

TYPES OF NUMBERS

1. Natural numbers

They are numbers(1,2,3,4,5,.....). They are also called positive integers, counting numbers or natural numbers.

2. Whole numbers

This is a set of natural numbers, plus 0

$$\{0,1,2,3,4,5,6, \dots \}$$

3. Integers

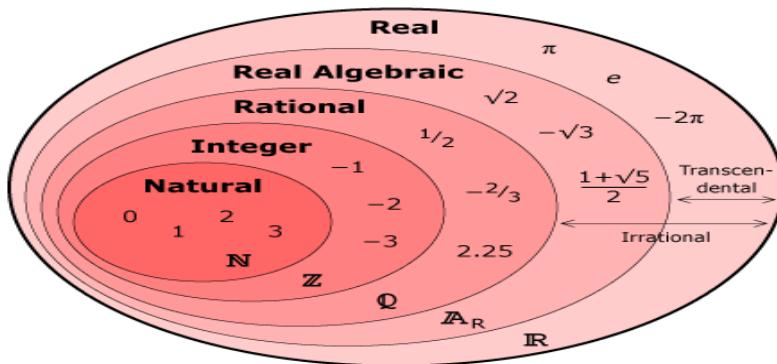
This is a set of all whole numbers plus all the negatives of the natural numbers, i.e. $\{-2, -1, 0, 1, 2, 3, 3, \dots\}$

4. Fraction

Fractions are parts of whole numbers. These indicate that an integer is being divided by another integer. Examples $\frac{1}{2}, \frac{3}{4}, \frac{5}{11}$ etc

5. Decimals

Decimals are numbers based on the decimal system or the 10s system. Decimals use a dot or a point to separate the ones place from the tenths place in a number. There are one or more digits to the right-hand side of the dot or point called the decimal point.



6.

EVEN NUMBERS

Integers which are divisible by 2 are known as **even** numbers. In other words, numbers whose unit place is 0, 2, 4, 6, 8 are divisible by 2 and such numbers are **even** numbers.

OLD NUMBERS

Integers which are not divisible by 2 are known as odd numbers. They have 1, 3, 5, 7, 9 at their unit place.

PRIME NUMBER

Prime numbers are those numbers which has only two factors ie. 1 and the number itself. For example:

- a. 2 is a prime number as its are 1 and 2.
- b. 7 is a prime number as its factors are 1 and 7.

A prime number is a number which can be divisible only by itself and the number 1.

What is the difference between prime and odd numbers?

- All the prime numbers are odd numbers but all the odd numbers are not prime numbers.
- A prime number is a number which can be divisible only by itself and the number 1.

Examples of prime numbers are 1, 2, 3, 5, 7, 11, 13, 17 etc

- An odd number is a number which cannot be divided by the number 2. For example 2 is not odd as it is divisible by 2 but 7 is an odd number as it is not divisible by 2 and leaves remainder 1.

Examples of odd number are 1, 3, 5, 7, 9, 11 etc

Composite numbers

Composite numbers are those natural numbers which have more than two factors. Such numbers are divisible by other numbers as well.

For example 4, 6, 8, 10, 14, 500, 6000 etc

NUMBER LINE

A number line is a straight line with a zero point in the middle with positive and

negative numbers listed on either side of zero and going on definitely.

RULES FOR INTEGERS

RULES FOR INTEGERS (SIGNED NUMBERS)			
ADDITION		SUBTRACTION	
+ and + = +		ADD THE OPPOSITE!	
- and - = -		(Change the subtraction sign to an addition sign. Change the sign of the second number. Now follow the Addition rules!)	
+ and - = +			
+ and - = -			
MULTIPLICATION AND DIVISION			
+ and + = +		+ and - = -	
- and - = +		- and + = -	

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Integer Rules

Add

Same Sign

Add and take
the sign

$$(-3) + (-4) = -7$$

$$3 + 7 = 10$$

Different Sign

Subtract and take
the sign of largest
absolute value

$$(-5) + 9 = 4$$

$$(-13) + 8 = -5$$

Subtract

Change to add the opposite

$$(-3) - (+4) = (-3) + (-4)$$

$$(-5) - (-9) = (-5) + (+9)$$

Multiply/Divide

Same Sign
Positive answer

Different Sign
Negative answer

Rules on multiplication and division

- When you multiply two integers with different signs, the result is always negative. Just multiply the absolute values and make the answer negative.
- When you divide two integers with the same sign, the result is always positive. Just divide the absolute values and make the answer positive.

Rules on addition and subtraction

- To add integers having the same sign, keep the same sign and add the absolute value of each number.
- To add integers with different signs, keep the sign with the largest absolute value and subtract the smallest absolute value from the largest. Subtract and integers by adding its opposite.

EXAMPLES

a. What is $5 + (-2)$?

$+(-)$ are **Unlike** signs (they are not the same), so they become a **negative sign**.

$$5 + (-2) = 5 - 2 = 3$$

b. What is $25 - (-4)$?

$-(-)$ are **like** signs, so they become a **positive sign**.

$$25 - (-4) = 25 + 4 = 29$$

c. What is $-6 + (+3)$?

$+(+)$ are **like** signs, so they become a positive sign.

$$-6 + (+3) = -6 + 3 = -3$$

NUMBER LINE AND INTEGERS

Integers can be represented on the number line. On a number line, the positive numbers are to the right of zero and the negative numbers are to the left of zero.

RULES ON NUMBER LINE FOR ADDING AND SUBTRACTING INTEGERS

1. Add a positive integer by moving to the right on the number line.
2. Add a negative inter by moving to the left on the number line.

3. Subtract an integer by adding its opposite.

NUMBER LINE GRAPH

Graphing on a number line. Integers and real numbers can be represented on a number line. The point on this line associated with each number is called the graph of the number. Notice that number lines are spaced equally or proportionally.

INTERVALS

An interval consists of all the numbers that lie within two certain boundaries. If the two boundaries, or fixed numbers, are included, then the interval is a **closed interval**. If the fixed numbers are not included, then the interval is called an **open interval**.

If the interval includes only one of the boundaries, then it is called a **half-open interval**.

EXAMPLES OF USING BODMAS/ BEDMAS FOR ORDER OF

OPERATIONS

	Do first
B racket	
O rders of	
D ivision	
M multiplication	
A ddition	
S ubtraction	Do last

BEDMAS

BEDMAS is an acronym to help remember an order of operations in algebra basics.

When you have math problems that require the use of different operations (**multiplications, divisions, exponents, brackets, subtraction, addition**) order is necessary and mathematics have agreed on the BEDMAS order. Each letter of BEDMAS refers to one part of the operation to be used.

Each letter stands for

- **B**- Brackets
- **E**- Exponent
- **D**- Division
- **M**- Multiplication
- **A**- Addition
- **S**- Subtraction.

There are a couple of things to remember when applying the BEDMAS order of operations. Brackets always come first and exponents come second. When working with multiplication and division, you do whichever comes first as you work from left to right. If multiplication comes first, do it before dividing. The same holds true for addition and subtraction, when the subtraction comes first, subtract before you add.

It may help to look at BEDMAS like this:

- Brackets
- Exponents
- Division or Multiplication
- Addition or Subtraction

Example 1

$20 - [3x (2 + 4)]$ Do the inside bracket first

= $20 - [3 \times 6]$ Do the remaining bracket

= $20 - 18$ Do the subtraction

= 2

Example 2

$(6 - 3)^2 - 2 \times 4$ Do the bracket first

= $(3)^2 - 2 \times 4$ Calculate the exponent

= $9 - 2 \times 4$ Now multiply

= $9 - 8$ Now subtract

= 1

Example 3

$2^2 - 3(10 - 6)$ Calculate inside the bracket

= $2^2 - 3 \times 4$ Calculate the exponent

= $4 - 3 \times 4$ Do the multiplication

= $4 - 12$ Do the subtraction

= -8

TYPES OF FRACTIONS

1. Proper fraction

This is fraction where the numerator (top number) is smaller than Denominator(bottom number) e.g. $\frac{1}{3}, \frac{3}{8}, \frac{5}{13}$

2. Improper fraction

The numerator is bigger than the denominator e.g. $\frac{11}{3}, \frac{23}{11}, \frac{15}{7}$

3. Mixed numbers

It is a fraction which has the whole number and a fraction e.g.

$4\frac{1}{3}, 7\frac{3}{8}, 15\frac{5}{13}$

NUMBERING SYSTEM

Explain four types of numbering system.

a. The Decimal Numbering System

The decimal numbering system has values 0 to 9 (0, 1, 2, 3, 4, 5, 6, 7, 8 and 9). The numbers are to the base 10. These are numbers in everyday use.

b. Octal Numbering System

These are numbers to base 8 as there 8 values used in the numbering system (0, 1, 2, 3, 4, 5, 6 and 7)

c. Hexadecimal Numbering System

These are numbers to the base 16 and uses 16 symbols (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. The letters A, B, C, D, E are equivalent to 10, 11, 12, 13, 14 and 15 decimals)

d. The Binary Numbering System

These are numbers to the base 2 and have values 0 to 1 (0 and 1 only).

Binary codes- refer to the binary notation used in computer system.

BINARY, OCTAL, DECIMAL AND HEXADECIMAL NUMBERS

Decimal	0	1	2	3	4	5	6	7	8
Hex	0	1	2	3	4	5	6	7	8
Octal	0	1	2	3	4	5	6	7	10
Binary	0	1	10	11	100	101	110	111	1000

Decimal	9	10	11	12	13	14	15
Hex	9	A	B	C	D	E	F
Octal	11	12	13	14	15	16	17
Binary	1001	1010	1011	1100	1101	1110	1111

DECIMAL NUMBERS

Decimal number system uses 10 as a base and the numbers range from 0 to 9.

BINARY NUMBERS

Binary is a based 2 number system that has two symbols 1 and 0. Each successive digit represents a power of 2.

HOW TO CONVERT BINARY NUMBERS TO DECIMAL AND VICE VERSA

CONVERTING FROM DECIMAL NUMBERS TO BINARY NUMBER

EXAMPLES

Convert to the following binary numbers to decimal.

- a. 10011_2
- b. 1001_2
- c. 1100_2

- a. 10011_{112}

$$\begin{aligned}
 &= 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\
 &= 16 + 0 + 0 + 2 + 1 \\
 &= 19
 \end{aligned}$$

b. 1001_{112}

$$\begin{aligned}
 &= 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\
 &= 8 + 0 + 0 + 1 \\
 &= 9
 \end{aligned}$$

c. 1100_{112}

$$\begin{aligned}
 &= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 \\
 &= 8 + 4 + 0 + 0 \\
 &= 13
 \end{aligned}$$

Convert $(36)_{10}$ to binary

Divisor	Dividend	Quotient	Remainder
2	36	18	0 ↑
2	18	9	0
2	9	4	1
2	4	2	0
2	2	1	0
2	1	0	1

The binary equivalent of $(36)_{10}$ is $(100100)_2$

Convert $(11010)_2$ to decimal (base 10)

Binary Number	1	1	0	1	0
	2^4	2^3	2^2	2^1	2^0
	16×1	8×1	4×0	2×1	1×0
$= 16 + 8 + 0 + 2 + 0$					
$= 26$					

The decimal equivalent of $(11010)_2$ is $(26)_{10}$

OCTAL NUMBERS

These are numbers to base 8 as there 8 values used in the numbering system (0,1,2,3,4,5,6 and 7)

Conversion of Decimal to Binary

This involves the following steps

1. Begin by dividing the decimal number by 8, the base of a octal number system.
2. Note the remainder separately as the rightmost digit of the binary equivalent.
3. Continually repeat the process of dividing by 8 until the quotient is zero and keep writing the remainders after each step of division.
4. Finally, when no more division can occur, write down the remainders in reverse order (last remainder written first).

Example

Convert $(359)_{10}$ to octal

Divisor	Dividend	Quotient	Remainder
8	359	44	7
8	44	5	4
8	5	0	5

The octal equivalent of $(359)_{10}$ is $(547)_8$

Convert 1456_{10} to base 8

Divisor	dividend	Quotient	Remainder
8	1456	182	0
8	182	22	6
8	22	2	6
8	2	0	2

Now we write the remainder upwards, that is 1456_{10} is 2660_8

a. 11_{10} to base 8

Divisor	Dividend	Quotient	Remainder

8	11	1	3
8	1	0	1

Now we write the remainder upwards, that is 11_{10} is 13_8

b. 64_{10}

Divisor	Dividend	Quotient	Remainder
8	64	8	0
8	8	1	0
8	1	0	1

Now we write the remainder upwards, that is 64_{10} is 100_8

CONVERSION OF OCTAL TO DECIMAL

In the octal to decimal conversion, each digit of the octal number is multiplied by its weighted position, and each of the weighted values is added together to get the decimal number.

Example

Convert $(456)_8$ to decimal

Binary Number	4	5	6
	8^2	8^1	8^0
	64×4	8×5	1×6
	256	40	6

Sum of the weight of all bits = $256 + 40 + 6$

$$= 302$$

The decimal equivalent of $(456)_8$ is $(302)_{10}$

SOLUTIONS

CONVERT OCTAL NUMBERS TO DENARY NUMBERS

a. Convert 100_8 to base 10

Octal number	1	0	0
	8^2	8^1	8^0
	$1 \times 8^2 = 64$	$0 \times 8^1 = 0$	$0 \times 8^0 = 0$
Denary number	$64 + 0 + 0 = 64_8$		

b. Convert 5406_8 to base 10

Octal number	5	4	0	6
	8^3	8^2	8^1	8^0
	$1 \times 8^2 = 64$	$0 \times 8^2 = 0$	$0 \times 8^1 = 0$	
Denary number	$64 + 0 + 0 = 64_8$			

Exercise

1. Convert $(010111)_2$ to octal
2. The binary equivalent of $(231)_8$
3. Convert $(231)_8$ to binary

HEXADECIMAL NUMBERS

These are numbers to the base 16 and uses 16 symbols (0,1,2,3,4,5,6,7,8,9 A,B, C, D, E, F. The letters A, B, C , D , E are equivalent to 10,11,12,13,14 and 15 decimals)

CONVERSION FROM DECIMAL TO HEXADECIMAL

To convert a decimal number into its hexadecimal equivalent, the same procedure is adopted as decimal to binary conversion but the decimal number is divided by 16 (the base of the hexadecimal number system)

Example

Convert $(5112)_{10}$ to hexadecimal

Divisor	Dividend	Quotient	Remainder
16	5112	319	8
16	319	19	F
16	19	1	3
16	1	0	1

The hexadecimal equivalent of $(5112)_{10}$ is $(13F8)_{16}$

CONVERSION OF HEXADECIMAL TO DECIMAL

Convert $(B14)_{16}$ to decimal

Binary Number	B=11	1	4
Weight of each bit	16^2	16^1	16^0
Weighted value	256×11	16×1	1×4

Solved multiplication	2816	16	4
Sum of the weight of all bits = $2816 + 16 + 4$ = 2836			

The decimal equivalent of $(B14)_{16}$ is $(2836)_{10}$

Convert $(11001011)_2$ to hexadecimal.

Binary Number	1100	1011
Decimal Number	12	11
Hex Number	C	B

The hex equivalent of $(11001011)_2 = (CB)_{16}$

Conversion of Hexadecimal to Binary

This involves converting each hexadecimal digit to its 4-bit binary equivalent and combining the 4-bit sections by removing the spaces to get the binary number.

BINARY ARITHMETIC

ADDITION OF BINARY NUMBERS

In binary addition, 1 plus 1 = 10 and 10 plus 10 = 100.

INPUT		OUTPUT	
X	Y	SUM(S)	CARRY(C)
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

SUBTRACTION OF BINARY NUMBERS

INPUT		OUTPUT	
X	Y	SUM(S)	CARRY(C)
0	0	0	0
0	1	1	1

1	0	1	0
1	1	0	0

The following steps are involved

- First, for the least significant bit (the right most), $1-1$ is 0.
- For next bit, $0-1$ cannot be computed since the subtrahend is smaller than the minuend. The operation is $10-1=1$ which in the decimal number system is $2-1=1$.
- For the third bit, since we borrowed 1 for the second bit, we have $0-0$ that is 0.
- For the fourth bit again, we cannot perform the subtraction. However, the fifth bit in the minuend is zero, so we must borrow from the sixth bit. This makes bit 10. Now the subtraction in binary is $10-1=1$ which is the result of the fourth bit.
- Since we borrowed 1 from the sixth bit for the fourth bit, so for the sixth bit, the subtraction is $0-0=0$.

OCTAL ADDITION

During the process of addition, if the sum is less than or equal to 7, then it can be directly written as an octal digit. If the sum is greater than 7, then subtract 8 from that particular digit and carry 1 to the next digit position

Add the octal numbers 5647 and 1425.

$$\begin{array}{r}
 & 5 & 6 & 4 & 7 \\
 + & 1 & 4 & 2 & 5 \\
 \hline
 & 7 & 10 & 7 & 12 \\
 & & 8 & & -8 \\
 \hline
 & 7 & 2 & 7 & 4
 \end{array}$$

OCTAL SUBTRACTION

During octal subtraction, instead of 1, we will borrow 8 and 8 and the rest of the steps are similar to that of binary subtraction.

Example

Subtract 677_8 from 770_8

$$\begin{array}{r}
 & 7 & 7 & 0 \\
 - & 6 & 7 & 7 \\
 & 0 & 7 & 1
 \end{array}$$

Thus the difference is 71_8

HEXADECIMAL ADDITION

During the process of addition, observe if the sum is 15 or less, then it can be directly expressed as a hexadecimal digit.

If the sum is greater than 15, then subtract 16 from that particular digit and carry 1 to the next digit position.

Example

Add the hex numbers 76_{16} and 45_{16}

$$\begin{array}{r}
 7 \quad 6 \\
 + \quad 4 \quad 5 \\
 \hline
 11 \quad 11 \\
 \hline
 B \quad B
 \end{array}$$

PERFORMING SUBTRACTIONS AND ADDITION IN DIFFERENT BASES

Examples

1. Subtract 1264_b from 30155_5 leave your answer to base 10.

Changing the given number to base ten

$$\begin{aligned}
 1264_8 &= 1 \times 8^3 + 2 \times 8^2 + 6 \times 8^1 + 4 \times 8^0 \\
 &= 1 \times 512 + 2 \times 64 + 6 \times 8 + 4 \times 1 \\
 &= 512 + 128 + 48 + 4 \\
 &= 692_{10}
 \end{aligned}$$

$$\begin{aligned}
 30155_5 &= 3 \times 6^4 + 0 \times 6^3 + 1 \times 6^2 + 5 \times 6^1 + 5 \times 6^0 \\
 &= 3 \times 1296 + 0 \times 216 + 1 \times 36 + 5 \times 6 + 5 \times 1 \\
 &= 3888 + 0 + 36 + 30 + 5 \\
 &= 3959_{10}
 \end{aligned}$$

Then subtract 692_{10} from 3959_{10}

	3	9	5	9
--	---	---	---	---

-		6	9	2
	3	2	6	7
$\therefore 3267_{10}$				

2. Evaluate $3452_6 + 7634_8$, convert your answer to base 5.

Changing the given number to base ten

$$\begin{aligned}
 3452_8 &= 3 \times 6^3 + 4 \times 6^2 + 5 \times 6^1 + 2 \times 6^0 \\
 &= 3 \times 216 + 4 \times 36 + 5 \times 6 + 2 \times 1 \\
 &= 648 + 144 + 30 + 2 \\
 &= 824_{10}
 \end{aligned}$$

$$\begin{aligned}
 7834_8 &= 7 \times 8^3 + 8 \times 8^2 + 3 \times 8^1 + 4 \times 8^0 \\
 &= 7 \times 512 + 8 \times 64 + 3 \times 8 + 4 \times 1 \\
 &= 3584 + 512 + 24 + 4 \\
 &= 4124_{10}
 \end{aligned}$$

Then add 3452_6 to 7634_8

		8	2	4
+	4	1	2	4
	4	9	4	8

Convert 4948_{10} to base 5

Divisor	Dividend	Quotient	Remainder
5	4948	989	3
5	989	197	4
5	197	39	2
5	39	7	4
5	7	1	2
5	1	0	1

$\therefore 124243_5$

What is 1111_2 in decimal (base10)?

- The “1” on the left is in the “ $2 \times 2 \times 2$ ” position , so that means $1 \times 2 \times 2 \times 2 (= 8)$

- The next “1” is in the “ $1 \times 2 \times 2 (=4)$ position so that means $1 \times 2 \times 2 (=4)$.
- The next “1” is in the “2” position so that means $1 \times 2 (=2)$.
- The last “1” is in the “1”position so that means 1

$$1 \times 2^3 + 2^2 + 2^1 + 2^0 = 8 + 4 + 2 + 1$$

$$= 15_2$$

Solution :

$$\begin{array}{r}
 2 \mid \underline{19} \\
 2 \mid \underline{9} \quad 1 \\
 2 \mid \underline{4} \quad 1 \\
 2 \mid \underline{2} \quad 0 \\
 2 \mid \underline{1} \quad 0 \\
 \hline & 0 \quad 1
 \end{array}
 \qquad
 \begin{array}{r}
 .25 \times 2 \\
 \hline
 0.50 \times 2 \\
 \hline
 1.00
 \end{array}$$

$$(19)_{10} = (10011)_2$$

$$(.25)_{10} = (.01)_2$$

$$\therefore (19.25)_{10} = (10011.01)_2$$

For example, consider the binary number 1101. This number is:

$$\begin{array}{cccc}
 1 & 1 & 0 & 1 \\
 \uparrow & \uparrow & \uparrow & \uparrow \\
 & 2^3 & 2^2 & 2^1 & 2^0 \\
 \text{position} & \text{(i.e., eights position)} & \text{position} & \text{(i.e., fours position)} & \text{position} \\
 & & & \text{(i.e., twos position)} & \text{position} \\
 & & & & \text{(i.e., ones position)}
 \end{array}
 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 13_{10}$$

Convert A37E₁₆ to base 10.

A37E

$$\begin{array}{r}
 14 * 16^0 = 14 \\
 7 * 16^1 = 112 \\
 3 * 16^2 = 768 \\
 10 * 16^3 = 40960 \\
 \hline
 \text{total is } 41854
 \end{array}$$

Convert 10011011_2 to base 10.

$$\begin{array}{r}
 10011011 \\
 \\
 1 * 2^0 = 1 \\
 1 * 2^1 = 2 \\
 0 * 2^2 = 0 \\
 1 * 2^3 = 8 \\
 1 * 2^4 = 16 \\
 0 * 2^5 = 0 \\
 0 * 2^6 = 0 \\
 1 * 2^7 = 128 \\
 \hline
 \text{total is } 155
 \end{array}$$

Convert 47_{10} to binary

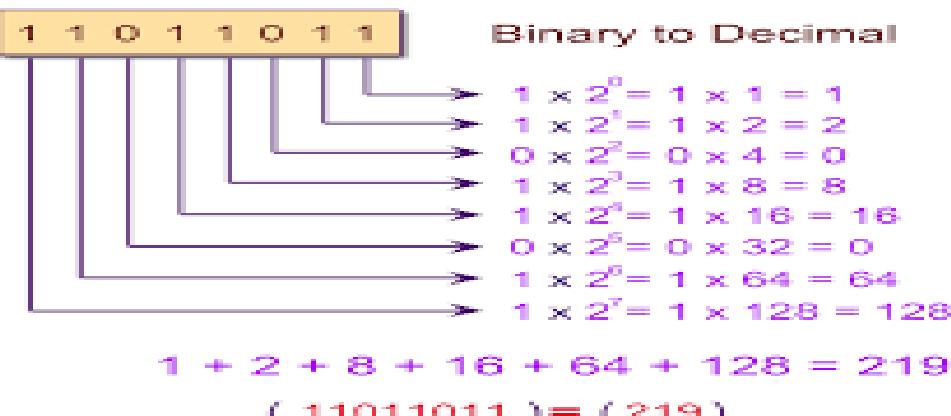
Decimal to Binary

$$\begin{array}{r}
 2 | 47 \\
 2 | 23 \\
 2 | 11 \\
 2 | 5 \\
 2 | 2 \\
 2 | 1 \\
 \hline
 0
 \end{array}
 \quad
 \begin{array}{r}
 1 \\
 1 \\
 1 \\
 0 \\
 1 \\
 \\
 2^0 \\
 2^1 \\
 2^2 \\
 2^3 \\
 2^4
 \end{array}$$

$$(47)_{10} = (101111)_2$$

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Convert 11011011_2 base 10.



SET THEORY

A set is a collection of objects called members or elements that is regarded as being a single object.

SUBSET

A is a subset of B if and only if every element of A is in B.

A is a subset of B if and only if every element of A is in B.

Example

Is A a subset of B, where A = {1,3,4} and B = {1,4,3,2}?

1 is in A, and 1 is in B as well, so far so good.

3 is in A and 3 is also in B

4 is in A and 4 is in B.

That's all the elements of A , and every single one is in B , so we are done.

Yes , A is a subset of B

EMPTY (or NULL/VOID) SET

It is a set which does not contain any elements. There isn't any element. Not one. Zero. It is represented by \emptyset or by {} (*a set with no elements*)

FINITE SET

It is a set which contains a definite number of elements. Empty set is also called a finite set.

Example

a. The set of all colours in the rainbow

b. $N = \{x : x \in n, x < 7\}$, This mean $N = \{1, 2, 3, 4, 5, 6\}$

c. $P = \{2, 3, 5, 7, 11, 13, 17, \dots, 97\}$

INFINITE SET

It is a set whose elements cannot be listed, i.e., set containing never-ending elements.

Examples

a. $A = \{x : x \in N, x > 1\}$

b. Set of all prime numbers

CARDINAL NUMBER OF A SET

It is the number of distinct elements in a given set A. It is denoted by $n(A)$.

Examples

a. $A = \{x : x \in n, x < 7\}$. Find $n(A)$

$A = \{1, 2, 3, 4, 5, 6\}$. Therefore $n(A) = 6$

b. $B = \text{set of letters in the word Algebra}$

$B = \{A, L, G, E, B, R, A\}$, Therefore $n(B) = 7$

EQUIVALENT SETS

Two sets A and B are said to be equivalent if their cardinal number is the same i.e $n(A) = n(B)$. The symbol for denoting an equivalent set is ' \leftrightarrow '.

Example

$A = \{1, 2, 3\}$. Here $n(A) = 3$

$B = \{p, q, r\}$ Here $n(B) = 3$

There, $A \leftrightarrow B$

EQUAL SETS

Two sets are equal if they have precisely the same members.

Example

Are A and B equal where A is the set whose members are the first four positive whole numbers, $B = \{4, 2, 1, 3\}$.

Let's check. They both contain 1. They both contain 2, And 3, And 4. And we

have checked every element of both sets, so **Yes, they are equal.**

Therefore $A = B$

Are these sets equal?

$A \text{ is } \{1, 2, 3\}$.

$B \text{ is } \{3, 1, 2\}$.

Yes, they are equal. They both contain exactly the members 1, 2 and 3. It doesn't matter where each member appears, so long as it is there.

INTERSECTION OF SETS

Intersection of two given sets is the largest set which contains all the elements that are common to both the sets. To find the intersection of two given sets A and B is a set which consists of all the elements which are common to both A and B. The symbol for denoting intersection is ' \cap '.

EXAMPLES

1. If $A = \{2, 4, 6, 8, 10\}$ and $B = \{1, 3, 8, 4, 6\}$. Find intersection of two set A and B.

$A \cap B = \{4, 6, 8\}$.

Therefore, 4, 6 and 8 are the common elements in both the sets.

2. If Set $A = \{4, 6, 8, 10, 12\}$, set $B = \{3, 6, 9, 12, 15, 18\}$ and Set $C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

- a. Find the intersection of sets A and B.
- b. Find the intersection of two set B and C.
- c. Find the intersection of the given sets A and C.

a. The set of all the elements which are common to both set A and B is $A \cap B = \{6, 12\}$

b. Set of all the elements which are common to both set B and set C is $B \cap C = \{3, 6, 9\}$

c. Intersection of the given sets A and C is $A \cap C = \{4, 6, 8, 10\}$

Set of all the elements which are common to both set A and set C.

UNION OF SETS

Union of two given sets is the smallest set which contains all the elements of both the sets.

For two sets A and B, $n(A \cup B)$ is the number of elements present in either of the sets A or B. To find the union of two given sets A and B is a set which consists of all elements of A and all the elements of B such that no element is repeated. The symbol for denoting union of sets is \cup .

EXAMPLES

2. Given $A = \{2, 4, 5, 6\}$ and set $B = \{4, 6, 7, 8\}$. Find $A \cup B$.

Taking every element of both the sets A and B, without repeating any element, we get a new set, $A \cup B = \{2, 4, 5, 6, 7, 8\}$. This new set contains all the elements of set A and all the elements of set B with no repetition of elements and is named as **union of set A and B**.

3. If $A = \{1, 3, 5, 7, 9\}$ and $B = \{3, 7, 8, 9\}$. Find union of set A and B.

$A \cup B = \{1, 3, 5, 7, 8, 9\}$. No element is repeated in the union of two sets. The common elements 3, 7 are taken only once.

4. Let $X = \{a, e, i, o, u\}$ and $Y = \{\emptyset\}$. Find the union of two given sets X and Y.

$X \cup Y = \{a, e, i, o, u, \emptyset\}$. Therefore, union of any set with empty set is the set itself.

5. If set $P = \{1, 2, 3, 4, 5\}$, set $Q = \{1, 2, 3, 4, 6\}$ and set $R = \{1, 2, 3, 7\}$.

- Find the union of sets P and Q.**
- Find the union of two set P and R.**
- Find the union of the given sets Q and R.**

Solution:

a. Union of sets P and Q is $P \cup Q$

The smallest set which contains the elements of set P and all the elements of set Q is $P \cup Q = \{1, 2, 3, 4, 5, 6\}$

b. Union of two set P and R is $P \cup Q$

The smallest set which contains all the elements of set P and all the elements of set R is $P \cup Q = \{1, 2, 3, 4, 5, 6, 7\}$

c. Union of the given sets Q and R is $Q \cup R$

d. The smallest set which contains all the elements of set Q and all the elements of set R is $Q \cup R = \{0, 2, 3, 4, 6, 8, 9, 12\}$

INTERSECTION OF SETS

$n(A \cap B)$ is the number of elements present in both the sets A and B .

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

For three sets A, B and C:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Examples

Let A and B be two finite sets such that $n(A) = 20$, $n(B) = 28$ and $n(A \cup B) = 36$, find $n(A \cap B)$.

Using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

$$\begin{aligned} &= 20 + 28 - 36 \\ &= 48 - 36 \\ &= 12 \end{aligned}$$

If $(A - B) = 18$, $n(A \cup B) = 70$ and $(B - A) = 25$, then find $n(B)$.

Using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$\begin{aligned} 70 &= 18 + 25 + n(A \cap B) \\ 70 - 43 &= \\ 70 &= 43 + (A - B) \\ n(B - A) &= 70 - 43 \\ n(B - A) &= 27 \end{aligned}$$

$$\begin{aligned} \text{Now } n(B) &= n(A \cap B) + (B - A) \\ &= 25 + 27 \\ &= 52 \end{aligned}$$

In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person like at least one of the drinks. How many like both cold drinks and hot drinks?

Let A= set of people who like cold drinks

B =set of people who like hot drinks

$$(A \cup B) = 60, n(A) = 27, n(B) = 42 \text{ then}$$

$$(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$60 = 27 + 42 - n(A \cap B)$$

$$60 = 69 - n(A \cap B)$$

$$60 - 69 = -n(A \cap B)$$

$$-9 = -n(A \cap B)$$

$$n(A \cap B) = 9$$

There are 35 students in art class and 57 students in dance class.

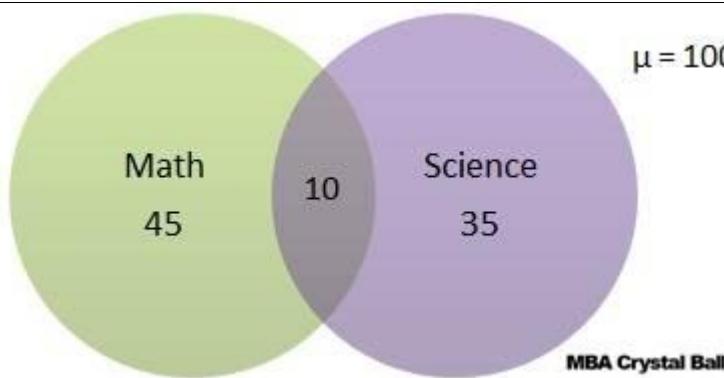
- a. Find the number of students who are either in art class or in dance class when two classes meet at the same hour.**

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 35 + 57 - \emptyset \\ &= 92 \end{aligned}$$

- b. Find the number of students who are either in art class or in dance class when two classes meet at different hours and 12 students are enrolled in both activities.**

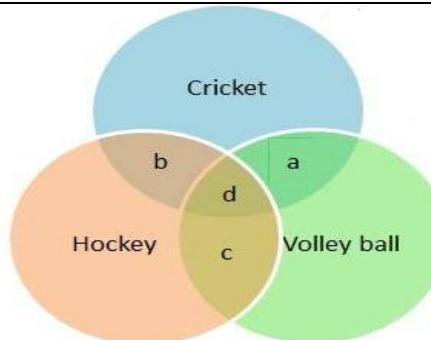
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 35 + 57 - 12 \\ &= 92 - 12 \\ &= 80 \end{aligned}$$

In a class of 100 students, 35 like science and 45 like math. 10 like both. The information is represented in the Venn diagram. How many like either of them and how many like neither.



$$\begin{aligned}
 n(S \cup M) &= n(S) + n(M) - n(S \cap M) \\
 &= 45 + 35 - 10 \\
 &= 70
 \end{aligned}$$

Among a group of students, 50 played cricket, 50 played hockey and 40 played volleyball. 15 played both cricket and hockey, 20 played both hockey and volleyball, 15 played cricket and volleyball and 10 played all three. If every student played at least one game, find the number of students and how many played only cricket, only hockey and only volleyball?



$$n(C) = 50, n(H) = 50, n(V) = 40$$

$$n(C \cap H) = 15, n(H \cap V) = 20, n(C \cap V) = 15, n(C \cap H \cap V) = 10$$

Now

$$n(C \cap H) = b + d = 15,$$

$$n(H \cap V) = c + d = 20,$$

$$n(C \cap V) = a + 15,$$

$$n(C \cap H \cap V) = 10 = d,$$

Therefore, $a = 15 - 10 = 5$ [cricket and volleyball only]

$$b = 15 - 10 = 5 \text{ [cricket and hockey only]}$$

$$c = 20 - 10 = 10 \text{ [hockey and volley ball only]}$$

Number of students who played only hockey = $n(H) - [b + c + d]$

$$= 50 - [5 + 10 + 10]$$

$$= 25$$

Number of students who played only volley ball = $n(V) - [a + c + d]$

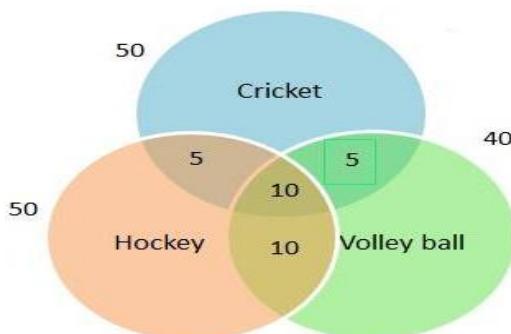
$$= 40 - [10 + 5 + 10]$$

$$= 15$$

Number of students who played only cricket = $n(C) - [a + b + d]$

$$= 50 - [5 + 10 + 10]$$

$$= 25$$



COMPLEMENT OF SETS

In set theory, the complement of a set A refers to the elements not in A. When all sets under consideration are considered to be subsets of a given set $U =$, the absolute complement of A is the set of elements in U but not in A.

The complement of set A, denoted by A' , is the set of all elements in the universal set that are not in A. It is denoted by A' .

EXAMPLES

If $A = \{1, 2, 3, 4\}$ and $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, find A complement (A').

Complement of set A contains elements present in universal set but not in set A.

Elements are 5, 6, 7, 8

$\therefore A$ complement, $A' = \{5, 6, 7, 8\}$

EXERCISE

1. There are 30 students in a class. Among them, 8 students are learning both English and French. A total of 18 students are learning English. If every student is learning at least one language, how many students are learning French in total?
2. In a class of 100 students, 35 like science and 45 like math. 10 like both. How many like neither. Use the Venn diagram below to find the answer.
3. There are 30 students in a class. Among them, 8 students are learning both English and French. A total of 18 students are learning English. If every student is learning one language, how many students are learning French in total. Use the Venn diagram below that illustrates the information to
4. Use Venn diagram: In a school, there are 20 teachers who teach Mathematics or English. Of these, 12 teach Mathematics and 4 teach both English and Mathematics . How many teach English only.

MULTIPLICATION AND DIVISION OF ALGEBRAIC EXPRESSION

MULTIPLYING ALGEBRAIC EXPRESSIONS

We can multiply two algebraic terms to get a product, which is also an algebraic term.

Example

1. Evaluate $3pq^3 \times 4qr$

Solution

$$\begin{aligned}3pq^3 \times 4qr &= 3 \times p \times q \times q \times q \times 4 \times q \times r \\&= 3 \times 4 \times p \times q \times q \times q \times q \times r \\&= 12 \times p \times q^4 r \\&= 12pq^4r\end{aligned}$$

LIKE TERMS AND UNLIKE TERMS

In algebra, terms that have the same letter or letters are called like terms. They must have the same letters in any order but they may have different numerical coefficients.

The process of adding or subtracting like terms is called collecting like terms.

EXAMPLES

Simplify the following algebraic expression

1. $5px^2 - qz - 3px^2 + 4qz = 5px^2 - 3px^2 + 4qz - qz$
= $2px^2 + 3qz$
2. $8x - 2y + 5x + 10y = 8x + 5x + 10y - 2y$
= $13x + 8y$
3. $4ab - 2bc - 3ab + 3cd = 4ab - 3ab + 3cd - 2bc$
= $ab - 2bc + 3cd$
4. $9xy + 5x - 4xy - 2x = 9xy - 4xy + 5x - 2x$
= $5xy + 3x$

ALGEBRA EXPANSION

Expanding means removing the () but we have to do it the right way () are called ‘parentheses’ or “brackets”. Whatever is inside the () needs to be treated as “package”. So when simplifying: multiply by everything inside the “package”.

EXAMPLES

Expand the following

- a. $5(y + 3) = 5y + 15$
- b. $a(y + 3) = ay + 3a$
- c. $-3(y + 3) = -3y - 9$
- d. $-b(x - y) = -bx + by$

EXERCISE

1. Expand the following brackets

- | | | |
|----------------|-------------------------------------|-----------------------------------|
| 1. $5(y + 3)$ | 7. $3(7a + 2)$ | 13. $3(9 - 2a)$ |
| 2. $9(x - 1)$ | 8. $4(3y + 2)$ | 14. $9(2x - 5)$ |
| 3. $4(a + 2)$ | 9. $7(9 - 2c)$ | 15. $5(2p - 1)$ |
| 4. $8(w + 10)$ | 10. $11(2k - 5)$ | 16. $4(3y + 2)$ |
| 5. $3(x - 7)$ | 11. $3(15w - 7)$ | 17. $9(2x - 5)$ |
| 6. $2(8 - t)$ | 12. $20(6a + 5c)$ | |

2. Expand the following brackets

- | | | |
|-----------------|-------------------|-------------------|
| 1. $a(y - 3)$ | 7. $p(7a + 2)$ | 13. $m(9n - 2a)$ |
| 2. $c(x - 1)$ | 8. $y(3y + 2)$ | 14. $3x(2x + 5)$ |
| 3. $b(a + 2)$ | 9. $2a(9 - 2c)$ | 15. $5g(2p + 4y)$ |
| 4. $2a(w + 10)$ | 10. $3r(2k + 5)$ | 16. $t(3y + 2)$ |
| 5. $3p(x + 7)$ | 11. $3y(15w + 7)$ | 17. $w(2x + 6)$ |
| 6. $2t(8 - t)$ | 12. $2v(6a + 5c)$ | |

3. Expand the following brackets

- | | | |
|----------------|-----------------|------------------|
| a. $-5(y + 3)$ | f. $-2(8 - t)$ | k. $-3(15w + 7)$ |
| b. $-9(x - 2)$ | g. $-3(7a + 2)$ | l. $-2(6a + 5c)$ |
| c. $-4(a - 2)$ | h. $-4(3y - 2)$ | m. $-5(2p - 1)$ |
| d. $-8(w + 2)$ | i. $-7(9 + 2c)$ | n. $-4(3y + 2)$ |
| e. $-3(x + 7)$ | j. $-2(2k - 5)$ | o. $-9(2x - 5)$ |

Expand the following algebraic expressions

$$\begin{aligned} 1. \quad (2x + 3)(x - 1) &= 2x^2 - 2x + 3x - 3 \\ &= 2x^2 + x - 3 \end{aligned}$$

$$\begin{aligned} 2. \quad (x + 3)(x + 2) &= x(x + 2) + 3(x + 2) \\ &= x^2 + 2x + 3x + 6 \\ &= x^2 + 5x + 6 \end{aligned}$$

$$\begin{aligned} 3. \quad (3x + 2)(x + 3) &= 3x(x + 3) + 2(x + 3) \\ &= 3x^2 + 9x + 2x + 6 \end{aligned}$$

RULES OF ALGEBRA EXPANSION

1. $(a + b)(a - b) = a^2 - b^2$
2. $(a + b)^2 = a^2 + 2ab + b^2$
3. $(a - b)^2 = a^2 - 2ab + b^2$

Expand the following

- | | |
|-----------------------|-----------------------|
| 1. $(x - 3)(x - 4)$ | 4. $(3y - 3)(2y + 2)$ |
| 2. $(x + 5)(x - 4)$ | 5. $(x - 1)(x + 1)$ |
| 3. $(2x + 1)(3x - 2)$ | 6. $(x - 4)(x + 4)$ |

7. $(y - 2)(y + 2)$

9. $(x + 4)(x + 3)$

8. $(x - 4)(x - 4)$

CO-ORDINATE GEOMETRY

GRADIENT AND EQUATION OF A STRAIGHT LINE IN THE FORM OF $Y = MX + C$

Gradient is a measure of how sloopy line on a plane is with respect to the horizontal.

In Cartesian plane, a gradient of a line is defined as the change in y-value divided by the change in x-value.

$$\text{Gradient} = \frac{\text{Change in } y\text{-values}}{\text{Change in } x\text{-value}}$$

Find the gradient of the straight line joining P (x_1, y_1) and Q(x_2, y_2).

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

Examples

Find the gradient of the line segment joining the following points

a. A= (1,1) and B =(5,2)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Gradient} = \frac{2-1}{5-1}$$

$$= \frac{1}{4}$$

b. P=(-2,3) and Q = (-2,1)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Gradient} = \frac{1-3}{-2--2}$$

$$= \frac{2}{0}$$

= Undefined (We cannot divide by zero)

EXERCISE

Find the gradient and the equation of the line joining the following points.

c. A= (4,3) and B =(7,5)

d. P=(8,1) and Q = (4,2)

- e. M= (0, -5) and N = (-1, -3)
- f. X= (-2,3) and Y = (-3,1)
- g. B= (-2,3) and C = (-3,1)

GRADIENT AND Y-INTERCEPT

Given the straight lines whose equation is:

a. $y = \frac{2}{5}x - 3$

Y = mx + c where m = gradient and c = y-intercept

Gradient = $\frac{2}{5}$

Y- intercept = -3

b. $2x + 3y = 7.$

First make y the subject of the formula where Y = mx + c where m = gradient and c = y-intercept

$3y = 7 - 2x$

$y = \frac{7}{3} - \frac{2}{3}x$

\therefore Gradient = $\frac{-2}{3}$

\therefore Y-intercept = $\frac{7}{3}$

EQUATION OF A LINE FROM ORDERED PAIRS (X, Y)

Example

Find the gradient and the equation of the line joining points A= (-1,3) and B=(3,5).

Equation of the straight line takes the form of Y = mx + c where m = gradient and c = y-intercept

Gradient, $m = \frac{y_2 - y_1}{x_2 - x_1}$

Gradient, $m = \frac{5 - 3}{3 - -1}$
 $= \frac{2}{4}$
 $= \frac{1}{2}$

To find c, $Y = \frac{1}{2}x + c$ passes through the point B(3,5)

$$5 = \frac{1}{2}(3) + c$$

$$5 = \frac{3}{2} + c$$

$$5 - 1.5 = c$$

$$1.5 = c$$

Equation of the straight line, $Y = \frac{1}{2}x + 3.5$

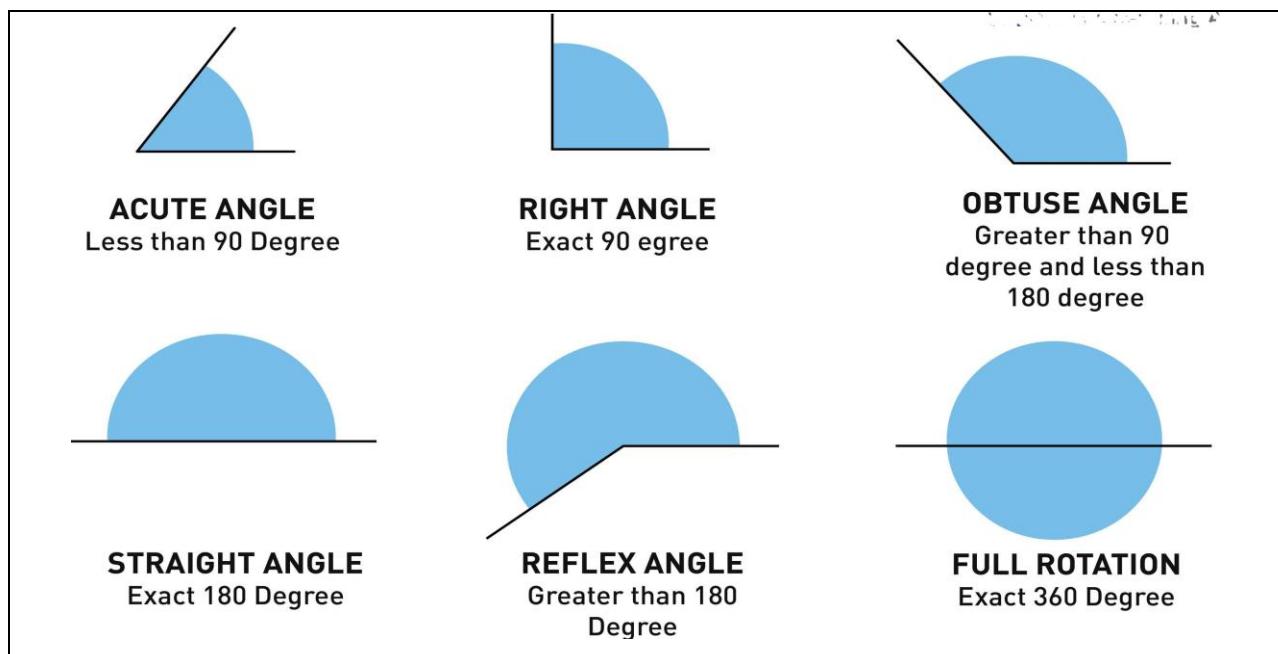
EXERCISE

Find the gradient and the equation of the line joining the following points.

- h. A= (-1,3) and B =(3,5)
- i. P=(1,1) and Q = (2,4)
- j. M= (2,6) and N = (3,8)
- k. X= (2,6) and Y = (3,8)
- l. B= (-3,2) and C = (-1,3)

GEOMETRY- TYPES OF ANGLES

1. **Acute angle-** This is angle that is less than 90° . In other words, an acute angle is an angle measuring between 0 and 90 degrees.
2. **Right angle-** A right is an angle measuring 90 degrees. Two lines or lines segments that meet at a right angle are said to be perpendicular.
3. **Obtuse angle-** An obtuse angle is an angle measuring between 90 and 180 degrees.
4. **Straight angle - A straight angle is a straight line and it measures 180 degrees.**
5. **Reflex angle-** A reflex angle is an angle measuring between 180 and 360 degrees.
6. **Full rotation or angle at a point-** Angles at a point add up to 360 degrees



COMPLEMENTARY ANGLES

- Complementary angles are two angles that have a sum of 90 degrees.
- To determine the complement, subtract the given angle from 90.
- There are two types of complementary angles, adjacent and non adjacent
- Adjacent complementary angles are the angles that share a common side and vertex and are side by side and the right angle is divided into 2 angles that are adjacent to each other creating a pair of adjacent, complementary angles as shown below.
- Non adjacent - The angles do not share a common side.

Examples

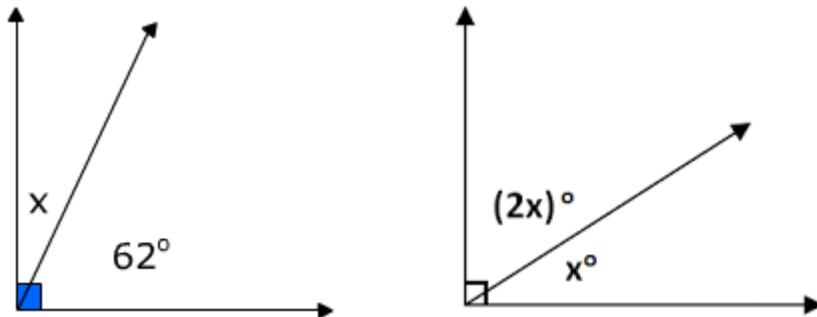
This first diagram shows complementary angles that are adjacent, meaning that the angles share a side and a vertex, or the corner point of the angle.

Since $65+25 = 90$, angles STA and ATR are complementary.

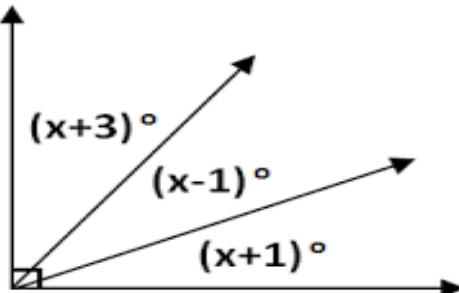
The next diagram shows two complementary angles that are not adjacent, but are in the same figure. Angles GDO and DOG are complementary because $70 + 20 = 90$. Angle DOG is a right angle as indicated by the little box we see at the vertex of DOG.

EXERCISE

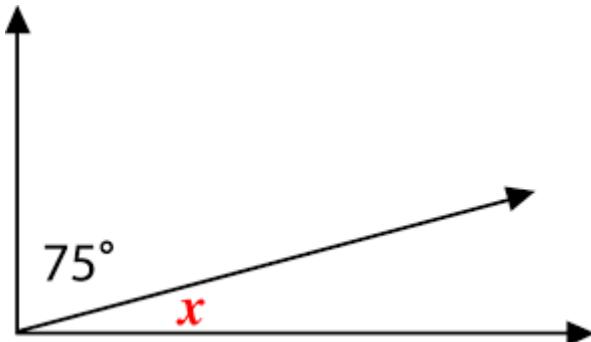
- 1.** Find the value of x degrees and the angles in the figure below.



- 2.** Find the value of x degrees and the angles in the diagram below.



- 3.** x and y are complementary angles. Given $x = 35^\circ$, find the value y .



- 4** Find the complementary angle of each of the following angles

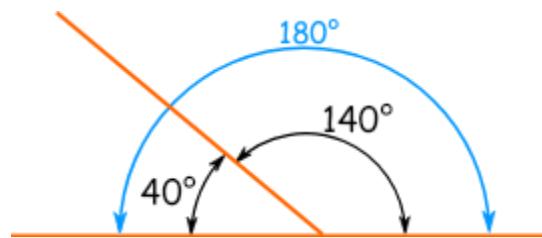
- 7°
- 38°

SUPPLEMENTARY ANGLES

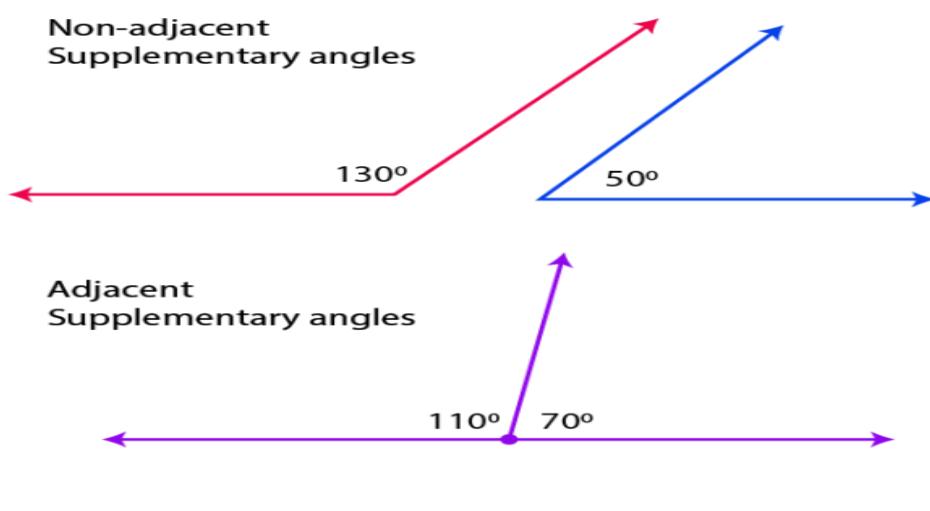
Supplementary angles are two angles that add up to 180 degrees on a straight line. They don't have to be next to each other, just so long as the total is 180 degrees.

PROPERTIES OF SUPPLEMENTARY ANGLES

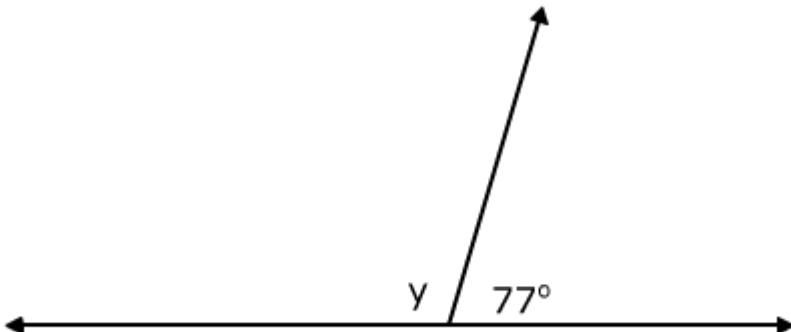
- The two angles are said to be supplementary angles when they add up to 180 degree. The two angles together make a straight line but the angles don't have to be together.



- S of supplementary angle stands for "Straight "line. This means that it forms 180 degrees
- Supplementary angles may be classified as either adjacent or non adjacent. The adjacent supplementary angle shares the line segment or arm with each other whereas the non- adjacent supplementary angles do not share the line segment or arm.

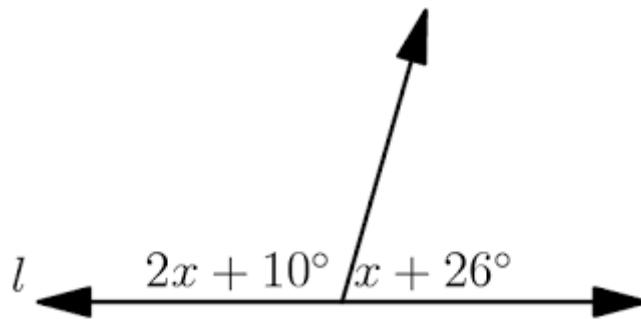
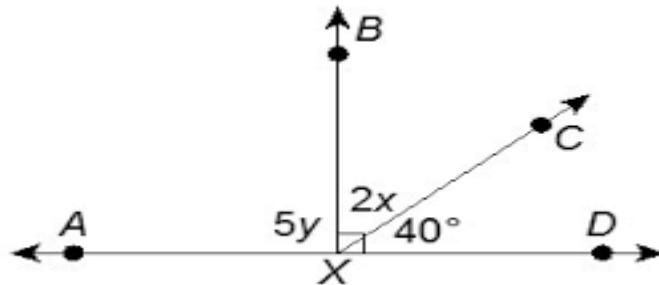
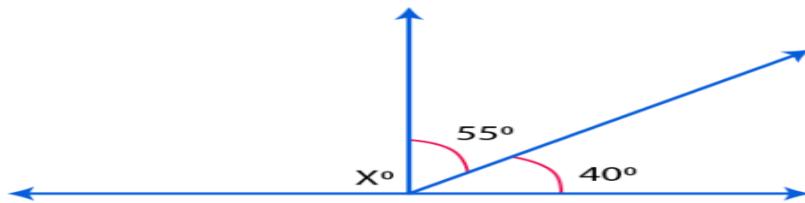


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EXERCISE

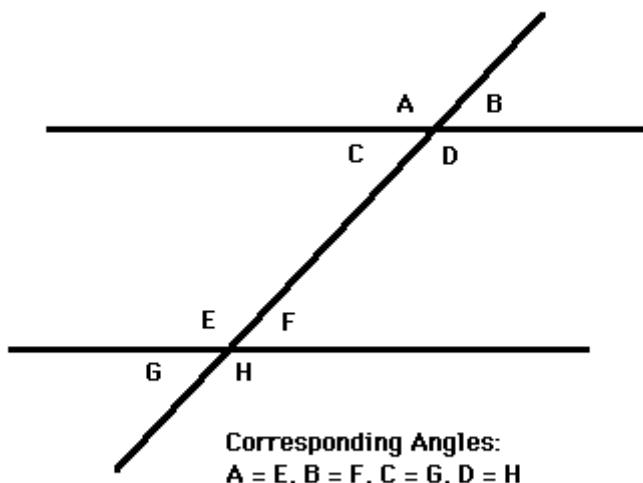
1. Find the value of unknown angles in the diagrams below.



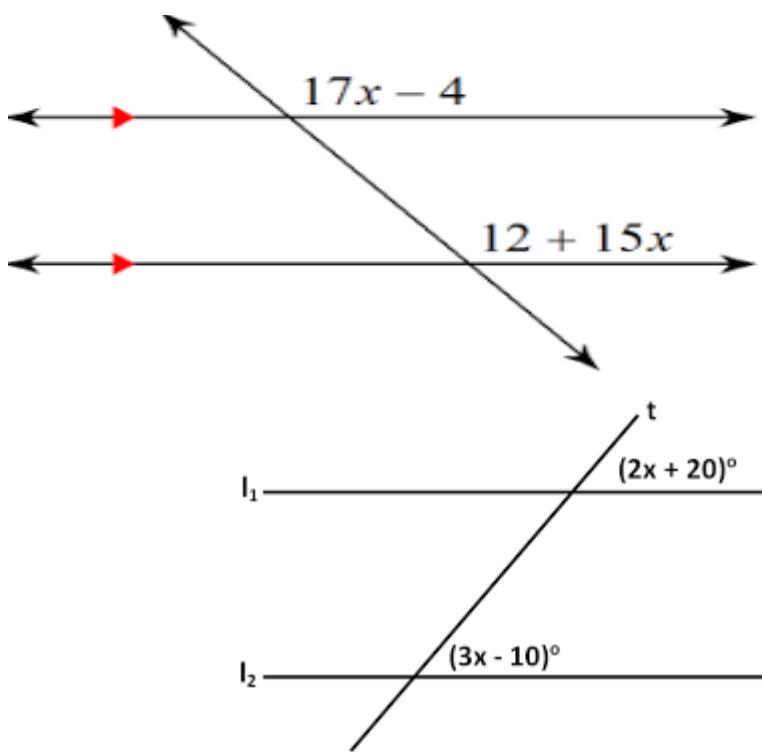
2. x and y are supplementary angles. Given $x = 72^\circ$, find the value of y.
3. Find the measure of the supplementary angle for each of the following angles
- 124°
 - 75°

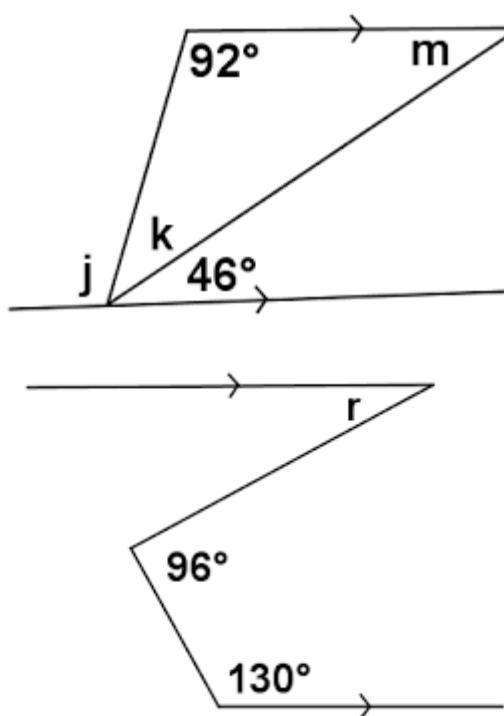
PARALLEL LINES

Corresponding Angles



MathATube.com

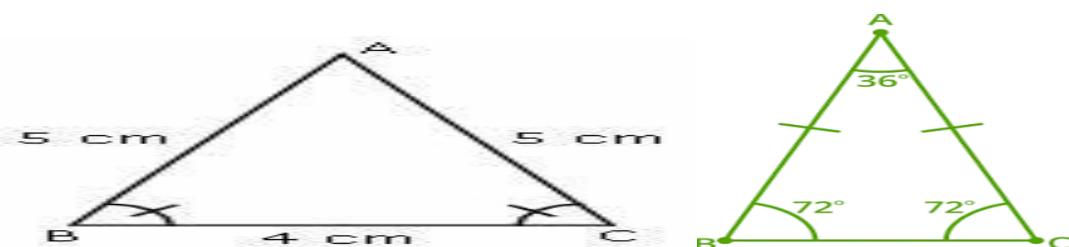




TYPES OF TRIANGLES

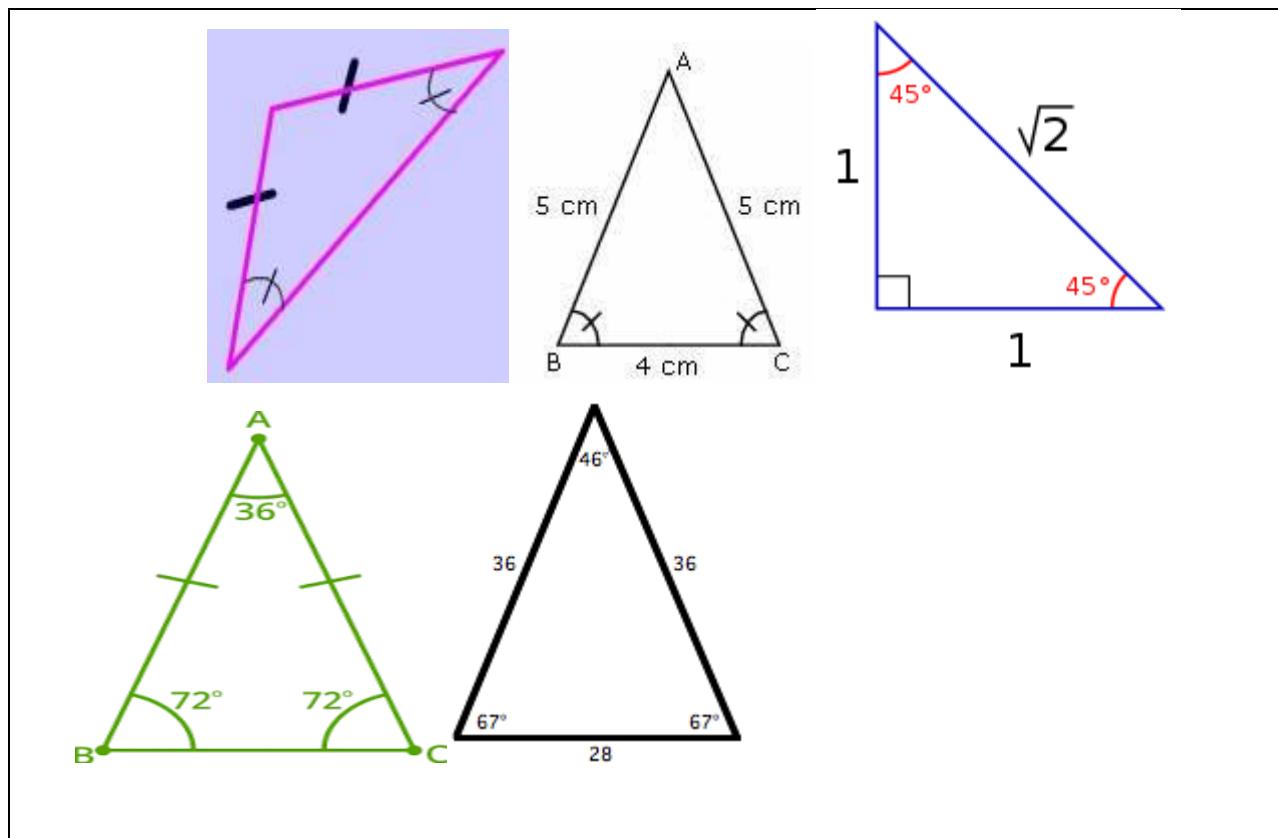
1. ISOSCELES TRIANGLE

Isosceles triangle is a triangle which has two equal sides and two equal angles. In other words, an isosceles definition states it as a polygon that consists of two equal sides, two equal angles, three edges, three vertices and the sum of internal angles of a triangle equal to 180° .



In the triangle ABC above, sides $AB = AC$ and angle $ABC = \text{angle } ACB$.

Other examples



ISOSCELES TRIANGLE PROPERTIES

1. Two sides are congruent to each other.
2. The third side of an isosceles triangle which is unequal to the other two sides is called the base of the isosceles triangle.
3. The two angles opposite to the equal sides are congruent to each other. That means it has two congruent base angles and this is called an isosceles triangle base angle theorem. The angle which is not congruent to the two congruent base angles is called an apex angle.
4. The altitude from the apex of an isosceles triangle bisects the base into two equal parts and also bisects its apex angle into two equal parts.
5. The altitude from the apex of an isosceles triangle divides the triangle into two congruent right angled triangles.
6. Area of Isosceles triangle = $\frac{1}{2} \times \text{base} \times \text{altitude}$.
7. Perimeter of Isosceles triangle = sum of all the three sides.

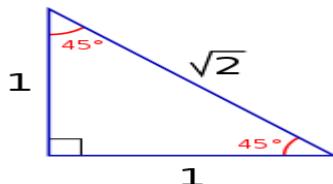
RIGHT ISOSCELES TRIANGLE

This triangle which has a right angle (90°) and also two equal angles.

PROPERTIES OF ISOSCELES RIGHT ANGLED TRIANGLE

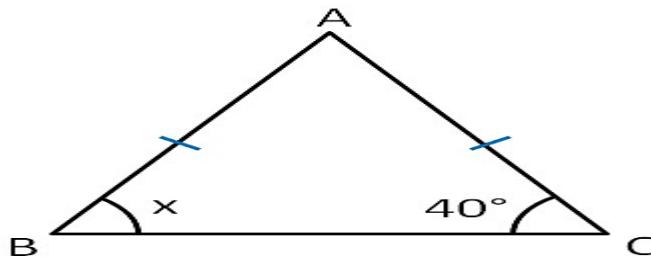
1. One angle is a right angle and the other two angles are both 45° degrees.
2. The longest side is the hypotenuse and is opposite the right angle.
3. The opposite and adjacent sides are equal.
4. The area is half product of the opposite and adjacent sides.
5. The centre of the circumscribing circle is the midpoint of the hypotenuse.

Example of the diagram showing right isosceles triangle

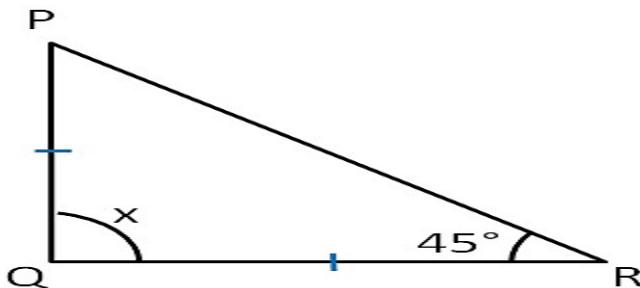


EXERCISE A

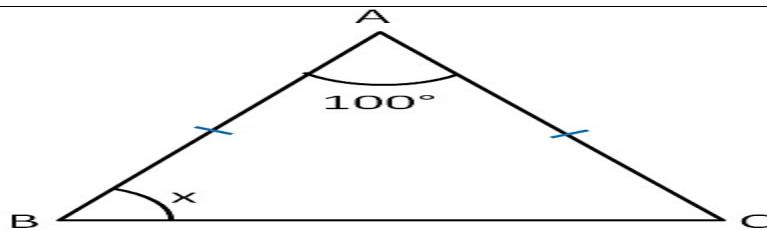
- a. Find the angle x and angle A in the diagram below.



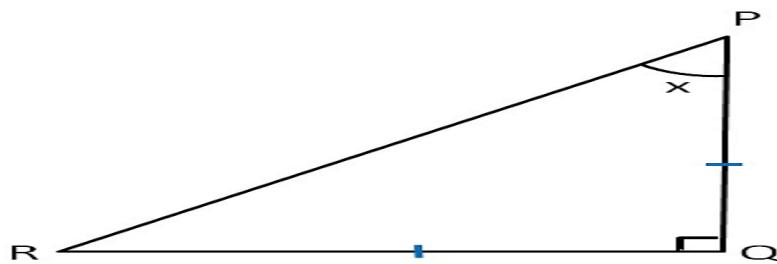
- b. Find angle X in the figure below.



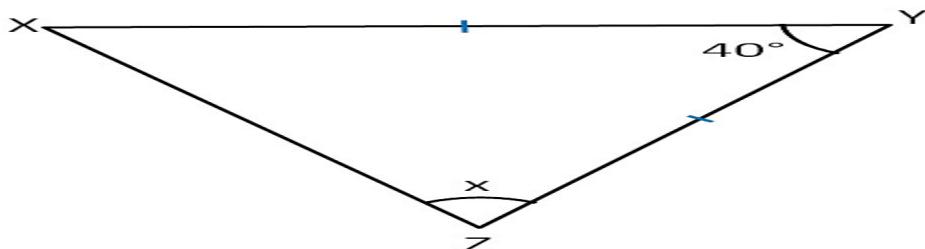
- c. Find angle x below



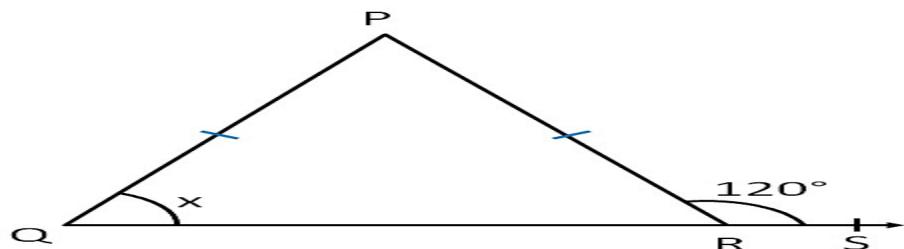
d. Find angle x below



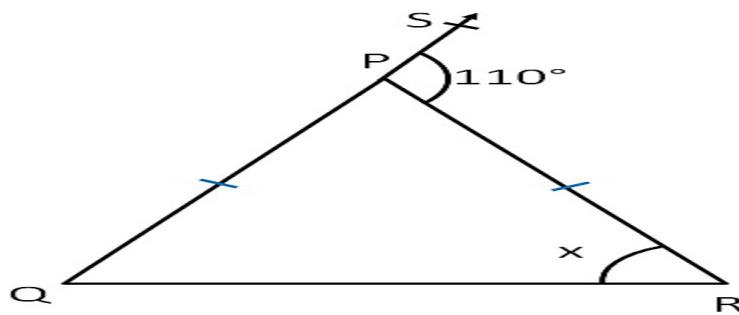
e. Find angle x below



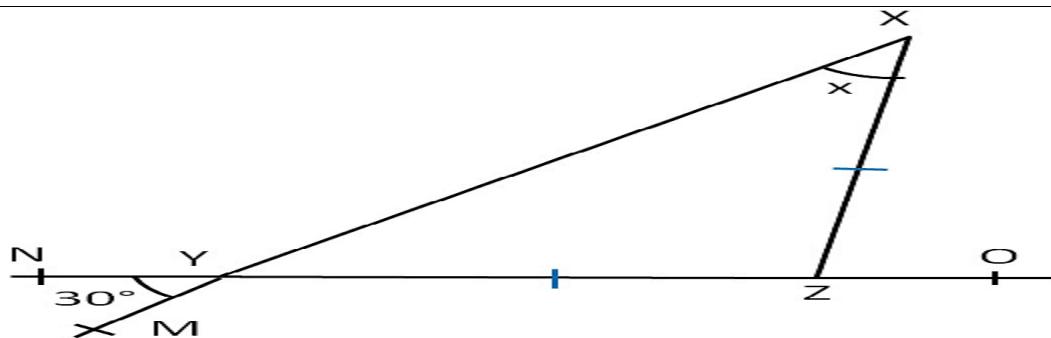
f. Find angle x below



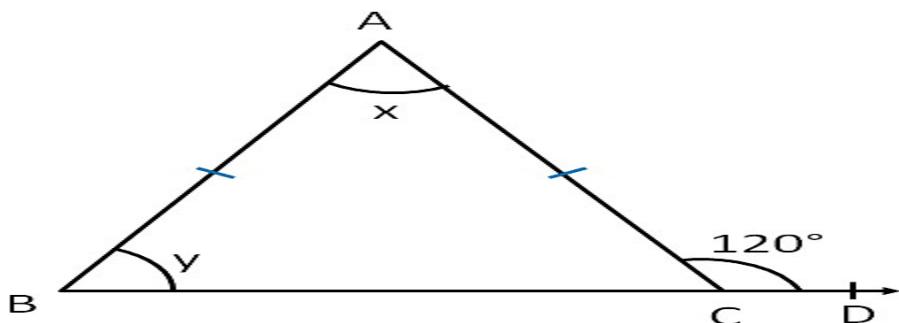
g. Find angle x below



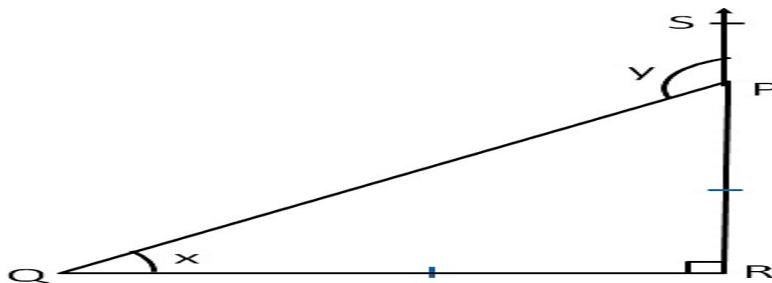
h. Find angle x below



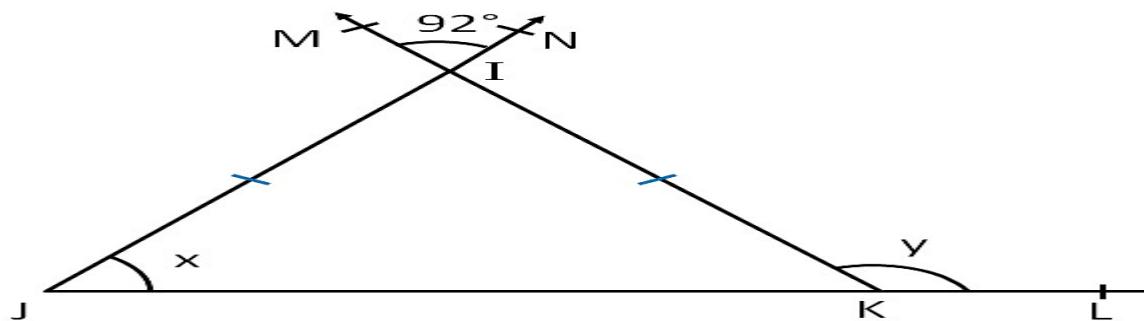
i. Find angle x and y below



j. Find angle x and y below

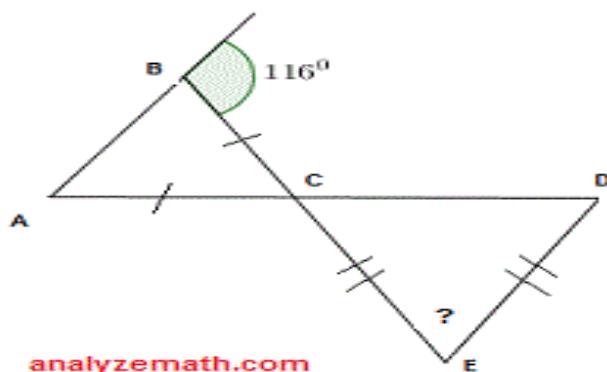


k. Find angle x and y below

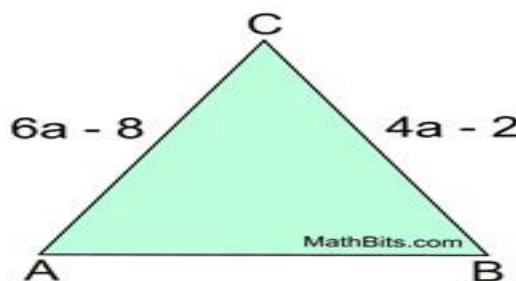


l. Find x and y from the given figure below

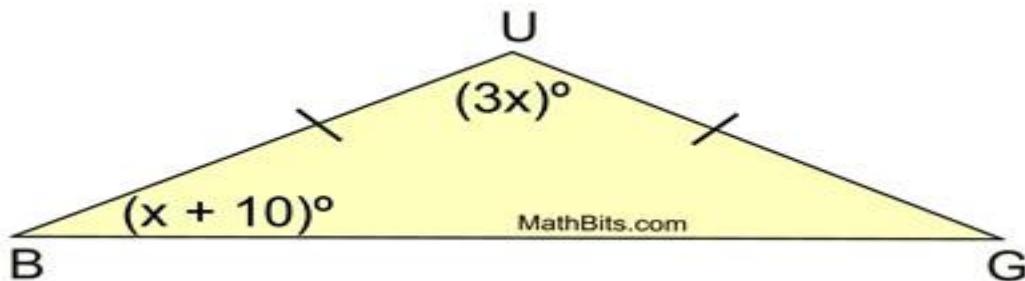
m. ABC and CDE are isosceles triangles. Find the size of angle CED.



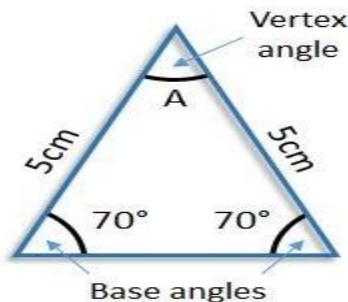
- n. Triangle ABC is isosceles where AC = CB. $AC = 6a - 8$ and $CB = 4a - 2$. Find AC.



- o. Triangle BUG is isosceles. Angle BUG = $3x$ and angle UBG = $x + 10$. Find angle BUG.

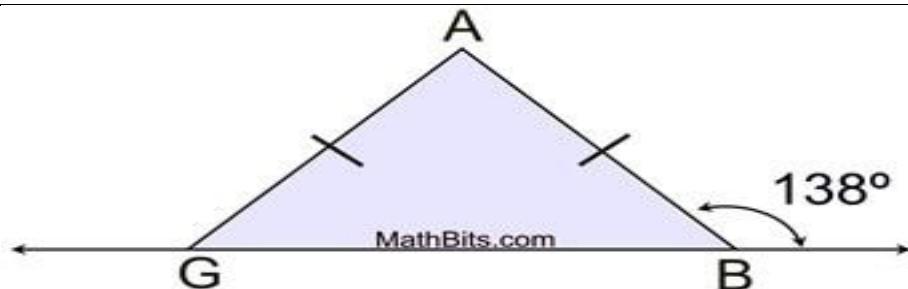


- p. Find the value of the vertex angle in the diagram below.

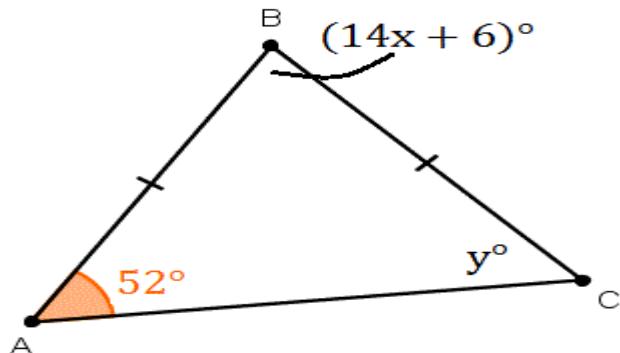


- q. $\triangle GAB$ is isosceles. The exterior angle at point B measures 138° .

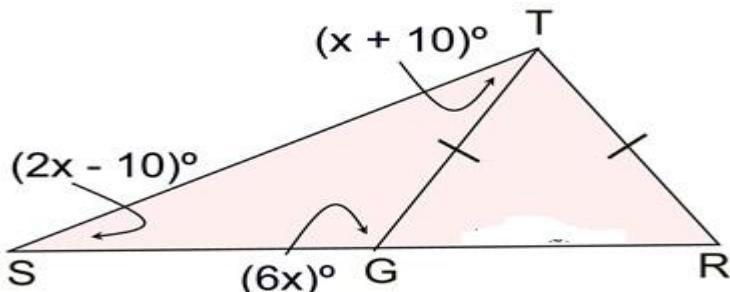
Find $< A$.



r. Find the value of x and y in the following diagram below.

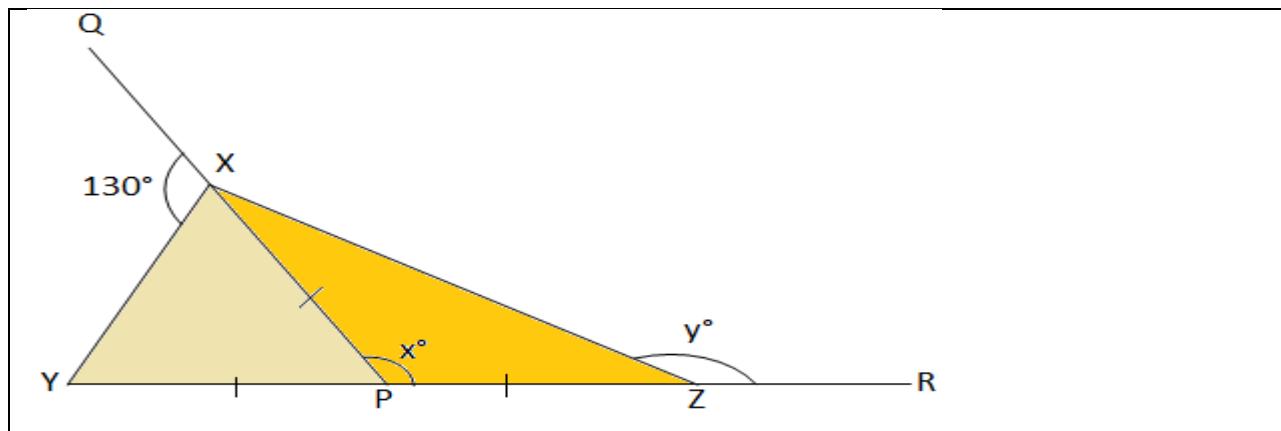


s. $\angle T = \angle R$, $\angle S = 2x - 10$; $\angle STG = x + 10$ and $\angle SGT = 6x$. Find $\angle S$ and $\angle TGR$.



EXERCISE B

2. If an isosceles triangle has lengths of two equal sides as 5cm and base as 4cm and an altitude is drawn from the apex to the base of the triangle. Then find its area and perimeter.
3. In the figure below, find the values of x and y .



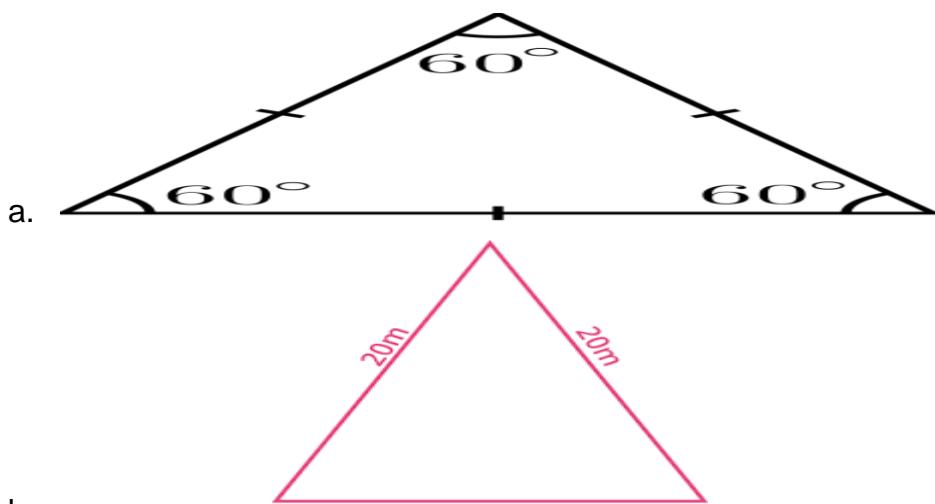
EQUILATERAL TRIANGLE

Equilateral triangle is a triangle which has three congruent sides (three equal sides) and three congruent angles (three equal angles). This means that all three sides are equal and all three angles are equal. Each angle is 60° . The Equilateral triangle is shown by the diagram below. In the triangle ABC above, sides AB=BC=CA and angle ABC= BCA=CAB = 60° .

PROPERTIES OF EQUALITERAL TRIANGLE

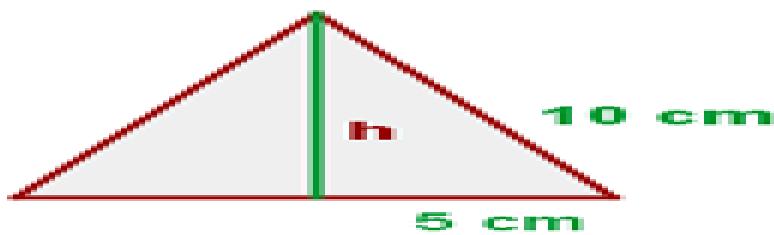
1. All three sides and three angles are equal.
2. The three angles are 60° each.

EXAMPLES



- c. Find the area of an equilateral triangle whose perimeter is 12cm
- d. Find the area of an equilateral triangle whose side is 7cm
- e. Find the area of an equilateral triangle whose side is 28cm.

- f. Calculate the height of an equilateral triangle with a side of 10 cm



4. SCALENI TRIANGLE

The scale triangle has no congruent sides (No equal sides and No equal angles). In other words, each side must have a different length. The scalene triangle is shown in the figure below.

PROPERTIES OF SCALE TRIANGLES

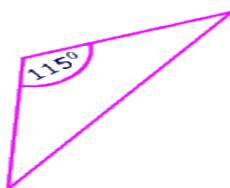
1. All sides of the scalene triangle are unequal
2. All angles of a scalene triangle are unequal.

RIGHT SCALENE TRIANGLE

This is a triangle which has one right angle and no equal sides.

5. OBTUSE TRIANGLE

The obtuse triangle is a triangle which has an obtuse angle (an obtuse angle has more than 90°). In the picture below, the shaded angle is the obtuse angle that distinguishes this triangle. Since the total degrees in any triangle is 180° , an obtuse triangle can only have one angle that measures more than 90° .



6. Acute triangle

The Acute Triangle has three acute angles (an acute angle measures less than 90°)

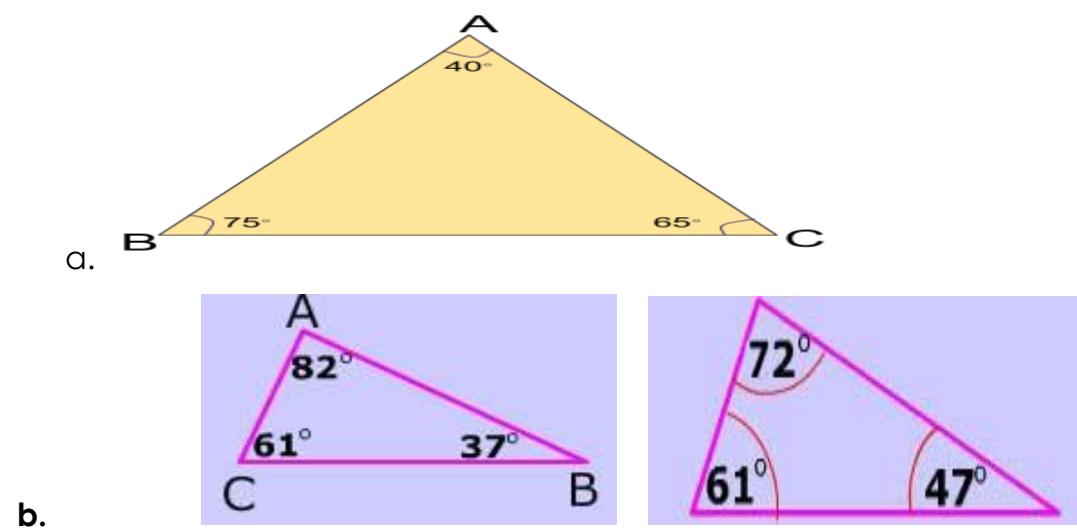
PROPERTIES OF ACUTE TRIANGLE

1. The interior angles of a triangle are always less than 90°

2. In an acute triangle, the line drawn from the base of the triangle to the opposite vertex is always perpendicular.
3. A triangle has three vertices.
4. The interior angles of a triangle are formed when two edges of a triangle meet.

Example

In the figure below angles A, B and C are all less than 90 degrees.



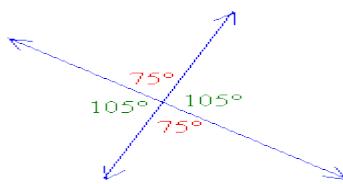
EXAMPLES OF THE PYTHAGOREAN THEOREM

When you use the Pythagorean Theorem, just remember that the hypotenuse is always 'c' in the formula above. Look at the following examples to see pictures of the formula.

VERTICALLY OPPOSITE ANGLES

Vertical angles are opposite angles are equal to each other. They are two pairs of angles which are formed by two intersecting lines.

The figure below shows the vertically opposite angles.



When two lines intersect they form two pairs of opposite angles, A + C and B + D. Another word for opposite angles are vertical angles.

Vertically angles are always congruent which means that they are equal.

EXERCISE

Given the diagram below, determine the values of the angles x, y and z.

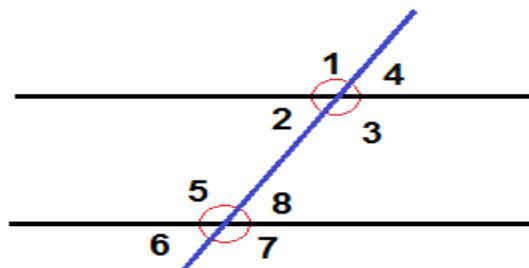
PARALLEL LINES

When parallel lines get crossed by another line which is called transversal, you can see that many angles are the same as in this example

Angles that are in the area between the parallel lines like angle 2 and 8 above are called **interior angles** whereas the angles that are on the outside of the parallel lines like 1 and 6 are called exterior angles.

Angles that are on the opposite sides of the transversal are called **alternate angles**, for example 1 and 8.

When a transversal intersects with two parallel lines eight angles are produced.



1. **Corresponding angles**- They are equal

Corresponding angles are congruent-equal. All angles that have the same position with regards to the parallel lines and the transversal are corresponding pairs.

Examples

- a. 3 and 7
- b. 4 and 8
- c. 2 and 6
- d. 1 and 5

2. **Alternate angles**- When a line intersects a pair of parallel lines alternate

angles are formed. Alternate interior angles are equal to each other.

Alternate angles are angles that are on the opposite sides of the transversal.

They are equal angles.

Examples

- a. 2 and 8
- b. 3 and 5

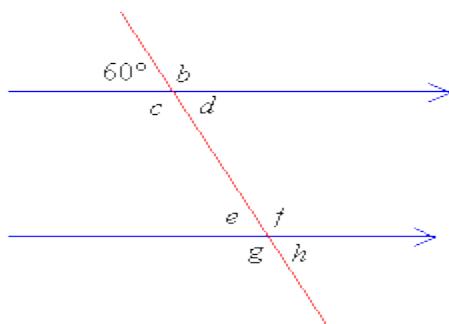
3. **Allied angles (Interior angles)** - They are angles that add up to 180 degrees.

Examples

- a. $3 + 8 = 180$ degrees (Allied angles)
- b.** $2 + 5 = 180$ degrees (Allied angles)

EXAMPLE

Given the diagram below, determine the values of the angles b, c, d, e, f, g and h.



Solution

Step 1: b is a supplement of 60 degrees

Therefore $b + 60 = 180$

$$\begin{aligned} b &= 180 \text{ degrees} - 60 \text{ degrees} \\ &= 120 \text{ degrees} \end{aligned}$$

Step 2 : b and c are vertical angles

Therefore, $c = b = 120$

Step 3 : d and 60 degrees are vertical angles

Therefore, $d = 60$ degrees

Step 4: d and e are alternate interior angles

Therefore, $e = d = 60$ degrees

Step 5 : f and e are supplementary angles.

Therefore, $f + 60 = 180$ degrees.

Step 6: f and e are supplementary angles

Therefore, $f + 60 = 180$

$$\begin{aligned}f &= 180 \text{ degrees} - 60 \text{ degrees} \\&= 120^\circ\end{aligned}$$

REGULAR POLYGON

Regular polygon is a polygon that is equiangular (all angles are equal in measure) and equilateral (all sides have the same length).

A polygon can either be a convex or a concave.

A convex polygon has all angles less than 180 degrees

A concave polygon has at least one angle greater than 180 degrees.

EXAMPLES OF REGULAR POLYGON

Name of the polygon	Number of sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10

SUM OF EXTERIOR ANGLES

The sum of interior angle of a convex polygon with n sides can be found using the following formula :

$$\text{The sum of Interior angle} = (2n - 4)90^\circ$$

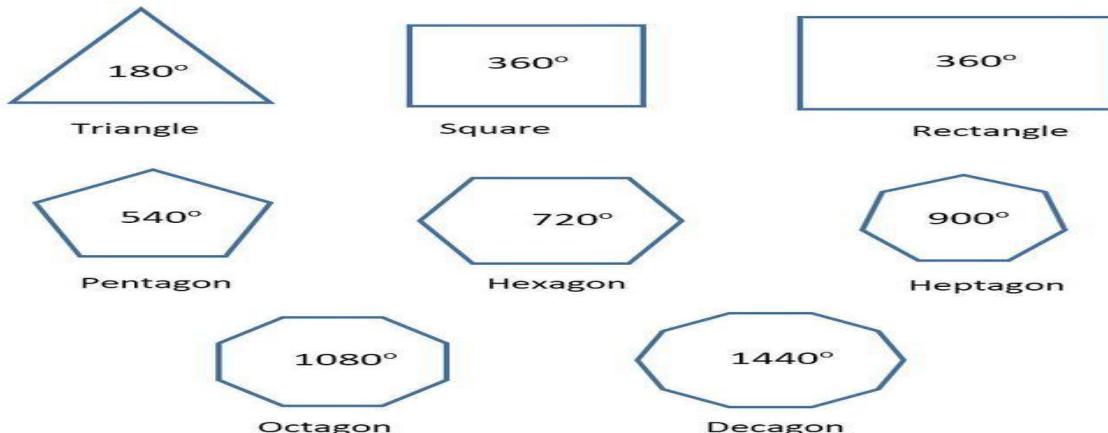
$$\text{OR The sum of interior angles} = (n - 2)180^\circ$$

EXAMPLES

The value of the interior angles of the regular polygon can be found using or calculated the following formula :

$$\text{Interior angle} = \left(1 - \frac{2}{n}\right) \times 180 \text{ degrees or}$$

$$= \frac{180(n-2)}{n} \text{ degrees}$$



EXAMPLES

Find the angle sum of each of the following regular polygons

Regular polygon	formula= $(n - 2)180^\circ$
a. Regular triangle (3 sided polygon)	$= (3 - 2)180^\circ = 180^\circ$
b. Regular quadrilateral (4 sided polygon)	$(4 - 2)180^\circ = 360^\circ$
c. Regular Pentagon (5 sided polygon)	$(5 - 2)180^\circ = 540^\circ$
d. Regular Hexagon (6 sided polygon)	$(6 - 2)180^\circ = 720^\circ$
e. Regular Heptagon (7 sided polygon)	$(7 - 2)180^\circ = 900^\circ$
f. Regular Octagon (8 sided polygon)	$(8 - 2)180^\circ = 1080^\circ$

INTERIOR ANGLE

The value of the interior angles of the regular polygon can be found using or calculated the following formula :

$$\text{Interior angle} = \left(1 - \frac{2}{n}\right) \times 180 \text{ degrees or}$$

$$= \frac{180(n-2)}{n} \text{ degrees}$$

EXAMPLE

Find the value of the interior angle of each of the following regular polygon

Number of sides	Formula	Calculations
a. 3 sided polygon	$\left(1 - \frac{2}{n}\right) \times 180$	$\left(1 - \frac{2}{3}\right) \times 180 = 60^\circ$
b. 4 sided polygon	$\left(1 - \frac{2}{n}\right) \times 180$	$\left(1 - \frac{2}{4}\right) \times 180 = 90^\circ$
c. 5 sided polygon	$\left(1 - \frac{2}{n}\right) \times 180$	$\left(1 - \frac{2}{5}\right) \times 180 = 108^\circ$
d. 6 sided polygon	$\left(1 - \frac{2}{n}\right) \times 180$	$\left(1 - \frac{2}{6}\right) \times 180 = 120^\circ$
e. 7 sided polygon	$\left(1 - \frac{2}{n}\right) \times 180$	$\left(1 - \frac{2}{7}\right) \times 180 = ?$
f. 8 sided polygon	$\left(1 - \frac{2}{n}\right) \times 180$	$\left(1 - \frac{2}{8}\right) \times 180 = 60^\circ$
g. 9 sided polygon	$\left(1 - \frac{2}{n}\right) \times 180$	$\left(1 - \frac{2}{9}\right) \times 180 = 60^\circ$
h. 10 sided polygon	$\left(1 - \frac{2}{n}\right) \times 180$	$\left(1 - \frac{2}{10}\right) \times 180 = 60^\circ$

Exercise

The angles of a triangle have the ratio 3:2:1. What is the measure of the smallest angle?

EXERCISE

Find the value of each interior angle of a regular polygon with the following number of sides

- | | |
|-------|-------|
| a. 15 | d. 22 |
| b. 11 | e. 18 |
| c. 15 | f. 24 |

EXTERIOR ANGLE

The exterior angle is the angle between any side of a shape and a line extended from the next side.

All the Exterior Angles of a polygon add up to 360° .

Each exterior angle can be found by the formula = $\frac{360^\circ}{n}$

EXAMPLE

What is the exterior angle of the following regular polygons?

Regular polygon	Exterior angle
g. Regular triangle (3 sided polygon)	$\frac{360}{3} = 120^{\circ}$
h. Regular quadrilateral (3 sided polygon)	$\frac{360}{4} = 90^{\circ}$
i. Regular Pentagon (3 sided polygon)	$\frac{360}{5} = 72^{\circ}$
j. Regular Hexagon (3 sided polygon)	$\frac{360}{6} = 60^{\circ}$

EXERCISES

1. Find the sum of the interior angles of a regular polygon with the following numbers of sides

a. 10	e. 20
b. 14	f. 13
c. 4	g. 16
d. 18	h. 12

EXAMPLE

The sum of interior angle of a regular polygon is 1800° . Find the number of sides in the polygon.

$$(2n - 4)90^{\circ} = 1800^{\circ}$$

$$2n - 4 = \frac{1800}{90} = 20$$

$$2n = 24$$

$$n = 12$$

A polygon with 12 sides is called a dodecagon.

EXERCISE

How many sides does a polygon have if the sum of the interior angle is

- | | |
|-------------------|------------------|
| a. 3240° | d. 540° |
| b. 2520° | e. 720° |
| c. 2340° | |
| f. 900° | |

EXAMPLE

Four angles of a hexagon are $(x + 18)^\circ$, $(2x - 10)^\circ$, $(3x + 25)^\circ$ and $(x + 42)^\circ$. The fifth and sixth angles are $(70)^\circ$ and $(50)^\circ$ respectively. Find the last and smallest angles of hexagon.

$$\text{Sum of interior angle of hexagon} = (2n - 4)90^\circ$$

$$= (2 \times 6 - 4)90^\circ$$

$$= (12 - 4)90^\circ$$

$$= 8 \times 90^\circ$$

$$= 720^\circ$$

$$(x + 18)^\circ + (2x - 10)^\circ + (3x + 25)^\circ + (x + 42)^\circ + (70)^\circ + 50^\circ = 720^\circ$$

$$7x + 195^\circ = 720^\circ$$

$$7x = 720^\circ - 195^\circ$$

$$7x = 525^\circ$$

$$x = 195^\circ$$

The angles of the hexagon are, $(2x - 10)^\circ$, $(3x + 25)^\circ$ and $(x + 42)^\circ$. The fifth and sixth angles are $(70)^\circ$ and $(50)^\circ$, that is

$$93^\circ, 140^\circ, 259^\circ, 117^\circ, 70^\circ \text{ and } 50^\circ$$

EXERCISE

- Find the largest and smallest angles of a hexagon whose angles are $(x + 20)^\circ$, $(2x - 40)^\circ$, $(2x + 30)^\circ$, $(x - 60)^\circ$, $(2x - 124)^\circ$ and 110°

$$(x + 20)^\circ + (2x - 40)^\circ + (2x + 30)^\circ + (x - 60)^\circ + (2x - 124)^\circ + 110^\circ = 720$$

$$8x + 160 - 224 = 720$$

$$8x - 64 = 720$$

$$8x = 784$$

$$x = 98$$

The angles are $(98 + 20)^\circ$, $(2 \times 98 - 40)^\circ$, $(2 \times 98 + 30)^\circ$, $(98 - 60)^\circ$, $(2 \times 98 - 124)^\circ$, 110°

118, 156, 226, 38, 72, 110

The largest and smallest angles are 226 degrees and 38 degrees

- The exterior angle of a regular polygon is 20° . How many sides does it have?

$$\begin{aligned}\text{Number of sides} &= \frac{360^\circ}{20^\circ} \\ &= 18\end{aligned}$$

3. The exterior and interior angles of a regular polygon are in the ratio 1 : 9.

Calculate the exterior angle and hence the number of sides.

Sum of exterior and interior angles:

$$\begin{aligned}9x + x &= 180 \text{ (Supplementary angles)} \\ 10x &= 180 \\ x &= 18 \text{ degrees} \\ 9x &= 9 \times 18 \\ &= 162\end{aligned}$$

4. Calculate the number of sides of a regular whose interior angles are each 160° .

$$\begin{aligned}\left(1 - \frac{2}{n}\right) \times 180 &= 160 \\ 1 - \frac{2}{n} &= \frac{8}{9} \\ 1 - \frac{8}{9} &= \frac{2}{n} \\ \frac{1}{9} &= \frac{2}{n} \\ n &= 18\end{aligned}$$

5. An interior angle of a regular polygon is 140° greater than the exterior angle.

Calculate the size of each exterior and the number of sides of the polygon.

Let exterior angle be x

The Interior angle = $140 + x$

$$x + 140 + x = 180 \text{ (supplementary angles)}$$

$$2x + 140 = 180$$

$$2x = 180 - 140$$

$$2x = 40$$

$$x = 20 \text{ degrees}$$

6. Each interior angle of a regular polygon is five times each exterior angle.
 - a. What is the size of exterior angle?
 - b. How many sides does the polygon have?

Let exterior angle be = x

Then Interior angle = $5x$

$$5x + x = 180$$

$$6x = 180$$

$$x = 30$$

$$\text{Number of sides of the polygon} = \frac{360}{30}$$

$$= 12$$

RECTANGLE

PERIMETER OF THE RECTANGLE

The perimeter of the rectangle is the distance around the rectangle. If you started at one corner and walked around the rectangle, you would walk $L+W+L+W$ units or two lengths and two widths.

Then the perimeter of the rectangle

$$P = L+W+L+W$$

$$P = 2L+2W$$

PROPERTIES OF RECTANGLES

1. Rectangles have four sides and four right angles
2. The lengths of opposite sides are equal.
3. The perimeter, P , of a rectangle is the sum of twice the width.
4. The area of a rectangle is the length times the width.

$$A = L \times W$$

EXAMPLES

1. Find the perimeter and area of the rectangle of length 17cm and breadth 13cm.

$$\begin{aligned}\text{Perimeter of the rectangle} &= 2(L+B) \\ &= 2(17+13)\end{aligned}$$

$$= 2 \times 30\text{cm}$$

$$= 60\text{cm}$$

Area of the rectangle, $A = \text{Length} \times \text{Breadth}$

$$= 33 \times 13\text{m}^2$$

$$= 221\text{m}^2$$

2. Find the breadth of the rectangular plot of land whose area is 660m^2 and whose length is 33. Find its perimeter.

$$\text{Breadth of the rectangular plot} = \frac{\square \square \square \square}{\square \square \square \square h}$$

$$= \frac{660\text{m}^2}{33\text{m}}$$

$$= 20\text{m}$$

$$\text{Perimeter of the rectangular plot} = 2(L+B)$$

$$= 2(22+20)\text{m}$$

$$= 2 \times 53\text{m}$$

$$= 106\text{m}$$

3. Find the area of the rectangle if its perimeter is 48cm and its breadth is 6cm.

$$\text{Perimeter of the rectangle} = 2(L+B)$$

$$48\text{cm} = 2(L+6)$$

$$\frac{48}{2} = L + 6$$

$$24 = L + 6$$

$$\text{Length, } L = 24-6$$

$$L = 18\text{cm}$$

$$\text{Now, area of the rectangle} = L \times B$$

$$= 18\text{cm} \times 6\text{cm}$$

$$= 108\text{cm}$$

4. Find the breadth and perimeter of the rectangular if its area is 96m^2 and the length is 12cm.

$$\text{Area of the rectangle} = \text{Length} \times \text{Breadth}$$

$$96 = 12 \times \text{Breadth}$$

$$\begin{aligned}\text{Breadth, } B &= \frac{96}{12} \\ &= 8\text{cm}\end{aligned}$$

Therefore, the Perimeter of the rectangle = $2(L+B)$

$$\begin{aligned}&= 2(12+8)\text{cm} \\ &= 2 \times 20 \\ &= 40\text{cm}\end{aligned}$$

5. The length and breadth of a rectangular courtyard is 75m and 32m. Find the cost of leveling it at the rate of K3 per square meter. Also, find the distance covered by a boy to take 4 rounds of the courtyard.

$$\begin{aligned}\text{Perimeter of the courtyard} &= 2(L+B) \\ &= 2 (75 + 32)\text{m} \\ &= 2 \times 107\text{m} \\ &= 214\text{m}\end{aligned}$$

Distance covered by the boy in taking 4 rounds = $4 \times 214\text{m}$
= 856m

$$\begin{aligned}\text{Area of the courtyard} &= 75\text{m} \times 32\text{m} \\ &= 2400\text{m}^2\end{aligned}$$

For 1m^2 , the cost of leveling = K3

$$\begin{aligned}\text{For } 2400\text{m}^2, \text{ the cost of leveling} &= K3 \times 2400 \\ &= K7200\end{aligned}$$

6. A floor of the room 8m and 6m wide is to be covered by square tiles. If each square tile is 0.8m, find the number of tiles required to cover the floor. Also, find the cost of tiling at the rate of K7 per tile.

$$\begin{aligned}\text{Area of the room} &= 8\text{m} \times 6\text{m} \\ &= 48\text{m}^2\end{aligned}$$

$$\begin{aligned}\text{Area of the tile} &= 0.8 \times 0.8\text{m}^2 \\ &= 0.64\text{m}^2\end{aligned}$$

$$\text{Number of one square tile} = \frac{\text{Area of the room}}{\text{Area of the tile}} = \frac{48}{0.64} = 75$$

$$= \frac{48}{0.64} \\ = 75 \text{ tiles}$$

For one tile, the cost of tiling is K7

$$\text{For 7 tiles, the cost of tiling} = (7 \times 7) \\ = K525$$

7. The length and breadth of the rectangle park are in the ratio 5:4 and its area is 2420 m^2 , find the cost of fencing the park at the rate of K10 per meter.

Let the common ratio be x

Then the length of rectangular park = $5x$

Breadth of rectangular park = $4x$

$$\text{Area of rectangular park} = 5x \times 4x \\ = 20x^2$$

$$\text{According to the question} \quad 20x^2 = 2420$$

$$\frac{20x^2}{20} = \frac{2420}{20} \\ x^2 = 121$$

$$x = 11$$

$$\text{Therefore, } 5x = 5 \times 11$$

$$5x = 55$$

$$4x = 4 \times 11$$

$$4x = 44$$

$$\text{The perimeter of the rectangular park} = 2(L + B)$$

$$= 2(55 + 44) \\ = 2 \times 99 \\ = 198 \text{ CM}$$

$$\text{For 1 meter, the cost of fencing} = \text{K10}$$

$$\text{For 198m, the cost of fencing} = 198 \times 10 \\ = \text{K1980}$$

8. A wire in the shape of rectangle of length 25 cm and 17 breadth 17 cm is rebent to form a square. What will be the measure of each side?

$$\text{Perimeter of rectangle} = 2(25 + 17)\text{cm}$$

$$= 2 \times 42$$

$$= 84\text{cm}$$

$$\text{Perimeter of the square of side } x\text{cm} = 4x$$

$$\therefore \text{Perimeter of rectangle} = \text{Perimeter of square}$$

$$84\text{cm} = 4x$$

$$x = 21.$$

$$\therefore \text{Each side of square} = 21\text{cm}$$

9. How many envelopes can be made out of a sheet of paper 1000cm by 75cm, supposing 1 envelope requires 20cm by 5cm piece of paper.

$$\text{Area of the sheet} = 100 \times 75 \text{ cm}^2$$

$$= 7500 \text{ cm}^2$$

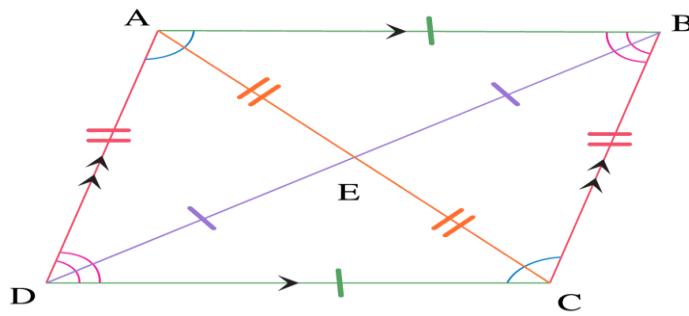
$$\text{Area of envelope} = 20 \text{cm} \times 5\text{cm}$$

$$= 100 \text{ cm}^2$$

$$\therefore \text{Number of envelopes that can be made} = \frac{\text{Area of sheet}}{\text{Area of envelope}}$$

$$= \frac{7500 \text{ cm}^2}{100 \text{ cm}^2}$$

PARALLELOGRAM

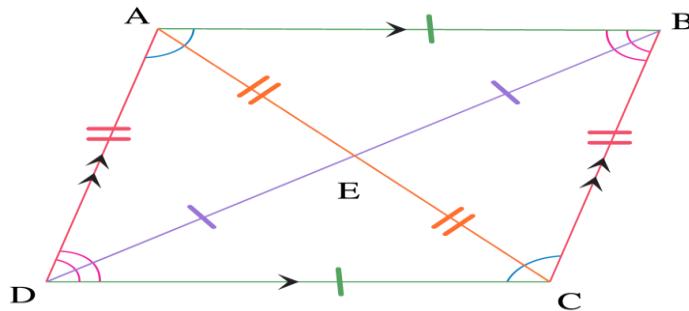


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1. Both pairs of opposite sides of parallelogram are parallel
2. Both pairs of opposite sides of parallelogram are congruent
3. Both pairs of opposite angle are congruent or equal.

4. Consecutive angles (interior angles) of parallelogram are supplementary
5. Diagonals bisect each other(cut each into equal parts)

PROPERTIES OF PARALLELOGRAMS EXPLAINED



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1. Opposite sides are parallel.

Segment AB is parallel to segment DC, and segment AD is parallel to segment BC.

2. Opposite sides are congruent

Segment AB is congruent to segment DC and segment AD is congruent to segment BC.

3. Opposite angles are congruent or equal

Angle A is congruent or equal to angle C and angle D is congruent to angle B.

4. Same-side interior or consecutive angles are supplementary.

Angles A and D are supplementary, angles A and B are supplementary, angles B and C are supplementary, and angles D and C are supplementary

5. Each diagonal of a parallelogram separates it into two congruent triangles

Triangle DAB is congruent to triangle DCB

6. The diagonals of a parallelogram bisect each other.

Segment AE is congruent to segment CE and segment DE is congruent to BE.

PERIMETER AND AREA OF THE PARALLELOGRAM

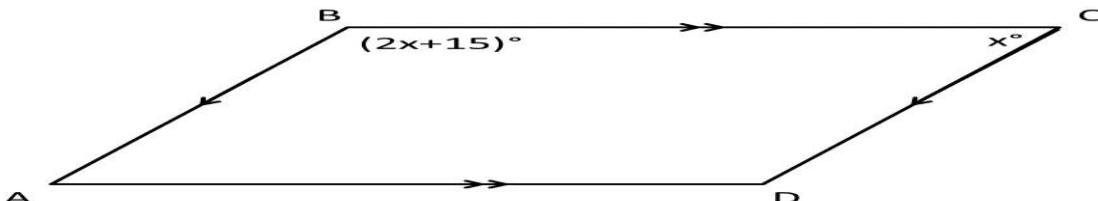
Perimeter of a parallelogram is the total distance of boundaries of the parallelogram.

Perimeter of the parallelogram = $2(a+b)$

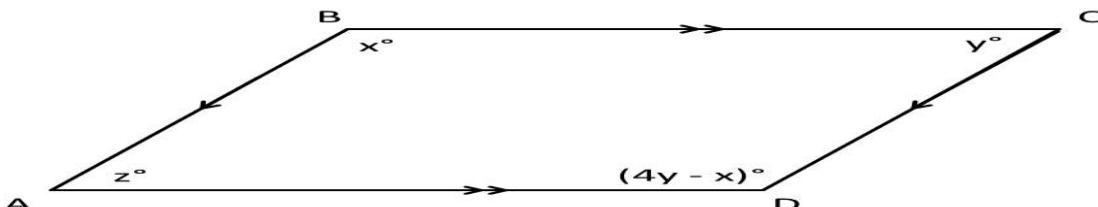
Area of the parallelogram = Base x Height

EXERCISE

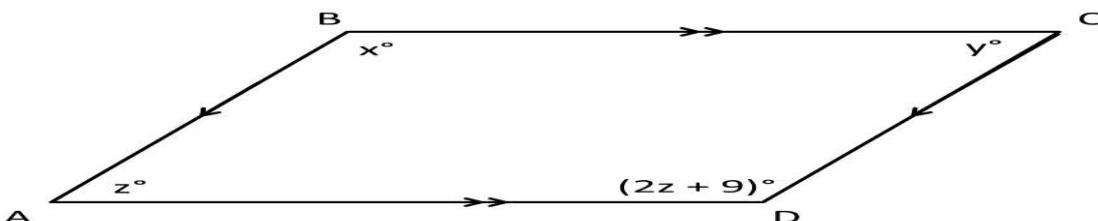
- 1.** In the parallelogram ABCD below, angle B = $(\square \square + \square \square)^\circ$ and angle \square° . Find \square° .



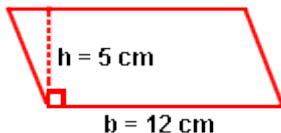
- 2.** ABCD below is a parallelogram. Find z.



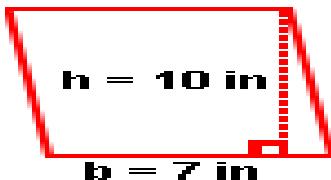
- 3.** ABCD below is a parallelogram. Find y



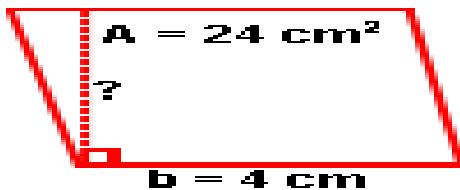
- 4.** Find the area of a parallelogram with a base of 12cm and a height of 5cm.



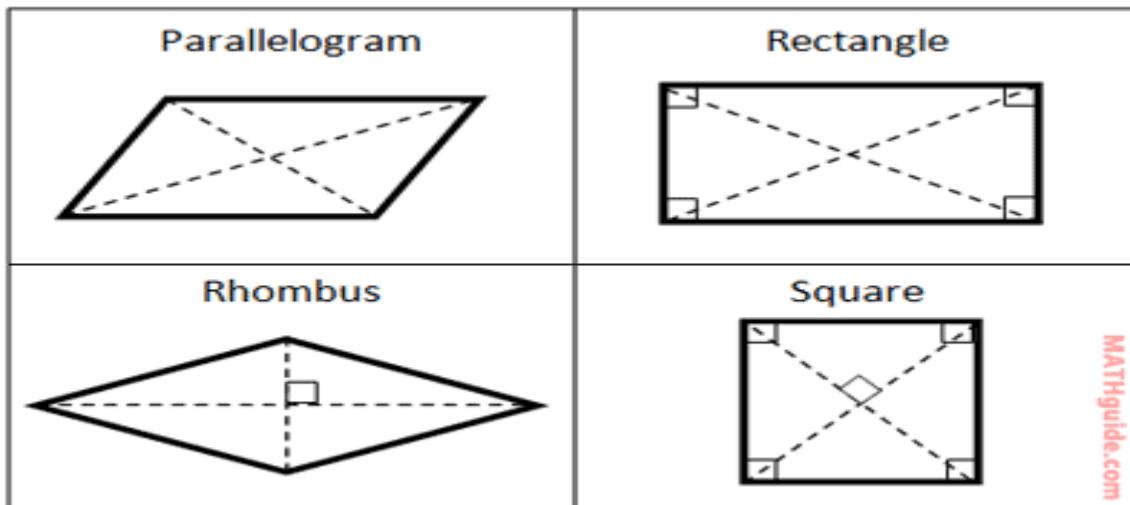
- 5.** Find the area of the parallelogram with a base of 7 m and a height of 19m.



6. The area of the parallelogram is 24 square centimeters and the base is 4 centimeters. Find the height.



QUADRILATERALS



COMMERCIAL ARITHMETIC

CASH AND TRADE DISCOUNT

A trade discount is a discount given by the seller to the buyer as a deduction in the list price of the commodity. A trade discount is one that is allowed by the wholesaler to the retailer calculated on the list price of the product.

Cash discount is a deduction in the amount of invoice allowed by the seller to the buyer in return for immediate payment in order to facilitate bulk sales. It is allowed by the sellers to the customers at the time of making the payment of the purchases as a reduction in the invoice price of the commodity to facilitate a prompt payment and thereby to avoid the credit risk.

EXAMPLE

The selling price of the Television Set is K2, 900. The manufacturer offers its dealer a 40% trade discount. What are the amount of trade discount and net price?

Trade discount = List price x Trade discount rate

$$= K2,900 \times \frac{40}{100}$$

$$= K1160$$

$$\begin{aligned} \text{Net Price} &= K2,900 - K1160 \\ &= K1740 \end{aligned}$$

The price of the computer after the discount is K18, 000. With 30% trade discount is offered; find the list price of the bag.

The list price of the computer = Net Price + Trade discount

$$= K18,000 + (30\% \times K18,000)$$

$$= K18,000 + K5,400$$

$$= K23,400$$

Two shops are offering the same model of TV set for sale which had an original price of K36, 000.

Shop A is offering a discount of 8% followed by a special offer of 3% off that discounted price.

Shop B is offering a single discount of 11%.

Required:

What saving would be made by buying the set at the cheaper price?

Cost at Shop A = 92 x 36000

100

=K33120 before offer

= **K33120 x 97**

100

= **K32126.40 after offer**

Cost at Shop B = 89 x 36000

100

= **K32040.00**

.: Saving by buying at Shop B

$$= K32126.40 - K32040.00$$

$$= K86.40$$

A businessman purchased a computer that was discounted at the rate of 8%. If the discounted price was K79, 650, find the original price of the computer, to the nearest Kwacha

K79,650 is 92% of the original price

$$\text{Original price} = \frac{\text{Discounted Price}}{0.92}$$

$$= K86,576.07$$

$$= K86,576$$

Two shops are offering the same model of TV set for sale which had an original price of K36, 000.

Shop A is offering a discount of 8% followed by a special offer of 3% off that discounted price.

Shop B is offering a single discount of 11%.

Required:

What saving would be made by buying the set at the cheaper price?

Cost at Shop A = 92 x 36000

101

=K33120 before offer

= K33120 x 97

100

= K32126.40 after offer

Cost at Shop B = 89 x 36000

101

= K32040.00

∴ Saving by buying at Shop B

$$= K32126.40 - K32040.00$$

$$= K86.40$$

A food retailer purchases rice at a cost of K160 per kg and sells it at K360 per kg.

Required:

- a. Calculate the gross profit received as a % of the selling price.
- b. If the retailer offered 10% discount on the selling price, what would be the new selling price of the rice? (Give your answer to two decimal places).

$$\text{Gross profit\%} = \frac{\text{Selling Price} - \text{Cost Price}}{\text{Cost Price}} \times 100$$

$$= 125\%$$

$$\text{New selling price at 10\% discount} = 360 \times 90\%$$

$$= K 324$$

1. A sales assistant is paid a basic wage of K100 per week plus commission of 5% of the value of the goods sold. What will be the earnings in a week in which the assistant sells goods to the value of £1,890?

$$\text{Earnings} = \text{Basic wage} + \text{commission}$$

$$= £100 + 5\% \times K1,890$$

$$= £100 + 94.50$$

$$= £194.50$$

Exercise

2. Two shops, A and B, are selling similar TV sets at a price of K96,000. Shop A is offering a discount of 8% followed by a special offer at 3% off the discounted price. Shop B is offering a single discount of 11%.

Required:

What saving would be made by buying the set at the cheapest price?

3. A food wholesaler purchased some fresh vegetables for K1,700 and sold them for K2,000.
 - a. Calculate the percentage profit received.
 - b. If the wholesaler offered a 17.65% discount on the selling price, what would be the new selling price of the fresh vegetables?

COMMISSION

Commission the reward paid employees for selling goods and services.

EXAMPLE

A certain girl who sells cell phone airtime gets a 5% on sales. If she sold K25,000 worth of airtime, how much commission does she get?

$$\begin{aligned}\text{Commission} &= \text{percentage} \times \text{total sales} \\ &= 5\% \text{ of K125,000} \\ &= 0.05 \times \text{K125,000} \\ &= \text{K1,250}\end{aligned}$$

The gets a commission of K1,250.

4. A used car salesperson can be paid using two methods

Method X uses straight commission of 3.5% of the selling price of all vehicles sold.

Method Y uses a fixed amount of K2, 500 per week plus commission of 1.5% of the selling price of all vehicles sold. If the total selling price of cars sold in each week is on average K200, 000, calculate which method of commission the salesperson would prefer.

5. A salesperson selling used cars can be paid using two methods of commission.

Method X uses straight commission of 3.5% of the selling price of all the vehicles sold. **Method Y** uses a fixed amount of K25, 000 per week plus a commission of 1.5% of the selling price of all the vehicles sold.

Required:

If the total selling price of cars sold in each week is, on average, K2 million, advise the salesperson which method of commission to choose.

6. A salesperson earns a basic wage of K1, 200 per month, plus commission on the value of goods sold in that month. In a particular month, the total value of goods sold by the salesperson was K18, 000 and the total wage received was K1, 470. Calculate the rate of commission (as a percentage) that the salesperson earns on the value of goods sold each month.

ELECTRICAL BILLS

- Electricity bills are charged per kwh (kilowatt-hours) which is the unit of

electricity.

- The difference between the readings of the first and last days of the month gives the number of units used.
- There is also a constant amount called the service charge which is added to every monthly bill whether you used electricity or not

Monthly	electricity	bill	=
(□ □ □ □ □ □ □ □ □ □ □ □ □ □ □ □) + □ □ □ □ □ □ □ □ □ □ □ □ □ □ □			

EXAMPLE

Study the table below and use it to answer the question that follows.

Name	old meter reading	New meter reading	No of units used	Service charge	Cost per unit
Mr. Sauka	9058	11003	1945	K750	K30
Mr. Mato	41	16609	2500	K1200	K25

Find the monthly bill for

- a. Mr. Sauka

Monthly	electricity	bill	=
(□ □ □ □ □ □ □ □ □ □ □ □ □ □ □ □) + □ □ □ □ □ □ □ □ □ □ □ □ □ □ □			
Number of units = New reading – Old reading			
= (□ □ □ □ □ □ □ □)			
= 1945			
Monthly electricity bill = (□ □ □ □ □ □ □) + □ □ □ □ □			
= K58,350 + K750			
= K59,100			

- b. Mr. Mato

Monthly	electricity	bill	=
(□ □ □ □ □ □ □ □ □ □ □ □ □ □ □ □) + □ □ □ □ □ □ □ □ □ □ □ □ □ □ □			
Number of units = New reading – Old reading			
= (□ □ □ □ □ □ □ □)			
= 12500			

Monthly electricity bill = (₦ 5,350,000) + ₦ 750
= ₦ 58,350 + ₦ 750
= ₦ 59,100

EXERCISE

Calculate the electricity bill for Professor Mvula if her new reading is 43,238 and the old reading was 42,782. She pays a fixed charge of K678 plus K22 per kilowatt-hour of electricity used.

WATER BILLS

- Water bills are calculated basing on the number of units used as well as the service charge in place.
 - However, the service charge is higher in low density areas like Nyambadwe in Blantyre, Chimaliroin Mzuzu and Area 10 in Lilongwe than in high density areas like Ndirande in Blantyre, Masasa in Mzuzu and Chilinde in Lilongwe.
 - Calculations for water bills are the same as for those of electricity.

Monthly water bill = (□ □ □ □ □ □ □ □ □ □ □ □ □ □ □) + □ □ □ □ □ □ □ □ □ □ □ □ □ □ □ □ □ □

Number of units = New reading – Old reading

EXAMPLE

Mr. St Gideon stays in a low -density area where the service charge is K2, 000 and the water is charged at K39 per kl. Work out his bill if his consumption is 82kl.

SOLUTION

Monthly water bill = (□ □ □ □ □ □ □ □ □ □ □ □ □ □) + □ □ □ □ □ □ □ □

Number of units = (□ □ □ □ □) + K2,000

= K3198 + K2,000

=K5 198

TELEPHONE BILLS

- Telephone bills are calculated basing on the number of units used as well as the service charge in place.
 - Calculations for telephone bills are the same as for those of electricity.

- **Monthly telephone bill = (No. of units x cost per unit) + Rental charge**
- Number of units = New reading – *Old reading*
- In telephone bills the service charge is called the rental charge.

EXAMPLE

Study the table below. Study it and answer the questions that follow.

	No. of Units	Cost per unit	Rental charge
Mr. Lungu	608	K36	K317

Required

Calculate the telephone bill for Mr. Lungu.

SOLUTION

$$\begin{aligned}
 \text{Monthly telephone bill} &= (\text{No of units} \times \text{cost per unit}) + \text{Rental charge} \\
 &= (608 \times 36) + \text{K317} \\
 &= \text{K22608} + \text{K317} \\
 &= \text{K22925}
 \end{aligned}$$

VALUE ADDED TAX (VAT)

- Value Added Tax is a tax levied on the difference between a commodity's price before taxes and its cost of production. It is the component of Customs duty.
- VAT is an indirect sales tax on the domestic consumption of goods and services except those that are zero rated such as food and essential drugs.
- VAT is charged as an addition to the selling price of a business's goods and services.
- Businesses have to pay the VAT they receive from customers to customs and excise department. The standard rate of VAT is set by parliament and it is currently at 16.5%

EXAMPLES

1. Calculate VAT payable on an article costing K8,000 before VAT is added if the VAT is added at 16.5%

$$\text{VAT at } 16.5\% = \text{K8,000} \times \frac{16.5}{100}$$

$$= K1,320$$

2. Mr. Gideon pays K160, 675 inclusive of 16.5% VAT for a deep freezer. How much would the deep freezer cost without VAT?

$$\text{Cost of deep freezer without VAT} = K160,675 - \left(\frac{16.5}{100} \times K160,675 \right)$$

$$= K26,511.38$$

$$\text{The cost without VAT} = K160,675 - K26,511.38$$

$$= K134,163.62$$

TAXATION- PAY AS YOU EARN CALCULATION

What does a salary comprise of?

It comprise of

- a. **Basic pay-** this is the amount that is based on some form of fixed pay structure within which each job is allocated to a certain pay level
- b. **Gross pay-** this is the amount that includes basic pay plus allowances.
- c. **Net pay-** this is the left over or take home pay an employee actually receives and is calculated by subtracting total deductions from the gross pay.

EXAMPLE

Suppose Mr. Philip receives K720, 000 as his annual salary. Calculate his net pay after deductions of tax using the following MRA Tax schedule.

MRA TAX SCHEDULE

1 st k20,000	Free or Zero
Next k3,000	15%
Excess over	30%

SOLUTION

Annual salary K720,00

$$\text{Monthly pay} = \frac{k720,000}{12 \text{ months}}$$

$$= K60,000$$

AMOUNT	TAX	REMAINDER

1 ST K20,000	-	O	(60,000-20,000)= K40,000
NEXT K3,000	3000 X 15%	K450	(40,000-3,000) = K37,000
EXCESS K37,000	37,000 X30%	K11,100	(37,000-37,000) = 0
	TOTAL TAX	K11,550	

Tax calculated = K11,550

Net Pay = Gross pay – Total deductions (Tax)

$$=K60,000 - K11,550$$

$$= K48,450$$

EXAMPLE TWO

Suppose Mr. Philip receives K720, 000 as his annual salary. Calculate his net pay after deductions of tax using the following MRA Tax schedule.

MRA TAX SCHEDULE

1 st k20,000	Free or Zero
Next k3,000	15%
Excess over	30%

In addition to annual pay of K720, 000, Mr. Philip receives nontaxable allowances as follows: overtime for May, 2015, K10, 000, monthly house allowances K25,000, he is deducted a loan K90,000 on a monthly instalments of K24,000, pension scheme K5,000 and NICO Insurance premium of K8,000.

Required

- Calculate the gross pay and net pay for Mr. Philip for month of May, 2015.
- Prepare Mr. Philip's pay slip for the month of May, 2015.

SOLUTION

Annual salary K720,00

$$\text{Monthly pay} = \frac{k720,000}{12 \text{ months}}$$

$$= K60,000$$

Gross pay = K60,000 + non taxable allowances

$$= K60,000 + K10,000 + K25,000$$

$$= K95,000$$

Calculation of P.A.Y.E			
AMOUNT	TAX		REMAINDER
1 ST K20,000	-	O	(60,000-20,000) = K40,000
NEXT K3,000	3000 X 15%	K450	(40,000-3,000) = K37,000
EXCESS K37,000	37,000 X30%	K11,100	(37,000-37,000) = 0
	TOTAL TAX	K11,550	

Tax calculated = K11,550

Net Pay = Gross pay – Total deductions

$$\begin{aligned}
 &= K95,000 - (K11,550 + K5,000 + K8,000 + K24,000) \\
 &= K95,000 - K48,550 \\
 &= K48,450
 \end{aligned}$$

PREPARATION OF PAY SLIP

Name of Co	Mr. Philip	ID T/044	District-Mzimba	Date 28/02/2015
CODE	DESCRIPTION	PAY	DEDUCTIONS	BALANCE
01	Basic pay	K60,000		
02	Overtime	K10,000		
03	House allowance	K25,000		
04	Loan repayment		K24,000	
05	Pension		K5,000	
06	Insurance Premium		K8,000	
07	Tax (PAYE)		K11,550	
		Gross pay K95,000	DEDUCTION K48,550	NET PAY K46,450

EXAMPLE THREE

Suppose that Mr. Thom of General Dealers Ltd gets K240, 000 as annual salary. In August, 2014, he had K500 accrued loan, K250 social club charges. Assuming that MRA has the following tax schedule.

MRA TAX SCHEDULE

1 st k20,000	Tax free
Next K1,500	10%
Next K1,500	15%
Excess over	30%

- a. Determine Mr. Thom's net pay for the month of August if General Dealers Ltd regards K4,000 for his house allowance as taxable.
- b. Prepare Mr. Thom's pay slip for the month of August, 2014.

SOLUTION

Annual salary K240,000

$$\text{Monthly pay} = \frac{K240,000}{12 \text{ months}} \\ = K20,000$$

$$\begin{aligned} \text{Gross pay} &= K20,000 + \text{non taxable allowances} \\ &= K20,000 + K4,000 \\ &= K24,000 \end{aligned}$$

Calculation of P.A.Y.E

AMOUNT	TAX		REMAINDER
1 ST K3,000	-	O	(24,000-3,000) = K21,000
NEXT K1,500	1,500 X 10%	K150	(21,000-1,500) = K19,500
NEXT K1,500	1,500 x 20%	K3,00	(19,500-1,500) = K18,000
EXCESS K37,000	18,000 X30%	K5,400	(18,000-18,000) = 0
	TOTAL TAX	K5,850	

Tax calculated = K5,850

Net Pay = Gross pay – Total deductions

$$\begin{aligned} &= K95,000 - (K5,850 + K500 + K250) \\ &= K95,000 - K6,600 \\ &= K17,400 \end{aligned}$$

PREPARATION OF PAY SLIP

Name of Co	Mr. Thom	ID T/044	District-Mzimba	Date 28/02/2015
CODE	DESCRIPTION	PAY	DEDUCTIONS	BALANCE
01	Basic pay	K20,000		
02	House allowance	K4,000		
03	Personal Loan		K500	
04	Social club charges		K250	
05	Tax (PAYE)		K5,850	
		Gross pay K24,000	DEDUCTION K6,600	NET PAY K17,400

CUSTOMS AND EXCISE DUTY

Customs duty is a tax charged on the importation of goods. The price of the taxed commodity will be higher as a result of the tax. Calculate the custom duty the trader paid.

EXAMPLE

A trader was charged 20% as custom duty on second hand clothes which he bought for K54, 000. Calculate the custom duty the trader paid.

Custom duty = 20% of K54,000

$$= \frac{20}{100} \times K54,000$$

$$= K10,800$$

The trader paid Custom duty of K10,800

EXAMPLE

A business person paid customs duty of K15, 000. If he bought goods amounting to K125, 000, what was the rate of the duty?

$$\text{Rate of custom duty} = \frac{15,000}{125,000} \times 100 \\ = 12\%$$

STATISTICS

MEAN

What is the mean?

- The mean is the average of a set of numbers.
- It is found by adding up the set of numbers and then dividing the total by the number of data points in the set.

HOW TO FIND THE MEAN

The "Mean" is computed by adding all of the numbers in the data together and dividing by the number elements contained in the data set.

$$\text{Mean} = \frac{\sum x}{N}$$

Example:

Data Set = 2, 5, 9, 3, 5, 4, 7

Number of Elements in Data Set = 7

$$\text{Mean} = (2 + 5 + 9 + 3 + 5 + 4 + 3) / 7 = 5$$

EXAMPLES

Find the mean of 5, 7, 8 and 4.

Step1: Add up the numbers to give a total of $8 + 5 + 7 + 13$

$$\text{Total} = 5+7+8+4$$

$$= 24$$

$$\text{Mean} = \frac{24}{4}$$

$$= 6$$

Median

The "Median" is the middle value of a set of **ordered** numbers. The "Median" of a data set is dependent on whether the number of elements in the data set is odd or even. First reorder the data set from the smallest to the largest then if the number of elements are odd, then the Median is the element in the middle of the data set.

If the number of elements is even, then the Median is the average of the two

middle terms.

Examples: Odd Number of Elements

Data Set = 2, 5, 9, 3, 5, 4, 7

Reordered = 2, 3, 4, 5, 5, 7, 9

Median = 5

Example: Even Number of Elements

Data Set = 2, 5, 9, 3, 5, 4

Reordered = 2, 3, 4, 5, 5, 9 - the middle terms are 4 and 5

Median = $(4 + 5) / 2 = 4.5$ - the median is the average of the two middle terms

Mode

The "Mode" for a set of data is the value that occurs most often. The "Mode" for a data set is the element that occurs the most often.

It is not uncommon for a data set to have more than one mode.

This happens when two or more elements occur with equal frequency in the data set. A data set with two modes is called bimodal.

A data set with three modes is called trimodal.

Examples: Single Mode

Data Set = 2, 5, 9, 3, 5, 4, 7

Mode = 5

Examples: Bimodal

Data Set = 2, 5, 2, 3, 5, 4, 7

Modes = 2 and 5

Examples: Trimodal

Data Set = 2, 5, 2, 7, 5, 4, 7

Modes = 2, 5, and 7

Range

The "Range" is the difference between the largest value and smallest value in a set of data.

First reorder the data set from smallest to largest then subtract the first element

from the last element.

Example:

Data Set = 2, 5, 9, 7, 5, 4, 3

Reordered = 2, 3, 4, 5

Range = $(9 - 2) = 7$

Worked Example 2

Five people play golf and at one hole their scores are 3, 4, 4, 5, and 7

For these scores, find

- (a) The mean
- (b) The median
- (c) The mode
- (d) The range.

- a. The numbers are already in order and the middle number is 4. So median = 4
- b. The score 4 occurs most often, so,
mode = 4
- c. The range is the difference between the smallest and largest numbers, in this case 3 and 7, so
range = $7 - 3$
 $= 4$

Calculate the , median and mode for the following set of numbers:

5,3,6,5,4,5,2,8,6,5,4,8,3,4,5,4,8,2,5,4

x	frequency (f)	cumulative frequency	fx
2	2	2	4
3	2	4	6
4	5	9	20
5	6	15	30
6	2	17	12
8	3	20	24

	$\sum f = 20$		$\sum fx = 96$
Mode = 5 Median = $\frac{20}{2} = 10$ Mean, $\bar{x} = \frac{\sum fx}{\sum f} = \frac{96}{20}$ $= 4.8$			

EXERCISE

1. Find the mean, median, mode and range of each set of numbers below.
 - a. 3, 4, 7, 3, 5, 2, 6, 10
 - b. 8, 10, 12, 14, 7, 16, 5, 7, 9, 11
 - c. 17, 18, 16, 17, 17, 14, 22, 15, 16, 17, 14, 12
 - d. 108, 99, 112, 111, 108
 - e. 64, 66, 65, 61, 67, 61, 57
 - f. 21, 30, 22, 16, 24, 28, 16, 17
2. In a survey of 10 households, the number of children was found to be 4, 1, 5, 4, 3, 7, 2, 3, 4, 1.
 - a. State the mode.
 - b. Calculate and also
 - c. the mean number of children per household
 - d. The median number of children per household.
 - e. A researcher says: "The mode seems to be the best average to represent the data in this survey." Give ONE reason to support this statement.
3. Eight people work in an office. They are paid hourly rates of £12, £15, £15, £14, £13, £14, £13, £13

Find

- a. the mean
- b. the median
- c. the mode.

4. In a singing contest, the scores awarded by eight judges were:

5.9 6.7 6.8 6.5 6.7 8.2 6.1 6.3

Using the eight scores, determine:

- a. the mean
 - b. the median
 - c. the mode
4. The marks of a student on seven examinations were 84, 91, 72, 84, 68, 87 and 78. Find
- a. the mean
 - b. the median of the marks.
 - c. the mode
5. Find the :
- a. mean
 - b. median
 - c. mode of the following numbers:
5,3,6,4,5,2,8,8,6,5,4,8,3,4,5,4,8,2,5,4,5.
6. Find the mean, the median and the mode of the following numbers
- a. 3,5,2,6,5,9,5,9,5,2,8,6
 - b. 51,6,48,7,50,3,49,5,48,5
7. Calculate the mean of the numbers 2, 4, 6, 8, 3, 7.

EXAMPLE

The goals scored by a hockey team in a season are shown in the table below.

Scores	Frequency
0	1
1	2
2	3
3	4
4	1
5	1

Find

- the mean
- the median
- the mode

SOLUTION

Score (x)	(f)	(Fx)	Cumulative frequency
0	1	0	1
1	2	2	3
2	3	6	6
3	4	12	10
4	1	4	11
1	1	5	12
	$\sum f = 12$	$\sum fx = 29$	

$$\text{Mean} = \frac{\sum fx}{N} = \frac{29}{12}$$

$$= 2.42$$

$$\text{Position of the median} = \frac{12+1}{2}$$

$$= \frac{13}{2}$$

$$= 6.5^{\text{th}}$$

$$\therefore \text{Median} = \frac{2+3}{2}$$

$$= 2.5$$

Mode = 3 (the value with the highest frequency)

8. 35 girls in Form R were asked to write down the number of brothers each one of them had. The following information was gathered.

12 girls each had 5 brothers, 8 girls each had four brothers, 7 girls each had 1 brother and 3 had no brothers. Find the mean, median and the mode number of brothers.

9. The table below shows the number of goals scored in 52 football matches.

Score	0	1	2	3	4	5	6
-------	---	---	---	---	---	---	---

Frequency	5	10	15	10	6	5	1
-----------	---	----	----	----	---	---	---

Find the mean, mode and median score.

10. From 40 potato plants, the yield of large potatoes was as shown in the table below. Copy and complete the table and calculate the mean, the median state the mode.

Number of large potatoes	0	1	2	3	4	5		
Frequency (f)	2	5	8	4	16	6		$\sum f =$
(fx)	0	5						$\sum fx$

11. The mean masses of four groups students were as follows:

Groups	A	B	C	D
Number of students	15	10	10	18
Mean mass (kg)	72	66.5	68.5	63

Find the mass of all students.

12. In a set of 100 numbers, 20 were 4's, 40 were 5's, 30 were 6's and the remainder were 7's. Make a frequency table for the information and calculate the mean.

BAR GRAPH

A Bar Graph (also called Bar Chart) is a graphically display of data using a graphically display of data using bar. In other words, bar graph is a graph that presents categorical data with rectangular bars with heights or lengths proportional to the values that they represent. The bars can be plotted vertically or horizontally. A vertical bar chart is sometimes called a column chart.

EXAMPLE

In a firm of 400 employees, the percentage of monthly salary saved by each employee is given in the following table.

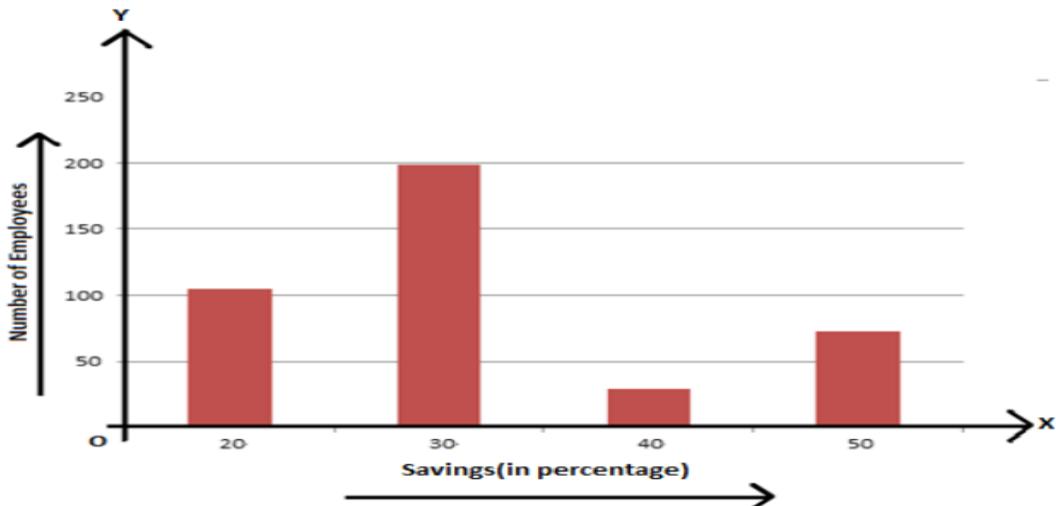
Savings (in percentage)	Number of employees (frequency)
20	105
30	199

40	29
50	73
Total	400

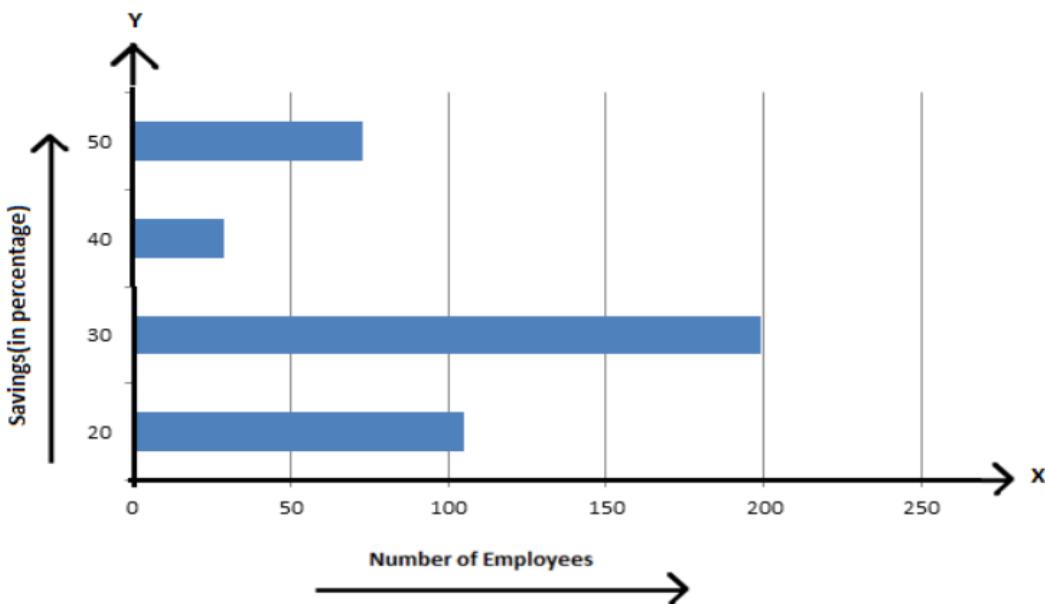
Represent it through a bar graph.

SOLUTION

Table 1 Vertical Bar Graph



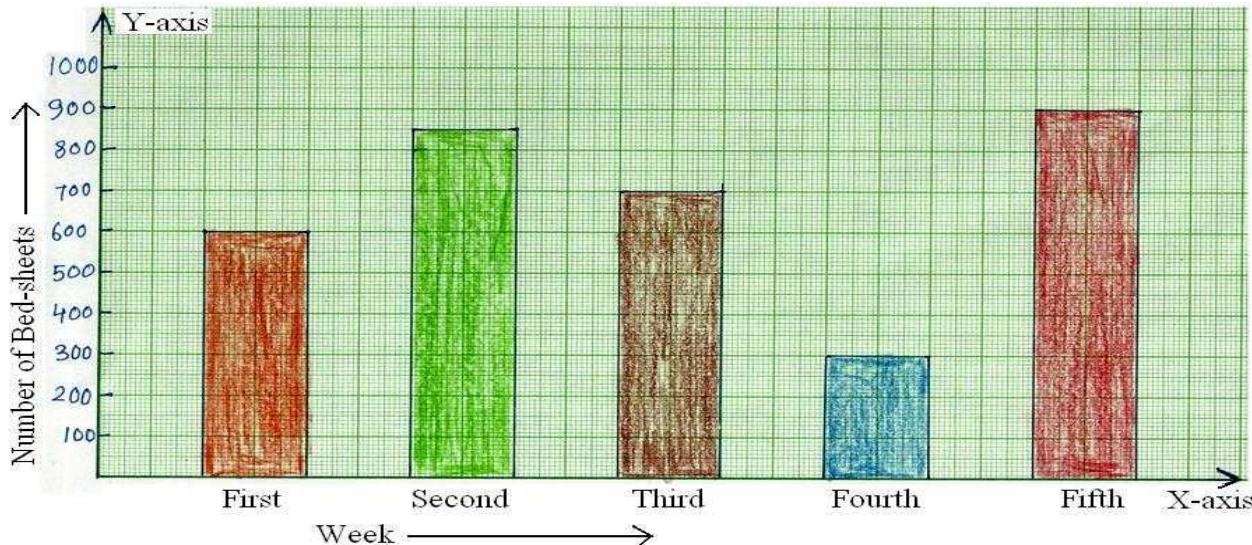
This can be also shown using a horizontal bar graph as follows:



The number of bed-sheets manufactured by a factory during five consecutive weeks is given below

Week	First	Second	Third	Fourth	Fifth
No.o. bed sheets	600	850	700	900	

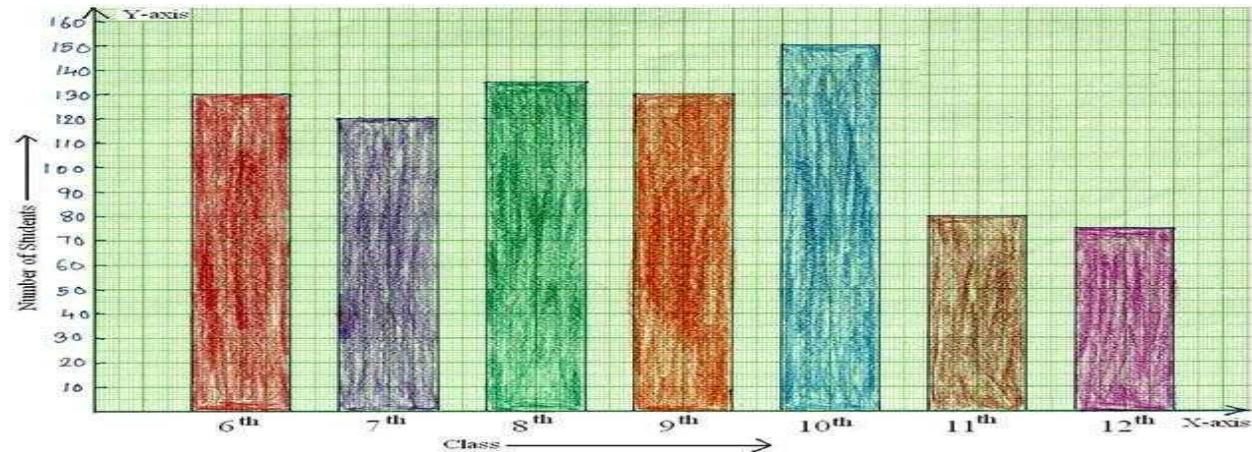
Draw the bar chart representing the above data.



The number of students in 7 different classes is given below. Represent this data on the bar graph.

Class	6 th	7 th	8 th	9 th	10 th	11 th	12 th
Number of students	130	120	135	130	150	80	75

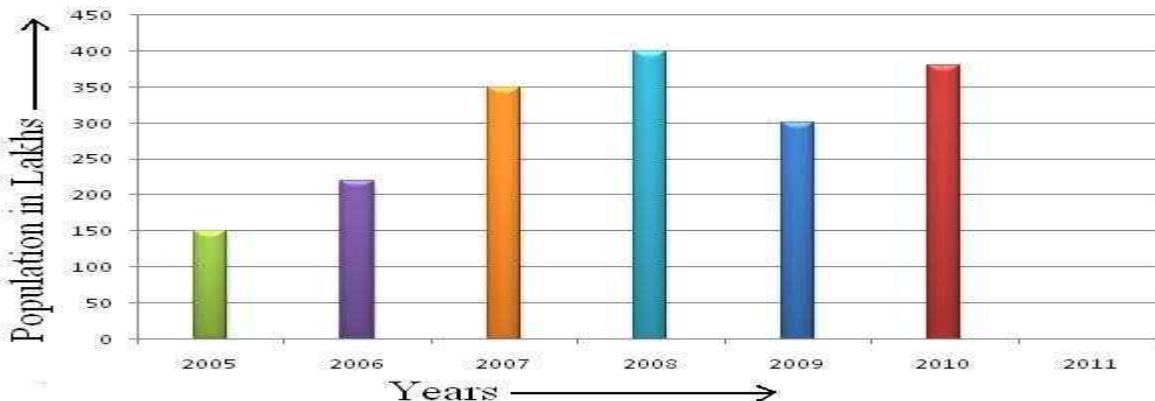
Represent this data on the bar graph.



The number of trees planted by Students at Target Secondary School in different years is given below.

Year	2005	2006	2007	2008	2009	2010
Number of trees planted	150	220	350	400	300	380

Draw the bar graph to represent the data.



EXERCISE

Imagine you just did a survey of your friends to find which kind of movie they liked best.

Comedy	Action	Romance	Drama	Scifi
4	5	6	1	4

Required

Represent the data on the bar graph.

A table below shows a survey of 145 people asked them.

Fruit	Apple	Orange	Banana	Kiwifruit	Blueberry	Grapes
People	35	30	10	25	40	5

Required

Represent the data on the bar graph. Which is the nicest fruit?"

PIE CHART

WORKED EXAMPLE 1

The mean of a sample of 6 numbers is 3.2. An extra value of 3.9 is included in the sample. What is the new mean?

Solution

Total of original numbers = 6×3.2

$$= 19.2.$$

$$\text{New total} = 19.2 + 3.9$$

$$= 23.1$$

$$\text{New mean} = 23.1 / 7$$

$$= 3.3.$$

WORKED EXAMPLE 2

The mean number of a set of 5 numbers is 12.7. What extra number must be added to bring the mean up to 13.1?

Solution

$$\text{Total of the original numbers} = 5 \times 12.7$$

$$= 63.5.$$

$$\text{Total of the new numbers} = 6 \times 13.1$$

$$= 78.6.$$

$$\text{Difference} = 78.6 - 63.5$$

$$= 15.1$$

WORKED EXAMPLE 3

Rohan's mean score in three cricket matches was 55 runs.

- How many runs did he score altogether?
- After four matches his mean score was 61 runs.
- How many runs did he score in the fourth match?

Solution

$$\text{a. Mean} = 55 = \frac{\text{Total score}}{3}$$

$$\text{So total scored} = 3 \times 55 = 165$$

$$\text{b. Total scored} = 4 \times 61 = 244$$

$$\text{Fourth match score} = 244 - 165$$

$$= 79$$

PIE CHART

A pie chart is a circular statistical graphic which is divided into slices to illustrate numerical proportion. In a pie chart, the arc length of each slice is proportional to the quantity it represents.

CONSTRUCTION OF PIE CHART

A pie chart is a circular graph which is used to represent data. In this:

- Various observations of the data are represented by the sectors of the circle.
- The total angle formed at the centre is 360° .
- The whole circle represents the sum of the values of all the components
- The angle at the centre corresponding to the particular observation component is given by

$$\frac{\text{Value of the component}}{\text{Total value}} \times 360^\circ$$

- If the values of observation/components are expressed in percentage, then the centre angle corresponding to particular observation/component is given by

$$\frac{\text{Percentage value of component}}{100} \times 360^\circ$$

HOW TO CONSTRUCT A PIE CHART?

Steps of construction of pie chart for a given data:

- Find the central angle for each component using the formula.
- Draw a circle of any radius
- Draw a horizontal radius
- Starting with the horizontal radius, draw radii, making central angles corresponding to the values of respective components.
- Repeat the process for all the components of the given data.
- These radii divide the whole circle into various sectors.
- Now, shade the sectors with different colours to denote various components.
- Thus, we obtain the required pie chart.

EXAMPLE

- The following table shows the numbers of hours spent by a child on different events on a working day. Represent the adjoining on a pie chart.

Activity	Number of hours
School	6

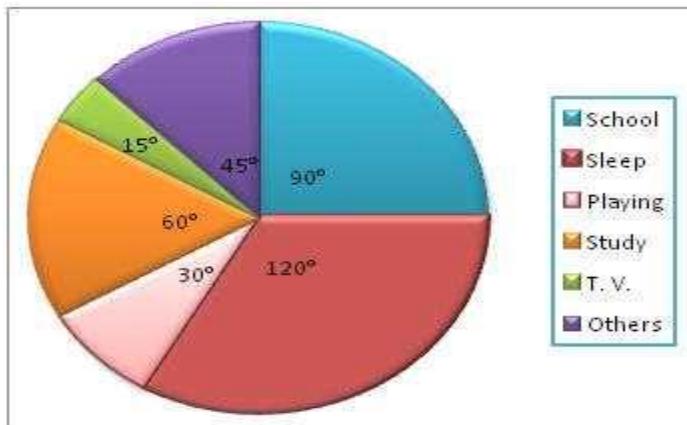
Sleep	8
Playing	2
Study	4
Television	1
Others	3

SOLUTION

The central angles for various observations can be calculated as:

Activity	Number of hours	Measure of central angle
School	6	$\left(\frac{6}{24} \times 360^\circ\right) = 90^\circ$
Sleep	8	$\left(\frac{8}{24} \times 360^\circ\right) = 120^\circ$
Playing	2	$\left(\frac{2}{24} \times 360^\circ\right) = 30^\circ$
Study	4	$\left(\frac{4}{24} \times 360^\circ\right) = 60^\circ$
Television	1	$\left(\frac{1}{24} \times 360^\circ\right) = 15^\circ$
Others	3	$\left(\frac{3}{24} \times 360^\circ\right) = 45^\circ$

The pie chart from the above calculations



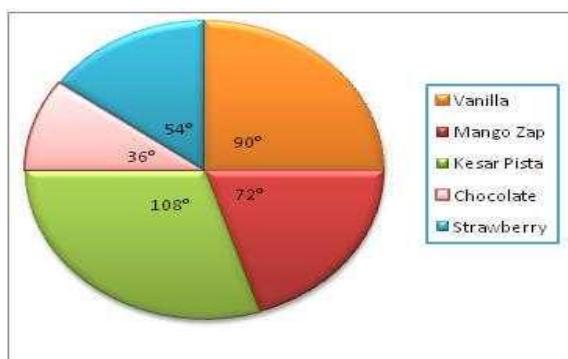
2. The favourite flavours of ice-cream for the children in a locality are given in percentage as follow. Draw the pie chart to represent the given information.

Flavours	% of students prefer the flavours
Vanilla	25%
Strawberry	15%
Chocolate	10%
Kersa- Pista	30%
Mango Zap	20%

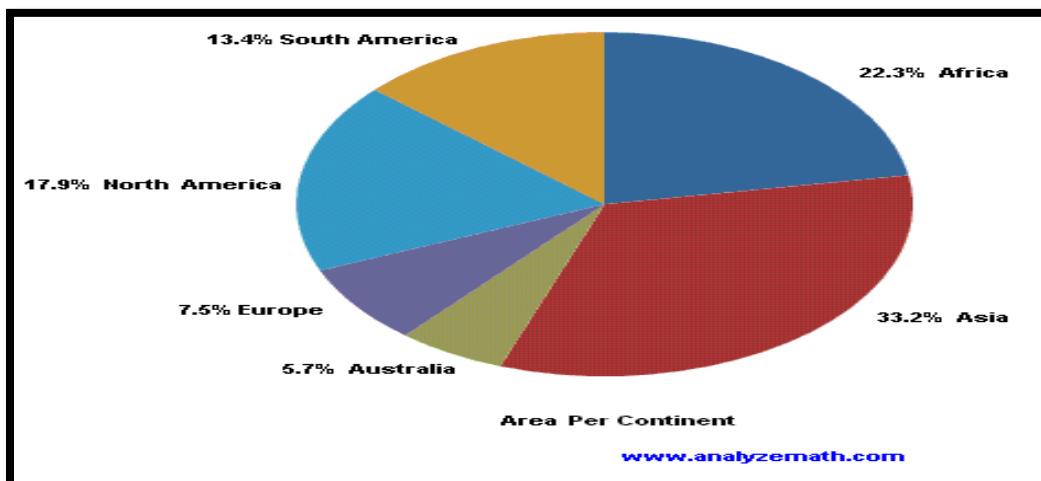
SOLUTION

Flavours	% of students prefer the flavours	% of students prefer the flavours
Vanilla	25%	$\left(\frac{25}{100} \times 360^\circ\right) = 90^\circ$
Strawberry	15%	$\left(\frac{15}{100} \times 360^\circ\right) = 54^\circ$
Chocolate	10%	$\left(\frac{10}{100} \times 360^\circ\right) = 36^\circ$
Kersa- Pista	30%	$\left(\frac{30}{100} \times 360^\circ\right) = 108^\circ$
Mango Zap	20%	$\left(\frac{20}{100} \times 360^\circ\right) = 72^\circ$

PIE CHART



The total area of Asia, Africa, North America, South America, Europe and Australia is 134 million square kilometers. The pie chart below shows the percentages of each continent.



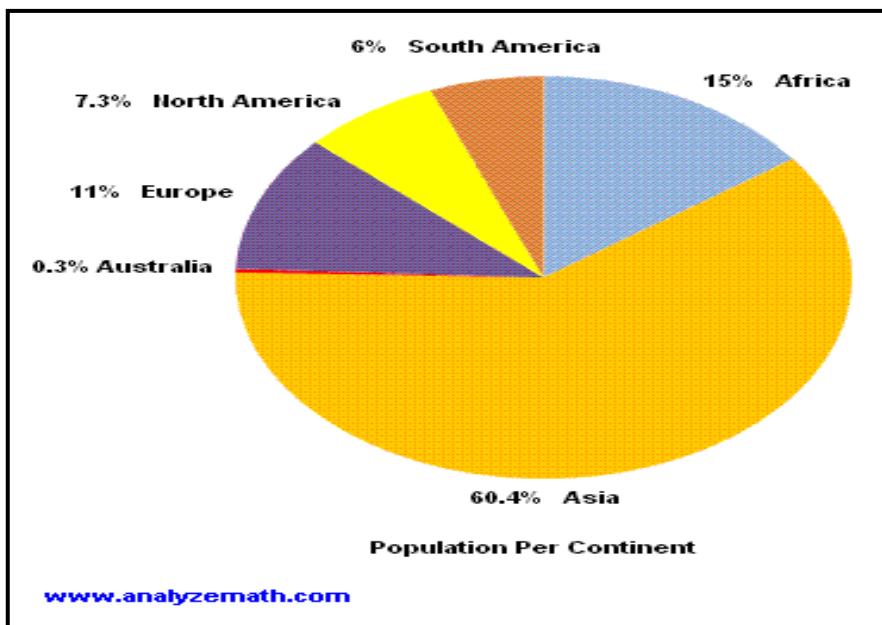
- What is the area of Asia?
- What is the area of Europe?

c. How much bigger is Africa than Europe?

SOLUTION

- a. $33.2\% \times 134 = 44.5$ million kilometers
- b. $7.5\% \times 134 = 10.5$ million square kilometers
- c.** $(22.3 - 7.5\%) \times 134 = 19.8$ million square kilometer

The pie chart below shows the percentages of the world population in each continent. The present world population is about 7 billion.



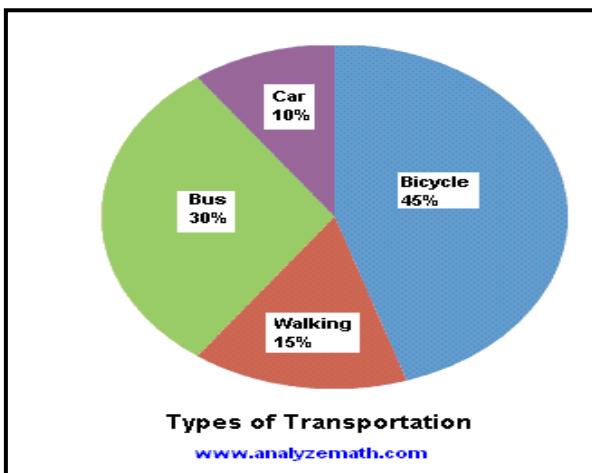
Required

- a. How many people live in Africa?
- b. How many people do not live in Asia?
- c. How many more people live in North America than in South America?

SOLUTION

- a. $15\% \times 7 = 1.05$ billion
- b. $7 - 60.4\% \times 7 = 2.772$ billion
- c. $(7.3\% - 6\%) \times 7 = 91$ million.

The pie chart below shows the percentages of types of transportation used by 800 students to come to school.



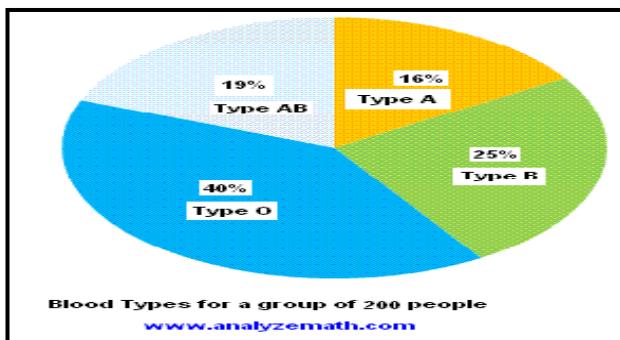
Required

- How many students, in the school, come to school by bicycle?
- How many students do not walk to school?
- How many students come to school by bus or in a car?

SOLUTION

- $45\% \times 800 = 360$ Students
 - $(100\% - 15\%) \times 800 = 680$ students
 - $(30\% + 10\%) \times 800 = 320$ students

The pie chart below shows the percentages of blood types for a group of 200 people.



Required

- How many people, in this group, have blood type AB?
- How many people, in this group, do not have blood type O?
- How many people, in this group, have blood types A or B?

SOLUTION

- | |
|---|
| a. $19\% \times 200 = 38$ people |
| b. $(100\% - 40\%) \times 200 = 60\% \times 200 = 120$ people |
| c. $(16\% + 25\%) \times 200 = 41\% \times 200 = 82$ people |

Imagine you survey your friends to find the kind of movie they like best.

Comedy	Action	Romance	Drama	Scifi
4	5	6	1	4

Required.

Represent the data by the pie chart.

SOLUTION

Comedy	Action	Romance	Drama	Scifi	Total
4	5	6	1	4	20

Next divide each value by the total and multiply by 360° to get a percent.

Comedy	Action	Romance	Drama	Scifi	Total
4	5	6	1	4	20
$\left(\frac{4}{20} \times 360^\circ\right)$ $= 72^\circ$	$\left(\frac{5}{20} \times 360^\circ\right)$ $= 90^\circ$	$\left(\frac{6}{20} \times 360^\circ\right)$ $= 108^\circ$	$\left(\frac{1}{20} \times 360^\circ\right)$ $= 18^\circ$	$\left(\frac{4}{20} \times 360^\circ\right)$ $= 72^\circ$	

EXERCISES

The table below shows the number of cars of each colour that passed through Dunduzu Roadblock on Republic Day.

Colour	Number of cars
Blue	75
Red	45
Green	150
White	300
Black	30

Required

Represent this information in a pie chart

FREQUENCY POLYGON

A frequency polygon is a graph constructed by using lines to join the midpoints of each interval, or bin. The heights of the points represent the frequencies.
--

A frequency polygon can be created from the histogram or by calculating the midpoints of the bins from the frequency distribution table.

CONSTRUCTION OF FREQUENCY POLYGON USING HISTOGRAM

Step 1

Obtain the frequency distribution from the given data and draw a histogram.

Step 2

Join the mid-points of the tops of adjacent rectangles of the histogram by means of the line segments.

Step 3

Obtain the midpoints of two assumed class intervals of zero frequency, one adjacent to the first bar on its left and another adjacent to the last bar on its right. These class intervals are known as imagined class interval.

Step 4

Complete the polygon by joining the midpoint s of first and last class intervals to the midpoint of the imagined class intervals adjacent to them.

EXAMPLE

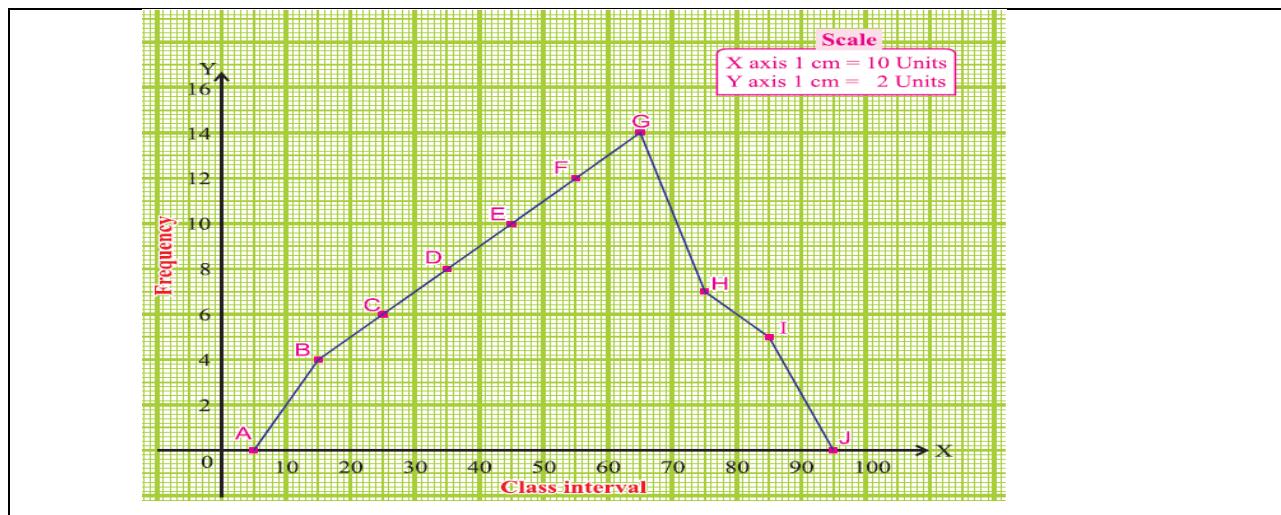
Draw a frequency polygon imposed on the histogram for the following distribution.

Class interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	4	6	8	10	12	14	7	5

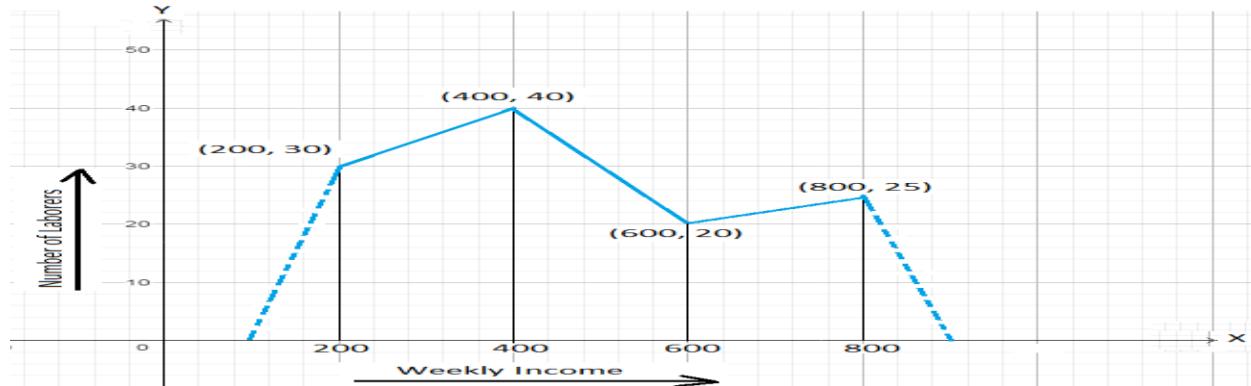
SOLUTION

- Mark the class intervals along the x-axis and the frequencies along the Y-axis with appropriate scale shown in the figure given below.
- Draw a histogram for the given data. Now mark the midpoints of the upper sides of the consecutive rectangles.
- Also we take the imagined class intervals (-10) -0 and 60-70

THE FREQUENCY POLYGON



The following frequency polygon displays the weekly incomes of labourers of a factory.



Required

- Find the class interval whose frequency is 25.
- How many labourers have a weekly income of at least K500 but not more than K700?
- What is the range of weekly income of the largest number of labourers?
- Prepare the frequency distribution table.

SOLUTION

- The frequency 25 corresponds to the class mark 800.
The common width of class intervals = $400 - 200 = 200$
So, the class interval is $(800 - \frac{200}{2}) - (800 + \frac{200}{2})$, that is 700-900.
- The number of labourers has to fall in the class interval 500-700 whose class mark is 600. The frequency corresponding to the mark 600 is 20. Hence, the required number of labourers is 20.

c. The largest number of labourers belongs to the class interval whose mark is 400. The corresponding class interval is $(800 - \frac{200}{2}) - (800 + \frac{200}{2})$, so the largest numbers of labourers have a weekly income of at least K300 but less than K500.

d. Frequency table

Weekly income (IN\$)	Number of labourers
100-300	30
30-500	40
500-700	20
700-900	25



HISTOGRAM

Histogram is a graphical display of data using bars of different heights. It is similar to a bar chart but a histogram groups numbers into ranges. The height of each bar shows how many fall into each range. Histogram shows continuous data. The continuous data takes the form of class intervals.

In other words, histogram is a graphical representation of a frequency distribution with class intervals or attributes as the base and frequency as the height.

A histogram consists of rectangles, each of which has breadth equal or proportional to the size of the concerned call interval, and height equal or proportional to the corresponding frequency.

PARTS OF A HISTOGRAM

1. **The title:** The title describes the information included in the histogram.
2. **X-axis:** The X-axis are intervals that show the scale of values which the

measurements fall under.

3. **Y-axis:** The Y-axis shows the number of times that the values occurred.
4. **The bars:** The height of the bar shows the number of times that the values occurred within the interval, while the width of the bar shows the interval that is covered.

CONSTRUCTION OF A HISTOGRAM

To make a histogram, follow these steps

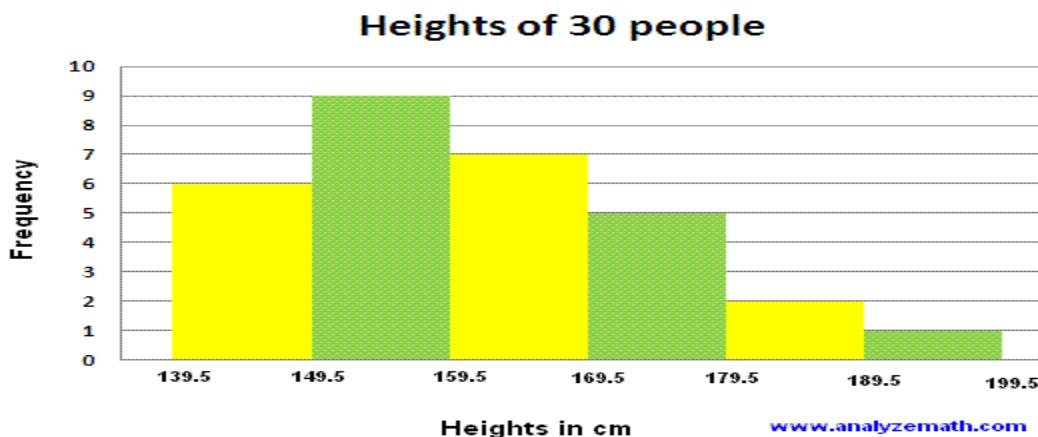
- a. On the vertical axis, place frequencies. Label this axis "Frequency".
- b. On the horizontal axis, place the lower value of each interval
- c. Draw a bar extending from the lower value of each interval to the lower value of the next interval,

FREQUENCY HISTOGRAM

A frequency Histogram is a special graph that uses vertical columns to show frequencies (how many times each score occurs).

EXAMPLES

The histogram below shows the heights in cm distribution of 30 people.



Required

- a. How many people have heights between 159.5 AND 169.5 cm?
- b. How many people have heights less than 159.5cm?
- c. How many people have heights more than 169.5cm?
- d. What percentage of people has heights between 149.5 and 179.5cm?

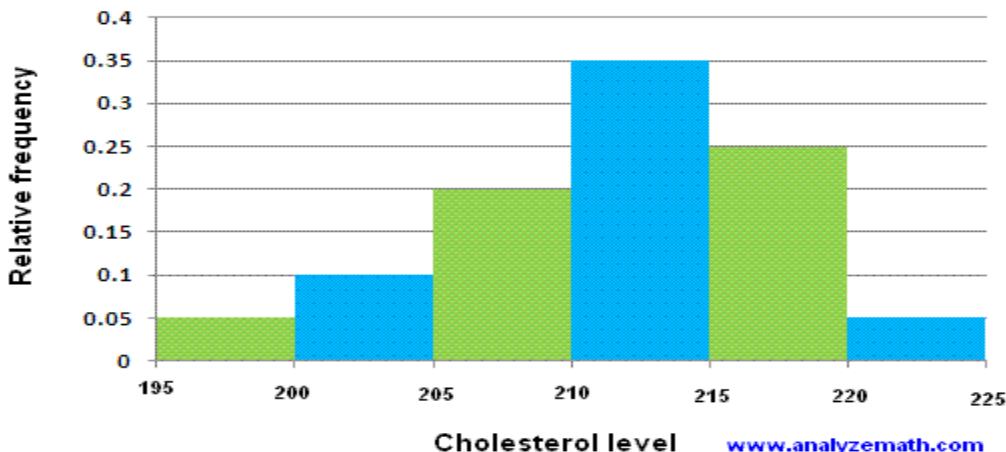
SOLUTION

- a. 7 people

- b. $9 + 6 = 15$ people.
- c. $5+2 + 1 = 8$ people
- d. $(9+7+5)/30 = 0.7 \times 100\%$
=70%

EXAMPLE

The histogram below shows the level of cholesterol in mg per dl of 200 people.



- a. How many people have a level of cholesterol between 205 and 210 mg per dl?
- b. How many people have a level of cholesterol less than 205m per dl?
- c. What percentage of people have cholesterol more than 215mg per dl?
- d. How many people have a level of cholesterol between 205 and 220mg per dl?

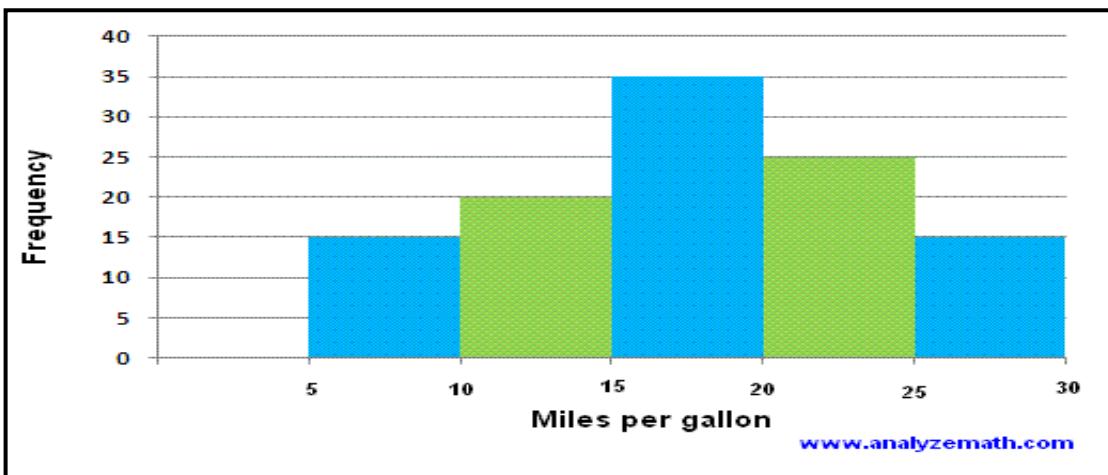
SOLUTION

Note that the relative frequency is shown on the vertical axis

- a. $0.2 \times 200 = 40$ people
- b. $(0.05 + 0.1) \times 200 = 30$ people
- c. $(0.25 + 0.05) = 30\%$
- d.** $(0.2 + 0.35 + 0.25) \times 200 = 160$ people

EXAMPLE

The histogram below shows the efficiency level (in miles per gallons) of 110 cars.



Required

- How many cars have efficiency between 15 and 20 miles?
- How many cars have efficiency more than 20 miles per gallon?
- What percentage of cars has efficiency less than 20 miles?

SOLUTIONS

- 35 cars
- $25 + 15 = 40$ cars
- $(15 + 20 + 35) / 110 = 0.626 \times 100\%$
 $= 62.6\%$