (e) 0.000 000 510 4 km (f) 680 000 *l*
6. Write the following in the normal form:
   (a) 2.13×10^6 g (b) 6.4×10^{-7} kg (c) 4.0×10^2 m
   (d) 1.1×10^{-9} g/cm^3 (e) 6×10^{-6} s (f) 9.1×10^{-31} kg
7. How many bytes are in the following:
   (a) 6 GB (b) 120 KB (c) 3TB
8. How many grams are there in:
   (a) 6×10^{12} kg (b) 6×10^{-12} mg (c) 8.4×10^6 tonnes
9. The diameter of the HIV virus is about 1 nm. Express this in mm?
10. Express the following using prefixes.
    (a) 4000 m (b) 1 600 000 g (c) $\frac{1}{10\ 000}$ g

1.4 Measurement of length

In Form 1, we learnt that length is measured in *metres*. We learnt how to measure length using instruments like a tape measure, metre rule etc. These instrument measure relatively large lengths.

In this section, we shall learn how to measure very small length: using *vernier calliper* and *micrometer screw* gauge.

Challenges of using a metre rule to measure length

When using a rule with a centimetre scale for measuring length, the second decimal place of the measurement has to be estimated. Fig. 1.1 show the reading on a centimetre scale.

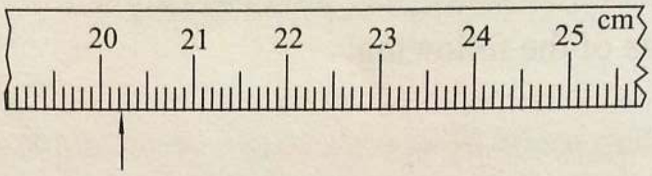


Fig. 1.1: Reading a cm rule.

The measurement 20.2 cm can be read directly. However, the length is more than 20.2 cm and less than 20.3 cm. Thus the correct reading, lies between 20.2 cm and 20.3 cm. In order to obtain the actual measurement, the second decimal place has to be estimated by the eye. To do this we observe that the reading is less than 20.25 cm. Therefore, the value may be 20.24 cm or 20.23 cm depending on the persons ability to estimate the fraction of the division. This problem may be overcome by the use of a vernier scale.

Vernier calliper

A vernier calliper was invented in the 17th century by a French technician called *Pierre Vernier*. Vernier calliper is used to measure short distances such as diameters of a small ball bearing, thin rod, internal diameter of a tube etc. Fig.1.2 shows the photograph of a vernier calliper.

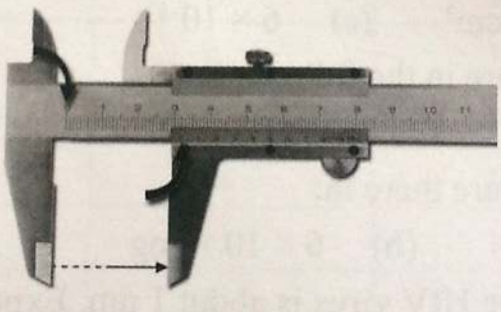


Fig. 1.2: Vernier calliper

Fig. 1.3 shows the parts of a vernier calliper.

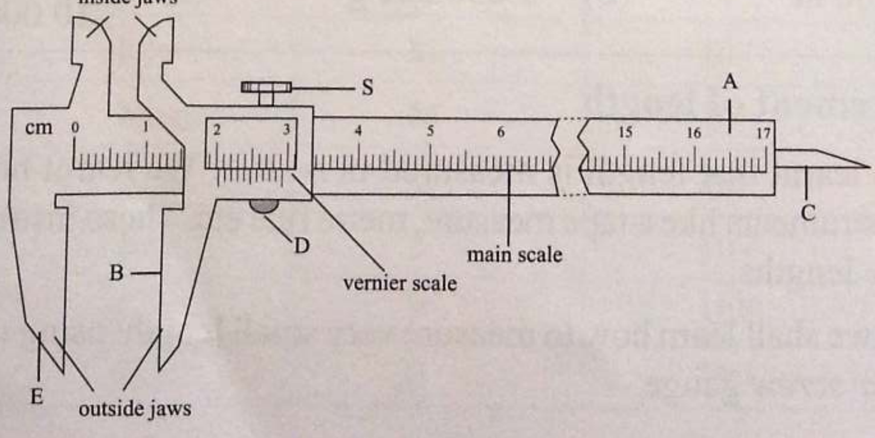


Fig. 1.3: Parts of a vernier calliper.

The calliper consists of a steel rigid frame A onto which a linear scale is engraved. This scale is called the *main scale* and it is calibrated in centimetres and millimetres. It has a fixed jaw E at one end and a sliding jaw *B* centrally aligned by a thin flat bar *C*. The spring-loaded button *D* is used to prevent the sliding jaw from moving unnecessarily. The sliding jaw carrying a vernier scale can move along the main scale and can be fixed in any position along the main scale by screw *S*.

The outside jaws are used to take external length measurements of objects. The inside jaws are used to take internal length measurement of an object. The sliding flat bar *C* is used to find the depth of blind holes.

Using a vernier scale

Least count of a vernier calliper

The vernier scale has a length of 9 mm. It is divided into ten equal divisions. Therefore, each division has a length of 0.9 mm. The difference between 1 division on the main scale and 1 division in the vernier scale is (1– 0.9) mm. The smallest reading called the *least count* (LC) that can be read from a vernier calliper is 1 mm – 0.9 mm = 0.1 mm or 0.01 cm .

The second decimal value in a reading is obtained by identifying a mark on the vernier scale which coincides with a mark on the main scale called the *vernier coincidence (VC)* and multiplying it with 0.01 cm. i.e second decimal value = (VC × LC).

How to read the vernier calliper

Activity 1.1: To measure the external diameter of a cylindrical object

- In pairs, place the object to be measured between the outside jaws as shown in Fig. 1.4. Slide the jaw until they touch the rod.

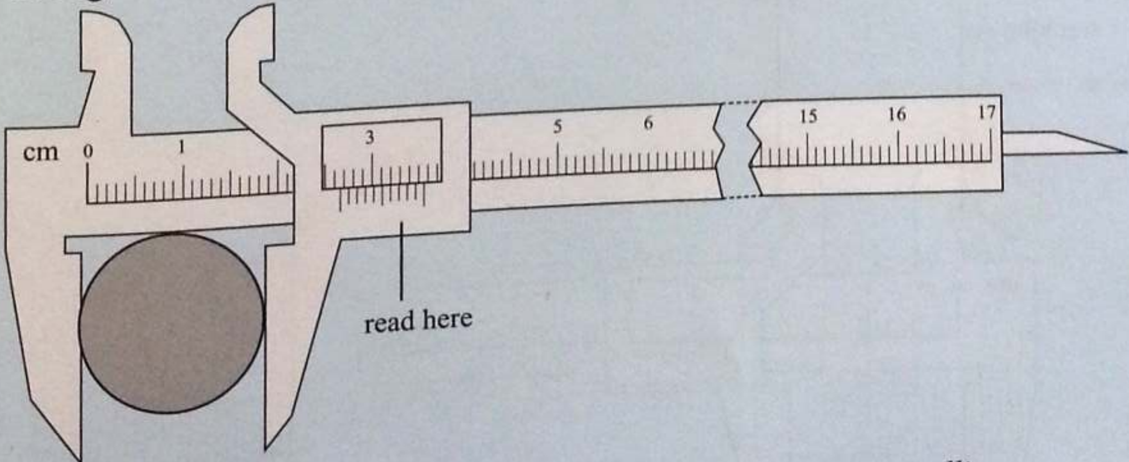


Fig. 1.4: Measurement of external diameter using a vernier calliper

- Record the readings on the main scale and the vernier scale. The main scale reading is the mark on the main scale that is immediately before the zero mark of the vernier scale.

- Multiply the vernier scale reading by 0.01 cm.
- Add the main scale reading (in cm) and the vernier scale reading (in cm) to get the diameter of the rod.

Discussion

On the vernier shown in Fig. 1.4, the main scale reading (MSR) is 2.6 cm. However, to get the second decimal value we make use of the vernier scale. The vernier scale mark that coincides exactly with a main scale mark gives the vernier coincidence (VC).

In this case, the 6^{th} division coincides with the main scale division.

Therefore, the external diameter of the cylindrical object is

$$\begin{aligned} MSR + (VC \times LC) &= 2.6 + (6 \times 0.01) \\ &= 2.66 \text{ cm.} \end{aligned}$$

Activity 1.2: To measure internal diameter of a test tube using a vernier calliper

- Insert the inside jaws of a vernier calliper into the test tube.
  Move the sliding jaws until the jaws just touch the inside walls of the test tube as shown in Fig. 1.5 (a).
- Take and record the readings on the main scale and the vernier scale as shown on Fig. 1.5 (b). Use these readings to determine the internal diameter of the test tube.

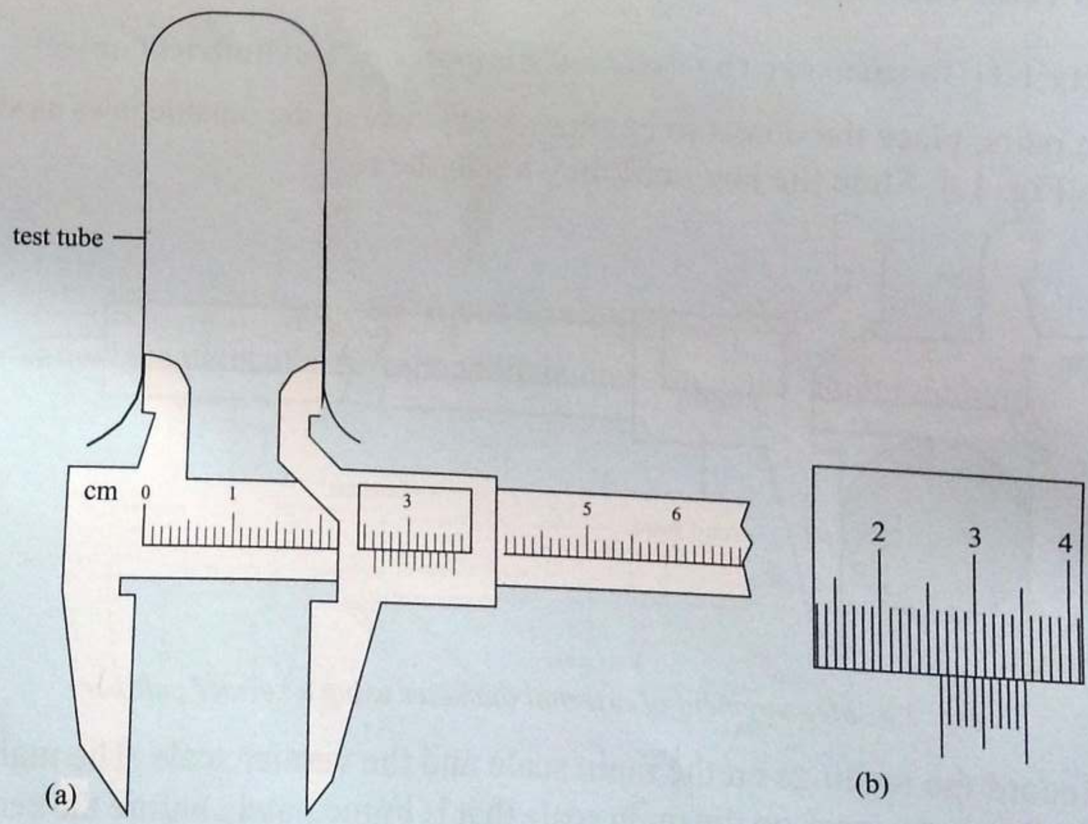


Fig. 1.5: Measurement of internal diameter using a vernier calliper.

Discussion

We can determine the diameter of the test tube shown in Fig. 1.5 as follows:

The internal diameter of the test tube $= MSR + (VC \times LC)$

$= 2.6 + (2 \times 0.01)$

$= 2.62$ cm

Activity 1.3: To determine the depth of a blind hole.

- Bring the end of the main scale to the edge of the blind hole when the jaws are fully closed. Slide the jaw outwards such that the sliding bar C gets into the blind hole until it touches the end of the hole.
- Take the reading of the vernier calliper X. This is the depth, X, of the blind hole, (Fig. 1.6).

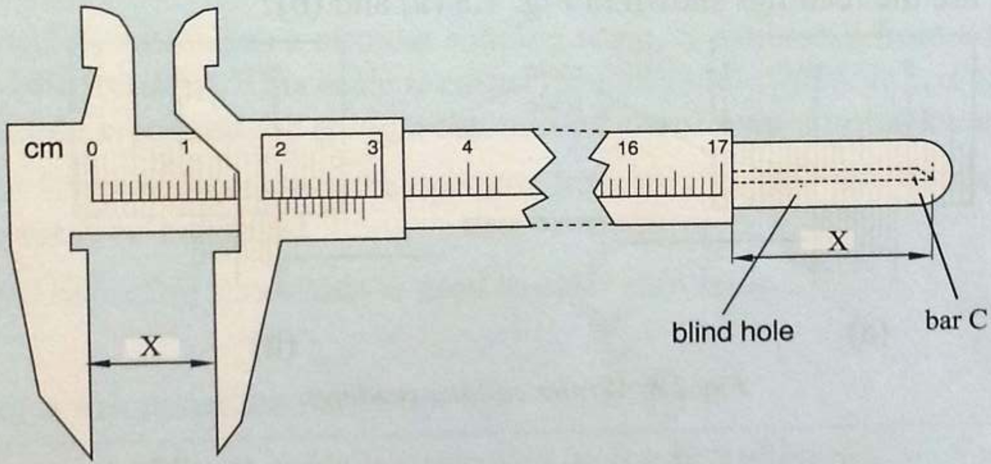


Fig. 1.6: Determination of depth of a blind hole.

From figure, Depth of blind hole $= MSR + (VC \times LC)$

$= 1.9 + (7 \times 0.01) = 1.97$ cm

Example 1.2

What are the readings shown by the vernier calliper in Fig. 1.7(a) and (b)?

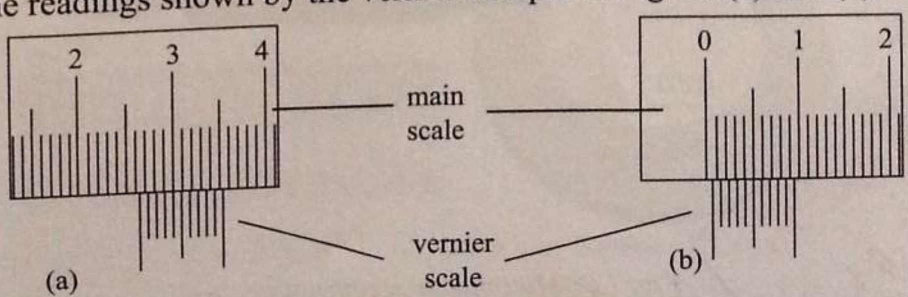


Fig. 1.7: Vernier calliper readings

Solution

(a) Main scale reading = 2.6 cm
Vernier scale reading = 0.04 cm +
Reading = 2.64 cm

(b) Main scale reading = 0.00 cm
Vernier scale reading = 0.05 cm +
Reading = 0.05 cm

Exercise 1.2

1. Explain the advantages of using a vernier calliper over a metre rule in measuring the diameter of a small ball.
2. With the aid of a well labelled diagram describe the main features of a vernier calliper.
3. What are the readings shown in Fig. 1.8 (a) and (b)?

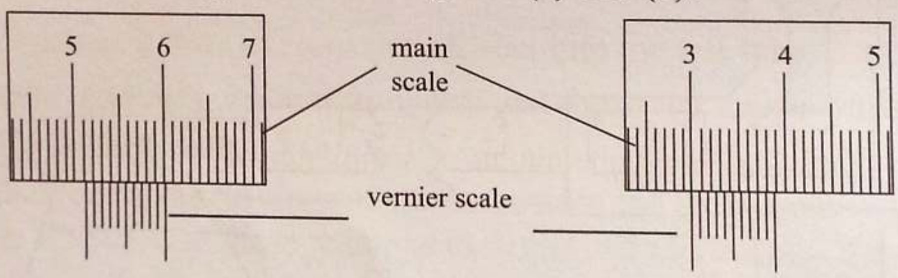


Fig. 1.8: Vernier calliper readings

Micrometer screw gauge

A micrometer screw gauge is an instrument for measuring very short length such as the diameters of wires, thin rods, thickness of a paper etc. It was first made by an astronomer called *William Gascoigne* in the 17^{th} century.

Fig. 1.8 shows the photograph of a micrometer screw gauge.

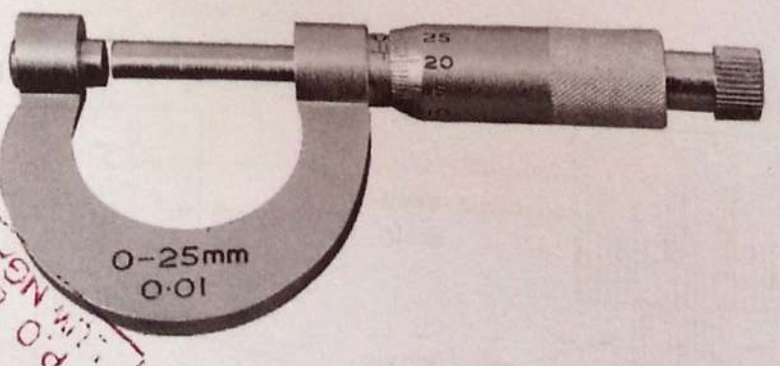


Fig. 1.9: Micrometer screw gauge

Fig. 1.10 shows the parts of a micrometer screw gauge.

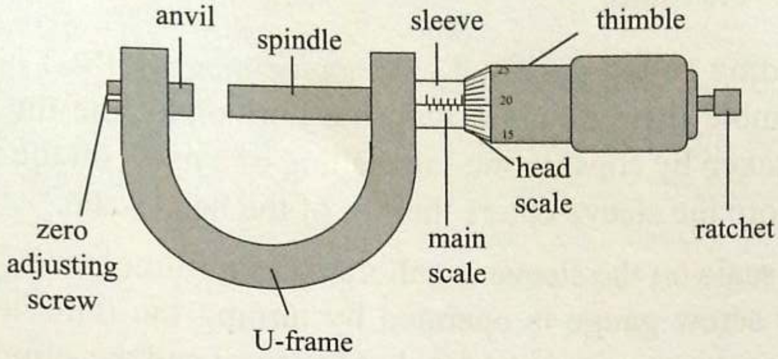


Fig. 1.10: Parts of a micrometer screw gauge

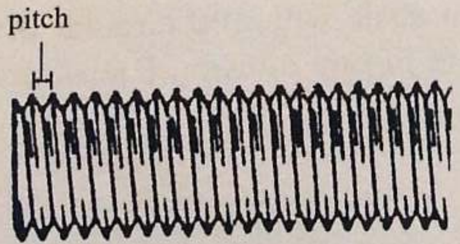
A micrometer screw gauge consists of the following:

- U-frame which holds an *anvil* at one end and a *spindle* at the other end.
- *Sleeve*, which has a linear *main scale (sleeve scale)* marked in millimetres or halve millimetres.
- *Thimble*, which has a circular rotating scale, is calibrated from 0 to either 50 or 100 divisions. This scale is called the *head scale (thimble scale)*. When the thimble is rotated the spindle can move either forward or backwards.
- *Ratchet* which prevents the operator from exerting too much pressure on the object to be measured.
- *Zero adjusting screw* that is used to clear zero error.

Reading a micrometer screw gauge

The movement of the thimble is controlled by a screw of known pitch as shown in Fig. 1.11. There are two common types of pitches on the thimble namely; 1 mm and 0.5 mm.

When the pitch is 1 mm, the thimble has 100 divisions called *head scale divisions*. In this case each division represents 0.01 mm. This is the least count (LC) of this screw gauge.



$$LC = \frac{1\ \text{mm}}{100} = 0.01\ \text{mm}.$$

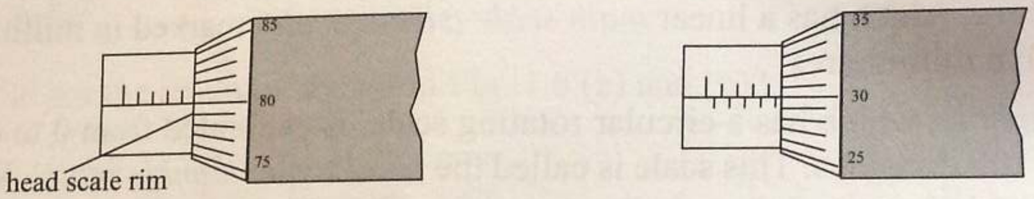
Fig. 1.11: The pitch of a thimble.

Similarly, if the pitch is 0.5 mm the thimble has 50 division. Each divisions represents 0.01 mm, i.e.

$$LC = \frac{0.5 \text{ mm}}{50} = 0.01 \text{ mm.}$$

The thimble reading called *the head scale coincidence* (HSC) is the value of the mark on the thimble that coincides with the horizontal line on the sleeve. Main scale reading is taken by considering the reading of a mark on the fixed scale that is immediately before the sleeve enters the rim of the head scale.

The linear main scale on the sleeve is calibrated in millimetres or half millimetres. The micrometer screw gauge is operated by turning the thimble until the object whose measurement is required just touches the anvil and the spindle. The ratchet is then rotated to press the object gently between the anvil and the spindle. When the correct pressure has been exerted on the object, a clicking noise is heard indicating that the reading can now be taken. See Fig. 1.12.



Thimble with 100 divisions

(a)

Thimble with 50 divisions

(b)

Fig. 1.12: Thimble divisions.

In Fig. 1.12(a), the least count = 0.01 mm.

The micrometer screw gauge reading $= \text{MSR} + (\text{HSC} \times \text{LC})$

$= 4.0 + (80 \times 0.01)$

$= 4.80$ mm.

In Fig. 1.12(b), LC $= 0.01$ mm. The micrometer screw gauge reading

$= \text{MSR} + (\text{HSC} \times \text{LC})$

$= 4.5 + (30 \times 0.01) = 4.80$ mm.

Example 1.3

A micrometer screw gauge has a circular scale with 100 circular divisions and screw pitch of 1.00 mm, find the length of one division (least count) on the thimble scale.

Solution

100 divisions have a length of 1.00 mm

$\therefore$ 1 division has a length of $\dfrac{1.00}{100} = 0.01$ mm

Activity 1.4: To determine the diameter of a ball bearing using a micrometer screw gauge.

- Clean the faces of the spindle and the anvil to remove any dirt.
- Close the gap between the anvil and the spindle to check for zero error. In case of any error remove it by rotating the zero adjustment screw clockwise or anticlockwise as the case may demand. Alternatively you may note the error as a negative or a positive value and add it to or subtract it from the final reading accordingly.
- Turn the spindle to open a suitable gap for holding the ball bearing in between the anvil and the spindle.
- Close the spindle to the correct tightness (Fig. 1.13).
- Take the readings on the main scale and the thimble scale.

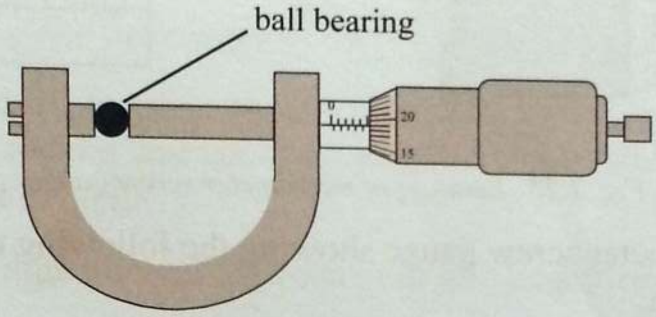


Fig.1.13: Using a micrometer screw gauge

- Repeat the experiment by taking two more measurements. Obtain the average value.
- Multiply the thimble scale reading by 0.01 mm.
- Add the main scale reading (in mm) and the thimble scale reading (in mm) to get the diameter of the ball bearing.

Example 1.4

What is the diameter of the ball bearing shown in Fig. 1.14?

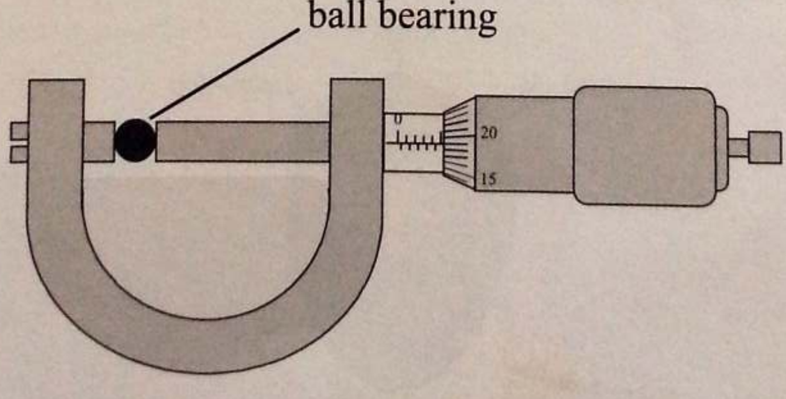


Fig. 1.14: Determining diameter of a ball bearing

Solution

Main scale reading = 5.0 mm

Head scale coincidence = 19 divisions

Head scale reading = $19 \times 0.01 = 0.19$ mm

Full reading = $5.0 + 0.19 = 5.19$ mm

The diameter of the ball bearing is 5.19 mm

Exercise 1.3

1. State the value of the readings shown by the micrometer screw gauges in Fig 1.15.

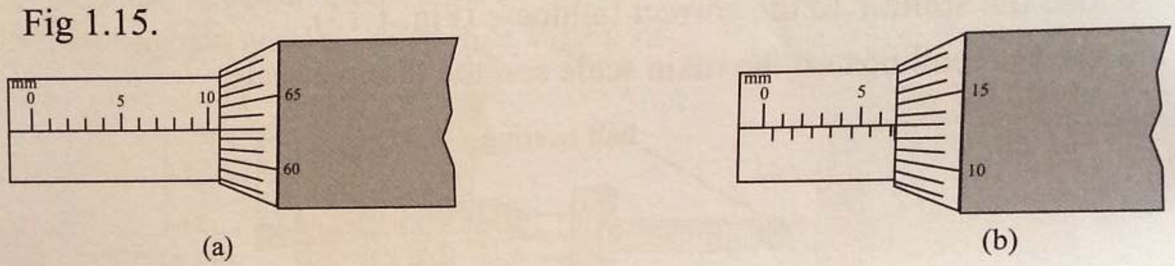


Fig. 1.15: Reading of micrometer screw gauge.

2. Draw a micrometer screw gauge showing the following reading if the screw pitch is 0.5 mm.

   (a) 18.56 mm (b) 2.36 mm (c) 5.72 mm

3. Repeat Question 2 above for a micrometer screw gauge of pitch 1 mm.

1.5 Measurement of time

In Form 1, we learnt that time is the interval between two distinct events. The SI unit of time is the second and its symbol is *s*. We also learnt how to measure time using instruments such as stopclock/watches and digital clocks. In this section we shall review how to measure relatively long time intervals and learn how to measure very small time intervals using digital stopwatches, an oscillating pendulum and CRO. Fig. 1.16 shows a digital stopwatch that is commonly used in our laboratories.

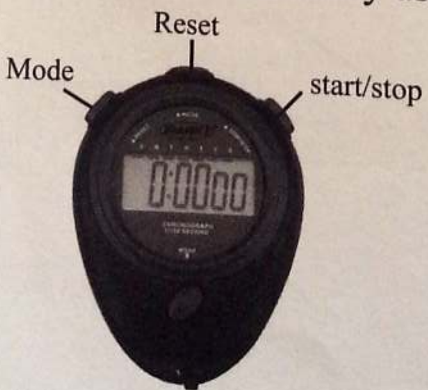


Fig. 1.16: A digital stopwatch

Digital stopwatches can measure very small time intervals. They can display, hours, minutes, seconds and milliseconds as shown in Fig. 1.16.

Activity 1.5: Using a digital stopwatch

- Start the stopwatch and time how long it takes to write a certain sentence e.g.
  - Stop environmental pollution.
  - Our environment is our livelihood
  - AIDS is incurable
  - Avoid unprotected sex.
- Stop the watch, reset and repeat the activity about four times. Find your average time for writing the sentence.

Activity 1.6: Timing the heartbeat

- Place your palm on the side of your chest and feel your heartbeat. Start the stopwatch. By feeling and counting your heartbeats determine the number (n) of heartbeat in 60 s.
- Determine the time interval between your two heartbeats as $\frac{60}{n}$
- When breathing normally, you should get $\frac{60}{72} = 0.0833$ s

Activity 1.7: Timing the reading of different words

- Using a stopwatch, determine the time in milliseconds (ms) you take to mention the words; electricity, Malawi, patriotic, responsible etc.

Example 1.5

The heart of a very fat student was beating at 65 beats per minute. Find the time interval for one beat. What can you advice the person (Hint: the norml heartbeat rate is 72 beats per minute).

Solution

85 beats takes 60 seconds

1 beat will take ?

$\frac{1 \times 60}{85} = 0.706$ s

The time for one heartbeat is 0.925 s. The person should visit a doctor for checkup.

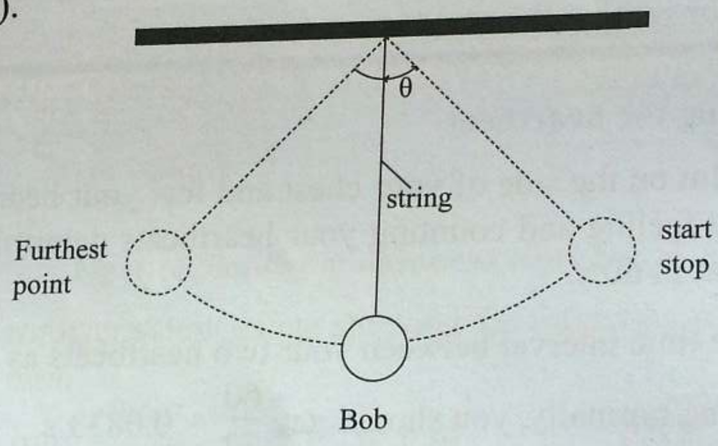
Always exercise your body and eat properly to avoid most lifestyle diseases.

Measuring time interval using a simple pendulum

A simple pendulum is a small mass called *bob* suspended by a light inextensible string from a fixed pivot, it undergo oscillatory motion (repetitive to and from motion). The following activity will help us estimate the time taken for one complete oscillation of a pendulum.

Activity 1.8: To measure time interval using a pendulum

1. Suspend pendulum bob on a clamp using a string as shown in Fig. 1.17
2. Displace the pendulum bob through a small angle ($\theta < 10^{o}$) and release it (Fig. 1.17).



1.17: An oscillating pendulum

3. Use the stopwatch to time 20 oscillations of the pendulum.
4. Repeat steps 2 and 3 two more times and get the average value of t.
5. Determine the time for one oscillation.

From Activity 1.5, we can determine the time for one oscillation by finding the periodic time. The periodic time (T) is the time it takes for an oscillating object to make one complete cycle i.e

$$\text{Period}, T = \frac{t}{\text{No. of oscillation}}$$

$$= \frac{t}{20} \text{ (s)}$$

Example 1.6

A pendulum makes 20 oscillations in 14.16 s. What is the time for one oscillation.

$$T = \frac{t}{20} = \frac{14.16}{20}$$

$$= 0.708 \text{ s}$$

Measuring time using cathode ray oscilloscope (C.R.O.)

A cathode ray oscilloscope (C.R.O.) uses cathode rays to display waveforms on a fluorescent screen. Fig. 1.18 is a picture of a cathode ray oscilloscope (CRO).

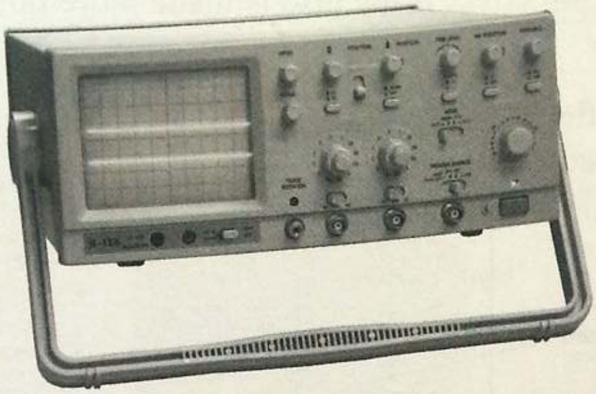


Fig. 1.18: A CRO

Fig. 1.19 shows the main internal features of a cathode ray oscilloscope.

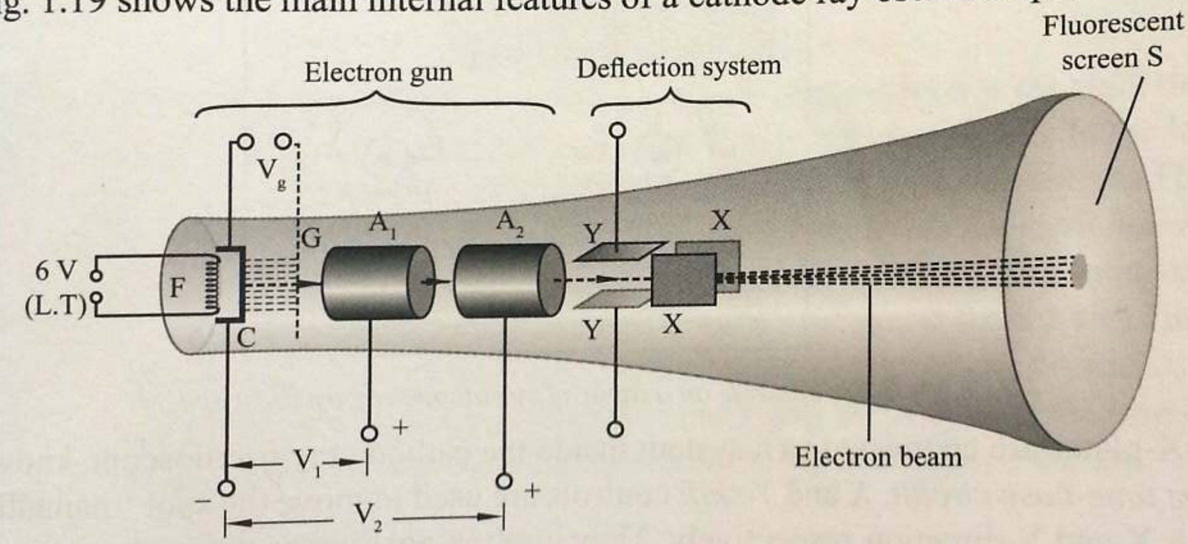


Fig. 1.19: Features of a cathode ray oscilloscope

The *electron gun* consists of a *filament F*, which is surrounded by a *cathode C*, *two anodes* A_1 and A_2 and a third electrode called the *control grid G*.

The *deflecting system* consists of two pairs of plates: a horizontal pair called *the Y-plates* and a vertical pair called *the X-plates*. At the end of the evacuated glass tube is the *fluorescent screen S* coated with a fluorescent material like phosphor and zinc sulphide.

When the cathode is heated by the current from a low tension (L.T) supply, free electrons are thermionically emitted from its surface. The emitted electrons are *accelerated* and *focused* by anodes A_1 and A_2 maintained at a positive voltage with respect to the cathode. The shapes and potential of the anodes are so chosen that the electric fields between them converge the beam into a fine spot on the fluorescent screen S.

The brightness of the spot on the screen is controlled by the control grid. If the grid is made more negative in potential with respect to the cathode, the number of electrons per second passing through the grid decreases and the spot becomes less bright. The reverse is the effect if the grid is made more positive in potential with respect to the cathode.

The Y-plates of the deflection system are connected internally to the input terminals (one of which is earthed) of the cathode ray oscilloscope (Fig. 1.20).

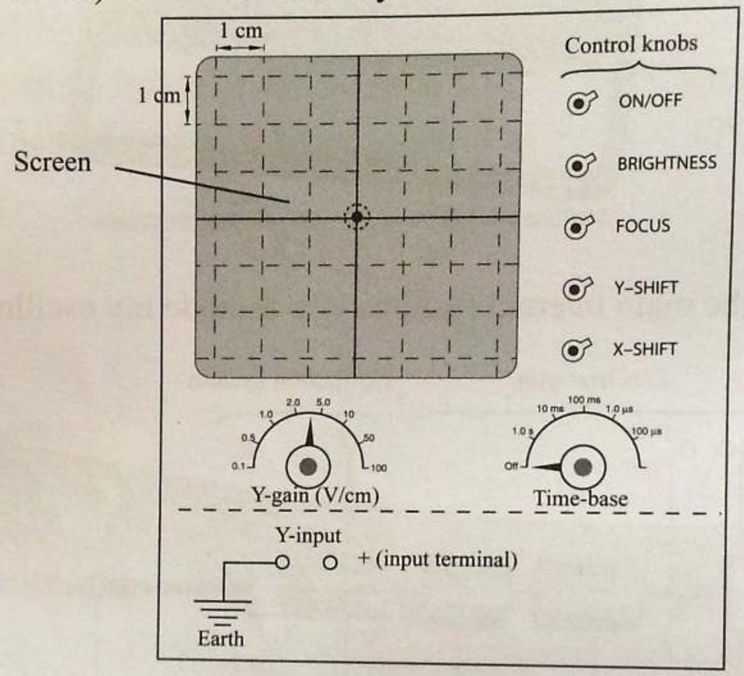


Fig. 1.20: Basic controls on a laboratory cathode ray oscilloscope

The X-plates are connected to a system inside the cathode ray oscilloscope known as *the time-base circuit. X* and *Y-shift* controls are used to move the spot 'manually' in the X and Y direction respectively. They apply a positive or a negative voltage to one of the deflecting plates according to the shift required. The *Y-gain* control is an amplifier control. The input voltages go through an amplifier before reaching the Y-plates. The amplification is altered by the Y-gain control which is calibrated in volts per division. These divisions are usually marked in centimetres on the plastic filter fitted in front of the screen.

The procedure for operating a cathode ray oscilloscope is as follows:

1. Switch on the oscilloscope and make sure that the time base knob is in the 'off' position.
2. Adjust the X-shift and Y-shift till the spot appears.
3. Set the spot to the centre of the screen by adjusting the X and Y-shift controls.
4. Adjust the focus and brightness control to obtain a sharply focused bright spot.

The Y-deflection plates

When the time base is switched 'off' and a potential difference is applied to the Y-plates, the electron spot is deflected up or down along the y-axis. Sometimes the deflection produced on the y-axis of the screen may be too small. This deflection can be adjusted with the help of Y-gain knob calibrated in volts per centimetre. The Y-gain is merely a scale used on the y-axis.

The X-deflection plates

The X-plates are internally connected to the time-base circuit, which applies a saw-tooth voltage to the X-plates as shown in Fig. 1.21.

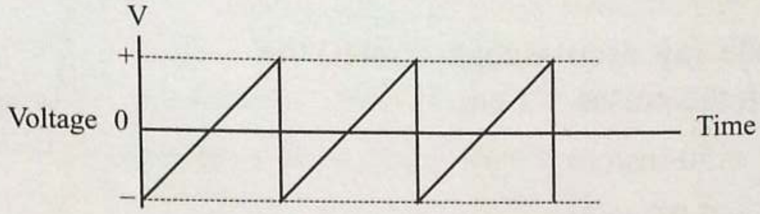


Fig. 1.21: Saw-tooth voltage waveform

The electron beam is moved from the left hand side of the screen to the right during the time that the voltage rises to maximum and then is returned rapidly to the left as the voltage returns to zero. This mechanism is called *the fly back mechanism*. The fly back mechanism 'sweeps' the spot along the X-axis from *left to the right hand side* of the screen. The frequency of motion of the electron beam along the X-axis can be adjusted with the help of the time-base knob and can be varied from 1 s to 1 μs per division. For example, if the time-base is set at 1 s/cm (1 second per division), then the electron spot takes 1 s to move through 1 cm along the X-axis. If the time-base is set at 1 ms/cm, then the time taken is 0.001 s for the spot to move 1 cm etc.

Measurement of small time interval

If two events occur within a short interval of time, say a student strikes the skin of a drum twice consecutively, one after another, the sound waves produced by the drum can be picked by a microphone. The microphone converts the sound energy into electrical pulses. These signals from the microphone are fed to the input terminals of the cathode ray oscilloscope whose time-base is on. The two 'pulses' produced are seen on the cathode ray oscilloscope's screen as shown in Fig. 1.22.

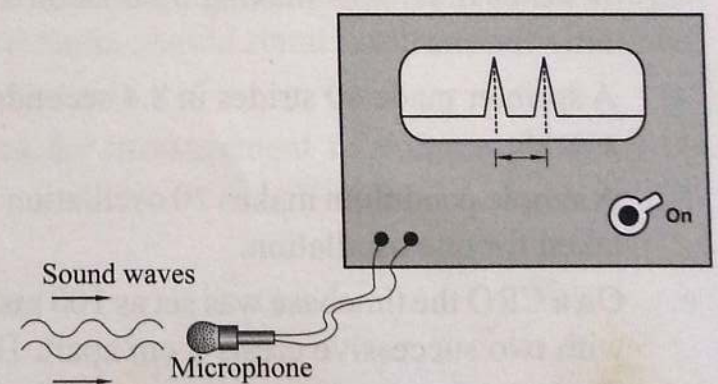


Fig. 1.22: Measurement of small time intervals

Knowing the time base of the cathode ray oscilloscope and the distance x between the two pulses, the time intervals between the two signals can be calculated.

Example1.7

Two Malawian athletes completed a race very close to each other in the 2004 Olympic games held in Athens, Greece. A cathode ray oscilloscope, with a time-base set at 50 ms/cm had been deployed to measure the time interval between the athletes. Two 'pulses' produced on the screen are shown in Fig. 1.23. Calculate the time interval separating the two athletes.

Solution

From the cathode ray oscilloscope screen, the distance between the pulses = 2 cm

The time-base = 50 ms/cm

= 0.05 s/cm

Since the distance between the pulses is 2 cm, the time interval between them

= 2 × 0.05 s

= 0.10 s

∴ the time interval between the two athletes is 0.10 s.

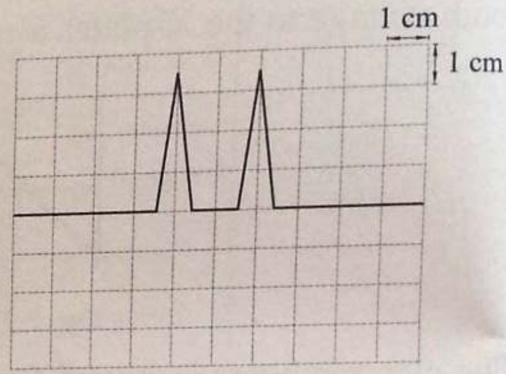


Fig. 1.23

Exercise 1.4

1. Define the terms:
   (a) time
   (b) period.
2. Describe an experiment to measure the time interval for one heartbeat.
3. A ticker timer was making 50 dots on a ticker tape in 30 seconds. Find the time for one dot.
4. A sprinter made 60 strides in 8.4 seconds. Determine the time taken to make a stride.
5. A simple pendulum makes 20 oscillation in 26.13 seconds. Calculate the time taken for one oscillation.
6. On a CRO the timebase was set as 100 ms/cm. A wave was formed on a screen with two successive crests 5 cm apart. Determine the time interval between the crests.

7. The heartbeat of a person is recorded on a cathode ray oscilloscope as shown in Fig. 1.24. Calculate the average heartbeat per minute, if the time-base setting is 500 ms/cm. (Hint: $2\lambda = 3.6$ cm)

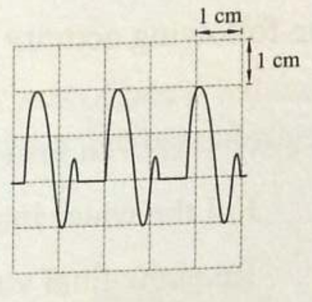
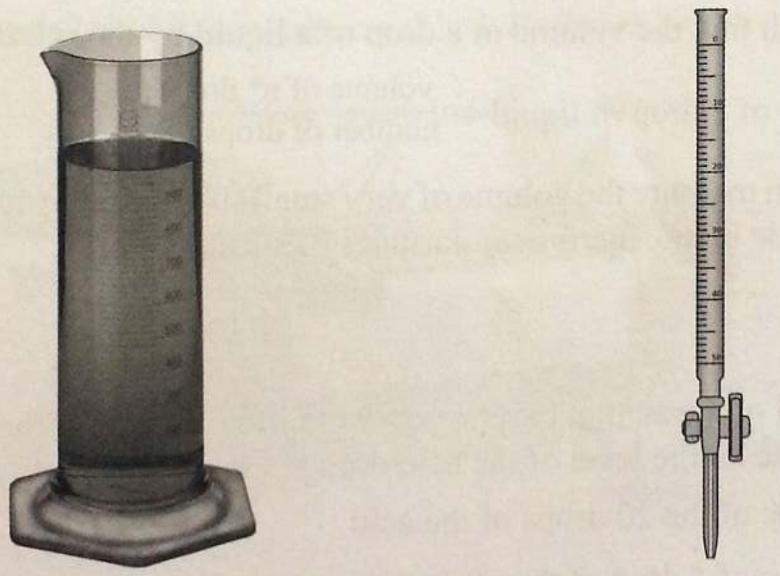


Fig. 1.24

1.6 Measurement of volume

We already know that volume is the amount of space occupied by an object. The SI unit of the volume of a substance is the cubic metre (m^3). Volume is measured using instruments like measuring cylinders, burette, pipette and measuring flask.

Fig. 1.25 shows the measuring cylinder and a burette.



(a) Measuring cylinder (b) Burette

Fig. 1.25: Measuring of small volume

Note that while taking readings using measuring instruments, they must be upright. For example, measuring cylinders and flasks should stand on a horizontal surfaces to reduce errors from tilting.

To choose the appropriate instrument for measurement of volume, consider the following.

- The nature of the substance.
- The amount of the substance.

The following activity will help us find the volume of a drop of water.

Activity 1.9: To find a volume of a drop of water

- Fill the water into a burette to a level V_1. Run out 10 drops of water.
- Read the final volume, V_2 of the burette.
- Determine the volume of 10 drops as $V_2 - V_1$
- Determine the volume of one drop.

From Activity 1.6, we can determine the volume of one drop as follows:

The volume of 10 drops of water = $(V_1 - V_2)$ cm^3

Thus, the volume of 1 drop of water = $\frac{V_2 - V_1}{10}$ cm^3

Generally, we can find the volume of a drop of a liquid by the relation

$$\text{Volume of 1 drop of liquid} = \frac{\text{volume of n}^{\text{th}}\text{ drops}}{\text{number of drops (n)}}$$

Therefore, we can measure the volume of very small substances by taking the overall volume and divide by the number of particles of substances.

Example 1.8

The initial level of the acid in burette was 9 cm^3. If 20 drops of the acid were run out of the burette an the level of the acid dropped to 21 cm, find:

(a) The volume of the 20 drops of the acid

(b) The volume of 1 drop of the acid

Solution

(a) Volume of the 20 drops = Final reading – initial reading

$$= 21 - 9 = 12 \text{ cm}^3$$

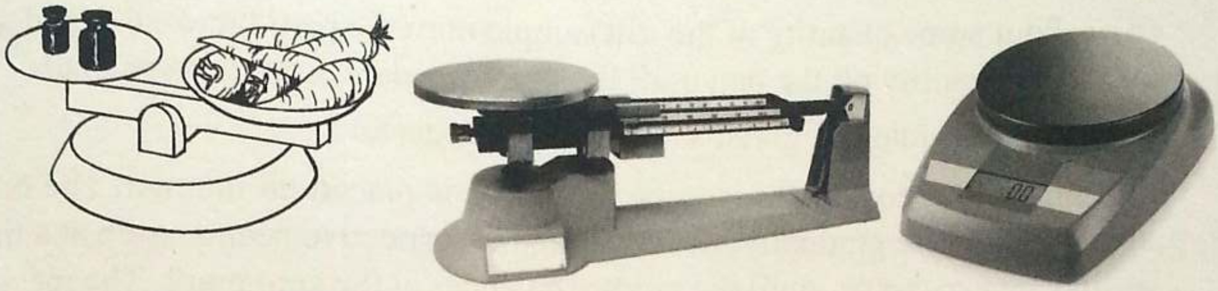
(b) The volume of 1 drop of the acid $= \frac{\text{volume of 20 drops}}{\text{number of drops}}$

$$= \frac{12}{20} = \frac{3}{5}$$

$$= 0.6 \text{ cm}^3$$

1.7 Measurement of mass

Mass is measured using the *beam balance*. There are many kinds of beam balances used for measuring mass. They include traditional beam balance, triple beam balance and digital beam balance. (Fig. 1.26)



(a) Traditional beam balance (b) Triple beam balance (c) Digital beam balance

Fig. 1.26: Different types of balances

In most school's laboratory, the mass of substances are measured using the *triple beam balance and digital beam balance*. Fig 1.27 shows a triple beam balance.

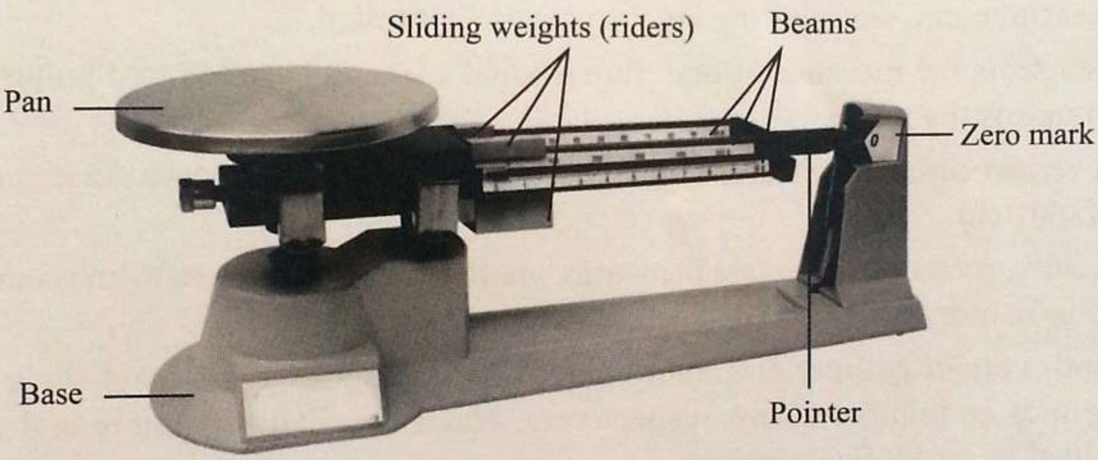


Fig. 1.27: Triple beam balance

The triple beam balance consists of three beams. The first beam is calibrated in 100-gram marks, the second one in 10-gram marks and the third one in 1- gram marks. On each beam, there is a corresponding slide weight.

Before using the triple balance to measure mass, the beam block is balanced horizontally at the balance point by moving all the slide weights to the zero marks on the respective beams. When balanced, the pointer attached to the beam block points to the zero mark.

A triple balance can be used to:

(a) Measure the required mass of a quantity from the given sample.

For example to measure 126 grams of salt from a sample of the salt provided,

(i) First adjust slide weight as follows:

- Move the slide weight on the 100 g- beam to the 100 g-mark.
- Move the slide weight on the 10 g - beam to the 20 g-mark.
- Move the slide weight on the 1 g - beam to the 6 g-mark.

(ii) Pour some quantity of the salt sample onto the pan. Increase or reduce the quantity on the pan until the pointer balances at the zero mark.

(b) Determine the mass of given sample of a substance.

The quantity whose mass is to be measured is placed on the pan. The three slide weights are gradually moved along the respective beams, each at a time as may be necessary, until the pointer balances at the zero mark. The mass of the sample is determined by adding the readings obtained on the three beams.

Unit summary

- The choice of a measuring instrument to be used depends on the type of measurement, size and the level of accuracy needed.
- Length is the measure of one dimensional space in between two points. Its SI unit is the metre.
- A vernier calliper measures both internal and external diameters to an accuracy of 0.01 cm.
- A micrometer screw gauge measures small external diameters to an accuracy of 0.01 mm.
- Both vernier calliper and micrometer screw gauges have a main scale and vernier or thimble scales respectively which are read separately and then added to give a final reading.
- Time is the interval between two distinct events. Its SI unit is the second.
- The instruments used to measure time are stop clock/watches.
- A simple pendulum is a small mass called a bob suspended on a fixed pivot by a light inextensible string.

Unit Test 1

1. Define the following terms:

(a) Length

(b) Frequency

2. Write down the most appropriate instrument to use in the measurements of the following:

   (a) Thickness of a mobile telephone scratch card.
   (b) Diameter of a thin metal rod.
   (c) Thickness of this book.
   (d) Diameter of a thin wire
   (e) The width of the rugby pitch.
   (f) The thickness of your hair.

3. A metal rod has a diameter of 2.1 cm. How many times would a thread of 100 cm be wound around the metal rod?

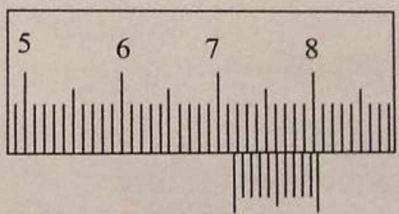
4. With the aid of a diagram explain how a vernier calliper may be used to measure the depth of a blind hole.

5. Express the following in standard form.

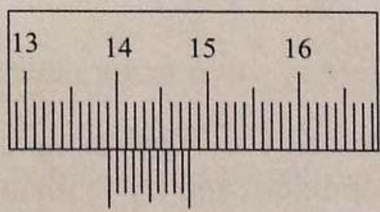
   (a) 436 000 cm   (b) 0.00001614 kg

6. The population of people in a certain country was 2 707 509 people. Express this in standard form.

7. What is the reading shown in the vernier callipers in Fig. 1.28.



(a)



(b)

Fig. 1.28

8. (a) What is the function of the ratchet of a micrometer screw gauge?
   (b) Explain how to make a correct reading on a micrometer screw gauge.
   (c) What is the reading shown on the micrometer screw gauge in Fig. 1.29?

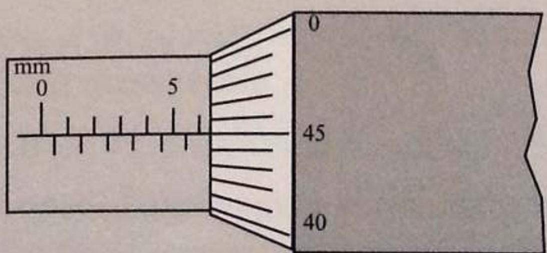


Fig.1.29

9. Draw a micrometre screw gauge showing the following reading given that the screw pitch is 1 mm.

   (a) 3.69 mm

   (b) 2.45 mm

   (c) 0.436 cm

10. Draw the diagram of a micrometer screw gauge showing a reading of 4.41 mm.

11. Convert the following:

   (a) 2.5 hours into seconds

   (b) 0.71 weeks into hours

   (c) 23 614.6 seconds into hours

   (d) 0.008 years into days

13. Give the readings shown in the watches in Fig. 1.30 below.

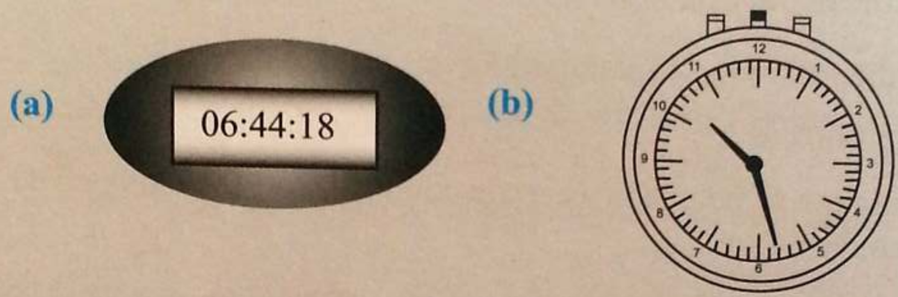


Fig. 1.30

14. Give any two factors that affect the period of pendulum.

15. Describe an experiment to determine factors that affect the period of a simple pendulum and try to identify what is to be kept constant if any.

UNIT

2 Scientific Investigations

Success Criteria

By the end of this unit, you must be able to:

- Design a scientific investigation.
- Carry out a scientific investigation.
- Analyse data from a scientific investigation.
- Communicate results from experiments.
- Evaluate a scientific investigation.

Introduction

In lower forms, we were introduced to scientific investigation. We looked at the definition of scientific investigation and the steps followed when conducting such an investigation. In this unit, we shall design and carry out a scientific investigation. We will also learn how to analyse the data obtained through scientific investigation, communicate the results obtained and evaluate the investigation.

2.1 Scientific investigation design

Scientific research originates from curiosity or the need to solve a problem. The researcher may initiate research to solve a problem he/she has identified or may be commissioned to do so by an interested party to solve a particular problem. For the researcher to conduct a standard scientific research work, he/she must follow some standard steps and guidelines. These guidelines and steps are referred to as *scientific investigation design.*

Scientific investigation design is *a systematic plan to study a scientific problem.* It specifies among other things the *study type* (descriptive, experimental, correlational, review etc.), *methods of data collection* and *analysis tools* to be used.

The following are general guidelines for a good scientific investigation design.

The design plan should:

1. State clearly the topic or problem to be solved. The statement should be precise and focused to make it manageable.
2. Suggest a hypothesis on the problem to be solved.

   A hypothesis is a proposed explanation given on the basis of limited evidence as a starting point for further investigation.

   For example, in an investigation to determine how electrical conductivity of a uniform cylindrical material varies with its diameter, one can formulate a hypothesis like,

"The electrical conductivity of a uniform cylindrical conductor is directly proportional to the diameter".

Such a hypothesis would have to be proofed true or disapproved by conducting the investigation.

3. Outline SMART objectives for the investigation.

S – specific

M - measurable

A – achievable

R – realistic

T – time bound

An example of a SMART objective is, " *To determine how the volume of a fixed mass of a gas varies with pressure at a constant temperature*"

4. Identify the variables involved in the investigation.

There are three types of variables:

(a) *Independent variables:* An independent variable is the quantity that the researcher has control over i.e. what one can vary or manipulate in order to produce a change in another variable (called dependent variable).

For example, in an investigation to determine how pressure affects the volume of a fixed mass of a gas at constant temperature, one would vary the pressure; this would produce a change in the volume of the gas. In this investigation, pressure would be the independent variable.

(b) *Dependent variables:* A dependent variable is the quantity that changes as a result of the change in the variable that is varied by the researcher or otherwise. In other words, the dependent variable responds to the change in the independent variable.

For *example*, in the investigation to determine how pressure affects the volume of a fixed mass of a gas at constant temperature described above, pressure is the independent variable while volume is the dependent variable.

Note that there cannot be a dependent variable without an independent variable in a scientific experiment.

(c) Controlled variables: A controlled variable is the quantity that the researcher wants to remain constant so that it does not affect the dependent variable.

The researcher performs a deliberate action to keep the controlled variable constant as he/she varies the independent variable, in order to determine the corresponding change in the dependent variable. For example, in the investigation to determine how pressure affects the volume of a fixed mass of a gas at constant temperature described above, the mass and temperature of the gas are the controlled variables.

5. Identify apparatus/resources to be used to carry out the research. Such apparatus should give the readings and results to the expected degree of accuracy to minimize errors.
6. Outline the procedure of carrying out the experiment. The procedure should include logical steps, set up diagrams and precautionary measures where necessary.
7. Identify the appropriate methods of collecting data. The methods should be efficient and apply standard procedures, enhance collection of primary data. They should also be cost effective.
8. Specify or identify any possible constraints or risks, and suggest ways to handle them.
9. Suggest methods and tools of data analysis that will be used. Such tools include graphs and charts.
10. Incorporate the use of secondary sources of information to validate the results obtained. Such sources include books, magazines, journals and internet links.

Activity 2.1

Prepare a scientific investigation design that you would use to investigate how the temperature of a fixed mass of water varies with time from 0°C until it boils completely.

Exercise 2.1

1. In an investigation to determine how the electrical resistance of a material varies with the length of the material, identify:
   (a) the independent variable.
   (b) the dependent variable.
   (c) two controlled variables.
2. Outline the procedure you would use to investigate the effect of temperature on the density of a liquid.

2.2 Carrying out a scientific investigation

After preparing the design for a scientific investigation, the next stage is carrying out the investigation.

To get appropriate and accurate data when carrying out a scientific investigation, the following should be considered:

- Carry out the outlined procedures and trials to get the appropriate data. Make sure the procedures followed give accurate results. In carrying out the procedures observe *health, safety and environmental measures*.
- Collect and record the data to the right precision as you carry out the procedures. Methods of recording data include tables, charts, photographing and recording of sound.

For example, Table 2.1 shows one way of recording the data obtained in the investigation to determine how pressure affects the volume of a fixed mass of a gas at constant temperature.

Mass of gas = 0.5 g

Temperature = 25°C

Table 2.1

Pressure (x 10^5 Pa)	Volume (cm^3)
1.00331	7.339
1.02180	7.241
1.03907	7.143
1.02565	7.038
1.07390	6.880
1.08481	6.830
1.09388	6.780
1.11137	6.670
1.12196	6.630

2.3 Data analysis in a scientific investigation

The raw data collected and recorded need to be analysed in order to give meaningful information. Data analysis may involve:

- Organizing the data and studying the trend to determine how it is varying or if it remain constant.
- Drawing graphs and charts to show the trend in the set of data.

- Calculating required values that are representation of the data.
- Interpreting the graphs, charts and the calculated values to derive meaningful information.
- Identifying sources of error in the experiment.

Let us look at some guidelines on how to draw graphs

Drawing graphs

A graph is a diagram showing the relationships between two or more quantities. It has two axes: horizontal axis, generally known as the *x-axis* and the vertical axis that is generally known as the *y-axis*.

Features of a graph

The following are the main features of a graph:

- Title
- Axes
- Scales
- Plotting points
- Line/curves of best fit
- Slope (gradient)

Title

The title indicates the variables whose relationship is represented in the graph. The first quantity in the statement of the title should be represented on the vertical axis and the other one on the horizontal axis.

Under normal circumstances, the dependant variable takes the vertical axis whereas the independent variable takes the horizontal axis.

For example, in the graph titled, *"A graph of m against T^2"* the variable *m* is represented on the vertical axis and T^2 on the horizontal axis.

Axes

- The axes are drawn perpendicular to each other with arrows indicating the direction of the increasing values.
- The axes should be labelled with the quantity and units represented on them.
- Both axes must have a starting value at the point of their intersection. The starting point is zero in most cases but may also be another value so that the graph may fit on the graph paper provided.

For example, Fig. 2.1 shows a graph of m against T^2 with the origin as the starting point for both axes.

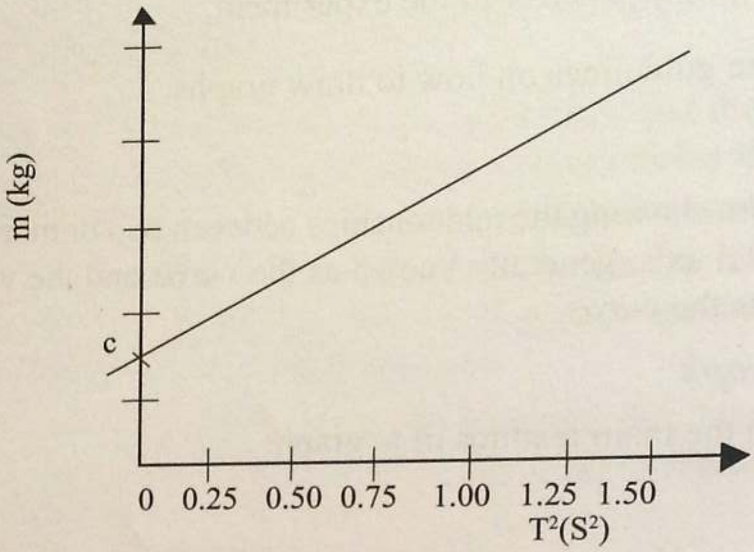


Fig. 2.1: A graph of m against T^2

From Fig. 2.1 point c is the intercept of the graph on the m-axis.

Scales

The choice of the correct scale enables us to plot all the points obtained or given, to get a graph of a reasonable size i.e. neither too small nor too big. The scale on the vertical axis should not necessary be the same as that on the horizontal axis. However, the scale chosen on each of the axes must be uniform.

It is also important to choose a convenient scale which enables one to represent and read all possible values on the grid. A standard graph paper used should have 1 cm grid.

Examples of convenient scales are:

1 cm represents 1 unit, 1 cm represents 2 units, 1 cm represents 5 units, 1 cm represents 10 units, 1 cm represents 20 units etc.

Examples of incovinient scales are

1 cm represents 3 units, 1 cm represents 6 units, 1 cm represents 7 units, 1 cm represents 9 units, 1 cm represents 30 units etc.

Example on how to determine the Scale

Consider the data in Table 2.2, on an experiment to determine the refractive index of a glass block.

Table 2.2

i (°)	x (cm)	y (cm)	sin i	$\frac{x}{y}$
10	0.9	6.5	0.174	0.14
20	1.5	6.6	0.342	0.23
30	1.9	6.7	0.500	0.28
40	3.0	7.1	0.642	0.42
50	3.9	7.5	0.766	0.52
60	4.5	7.8	0.866	0.58

We are required to plot a graph of *sin i* against $\frac{x}{y}$.

We can *roughly* determine the scale using the expression 1 cm represents $\frac{v_h - v_l}{n}$ units where
v_h - highest value in the column.

v_l - lowest values in the column.

n - length in centimeters along the axis where the points will be plotted.

For this graph *sin i* will be represented on the vertical axis. Taking a length of 10 cm to represent all the values on the vertical axis, we can roughly determine the scale on the vertical axis as follows:

$$\frac{v_h - v_l}{n} = \frac{0.866 - 0.174}{10} = 0.069$$

A scale of 1 cm represents 0.069 is not a very convenient to use. We need to round off the value 0.069 to get a convenient scale near the same value. Doing this, we get a convenient scale of *1 cm represents 0.10 units*. Thus, we will graduate the vertical axis in equal intervals of 0.10 cm i.e. 0.10 cm, 0.20 cm, 0.30 cm, 0.40 cm, 0.50 cm, 0.60 cm and so on.

In a similar way, we will obtain a scale of 1 cm represents 0.05 units on the horizontal axis. Drawing the two axes on a grid using the scales choosen we obtain Fig. 2.2.

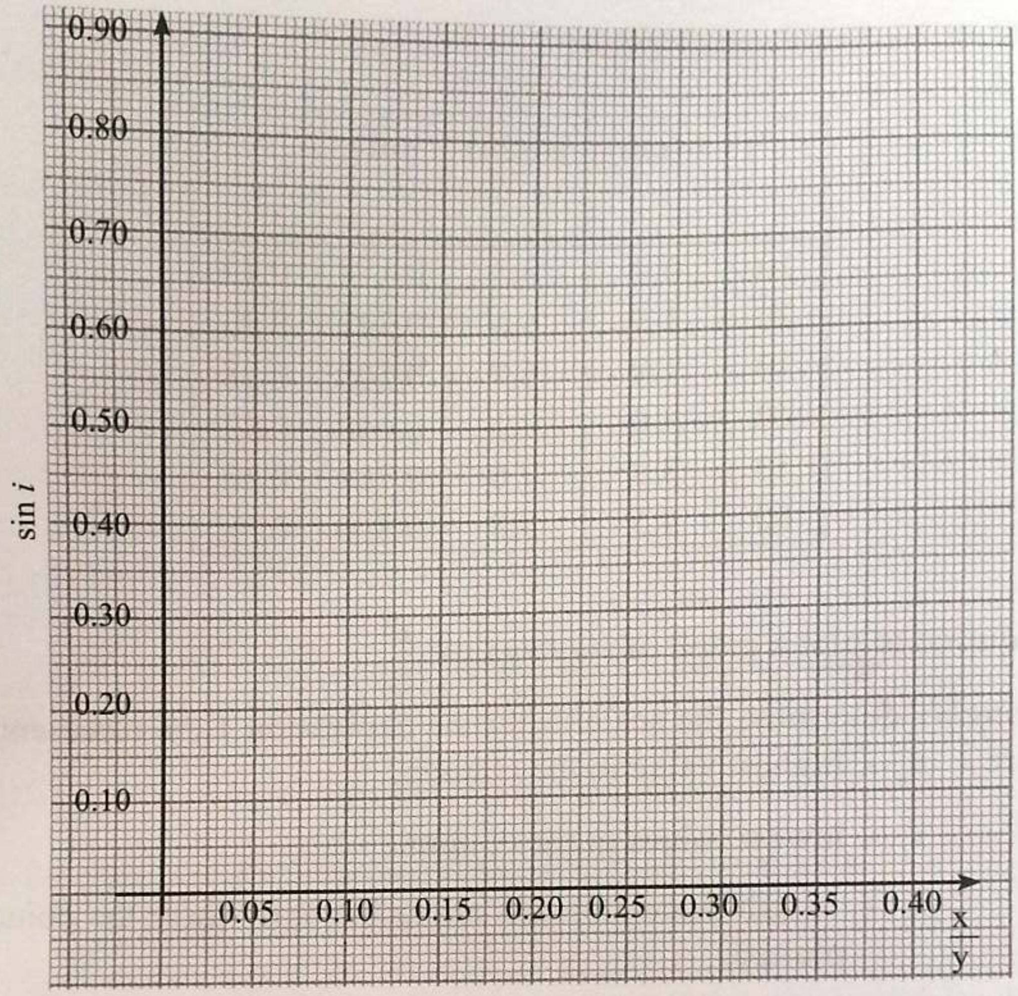


Fig. 2.2: A graph of sin i against $\frac{x}{y}$

Note: The decimal values can be represented in standard form on the axes.

Plotting points

The points are plotted using a small cross (×) or an encircled dot (●). These plotting marks should not be too big, otherwise they will not be accurate.

For ease of plotting, one should determine in advance what unit one small division on a given axis represents using the scale on that axis.

For example, with a scale of 1 cm represents 0.10 units on the horizontal scale in a centimetre grid with 5 small division in a 1 cm interval,

$$\text{One small square represents } \frac{0.10}{5} = 0.02 \text{ units}$$

All the points should be plotted on the grid as accurately as possible guided by the scale.

It is important to confirm the plotting for each point.

Line / curve of best fit

Once the collected data is represented with dots or crosses on grid, they may show a pattern. Quite a number of relationships between physical quantities in real life may lead to a pattern of dots that may suggest a straight line or curve which omits almost equal number of points on both sides of it. In such cases, we join the dots with a straight line using a ruler, such that the line joins the greatest number of points possible. Such a line is known as the *line of best fit*.

It is advisable to draw this line using a transparent rule so as to see the points clearly.

In case the pattern of dots suggest a curve, they should be joined with a smooth curve with a free hand to include as many points as possible (Fig. 2.4).

The graphs in Fig 2.3 and Fig. 2.4. shows the line and curve of best fit respectively.

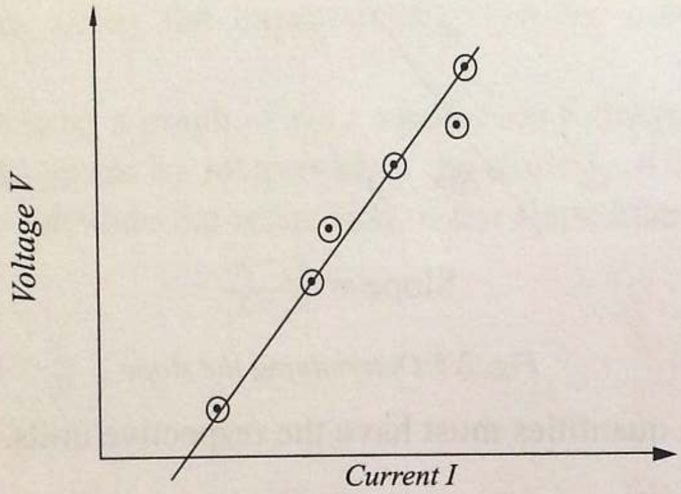


Fig. 2.3: A graph of V against I

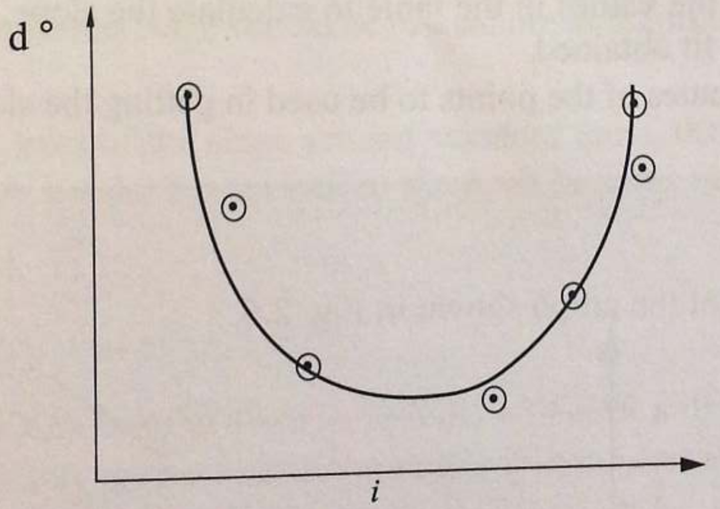


Fig. 2.4: A graph of d against i

Note:

By drawing the line or curve of best fit we minimise the errors due to some of wrong measurements taken. Such values are omitted on both sides of the line or curve. This process is known as to *even out errors*.

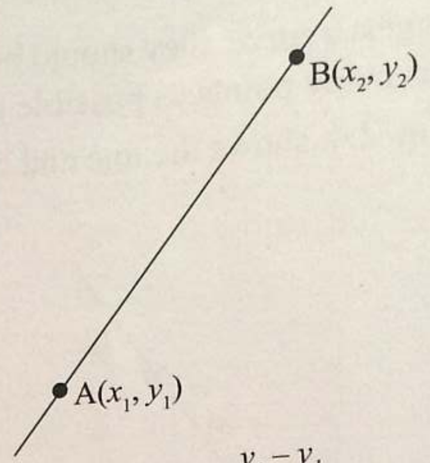
Slope (gradient)

The slope is a measure of the extent to which the variable in the vertical axis changes in relation to the change in the variable on the horizontal axis.

It is given by the following expression.

$$\text{Slope} = \frac{\text{change in the quantity on the vertical axis}}{\text{change in the quantity on the horizontal axis}}$$

Using two points on the line of best fit $A(x_1, y_1)$ and $B(x_2, y_2)$, we get the slope of the line in Fig. 2.3 as follows.



$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Fig. 2.5: Determining the slope

The changes in both quantities must have the respective units.

Note:

- The slope triangle must touch the line of best.
- Do not use the values in the table to calculate the slope. Use the points on the line of best fit obtained.
- Read cordinates of the points to be used in getting the slope correctly from the graph.

Example 2.1

Find the slope of the graph shown in Fig. 2.6.

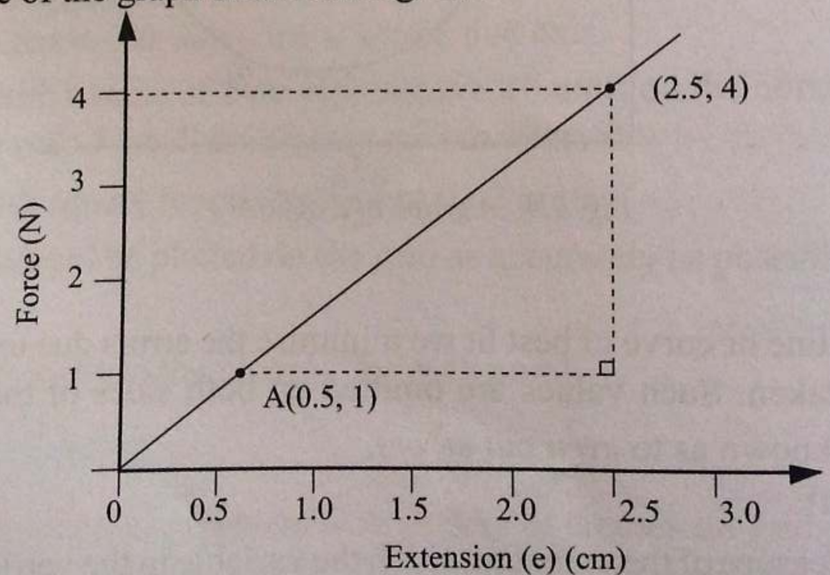


Fig. 2.6: A graph of force against extension

$$\text{Slope} = \frac{\text{Change in Force}}{\text{Change in extension}}$$

$$= \frac{(4-1)\text{ N}}{(2.5-0.5)\text{ cm}} = \frac{3\text{ N}}{2\text{ cm}} = 1.5\text{ N/cm}$$

Determination of key quantities

In some experiments, the slope alone may not lead us to making the anticipated conclusion from the experiment. In such cases, we might be required to calculate some other quantities using the expressions given by substituting values for quantities.

For example, after plotting a graph of *sin r* against *sin i* the refraction index, *n*, of the material would be given by reciprocal of the slope *S*, of the graph. Thus, one would be required to calculate the reciprocal of the slope after first calculating the slope *S*

i.e. $\boxed{n = \frac{1}{S}}$

Note:

- *n* should be expressed to the same number of significant figure as *S*.
- The working at this stage should be carried out in S.I units where the units exist.
- In case the units of the slope are not standard units, they should first be converted to standard units before they can be used in any formula or expression given.

Determination of experimental errors

An error is the variance between a measurement and the true or accepted value

Other terms that are closely related to errors are *uncertainty, precision and accuracy*.

- *Uncertainty* is the interval around a value such that any repetition of the measurement will produce a new result that lie within this interval.
- *Precision* is the degree to which repeated measurements under unchanged conditions show the same results.
- *Accuracy* is the degree of closeness of a measurement to the actual value.

Experimental errors are classified into:

- Absolute errors
- Relative errors

- Parallax errors
- Random errors
- Systematic errors

Though there are many other types of errors, we will limit our discussion to the ones in the list above.

Absolute error

An absolute error is the difference in magnitude between the value of the measurement obtained and the actual value. It is the deviation of the measurement obtained from the actual value.

For example, the actual length of a table is 2.3 m.

A student measured and obtained the length of as 2.35 m.

The absolute error in the measurement is 2.35 m – 2.3 m = 0.05 m

Sometimes, absolute error is taken as half of the least value (count) that can be measured using the given instrument.

For example, if the length of a table is measured in centimeters as 300 cm, the least count is 1 cm.

Therefore, the absolute error $= \frac{1}{2} \times 1 \text{ cm} = 0.5 \text{ cm}$

Example 2.2

Find the absolute error in 107.4 kg

Solution

The least unit of measurement is 0.1 kg

The absolute error $= \frac{1}{2} \times 0.10 \text{ kg} = 0.05 \text{ kg}$

Minimizing absolute error

Absolute error is a procedural error that can be corrected by being more accurate when taking the measurements.

Relative errors

Relative error is the ratio of the absolute error to the true value, usually expressed as a percentage.

$$\textit{Relative error} = \frac{\text{Absolute error}}{\text{Actual (True) value}} \times 100\%$$

Minimizing relative error

Relative error is automatically minimized when absolute error is minimized.

Example 2.3

A student measured the length of her a textbook as 12.1 cm. Calculate the relative error, if the actual length the textbook is 12.0 cm.

Actual length = 12.0 cm

Measured length = 12.1 cm

Absolute error = 12.1 cm - 12 .0 cm = 0.1 cm

Relative error $= \frac{0.1}{12} \times 100\%$

$= 0.83\%$

Systematic errors

A systematic error is an error that is constant (error value remain the same) in a series of repetitions of the same experiment or observation. These errors arise from the errors in measuring instruments for example the zero errors and calibration errors of the measuring instruments.

Minimizing systematic errors

Systematic errors are minimized by correcting the zero errors in the measuring instruments using error-free instruments.

Zero errors

Zero error is the reading that an instrument gives when it is supposed to give a reading of zero.

Sources of zero errors

- Using a maladjusted instrument i.e. an instrument that has not been reset to zero before using it to measure.
- Using wrongly calibrated instruments.
- Using a damaged measuring instrument.

Minimizing Zero Error

- Resetting measuring instruments to zero reading before using them.
- Using instrument that are correctly calibrated.

Note:

When the zero error is negative, we determine the actual reading by adding the error to the reading given by the instrument.

When the error is positive, we determine the actual reading by subtracting the negative error from the reading given by the instrument.

Example 2.4

A voltmeter had a zero error of -0.2 V. It was used to measure voltage across the terminals of a dry cell and gave a reading of 1.3 V. What was the actual voltage of the dry cell?

Solution

Actual voltage = Reading – zero error

= 1.3 V - (-0.2 V)

= 1.3 V + 0.2 V

= 1.5 V

Environmental errors

Environmental errors are those that arise due to conditions that are external to the measuring instrument. Such conditions include temperature, humidity pressure, magnetic and electric fields.

Minimizing environmental errors

Environmental errors are minimized by controlling the external environment where possible, or performing the experiment in an environment free of the interfering factors.

Random errors

Random errors are errors that arise from the inconsistency in the repeated measurements of a constant quantity. They are caused by the unpredictable fluctuations in the readings of a measuring instrument and also from the inaccurate taking of readings from a measuring instrumental.

Minimizing random errors

Random errors are minimized by taking the average of a number of repeated observations.

Parallax errors (observational errors)

Parallax errors arise from reading the wrong value on an instrument as a result of either wrong positioning of the eye relative to the correct reading on the instrument or poor vision by the observer.

Minimizing parallax errors

The observer should always position the eye perpendicular to the correct value mark on the instrument. If the instrument is placed horizontally, the observer's eye should be vertically above the correct mark on the instrument (Fig. 2.7). If the instrument is placed vertically, the observer's eye should be such that the line from the eye to the mark is perfectly horizontal.

Fig. 2.7 shows three possible eye positions when reading the length of a block on a metre rule.

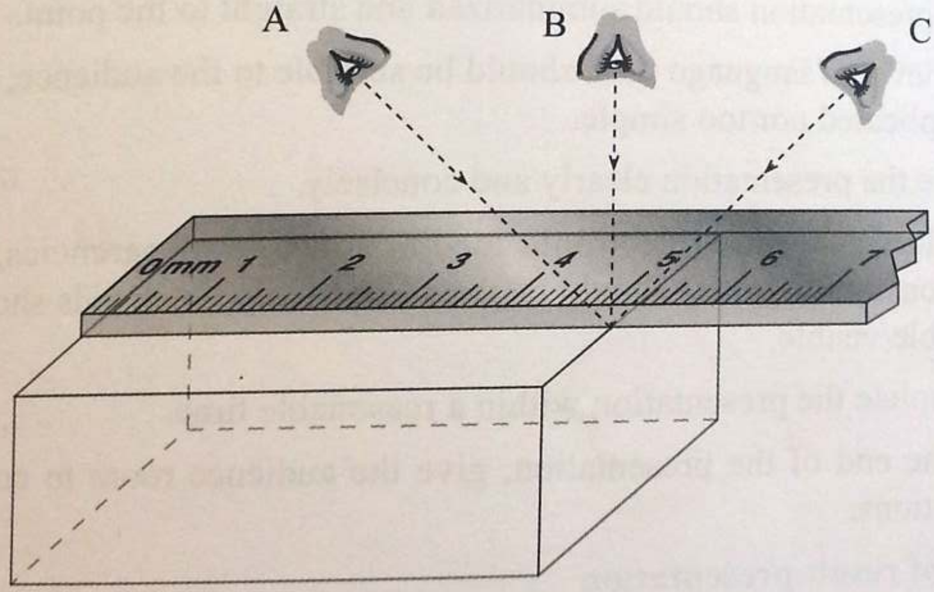


Fig. 2.7: Reading a metre rule

The reading taken while at position A is 4.8 cm (wrong)
The reading taken while at position B is 5.0 cm (correct)
The reading taken while at position C is 5.2 cm (wrong)

Drawing conclusions from an investigation

A conclusion is a summary of what was established though the investigation. It can be a statement to the effect that the quantities or objects considered in the investigation obey a certain law, condition etc. or not.

At this stage the researcher also compares the *hypothesis* with the conclusion, and gives a statement confirming the hypothesis as true or disapproving it all together.

The following is an example of a conclusion:

"From the results of this investigation, we have established that the volume of fixed mass of a gas at constant temperature is directly proportional to its pressure."

2.4 Presentation of the results of a scientific investigation

In most cases, the findings of a scientific investigation have to be communicated in a formal way to the interested parties.

The following are some *important guidelines* on how to present the results of a scientific investigation:

1. Identify the audience to whom the presentation is intended. This will guide you in the choice of the presentation method(s) and tools to use.
2. The presentation should be well organised. It should have a title, objective, hypothesis, introduction, experiment and conclusion(s).
3. The presentation should summarized and straight to the point.
4. The level of language used should be suitable to the audience; be neither too complicated nor too simple.
5. Make the presentation clearly and concisely.
6. Audio/visual aids (electronic media, slides, transparencies, illustrations, demonstrations, etc.) should be used effectively. Such aids should be clearly audible/visible.
7. Complete the presentation within a reasonable time.
8. At the end of the presentation, give the audience room to comment or ask questions.

Methods of result presentation

Methods of presenting the findings of a scientific investigation include:

- Oral presentation
- Power point presentation
- Use of posters
- Video conferencing
- Scientific journals, publications, and reports

The presenter should select the most appropriate method of presentation depending on the nature of research.

2.5 Evaluating a scientific investigation

After completing a scientific investigation, the researcher should evaluate the entire process of the investigation against the objectives outlined before the commencement of the investigation. In so doing, the researcher should:

- Determine whether the objectives of the investigation were met or not.
- Highlight the challenges that affected particular stages of the investigation.
- Discuss ways of addressing the challenges encountered during the investigation.
- Point out areas that would require further investigation
- Suggest the way forward.

Finally, the researcher should submit the report for peer review before the report is adopted.

2.6 Lab report

A lab report is how you explain what you did in an experiment, what you learned, and what the results meant. Though different lab reports may contain detailed information, a simple one should have the following standard format.

- Title
- Aim
- Introduction
- Materials
- Methods
- Results
- Discussion
- Conclusion
- References

1. The title

There are two kinds of titles; title page and title.

a) The title page

Not all lab reports have the title page, but if your teacher wants one, it should be a single page that states:

- The title of the experiment.
- Your name and the names of any lab partners.
- Your teacher's name.
- The date the lab was performed or the date the report was submitted.

b) The title

The title says what you did. It should be brief (aim for ten words or less) and describe the main point of the experiment or investigation. For example, the title for experiment in page 46 would be: "Investigating the relationship between the periodic time and mass." If you can, begin your title using a keyword rather than an article like 'The' or 'A'.

2. The Aim

This is the purpose of carrying out the experiment. It may be one or many. For instrument-based practicals, the apparatus used are normally mentioned. For instance, the aim for determining the relationship between the periodic time and mass in page 46 is: "To investigate the relationship between the periodic time and mass on an oscillating spring."

3. Introduction

Usually the Introduction is one paragraph that states the objective of the experiment and provides the reader with background to the experiment. Sometimes an introduction may contain a brief summary on how the experiment was performed, state the findings of the experiment, and list the conclusions of the investigation. Also, this would be where you state your hypothesis.

Example: The purpose of this experiment was to investigate the relationship between periodic time and mass. We used the oscillating spring and determined that the periodic time of an oscillating spring is directly proportional to the mass of the spring. The scientists, Galileo Galilei reportedly made a breakthrough discovery about the simple pendulum, which later would help in determining this relationship.

4. Materials

It is usually a simple list of everything needed to complete your experiment. Ensure that it is accurate and complete.

5. Methods

These are steps/procedures that show how you carried out the experiment. It is normally given out as part of the practical notes and very rarely would you be required to rewrite it, although you may have to note any alterations. In case you are required to provide the methods, then describe the steps you completed during your investigation. This is your procedure. Be sufficiently detailed but precise so that anyone can read this section and duplicate it. Write it as if you were giving direction for someone else to do the experiment. Therefore your steps should be clear. It may be helpful to provide a diagram of your experimental setup.

6. Results

The results section contains the raw data that you obtained from carrying out the experiment. It should show the summarized data from the experiments without discussing their implications or giving any interpretation. The data should be organized into tables, figures, graphs, photographs and so on. Graphs and figures must both be labelled with a descriptive title. Label the axes on a graph being sure to include units of measurement.

7. Discussion

This is the most important part of your report, because here, you show that you understood the experiment beyond the simple level of completing it. It is where you explain, interpret and analyse the data and determine whether or not a hypothesis was accepted or rejected. In writing this section, you should explain the logic that allows you to accept or reject your original hypotheses. This is also the section where you would discuss any mistakes you might have made while conducting the investigation. You may also wish to describe ways the experiment might have been improved and then conclude by suggesting future experiments that might clarify areas of doubt in your results.

8. Conclusion

In this section, you are required to sums up what happened in the experiment, whether your hypothesis was accepted or rejected, and what this means. Most of the time the conclusion is a single paragraph.

9. References

This is a part where you list all references you used in your research on the process of carrying out your experiment and writing a report. If for instance your research was based on someone else's work or if you cited facts that require documentation, then you should list the references. Note that even the lab manual you used should be included in the list.

A SAMPLE OF A SCIENTIFIC INVESTIGATION

Aim: To investigate the relationship between the periodic time and the mass on an oscillating spring

Hypothesis: *The periodic time of an oscillating spring is directly proportional to the mass on the spring.*

Apparatus required

- A retort stand
- Clamp and boss
- A spiral spring fitted with a pointer
- Six, 100 g masses
- A stopwatch

Procedure

1. Suspend a load of 100 g mass at the end of the spring as shown in Fig. 2.8.

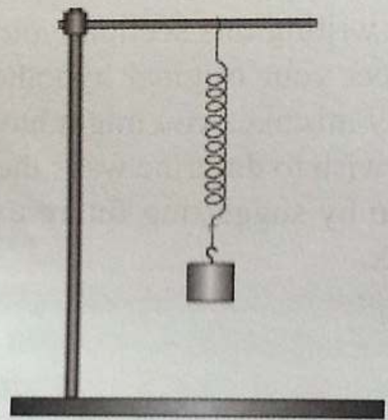


Fig. 2.8: Suspended load

2. Now give the mass a small vertical displacement and release so that systems perform vertical oscillation. Time 20 oscillations and record as Trial 1 in Table 2.3. repeat the timing for the same mass and record the time as Trial 2. Determine the mean periodic time T.

3. Enter your results in Table 2.3. Repeat the experiment for the other values of masses and complete the table.

Table 2.3

Mass, m (kg)	Time, t, for 20 oscillations (s)			T(s)	T^2 (s^2)
	Trial 1	Trial 2	*Mean*		
0.200				–	
0.300					
0.400					
0.500					
0.600					

4. On the grid provided, plot a graph of T^2 (y-axis) against m.

5. Use the graph to test the hypothises and draw a conclusion.

Recording of results

The result obtained was recorded as shown in Table 2.4.

Table 2.4

Mass, m (kg)	Time, t, for 20 oscillation (s)			Periodic time, $T = \frac{t}{20}$ (s) $= \frac{t}{20}$ (s)
	Trial 1	Trial 2	Mean	
0.100	20.8	21.2	21.0	1.05
0.200	29.8	29.4	29.6	1.48
0.300	36.7	36.9	36.8	1.84
0.400	42.5	43.1	42.8	2.14
0.500	47.8	48.6	48.2	2.41
0.600	52.5	53.5	53.0	2.85

Data analysis

To see the relationship between the mass and periodic time (T), we need to draw a graph. In this experiment, when mass is varied, different values of periodic times are obtained and recorded down in Table 2.4.

Thus, mass is the independent variable while T is the depended variable. We need to draw a graph of T (s) against mass, m (kg).

Choosing of scale:

T (s) axis; the values range from (0 s to 3.80 s)

One suitable scale is *1 cm represents 0.25 s*

Mass, m (kg) axis; the values range from (0 kg to 0.70 kg)

One suitable scale is *1 cm represents 0.05 kg*

Plotting the values of *T (s)* against mass, *m* (kg), we see that the points are not in a straight line i.e. they form a curve.

Joining the plotted points using a smooth curve, we get the curve as shown in Fig. 2.9.

Fig. 2.9: A graph of periodic time against mass

Discussion of results

Looking at the graph in Fig. 2.9, the curve suggests that T is not directly proportional to mass. This disapproves the hypothesis.

Further analysis

Though our hypothesis has been disapproved by the results of the experiment, as scientists we should be curious enough to go further and try establishing other relationships between our variables using the data we obtained. Some of the greatest laws and discoveries we apply today are as result of further and persistent investigations by some scientists.

Let us first determine the values of T^2 and make a new table as shown in Table 2.5.

Table 2.5

Mass, *m* (kg)	0.10	0.20	0.30	0.40	0.50	0.60	0.70
$T^2(s^2)$	1.10	2.20	3.40	4.60	5.80	7.00	8.00

Choosing the scale:

m (kg) axis the values range from (0 kg to 0.70)

One suitable scale is *1 cm represent 0.070*

T^2 *(s) axis*; the values range from (0 kg to 8.00)

One suitable scale is *1 cm representts 0.50* s^2

Plotting a graph of T^2s^2 against *m* (kg), we get a straight line as shown in Fig.2.10.

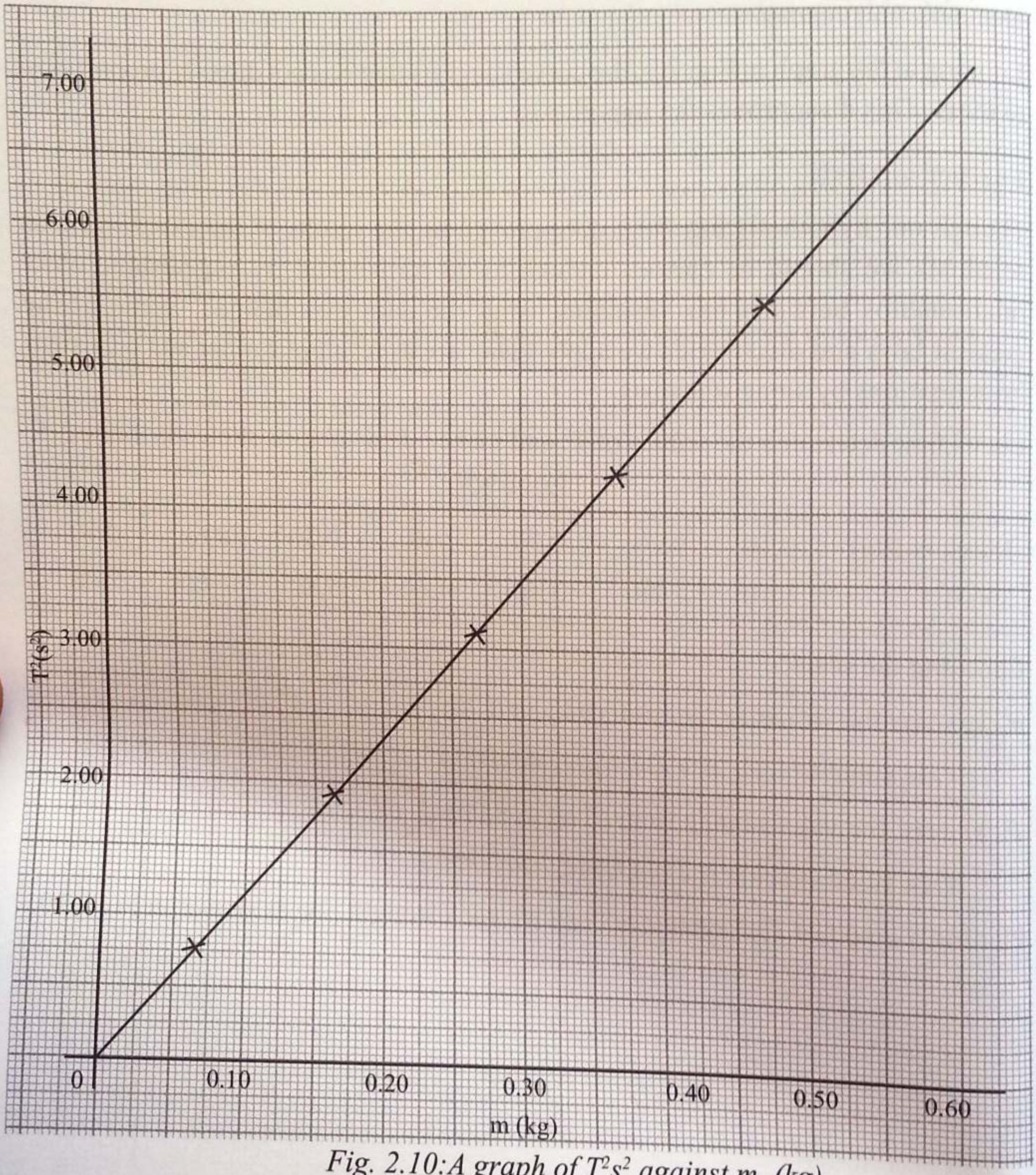


Fig. 2.10:A graph of T^2s^2 against m (kg)
m (kg)

Discussion of result

The graph of $T^2(s^2)$ against *m* is a straight line as shown in Fig. 2.10

Possible sources of errors are shown in Table 2.6

Table 2.6

Sources of error	*Reducing errors*
Error when reading taking time on the stopwatch	Taking and averaging repeated time readings
Reading errors	Taking readings carefully
Zero errors in the stopwatch	Zeroing the stopwatch

Conclusion

From the results of this experiment, we conclude that the square of periodic time of an oscillating spring is directly proportional to the mass of the spring.

Unit Summary

- Scientific investigation design is a set of analytical techniques and perspectives for performing research.
- An independent variable is the quantity that is varied by the researcher in an experiment in order to produce a corresponding change in another quantity called the dependent variable. The controlled variables are kept constant in an experiment so that the results are valid.
- A dependent variable is the quantity that changes as a result of the change in the variable that is deliberately varied by the researcher.
- A controlled variable is the quantity that is kept constant so that it does not affect the dependent variable in an experiment.
- Absolute error is the magnitude of difference between the actual and the individual measurement of any quantity in question.
- Relative error is the ratio of measurement (absolute) error to a true value and is normally expressed as a percentage.
- Systematic error is the error that is constant in a series of repetitions of the same experience or observation.

Unit Test 2

1. What is a hypothesis?
2. Differentiate between dependent and independent variables.
3. Explain each of the following type of errors briefly:

   (a) Parallax errors (b) Zero error (c) Systematic errors

4. A carpenter measured the length of a small piece of timber as 24.6 cm. Calculate the relative error in the measurement if the true length is 24.5 cm.
5. In groups of 5, conduct scientific investigations on the following and write a report for each case:

   (a) To determine the relationship between the forces exerted on a spring and the extension (increase in length) it produces on the spring.

   (b) To determine the relationship between the square of the periodic time of a simple pendulum and the length of the pendulum.

6. Propose two constraints that one may face when carrying a scientific investigation on the effects of global warming to climate pattern.
7. An experiment was performed to determine the length of a pendulum. The results in Table 2.6 were obtained

Table 2.7

x (cm)	15.0	20.0	25.0	35.0	35.0	40.0
y (cm)	2.9	4.7	7.1	10.0	14.0	18.7

   (a) Complete the table by including a column of values of x^2

   (b) Plot a graph of x^2 against y.

   (c) Find the slope, S, of the graph.

   (d) Calculate the length, l, of the pendulum given that $l = \frac{S}{2}$.

   (e) Find the uncertainity in the slope.

8. Name two methods of communicating the result of a scientific investigation and explain their advantage.

Properties of Matter

Outcome

The students will be able to relate the behaviour of matter from microscopic level to the macroscopic level when subjected to different environments and creatively apply these properties to bring about technological developments and at the same time examine the ethical and moral implications of using and applying science.

UNIT

3 Kinetic Theory of Matter

Success Criteria

By the end of this unit, you must be able to:

- Describe the kinetic theory of solids, liquids and gases.
- Explain the cause of gas pressure.
- Explain the relationship between average molecular speed and temperature.
- Explain the meaning of the term absolute temperature.

Introduction

We have already learnt that matter is anything that has mass and occupies space. It exists in three states namely: solid, liquid and gas.

Evidence of movement of the particles of matter is observed in real life settings in processes such as dissolving of substances like salt and sugar, spreading of smell e.g. perfume through diffusion, inflating tyres/balls, bursting of gas cylinders in case of fire etc.

In this unit, we will focus on the movement of these tiny particles. Their movement is summed up in a model known as the *kinetic theory of matter*.

3.1 The kinetic theory of matter

The word 'kinetic' is derived from the Greek word 'Kineo' which means "I move". Particles in substances are in constant motion; they possess kinetic energy, which is the energy due to movement. Therefore, kinetic theory of matter states that *matter is made up of tiny discrete individual particles that are continuously in random motion*.

The theory explains how the particles are packed in solids, liquids or gases; the attractive forces between them and the effect of temperature on them. The arrangement of particles of matter and the way they move determines the state of a substance i.e. whether to be in solid, liquid or gaseous state.

Kinetic theory of solids

Solids have strong cohesive forces of attraction between its particles or molecules. Particles are closely packed in fixed positions by the attractive forces (Fig. 3.1). This make solids to have the highest density compared to liquids and gases and also to have definite shapes.

A large force is needed to change the size and the shape of a solid. Also, for a solid to melt into a liquid, it requires a lot of heat energy since the cohesive forces between particles are strong.

The particles or molecules in solids are continuously vibrating in a fixed or mean position (See Fig. 3.1).

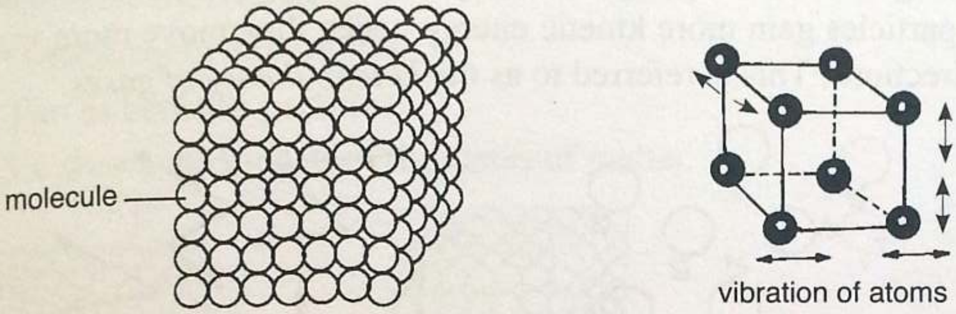


Fig.3.1: Arrangement of particles in solids

When a solid is heated, the heat energy absorbed by the particles increases their kinetic energy. This makes the particles to vibrate more vigorously but in their fixed position. Increase in heat energy increases the kinetic energy in the particles and reduces or weakens the cohesive forces between molecules. At a certain point, the intermolecular forces are so weak that they allow matter to flow. This point is referred to as the melting point. The process of a solid changing to liquid is called melting.

Kinetic theory of liquids

- Liquids have moderate intermolecular forces of attraction between particles. As such their particles are loosely packed. This explains why liquids have lower density compared to solids.
- The particles of a liquid are free to move randomly i.e., flow while sticking together (Fig. 3.2). Thus, liquids do not have definite shape but take the shape of the container.

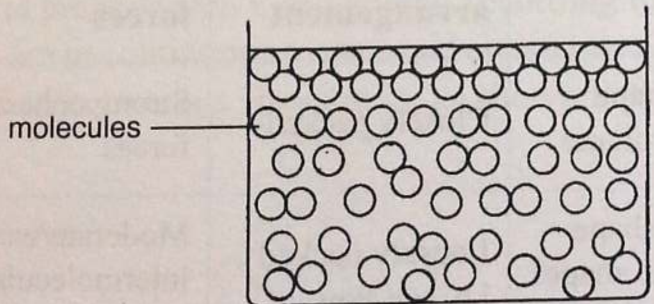


Fig. 3.2: Arrangement of particles in liquids

Particles or molecules of liquids move freely and randomly. When the temperature is raised, molecules acquire more kinetic energy and hence move faster. This increased kinetic energy of liquid molecules weakens the intermolecular forces between its particles. A further increase in kinetic energy makes the molecules to escape through the surface of a liquid i.e. change into steam or gaseous state. The process of liquid changing of gas is known as evaporation.

Kinetic theory of gases

In gases, the intermolecular force are so weak to be considered. Weak intermolecular forces only exist upon collision. As such, a gas has no definite shape but fills up the container.

Molecules of gases move rapidly and freely around (Fig. 3.3). A rise in temperature makes the particles gain more kinetic energy hence they move more vigorously in different directions. This is referred to as the *kinetic theory of gases*.

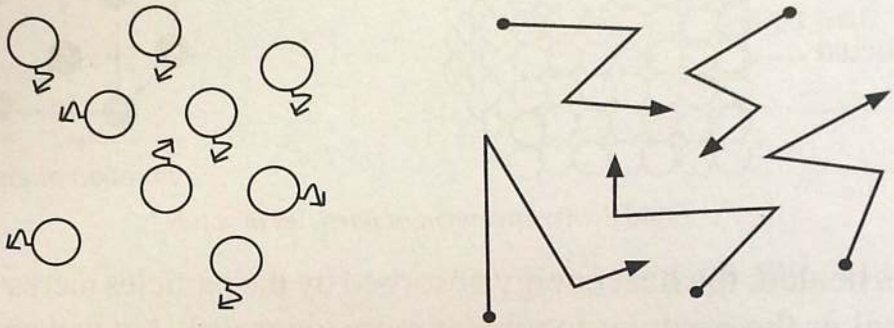


Fig. 3.3: Arrangement and movement of particles in gases

This vigorous movement of gases when directed to one point gives produces large forces. This is what makes it possible for steam to rotate turbines in the geothermal electric plants.

Table 3.1 shows the property, particle arrangement, strength of forces and particles movement of the three states of matter.

Table 3.1

State	Property	Particles arrangement	Strength of forces	Particles movement
Solid	Definite shape Size and volume	Closely packed	Strong cohesive forces	Vibrating at a fixed position
Liquid	Indefinite shape i.e. take the shapes of the container.	Loosely packed i.e. can flow	Moderate/weak intermolecular forces	Move freely and easily
Gases	Indefinite shape Fills up the whole space	Well spaced out	Very weak intermolecular forces only during collisions	Move randomly and at higher speed.

Exercise 3.1

1. In your own words, describe the kinetic theory of matter.
2. Give the difference between solids and gases under the following sub-headings
   (a) Particles movement.
   (b) Arrangement of particles.
   (c) Forces between particles.
3. Fig. 3.4 shows the models of the states of matter.

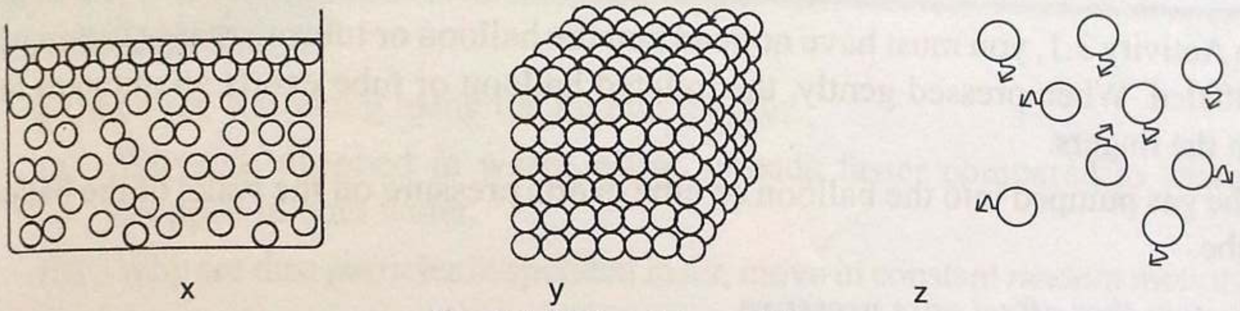


Fig. 3.4: Matter states model

   (a) Identify from the models:
   (i) State x (ii) State y (iii) State z
   (b) All the states in 3(a) above were subjected to heat. Explain in which state will particles move faster.
4. State two differences between solids and liquids using the kinetic theory.
5. Explain the effect of temperature on solid, liquid and gas.

3.2 The causes of gas pressure

It is comfortable to ride a bicycle with inflated tyres than on flat tyres. What exactly happens when a gas is pumped into a container? According to the kinetic theory of gases, gas molecules are in continuous motion and constantly collide with each other and with the walls of any container they are in, as shown in the Fig. 3.5.

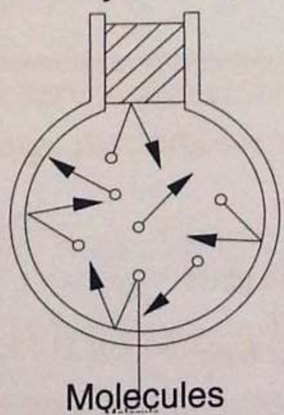


Fig. 3.5: Gas pressure

The container experiences an outward force or push each time a molecule strikes it and bounces off. Many such molecules hit the container per second producing a steady outward force on a given area. Hence, gas molecules exert pressure on the walls of the container which is called *gas pressure*. Activity 3.1 will help us understand how gas molecules moving randomly cause gas pressure.

Activity 3.1

- Blow into the balloon or bicycle tube using your mouth or bicycle pump respectively. What do you notice about the shape and size of the balloon/tube?
- Press the balloon/tube gently with your fingers. How do you feel?

In Activity 3.1, you must have noticed that the balloon or tube increases in size when inflated. When pressed gently, the inflated balloon or tube exerts an outward force on the fingers.

The gas pumped into the balloon or tube exerts pressure on the walls of the balloon/tube.

Factors that affect gass pressure

There are many factors that affect gas pressure, these include:

- Number of particles (molecules) in a container.
- Temperature of the gas.

Activity 3.2 will help us investigate how temperature affects gas pressure.

Activity 3.2: To investigate how temperature affects gas pressure

1. Fit the balloon tightly into the mouth of the a thin walled flask using the rubber band Fig. 3.6(a).
2. Place the flask on a tripod stand and heat slightly (Fig. 3.6(b). Observe what happens.

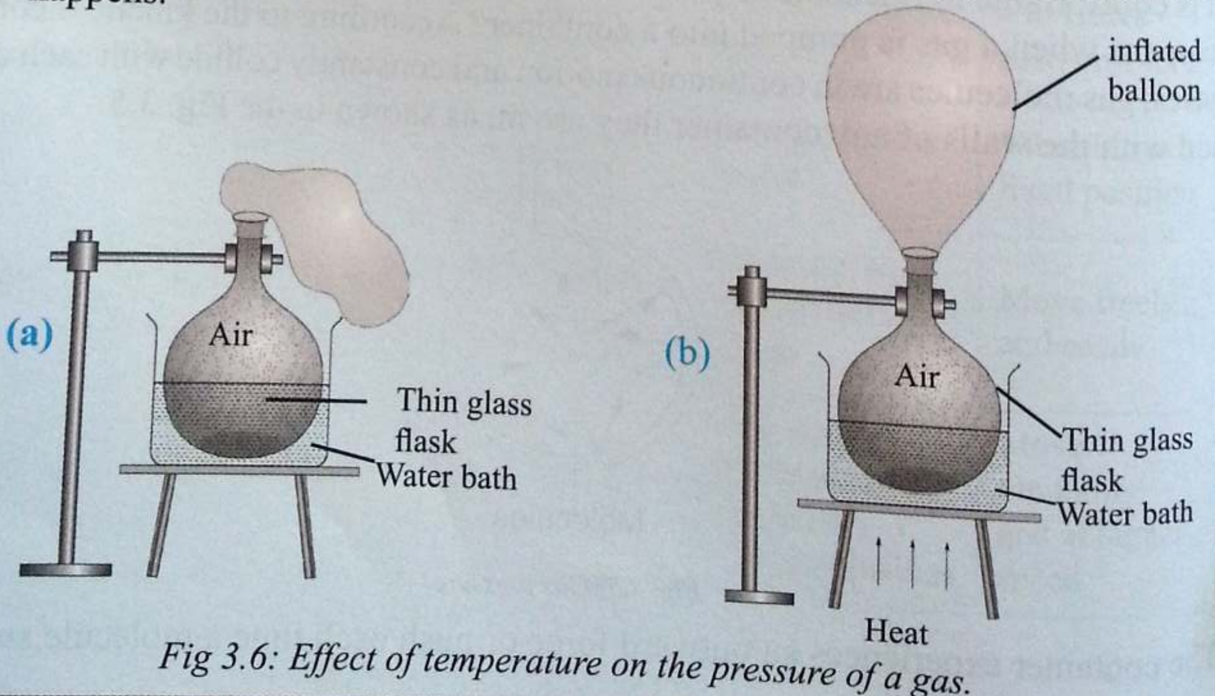


Fig 3.6: Effect of temperature on the pressure of a gas.

3. What conclusion do you draw from this observation?

In Activity 3.2, you must have observed that the balloon becomes inflated. According to kinetic theory of matter, when the temperature of a gas increases, the molecules move faster. As a result, they strike the wall of the container with greater force and more frequently. Hence, the pressure on the walls of the container increases. This makes the volume of the balloon to increase. Therefore, when the temperature of a gas increases, the pressure also increases.

Exercise 3.2

1. Explain the following using the kinetic theory:
   (a) Why ink dropped in warm water spreads faster compared to when dropped in cold water.
   (b) Why are dust particles suspended in air, move in constant random motion.
2. Explain two factors that affect gas pressure.
3. Explain the effect of temperature on gas pressure.

3.3 Effect of temperature on the molecular speed

As we learnt earlier, molecules or particles of fluids or solids are always moving randomly or vibrate about a mean position. We also know that at higher temperature, the particles or molecules will move faster. Activities 3.3 and 3.4 will help us to establish the effects of temperature on the average molecular speed of a gas.

Activity 3.3

- Pour water in a beaker. Place the beaker on to the tripod stand and heat it for sometimes.
- Observe the movement of bubbles with time. What do you notice about the speed of the bubbles with time?

In Activity 3.3, you must have observed that at the start, the bubbles were moving but slowly. On continuing heating, the bubbles started moving faster and more vigorously.

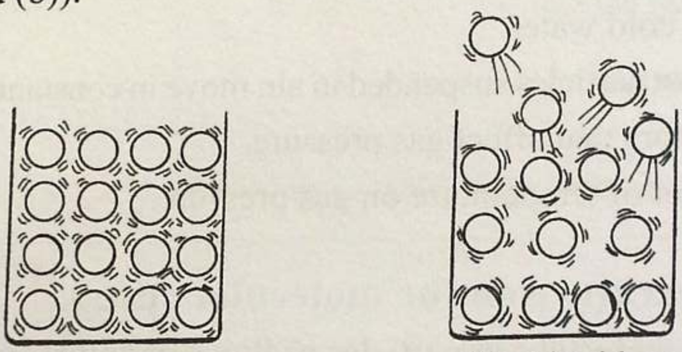
Activity 3.4

- Take the candle wax and heat it for sometimes.
- What happens to the candle wax as you heat it?

In Activity 3.4, you must have observed that after heating for some time, the candle wax melts and starts to flow.

From the two activities, we conclude that increase in temperature increases the motion of particles or molecules in matter. From the kinetic theory of matter, particles or molecules are in constant random motion. If say a solid, liquid and gas is subjected to heat energy, the particles or molecules acquire more kinetic energy. Increased kinetic energy overcomes the attraction forces holding the molecules or particles together. This makes the internal energy to raise. The bonding between molecules in solids is weakened hence they change into a liquid and start to flow. A further increase in temperature causes the attraction forces to weaken further. Those particles with higher kinetic energy start to move to the surface and escape to the surrounding. This is how evaporation occurs.

On cooling, molecules or particles loose kinetic energy which in turn reduces their internal energy. This will consequently reduce the average molecular speed (Fig. 3.7 (a) and (b)).



(a) Liquid particles moving randomly but sticking together.

(b) Liquid particles overcoming forces of attraction and escaping into the air.

Fig 3.7: Behaviour of liquid particles.

The above changes of state can be summarised in Fig. 4.8.

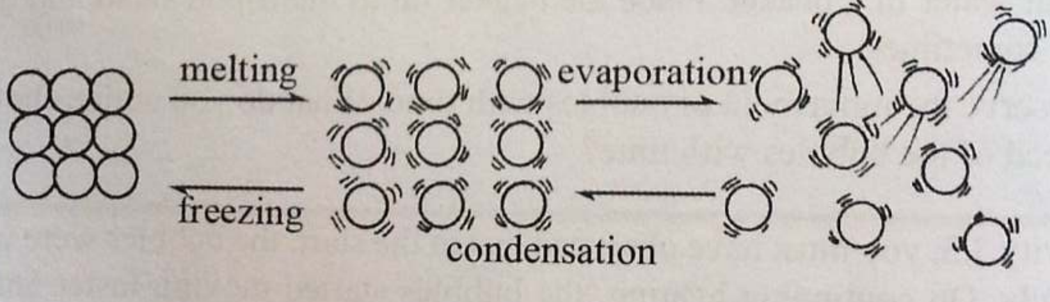


Fig 3.8: Effect of temperature on solids and liquids.

When a liquid gets hot enough, most of the particles acquire enough energy and speed to overcome the forces of attraction between them and escape into the air. This process is known as boiling and takes place at a fixed temperature. During boiling, big bubbles of gas form inside the liquid as the particles break away from each other.

From this discussion, we can summarise the effect of temperature on the molecular motion of particles as follows:

- The speed of molecules or atoms increases as the temperature increases.
- The spaces between particles also increases with increase in temperature.

Absolute zero temperature

As we have already learnt, kinetic energy enhances the speed of the molecules of a gas. The speed increases as temperature increases making the molecules to move about more vigorously and causing the gas to expand.

As the temperature is lowered, the speed of vibration and hence the movement of molecules keeps on reducing making the gas to start contracting. At some very low temperature, we might expect the molecules to stop moving altogether (zero energy) and the gas sample to have zero volume. Such a temperature at which the molecules of a gas would have kinetic energy and zero volume is know as *absolute zero temperature*. However this is not the case. Why?

This is because all real gases condense to a liquid or solid (theoretically) at temperatures higher than absolute zero. Hence, absolute zero is an ideal (theoretical) value. It is estimated at –273°C which correspond to 0K. (Kelvin Scale).

Exercise 3.3

1. Differentiate between:
   (a) Melting and freezing.
   (b) Evaporation and boiling.
2. Define the term 'absolute' temperature.
3. What is the effect of temperature on the molecular motion of particles?
4. Explain why air bubbles in boiling water move faster than in warm one.
5. If a solid is exposed to heat energy what happens to its internal energy.
6. Give the relationship between temperature and average molecular speed.

Unit summary

- Kinetic theory states that matter is made up of tiny discrete individual particles that are in a continuous state of random motion.

Table 3.2: Kinetic theory of solids, liquids and gases.

Description	Solid	Liquid	Gas
Intermolecular spacing	Very closely packed	Closely packed	Far apart
Movement	Vibrates in fixed positions	Particles are free to move throughout the liquid body	Move at high speed freely and randomly
Intermolecular forces	Strong	Moderate	Weak

- Gas pressure increases with temperature. At high temperature the gas molecules move faster and knock on the container walls with greater force.
- Absolute temperature is the lowest possible temperature that can be reached by a gas.

Unit Test 3

1. Matter is anything that occupy space and has ________.
   - A. Volume
   - B. Weight
   - C. Density
   - D. Mass
2. Which state of matter has the weakest intermolecular forces?
   - A. Solid
   - B. Liquid
   - C. Gas
   - D. Plasma
3. What happens to the kinetic energy of the particle when the temperature of a body is reduced?
   - A. Reduces
   - B. Increases
   - C. Remains constant
   - D. Increases then decreases
4. When changing a solid to liquid then gas, which physical quantity does not change?
   - A. Volume
   - B. Mass
   - C. Density
   - D. Weight

5. Which property makes a gas to be suitable for use in inflating a football?

   A. Takes the shape of container

   B. Can flow

   C. Intermolecular forces are weak

   D. Move random

6. State whether each of the following describes a solid, a liquid or a gas.

   (a) Particles vibrate but can change positions.

   (b) Particles vibrate but cannot change positions.

   (c) No fixed shape or volume.

   (d) Fixed volume but no fixed shape.

   (e) Almost no attraction between particles.

7. How does the motion of particles in a gas change when its temperature is raised.

8. Use the kinetic theory to describe the difference between:

   (a) Solids and liquids

   (b) Gases and liquids

9. Explain the following in terms of molecules:

   (a) Why the pressure of a gas in a sealed gas cylinder increases when it is heated.

   (b) Why diffusion in liquids is slower than in gases.

10. Explain each of the following statements using the kinetic theory of matter.

    (a) One can smell cooked fish aroma while outside the house.

    (b) Salt dissolves faster in hot water than cold water.

    (c) The temperature of methylated spirit decreases when air is blown over it.

UNIT

4 Thermometry

Success Criteria

By the end of this unit, you must be able to:

- Differentiate types of temperature scales.
- Describe how various thermometers function.

Introduction

In Form 2, we learnt that matter expands when heated and contracts when cooled. In this unit, we will learn about instruments that are used to measure the temperature of a body when it is heated or cooled.

4.1 Temperature scales

A temperature scale is a range of values for measuring the degree of hotness or coldness of a body referred to as temperature. Temperature is commonly expressed in degrees *celsius* (also called *degrees centigrade*) using the celsius scale. However, the SI unit for temperature is the *kelvin(K)* which is measured using the *kelvin scale*. This is the unit that is used in scientific work. Let us discuss each of these scales in details.

The celsius scale

This scale uses the *degree celsius* (°C) as the unit of measuring temperature. Two values in this scale are fixed such that the temperature at which pure ice melts is 0°C and boiling point of pure water is 100°C (under standard atmospheric pressure of 101 325 Pa).

These two fixed points are called the *lower* and *upper fixed points* of the celsius scale respectively. The region between these two points on the scale is graduated into 100 equally spaced temperature marks. Temperatures below 0°C have negative (–) values.

The kelvin scale

This scale uses *kelvin (K)* as the unit of measuring temperature. It uses the absolute zero (-273°C) as its reference point. Thus, 0 K on kelvin scale is equivalent to –273°C on the celsius scale. It is worth noting that a temperature change of 1 K is equal in size to a change of 1°C.

Relationship between Celsius and Kelvin scale

To convert temperature from degrees celsius (°C) to kelvin temperature (K) , we add 273 to degrees celsius temperature i.e.

Temperature in K = temperature in °C + 273

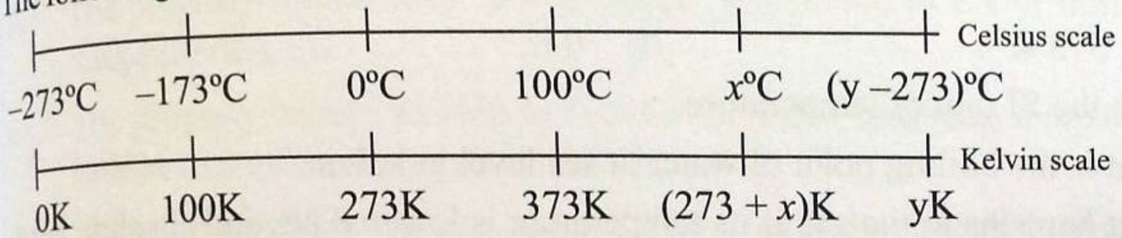
$$T = (\theta + 273)K$$

To convert kelvin (K) temperature to degrees celsius (°C) temperature, we subtract 273 from kelvin temperature i.e.

Temperature in °C = (Temperature in K – 273)

$$\theta = (T - 273)°C$$

The following is a summary of the relationship between Kelvin scale and Celsius scales.



Where; x is any value of temperature in degrees celcius and y is any value of temperature in kelvin.

Example 4.1

What is the lower fixed point (L.F.P) in kelvin?

Solution

Lower fixed point is 0°C. To convert °C to kelvin, add 273.

Therefore, L.F.P = (0°C + 273)K = 273 K.

Example 4.2

Express the room temperature of 27°C in kelvin.

Solution

To convert °C to kelvin, add 273.

Therefore, room temperature is (27 + 273)K = 300 K.

Example 4.3

Convert 327 K to degrees celsius.

Solution

To convert kelvin to degrees celsius, subtract 273.

Therefore, 327 K = (327 – 273) °C = 54°C.

Exercise 4.1

1. Convert each of the following into kelvin scale.
   (a) 34 °C (b) -371 °C
   (c) 17 °C
2. Convert each of the following into degrees Celsius scale.
   (a) 314 K (b) -6 K
   (c) 273 K (d) 45 K
   (e) 573 K (f) 0 K
3. State the SI unit of temperature.
4. What is the boiling point of water at sea level in kelvin.
5. What happens to the gas if its temperature is lowered beyond absolute zero temperature.
6. State one difference between kelvin and Celsius scale.

4.2 Thermometers

A *thermometer* is an instrument for measuring temperature. In the construction of a thermometer, a *thermometric substance* is chosen first. Then, a *temperature scale* is defined by means of two *fixed points; lower forced point* and *upper forced point.*

There are various types of thermometers in use. The liquid-in-glass thermometer is the most common one. The others include electrical resistance thermometers, digital thermometers, constant volume gas thermometers and the thermocouple thermometers. The main difference between them is in the property of the thermometric substance which they use.

Thermometric substances

Thermometric substances are substances that are used in the thermometers as thermometric solids, liquids or gases. Their properties change uniformly with temperature.

Thermometric properties

As mentioned earlier, different thermometers use thermometric substances with different properties. For example, in mercury-in-glass thermometer, the thermometric substance used is mercury and the property is the uniform expansion and contraction of mercury on heating and cooling respectively. Platinum resistant thermometers use the property of uniform increase in resistance of platinum. Gas thermometers use the property of uniform change in the volume of a gas. Some important characteristics of the temperature - measuring property of a thermometric substances are:

1. The property should remain constant, if temperature is constant.
2. The property should change uniformly with change in temperature.
3. The property should change uniformly for every 1°C change in temperature.
4. The property should acquire thermal equilibrium as quickly as possible, when temperature measurements are needed.
5. The property should cover a wide range (should not freeze or boil at normal temperatures).
6. The property should be able to register the rapid changing temperature (e.g. sudden explosion due to chemical reactions).
7. The property should have a large change even if the change in temperature is small.
8. The property should be such that the temperature can be taken easily without waiting for a long time.

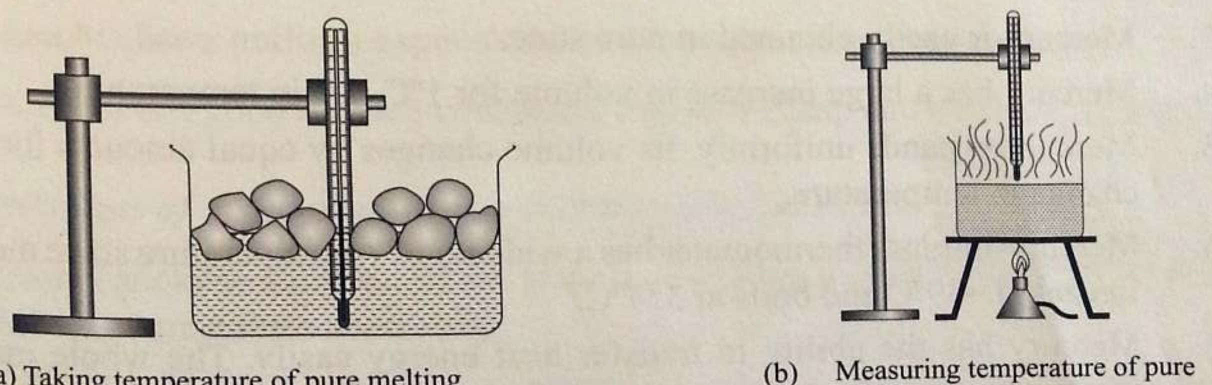
Liquid-in-glass thermometers

A liquid-in-glass thermometer uses either mercury or coloured alcohol as the thermometric substance.

Working of liquid-in-glass thermometer

The end of the glass tube containing the liquid is dipped into or placed on the substance whose temperature is to be determined. The liquid in the tube expands when the substance transfer heat to it. This gives a higher temperature reading. When the substance is cold, the liquid contracts and the thermometer shows a lower temperature reading.

Fig. 4.1 (a) and (b) shows how to measure the temperatures of pure ice and boiling water respectively.



(a) Taking temperature of pure melting ice at 0°C

(b) Measuring temperature of pure boiling water

Fig. 4.1: Measure of ice and boiling water.

Let us discuss in detail the two types of liquid-in-glass thermometer:

Mercury-in-glass thermometer

This thermometer consists of *a thin walled bulb*, containing mercury and a thin *capillary tube (bore)* of uniform cross-sectional area. There is a space above the mercury thread which is usually evacuated to avoid excess of pressure being developed when mercury expands (Fig 4.2).

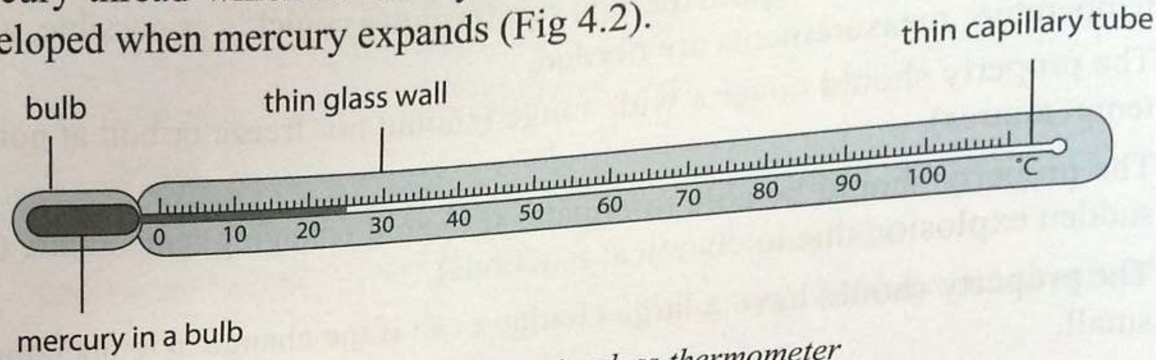


Fig. 4.2: Mercury-in-glass thermometer

Some important precautions are taken in the construction of this type of thermometer. They include:

1. The walls of the bulb should be thin. This is to ensure that the mercury can be heated easily.
2. The quantity of mercury in the bulb should be small so that the mercury takes little time to warm up.
3. The thin capillary tube should be of uniform cross-section so that the mercury level changes uniformly along its length.

Advantages of using mercury as a thermometric substance

1. Mercury is a shiny opaque liquid. The position of the mercury meniscus is seen easily and readings taken without strain.
2. Mercury does not wet glass. Hence it does not stick to the sides of the capillary tube.
3. Mercury is easily obtained in pure state.
4. Mercury has a large increase in volume for 1°C rise in temperature.
5. Mercury expands uniformly. Its volume changes by equal amounts for equal change in temperature.
6. Mercury-in-glass thermometer has a wide range of temperature since mercury freezes at –39°C and boils at 357°C.
7. Mercury has the ability to transfer heat energy easily. The whole mass of mercury in the bulb attains the temperature of the substance in which the bulb is placed easily.

Disadvantages of using mercury as a thermometric substance

1. Usually, it is only the bulb which is in contact with the body when taking the temperature. A large portion of the stem is not in contact with the body.
2. There is a change in internal pressure due to the different positions of the thermometer. The reading of the mercury level is low when the tube is vertical as compared to the reading in the horizontal position.
3. Mercury takes some time to contract to the original volume. The same thermometer cannot be used to measure a low temperature soon after a high temperature.
4. There may be non-uniformity in the capillary bore of the tube.
5. This thermometer is not suitable to measure temperatures below –39°C.

Alcohol-in-glass thermometer

The alcohol-in-glass thermometer uses coloured alcohol instead of mercury. Volume of alcohol changes uniformly and easily when heated. The change in volume of alcohol is about six times more than that of mercury for the same change in temperature.

The range of temperatures that can be measured with this thermometer is limited, as alcohol boils at 78°C. However this thermometer is ideal for measuring low temperatures since alcohol freezes at –112°C .

Advantages of using alcohol as a thermometric substance

1. Alcohol has a very low freezing point of –112°C hence its suitable in thermometers to record very low temperatures.
2. Alcohol can be coloured brightly (by adding a dye, generally red dye). This makes it clearly visible through glass.
3. Alcohol has a uniform expansion and contraction than even mercury.
4. Alcohol is a good thermal conductor, it is also cheap and easily available.

Disadvantages of using alcohol as a thermometric substance

1. Alcohol sticks to the walls of the glass thus wetting it. This makes it difficult to read the temperature accurately.
2. Alcohol has a low boiling point of 78°C, therefore it cannot be used to measure high temperature.

Clinical thermometer

A clinical thermometer is an instrument designed for measuring the human body temperature. It consists of a thin walled bulb containing mercury. The capillary bore is very narrow and of uniform diameter.

This thermometer has a narrow *constriction* in the tube just above the bulb. The thermometer has a limited range from about 35°C to about 43°C (Fig. 4.3).

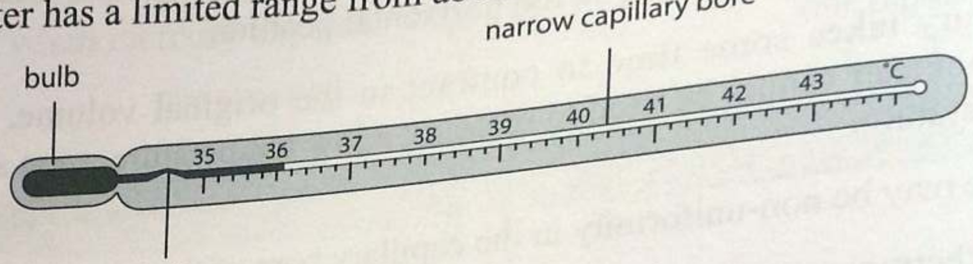


Fig. 4.3: Clinical thermometer

Working of the clinical thermometer

When the thermometer is in contact with a human body, the heat fom the body reaches the mercury in the bulb hence it expands. It forces its way through the constriction to the narrow bore. When the thermometer is removed from the body, the mercury in the bulb cools down and contracts. The mercury thread is broken at the constriction. Hence the mercury in the tube stays back.

The reading of the thermometer on the stem can be taken without any hurry since the constriction holds the mercury thread past it. After use, the mercury in the tube can be forced through the constriction back to the bulb by flicking the thermometer vigorously.

The normal human body temperature is 36.9°C.

Six's maximum and minimum thermometer

This thermometer is used to measure both the maximum and minimum temperature of a place during a day. It was invented by a physicist called *John Six*. The thermometer consists of a U-shaped tube connected to two bulbs. The U-tube contains mercury. The two bulbs contain alcohol, which occupies the full volume of one of the bulbs. The other bulb has a space above alcohol. There are two indices fitted with light fine springs (See Fig. 4.4).

Working of the Six's thermometer

When temperature rises, alcohol occupying the full volume of bulb A expands and forces mercury in the U-tube to rise on the right hand side. Mercury, in turn, pushes the index I_2 upwards. The maximum temperature can be noted from the lower end of index I_2(See Fig. 4.4).

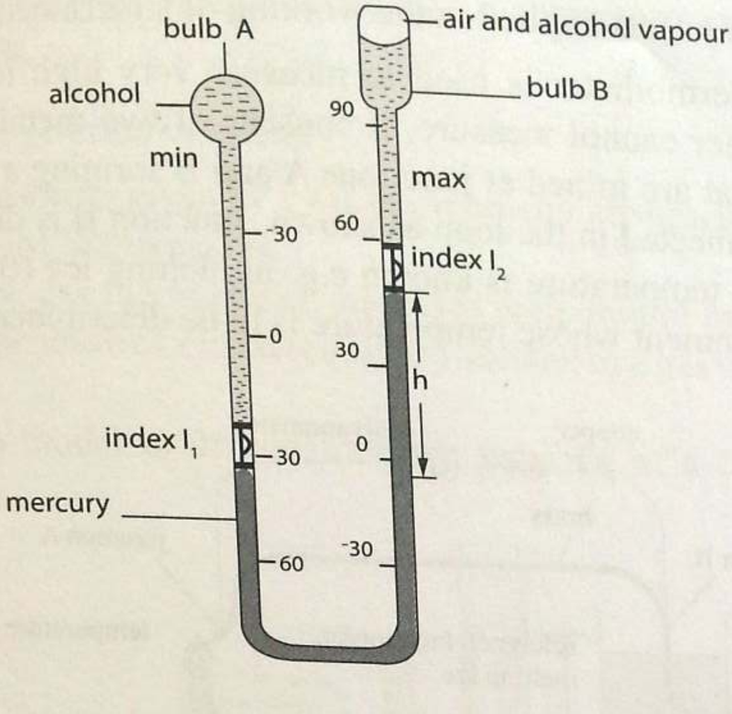


Fig. 4.4: Six's maximum and minimum thermometer

When the temperature falls, alcohol in bulb A contracts. Due to the pressure difference in the two arms of the U-tube, mercury level will rise on the left hand side of the U-tube pushing the index I_1 upwards. The index I_2 on the right hand side is left behind (held by the fine spring) to register the maximum temperature. The lower end of index I_1, touching the mercury meniscus gives the minimum temperature.

The two steel indices can be reset with the help of a magnet.

Thermocouple thermometer

Thermocouple thermometers work using the thermoelectric effect that was discovered by a German physicist called *Thomas Seebeck* (1170-1831). Thomas investigated the relationship between heat flow and electric current and discovered that when the two ends of a metal piece are at different temperatures, electric current flows from the hot end to the cold end. The amount of current that flows depends on the temperature difference between the two ends. He also noted that different amounts of electric current flow through different types of metals of the same dimensions (thickness and length) even when one end of each of the pieces is heated to the same temperature.

As such, when two different types of wires are joined together at the ends forming a loop, and then one of the joined ends (junction) heated to a higher temperature, electric current flows through the loop in a complete circuit.This current can be measured by connecting a galvanometer/millimeter at some point in the loop (See Fig. 4.5). The amount of current that flows in the circuit is a measure of the temperature difference between the two joined ends of the loop. The value of the current obtained can be used to determine the temperature at one end if the temperature at the other end is known. Those are the facts applied in the working of a thermocouple thermometer.

A thermocouple thermometer is used to measure very high temperature which ordinary thermometer cannot measure. It consists of two metallic conductors e.g copper and brass that are joined at junctions A and B forming a loop (Fig. 4.5). A galvanometer is connected in the loop as shown. Junction B is dipped/placed in an environment whose temperature is known e.g. in melting ice (0°C). Junction A is placed in the environment whose temperature is to be determined e.g. in a burning flame, furnace etc.

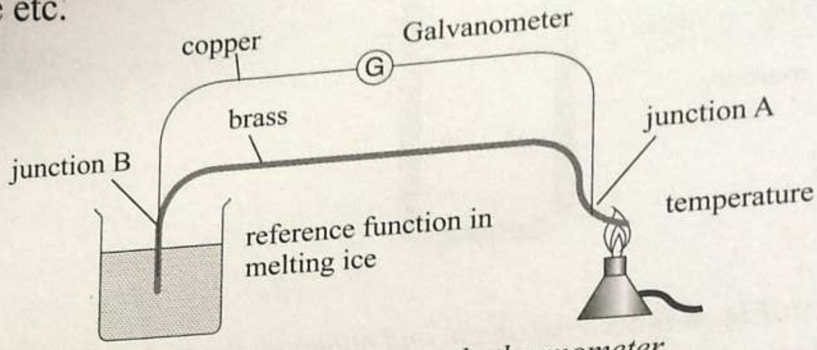


Fig. 4.5: thermocouple thermometer

Calibrating a thermocouple thermometer

By maintaining junction B at an environment of known temperature e.g. melting ice (0°C), junction A is dipped on various environments of known temperatures e.g. boiling point of pure water (100°C), boiling point of mercury (356.7°C) etc., and the corresponding currents flowing in the loop is recorded. Using the corresponding temperature differences and current values obtained, a temperature scale is developed and calibrated accordingly for use in determining unknown temperature values in other environments.

Using a thermocouple thermometer to measure temperature

Junction B is dipped/placed in an environment of known temperature that was used in developing the temperature scale. Junction A is placed in the environment whose temperatureis to be determined. The current that flows in the loop is recorded and used to determine the temperature in the environment at junction A using the temperature scale of the thermocouple.

Uses of thermocouple

- Thermocouple is used in industries e.g. in kilns, gas turbine exhaust to determine if the required temperature is reached.
- Thermocouples can be used in homes, offices and business places as the temperature sensors in thermostats.
- Thermocouples are used as flame sensors in safety devices.
- A thermoelectric thermometer is also used in the same way as a thermocouple

and can measure very high temperatures e.g. temperatures of metals with high melting points.

Constant -volume gas thermometer

The temperature readings given by a gas thermometer are nearly independent of the substance used in the thermometer.

A constant-volume gas thermometer measures temperature by making use of the change in the temperature of a fixed (constant) volume of a gas when the temperature changes.

Fig. 4.6 shows a model to demonstrate the working of a constant-volume gas thermometer.

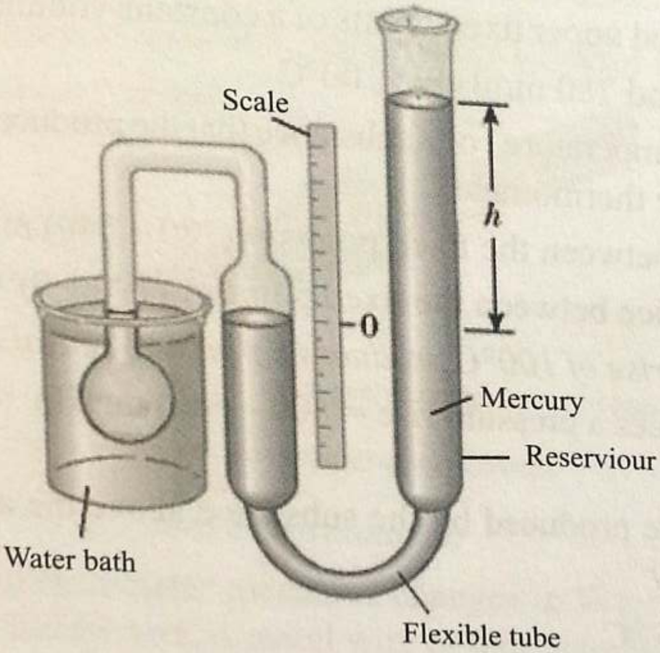


Fig. 4.6: A constant-volume gas thermometer model.

It is composed of bulb that is filled with a fixed amount of a gas. The bulb is connected to a mercury manometer that measures the gas pressure. The smaller column of the manometer is partially filled with mercury and is connected to the longer column called the reservoir) by a flexible tube.

Working of a constant-volume gas thermometer

Before using the thermometer, the height of the mercury in the smaller column is set to a reference point O on a fixed millimeter scale like a ruler. It must be maintained at this level so that the volume of the gas remains constant.

To use the thermometer, the gas bulb is placed in the environment whose temperature is to be measured for example in a water bath. If the environment is hotter, the temperature of the gas rises. This increases the pressure of the gas and also tends to increase the volume of the gas through expansion (The reverse happens when the environment is cooler). Due to the increase in these two quantities, mercury rises higher up in the longer column (reservoir) of the manometer. To ensure that the *volume of the gas remains constant*, the reservoir is physically raised up or down for the mercury in the smaller column to flow back to the reference point O on the scale. The height difference h between the mercury levels in the two smaller columns give the pressure of the gas in the bulb at the temperature θ°C of the given environment. A practical constant-volume gas thermometer has two fixed points: the lower and upper fixed points that are set using known values of temperature for example melting point of pure ice (0°C) and boiling point of pure water (100°C). Using these points we are able to determine the temperature of other substances or environments using the thermometer.

The following illustration will help us to understand how to do that.

Suppose the lower and upper fixed points of a constant-volume gas thermometer are:

540 mmHg at 0°C and 780 mmHg at 100°C

What would be the temperature t of a substance that the produces a pressure difference of 600 mmHg on the thermometer?

Pressure difference between the fixed points = (780 – 540) mmHg = 240 mmHg.

Temperature difference between the fixed points = (100 – 0)°C= 100°C.

Thus, a temperature rise of 100°C produces a pressure rise of 240 mmHg in the gas.

The substance produces a pressure rise =(600 – 540) mmHg = 60 mmHg above the lower fixed point.

Thus, the temperature produced by the substance above the above the lower fixed point is then given by

$$\frac{60}{240} \times 100°C = 25\ °C$$

$$\text{Since } 25\ °C = t - 0\ °C,$$

$$\text{then, } t = 25\ °C.$$

Example 4.4

The pressure in a constant - volume gas thermometer is 755 mmHg at 0° C of ice and 790 mmHg at 100°C. What is the temperature when the pressure is 765 mmHg?

Solution

Pressure difference between fixed points = (790 – 755) mmHg

= 35 mmHg

Temperature difference between fixed points = 100°C – 0°C

= 100°C

Pressure difference between 765 mmHg and 755mmHg = (765 – 755) mmHg

= 10 mmHg

Temperature when pressure is 755 mmHg is given by

$$= \frac{10}{35} \times 100°C = 28.6°C$$

Electrical resistance thermometer

Electrical resistance thermometer is a device used to measure temperature by the change of electrical resistance of a metallic sensor. The thermometer works using the fact that electrical resistance increases with increase in temperature. The most accurate and commonly used resistance thermometers are standard platinum resistance thermometers (SPRTs) that use platinum wire sensors.

Fig. 4.7 shows an example of resistance temperature detector (thermometer).

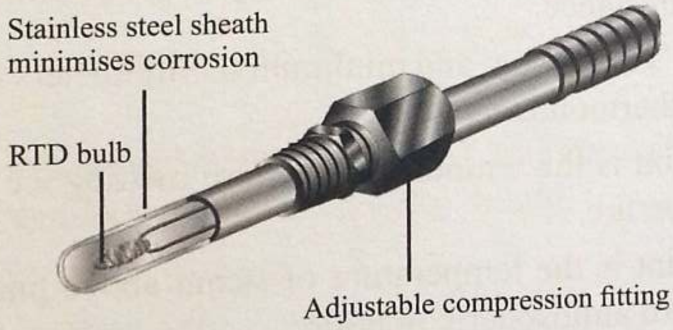


Fig. 4.7: Resistance temperature detector (RTD)

Working of an electrical resistance thermometer

Electrical resistance thermometer measures changes in the electrical resistance of metallic materials or thermistors. A metal wire or thermistor is housed in a thin rod. It is connected in the circuit whose temperature is to be measured. The change in electrical resistance is indicated as a temperature reading on a digital display screen.

Exercise 4.2

1. What is a thermometer?
2. Name two types of thermometers that use liquids to function.
3. What is the function of the construction in a clinical thermometer.
4. Explain how a clinical thermometer works.
5. The pressure in a constant-volume gas thermometer is 0.800 atm at 100°C and 0.421 atm at 0°C. Calculate:
   (a) The temperature when the pressure is 0.200 atm
   (b) The pressure at 250°C
6. Briefly explain how electrical resistance thermometer works.

7. State three advantages of using mercury in a liquid - in-glass thermometer over alcohol.
8. State and briefly explain three features of liquid - in - glass thermometer.

Unit summary

- Temperature is the degree of hotness or coldness of a body or a place. It is also the average kinetic energy of the molecules of a substance.
- The SI unit of temperature is kelvin (K).
- A thermometer is an instrument used to measure the temperature of a body.
- Liquid-in-glass thermometers commonly use mercury or alcohol as their thermometric substance.
- Clinical and six's maximum and minimum thermometers are special types of liquid-in-glass thermometers.
- Lower fixed point is the temperature of pure melting ice at 0°C at standard atmospheric pressure.
- Upper fixed point is the temperature of steam above pure boiling water at 100°C at standard atmospheric pressure.
- Different thermometers use thermometric substances of different properties.
- Absolute zero is the temperature at which a gas appears to have zero volume.
- Temperature in kelvin = (temperature in °C + 273).
  Temperature in °C = (temperature in K – 273).
- Thermocouple is the type of thermometer used to measure high temperature. Consists of two dissimilar metallic conductors joined at a junction.
- Electrical resistance thermometer is a thermometer that measures temperature by changes in the resistance of a spiral of platinum wire.

Unit Test 4

1. A mercury thermometer without a scale is placed in pure melting ice. What does the level of mercury in the thermometer represent?
   A. Upper fixed point
   B. Lower fixed point
   C. Boiling point of water
   D. Melting point of water
2. What must expand to show the rise in temperature in a mercury-in-glass thermometer?
   A. The vacuum
   B. The mercury

C. The glass stem D. The glass bulb

3. To make a liquid-in-glass thermometer sensitive to a small change of temperature, we must have;
   A. a bulb with a thin glass wall
   B. a strong liquid in the bulb
   C. a very narrow bore
   D. a stem with a thick glass wall
4. The normal human body temperature is 37°C. In kelvin scale it is;
   A. 37 K B. 310 K
   C. 236 K D. 98 K
5. The boiling point pure of water in kelvin scale is;
   A. 273 K B. 373 K
   C. 100 K D. 0 K
6. Which of the following statements is NOT true for a clinical thermometer? It is desirable that a clinical thermometer should;
   A. have a very small range
   B. be very sensitive
   C. take time to acquire its maximum reading
   D. retain the reading until shaken
7. What is a thermometer? Name two types of thermometers.
8. Explain the meaning of the following terms: upper fixed point, ice point, steam point.
9. State three characteristics of a good thermometric substance.
10. State the two special features of a clinical thermometer and explain their roles.
11. State one advantage of an alcohol-in-glass thermometer as compared to mercury-in-glass thermometer.
12. Estimate the room temperature and express it in kelvin.
13. A thermometer reads 2°C in pure melting ice and 103°C in steam. What is the error when the temperature rise is calculated?
14. Explain how you can use a liquid-in-glass thermometer to determine the boiling point of water.
15. Explain how six's maximum and minimum thermometer works.
16. State two uses of a thermocouple thermometer.
17. What is an electrical resistance thermometer?

UNIT

5 Pressure

Succes Criteria

By the end of this unit, you must be able to:

- Define pressure
- Determine the pressure exerted by regular solids
- Describe the experiments to investigate factor affecting pressure in liquids
- State Pascal's principle of transmission of pressure in fluids
- Explain atmospheric pressure
- Describe the applications of pressure in fluids
- Explain Archimedes principle

5.1 Definition and units of pressure

We use the word *pressure* almost daily without paying much attention to its exact meaning. In hospitals the nurses and doctors often talk about *blood pressure.* In petrol stations, we add *air pressure* in car tyres. In the kitchen we use *pressure cookers.* Psychologists and counsellors talk about *peer pressure*. A worker would talk about *pressure at work,* etc. In physics, we define pressure in terms of force and area.

Pressure is defined as *force acting normally (perpendicularly) per unit area.*

$$\text{Pressure (P)} = \frac{\text{Force (F)}}{\text{Area (A)}}$$

$$P = \frac{F}{A}$$

Activity 5.1 will help us to show that pressure depends on the area of contact.

Activity 5.1: To show that pressure depends on the area of contact of a solid

- Take a piece of bar soap. Place a nail on it with the blunt end resting as shown in Fig. 5.1(a).
- Place a stone on the sharp end of the nail. Observe what happens to the nail.
- Repeat the activity with the sharp end of the nail on the soap (Fig. 5.1(b)).
- Compare the penetration of he nail in the soap in the two cases.

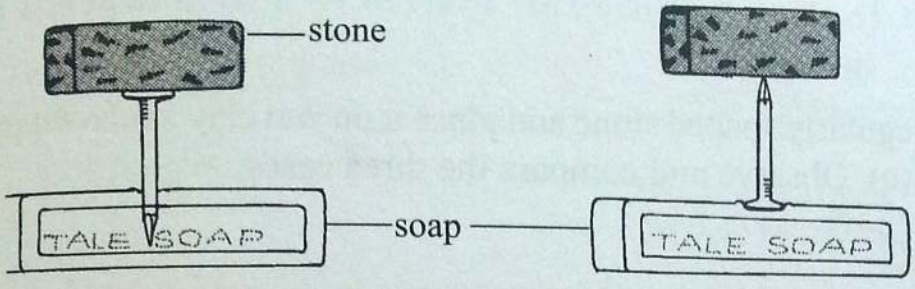


Fig. 5.1: Pressure depends on area

In Activity 5.1, you must have observed that in the setup in Fig. 5.1 (a), the nail penetrates a longer distance inside the soap while in Fig. 5.1(b) the penetration is less.

The pressing is larger over a small area than over a large area. The pressing ability of a force is called *pressure.*

If the force applied is more, the pressing ability is more and vice versa.

$$\text{Greatest pressure} = \frac{\text{Force}}{\text{minimum area}} \text{ and}$$

$$\text{Least pressure} = \frac{\text{Force}}{\text{maximum area}}$$

A solid with a small area of contact, exerts more pressure while that with a large area of contact, exerts less pressure on any surface when pressed.

Units of pressure

The SI unit of pressure is the *Pascal (Pa)*. Since the SI unit of force is newton (N) and that of area is square metre (m^2), the other unit of pressure is *newton per square metre (N/m^2)*. 1 pascal is defined as *one newton per square metre* i.e.

$$1 \text{ Pa} = 1 \text{N/m}^2$$

5.2 Pressure in solids

We note from the formular $P = \frac{F}{A}$, that pressure is proportional to F and $\frac{1}{A}$. Hence, we can conclude that pressure in solids depends on the force exerted by the solid and the area of contact with the solid.

Activity 5.2 will help us understand the factors that affect pressure exerted by solids.

Activity 5.2: To show that pressure exerted by a solid depends on the area in contact

- Take a regularly shaped stone and place it on wet clay as shown in Fig. 5.2(a), (b) and (c). Observe and compare the three cases.

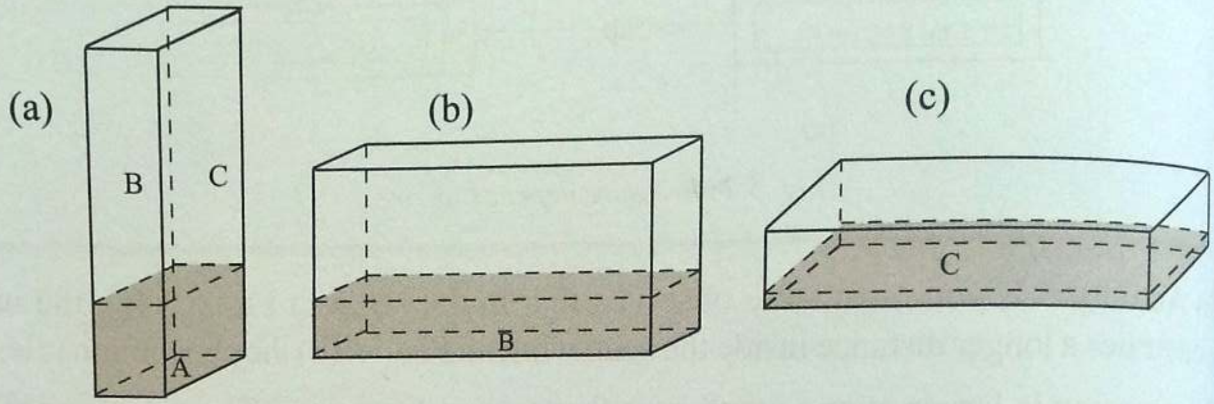


Fig. 5.2: Stone on wet clay

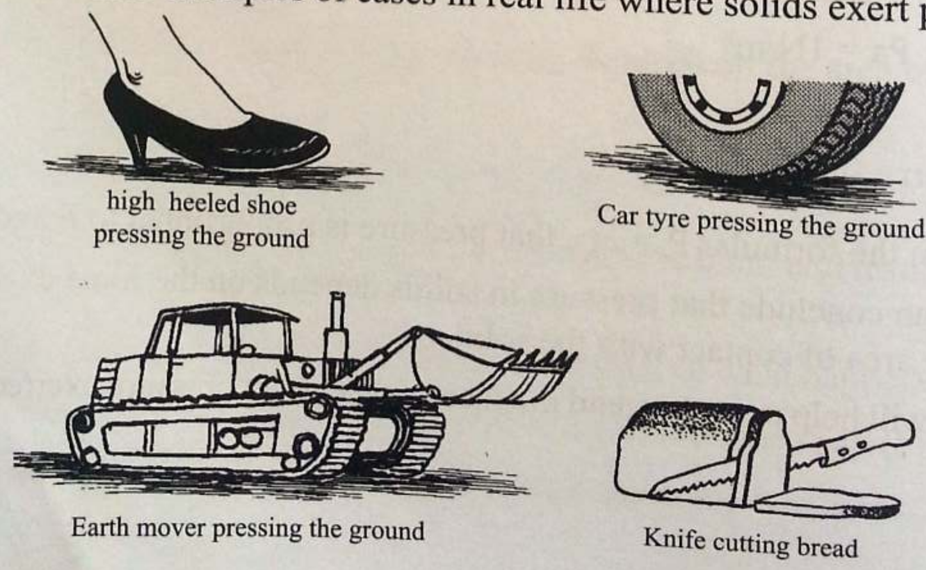
In Activity 5.2, you must have noted that when the stone rests on side A, it sinks more than when it rests on side B or C. It sinks least when it rests on side C.

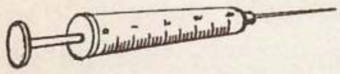
The weight (force) of the stone is the same in all the cases. When the stone is resting on side A, its weight is distributed over a smaller area than when resting on sides B and C. The pressure exerted by the stone is therefore maximum when resting on side A.

We therefore conclude that the larger the area, the smaller the pressure exerted on it and vice versa. Thus, the presssure exerted by solids is inversely proportional to the area of contact.

If Activity 5.2 was repeated with heavier regular-shaped stone than the one used, it will be noted that the stone sinks further in all resting sides: A, B and C than before. This shows that the pressure exerted by a solid is directly proportional to the force applied.

Fig. 5.3 shows more examples of cases in real life where solids exert pressure.





Piston pressing liquid in a syringe

Foot of webbed birds pressing the ground

Fig. 5.4: Examples of cases where solid pressure is exerted.

Example 5.1

Express a pressure of 35.6 N/m^2 in pascals

Solution

$$1 N/m^2 = 1 Pa$$

$$\therefore \quad 35.6 N/m^2 = \frac{35.6 \cancel{N/m^2} \times 1 Pa}{1 \cancel{N/m^2}} = 35.6 Pa$$

Example 5.2

A rectangular solid glass of density 2.5 g/cm^3 has dimensions 10 cm × 40 cm × 30 cm. The block rests on a horizontal flat surface. Calculate the:

(a) minimum pressure (b) maximum pressure it can exert.

Solution

(a) For minimum pressure, the area must be maximum.

$$\text{Maximum area} = (\frac{40}{100} \times \frac{30}{100}) m^2 = \frac{1\,200}{10\,000} m^2 = 0.12 m^2$$

$$\text{Density of block} = 2.5 g/cm^3$$

$$\text{Volume of block} = 10 \times 40 \times 30$$

$$= 1\,200 cm^3$$

$$\text{Mass of block} = \text{density} \times \text{volume}$$

$$= (2.5 \times 1\,200)g = 3\,000 g$$

$$= 3 kg$$

$$\text{Weight} = \text{mass} \times \text{pull of gravity}$$

$$= 3 kg \times 10 N/kg$$

$$= 30 N$$

$$\text{Minimum pressure} = \frac{\text{Force}}{\text{Area (maximum)}}$$

$$= \frac{30\text{ N}}{0.12\text{ m}^2} = 250\text{ Pa}$$

(b) For maximum pressure, the area considered must be minimum

$$\text{Minimum area} = \left(\frac{10}{100} \times \frac{30}{100}\right)\text{ m}^2 = \frac{300}{10\,000}\text{ m}^2 = 0.03\text{ m}^2$$

$$\text{Maximum pressure} = \frac{\text{Force}}{\text{Area (minimum}}$$

$$= \frac{30\text{ N}}{0.03\text{ m}^2}$$

$$= 1\,000\text{ Pa}$$

Exercise 5.1

1. Define pressure and state its SI unit.
2. Explain the following statements in terms of pressure:
   (a) Large flat feet enable elephants to move freely over soft ground.
   (b) It is painful if one tries to lift a heavy load by a thin string.
3. Sinking of tyres of a car into soft, damp sand patch may be prevented by letting off some air in each tyre. Explain.
4. Three similar blocks A, B and C are placed on a horizontal surface as shown in Fig. 5.4. Which block exerts the minimum pressure on the surface? Explain your answer.

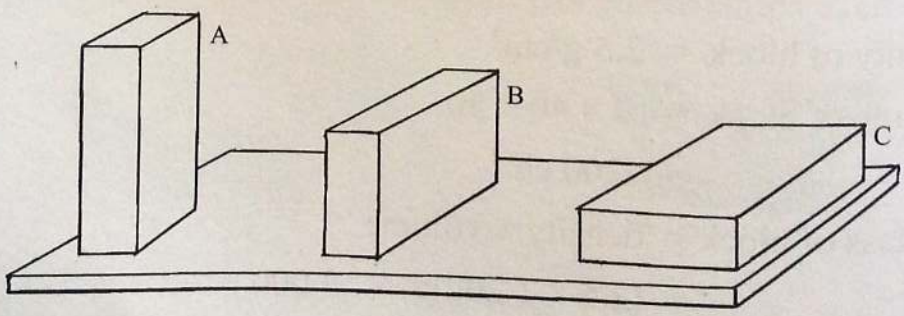


Fig. 5.4

5. A pick-up carrying stones weighs 20 000 N. The weight is evenly spread across the four tyres. The area of contact of each tyre with the ground is 0.025 m². Calculate the pressure exerted by each tyre on the ground.

6. (a) A thumb is used to push a thumb pin into a piece of wood. Explain, in terms of pressure, why the pressure on wood is greater than the pressure on the thumb.

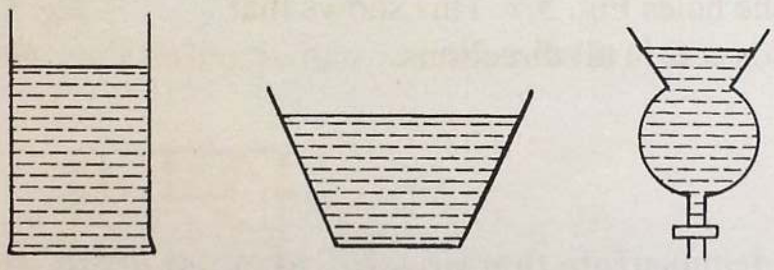
   (b) Find the pressure exerted on a thumb if the force the thumb exerts on a pin of area 5 mm^2 is 45 N.

7. An elephant of mass 2 000 kg has feet of average area of 150 cm^2. A vulture of mass 20 kg walks beside the elephant on a muddy area. The average area of the feet of the vulture is 1.5 cm^2. Which one is likely to sink? Explain your answer.

8. A container has a mass of 70 kg. Calculate the pressure it exerts on the ground when the area in contact with the ground is 0.14 m^2.

5.3 Pressure in liquids

Liquids, unlike solids, do not have definite shapes. They take the shape of the container in which they are in as shown in Fig. 5.5.



(a) (b) (c)

Fig 5.5: Liquids take the shape of a container.

Like solids, liquids exert pressure. A solid exerts pressure only on the area in contact with its surface but a liquid exerts pressure on all the walls of the container. Activities 5.3 to 5.5 will help us study pressure in liquids.

Activity 5.3: To demonstrate liquid pressure

- Pierce a hole at the bottom of a container. Using one of your finger, block the hole and put some water in the container. What does your finger feel?
- Remove the finger from the hole. What happens to the water inside the container?

In Activity 5.3 you must have observed that when the water was put into the container the finger felt being pushed. This shows that there is a force in the water that pushes the finger. The force pushes the water out of the container through the hole when the finger is removed as shown in Fig. 5.6.

Fig. 5.6: Water coming out of the container forcefully

Activity 5.4: To show that liquid pressure acts in all directions

- Take a small polythene bag and half fill it with water. Grip it tightly round the neck.
- Make a few pin holes on different parts of the bag. What happens to the water inside the bag? Explain.

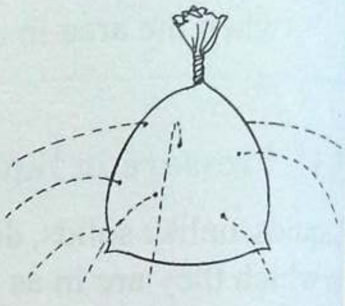


Fig. 5.7: Water coming out through all the hole

In Activity 5.4, you must have noted that water comes out through all the holes Fig. 5.7. This shows that pressure in liquids acts in all directions.

Activity 5.5: To demonstrate that pressure at equal depth, acts equally in all directions

- Take a can and make similar holes all round the can at the same height.
- Fill the can with water (Fig. 5.8). Compare the distances travelled by the water coming out of the three holes and observe what happens.

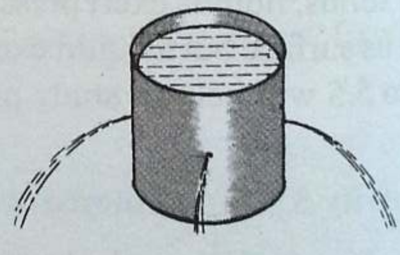


Fig. 5.8: Pressure acts equally at the same level

In Activity 5.5, you must have observed that water travel equal horizontal distances from the can.

We conclude that the pressure exerted by a liquid at the same depth is the same in all directions.

Factors affecting pressure in liquid

Pressure in liquid depends on the following:

- The height of the liquid column (depth)
- Density of the liquid.
- The gravitational field strength of a place

Experiment 5.1 will help us to study these factors

Experiment 5.1: To demonstrate the factors that affect pressure in liquids

(a) Depth

Apparatus

- A cylindrical tin
- Water
- A nail
- Brine, paraffin

Procedure

1. Make three similar holes on the sides of a long container, one almost at the bottom, another at the middle and the third almost at the top (Fig. 5.9).
2. Pour water in the container and keep the water at the same level by pouring more water into the container.
3. Compare the horizontal distances travelled by water from the three holes.

Observation

Water from hole A travels furthest while water from hole C covers the least distance (Fig. 5.9).

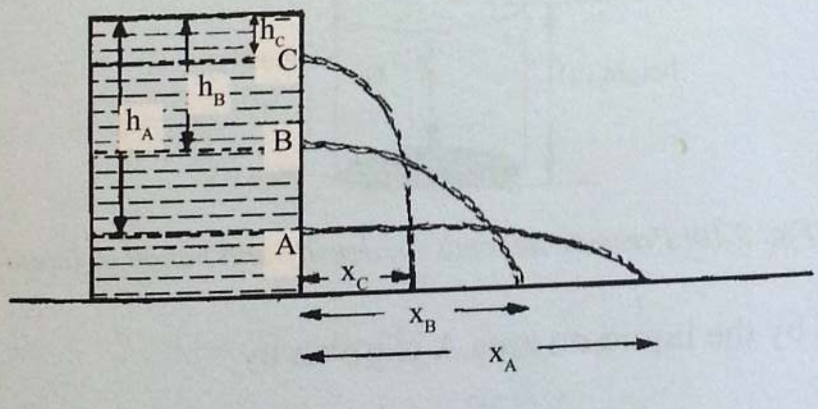


Fig. 5.9: Pressure depends on depth

Conclusion

Pressure in liquids depends on the height of the column of the liquid.

(b) Density

Repeat Experiment 5.1 with three liquids of different known densities and observe the horizontal distance covered.

Observation

The liquid of highest density covers the longest horizontal distance.

Conclusion

Pressure in liquids depends on the density (ρ) of the liquid.

From Experiment 5.1 we conclude that:

- Pressure in liquid depends on the height of the column of the liquid. i.e. increase in the depth of the liquid leads to increase in pressure.
- Pressure is directly proportional to the density of the liquid.
- It has been proven experimentally that pressure is also directly proportional to gravitational field strength.

Formula for pressure in liquids

Consider a liquid column (h) acting on an area (A) as shown in Fig. 5.10. Force (F) acting on area (A) is due to the *weight* (mg) of liquid above it. Given that the volume of the liquid is V and its density ρ then;

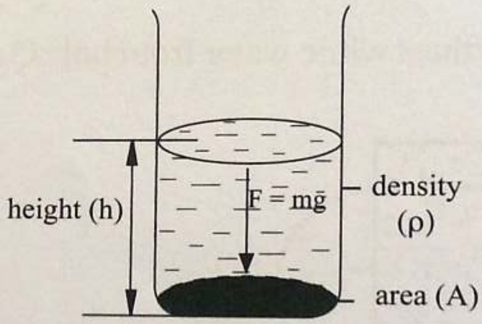


Fig. 5.10: Pressure depends on density and height of liquid.

Pressure exerted by the liquid on area A is given by

$$P = \frac{F}{A}$$

But $F = mg$ (weight of liquid above A).

$$\therefore \quad P = \frac{mg}{A}$$ where $m = \rho V$ and since $V = Ah$, $m = \rho Ah$

$$\therefore \quad P = \frac{\rho \cancel{A} hg}{\cancel{A}} = \rho gh$$

> Pressure in liquid = density (ρ) × gravitational field strength (g) × height (h)
>
> Hence, $P = \rho gh$

From the above formula, we can see that pressure is directly proportional to height (h), density (ρ) and the pull of gravity (g).

It is worthwhile to note that pressure in liquids does not depend on the cross sectional area, A, and the shape of the container. Fig. 5.11 shows tubes of different shapes and different cross-sectional area. Note that the height of the liquid column, (h), is the same in all the tubes, hence they exert equal pressure.

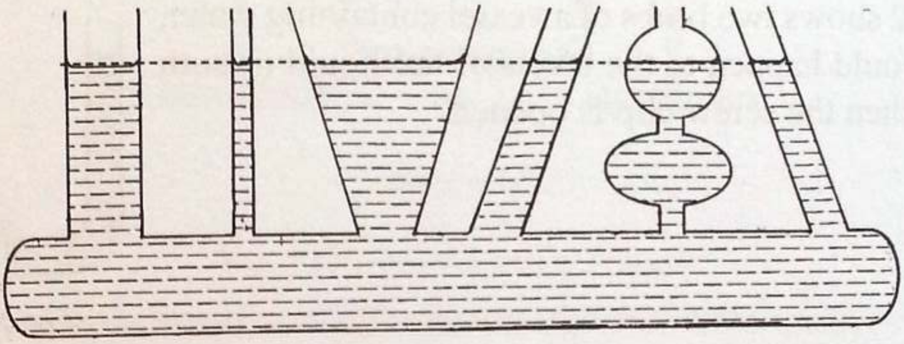


Fig 5.11: Pressure does not depend on the shape of the container

Example 5.3

Calculate the pressure at the bottom of a beaker when it is filled with water to a height of 12 cm. Take the density of water as 1 g/cm^3 and g = 10 N/kg.

Solution

Density of water = 1 g/cm^3 = 1 000 kg/m^3

Column of water = 12 cm = 0.12 m

Pressure = ρgh

= 1 000 × 10 × 0.12

= 1 200 N/m^2

Exercise 5.2

1. (a) Name three factors which affect the pressure in a liquid.
   (b) Describe experimentally how one of the factors in Question 1 (a) affects pressure in liquid. What precaution(s) would be necessary?
2. Explain the following statements:
   (a) The wall of a reservoir is thicker at the bottom than at the top.
   (b) Submarines, divers and even common fish cannot descend in water beyond a certain depth.

3. (a) A column of glycerine 8.20 m high, a column of sea water 10.08 m high, column of mercury 0.76 m high and column of fresh water 10.34 m high exert the same pressure at the bottom of a container. Arrange these substances in decreasing order of their densities.

(b) Calculate the pressure exerted by the column of mercury of Question 3(a) given that the density of mercury is 13.6 g/cm^3.

4. Fig. 5.12 shows two limbs of a vessel containing water. What would happen to the levels of the liquid in both limbs when the screw clip is opened?

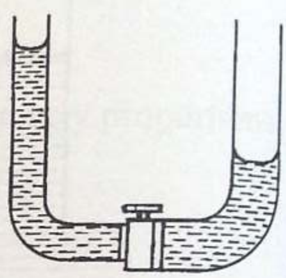


Fig. 5.12

5.4 Transmission of pressure in fluids

Fluids are substances which are capable of flowing freely. This include all liquids and gases.

Experiment 5.2: To show that pressure is equally transmitted through a liquid

Apparatus

- Two syringes A and B
- Two stands
- Two masses, m_1 and m_2
- A plastic tube

Procedure

1. Measure the radius of two syringes A and B and find their cross-sectional areas A_1 and A_2 respectively.
2. Connect them as shown in Fig. 5.13. Place a 100 g mass (m_1) on A and observe what happens to both pistons.
3. Add some mass, m_2 on piston B until the two pistons are at the same level.

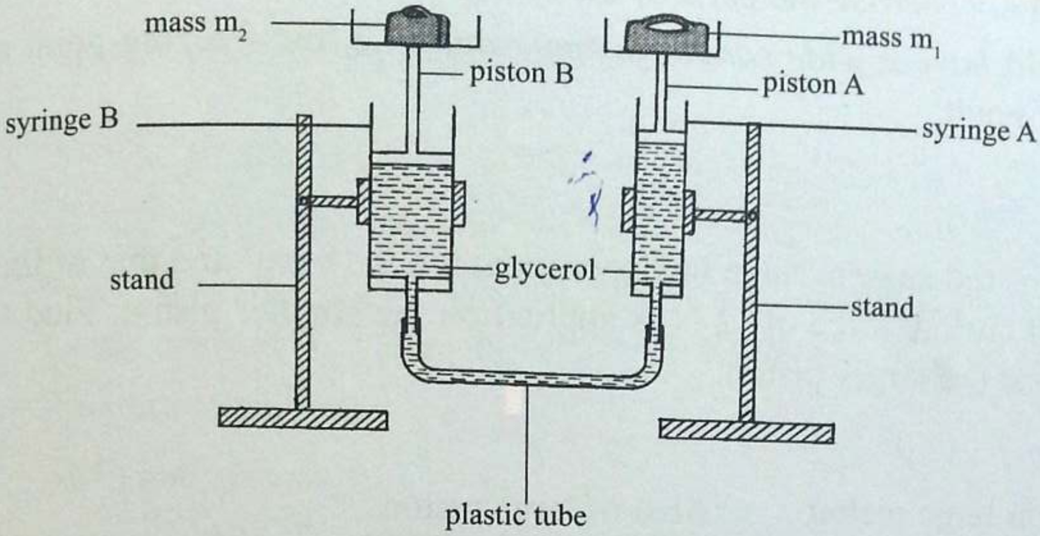


Fig. 5.13: Transmission of pressure

4. Repeat the experiment with different masses on A and find the corresponding masses on B. Record the masses on A and B in a table 3.1.

Table 5.1

Mass m_1 (g)	Weight of m_1 (N)	Mass of m_2 (g)	Weight of m_2 (N)	$P_1 = \frac{F_1}{A_1}$ (Pa)	$P_2 = \frac{F_2}{A_2}$ (Pa)

- Calculate the corresponding weight of each mass and then calculate the pressure in each case.

Discussion

When the two pistons are at the same level, the pressure exerted by the force due to masses A and B is equal. Pressure applied at one point in a liquid is transmitted equally to all other points of an enclosed liquid.

Pressure, P_1 = Pressure, P_2

$$\therefore \quad \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

From this experiment, we can conclude that a small force applied on a small piston produce a large force on a large piston. This is *the principle of transmission of pressure in liquids* or *Pascal's hydraulic principle.*

Pascal's hydraulic principle states that *pressure applied at a point in a fluid at rest is transmitted equally to all parts of the fluid.*

For Pascal's principle to hold, the fluid used should have the following properties:

- It should be *incompressible.*

- It shoul *not corrode* the parts of the system.
- It should *have a wide range of temperature* i.e low freezing point and high boiling point.

Example 5.4

The area of the large syringe in Experiment 5.2 is 18 cm^2 and that of the smaller one is 3.0 cm^2. A force of 2 N is applied on the smaller piston. Find the force produced at the larger piston.

Solution

$$\frac{\text{Force on large piston}}{\text{Force on small piston}} = \frac{\text{Area of large piston}}{\text{Area of small piston}}$$

$$\frac{\text{Force on large piston}}{2\text{ N}} = \frac{18\text{ cm}^2}{3.0\text{ cm}^2}$$

$$\text{Force on large piston} = 2\text{ N} \times \frac{18\text{ cm}^2}{3.0\text{ cm}^2}$$

$$= 12\text{ N.}$$

Example 5.5

Fig. 5.14 shows a hydraulic lift.

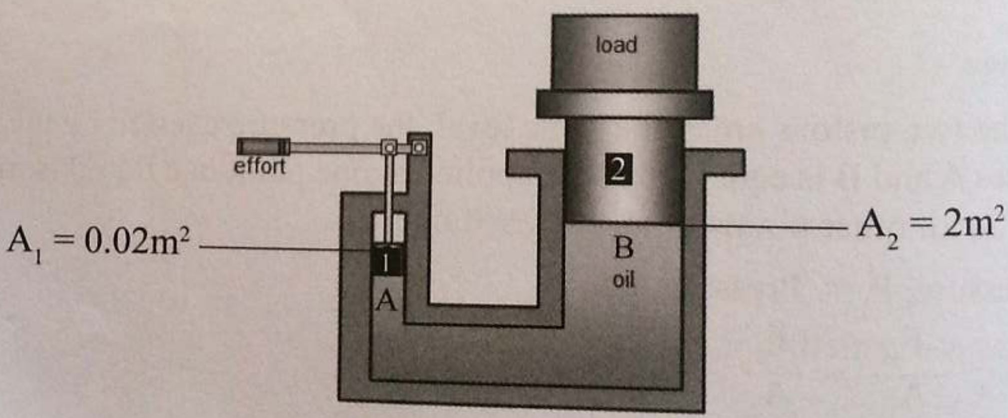


Fig. 5.14: A hydraulic lift

(a) Determine:

(i) Pressure exerted on the oil by piston 1 at point A if a force of 250N is applied on the handle.

(ii) Pressure at point B.

(iii) Force exerted on piston 2 by the oil.

(b) State three properties of the oil that makes it suitable for use in the hydraulic lift.

Solution

(a) (i) Pressure exerted by piston 1 on the oil

$$P_1 = \frac{F_1}{A_1} = \frac{250}{0.02 m^2}$$

$$= 12\,500 \text{ N/m}^2$$

(ii) Pressure exerted on to the oil by piston 1 at point A is transmitted by the oil to point B. Hence $P_B = P_A = 12\,500 \text{ N/m}^2$

(iii) Pressure at point B

By Pascals principle, $P_1 = P_B$

Force exerted on the piston 2 by the oil

$$P_1 = P_B = P_2 = \frac{F_2}{A_2}$$

$$F_2 = P_2 \times A_2 = 12\,500 \text{ N/m}^2 \times 2 \text{ m}^2$$

$$= 25\,000 \text{ N}$$

(b) Properties of oil as a hydraulic fluid are:

- Oil is imcompressible.
- Oil has a high boiling point and low freezing point.
- Oil does not corrode the parts of the system.

Exercise 5.3

1. State Pascal's principle of transmission of pressure.
2. Explain how hydraulic lifts work.
3. Why is air not commonly used as the fluid in hydraulic lift?
4. Describe an experiment to show that pressure at a point in a liquid is distributed equally in all directions.
5. A model of hydraulic lift has effort and load pistons of diameter 1 cm and 10 cm respectively. Find the force needed to raise a load of 50 N.
6. A hydraulic car jack has pistons of diameter 2 cm and 20 cm. Find the weight of a car that can be lifted by a force of 350 N.
7. Fig. 5.15 is a hydraulic system used in a garage to lift vehicles under repair.

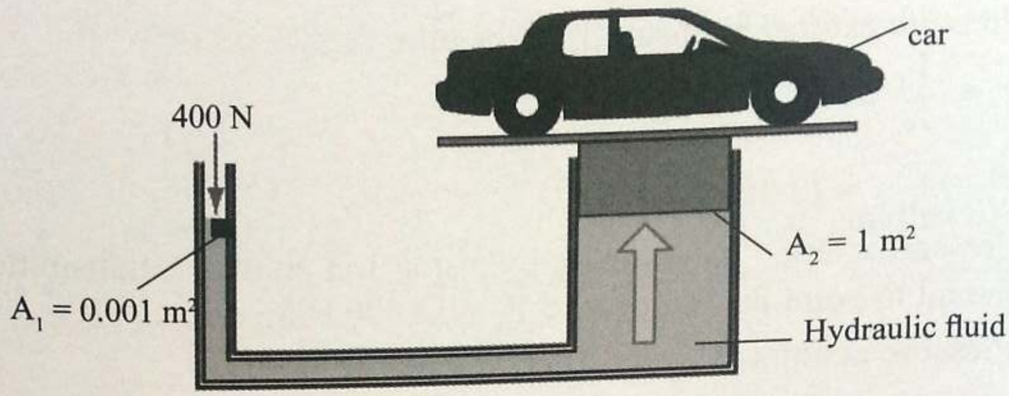


Fig. 5.15

(a) Explain why air is not used as a hydraulic fluid.

(b) Determine:

(i) The pressure exerted by piston of area A_1 on the fluid

(ii) Pressure at point Y.

(iii) The force needed to lift the car.

5.5 Atmospheric pressure

A gas exerts pressure on the walls of a container. Since air is a mixture of gases, it also exerts pressure. The earth's surface is surrounded by the thick layer of air. We live under a vast *column* of air called *atmosphere*. The density of air varies from the earth's surface to the outer space. Air is more dense at sea level than high up in the mountains (Fig. 5.16). The pressure exerted by air is called *atmospheric pressure*

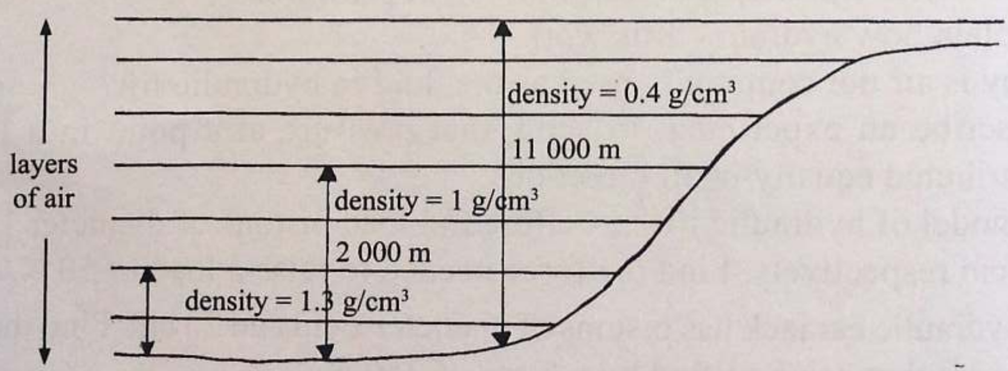


Fig. 5.16: Atmospheric pressure varies with altitude

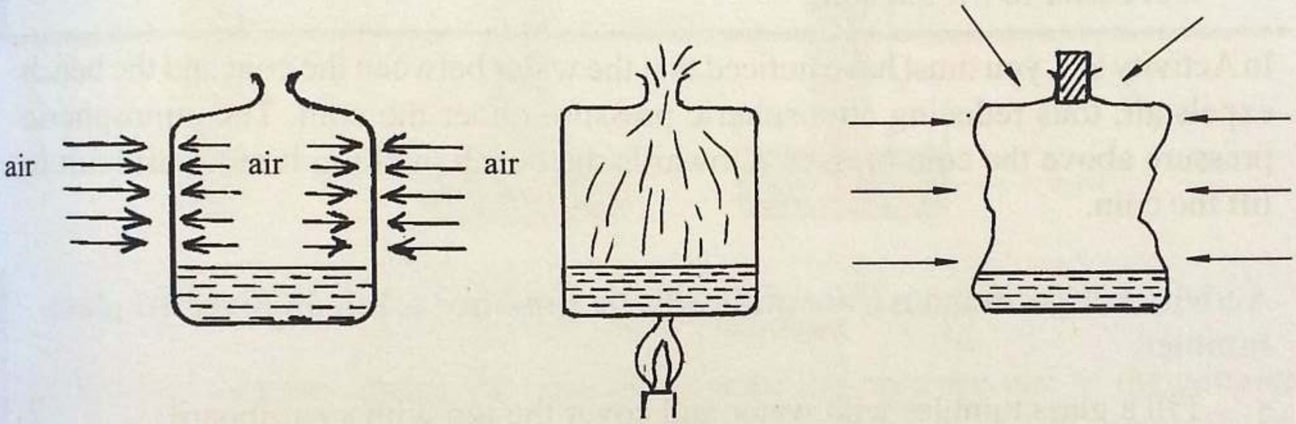
Effects of atmospheric pressure

Activities 5.6 to 5.9 will help us to study the atmospheric pressure.

Activity 5.6: To demonstrate that air exerts pressure using a can

- Pour some water in a large thin-walled can. Boil off the water in the can and immediately cork the can.
- Allow the can to cool and observe what happens.

Before heating the can in Activity 5.6 (Fig. 5.17 (a)), the air inside and outside the can exerts equal but opposite force on the walls of the container. On heating (Fig. 5.17(b)), the steam that is formed expels the air inside the can.



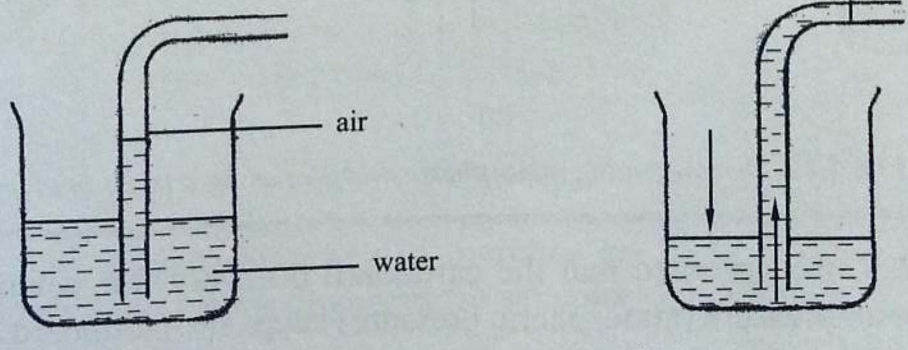
(a) *(b) on heating* *(c) on cooling*

Fig. 5.17: Showing crashing can

After corking and cooling the can, a partial vacuum is formed inside the can. On cooling (Fig. 3.19c), the pressure from the air outside the can which is greater than the pressure of the air inside the can makes the can to crush or collapse. We conclude that air exerts pressure.

Activity 5.7: To demonstrate effect of pressure in using a straw

- Arrange the apparatus as shown in Fig. 5.18(a).
- Suck out some air gently from the tube and observe what happens to the water level inside and outside the tube.



(a) (b)

Fig. 5.18: Demonstrating effect of atmospheric using a straw

In activity 5.7, you must have noticed that the water level inside the tube rises (Fig. 5.18 (b)).

When some air inside the tube is sucked out, the pressure inside the tube becomes less than the atmospheric pressure. Hence atmospheric pressure pushes water up the tube.

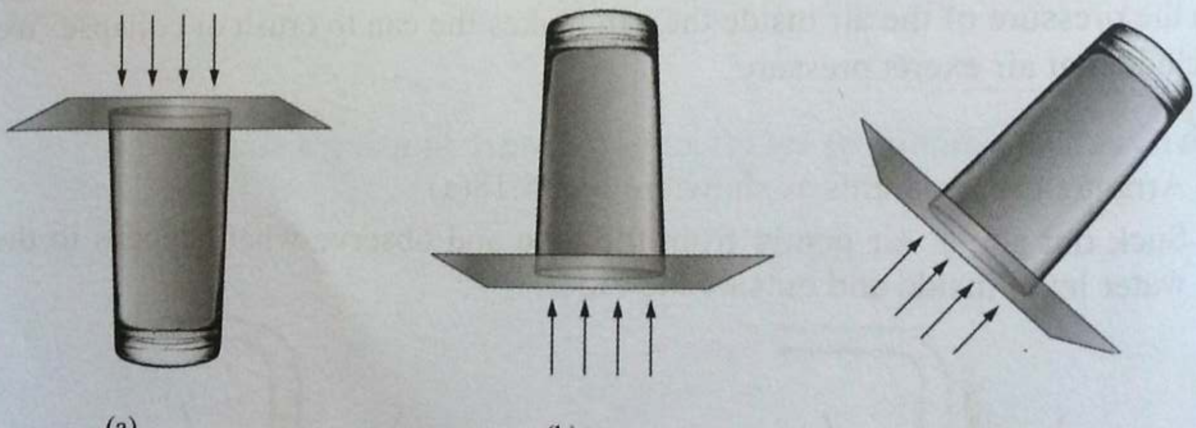
Activity 5.8: To demonstrate atmospheric pressure by lifting a coin from a bench

- Place a coin on top of a bench and lift it up.
- Repeat the experiment but with a drop of water under the coin. In which case is it easier to lift the coin?

In Activity 5.3, you must have noticed that the water between the coin and the bench expels air, thus reducing atmospheric pressure under the coin. The atmospheric pressure above the coin presses it towards the bench, making it more difficult to lift the coin.

Activity 5.9 To demonstrate atmospheric pressure using an inverted glass tumbler

- Fill a glass tumbler with water and cover the top with a cardboard.
- Invert the tumbler while holding the cardboard, suddenly remove the hand holding the cardboard. Observe what happens (Fig. 5.19).
- Suddenly open the ends of the tube at the same time. What happens to the water inside the tube?



(a) (b) (c)

Fig. 5.19: Demonstrating atmospheric pressure using a glass tumbler

In Activity 5.9, we observed that the cardboard does not fall when the glass is inverted. The air pressure (atmospheric pressure) keeps the cardboard in place. The water does not flow out.

Measurement of atmospheric pressure due to gases

Atmospheric pressure is measured using an instrument called *a barometer.*

A simple mercury barometer

A clean dry thick-walled glass tube one metre long is filled with mercury to its brim (Fig. 5.20(a)). The open end is closed and then inverted into a bowl containing mercury (Fig. 5.20(b)). Keeping the tube vertical and the closed open end still inside the mercury, the cover is removed (Fig. 5.20(c)). This set-up constitutes a simple mercury barometer.

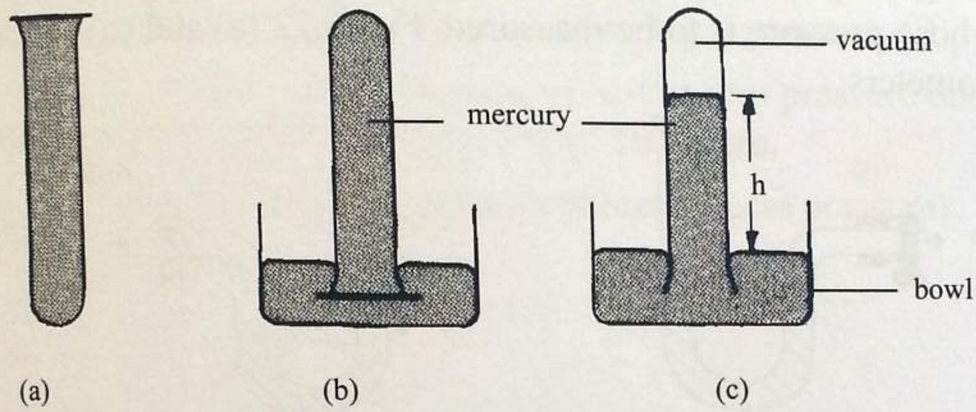


Fig. 5.20: Mercury barometer.

The level of mercury inside the tube drops until the pressure due to the column of mercury equals the atmospheric pressure. The height, *h*, is a measure of the atmospheric pressure. The length of column, *h*, depends on the liquid used and the altitude of the place. At sea level, mercury stands at a height of 760 mm. We say that the atmospheric pressure at sea level is equal to a column of 760 mm of mercury. If water is used instead of mercury the height of the column of water is about 10 m.

From the mercury barometer readings, the atmospheric pressure is given by:

$$\text{Atmospheric pressure, } P_A = h \times \rho_{mercury} \times \text{gravitational acceleration}$$
$$= h\rho g$$

Example 5.6

Fig. 5.21 shows a mercury barometer used to measure the atmospheric pressure on the bank of a river.

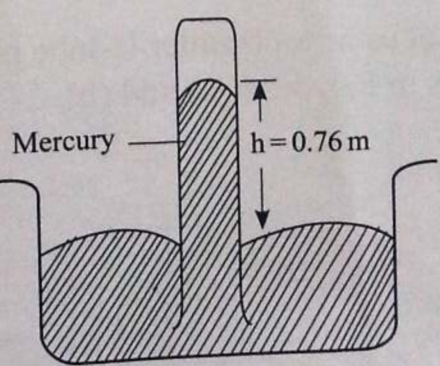


Fig. 5.21: Measuring atmospheric pressure

Taking g = 10 N/kg and density of mercury as 13 600 kg/m^3. Calculate the atmospheric pressure on the bank of the river.

Solution

$$P = h \times \rho \times g$$

$$= 0.75\ m \times 13\ 600\ kg/m^3 \times 10\ N/kg$$

$$= 102\ 000\ N/m^2$$

Manometer

A manometer is an instrument used to measure gas pressure. It consists of a transparent tube containing a liquid. One end is left open while the other is usually connected to the gas whose pressure is to be measured. Fig. 5.22 (a) and (b) are examples of U-tube manometers.

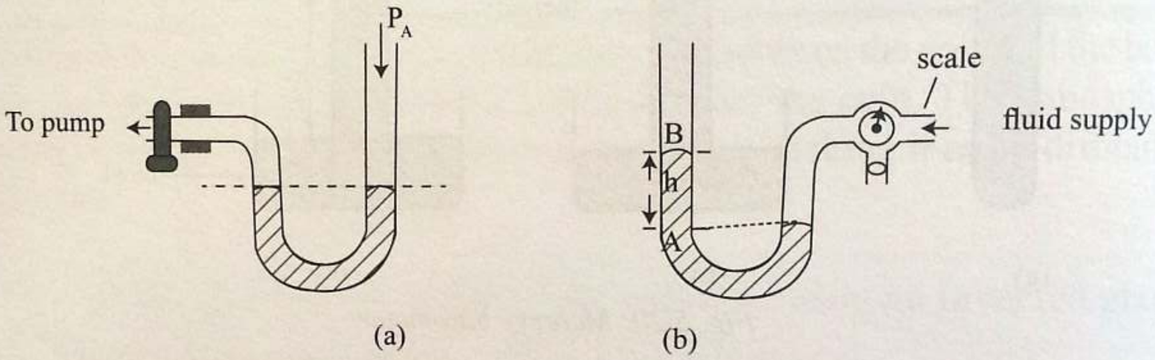


Fig. 5.22: Examples of manometers

Consider the U-tube manometer shown in Fig. 5.33. The level of mercury is same in both arms. This implies that the gas pressure (P_g) is equal to the atmospheric pressure (P_A). i.e $P_A = P_g$

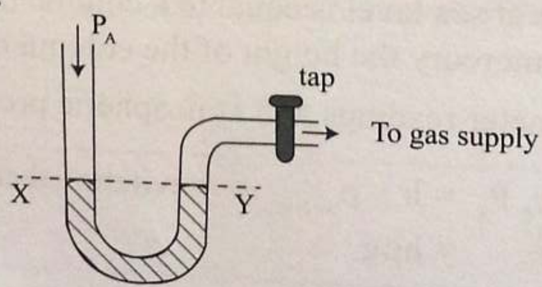


Fig. 5.33: U-tube manometer

Let us now consider U-tube manometers with different levels of liquid in both arms as in Fig. 5.24 (a) and (b).

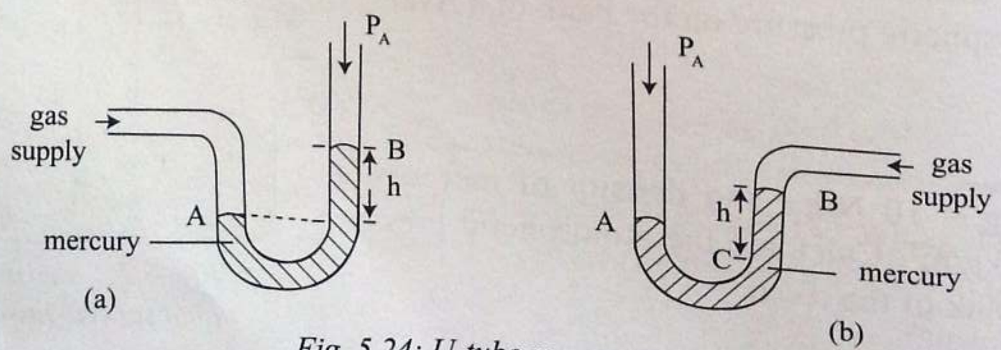


Fig. 5.24: U-tube manometers

In Fig. 5.26(a), the atmospheric pressure P_A and the pressure due to mercury column are suported by the gas pressure P_g i.e gas pressure is more than P_A (atmospheric pressure) $P_g > P_A$. Therefore,

$$P_g = P_A + \text{pressure due to mercury column}$$
$$= P_A + h\rho g, \text{ where is the density of mercury.}$$

Thus, $\boxed{P_g = P_A + h\rho g}$

In Fig. 5.26(b) the atmospheric pressure, P_A is supporting the gas pressure and pressure due to mercury column in the opposite arm, B. $P_A >$ gas pressure, $(P)_g$.

Pressure at point C is due to the gas pressure and excess pressure due to mercury column of length (h) i.e excess pressure = $h\rho g$. Therefore,

$$P_A = P_g + \text{pressure due to mercury column (excess pressure)}$$
$$= P_g + h\rho g$$

Thus, $\boxed{P_g = P_A - h\rho g}$

Example 5.7

The Fig. 5.25 is a U-tube manometer used to determine the pressure of the gas used to inflate a tyre. The density of mercury is 13 600 kg/m³ and g = 10 N/kg and atmospheric pressure P_A is 1.02×10^5 Pa.

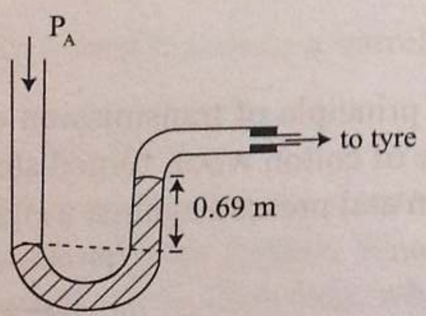


Fig. 5.25: U-tube manometer

Calculate the pressure of the gas.

Solution

From $P_A = P_g + h\rho g$

$\therefore \quad P_g = P_A - h\rho g$

$$= 1.02 \times 10^5 \text{ Pa} - 0.69 \text{ m} \times 13\,600 \text{ kg/m}^3 \times 10 \text{ N/kg}$$
$$= 1.02 \times 10^5 - 93\,840 \text{ Pa}$$
$$= 8\,160 \text{ Pa}$$

$\therefore$ The pressure of the gas used to inflate the tyre is 8 160 Pa.

5.6 Applications of pressure in fluids

Hydraulic brakes

Hydraulic brakes use the principle of transmission of pressure. When a small force is applied on the brake pedal of a car, it pushes the piston in *the master cylinder*. This produces pressure that is equally transmitted to the pistons in the *slave cylinder* (Fig. 5.26).

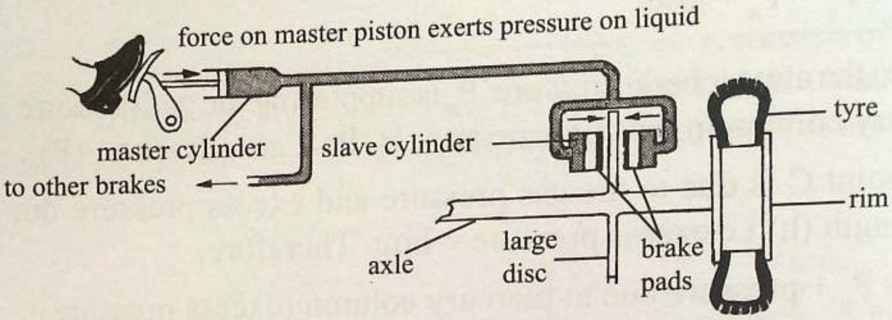


Fig. 5.26: Hydraulic brakes

The pressure forces the brake pads to come into contact with the large disk and slows or stops the car. On releasing the foot pedal, the unbalanced pressure in the slave cylinder forces the liquid back into the master cylinder. Consequently the disk is released.

The pistons in the master cylinder are usually smaller in diameter than the pistons in the slave cylinder, hence a small force applied on the brake pedal produces a large force on the pistons in the slave cylinder.

Hydraulic press

The hydraulic press uses the principle of transmission of pressure. The material to be compressed, such as a bale of cotton wool, forged steel, a bound book, oil seeds, etc. is placed above the piston and pressed against a rigid plate E (Fig. 5.27).

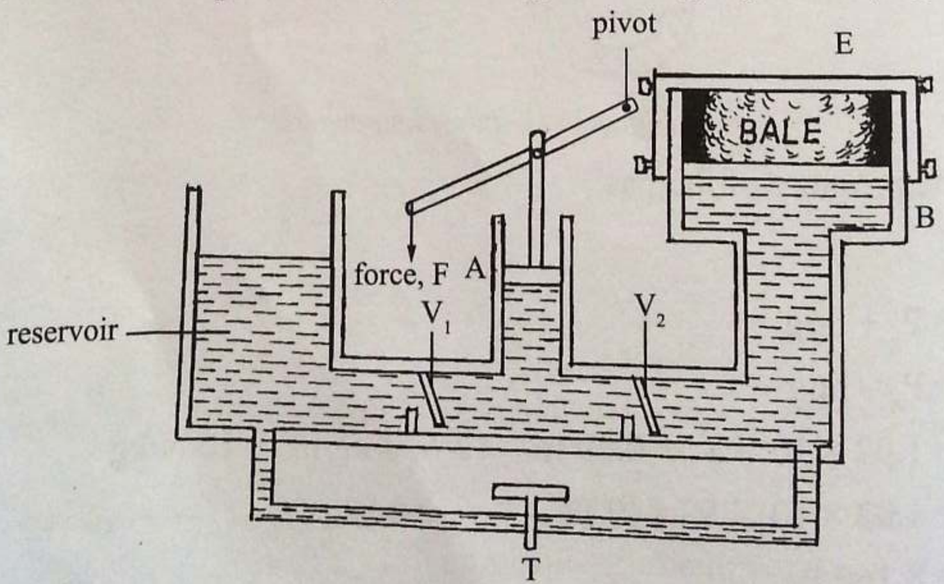


Fig. 5.27: Hydraulic press

When a force F is applied on piston A, valve V_1 closes while valve V_2 opens. The pressure is transmitted equally to B through the liquid. This produces a large force that presses the bale against a rigid plate (E) that is fixed above piston B. On removing the force on A, the valve V_1 opens and V_2 closes. As a result more fluid enters through V_1. Since V_2 is closed, the bale is under constant pressure. The process is repeated and an enormous pressure on the bale is created. By opening the tap T, the liquid in B can be returned to the reservoir and the compressed bale can be removed from the press.

Drinking straw

Sucking through a straw reduces the air pressure inside the straw. Atmospheric pressure forces the liquid into the mouth through the straw (Fig. 5.28).

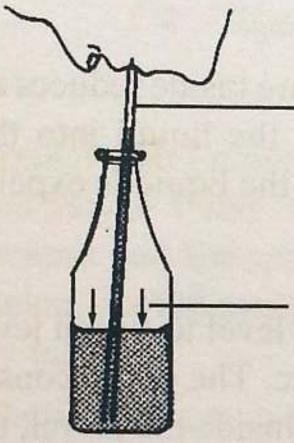


Fig. 5.28: Drinking straw

Syringe

A syringe consists of a tight-fitting piston in a barrel (Fig. 5.29(a)). It is used by doctors to give injections.

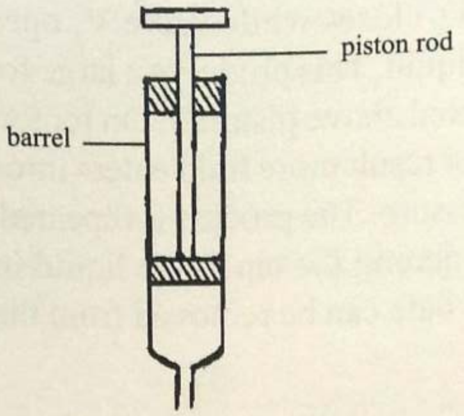
Working of a syringe

Pull and push the piston of the syringe when the nozzle is open. Repeat the procedure with the nozzle closed with one of your fingers. What do you observe? When the nozzle is open the piston moves freely. However, when it is closed the movement of the piston is restricted.

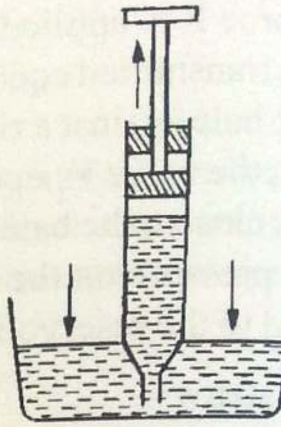
When the piston is pulled with the nozzle open, space is created in the barrel thus lowering the pressure inside. Air from outside is pushed in by the atmospheric pressure. Since the barrel is also open to the outside, both the top and the bottom of the piston is under the same force but in different directions. Hence the piston moves freely. The same happens when the piston is pushed, only that the pressure increases inside the tube and is balanced by the atmospheric pressure once the air is pushed out of the barrel by the piston.

When the nozzle is closed and the piston is pushed, the pressure inside increases and the movement of the piston is restricted.

Repeat the experiment with the nozzle inside a liquid (Fig. 5.29(b)).



(a) Outside a liquid



(b) Inside a liquid

Fig 5.29: A syringe.

When the piston is pulled (upstroke) the pressure inside reduces and the atmospheric pressure on the surface of the liquid pushes the liquid into the barrel. During a downstroke, the pressure inside increases and the liquid is expelled from the barrel.

Lift pump

A lift pump is used to raise liquids from a low level to a high level e.g raising water from a well, drawing paraffin from a drum, etc. The pump consists of a cylindrical metal barrel with a delivery tube (Fig. 5.30). Inside the barrel, there is a piston and two valves. Before starting the pump, the liquid to be drawn is poured on top of the piston in order to have a good air tight seal round the piston and valve 2.

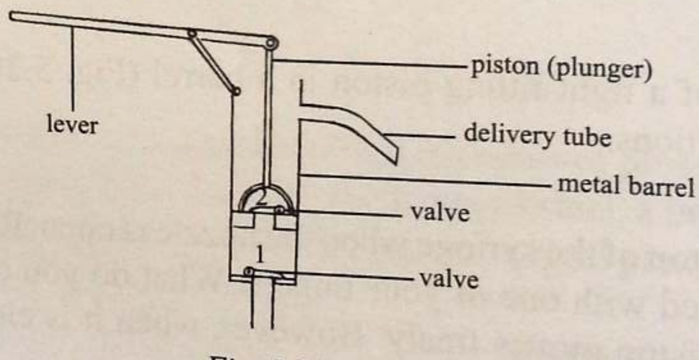


Fig. 5.30: A lift pump

The pump is operated by means of a lever which moves the plunger up and down the barrel.

Upstroke

During the upstroke, the air between valve 1 and 2 expands and its pressure reduces below the atmospheric pressure. The atmospheric pressure on the water surface forces the water up past valve 1 into the space between valves 1 and 2 as shown in Fig. 5.31(a). At the same time valve 2 closes due to its weight.

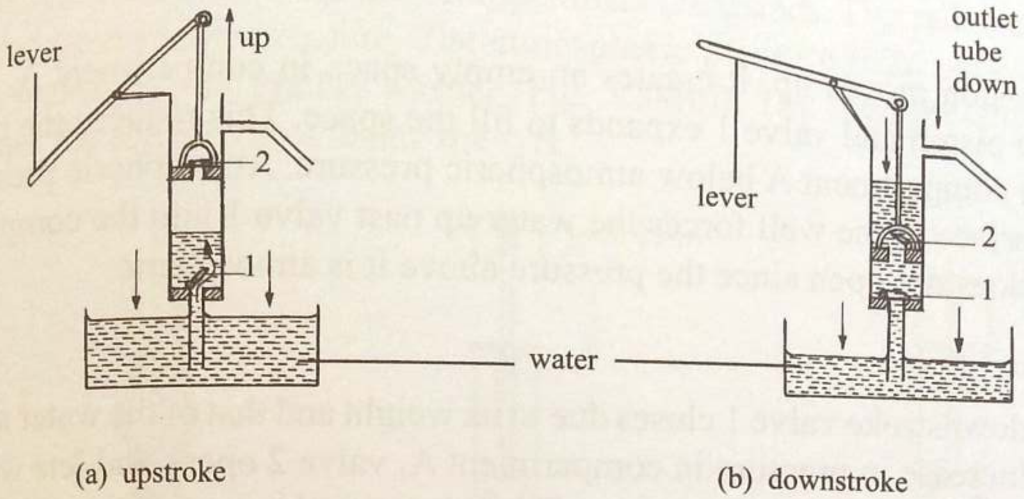


Fig. 5.31: Working of a lift pump

Downstroke

During the downstroke, valve 1 closes due to its weight and the weight of the water in the space between valves 1 and 2. As the plunger moves down, it forces valve 2 to open. Water escapes into the space above valve 2 and out through the pipe to the required destination. As the process is repeated, the plunger lifts the water out through the delivery tube.

Since the atmospheric pressure can only support a water column of about 10 m, this pump cannot raise water above a height of 10 m. The situation becomes even worse when the pump is used in areas well above sea level where the atmospheric pressure is less.

Force pump

A force pump consists of two valves, a solid piston, two compartments A and B and the outlet tube. The pump is operated by moving the solid piston up and down using a lever system (Fig. 5.32).

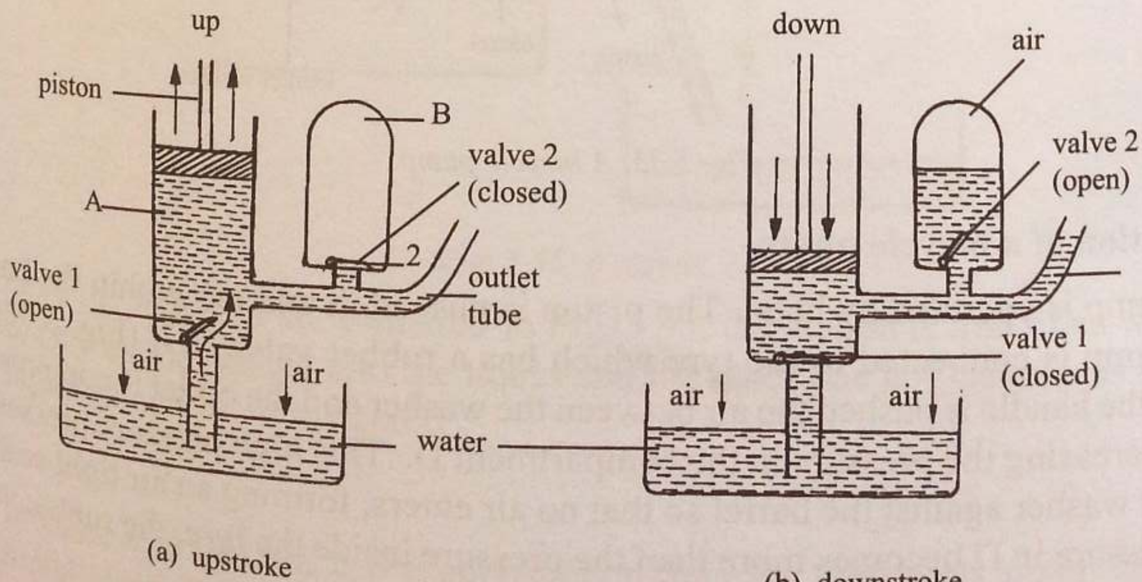


Fig. 5.32: Working of a force pump.

Upstroke

When the piston moves up, it creates an empty space in compartment A. The air between the piston and valve 1 expands to fill the space. This reduces the pressure of the air in compartment A below atmospheric pressure. Atmospheric pressure on the water surface in the well forces the water up past valve 1 into the compartment A. Valve 2 does not open since the pressure above it is atmospheric.

Downstroke

During the downstroke valve 1 closes due to its weight and that of the water above it. Due to the increase in pressure in compartment A, valve 2 opens and lets water out through the outlet tube via compartment B. Air is trapped in compartment B. During the next upstroke, valve 2 closes and the pressure due to the trapped compressed air forces water out through the outlet tube. A continuous flow is thus maintained.

The height to which water can be pushed by the pump does not depend, unlike lift pump, on the atmospheric pressure, but on the amount of the force exerted during the downstroke and the material of compartment B i.e. whether the material can withstand high pressure created by the long column of water in compartment B.

Bicycle pump

A bicycle pump consists of a flexible leather washer at one end of a piston inside a barrel. The pump is used to inflate tubes or balloons whose material are not too hard to expand. The flexible leather washer acts as a valve as well as a piston. Fig. 5.33 shows the parts of a bicycle pump.

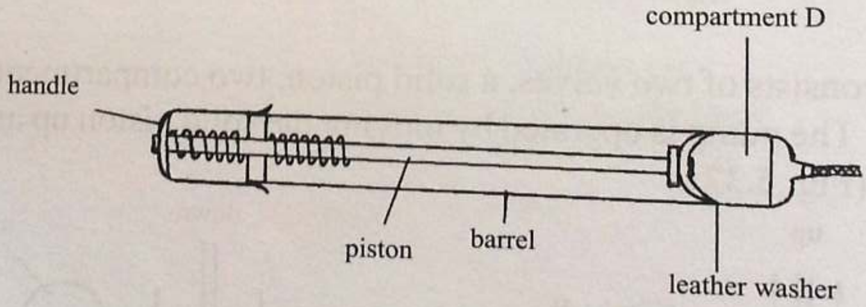
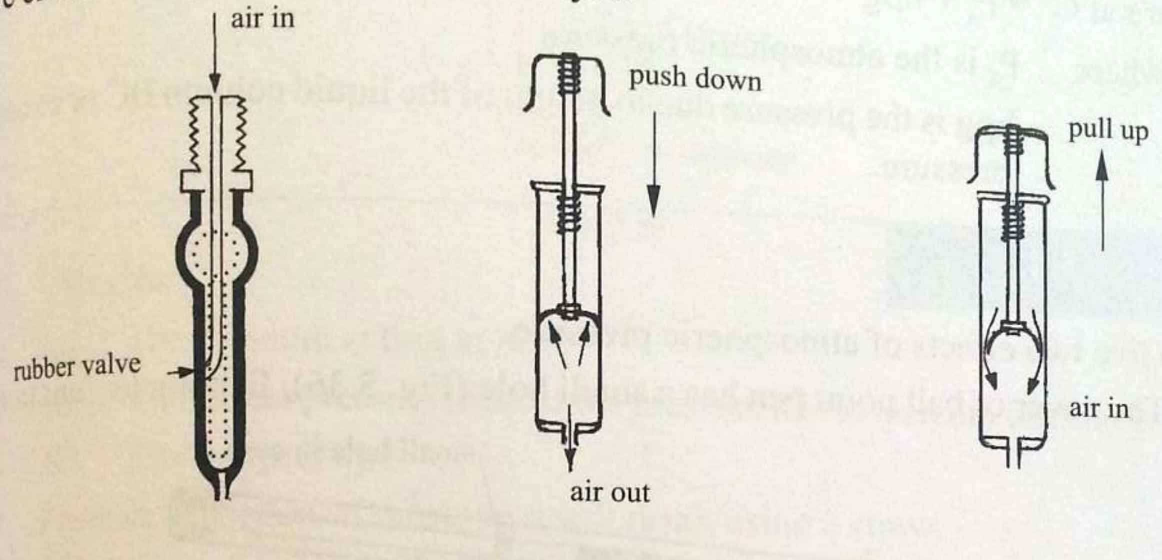


Fig. 5.33: A bicycle pump.

Operation of a bicycle pump

The pump is operated by hand. The piston is pushed in and out within the barrel. The pump is connected to the tyre which has a rubber valve in it (Fig. 5.33(a)). When the handle is pushed the air between the washer and the outlet is compressed, thus increasing the pressure in the compartment D. This high pressure presses the leather washer against the barrel so that no air enters, forming an air tight seal. As the pressure in D becomes more than the pressure inside the tyre, the rubber valve opens and air is forced into the tyre (Fig. 5.34(b)).

When the handle is pulled, the air in compartment D expands. This reduces its pressure below the atmospheric pressure. The atmospheric pressure from outside forces air into the barrel past the leather washer (Fig. 5.34(c)). The high pressure inside the tyre closes the rubber valve inside the tyre.



(a) Rubber tyre valve (b) Handle being pushed (c) Handle being pulled

Fig. 5.34: Operations of a bicycle pump.

Siphon

A flexible tube may be used to empty fixed containers e.g. petrol tanks in cars, which are otherwise not easy to empty directly. When used in this manner, the flexible tube is called a *siphon*. (Fig. 5.35)

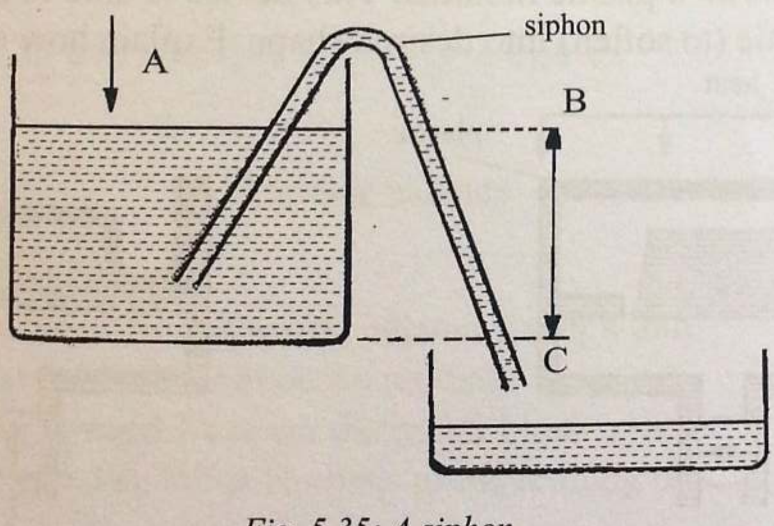


Fig. 5.35: A siphon

To empty the liquid in the container in Fig. 5.35, the siphon is first filled with the liquid. One end is pushed into the liquid and the other one left hanging as shown. The liquid comes out of the end C.

The working

The pressure at points A and B is atmospheric since the two points are on the same horizontal level. The pressure at C in the liquid is greater than the atmospheric pressure by the pressure due to the height of liquid column, BC. This difference in pressure causes the water to flow out of the container.

Pressure at C $= P_A + h\rho g$

where P_A is the atmospheric pressure

$h\rho g$ is the pressure due to height of the liquid column BC or excess pressure.

Exercise 5.4

1. Give two effects of atmospheric pressure.
2. The cover of ball point pen has a small hole (Fig. 5.36). Explain its function.

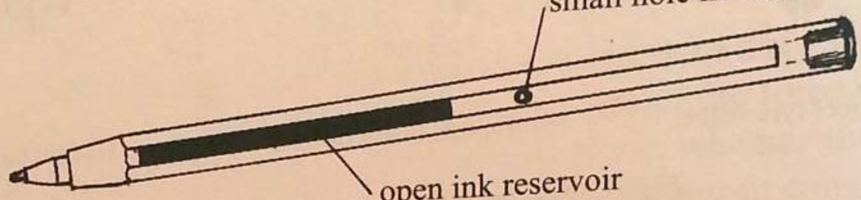


Fig. 5.36

3. A mercury barometer reads 0.76 cmHg. If water is used instead of mercury, find the reading on the barometer. (Take ρ_{water} = 1 000 kg/cm³ and g = 10 N/kg)
4. Fig. 5.37 shows a plastic moulder. This device is able to mould previously heated plastic (to soften) into desired shape. Explain how it works.

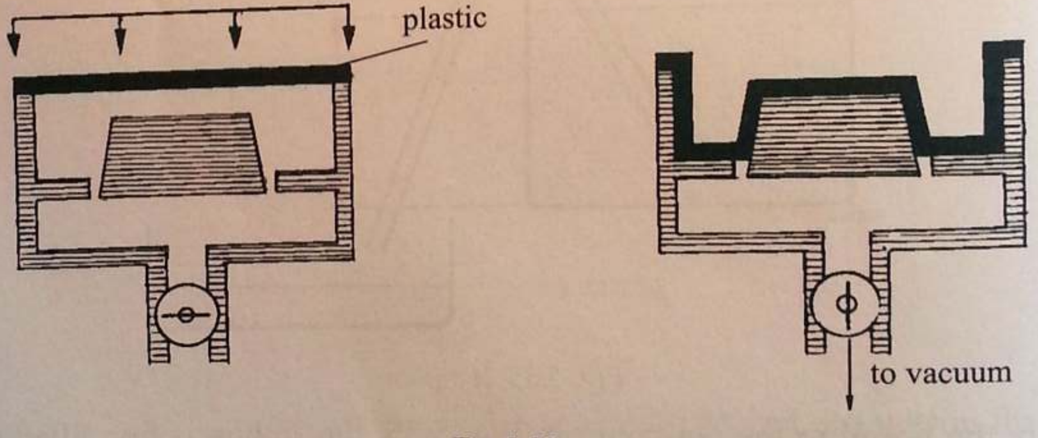


Fig 5.37

5. A mercury barometer in Lusaka reads 747 mm. Calculate the atmospheric pressure in pascals given that g = 10 N/kg and density of mercury is 13 600 kg/m³.
6. Fig. 5.38 is a U-tube manometer used to measure gas pressure. The atmospheric pressure is 1.02×10^5 N/m² and the density of mercury is 13.6 g/cm³.

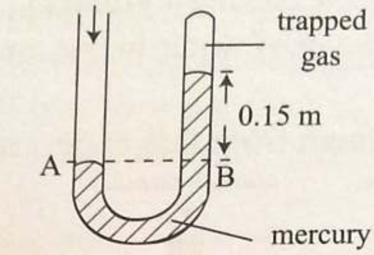


Fig. 5.38

Calculate:

(a) The pressure acting at A.

(b) The excess pressure (hint: is the pressure due to mercury column height).

(c) The gas pressure.

7. Explain the action of drinking a soft drink using a straw.
8. In a bicycle pump the flexible leather washer acts as a piston as well as a valve. Explain.
9. What is the function of the compressed air in the force pump?

5.7 Upthrust and Archimedes' principle

Upthrust

Experiment 5.3: To demonstrate upthrust in liquids

Apparatus

- A metal block
- A spring balance

Procedure

1. Hang a metal block from a spring balance using a thin thread. The spring balance shows a reading.
2. Now apply an upward force on the metal block using your hand (Fig. 5.39). What happens to the reading on the balance?
3. Release the block and allow it to settle. Note its reading (Fig. 5.40(a)).
4. Fill some water into a measuring cylinder. Note the level of the water. Lower the metal block gently into the water.
5. Note the spring balance readings when the cylinder is partially immersed and when it is completely immersed in water (Fig. 5.40 (b) and (c)).

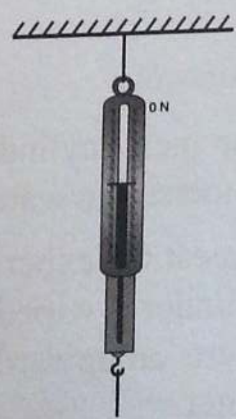


Fig. 5.39: Reading reduced due to applied force

6. Make sure that the metal block is hanging freely (It should not touch the sides or the base of the measuring cylinder). What happens to the spring balance reading and the level of water in the measuring cylinder as the block is lowered?

What conclusions can you drawn from this experiment?

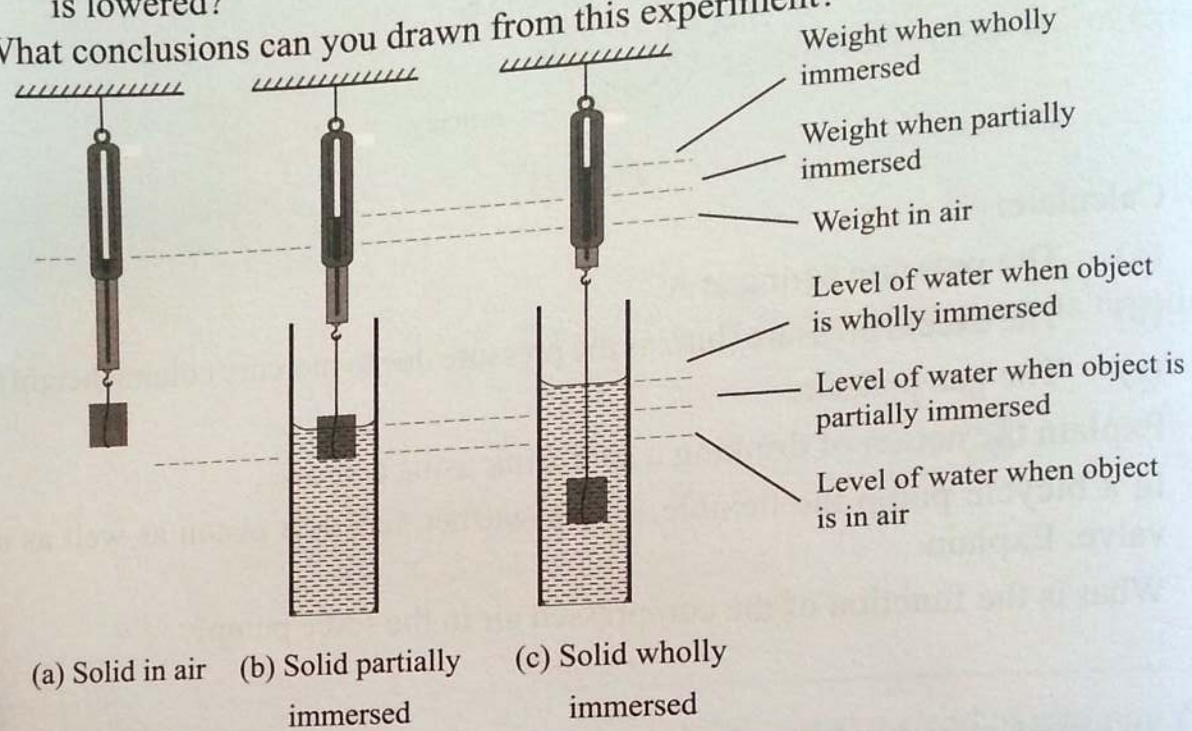


Fig. 5.40: Weight in air and water

Observation

The spring balance reading reduces due to the force applied upward by the hand in step 2 force.

The spring balance reading reduces and the level of the water in the measuring cylinder rises in steps 5 and 6.

Discussion

The metal cylinder experiences an upward force when it is partially or wholly immersed in water and it displaces some water.

Repeat the experiment with kerosene in place of water. Make sure that the metal cylinder is wiped dry before immersing it in kerosene. What happens? Kerosene exerts an upward force on the metal cylinder which is less than the force due to water.

Conclusion

When a body is partially or wholly immersed in a liquid it experiences an upward force. This upward force is called *upthrust*.

Magnitude of the upthrust due to a liquid

Compare the spring balance readings in Experiment 5.3, when the cylinder was weighed in air (W_{air}) and when it was weighed completely immersed in water (W_{water}). The cylinder appears to weigh less in water than in air (Fig. 5.41). The difference between the weight in air and weight in water (a liquid) is known as *apparent loss in weight* of the body.

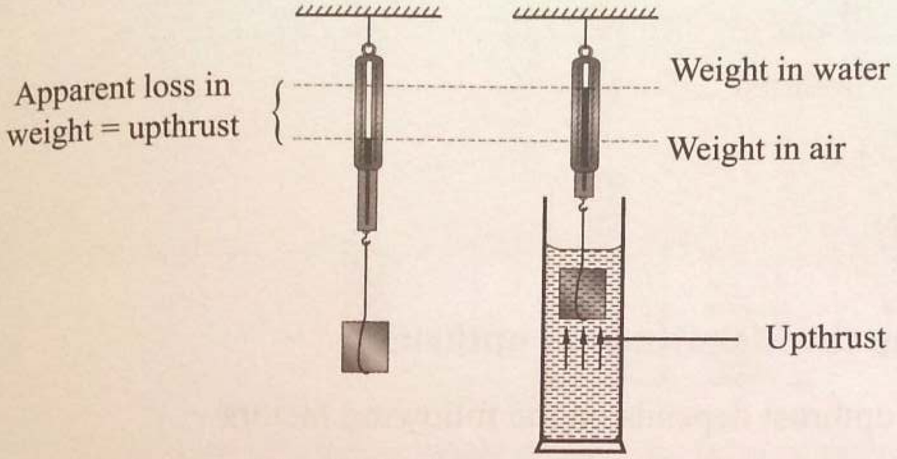


Fig. 5.41: Apparent loss in weight

The magnitude of the apparent loss of weight of a body in a liquid is equal to upthrust exerted by the liquid on the body i.e.

> Apparent loss in weight = upthrust = $W_{air} - W_{liquid}$

Example 5.8

A body of mass 4 kg weighs 30 N in a liquid. Find the upthrust on the body due to the liquid.

Solution

Weight in air = mg = 4 × 10 = 40 N

Weight in liquid = 30 N

Upthrust $= W_{in\ air} - W_{in\ liquid}$

$= (40 - 30)$ N

$= 10$ N

Example 5.9

A body weighs 3.5 N in air. When the body is completely immersed in water the upthrust on the body is 1.6 N. Find the weight of the body in water.

Solution

$$\text{Upthrust} = W_{air} - W_{water}$$

$$W_{water} = W_{air} - \text{upthrust}$$

$$= 3.5 - 1.6$$

$$= 1.9 \text{ N}$$

Factors affecting the magnitude of upthrust

Magnitude of the upthrust depends on the following factors:

- *Density of the liquid.*

  As the density of the liquid increases, the upthrust increases and vice versa i.e a denser liquid exerts greater upthrust on an object thana less dense liquid.

- *The volume of the body immersed in the liquid.*

  The greater the height, and hence the volume of the portion of the object submerged into liquid, the greater the upthrust exerted on the body.

Upthrust and the weight of the liquid displaced

Experiment 5.4: To investigate the relationship between upthrust and the weight of water displaced

Apparatus

- An iron bar
- An overflow can (eureka can)
- A compression balance

Procedure

1. Weigh a uniform iron bar in air. Fill an overflow can (eureka can) with water. Allow the excess water to flow out through the spout.
2. Place an empty beaker on a compression balance under the spout and record its weight.

3. Immerse a quarter of the length (0.25 l) of the iron bar into water (Fig. 5.42). What happens to the water in the eureka can?

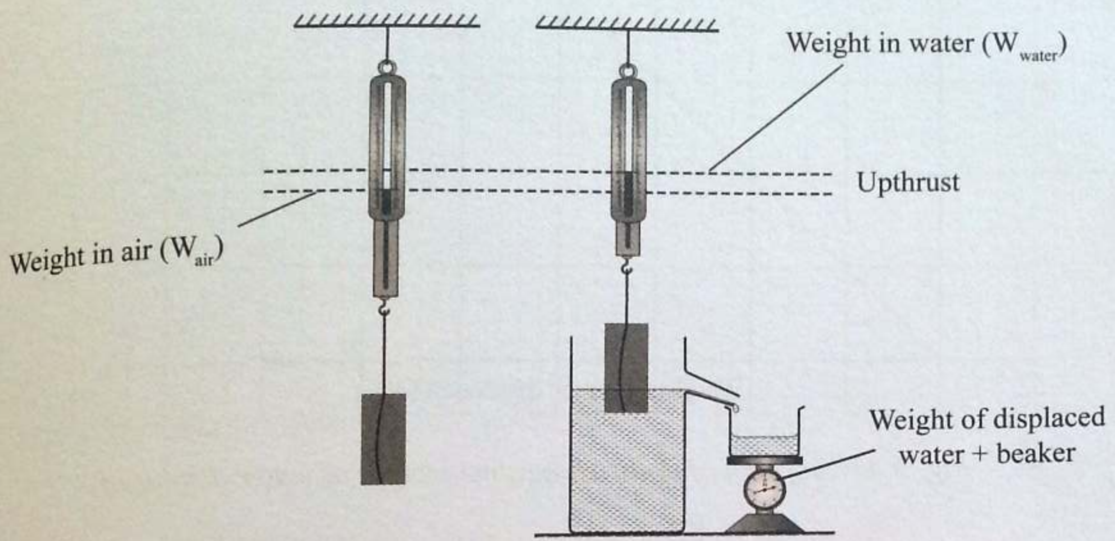


Fig. 5.42: Effect of the weight of liquid displaced on upthrust

4. Record the upthrust and the weight of the liquid displaced as read from the spring balance and compression balance respectively.
5. Repeat the experiment with half length 0.5 l, three quarter length 0.75 l and full length l, of the iron bar immersed in the water. Tabulate your results as in the Table 5.2.

Table 5.2

Portion immersed	**Upthrust = $W_{air} - W_{water}$ (N)**	**Weight of displaced water (N)**
0		
0.25 l		
0.5 l		
0.75 l		
1l		

Plot a graph of upthrust against weight of the water displaced.

Observation

The iron bar displaces some water which goes into the beaker.

Discussion

The graph is a straight line graph passing through the origin (Fig. 5.43). This shows *that upthrust is directly proportional to the weight of liquid displaced.*

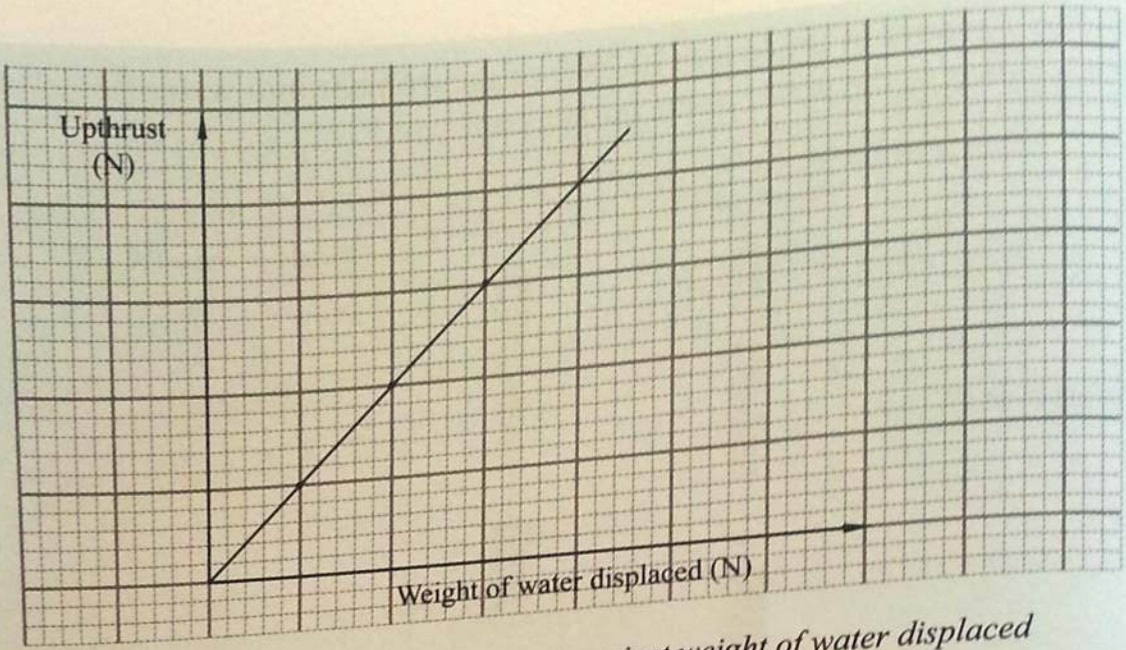


Fig. 5.43: A graph of upthrust against weight of water displaced

The slope of graph is 1.

Therefore upthrust is equal to the weight of water displaced.

Similar experiment with other liquids show similar results i.e.

Upthrust = weight of liquid displaced

Experiments involving gases instead of liquids give results similar to ones obtained using liquids.

Therefore for all fluids

upthrust = weight of fluid displaced.

Archimedes' principle

Experiment 3.4 is a verification of what is called *Archimedes' principle,* which states states that:

When a body is wholly or partially immersed in a fluid, it experiences an upthrust which is equal to the weight of the fluid displaced.

Upthrust = Weight of fluid displaced = Apparent loss in weight

Note

1. The same effect is observed for a partially immersed cube.
2. The forces on the sides are equal but act in opposite directions hence the net force on the sides of the cube is zero.

Example 5.10

A concrete block of mass 2.7×10^3 kg and volume $0.9\ m^3$ is totally immersed in sea water of density $1.03 \times 10^3\ kg/m^3$. Find:

(a) Weight of the block in air.

(b) Weight of the block in sea water.

Solution

(a) Weight in air $= mg$

$= 2.7 \times 10^3 \times 10$

$= 2.7 \times 10^4$ N

(b) Volume of water displaced $=$ volume of the block

$= 0.9\ m^3$

Weight of water displaced $= V\rho g = 0.9 \times 1.03 \times 10^3 \times 10$

$= 9.27 \times 10^3$ N

Upthrust $=$ weight of water displaced

$= 9.27 \times 10^3$ N

upthrust $= W_{air} - W_{liquid}$

$\therefore W_{liquid} = W_{air} -$ upthrust

$= (27 \times 10^3) - (9.27 \times 10^3)$

$= 17.73 \times 10^3$ N

$= 1.77 \times 10^4$ N

Example 5.11

A beaker of mass 150 g containing 700 g of water of density 1 000 kg/m^3 rests on a compression balance calibrated in newton. A stone of mass 200 g and of density $2.7 \times 10^3\ kg/m^3$ is hung from a light thread attached to a spring balance as shown in Fig. 5.44. Calculate the readings on (i) the spring balance and (ii) the compression balance when the stone is,

(a) hanging in air just above the water surface.

(b) fully immersed in water without touching either the sides or the bottom of the beaker.

(c) resting at the bottom of the beaker and the thread in slack. (Take $g = 10\ m/s^2$)

Solution

(a) (i) Spring balance reading

$= \text{weight of the stone in air}$

$= mg = 0.2 \times 10$

$= 2\ \text{N}$

(ii) Compression balance reading

$= \text{weight of water} + \text{weight of beaker}$

$= (m_{water} + m_{beaker}) \times g$

$= (0.70 + 0.150) \times 10$

$= 8.5\ \text{N}$

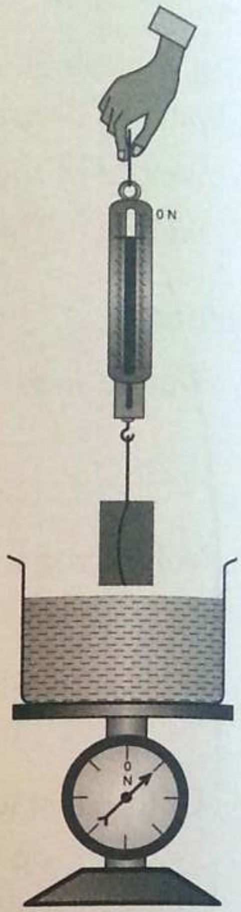


Fig. 5.44

(b) (i) Spring balance reading

$= \text{weight of stone in water}$

$= (\text{weight of stone in air}) - (\text{upthrust})$

Upthrust= weight of water displaced

$= V_{solid} \times \rho_{water} \times g$

$$= \frac{mass_{solid}}{\rho_{solid}} \times \rho_{water} \times g$$

$$= \frac{0.2}{2.7 \times 10^3} \times 1.0 \times 10^3 \times 10 = \frac{2}{2.7}$$

$= 0.74\ \text{N}$

$\therefore$ Spring balance reading $= 2 - 0.74$

$= 1.26$ N (This is the weight of stone in water)

(ii) Compression balance reading

$=$ (weight of beaker and water) + weight of water displaced (upthrust)

$= 8.5 +$ weight of displaced water

$= 8.5 +$ upthrust $= 8.5 + 0.74$

$= 9.24\ \text{N}$

(c) (i) Spring balance reading $= 0$ N

(ii) Compression balance reading

$=$ compression balance reading in b(ii) + weight of stone in water

$= 9.24 + 1.26$

$= 10.5\ \text{N}$

Exercise 5.5

1. (a) What do you understand by the term upthrust in fluids?
   (b) State the Archimedes' principle.
   (c) With the aid of a well labelled diagram describe an experiment to verify Archimedes' principle.

2. A copper solid of mass 250 g and density 8 900 kg/m^3 is fully immersed in a fluid of density 890 kg/m^3 as shown in Fig. 5.45.
   (a) Draw the diagram and indicate the forces acting on the solid.
   (b) Find the tension in the string.
   (c) Explain what will happen to the tension in the string if a liquid of higher density is used.

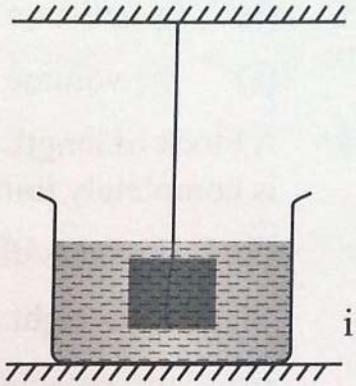


Fig. 5.45

3. A piece of metal of mass 800 g and volume 20 cm^3 is suspended from a spring balance and is completely immersed in a liquid of density 790 kg/m^3. Determine the reading of the spring balance.

4. A metal block of density 7.8 g/cm^3 weighs 117 N in air. Find the weight of the block when wholly immersed in water of density 1.0 g/cm^3.

5. A block of mass 250 g is supported in air from a spring balance. The compression balance reads 0.5 N (Fig. 5.46). The block is then lowered such that it is fully immersed in the liquid of density 1.03×10^3 kg/m^3. A volume of 100 cm^3 of the liquid is collected in the cylinder.

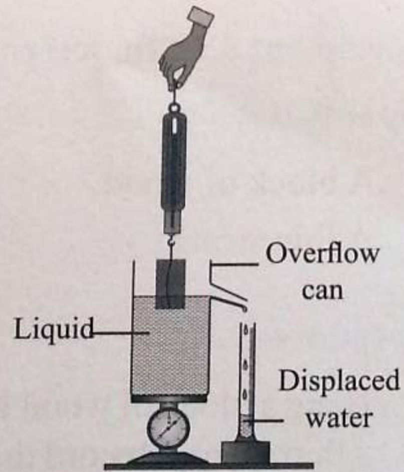


Fig. 5.46: Displacement method

   (a) Calculate the density of the block.
   (b) What is the spring balance reading?
   (c) What is the compression balance reading?
   (d) Calculate the reading of the spring balance when the bock is immersed.

6. A metal cube of mass 640 g and density 2.7 g/cm^3 is suspended half immersed in a liquid of density 0.8 g/cm^3 using a thread. Find the tension in the thread.

7. A cube of side 5 cm weighs 18.7 N in air. Calculate
   (a) the density of the material making the cube.

(b) its apparent weight when completely immersed in a liquid of density 0.80 g/cm^3.

8. A spring has unstretched length of 185 mm. When a piece of metal of mass 1.5 kg is attached to the spring, the spring extends to a length of 204 mm. When the metal is completely immersed in water of density $1\,000 \text{ kg/m}^3$ its length becomes 175 mm. Find

   (a) the force needed to produce an extension of 5 mm,

   (b) upthrust on the metal,

   (c) the volume of the metal.

9. A block of length 30 cm, cross-sectional area of 3 cm^2 and density 1.3 g/cm^3 is completely immersed in a liquid of density 1.03 g/cm^3. Find

   (a) the mass of the block,

   (b) the weight of the block in the liquid,

   (c) the apparent weight of the block if three quarters of it is immersed.

5.8 The law of flotation

Let us consider a *special case of Archimedes' principle* in which an object is floating on a fluid.

Experiment 5.5: To determine upthrust for a floating body

Apparatus

- A block of wood
- A thin string
- A spring balance

Procedure

1. Hang a block of wood from a spring balance using a thin string. Record the weight of the block in air.
2. Lower the block gradually into an overflow can filled with water. Collect the displaced water using a measuring cylinder as shown in Fig. 5.47. What happens to the apparent weight of the wooden block?

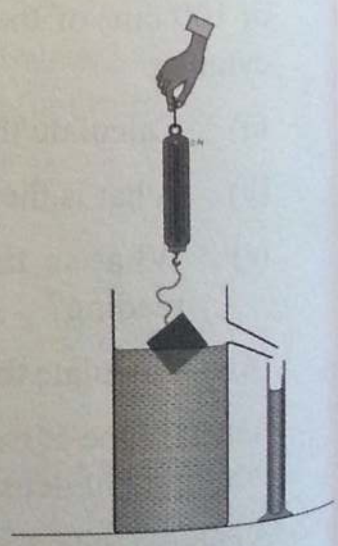


Fig. 5.47

Discussion

The apparent weight gradually reduces and finally falls to zero when the string becomes slack. At this moment the block is said to be floating. Since the tension in the string is zero, it is only the upthrust that is supporting the whole weight of the block.

Determine the weight of the water displaced (upthrust). Compare the weight of the water displaced and the weight of the block in air. The two are found to be equal. Similar results are obtained when the experiment is repeated with other liquids and gases.

The above results can be summarised in what is known as the *law of flotation*. The law states that:

A freely floating body displaces a fluid of weight which is equal to its own weight

Weight of a floating body = weight of fluid displaced

The law of flotation is a special case of the Archimedes' principle in that *the net force on the body is zero.*

Example 5.12

An object floats in sea water of density 1.03×10^3 kg/m^3. It displaces 260 m^3 of sea water. Find the mass of the object.

Solution

Weight of body = weight of sea water displaced

Mass of body = mass of sea water displaced

= density × volume

$= (1.03 \times 10^3 \times 260)$ kg

$= 2.68 \times 10^5$ kg.

Example 5.13

A cube of wood of mass 90 g and side 6 cm floats in water as shown in Fig. 5.48. What is the length of the cube under water? (Density of water = 1.0 g/cm³)

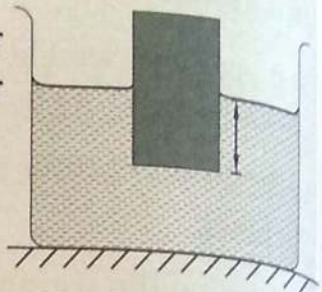


Fig. 5.48: A cube immersed in water

Solution

Let the volume be V cm³

Weight of block = weight of water displaced

Mass of block = mass of water displaced

= volume × density

$$90 = V \times 1.0$$

$$\therefore \quad V = 90 \text{ cm}^3$$

Cross-sectional area A = 36 cm²

$$V = l \times A = 90 \text{ cm}^3$$

$$\therefore \quad l = \frac{90}{36} = 2.5 \text{ cm}$$

Example 5.14

A glass block of mass 500 g and density 2.5 g/cm³ floats in a liquid with 18.5% of its volume submerged in the liquid. Calculate the volume of water displaced.

Solution

$$\text{Volume of glass} = \frac{\text{mass of glass}}{\text{density of glass}}$$

$$= \frac{500}{2.5}$$

$$= 200 \text{ cm}^3$$

$$\text{Volume under water} = \frac{18.5}{100} \times 200$$

$$= 37.0 \text{ cm}^3$$

5.9 Applications of the law of floatation

Floating of ships

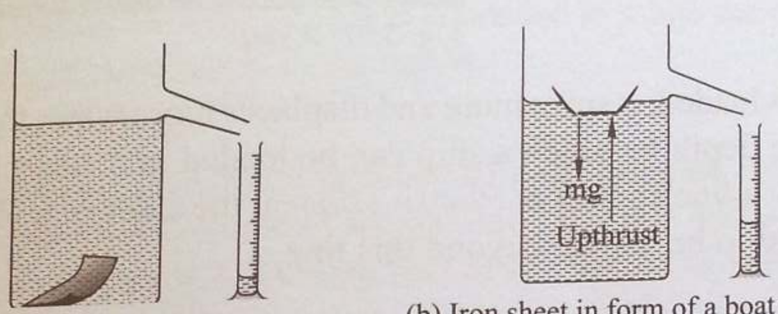
Experiment 5.6: To investigate why the ship made from iron floats but a piece of iron sinks in water

Apparatus

- A thin sheet of iron
- An overflow can
- Water

Procedure

1. Take a thin sheet of iron and place it on water contained in an overflow can and collect the water displaced (Fig. 5.49 (a)). What happens?
2. Measure and compare the weight of water displaced and the weight of the iron sheet.



(a) iron sheet sinks in water (b) Iron sheet in form of a boat floats

Fig. 5.49: Demonstrating application of the laws of floatation.

Observation

The weight of the displaced water is less than the weight of the iron sheet. Hence there is a resultant downward force that pulls the iron sheet to the bottom of the overflow can (Fig. 5.49 (a)). Also the density of iron is more than the density of water, hence iron sinks in water.

Now shape the iron sheet into a shape of a boat (Fig. 5.49 (b)) and repeat the experiment. What happens now?

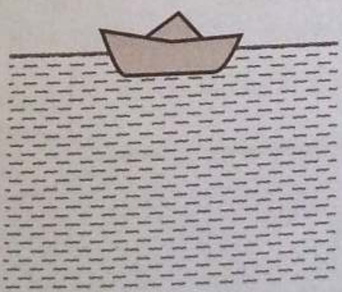


Fig. 5.50: A ship made of metal floats in water

The boat 'floats' on water displacing some water (See Fig 5.50).

Measure and compare the weight of the boat and the weight of the water displaced. The two weights are found to be equal.

Discussion

The upthrust is enough to support the weight of the boat. The boat is hollow i.e. it consists of steel and air. Its average density is less than the density of water. The hollow steel displaces more volume of water than its own volume. The same argument may be employed to the floating of a ship (Fig. 5.51).

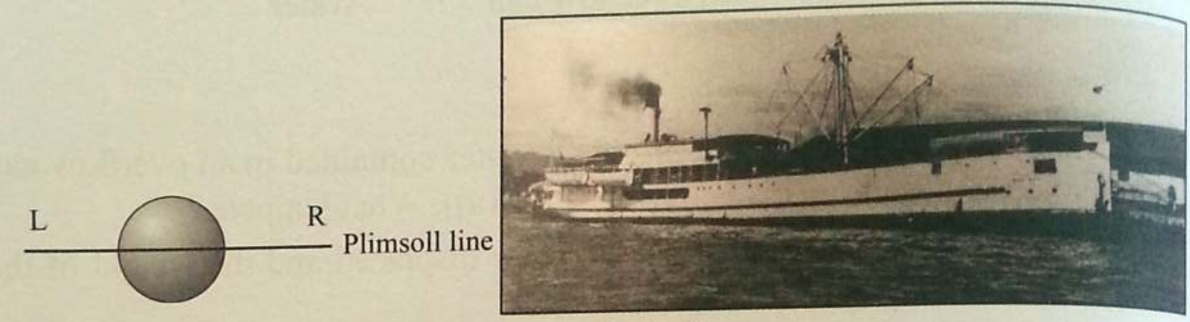


Fig. 5.51: A ship

When a ship is loaded, it sinks more and displaces more water to balance the added load. The safe depth to which a ship can be loaded in different seas and seasons are marked by a line known as *Plimsoll line* on the sides of a ship (Fig. 5.51). No ship is allowed to be loaded beyond this line.

Submarines

Submarines are types of ships which can float and sink in water. Submarines have internal tanks called *ballast tanks* which can be filled with water or air (5.52). A submarine is made to sink by admitting water into the ballast tanks (Fig. 5.52 (a)). It can be made to float by expelling water from the tank by compressed air. This makes the average density of the submarine less than that of sea water and hence the submarine floats (Fig. 5.52 (b)).

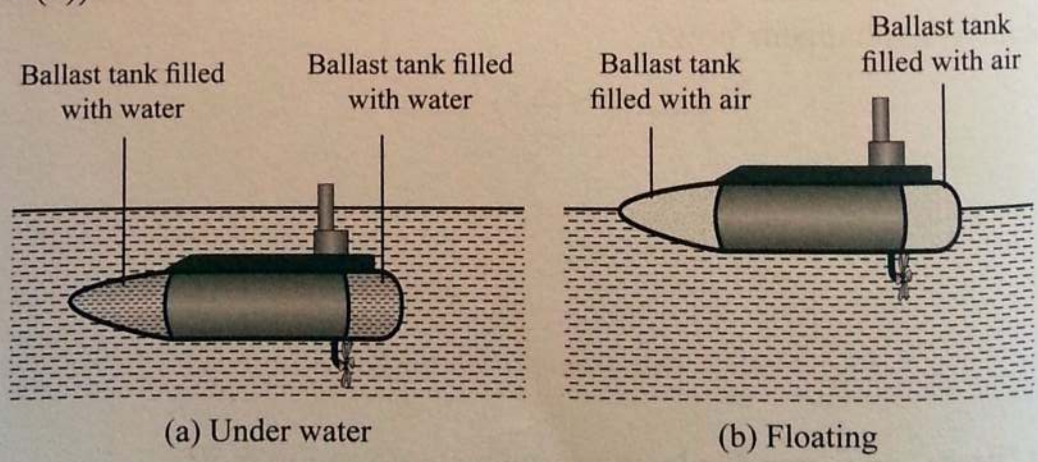


Fig. 5.52 : Submarines

Exercise 5.6

1. Fig. 5.53 shows a hollow spherical body of volume 20 litres and mass 7.5 kg anchored by a light cable in a liquid of density 1030 kg/m^3. The sphere has 20% of its volume in air.
   (a) Copy the diagram and indicate all the forces acting on it.
   (b) Find the tension in the cable.

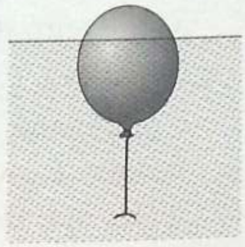


Fig. 5.53

2. A block of wood weighs 2.78 N in air. If the density of wood is 0.75 g/cm^3 determine;
   (a) the volume of the block immersed when it floats in a liquid of density 0.89 g/cm^3
   (b) the minimum vertical force that is needed to make the block just to be submerged completely in the liquid.
3. State the law of flotation.
4. A weather forecasting balloon is made of an extensible material of mass 9.5 kg. The maximum volume of the balloon at s.t.p is 450 m^3. The balloon is filled with hydrogen that takes up half of its maximum volume.
   (a) Find the heaviest load that this balloon can lift,
   (b) Find the density of the surrounding air when the balloon carrying weather equipment of mass 7 kg stops moving,
   (c) Name two factors that determines the height the balloon can rise.
   (Density of air = 1.29 kg/m^3 and density of hydrogen = 0.09 kg/m^3)
5. A fishing boat is at rest on sea water with a steel anchor over the side. Explain what would happen to the volume of displaced water by the boat when
   (a) the anchor is lowered and left hanging in water,
   (b) rests on the bed at the sea.

5.10 Applications of Archimedes' principle and relative density

We use the Archimedes' principle to measure the relative density of substances. The following Activities will help us to understand how to do this.

Activity 5.10

- Fill an overflow can with water up to the spout.
- Measure the mass of the given solid (e.g. a metal cube whose volume is known).
- Place a previously weighed dry clean beaker under the spout.

- Lower the solid gently into a overflow can (Fig. 5.54).
- Make sure that the solid is wholly immersed in the liquid.
- Collect the water displaced using a dry clean beaker. How does the volumes of the displaced water and the solid compared?
- Determine the mass of the displaced water.

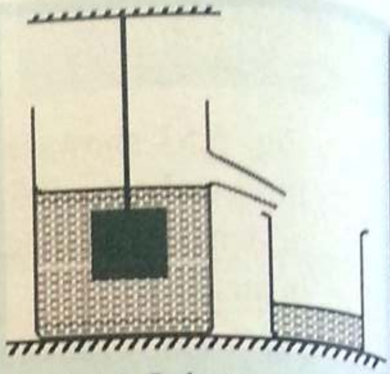


Fig. 5.54: Relative density of a solid by Archimedes' priciple

In Activity 5.10, you must have noticed that when a solid is wholly immersed in water, the volume of the solid is equal to the volume of water displaced.

Taking the density of water as 1 g/cm^3, the volume of water displaced is equal to the mass of the water displaced i.e,

$$\therefore \text{ Relative density} = \frac{\text{mass of solid in air}}{\text{mass of an equal volume of water}}$$

$$= \frac{\text{mass of solid in air}}{\text{mass of water displaced}} = \frac{\text{weight of solid in air}}{\text{weight of water displaced}}$$

weight of water displaced = apparent loss in weight = upthrust (Archmedes' principle)

$$\text{Relative density} = \frac{\text{weight of solid in air}}{\text{upthrust in water}}$$

The ratio tells us how many times a substance is more or less dense than water. Relative density has no units since it is a ratio.

Example 5.15

Concentrated sulphuric acid has a density of 1.8 g/cm^3. Find its relative density.

Solution

$$\begin{array}{l}\text{Relative density} \\ \text{(concentrated sulphuric acid)}\end{array} = \frac{\text{density of concentrated sulphuric acid}}{\text{weight of water displaced}} = \frac{1.8\ g/cm^3}{1\ g/cm^3}$$

$$= 1.8$$

This means that 1 cm^3 of sulphuric acid is 1.8 times denser than 1 cm^3 of water. Hence sulphuric acid is more dense than water.

Note: The density of a material in grams per cubic centimetres is thus numerically equal to its relative density.

Example 5.16

A body of mass 3 kg weighs 22 N in kerosene and 20 N in water. Find

(a) the relative density of kerosene.

(b) the density of kerosene in kilograms per cubic metres.

Solution

(a) Weight of solid in air = 3 × 10 N

Loss of weight in kerosene = 30 – 22 = 8 N

Loss of weight in water = 30 – 20 = 10 N

$$\text{Relative density of kerosene} = \frac{\text{upthrust in kerosene}}{\text{upthrust in water}} = \frac{8}{10} = 0.8$$

(b) $$\text{Relative density} = \frac{\text{density of substance}}{\text{density of water}}$$

$$0.8 = \frac{\text{density of substance}}{1\ \text{g/cm}^3}$$

$$\begin{aligned}\text{Density of kerosene} = 0.8 \times 1/\text{gcm}^3 &= 0.8\ \text{g/cm}^3 \\ &= 0.8 \times 10^3\ \text{kg/m}^3 \\ &= 800\ \text{kg/m}^3\end{aligned}$$

Example 5.17

A piece of metal of mass 135 g weighs 0.80 N in water and 0.85 N in palm oil. Find

(a) relative density of palm oil.

(b) density of the metal.

Solution

$$\begin{aligned}\text{Weight of metal in air} = mg &= 0.135\ \text{kg} \times 10\ \text{N/kg} \\ &= 1.35\ \text{N}\end{aligned}$$

(a) $$\text{Relative density of liquid} = \frac{\text{upthrust in liquid}}{\text{upthrust in water}} = \frac{1.35 - 0.85}{1.35 - 0.80}$$

$$= \frac{0.50}{0.55} = 0.91$$

(b) Relative density of solid $= \dfrac{\text{weight in air}}{\text{upthrust in water}}$

$$= \frac{1.35}{0.55} = 2.45$$

$$\text{Relative density} = \frac{\text{density of substance}}{\text{density of water}}$$

$$\therefore \text{ Density of solid} = \text{relative density} \times \text{density of water}$$

$$= 2.45 \times 1 \text{ g/cm}^3$$

$$= 2.45 \text{ g/cm}^3$$

Hydrometer

The method of measuring relative density of liquids we have learnt so far involves the measurement of mass and/or volume of a substance. It is then followed by the use of certain equations to calculate the relative density. These methods are slow and cumbersome. This can be avoided by using an instrument called a *hydrometer*. A hydrometer is an instrument for measuring the relative density of a liquid directly. This instrument is quick and convenient to use.

A hydrometer consists of an upper stem and lower bulb. Both the stem and the bulb contain air.

Upper stem

The upper stem consists of a hollow narrow glass tube and a scale on the inside of the tube. The thin stem gives the instrument a greater sensitivity. The scale is graduated in relative densities not evenly spaced out. The scale calibration starts from the top going downwards, towards the bulb.

Lower bulb

The lower bulb consists of a wide bulb which is loaded with lead shots to keep the hydrometer upright in the liquid. Fig. 5.55 shows a hydrometer floating in a liquid. The bulb is wide so as to displace a reasonable amount of the liquid making it experience a reasonable amount of upthrust. A hydrometer will sink more in a liquid of less relative density and less in liquid of more relative density.

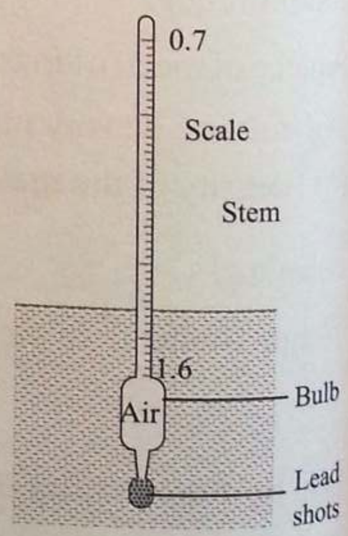


Fig. 5.55: A hydrometer floating in a liquid

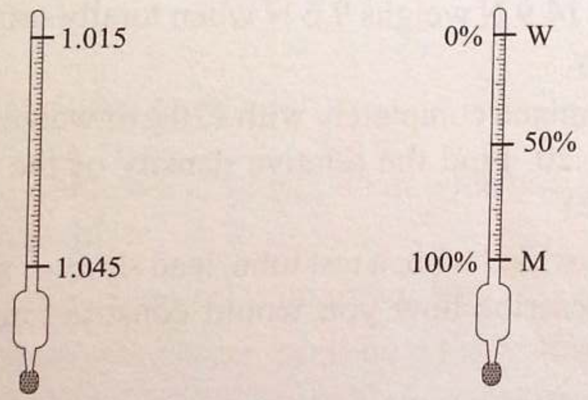
Types of hydrometers

Hydrometers are usually made in sets, each covering a given range depending on use e.g. 0.7–1.0; 1.0–1.3; 1.3–1.6; 1.6–2.0. For a given hydrometer, the bulb must sink so that the stem is above the liquid surface. Types of hydrometers include *lactometer*, *spirits hydrometer* and *car acid hydrometers* among others.

Lactometer

Lactometer is a hydrometer that is used to test the purity of milk. i.e. to check whether any water has been added to milk. A lactometer has a range from 1.015 to 1.045 (Fig. 5.56 (a)). Pure milk has a relative density of 1.030.

The graduations in a lactometer are usually marked in percentage purity of milk. This ranges from 0% to 100%. The mark W represents zero purity of pure milk and M for 100% purity of pure milk (Fig. 5.56 (b)).



(a) Range in a Lactometer (b) % purity of milk graduation

Fig. 5.56: Lactometer

Spirits/wines/beer hydrometer

This hydrometer determines the percentage of alcohol in beers, wines and spirits.

A car acid hydrometer

This hydrometer is used to test the state of charge of acid batteries. This is done by measuring the relative density of sulphuric acid in the battery.

The hydrometer is enclosed in a glass tube fitted with a rubber bulb (Fig. 5.57).

The rubber bulb is squeezed to expel the air from the glass tube so as to reduce the pressure. On releasing the bulb the atmospheric pressure forces the acid up into the glass tube. The hydrometer is made to float and relative density can be read. When fully charged the

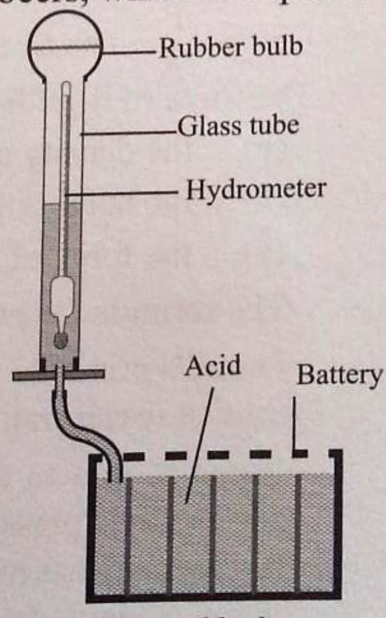


Fig. 5.57: Acid hydrometer

relative density is 1.25. If the relative density of the acid is 1.18, the battery needs recharging. In most cases the hydrometer has a red band around the top of the stem. If the acid level reaches this band, the battery requires recharging.

Exercise 5.7

1. Define the term relative density.
2. A body of weight 2.2 N weighs 1.5 N in water, 1.48 N in a liquid A and 1.58 N in liquid B. If the volume of the body is 0.02 m^3, calculate the:
   (a) Density of the body,
   (b) Relative density of liquid A,
   (c) Density of liquid B.
2. A solid of weight 14.9 N weighs 9.6 N when totally immersed in water. Find its relative density.
4. 80 g of a liquid is mixed completely with 120 g of water. If the relative density of the liquid is 1.20. Find the relative density of the mixture. (assume no change in volume)
5. You have been provided with, a test tube, lead shots or sand, a cylinder, water and kerosene. Describe how you would construct and calibrate a simple hydrometer.
6. Draw and label a diagram of a hydrometer. Explain why the stem is narrow.

Unit summary

- Pressure is defined as a force acting perpendicularly per unit area.
- $P = \frac{F}{A}$. The SI unit of pressure is pascal (Pa). 1 pascal is equal to 1 N/m^2.
- **Pressure in liquids depend on:**
  (a) the density of liquid (ρ)
  (b) the height of column of liquid, h
  (c) the force of gravity
- The formula for pressure in a liquid is given by, $p = h\rho g$
- Pascal's principle states that the pressure applied at one point in an enclosed fluid at rest is transmitted equally to all parts of the fluid.
- Pressure due to air column acting on the surface of the earth is called atmospheric pressure.
- Air pressure has many applications in every day life situations. e.g in a bicycle pump, siphon, drinking straw, syringe, force and lift pumps, etc.

- The upward force due to a fluid is called upthrust.
- Upthrust, $U = W_{air} - W_{liquid}$
- The magnitude of upthrust depends on: The density of the fluid and the volume of body immersed.
- Archimedes' principle states that when a body is wholly or partially immersed in a fluid, it experiences an upthrust which is equal to the weight of the fluid displaced.
- Law of flotation states that a floating body displaces a fluid of weight equal to its own weight.

Unit Test 5

1. Define pressure and give its SI unit.
2. A block of iron measures 6 cm long, 8 cm wide and 1.0 cm deep and has a mass of 360 g. Calculate
   (a) The block's minimum and maximum area.
   (b) The greatest pressure it can exert on a flat horizontal surface.
3. A car's four tyres, each of area 0.06 m^2, are in contact with the road. Calculate the pressure exerted by each tyre if the car has a mass of 1 200 kg.
4. A metal cube of mass 68 kg exerts a pressure of 17 000 Pa on a flat horizontal ground. Calculate:
   (a) The area in contact with the ground.
   (b) The dimension of the cube.
5. Explain the following:
   (a) A hippopotamus walks in a swampy area without sinking but a person who is lighter sinks.
   (b) Trucks carrying heavy cargoes have many tyres.
6. In Hare's apparatus, the height of water column in one tube is 80 cm and the oil column in the other tube is 1.0 m. Calculate the density of oil used, if density of water is 1 000 kg/m^3.

7. Fig. 5.58 shows a hydraulic machine. What load can the machine lift if an effort of 15 N is applied in the diagram?

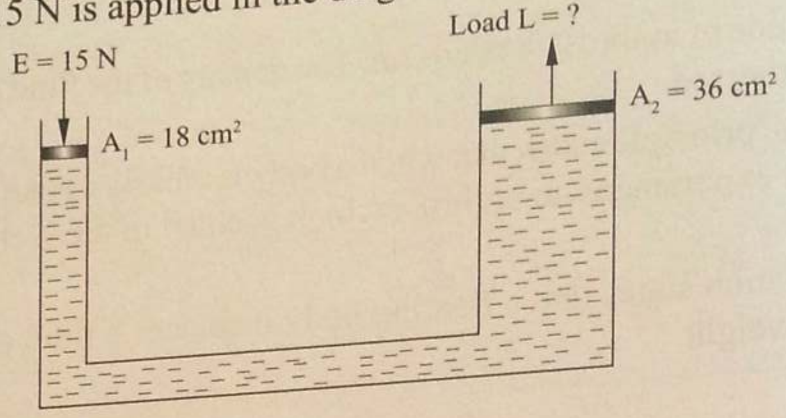


Fig. 5.58

8. A u-tube manometer sealed at on end is used to measure gas pressure (see Fig. 5.59).

   If the atmospheric pressure P_A is 1.02×10^5 N/m², density of mercury is 13 600 kg/m³ and g = 10 N/kg.

   Calculate the pressure of the trapped gas. (Take density of water as 1 000 kg/m³ or 1g/cm³).

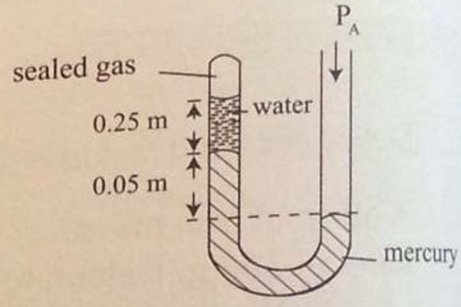


Fig. 5.59

9. **(a)** State the law of flotation.
   (b) Describe how the Archimedes' principle can be verified experimentally.
10. A body of mass 0.75 kg is suspended in water using a light thread. If the tension in the thread is 5.45 N. Find volume of the solid.
11. A specimen of an alloy is obtained by mixing silver and gold. The alloy weighs 35.2 N in air and 33.13 N in water. Determine the upthrust due to water.
12. The upthrust on a body that is immersed in water is 255 N. If the body weighs 945 N in air, calculate the weight of the object while in water.
13. A body of mass 26 kg weighs 94.6 N in a liquid. Find the upthrust on the body while in the liquid.
14. A building block of mass 9.17×10^4 kg and volume 1.1 m³ is totally immersed in a salty water lake. The density of the salty water is 1.03×10^3 kg/m³. Find the:
   (a) Weight of the block in air,
   (b) Weight of the block in sea water.

15. Fig. 5.60 shows a uniform metre rule balanced by rectangular glass block that is totally immersed in oil of relative density 0.89. The glass block has a volume of $2.1 \times 10^{-2}\,m^3$.

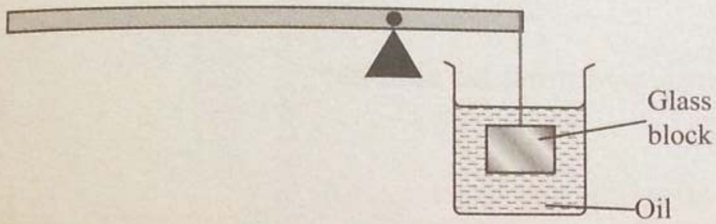


Fig. 5.60

Find the position of the pivot if the mass of the rule is 6.76 kg and density of glass block is 2 500 kg/m^3

16. Fig. 5.61 shows a hollow cylindrical tube of mass 26 g closed on one end. If the tube has a cross-sectional area of 2.5 cm^2,

(a) find the mass to be added into the tube to make it float with its axis vertical and with 15 cm of its length submerged in water.

(b) what length would be immersed, if the tube is transferred into a liquid of relative density 1.2?

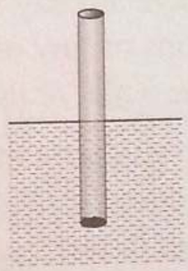


Fig. 5.61

17. A metal sphere floats in water with $\frac{7}{8}$ of its volume submerged. The sphere is transferred in a liquid where it floats with $\frac{3}{4}$ of its volume submerged.

(a) Explain why the sphere should be hollow.

(b) Calculate the relative density of the liquid.

Success Criteria

By the end of this unit, you must be able to:

- Discuss gas laws
- Explain applications of gas laws

Introduction

We have already learnt that according to kinetic theory of matter, a gas is made up of millions of tiny moving particles. They move about freely at high speed and bounce off the walls of the container. The higher the temperature, the faster they move.

When studying the behaviour of a fixed mass of a gas, three important quantities are considered: *pressure, volume, and temperature*. Depending on the circumstances, a change in one quantity produces a change in one or both of the other two.

In this unit, we will study the laws that relate to these changes. These laws are known as *Gas laws*. It is worth noting that strictly speaking, gas laws only apply to an ideal gas which has no attractions between its molecules. Real gases only behave as ideal gases when at high temperature and low pressure. Room temperature and pressure (25°C and 760 mm Hg) is just sufficient for real gases to obey gas laws.

6.1 Boyle's law

When the nozzle of a bicycle pump is closed with a finger and the piston slowly pressed inwards, the pressure on the finger becomes increasingly unbearable. At a certain point, the gas just escapes. Let us investigate the pressure and volume of a fixed mass of a gas, by carrying out experiment 6.1.

Experiment 6.1: To investigate how the volume and pressure of a fixed mass of gas are related at constant temperature

Apparatus

- Foot pump
- strong glass tube
- reservoir fitted with a gas tube closed with a tap
- bourdon gauge
- calibrated volume scale
- oil

Procedure

1. Arrange the apparatus as shown in Fig. 6.1.

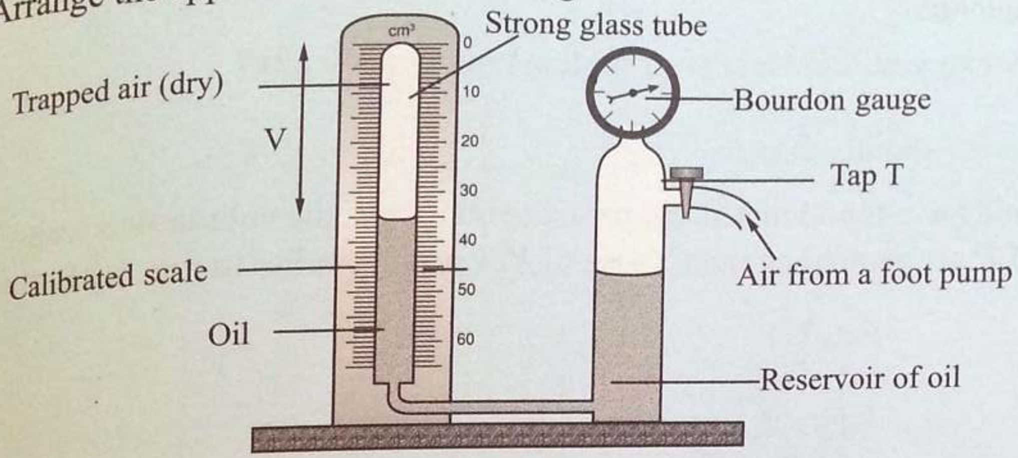


Fig. 6.1: Investigating Boyle's law

2. Open tap T. Read and record the volume of the trapped air in the glass tube. Read and record the pressure of the air above the oil in the reservoir from Bourdon gauge. Since the pressure above the oil in the reservoir is transmitted through the oil to the trapped air, the pressure of the trapped air is the same as the pressure above the oil in the reservoir.
3. Repeat the experiment by gradually increasing the pressure (using a foot pump) of the air trapped in the glass tube to 2, 3, 4, 5 and 6 kPa.
4. Repeat the step 2 by decreasing the pressure to 5, 4, 3, 2, and 1 kPa by allowing some air to escape through tap T. Close the tap each time the desired pressure has been attained.
5. Record the pressure and the corresponding volume in a table (Table 6.1). Calculate the average volume and complete Table 6.1.

Table 6.1

Increasing pressure		**Decreasing pressure**		**Average volume, V**
Pressure (p) (kPa)	Volume (V_1) (cm³)	Pressure (kPa)	Volume (V_2) (cm³)	$V = \dfrac{V_1 + V_2}{2}$ (cm³)
1		1		
2		2		
3		3		
4		4		
5		5		
6		6		

6. Plot a graph of pressure, P against volume, V. From the graph calculate the product PV. What is the relationship between pressure and volume?

7. What can you say about the product of pressure and volume?
8. Now, plot a graph of V against $\frac{1}{P}$. From the graph determine what the gradient is equal to.
9. How can you represent your findings mathematically?

Observations and discussions

You should have found that as the pressure increases, the volume decreases. When values of P are plotted against those of V, a graph similar to Fig. 6.2 is obtained.

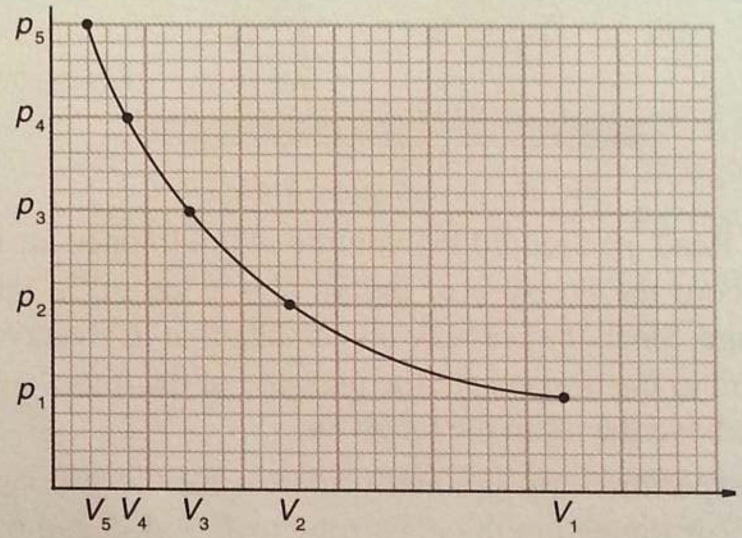


Fig. 6.2: Graph of pressure against volume

The product PV at any chosen set of values is found to be a constant.

Thus, the product can be written as:

$V \propto \frac{1}{P}$ which implies that ;

$$P_1 V_1 = P_2 V_2 = P_3 V_3$$

i.e. PV = constant

A graph of pressure P, against $\frac{1}{V}$ is a straight line through the origin (See Fig. 6.3).

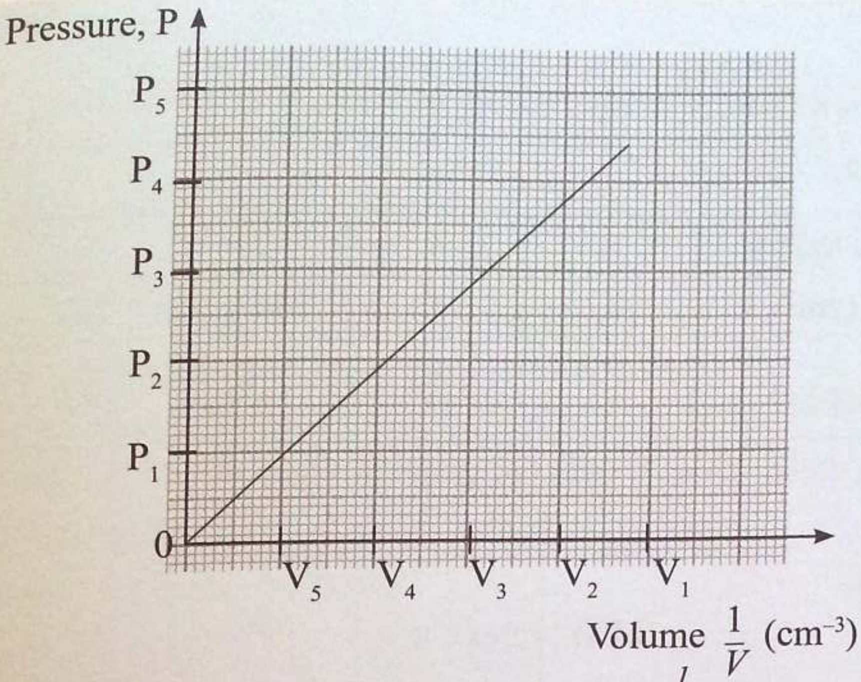


Fig. 6.3: A graph of pressure, P against $\frac{1}{v}$

Where the gradient, $S = \dfrac{\Delta P}{\Delta \frac{1}{V}} = \Delta P \Delta V$

The gradient is equals to PV which is a constant.

Conclusion

The volume of a fixed mass of a gas is inversely proportional to its pressure i.e.

$V \propto \frac{1}{P}$. This relationship is summarised in Boyle's law.

Boyle's law states that *the volume of a fixed mass of a gas is inversely proportional to the pressure, provided that the temperature remains constant. i.e. PV = constant.* Mathematically, Boyle's law is represented as $P_1V_1 = P_2V_2$.

Example 6.1

Table 6.2 shows the results obtained in an experiment to investigate the relationship between the pressure and the volume of a given mass of gas at constant temperature. Plot a graph of $\frac{1}{P}$ against V. What conclusion can be drawn?

Table 6.2

Pressure (kPa)	5	4	3	2	1
Volume (cm^3)	11.3	14.1	18.8	28.2	56.3

Solution

Table 6.3 shows corresponding values of V and $\frac{1}{P}$.

Table 6.3

Pressure (kPa)	5	4	3	2	1
Volume (cm^3)	11.3	14.1	18.8	28.2	56.3
$\frac{1}{P}$ **$(kPa)^{-1}$**	0.2	0.25	0.33	0.5	1

On plotting these values we get the graph in Fig. 6.4.

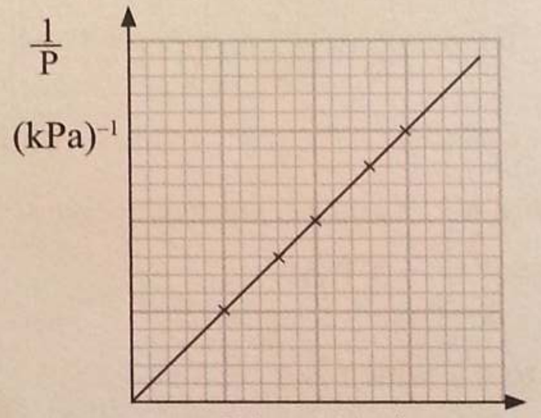


Fig. 6.4: Graph of $\frac{1}{P}$ against V.

The graph is a straight line passing through the origin. This shows that the volume is inversely proportional to the pressure.

$$V \alpha \frac{1}{P} \text{ or } PV = \text{constant (gradient)}$$

Example 6.2

Oxygen is compressed at constant temperature until its pressure rises from 82 cmHg to 140 cmHg. If the final volume of oxygen is 50 cm^3, find the initial volume of oxygen.

Solution

Initial volume V_1 = ?, Initial pressure P_1 = 82 cmHg

Final volume V_2 = 50 cm^3, Final pressure P_2 = 140 cmHg

From Boyle's law; $P_1V_1 = P_2V_2$

$$V_1 = \frac{P_2V_2}{P_1} = \frac{140 \times 50}{82}$$

$$= 85.37\ cm^3$$

Example 6.3

Air is trapped inside a glass tube by a thread of mercury 240 mm long. When the tube is held horizontally, the length of the air column is 200 mm (Fig. 6.5).

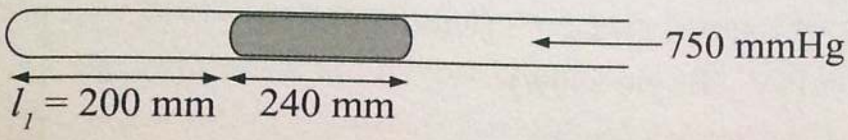


Fig. 6.5: Mercury thread in horizontal glass tube

Assuming that the atmospheric pressure is 750 mmHg and the temperature is constant, calculate the length of the air column when the tube is held,

(a) Horizontally with the open end.

(b) Vertically with the open end down.

(c) Explain why the mercury does not fall out in (b).

Solution

(a) Atmospheric pressure P_a =750 mmHg

Cross-sectional area of the tube = A

Volume (V) of trapped air = length (l) × cross-sectional Area of tube (A) i.e

$$V = lA$$

When the tune is horizontal (Fig. 6.5),

Pressure (P_1) of trapped air is P_1 = Pa = 750 mmHg

Length (L_1) of trapped air is L_1 = 200 mmHg

(b) When the tube is vertical with open end (Fig. 6.6), Pressure (P_2) of trapped air is the sum of atmospheric pressure and the pressure due to mercury column

i.e. $P_2 = P_a + P_{Hg}$ = (750 + 240) mmHg = 990 mmHg

Length of trapped air L_2 = ? (unknown)

But $P_1V_1 = P_2V_2$ *(Boyle's law.)*

$\Rightarrow P_1 l_1 A = P_2 l_2 A$

$P_2 l_2 = P_1 l_1$ (dividing by A both sides)

$$l_2 = \frac{P_1 l_1}{P_2} = \frac{750 \times 240}{990}$$

$$= 182 \text{ mm}$$

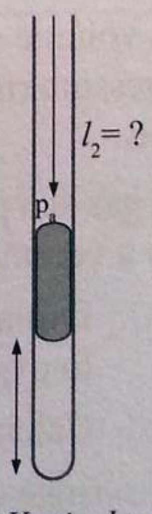


Fig. 6.6: Vertical tube open end up

(b) When the tube is vertical with open end down (Fig. 6.7). Pressure (P_3) of trapped air is the differnece of atmospheric pressure and the pressure due to mercury column i.e,

i.e $P_3 = P_a - P_{Hg} = (750 - 240)$ mmHg = 510 mmHg

Length L_3 of trapped air L_3 = ? (Unknown)

But $P_1V_1 = P_3V_3$ (Boyle's law)

$PlA = P_3 l_3 A$ (*Boyle's law.*)

$P_2 l_2 = P_3 l_3$

$$l_3 = \frac{P_1 l_1}{P_3}$$

$$= \frac{750 \times 240}{510}$$

$$= 353 \text{ mm}$$

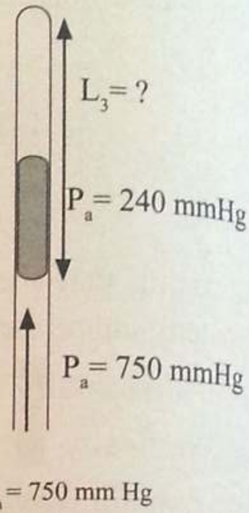


Fig. 6.7: vertical tube open end down

(c) Mercury does not fall out because the atmospheric pressure (750 mmHg) is greater than the pressure exerted by the trapped air.

Exercise 6.1

1. State Boyle's law. Explain why this law may not apply in solids.
2. Give one physical condition necessary for the expression PV = constant to be true.
3. Sketch graphs of the following when the temperature is kept constant.
   (a) P against $\frac{1}{V}$
   (b) P against V
4. A volume of 50 cm^3 of a gas is compressed at constant temperature until its pressure rises from 90 cmHg to 150 cmHg. Calculate the final volume of the gas.
5. A gas at a pressure of 200 kPa is allowed to expand from a volume of 75 cm^3 to a volume of 375 cm^3 under constant temperature.
   (a) Explain how the temperature can be kept constant as the gas is allowed to expand.
   (b) Calculate the final pressure of the gas.
6. A syringe contains 50 cm^3 of a gas at atmospheric pressure. The piston of the syringe is moved outwards so that the pressure falls to 7×10^4 N/m^2.

Calculate the volume of the gas if the temperature of the gas remains the same. (1 atm = 1×10^5 N/m^2).

7. An air bubble of volume 2.5 cm^3 is released at the bottom of the pond. At the surface of the pond its volume is found to be 5.0 cm^3.
   (a) Explain why the bubble rises up.
   (b) Why did the volume of the bubble change?
   (c) Calculate the depth of the pond. Assume the temperature of the bubble did not change and the atmospheric pressure is equivalent to a column of 10 m of the water and density of water 1 000 kg/m^3.

8. The readings shown in the Table 6.4 were obtained in an experiment to investigate the relationship between the volume and the pressure of a fixed mass of a gas.

Table 6.4

Pressure (N/m^2)	500	400	300	200	
Volume (cm^3)		5			20

   (a) Copy and complete the table.
   (b) Plot a graph of pressure against $\frac{1}{\text{volume}}$.
   (c) State the relationship which this graph shows between pressure and volume.

6.2 Charles' law

We have already learnt that air expands (increases in volume) when the temperature is raised. In this section, we will investigate the relationship between temperature and volume but at a constant pressure. This is the *Charle's law*.

Charles' law states that *the volume of a fixed mass of gas at constant pressure is directly proportional to its absolute temperature.*

The following experiment will help us verify the Charles' law.

Experiment 6.2: To investigate the relationship between the volume of a given mass of a gas and its temperature at constant pressure.

Apparatus

- millimetre scale
- water in a beaker
- wooden stirrer
- rubber band
- source of heat
- concentrated sulpuric acid
- capillary tube
- thermometer.

Procedure

1. Trap some air in a capillary tube closed at one end and open at the other with the help of a drop of concentrated sulphuric acid. Note that concentrated sulphuric acid dries the air and also acts as an index.

2. Fasten the tube and a thermometer to a 30 cm ruler with an elastic rubber band at each end.

3. Adjust the tube on the scale so that the bottom of air column coincides with zero of the scale (Fig. 6.8).

4. Dip the thermometer, the scale and the capillary tube in a tall beaker containing water (Fig. 6.8). Make sure that the air column is *under the water* and the open capillary tube end is exposed in the air.

5. Heat the water slowly as you stir gently so that the temperature of the air will be the same as the thermometer reading.

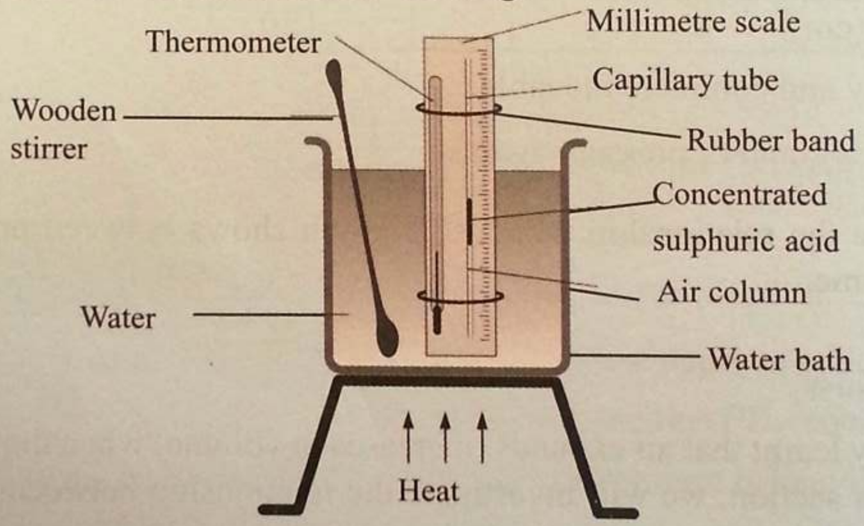


Fig. 6.8: Apparatus for verifying Charles' law

6. Record the length of the air column at different temperatures in a tabular form (Table 6.5).

Table 6.5

Temperature (°C)	20	30	40	50	60	70	80
Length (cm)							
Volume (cm³)							

7. Measure the diameter of the tube and calculate the volume of air in the capillary tube for the various temperatures.

8. Plot a graph of volume of the air against the temperature.

9 (a) Describe the graph

(b) What conclusion can you draw from the graph?

(c) What is the mathematical representation of your conclusion?

Observation

When the temperature rises, the height of concentrated sulphuric acid increases.

Discussions

A straight line graph is obtained with an intercept on the volume axis (Fig. 6.9).

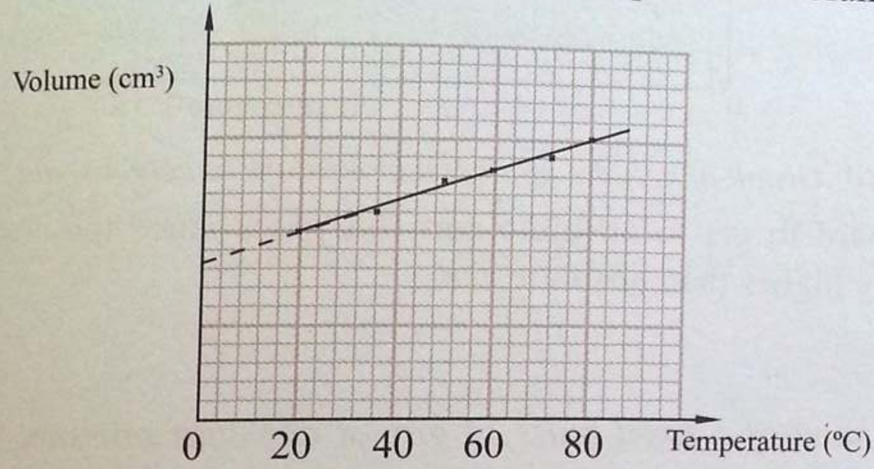


Fig. 6.9: Graph of volume against temperature

This graph shows that: *when the pressure of a fixed mass of a gas is kept constant, the volume changes linearly as the temperature increases or decreases.* For a capillary tube with uniform cross-sectional area, its length is directly proportional to volume of the air trapped inside.

If a graph of length, l, is plotted against temperature, a similar graph is obtained.

Use the values obtained in Experiment 6.2 to complete Table 6.6. Convert the celsius temperature to kelvin temperature.Draw a graph of volume of a gas against its kelvin temperature at constant pressure.

Table 6.6

Temperature (°C)	20	30	40	50	60	70
Temperature (K)	293	303	—	—	—	—
Volume (cm^3)						

The graph is a straight line passing through the origin (Fig. 6.10). This shows that the volume of the gas is directly proportional to its absolute temperature.

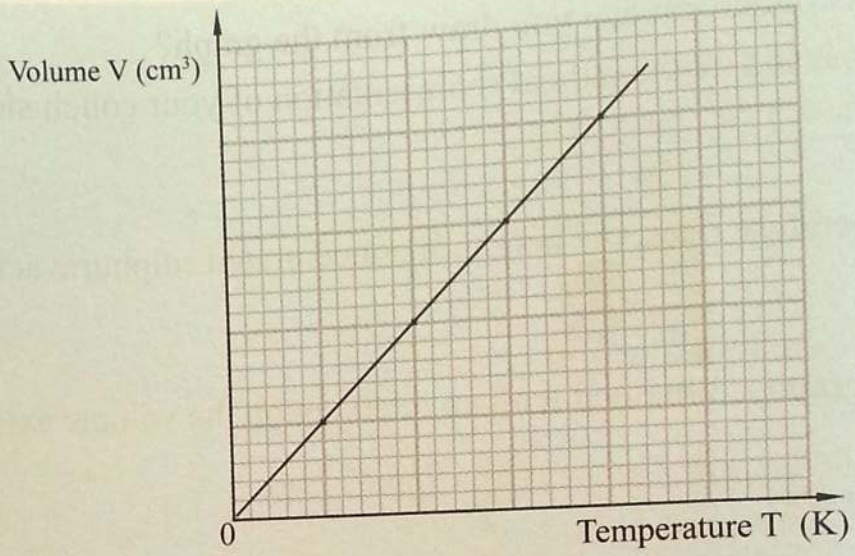


Fig. 6.10: Graph of volume against temperature in kelvin scale

Note: It is very hard to get to absolute zero for gases since they condense at temperatures fairly higher than absolute zero.

Conclusion

Generally, *the volume of a fixed mass of gas at constant pressure is directly proportional to its absolute temperature.* This is known as Charles' law.

Mathematically; Charles' law is represented as follows

$V \propto T$ (at a constant pressure and fixed mass)

$\therefore$ $V = KT$ where K is a constant

$\frac{V}{T}$ = a constant.

Let us now express the gradient of the line of the graph shown in Fig. 6.11 in terms of V_1, T_1 and V_2, T_2

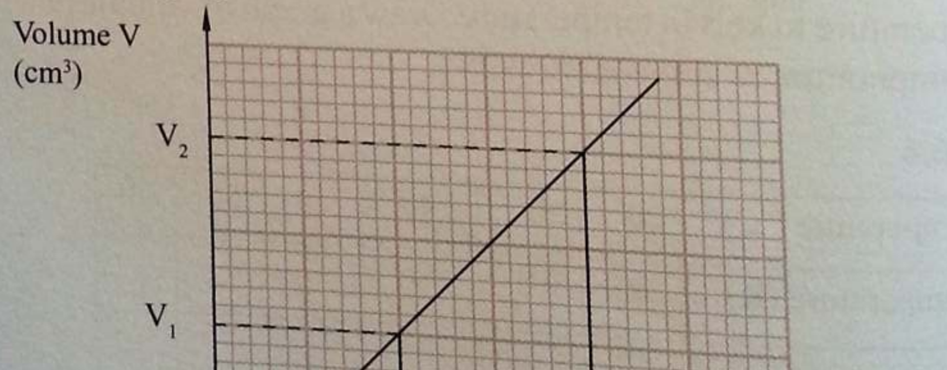


Fig. 6.11: The graph of volume against temperature.

Using points (0,0) and (T_1, V_1) or (T_2, V_2),

$$\text{Gradient} = \frac{V_1 - 0}{T_1 - 0} = \frac{V_1}{T_1}$$

$$\text{Hence, gradient} = \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\boxed{\therefore \frac{V_1}{T_1} = \frac{V_2}{T_2} = \text{constant. (Charles' law)}}$$

Note: Always make sure that T is expressed in absolute temperature (kelvin)

Example 6.4

A gas at 0 °C was found to occupy a volume of 100 cm^3. What will be the volume of the gas at 50 °C? Assume the pressure of the gas to be constant.

Solution

$V_1 = 100\ cm^3$; $V_2 = ?$; $T_1 = 0\ °C = 273\ K$; $T_2 = 50\ °C = 323\ K$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow V_2 = \frac{V_1T_2}{T_1}$$

$$= \frac{100 \times 323}{273}$$

$$= 118.3\ cm^3$$

Example 6.5

A volume of 400 cm^3 of carbon dioxide at 27 °C is heated at constant pressure to a temperature of 300 °C. Calculate the new volume of the gas.

Solution

$T_1 = 27\ °C = 27 + 273 = 300\ K$; $V_1 = 400\ cm^3$; $V_2 = ?$

$T_2 = 300 + 273 = 573\ K.$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow V_2 = \frac{V_1T_2}{T_1} = \frac{400 \times 573}{300}$$

$$= 764\ cm^3$$

Exercise 6.2

1. What are the advantages of using concentrated sulphuric acid in trapping the air in the capillary tube in the experiment to verify Charles' law?
2. State four precautions that need to be taken when investigating the relationship between the volume and the temperature of a fixed mass of a gas.
3. Explain the term absolute zero temperature.
4. In an experiment to investigate Charles' law, the following results were obtained as shown in Table 6.7.

Table 6.7

Volume (cm^3)	6.7	7.5	8.1	8.5	8.7
Temperature (°C)	10	40	60	75	85

   (a) Plot a graph of volume against temperature.
   (b) Determine the gradient of the line.
   (c) What information can you deduce about the way the gas expands.
5. What is an ideal gas?
6. (a) State Charles' law.
   (b) A fixed amount of oxygen has a volume of 5 m^3 at 27 °C. Calculate the volume of the oxygen at 77 °C when the pressure remains constant.

6.3 The Pressure law

We have so far investigated the relationship between the pressure and volume of a gas at constant temperature and between volume and temperature at constant pressure. But what is the relationship between pressure and temperature at constant volume, otherwise known as the pressure law?

The following experiment will help us investigate the pressure law.

Experiment 6.3: To investigate the relationship between the pressure of a given mass of a gas with temperature when volume is kept constant

Apparatus

- pressure gauge
- source of heat
- thermometer
- flask
- stirrer
- tripod stand
- water

Procedure

1. Trap air in a flask and connect it to a pressure gauge. Immerse the flask fully in the water bath (Fig. 6.12).

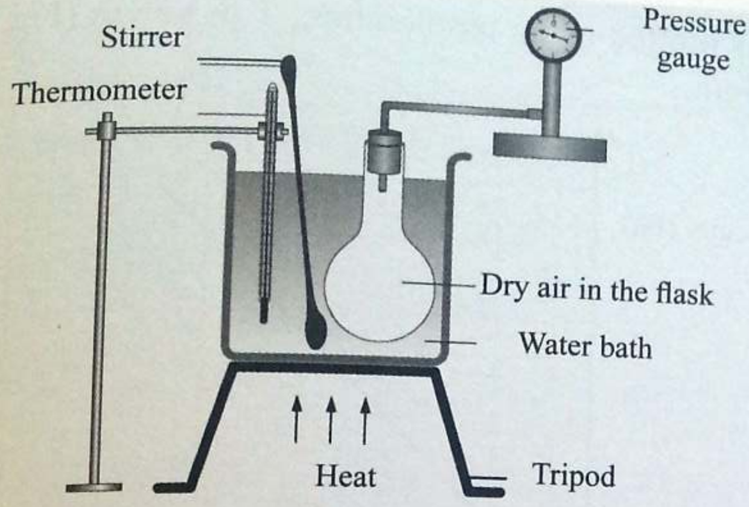


Fig. 6.12: Apparatus to verify pressure law

2. Stir the water gently and wait for some time before taking the readings. This ensures that the temperature of the surrounding is the same as the temperature of the air inside the flask.
3. Heat the water. Read and record the temperature of the air inside the flask and its pressure. The volume of the gas remains constant as it is not allowed to expand. Repeat the experiment and fill in Table 6.8 below.

Table 6.8

Temperature (°C)	20	30	40	50	60	70	80
Pressure (Pa)							

4. Plot a graph of pressure against temperature.

Discussion

A straight line graph is obtained with an intercept on the pressure axis (Fig. 6.13). When the line is extrapolated, it meets the temperature axis at approximately -273°C.

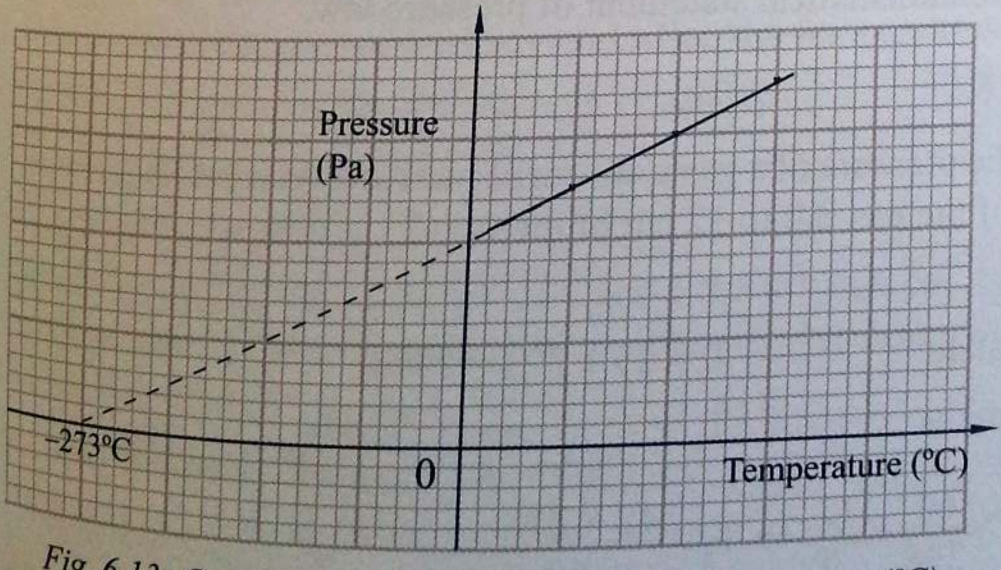


Fig. 6.13: Graph of pressure (Pa) against temperature (°C)

When the graph is plotted using temperature, T in kelvin (Fig. 6.14), the graph starts from the origin.

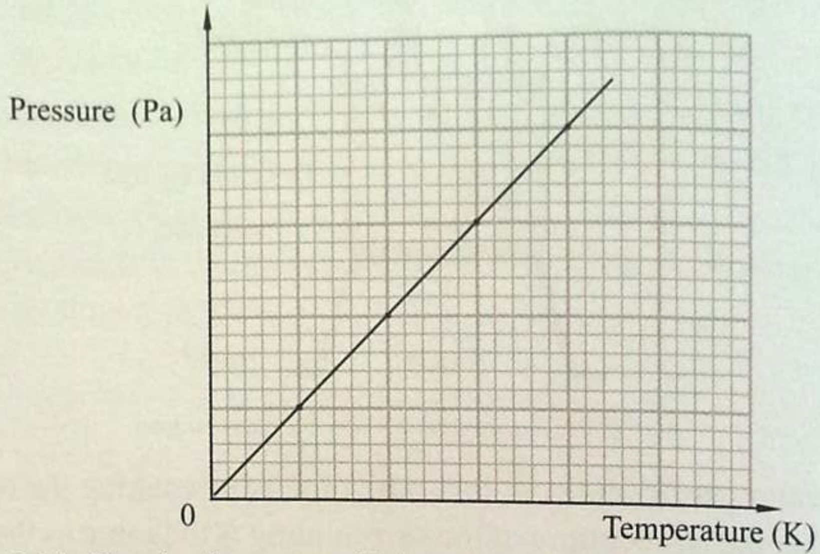


Fig. 6.14: Graph of pressure (Pa) against absolute temperature (°C)

Conclusion

We can conclude that *the pressure of a fixed mass of gas is directly proportional to the absolute temperature provided the volume remains constant*. This is the pressure law.

Pressure law states that the pressure of a fixed mass of gas is directly proportional to its absolute temperature provided the volume is kept constant.

Mathematically:

Pressure (P) ∝ absolute temperature (T)

$$P \propto T$$

$$\frac{P}{T} = \text{constant or } \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

This is the mathematical statement of pressure law.

Example 6.6

At 25°C the pressure of a gas is 60 cm of mercury. At what temperature would the pressure of the gas fall to 12 cm of mercury?

Solution

$P_1 = 60$ cmHg $\qquad T_1 = 25 + 273 = 298$ K

$P_2 = 12$ cmHg $\qquad T_2 = ?$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \frac{60}{298} = \frac{12}{T_2} \Rightarrow T_2 = \frac{12 \times 298}{60}$$

$$= 59.6 \text{ K}$$

Example 6.7

A container is filled with air at 2°C and 2 atmospheric pressure. What will be the pressure in the container if the temperature rises to 90°C?

Solution

$T_1 = 2 + 273 = 275$ K $\quad\quad P_1 = 2$ atmosphere

$T_2 = 90 + 273 = 363$ K $\quad\quad P_2 = ?$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} = \quad \frac{2}{275} = \frac{P_2}{363} \Rightarrow P_2 = \frac{2 \times 363}{275}$$

$$= 2.64 \text{ atmosphere}$$

Exercise 6.3

1. Which scientific process takes place when the temperature of a gas is lowered beyond -273°C.
2. A fixed mass of gas exerts a pressure of 20 mmHg at 20°C. If its volume is kept constant, find the pressure exerted at 0°C.
3. Why is the absolute zero regarded by the scientist as very important temperature point?
4. In an experiment, a Form 3 student, obtained the readings shown in table 6.9.

Table 6.9

Temp (°C)	–50	–25	0	25	75
Pressure (KPa)	80	90	95	110	125

(a) Plot a graph of pressure against temperature.

(b) From the graph determine the absolute zero temperature.

(c) What would the pressure gauge read when the temperature is 160°C.

(d) What assumption have you made in (c) above.

6.4 The combined gas law

The three gas laws; Boyle's law, Charles' law and Pressure law can be combined to form an ideal gas law. The ideal gas law relates the changes in pressure, volume and the absolute temperature.

From Boyle's law, temperature is constant, we get

$PV = \text{Constant}$ .... (a)

From Charles' law, pressure is constant, we get

$\frac{V}{T} = \text{Constant}$ .... (b)

Pressure law, volume is constant so

$\frac{P}{T} = \text{Constant}$ .... (c)

Combining the LHS of a, b, c and equating it to constant, we get $\frac{PV}{T} = \text{Constant}$.

This is the general equation connecting P, V and T.

If we consider the initial and the final values of P, V and T, then:

$$\boxed{\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}}$$

This is called the *ideal gas law.*

Standard temperature and pressure (s.t.p)

The volume occupied by a fixed mass of a gas depends on its pressure and its temperature. Consider two gases in containers A and B (Fig. 6.15).

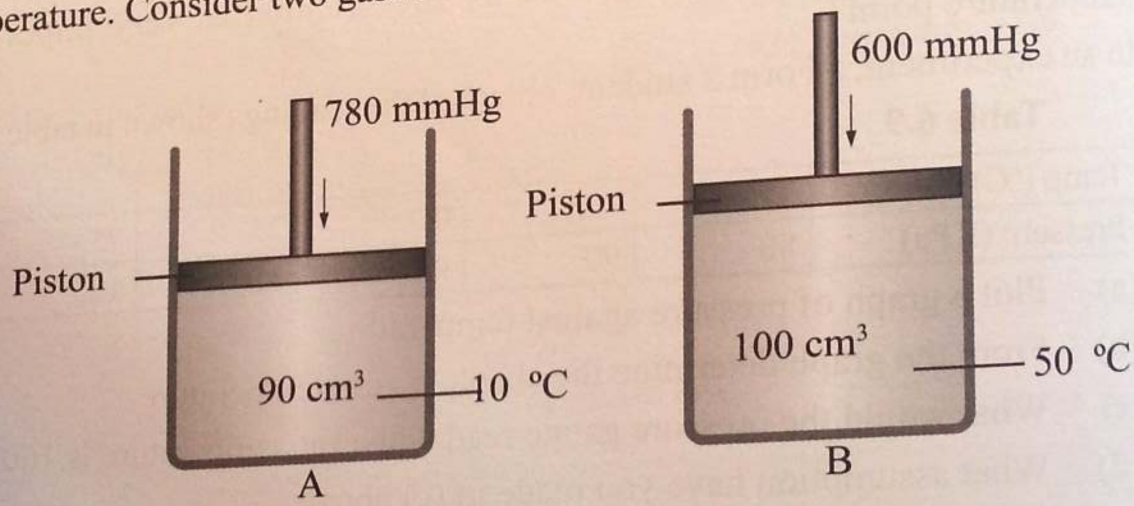


Fig. 6.15: A fixed mass of a gas at different conditions

The gas in A occupies a smaller volume than in B. This comparison is misleading in that the conditions of these two gases are different. In order to compare the volumes of gases, we need to have them at the same temperature and pressure. 0 °C temperature and 760 mmHg pressure are chosen as the standard states of the gas. *The 0 °C temperature and 760 mmHg pressure are called standard temperature and pressure (s.t.p).* 760 mm Hg can be written in short as 1 atm.

Example 6.8

Compare the volumes of the gases in A and B in Fig. 2.14 at (s.t.p).

Solution

Gas in A

$P_1 =$ 780 mmHg, $P_2 = 760$ mmHg

$V_1 = 90\ cm^3$, $V_2 = ?$

$T_1 = 10\ ^\circ C + 273$, $T_2 = 273$ K

$= 283$ K

$$V_2 = \frac{P_1V_1T_2}{T_1p_2}$$

$$\text{For A, } V_2 = \frac{780 \times 90 \times 273}{283 \times 760}$$

$$= 89.10\ cm^3$$

Gas in B

$P_1 = 600$ mmHg, $P_2 = 760$ mmHg

$V_1 = 100\ cm^3$, $V_2 = ?$

$T_2 = 273$ K

$T_1 = 50\ ^\circ C + 273$

$= 323$ K

$$\text{For B, } V_2 = \frac{600 \times 100 \times 273}{323 \times 760}$$

$$= 66.73\ cm^3$$

The volume of A is greater than that of B.

Example 6.9

A quantity of a gas occupies a volume of 4 m^3. The pressure of this gas is 3 atmospheres (atms) when its temperature is 27°C. What will be its pressure if it is compressed into half the volume and heated to a temperature of 127°C.

Solution

Using the information given in the question,

$P_1 = 3$ atm $P_2 = ?$

$V_1 = 4\ m^3$ $V_2 = 2\ m^3$ half the original volume

$T_1 = (273 + 27)$ K $= 300$ K $T_2 = (273 + 127)\text{K} = 400$ K

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1} \Rightarrow \frac{P_2 \times 2\ m^3}{400\ K} = \frac{3\ atm \times 4\ m^3}{300\ K}$$

$$\frac{P_2 \times 2}{400} = \frac{3 \times 4}{300}$$

$$P_2 = \frac{400 \times 3 \times 4}{300 \times 2} = 8\ atm$$

Therefore, $P_2 = 8$ atms

Example 6.10

When an inverted beaker is placed on the surface of water, it contains 200 cm^3 of trapped air at atmospheric pressure. What will be the volume of the air when the beaker is taken 30 m beneath the water surface?

Assume that the temperature is constant, and that atmospheric pressure can support a column of water 10 m high.

Solution

P_1 = atmospheric pressure
= 10 m of water
= 1 atm

V_1 = 200 cm^3

P_2 = atmospheric pressure + pressure due to 30 m of water
= 1 atm + 3 atm (when 10 m = 1 atm)

V_2 = ?

T_1 is not given but is equal to T_2

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

Cancelling T_1 and T_2 (because they are equal) and substituting values gives,

$$4 \text{ atm} \times V_2 = 1 \text{ atm} \times 200$$

$$\Rightarrow V_2 = \frac{1 \times 200}{4}$$

$$V_2 = 50 \text{ cm}^3$$

Exercise 6.4

1. The volume of a given mass of air at 28°C and 74 cmHg pressure is 150 cm^3. Find its volume at 76 cmHg pressure and temperature at -25°C.
2. 100 cm^3 of hydrogen gas is collected at 19°C and 75 cm of mercury pressure. What is its pressure if 50 cm^3 of hydrogen gas is collected at 25°C.
3. Sketch a graph of:
   (a) PV against T (in kelvin).
   (b) PV against P for an ideal gas.
4. A pressure of 2 atmospheres compresses a gas to a volume is 12 cm^3 at 6°C. Find the volume of the gas when pressure is 0.5 atmosphere at 100°C.
5. The volume of a fixed mass of a gas is 140 cm^3 at 300°C and 50 cmHg. If the volume is tripled. What is its pressure at 273°C.

6.5 Kinetic theory of gases

We use kinetic theory to explain the behaviour of gases. In this theory, gases are considered to consist of many tiny particles which possess kinetic energy and are always in motion. In order to completely explain the observed behaviour of gases the following assumptions are made.

Assumptions made in kinetic theory of gases

1. A gas is made up of tiny, identical, solid particles which are in constant, rapid and random motion.
2. The particles move in a straight line and their motion is only affected by collision.
3. All collisions are perfectly elastic i.e. kinetic energy before collision is equal to kinetic energy after collision.
4. The time that the particles are actually in contact with each other or with the walls of a container is negligible compared with the time between two successive collisions with the walls.
5. The forces of attraction between the particles is negligible.
6. The actual volume of the particle is negligible compared with the space in which the particles move.

A gas that would obey all the above assumptions is called an ideal gas. An ideal gas is defined as one that obeys the general gas law $\frac{PV}{T}$ = constant, exactly and under all conditions of temperature and pressure. In practice, however, gases only obey the ideal gas law approximately at low pressure and high temperatures. At low temperature and high pressure all gases deviate appreciably from the ideal gas equation.

Kinetic theory and the gas laws

Explaining gas laws using kinetic theory of gases

The molecules of a gas are in constant, rapid and random motion as shown in Fig. 6.16.

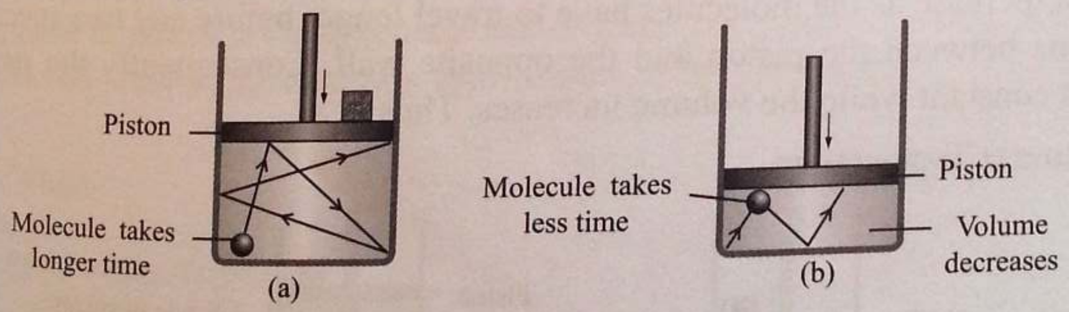


Fig. 6.16: Motion of gas molecules

Each time the molecules strike the walls of the container they rebound. The molecules therefore experience a change in momentum. This change in momentum of the gas molecules is the cause of the force on the walls of the container (Fig. 6.17).

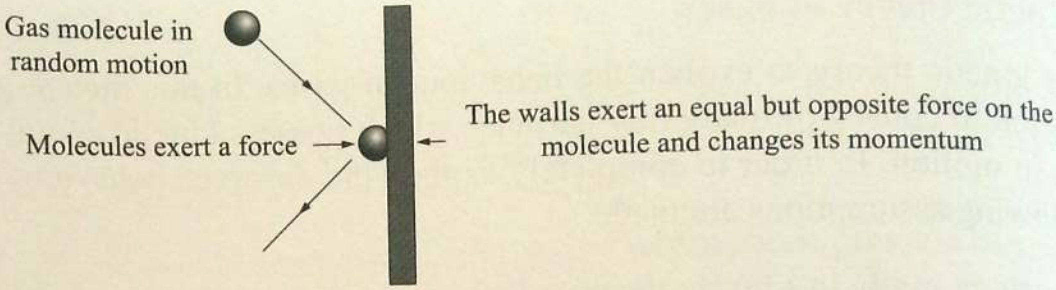


Fig. 6.17: Force on the walls of a container

The force on the walls of the container gives rise to the gas pressure. The number of collisions per second is a measure of the gas pressure. When the number of collisions per second is high, the gas pressure is high and vice-versa.

Boyle's law

In Boyle's law, as the temperature of the gas is constant, the average kinetic energy of the gas molecules is constant. When the volume decreases, the number of collisions per second increases as the molecules take less time between any two collisions (Fig. 6.16 (b)). The gas pressure therefore increases. When the volume is increased the number of collisions per second decreases since the molecules spend more time in between any two collisions. This results in a decrease in pressure as observed in the Boyle's law.

$$\text{Volume } \alpha \ \frac{1}{\text{Pressure}}$$

Charles' law

When a gas is heated, at constant pressure as shown in Fig. 6.18, the energy possessed by the gas molecules is used in moving the piston up (doing work) and in increasing the internal energy of the molecules. Hence the temperature rises. The increase in internal energy or kinetic energy of the molecules results in the molecules moving faster. Although the molecules move faster, the number of collisions per second does not increase as the molecules have to travel longer before any two successive collisions between the piston and the opposite wall. Consequently the pressure remains constant while the volume increases. Thus,

Volume α Temperature

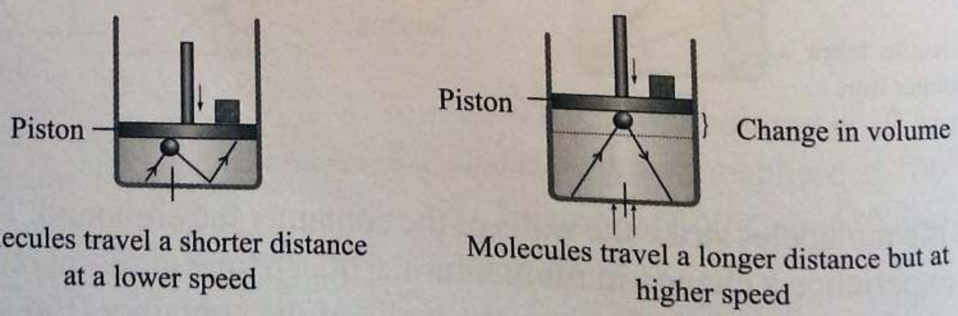


Fig. 6.18: Gas heated at constant pressure

Pressure law

When a gas is heated at constant volume (Fig. 6.19), the energy given to the gas molecules is used only in increasing the internal energy i.e. kinetic energy as there is no external work done.

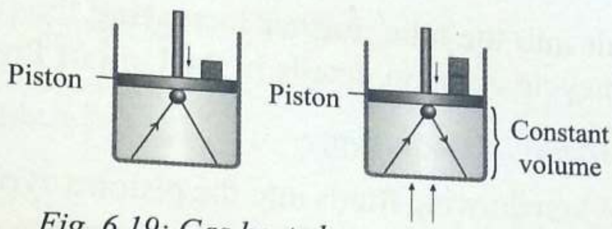


Fig. 6.19: Gas heated at constant volume

The temperature of the gas increases. The increase in kinetic energy makes the molecules move faster. This results in more collisions per second and hence increase in pressure. Thus,

Pressure α Temperature

Exercise 6.5

1. State the basic assumptions made in kinetic theory.
2. Explain using the kinetic theory of gases why the temperature of a gas increases when the bicycle pump is used.
3. Using the kinetic theory of gases, explain how a gas exerts pressure.
4. A tin containing air is corked when the atmospheric pressure is 750 mmHg and the temperature is 23 °C. The tin is then heated. The cork blows out when the pressure in the flask exceeds atmospheric pressure by 160 mmHg. Calculate the temperature of the flask at which the cork blows out.
5. (a) Explain how you would determine the absolute zero temperature of a given mass of air experimentally at
   (i) constant volume (ii) constant pressure

6.6 Application of gas laws

The concept of gas laws is widely used in our daily life. We encounter the knowledge of gas laws in industries and general use of some equipments. In this section, we shall discuss some of the common applications of the gas laws.

Boyle's law

Working of a bicycle pump

The working of the bicycle pump is governed by the Boyle's law. The washer in the pump allows air to enter the barrel during the upstroke, but during the downstroke, the air does not escape to the outside. The air is compressed in the barrel. This leads to the volume of air decreasing as the pressure increases. When the pressure of the air in the pump is greater than that in the inner tube, the valve opens and allow more air into the tube, further increasing the pressure. We learnt about the working of a bicycle pump in details in the Unit of Pressure.

Action of the syringe

When drawing fluids into the piston a syringe is pulled making the volume inside the syringe to increase. This leads to corresponding decrease in air pressure. The presure outside the syringe is greater forces the liquid fluid into the syringe. When the syringe is on reverse the volume reduces and pressure inside the syringe increases forcing the liquid out. We learnt about the working of the syringe in details in the Unit of Pressure.

Action of the lungs

During inhaling the lungs expand increasing the volume as the pressure inside them decreases. The pressure outside being greater, forces air into the lungs.

During exhaling, the lungs contract increasing the pressure as the volume decreases, forcing the air out.

Charles' law

An increase in temperature of a fixed gas increases its volume when pressure is not changed.

- Slightly inflated rubber tyres left in bright sunlight swells up. This is the reason why motorists are discouraged from over inflating their tyres because they can burst on hot days.
- A football inflated inside and taken outdoors on a cold day shrinks slightly obeying Charles' law.
- To fly hot-air balloons, balloonists apply Charles' law. As the air inside the balloon is heated, its volume increases. The density of the balloon reduces as the air inside is heated and expands. This enables the balloon to fly.

Pressure law

From pressure law, increase in temperature leads to an increase in pressure if the volume and the amount of gas remains constant.

- The pressure gauge in steel-belted automobiles tyres reads a higher value when the car is travelling on a hot path than when it is moving on a cold path.
- An aerosol can thrown in the fire explodes; Since the pressure inside the can increases with an increase in temperature.

Making a manometer

A manometer is a device designed for measuring the pressure difference between two systems. A liquid column manometer uses a tube filled with a liquid to measure the pressure difference between the two ends of the tube.

The following project will help us make a manometer.

Project: How to make your own Manometer

Materials

- 60-cm long thin wooden plank
- 1 m long, transparent, flexible vinyl plastic tubing
- Tube fasteners
- Nails
- Hammer
- Plumb line
- Ruler
- Tape
- 500 ml beaker
- 100 ml water
- 5 ml red food dye

Procedure

1. Mount the plastic tubing carefully on the wooden plank using the tube fasteners such that it forms a smooth, even "U" bend on one end without becoming kinked (Fig 6.20). Ensure that one arm of the now U-shaped tube is longer than the other.
2. Using the plumb line, position the plank of wood vertically against a support like a soft board. Hammer a nail through the plank or use some other means to attach it securely to the board (Fig. 6.20).
3. Pour the 100 ml of water into the beaker. Add some drops of the red food dye into the water to make it turn red hence visible, and mix thoroughly. Pour the coloured water carefully into the U-shaped tube.
4. Place the ruler on the plank of wood along the longer arm of the U-shaped tube and line up the zero mark of the ruler with the meniscus of the water in the tube. Secure the ruler firmly on the plank using the tape (Fig. 6.20).

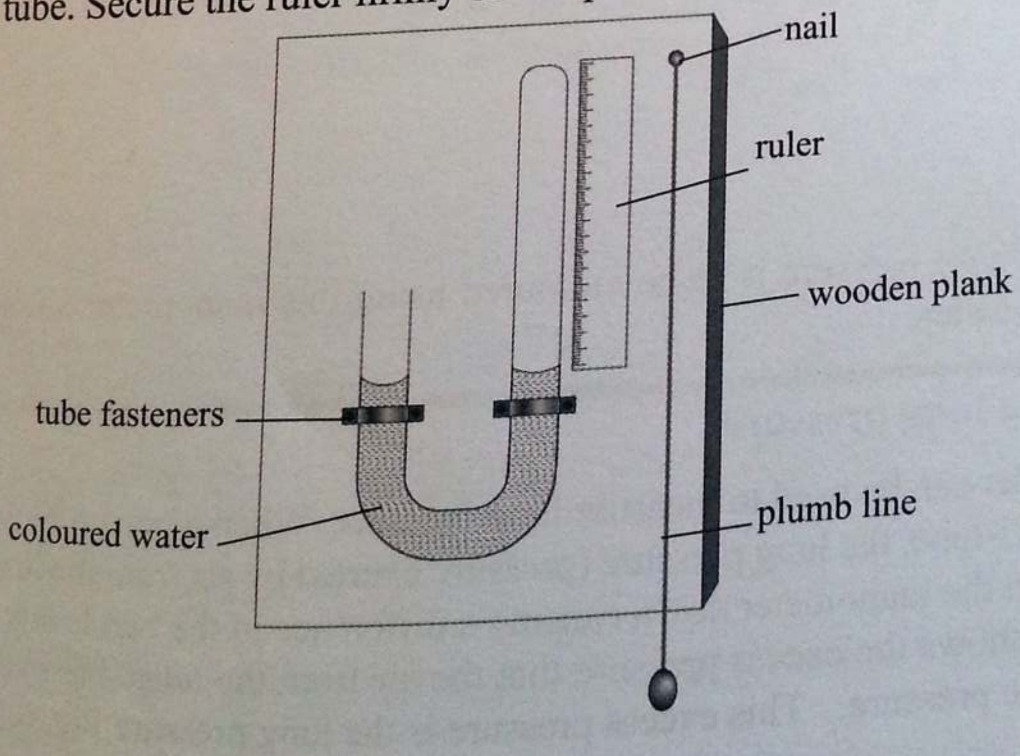


Fig. 6.20: Making a manometer

5. To measure the pressure of a gas, connect the tube from the outlet of the gas container to the mouth of the shorter side of the U-shaped tube (Fig. 6.21).

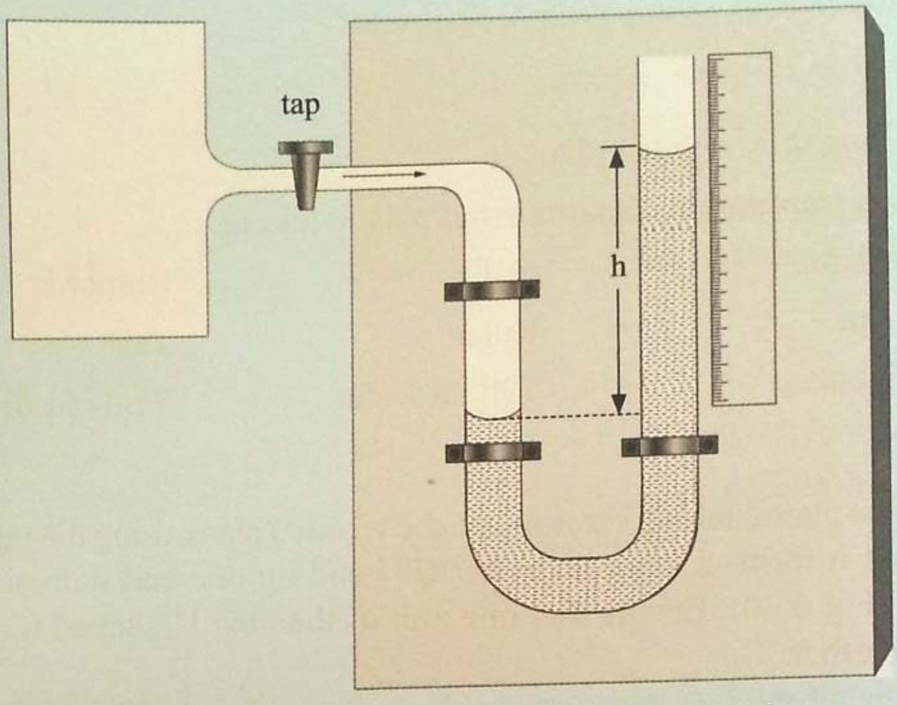


Fig. 6.21: Connecting a manometer to a gas supply

The gas pressure will cause the water to rise up in the longer side of the U-shaped tube. The height difference between the levels of water in the two columns inside the tube and atmospheric pressure are used to determine the gas pressure as earlier discussed in the unit of pressure.

Pressure of gas = Atmospheric pressure + Pressure due to the water column

$$P_{gas} = P_{atm} + P_{water}$$

$$P_{gas} = P_{atm} + \rho gh$$

Note

The gas whose pressure is to be measured using this manometer should not be soluble in water.

Measuring lung pressure

A manometer can be used to measure lung pressure. When a person blows into one arm of the U-tube, the lung pressure (pressure exerted by air from the lungs) pushes the liquid in the manometer down causing a difference in the two levels, the height difference shows the excess pressure that the air from the lungs has exerted above atmospheric pressure. This excess pressure is the lung pressure Fig. 6.22.

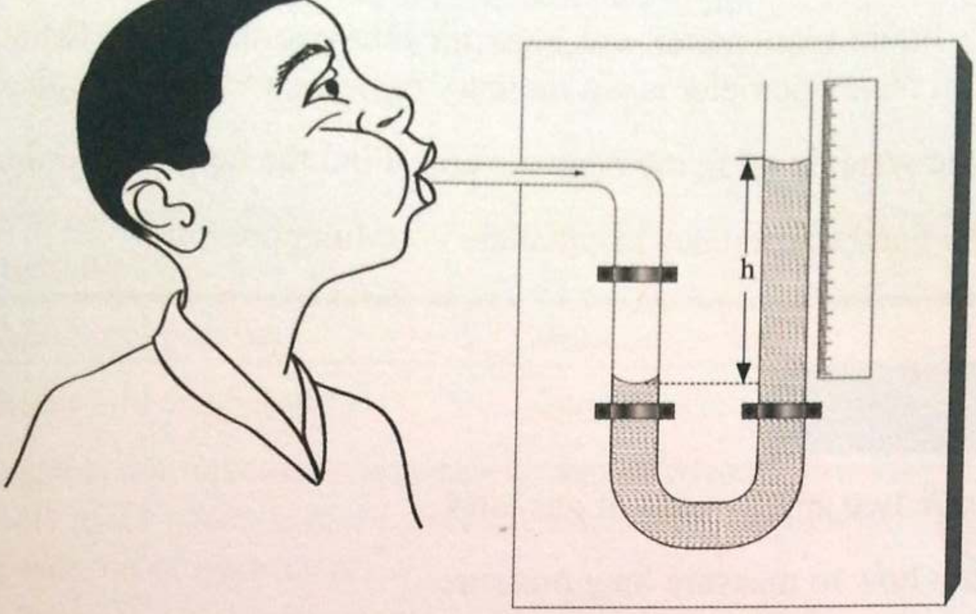


Fig. 6.22 Measuring lung pressure

The lung pressure supports the pressure due to the water column in the manometer and atmospheric pressure. To find the actual lung pressure, add atmospheric pressure to this excess pressure.

Lung pressure P_l = Atmospheric pressure (P_a) + Pressure due to water column ($h\rho g$)

$$P_l = P_a + h\rho g$$

Example 6.11

A Form 3 student blew into one arm of water manometer. The water in the other arm rose by 20 cm. Given that the density of water is 1.0 g/cm³ and the atmospheric pressure is 101 000 N/m², calculate the lung pressure of the student.

Solution

Data

$h = 20$ cm; $P_{atm} = 101\ 000$ N/m²; $\rho = 1.0$ g/cm³ $= 1000$ kg/m³: $P_2 = ?$ $g = 10$ N/kg

$P_2 = P_{atm} + h\rho g$

$= 101\ 000 \text{ Pa} + \left(\frac{20}{1\ 00} \times 1000 \times 10\right) \text{Pa}$

$= 101\ 00 \text{ Pa} + 2000 \text{ Pa}$

$= 103\ 000 \text{ Pa}$

Activity 6.1

- Take a liquid manometer and blow air into one open arm (Make sure the liquid in the manometer is not mercury because, mercury is poisonous.)
- Note the water level in the opposite arm. Find the height difference.
- Use the height difference to calculate your lung pressure.

Exercise 6.6

1. Describe two applications of gas laws.
2. Explain how to measure lung pressure.
3. A student blew air into one arm of a water manometer and the liquid in the other arm rose to a height 5 cm. If the density of water is 1.0 g/cm^3 and the atmosphoric pressure of the place = 100 000 Pa, calculate the lung pressure in Pa.

Unit Summary

- The state or condition of a gas is fully described by its pressure, volume and temperature.
- Boyle's law states that PV = constant, if temperature is constant.
- Charles' law states that $\frac{V}{T}$ = constant, if pressure is constant.
- Pressure law states that $\frac{P}{T}$ = constant, if volume is constant.
- General gas law is $\frac{PV}{T}$ = constant; also called ideal gas equation.
- Kinetic theory helps to explain the behaviour of gases at molecular level.
- Absolute zero temperature is the theoretical temperature when the volume or pressure of a gas is zero.
- A manometer can be used to measure lung pressure.

Unit Test 6

1. Boyle's law relates pressure of a fixed mass of a gas and ________.

   A. Temperature   **B.** Volume

   C. Mass   **D.** Absolute zero

2. Name two physical quantities of a gas that need to be fixed or constant during verification of Charles' law.

   A. Mass and pressure   **B.** Mass and volume

   C. Mass and temperature   **D.** Temperature and pressure

3. Hydrogen is compressed at constant temperature until its pressure rises from 3 pascals to 15 pascals. If the final volume of hydrogen is 100 cm^3, find the initial volume of hydrogen.

4. A gas at 5°C occupied a volume of 215 cm^3. What will be the volume of the gas at 63°C. Assume the pressure of the gas to be constant.

5. Give a mathematical relationship of volume, pressure and temperature of ideal gas.

6. Convert –273°C to kelvin scale.

7. If you compress a balloon, the pressure inside increases. How does kinetic theory explain this?

8. According to Boyle's law, the pressure of an ideal gas is proportional to the volume.

   (a) Write this in symbols.

   (b) State two conditions for Boyle's law to be true.

9. Under what conditions do real gases behave like ideal gases.

10. The readings in Table 6.10 are for a fixed mass of gas at constant volume.

Table 6.10

Pressure/atm	0.78	0.96	1.13	1.13	1.48
Temperature °C	–50	0	50	100	140

   (a) Plot a graph of pressure (y-axis) against temperature (x-axis).

   (b) Extrapolate your graph and estimate a value for absolute zero.

   (c) Does the graph obey the pressure law? Give a reason.

11. A gas occupies a volume of 6 m^3 at a temperature of 27°C at atmospheric pressure. Calculate the volume of the gas at the same pressure if the temperature changes to;

(a) 67°C (b) -273°C

12. At the beginning of a journey, the pressure of the air in a car tyre was found to be 276 kPa at 23°C. Immediately after the journey, the pressure was 300 kPa. What is the temperature of the air in the tyre after the journey? Assume the volume of air inside remained constant.

13. A gas occupies a volume of 2 m^3 when its pressure is 1 140 mm Hg and its temperature is 27°C. What volume would it occupy at standard temperature and pressure (0°C and 760 mm Hg).

Mechanics

Outcome

The student will be able to appreciate and demonstrate the use of appropriate quantities to explain various states of motion including the energy interactions and the changes that take place due to force.

UNIT

7 Scalars and Vectors

Sucess criteria

By the end of this unit, you must be able to:

- Define scalar and vector quantities.
- Represent vectors.
- Add and subtract vectors.
- Resolve vectors.

Introduction

Quantities are part and parcel of our daily lives. For example, hardly does a day pass without us walking some distance in a particular direction, lifting an object with some force, buying items whose masses or capacities are measured in kilograms and litres respectively, doing an activity in a duration of time e.g. taking lessons in school and so on. Some of these quantities have both *magnitude* and *direction* while others have magnitude only.

In this unit, we will discuss these two types of quantities and how quantities of the same kind can be combined together through addition or subtraction.

7.1 Scalar and vector quantities

Scalars quantities

In science, there are quantities which can be described by *size (magnitude)* only. For example, the time taken by a runner to cover a 100 m race is 12.0 s. Here we have only a numerical value attached to the units of time. Such quantities, are called *scalars.* Examples of other scalar quantities include *pressure*, *temperature*, *speed*, *distance*, *mass*, *density*, *area*, *energy* and *volume*.

Vectors quantities

Other quantities have *both magnitude* and *direction*. Such quantities are called *vectors.* Force is an example of a vector quantity. It has a numerical value and acts in a specified direction.

Other examples of vector quantities include: *displacement*, *velocity, acceleration*, *momentum*, *magnetic field* and *temperature gradient*.

7.2 Representing vectors

A vector is normally represented by a line with an arrow head (→). The length of the line represents the magnitude and the arrow head shows the direction. The length may be drawn to a given scale.

Consider a cyclist moving at a velocity of 10 m/s due east, and a motorist moving at a velocity of 20 m/s due west. On paper, their velocities can be represented as shown in Fig. 7.1 (a) and (b) respectively.

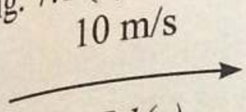


Fig. 7.1(a)

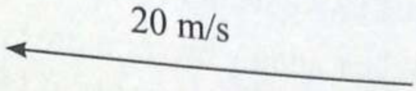
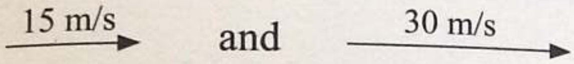


Fig. 7.1 (b)

Key properties of vectors

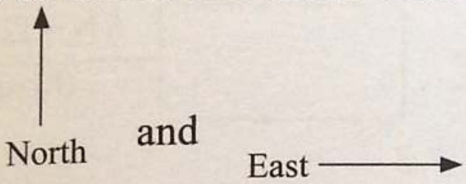
1. *They have size (magnitude)*. The length of the line represents the magnitude.

   Examples:



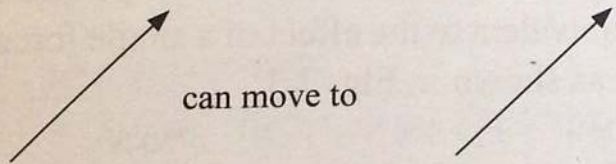
2. Vectors have direction. The arrow head shows the direction.

   Examples:



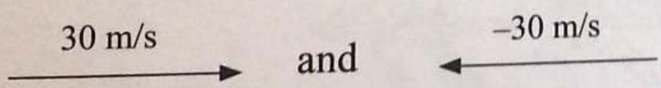
3. In vector diagrams, a vector can be moved from one position to another as long as the magnitude and direction are maintained.

   Example:



4. When the direction of a vector is reversed, it is assigned a negative value.

   Examples:



Exercise 7.1

1. Draw the following vectors using the scale given.
   (a) Force of 15 N (1 cm represents 5 N)
   (b) Velocity of 100 m/s (1 cm represents 20 m/s)
   (c) Displacement of 1000 m (1 cm represents 200 m)
2. Differentiate between vector and scalar quantites and give two examples each.
3. Velocity and speed are not the same. Explain this statement.

7.3 Addition and subtraction of vectors

Vector addition

We already know how to add scalar quantities. For example 2 m + 2 m = 4 m. Similarly, vector quantities can be added. We use the term *resultant* to mean *the sum* *of two or more vector quantities* i.e. a single vector equivalent to the other vectors combined together.

Adding parallel vectors

When adding two or more vector quantities, we consider both their magnitudes and directions. For example, if two forces act in the same direction as shown in Fig. 7.2 (a) and (b), their resultant force can be obtained as follows:

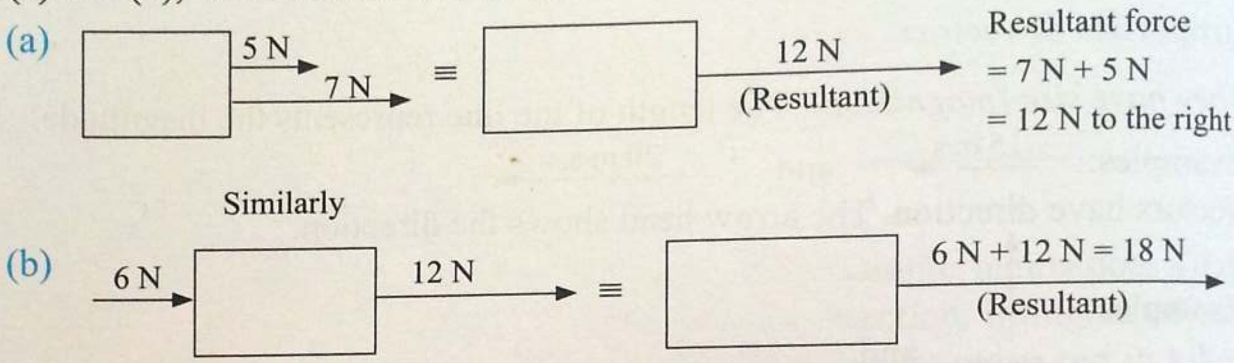


Fig. 7.2

It is worth noting that, two vectors acting at a point can be replaced by a single vector with the same effect. For example, the effect of two people applying forces of 750 N and 400 N on a cart is equivalent to the effect of a single force of 750 N applied on the same cart by a donkey as shown in Fig. 7.3.

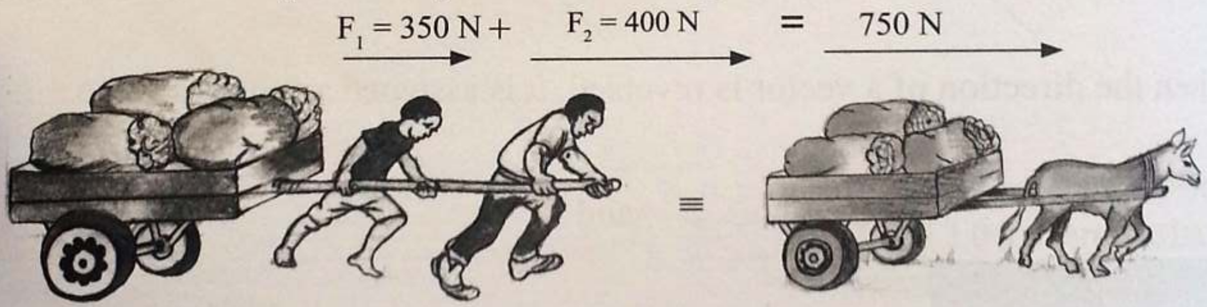


Fig 7.3: Addition of vector quantities in the same direction

Thus, when two vectors in the same direction are added, the resultant vector is the sum of their magnitudes in the same direction.

Addition of vectors using the triangle rule

We have so far added vectors that are in the same direction. Let us learn how to add vectors which are at an angle to each other. This process is known as *composition of vectors*.

One method of doing this is the *triangle rule* which state that; *If two sides of a triangle completely represent two vectors both in magnitude and direction taken in same order, then the third side taken in opposite order represent the resultant of the two vectors.*

In this method the vectors are redrawn head to tail and the resultant is the line that completes the triangle.

Example 7.1

Harry moved from point A to a point B, 40 m East and then 30 m North to point C. What was his displacement?

Solution

In working out this problem, we can use trigonometry or scale drawing.

(a) Using trigonometry

Sketching the vectors, we get a right-angled triangle ABC (Fig. 7.4).

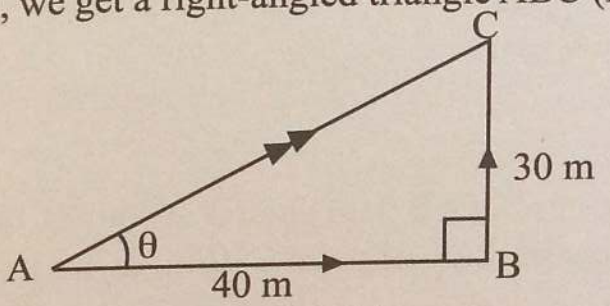


Fig 7.4

AC represents his displacement

$AC = \sqrt{(40\text{ m})^2 + (30\text{ m})^2}$ (Pythagoras' Theorem)

$= 50\text{ m}$

$\tan\theta = \frac{BC}{AB} = \frac{30}{40} = 0.75 \Rightarrow \theta = 38.67^\circ$

His displacement is 50 m at an angle of 38.7° to AB.

(b) By scale drawing

(i) Choose a scale e.g. 1 cm represents 10 m. Hence, a length of 4 cm will represents 40 m and 3 cm will represent 30 m.

(ii) Draw AB = 4 cm.

(iii) Draw angle $\angle ABC = 90^\circ$ using a ruler and a compass or a protractor.

(iv) Join A to C with a straight line (Fig. 7.5).

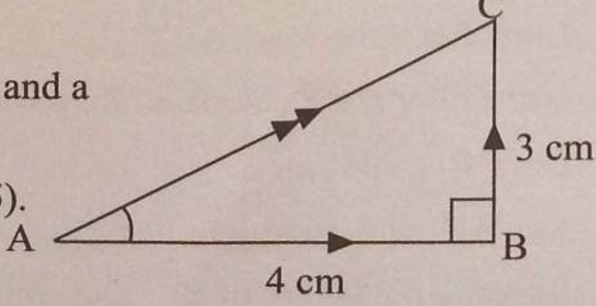


Fig. 7.5

(v) Measure the length AC and $\angle CAB$ to obtain the actual displacement. $AC = 50$ m and $\angle CAB = 38.7^\circ$.

You will get a displacement of 50 m at an angle of 38.7° to AB.

Example 7.2

Two forces of 50 N and 40 N acts at a point A as shown in Fig. 7.6. Find the resultant force.

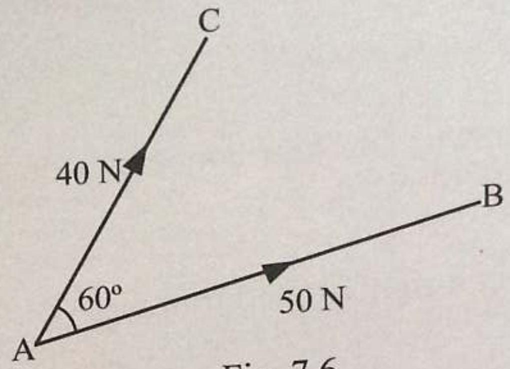


Fig. 7.6

Solution

Sketch vector AC at the head of AB with point A at point B to obtain triangle ABC′ where point C′ is the new position of point C (Fig. 7.7).

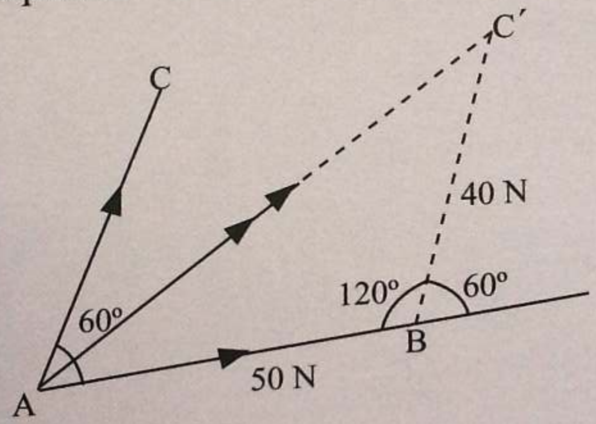


Fig. 7.7

- AC′ is the resultant force
- BC′ is parallel to AC
- ∠ABC = 180° – 60° = 120°

To accurately locate the resultant force AC′, let us use a scale of 1 cm represents 10 N to draw Δ ABC′.

- Draw AB = 5 cm.
- Use a ruler and a compass or a protractor to draw < ABC′ = 120°.
- Draw BC′ = 4 cm.
- Join AC′ with a straight line.
- Measure the length of AC′ and convert this length into newtons using the scale
- Measure < BAC′.

We obtain Δ ABC′ in Fig. 7.8

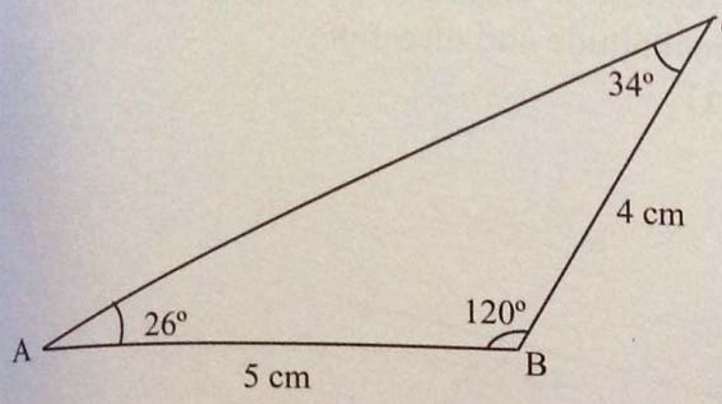


Fig. 7.8

AC′ = 7.9 cm = 79 × 10 N = 79 N
<BAC′ = 26°
Therefore, the resultant force is 79 N at an angle of 26° to the force of 50 N.

Note that the triangle rule can also be used when adding more than two vectors. For example, when adding three vectors say F_1, F_2, and F_3, determine the resultant of any two vectors say $F_{12} = F_1 + F_2$, then find the resultant of the 3rd vector whith the resultant of the first two vectors, say $F_{123} = F_{12} + F_3$.

Exercise 7.2

1. Find the resultant of the forces in Fig. 7.9.

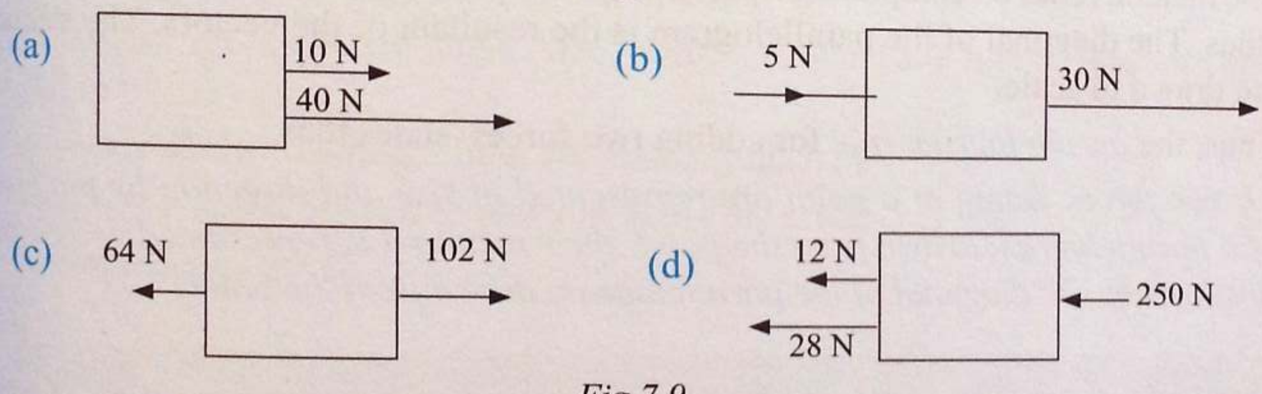


Fig 7.9

2. State the triangle rule for adding vectors.
3. Using the triangle rule and scale drawing find the resultant of the forces in Fig. 7.10 (a) and (b) by scale drawing. Use a scale of 1 cm represents 4 N.

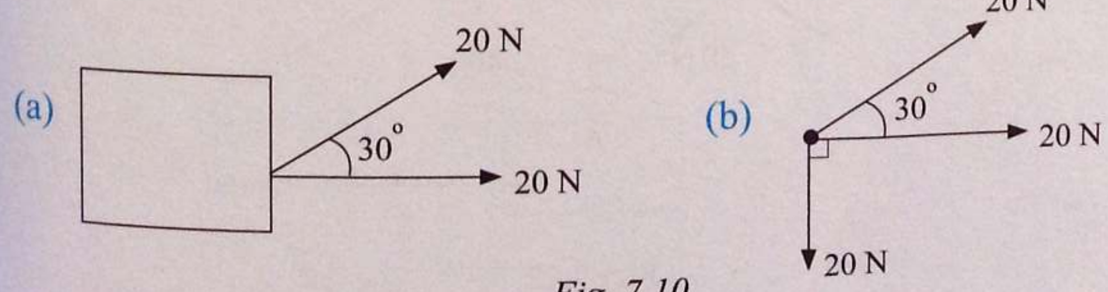


Fig. 7.10

4. A housefly moved along two sides of a rectangular table top. It first moved 0.8 m from one corner to another then 1.2 m to the third corner. Describe fully it's displacement in terms of the magnitude and direction.
5. Find the resultant force in Fig. 7.11.

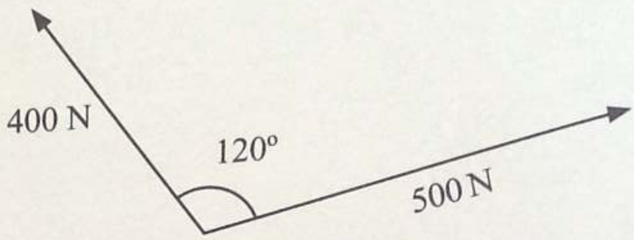


Fig. 7.11: Forces acting at an angle

6. A stone is catapulted 300 m at an angle of 40° above the horizontal. Find:
   (a) its vertical displacement
   (b) Its horizontal displacement

The parallelogram rule

The parallelogram rule is another method of finding the resultant for vectors that are at an angle to each other.

The method relies on completing a parallelogram made by the two vectors as adjoining sides. The diagonal of the parallelogram is the resultant of the vectors. The vectors are drawn to scale.

Thus, the *parallelogram rule* for adding two forces states that

"If two forces acting at a point are represented in size and direction by the sides of a parallelogram drawn from the point, their resultant is represented in size and direction by the diagonal of the parallelogram drawn from the point."

Example 7.3

Find the resultant force acting on the portrait suspended using two ropes as shown in Fig 7.12.

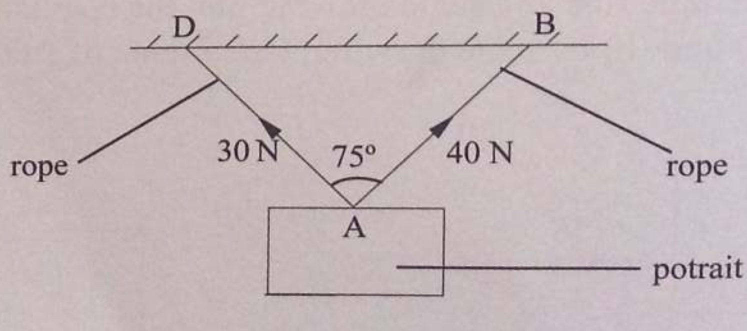


Fig. 7.12

To find the resultant of the two forces, we first sketch a parallelogram of which the sides of 30 N and 40 N are adjoining (Fig. 7.13).

Sketch

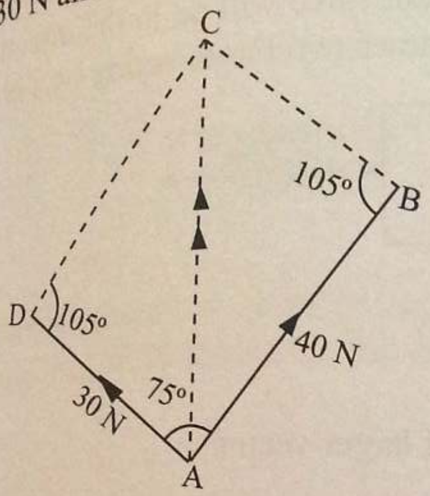


Fig. 7.13

ABCD is the parallelogram
AD // to BC
AB // to DC
Angle ADC = 180° – 75 = 105°
Angle ABC = <ADC = 105
AC is the resultant force.

To locate the resultant force AC, we draw parallelogram ABCD as follows using a scale of 1 cm to represent 10 N.

- Draw AB = 4 cm.
- Draw angle BAD = 75° and measure AD = 3 cm.
- Draw angle ABC = 105° and prolong line BC.
- Draw line DC such that angle ADC = 105°. Prolong the line to intersect line BC at C.
- Join A to C with a straight line. We obtain Fig. 7.14.
- Measure AC and convert to newtons using the scale.
- Measure angle BAC.

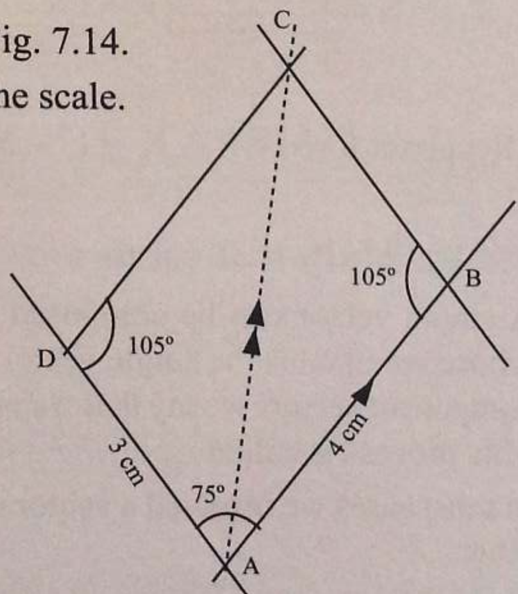


Fig. 7.14

AC is the resultant force.

AC = 5.6 cm
= 5.6 × 10 N
= 56 N

<BAC = 44°

Thus, the resultant force is 56 N acting at an angle of 44° to the 30 N

Subtraction of vectors

Vectors can also be subtracted. Consider two people in a tug-of-war. They are pulling the rope into opposite direction. The resultant force will be in the direction of the person pulling with more force. Fig. 7.15 shows two forces acting on a body.

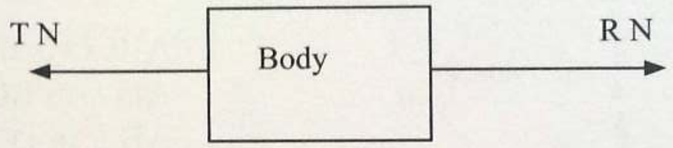


Fig. 7.15

The resultant vectors is given by

Resultant = (R –T) N in the direction of larger vector.

Example 7.4

Two forces of 5 N and 7 N act in opposite directions on a block. Find the resultant force.

Solution

We represent the vectors in diagram as shown in Fig. 7.16.

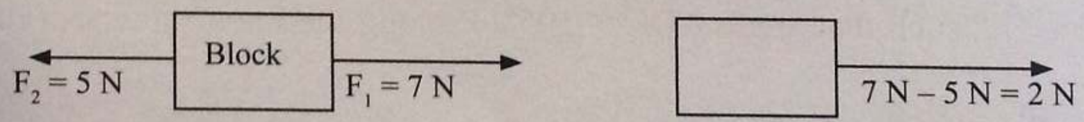


Fig. 7.16

Resultant force = $F_1 - F_2 = (7 - 5)$ N = 2 N

7.4 Resolution of vectors

A single vector can be composed of two vectors acting in different direction but whose resultant is the single vector. When we 'breakdown' the single vector into its component vectors we say that a single vector has been *resolved* into two components. This process is called *resolving a vector*.

In most cases we resolved a vector into components that are at right angle with each other.

Consider a force F applied at an angle θ to the horizontal (see Fig. 7.17).

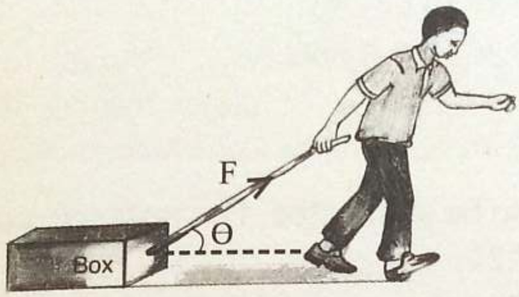


Fig. 7.17

The box "experiences" two effects: being lifted from the ground and being dragged horizontally. Fig. 7.18 shows the forces involved in dragging the box.

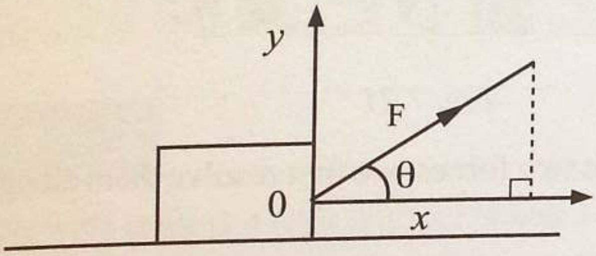
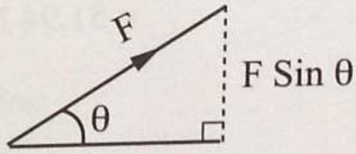


Fig. 7.18

x and y are the resolved components of F. x represents the horizontal component and y vertical component of F respectively. Using trigonometry, the values of x and y can be determined by the following equations



$y = F\ \sin\theta$ (vertical component of F)

$x = F\ \cos\theta$ (vertical component of F)

Fig. 7.19

Example 7.5

A box is being dragged by a force of 50 N through a string as shown in Fig 7.20.

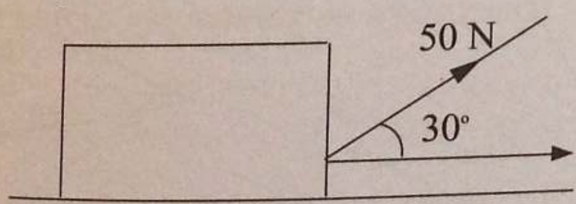


Fig. 7.20

Determine the vertical and horizontal components of the force.

Solution

$$
\begin{aligned}
\text{Vertical component } y &= 50 \sin 30^\circ \\
&= 50 \times 0.5 \\
&= 25\ \text{N}
\end{aligned}
$$

horizontal component $x = 50 \cos 30°$
$= 50 \times 0.866$
$= 43$ N

Resolved vectors can also be subtracted. For instance, consider two people pulling a box as shown in Fig. 7.21.

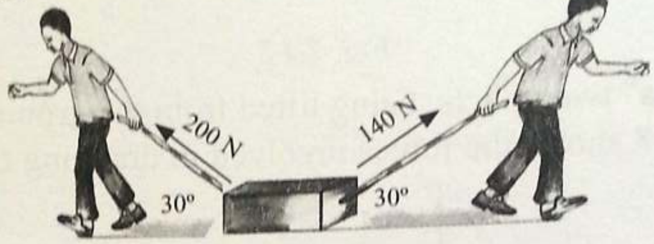


Fig. 7.21

To find the resultant forces of the two forces, we first resolve them along the horizontal component.

Horizontal component of 140 N = 140 cos 30° to the right

Horizontal component of 200 N = 200 cos 30° to the left

Resultant force = 200 cos 30° N – 140 cos 30°N

= 51.94 N

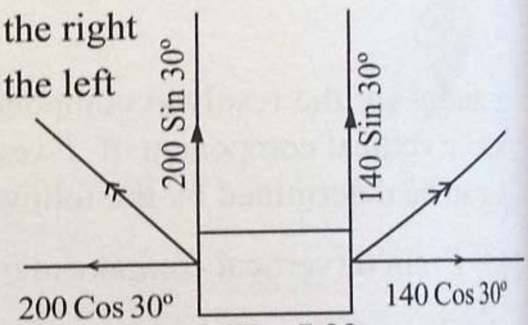


Fig. 7.22

To find the difference of two vectors at an angle to each other, we reverse the direction of one of them then use either the triangle of parallelogram rules to add the vectors Fig. 7.21(b).

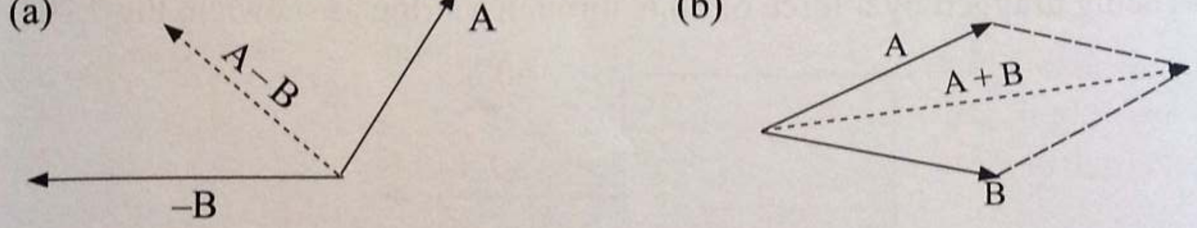


Fig. 7.23

Example 7.6

Two forces of 25 N and 20 N acts on a block at 25° each in opposite direction as shown in Fig. 7.24. Find the resultant force if the block is moving horizontally on a frictionless surface.

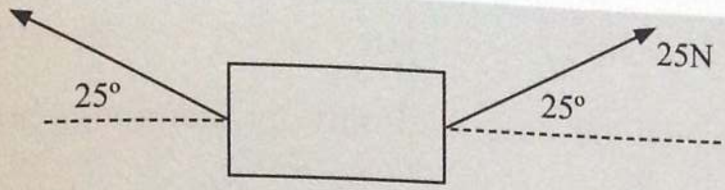


Fig. 7.24

Solution

Resultant force = 25 cos 25° N – 20 cos 25° N
= 22.66 N – 18.13N
= 4.53 N to the right

Exercise 7.3

1. Two people were pulling a rope with 29 N and 31 N in opposite direction. Find the resultant force.
2. Fig. 7.25 shows a metal ball being pulled by two strings at a point with an angle of 60° between them.

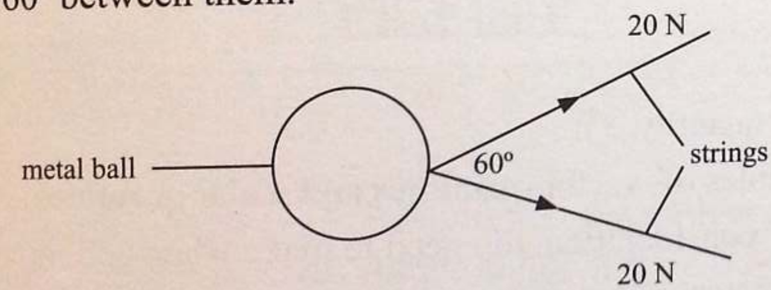


Fig. 7.25

Find the resultant force and the direction moved by the metal ball.
3. Calculate the resultant force in each of the following cases. (Fig. 7.26(a) and (b)). Note that the blocks are moving on a horizontal frictionless surface.

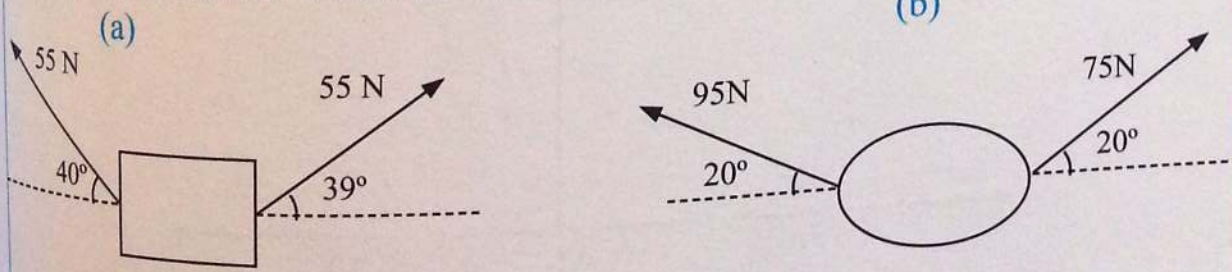


Fig. 7.26

4. A missile is projected into the air at a velocity of 400 m/s and at an angle of 60° to the horizontal ground. Calculate its
   (a) initial vertical velocity
   (b) initial horizontal velocity

Unit summary

- Quantities which are described only by magnitude are known as scalar quantities.
- Quantities which are described by both magnitude and direction are known as vectors quantities.
- When two or more vectors are added, the result is a single vector called the resultant.
- The process of splitting a single vector into two components at an angle is called resolving a vector.
- Methods of adding vectors include:

  Triangle rule: The vectors are joined head to tail and the resultant is the line that completes the triangle. The two vectors must be drawn to the same scale and their direction accurately located.

  Parallelogram rule: If two vectors are represented in size and direction from a common point, their resultant is represented in size and direction by the diagonal of the parallelogram drawn from the point.

Unit Test 7

1. Define a vector quantity.
2. Give four examples of vector quantities and scalar quantities.
3. State one major consideration you need to make when adding vectors.
4. When adding vectors, what name is given to the single vector that has the same effect as the other vectors?
5. Three forces, 3 N, 6 N and 8 N are acting at a point Q as shown in Fig. 7.27.

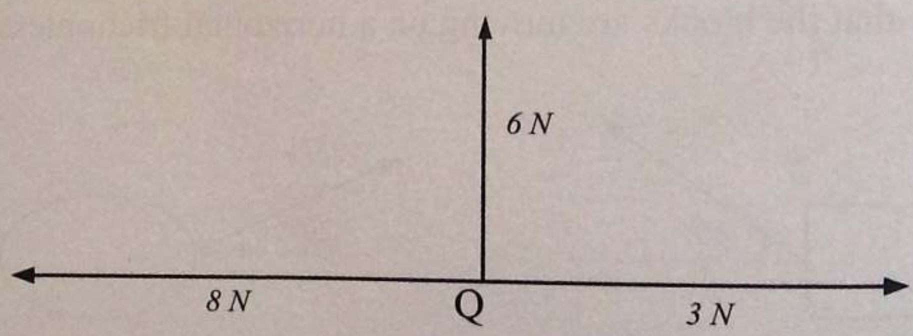


Fig. 7.27

(a) What is the magnitude of the resultant force?

(b) What angle does the resultant make with the force of 8 N

6. Use the parallelogram rule to find the resultant of the velocities shown in Fig. 7.28.

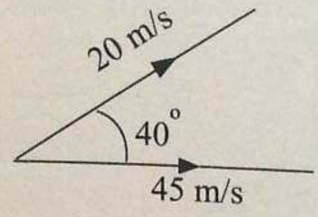


Fig 7.28

7. Two forces $F_1 = 10$ N and $F_2 = 5$ N are acting at the point P in the directions PY and PZ respectively as shown in Fig. 7.29.

Find by scale drawing or otherwise,

(a) the magnitude of the resultant force.

(b) the angle between the resultant and F_1.

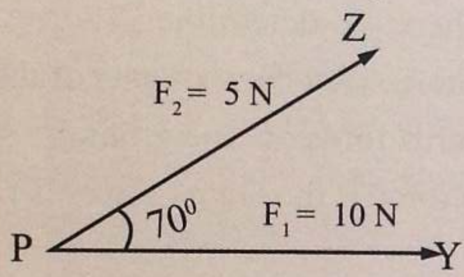


Fig 7.29

8 A body of mass M kg is held in equilibrium by a string fixed to the wall and a horizontal spring balance as shown in Fig. 7.30.

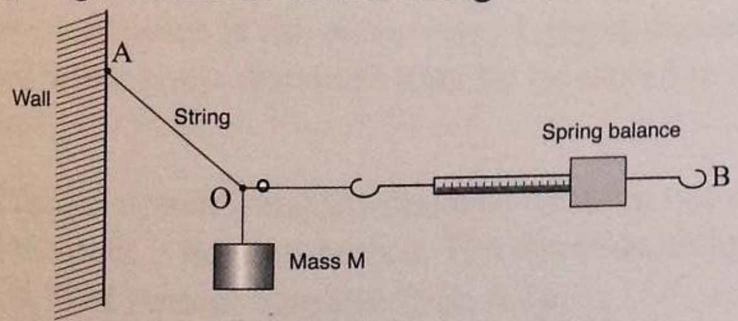


Fig. 7.30

If the balance reads 20 N:

(a) Draw and indicate the direction of the forces activating at O.

(b) Determine the:

(i) tension in the string

(ii) mass M of the body.

9. A lawn mower of mass 30 kg is being pushed with a force of 150 N as shown in Fig. 7.31.

Fig. 7.31

By scale drawing or otherwise determine:

(a) The vertical and horizontal components of the force.

(b) The total downwards force on the ground.

(c) Total downwards force if the lawn mower is pulled with the same force instead.

10. A deep sea diver dives at an angle of 30° to the horinzontal and follows a straight-line path for a distance of 220 m. How far is the diver from the surface of the water after covering that distance.

UNIT 8

Linear Motion

Success Criteria

By the end of this unit, you must be able to:

- Describe distance, displacement, speed, velocity and acceleration.
- Conduct experiments to determine velocity and acceleration.
- Determine acceleration due to gravity.
- Explain motion-time graphs.
- Apply the equations of uniformly accelerated motion.

Introduction

In our daily lives, we come across various objects in motion. People, animals and machines are from time to time involved in motion in different directions. In this unit, we are going to study linear motion i.e. motion of objects in a straight line. We shall pay attention to the time taken, distance covered, speed, velocity and acceleration of the motion and the relationships between them.

8.1 Distance, displacement, speed, velocity and acceleration

Distance

Distance is *the total length of the path between two points*. It is measured in units of length. The *SI unit* of distance is the *metre (m)*. Long distances may be measured in kilometres (km) while short distances may be measured in centimetres (cm) or millimetres (mm).

It should be noted that in determining the distance between two points, the direction at any point along the path is not considered. The direction along the path may keep on changing (Fig. 8.1) or remain constant (Fig. 8.2).

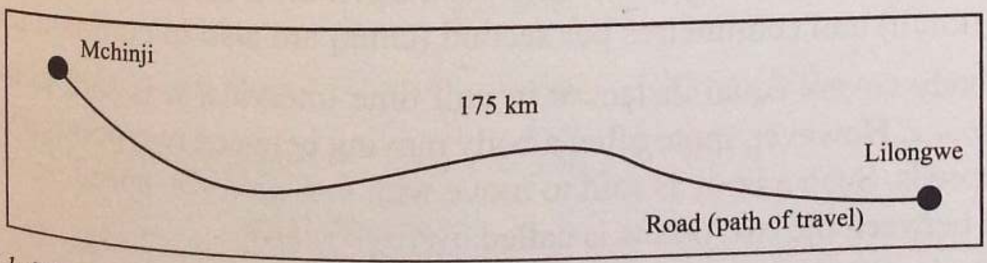


Fig.8.1: Distance between Lilongwe and Mchinji is 175 km (direction keeps on changing)

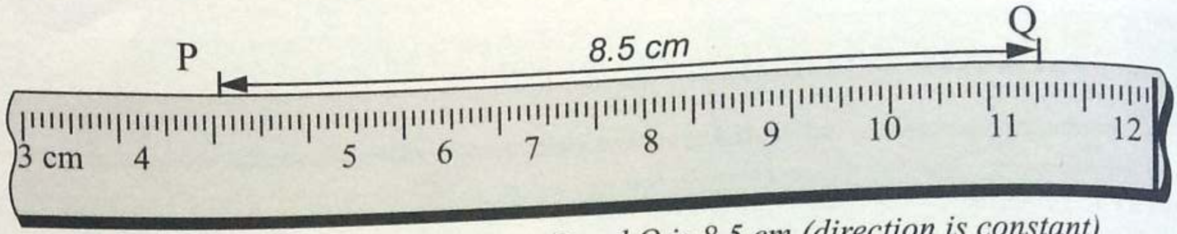


Fig. 8.2: Distance between points P and Q is 8.5 cm (direction is constant)

Displacement

Displacement is the straight line (shortest distance) between two points in the direction of motion. To fully describe the displacement, you need to specify how far you have travelled from where you started as well as in what direction you have travelled. For example, Mchinji is 100 kilometres North of Lilongwe. An arrowhead in displacement diagrams indicates the direction of motion (Fig. 8.3).

Displacement is defined as the distance covered in a stated direction from a point. The *SI unit* of displacement is the *metre* (m).

displacement = distance in a stated direction from a reference point

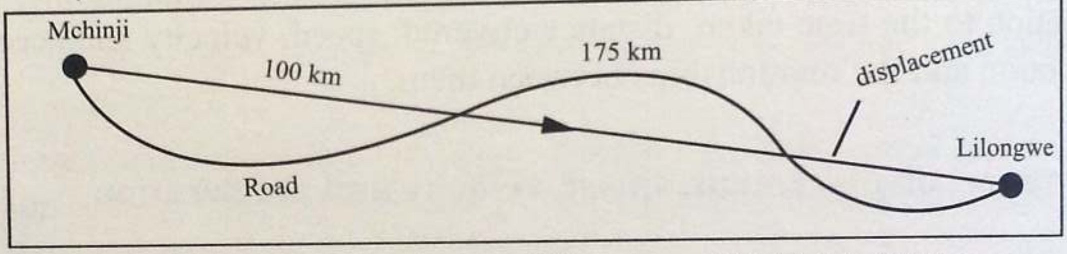


Fig. 8.3: Displacement between Lilongwe and Mchinji is 190 km

Speed

The speed of a body is the distance *moved by the body per unit time*. In this motion, direction is not considered. Thus,

$$\text{speed} = \frac{\text{distance moved}}{\text{time taken}}$$

The SI unit of speed is metres per second (m/s). Other units of speed such as kilometres per hour (km/h) and centimetres per second (cm/s) are also in common use.

When a body covers equal distances in unit time intervals, it is said to move with *uniform speed*. However, quite often a body moving between two points does so with varying speeds. Such a body is said to move with *non-uniform* speed. In such a case the speed between the two points is called *average speed.*

$$\text{Average speed} = \frac{\text{total distance moved}}{\text{total time taken}}$$

Example 8.1

What is the speed of a racing car in metres per second if the car covers 360 km in 2 hours?

Solution

$$\text{Speed} = \frac{\text{distance moved}}{\text{time taken}}$$

$$= \frac{360\text{ km}}{2\text{ h}} = 180\text{ km/h}$$

$$= \frac{180 \times 1\,000\text{ m}}{1 \times 60 \times 60\text{ s}} = 50\text{ m/s}$$

Example 8.2

A car moving along a straight road ABC as shown in Fig. 8.4 maintains an average speed of 90 km/h between points A and B and 36 km/h between points B and C.

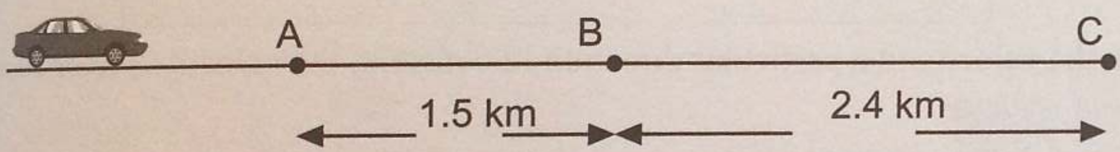


Fig. 8.4

Calculate the:

(a) Total time taken in seconds by the car between points A and C.

(b) Average speed in metres per second of the car between points A and C.

Solution

(a) $\text{Average speed} = \dfrac{\text{total distance}}{\text{time taken}}$

$$\text{total time between A and B} = \frac{\text{total distance}}{\text{average speed}}$$

$$= \frac{1.5}{90}\text{ h} = \frac{1.5}{90} \times 60 \times 60\text{ s} = 60\text{ s}$$

$$\text{Total time between B and C} = \frac{2.4}{36} \times 60 \times 60\text{ s} = 240\text{ s}$$

$$\text{Total time between A and C} = 60\text{ s} + 240\text{ s} = 300\text{ s}$$

(b) $\text{Average speed} = \dfrac{\text{total distance}}{\text{time taken}}$

$$= \frac{(1.5 + 2.4) \times 1\,000\text{ m}}{300\text{ s}}$$

$$= 13\text{ m/s}$$

Velocity

The speed of a body in a specified direction is called velocity or velocity is the rate of change of distance in a particular direction. Therefore,

$$\text{velocity} = \frac{\text{distance moved in a particular direction}}{\text{time taken}}$$

Velocity is defined *as the displacement covered in unit time* or *the rate of change of displacement.*

$$\text{velocity} = \frac{\text{displacement}}{\text{time taken}}$$

In some cases the velocity of a moving body keeps on changing. In such cases, it is better to consider the average velocity of the body.

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{time taken}}$$

When the velocity in a particular direction in constant, the velocity is referred to as *uniform velocity*.

For example Table 8.1 below shows the displacement of a car and the corresponding time taken.

Table 8.1

Displacement (m)	0	4	8	12
Time taken (s)	0	2	4	6

The velocity after every two seconds is 2 m/s, hence velocity of this car is uniform.

The SI unit of velocity is metres per second (m/s). When stating or describing the velocity of an object, the direction of velocity should always be indicated. In doing so, we state direction say north, south, upwards, downwards, etc. A negative sign in a value of velocity is commonly used to indicate movement in the reverse direction.

Example 8.3

A car travelled from town A to town B 200 km east of A in 3 hours. The car changed direction and travelled a distance of 150 km due north from town B to town C in 2 hours. (Fig. 8.5). Calculate the average

(a) speed for the whole journey.

(b) velocity for the whole journey.

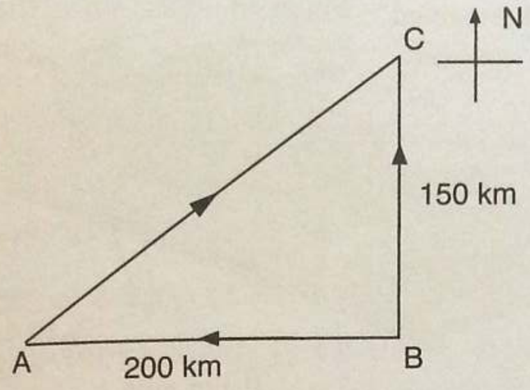


Fig. 8.5: Displacement from A to C.

Solution

(a) Average speed $= \dfrac{\text{total distance}}{\text{time taken}}$

$$= \frac{(200 + 150)\ \text{km}}{(3 + 2)\ \text{h}}$$

$$= \frac{350}{5}\left(\frac{\text{km}}{\text{h}}\right)$$

$$= 70\ \text{km/h}$$

(b) Average velocity $= \dfrac{\text{displacement, AC}}{\text{time taken}}$

$$= \frac{\sqrt{200^2 + 150^2}}{3 + 2}$$

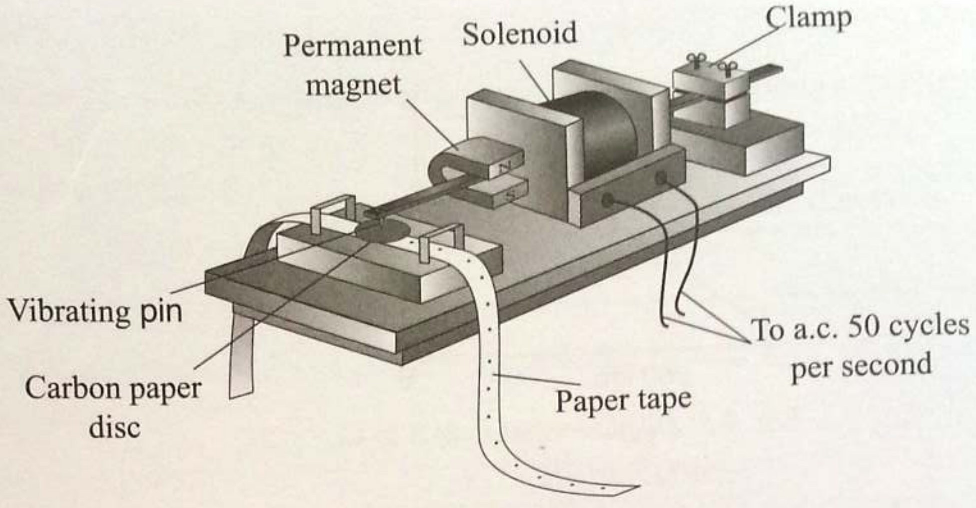
$$= \frac{250}{5}$$

$= 50$ km/h, Direction is from A to C

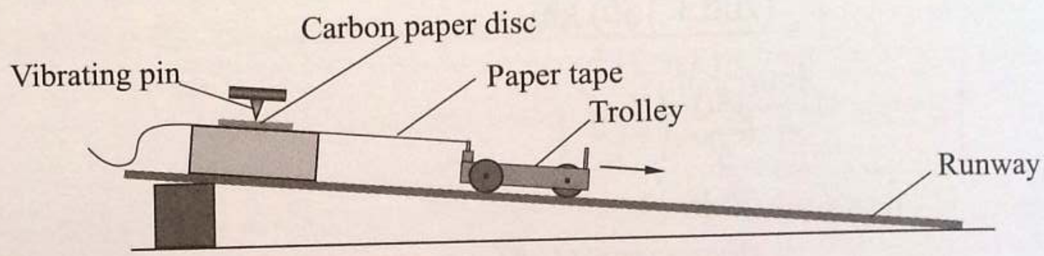
Measurement of velocity

Velocity of a moving object can be determined in a number of ways. One of the methods is to use a ticker-tape timer.

The ticker-tape timer (Fig. 8.6 (a)) is an electrical vibrator which moves a metal pin up and down 50 times every second. This means that the time taken for one complete vibration is $\frac{1}{50}$ s $= 0.02$ s. Each time the pin moves downwards it presses on a carbon paper disc and makes a dot on the paper tape which passes underneath the carbon. The tape is attached to a moving body e.g. a trolley as shown in Fig. 8.6 (b). Each successive pair of dots represents a time interval of 0.02 s. The distance between any two successive pair of dots is the distance the object has moved in 0.02 s. The tape, therefore, records the distance moved and the time taken by a moving body.



(a) A ticker-tape timer



(b) Tape and trolley

Fig. 8.6: A paper tape attached to a moving trolley

Experiment 8.1: To determine velocity of a trolley

Apparatus

- A long tape
- Ticker-tape timer
- A runway
- Carbon disc
- A trolley

Procedure

1. Pass a long tape under the carbon disc of the ticker-tape timer and attach it to a trolley. Place the trolley on a horizontal runway.
2. Displace the trolley slightly along the runway. What happens to it? (It moves a short distance and then stops due to friction).
3. Tilt the runway gradually until the trolley just moves the whole length of the runway without stopping.
4. Fix the runway in this position with a support as shown in Fig. 8.7. The runway is now said to be *friction compensated*. This means that the force of friction does not play any part in the experiment.

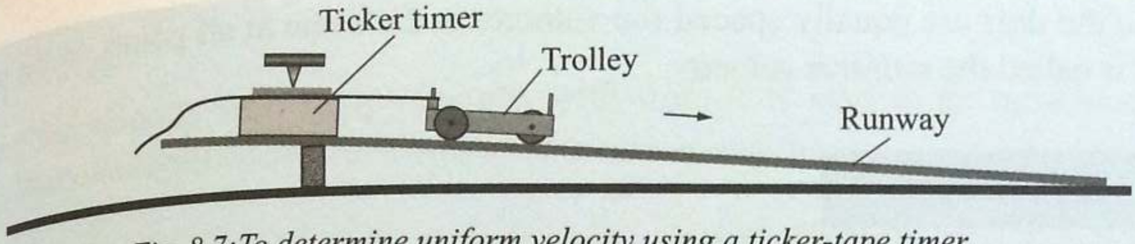


Fig. 8.7: To determine uniform velocity using a ticker-tape timer.

5. Now place the trolley at the top of the runway and start the ticker-tape timer.
6. Release the trolley to move down the runway. Remove the tape from the trolley and examine the dots made on the tape.
7. Cut the tape into portions containing 5 spaces. Stick the portions side by side to obtain a tape chart as shown in Fig. 8.8.

Discussion

Each portion represents the distance travelled in 0.10 s (5 × 0.02 s). The constant height of the portions means that the tape was covering equal distance in equal time i.e. moving with uniform velocity as indicated by line AB. Measure the height OA.

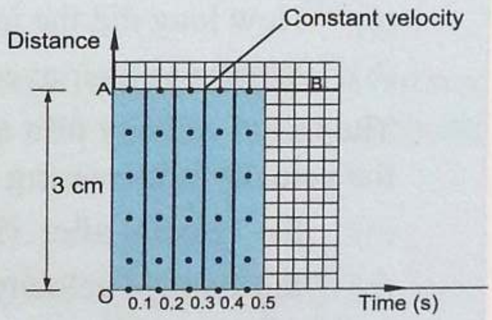


Fig. 8.8: Tape chart (Distance axis not to scale)

$$\text{Velocity} = \frac{OA}{\text{Time taken}} = \frac{3.0}{0.1} = 30 \text{ cm/s}$$

How to determine velocity using a ticker-tape timer

Consider the ticker-tape shown in Fig. 8.9.

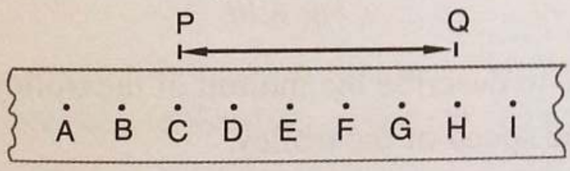


Fig. 8.9: Ticker timer tape (velocity)

Measure the distances AB, BC, CD, DE, EF, FG, GH, HI. What can you say about these distances? (All these distances are equal. They are distances the tape moves in equal intervals of time). What is the distance PQ? (3.0 cm). What is the time taken by the tape to travel the distance PQ? (As there are 5 spaces between P and Q, the time taken by the tape to travel the distance PQ is $5 \times 0.02 \text{ s} = 0.10 \text{ s}$).

Calculate the velocity between points P and Q.

$$\text{Velocity of the tape} = \frac{\text{displacement PQ}}{\text{Time taken}} = \frac{3.0}{0.10} = 30 \text{ cm/s}$$

Since the dots are equally spaced the velocity is the same at all points on the tape. This is called the *uniform velocity*.

Exercise 8.1

1. Distinguish between:
   (a) Speed and velocity
   (b) Distance and displacement
2. A student cycles to school 2.5 km away in 5 minutes. What is the student's average speed in (a) metres per second (b) kilometres per hour.
3. Fariba and Mohammed decided to walk to a picnic site 12 km away. They walked the first 6 km at an average speed of 6 km/h and the rest at 5 km/h.
   (a) How long did the journey take?
   (b) What was their average speed for the journey?
4. The initial velocity of a motor cyclist riding on a straight road is 10 m/s. If the velocity is increasing by 5 m/s every second, find;
   (a) the velocity after (i) 1 s (ii) 2 s (iii) 5 s
   (b) the average velocity in 5 s?
5. Fig. 8.10 shows a section of the motion of a trolley on a ticker-tape. The ticker-tape timer makes 50 dots in one second.

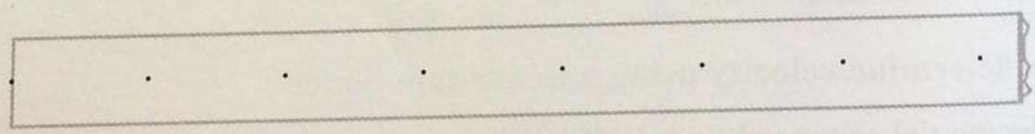


Fig. 8.10

   (a) Use the tape to describe the motion of the trolley.
   (b) Calculate the speed of the trolley.
6. Fig. 8.11 shows the motion of a trolley obtained using a ticker-tape timer which makes 50 dots in one second.

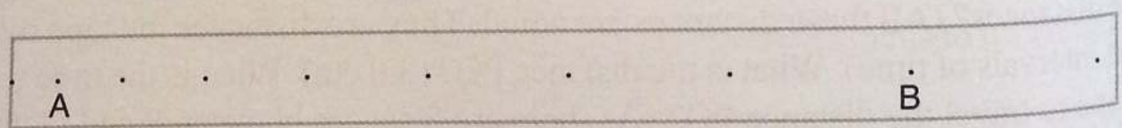


Fig. 8.11

   (a) Find the velocities at points A and B.
   (b) Given that the motion of the trolley has a constant acceleration use your answer to part (a) to determine the acceleration of the trolley.

Acceleration

When the velocity of a body changes with time it is said to be *accelerating*. Acceleration is defined as *the rate of change of velocity.*

$$\text{Acceleration} = \frac{\text{change in velocity of body}}{\text{time taken}}$$

The SI unit of acceleration is *metres per square second* or *m/s²*.

If the acceleration of a body is 4 m/s^2, it means that its velocity is increasing by 4 m/s every second. When the velocity of a body decreases, it is said to be decelerating or retarding. *Deceleration or retardation is negative acceleration*. This is usually shown with a negative sign before the value e.g – 4 m/s^2, deceleration at 4m/s^2. A body moving with uniform velocity has zero acceleration since there is no change in velocity.

When the rate of change of velocity with time is constant, the acceleration is referred to as *uniform acceleration*. Consider a body moving with velocity in time as shown in Table 8.2.

Table 8.2

Velocity (m/s)	0	5	10	15
Time taken (s)	0	2	4	6

The velocity increases by 5 m/s for every 2 seconds. Thus, the body is said to be accelerating uniformly at 2.5 m/s^2.

Example 8.4

A car accelerates from rest to a velocity of 20 m/s in 5 s. Thereafter, it decelerates to a rest in 8 s. Calculate the acceleration of the car (a) in the first 5 s, (b) in the next 8 s.

Solution

(a) $\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$

$= \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$

(rest means velocity is zero)

$= \frac{20 - 0}{5}$

$= 4\ m/s^2$

(b) Acceleration

$\frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$

$= \frac{0 - 20}{8}$

$= \frac{-20}{8} = -2.5\ m/s^2$

or deceleration of 2.5 m/s^2

Experiment 8.2: To determine acceleration of a trolley

Apparatus

- A long tape
- Carbon disk
- A runway
- Ticker-tape timer
- A trolley

Procedure

1. Set up the apparatus as in Experiment 8.1 (See Fig.8.7)
2. First ensure that the runway is friction compensated as in Experiment 8.1.
3. Now increase the angle of inclination using the wooden block until the trolley is seen to be moving with increasing speed down the runway.
4. Attach a long paper tape to the trolley.
5. Release the trolley and start the ticker-timer. What do you notice about the separation of adjacent dots on the tape?

Observation

It can be seen that the separation of the dots increases with time as shown in Fig. 8.12. Cut the tape into portions containing 5 spaces each. Since the time between two successive dots is 0.02 s, the time between each 5 spaces length is 0.10 s.

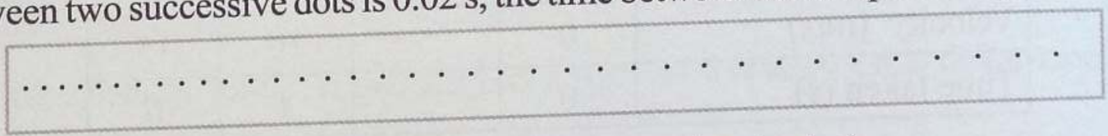


Fig. 8.12: Ticker-tape for an accelerating body

Stick the portions side by side in the right order to obtain a tape chart as shown in Fig. 8.13. What do you notice about the lengths of the tape portion?

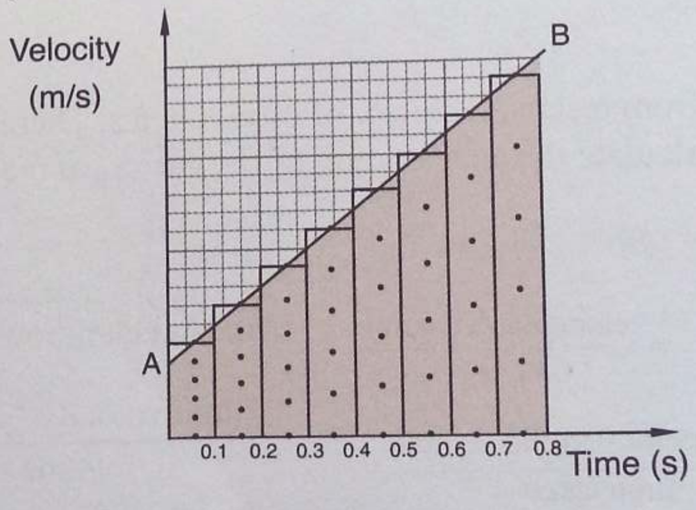


Fig. 8.13: Tape chart for constant acceleration

Discussion

Each portion represents the distance travelled in 0.10 s. The tape chart shows that the velocity of the trolley down the runway was increasing. This means that the trolley was accelerating. Determine the gradient of line AB. The straight line AB has a constant gradient which shows that the acceleration was constant. The gradient of the line is equal to the acceleration of the trolley.

8.2 Acceleration due to gravity

You may have noticed that when an object is thrown vertically upwards, it starts with a certain speed which decreases as it moves up. At some point the speed of the object becomes zero and the object starts falling back to the earth. Try this activity by using objects such as polystyrene balls or paper balls. Consult your teacher if you have to use an object such as a stone. Take care that you are not hit by the falling objects.

If an object is dropped from the top of a building or a tree, it starts from rest (with zero velocity) and its velocity increases as it falls. These observations show that objects experience an acceleration towards the earth as they fall. This acceleration due to the pull of the earth on the objects is called *acceleration due to gravity (g)*. All freely falling objects near the earth's surface are attracted towards the centre of the earth with this acceleration. It is known as *the acceleration of free fall* i.e. when a body is just allowed to fall freely in air.

Determination of acceleration due to gravity using an electromagnet

Applying the equation $s = ut + \frac{1}{2}at^2$ for a body in free fall, the distance x fallen by the body in time t is given by $x = ut + \frac{1}{2}gt^2$ where g is the acceleration due to gravity.

Since $u = 0$ for free fall,

then $x = \frac{1}{2}gt^2$

Thus, $g = \frac{2x}{t^2}$.

If we can measure x and t accurately we can calculate the value of g. The electric timer shown in Fig. 8.14 can measure time to an accuracy of 0.001 s. It is able to measure the short time t, taken by the ball to fall through distance, x.

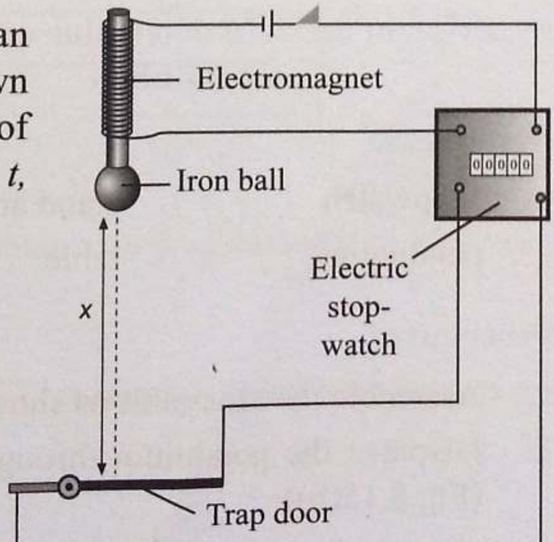


Fig. 8.14: Acceleration due to gravity using an electromagnet

When the switch is in the position shown, the electromagnet holds the ball. When the switch is opened, the ball falls and the clock starts counting at the same time. The clock stops when the ball hits the trap door and breaks the circuit. The time recorded is the time *t*, taken by the ball to fall through the distance *x*. Using the equation $x = \frac{1}{2} gt^2$, *g* can be calculated,

i.e $g = \frac{2x}{t^2}$

For convenience in calculations, a value of $g = 10 \text{ m/s}^2$ is usually used for objects near to the Earth's surface. On the moon, the acceleration of free fall is only 1.6 m/s^2. One can jump much higher on the moon's surface than on the earth's surface. It is important to note that *g* has two meanings: *the gravitational field strength* (10 newtons per kilogram 10 N/kg) and *the acceleration of free fall* (10 metres per second per second (10 m/s^2))

Let us consider a steel ball dropped from a tall building when there is no air resistance. With a gravitational acceleration of 10 m/s^2, its speed increases as follows;

At t = 0 s,the downward speed is 0 m/s

After t = 1 s,the downward speed is 10 m/s

After t = 2 s,the downward speed is 20 m/s

After t = 3 s,the downward speed is 30 m/s

After t = 4 s,the downward speed is 40 m/s

Conversely, when a body is projected vertically upwards, it decelerates at $g = {}^{-}10 \text{ m/s}^2$ until it reaches a velocity of zero at maximum height.

Experiment 8.3: To determine acceleration due to gravity using a simple pendulum

Apparatus:

- Stopwatch
- pendulum
- stand and clamp
- table

Procedure:

1. Assemble the apparatus as shown in Fig. 8.15 (a).
2. Displace the pendulum through a small angle θ (θ < 10°) and release it (Fig.8.15(b)).

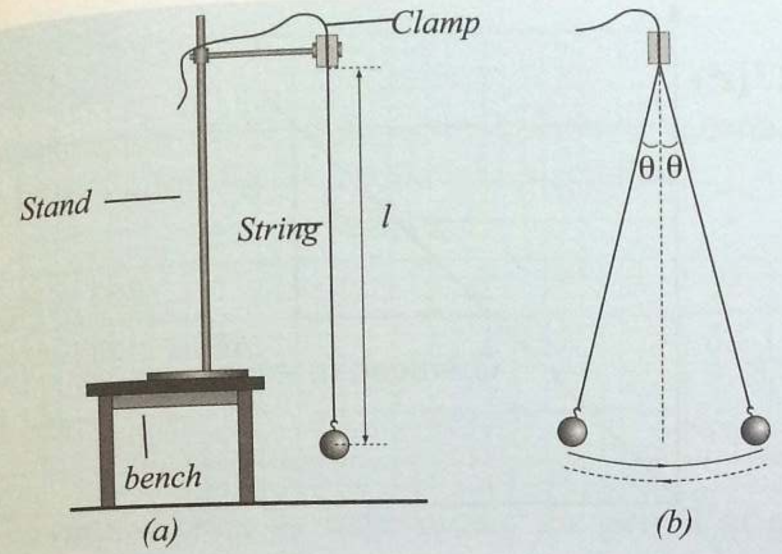


Fig. 8.15: To determine the acceleration due to gravity using a simple pendulum

3. Use a stopwatch to time 20 oscillations (complete cycles) of the pendulum. Repeat the experiment a second time and calculate the average time for 20 oscillations. Repeat the process for at least six different lengths. Record your results in a table (see Table 8.3).

Table 8.3

Length, l (m)	Time for 20 oscillations (s) trial 1 t_1	trial 2 t_2	Average time t for 20 oscillations (s)t $=\frac{t_1+t_2}{2}$	Periodic time T (s) $T=\frac{t}{20}$	T^2 (s^2)
0.60					
0.70					
0.80					
0.90					
1.00					
1.10					

Draw a graph of T^2 against l.

Draw the line of best fit through the points. Determine the gradient, m of the line.

Discussion

On plotting, the values, we get a graph of T^2 against l similar to the one in Fig. 8.16.

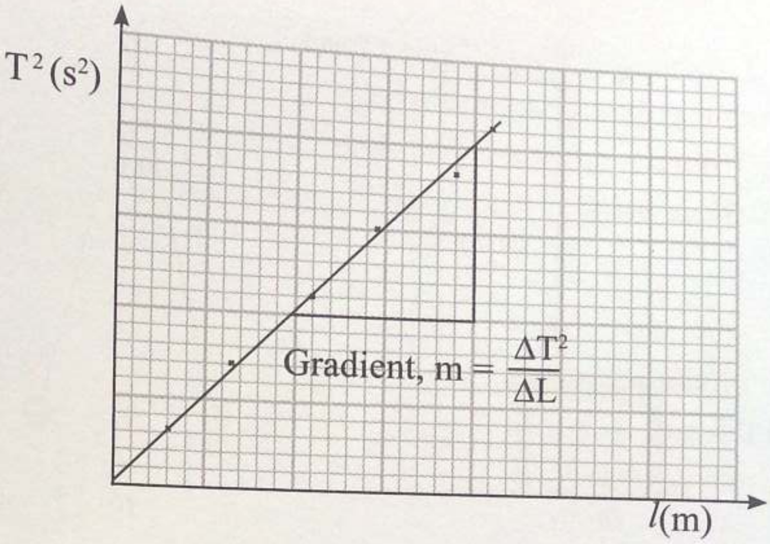


Fig. 8.16: Graph of time, T^2, against length, l,

For a simple pendulum oscillating with a small amplitude, the period (T) is given as $T = 2\pi\sqrt{\frac{l}{g}}$,

Where, T is the period, l, the length of the pendulum and g the acceleration due to gravity.

Squaring both sides of the equation, $T = 2\pi\sqrt{\frac{l}{g}}$, we get

$$T^2 = \frac{4\pi^2 l}{g^2} \quad \ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Making g the subject of the formula in equation (i), we get

$$g = \frac{4\pi^2 l}{T^2}$$

Therefore, acceleration due to gravity is given by $g = \frac{4\pi^2 l}{T^2}$.

If the values of T^2 and the corresponding values of l obtained through an experiment are plotted in a graph, we can determine the value of g as follows:

Comparing equation (i) and the general equation of the straight line i.e.

$T^2 = \frac{4\pi^2 l}{g}$ and $y = mx + c$,

Where T^2 and l correspond to y and x respectively, we note that

$\frac{4\pi^2 l}{g^2} = m$ (the gradient of the graph).

Making g the subject, we get that g is given by

$$g = \frac{4\pi^2 l}{m} \qquad \text{i.e.} \quad g = \frac{4\pi^2 l}{\text{gradient of the graph}}$$

Exercise 8.2

1. The following results (Table 8.4) were obtained in a certain experiment to determine factors affecting the period of a pendulum.

Table 8.4

Length (l) (cm)	0	5	10	15	20	25	30	35	40
Time (t) for 10 oscillations	0	1.7	3.2	4.4	5.5	6.5	7.4	8.2	8.6
Period $T(t) = \frac{t}{10}$ (s)									

(a) Complete the table by determining the period of each time for 10 oscillations.

(b) Draw a graph of period, (T) in seconds against length, l, (cm).

2. A form three student performed an experiment to dertmine the acceleration due to gravity by timing on oscillating pendulum and obtained the following results (Table 8.5).

Table 8.5

Length of pendulum (m)	0.12	0.13	0.14	0.15	0.16	0.17	0.18
Time (t) for 20 oscillations	30.5	32.0	36.7	40.2	45.0	48.6	50.3

(a) Explain how the length of the pendulum is measured.

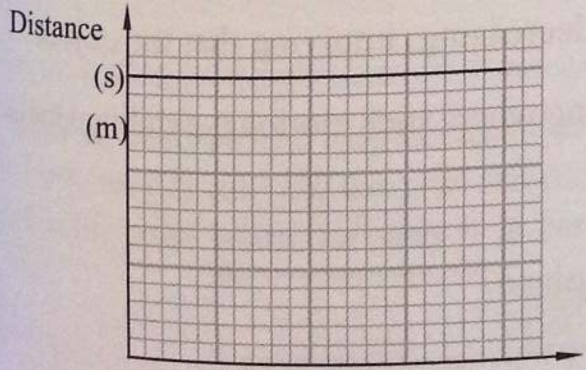
(b) Plot a graph of T^2 against L and determine the acceleration due to gravity.

(c) List the precautions you would take in this experiment to ensure accurate results.

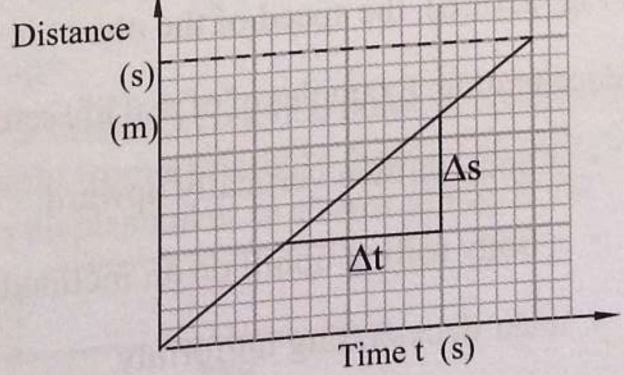
8.3 Motion-time graphs and their interpretation

Distance – time graph

Fig. 8.17 shows two distance-time graphs for two bodies: one at rest (Fig. 8.17(a)) and the other moving at constant velocity (Fig. 8.17(b))



(a) A Body at rest



(b) Moving body with constant velocity

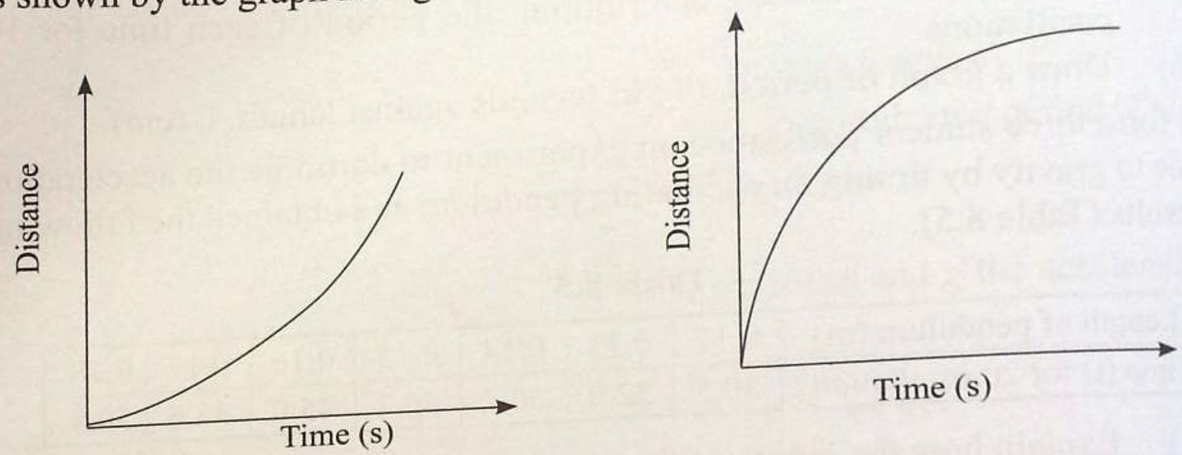
Fig. 8.17: Distance-time graph

The graph in Fig. 8.17(a) shows that distance covered by the body is not changing with time. The body is therefore at rest (stationary).

The graph in Fig. 8.17(b) shows that the distance covered by the body is increasing with time.

The gradient of the graph is $\frac{\Delta s}{\Delta t}$ and represents the speed of the object. Thus, the graph represents the motion of the body moving with constant (uniform) speed.

In some cases, the distance travelled by the object increases or decreases with time as shown by the graph in Fig. 8.18.



(a): Speed increasing with time

(b): Speed decreases with time

Fig. 8.18: Distance - time graph

In Fig. 8.18(a) the gradient representing speed is increasing, implying that the object is accelerating. Examples of real life settings where such motion is exhibited include:

- a body rolling down an inclined plane.
- a car accelerating uniformly from rest.

In Fig. 8.18(b), the speed of the object is decreasing, implying that the object is decelerating. Examples of of real life setting where such motion is exhibited include:

- a body thrown vertically upward.
- a body rolling uphill on an inclined plane.
- a car decelerating uniformly.

Example 8.5

Fig. 8.19 shows a distance-time graph for a motorist. Study it and answer the questions that follow.

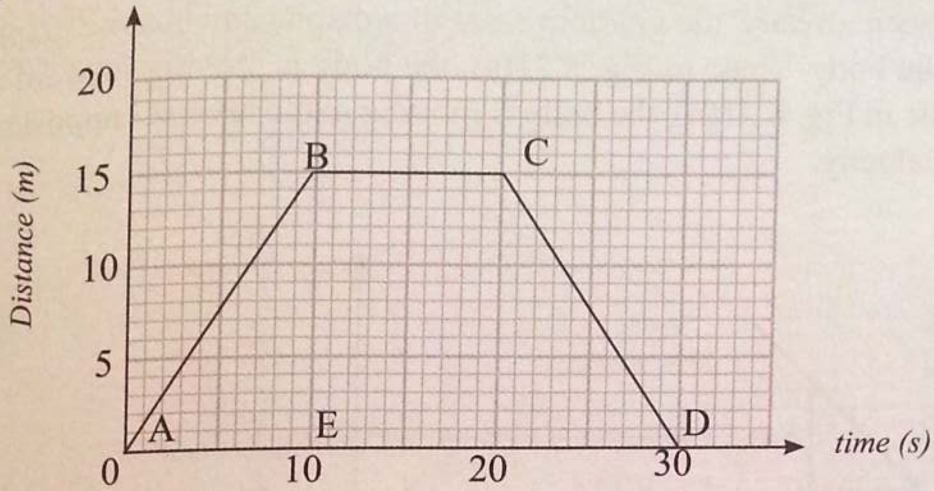


Fig. 8.19: Distance - time graph

(a) How far was the motorist from the starting point after 10 seconds?

(b) Calculate the average speed of the motorist for the first 10 seconds.

(c) Describe the motion of the motorist in regions (i) BC (ii) CD

Solutions

(a) By reading directly from the graph, distance travelled in 10 s = 15 m.

(b) Slope of the graph = speed of the motorist.

$$\text{Slope} = \frac{\text{change in distance}}{\text{change in time}} = \frac{15-0}{10-0} = 1.5\text{mls}$$

(c) (i) At BC, distance does not change but time changes, hence the body is at rest (stationary).

(ii) At CD, the motorist is moving at a constant speed (towards the starting point).

Interpretation of displacement-time graph

In order to describe the displacement of a body, a reference point is considered . The body may be moving towards left or right from a reference point. The reference point is the point when the body is at zero displacement as shown in Fig. 8.20.

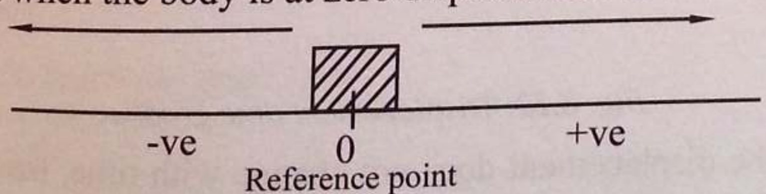
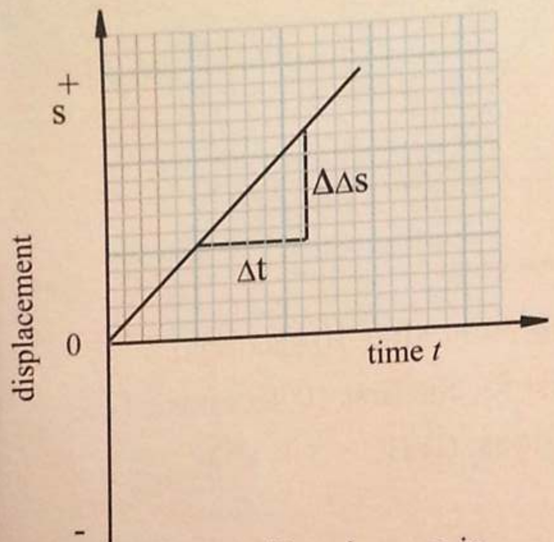


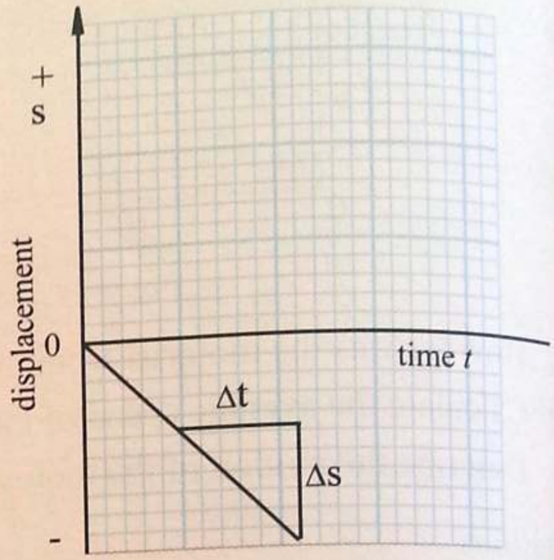
Fig. 8.20: Moving object

Let us consider a body moving in such a way that its displacement changes uniformly with time. Depending on the direction taken, two graphs can be drawn as shown in Fig. 8.21(a) and (b).

As we have seen already, the gradient Δs/Δt of a displacement-time graph gives the velocity of the body. Thus, in Fig. 8.21(a), the body is moving forward at constant velocity while in Fig. 8.21(b), the body is moving in the reverse (opposite direction) at constant velocity.



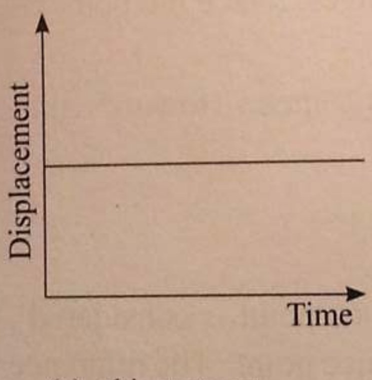
(a) A positive uniform increase in displacement with time



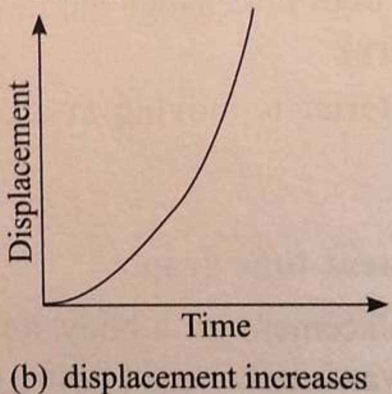
(b) A negative uniform increase in displacement with time

Fig. 8.21: Displacement-time graphs for a moving object

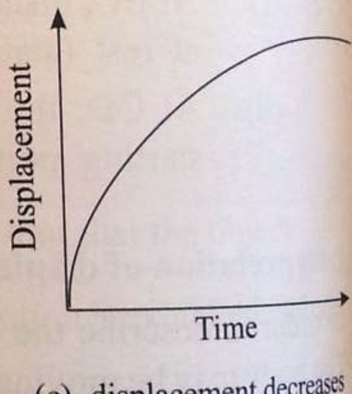
Let us now sketch displacement-time graphs for a body at rest and one whose rate of change of displacement with time (velocity) is not constant (Fig. 8.22).



(a) object at rest.



(b) displacement increases non-linearly with time.



(c) displacement decreases non-linearly with time.

Fig. 8.22: Displacement time graphs

In Fig. 8.22(a), the displacement does not change with time, hence the body is at rest. In Fig 8.22 the gradient (velocity) is increasing hence the body is accelerating. In Fig.8.22(c), velocity is decreasing hence the body is decelerating.

Speed-time graph

(a) Object at rest

At rest, the object is covering no distance since there is no movement. Therefore, the speed of the object is zero as shown in Fig. 8.23.

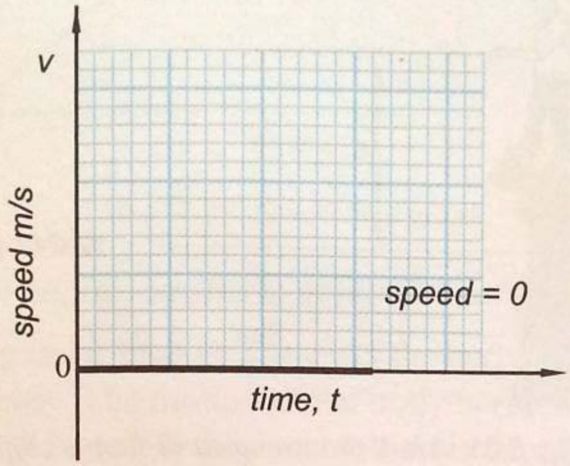


Fig. 8.23: speed-time graph for a body at rest

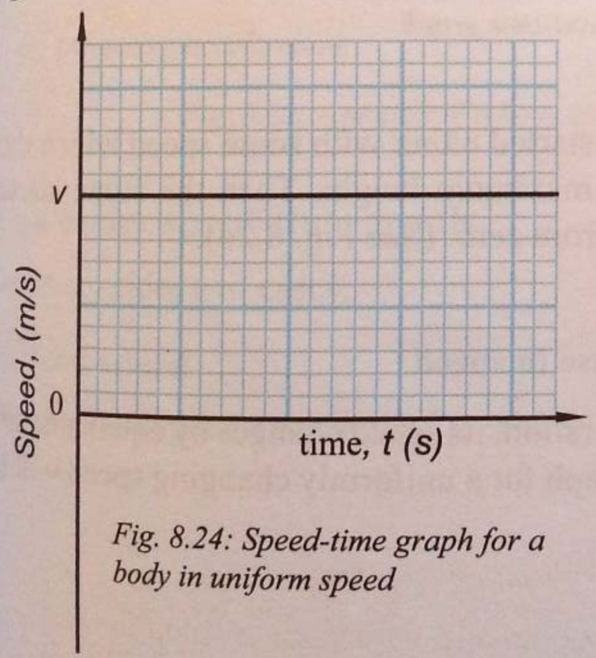
The gradient in a speed time graph gives us

$$\frac{\text{Change in speed}}{\text{Change in time}} = \text{acceleration.}$$

In this case (when the object is stationary), the gradient is zero and so acceleration is zero.

A body moving with uniform speed

Fig. 8.24 shows a motuion of a body of moving with the uniform speed.



Gradient = 0

acceleration in this case is zero

$a = 0\ m/s^2$

Fig. 8.24: Speed-time graph for a body in uniform speed

A body moving with non-uniform speed

Consider a ball thrown vertically upwards with an initial speed u from the top of a cliff which is s metres from the water level. See Fig. 8.25.

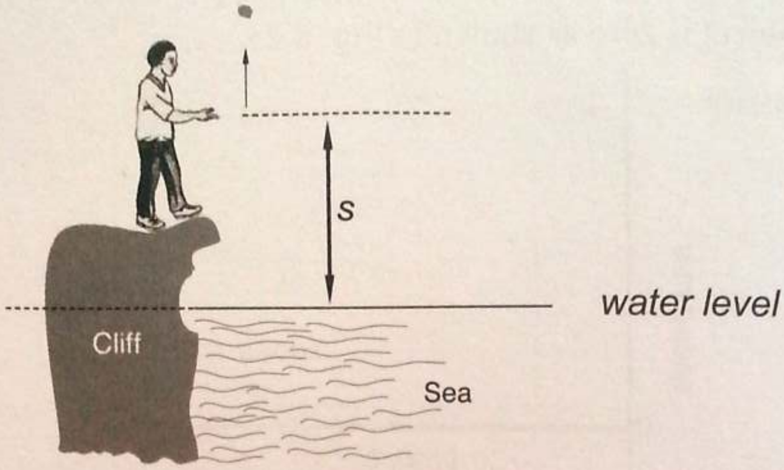


Fig. 8.25: A ball thrown upwards from a cliff

If we take the sea level as our reference point and upwards as the positive direction, the motion graphs are as follows;

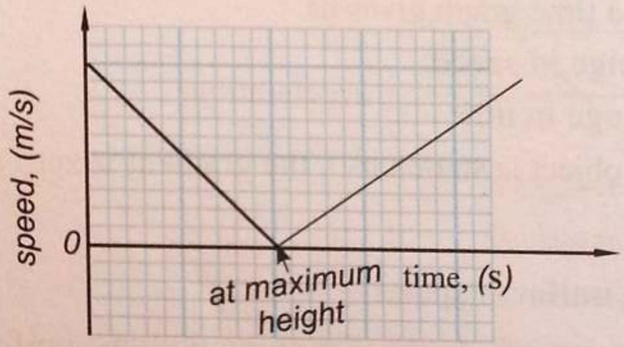


Fig. 8.26: Speed-time graph

The stone was thrown upwards and so it started either with some speed which then started decreasing to zero speed at the maximum height. Then the stone started dropping as its speed started increasing from zero. (See Fig. 8.26).

An object moving with uniform increase in speed

When a body moves with uniform acceleration, its speed changes by equal amounts in equal interval time. The speed-time graph for a uniformly changing speed will be straight line as shown in Fig. 8.27.

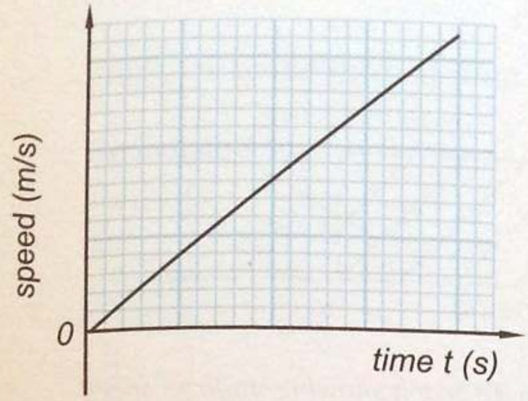
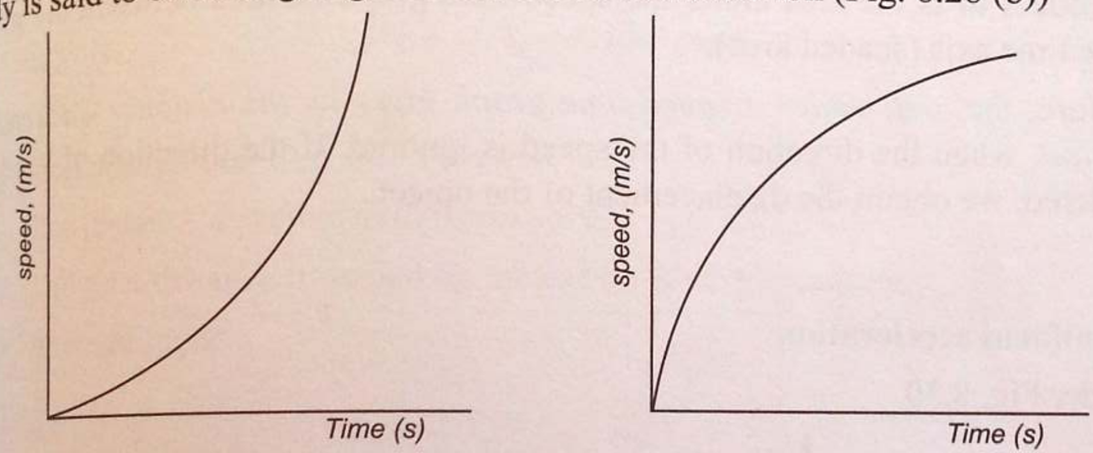


Fig. 8.27: Speed-time graph

(e) A body moving with non-uniform acceleration/deceleration

Consider a body falling in air then in water. As the body falls through air, the speed increases but not regularly. The motion of the body is represented by a speed - time graph as shown in Fig. 8.28 (a). If the acceleration is decreasing non-uniformly the body is said to be undergoing non-uniform random motion (Fig. 8.28 (b))



(a) Increasing acceleration *(b) Decreasing acceleration (deceleration)*

Fig. 8.28: A motion of a body mocing with non-uniform motion

Area under speed-time graph

Let us consider two cases:

Uniform speed

The graph of speed against time is shown in Fig. 8.29.

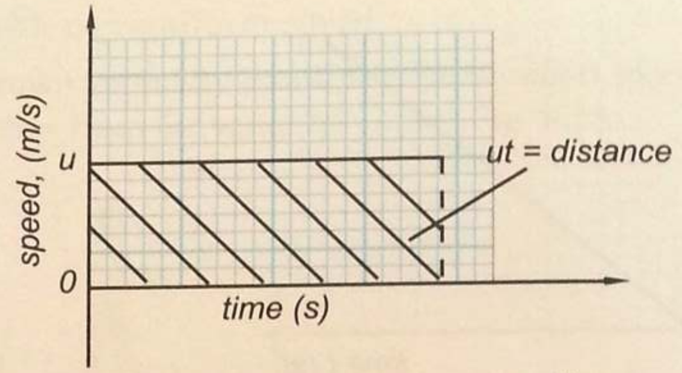


Fig. 8.29: Speed-time graph showing uniform speed

Consider a body moving at constant speed of *u* m/s for a time, *t* seconds. The distance x travelled by a body moving with a speed u for a time *t* is given by;

distance (x) = constant speed (u) × time (t)

$$x = ut$$

The product **ut** is the area under the speed-time graph i.e area between the graph and the time axis (shaded area).

Therefore, *the area under a speed-time graph gives us the distance covered by the object,* when the direction of the speed is ignored. If the direction of speed is considered, we obtain the displacement of the object.

(ii) Uniform acceleration

Consider Fig. 8.30

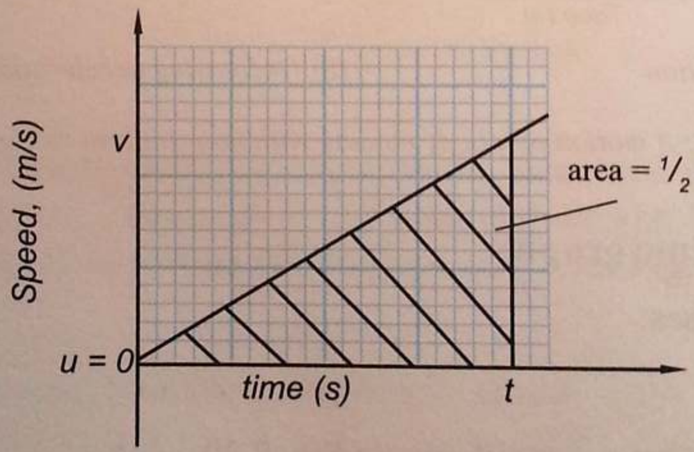


Fig. 8.30: Speed-time graph showing uniform acceleration

Distance covered = $\frac{1}{2}$ v × t = Area under the velocity time graph

Example 8.6

Fig. 8.31 shows a graph of speed against time of a motion of a car travelling on the road.

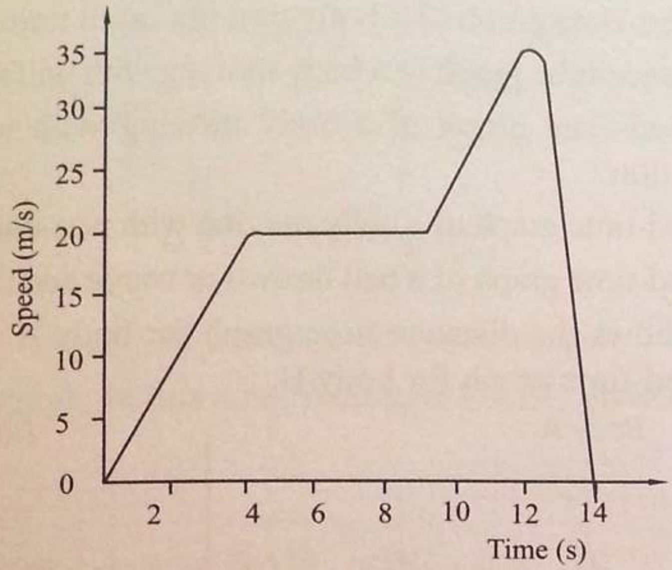


Fig. 8.31: Speed - time graph

Determine

(a) The acceleration of a car in the first 4 s.

(b) The distance travelled at uniform speed.

(c) The total distance travelled by the car for a whole journey.

(d) Average speed.

Solution

(a) $a = \dfrac{\text{change in velocity}}{\text{time taken}} = \dfrac{\Delta v}{\Delta t} = \dfrac{20}{4} = 5 \text{ m/s}^2$

(b) Distance travelled = Area under the graph

$$= \text{speed (m/s)} \times \text{time (s)}$$

$$= 20 \times 4 = 80 \text{ m}$$

(c) Total distance covered = Total area under the graph

$$= \left(\frac{1}{2} \times 4 \times 20\right) + (20 \times 4) + \frac{1}{2}(35 + 20)4 + \frac{1}{2}(35 + 20)$$

$$= 40 + 80 + 110 + 35 = 265 \text{ m}$$

(d) Average speed $= \dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{265}{10} = 26.5 \text{ m/s}$

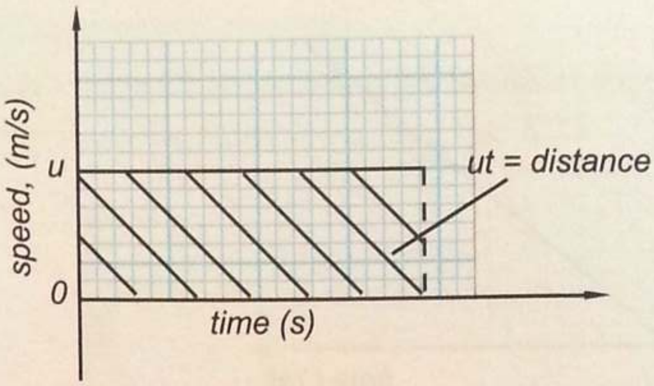


Fig. 8.29: Speed-time graph showing uniform speed

Consider a body moving at constant speed of *u* m/s for a time, *t* seconds. The distance x travelled by a body moving with a speed u for a time *t* is given by;

distance (x) = constant speed (u) × time (t)

$$x = ut$$

The product **ut** is the area under the speed-time graph i.e area between the graph and the time axis (shaded area).

Therefore, *the area under a speed-time graph gives us the distance covered by the object,* when the direction of the speed is ignored. If the direction of speed is considered, we obtain the displacement of the object.

(ii) Uniform acceleration

Consider Fig. 8.30

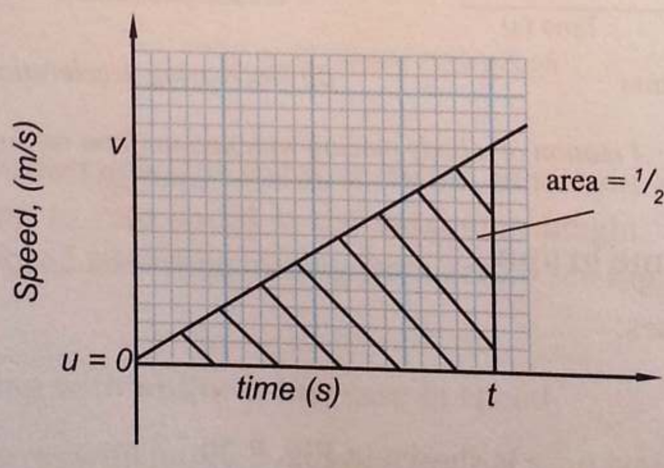


Fig. 8.30: Speed-time graph showing uniform acceleration

Distance covered $= \frac{1}{2} v \times t$ = Area under the velocity time graph

Example 8.6

Fig. 8.31 shows a graph of speed against time of a motion of a car travelling on the road.

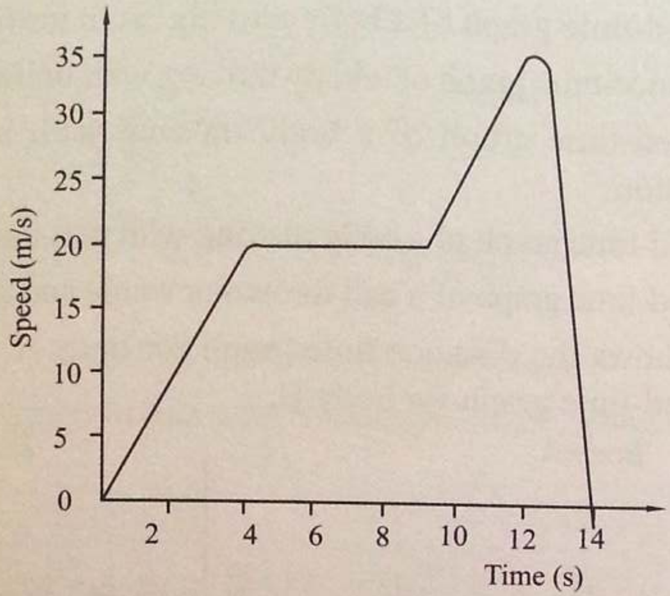


Fig. 8.31: Speed - time graph

Determine

(a) The acceleration of a car in the first 4 s.

(b) The distance travelled at uniform speed.

(c) The total distance travelled by the car for a whole journey.

(d) Average speed.

Solution

(a) $a = \dfrac{\text{change in velocity}}{\text{time taken}} = \dfrac{\Delta v}{\Delta t} = \dfrac{20}{4} = 5 \text{ m/s}^2$

(b) Distance travelled = Area under the graph

$= \text{speed (m/s)} \times \text{time (s)}$

$= 20 \times 4 = 80 \text{ m}$

(c) Total distance covered = Total area under the graph

$= (\frac{1}{2} \times 4 \times 20) + (20 \times 4) + \frac{1}{2}(35 + 20)4 + \frac{1}{2}(35 + 20)$

$= 40 + 80 + 110 + 35 = 265 \text{ m}$

(d) Average speed $= \dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{265}{10} = 26.5 \text{ m/s}$

Exercise 8.3

1. Sketch the following graphs.
   (i) The speed-time graph of a body moving with uniform speed.
   (ii) The distance-time graph of a body moving with uniform speed.
   (iii) The speed-time graph of a body moving with uniform (constant) acceleration.
   (iv) The speed-time graph of a body moving with non-uniform acceleration.
   (v) The speed-time graph of a ball thrown upwards and then caught again.
2. Fig. 8.32 (a) shows the distance-time graph for body A while Fig. 8.32 (b) shows the speed-time graph for body B.

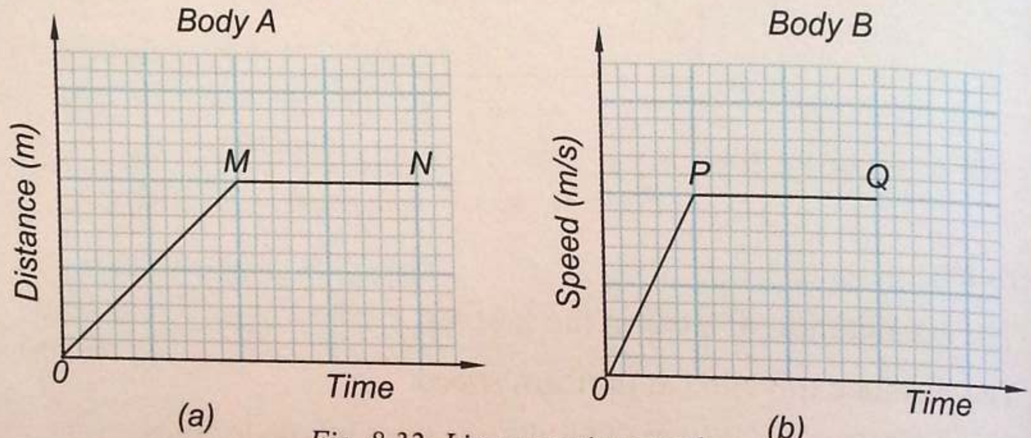


Fig. 8.32: Linear motion graphs

   Describe fully the motion of the bodies in the regions:
   (a) OM (b) MN (c) OP (d) PQ
3. (a) Sketch a velocity-time graph for a car moving with uniform acceleration from 10 m/s to 30 m/s in 20 s.

Velocity – time graphs

A velocity-time graph tells us how the speed and direction of an object changes with time. Where there is no change in direction a velocity–time graph looks the same as a speed-time graph. On a velocity – time graph, the gradient of the line is numerically equal to the acceleration. The gradient tells us how much extra speed is gained every second. The area under a velocity–time graph gives the distance covered by the object.

A body moving at constant velocity

The velocity-time graph for a body moving at constant velocity shown in Fig.8.33.

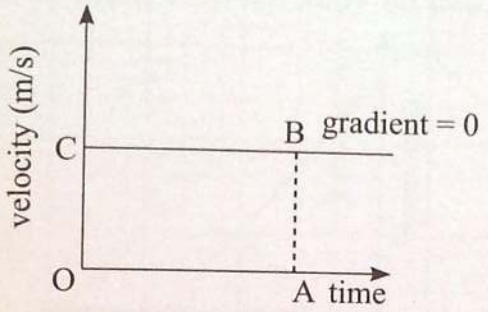


Fig. 8.33: Motion with constant velocity

The area under the graph, in this case, rectangle OABC gives the distance covered by the body.

$$\boxed{\text{Distance covered} = \text{Area of OABC} = OC \times OA}$$

A body moving with uniform acceleration from rest

Consider a car that started from rest and increased its velocity regularly. Its velocity-time graph is shown in Fig. 8.34.

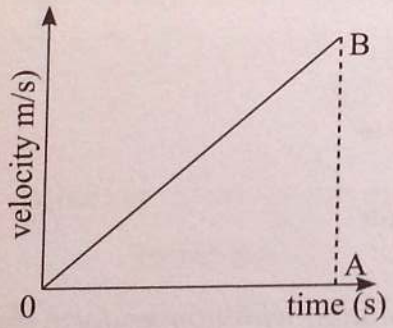


Fig. 8.34: Motion with uniform acceleration.

The gradient of velocity time graph represent the acceleration i.e.

$$\text{gradient} = \frac{\text{change in velocity } (\Delta V)}{\text{time taken } (\Delta t)} = \text{acceleration}$$

The rate of change of velocity (acceleration) is uniform, hence the graph is a straight line.

The area under the graph i.e area of ΔOAB, gives the distance covered by the car

$$\text{Distance covered} = \text{Area of } \Delta OAB = \tfrac{1}{2}(OA \times AB.)$$

A body decelerating uniformly

Consider a car moving at a particular velocity. If the brakes are applied such that it decelerates uniformly to rest, its velocity-time graph is as shown in Fig. 8.35.

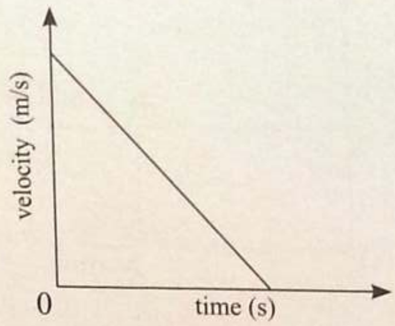
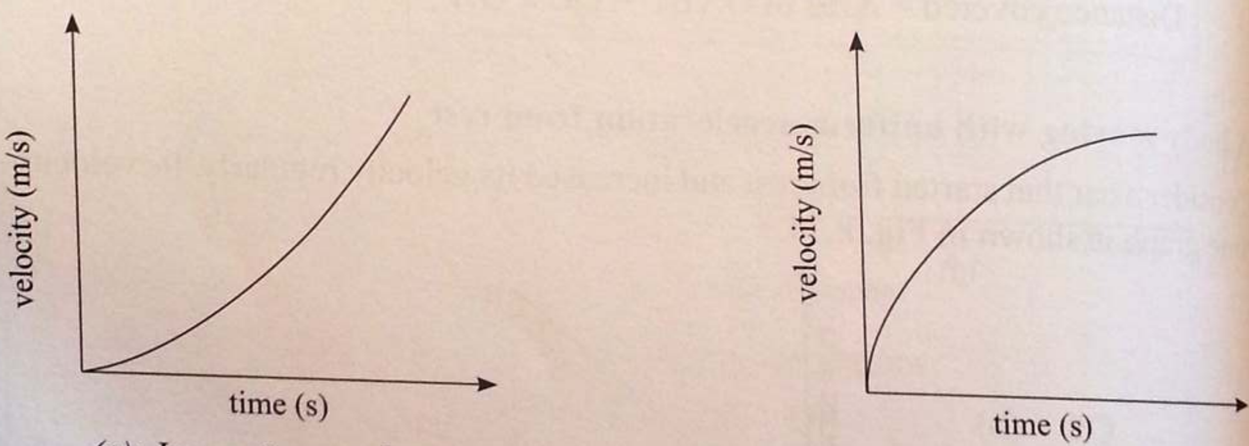


Fig. 8.35: Velocity-time graphs

A body moving with non-uniform acceleration

In some situations, acceleration is not uniform; it may be increasing or decreasing. This can be represented by the graphs shown in Fig. 8.36 (a) and (b).



(a): Increasing acceleration

(b): Decreasing acceleration

Fig. 8.36:Motion with non-uniform deceleration.

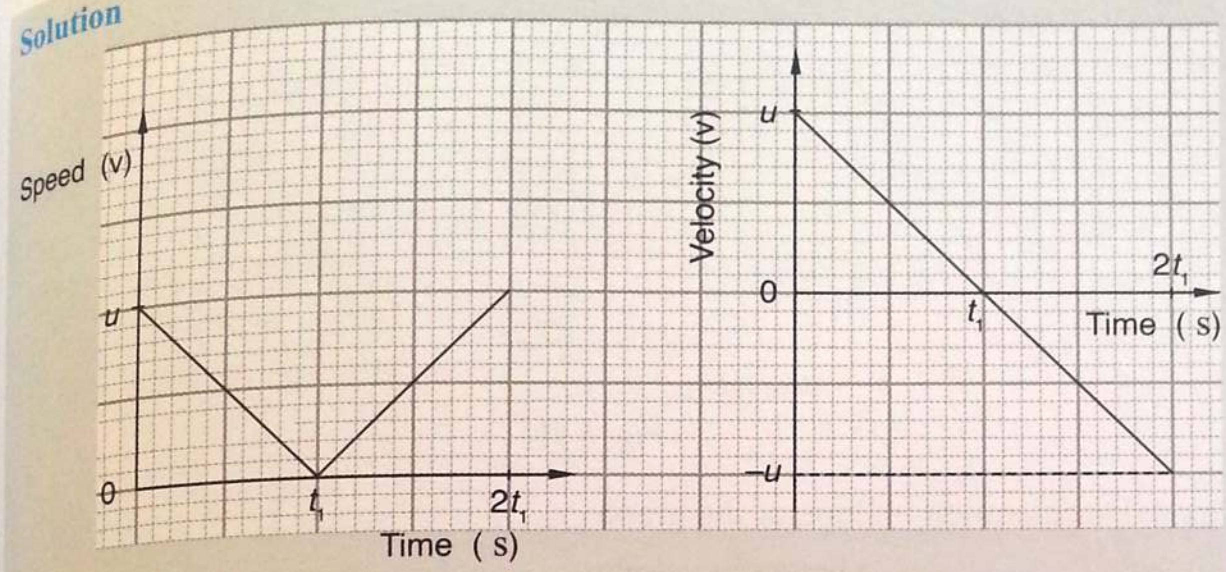
Example 8.7

A stone is thrown vertically upwards with an initial velocity u. Sketch;

(a) Its speed-time graph.

(b) The velocity-time graph for the stone from its release up to the time it comes back to its original position.

Solution



(a) Speed-time graph (b) Velocity-time graph

Fig. 8.37: Motion of the body under gravity

The time taken to reach the maximum height is t_1

The time taken by the body to fall back to its starting point is also t_1

The total time of flight is $2t_1$

Example 8.8

Table 11.12 shows the data collected to study the motion of cyclist.

Table 8.6

Velocity (m/s)	0	3	6	6	6	6
Time (s)	0	2	4	6	8	10

(a) Plot a graph of velocity (y-axis) against time (x–axis).

(b) Use your graph to determine the:

(i) The acceleration of the cyclist in the first four seconds.

(ii) Distance travelled by the cyclist in the entire period.

Solution

(a)

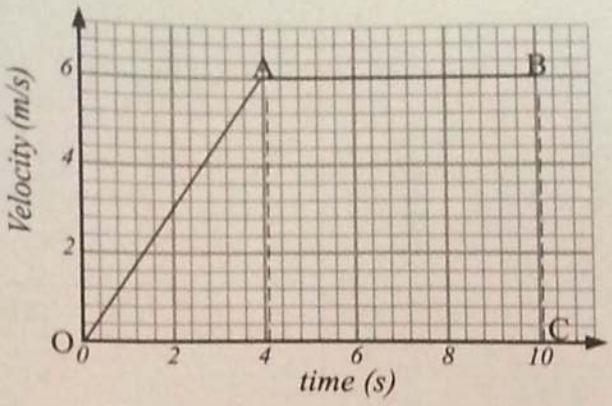


Fig. 8.38

(b) (i) acceleration = slope of the graph

$$= \frac{\text{change in velocity}}{\text{change in time}}$$

$$= \frac{6-0}{4-0} = \frac{6}{4} = 1.5 \text{ m/s}^2$$

(ii) Distance travelled in the entire period

$= $ area under the graph between t = 0 s and t = 10 s

$= $ area of the trapezium OABC.

$= \frac{1}{2}$ (sum of parallel sides) × height

$= \frac{1}{2}(6 + 10) \times 6 = 48$ m

Total distance covered = 48 m.

Exercise 8.4

1. Define the term acceleration.
2. A bus changes its speed from 180 m/s to rest in 10 s. Calculate the
   (a) deceleration of the bus.
   (b) displacement of the bus.
3. (a) Sketch a velocity-time graph for a car moving with uniform acceleration from 10 m/s to 30 m/s in 20 s.
   (b) Use the graph to find the acceleration of the car and the total distance travelled by the car.

4. Fig. 8.39 shows the velocity-time graph of a car.

Use the graph to find:

(a) acceleration of the car.

(b) deceleration of the car.

(c) total displacement of the car.

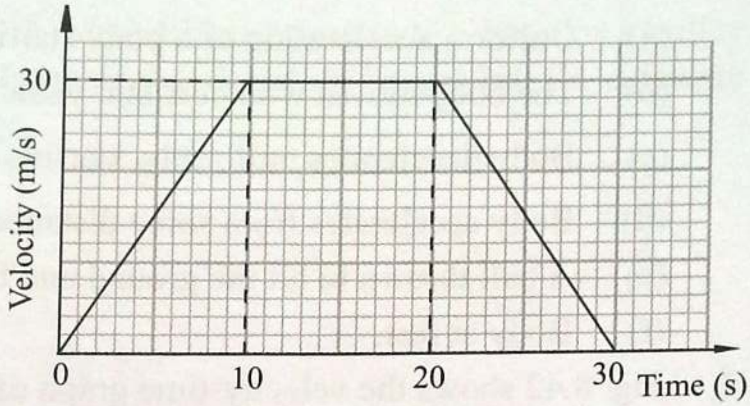


Fig. 8.38

5. The graph in Fig. 8.40 shows the motion of a body falling freely under gravity.

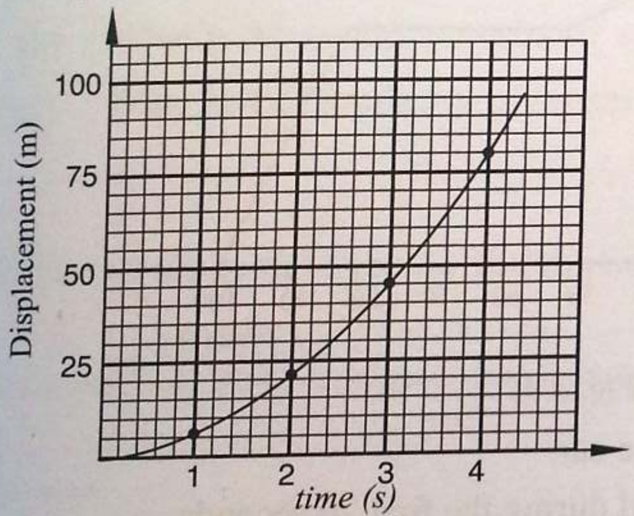


Fig. 8.40

(a) Determine the values of s at $t = 1, 2, 3$ and 4 s.

(b) Draw a graph of s against t^2.

(c) Use your graph in (b) to find the value of g.

6. The sketches in Fig. 8.41 represent the motion of bodies in a straight line. Match the graph with appropriate description given below, the graphs.

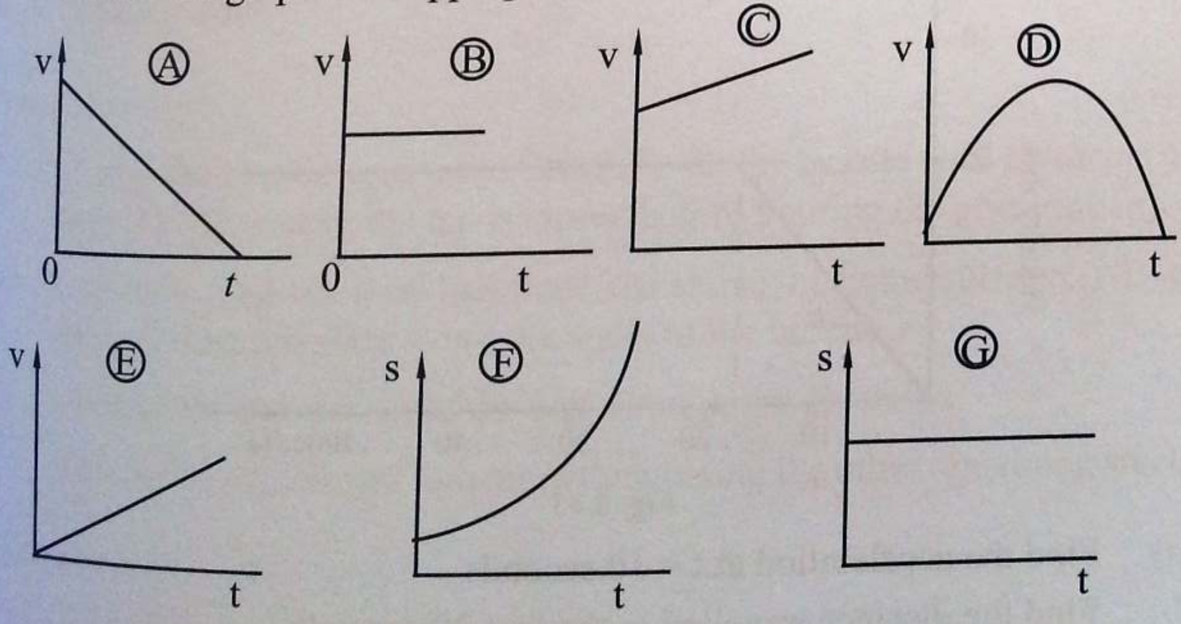


Fig. 8.41

(a) Uniform acceleration of a body starting from rest.

(b) The body moves with constant velocity.

(c) Body decelerates uniformly, starting with an initial positive velocity.

(d) Body accelerates from some distance away from reference point.

(e) A ball thrown to hit the ground and bounces back.

(f) Body at rest.

7. Fig. 8.42 shows the velocity-time graph of a motorcar during a short drive.

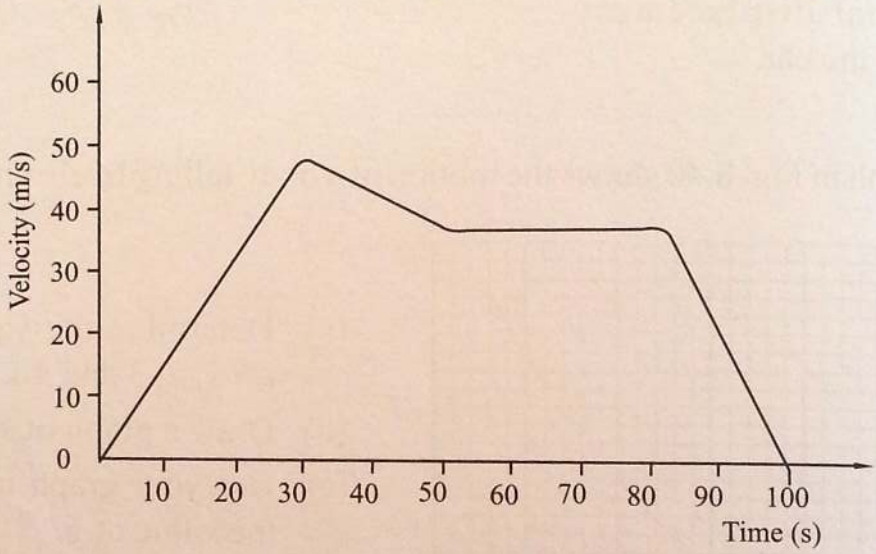


Fig. 8.42

(a) Describe the motion of the car.

(b) Find the distance travelled during the first 20 seconds.

(c) Find the retardation at time, 90 seconds.

8. Figure 8.43 shows velocity-time graph for a certain body.

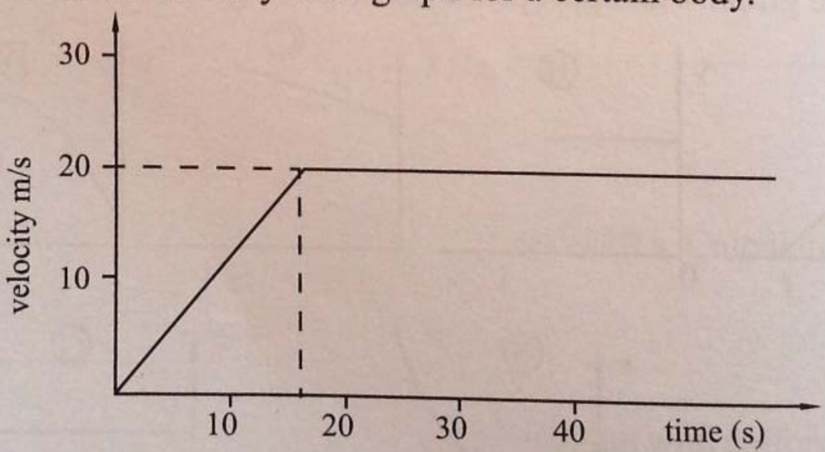


Fig. 8.43

(a) Find the acceleration at t = 10 seconds

(b) Find the distance travelled in the first 30 seconds

9. Figure 8.44 represents a part of a tape pulled through a tick-timer by a trolley moving down a plane. If the frequency of the ticker-time is 100 Hz, calculate the acceleration of the trolley.

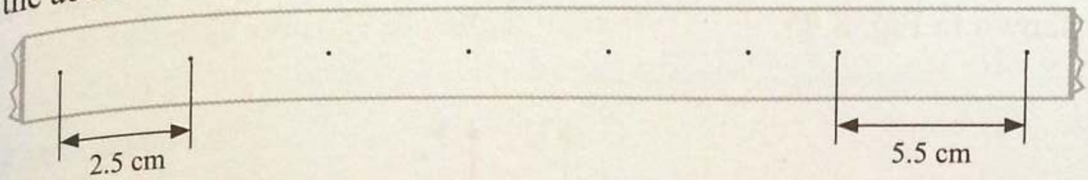


Fig. 8.44

10. Draw a graph of velocity gainst time for a car which starts with an initial velocity of 20 m/s and accelerates uniformly at 4 m/s^2 for 8 seconds, then slows down to rest in 20 seconds.
   (a) How far does the car travel?
   (b) What is the maximum velocity attained by the car?
   (c) What is the retardation of the car as it comes to rest?

8.4 Effect of air resistance on the motion of a body falling through air

Experiment 8.4: To investigate the motion of a body falling in a fluid.

Apparatus

- Burette
- 3 steel balls
- clamp funnel
- glycerine
- complete stand

Procedure

1. Clamp the burette vertically. Carefully fill the burette with glycerine using a funnel (**NB:** ensure the tap is closed before pouring the glycerine in).
2. Carefully drop one steel ball from just above the liquid surface. (NB: ensure the ball does not slide along the walls of the burette.)
3. Observe the movement of the ball through the glycerine.
4. Repeat this experiment two more times using the other remaining steel balls.

Observation

Initially the speed of the ball increases and eventually becomes constant.

Discussion

When an object is falling in air or any other fluid, three forces act on it. These forces are: *weight of the body (W)*, *upthrust (U)* and *viscous drag* (fluid friction F_r) as shown in Fig. 8.45.

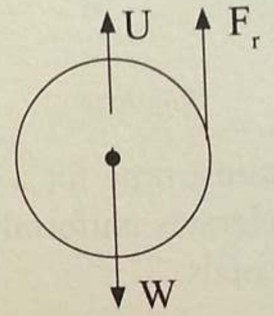


Fig. 8.45

The viscous drag increases with increase in speed of the falling body.

Initially, $W > (U + F_r)$ hence the body accelerates downwards. As viscous drag (F_r) increases, it reaches a point where $U + F_r = W$

There being no resultant force, the body moves at uniform (constant) velocity. This constant velocity is called terminal velocity (V_t). *Terminal velocity is therefore defined as the maximum downward velocity possible for a particular object falling through a fluid when nil forces on the body are zero.*

If the velocity of the object is plotted against time, a graph similar to the one in Fig. 8.46 is obtained.

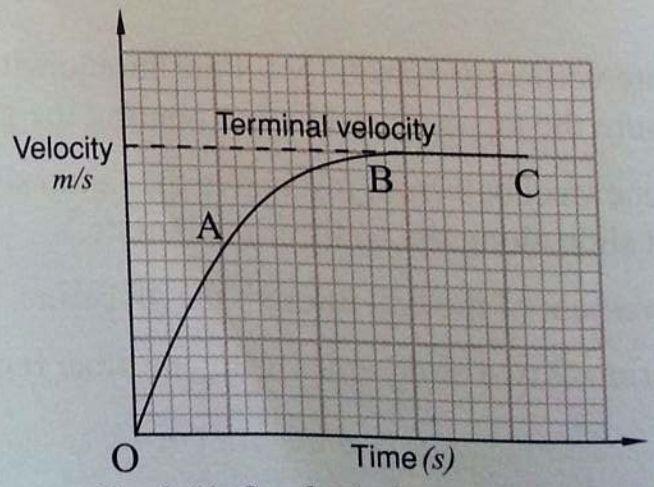


Fig. 8.46: Graph of velocity against time

The motion of the body in the graph can be described using the following regions:

OA – velocity increasing with time (constant acceleration)

AB – acceleration of the body decreasing to zero upto terminal velocity.

BC – body at constant velocity (terminal velocity).

Think about it!

How can a parachutist land from the sky without getting hurt yet we cannot jump from a roof-top safely?

For a body falling freely in a vacuum, the motion is due to its weight only. i.e there is no air ressitance and upthrust.

The velocity of the body is increasing with time. The body has uniform acceleration, a which is equal to g. The velocity-time graph of a motion of the body moving with uniform acceleration is shown in Fig. 8.47.

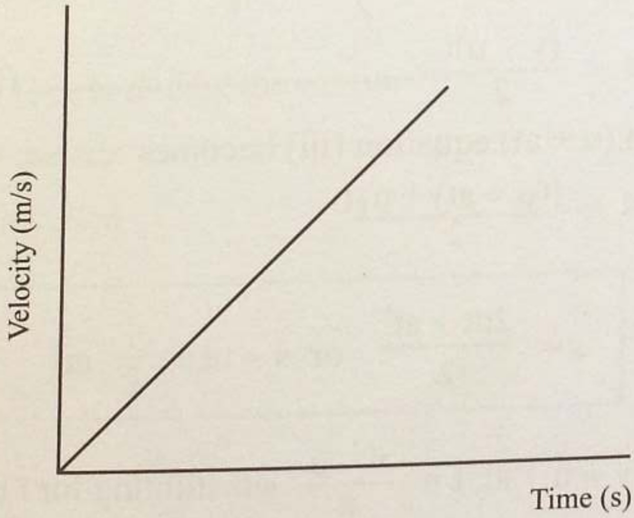


Fig. 8.47: Velocity-time graph of a motion of the body falling in vacuum

8.5 Equations of uniformly accelerated motion

Consider a body moving along a straight line with uniform (constant) acceleration (See Fig. 8.48).

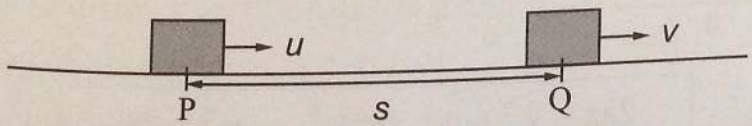


Fig. 8.48: A body moving at uniform acceleration.

Let the initial velocity at point, P be u and the final velocity at Q be v. If the distance travelled PQ, in time, t is s, then:

1. acceleration, $a = \frac{\text{change in velocity}}{\text{time taken}}$

$$= \frac{\text{Final velocity (v) – Initial velocity (u)}}{\text{time taken}}$$

$$\boxed{a = \frac{v - u}{t} \quad \text{or} \quad v = u + at}$$

2. average velocity $= \frac{\text{displacement}}{\text{time taken}} = \frac{s}{t}$ .... (i)

Also,

average velocity $= \frac{\text{Initial velocity + final velocity}}{2}$

$$= \frac{u + v}{2} \quad \text{or} \quad \frac{v + u}{2} \quad \text{.......................} \quad \text{(ii)}$$

Equating equation (i) and (ii) i.e $\frac{v + u}{2} = \frac{s}{t}$

$$s = \frac{(v + u)t}{2} \quad \text{......................................} \quad \text{(iii)}$$

substituting v with (u + at) equation (iii) becomes

$$s = \frac{\{(u + at) + u\}t}{2}$$

$$\boxed{s = \frac{2ut + at^2}{2} \quad \text{or} \quad s = ut + \frac{1}{2}at^2}$$

3. From equation $v = u + at$, $t = \frac{v - u}{a}$ substituting for t in equation $\frac{s}{t} = \frac{v + u}{2}$

we get $\frac{as}{v - u} = \frac{v + u}{2}$.

$$\therefore 2as = (u + v)(v - u)$$

$$2as = v^2 - u^2$$

$$2s = \frac{uv - u^2 + v^2 - uv}{a}$$

$$\boxed{2as = v^2 - u^2 \quad \text{or} \quad v^2 = u^2 + 2as}$$

The equations $v = u + at$, $s = ut + \frac{1}{2}at^2$ *and* $v^2 = u^2 + 2as$ are referred to as the equations of uniformly accelerated motion.

Example 8.9

A stone is thrown vertically upwards with a velocity of 20 m/s. After 4 s the stone returns to the same position with a velocity of 20 m/s downwards. Determine the acceleration of the ball during the 4s seconds.

Solution

$$u = +20,\ v = -20, \quad a = \frac{v - u}{t} \Rightarrow a = \frac{-20 - 20}{4} = \frac{-40\ \text{m/s}}{4\ \text{s}}$$

$$= -10\ \text{m/s}^2$$

Example 8.10

A car on a straight road accelerates from rest to a speed of 30 m/s in 5 s. It then travels at the same speed for 5 minutes and then brakes for 10 s in order to stop. Calculate the:

(a) acceleration of the car during the motion.

(b) deceleration of the car.

(c) total distance travelled.

Solution

(a) From $v = u + at$, $a = \frac{v - u}{t}$

$$= \frac{30 - 0}{5}$$

$$= 6\ \text{m/s}^2$$

(b) $a = \frac{v - u}{t}$

$$= \frac{0 - 30}{10} = {}^{-}3\ \text{m/s}^2$$

$\therefore$ deceleration is 3 m/s^2

(c) There are three distinct sections of the journey

(i) Section of acceleration: $a = 6\ \text{m/s}^2$

The distance, $s = ut + \frac{1}{2}at^2$

$$= 0 + \frac{1}{2} \times 6 \times 5^2$$

$$= 75\ \text{m}$$

(ii) Section of constant velocity:

$$a = 0$$

$$\text{Distance, } s = 30 \times (5 \times 60)$$

$$= 9\,000 \text{ m}$$

(iii) Section of deceleration a $= -3 \text{ m/s}^2$ (from part (b))

$$\text{Distance, } s = ut + \frac{1}{2} at^2$$

$$= 30 \times 10 + \frac{1}{2} \times (-3) \times 10^2$$

$$= 300 - 150 = 150 \text{ m.}$$

$$\text{Total distance travelled} = 75 + 9\,000 + 150$$

$$= 9\,225 \text{ m}$$

Example 8.11

The driver of a bus initially travelling at 72 km/h applies the brakes on seeing a crossing herd of cattle. The bus comes to rest in 5 seconds.

Calculate:

(a) the average retardation of the bus.

(b) the distance travelled in this interval.

Solution

(a) Velocity of 72 km/h in m/s $= \dfrac{72 \text{ km/h} \times 1\,000 \text{ m}}{1 \times 3\,600 \text{ s}}$

$$= 20 \text{ m/s}$$

The quantities given are $u = 20$ m/s, $v = 0$ m/s and $t = 5$ s

From, $a = \dfrac{v - u}{t}, \Rightarrow a = \dfrac{0 \text{ m/s} - 20 \text{ m/s}}{5} = {}^{-}4 \text{ m/s}^2$

Since the question talked of average retardation, ignore the negative sign hence average retardation $= 4 \text{ m/s}^2$

(b) From, $s = ut - \dfrac{1}{2} at^2$,

$$s = 20 \times 5 - \frac{1}{2} \times 4 \times 5^2$$

$$= 100 - 50$$

$$= 50 \text{ m}$$

or From, $v^2 = u^2 + 2as$, $0^2 = 20^2 - 2 \times 4 \times s$

$$0 = 400 - 8s$$

$$\Rightarrow = \frac{400}{8}$$

$$s = 50 \text{ m}$$

Exercise 8.5

1. Define the term uniform acceleration of a body.
2. A racing car accelerates on a straight section of a road from rest to a velocity of 50 m/s in 10 s. Find:
   (a) The acceleration of the car.
   (b) The distance travelled by the car in 10 s.
3. A motorcyclist accelerates from 10 m/s to 30 m/s in 10 s. Calculate:
   (a) The acceleration of the motorcyclist.
   (b) The displacement of the motorcyclist.
4. An object travelling at 10 m/s decelerates at 2.0 m/s^2. How long does the object take before coming to rest? Calculate the distance travelled by the object before it comes to rest.
5. Fig. 8.49 shows a toy car accelerating down a straight incline.

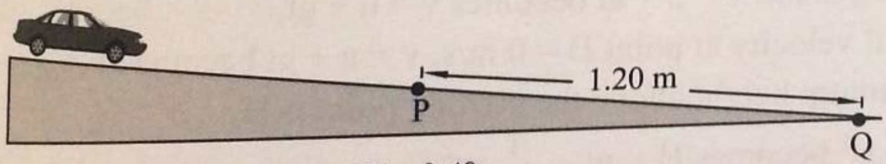


Fig. 8.49

   (a) Explain how you would determine the average speed with which the toy car passes point P.
   (b) It is found that the instantaneous speed of the car is 1.50 m/s at point P and 1.60 m/s at point Q. Point Q is 1.20 m from point P. Calculate the acceleration of the car between points P and Q.
6. A cyclist starts from rest and accelerates at 1.0 m/s^2 for 30 s. The cyclist then travels at a constant speed for 1 minute and then decelerates uniformly and comes to a stop in the next 30 s.
   (a) Find the maximum speed attained in:
      (i) metres per second.
      (ii) kilometres per hour.
   (b) Calculate the total distance covered in metres.

Equations of motion under gravity

Consider a stone thrown vertically upwards from point A in Fig. 8.50 with an initial speed u. It experiences a deceleration of 10 m/s^2 i.e. (ignoring air resistance) until its speed reduces to zero at the maximum height reached at point B. The stone then falls downwards accelerating due to gravity at 10 m/s^2.

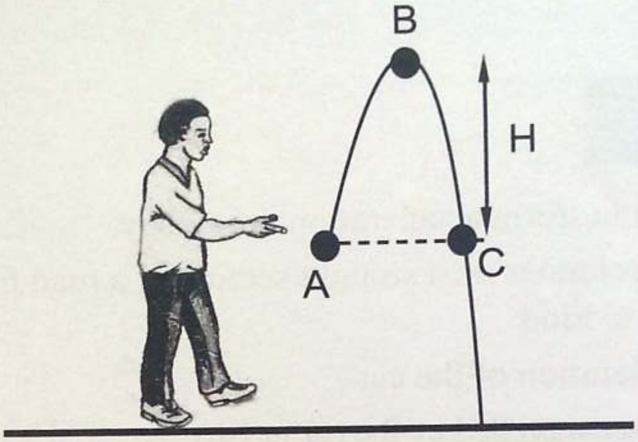


Fig. 8.50: A stone thrown vertically upwards

When an object is thrown upwards, its acceleration due to gravity becomes negative and when moving downwards, its acceleration is positive.
How do the equations of linear motion already derived, apply to motion under gravity?
For motion upwards from point A to point B, acceleration, $a = g = -10\ m/s^2$.
Thus, the equation $v = u + at$ becomes $v = u - gt$.
For motion downwards, (point B to point C), acceleration, $a = g = 10\ m/s^2$.
Thus, the equation $v = u + at$ becomes $v = u + gt$.
Since initial velocity at point B = 0 m/s, $v = u + gt$ becomes $v = gt$.
If the maximum height above the starting point is H,
$s = ut + \frac{1}{2}at^2$ becomes $H = ut - \frac{1}{2}gt^2$ for motion upwards while
for motion downwards $H = \frac{1}{2}gt^2$ since initial velocity u = 0 m/s.

For motion upwards, $v^2 = u^2 + 2as$ becomes, $u^2 = -2gH$ and $v^2 = 2gH$ for motion downwards.

Example 8.12

A body is thrown vertically upwards with an initial velocity of 20 m/s. Given that the gravitational acceleration $g = 10\ m/s^2$, find:

(a) the time the body takes to reach the maximum height;

(b) the maximum height, H reached above the starting point;

(c) the total time of flight.

Solution

(a) For motion upwards $u = 20$ m/s, $v = 0$ m/s and $g = 10$ m/s^2

Using the equation,
$$v = u - gt$$
$$0 = 20 - 10t$$
$$10t = 20$$
$$t = 2 \text{ seconds}$$

(b) Using the equation,
$$H = ut - \frac{1}{2}gt^2$$
$$= 20 \times 2 - \frac{1}{2} \times 10 \times 2 \times 2$$
$$= 40 - 20$$

maximum height $= 20$ m

(c) The total time of flight = time taken from starting point to maximum height + time taken from the maximum height to the starting point.

From (i) above time for upward motion = 2 seconds

Using the equation $H = \frac{1}{2}gt^2$, time for downward motion will be,

$$H = \frac{1}{2}gt^2, \ H = 20 \text{ m from (ii) above}$$
$$20 = \frac{1}{2} \times 10 \times t^2$$
$$20 = 5t^2$$
$$t^2 = 4$$
$$t = 2 \quad \therefore t = 2 \text{ seconds}$$

total time of flight $= 2 + 2 = 4$ seconds

Exercise 8.6

(Take acceleration due to gravity on the earth's surface as 10 m/s^2)

1. Define the terms acceleration due to gravity.
2. A stone is let to fall vertically down from a window on the 10th floor of a building 40 m above the ground. Find the time taken by the stone to reach the ground.

3. An object is thrown vertically upwards with an initial velocity of 50 m/s. Find
   (a) the maximum height reached by the object.
   (b) the time taken to reach this height. (neglect the effect of air resistance).
4. Describe a laboratory experiment by which you would measure the acceleration due to gravity. Show how the acceleration is obtained from your results.
5. A mass is dropped from a height of 50 m above the moon's surface. How long does the mass take to fall to the moon's surface if the moon's acceleration due to gravity is 1.7 m/s^2? Calculate the difference in time for the same mass when dropped 50 m above the earth's surface (neglect air resistance).
6. The diagram in Fig. 8.51 shows the motion of an object projected vertically upwards on point A from the top of a cliff with a velocity of 60 m/s.

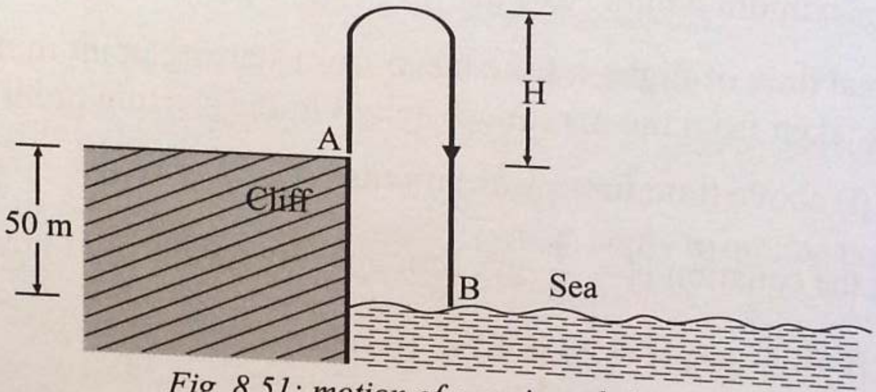


Fig. 8.51: motion of a projected object

Ignoring air resistance and taking g = 10 m/s^2, find the
   (a) maximum height, H reached by the object.
   (b) time of flight of the object up to the point, B where it hits the sea.

Unit summary

- Distance is the total length of the path travelled.
- Displacement is the shortest distance between two points in the direction of motion.
- Speed is the distance moved by the body per unit time.

$$\text{Speed} = \frac{\text{distance moved}}{\text{time taken}}$$

- A body covering equal distances in unit time intervals is said to move with uniform speed.
- Velocity is the rate of change of displacement.

  $$\text{Velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$

- Instantaneous velocity is the velocity of a body at a specific moment in time.
- Acceleration is the rate of change of velocity

  $$\text{Acceleration} = \frac{\text{change in velocity}}{\text{change in time}}$$

- The gradient of a velocity-time graph represents acceleration.
- The area under a velocity-time graph represents displacement.
- The gradient of a displacement-time graph represents velocity
- Equations of motion:

  $$v = u + at \quad ; \quad s = ut + \frac{1}{2}at^2 \quad ; \quad v^2 = u^2 + 2as$$

- If the motion is due to gravity, the equations of motion becomes:

  $$v = u - gt$$

  $$H = ut - \frac{1}{2}gt^2$$

  $$v^2 = u^2 - 2gH$$

- All bodies near the earth's surface experience acceleration due to gravity. Its value is 9.8 m/s^2 or approximately 10 m/s^2 near the earth and is directed towards the earth's centre.
- Terminal velocity is the maximum downwards velocity attained by a body falling freely through a fluid.

Unit Test 8

(Where necessary take $g = 10\ m/s^2$)

1. A trolley pulled the ticker-tape in Fig. 8.52 through a ticker timer which made 100 dots per second. What is the speed of the trolley?

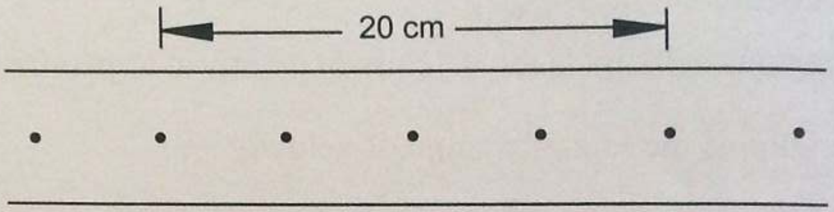


Fig. 8.52

A. 200 cm/s B. 25 cm/s C. 500 cm/s D. 100 cm/s

2. Uniform velocity means that the rate of change of
   A. acceleration with time is constant.
   B. displacement with time is constant.
   C. velocity with time is constant.
   D. distance with time is constant

3. The rate at which distance covered by a body changes with time is known as

A. speed B. velocity C. acceleration D. displacement

4. A cyclist travelling at a uniform acceleration of $2.5\ m/s^2$ passes through two points P and Q in a straight line. Her speed at point P is 20 m/s and the distance between the points is 100 m. Calculate her speed at point Q.

A. 22.8 m/s B. 30.0 m/s C. 15.8 m/s D. 900 m/s

5. A car increases its speed steadily from 8.0 m/s to 30 m/s in 10 s. How far does it travel in this time?

A. 80 m B. 190 m C. 110 m D. 220 m

6. A student runs 100 m race in 12.0 s. What is the average velocity of the student?

7. A racing cyclist starts from rest and accelerates uniformly to a velocity of 20 m/s in 4 s.
   (a) What is the acceleration of the cyclist?
   (b) What is the distance covered in the 4 s?

8. A cricketer throws a ball vertically upwards with an initial velocity of 50 m/s.
   (a) What is the time taken to reach the highest point?

(b) Calculate the maximum height reached.

9. Which one of the following motion-time graphs and acceleration-time graphs represents a body moving with uniform acceleration from rest? (Fig. 8.53)

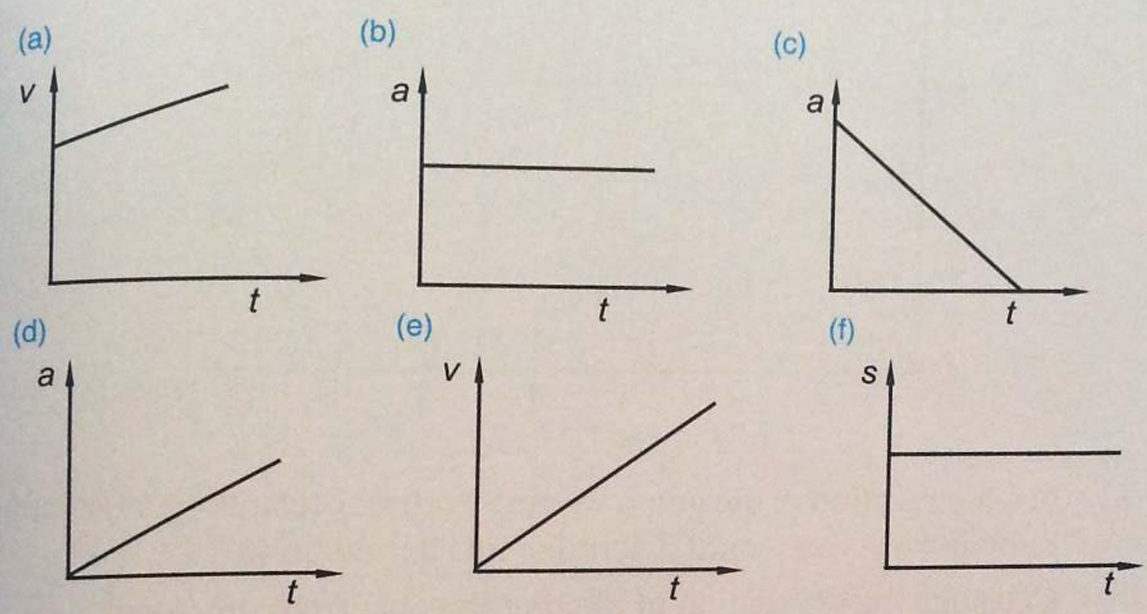


Fig. 8.53

10. The velocity-time graph in Fig. 8.54 shows the movement of a toy car on a straight path. Use the information to find:

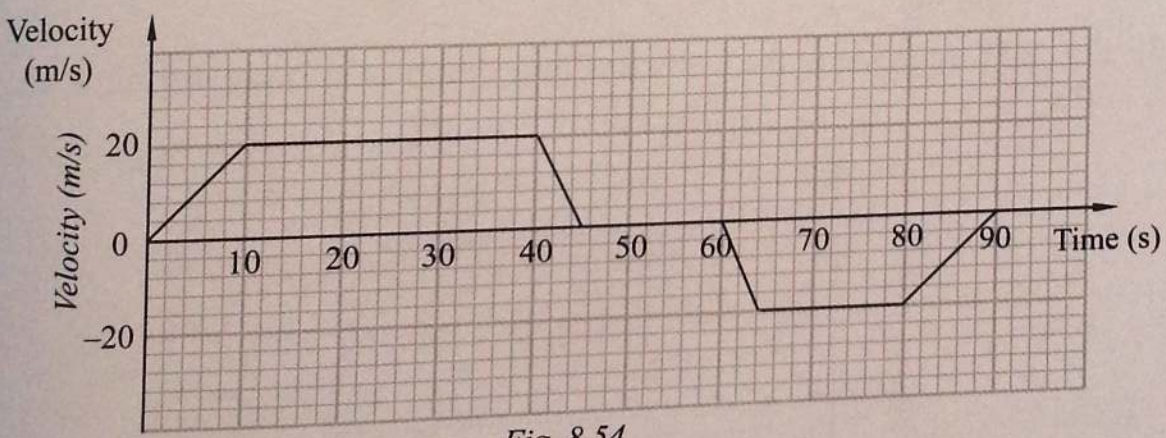


Fig. 8.54

(a) the initial acceleration of the car.

(b) the total time the car was not moving.

(c) the total distance travelled by the car.

(d) the displacement of the car from the starting point.

11. A lift carrying people starts from the third floor and stops on the sixth floor of a building after 20 s. Sketch a velocity-time graph of the motion of the lift. Show how you would use your sketch to determine the distance between the third and the sixth floor of the building.

12. Fig. 8.55 shows the motion of a motorcyclist on a straight road. Use the information on the graph to answer the following questions.

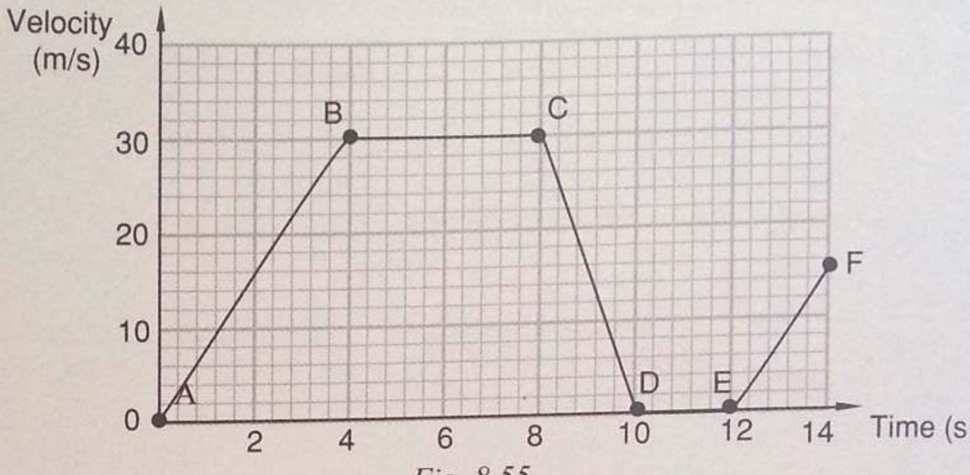


Fig. 8.55

(a) In which section of the graph was the cyclist accelerating most rapidly? Explain how you would determine this acceleration?

(b) Calculate the retardation of the motorcyclist from the graph.

(c) Which part of the graph shows that the motorcyclist was stationary and for how long?

(d) Use the graph to find the distance travelled by the motorcyclist before stopping.

13. Fig. 8.56 represents the velocity-time graph of a body during a period of 30 s.

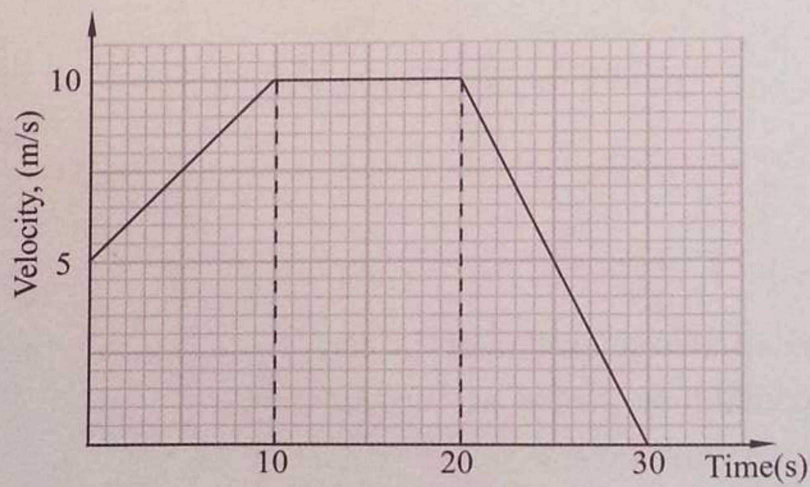


Fig. 8.56

(a) Use the equations of motion to find the displacement of the body in 30 s.

(b) Use the graph to determine the displacement of the body in 30 s.

(c) What is the retardation of the body?

14. Fig. 8.57 shows a displacement-time graph of the motion of a body over a period of 14 s. Use the graph to determine:-

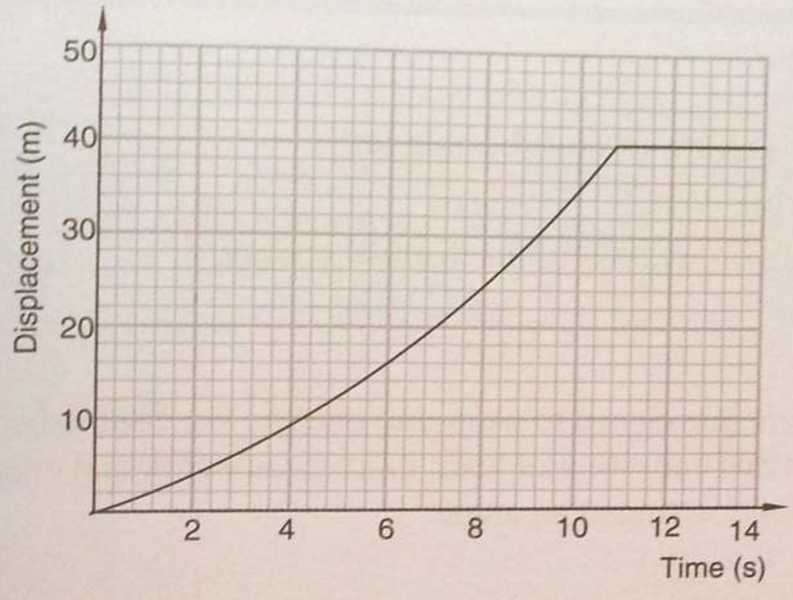


Fig. 8.57

(a) the velocity when t = 3 s and t = 7 s.

(b) the acceleration of the body between 3 s and 7 s.

(c) the time, in seconds, the body was stationary.

UNIT

9 Work and Energy

Success Criteria

By the end of this unit, you must be able to:

- Calculate work done
- Explain the conservation of mechanical energy

Introduction

In Form 1, we were introduced to work and energy. We established the relationship between them. In this unit, we will learn more about work and energy including the work done by a force acting at an angle.

9.1 Work

Remember work is the product of the force and the distance moved in the direction of the force. i.e.

Work done = Force × distance moved in the direction of the applied force

$$W = F \times d$$

Its SI unit is the *joule (J)*. Other units of work done are *kilojoule (kJ), megajoules (MJ)* etc.

Work done by an object on another

Activity 9.1: To demonstrate the work done on an object by another

- Place two identical balls A and B, 2 m apart on a smooth flat surface as shown in Fig. 9.1.

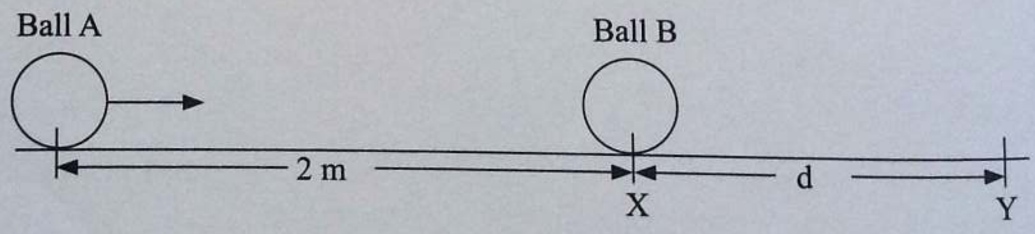


Fig. 9.1: To demonstrate work done by an object on another

- Roll ball A towards the stationary ball B and observe what happens.

In Activity 9.1, you must have noticed that ball A hits ball B and ball B started moving from point *X* towards *Y*. Ball A exerts a force, *F* on ball B which makes ball B to move through a distance '*d*' in the direction of the force. We say that ball A has done some work on ball B. Therefore, work done on ball B by ball A is given by:

Work done $= \text{Force} \times \text{distance in the direction of force}$

$= F \times d$

Example 9.1

A stationary trolley was hit by another trolley on the same horizontal flat surface with a force of 20 N. If the stationary trolley moved 15 m in the direction of the force, calculate the work done.

Solution

Work done $= F \times d = 20\ N \times 15\ m$

$= 300\ J$

Example 9.2

A towing truck was used to tow a broken car through a distance of 30 m. The tension in the towing chain was 2 000 N. If the total friction is 150 N, determine

(a) Work done by the pulling force.

(b) Work done against friction.

(c) Useful work done.

Solution

Fig. 9.2 shows the forces acting on the two cars.

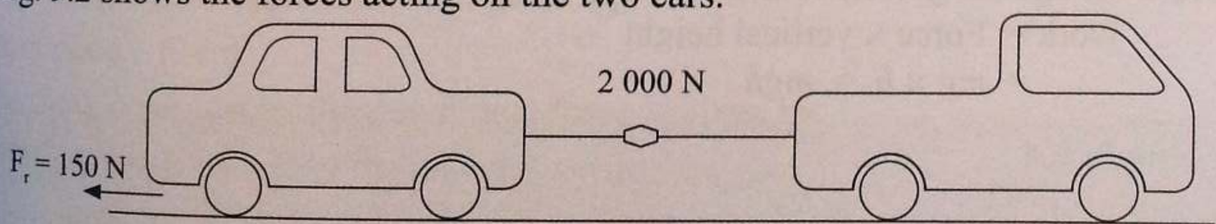


Fig. 9.2: Diagram of cars

(a) Work done by the pulling force

$W = Fd$

$= 2\ 000\ N \times 30\ m$

$= 60\ 000\ J$

(b) Work done against friction

$W = F_r d$ (F_r is the frictional force)

$= 150\ N \times 30\ m$

$= 4\ 500\ J$

(c) Useful work done

Useful work done $= Fd - F_r d$

$= (60\ 000 - 4\ 500)\ J$

$= 55\ 500\ J$

Example 9.3

A force of 200 N was applied to move a log through a distance of 10 m. Calculate the work done on the log.

Solution

$$W = F \times d$$
$$= 200\text{ N} \times 10\text{ m}$$
$$= 200\text{ J}$$

Work done against the force of gravity

The gravitational force (weight) acting on a body of mass m is equal to the product of mass and acceleration due to gravity, g, i.e. $w = mg$.
Thus, to lift a body, work has to be done against the force of gravity (Fig. 9.3).

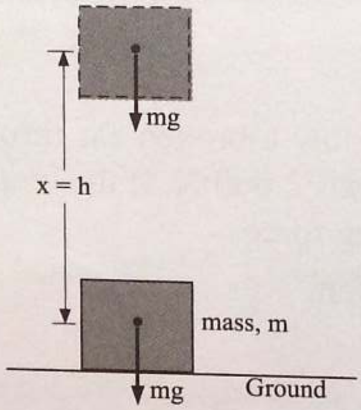


Fig 9.3: Work done against gravity

Work done against gravity to lift a body through height is given by

$$\text{Work} = \text{Force} \times \text{vertical height}$$
$$= mg \times h = mgh$$

Example 9.4

Calculate the work done by a weight lifter in raising a weight of 400 N through a vertical distance of 1.4 m.

Solution

$$\text{Work done against gravity} = \text{Force} \times \text{displacement}$$
$$= mg \times h = 400\text{ N} \times 1.4\text{ m}$$
$$= 560\text{ J}$$

Exercise 9.1

1. Define work done and give its SI unit.
2. Calculate the work done by a force of 12 N when it moves a body through a distance of 15m in the direction of that force.

3. Determine the work done by a person in pull a bucket of mass 10 kg steadily from the well through a distance of 15 m.
4 A car moves with uniform speed of 20m/s fo 20 s. The net resistive force acting on the car is 3 000 N.
   (a) What is the distance travelled by the car?
   (b) Calculate the work done by the car to overcome the resistive force.
   (c) Calculate the power developed.
5. A student of mass 50 kg climbs a staircase of vertical height 6 m. Calculate the work done by the student.
6. A block was pushed by a force of 20 N through 9 m. Calculate the work done.

Work done along inclined plane

Consider an inclined plane as shown in Fig. 9.4. A body of mass *m* moved up by a force *F* through a distance *d*.

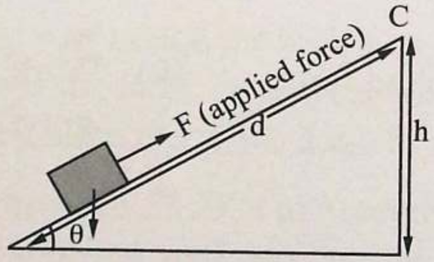


Fig. 9.4: Work done along inclined plane.

Work done by the applied force is given by

Work done = $F \times$ d

The work done against the gravitional force is given by

Work done = weight of the object × vertical height

= mgh

In case the inclined plane is frictionless force,

Work done by the applied force = work done against gravity

In case there is some frictional force opposing the sliding of the object along the plane,

Work done by the applied force > Work done against gravity

Work done against friction = Work done by applied force – work done against gravity

$$\text{Efficiency of the system} = \frac{(\text{Work done against gravity})}{(\text{work done by the applied force})} \times 100\%$$

Example 9.5

A box of mass 100 kg is pushed by a force of 920 N up an inclined plane of length 10 m. The box is raised through a vertical distance of 6 m (Fig. 9.5).

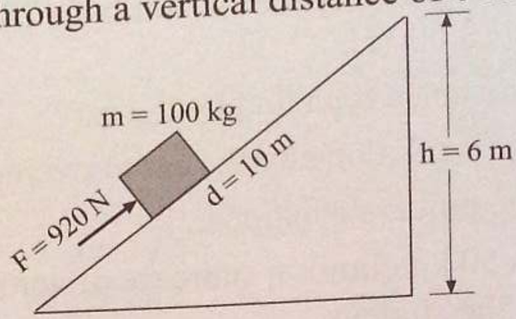


Fig 9.5: Inclined plane

(a) Calculate:

  (i) the work done by the applied force,

  (ii) the work done against the gravitational force.

(b) Why do the answers to (i) and (ii) in part (a) differ?

Solution

(a) (i) Work done by the applied force

$= F \times d$

$= 920\ N \times 10\ m$

$= 9\ 200\ J$

(ii) Work done against the gravitational force

$= mg \times h$

$= 100 \times 10 \times 6$

$= 6\ 000\ J$

(b) The difference in work done $= 9\ 200\ J - 6\ 000\ J$

$= 3\ 200\ J$

This work done is used to overcome the friction between the box and surface of the incline plane. The useful work done is 6 000 N.

Exercise 9.2

1. A box of mass 50 kg is pushed with a uniform speed by a force of 200 N up an inclined plane of length 20 m to a vertical height for 8 m (Fig. 9.6).

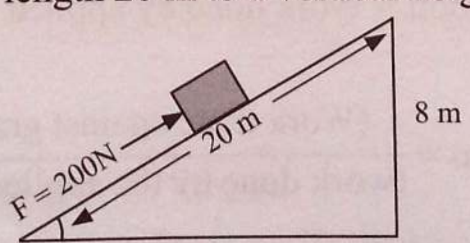


Fig. 9.6: Work done along inclined plane

Calculate the:

(a) Work done to move the box up the inclined plane.

(b) Work done if the box was lifted vertically upwards.

2. A body of mass 85 kg is raised through a vertical height 6 m through an inclined plane as shown in Fig. 9.7.

Calculate the:

(a) Slant distance.

(b) Work done by the force 150 N.

(c) Work done, if the body was lifted vertically upwards.

(d) Work done against friction.

(e) Frictional force between the body and the track.

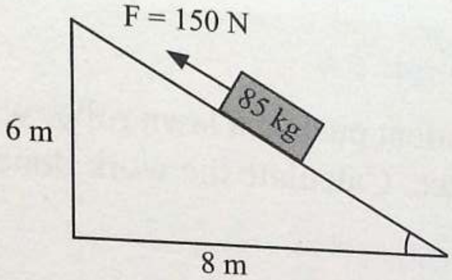


Fig. 9.7: Work done on an inclined plane

3. A block of mass 60 kg was raised through a vertical height 7 m. If the slant height of a frictionless track is 21 m, and the force used to push the block up the plane is 800 N, Calculate the work done in pushing the block.

4. A car engine offers a thrust of 2500 N to ascend a sloppy road for 1.1 km. At the top of the slope, the driver realized that the attitude change was 200 m. If the mass of the car is 1.2 tonnes, calculate;

(a) Work done by the car engine.

(b) Work done against resistance.

9.2 Work done by a force acting at an angle

Consider a person pushing a land mower (Fig. 9.8). Force is applied on the land mower at an angle θ from the direction of the motion.

Fig. 9.8: Land mower

To calculate the work done by the user to move the mower, we must first find the horizontal component of the applied force.

Assuming the applied force is F, the horizontal component will be:

$$\text{Horizontal component} = F\cos\theta$$

If the land mower is moved through a distance d, the work done is given by

$$\text{Work done} = F\cos\theta \times d$$

Example 9.6

A student pushed a lawn roller with a force of 800 N at an angle of 39° to the lawn surface. Calculate the work done if the roller is pushed for 30 m.

Solution

Work done $= F\cos\theta \times d$

$= 800 \times \cos 39^0 \times 30$

$= 18\,651.5$ J

Example 9.7

A person pushed a block along a horizontal rough surface as shown in Fig. 9.9. the surface offered a friction force of 30 N against the motion of the block. The block moved through a distance of 12 m.

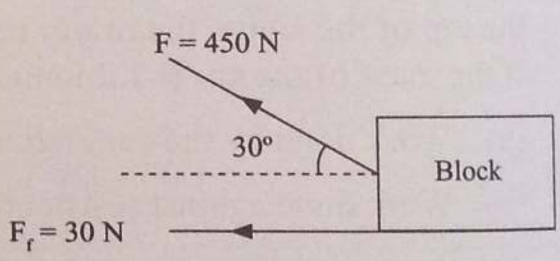


Fig. 9.9

Calculate the:

(a) Work done by the applied force

(b) Work done against friction

(c) Useful work done

Solution

(a) Work done by the applied force

$= F\cos\theta \times d$

$= 450 \times \cos 30^0 \times 12$

$= 4675.5$J

(b) Work done against friction

$= F_f \times d$

$= 30 \times 12$

$= 360$ J

(c) Useful work done $= F\cos\theta \times d - F_f d$

$= 4675.5 - 360$

$= 4315.5$J

Exercise 9.3

1. A man moved a land mower through a distance of 20 m on the horizontal ground. If he applied a force of 235 N at an angle of 60° to the horizontal, calculate the work done in moving the mower horizontally.
2. Fig. 9.10 is a block of wood which is being pulled on a horizontal ground.

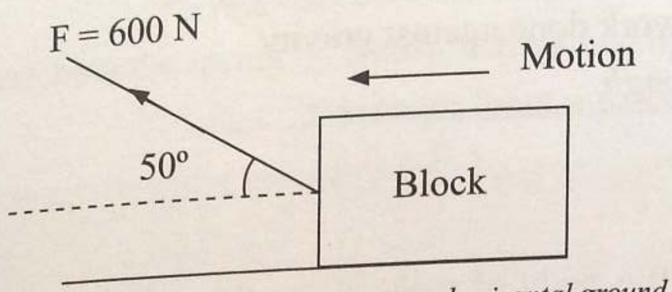


Fig. 9.10: A block moving on a horizontal ground

If the force is applied at an angle 50° to the horizontal to move the block through a distance of 30 m, calculate the work done to move the block.

3. A person is pushing a travelling bag through 10 m on the track as shown in Fig. 9.11.

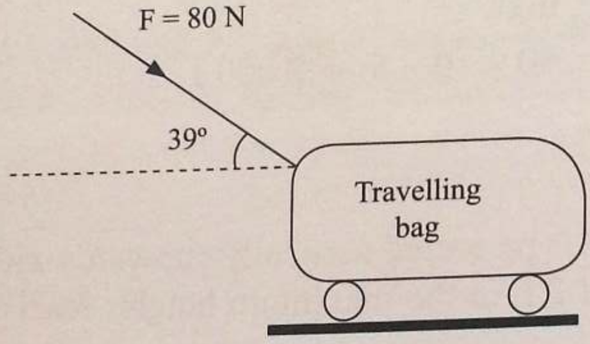


Fig. 9.11: A bag being pushed on a horizontal surface

If there is a frictional force of 9 N between the track and the wheels, calculate:

(a) The work done to moving the bag horizontally
(b) The work done against friction
(c) The useful work done

9.3 Mechanical energy

In mechanics, bodies are said to possess *mechanical energy*. Mechanical energy is classified into two types, namely *potential energy (P.E)* and *kinetic energy (K.E)*.

Potential energy (P.E)

There are two types of potential energy: *gravitational potential energy* and *elastic potential (strain) energy*.

Gravitational potential energy is the energy possessed by a body by virtue of its vertical position.

When a body of mass *m* is raised through a vertical height, *h* the work done against gravity is equal to the potential energy gained by the body i.e.

$$\text{P.E} = \text{work done against gravity}$$
$$\text{P.E.} = mgh$$

Example 9.8

A crane is used to lift a body of mass 40 kg through a vertical height of 5 m. Calculate the gravitational P.E stored in the body.
(Take g = 10 m/s^2)

Solution

$$\begin{aligned}\text{Gravitational P.E} &= \text{Work done against gravity}\\ &= \text{mgh}\\ &= 40 \times 10 \times 5 = 2\,000 \text{ J}\end{aligned}$$

Example 9.9

A stone of mass 100 g is projected vertically upwards with a velocity of 20 m/s. Find the gravitational P.E at the maximum height reached by the stone. (Take g = 10 m/s^2)

Solution

From the equation motion,
$$\begin{aligned} v^2 &= u^2 + 2as\\ 0 &= 20^2 + 2(^-10)h\\ 0 &= 400 - 20h\\ \therefore\quad h &= 20 \text{ m}\end{aligned}$$

$$\text{Gravitational P.E} = \text{mgh} = 0.100 \times 10 \times 20 = 20 \text{ J}$$

Elastic potential or *strain energy* is the energy passed by or possessed by a compressed or stretched spring. This energy is due to the *state of strain* of the object.

Fig. 9.12 shows a force *F* that produces an extension *e* in a spring.

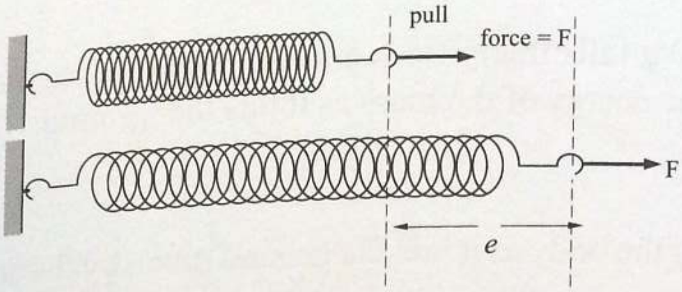


Fig.9.12: Elastic potential energy

$$\text{Work done in stretching the spring} = \text{Elastic P.E gained by the spring}$$
$$= \text{average force} \times \text{extension}$$
$$= \left(\frac{0+F}{2}\right) \times e$$
$$= \frac{1}{2} Fe$$

Work done is stored as elastic potential energy.

Note: Since the force is not uniform (F increases from 0 to F) we should use the *average force* in calculating the work done.

Example 9.10

Calculate the elastic gravitational P.E stored in a spring when stretched through 4 cm by a force of 2 N.

Solution

Elastic P.E $= \frac{1}{2}$ Fe $= \frac{1}{2} \times 2 \times (0.04) = 0.04$ J

Kinetic energy

Kinetic energy (K.E) is the energy possessed by a body due to its motion.

A body of mass m moving at a speed of v, it posses K.E given by K.E $= \frac{1}{2} mv^2$

Example 9.11

Calculate the kinetic energy of a car of mass 1 200 kg moving with a velocity of 40 m/s.

Solution

$$\text{K.E} = \frac{1}{2}\text{mv}^2 = \frac{1}{2} \times 1\,200 \times 40^2 = 960\,000 \text{ J}$$
$$= 9.6 \times 10^5 \text{ J}$$

Example 9.12

A body of mass 400 g falls freely from a tower and reaches the ground after 4 s. Calculate the kinetic energy of the mass as it hits the ground. (Take $g = 10\ m/s^2$)

Solution

The final velocity of the body as it hits the ground given by the equation of motion,

$$v = u + at = u + gt = 0 + 10 \times 4$$
$$= 40\ m/s$$

$$K.E = \frac{1}{2}mv^2 \text{ (as the body hits the ground)}$$
$$= \frac{1}{2} \times 0.400 \times 40^2 = 320\ J$$

Example 9.13

A car of mass 1 000 kg travelling at 36 km/h is brought to rest by applying brakes. Calculate the distance travelled by the car before coming to rest, if the frictional force between the wheels and the road is 2 000 N.

Solution

$$v = 36 \text{ km/h to m/s}$$
$$= 36 \times \frac{1\,000}{60 \times 60}\ m/s$$
$$= 10\ m/s$$

$$K.E = \text{work done against friction}$$
$$\frac{1}{2}mv^2 = F \times d$$
$$\Rightarrow \frac{1}{2} \times 1000 \times 10^2 = 2000 \times d$$
$$\Rightarrow 50\,000 = 2000\ d$$
$$\therefore\ d = \frac{50\,000}{2\,000} = 25\ m$$

The stopping distance is 25 m.

Exercise 9.4

1. Explain what is meant by gravitational potential energy.
2. A force of 8.5N stretches a certain spring by 6 cm. How much work is done in stretching the spring by 10 cm.
3. A body is acted on by a varying force, *F* over a distance of 7 cm as shown in Fig. 9.13.

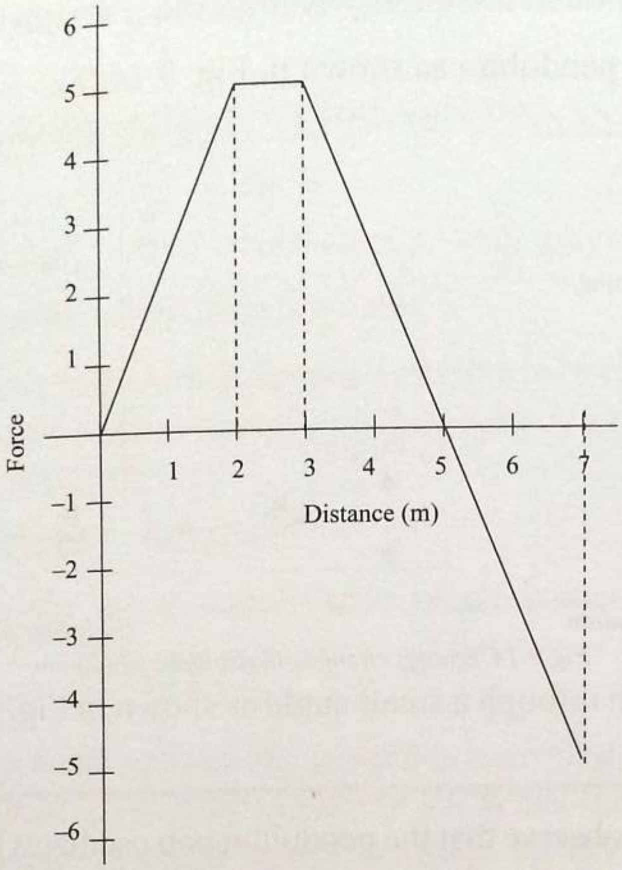


Fig.9.13

Calculate the total work done by the force.

5. An object of mass 3.5 kg is released from a height of 7.0 m above the ground.
   (a) Calculate the gravitational potential energy of the object release.
   (b) Calculate the velocity of the object just before it strikes the ground. What assumption have you made in your calculation.

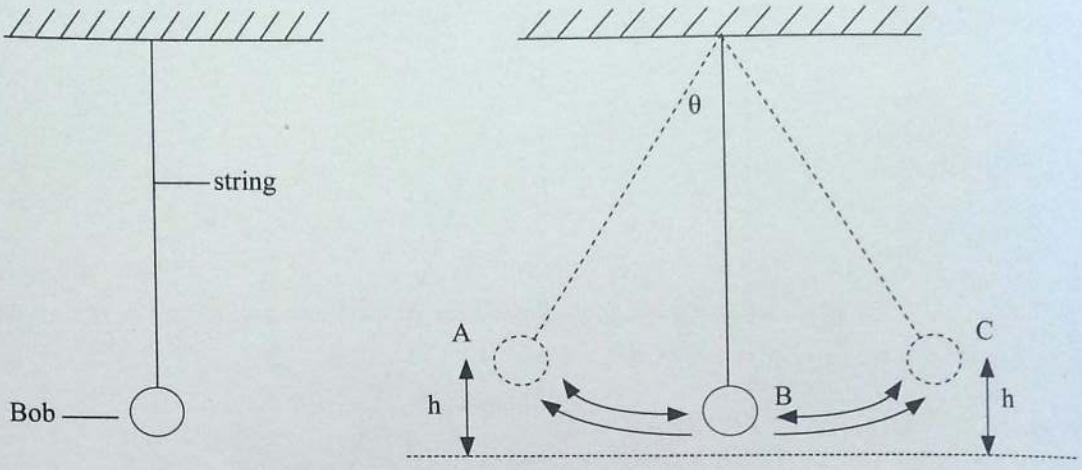
9.4 Conservation of mechanical energy

As we already know, energy can be converted from one form to another. During conversion energy is conserved. This is summarised in the law of conservation of energy states that *energy can neither be created nor destroyed but can only be converted from one form to another*.

In this section, we will discuss the law of conservation of mechanical energy. The following activities will help us demonstrate conservation of mechanical energy.

Activity 9.2: To demonstrate energy changes in a simple pendulum

- Set up a simple pendulum as shown in Fig. 9.14 (a).



(a) *Stationary pendulum* (b) *Swing pendulum (Fig. 9.8b)*

Fig.9.14:Energy changes in a simple pendulum

- Displace the bob through a small angle as shown in Fig. 9.14 (b) and observe what happens.

From Activity 9.2, we observe that the pendulum bob oscillates from A to C then, back to A through B. We also notice that at points A and C (at maximum height) the bob is momentarily stationary and at point B the bob, appears to move at maximum speed. The energy changes between A to C are

At A		At B		At C
Max. Potential energy (P.E)	$\longrightarrow$	max. kinetic energy (K.E)	$\longrightarrow$	max. potential energy (P.E)

Between points A and B, and between points B and C, the bob posses both P.E and K.E. Therefore, the total mechanical energy E is given by;

$$E = P.E + K.E = \text{constant at any given point}$$

Activity 9.3: To demonstrate energy changes in a loaded spring

- Set up a loaded spring as shown in Fig. 9.15.
- Displace the spring vertically and observe what happens.

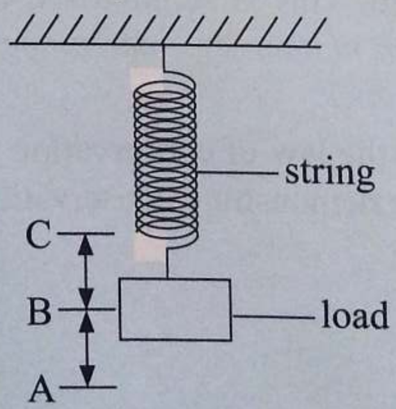


Fig. 9.15: Energy changes in a loaded spring

From Activity 9.3, we observe that the spring oscillates from A to B to C to B then to A.

Energy changes in the block as the spring oscillates are

At A	At B	At C
Max. elastic potential energy	Max. kinetic energy	Max. gravitational potential energy

(We assume that no energy is lost from a system).

From this activities, we conclude that for any system where energy is not lost to the environment, the sum of potential energy (P.E) and Kinetic Energy (K.E) is the total mechanical energy at any point i.e.

Total mechanical energy (E) = P.E + K.E

This is summarized in the law of conservation of mechanical energy which states that *for any mechanical system total energy in it is conserved i.e. energy can neither be created nor destroyed but only be converted into one another.*

For example, an object lifted up from the ground to the top of a cliff as shown in the Fig.9.16 gains gravitational potential energy (P.E). When the object falls from the top to the ground, it looses potential energy to kinetic energy (K.E).

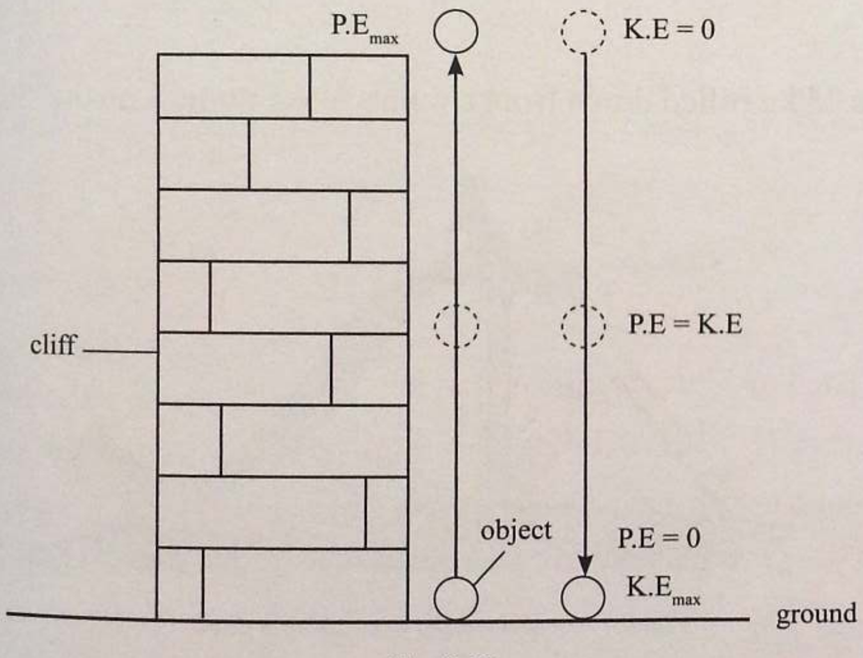


Fig. 9.16

The energy changes between the top of the cliff and the ground as an object falls is

$$\text{loss in } P.E_{max} = \text{gain in } K.E_{max}$$

$$\text{Thus, } mgh = \frac{1}{2} mv^2_{max}$$

$$v_{max} = \sqrt{2\,gh}_{max}$$

v is the maximum speed attained by a falling object immediately before it touches the ground.

Example 9.14

What is the velocity of an object dropped from a cliff of height 20 m just before it strikes the ground? $g = 10\ m/s^2$.

Solution

P.E lost = K.E gained

$$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2\,gh}$$
$$= \sqrt{2 \times 10 \times 20}$$
$$= \sqrt{400}$$
$$= 20\ m/s$$

Note that the total sum of P.E and K.E at each point is equal to mechanical energy M.E given as: M.E = P.E + K.E

Example 9.15

A girl of mass 25 kg rolled down from a point, A to a point B on the slide as shown in Fig. 9.17.

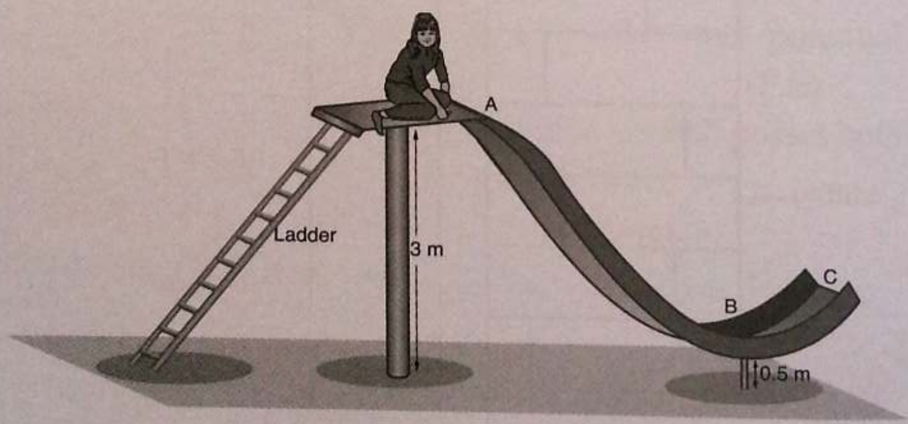


Fig. 9.17: A girl rolling down from a point.

If the pull of gravity is 10 kg/N. Calculate the:

(a) potential energy of the girl at point A

(b) kinetic energy midway between A and B

(c) speed attained by the girl just before reaching point B

Solution

(a) P.E at point A $= mgh$

$= 25 \times 10 \times 3$

$= 750$ J

(b) K.E midway of A to B

P.E lost = K.E gained

P.E midway A to B $= \frac{1}{2}(mhg)$

$= \frac{1}{2} \times 750$ J

$= 375$ J

Therefore K.E midway AB = 375 J

(c) Speed attained

P.E lost = K.E gain

$mgh = \frac{1}{2}mv^2$

$v = \sqrt{2gh}$

$= \sqrt{(2 \times 10 \times 0.5)}$

$= 3.16$ m/s

Example 9.16

A 150 g ball falls vertically downwards from a height of 1.8 m on a horizontal plate. On hitting the plate, the ball rebounds to a height of 1.25 m. Find the:

(a) velocity of the ball just before hitting the plate,

(b) kinetic energy of the ball as it hits the plate,

(c) rebound velocity,

(d) kinetic energy of the ball as it leaves the plate. (Take $g = 10\ m/s^2$)

Solution

(a) Initial velocity $u = 0$

Distance $s = 1.8$ m

Acceleration $g = 10\ m/s^2$

Final velocity $v = ?$

From $v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times 10 \times 1.8$

$= 36$

$v = 6$ m/s

The velocity of the ball just before hitting the plate is 6 m/s.

(b) $k.e = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.150 \times 6^2 = 2.7$ J

(c) Final velocity $v = 0$

Acceleration $a = {}^-g = {}^-10\ m/s^2$

Distance $h = 1.25$ m

Initial velocity $u = ?$

From $v^2 = u^2 + 2as$

$0 = u^2 + 2(-10)1.25$

$0 = u^2 - 25$

$u^2 = 25$

$u = 5$ m/s

The rebound velocity is 5 m/s

(d) k.e on rebound $= \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.15 \times 5^2$

$= 1.875$ J

Example 9.17

A ball is thrown vertically upwards with a velocity of 15 m/s. Calculate the maximum height reached by the ball. (Take $g = 10\ m/s^2$)

Solution

By law of conservation of energy,

$$K.E_{lost} = P.E_{gained}$$
$$\frac{1}{2}mv^2 = mgh$$
$$v^2 = 2gh$$
$$15^2 = 2 \times 10 \times h$$
$$225 = 20h$$
$$h = 11.25\ m$$

The maximum height reached by the ball is 1.25 m.

9.5 Energy – work theorem

Consider the resultant (net) force F acting on a rigid body of mass M over a distance S (Fig 9.18).

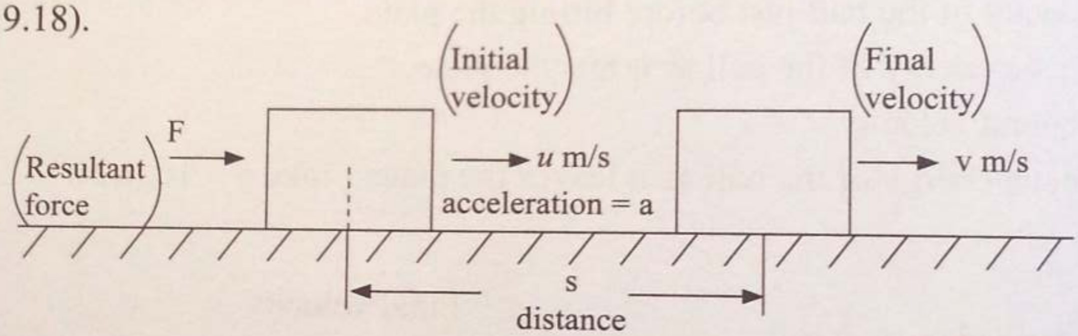


Fig. 9.18: Resultant force or a rigid body

The force causes the speed of the body to change from an initial value *u*m/s to a final value v m/s. i.e., the body accelerates with a value $a\ m/s^2$ from u m/s to v m/s. Consequently, the work done on the body by the net force is transformed into kinetic energy of the body i.e. its K.E changes from an initial value of $\frac{1}{2}mu^2$ to a final value of $\frac{1}{2}mv^2$. We say that the *net work done by the net force on the rigid body is equal to the change in kinetic energy* of the body. This is known as the energy-work theorem and is represented mathematically as

Work done by the net force = Change in KE

$$W = \Delta KE$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

or $Fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ (sine $W = F \times s$)

Derivation of the energy-work theorem

Consider Fig 9.18

Net work done by the resultant force W = F × s

According to the newton second law of motion,

The resultant force F is given

$$F = ma$$

Applying the 3rd equation of motion on the block on Fig. 9.19, we can get an expression for a in terms of u, v and s, i.e

$$v^2 + u^2 + 2as = a = \frac{v^2 - u^2}{2s}$$

Work done by the resultant force can be expressed as follows

$$W = F \times s = mas = m\left(\frac{v^2 - u^2}{2\cancel{s}}\right)\cancel{s} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

In short,

$$W = F \times s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Note

- Note that force is a vector quantity hence a force applied to slow the speed of the object is in the opposite direction (negative) and the net work it does is also negative.
- The energy-work theorem does not apply if the object being acted on by the net force is not rigid as some of the energy is used to deform the object.

Example 9.18

A constant force of 20 N is applied on a trolley of mass 0.5 kg on flat frictionless surface. If the trolley starts from rest and moves a distance of 1.5 m, find the final velocity of the trolley.

Solution

Net work = change in KE energy

$$F \times s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\text{or } F \times s = \frac{1}{2}m(v^2 - \frac{1}{2}u^2)$$

Substituting for each of the values we get

$$20\text{ N} \times 1.5\text{ m} = \frac{1}{2} \times 0.5(v^2 - 0^2)$$

$$30\text{ N} = 0.25v^2$$

$$v^2 = \sqrt{\frac{30}{0.25}} = \sqrt{120}$$

$$= 10.95\text{ m/s}$$

Example 9.19

The frictional force between the wheels of a motorbike and the ground is 150 N. The air resistance acting on the motorbike and the cyclist is 90 N. A driving force of 960 N is applied on the motorbike by the engine over a distance of 50 M. If the total mass of the motorbike and the cyclist is 180 kg, and it started from rest, find the:

(a) resultant force on the motorbike

(b) final velocity of the motorbike

(c) acceleration of the motorbike

Solution

(a) Resultant force = Applied force – sum of the resisting forces.

$$F = 960\text{ N} - (150 + 90)\text{ N}$$
$$= 720\text{ N}.$$

(b) Final velocity (v) of the motorbike is given by the work energy theorem

Net work done = Change in KE

$$F \times s = \frac{1}{2}m(v^2 - u^2)$$

$$720\text{ N} \times 50\text{ m} = \frac{1}{2} \times 180(v^2 - 0)$$

$$36\,000 = 90v^2$$

$$v^2 = \frac{36000}{90} = 400\text{ m/s}$$

Therefore, $v = 20$ m/s

(c) Acceleration of the motorbike

Method 1

$$F = ma$$

$$a = \frac{F}{m}$$

$$= \frac{720\text{ N}}{180\text{ kg}} = 4\text{ m/s}^2$$

Method 2

$$u = 0,\ v = 20\text{ m/s},\ s = 50\text{ m}$$

$$v^2 = u^2 + 2as$$

$$20^2 = 0 + 2a \times 50$$

$$400 = 100a$$

$$a = 4\text{ m/s}^2$$

Exercise 9.5

1. Give one example of a body with potential energy due to its state.
2. A stone is dropped from a height of 70 m. Find the velocity with which it strikes the ground.
3. A 150 g ball falls vertically down from a height of 1.8 m on a horizontal plate. On hitting the plate the ball rebounds to a height of 1.25 m. Find
   (a) the kinetic energy of the ball as it hits the plate.
   (b) its velocity just before on hitting the plate.
   (c) the kinetic energy of the ball as it leaves the plate.
   (d) the rebound velocity.
4. A bullet of mass 30 g is fired at 180 m/s into a block of wood of mass 2 kg hanging freely as shown in Fig. 9.19. The bullet remains embedded in the wood.

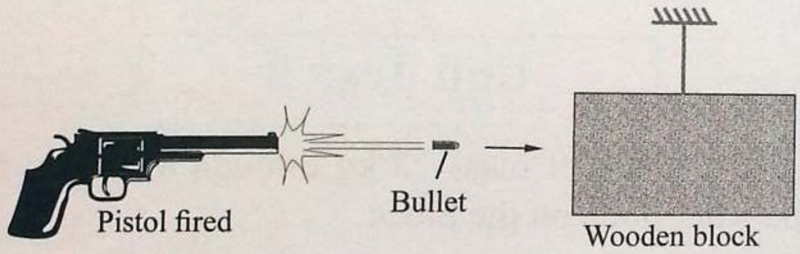


Fig. 9.19:

   (a) Find the speed of the block just after impact.
   (b) What is the kinetic energy of the bullet before impact?
   (c) Calculate the kinetic energy of the wood and bullet just after impact.
   (d) How much kinetic energy is lost during the impact?
   (e) What happens to the lost kinetic energy?
   (f) Find the vertical height through which the block rises.
5. A stone of mass 8 kg moves through a horizontal distance 20 cm from rest. If the force acting on the stone is 10 N, calculate:
   (a) the work done by the force
   (b) the K.E gained by the stone
   (c) the final velocity of the stone
6. A car of mass 1800 kg was moving at a constant velocity of 20 m/s. The driver stepped on the brakes that applied a constant retarding force of 360 N on the wheels. After how far will the car move
   (a) by the time its speed is reduced by halfsx
   (b) to come to rest

Unit Summary

- Work is defined as *the product of force and distance moved in the direction of the force.* $W = F \times d$. SI unit of work is the joule (J): 1 J = 1 Nm
- Energy *is the ability to do work.* SI unit of energy is the joule (J).
- Potential energy (P.E) is the energy due to state or position of an object. P.E = mgh
- Kinetic energy (K.E) is the energy due to the motion of the body:

$$K.E = \frac{1}{2} mv^2$$

- Energy can neither be created nor destroyed. It can only be changed from one form to another.
- A device which can change energy from one form to other is called a transducer.

Unit Test 9

1. A motor raised a block of mass 72 kg through a vertical height of 2.5 m. Calculate the work done on the block.
2. A person of mass 40 kg runs up a flight of 50 stairs each of height 20 cm. Calculate:
   (a) the work done.
   (b) explain why the energy the person uses to climb up is greater than the calculated work done.
3. A fork-lift truck raises a 400 kg box through a height of 2.3 m. The case is then transported horizontally by the truck at 3.0 m/s onto the loading platform of a lorry.
   (a) What minimum upward force should the truck exert on the box?
   (b) How much P.E. is gained by the box?
   (c) Calculate the K.E of the box while being transported
   (d) What happens to the K.E once the truck stops?
4. (a) Name four forms of energy.
   (b) A tennis ball of mass 20 g is released from rest at a height of 4.0 m above the ground. On hitting the ground the ball rebounces to a height of 3 m.
      (i) Find the energy loss. (ii) Explain what happens to this energy.

5. A ball of mass 0.3 kg is thrown vertically upwards with a velocity of 15 m/s.
   (a) Find the:
      (i) Initial kinetic energy of the ball as it is released.
      (ii) Final kinetic energy of the ball when at maximum height.
      (iii) Kinetic energy and potential energy when the ball reaches 80% of its maximum height.
   (b) Explain what happens to the energy when the ball hits the ground.
6. Fig. 9.20 shows a block of mass 0.35 kg resting on a frictionless surface and attached to a spring of spring constant 200 N/m. The spring is compressed 50 mm and then released.

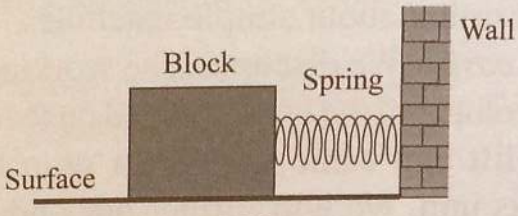


Fig. 9.20

   (a) Explain why the block moves when the spring is released.
   (b) Calculate the maximum speed of the block after releasing the spring.

10 Machines

Success Criteria

By the end of this unit, you must be able to:

- Describe what machines are.
- Explain efficiency, mechanical advantage and velocity ratio of a machine.
- Calculate efficiency, mechanical advantage and velocity ratio of machines.

10.1 Review of the definition of a machine

In Form 2, we started learning about simple machines. We defined a machine as a *device that makes work easier.* We discussed the working of levers, inclined planes and pulley systems. We compared the effort applied on them against the corresponding loads they are able to lift and came up with a ratio that known as mechanical advantage (M.A). In this unit, we will further our understanding of machines, by first reviewing mechanical advantage then will discuss other concepts on machines namely *mechanical advantage*, *velocity ratio* and *efficiency* of various machines.

10.2 Mechanical advantage, velocity ratio and efficiency

Mechanical advantage

Mechanical Advantage (M.A) is *the ratio of the load to the effort.*

$$\text{Mechanical advantage} = \frac{\text{load}}{\text{effort}} \text{ or M.A.} = \frac{L}{E}$$

Velocity ratio

Velocity ratio, V.R, of a machine *is the ratio of the velocity of the effort to the velocity of the load.* Since the effort and the load move for the same amount of time, we define velocity ratio as the ratio of the distance moved by the effort to the distance moved by the load i.e.

$$\text{Velocity ratio (V.R).} = \frac{\text{distance moved by the effort}}{\text{distance moved by the load}}$$

Efficiency

No machine is perfect. In every machine, some energy is wasted in overcoming friction. Thus,

Energy input = energy output + energy lost against friction

The output of a machine is therefore, always less than the input. *The ratio of the energy output of a machine to its energy input is called the efficiency.*

The efficiency of a machine is defined by the equation,

$$\text{efficiency} = \frac{\text{energy output}}{\text{energy input}} \times 100\%$$

Relationship between mechanical advantage, velocity ratio and efficiency

$$\text{Efficiency} = \frac{\text{energy output}}{\text{energy input}} \times 100\%$$

$$= \frac{\text{useful work output}}{\text{work input}} \times 100\%$$

$$= \frac{\text{load} \times \text{distance moved by the load}}{\text{effort} \times \text{distance moved by effort}} \times 100\%$$

$$= \frac{\text{load}}{\text{effort}} = \frac{\text{distance moved by the load}}{\text{distance moved by effort}} \times 100\%$$

$$= M.A \times \frac{1}{\text{V.R}} \times 100\%.$$

$$\text{Therefore, Efficiency} = \frac{\text{M.A}}{\text{V.R}} \times 100\%$$

Example 10.1

A machine is used to lift a load of 400 N with an effort of 80 N. Calculate:

(a) The mechanical advantage of the machine,

(b) The efficiency of the machine, if its velocity ratio is 8.

Solution

(a) $\text{M.A} = \frac{\text{load}}{\text{effort}} = \frac{400}{80} = 5$

(b) $\text{Efficiency} = \frac{\text{M.A}}{\text{V.R}} \times 100$

$$= \frac{5}{8} \times 100\% = 62.5\%$$

Example 10.2

An effort of 250 N raises a load of 900 N through a distance of 5 m. If the effort moves through 25 m, calculate:

(a) the work done in raising the load,

(b) the work done by the effort,

(c) the efficiency of the machine.

Solution

(a) Work done in raising the load,

$$= \text{load} \times \text{distance moved by load}$$

$$= 900 \times 5 = 4\,500 \text{ J}$$

(b) Work done by the effort,

$$= \text{effort} \times \text{distance moved by the effort}$$

$$= 250 \times 25 = 6\,250 \text{ J}$$

(c) Efficiency $\dfrac{\text{work output}}{\text{work input}} \times 100\%$

$$= \frac{4\,500}{6\,250} \times 100\% = 72\%$$

10.3 Types of simple machines

Simple machines may be classified into two groups namely: *the force multipliers* and *the distance or speed multipliers*.

Force multipliers are those that allow a small effort to move a large load e.g. levers, screw jack of a car, hydraulic machine or press.

Distance or speed multipliers are those that allow a small movement of the effort to produce a large movement of the load, e.g. bicycle gears, levers like fishing rod etc.

Let us look at how to determine M.A, V.R and efficiency of some simple machines.

Levers

A lever is a rigid bar capable of rotation about a fixed point called the pivot or fulcrum.

There are three types of levers depending on the position of the pivot with respect to the load to be overcome and effort applied.

1. The pivot is between the load and the effort (Fig. 10.1(a). Examples include a crowbar, a pair of scissors, claw hammer, pliers, see-saw, spanner, etc.
2. Load is between the pivot and effort (Fig. 10.1(b)). Examples include a wheelbarrow and bottle opener.
3. Effort is between the pivot and the load (Fig. 10.1(c)). Examples include a fishing rod, tweezers, forceps and supporting a load with a forearm.

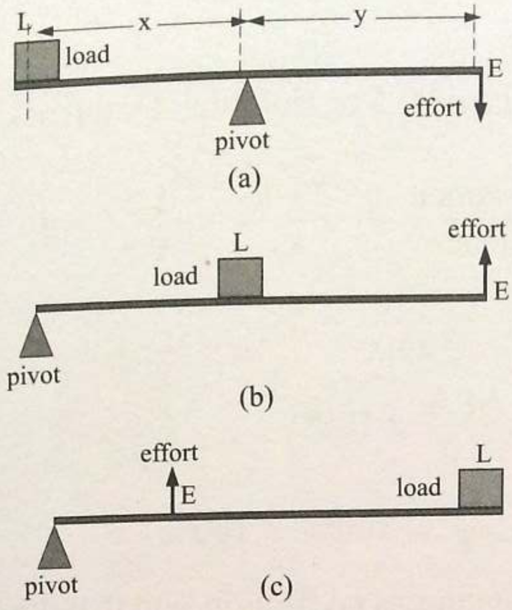


Fig. 10.1: Types of levers

In Fig. 10.1(a), x is called the load arm and y the effort arm.

Example 10.3

A worker uses a crowbar 2.0 m long to lift a rock weighing 750 N with an effort of 250 N as shown in Fig. 10.2.

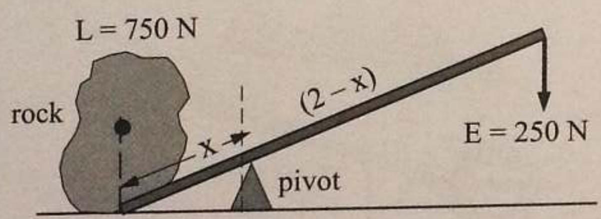


Fig. 10.2

(a) Determine the position of the pivot.

(b) Calculate: (i) velocity ratio ,

(ii) mechanical advantage,

(iii) efficiency of the machine.

(c) Comment on your answer in (b) (iii).

Solution

(a) By the principle of levers,

load × load arm = effort × effort arm

Let the distance between the load and the pivot point be x

The distance from pivot point to the effort is $(2 - x)$

$\therefore \quad 750 \times x = 250 \times (2 - x)$

$750\,x = 500 - 250\,x$

$$1\,000\,x = 500$$
$$x = 0.5 \text{ m}$$

The pivot should be placed 0.5 m from the load (rock).

(b) **(i)** $\text{V.R} = \dfrac{\text{effort distance}}{\text{load distance}} = \dfrac{2 - x}{x} = \dfrac{1.5}{0.5} = 3$

(ii) $\text{M.A} = \dfrac{\text{load}}{\text{effort}} = \dfrac{750}{250} = 3$

(iii) $\text{Efficiency} = \dfrac{\text{M.A}}{\text{V.R}} \times 100\%$

$$= \frac{3}{3} \times 100\% = 100\%$$

(c) We have assumed that there is no friction and that the crowbar has no weight.

Example 10.4

In a wheelbarrow, an effort of 50 N is applied to lift a load as shown in Fig. 10.3. Calculate the load.

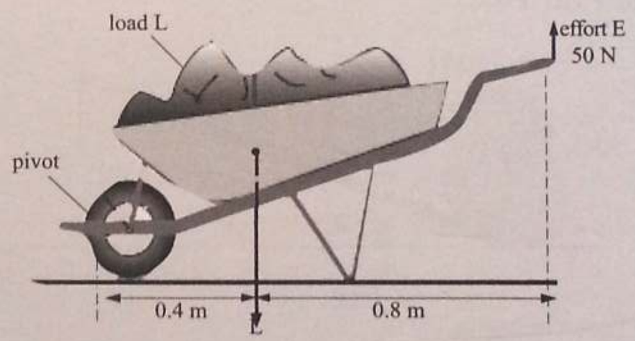


Fig.10.3

Solution

By the principle of levers,

load × load arm = effort × effort arm

$$L \times 0.4 = E \times (0.4 + 0.8)$$
$$L \times 0.4 = 50 \times 1.2$$
$$L = \frac{50 \times 1.2}{0.4} = 150 \text{ N}$$

The load is 150 N.

Example 10.5

A human forearm supports a load of 100 N as shown in Fig.10.4

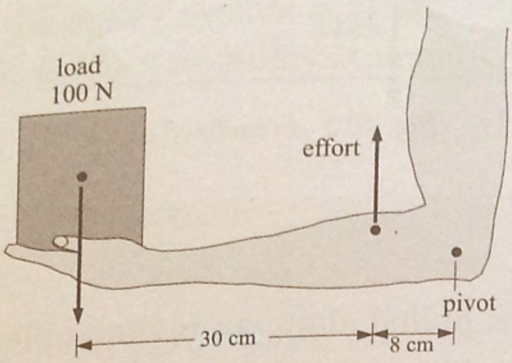


Fig. 10.4

Calculate:

(a) the effort used to support the load,

(b) the mechanical advantage of the forearm.

(c) Comment on your answers.

Solution

(a) By the principle of levers,

load × load arm = effort × effort arm

$100 \times (30 + 8) = \text{effort} \times 8$

$$\text{Effort} = \frac{100 \times 38}{8}$$

$= 475$ N

(b) $\text{M.A} = \frac{\text{load}}{\text{effort}} = \frac{100}{475} = 0.21$

(c) A large effort is needed to overcome a small load, hence the mechanical advantage is less than 1. However, the load moves through a larger distance than the effort moves. This is the case of a speed or distance multiplier lever.

Inclined plane

An inclined plane is a slope or ramp that enables us to raise heavy loads to a certain vertical height, pushing or pulling them along the plane more easily than lifting them vertically to the same height (See Fig. 10.5).

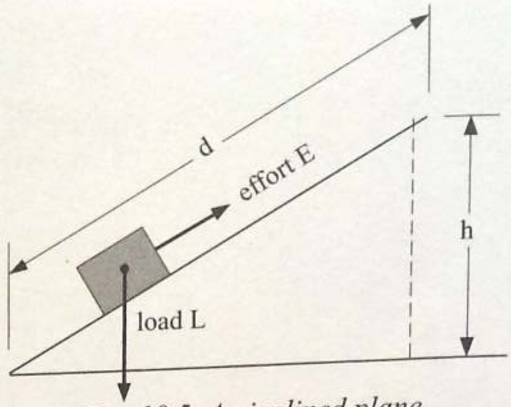


Fig. 10.5 An inclined plane

Example 10.6

A roller of mass 200 kg is pulled along an inclined plane by a force of 1 500 N as shown in Fig. 10.6

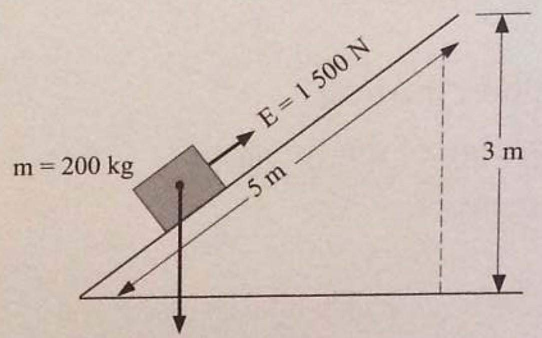


Fig. 10.6

Calculate:

(a) mechanical advantage,

(b) velocity ratio,

(c) efficiency of the inclined plane.

Solution

(a) $\text{M.A} = \dfrac{\text{load}}{\text{effort}} = \dfrac{2\,000}{1\,500} = 1.33$

(b) $\text{V.R} = \dfrac{\text{distance moved by effort}}{\text{distance moved by load}}$

$= \dfrac{5}{3} = 1.67$

(c) $\text{Efficiency} = \dfrac{\text{M.A}}{\text{V.R}} \times 100$

$= \dfrac{2\,000}{1\,500} \times \dfrac{3}{5} \times 100 = 80\%$

Pulleys

A *pulley* is a grooved wheel which turns on an axis or axle fixed to a block. A string is made to pass over the grooved wheel or rim. If the block of the pulley is fixed, the system is known as a *fixed pulley*. Whereas if the block is movable, the system is known as *movable pulley*. If two pulleys are set such that one is fixed and the other is allowed to move, the pulley system is called a *block and tackle*. Fig. 10.7 (a), (b) and (c) show the three types of pulley systems.

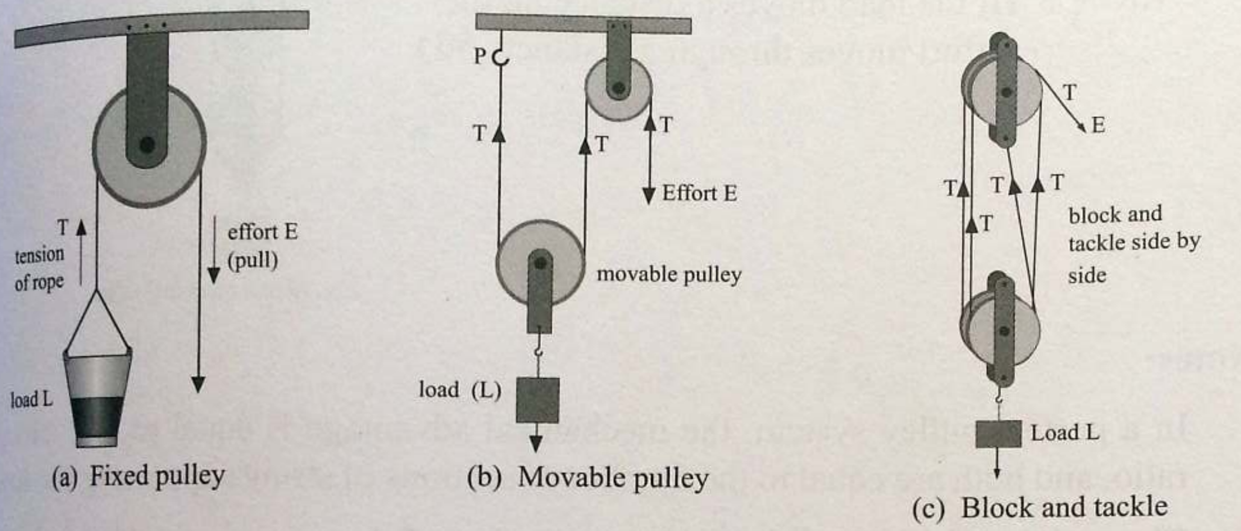


Figure 10.7

Pulleys are used to change the direction of a force and to gain a mechanical advantage greater than 1.

Assuming that the rope and the lower pulleys are weightless and that the system is frictionless, tension in every part of the rope is the same.

For a single fixed pulley (Fig 10.7 (a))

load, L = tension in the rope, T = effort, E

$\therefore E = L$

For a single moveable pulley (Fig. 10.7(b))

$2T = L$ and $T = E$

$\therefore 2E = L.$

For the block and tackle (Fig. 10.7(b))

$4T = L$ and $T = E$

$\therefore 4E = L.$

Mechanical advantage and velocity ratio of a block and tackle

In Fig. 10.8 shows a block and tackle pulley system. Its mechanical advantage and velocity ratio is calculated as follows

$$5T = L \text{ and } T = E$$

$$\therefore 5E = L$$

$$M.A = \frac{L}{E} = \frac{5E}{E} = 5$$

V.R = 5 (If the load moves a distance, d, the effort moves through a distance, 5d).

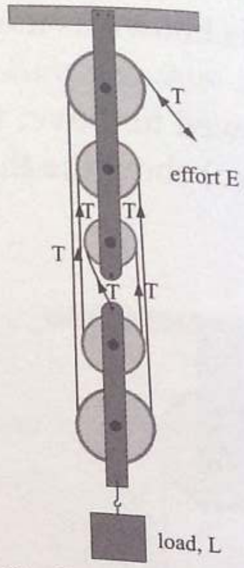


Fig. 10.8: A block and tackle

Notes:

1. In a perfect pulley system, the mechanical advantage is equal to the velocity ratio, and both are equal to the number of sections of string supporting the load.
2. The weight of the block in the lower section of the system and friction in the pulleys reduces the mechanical advantage of the system. Hence, mechanical advantage is usually less than velocity ratio.
3. The V.R of a pulley system is numerically equal to the number of strings section supporting the load.

Example 10.7

For each of the pulley systems in Fig. 10.9(a), (b) and (c),

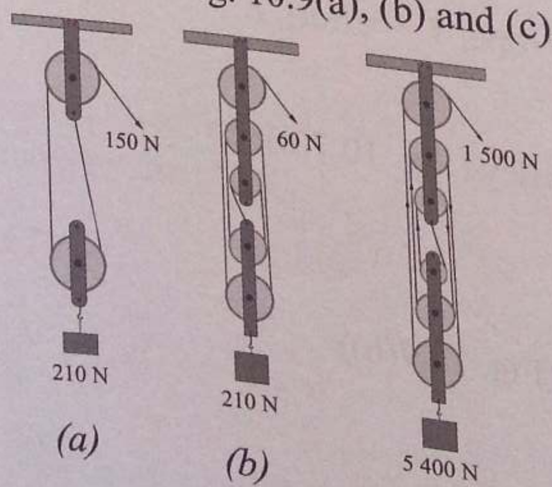


Fig. 10.9

Calculate:

(a) velocity ratio,

(b) mechanical advantage,

(c) efficiency.

Solution

(a) (i) V.R = 2, the number of string sections supporting the lower pulley.

(ii) $M.A = \frac{\text{load}}{\text{effort}} = \frac{210}{150} = 1.4$

(iii) $\text{Efficiency} = \frac{M.A}{V.R} \times 100\%$

$= \frac{1.4}{2} \times 100\% = 70\%$

(b) (i) V.R = 5

(ii) $M.A = \frac{210}{60} = 3.5$

(iii) $\text{Efficiency} = \frac{M.A}{V.R} \times 100\%$

$= \frac{3.5}{5} \times 100 = 70\%$

(c) (i) V.R = 6

(ii) $M.A = \frac{5\,400}{1\,500} = 3.6$

(iii) $\text{Efficiency} = \frac{M.A}{V.R} \times 100\%$

$= \frac{3.6}{6} \times 100\% = 60\%$

Exercise 10.1

1. (a) Define the following terms as applied to machines.
   (i) Mechanical advantage, (ii) Velocity ratio,
   (iii) Efficiency.
   (b) Deduce the relationship between the three terms.
2. A machine of efficiency 80% is used to lift a load of 480 N with an effort of 60 N. Calculate the velocity ratio of the machine.
3. An effort of 400 N raises a load of 1200 N through 4 m in a machine of efficiency 60%. Calculate the distance through which the effort is moved.
4. Which of the following is better to use and why?
   (a) A pair of scissors with long blades or short blades.
   (b) A pair of tongs with a long handle or short handle.
   (c) A wheelbarrow with a short effort arm or a long effort arm.

5. (a) Calculate the minimum force that a gardener needs to exert to hold the legs of the wheelbarrow off the ground (Fig. 10.10).

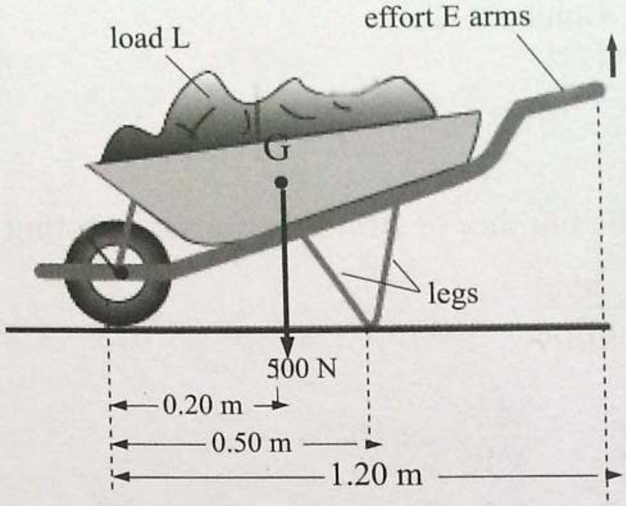


Fig. 10.10

(b) State and explain the effect of increasing the effort arms by 5 cm.

6. A load of 3 000 N is pulled along an inclined plane of length 8 m by a force of 2 100 N. Calculate the efficiency of the system if the load is raised through a vertical distance of 4 m.

7. A worker wants to load a drum of weight 1 000 N on a truck whose floor is 1 m above the ground. The man rolls the drum up a plank 3 m long with a force of 400 N.

(a) Calculate:

(i) the work output, (ii) the work input,

(iii) efficiency of the machine.

(b) Compare your answer in (a) (iii) with the efficiency obtained by calculating the velocity ratio and mechanical advantage of the machine.

8. In the pulley system shown in Fig. 10.11, an effort of 120 N is required to lift a load of 200 N. Calculate

(a) the distance through which the effort moves when the load is lifted through 1.2 m,

(b) the work done by the effort,

(c) the work done on the load,

(d) the efficiency of the system.

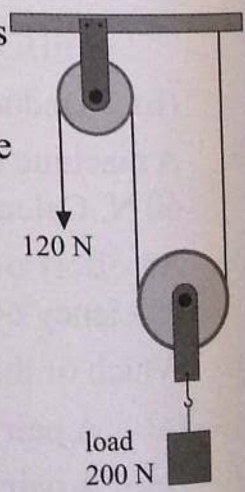


Fig. 10.11

9. In the pulley system shown in Fig. 10.12, an effort of 20 kN is required to lift a load of 32 kN.

Calculate the:

(a) velocity ratio,

(b) mechanical advantage,

(c) efficiency of the system.

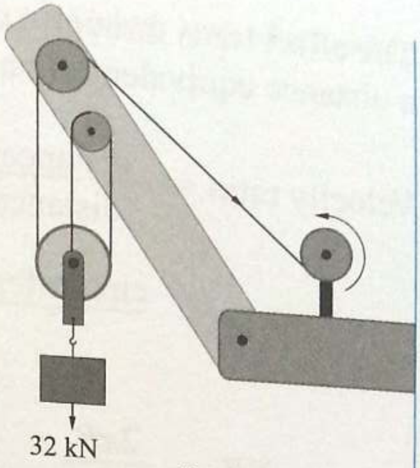


Fig. 10.12

10. Fig. 10.13 shows a block and tackle pulley system of efficiency 80%. It is used to raise a load through a height of 20 m with an effort of 100 N.

(a) What is the velocity ratio of the system?

(b) Calculate:

(i) the load raised,

(ii) the work done by the effort,

(iii) the energy wasted.

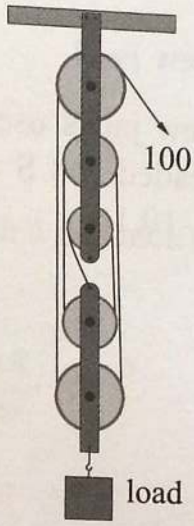


Fig.10.13

Screws and bolts

Screws and bolts are used daily to hold things together. Fig. 10.14 shows a screw and a bolt.

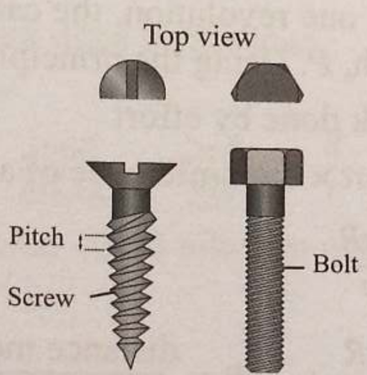


Fig. 10.14: Screws and bolts

The distance between the two successive threads is called *pitch*. When the screw is turned through one revolution by a force applied at the screw head, the lower end moves up or down through a distance equal to its pitch. The working of screws and bolts is based on the principle of an inclined plane.

As the bolt is turned through one revolution, the screw moves one pitch up or down.

The effort turns through a circle of radius R as the load is raised or lowered through a distance equivalent to one pitch (Fig. 10.15).

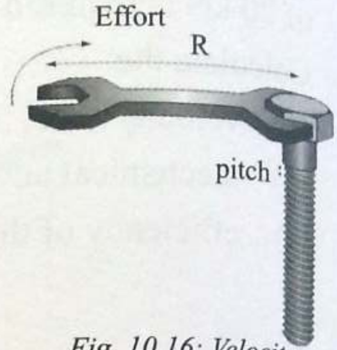


Fig. 10.16: Velocity ratio for a bolt

$$\text{Velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$= \frac{\text{circumference of a circle, } R}{\text{pitch } (p)} = \frac{2\pi R}{p}$$

$$\text{V.R} = \frac{2\pi R}{p}$$

Screw jack

Screw jacks use the principle of inclined plane. A screw jack consists of a long threaded rod S passing through a threaded block brace and a base as shown in Fig. 10.16.

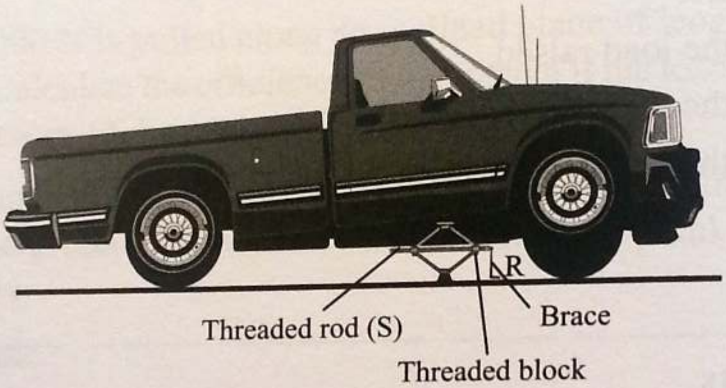


Fig. 10.16: A screw jack

When the brace turns through one revolution, the car is raised or lowered through a distance equal to pitch length, P. Using the principle of conservation of energy,

work done on load = work done by effort

load × pitch = effort × circumference of a circle

$$\frac{\text{load}}{\text{effort}} = \frac{2\pi R}{p}$$

$$\text{mechanical advantage} = \frac{2\pi R}{p}\ ; \quad V.R = \frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{2\pi R}{p}$$

The mechanical advantage is greatly affected by friction on the threads.

The wheel and axle

A screw driver, a steering wheel of a car, a box spanner, a brace and a windlass are some common examples of a *wheel and axle* machine. Fig. 10.17 shows an effort being applied on a large wheel while the load is raised by a string wound round in the opposite direction on the axle.

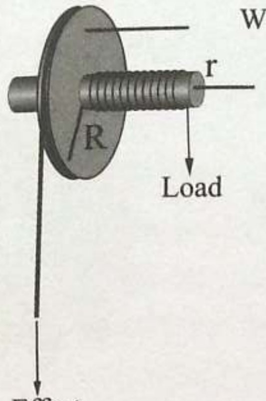


Fig. 10.17: Wheel and axle

Velocity ratio and mechanical advantage of wheel and axle

For one complete revolution of the wheel, the effort moves through a distance $2\pi R$ while the load moves through a distance of $2\pi r$.

$$\text{Velocity ratio} = \frac{\text{distance covered by effort}}{\text{distance covered by load}} = \frac{2\pi R}{2\pi r} = \frac{R}{r}$$

$$\therefore \text{ Velocity ratio} = \frac{\text{radius of wheel}}{\text{radius of the axle}}$$

$$\text{V.R} = \frac{R}{r}$$

The mechanical advantage may be calculated using the principle of conservation of energy i.e.

Work done by the effort = Work done on the load

Effort × $2\pi R$ = load × $2\pi r$

$$\text{Mechanical advantage} = \frac{\text{load}}{\text{effort}} = \frac{2\pi R}{2\pi r} = \frac{R}{r}$$

This holds only if the machine is 100% efficient which is never the case.

$$M.A = \frac{R}{r}$$

Gears

Gears are toothed wheels of different diameters. They turn together with their axles (Fig. 10.18 (a)). Gears are designed in such a way that they are able to raise or lower the speed of the rotation. In gears, the effort is applied on one axle of the wheel while the load to be overcome acts on a different axle of a gear wheel. The gear wheel which provides the effort is called *input gear wheel or the driving wheel* while the gear wheel that works against the load is called *the output gear wheel* or *the driven wheel*. When gears are fitted into each other they are said to be *engaged* or *meshed*. (See Fig. 10.18 (b).

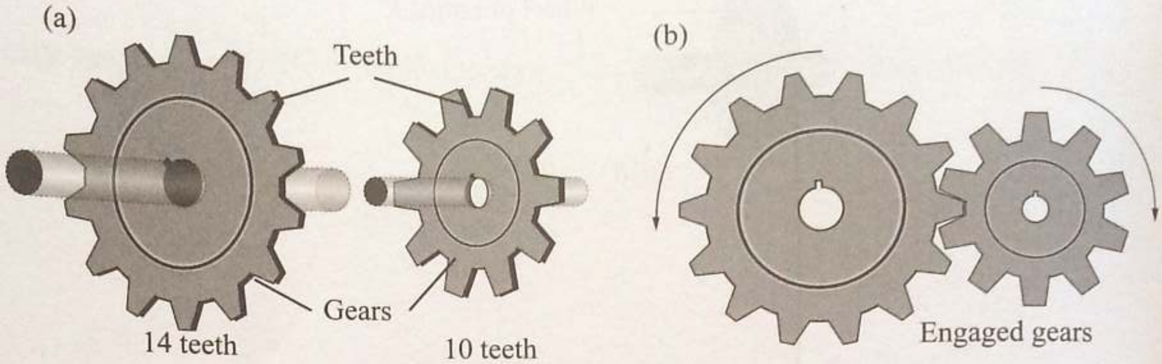


Fig. 10.18: Gears

Experiment 10.1: To demonstrate the slowing down and the speeding up effect of gears

Apparatus

- Two container tops with grooves
- A piece of wood

Procedure

1. Use two container tops A and B with grooves. Choose top A with half the diameter of top B.
2. Locate the centre of each container top and fix a piece of wood at the centres such that the grooves fit into each other as shown in the Fig. 10.19.

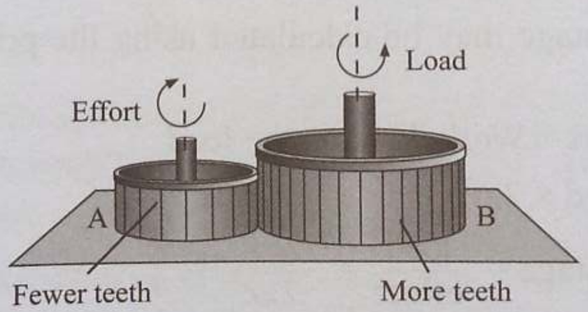


Fig. 10.19: slowing and speeding gears

3. Now, Steadily turn top A through one complete revolution and note how many revolutions the top B makes.

Top A is the *driving wheel* while top B is the *driven wheel*. Turn top A and observe what happens to top B.

Discussion

When top A is turned Top B also turns.

When top A covers one revolution, top B makes only a half revolution. The fast motion of A produces a slow motion in B. This is the case of *slowing down of rotation*.

Repeat the experiment but move top B. Compare the motion of B and A. In this case when B makes one complete revolution, A makes 2 revolutions. This is the case of *speeding up of rotation*. Note the direction of rotation. One top rotates clockwise when the other top is turning in an anticlockwise direction.

From this experiment, we can conclude that the larger gear wheel, with more teeth always turns slowly. This means that there is an inverse relationship between the number of teeth and the speed of the rotation.

$$\frac{\text{number of teeth on smaller wheel}}{\text{number of teeth on larger wheel}} = \frac{\text{speed of rotation of larger wheel}}{\text{speed of rotation of smaller wheel}}$$

Since the time of rotation is the same i.e. the action takes place at the same time and the effort is on the driving wheel,

$$\text{velocity ratio} = \frac{\text{speed of rotation of driving wheel}}{\text{speed of rotation of driven wheel}}$$

$$= \frac{\text{revolutions per second of the driving wheel}}{\text{revolutions per second of the driven wheel}}$$

or

$$\text{velocity ratio} = \frac{\text{number of teeth on the driven wheel (load)}}{\text{number of teeth on the driving wheel (effort)}}$$

Depending on the number of teeth on the driving wheel, the velocity ratio may be greater or less than 1. To obtain a velocity ratio greater than 1 a small gear wheel is used to drive a larger one (Fig. 10.20 (b).

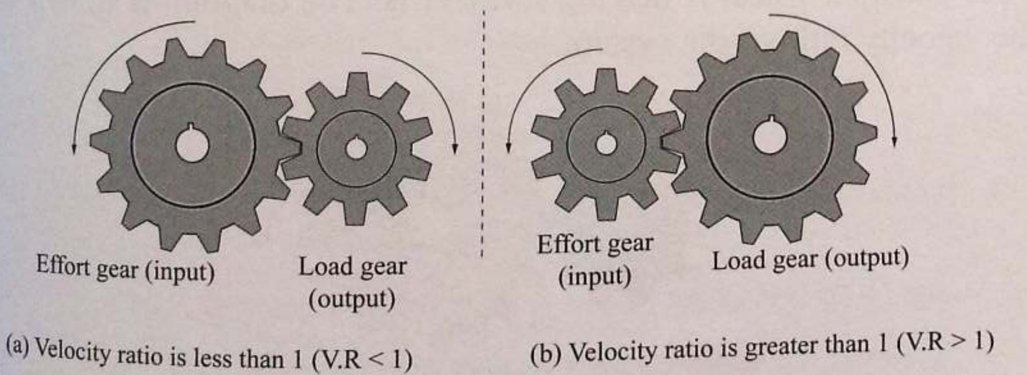


Fig. 10.20: Velocity ratio depends on the number of teeth in the driving wheel

In cars, the velocity ratio can be changed at will. The gear box has a set of gears that can be combined in a number of ways. In climbing up a hill, the mechanical advantage and velocity ratio have to be high.

In such a case a small gear wheel should drive a large gear wheel. On a flat road or going down the hill, the weight of the car is not a part of the load. Hence we need low mechanical advantage and low velocity ratio.

Chains and belts

A wheel can be driven by another wheel although they are not in direct contact e.g. in conveyor belts as shown in Fig. 10.21.

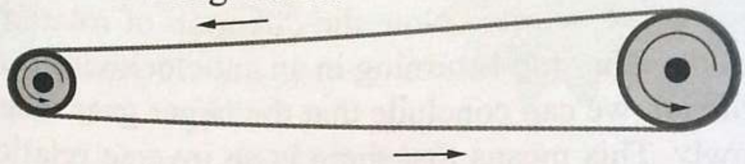


Fig. 10.21: Chain or belt

The velocity ratio is the same as that which would be provided by a pair of gear wheels with the same radii but in contact. The only difference is that the two wheels rotate in the same direction unlike the gear wheels.

A toothed wheel can also be driven by another one not in contact e.g. bicycle chain (Fig. 10.22). The velocity ratio is obtained in the same way as in the gears system.

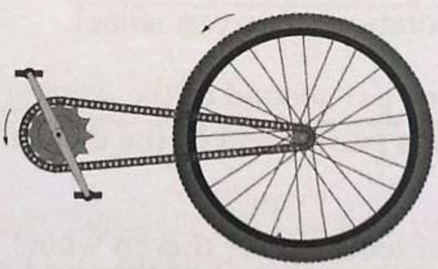


Fig. 10.22: Bicycle wheels

Example 10.8

Fig. 10.23 shows a wheel A driving a wheel B. The diagram is drawn to scale. Find the velocity ratio of the system.

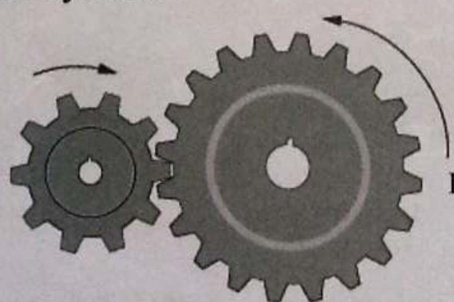


Fig. 10.23

Solution

$$\text{velocity ratio} = \frac{\text{number of teeth driven (B)}}{\text{number of teeth driving (A)}} = \frac{20}{10} = 2$$

Example 10.9

A bicycle has a driving gear wheel of radius 10 cm with 24 teeth. The driven rear wheel has a radius of 40 cm mount on a rear gear with 8 teeth, determine:

(a) (i) The velocity ratio for the gear wheel system.

(ii) The mechanical advantage.

(iii) Efficiency.

(b) What is the effect of increasing the:

(i) Number of teeth on the driving wheel.

(ii) Radius of the driven wheel.

(c) Why is it unrealistic to have both wheels having the same radius and equal number of teeth?

Solution

(a) (i)

$$\text{velocity ratio} = \frac{\text{number of teeth on the driven wheel}}{\text{number of teeth on the driving wheel}} = \frac{8}{24} = \frac{1}{3}$$

(ii)

$$\text{Mechanical advantage} = \frac{\text{Radius of the driving wheel}}{\text{Radius of the driven wheel}} = \frac{R}{r} = \frac{10}{40} = 0.25$$

(iii)

$$\text{efficiency} = \frac{\text{mechanical advantage}}{\text{velocity ratio}} \times 100\% = \frac{0.25}{\frac{1}{3}} \times 100\% = 75\%$$

(b) (i) The velocity ratio will be decreased and the arrangement will speed up the rotation.

(ii) The velocity ratio will be increased but the speed will reduce since

$$\text{velocity ratio} = \frac{\text{speed of the driving wheel}}{\text{speed of the driven wheel}}$$

(c) There will be no increase on mechanical advantage. This means that the effort applied must be very large even on a flat ground. The friction will cause the load to be less than effort.

Example 10.10

Fig. 10.24 shows three wheels A, B and C. A has 30 teeth, B has 10 teeth and C has 40 teeth. If A makes 12 revolutions, how many revolutions will C make?

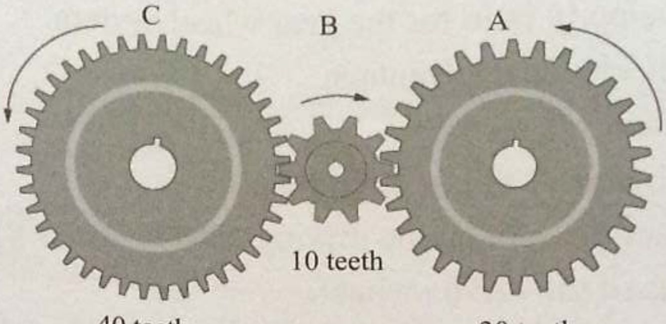


Fig. 10.24

Solution

$$\text{Velocity ratio of A and B} = \frac{\text{number of teeth driven wheel B}}{\text{number of teeth driving wheel A}} = \frac{10}{30} = \frac{1}{3}$$

$$\text{Also velocity ratio} = \frac{\text{speed of rotation of driving wheel A}}{\text{speed of rotation of driven wheel B}} = \frac{1}{3} = \frac{12}{\text{speed of wheel B}}$$

$\therefore$ speed of B $= 12 \times 3 = 36$ revolutions.

$$\text{velocity ratio of B and C} = \frac{\text{number of teeth on wheel C}}{\text{number of teeth on wheel B}} = \frac{40}{10} = 4$$

$$\text{Also velocity ratio} = \frac{\text{speed of rotation of wheel B}}{\text{speed of rotation of wheel C}}$$

$$4 = \frac{36}{\text{speed of rotation of wheel C}}$$

$$\therefore \quad \text{speed of wheel C} = \frac{36}{4} = 9 \text{ revolutions}$$

Hydraulic machine

A hydraulic machine consists of two pistons of different diameters and a liquid contained in a vessel. The bigger piston is called *the ram piston* while the smaller one is called *the pump piston* (Fig. 10.25).

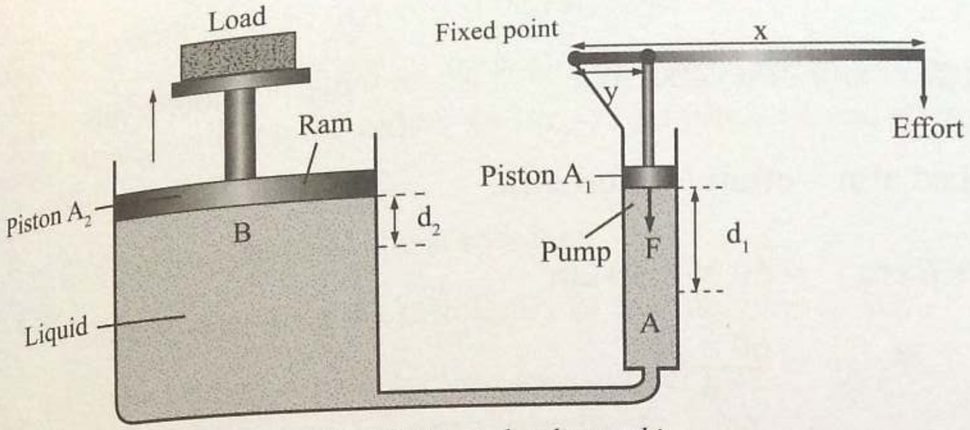


Fig. 10.25: Hydraulic machine

In many hydraulic machines, the effort is provided by the use of a lever system as shown in Fig. 10.25. The lever system multiplies the effort applied.

The force, F, on the smaller piston is calculated by applying the principle of moment

$$\therefore \quad E \times x = y \times F$$

$$F = \frac{E \times x}{y}$$

Hence the hydraulic machine has velocity ratio and mechanical advantage due to itself and due to the lever system.

Suppose while pressing down on A_1, the piston moves a distance d_1 and the piston at A_2 moves up a distance d_2. The volume of the liquid swept from piston A_1 is equal to the volume of the liquid moved to A_2.

Volume from $A_1 = A_1 d_1$

Volume moved to $A_2 = A_2 d_2$

$\therefore \quad A_1 d_1 = A_2 d_2$

velocity ratio $= \dfrac{d_1}{d_2} = \dfrac{A_2}{A_1} = \dfrac{\pi R^2}{\pi r^2} = \dfrac{R^2}{r^2}$, where R and r are the radii of the pistons A_2 and A_1 respectively.

In practice, owing to friction and weight of the moveable parts e.g. ram, the mechanical advantage of the machine is less than the velocity ratio.

Example 10.11

Refer to hydraulic press in Fig. 10.25 on page 219. Given that x = 30 cm, y = 6 cm, effort = 60 N area $A_1 = 4\ cm^2$, area $A_2 = 12\ cm^2$. Calculate

(a) the force, F_A, exerted on the liquid at A.

(b) velocity ratio.

(c) maximum load that could be raised at B.

Solution

(a) By the principle of levers,

load × load arm = effort × effort arm.

$$F_A \times 6\text{ cm} = 60\text{ N} \times 30\text{ cm}$$

$$F_A = \frac{60 \times 30}{6}$$

$$= 300\text{ N}$$

(b) Velocity ratio $= \frac{A_2}{A_1}$

$$= \frac{12}{4}$$

$$= 3$$

(c) Pressure at A = pressure at B

$$\text{Pressure at A} = \frac{300}{4 \times 10^{-4}}$$

Pa ; Pressure at B $= \frac{\text{Load}}{12 \times 10^{-4}}$ Pa.

$$\therefore \quad \frac{\text{Load}}{12 \times 10^{-4}} = \frac{300}{4 \times 10^{-4}}$$

$$\text{Load} = \frac{12 \times 300}{4} = 900\text{ N}$$

Alternatively;

$$\text{M.A} = \frac{F_L}{F_E}$$

assuming perfect machine;

$$\text{M.A} = \text{V.R}$$

$$\frac{L}{300} = 3$$

$$L = 900\text{ N}$$

Exercise 10.2

1. A screw has a head with a radius of 1.4 cm and pitch of 0.1 cm. Calculate the:
   (a) screw's velocity ratio,
   (b) mechanical advantage, if energy is conserved.
2. A steering wheel of a car has a diameter of 40 cm and its axle has a radius of 5 cm. Calculate the velocity ratio of this system.
3. In a gear system, a driving wheel has 32 teeth while the driven wheel has 74 teeth. Calculate the velocity ratio of the gear system.
4. Refer to hydraulic press in Fig. 10.25. Given that y = 12 cm, x = 40 cm effort = 80 N, area $A_1 = 2\text{ cm}^2$, area, $A_2 = 9\text{ cm}^2$, calculate:
   (a) the force, F_A exerted on the liquid at A,
   (b) the velocity ratio,

(c) the maximum load that could be raised at B.

5. An effort of 50 N is applied to the brace of a car's screw jack whose handle moves through a circle of radius 14 cm. The pitch of the screw is 2 mm. Calculate:

(a) The velocity ratio of the screw jack,

(b) The load raised , if the efficiency of the machine is 30%.

6. Fig. 10.26 shows the cross-section of a wheel and axle, of radius 6 cm and 1.5 cm respectively, used to lift a load of 150 N.

Calculate the efficiency of the machine.

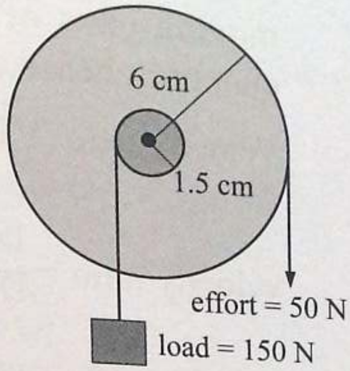


Fig. 10.26

7. (a) Draw a set of gears that may be used to lift a load attached to an axle of 10 teeth gear by applying an effort to the axle of 20 teeth gear.

(b) When the load is raised a distance of 6 m, the axle of the 20 teeth gear has to be rotated 15 times. How many times does the axle of 10 teeth rotate.

(c) What distance does the effort move if both axles are of the same diameter?

(d) The machine is 90% efficient. Calculate the effort needed to lift a load of 400 N.

8. Fig. 10.27 shows a hydraulic machine. What load can be lifted by the machine if an effort of 15 N is applied as shown in the diagram?

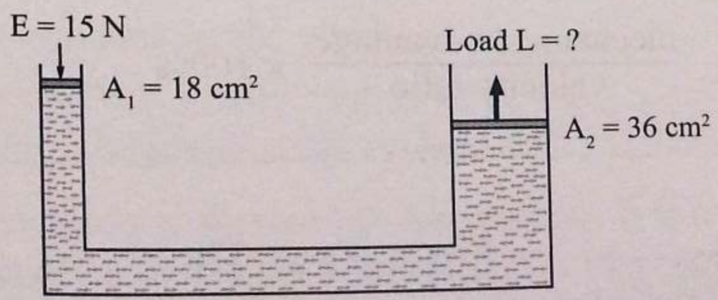


Fig.10.27

Unit summary

- A machine is a device that makes work easier.
- Mechanical advantage (M.A) is defined as the *ratio of load to effort.*

  $$\text{mechanical advantage} = \frac{\text{load}}{\text{effort}}.$$

  The mechanical advantage of a machine depends on loss of energy of the moving parts of a machine. Mechanical advantage is a ratio of similar quantities hence it has no units.
- Velocity ratio (V.R) is defined *as the ratio of distance the effort moves to that moved by the load.*

  $$\text{velocity ratio} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}.$$

- Velocity ratio is a ratio of similar quantities hence it has no units.
- Theoretical value of velocity ratio is obtained from the dimensions of the machine e.g. in pulleys–number of the sections of string supporting the load.

Machine	VR
Inclined plane	$\frac{1}{\sin\theta}$
Screw Jack	$\frac{2\pi r}{\text{pitch, P,}}$
Wheel and axle	$\frac{\text{Radius of wheel, R}}{\text{Radius of axle, r}} = \frac{R}{r}$

- $$\text{efficiency} = \frac{\text{work output}}{\text{work input}} \times 100\%$$

  $$= \frac{\text{mechanical advantage}}{\text{velocity ratio}} \times 100\%$$

Unit Test 10

1. Define the following terms;

(a) machine **(b)** efficiency

(c) mechanical advantage

2. A student wanted to put 10 boxes of salt at the top of the platform using an inclined plane (Fig. 10.28).

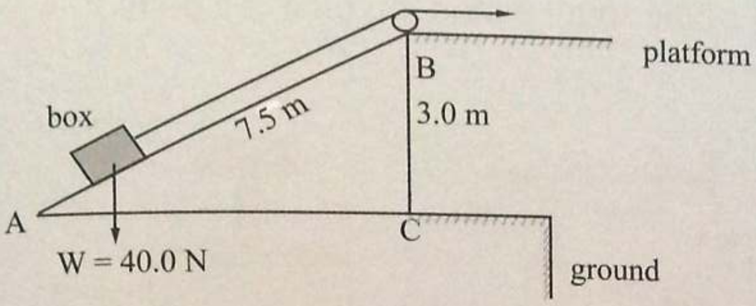


Fig. 10.28

If the resistance due to friction is 10 N. Calculate:

(a) The work done in moving the 10 boxes.

(b) The efficiency of this arrangement.

(c) The effort required to raise one box to the platform.

3. A pulley system has a velocity ratio of 4. In this system, an effort of 68 N would just raise a load of 217 N. Find the efficiency of this system.

4. A crane just lifts 9 940 N when an effort of 116 N is applied. The efficiency of the crane is 75%. Find its

(a) mechanical advantage

(b) velocity ratio

5. Fig. 10.29 shows a pulley system. An effort of 113 N is required to lift a load of 180 N.

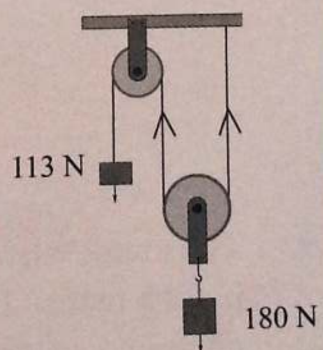


Fig 10.29

(a) What distance does the effort move when the load moves 1 m?

(b) Find the work done by the effort.

(c) Find the work done on the load.

(d) Calculate the efficiency of the system.

6. A block and tackle pulley system has five pulleys. It is used to raise a load through a height of 20 m with an effort of 100 N. It is 80% efficient.

(a) Is the end of the string attached to the upper or lower block of pulleys if the upper block has three pulleys? Show it in a diagram.

(b) State the velocity ratio of the system.

(c) Calculate the load raised.

(d) Find the work done by the effort.

(e) Find the energy wasted.

7. Fig. 10.30 shows a hydraulic press system using a lever of negligible mass, on the side of the small piston, pivoted at a point P. A force of 50 N is applied at R.

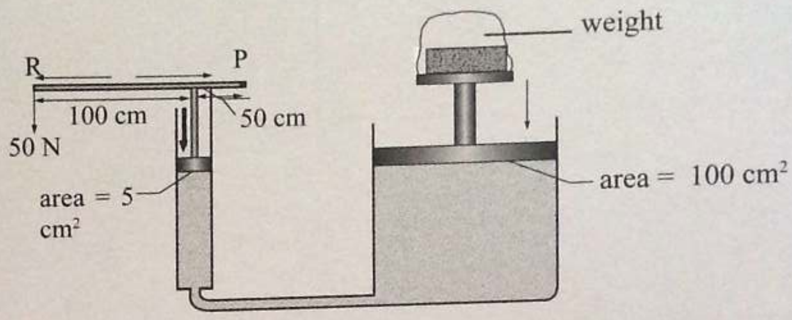


Fig. 10.30

Calculate:

(a) The force exerted by the small piston on the liquid,

(b) The pressure of the liquid below the small piston,

(c) The weight balanced by the large piston.

8. Fig. 10.31 shows three wheels A, B and C of a gear system. A has 30 teeth, B has 10 teeth and C has 40 teeth. If the driving wheel A makes 12 revolutions, how many revolutions will C make?

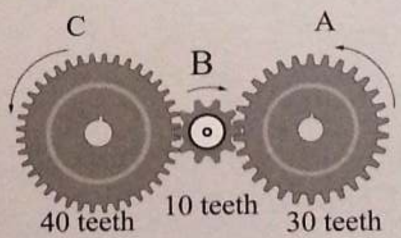


Fig. 10.31

9. The crank wheel (driving wheel) of a bicycle has 60 teeth and the free wheel has 15 teeth. If the rear wheel has a diameter of 84 cm and the pedal turns once in a second (Fig. 10.32), calculate the speed of the bicycle.

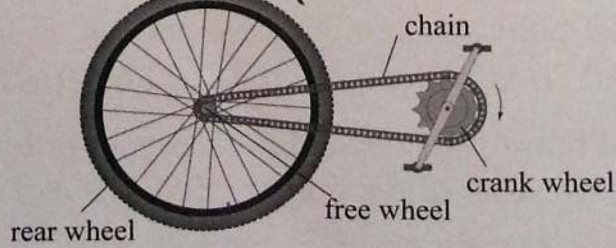


Fig. 10.32

Electricity and Magnetism

Outcome

The students will be able to understand the laws and principles of electricity, magnetism and then apply this knowledge in areas such as analogue and digital electronic systems.

UNIT

11 Electric Current and Potential Difference

Success Criteria

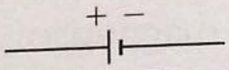
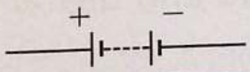
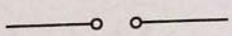
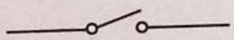
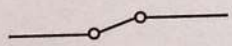
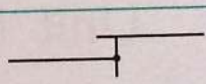
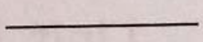
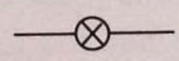
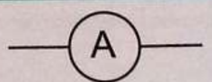
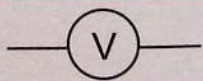
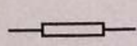
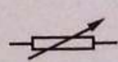
By the end of this unit, you must be able to:

- Describe electrical current.
- Describe potential difference.

11.1 Basic circuit symbols

When drawing diagrams of the electric circuits, we represent the actual components with symbols. Table 10.1 shows some symbols used in electric circuit diagrams.

Table 11.1: Circuit components and their symbols

Name of component	Symbol
Cell	
Battery	
Power supply	
Open switch	
Close switch	
Wires joined	
Connecting wires	
Lamp	
Ammeter	
Voltmeter	
Resistor	
Variable resistor	

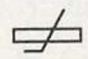
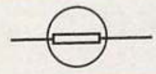
Fuse	
Thermistor	
Diode	
Light dependent resistor	
Light emitting diode LED	

Fig 11.2 shows an example of a diagram with electric symbols.

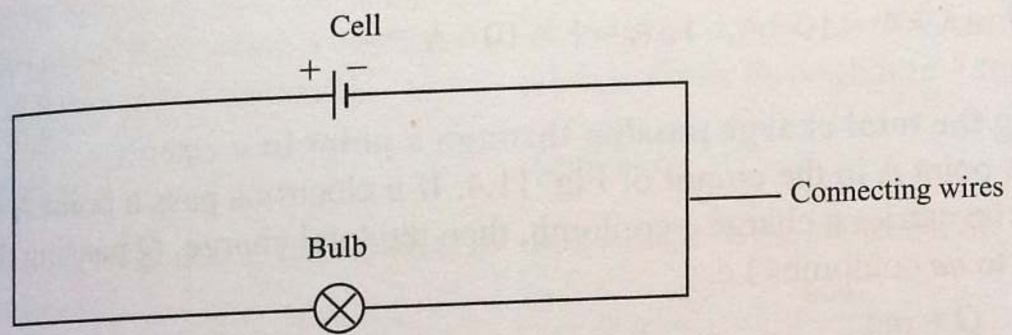


Fig. 11.2: Simple electric circuit using symbols

11.2 Electric current

The movement of charged particles called *electrons* constitutes an *electric current.* The conducting path through which the electrons move is called *an electric circuit.* Fig. 11.3 shows a simple electric circuit *path involving* wires, a bulbs cell and a switch.

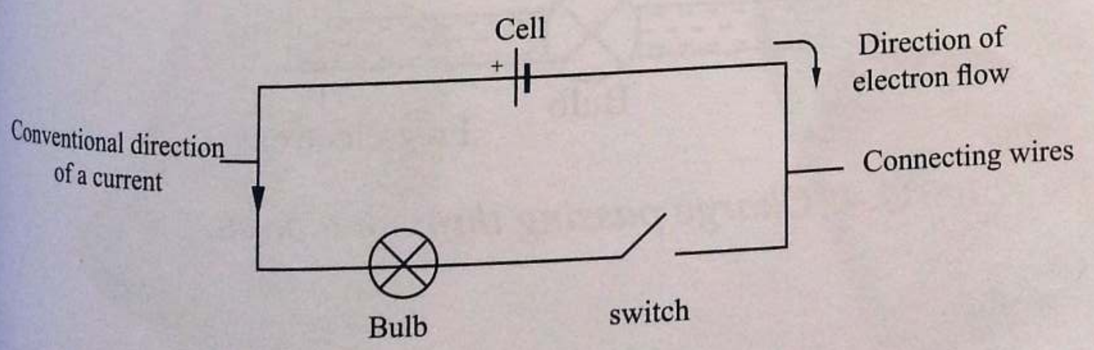


Fig. 11.3: An electric circuit.

When the circuit is complete, electrons flow from the negative terminal of the cell round the circuit then through the positive terminal of the cell. The opposite direction to the flow of the electrons is the conventional direction of the flow of electric current. An electric current is therefore defined as the *rate of flow of charges*. i.e.

$$\text{Current } (I) = \frac{\text{Charge (Q)}}{\text{Time (t)}}$$

$$I = \frac{Q}{t}$$

The SI unit of current is the *ampere* (A). The ampere is the amount of charge per second. i.e. Ampere $= \frac{\text{coulomb}}{\text{second}}$. Smaller currents can be measured in *milliamperes* (mA) and microamperes (μA).

$$1 \text{ mA} = 1 \times 10^{-3} \text{ A}, \quad 1 \mu\text{A} = 1 \times 10^{-6} \text{ A}$$

Determing the total charge passing through a point in a circuit

Consider a point A in the circuit of Fig. 11.4. If n electrons pass a point A and that each electron carries a charge e coulomb, then the total charge, Q passing the point A is equal to ne coulombs i.e.

$$Q = ne;$$

$$\text{But } Q = It$$

$$\text{Therefore } I = \frac{ne}{t}$$

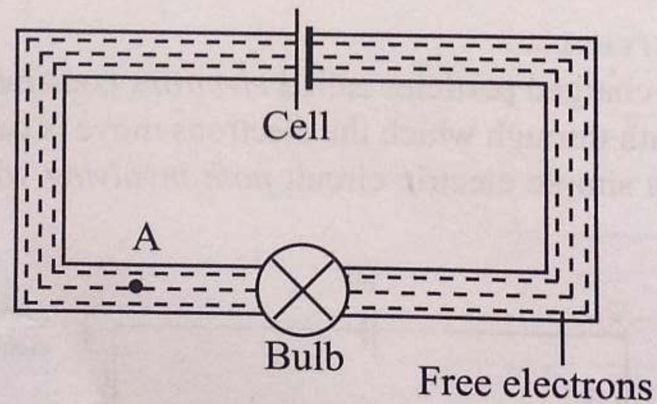


Fig. 11.4: Charge passing through a point.

Experiments show that the charge of an electron is equal to 1.6×10^{-19} coulomb.

Example 11.1

Calculate the amount of charge that passes through a point in a circuit in 3 seconds, if the current in the circuit is 0.5 A.

Solution

Charges $Q = It$

$= 0.5 \times 3 = 1.5\ C$

Example 11.2

How long would it take for a charge of 1200 μ C to flow when a current of 0.01A is flowing in a circuit?

Solution

From Q = It, we make, t, the subject of the formular

$$t = \frac{Q}{t} = \frac{1200 \times 10^{-6}C}{0.01A}$$

$= 0.12$ seconds

Example 11.3

Find the amount of current passing through a lamp, if 600 coulomb of charge flows through it in 4 minutes

Solution

It, we make, t, the subject of the formular

$$I = \frac{Q}{t} = \frac{600}{(4 \times 60)s}$$

$= 2.5A$

Measurement of electric current

Current through electric circuits is measured using an instrument called an *ammeter*. Fig. 11.5(a) shows an analogue ammeter. It has two positive terminals and one negative terminal. Fig. 11.7 (b) shows a digital ammeter.

(a) An analogue ammeter

(b) A digital ammeter

Fig 11.5: Ammeters

An analogue ammeters has two scales (Fig 11.6(a)). The magnitude of the current determines the scale to be used. The symbol of an ammeter is as shown in Fig. 11.8.

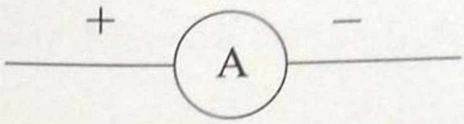


Fig 11.6: A symbol for an ammeter

An ammeter is connected in series with the circuit components for which current is to be measured (Fig. 11.7). An ideal ammeter has almost zero resistance. This ensures that the most current that passes through the device also passes through the ammeter.

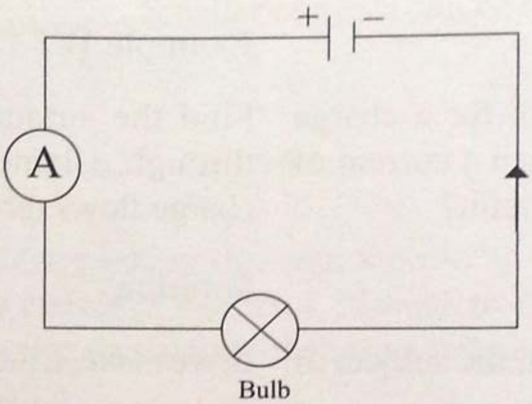


Fig 11.7: An ammeter connected in series with a bulb

How to read an analogue ammeter

Connect the ammeter scale in Fig. 11.8, circuit with the negative terminal of the power source leading to the negative terminal ammeter (referred to as the common terminal - usually black) and positive terminal leading to the 1 A or 5 A terminal positive terminal - usually red or brown in colour) depending on the amount of current to be measured.

Fig 11.8 shows the scale on an ammeter that measures current in the range 0 – 1 A, or 0 – 5 A.

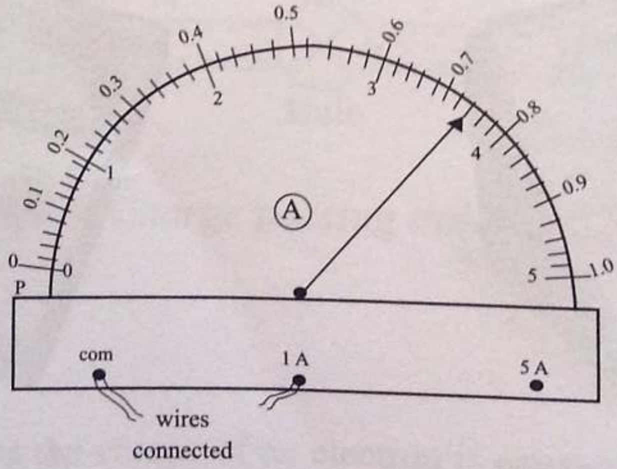


Fig. 11.8: An ammeter with a scale 0 – 1 A, 0 – 5 A

When connected to the 1 A terminal, the upper scale running fronm 0 - 1 A should be used.

We determine the current represented by each smallest division on the upper scale, as follows:

$$5 \text{ divisions correspond to } \ldots\ldots\ 0.1 \text{ A}$$

$$1 \text{ division } \ldots\ldots\ldots\ldots\ldots\ldots \frac{0.1\text{A}}{5} = 0.02 \text{ A}$$

In Fig. 11.8, the pointer is on the 2nd mark after the 0.7 a mark, hence the ammeter reading is:

$$0.7 \text{ A} + (2 \text{ divisions} \times 0.02\text{A}) = 0.7\text{A} + 0.04 \text{ A} = 0.74 \text{ A}$$

Example 11.4

What is the reading shown by the pointer in Fig. 11.9, if the full scale range is

(a) 0–100 mA,

(b) 0–250 mA?

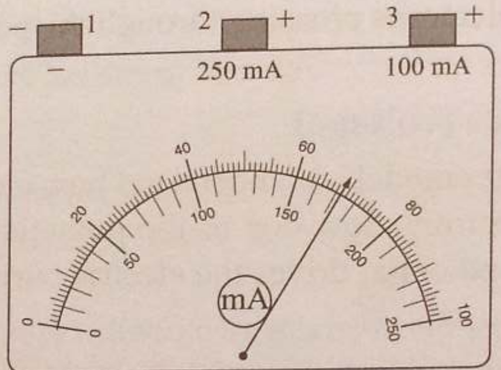


Fig. 11.9: Ammeter reading scale

Solution

(a) Full scale deflection = 100 mA. (Use the upper scale)
The pointer is at 69th division.
The reading is 69 mA.

(b) Full scale deflection = 250 mA. (Use the lower scale)
The pointer is between 170th and 180th divisions. There are 4 divisions of the upper scale corresponding to 10 mA in the lower scale.

$\therefore$ 1 division represents 2.5 mA.

Reading = 170 mA + 2.5 mA = 172.5 mA.

The reading is 172.5 mA.

Exercise 11.1

1. Define the term electric current and state its SI unit.
2. A car battery circulates charge round a circuit for 5 minutes. If the current is held at 15A, what quantity of charge passes through the wire?
3. A charge of 40 coulombs flows through a point on a conducting wire in 15 s. Calculate the current flowing in the conductor.
4. Calculate the number of electrons which carry a charge of 1.0 C. (charge of an electron, 1.6×10^{-19}C.)
5. If a charge of 1.5 C crosses through a point in a circuit in 0.5 s, calculate the current in the circuit.
6. If the current in a circuit is 2 A, calculate:
   (a) the charge that crosses a point in the circuit in 0.6 s,
   (b) the number of electrons crossing through the point.

11.3 Potential difference (voltage)

In Form 1, we used "a water model" to understand how electrons flow in an electric circuit. We learnt that electrons flow due to the potential difference between two points. This difference in potential drives the electric current round the circuit.

Electric potential difference between any two points is the work done in moving one coulomb of charge from one point to the other.

The SI unit of potential difference (p.d) is the volt (V)

$$\text{Volt} = \frac{\text{joule}}{\text{coulomb}}$$

$$\text{Thus, P.d (V)} = \frac{\text{work done (J)}}{\text{charge moved (C)}}$$

The volt

In Fig. 11.8(a), points A and B are at a potential difference of one volt if the work done in moving one coulomb from A to B is one joule.

1 volt is therefore defined as *the potential difference of 1 joule is needed to move one coulomb of charge from one point to another*.

The potential difference between the terminals of a cell indicates the energy given to each coulomb of charge in the circuit. For example, a battery with a potential difference of 6 V gives 6 joules of potential energy to each coulomb of charge in the circuit. This energy is then converted into other forms of energy e.g. light and heat in bulbs (Fig. 11.10 (b)).

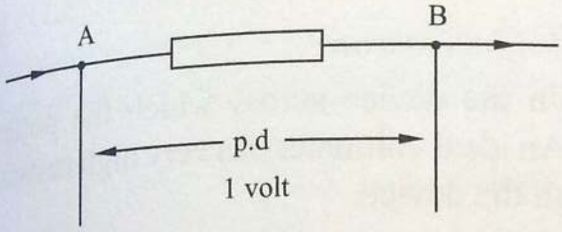


Fig. 5.4: Definition of a volt.

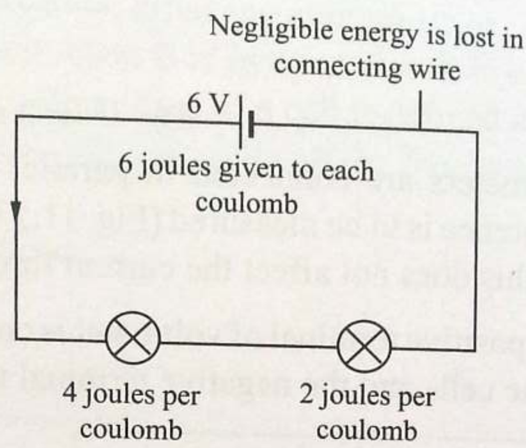


Fig. 11.10: Energy carried by a coulomb of charge.

From the definition of the volt, we see that if a charge of 2 coulombs moves between two points with a p.d of 1V, 2J of energy is released. Therefore, if a charge of Q coulombs move between two points

Energy released in joules, $W = QV$

But we know that $Q = I \times t$

Therefore, $W = ItV$

Example 11.5

A charges of 10 coulombs were moved from point A to B with of potential difference 3V. Calculate the energy needed move to the charge.

Solution

$$W = QV = 10\text{ C} \times 3\text{V} = 30\text{ J}$$

Measurements of potential difference

The potential difference between two points is measured using an instrument called the *voltmeter*. Fig. 11.11(a) and (b) show an analogue and digital ammeters respcetively.

Fig. 11.11: Voltmeters

The symbol of the voltmeter is as shown in Fig. 11.12

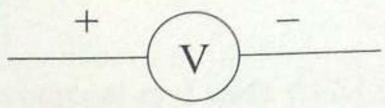


Fig. 11.12: A symbol for a voltmeter

Voltmeters are connected in parallel with the device across which the potential difference is to be measured (Fig. 11.13). An ideal voltmeter has very high resistance but this does not affect the current through the device.

The positive terminal of voltmeter is connected to the wire from the positive terminal of the cells and the negative terminal to the wire leading to negative terminal.

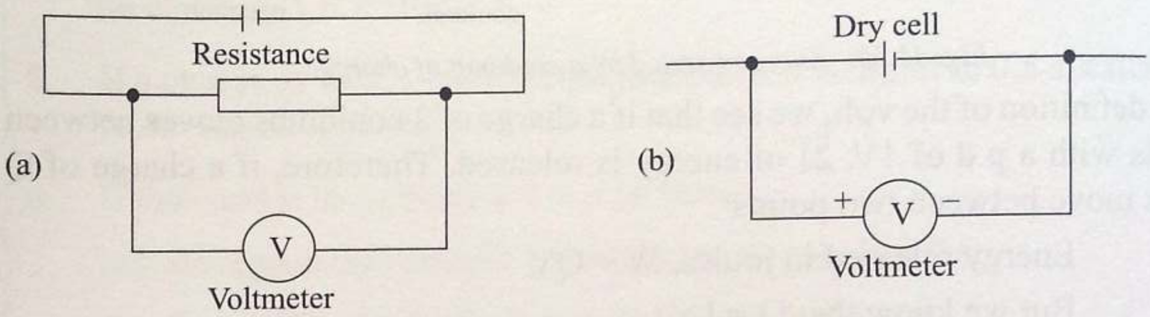


Fig. 11.13: Connecting of a voltmeter in a circuit

Note:

- If the pointer of the ammeter or voltmeter moves in an anticlockwise direction, then interchange the wires on the terminals of an ammeter or voltmeter.
- The voltmeter scale is read in the same way as the ammeter.

Example 11.6

Identify the instrument in Fig. 11.14 and state the reading on it.

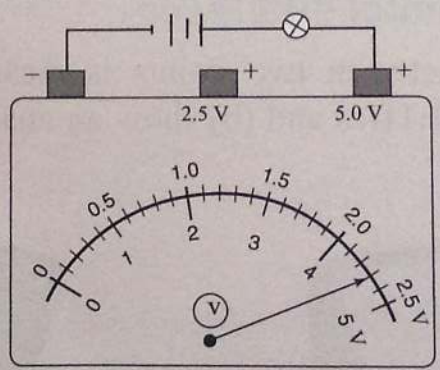


Fig. 11.14: Reading a voltmeter

Voltmeter

The +ve terminal is 5 V.

We read the lower scale.

= 4.6 V

Electromotive force

When a cell is not connected to any load, it does not drive any current; *open circuit* (Fig. 11.15). The p.d at the terminals of a cell when it is in open circuit is called *electromotive force or e.m.f.(E)* The electromotive force of a cell is defined as *the energy available between the terminals of the cell per coulomb for the complete circuit.*

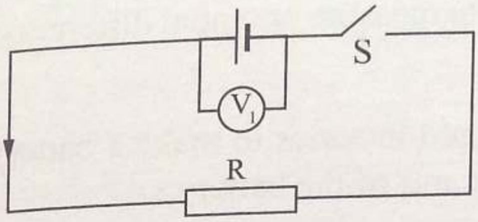


Fig. 11.15: Open and closed circuits

When a load of some resistance is connected to a cell in the circuit, the cell is said to be in a *closed circuit.*

The p.d at the terminals of a cell when it is in a closed circuit is called *terminal voltage.* The value of V in an open circuit is higher than the value of V_1 in a closed circuit.

Example 11.7

Four dry cells have e.m.f of 1.5V each. Find the total e.m.f when the cells are connected in series.

Solution

Total e.m.f $= V_1 + V_2 + V_3 + V_4$

$= 4 \times 1.5 = 6\text{ V}$

Example 11.8

In a circuit, 5 joules are used to drive 2 coulombs of charge across a bulb in a simple circuit. What is the potential difference across the bulb?

Solution

$$\text{p.d. (V)} = \frac{\text{work done}}{\text{charge}} = \frac{5\text{J}}{2\text{C}} = 2.5\text{ V}$$

Exercise 11.2

1. Define potential difference and state its SI unit.
2. In a circuit, 5 joules are used to drive 2 coulombs of charge across a bulb in a simple circuit. What is the potential difference across the bulb?
3. What instrument is used to measure potential difference? How is it connected in a circuit?
4. Six dry cells were arranged in series to make a battery. If the e.m.f of each cell is 1.5V, what is the e.m.f of the battery.
5. A battery cell uses 15J to drive a charge of 2.5 C through a resistor in a circuit. What is the e.m.f of the battery? (neglect internal resistance of the battery).
6. A dry cell is rated 1.5 V. How much energy does it use to drive a charge of 2.4 C round a circuit?
7. (a) What instrument is used to measure electrical current?
   (b) How is it connected in a circuit?
   (ic) Identify the instrument in Fig. 11.16 and state the reading on it.

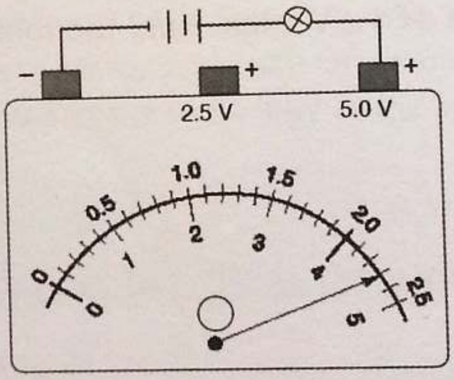


Fig. 11.16

Unit Summary

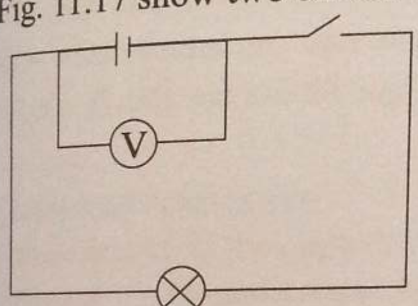
- Electric current is the rate of flow of charge.

$$\text{Current, I} = \frac{\text{charge, Q}}{\text{time, t}}$$

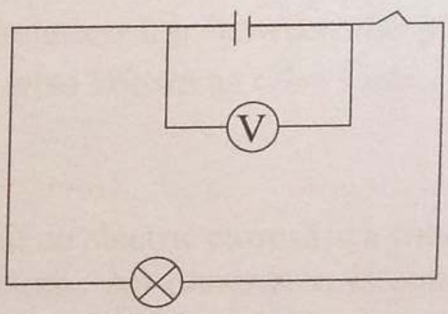
- An ammeter is an instrument used to measure electric current.
- Potential difference (p.d) between the terminals of a cell is the work done in transferring one coulomb of charge from one point to another.
- Electromotive force (e.m.f) of a cell is the voltage across its terminal when it is supplying no current in the circuit (an open circuit).

Unit Test 11

1. Define the following terms:
   (a) Electric current.
   (b) Potential difference.
   (c) Electromotive force.
2. A charge of 200 coulombs flows through a lamp in 10 minutes. Determine the current flowing through the lamp.
3. Find the amount of charge that will pass through a certain point in a circuit, if 5 mA flows through the point for 6 hours.
4. Twelve dry cells of e.m.f 1.5 V each are connected together. Find the e.m.f if the cells are connected in
   (a) Parallel (b) Series
5. Fig. 11.17 show two circuit diagrams.



(a) open circuit

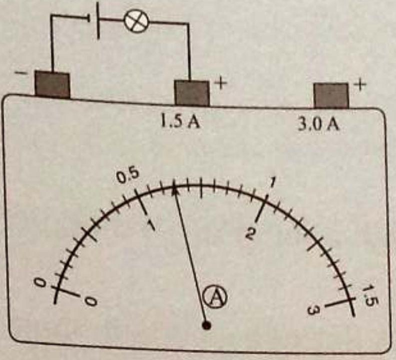


(b) Closed circuit

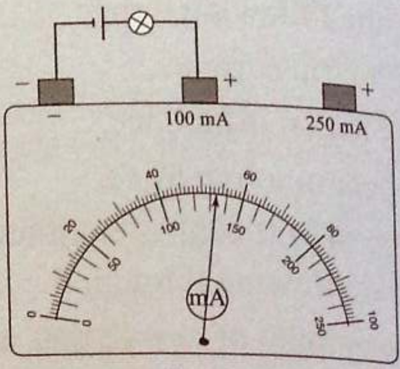
Fig. 11.17

What type of voltage is measured when:
   (a) the switch is open.
   (b) the switch is closed.
6. Describe the convention current in an electric circuit.
7. A current of 0.12A flows in a circuit for 9 minutes. How much charge passes through a given point in the circuit?
8. 1 800 coulombs of charge are passing through a point in an electric circuit in 15 minutes. Determine the amount of electric current in the circuit.
9. What instrument is used to measure electric current? How is it connected in a circuit?

10. Identify the instruments in Fig. 11.18 and state the reading on them in SI units.



(a)



(b)

Fig. 11.18

UNIT 12

Electrical Resistance

Success Criteria

By the end of this unit, you must be able to:

- Describe electrical resistance.

Introduction

In Form 1, we were introduced to electrical resistance. Do you recall how we defined electrical resistance? Tell your class what electrical resistance is. We also investigated the factors that affect electrical resistance of a conductor.

In this unit, we are going to advance our knowledge of electrical resistance. In particular, we will learn how to measure and determine the value of the electrical resistance of a conductor, and investigate the relationship between the potential difference(p.d) and current through a conductor, also known as *Ohm's law*.

12.1 Electrical resistance

Electrical resistance is the opposition to the flow of an electric current in a conductor. It is measured by an instrument know as ohmmeter. An ohmmeter, is connected across the component whose resistance is to be measured.

The symbol of ohmmeter is as shown in Fig.12.1.

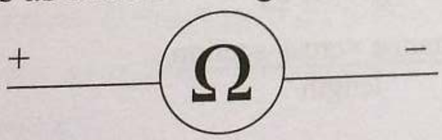


Fig.12.1 A symbol of an ohmmeter

Electrical resistance can also be measured using multimeter. This is a device used to measure voltage, resistance and current.

12.2 Factors affecting electrical resistance

In Form 1, we investigated factors that affect electrical resistance of a conductor. We observed that electrical resistance depends on:

- The length of a conductor.
- The cross sectional area of a conductor.
- The type of material a conductor is made of.
- Temperature.

(a) The length of a conductor

Electrical resistance of a conductor is directly proportional to the length i.e. A longer electrical conductor offers a more electrical resistance to the flow of current. We summarise this as:

$$R \propto l \text{ .... } (1)$$

(b) Cross-section area(thickness) of a conductor

Electrical resistance is inversely proportional to the cross-section area (thickness) of a conductor i.e.

$$R \propto \frac{1}{A} \text{ .... } (1)$$

Thicker conductors offers less electrical resistance than conductors with smaller cross-section area.

Combining equations 1 and 2, we conclude that

$$R \propto \frac{l}{A} \text{ .... } (1)$$

Mathematically

$R \propto \frac{l}{A}$ Where R is a resistance of a material

L is the length of a conductor

A is the cross-section areas of a conductor

is the resistivity.

Resistivity of a conductor is the ability of a material to offer opposition to the flow of current. It's symbol is ρ and given by

$$\text{Resistivity} = \frac{\text{Resistance} \times \text{cross-section}}{\text{length}}$$

$$\rho = \frac{RA}{L}$$

The SI unit of resistivity is the *ohm – metre (Ω m)*.

Example 12.1

A wire of length 2 m and diameter 0.35 mm has a resistance of 10 Ω. What is the resistivity of the material of which is made?

Solution

$L = 2\text{ m}$ diameter $= \frac{0.35\text{ mm}}{1000} = 0.00035$

$A = \pi r^2 = \frac{22}{7} \times (\frac{0.00035}{2})^2 = 9.625 \times 10^{-8}\text{m}^2$

$\rho = \frac{R \times A}{L}$

$\rho = \frac{10 \times 9.625 \times 10^{-8}}{2} = 4.8125 \times 10^{-7}\,(\Omega\text{ m})$

(c) Temperature

Electrical resistance of a good conductor of electricity increases with increase in temperature. This is because the vibration of atoms increase the collision per cross-section area of a conductor as the temperature increases. Hence, the opposition to the flow of electron increases.

(d) Type of material the conductor is made of

Electrical resistance also depend on the nature and type of material the conductor is made of.

Exercise 12.1

1. Explain three factors that affect the electrical resistance of a conductor.
2. An electrical conductor of length 100 cm has resistance 25 Ω. The conductor is cut into five equal pieces. Find the resistance of each piece.
3. What length of resistance wire of diameter 0.11 mm and resistivity $2.1 \times 10\ \Omega$ m, would you cut from a reel in order to make a 20 Ω resister?
4. Two material; A of resistivity $1.2 \times 10^{-6}\ \Omega$ m and B of resistivity $1.1 \times 10^{-7}\ \Omega$ m are used to make a resistor. Which material would advice to make a resister of higher resistance?

12.2 Ohm's Law

Ohm's law states that *the current (I) flowing in a conductor is directly proportional to the potential difference (V) across it, if the temperature and other physical quantities of the conductor remain constant.*

$$V \alpha I \Rightarrow \frac{V}{I} = \text{constant}$$

The following experiment will help us verify Ohm's law.

Experiment 12.1: To verify Ohm's law

Apparatus

- dry cells, cell holder
- a fixed resistor
- ammeter
- 100 cm of nichrome wire
- connecting wires
- voltmeter and a switch.

Procedure

1. Connect the setup as shown in Fig. 12.2

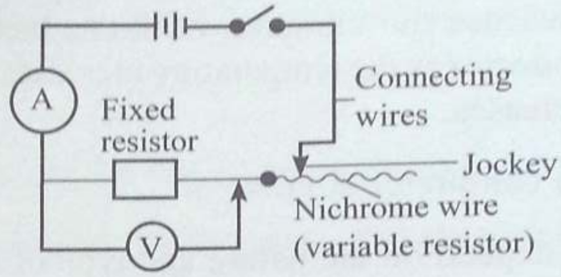


Fig. 12.2: Circuit to verify Ohm's law

2. Close the switch. Move the jockey along the nichrome wire. This varies the p.d and current across the fixed resistor.

   Note: that the nichrome wire is acting as the variable resistor. It can be replaced with a rheostat or any other suitable variable resistor.
3. By moving the jockey along the nichrome wire, set the p.d across the fixed resistor to 0.5 V. Record the corresponding value of current as indicated by the ammeter.
4. Increase the voltage across the fixed resistor in steps of 0.5 V, each time noting and recording the corresponding values of the current through the fixed resistor. Record your results in a table similar to Table 12.1.

Table 12.1

P.d across the resistor (V)	0.5						
Current through the resistor (A)							

5. Draw a graph of p.d against current.

Discussion

The results show that as the potential difference across the wire increases, the current through the wire also increases.

The graph of V against I is a straight line passing through the origin (Fig. 12.3). This shows that the current is directly proportional to the applied potential difference $I \propto V$ or say $V \propto I$.

The gradient is a constant. Thus, Ohm's law is verified.

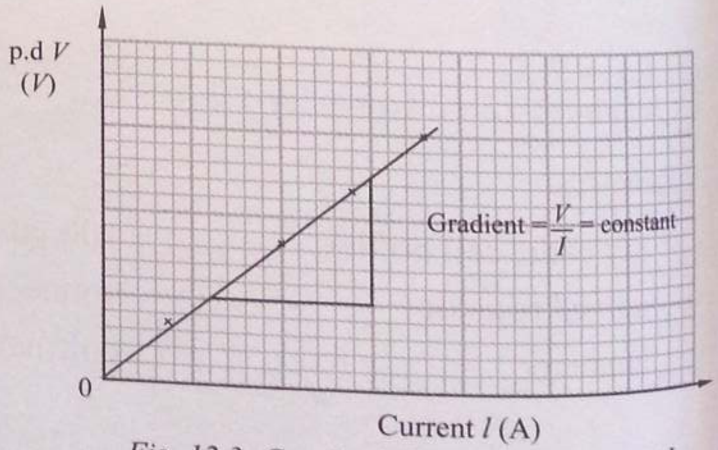


Fig. 12.3: Graph of p.d. (V) against current I

Ohm's law was named after a famous physicsts *George Simon Ohm*. He established that *it is obeyed by* most metals and some non metals like graphite. Conductors that obey ohms laws are called *ohmic conductors* while those which do not are *non-ohmic conductors*. The graph of potential differences, V, against current, I, for all ohmic conductors is a straight-line graph through the origin.

From Ohm's law $V \propto I \Rightarrow V = RI$ rearranged as $V = IR$ where R is the constant of proportionality. This constant R is the *resistance of the conductor*.

Resistance is therefore also defined as the ratio of p.d, V, across the ends of a conductor to the current, I, passing through it. The SI unit of resistance is the ohm (Ω).

From $V = IR$, then $R = \frac{V}{I}$ and $I = \frac{V}{R}$

Example 12.2

A p.d of 12 V is needed to drive a current of 2 A flow through a wire. Find the resistance of the wire.

Solution

$$R = \frac{V}{I} = \frac{12}{2} = 6\ \Omega$$

Resistance = 6 Ω (reads as 6 ohms)

Example 12.3

A current flows through a coil of wire of resistance 80 Ω when it is connected to the terminals of a battery. If the potential difference is 60 V. Find:

(a) the value of the current.

(b) the number of electrons that pass through the coil per second. Charge of an electron = 1.6×10^{-19} C.

Solution

(a) $R = 80\ \Omega \quad , V = 60\ V$

From Ohm's law,

$$I = \frac{V}{R} = \frac{60}{80} = 0.75\ A$$

(b) Let e = charge of each electrons

n = number of electrons.

t = time taken.

Total charge $Q = ne \Rightarrow I = \frac{ne}{t}$

$$n = \frac{I \times t}{e} = \frac{0.75 \times 1}{1.6 \times 10^{-19}}$$

$$= 4.69 \times 10^{18} \text{ electrons}$$

Exercise 12.2

1. State ohm's law.
2. A p.d of 12 V is required to drive a current of 1.5 A to flow through a filament. Find the resistance of the filament.
3. A resistor of value 20 Ω allows a current of 0.3 A to pass through. Calculate the voltage across the resistor.
4. Fig. 12.4 is an ohmmeter connected in a circuit.

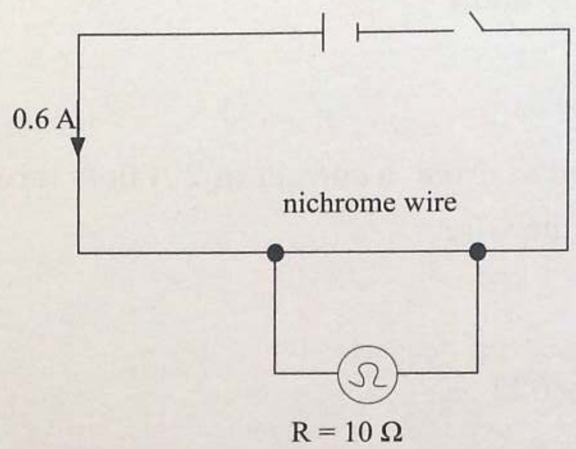


Fig. 12.4 A circuit diagram.

If the switch is closed, find the voltage across the nichrome wire.

12.3 Types of resistors

Materials that offer resistance to an electric current are called *resistors*. There are mainly two types of resistors, i.e. *fixed* and *variable* resistors.

Fixed resistors

The term *fixed resistor* refers to resistors whose resistance is almost constant. Fixed resistors are made from a variety of materials. Some are made from carbon and a carrier material. The two are mixed and baked. The baked mixture is then covered with a ceramic tube. These types of resistors are called *carbon resistors* (Fig. 12.5). These resistors are not bulky and are available in wide range of values. They are widely used in electric circuits e.g. in radio and T.V circuits.

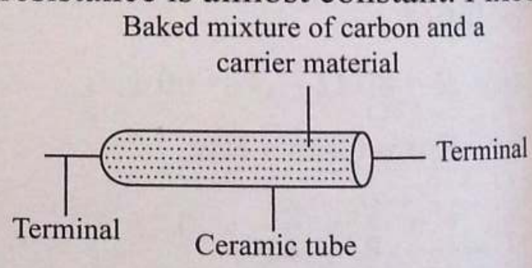


Fig. 12.5: A carbon resistor.

Colour code for carbon resistors

The value of the resistance for carbon resistors is indicated by four coloured bands painted round them (Fig. 12.6).

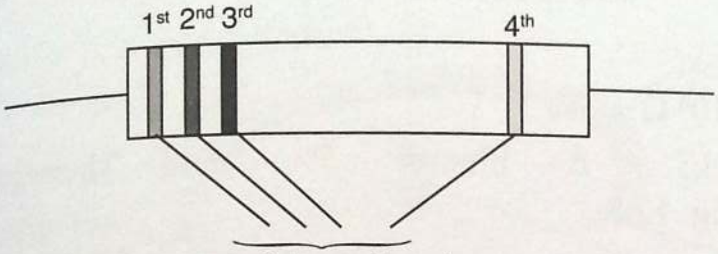


Fig. 12.6: Carbon resistor with coloured bands

Each colour indicates a given value e.g. brown indicates 1. Using this colour code, the value and the tolerance (accuracy) of the resistor can be worked out.

Table 12.2 gives the colour code for resistance while Table 12.3 gives the colour code for tolerance, which means the maximum error in the value of resistor.

Table 12.2: Colour codes for resistance values

Colour	Black	Brown	Red	Orange	Yellow	Green	Blue	Violet	Grey	White
Value	0	1	2	3	4	5	6	7	8	9

Table 12.3: Colour codes for tolerance

Colour	Red	Gold	Silver	No colour
±%	2	5	10	20

If the value of resistance is $ab \times 10^n$. The first colour band gives the first digit. The second band gives the second digit and the third band gives the power of ten. Tolerance, the maximum error in the value of the resistor is given by the fourth band. For example the value of the resistor shown in Fig. 12.7 is $56 \times 10^5\ \Omega$ with a tolerance of 10%.

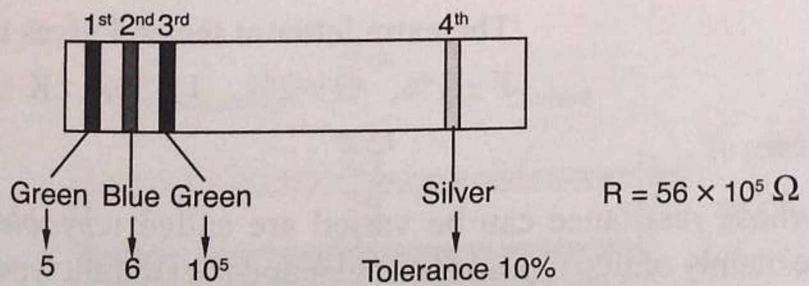


Fig. 12.7: Colour code for carbon resistors.

Example 12.4

A carbon resistor (Fig. 12.8)has a value of 20 MΩ ± 5%. What is the colour code for this resistor?

Solution

$20\ M\Omega \pm 5\% = 20 \times 10^6\ \Omega \pm 5\%$

2 — red; 0 — black; 6 — blue; 5% — gold. Therefore the colour code is as shown in Fig. 12.8.

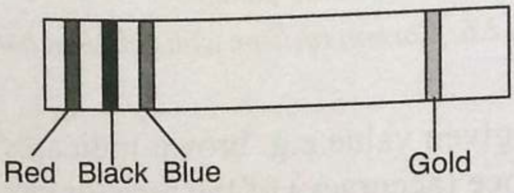


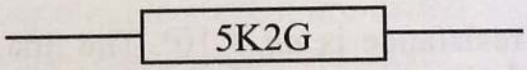
Fig. 12.8

Standard notation

To simplify usage of the coloured resistors, most of them have standard notation.

R27 means 0.27Ω
2R7 means 2.7Ω
3K0 means 3000Ω
5K6 means 5600Ω
47K means 47KΩ
2M2 means 2.2MΩ

In some fixed resistors, the resistance is printed on the resistor (Fig. 12.9).
For example,



So, resistance = 5.2KΩ or 5200

Fig. 12.9: Fixed resistor

The extra letter at the end gives the tolerance:
F ± 1%, G ± 2%, J ± 5%, K ± 10%, M ± 20%,

Variable resistor

Resistors whose resistance can be varied are called *variable resistors*. Variable resistors are mainly of two types i.e. circular and the straight type. The circuit symbol for a variable resistor are shown in Fig. 12.10.

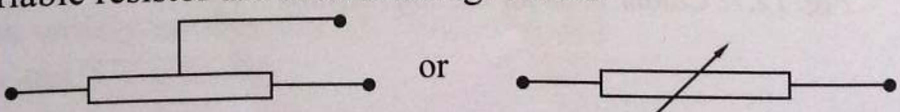


Fig. 12.10: Symbol for a variable resistor.

Circular types of a rheostat

Fig. 12.11 shows a circular variable resistor. The metal slide connects the rotating arm to terminal 2.

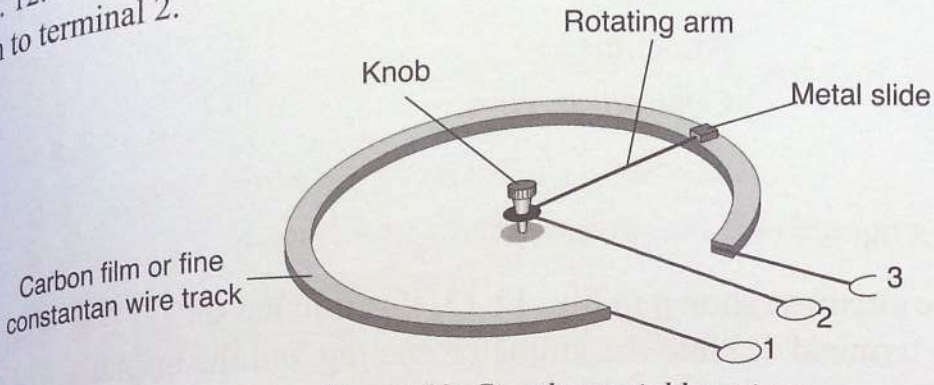


Fig. 12.11: Circular variable resistor.

A knob is connected to the rotating arm. The function of the knob is to turn the rotating arm. The circular track is made of a thin carbon film for small current control or a coil of constantan wire for large current control. When terminals 1 and 3 are connected to the circuit, you get a maximum resistance which cannot be changed. However, when terminals 2 and 3 are used the resistance in a circuit can be varied. Minimum resistance can be obtained when the rotating arm is turned fully in a clockwise direction.

These types of resistors are used in electrical devices such as radios, iron boxes and light bulbs, to control sound, heat and brightness respectively.

Straight variable resistors

These is the most commonly used in laboratories. Fig. 12.12 shows one type of straight variable resistor called a *rheostat*.

Terminals 1 and 3 provide a fixed resistance while terminals 1 and 2 provide a variable resistance. The resistance is maximum when the slide is pushed near terminal 2 and minimum when the slide is near terminal 1.

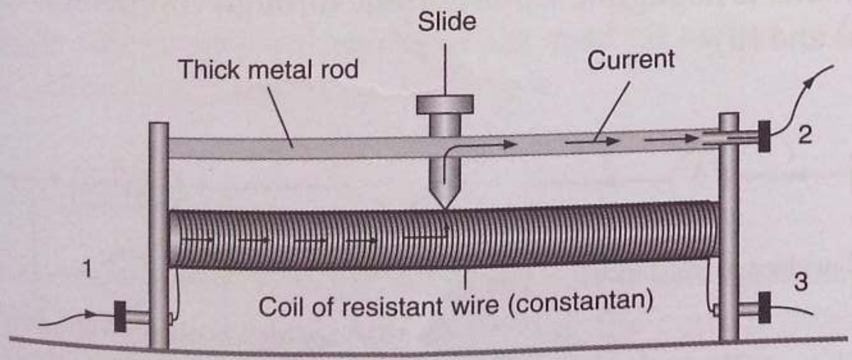


Fig. 12.12: A rheostat

Experiment 12.2: To vary the current in a circuit using a variable resistor

Apparatus

- A rheostat
- A bulb
- A dry cell
- An ammeter
- Connecting wire

Procedure

1. Connect the circuit as shown in Fig. 12.13. Position the slider (2) as close as possible to terminal 1. Note the ammeter reading and the brightness of the bulb.
2. Slowly move the slider of the rheostat towards the other end and note what happens to both the ammeter reading and the brightness of the bulb.
3. Repeat the experiment by connecting terminals 1 and 3 of the rheostat to the circuit.

Observation

When terminal 1 and 3 are connected the ammeter records the lowest value. The resistance of the resistor is maximum.

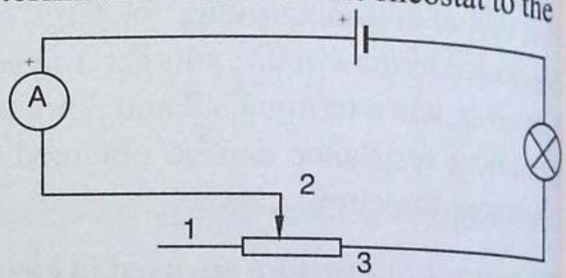
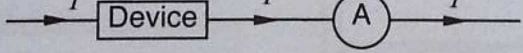


Fig. 12.13: Using a variable resistor

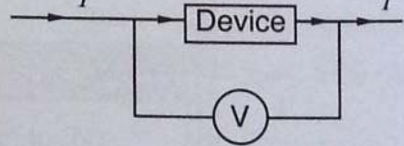
When the slider of the rheostat is moved towards the end, the ammeter reading and brightness of the bulb increases. This shows that the resistance of the circuit has decreased.

Discussion

Ammeters are designed to have negligible resistance so as not to interfere with the current to be measured. Similarly voltmeters are designed to have very high resistance so that a negligible current pass through them. The potential difference across an ammeter is negligible while current through voltmeter is also negligible (Fig. 12.14(a) and (b)).



(a) Ammeter of negligible resistance



(b) Very high resistance voltmeter

Fig. 12.14: Effect of ammeter and voltmeter in electric circuits

Exercise 12.3

1. Explain why carbon resistors are commonly used in radio and T.V circuits, despite the fact that they are not very accurate.
2. Describe how you would easily remember the order of the colours in the colour code.
3. Define electrical resistance and give its SI unit.
4. A p.d. of 12 V is needed to drive a current of 4 A through a conductor. Find the resistance of the conductor.
5. Work out the values of resistors shown in Fig. 12.15.

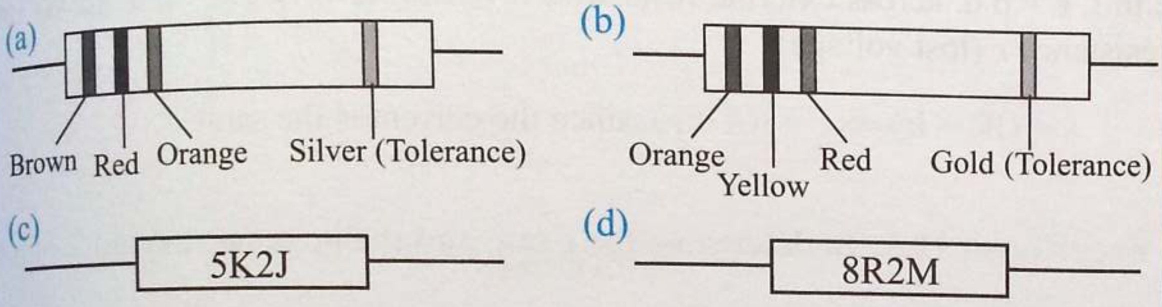


Fig. 12.15

7. Write the following using a colour code:
   (a) 63 000 Ω (b) 40 Ω (c) 1 000 Ω
8. Explain the term *'tolerance'* in relation to a carbon resistor.
9. The Table 12.4 shows the current (I) through a carbon resistor and the corresponding potential difference (V) applied across its ends.

Table 12.4

P.d V (V)	50	100	200	300	400
Current I (mA)	0.6	1.15	2.20	3.15	4.04

   (a) Draw a suitable graph to determine the resistance of the resistor. For an applied p.d. of 350 V. Write the colour code for this resistor.

9. Suggest one reason why colour codes are used on carbon resistors and not the actual value (numbers) of the resistance.

12.4 Internal resistance, r

There seems to be some lost 'voltage' when the cell is driving a current round the circuit. This is the voltage the cell uses in driving the current through itself. The resistance the cell offers to the flow of current inside the cell is called the *internal resistance* of the cell. It results from the properties of the substances used in the construction of the cell.

The total e.m.f available in the cell (Fig. 12.16) is used up in two ways: 1. Driving the current through the cell i.e. to overcome internal resistance (r).

2. Driving the current through the external resistance R.

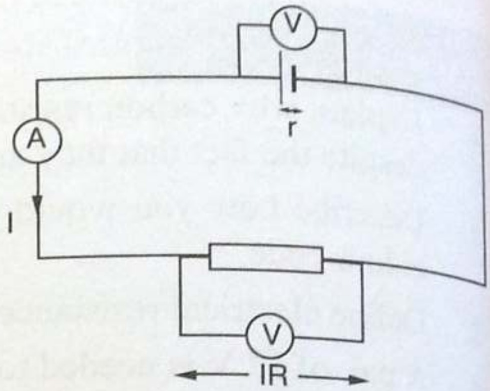


Fig. 12.16: Total e.m.f

Hence, the total electromotive force (e.m.f = ε) is given by

e.m.f, ε = p.d. across external resistance R (terminal voltage) + p.d. across internal resistance r (lost voltage)

$\varepsilon = (IR + Ir) \Rightarrow \varepsilon = I(R + r)$, since the current is the same.

Experiment 12.3: To determine the e.m.f. and the internal resistance of a cell

Apparatus

- A voltmeter
- A fixed resistor
- A cell
- A rheostat
- An ammeter

Procedure

1. Using a high-resistance voltmeter determine the value of the e.m.f. (e) of the cell as shown in Fig. 12.17(a). Connect the same cell to a known resistance R as in Fig. 12.17 (b).

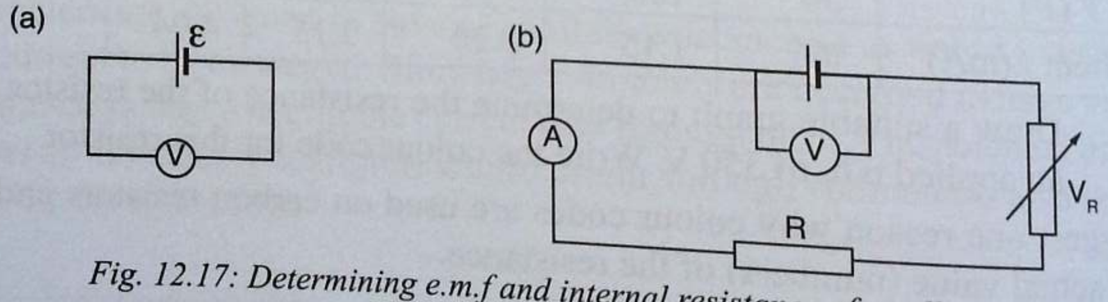


Fig. 12.17: Determining e.m.f and internal resistance of a cell.

2. Set the variable resistor at its maximum value. Record the ammeter and voltmeter readings (Table 12.5).

Table 12.5

Voltmeter (V)								
Ammeter (A)								

3. Adjust the variable resistor by moving the sliding contact to change the resistance in the circuit and each time recording the ammeter reading I and voltmeter reading V. What happens to the ammeter and voltmeter readings as the resistance is reduced?

Observation

The ammeter reading increases and voltmeter reading decreases. A graph of voltage (V) against current (I) is as shown in Fig. 12.18.

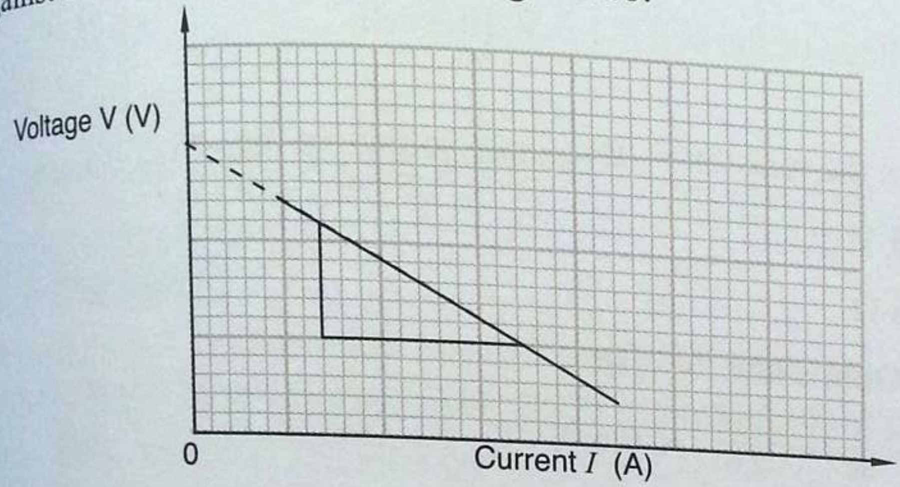


Fig. 12.18: Graph of voltage V against current I

Discussion

The line is produced so as to meet the voltage axis as shown in Fig. 12.18. This is the p.d. across the cell when the current is zero. The potential difference across the terminals of a cell when it is not driving a current is the e.m.f. of the cell. The value of the intercept therefore gives the e.m.f. (V_0) of the cell.

$$V_0 = IR + Ir$$

$$V_0 = V_R + Ir$$

$$V_R = -rI + V_0$$

When this equation is compared with the general equation of a straight line i.e. $y = mx + c$, then the gradient of the line is equal to $-r$. The minus sign represents a negative slope. The slope of the graph gives the value of the internal resistance, r.

Example 12.5

Fig. 12.19 shows an 'ideal' voltmeter connected across the terminals of a cell. The voltmeter reads 1.5 V when the switch is open and 1.3 V when the switch is closed.

(a) What is the e.m.f of the cell?

(b) What is the terminal voltage of the cell?

(c) Calculate

(i) the current in the circuit

(ii) the internal resistance of the cell.

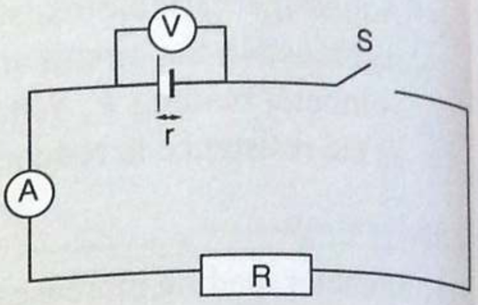


Fig.12.19

Solution

(a) e.m.f of the cell = 1.5 V (no current is drawn from the cell).

(b) Terminal voltage of the cell = 1.3 V (the cell is in use).

(c) (i) p.d across the resistor = terminal voltage of the cell

$$V = 1.3\text{ V}$$

$$R = 2.6\ \Omega$$

From Ohm's law, V = IR

$$I = \frac{V}{R} = \frac{1.3}{2.6} = 0.5\text{ A}$$

Current in the circuit is 0.5 A.

(ii) Lost voltage inside = 1.5 V – 1.3 V
= 0.2 V

$$V = Ir$$

$$\Rightarrow \rho = \frac{V}{I} = \frac{0.2}{0.5} = 0.4\ \Omega$$

The internal resistance of the cell is 0.4 Ω.

Example 12.6

The 'ideal' ammeter in Fig. 12.20 reads 0.20 A when the switch, S is closed.

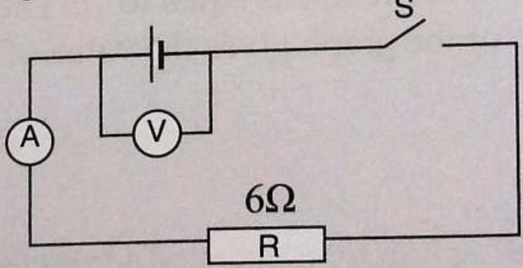


Fig.12.20

Determine the internal resistance of the cell.

Solution

e.m.f of the cell = 1.50 V.

p.d across the resistor $V = IR$

$= 0.2\text{ A} \times 6\ \Omega$

$= 1.20\text{ V.}$

$\therefore$ Terminal voltage $= 1.20$ V. $6\ \Omega$

Lost voltage $V = 0.30$ V.

$V = Ir$

$$\Rightarrow \rho = \frac{V}{I} = \frac{0.2}{0.5} = 1.5\ \Omega.$$

The internal resistance of the cell is 1.5 Ω.

Exercise 12.4

1. The p.d across the terminals of a cell is a 1.5 V when there is no current in the cell. When there is a current of 0.50 A in the circuit the p.d. falls to 1.3 V.
   (a) What is the e.m.f of the cell?
   (b) What is the terminal voltage at the cell?
   (c) Calculate the internal resistance of the cell.
2. Explain the term internal resistance of a cell. How does it arise?
3. The e.m.f. of a cell is given by the expression, $\varepsilon = I(R + r)$. Explain the meaning of each term in the expression.
4. Describe an experiment to determine
   (a) the internal resistance of a cell.
   (b) Table12.6 shows readings obtained in an experiment to determine the e.m.f, ε and the internal resistance, r of an accumulator.

Table 12.6

External resistance R (Ω)	0.35	1.3	2.75
Current $I(A)$	2.5	1.0	0.5

   (a) Draw a suitable circuit diagram that can be used to get the above results.
   (b) Plot a graph of $\frac{1}{I}$ against R.
   (c) Determine the values of r and ε.
5. A battery consisting of three cells in series, each of 1.5 V and internal resistance of 0.3 Ω is used to pass a current though a 1.8 Ω resistor. Calculate the current through the battery.

Unit Summary

- Electrical resistance is the opposition to the flow of electrical current in a conductor.
- Voltmeters are connected in parallel while ammeters are connected in series in electric circuits.
- Ohm's law states that the *current passing through a conductor is directly proportional to the potential difference applied across its ends provided the temperature* and other *physical properties of the conductor remain constant*: $V = IR$
- Non-ohmic conductors do not obey Ohm's law.
- Electromotive force (e.m.f) is the p.d at the terminals of a cell per coulomb of charge.
- Instruments used to measure electrical resistance are:
  - Ohmmeter
  - Multimeter
- Multimeter is a device used to measure voltage, resistance and current.
- Factors affecting electrical resistance include length, cross-section area, temeperature and nature of conductor.
- A resistor is a material that offers resistance to flow of electric current.
- There are two resistors: Fixed and variable resistors. In fixed resistor, the resistance is almost a constant while in variable resistor the resistance changes.
- Internal resistance is the resistance a cell offers to the flow of charges inside it.
- e.m.f, $E = I(R + r)$, where R = total external resistance, r = Internal resistance

Unit Test 12

1. Calculate the resistance of a coil of wire through which a current of 3A flows due to a potential difference of 12V.
2. In the circuit shown in Fig. 12.21, the 12 V lamp glows when switch S is closed.

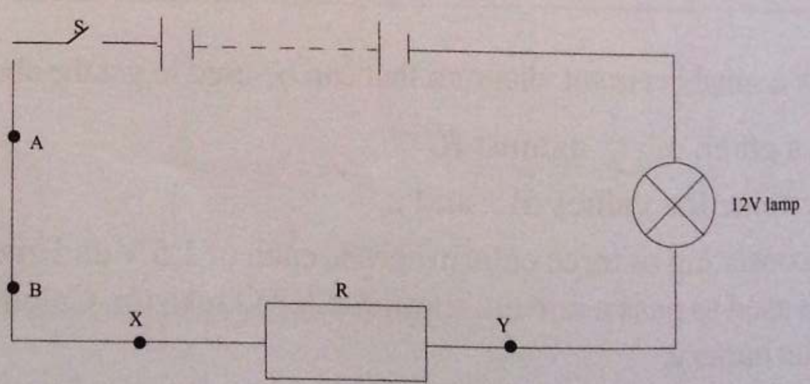


Fig. 12.21

What happens to the brightness of the lamp when another identical resistor R is connected between

(a) Points A and B

(b) Point X and Y. Give a reason

3. Figure 12.22 shows a battery with e.m.f. of 12V supplying power to two identical lamps.

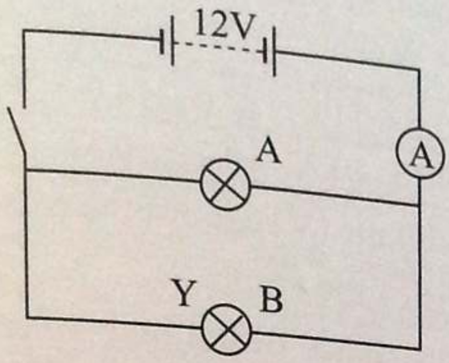


Fig. 12.22

The current in lamp B is 5A. Calculate

(a) the total current supplied by the battery when both lamps are on.

(b) the current in lamp A.

(c) the resistance of lamp A.

4. Figure 12.23 shows a battery of e.m.f. 6V supplying current to two fixed resistors.

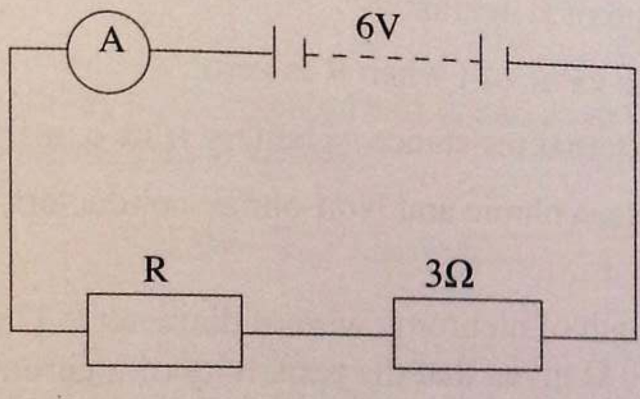


Fig. 12.23

When the current is switched on, the ammeter read 0.5A. Calculate the value of the unknown resistor R.

5. Write the following using colour code

(a) 2500 Ω (b) 80 Ω (c) 2000 Ω

6. Work out the values of the resistors shown in Fig. 12.24

(a) (b)

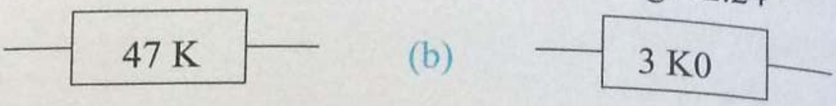


Fig. 12.24

7. List two factors that affect the resistance of the conductor.

8. The set of readings shown in Table 12.7 has been obtained in an experiment.

Table 12.7

P.d (V)	0.2	0.5	1.0	2.0	4.0	6.0	8.0	10.0	12.0
Current (A)	0.12	0.15	0.22	0.23	0.32	0.36	0.40	0.44	0.51

(a) Plot a graph of voltage (V) against the current I(A)

(b) Explain the shape of the graph.

(c) Determine the resistance of the bulb when the current is 0.25A and 0.60 A. Comment on the answer.

9. A battery is connected in series with an ammeter and a variable resistor R. The resistance R is varied and corresponding reading of the ammeter recorded in table 12.8.

Table 12.8

Resistance R (Ω)	1	2	3	4	5	6	10
Current I (A)	2.00	1.50	1.20	0.23	1.00	0.75	0.50

(a) Draw the circuit diagram used.

(b) Plot a graph of R against $\frac{1}{I}$.

(c) What is the value of I when R is zero.

(d) Find the internal resistance of battery if its e.m.f = 6.0 V

10. Distinguish between ohmic and Non-ohmic conductors. Give two examples of each.

11. Calculate the length of nichrome wire of diameter 0.32 mm needed to make a resistance of 40 Ω given that the resistivity of nichrome is 1.1×10^{-6} Ω m.

UNIT 13

Electric Circuit, Energy and Power

Success criteria

By the end of this unit, you must be able to:

- Analyse electric circuits.
- Determine electric power and energy.

Introduction

In Form 1, we learnt that electricity is a form of energy. When being used, it is converted to other forms of energy such as light, heat and sound. This takes place in devices such as bulbs, iron box and loud speaker respectively.

In this unit, we shall discuss electric circuits, rate of energy consumption by a device, total energy consumed in a given time and the costing.

13.1 Electric circuits

Arrangement of resistors in a circuit

In electrical circuits, several resistors may be combined. There are two ways of connecting these resistors namely *series* and *parallel* connections.

The *equivalent resistance* otherwise known as the *effective resistance* of two or more resistors is their combined resistance.

Resistors in series

Resistors are in series if they are connected end to end as shown in Fig. 13.1. Same (equal) current passes through each resistor.

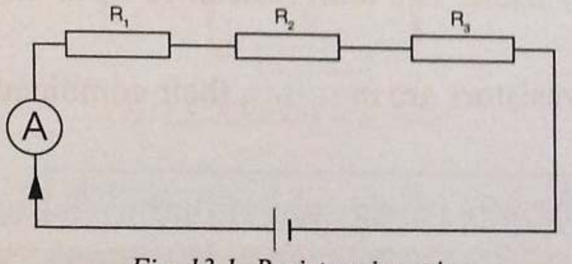


Fig. 13.1: Resistors in series

Experiment 13.1: To determine the equivalent resistance of resistors in series

Apparatus

- 2 resistors
- 3 voltmeters
- Ammeter
- 1 dry cell, switch
- connecting wires

Procedure

1. Connect the two resistors in series as shown in Fig 13.2.
2. Close the switch and note the ammeter and voltmeter readings.
3. Using Ohm's law, calculate R_1 and R_2.

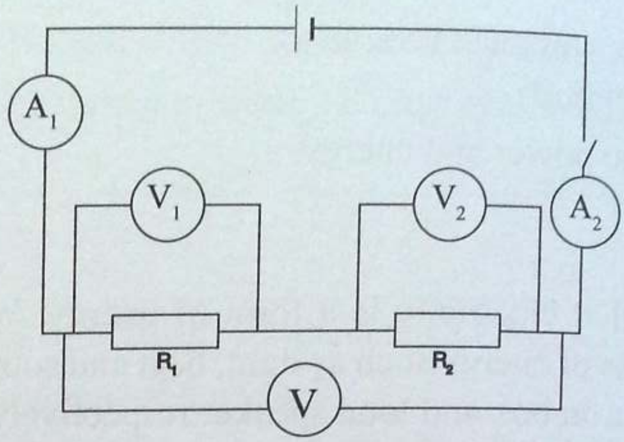


Fig. 13.2: Determine equivalent resistance of resistors in series

Observation

We notice that current through Ammeter, A_1, is equal to current through A_2.
i.e $I_1 = I_2$.

Also $\boxed{V = V_1 + V_2}$

Discussion

Current flowing through R_1 and R_2 is I

$V = V_1 + V_2$, But $V = IR$

$IR_s = IR_1 + IR_2$, where R_s is the effective resistance

$R_s = R_1 + R_2$

If two or more resistors are connected in series, they give a higher resistance than any of the resistors by itself. The total resistance R_S is the sum of the individual resistances.

Thus, if two or more resistors are in *series*, their combined resistance R_s is given by this equation.

$$\boxed{R_S = R_1 + R_2 .... + R_n}$$

Example 13.1

Work out the combined resistance for the resistors shown in Fig 13.1 (a) and (b).

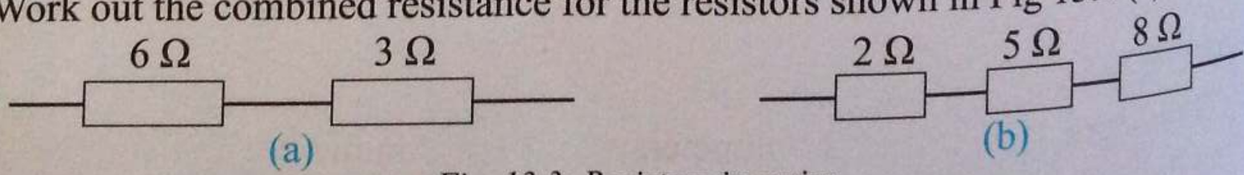


Fig. 13.3: Resistors in series

Solution

(a) $R_S = R_1 + R_2 = 6 + 3 = 9\ \Omega$ (read as 9 Ohms)

(b) $R_S = R_1 + R_2 + R_3 = 2 + 5 + 8 = 15\ \Omega$ (read as 15 Ohms)

Example 13.2

Find the value of resistor x if the combined resistance of the resistors in Fig. 13.4 is 21 Ω.

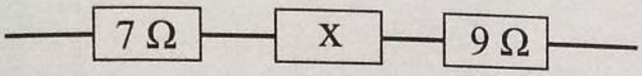


Fig. 13.4: Resistors in series

Solution

$$R_1 + R_2 + R_3 = R_s$$
$$7 + x + 9 = 21$$
$$x = 21 - 16$$
$$= 5\ \Omega$$

Resistors in parallel

In parallel connections, resistors are connected side by side and their corresponding ends joined together. The current divides itself into different paths depending on the resistance of the resistors.

Fig. 13.5 shows three resistors R_1, R_2 and R_3 connected in parallel.

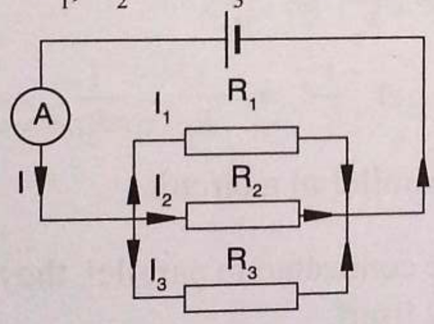


Fig13.5: Resistors in parallel

Experiment 13.2: **To determine the equivalent resistance when the resistors are connected in parallel**

Apparatus

- 3 resistors
- 4 voltmeters
- 1 dry cell,
- ammeter
- switch,
- connecting wires

Procedure

1. Arrange the three resistors in parallel as shown in Fig. 13.6.
2. Close the switch and note the ammeter and voltmeter readings.
3. Use Ohm's law to determine the values of resistances R_1, R_2 and R_3.

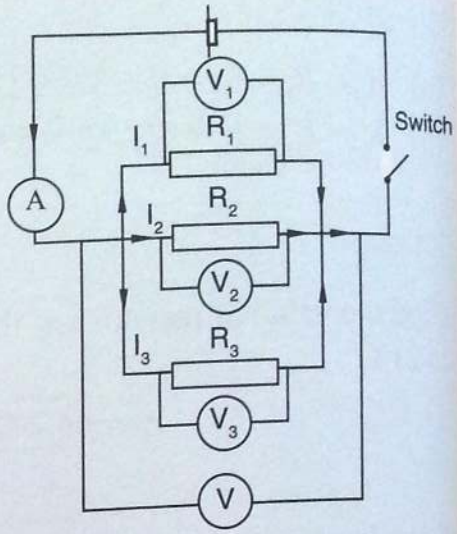


Fig. 13.6: Determining equivalent resistance of resistor in parallel

Observation

We observe that $\boxed{V_1 = V_2 = V_3 = V}$ and on calculation $\boxed{I = I_1 + I_2 + I_3.}$

Discussion

Voltage accross R_1, R_2 and R_3 is V. But, $I = I_1 + I_2 + I_3$

From Ohm's law $I = \frac{V}{R}$.

Therefore, $\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

Dividing throug by V. We get $\frac{1}{R_p} = \frac{1}{R_1} \quad \frac{1}{R_2} \quad \frac{1}{R_3}$ where R_p is the effective resistance of resistors in parallel in a circuit.

If two or more resistors are connected in parallel, they give a *lower* resistance than any one of the resistors by itself.

If two or more resistors are in parallel (Fig 13.6), their combined resistance R_p is given by this equation.

$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots\dots \frac{1}{R_n}}$$

Where, n = 1, 2, 3, ...

Example 13.3

Calculate the combined resistance in each case of the resistors in Figure 13.7.

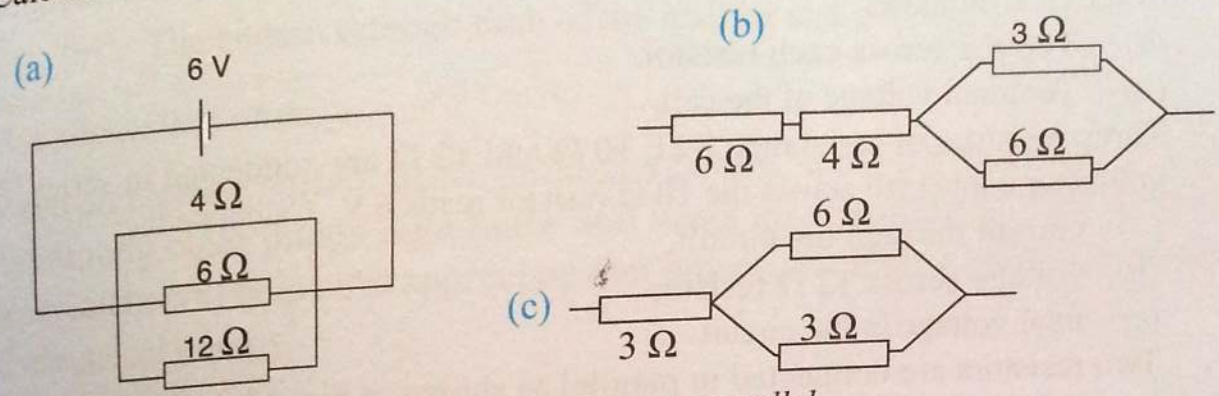


Fig.13.7: Resistors in parallel

Solution

(a) For resistors in parallel

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{12} + \frac{1}{6} + \frac{1}{4} + \frac{6}{12}$$

$$R_p = \frac{12}{6} = 2\,\Omega$$

(b) Starting with either the series or parallel

$R_s = 6 + 4 = 10\,\Omega$ (series)

$$\frac{1}{R_p} = \frac{1}{6} + \frac{1}{3} = \frac{6+3}{18} = \frac{9}{18}$$

$$R_p = \frac{18}{9}$$

$$R_p = 2$$

Combining R_s and R_p we get

$R_T = 10 + 2 = 12\,\Omega$

(c) Starting with parallel circuit, we get

$$\frac{1}{R_p} = \frac{1}{6} + \frac{1}{3} = \frac{6+3}{18} = \frac{9}{18}$$

$$R_p = \frac{18}{9}$$

(R_p is the equivalent resistance for the parallel connection)

and then combining with 3Ω $\quad R_s = 3\Omega \quad$ (R_s is the equivalent

$R_T = (3 + 2)\,\Omega = 5\Omega \quad$ resistance in series)

Exercise 13.1

1. A cell supplies a current of 0.24 A through two resistors 6 Ω and 3 Ω arranged in series. Calculate:
   (a) The p.d across each resistor.
   (b) Terminal voltage of the cell.
2. Three resistors of resistance 8 Ω, 10 Ω and 12 Ω are connected in series. A voltmeter connected across the 10 Ω resistor reads 6 V. Calculate the:
   (a) current through the circuit,
   (b) voltage across 12 Ω resistor
   (c) total voltage in the circuit.
3. Two resistors are connected in parallel as shown in Fig. 13.8. Calculate the:
   (a) Current that passes through R_1.
   (b) Terminal potential difference across the battery.

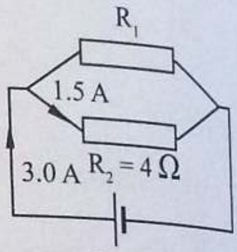


Fig. 13.8

4. Fig 13.9 are circuit diagrams

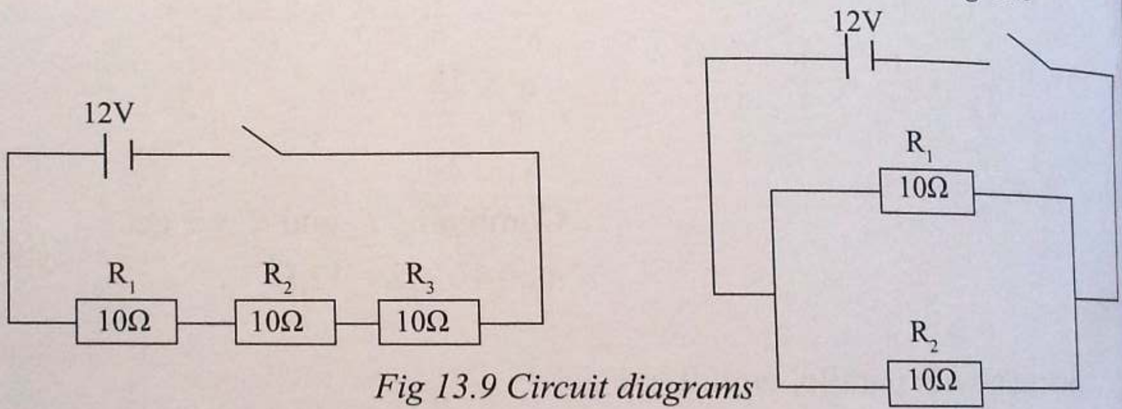


Fig 13.9 Circuit diagrams

Calculate:
   (a) the effective resistance in each resistors,
   (b) current across each resistors in each circuits (a) and (b).
5. A p.d. of 12V is applied across two resistors of 10Ω and 20Ω connected in series. Find:
   (a) the equivalent resistance for the circuit.
   (b) the total current in the circuit.
   (c) the current through each resistor,
   (d) the voltage drop across each resistor.
6. Show that the equivalent resistance R of R_1, R_2 and R_3 when connected in parallel is given by

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

7. Two resistors 4Ω and 2Ω in parallel are conected in series to a 3 Ω resistor and a cell of e.m.f 1.5V. Calculate:
   (a) The equivalent resistance for the circuit
   (b) The current through each of the resistor and p.d. across each.

13.2 Electrical energy

When an electric current flows through a circuit, it supplies electrical energy that among other things light bulbs and heats up a conductor. The following experiment will help us to appreciate that indeed an electric current is a source of electrical energy.

Experiment 13.3: To demonstrate that electric current is a source of electrical energy

Apparatus

- 2 immersion heaters of different sizes
- Thermometer
- Cold water
- Switch and connecting wires
- Stopwatch
- A small bucket
- Battery
- Variable resistor

Procedure

1. Dip an immersion heater into water in a bucket and switch on the power supply (Fig. 13.11).

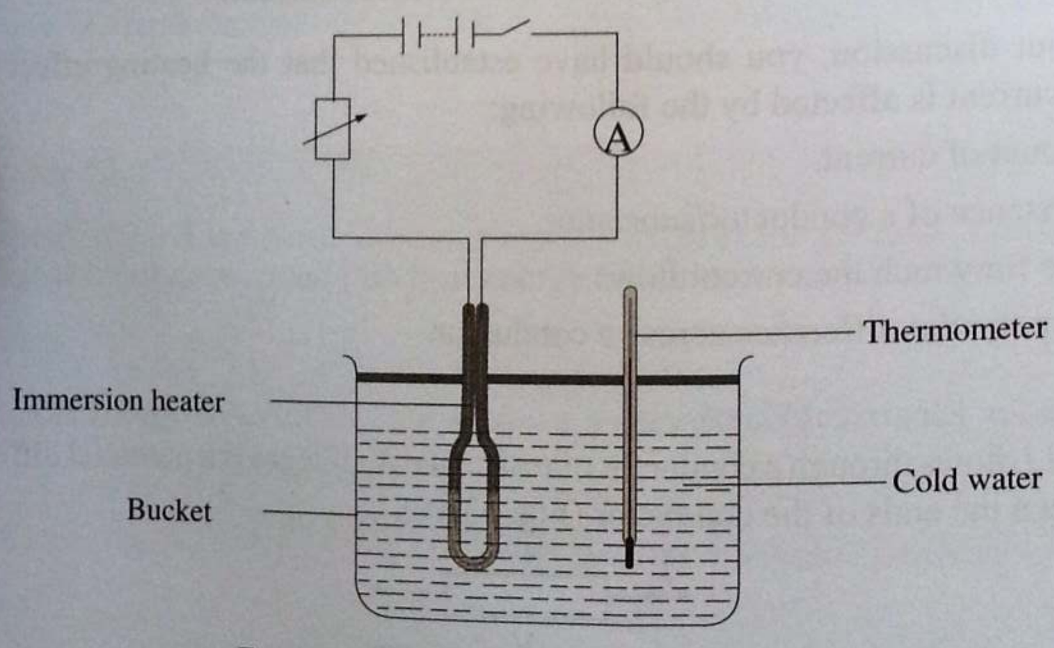


Fig. 13.11: A setup to demonstrate heating effect

2. After a couple of minutes, switch off the power supply and slightly touch the water in the bucket. What do you feel? Discuss.

   We can therefore conclude that an electric current has a heating effect on a substance. But what are the factors that affect the heating effect of an electric current?

3. Measure the temperature of the water using thermometer and record it down.

Factors affecting heating effect of an electric current

Experiment 13.4: To find out the factors affecting the heating effect of an electric current

Procedure

1. Repeat Experiment 13.3 by:
   (a) Increasing the amount of current (i.e reducing resistance using variables resistor) through the same coil for the same amount of time. Note down the temperature of the water. Compare with the reading obtained in Experiment 13.3. What do you notice? Discuss.
   (b) Passing the same amount of current through different coils of different resistance (lengths). Record down your observation.
   (c) Passing the same amount of current through the same coil for different time intervals such as 30 seconds, 60 seconds or 5 minutes. What do you notice?
2. Discuss in your group the observations you made in 1(a), (b) and (c) and suggest the factors that affect heating effect of an electric current.

From your discussion, you should have established that the heating effect of an electric current is affected by the following:

1. Amount of current.
2. Resistance of a conductor/substance.
3. Time for which the current flows.
4. The potential difference across a conductor.

Formula for Electrical Energy

A current I, flows through a conductor of resistance R. If there is a potential difference V, between the ends of the conductor (Fig. 13.10).

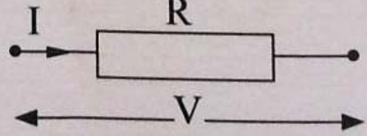


Fig. 13.10: Current, I passes through resistance, R when the potential difference is V

Potential difference is the work done per unit charge to drive it through the current, i.e

Potential difference, $V = \frac{\text{work done } W}{\text{charge } Q}$

$$V = \frac{W}{Q} \text{ or } W = VQ \text{ .... (i)}$$ where W is the electrical work that is converted into heat energy E.

But current is the rate of flow of charge.

$$\text{Current, } I, = \frac{\text{charge } (Q)}{\text{time } t} \text{ in short } I = \frac{Q}{t}$$

From equations (i) and (ii), we obtain $\boxed{E = VQ = V(It)}$

The SI unit for electrical energy is the joule (J)

Alternative equations for finding electrical energy, E are obtained by replacing V or I in the equation, $E = VIt$ as shown below:

From ohm's law $V = IR$

Substituting for V in equation $E = VIt$, we get,

$$E = (IR)It$$

∴ Electrical energy, $E = I^2Rt$

Also from Ohm's law $I = V/R$

Substituting for I in equation $E = VIt$, we get

$$E = V(V/R)t$$

$$\boxed{\therefore \text{ Electrical energy, } \quad E = \frac{V^2t}{R}}$$

Example 13.4

A current of 2.0 A is passed through a resistor of 20 Ω for 1.0 hour. Calculate the electrical energy converted into heat energy in the resistor.

Solution

Electrical energy $E = I^2Rt = (2.0)^2 \times 20 \times 1 \times 60 \times 60$

$= 288\,000$ J

$= 2.88 \times 10^5$ J

Example 13.5

An electric iron consumes 2.592 MJ of energy in 1 hour when connected to the mains power supply of 240 V. Calculate the current through the filament in the electric iron.

Solution

Energy consumed $VIt = 2.592$ MJ

$= 2.592 \times 10^6$ J

$= 2\ 592\ 000$ J

$\therefore\ 240 \times I \times (1 \times 60 \times 60) = 2\ 592\ 000$

$\therefore$ current, $I = 3$ A

The current through the filament is 3 A.

13.3 Electrical power

Power is the rate at which work is done i.e.

$$\text{Power} = \frac{\text{Work done}}{\text{time}}$$

For an electrical device,

$$\text{Power } P = \frac{\text{electrical energy transformed}}{\text{time}} = \frac{VIt}{t} = VI$$

The SI unit of power is Watts (W)

Alternative, equations for finding power P are obtained as follows:

$$\text{Power} = \frac{\text{Electrical energy}}{\text{time}} \quad \text{i.e } P = \frac{E}{t}$$

$$\text{but E} = I^2Rt \Rightarrow P = \frac{I^2Rt}{t} \quad \therefore \boxed{P = I^2R}$$

$$\text{Also } E = \frac{V^2t}{R} \Rightarrow P = \frac{V^2t}{R \times t} \quad \therefore \boxed{P = \frac{V^2}{R}}$$

Example 13.6

A torch bulb is labelled 2.5 V, 0.3 A. Calculate the power of the bulb.

Solution

Electrical power, $P = VI = 2.5 \times 0.3 = 0.75$ W

The power of the bulb is 0.75W.

Example 13.7

An electric bulb is labelled '40 W, 240 V'. Calculate:

(a) the resistance of the filament used in the bulb,

(b) the current through the filament when the bulb works normally.

Solution

(a) Power rating of the bulb is '40 W, 240 V'

$$P = VI$$

$$= V\left(\frac{V}{R}\right) = \frac{V^2}{R}$$

$$\therefore R = \frac{V^2}{P} = \frac{240^2}{40} = 1\,440\ \Omega$$

The resistance of the filament = 1 440 Ω

(b) Current, $I = \frac{V}{R} = \frac{240}{1440}$

$$= 0.167\text{A}$$

Example 13.8

In 5 seconds, an electric iron takes 10 000 joules of energy from the main supply. What is its power:

(a) in watts? **(b)** in kilowatts?

Solution

(a) Power $= \frac{\text{energy}}{\text{time}} = \frac{10\,000}{5} = 2\,000\text{W}$

(b) 1 000W = 1kW (Kilowatt)

2 000 = 2 kW

Example 13.9

What is the power dissipated in a 6 Ω resistor when the current through it is?

(a) 2A **(b)** 4A

Solution

(a) Power = $(\text{Current})^2 \times \text{Resistance} = 2^2 \times 6 = 24$ W

(b) $P = I^2R$ $= 4^2 \times 6 = 96$ W

Example 13.10

A 3 kW immersion heater is used to heat water. Calculate the electrical energy converted into heat energy in 40 minutes.

Solution

Electrical energy E $= $ Electrical power, $P \times$ time, t

$= (3 \times 1\ 000) \times (40 \times 60)$ J

$= 7\ 200\ 000$ J

$= 7.2 \times 10^6$ J

$= 7.2$ MJ

Exercise 13.2

1. Define electrical power as used in electrical circuit.
2. Calculate the (a) current through (b) resistance of the filament of
   (a) a bulb rated at 240 V, 60 W.
   (b) an electric kettle rated at 240 V, 2kW
3. (a) A washing machine is marked 240 V, 3 kW. What does this mean?
   (b) Calculate the electrical energy used up by this machine in 1 hour.
4. (a) Which bulb in Fig. 13.11 is the brightest? Explain your answer.
   (b) Both the bulbs in Fig. 13.11 are dimmer compared to the normal brightness, when each bulb is connected in turn to the same power supply. Why?
5. In the circuit shown in Fig. 13.12 each bulb is rated at '6V, 3W'.

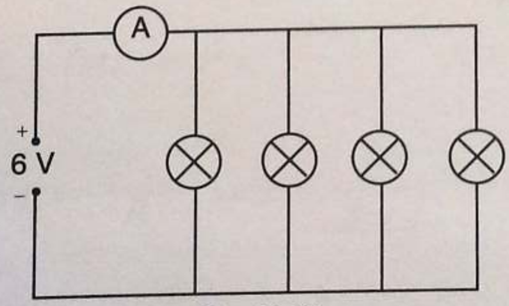


Fig. 13.12

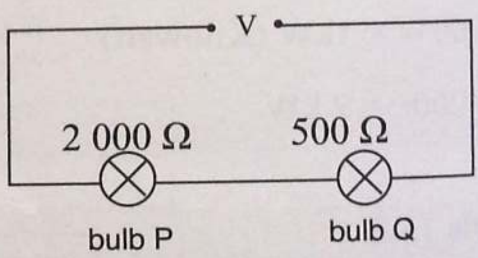


Fig. 13.11

   (a) Calculate the current through each bulb, when the bulbs are working normally.
   (b) How many coulombs of charge pass in 6 seconds through each bulb?
   (c) What would the ammeter read when all the bulbs are working normally?
   (d) Calculate the electrical power delivered by the battery.

6. A 2 kW immersion water heater is switched on for 3 hours. Calculate the amount of head given off by the heater.
7. If an electric heater consumes a current of 4 A when connected to a 240 V supply, what is its power.
8. How much current does a bulb rated 150 W and designed to supply of 240 V when operating normally?
9. Two light bulbs rated ar 250W, 240V and 100W, 240 V are connected to 240 V main supply in turn. By giving a reason, state which bulb will light brighter when connected in:
   (a) Parallel
   (b) Series

13.4 Power rating

Appliances such as toasters, irons, bulbs, hairdryers and TV's have a *power rating* marked on them, either in Watts or kilowatts.

Table 13.1 gives common house hold appliances and their approximate power rating.

Table 13.1

Appliance	Power rating
Filament lamps	25W – 150W
Refrigerator	150W
Television set	200W
Electric iron	750W

Appliance	Power rating
Room heater and grill	1 – 3 kW
Electric kettle	2 – 3 kW
Immersion heater	3 kW
Electric cooker	3 – 5 kW

If the power rating of a bulb is 100W, it converts 100 joules of electric energy every second into heat and light. Similary, a 40W bulb converts 40J of electrical energy every second into heat and light. In general, the larger the power rating the more energy it converts per second.

It is worth noting that energy saving bulbs (compact florescent lamps) have lower power rating yet they emit a lot of light. They are more efficient than ordinary bulbs.

A closer look at Table 13.1 reveals that for the same period,

- An electric iron consumes more power than a T.V.
- An electric heater consumes more than the iron.

Experiment 13.5: To investigate the power rating of various electrical components

Apparatus

- 0-5A Ammeter
- 0-5V Voltmeter
- Torch bulb
- light emitting diode
- dry cells
- connecting wires
- 5 cm nichrome
- 10 cm nichrome.

Procedure

1. Set up the circuit as shown in Figure 13.13
2. Connect a torch bulb across PQ
3. Record the ammeter and voltmeter reading on Table 13.2
4. Repeat procedure 2 and 3 by replacing the torch bulb with
   (a) 10 cm nichrome wire
   (b) 5 cm nichrome wire
   (c) light emitting diode (L.E.D)
5. Complete Table 13.2

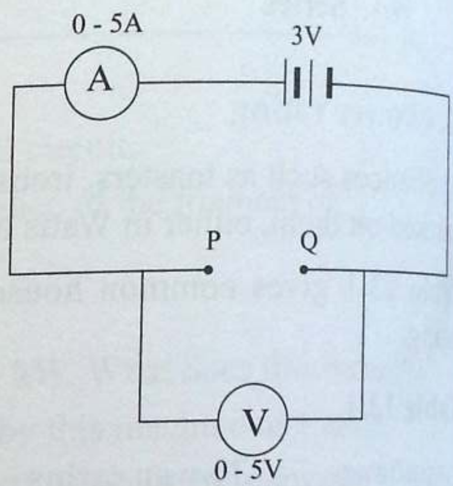


Fig 13.13: Electric circuit

Table 13.2

Component	Ammeter reading (*I*)	Voltmeter reading (*V*)	Power rating (*VI*)
Bulb			
10 cm nichrome wire			
5 cm nichrome wire			
LED			

6. Compare the values of *I*, *V* and *VI* obtained above. What do you notice?

Discussion

It is observed that for the same potential difference, different currents pass through the various electrical components, when each is connected in the circuit.

Conclusion

Different electrical components have different power ratings.

13.5 Unit of electrical energy

Electricity supply companies use the *kilowatt – hour* rather than the joule, as their unit of energy measurement:

1 kWh = One kilowatt of energy consumed by an appliance whose power is operating for 1 hour.

In symbols $1kWh = 1kW \times 1h$

Energy in kilowatts is calculated as follows :

$$Energy\ transformed = Power \times time$$

$$\boxed{Energy\ (kWh) = (kW) \times (hours)}$$

Example 13.11

If a 100W bulb is switched on for 10 hours, the energy supplied can be calculated in kWh as shown.

Solution

$$\begin{aligned} \text{Energy supplied} &= \text{Power} \times \text{time} \\ &= 0.1\ \text{kW} \times 10 \\ &= 1\ \text{kWh} \end{aligned}$$

Example 13.12

A heater rated 3 kW is used for 8 hours, what is the energy supplied in kWh.

$$\begin{aligned} E &= p \times t \\ &= 3\text{kW} \times 8\ \text{h} \\ &= 24\ \text{kWh} \end{aligned}$$

13.6 Calculating the cost of electricity

For purposes of measuring the amount of electrical energy a homestead, institution or company consumes in Malawi, Electricity supply Corporation of Malawi, (ESCOM) electricity meters are fixed on buildings. The reading on the meter gives the total energy supplied in *units* where.

1 unit = 1kWh (Kilowatt-hour)

The more the units consumed in a given period, the more the amount to be paid as in the bill.

Table 13.3 shows a sample of ESCOM bill.

Table 13.3

Electricity Supply Corporation of Malawi Ltd., Box 2047, Blantyre

Best Books Limited
P.O Box 10891
Lilongwe
House No./Location ___________

A/C Number ___________
Tariff ___________
Reference ___________

Electricity account covering period approximately one period preceding date of meter reading

DATE	DETAIL OF TRANSACTION			AMOUNT	BALANCE
01/02/15	Balance brought forward			0	
	Previous Reading (kwh)	Current A Reading (kwh)	Consumption		
03/02/15	28935	29297	362	9209.28	
SURTAX	16.5% of K9209.28			1519.53	
	Total current bill			10729.81	10729.81
The monthly total is for January 2015 and is payable 10/02/2015. Please ALWAYS VERIFY THE READING ON YOUR BILL AND HOUSE CARD. BILL INCLUDES 1% MERA LEVY AND 4.5% MAREP LEVY					
					AMOUNT DUE
					10729.81

A close look at the bill reveals that the consumer:

- Did not have a previous balance.
- Consumed 362 kWh of electric energy in the month of January 2015 calculated as the difference between the current and previous meter readings i.e (29297 – 28935) Kwh = 362 Kwh or units.
- The amount to be paid for a consumption of 362 Kwh is K9209.28. Hence the cost per unit (Kwh) 12 $\frac{K9209.28}{362} = K25.44$
  In short
  $$\textit{Units used} \times \textit{Unit cost} = 362 \times K25.44 = K9209.28$$
- The consumption cost of K9209.28 already includes ministry of Energy, Mines, Natural Resources and Energy (MAREP) levy at 4.5%; Malawi Energy Regulatory Authority (MERA) levy at 1% . (These were the rates at the time of writing this book).
- A SURTAX of 16.5%is then levied on the consumption cost of K9209.28 to get 16% × K 9209.28 = K 1519.53
- The total bill = K9209.28 + K1519.63 = K10729.81

Example 13.13

What is the cost of heating water in a tank with a 3 kW heater for 90 minutes, if the cost of electricity is K 25.4 per unit

$$\begin{aligned} \text{Units used (kWh)} &= \text{kW} \times \text{h} \\ &= 3\ \text{kW} \times 1.5\ \text{h} \\ &= 4.5\ \text{kWh} \end{aligned}$$

Consumption cost = Units used × price per unit = 4.5 × 25.4 = K114.3

Example 13.14

Calculate the total bill for of using a Television set rated 200 W, for 10 days, an electric kettle rated 2.5 kW for 6 hours and electric lamp rated 150W for 30 days at the rate K25.4 per unit and a SURTAX of 16.5% of the consumption.

Solution

Units used by the television set rated 200W are

$$= \frac{200}{1000}\ \text{kW} \times 10 \times 24\text{h} = 48\ \text{kWh}$$

Units used by the electric kettle rated 2.5 kW are

$$= 2.5 \times 6 = 15\ \text{kWh}$$

Units used by the electric lamp rated 150 W are

$$= \frac{150}{1000}\ \text{kW} \times 30 \times 24\text{h} = 108\ \text{kWh}$$

$$\begin{aligned} \text{consumption cost} &= \text{units used} \times \text{price per unit} \\ &= (48 + 15 + 108) \times \text{K}\ 25.4 \\ &= \text{K}\ 4343.4 \end{aligned}$$

Total bill = K4343.4 + 16.5% of K4343.4 = K5060.061

Conserve energy to save on consumption costs. Use energy saving bulbs and put off the switches during the day to conserve electrical energy.

13.7 Domestic wiring system

Live (L), neutral (N) and earth (E) wires

The domestic supply in Malawi is mainly 240 V ac with a frequency of 50 Hz. This is supplied by two cables from a local sub-station. One cable is called the *live wire* (L) and the other *neutral wire* (N). The live wire may be likened to the positive terminal of a cell or a battery and the neutral wire to the negative terminal (Fig. 13.14(a) and (b)).

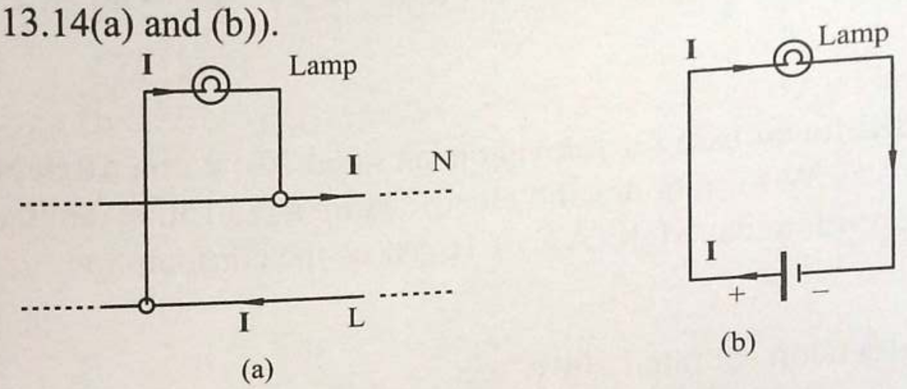


Fig. 13.14: Live and neutral wires

All electrical appliances need a live and a neutral wire to form a complete circuit from the power supply through the appliance and back to the power supply. The live wire delivers the current to the appliance. It is dangerous, because it is capable of giving electric shocks, if handled carelessly. *Switches in a circuit should be fitted in the live wire*, so that when the switch is off, the high voltage is disconnected from the appliance. The current returns to the supply through the neutral wire. Some electrical appliances have a third wire known as the *earth wire* (E) for safety.

Colour codes for the wires used in the house circuits

The insulation, usually of plastic, on the three wires of a cable is distinctively coloured to denote the live, neutral and earth wires. The basic idea of using different colours is to easily identify the wires so that correct connections are made with care. The present international convention is *brown for live, blue for neutral* and *green with yellow stripes for earth*. Electrical wiring should be checked to ensure that the earth lead goes to the metal case of the appliance.

(a) Fuses

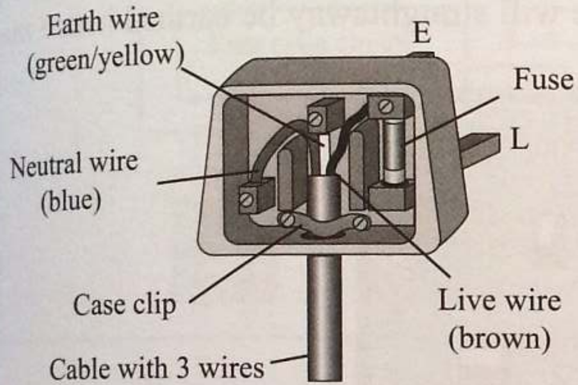
A fuse is a short thin piece of wire of low melting point. The wire melts as soon as the current through it exceeds its rated value. Fuses are usually fitted in all electrical circuits to prevent dangerously large current from flowing.

A melted or 'blown' off fuse stops the current and protects the electrical appliance against the risk of fire caused by the heat. *Just like the switch, the fuse should be fitted in the live wire.*

The fuse used should be of a value just higher than the normal current required by the appliance. Some common standard values of available fuses are 2 A, 5 A and 13 A, although 1 A, 3 A, 7 A and 10 A fuses are also made. If the power rating of an electrical appliance is '2 000 W, 250 V', the required current through it is 8 A. The correct fuse to protect the appliance is 10 A. Similarly, if the required current for an appliance is 4 A, the correct fuse to be used is 5 A.

Three pin plugs and sockets

Most of the modern electrical appliances like the electrical iron, kettle, toaster, electric geyser, immersion heater, refrigerator, hot plates, etc. are supplied with a 3-pin plug, while some systems like television set, record players, hair-blow dryer, key boards have only 2-pin plugs. A 3-pin plug has its three pins, usually marked with letters L, N and E standing for live, neutral and earth respectively (Fig.13.15 (a) and (b)).



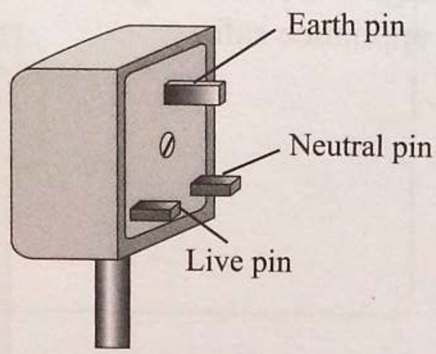


Fig. 13.15: 3-pin plug

Note that the earth pin is slightly longer than the other two pins and that the live pin is on the right hand side of the plug when connected to the socket.

Earth connection

The earth wire connects the metal case of an appliance (e.g. an electric iron) to the ground and prevents it from becoming *live*, if a fault develops. If, for example, the cable insulation wears out due to the heating effect of the current, there are chances that a few fine strands of the bare live wire could touch the metal case. When such a fault occurs, a current flows through the live wire and the earth wire in series. The fuse in the live wire will blow and cut off the power supply. If on the other hand, there was no earth wire connection, a person touching the metal case would get an electric shock.

In appliances like television set, record player, etc. the outer case is not metallic and hence 2-pin plugs are sufficient. It is dangerous to use the 2-pin plug with any appliance which has an outer metal case.

Short circuits

If a few strands of the fine bare live wire touch, by chance, those of neutral wire, a large current can flow between the live and the neutral wires of the supply cables. This is due to the fact that current tends to take the path of least resistance. This is called the *short-circuiting* of the appliance.

On such occasions the fuse usually blows off. Otherwise if no fuse is in use, the 'sparking' produced by the large current might burn the cable and there are risks of fire being produced.

In a socket for 3-pin plug, the holes for the live and the neutral pins are usually closed by an insulating material called a 'blind' (Fig. 13.16). This is a safety measure, especially to children who like to play with nearly everything and might cause short circuiting by putting in wires in the socket. The 'blinds' are opened by the longer earth pin of the 3-pin plug. The moment the earth pin touches and opens the socket, any leakage current through the metal case will straightaway be earthed hence making the appliance safe.

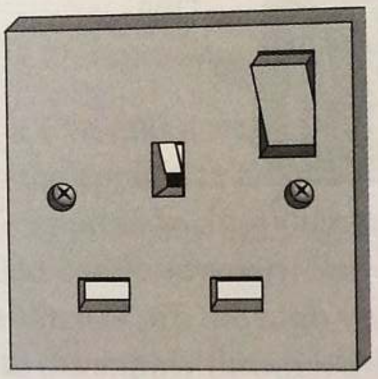


Fig. 13.16: A socket

Household wiring

Fig. 13.17 shows the arrangement of the domestic wiring system consisting of the following: the main fuse, electricity meter, consumer unit or the fuse box, lighting circuit and the ring main circuit.

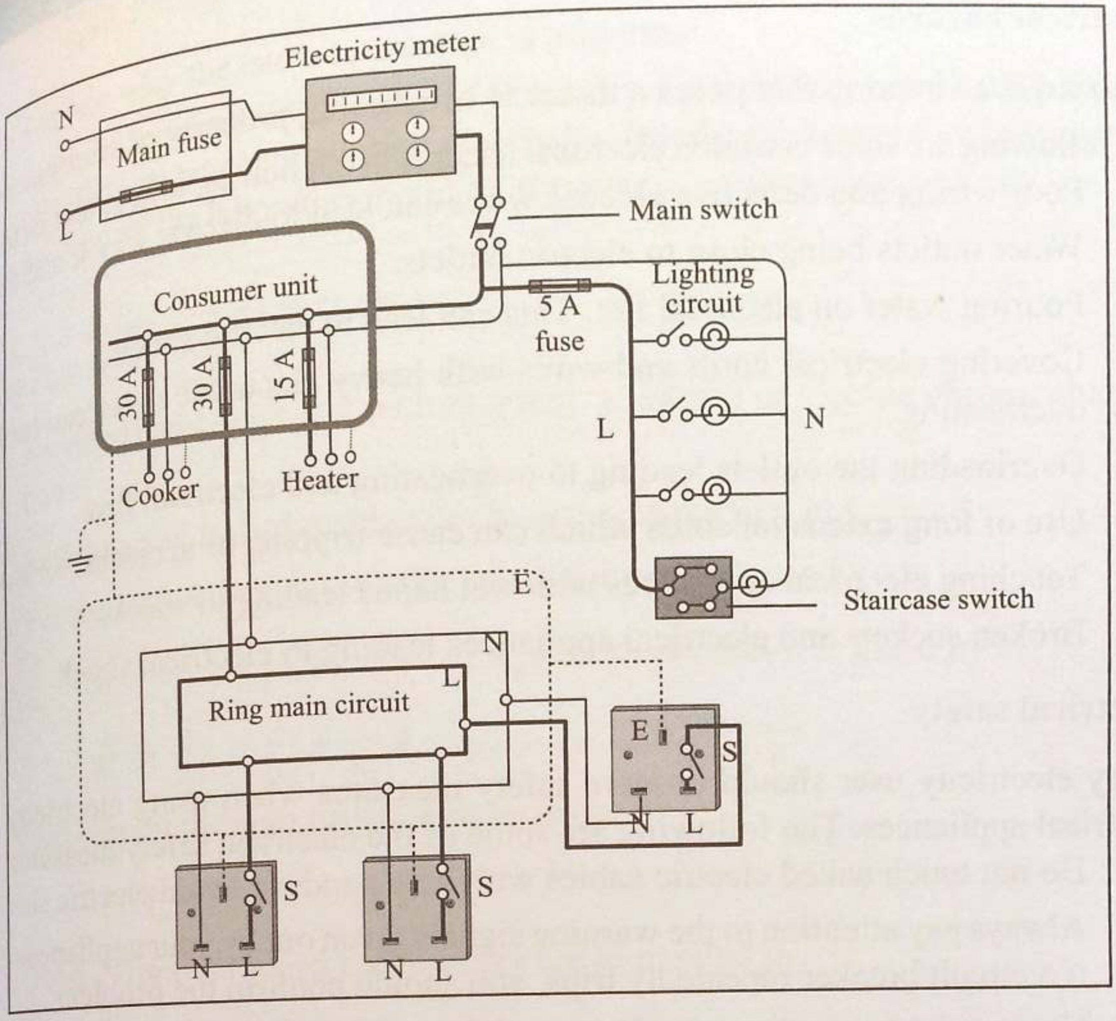


Fig. 13.17: Domestic wiring system

Every circuit is connected in parallel with the power supply, i.e. across the live and the neutral wires. Every circuit receives 240 V ac. There is no connection between the live and the neutral wires except through an electrical appliance.

The *electricity meter* records the electrical energy consumed in the whole house.

The *consumer unit* is a junction box which distributes current to several separate circuits. The consumer unit also houses the main switch which can switch off all the circuits in the house, if required.

The *lighting circuit* contains lights for different places and the 2-way switches for places like the staircases. Each lamp is connected in parallel at a suitable point along the cable. The lighting circuit does not require the earth connection, as the current is normally quite low.

The *ring main circuit* provides parallel circuit connections to each electrical appliance plugged into the sockets. Since the current drawn is high, the ring main circuit incorporates the earth wire connection.

Electrical hazards

A hazard is a situation that poses a threat to life, health, property or environment. The following are some common electrical hazards in our homes, offices and factories.

- Poor wiring and defective electric wires can lead to electric shock and fires.
- Water outlets being close to electric outlets.
- Pouring water on electrical fire. This can lead to electric shock.
- Covering electrical cords and wires with heavy electrical cover can lead to overheating.
- Overloading the outlets leading to overheating and electrical fire.
- Use of long extension cords which can cause tripping or accident.
- Touching electrical appliances with wet hands leading to shocks.
- Broken sockets and electrical appliances leading to electrical shock.

Electrical safety

Every electricity user should observe safety measures when using electricity and electrical appliances. The following are some of the electrical safety measures.

- Do not touch naked electric cables with bare hands to avoid electric shock.
- Always pay attention to the warning signals given out by your appliances e.g. if a circuit breaker repeatedly trips, you should confirm the problem.
- Use the right size circuit breakers and fuses to avoid overloading.
- Ensure that potentially dangerous electrical devices or naked wires are out of reach of children.
- You should avoid cube taps and other outlet-stretching devices.
- Always replace broken plugs and naked wires.
- Use the correct appliances in a socket to avoid overload.

Exercise 13.3

1. A bulb rated 200W is used for 12 hours. Calculate the energy it consumes in kWh.
2. Name any four devices that have electric heating elements.
3. If energy costs K25.4 per unit and the energy saving bulbs are used on average, for 5 hours per day, what will the annual saving be if the bulb is rated 5kWh?
4. State the internationally accepted colours for the live (wire), neutral and earth leads of a 3-core flex.
5. Define '*fuse*' and state its function in an electrical circuit.
6. Sketch and name a three pin plug.

7. Explain why the earth connection is important.
8. A laboratory building in one of Voluntary Counselling and Testing Centre (VCT) is to be supplied with electricity. Briefly explain how wiring would be done in the laboratory building for effective supply of electricity.

Unit Summary

- An electrical circuit is a path in which a voltage or electric current charges flow.
- Resistors can be arranged either in series or parallel in a circuit.
- For resistors in series the effective resistance is given by
  $R_e = R_1 + R_2 + R_3 + \ldots\ldots R_n$ and for parallel it is given by
  $$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots\ldots \frac{1}{R_n}$$
- Total current through resistors in series is
  $$I_s = I_1 = I_2 = I_3 = I_n$$
- Total current (I) when the resistors are in parallel is
  $$I_p = I_1 + I_2 + I_3 \ldots\ldots\ldots I_n$$
  where $I_1, I_2, I_3 \ldots\ldots I_n$ represent current through the different resistors in parallel.
- Power is the rate at which work is done
  $$P = VI$$
  $$\text{but } V = I^2R$$
  $$\therefore \quad P = I^2R$$
  $$\text{also } I = \frac{V}{R}$$
  $$\text{Hence } P = V \times \frac{V}{R}$$
  $$= \frac{V^2}{R}$$
- A fuse is a short thin wire of low melting point. It melts when a large current flows through it hence breaking the circuit.
- The earth wire is connected to the ground and prevents it from becoming live. This is important incase there is electrical fault.

Unit Test 13

1. A Form three student was provided with three resistors of value 4Ω, 6Ω and 8Ω.
   (a) Draw a circuit showing the resisitor in:
      (i) Series with each other.
      (ii) Parallel with 6 resistor.
   (b) Calculate total resistance of circuit in a(i) and (ii) assume a negligible internal resistance.
2. Two resistances of 12 Ω and 36 Ω are connected in parallel. Two such combinations are put in series. Calculate the effective resistance
3. Starting from electrical work done, W = VIt, show that electrical power (P) generated in a conductor is given by V^2/R, where the symbols have the usual meaning.
4. A current of 6 A is passed through a resistor of 30 m for 1 ½ hours. Calculate the electrical energy converted into heat energy in the resistor.
5. A family uses an electric iron box rated 1500W one hour per day, 3 bulbs each rated 100W four hours everyday and watch a television set rated 1800W for three hours per day. What would be their total bill for the month of June at a rate of K25.4 per unit and SURTAX at 16.5%.
6. What is the cost of lighting a 60 kW bulb for 2 days. If the cost of electricity is K 25.4 per unit (inclusive of all levies).
7. State four household electrical appliances where electrical energy is converted into heat energy.
8. The filament of a bulb is made of tungsten and the bulb contains a mixture of argon and nitrogen at a low pressure.
   (a) What is the purpose of the presence of the gases inside the bulb?
   (b) Why is tungsten a suitable material for the filament?
9. Electrical heaters are said to be environmentally friendlier than the heating devices which use firewood or charcoal. Explain this statement.
10. Find the equivalent resistance in the circuits shown in of Fig.13.18.

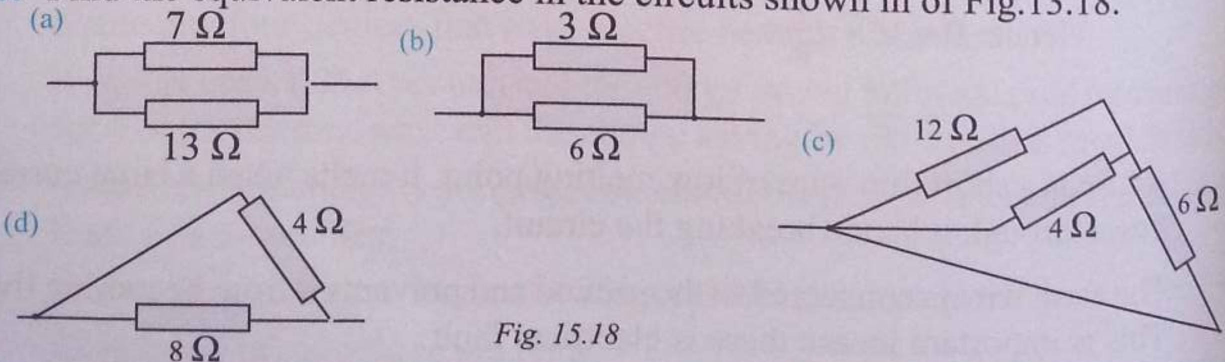


Fig. 15.18

Oscillations and Waves

Outcome

The students will be able to understand and apply waves and their properties in designing and developing various technologies in communication, medicine, musical and military equipments.

Unit 14: Oscillations and Waves

Unit 15: Sound

UNIT

14 Oscillations and Waves

Success Criteria

By the end of this unit, you must be able to:

- Explain oscillation in relation to a pendulum or a hanging mass on a spring.
- Describe a wave.
- Differentiate between a transverse wave and a longitudinal wave.
- Describe wave properties.
- Apply the wave equation in problem solving.

Introduction

When a stone is dropped in a pool of still water, ripples spread out in a circular form. This constitutes what is called *water waves*. There are many different types of waves. These include radio and T.V. waves which are useful in communication, microwaves used for cooking, water waves for production of electricity and sound waves used in ultra sounding in hospitals.

Since waves can do work as seen from the above examples, then waves are indeed a form of energy which when properly harnessed can provide a useful source of energy that is safe and environmentally friendly. Earthquakes produce shock waves that are very destructive in that they possess enormous and uncontrolled amount of energy that shakes and destroys buildings. A good example of shock waves is the wave known by the Japanese as *Tsunami*. These waves cause enormous damage to infrastructure and the environment.

In this unit, we shall study the production of waves and some common terms and properties used in describing wave motion. By understanding more about waves, more uses are made of them. The study of waves begin with the concept of oscillations.

14.1 Oscillations

Movements form a major part in our lives. Movements can be regular or irregular. Some movements follow a fixed path and keep repeating. These kinds of movements are important in our lives as shown in the following examples:

- A pendulum clock repeats movement to keep time.
- Wheels of bicycles and vehicles keep repeating their movements round circular path and this results in our moving faster and easily to other places.
- Heartbeats are also rhythmic movements that help us remain alive.
- Swings in children's playgrounds.

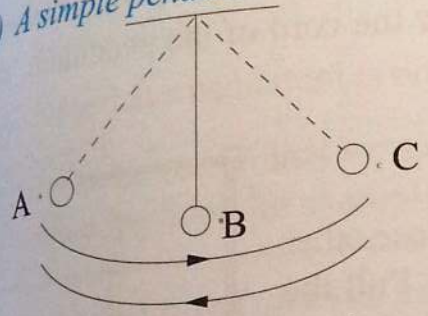
All these and many others are repetitive to-and-fro movements called oscillations. Therefore, *oscillations are repeated, regular movements that happen at a constant rate.*

14.2 Characteristics of an oscillation

- The *displacement, d*, of a vibrating body is the distance of that body from the mean/fixed position.
- The *amplitude, a*, of a vibration is the maximum displacement from the fixed/mean position in either direction.
- *Periodic time, T,* is the time taken to complete one oscillation or cycle.
- The *frequency, f,* is the number of complete oscillations (or cycles) made in one second.SI unit for frequency is the *hertz, Hz*. One hertz is defined as *one oscillation per second or one cycle per second*.

Consider the following cases of oscillations:

(a) A simple pendulum

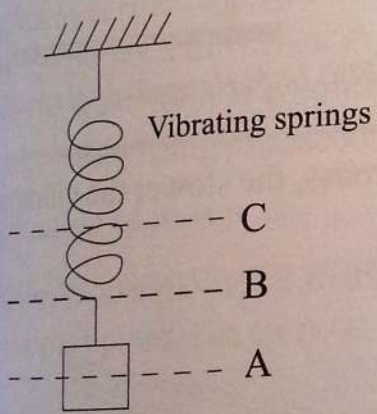


One oscillation is the movement
A → B → C → B → A

Amplitude is the distance BA or BC.

Fig. 14.1: A simple pendulum

(b) A vibrating spring

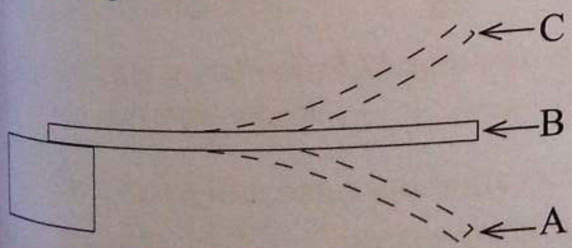


One oscillation is the movement
A → B → C → B → A.

The amplitude is the distance BA or BC.

Fig. 14.2: A vibrating spring

(c) A clamped metre rule



One oscillation is the movement
A → B → C → B → A.

The amplitude is the distance BA or BC.

Fig. 14.3: A clamped rule

14.3 Factors affecting oscillations

Activity 14.1:

Fig. 14.4 shows a simple pendulum consisting of a bob attached at the end of a light cord. The other end of the string is clamped rigidly in position.

1. Displace the bob slightly to one side then release it and observe what happens.
2. Increase the length of the cord and observe the change in vibration time.
3. Repeat this several times.

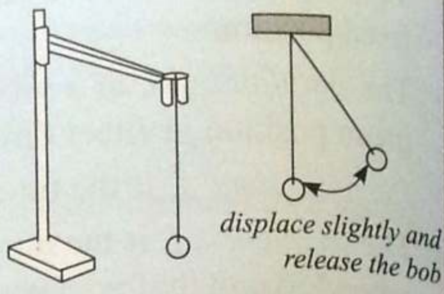


Fig. 14.4: Simple pendulum

From Activity 14.1, we observe that the longer the cord of the pendulum, the slower it oscillates.

Activity 14.2

1. Attach a mass to one end of a spiral spring whose other end is rigidly clamped in position Fig. 14.5. Pull the mass slightly downwards then release it and observe what happens.
2. Repeat this activity three times each time using a bigger mass.

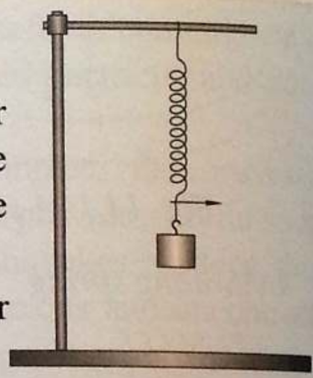


Fig. 14.5: Hanged spiral spring

From Activity 14.2, we observe that the bigger the mass, the slower the vibration of the spring.

Activity 14.3

1. Fix a mass at the end of a metre rule and clamp the other end as shown in Fig. 14.6.
2. Displace the free end of the rule then release and observe what happens. Repeat this activity by attaching another mass two more times.
3. Repeat this activity using half of the metre rule.

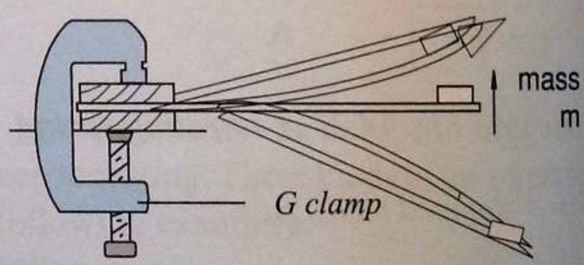


Fig. 14.6: Oscillations of a loaded metre rule.

From Activity 14.3, we observe that

(a) the bigger the mass, the slower the ruler swings.

(b) the longer the ruler, the slower it swings.

Discussion

All the above activities indicate that the frequency of a vibrating system is affected by:

(a) Length – the longer or larger the body the lower the frequency e.g a shortened guitar wire produces higher pitch.

(b) Mass – the bigger the mass /thickness the lower the frequency (longer periodic time). This is usually the case in guitar wires, where thinner ones give higher pitch.

Note that a pendulum is not affected by mass of the bob attached.

(c) It can be shown that increase in tension increases the frequency /pitch of a vibrating body for example a string /wire.

14.4 Formation of waves and pulses

Formation of wave motion

As learnt earlier, waves transfer energy but not matter. This energy is transferred through pulses and waves. A pulse is a sudden short-lived disturbance in matter. A wave or wave train is a continuous disturbance of the medium which arises due to regular pulses being produced. The following experiments demonstrate wave motions.

Formation of pulses

A pulse is a single wave disturbance that moves through a medium from one point to the next point. Let us now demonstrate the formation of pulse in Activity 14.4.

Activity 14.4: To demonstrate the formation of pulses using a rope

1. Fix one end of a rope to a wall. Hold the free end of the rope so that the rope is fully stretched.
2. Quickly move your hand upwards and then return to the original position as shown in Fig. 14.7(a). Observe what happens to the rope.
3. Now move your hand suddenly downwards and return to the original position as in Fig. 14.7(b). Observe what happens to the rope.

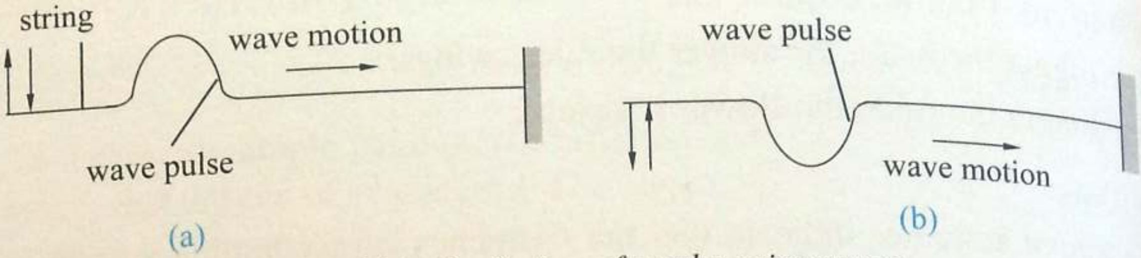


Fig. 14.7: Production of a pulse using a rope

In Activity 14.4, we notice that pulses that move from one end of the rope to the other are produced.

If the disturbance is continuous, waves or wave trains are formed.

When pulses are produced regularly, they give rise to a continuous wave motion. Waves or a *wave train* is a continuous disturbance of the medium which arises due to the regular pulses being produced. In Activity 14.4, when the hand *(source)* is moved continuously up and down or forward and backward, the particles of the rope or spring *(medium)* also move up and down or forward and backward. When the source is moved at regular intervals, the disturbance is also produced at regular intervals (Fig. 14.8).

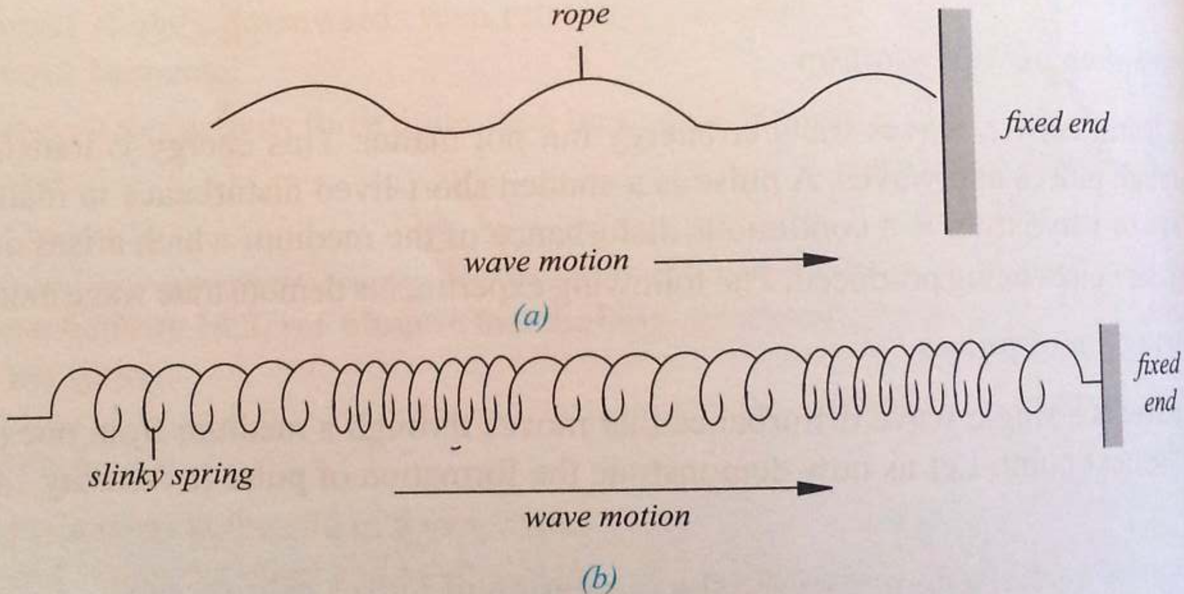


Fig. 14.8: Production of continuous pulses in a string and a slinky

Continuous disturbance of a medium at a point produce continuous waves or wave trains. The waves or wave trains produced are of two types: *transverse waves* and *longitudinal waves*.

Transverse waves

Transverse waves are mechanical waves in which the particles of the medium move in a direction perpendicular to the direction of travel of the wave. Therefore, in a transverse wave, the direction of disturbance is at right angles to the direction of travel of the wave.

Activity 14.5: To demonstrate a transverse wave

1. Tie one end of the rope to the fixed pole as shown in Figure 14.9.

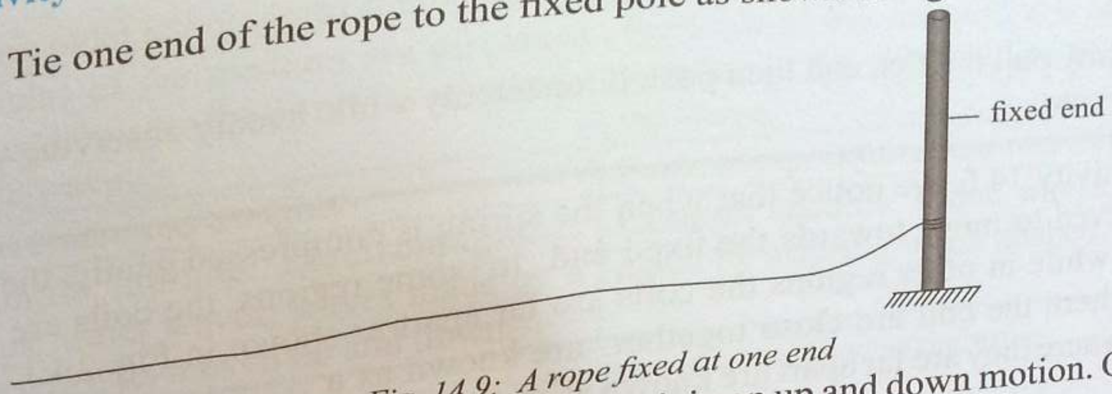


Fig. 14.9: A rope fixed at one end

2. Hold the free end of the rope and shake it in an up and down motion. Observe how the rope behaves.

From Activity 14.5, we notice that when the rope is shook up and down, it is seen to make rises and falls which move through the fixed end (Fig. 14.10).

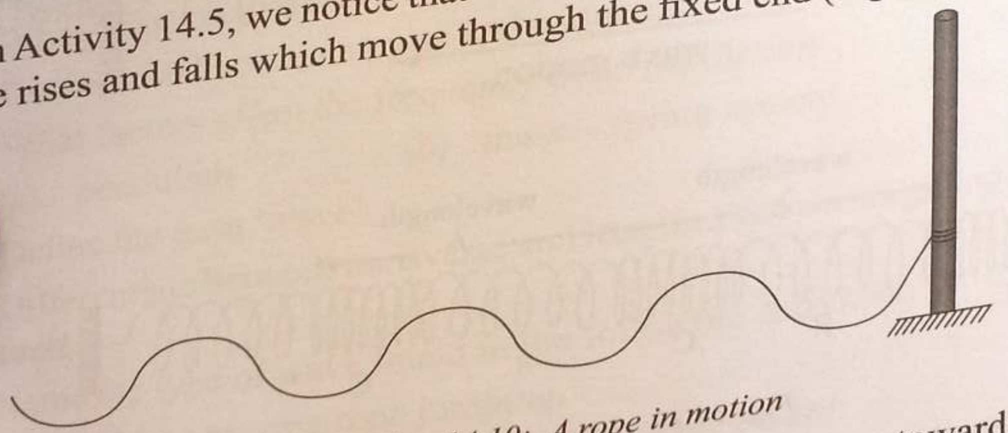


Fig. 14.10: A rope in motion

The rope particles are displaced up and down as they move towards the fixed end. These up and down disturbance are perpendicular to the direction of motion of the wave. The rises are known as crests while the falls are known as troughs (Fig. 14.11).

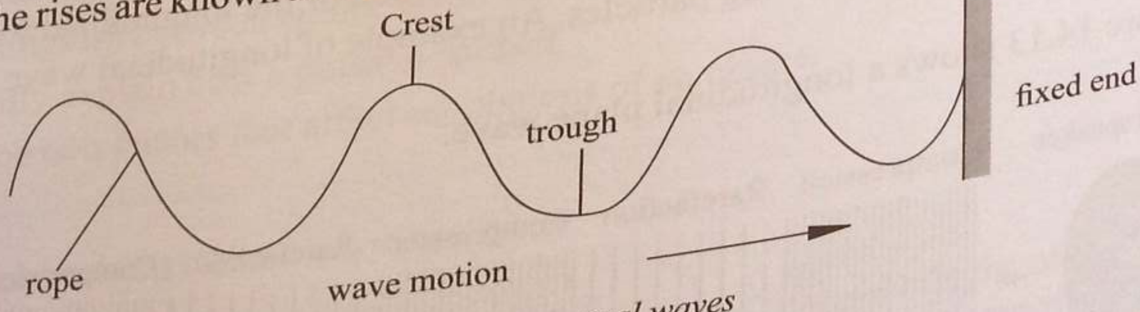


Fig. 14.11: Transersal waves

Longitudinal waves

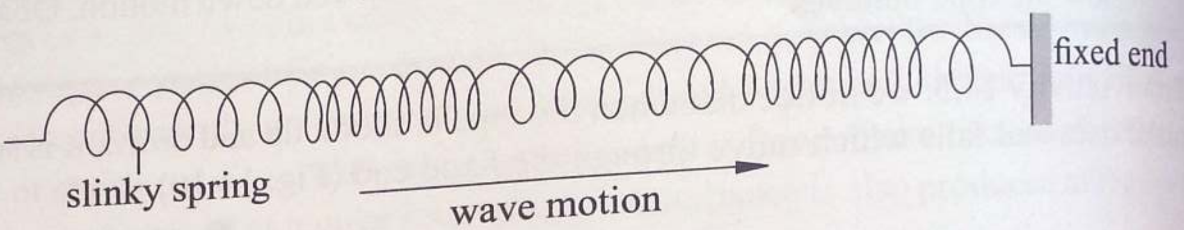
Longitudinal waves are mechanical waves in which particles of the medium move in direction parallel to the direction of the wave motion. The particles of the transmitting medium vibrates to and fro along the same line as that in which the wave is travelling.

Activity 14.6: To demonstrate a longitudinal wave

1. Place the helical spring to lie on a table and hold it firmly to the table on one end.
2. Gently pull the free end then push it repeatedly while keenly observing what happens.

From Activity 14.6, we notice that when the spring is compressed gently, the coils are observed to move towards the fixed end. In some regions, the coils are close together while in other regions the coils are far apart as shown in Fig. 14.12. The region where the coil are close together are known as a compressions while the regions where they are far apart are known as rarefactions. (See Fig 14.12 (a) and (b))

(a)



(b)

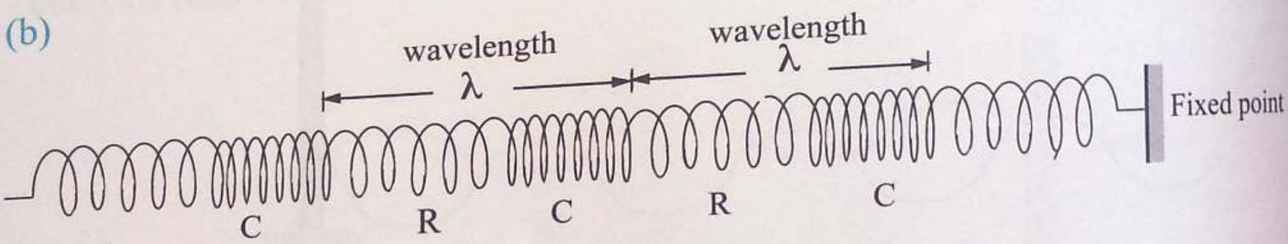


Fig. 14.12: Longitudinal wave

Thus, a longitudinal wave consists of compression and rarefactions. Compressions is a region on a longitudinal wave with a high concentration of vibrating particles. On the other hand a rarefaction is a region of the longitudinal wave with low concentration of vibrating particles. An example of longitudinal wave is sound waves.

Figure 14.13 shows a longitudinal plane wave.

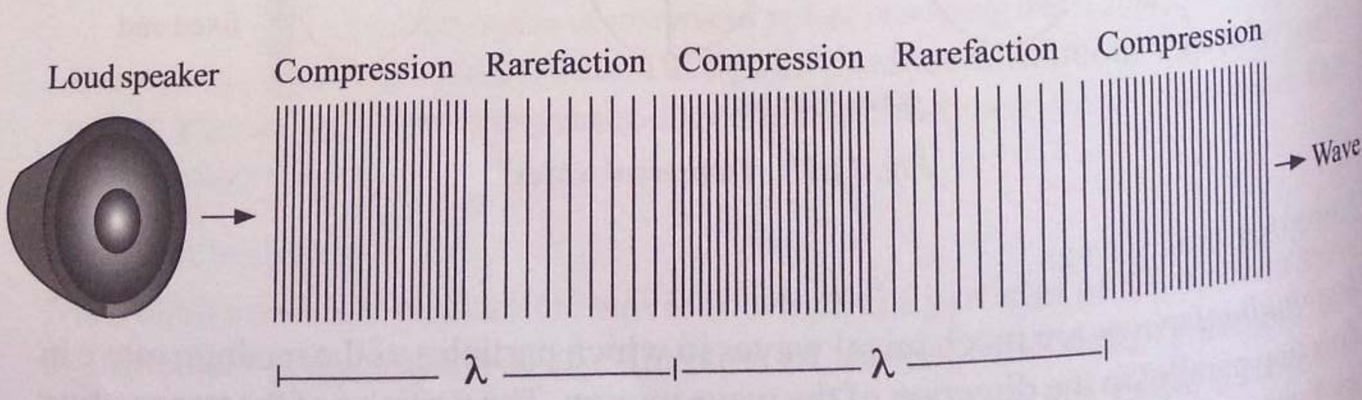


Fig. 14.13 shows a longitudinal plane waves

Differences between Transverse and Longitudinal waves

Transverse waves	Longitudinal waves
Particles of the medium are displaced perpendicular to the direction of motion of the wave.	Particles of the medium are displaced parallel to the direction of motion of the wave.
Form crests and troughs.	Form compressions and rarefactions.
Example include: Electromagnetic waves, water waves, waves made by a rope when its moved up and down.	Example include: sound waves, waves made by a spring when pushed.

Table 14.1: Difference between transverse and longitudinal waves

Exercise 14.1

1. What is an oscillation?
2. Distinguish between a pulse and wave train
3. What factors affect the frequency of an oscillating:
   (a) pendulum (b) mass – spring system
4. Define the term 'wave'.
5. Differentiate between transverse and longitudinal waves giving an example for each.
6. Name the type of wave found in the following activities:
   (a) Children playing rope jumping.
   (b) A spring being displaced forward and backward.
   (c) Waves due to dropping a stone into water on a basin.
   (c) A car moving on a bump.
7. Distinguish between compression and rarefaction
8. Briefly explain how a pulse in formed.
9. Name two factors that affect oscillations of an object.

14.5 Characteristics of wave motion

Wavelength of transverse waves

Consider a long rope with one of its ends rigidly tied to a peg while the other end is free. Produce a pulse by moving the hand upwards and notice the distance travelled by the disturbance. If the hand is moved up and down once through a complete cycle, the time taken by the hand is the *periodic time (T)*.

Fig. 14.14 shows a graph of displacement of particles against time. We see that the particles of the rope just vibrate up and down about their mean or rest position, but do not move with the wave. The disturbance is transferred from particle to particle. The distance travelled by the disturbance (wave energy) during each periodic time T is called the *wavelength*, λ, of the wave.

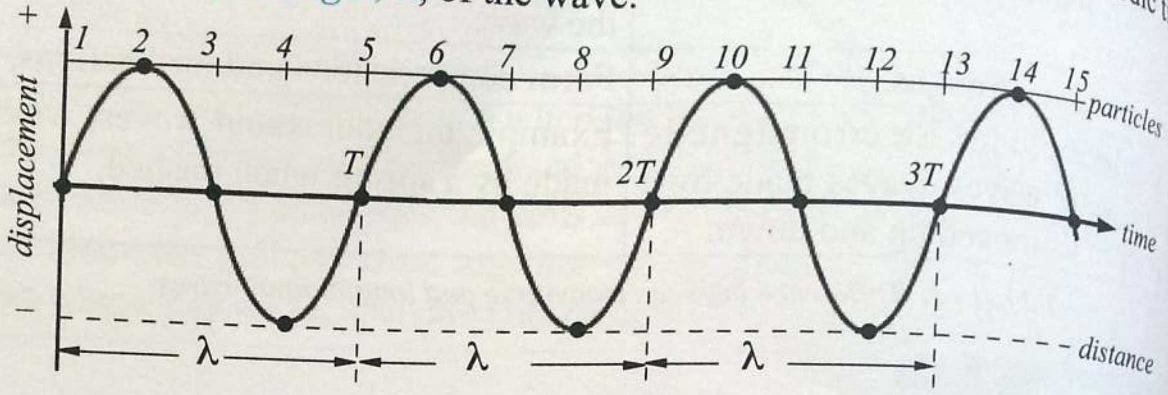


Fig. 14.14: Wavelength of a transverse wave.

From the graph (Fig. 14.14), particles 2, 6, 10, 14, etc. are at similar positions and move in the same direction. Such positions are called the *crests* of a wave. Similarly, particles 4, 8, 12 etc, are at similar positions and are moving in the same direction. Such positions are called the *troughs* of a wave.

Particles that are at similar positions and are moving in the same direction are said to be in *phase*.

A crest is the position of maximum positive displacement, and a trough is the position of maximum negative displacement as shown in Fig. 14.15.

The distance between two successive particles in phase such as two successive crests or troughs is equal to the wavelength of the wave.

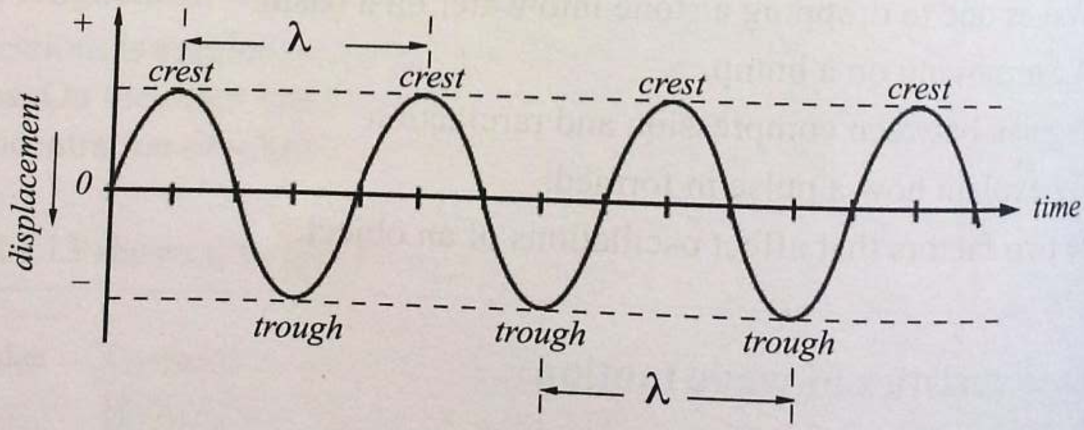


Fig. 14.15: Crest and troughs in a transverse wave

Wavelength of a longitudinal wave

Fig. 14.16 shows the energy propagation in a slinky spring.

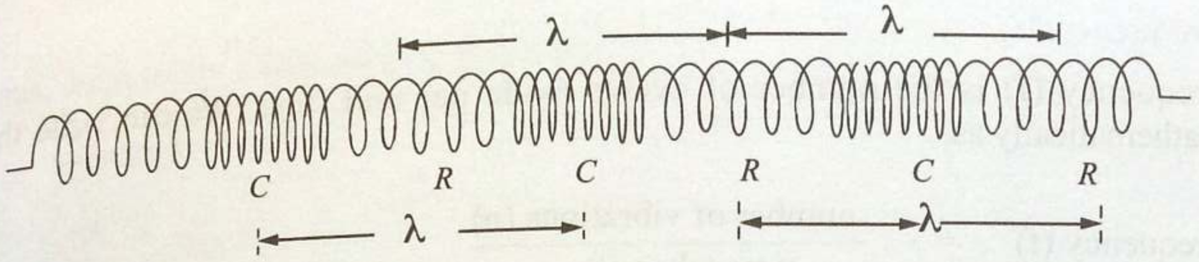


Fig. 14.16: Compressions and rarefactions in a longitudinal wave.

Just like the production of crests and troughs in a transverse wave, we have the regions of compressions (C) and rarefactions (R) in a longitudinal wave.

A *compression* is a region where the particles of the medium are closely packed. In this region, the pressure of the particles of the medium is high, hence the density is high.

A *rarefaction* is the region where the particles of the medium are spread out. In this region the pressure of the particles of the medium is low, hence the density is low.

The wavelength of a longitudinal wave can be described as the distance between two successive compressions or rarefactions.

Fig. 14.17 is a displacement -time graph for a wave. We will use it to describe other characteristics of waves.

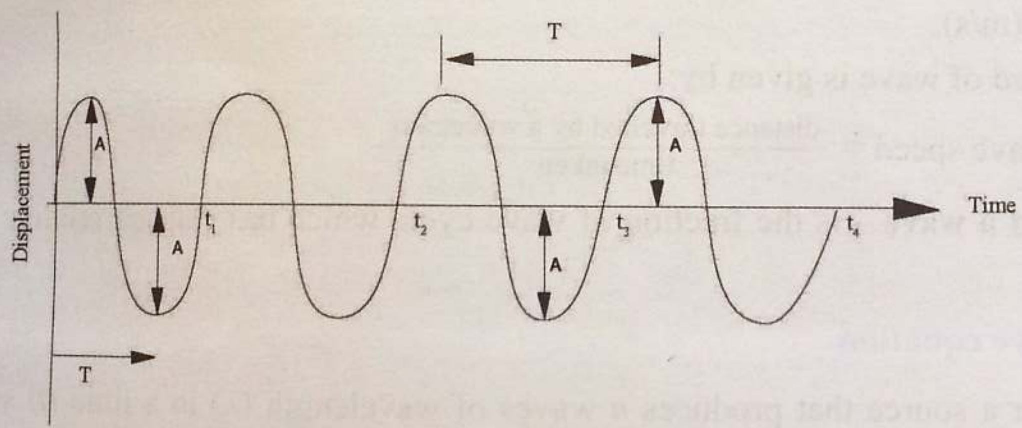


Fig. 14.17: Displacement – Time graph

Periodic time, T

The time taken for one vibration /oscillation. It is also the time taken to cover a distance of one wave length. Thus, the value of T in Fig. 14.17 is the periodic time. By definition, periodic time is the duration for one complete oscillation.

Amplitude, (A)

As a body or particles vibrate, they change position from the mean rest position. The position of a point from the resting position at any given time is called its *displacement*.

The maximum value of displacement is called *amplitude* (A) as shown on the Fig. 14.17.

Frequency, (f)

Frequency (f) is the number of cycles made per unit time. We can write this mathematically as,

$$\text{Frequency (f)} = \frac{\text{number of vibrations (n)}}{\text{time taken (t)}}$$

In symbols, $f = \frac{n}{t}$

If $n = 1$ (i.e 1 oscillation), then $t = T$ (periodic time)

$$\text{Hence } f, = \frac{1}{T} \text{ and } T = \frac{1}{f}$$

For example, if a newborn baby's heart beats at a frequency of 120 times a minute, its frequency is $f = \frac{120}{60} = 2\text{ Hz}$ and $T = \frac{I}{f} = \frac{I}{2} = 0.5\text{ s}$

Wave speed

This is the distance *covered by a wave per unit time*. It is measured in *metres per second*, (m/s).

The speed of wave is given by:

$$\text{Wave speed} = \frac{\text{distance travelled by a wavetrain}}{\text{time taken}}$$

Phase of a wave – is the fraction of wave cycle which has elapsed relative to the origin.

The wave equation

Consider a source that produces *n* waves of wavelength (λ) in a time *(t)* seconds (Fig. 14.18)

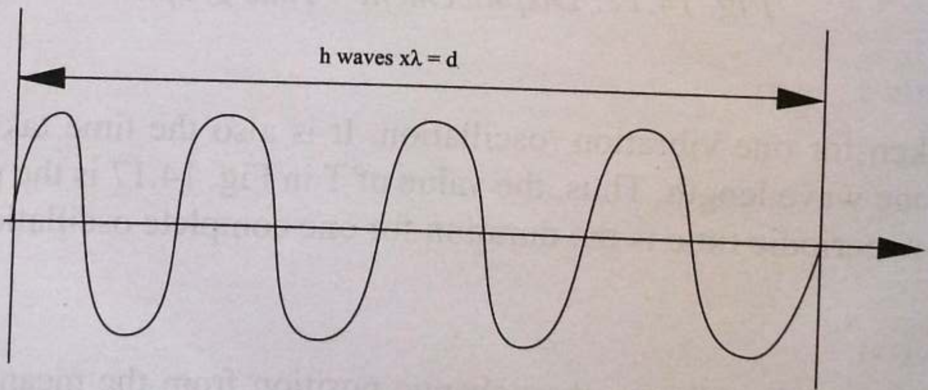


Fig. 14.18: Displacement – distance graph

The distance travelled by a wave train in one period time is the wavelength of a wave.

$$\text{Wave speed, } v = \frac{\text{Wavelength } \lambda}{\text{Periodic time, T}}$$

Thus, the velocity of the wave is given by:

$$\text{velocity (v)} = \frac{n\lambda}{T} \quad \text{..................................(i)}$$

$$\text{but } T = \frac{1}{f}$$

Substituting for T in (i), we get $v = \dfrac{\lambda}{\frac{1}{f}} = \lambda f$

The speed of a wave is given by: Frequency × wavelength

$$v = f\lambda$$

The equation $v = f\lambda$ is called the *wave equation*. This formula holds for all waves.

Example 14.1

A slinky spring is made to vibrate in a transverse mode with a frequency of 4 Hz. If the distance between successive crests of the wave train is 0.7 m calculate the speed of the waves along the slinky spring.

Solution

$\lambda = 0.7\text{m},\ f = 4\text{Hz}$

$$\begin{aligned}\text{Wave speed} &= \text{frequency} \times \text{wavelength}\\ &= 4 \times 0.4\\ &= 2.8 \text{ m/s}\end{aligned}$$

Example 14.2

Calculate the frequency of a wave if its speed is 30 cm/s and the wavelength is 6 cm.

Solution

$$\text{Wave speed} = \text{frequency} \times \text{wavelength}$$

$$v = f \times \lambda$$

$$f = \frac{v}{\lambda} = \frac{30}{6} = 5 \text{ Hz}$$

Example 14.3

A source of frequency 256 Hz is set into vibrations. Calculate the wavelength of the waves produced, if the speed of sound is 332 m/s in air.

Solution

$$v = f \times \lambda$$

$$\lambda = \frac{v}{f} = \frac{332}{256} = 1.30 \text{ m.}$$

Example 14.4

The speed of a certain wave in air is 3×10^8 m/s. The wavelength of that wave is 5×10^{-7}m. Calculate the frequency of that wave.

Solution

$$v = f\lambda$$

$$f = \frac{v}{\lambda} = \frac{3 \times 10^8}{5 \times 10^{-7}} = 0.6 \times 10^{15} \text{ Hz} = 6.0 \times 10^{14} \text{ Hz}$$

Example 14.5

Fig. 14.19 shows a wave produced in a string.

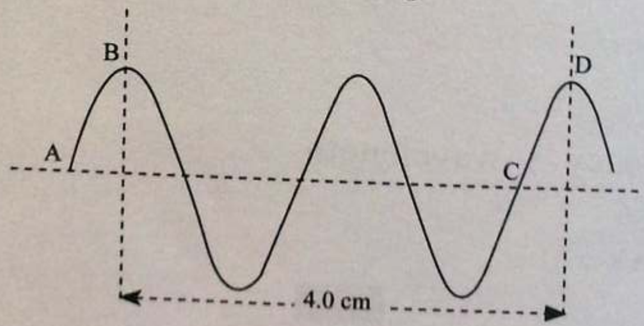


Fig. 14.19

(i) Calculate the wavelength of the wave.

$$\text{Wavelength } (\lambda) = \frac{\text{length of a number of waves}}{\text{number of waves}} = \frac{4 \text{ cm}}{2} = 2 \text{ cm}$$

(ii) If ten complete waves are produced in a duration of 0.25 seconds, calculate the speed of the waves.

$$f = \frac{10}{0.25} = 40 \text{ Hz}$$

$$v = f\lambda = 0.02 \times 40 = 0.8 \text{ m/s}$$

Example 14.6

Fig. 14.20 shows the displacement–time graph of a wave travelling at 200 cm/s.

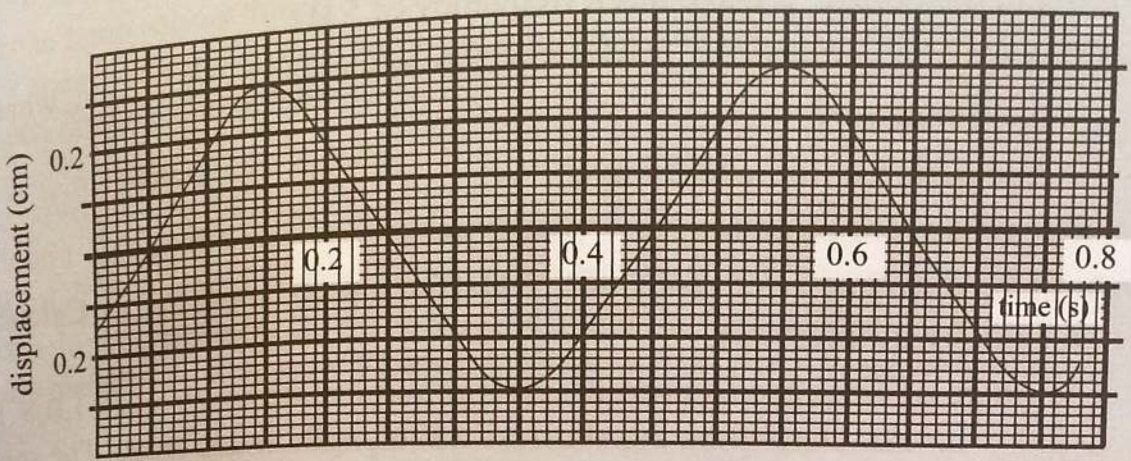


Fig. 14.20: Displacement – time graph

Determine the:

(a) amplitude (b) Period (c) frequency (d) wavelength

Solutions

(a) 0.3 cm (b) T = 0.4 s (c) $f = \frac{1}{T} = 2.5$ Hz (d) $\lambda = \frac{1}{f} = \frac{2.00}{2.5} = 0.8$ m

Example 14.7

A spring vibrates at the rate of 20 cycles every 5 seconds

(a) Calculate the frequency of the waves produced.

(b) If the wavelength of the waves is 0.01 m, find the speed of the waves.

Solutions

(a) 20 cycles = 5 seconds

4 cycles = 1 second

$\therefore$ f = 4 Hz

(b) $v = f\lambda$

$= 4 \times 0.01$

$= 0.04$ m/s

Exercise 14.2

1. Draw a wave and mark on it the wavelength and amplitude.
2. Explain the phrase 'a wave has a frequency of 5 Hz'.
3. A flag is fixed in an ocean. If two waves pass the flag every second, what is
   (a) its frequency?
   (b) the period of the water waves?
4. Derive the wave equation.
5. A sound wave has a frequency of 170 Hz and a wavelength of 2 m. Calculate the velocity of this wave.
6. The range of frequencies used in telecommunication varies from 1.0×10^6 to 2.0×10^7 Hz. Determine the shortest wavelength in this range. The speed of the wave is 3×10^3 m/s).
7. The speed of sound in air is 320 m/s. Calculate the frequency of sound when the wavelength of sound is 60 cm.

14.6 Properties of waves

Generally, waves exhibit similar properties. The properties of waves can be studied in the laboratory with the help of a ripple tank (See Fig. 14.21). The behaviour of water waves in the tank represent the behaviour of other types of waves e.g sound, light etc., when similar conditions are put in place as we shall learn in this section. In this section we will cover reflection, refraction, diffraction and interference.

The ripple tank

Fig. 14.21 shows a diagram of a ripple tank. It consists of a transparent tray containing water, having a lamp above and a paper screen below the tank.

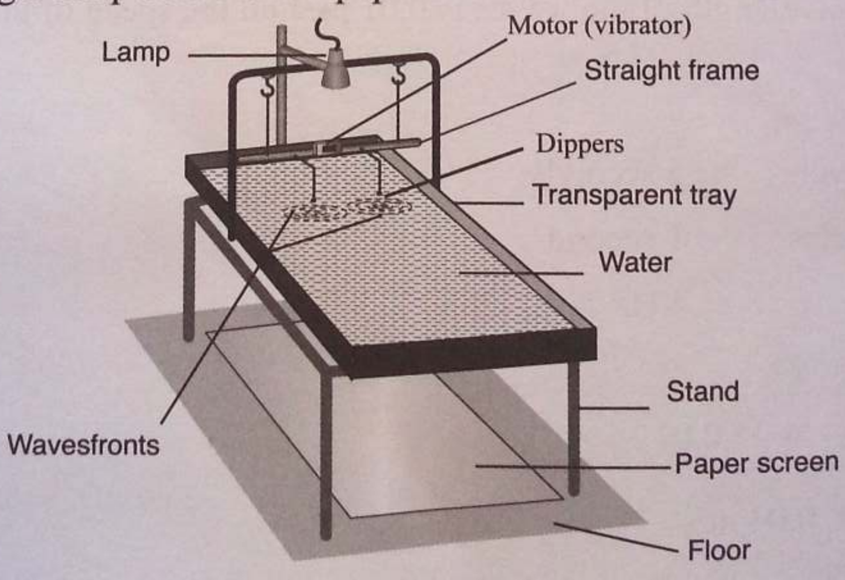


Fig. 14.21: Ripple tank

The motor is used to vibrate the straight block. When the block is used alone, it produces *straight wavefronts*. When dippers are connected, they produce *circular wavefronts* that help us study the behaviour of a source of water wave. When light from the lamp passes through the waves, images of the waves are projected on the paper underneath the tank. A series of alternating bright and dark bands (shadows) are seen on the paper screen.

Wave propagation

We learnt earlier that waves transfer energy form the source to other parts of the medium. This process is known as wave propagation. Wave propagation is the process in which waves travel from one point in the medium to another. Waves require medium to propagate. Electromagnetic waves can propagate in vacuum with a speed of 3×10^8 m/s.

The speed of the wave depend on the medium in which the wave is propagating. Waves propagate slower in air and liquid than in a solid.

Reflection of waves

Reflection is the bouncing back of waves when they meet an obstacle. Let us investigate how the water wavefronts are reflected using the following experiment.

Experiment 14.1: To investigate the reflection of plane wavefronts by a plane reflector

Apparatus

- Ripple tank

Procedure

1. Lower the frame on which a running motor is mounted so that it just touches the water surface.
2. Investigate the reflection of continuous plane wavefronts in a ripple tank.
3. Observe the shape, spacing and direction of both the incident and reflected wavefronts and draw the wave diagrams as shown in (Fig. 14.22).

Observation

In Fig. 14.22(a) the dotted lines show the path of the incident wavefronts, if the reflector had not been there.

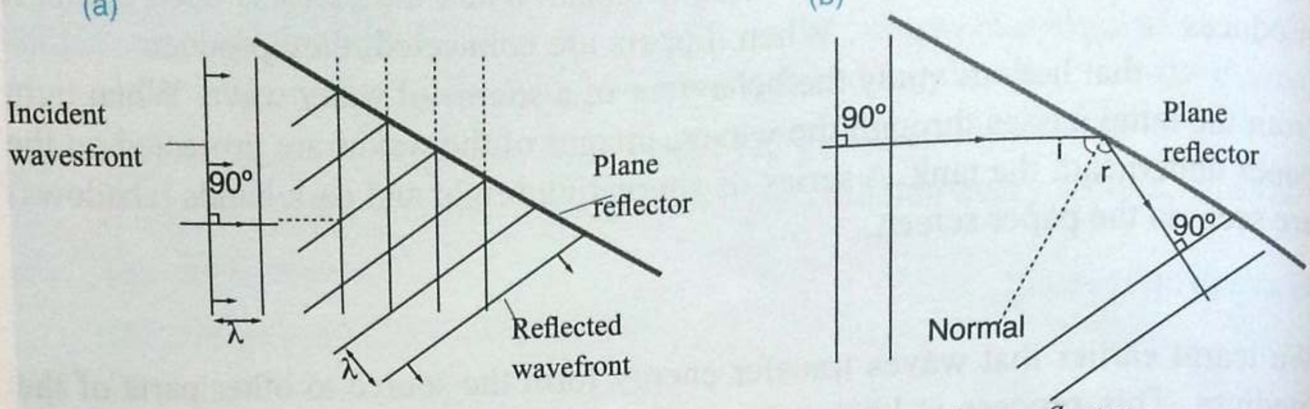


Fig. 14.22: Reflection of plane wavefronts by a plane reflector

Discussion

Waves bounce off a hard surface after striking it. This is known as *reflection*.

Note: The reflection of waves (sound, light ,water. e.t.c) obey laws of reflection as shown in Fig. 14.22 (b) above.

From experiment 14.1, we noticed that waves bounce off a hard surface after striking it. The reflected waves obey some laws called the laws of reflection of waves, which states that:

1. The incident wave, the reflected wave and the normal at the point of incidence all lie in the same plane.
2. The angle of incidence is equal to the angle of reflection.

Fig. 14.23 shows how waves are reflected on a plane surface.

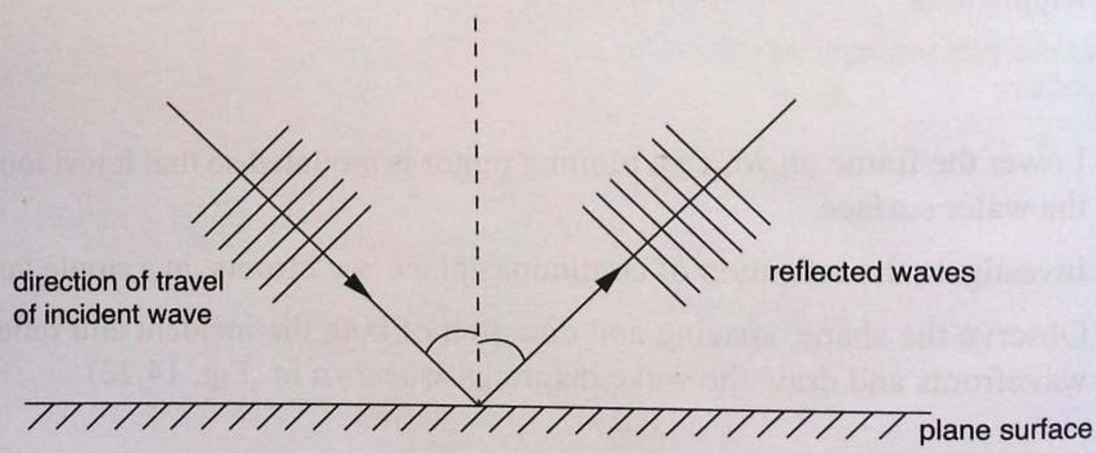


Fig. 14.23: Regular reflection of a parallel wave

If Experiment 14.1 is repeated using a single dipper fitted on the straight frame, the pattern of circular wavefronts shown in Fig. 14.24 is observed.

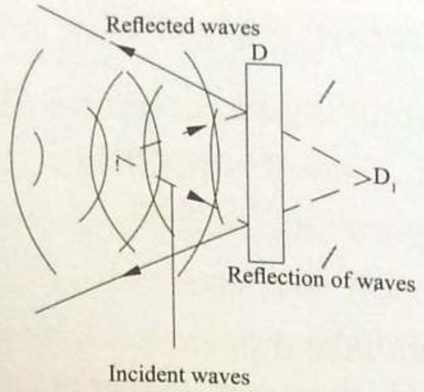
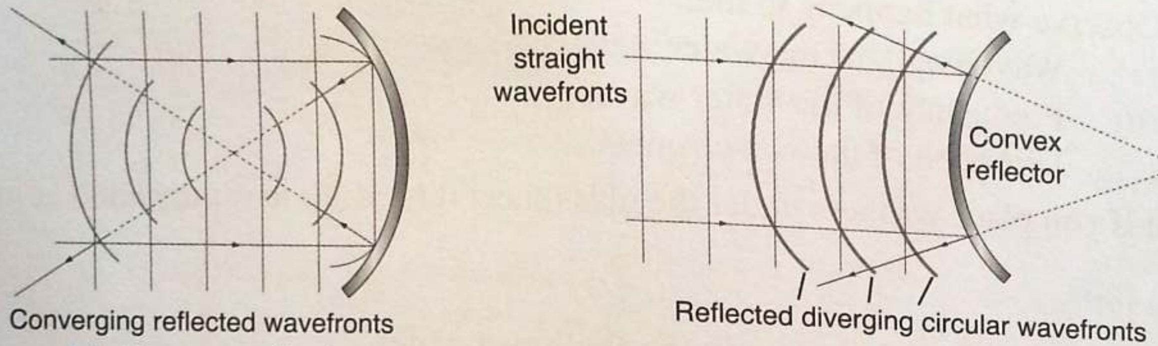


Fig. 14.24: Circular wavefronts

Similar experiments done using straight vibrator (straight waves) and curved reflectors produce the patterns below (Fig. 14.25).



(a) Concave reflector

(b) Concave reflector

Fig. 14.25: Concave and convex reflector

Refraction of waves

Activity 14.4

Dip a plastic ruler into a transparent container of clean water and view the ruler from the top and from the side of the container (Fig. 14.26).

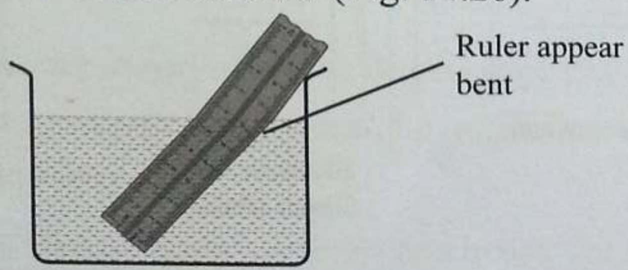


Fig. 14.26: Appearance of a ruler in water.

From Activity 14.4, we notice that, the ruler seems to be bent at the point where it enters into water. These observations are some optical illusions we observe in everyday life.

Refraction of waves is bending of waves when they pass from one medium to another. Also refraction of the wave is the change in the speed of the wave when it passes from one medium to another. This is demonstrated in the experiment 14.2.

Experiment 14.2: To show the refraction of water wavefronts

Apparatus

- Ripple tank
- thick glass
- water
- sheet

Procedure

1. Set up the ripple tank with the dipper.
2. Place a thick glass sheet in the water so that the depth of water over the glass is less than the depth elsewhere in the tank.
3. Repeat the procedure, this time, with the sheet of glass placed at an angle to the incident waves.
4. Observe what happens to the:
   (a) Wavelength of the water waves,
   (b) Frequency of the water waves,
   (c) Direction of the wavefronts.

Note: If you place washers under the glass sheet it is easier to remove and adjust.

Observations

(a) The wavelength decreases in the shallower water.

(b) The frequency does not change as water waves crosses from deep to shallow region.

(c) The direction of the waves change when the waves strike the shallower water at an angle. Fig. 14.27 (a) and (b) illustrates some of these observations.

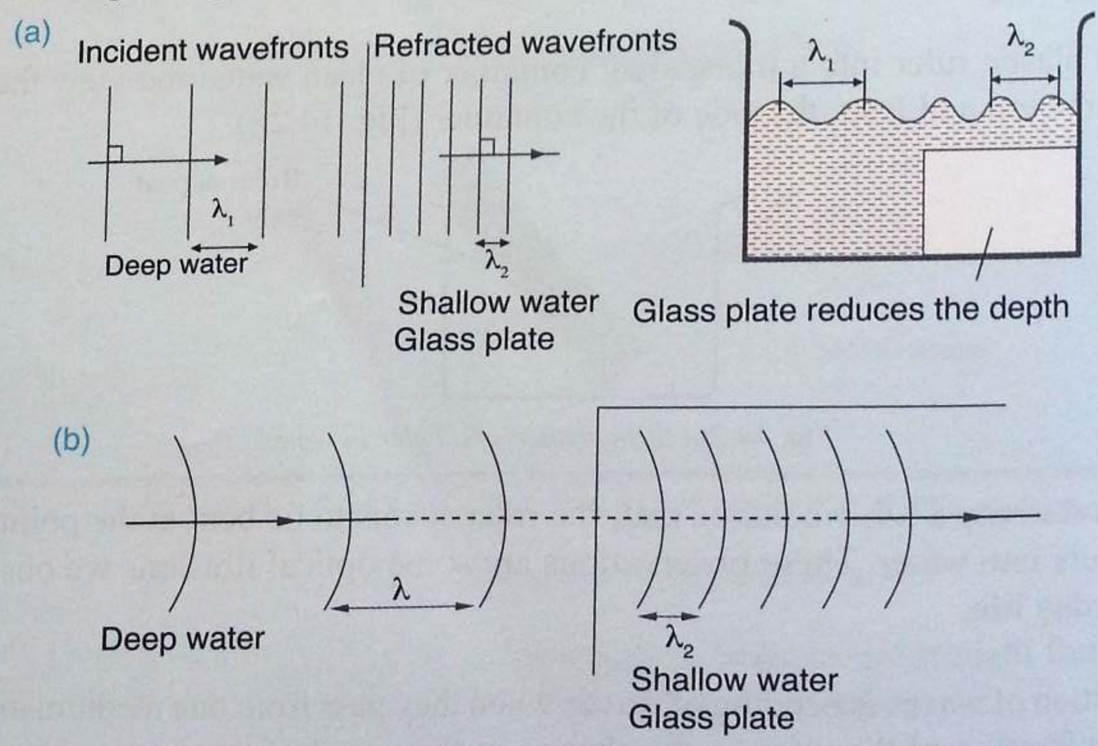


Fig. 14.27: Refraction of wavefronts

Discussion

It is observed that the wavefronts travel straight through but the wavelengths are different. The wavelength of the plane wavefronts (λ_2) in the shallow region is less than the wavelength (λ_1) in the deeper part. The wave is observed to be slower in the shallow end and faster in the deep end. The frequency of the waves does not change as the waves move into the shallow water. The speed (v) of water waves is given by $v = f\lambda$. As the wavelength λ_2 has decreased in the shallow regions, the speed v_2 of the waves must decrease. The *waves therefore travel more slowly in shallow water*.

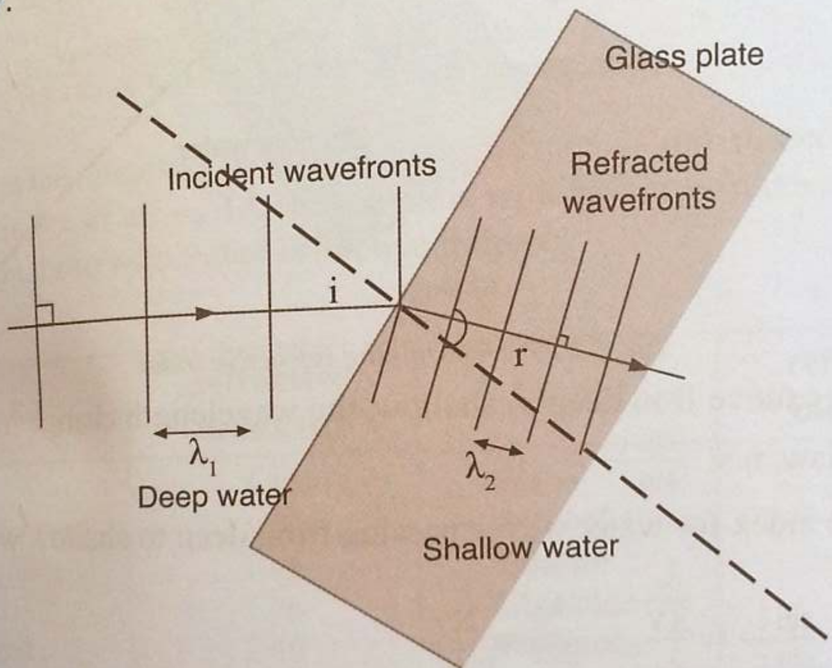


Fig. 14.28: Refraction of plane waves

When the glass is at an angle we observe that the direction of travel of the waves in the shallow region is *bent towards the normal*, i.e. refraction has taken place (Fig. 14.28).

The shallow water behaves like a denser medium and hence the speed of water waves is less. When light enters an optically more dense medium it bends towards the normal. The converse is also true. This is the reason why objects look broken when viewed through a transparent medium.

Refractive index of medium

Consider a wavefront travelling from deep to shallow region. Fig. 14.29.

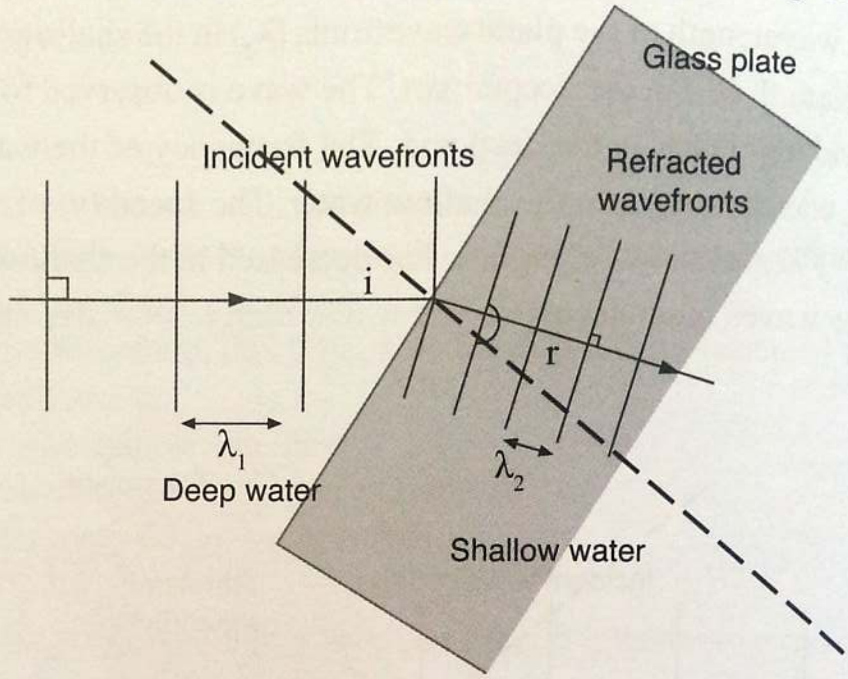


Fig. 14.29: Determining refractive index

When the wave move fron deep to shallow, the wavelength change from λ_1 to λ_2.

From Snell's law, $\eta = \dfrac{\sin i}{\sin r}$

The refractive index for water waves passing from deep to shallow water is

$$\eta = \frac{\sin i}{\sin r} = \frac{\frac{\lambda_1}{XY}}{\frac{\lambda_2}{XY}} \times \frac{\lambda_1}{XY} = \frac{XY}{\lambda_2}$$

$$= \frac{\lambda_1}{\lambda_2}$$

Experiments proves that frequency, f, of the waves remains unaltered. using wave equation, $v = f\lambda$,

Velocity in deep water $= v_1 = f\lambda_1$

Velocity in shallow water $= v_2 = f\lambda_2$

$${}_1\eta_2 = \frac{v_1}{v_2} = \frac{f\lambda_1}{f\lambda_2} = \frac{f\lambda_1}{f\lambda_2}$$

Therefore;

$$\text{Refractive index} = {}_1\eta_2 = \frac{V_1}{V_2} = \frac{\text{Velocity in deep water}}{\text{Velocity in shallow water}}$$

Where ${}_1\eta_2$ is the refractive index of a wave travelling from deep water to shallow water.

Generally, refractive index of a wave travelling from medium 1 to medium 2 is

$$_1\eta_2 = \frac{\text{Velocity of wave in medium 1}}{\text{Velocity of wave in medium 2}}$$

If the wavefront are travelling from medium 2 to medium 1, the refractive index is

$$_2\eta_1 = \frac{\text{Velocity of wave in medium 2}}{\text{Velocity of wave in medium 1}}$$

$$= \frac{1}{_1\eta_2}$$

Generally, when a wave is travelling from air to a more opticaly dense medium, refractive index of the medium is given by

$$_a\eta_m = \frac{\text{Velocity of wave in air}}{\text{Velocity of wave in a medium}} = \frac{V_{air}}{V_{medium}}$$

Table 14.4 gives the refractive indices of some substances with respect to air (taking the refractive index of air as 1.00). Materials with higher refractive indices bend wave more than those with lower refractive indices.

Table 14.4

Solid	**refractive index (η)**	**Liquid**	**refractive index (η)**
Ice	1.31	Water	1.33
Glass (crown)	1.50	Alcohol	1.36
Glass (flint)	1.65	Paraffin	1.44
Ruby	1.76	Glycerine	1.47
Diamond	2.40	Turpentine	1.47

Example 14.8

A light wave passing from air to glass is incident at an angle of 30°. Calculate the angle of refraction in the glass, if the refractive index of glass is 1.50.

Solution

Refractive index of glass $_a\eta_g = \frac{\sin i}{\sin r}$

$$\therefore \quad \sin r = \frac{\sin i}{\eta_g} = \frac{\sin 30°}{1.50} = \frac{0.50}{1.50} = 0.33$$

$$r = \sin^{-1}0.33$$

$$= 19.5°$$

$\therefore$ The angle of refraction in glass is 19.5°

Example 14.9

Calculate the refractive index of water, given that the velocity of a light wave in air is 3×10^8 m/s and velocity of a light wave in water is 2.25×10^8 m/s.

Solution

$$\eta_w = \frac{\text{velocity of light in air } (c)}{\text{velocity of light in water } (v)} = \frac{3 \times 10^8 \text{ m/s}}{2.25 \times 10^8 \text{ m/s}} = 1.33$$

Example 14.10

The velocity of a light wave in glass is 2.0×10^8 m/s. Calculate (a) the refractive index of glass and (b) the angle of refraction in the glass for a ray of light passing from air to glass at an angle of incidence of 40°.

Solution

(a) $${}_a\eta_g = \frac{C_{air}}{V_{Glass}} = \frac{3 \times 10^8 \text{ m/s}}{2 \times 10^8 \text{ m/s}}$$

$${}_a\eta_g = 1.50$$

(b) $${}_a\eta_g = \frac{\sin i}{\sin r} = 1.50$$

$$\sin r = \frac{\sin i}{1.50} = \frac{\sin 40°}{1.50} = 0.428$$

$$r = \sin^{-1} 0.428$$

$$= 25.4°$$

Example 14.11

A light wave passes from a liquid to air. Calculate the refractive index of the liquid, if the velocity of light in the liquid is 2.4×10^8 m/s, while in air is 3.0×10^8 m/s.

Solution

$$\text{Refractive index of a liquid} = \frac{\text{velocity of light in air}}{\text{velocity of light in liquid}}$$

$$= \frac{3 \times 10^8}{2.4 \times 10^8} = 1.25$$

Exercise 14.3

1. Define the term refractive index.
2. (a) State the laws of refraction of a wave.
   (b) Describe an experiment to determine the refractive index of a rectangular glass block.
3. A light wave travels through glass of refractive index 1.60 with a speed v m/s. Calculate the value of v, if the speed of light in air is 3.0×10^8 m/s.
4. The light wave passing from glass to air is monochromatic and has a frequency of 4×10^{14} and a wavelength of 5×10^{-7}m in glass
   (a) What is meant by monochromatic?
   (b) Calculate the velocity of light in glass
   (c) Calculate the velocity of light in air (refractive index of glass is 1.50)
5. The velocity of water in a shallow water is 2.5 m/s. Calculate the velocity of water wave in a deep water if the refractive index of the water is 1.32.

Diffraction of waves

You may have observed the following:

- Sound waves produced in one room spread into another through a door and a person can hear that voice from any point in the second room.
- Light from one room does not fill the next room through a door or a window. Sound waves were heard in the other room because they can spread around a small hole. This is known as the *diffraction of waves*.

To understand these effects, let us go through the experiment in Activity 9.3.

Experiment 14.3: To investigate diffraction of water waves

Apparatus

- Ripple tank

Procedures

1. Set up the ripple tank to produce continuous straight waves and place two straight metal barriers in the tank parallel to the wavefronts in such a way that they form a narrow slit between them.
2. Investigate the path of the continuous plane wavefronts and observe the shape, spacing and the direction of the wavefronts as they emerge from the slit formed by the metal barriers.

3. Vary the gap between the two barriers and observe the effects of the size of the gap.
4. Vary the wavelength of the waves by adjusting the motor speed on the vibrating wooden rod, and observe the effect of the different sizes of the gap on the waves.
5. Compare the emergent waves with the incident waves in each case.

Discussion

When the gap is very wide, wavefronts emerge almost straight as shown on Fig. 14.30(a). If the width of the gap is reduced a bit, there is a slight curvature at the edges as shown on (Fig. 14.30 (b)). When the gap is narrow (Fig. 14.30 (c)), the wavefronts become circular and they appear to diverge from a point in the gap. The circular wavefronts spread out around the edges of the gap.

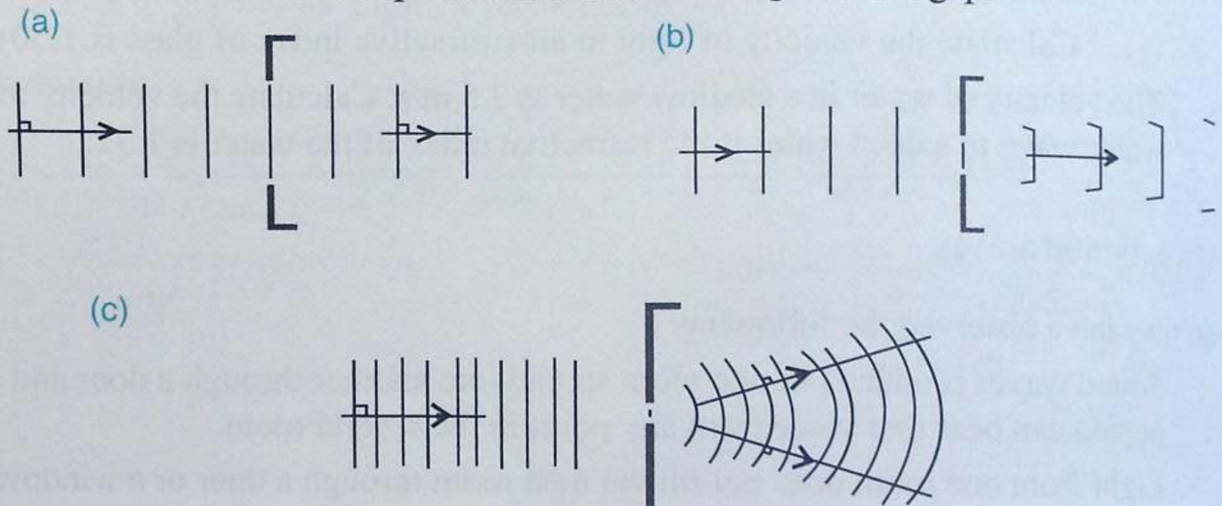


Fig. 14.30: Diffraction of straight wavefronts

The spreading of waves round the edge and corners of obstructions is called *diffraction*. Experiments show that the diffraction effect is most pronounced when the gap width is comparable to the wavelength of the waves.

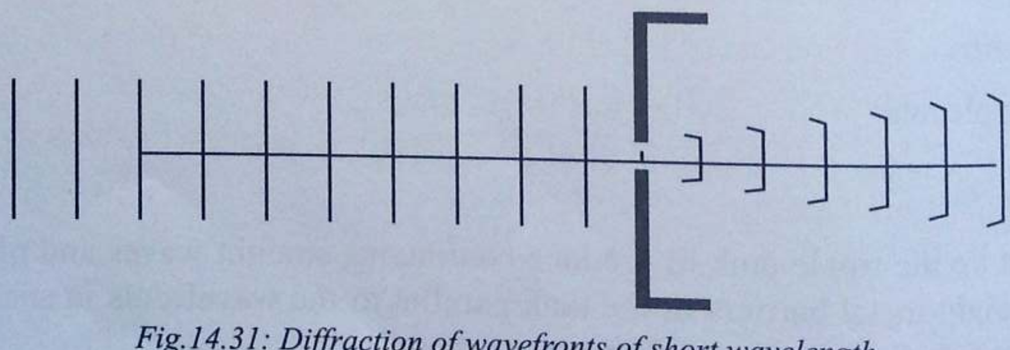


Fig.14.31: Diffraction of wavefronts of short wavelength

The wavelength of light is extremely small to the extent that the opening required to diffract light is very small. This means light moves unaffected (straight line) as shown in Fig. 14.30(a). This is the reason why wide openings like doors, windows etc. do not diffract light.

Diffraction of waves at a sharp edge appear as shown in Fig. 14.32.

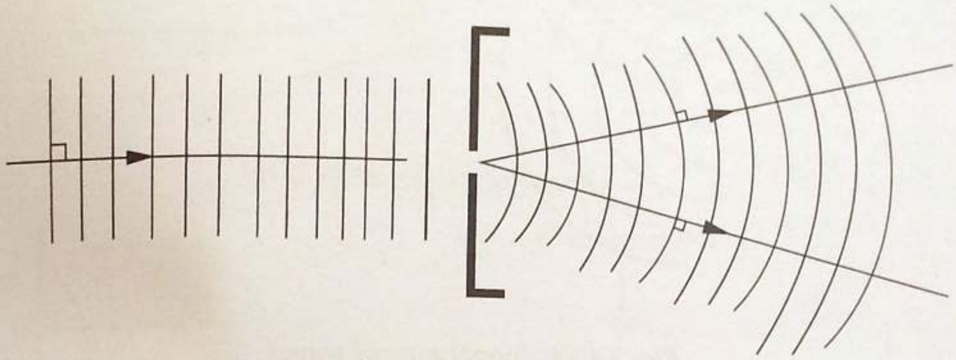


Fig. 14.32: Diffraction of waves at a sharp edge.

Interference of waves

When progressive (travelling) waves meet or interact in their medium of travel, they are said to interfere and the resulting observable effects are referred to as *interference patterns*. Interference is sometimes referred to as *superposition*.

Experiment 14.4: To demonstrate interference of water waves

Apparatus

- Ripple tank

Procedure

1. Fix two dippers, about 3 cm apart, on the wooden bar and raise it so that the dippers just touch the surface of the water in the ripple tank.
2. Start the motor to generate continuous circular waves on the surface of water. The two dippers, vibrating with the same frequency, produce waves that are *in phase* e.g. a trough and trough or crest and crest.
3. Observe what happens where the two sets of circular waves overlap and draw the wave diagrams.

Observations

Fig. 14.32 shows the interference pattern obtained.

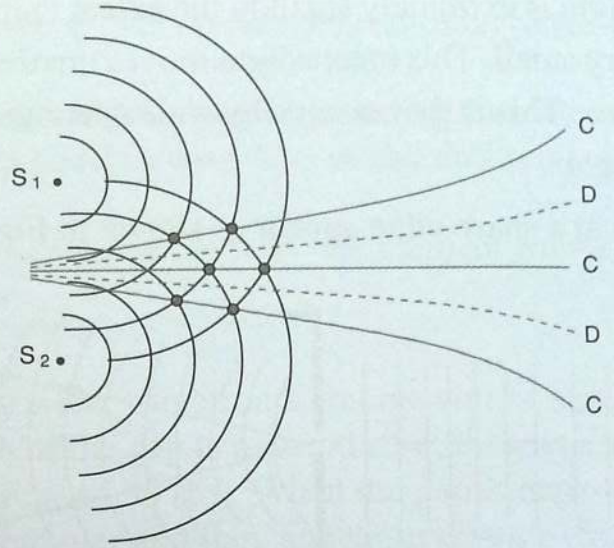


Fig. 14.33: Inteference of waves

Discussion

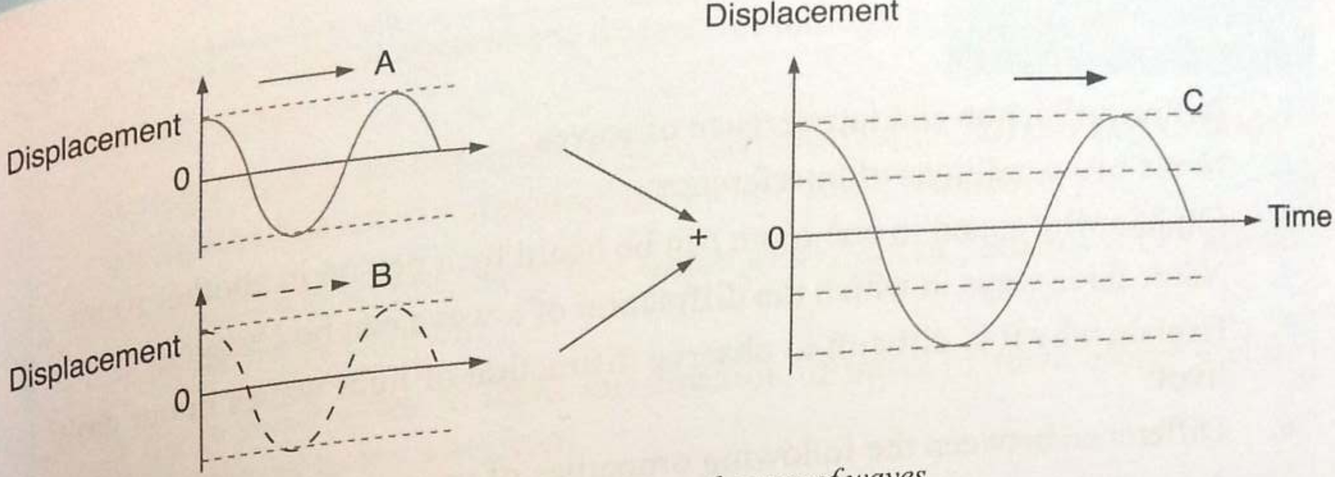
Along the solid lines marked C, the amplitude of the disturbance is maximum. A tiny piece of cork placed here shows maximum amplitude. Along this line, waves are meeting in phase and reinforcing each other. Along the dotted lines marked D (between the solid lines), the water is still. A piece of cork placed here remains undisturbed. Along this line, waves are meeting out of phase e.g. (trough and crest) and destroying each other.

The term *interference* is used to describe the effect of overlapping of the waves travelling through the same medium in the same direction.

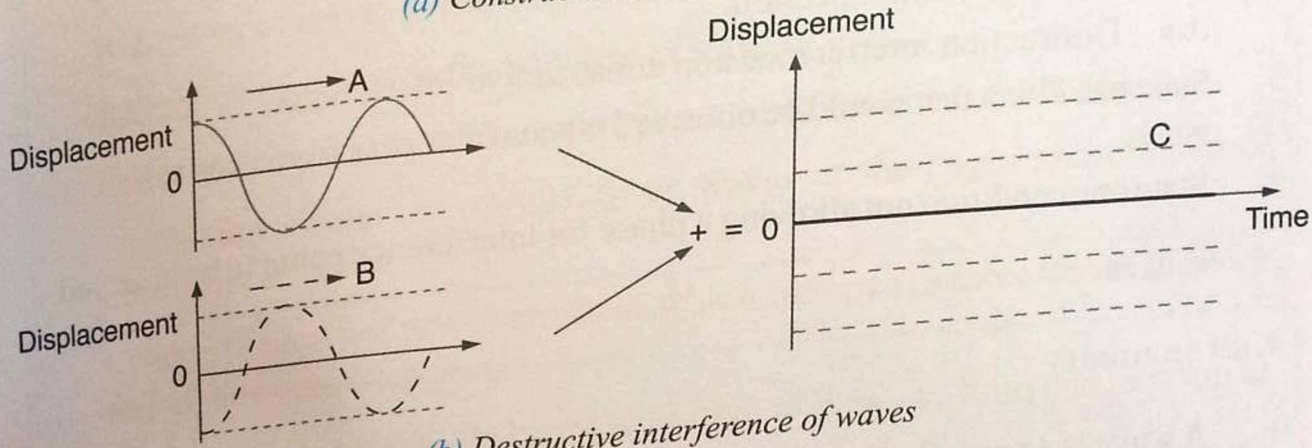
Constructive and destructive interference

Consider two identical waves having the same wavelength, amplitude and speed travelling through the same medium in the same direction. If the two waves are *in phase,* then the resultant amplitude is *higher* as shown on (Fig. 14.34 (a)) i.e., the waves reinforce each other. If they are *out of phase* (crest and trough), the resultant amplitude is smaller or is *zero* i.e., the waves cancel each other as shown in Fig. 14.34 (b).

In Fig. 14.34 (a) and (b), the approaching wave A is represented by a solid curve and the other wave B by a dotted curve. The resultant of the two waves is repeated as wave C.



(a) Constructive interference of waves



(b) Destructive interference of waves

Fig. 14.34: Constructive and destructive interference

Waves, which on meeting produce a larger displacement i.e. reinforce each other, are said to interfere *constructively* (constructive interference), while those which destroy each other are said to interfere *destructively* (destructive interference). Sources which produce waves *in phase* and of the same frequency are called *coherent sources*.

When the crests of the waves from two coherent sources meet, the amplitude is doubled and the superposition is said to be *constructive*. When a crest and a trough of waves of two coherent sources meet, the amplitude is zero and the superposition is said to be *destructive*.

When Experiment 14.4 is done with light, a series of dark and bright lines called *fringes* are observed. This experiment requires special apparatus since the distance between the two point sources have to be invisibly small.

Interference also occurs in sound waves. For example, when two speakers are placed several metres apart in a room, alternating loud and very low sounds are heard in adjacent regions as one walks across the room.

Exercise 14.4

1. Define diffraction and interference of waves.
2. Name two conditions of interference.
3. Explain why sound in one room can be heard by a person in another room.
4. Name three ways in which the diffraction of a wave can be changed.
5. Explain why it is difficult to observe diffraction of light waves in our daily lives.
6. Difference between the following properties of waves:
   (a) Refraction and reflection.
   (b) Destruction interference adn constructive interference.
7. State one effect that would be observed when water pass from deep to shallow water.
8. State one condition not allowing a phase for interference patter to be observed.

Unit summary

- A wave is a periodic disturbance that transfers energy in space from one point to another in a medium.
- When a rope fixed at one end is shaken up and down two waves trains are produced: transversal and longitudinal.
- In longitudinal waves, motion of the medium particles are displaced in the same direction as that of the travel of wave. In the transversal waves motion of the medium particles are displaced perpendicular to the direction of the travel of the wave.
- Wavelength is the distance between successive crests or troughs of a wave.
- Amplitude is the displacement of a particle from its mean or rest position.
- Period is the time taken for a wave to make one cycle.
- Frequency is the number of waves passing at a given point per second.
- Compression is a region in a longitudinal wave with high concentration of vibrating particles.
- Rarefaction is a region in a longitudinal wave with low concentration of vibration particles.

- A pulse is a single disturbance that moves through a medium from one point to the next point.
- A ripple tank is an apparatus used to demonstrate the various properties of waves like reflection, refraction, diffraction and interference.
- A wavefront is an imaginary line which joins a set of particles which are *in phase* in a wave motion.
- A ray is a line draw to show the direction of travel of wave energy and is perpendicular to the wavefront.
- Reflection of waves is the bouncing back of waves.
- Refraction of waves is the bending of waves.
- Water and sound waves like light waves, obey the laws of reflection.
- Diffraction means the spreading of waves at the edges and corners of obstructions.
- Coherent sources of waves are those which emit two or more waves *in phase* and of the same frequency.
- Refraction of a wave is the bending of light as it passes from one medium to another.
- Refraction of a wave takes place as the velocity of a wave is different in different media.
- Laws of refraction of a wave states that:
  1. The incident wave, the refracted ray and the normal to the surface at the point of the incidence are all in the same plane.
  2. The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media (this law is also known as Snell's law).
- According to Snell's law, $\frac{\sin i}{\sin r}$ = a constant known as the refractive index of the medium with respect to air, when the incident wave is in air.
- Refractive index of a medium (η) is defined as

$$\eta = \frac{\text{velocity of light in air } (c)}{\text{velocity of light in a medium } (v)}$$

Unit Test 14

1. What causes mirages on the tarmac road? (Hint: mirage is a phenomenal where on a sunny day tarmac road appear to have a pool of water at a distance).
   A. Reflection B. Refraction
   C. Diffraction D. Propagation
2. Two waves that are in phase, they form a type of interference called _____.
   A. Constructive B. Destructive
   C. Coherent D. Out of phases
3. When plane waves are reflected, the reflected waves take the shape of the reflecting surface. Draw the reflected waves emerging from a:
   (a) Convex reflector
   (b) Concave reflector
4. Copy and complete this paragraph about waves.
   When a wave enters a shallow region, it __________ down and bends towards the __________. This change of direction is called __________.
5. What happens to the following properties of waves after the waves move into shallow water.
   (a) Frequency (c) Wavefront direction
   (b) Speed (d) Wavelength
6. The figure 14.35 shows a displacement-time graph for a certain wave

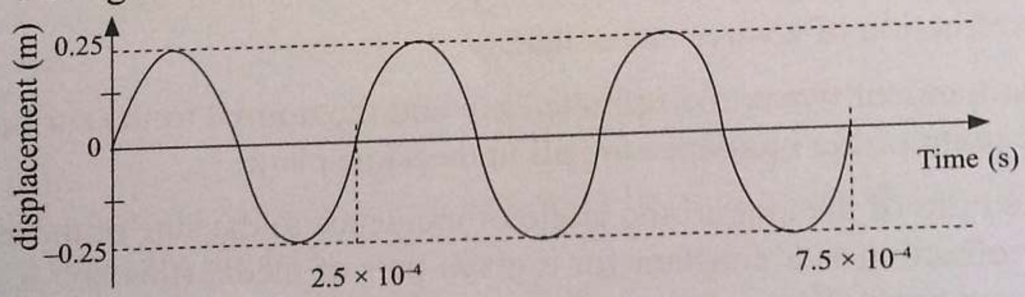


Fig. 14.35: Displacement - time graph

   (a) Identify the type of wave.
   (b) State the period of the wave.
   (c) Determine the frequency of the wave.
   (d) If the wave has a wavelength of 3.5 cm, what is its velocity?
7. A wave source generate 300 waves signals in a second. Each of the wave signals has a wavelength of 4.5 cm.
   (a) Determine the:
      (i) Frequency of the wave.

(ii) Period of the wave.

(iii) velocity of the wave.

(b) Determine the time taken by the generated waves to hit a barrier that is 250 m away from the wave.

8. Using specific properties of light, explain why it is a transverse wave.

9. Define the following terms and state its S.I units:

(a) Amplitude (b) Period

(c) Wavelength (d) Frequency

10. A radio station broadcasts on a frequency of 88.5 kHz producing signals of wavelength 3389.83 m. Determine:

(a) The period of its signals

(b) The velocity of radiowaves

(c) The velocity of radio Africa if its signals have a wavelength of 3405.22 m braodcasting on the frequency 88.5 kHz.

11. (a) Give the meaning of the symbols in the equation $v = f\lambda$.

(b) Calculate the wavelength of a wave if the speed is 45 m/s and the frequency is 5 Hz.

12. Radio wave travel with a speed of 3×10^8 m/s in air. If a radio station broadcasts at a wavelength 125 m, calculate the frequency of the transmitted waves.

13. Fig. 14.36 shows the drawing of a student in Form 3, who attempts to show diffraction pattern of the waves when they pass through the gap in the barrier.

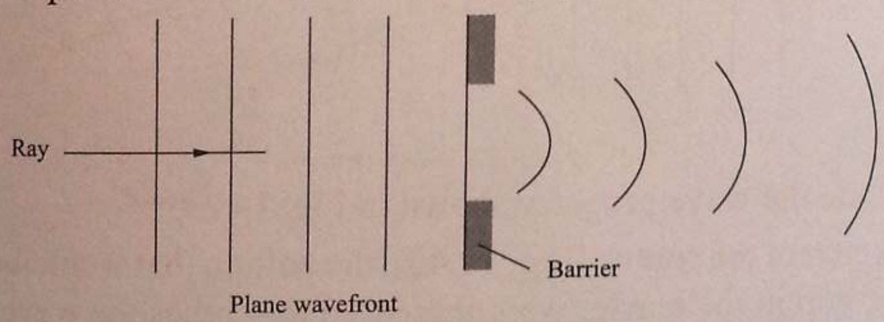


Fig. 14.36: Diffraction of waves

(a) Write down two things that are wrong with the wave pattern shown by the student.

(b) Sketch the correct wave pattern after diffraction in Fig. 14.37.

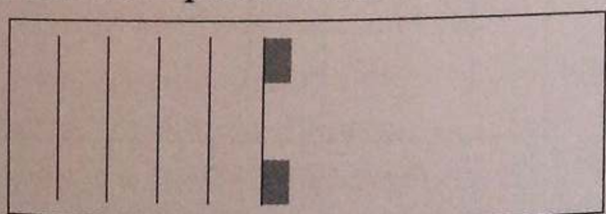


Fig. 14.37: Diffraction

(c) Calculate the frequency of the wavefronts if the speed of the water waves is 10.4 cm/s and the wavelength is 1.3 cm.

14. In a ripple tank, plane wavefronts are produced and a glass block of uniform thickness placed inside the tank as shown in Fig. 14.38.

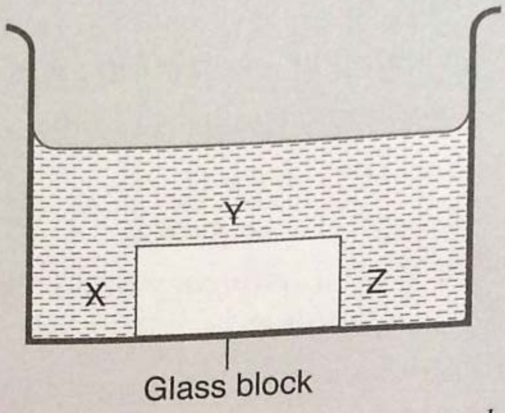


Fig. 14.38: A glass block in a tank

Draw a diagram to show the wavefronts as observed from above regions X, Y and Z.

15. Fig. 14.39 shows the passage of wavefronts through a gap in a barrier.

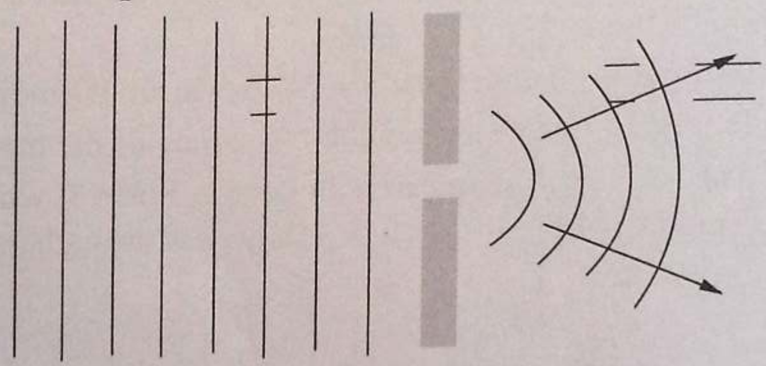


Fig. 14.39: Wave motion

(a) Name the wave property shown in Fig. 14.39.

(b) Sketch in the source (Fig. 14.40), the pattern that would be obtained, if the gap in the barrier were increased to 4 times the wavelength of the waves.

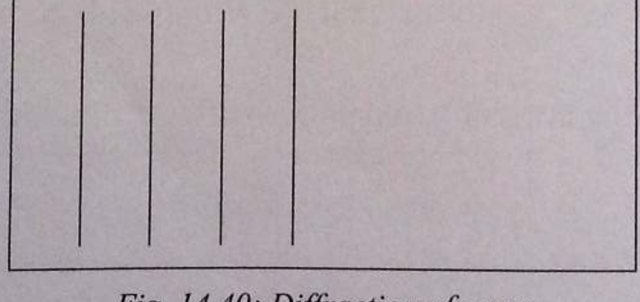


Fig. 14.40: Diffraction of wave

UNIT 15

Sound

Success Criteria

By the end of this unit, you must be able to:

- Describe experiment to show that sound is produced by vibrating bodies.
- Discuss free vibrations, forced vibrations, natural frequency and resonance.
- Explain the nature of sound waves.
- Explain the factors affecting the speed of sound.

Introduction

In Unit 14, we learnt that sound is an example of longitudinal waves. Since a wave is a form of energy, sound is thus a form of energy propagated in a longitudinal manner. In this unit, we shall study the production, propagation, characteristic and application of sound waves.

15.1 Production of sound

Sound is a form of wave caused by vibrating bodies.

Fig. 15.1 shows some of the sound producing instruments.

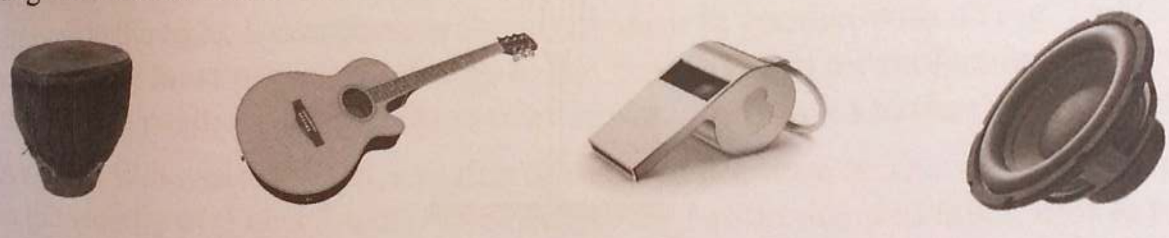


Fig. 15.1: Examples of sound producing instruments

The following activity will help us understand how sound is produced.

Activity 15.1 To demonstrate sound production

(a) Pluck a stretched metallic string or rubber band.
(b) Fix one end of a half-metre rule near the edge of one side of a table and press the free end downwards slightly and release.
(c) Blow a whistle or a flute.
(d) Hit a metallic rod against another.

(e) Hit the 'skin' of a drum gently with a piece of wood.
(f) Gently tap a glass beaker with a pen.

In each of the activites within Activity 15.1, sound is produced as the objects vibrate.

Activities 15.2-4: To show that a vibrating source produces energy

Apparatus

- A tuning fork
- A pith ball
- water in a container
- glass plate
- tooth brush bristle
- lamp soot

Activity 15.2

- Take a tuning fork and strike a hard rubber pad with one of the prongs on and observe what happens.
- Make one of the vibrating prongs of the tuning fork to touch a small pith ball suspended by a thread (Fig. 15.2) and see what happens.

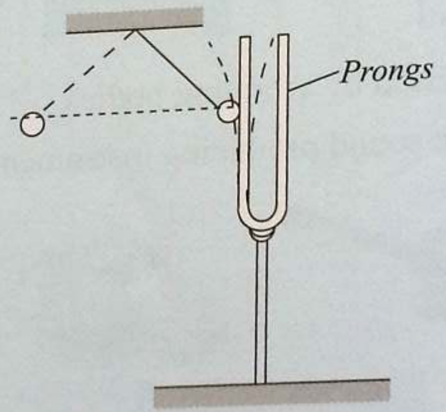


Fig. 15.2: Vibrating tuning fork displaces a pith ball.

Activity 15.3

- Dip the vibrating prongs in water in a container and observe what happens.

Activity 15.4

- Cover a glass plate with a uniform coating of lamp soot. Attach a short stiff hair of a tooth brush (bristle) to one of the prongs of a tuning fork. Set the tuning fork into vibration and let the bristle lightly touch the soot on the glass plate. Pull the glass plate gently along a straight line and observe what happens (Fig. 15.3).

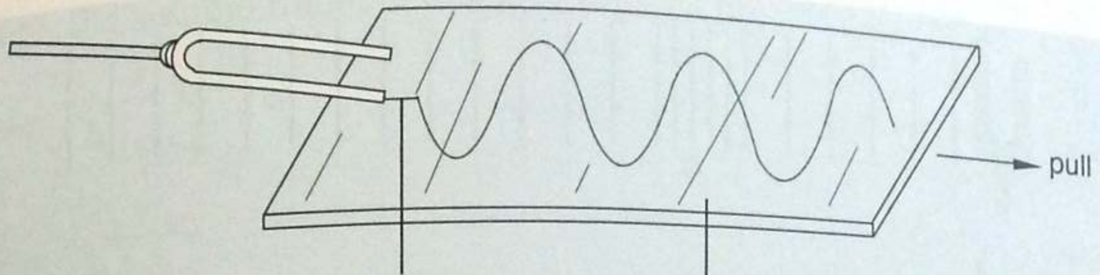


Fig. 15.3: Vibrating tuning fork on a glass plate.

Observation

Activity 15.2: The prongs start vibrating.

The pith ball is seen to be jerked to one side.

Activity 15.3: Water is violently agitated.

Activity 15.4: A wavy trace is seen on the glass plate.

Discussion and conclusion

In all activities 15.2-4, the vibrating prongs of the tuning fork produce energy. Therefore, *sound is a form of energy produced by vibrating objects.*

15.2 Nature of sound waves

Consider a tuning fork in a state of vibration as shown in Fig. 15.4. As prong X moves to the right, it compresses the layer of air in contact with it (Fig. 15.4(b)). The compressed layer passes the energy to the next layer of air molecules and returns to the original position. Thus a region of *compressions* moves to the right (Fig. 15.4(c)).

As prong X moves to the left, a region of reduced pressure or a *rarefaction* is produced in the vicinity of (Fig. 15.4(d)). The compressed air in the next layer, moves towards the left to 'equalise' the reduced pressure and hence produces another rarefaction to its right and so on. Thus a region of rarefaction moves to the right.

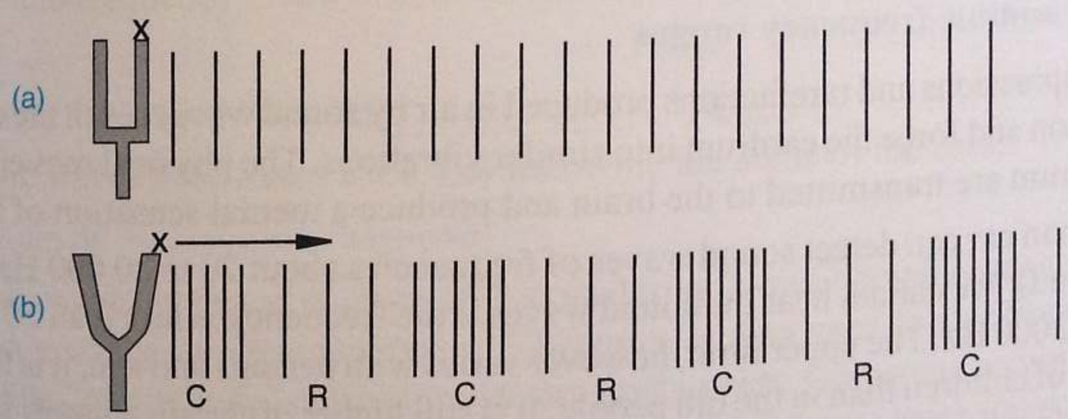


Fig. 15.4: Production of rarefactions and compressions in a sound wave.

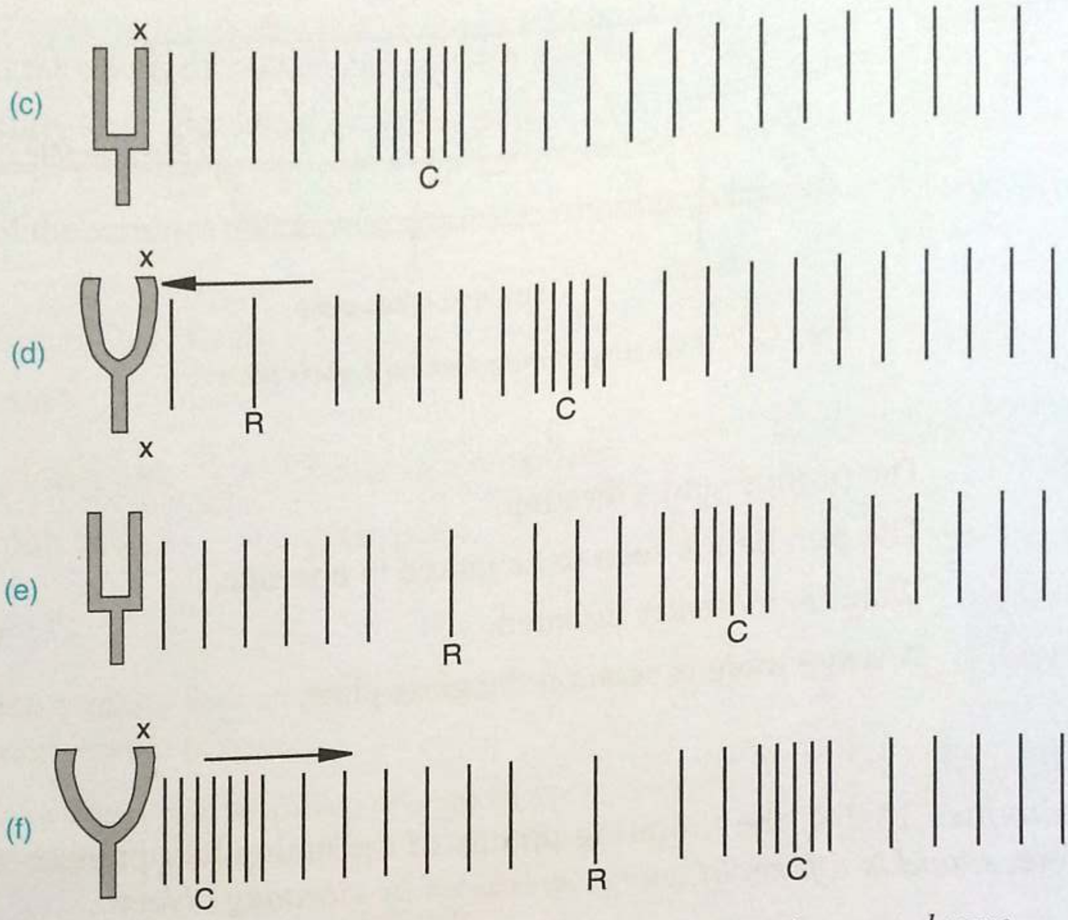


Fig. 15.4: Production of rarefactions and compressions in a sound wave.

Note: As long as the vibrations are periodical, the number of lines representing a compression must be equal to the number representing rarefaction and evenly spaced respectively.

Therefore, as the prong X vibrates to and fro, a series of compressions and rarefactions are produced. Each layer of air vibrates back and forth about its *mean* position along the direction in which propagation of energy takes place.

Thus, sound waves are *longitudinal waves*.

The wavelength of sound waves is the distance between two succussive compressions and rarefactions.

Human audible frequency ranges

The compressions and rarefactions produced in air by sound waves reach the eardrum of a person and force the eardrum into similar vibrations. The physical movements of the eardrum are transmitted to the brain and produce a mental sensation of hearing.

The human ear can detect sound waves of frequencies about 20 to 20 000 Hz (cycles per second). We cannot hear the sound waves if the frequency is less than 20 Hz or is above 20 000 Hz. The upper limit, however varies with persons and age, it is higher in the case of children than in the old people. It is still higher in certain animals like bats.

Just as other types of waves, sound waves obey the wave equation, $v = \lambda f$.

Therefore, the audible frequency ranges is given by;

$$f_{maximum} = \frac{v}{\lambda_{minimum}}$$

$$f_{minimum} = \frac{v}{\lambda_{maximum}}$$

Example 15.1

A certain animal can hear sound of wavelength in the range of 2 m to 10 m. Calculate its audible range of frequency. Take the speed of sound in air as 330 m/s.

Solution

$$f_{minimum} = \frac{v}{\lambda_{maximum}} = \frac{330 \text{ m/s}}{10 \text{ m}} = 33 \text{ Hz}$$

$$f_{maximum} = \frac{v}{\lambda_{minimum}} = \frac{330 \text{ m/s}}{2 \text{ m}} = 165 \text{ Hz}$$

Its audible frequency range is 33 Hz to 165 Hz

Ultrasonic sound

Ultrasonic sound is a sound wave that have a frequency above the normal human audible frequency range. Very high frequency waves can penetrate deep sea-water without loss of energy by diffraction. Examples of sources of ultrasonic sound is ship siren and some factory sirens.

Therefore, ultrasonic sound has a fundamental frequency that is above the human hearing range i.e. sound with fundamental frequency above 20 000 Hz.

The reverse of ultrasonic wave is the infrasonic. Infrasonic is a wave in which the fundamental frequency is lower than the human ear hearing range (audible range).

Uses of ultrasonic sound waves

Ultrasonic waves have many uses. The following are some of the uses:

1. In medical and surgical diagnosis

Ultrasonic waves are used in place of X-rays during X-radiography scanning parts of the body using an ultrasonic beam. Ultrasonic is also used to sterilize surgical instruments, jewellery and cleaning medicare instruments. Ultrasonic waves are also used to monitor patient's heart beats, kidney, growth of foetus (prenatal scanning) and destroy kidney stones.

2. In industries

Ultrasonic waves is used in cleaning of the machine parts in industries. Objects or parts with dirt are placed in a fluid through which ultrasonic waves are passed. The waves are used in analysing the uniformity and purity of liquids and solid particles.

3. In fishing

Ultrasonic waves are is used to locate shoals of fish in deep sea by the process called *echolocation* i.e. use of echo to locate an object. More interesting is that this method can detect different types of fish. This is because different fish reflect sound to different extents.

4. In security

Ultrasonic waves are used in security systems to detect even the slightest movement. Many buildings have ultrasonic motion sensors that detect motion.

Exercise 15.1

1. Define the term sound.
2. Describe an experiment to show how sound is produced.
3. Explain the following terms in respect to sound wave:
   (a) Compression
   (b) Rarefaction
4. Distinguish between utrasonic and infrasonic waves.
5. An animal has audible frequency range of 40 Hz to 20 000 Hz. Calculate the corresponding wavelengths of the frequencies.
6. Explain why a human being cannot hear sound above 20 000 Hz.
7. Explain how ultrasonic sound is used in:
   (a) Industry
   (b) Security

15.3 Characteristics of sound waves

The three main characteristics of musical sounds are:

Pitch

It is the characteristic of a musical sound which enables us to distinguish a sharp note from a hoarse one. For example, the voices of a women or of children, usually of high pitch than of men. Similarly, the note produced by the buzzing of a bee or the humming of a mosquito is of much higher pitch than the roaring of a lion, though the latter is much louder.

Pitch is purely qualitative and cannot be measured quantitatively. The greater the frequency of a vibrating body, the higher is the pitch of sound produced and vice versa. It should be noted that *pitch is not frequency; it is a characteristic depend on the frequency*. Frequency is a physical quantity and can be measured. Pitch cannot be measured.

The pitch of sound depends on the following two factors:

1. *Frequency of the sound produced*

   Pitch is directly proportional to the frequency.

2. *Relative motion between the source and the observer*

   When a source of sound is approaching, a listener or the listener approaches the pitch of sound appears to become higher. On the other hand, if the source is moving away from the listener or the listener moves away from the source, the pitch appears to become lower. (This effect is known as the *Doppler's effect*).

Intensity and loudness sound

Intensity of sound at any point is the quantity of energy received per second on a surface area of 1 m^2 placed perpendicular to the direction of propagation at those points. Thus, the intensity of sound is purely a physical quantity, quite independent of the ear and can be measured quantitatively. It is measured in *joules/second/m²*. ($Js^{-1}m^{-2}$)

The loudness of sound is the degree of sensation of sound produced in the ear. It depends on the intensity of sound waves producing the sound and the response of the ear. In general, *the sound waves of higher intensity are louder.*

Intensity of sound depends on the following factors:

1. *Amplitude of vibrating body*

   The intensity or loudness I, of sound is directly proportional to the square of the amplitude of the vibrating body.

If the amplitude of the vibrating body is doubled, the loudness of sound produced becomes two times greater.

2. *Distance from the vibrating body*

   The intensity or loudness of sound I, is inversely proportional to the square of the distance from the vibrating body.

$$\therefore \text{Intensity } \alpha \ \frac{1}{(\text{distance})^2}$$

If the distance from the source of sound is doubled, its intensity of sound becomes $\frac{1}{4}$ and so on.

3. *Surface Area of the vibrating surface*

Intensity directly proportional surface area of the vibrating body

This is because the greater the area of the vibrating surface, the larger the energy transmitted to the medium and the greater is the loudness of the sound produced.

4. *Density of the medium*

∴ The intensity of sound is directly proportional to the density of the vibrating medium

For example, an electric bell ringing in a jar filled with oxygen produces a much louder sound than the jar filled with hydrogen. Similarly, the intensity sound of a tuning fork is much higher when the stem of the fork is placed on the table than in air.

5. *Motion of the medium*

If wind blows in the direction in which the sound is travelling, the intensity of sound at a point in the direction of the wind increases and vice versa. Thus, if we shout on a windy day, the sound heard is much louder at a certain distance in the direction of the wind than at the same distance in the opposite direction.

Quality (Timbre) of sound

Quality is that characteristic of musical note which enables us to distinguish a note produced by one instrument from another one of the same pitch and intensity produced by a different instrument. If, for example, a note of a given pitch is successfully produced by a violin, a guitar or a piano, the ear can distinguish between the three notes.

For example, Fig. 15.5 represents two separate waves, one of which has the frequency twice that of the other. When the resultant of these two waves fall upon the ear, the ear is able to recognise the individual waves which have given rise to the resultant wave as they have different qualities (timbre).

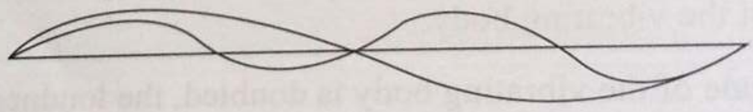


Fig. 15.5: Waves of two different frequencies

15.4 Free, forced and resonant vibrations

If we slightly displace a pendulum bob from its normal position and then let it go, it begins to vibrate to and fro with its natural frequency depending on its length and the acceleration due to gravity at that place. Similarly, a child's swing, a violin string etc. vibrates with their natural frequencies. Such vibrations which a body executes undisturbed by the influence of any other body or system are called *free vibrations* and the corresponding frequency of vibration is known as the *natural frequency* (f_0).

Sometimes, however, a body is made to vibrate with a frequency other than its own natural frequency. For example, when a tuning fork is set into vibrations and then its stem is pressed against the top surface of the table. The table begins to vibrate with a frequency equal to that of tuning fork. Such vibrations are called *forced vibrations*. Similarly all of us talk with our own natural frequency. If we talk with an "affected" tone by putting on an accent, we are forcing ourselves to talk with a forced or an artificial frequency. Thus, *when a body is compelled to vibrate with a frequency other than its own natural frequency, it is said to be executing forced vibrations*.

Activity 15.5: To demonstrate resonate vibration

Apparatus

- Four pendilum A, B, C and D. (A and B should be of the same length).
- Wooden metre rule.

Procedure

1. Suspend four pendulum A, B, C and D from a light wooden metre rule fairly well-damped at its ends (Fig 15.6)
2. Make A and B of the same length, C shorter and D longer..
3. Set A into oscillations, in a plane perpendicular to the plane containing the pendulums. What do you notice?

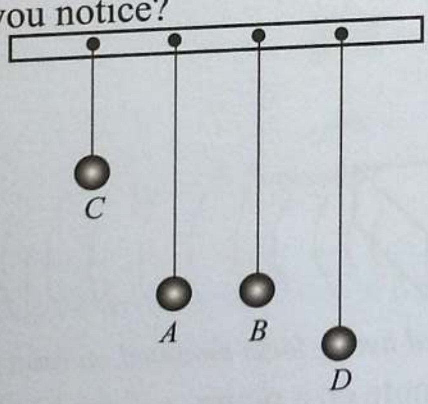


Fig. 15.6: Pendulums suspended at varying heights.

Observation and Discussion

As A starts oscillating, it exerts a periodic force on the metre rule which further transmits it to the other three pendulums.

Hence, soon after, B soon begins to vibrate with a large amplitude. The reason is that the impulses imparted to B through the metre rule arrive at intervals equal to its own natural frequency. The frequency of B is equal to the frequency of A and its amplitude is large. The vibrations of A are thus *resonant* vibrations; B is said to vibrate in *unison* with A. We say *resonance has* taken place. *Resonance is a phenomenon where one system in the vibrating state induces vibrations to another system both of which vibrate with the same natural frequency.*

The frequency of the pendulums C and D are different from that of A. The impulses do not reach them in away with their natural time periods. They first make rather irregular vibrations but eventually settle down to vibrate with the frequency of A. The pendulums C and D are said to undergo forced vibrations. These vibrations never attain large amplitude.

Some real life illustrations of resonance

The following are real life situation that explain resonance.

1. Soldiers crossing a suspension bridge are warned to "break" their steps and not to march shows across the bridge in step. If they march in step, their frequency may coincide with the natural frequency of the bridge settling it into large amplitude resonant vibrations and may even come crashing down.
2. Fig.15.7 shows two identical tuning forks A and B of same frequency mounted on resonating boxes, with their open ends facing each other (Fig. 15.7). If one of the tuning fork, say A, is set into vibrating and the vibrations stopped after a few seconds, by bonding its prong with the hand, the other fork B will be found to start vibrating though there is no direct contact between the two. The waves produced by A act on fork B and force it into vibrations. Since their frequencies are equal, resonance takes place.

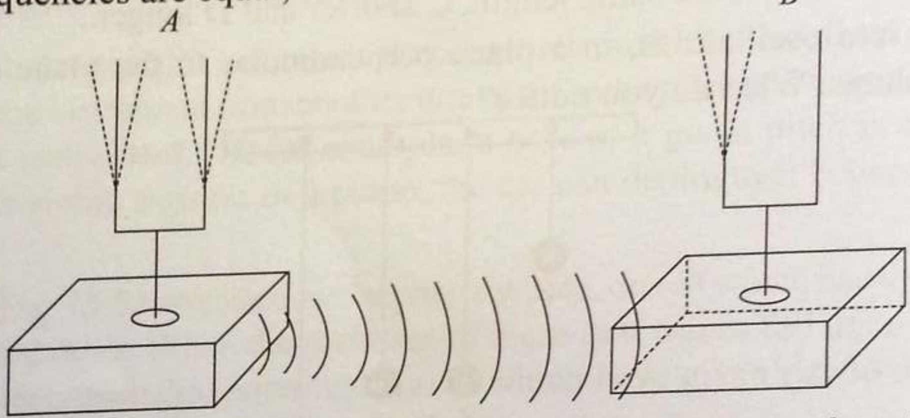


Fig. 15.7: Two identical tuning forks mounted on resonating boxes vibrate in unison.

3. If we play a particular note on a piano, a glass bottle or a piece of china-ware placed on the top of the piano or a nearby shelf is set into resonant vibrations and may even break if the amplitude of vibration is large.
4. When a car is running at a particular speed, a brisk rattling sound is heard, but the sound disappears if the speed changes. The sound is due to resonance taking place between the car engine and the rattling object.
5. Modern toys are constructed in a way that they are able to respond to a particular word of command. This is due to the resonant vibrations of a "disc" placed inside them when sound of a particular frequency falls on them.
6. In a radio or a transistor receiver set, a large current flows in a particular circuit called the "tuning" circuit, if the frequency of the electrical vibrations

of the circuit coincides with the frequency of one of the radio waves in the atmosphere. Different radio stations in the world broadcast news at different frequencies.

Sonometer

A sonometer or a monochord consists of a metal wire stretched across two wooden bridges A and B placed on a hollow wooden sound box, about a metre long. One end of the wire is tied to a peg at one end of the box. The other end passes over a smooth pulley fixed at the other end of the sound box and carries a hanger or a pen on which the desired weights may be placed (Fig. 15.8). The bridge B can slide along to have a suitable desired length of the wire between A and B. The tension, T, in the wire (T = mg) keeps the wire taut.

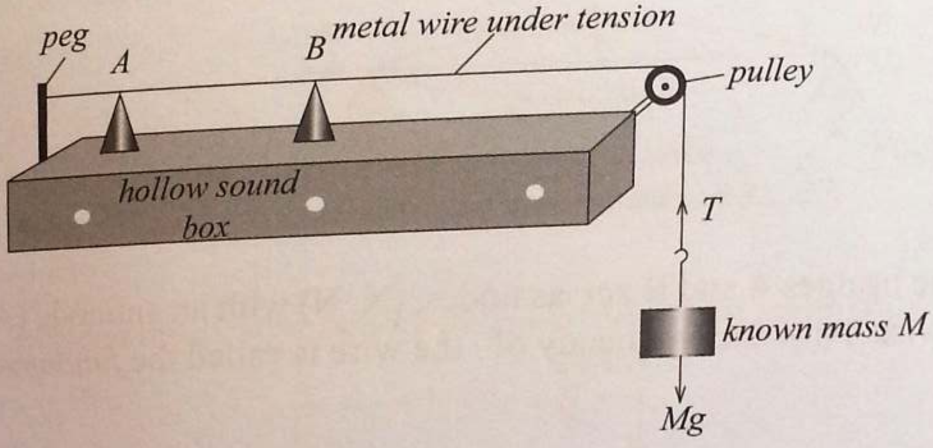


Fig. 15.8: Sonometer

Experiment 15.2: To demonstrate resonance with a sonometer

Apparatus

- A Sonometer
- A Tuning fork

Procedure

1. Set a tuning fork of known frequency into vibration.
2. Press the stem of vibration the tuning fork gently on the sonometer box. Observe the wire of the sonometer box.
3. Gradually alter the position of movable bridge and observe the vibrating wire.

Observation and discussion

The tuning fork in the vibrating state "force" the wire to vibrate. The vibrations are transmitted through the box to the wire.

The position of the movable bridge is gradually altered, so that the shortest vibrating length l, of wire is such that the frequency of the wire is equal to the frequency of the tuning fork for "resonance" to take place. The wire vibrates with a maximum amplitude.To check resonance in this position, a small piece of paper in the form of an inverted V can be place in the middle of the wire. The paper "rider" will vibrate considerably and is thrown off the wire.

When resonance takes place for the first time, the length l, of the wire $= \frac{l}{2}$ as shown in Fig. 15.9.

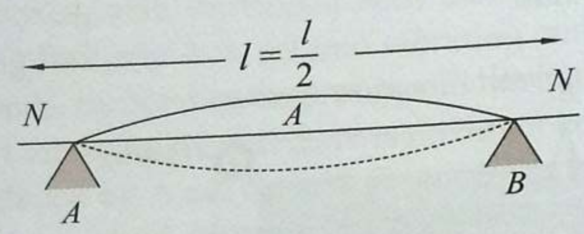


Fig. 15.9 : Fundamental frequency

The positions of the bridges A and B act as nodes, (N, N) with an antinode (A) in the middle of the wire. Now the frequency of the wire is called the *fundamental frequency*.

Note: Since $f_{wire} = f_{tuning\ fork}$, and $l = \frac{l}{2}$, we can calculate the speed of sound waves v in the wire. $v = f\lambda = f(2l)$.

Musical sounds

Musical sounds and noises

In general, sound may be roughly classified as either (a) musical sounds (b) noises. If we pluck the string of a guitar or a stretched sonometer wire or set a tuning fork into vibrations, the sound produced has a pleasant effect on our ears. If however, we listen to the slamming of a door, the sound produced by thunder clouds or the rattling sound of some parts of a car, the sounds produced have an unpleasant effect on the ears. A sound of *which appears pleasant to the ear is called a musical sound* whereas that *which produces an unpleasant or jarring effect on the ear is called a noise.*

The curves shown in Fig. 15.10 (a) and (b) bring out the difference between noises and musical sounds.

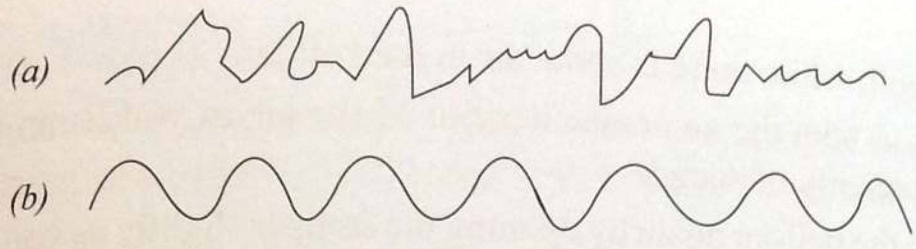


Fig. 15.10: (a) Noise (b) Musical sound

Musical sound is *regular and periodic* with pulses following each other very rapidly produce the sensation of a continuous sound. Noises, on the other hand, are generally *sudden* and have no regular period; and are usually complex in nature.

15.5 Propagation of sound

Sound waves cannot be transmitted through a vacuum. The transmission of sound waves requires at least a medium which can be a solid, liquid or a gas.

Experiment 15.3: To show that sound requires material medium to travel through

Apparatus

- Air tight bell jar
- Electric bell
- Power supply (battery) and connecting wire
- Vacuum pump

Procedure

1. Suspend the electric bell inside an air-tight bell jar as shown in the Fig. 15.11.

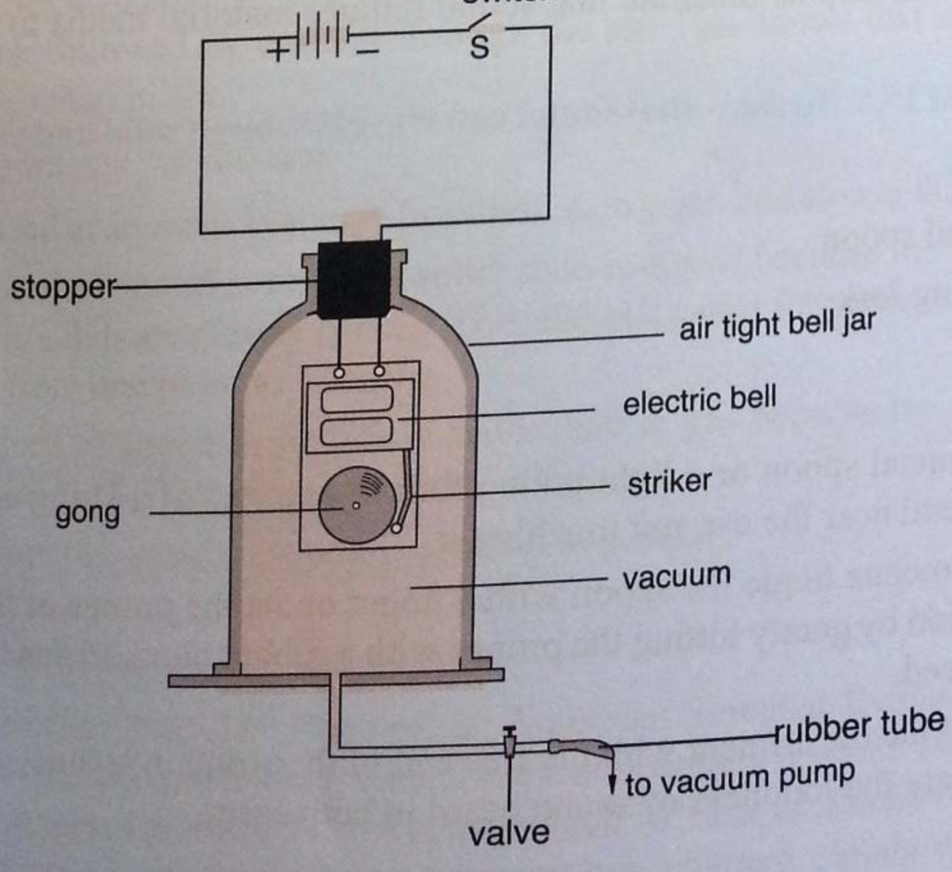


Fig. 15.11: Electric bell

2. Switch on the bell, while there is some air in the bell jar.
3. Start the pump to take the air molecules out of the jar as you listen to the change in the intensity of sound.
4. Return the air to the bell-jar again by opening the stopper slightly as you listen the change in sound.

Observation

When there is air in the jar the bell is heard ringing.

When the pump is switched on to remove the air, the sound dies down gradually and is eventually not heard at all.

When air is allowed to return to the jar, the sound is heard once again.

Conclusion

This experiment shows that a (medium) like air is necessary for propagation of sound. *Sound cannot travel through a vacuum.*

Speed of sound in solid, liquid and gas

The speed of sound is different in solids, liquids and gases. The arrangement of particles in matter determines how fast sound can travel in matter. The following experiment will help us illustrate that sound require material media to travel.

Experiment 15.4: To show that sound can travel through solid and liquid media

Apparatus

- A metal spoon
- A tuning fork

Procedure

1. Tie a metal spoon or a light tuning fork to one end of a string and hold the other end near the ear, not touching it.
2. Let someone tingle the spoon with a finger or set the prongs of the fork into vibration by gently hitting the prongs with a rubber bung. Listen to the sound produced.
3. Repeat the experiment with the free end of the string in contact with the ear. Compare the loudness of sound heard in both cases.

Observation

The loudness of sound heard is more when the string is in contact with the ear. The string transmits sound through it and does it better than air.

Discussion

This experiment shows that sound can travel through solids. Similarly if sound is produced inside water e.g in a swimming pool, it can be heard a short distance away.

From the experiments, have established that the speed of sound in water is about 1 500 m/s and in steel about 5 500 m/s.

Comparison of the speed of sound in solids, liquids and gases

The speed of sound varies in solid, liquid and gas. Activity 15.6 will help us to show the speed of sound in solid, liquid and gas.

Activity 15.6

Let one learner place the ear on one end of 20 m wooden plank, while another taps the plank once with a stone on the opposite end.

In Activity 15.6, two sounds, will be heard by the listener: one coming through the wooden plank followed by another through the air. This shows that sound travels faster in solids than in air.

Several experiments proved that

- The speed of sound is higher in liquids than in gases and slower than in solids.
- The speed of sound is faster in solid than in liquid because the particles or atoms in solids are closely packed. This makes it easier for particles to transmit sound from one point to another.
- The speed of sound in liquids is faster than in gas because the particles in liquids are relatively closer than those in gases.
- Therefore the speed of sound is slowest in gases.

Lightning and thunder

About the middle of the 18^{th} Century, an American Scientist Benjamin Franklin demonstrated that charged thunder clouds in the atmosphere produce thunderstorms. These thunderstorms produce a lot of sound which we hear as *thunder* on the earth. Due to the spark discharge occuring between two charged clouds or between a cloud and the earth, electric spark discharge called the *lightning* occurs. Though the sound due to thunder is produced first, we see the flash of lightning first and after a few seconds we hear the sound of thunder. This is due to the fact that light travels much faster than sound in air. Experiments have proved that the speed of light in air (or vacuum) is 3.0×10^8 m/s.

Take care!

Avoid walking in drain water or standing under tall trees when it is raining.

Example 15.2

The time interval between "seeing" the flash of lightning and "hearing" the sound of thunder clouds is 5 seconds.

(a) Calculate the distance between the thunder clouds and the observer on the earth.

(b) Explain why the calculated distance is only approximate. (Speed of sound in air = 330 m/s)

Solution

(a) speed of sound $= \dfrac{\text{distance}}{\text{time}}$

$$v = \frac{x}{t}$$

$$x = v \times t = 330 \times 5 = 1\,650 \text{ m/s}$$

$\therefore$ The distance between the thunder clouds and the observer is 1 650 m.

(b) The clouds may be moving.

Factors affecting the speed of sound in gases

Density

The higher the density of a gas, the lower the speed of sound. For example, the density of oxygen is 16 times higher than the density of hydrogen hence sound travels faster in hydrogen than in oxygen (speed of sound in hydrogen = $4 \times$ speed of sound in oxygen).

Humidity

Moist air containing water vapour is less dense than dry air. The density of water vapour is about 0.6 times that of dry air under the same temperature conditions. If the humidity of air increases, density of air decreases hence the speed of sound in air increases.

Early in the morning the percentage of humidity of air is more and sound travels faster in the morning air.

Pressure

The speed of sound is not affected by any change in pressure provided temperature is constant. For example, on a day when the temperature and humidity of air is the same in Lilongwe and a city at the sea level, the speed of sound is the same in the two cities, although the air pressure in Lilongwe is lower than that at the city situated at the sea level.

Temperature

A change in the temperature of a gas changes its density and hence affects the speed of sound through it. If temperature increases, the density of air decreases and hence the speed of sound increases. If temperature decreases the reverse is the effect.

Wind

Wind "drifts" air through which the sound waves travel. If air blows in the direction of sound, then the speed of sound increases. The speed of wind is added to the speed of sound in air, to get the resultant speed of sound. If wind blows in the opposite direction to that of sound, then the sound travels more slowly.

Table 15.1 summarises how the speed of sound in matter is related and their corresponding reasons.

Table 15.1

Matter	Speed of sound	Reason
Solid	Fastest	Particles are closely packed
Liquid	Medium	Particles loosely packed
Gas	Slowest	Particles are very far apart

Exercise 15.2

1. Explain why the speed of sound in solid is faster than the speed of sound in air.
2. Name two factors that affect the speed of sound in air.
3. State the characteristic of sound waves.
4. Explain why at night sound from a source is clear than during hot daytime.
5. Describe two factors that affect the pitch of sound.

6. Define the following terms:
   (a) Resonance
   (b) Quality
7. Distinguish between music and noise.
8. Explain the factors that affect the frequency of sound.
9. During thunder and lightening, there are two types of waves produced.
   (a) Name the two waves.
   (b) Which one reaches the ground first? Explain.
10. Sound is a longitudinal wave. How is it propagated? Describe an experiment to demonstrate the fact that sound is actually produced by vibrating body.

Reflection of sound waves

Just like light, sound waves undergoe reflection on stricking plane hard surface as well as curved surfaces.

Experiment 15.5: To investigate the laws of reflection of sound waves

Apparatus

- Two tubes A and B
- Hard drawing board
- Flat piece of metal
- Stopwatch

Procedure

1. Set up two tubes A and B, about 1.2 m long and 4 or 5 cm in diameter as shown in Fig. 15.12. Place a flat piece of a large metal or a hard drawing board facing the tubes about 10 cm from their ends.
2. Place a stop watch at the mouth of the tube A and place your ear at the end of the tube B. A soft board S is placed in between the two tubes to prevent the sound waves from the stopwatch to reach the ear directly.

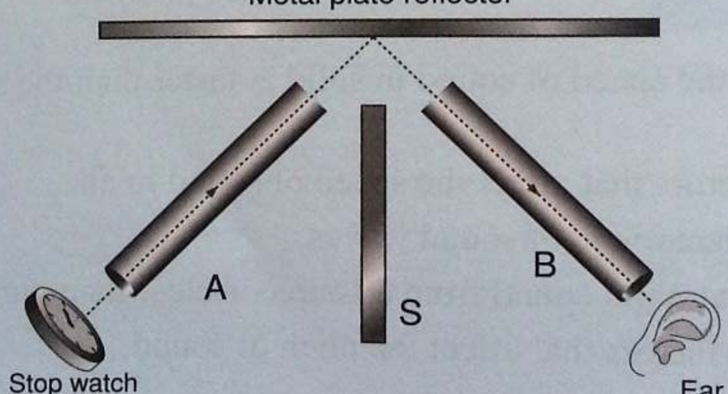


Fig. 15.12: Reflection of sound waves

3. Adjust the position of tube B until the sound heard is the loudest.
4. Measure angles of incidence i and reflection r. What do you notice? Explain.

Observation and Discussion

These angles are found to be approximately equal. Both the tubes containing the incidence waves and the reflected waves lie in the same plane as the normal to the reflecting surface. We can then conclude that *sound waves obey the laws of reflection as is the case with light wave.*

You should note that since audible sounds have large wavelengths, you need a reasonably large *reflector.*

When sound waves meet a boundary between one medium and another, a part of it is *reflected*, a part is *refracted* and the remaining part is *absorbed*. The relative amounts of these parts are determined by the size and the nature of the boundary under consideration. The proportion of energy reflected is greater in the case of hard substances such as stone and metal. An *echo*, a reflection of sound, is frequently heard in mountainous regions. There is very little reflection from cloth, wool and foam rubber. Sound which is incident on such soft materials is mainly transmitted through them or absorbed. In places where the effect of echo has to be illuminated, e.g. musical recording room and concert halls, soft materials are used to line the walls of the hall.

Uses of reflection of sound

1. Sound waves can be used to measure the speed of sound in air by reflecting sound at hard surfaces.
2. In public halls and churches, parabolic sound reflection is often placed behind the speaker. It reflects the sound waves back to the audience and thus increasing the loudness of the sound.
3. Sound waves undergo a total internal reflection just like light. Speaking metal tubes that are used to pass message on ships (Fig. 15.13) use total internal reflection of sound waves.

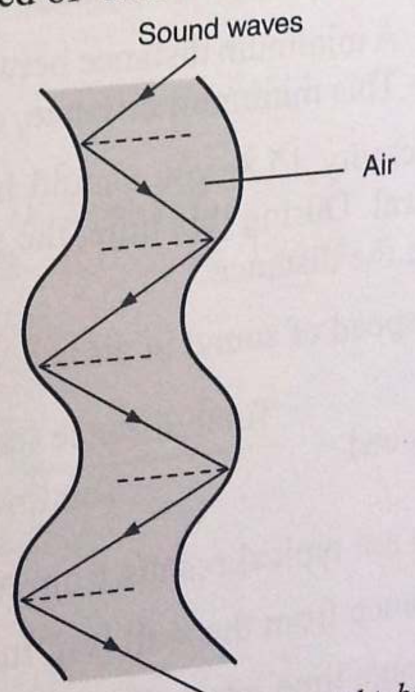


Fig. 15.13: 'Speaking' metal tubes

Determining speed of sound by echo method

Activity 15.7: To produce an echo

Stand about 100 m away from a cliff or a large hard surface such as the wall of a building and clap your hands. What do you hear.

In Activity 15.7, you will hear two sounds; the one you produce and the reflected sound.

The reflected sound produced is called an *echo*. An echo is a reflection of sound from a large hard surface.

Activity 15.8

- Stand about 100 m from an isolated, large hard surface or a stone wall.
- Shout loudly and start a stopwatch at the same time. Stop the watch on hearing the echo. Find the time interval between the production of the loud noise and hearing the echo.
- Repeat this a number of times and find the average time taken.

Note

For activity 15.8 to be more accurate:

1. A large obstacle, e.g. a cliff or a wall is needed. This is because the wavelength of sound waves is large.
2. A minimum distance between the source and the reflecting surface is required. This minimum distance, called persistence of hearing is about 17 m.

In Activity 15.8, you should have noticed that an echo is heard after some time interval. During this time, the sound travels to and from the hard surface covering twice the distance.

The speed of sound in air is given by the formula:

$$\text{Speed} = \frac{\text{Total distance travelled by sound}}{\text{Total time taken}}$$

Here are typical results from Activity 15.8:

Distance from the wall is d, metres.

Average time interval between the production of sound and hearing its echo is t seconds.

Total distance travelled by sound is $2d$ metres.

$$\text{Speed of sound} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{2 \times d\,(\text{m})}{t(\text{s})}$$

$\therefore$ The speed of sound in air is given by $\frac{2d}{t}$

Example 15.3

A girl standing 100 m from a tall wall and bangs two pieces of wood once. If it takes 0.60 s for the girl to hear the echo, calculate the speed of sound in air.

Solution

$$\text{Speed of sound, v} = \frac{\text{Total distance}}{\text{Total time taken}} = \frac{2d}{t}$$

$$= \frac{2 \times 100}{0.60} = \frac{200}{0.60}$$

$$= 330 \text{ m/s}$$

$\therefore$ the speed of sound in air is 330 m/s

Example 15.4

A man stands in front of a cliff and makes a loud sound. He hears the echo after 1.2 s. If the speed of sound in air is 330 m/s, calculate the distance between the man and the cliff.

Solution

Let the distance between the man and the cliff be x. (Fig. 15.14)

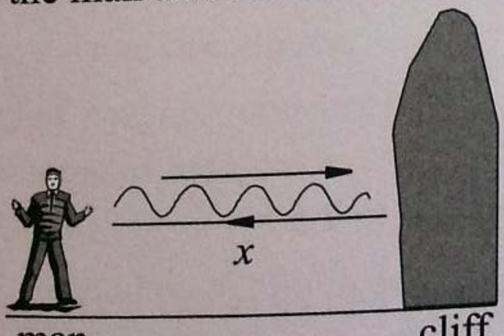


Fig. 15.14.

$$\text{Speed of sound} = \frac{\text{Total distance}}{\text{Total time}}$$

$$330 \text{ m/s} = \frac{2x}{1.2}$$

$$2x = 330 \times 1.2$$

$$= 396 \text{ m}$$

$$\therefore \quad x = 198 \text{ m}$$

The distance between the man and the cliff is 198 m.

Example 15.5

A man standing between two parallel cliffs fires a gun. He hears the first echo after 1.5 s and second echo after 2.5 s.

(a) What is the distance between the cliffs?

(b) When does he hear the third echo? (Take speed of sound in air to be 336 m/s).

Solution

(a) The sketch is as shown in Fig. 15.15.

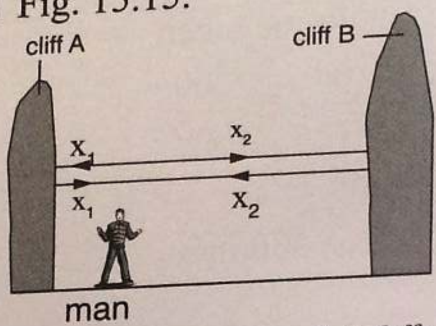


Fig. 15.15: A man between parallel cliffs

From Cliff A:

$$\text{Speed, } v, = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$v = \frac{2x_1}{1.5} \Rightarrow 2x_1 = 1.5 \times v$$

$$\therefore \quad 2x_1 = 1.5 \times 336$$

$$x_1 = 252 \text{ m}$$

From cliff B:

$$v = \frac{2x_2}{2.5} \Rightarrow 2x_2 = 2.5 \times v$$

$$\therefore \quad 2x_2 = 2.5 \times 336 = 840$$

$$x_2 = 420 \text{ m}$$

$\therefore$ The distance between the cliffs is 252 m + 420 m = 672 m

(b) The first echo (after 1.5 s) reaches cliff B and returns after 2.5 s. So the man hears the 3rd echo after 1.5 + 2.5 = 4 s.

Exercise 15.3

1. How is sound propagated?
2. Define the term echo.
3. A person stands infront of a wall and makes a loud sound. She hears the echo after 1.55. If the speed of sound is 333 m/s. Calculate the distance between the person and the cliff.
4. A person standing 150 m from the foot of a cliff claps and hears an echo after 0.9 s. What is the speed of sound in air?
5. A pupil, standing between two cliffs and 500 m from the nearest cliff clapped his hand, and heard the first echo after 3 s and the second echo 2 s later. Calculate,
   (a) The speed of sound in air,
   (b) The distance between the cliffs.
6. An echo of the sound produced by a whistle is heard after 0.50 s. If the speed of sound in air is 332 m/s, find the distance between the whistle and the reflecting surface.

15.6 Sound pollution

Sound is a very important form of energy. Human beings and animals use sound as a way of communication. But if sound is unorganised, it becomes *noise*. Any unwanted sound becomes a nuisance and leads to *pollution in form of noise*. Therefore, *sound pollution is a type of pollution caused by undesirable or unwanted sound*. Sound pollution can cause damage to the eardrum or hinder communication. Sources of sound pollution are: very high music from discos, concerts, celebrations, factory sirens etc. Everybody is encouraged to minimize sound pollution at all cost. The government through some agencies must prohibit sound pollution by enacting laws to govern this. The following are some of the ways used to minimize sound pollution.

1. Factories are encouraged to use sound sirens that are environmental friendly. Most of them use the normal fire alarms.
2. During construction of musical concert halls, the constructor should use materials that absorb most of incident waves of sound to avoid reverberation (reflected multiple sound).
3. Proper laws must be enacted by the government to reduce sound pollution.
4. Proper education of the citizens on sound pollution should be done to sensitize them on the importance of reducing sound pollution.

Listen to moderate music!
Loud music can affect your eardrum if you listen for long

Exercise 15.4

1. Explain what is sound pollution?
2. Sound wave just like light wave undergo reflection. Explain two uses of reflection of sound.
3. Explain two ways in reducing sound pollution.

Unit summary

- All vibrating bodies produce sound.
- Sound cannot travel through a vacuum. It needs a material medium like solid, liquid or gas.
- Sound waves are longitudinal in nature consisting of compressions and rarefactions.
- Human audible frequency range is between 20 Hz and 20 000 Hz.
- Speed of sound in air = 332 m/s at 0°C.
- Speed of light in vacuum = 3×10^8 m/s.
- Sound waves undergo reflection. Reflection is the bouncing back of sound wave when it strikes plan hard surface or curved surface.
- Echo is the reflection of sound from a large, rigid barrier like cliff, tall wall etc.
- Speed of sound in air can be determined by echo method.
- Density of air, humidity, temperature and wind affect the speed of sound.
- Pressure, amplitude of wave and loudness of sound do not affect the speed of sound.
- Ultrasonic waves are sound wave that have a fundamental frequency above the human audible range frequencies.
- When the frequency of a forced vibrations coincide with that of the forcing vibrations, resonance takes place.
- Resonance is a phenomenon where one system in the vibrating state induces vibrations in another system such that both vibrate with the same natural frequency. When resonance takes place, the amplitude of the forced vibrations is large.

- The three important characteristics of musical sound are the pitch, intensity and quality of sound.
- A sound which appears pleasant to the ear is called a musical sound and the one which produces a jarring effect on the ear is called noise.

Unit Test 15

1. Sound cannot pass through a
   A. solid
   B. liquid
   C. air
   D. vacuum
2. A normal human being can hear sound of frequency
   A. less than 20 Hz.
   B. between 20 Hz and 20 000 Hz.
   C. between 20 Hz and 200 Hz.
   D. above 20 000 Hz.
3. Which of the following is correct? Sound waves
   A. are transverse in nature.
   B. are longitudinal in nature.
   C. can never undergo diffraction.
   D. can never interfere with each other.
4. The speed of sound is NOT affected by
   A. pressure
   B. humidity.
   C. temperature
   D. wind.
5. Which statement is true about the music produced by the loudspeaker of a radio? When the music is made louder,
   A. the amplitude of sound decreases.
   B. the amplitude of sound increases.
   C. the speed of sound increases.
   D. the speed of sound decreases.
6. Suggest a simple experiment to establish each of the following:
   (a) Sound is produced by a vibrating body.
   (b) Sound cannot travel through vacuum.
7. State three factors which affect the speed of sound in air. Choose one of the factors and explain how it affects the speed of sound in air.

8. In which gas is the speed of sound greater, hydrogen or oxygen?

9. (a) Describe an experiment to show how echoes are produced.

   (b) The echo method can be used to determine the speed of sound in air.

      (i) What measurements would you take in order to do this?

      (ii) Show how you would calculate the speed of sound in air from your measurements.

      (iii) State a precaution to be taken to improve your result.

10. A person standing 150 m from the foot of a cliff claps and hears an echo after 0.9 s. What is the speed of sound in air?

11. A student, standing between two cliffs and 500 m from the nearest cliff clapped his hand, and heard the first echo after 3 s and the second echo 2 s later. Calculate:

   (a) The speed of sound in air.

   (b) The distance between the cliffs.

12. A worker uses a hammer to knock a pole into the ground (Fig. 15.16).

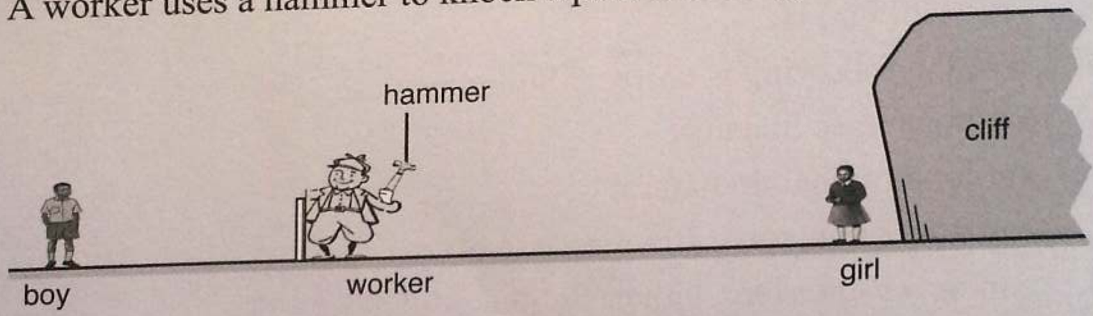


Fig. 15.16: A worker knocking hammer against the pole

   (a) A girl at the foot of the cliff hears the sound of the hammer after 2.0 s. Calculate the distance of the worker from the girl (speed of sound in air is 340 m/s)

   (b) A boy on the other side of the cliff observes that each time the hammer hits the pole, he hears two separate sounds, one after the other. Explain this observation. Given that the first sound is heard by the boy after 1.0 s, determine the:

      (i) Distance of the boy from the worker.

      (ii) Time interval between the two sounds.

13. A soldier standing between 2 cliffs fires a gun. She hears the first echo after 2 s and the next after 5 s.

(a) What is the distance between the two cliffs?

(b) When does she hear the third echo? (speed of sound in air = 336 m/s).

(c) Why is the 3[rd] echo faint than the 2[nd] one.

14. A student makes observations of a distant thunderstorm and finds the time interval between seeing the lightning flash and hearing the thunder as 4.0 s. Given the speed of sound in air = 340 m/s and speed of light in air = 3.0×10^8 m/s,

(a) Explain why there is a time delay?

(b) Calculate the distance between the thunder cloud and the student.

(c) Explain why the speed of light is not taken into account in this calculation.

(d) Calculate the frequency of the flash of light emitted if the mean wavelength of light emitted is 6.0×10^{-7} m.

15. In an athletics competition, the time keeper in a 100 m race starts the stopwatch on hearing the sound from the starter's pistol and records the time as 10.00 s. Calculate:

(a) The actual time taken by the athlete to cover the 100 m race.

(b) The average speed of the athletee.

(speed of sound in air = 340 m/s).

Glossary

Vernier callipers: an instrument used to measure short distances such as diameters of a small ball bearing, thin rod, internal diameter of a tube etc. to one tenth of a millimeter degree accuracy.

A micrometer screw gauge: an instrument for measuring very short length such as the diameters of wires, thin rods, thickness of a paper etc. to one hundredth of a millimeter degree accuracy.

Cathode ray oscilloscope (C.R.O.): an instrument that uses a cathode ray beam to display waveforms of electrical signals and to measure very small time intervals and voltages.

Independent variables: the quantity that the researcher varies to produce a change in another quantity.

Dependent variable: the quantity that changes as a result of the change in the independent variable.

Controlled variable: the quantity that is kept constant during a scientific investigation.

Pressure: force acting normally per unit area. Its SI unit is the Pascal or N/m^2.

Atmospheric pressure: pressure exerted by the vertical column of air in the atmosphere.

Barometer: an instrument for measuring atmospheric pressure.

Manometer: an instrument for measuring gas pressure.

Hydrometer: an instrument for measuring relative densities of liquids.

Kinetic theory of matter: a theory that states that matter is made up of tiny particles that are continuously in random motion.

Thermometer: an instrument for measuring temperature.

Boyle's law: the law that state that states that volume of a fixed mass of gas is inversely proportional to pressure at constant temperature.

Charles' law: the law that state that states that volume of a fixed mass of gas is directly proportional to temperature at constant pressure.

Pressure law: the law that state that states that pressure of a fixed mass of gas is directly proportional to temperature at constant volume.

Scalar: a quantity that has magnitude only.

Vector: a quantity that has magnitude and direction.

Speed: rate of change of distance.

Velocity: rate of change of displacement.

Acceleration: rate of change of velocity.

Gravitational potential energy: energy possessed by an object by the virtue of its position above the ground.

Kinetic Energy: energy possessed by objects in motion.

Velocity ratio (V.R): the ratio of the distance moved by the effort to the distance moved by the load in a machine.

Mechanical advantage (M.A): the ratio of load to effort applied on a machine.

Efficiency: the quantity (M.A/V.R) × 100% of a machine.

Electric current: the rate of flow of charge in a circuit.

Potential difference: the work done in moving one coulomb of charge from one point to the other.

Electrical resistance: the resistance of a conductor to the flow of electric current through it.

Kilowatt hour (kWh): Conventional units of measuring the electrical energy consumed.

Oscillations: repeated, regular movements that happen at a constant rate.

Amplitude (a): the maximum displacement of a particle from the mean position on either direction.

Period (T): the time taken to complete one oscillation or cycle.

Frequency (f): the number of cycles made per second.

Wavelength: the distance between two consecutive particles that are in phase.

Reflection: the bouncing back of waves when they strike an obstacle.

Refraction: the bending of waves when they pass from one medium to another.

Diffraction: spreading of waves around corners or as they pass through a narrow path.

Interference: Constructive or destructive interaction of two or more waves.

Ultrasonic sound: sound wave with a frequency above the normal human audible frequency range.

Resonance: is a phenomenon through which a body is made to vibrate with a frequency equal to its own natural frequency.

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