VECTORS

1989

The position vector of point A is $\underline{i} - 2\underline{j} + \underline{k}$ and that of B is $3\underline{i} + 4\underline{j} - 2\underline{k}$.

Find the position vector of the point P which divides AB in the ratio 2:1. 3

Write down the vector equation of the line AB in the form $r_1 = \ldots\ldots$ 4

Similarly write down the equation of the line CD where C is the point with position vectors $3\underline{i} + 2\underline{j} - \underline{k}$ and D is the point $5\underline{i} + \underline{j}$. 4

Calculate the position vector of the point where AB meets CD. 7

Given that $\underline{a} = 2\underline{i} + \underline{j} + 2\underline{k}$, $\underline{b} = \underline{i} + 3\underline{j} - 3\underline{k}$.

a. Write down the vector $3\underline{a} - 2\underline{b}$, in terms of $\underline{i}$, $\underline{j}$ and $\underline{k}$. 3

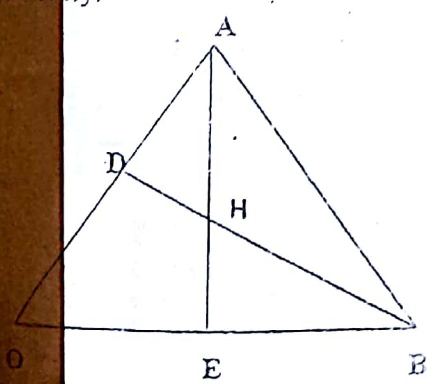
b. Calculate the modulus of $3\underline{a} - 2\underline{b}$. 2

c. Calculate the angle between $\underline{a}$ and $3\underline{a} - 2\underline{b}$, correct to the nearest degree. 4

1990 (sample paper)

Write down the vector of the line parallel to the vector $\underline{i} + 2\underline{j} - \underline{k}$ and which passes through the mid-point of PQ where $\underline{p} = \underline{i} - \underline{j} + 3\underline{k}$ and $\underline{q} = 3\underline{i} + 3\underline{j} + 5\underline{k}$ are the position vectors of points P and Q. Show that this line passes through the point (0, -3, 6). 10

In a triangle below, AD is the altitude from A to OB, and BE is the altitude from B to OA. AD and BE intersect at H The position vectors of A, B and H relative to the origin O are $\underline{a}$, $\underline{b}$ and $\underline{h}$ respectively.



By using the fact that OB is perpendicular to AH, show that $\underline{b}.(\underline{h}-\underline{a}) = 0$ 6

Show also that $\underline{a}.(\underline{h}-\underline{b}) = 0$ 4

Deduce from a and b that $\underline{h}.(\underline{b}-\underline{a}) = 0$ 5

Prove that OH is perpendicular to AB. 5

1991

If $\underline{a} = 2\underline{i} - \underline{j} + \underline{k}$, $\underline{b} = \underline{i} + 3\underline{j} - 2\underline{k}$, $\underline{c} = -2\underline{i} + \underline{j} - 3\underline{k}$ and $\underline{d} = 3\underline{i} + 2\underline{j} + 5\underline{k}$. Find the scalars p, q and r such that $\underline{d} = p\underline{a} + q\underline{a} + r\underline{c}$. 10

If $\overrightarrow{AB} = \underline{c}$ and $\overrightarrow{BC} = \underline{a}$ express $\overrightarrow{AC}$, $\overrightarrow{XB}$ and $\overrightarrow{BY}$ in terms of $\underline{c}$ and $\underline{a}$ where X, Y are the mid-points of AB, BC respectively. Hence prove that the line joining th mid-points of two sides of a triangle is parallel to the third side and has one half its magnitude. Illustrate your answer by use of a diagram. 8

1992

Given that $\underline{a}$ and $\underline{b}$ are position vectors of A and B respectively with respect to O where $\underline{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix}$

Find:

the mid-point of line AB,

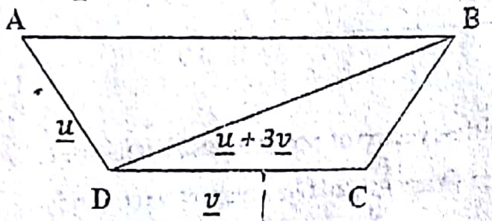
the vector equation of line through A and B. 5

Show that the point $\begin{pmatrix} 0 \\ -1 \\ 10 \end{pmatrix}$ lies on the line through A and B. 4

Find a unit vector parallel to the resultant of the vectors $\underline{a} = \underline{i} + 4\underline{j} - 3\underline{k}$ and $\underline{b} = 2\underline{i} + 2\underline{j} + \underline{k}$. 7

Find the angle between $\underline{p}$ and $\underline{p}$, where $\underline{p} = 2\underline{i} + 2\underline{j} - \underline{k}$ and $\underline{q} = 6\underline{i} - 3\underline{j} + 2\underline{k}$. 8

In figure below, $\overrightarrow{DA} = \underline{u}$, $\overrightarrow{DB} = \underline{u}+3\underline{v}$ and $\overrightarrow{DC} = \underline{v}$.



Prove that ABCD is a trapezium. 5

1995

Let OACB be a parallelogram. Using *vector method,* prove that the diagonals bisect each other. 9

Given that the position vectors $\underline{A}$, $\underline{B}$ and $\underline{C}$ are $2\underline{i}+3\underline{j}-2\underline{k}$, $-\underline{i}+2\underline{j}-4\underline{k}$ and $-\underline{i}+5\underline{k}$ respectively, find:

the position vector $\underline{M}$ where M is the mid-point of $\overrightarrow{AB}$,

the position vector of N where $\overrightarrow{BN} = \frac{2}{3}\overrightarrow{BC}$,

the angle MON. 9

If $\overrightarrow{XY} = \underline{x}$ and $\overrightarrow{YZ} = \underline{z}$, show that the area of triangle XYZ is given by $\frac{1}{2}\sqrt{(|x|^2|z|^2 - (xz))^2}$.

Hence, find that area of triangle XYZ if the coordinates of X, Y and Z are (3, 2) (-1, -1) and (5, -3) respectively. 12

Given that the position vector of A and B are $3\underline{i}-4\underline{j}$ and $2\underline{i}-3\underline{j}$ respectively,

Find the equation of the line passing through A and B.

Given also that the position vector of C is $-\underline{i}+\underline{j}$, find the equation of the line through C parallel to the vector $-3\underline{i}+2\underline{j}$.

Find the point of intersection of the lines. 8

1996

If $\underline{x} = 2\underline{i} - 3\underline{j} + \underline{k}$ and $\underline{y} = 6\underline{i} + \underline{j} - 2\underline{k}$, find u and v if the vector $\underline{i} + v\underline{j} + u\underline{k}$ is perpendicular to both $\underline{x}$ and $\underline{y}$. 4

The lines with vector equations:

$r_1 = \begin{pmatrix}1\\-5\\2\end{pmatrix} + t\begin{pmatrix}-3\\4\\0\end{pmatrix}$ and $r_2 = \begin{pmatrix}3\\-13\\1\end{pmatrix} + s\begin{pmatrix}4\\0\\k\end{pmatrix}$ meets at Q.

Find:

the value of k,

the position vector of Q and

the distance of Q from the origin 7

ABCD is a quadrilateral and the points W, X, Y and Z are the mid-points of AB, BC, CD and DA respectively. If $\overrightarrow{AB} = \underline{b}$, $\overrightarrow{BC} = \underline{c}$, $\overrightarrow{CD} = \underline{d}$ and $\overrightarrow{DA} = \underline{a}$, prove that WXYZ is a parallelogram and the perimeter of WXYZ is equal to the sum the diagonals of ABCD. 12

Relative to the origin O, the points A, B, and C have position vectors $\begin{pmatrix}2\\-2\end{pmatrix}$, $\begin{pmatrix}3\\-1\end{pmatrix}$ and $\begin{pmatrix}2\\4\end{pmatrix}$ respectively.

Find in vector form, an equation of the straight line through A and B.

Find a vector equation for the straight line through C which is perpendicular to AB.

Given that the position vector for point F is $5\underline{i} + \underline{j}$, show that the line in b(ii), passes through the point F. 8

1997

Triangle ABC has vertices A (1, 2, -2), B (3, 1, 1) and C (2, 1, -3)

a. Find the vectors $\overline{AB}$ and $\overline{AC}$. 2
b. Show that angle A is a right angle. 2
c. Find the area of triangle ABC. 3
d. Find the vector equation of the line through B and C. 2

Triangle OAB has vertices O (0, 0), A (1, 2) and B (4, 0). Point C, D and E are the mid-points of OB, AB, and OA respectively. Show that AC, OD and BE intersect in one point and find the coordinates of the point. 11

1998

Given that $\underline{u} = 3\underline{i} - 4\underline{j} + \underline{k}$ and $\underline{v} = 2\underline{i} + 3\underline{j} - 4\underline{k}$.

Write down the expression for $2\underline{u} - 3\underline{v}$. 3

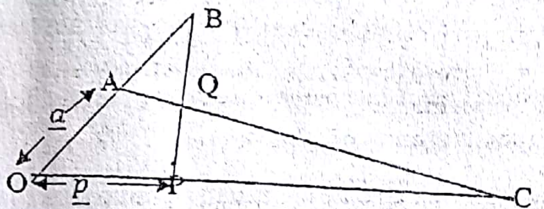
Calculate the modulus of $2\underline{u} - 3\underline{v}$. 2

Find the value of x if the vector $2\underline{i} - x\underline{j} + 7\underline{k}$ is perpendicular to $2\underline{u} - 3\underline{v}$. 3

Calculate the unit vector in the direction of $2\underline{u} - 3\underline{v}$. 2

The position vectors of the points A and B relative to the origin O are $\begin{pmatrix}2\\4\\-10\end{pmatrix}$ and $\begin{pmatrix}6\\-4\\-2\end{pmatrix}$ respectively.
Find the vector equation of the line AB.
Calculate the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$.
10

In figure below $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OP} = \underline{p}$, $\overrightarrow{OC} = 2\underline{p}$ and $\overrightarrow{OB} = \frac{3}{2}\underline{a}$



If $\overrightarrow{BQ} = r\overrightarrow{BP}$ and $\overrightarrow{AQ} = t\overrightarrow{AC}$, find two distinct expressions for $\overrightarrow{OQ}$ in terms of $\underline{a}$, $\underline{p}$, r and t.
10

1999

If $\underline{a} = \begin{pmatrix}3\\-2\lambda\\5\lambda\end{pmatrix}$ and $|a| = \sqrt{67}$, find the values of λ.
3

The angle between $\underline{i} + \underline{j}$ and $\underline{i} + \underline{j} + p\underline{k}$ is $\frac{\pi}{4}$. Find the values of p.
5

Given that vector $\underline{v}$ is parallel to the vector $8\underline{i}+\underline{j}+4\underline{k}$ and is equal in magnitude to the vector $\underline{i}-2\underline{j}+2\underline{k}$. Find $\underline{v}$.
4

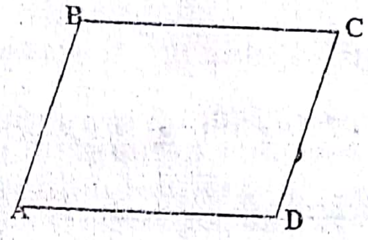
You have lines $\underline{a}$ and $\underline{b}$, determine whether the two lines intersect and if they do, find the point of intersection.

i. $\underline{a} = \begin{pmatrix}1\\1\\5\end{pmatrix} + t\begin{pmatrix}2\\-4\\-1\end{pmatrix}$ and $\underline{b} = \begin{pmatrix}4\\-1\\4\end{pmatrix} + s\begin{pmatrix}-1\\6\\1\end{pmatrix}$

ii. $\underline{a} = \begin{pmatrix}3\\2\\0\end{pmatrix} + t\begin{pmatrix}1\\-4\\1\end{pmatrix}$ and $\underline{b} = \begin{pmatrix}4\\3\\-2\end{pmatrix} + s\begin{pmatrix}-1\\1\\3\end{pmatrix}$
9

Let $\underline{a} = x\underline{i} + y\underline{j}$ be a vector in the first quadrant and suppose the smallest angle between the x-axis and $\underline{a}$ is θ, show that $\underline{a} = |\underline{a}|(\cos\theta\underline{i} + \sin\theta\underline{j})$.
2

Use the dot product to prove that the diagonals of the rhombus below are perpendicular to each other.



2000

The position vector of points A and B relative to the origin O are $\begin{pmatrix}2\\1\\0\end{pmatrix}$ and $\begin{pmatrix}3\\2\\1\end{pmatrix}$ respectively. Find

a. the length of the vector $\overline{AB}$. 3
b. the vector equation of the line $\overline{AB}$ 3

With respect to the origin, O, the position vectors of points L and M are $6\underline{i}+3\underline{j}+2\underline{k}$ and $2\underline{i}+2\underline{j}+\underline{k}$ respectively.
Calculate angle LOM.
A point N is on the line LM produced such that the angle MON is 90^0. Find the vector equation of the line ML.
Calculate the position vector of N. 11

Collinear points P, Q and R have position vectors $\underline{p}$, $\underline{q}$ and $\underline{r}$ respectively. If $\overline{PR} = 3\overline{PQ}$.
Express $\underline{r}$ in terms of $\underline{p}$ and $\underline{q}$.
Show that if $\underline{p} = m\underline{q} + n\underline{r}$, then $m + n = 1$.
9

The edges $\overline{OP}$, $\overline{OQ}$ and $\overline{OR}$ of a tetrahedron OPQR are vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ respectively. If $\overline{OP}$ is perpendicular to $\overline{QR}$.

i. Prove that $\underline{a}.(\underline{b} - \underline{c}) = 0$.
ii. If $\overline{OQ}$ is perpendicular to $\overline{RP}$, prove that $\overline{OR}$ is perpendicular to $\overline{PQ}$.
11

2001

Given that P (2, 1, 3) and Q (6, 4, 5) are any two points.
Express $\overline{PQ}$ in terms of $\underline{i}$, $\underline{j}$ and $\underline{k}$. 4
Find the unit vector parallel to $\overline{PQ}$. 4
Find the point R on $\overline{PQ}$ such that PR:PQ = 1:5.
6

A line joining P (2, 8) to Q (4, 0) intersects with the line joining R (2, 2) to S (6, 4). Find using vectors the coordinates of the point of intersection.
11

In a parallelogram ABCD, X is the mid-point of AB and Y divides BC in the ratio 1:2, If $\overrightarrow{AB} = \underline{a}$, $\overrightarrow{AD} = \underline{b}$, $\overrightarrow{AR} = \lambda\overrightarrow{AY}$, $\overrightarrow{DR} = \mu\overrightarrow{DX}$ and DX intersect AY at R. Calculate the ratio of DR to DX. 9

2002

Find the scalar product of the following vectors;

a. $\underline{a} = \underline{i} + \underline{j}$, $\underline{b} = 3\underline{i} + 4\underline{j}$ 3

b. $\underline{a} = \binom{1}{3}$, $\underline{b} = \binom{-2}{-6}$ 3

In triangle PQR, $\overrightarrow{PQ} = \underline{q}$, $\overrightarrow{PR} = \underline{r}$, and M is the mid-point of PQ. Express each of the following in terms of q and $\underline{r}$:

i. $\overrightarrow{PM}$

ii. $\overrightarrow{MR}$

iii. $\overrightarrow{RM}$ 6

Find the unit vector which is in opposite direction to the sum of the vectors $(3\underline{i} + 2\underline{j} + \underline{k})$ and $(-5\underline{i} - 3\underline{j} + 6\underline{k})$. Prove that this unit vector is perpendicular to $(9\underline{i} - 4\underline{j} + 2\underline{k})$. 7

The vertices of a triangle ABC have position vectors $(2\underline{i} - 3\underline{j})$, $(\underline{i} + 2\underline{j})$ and $(4\underline{i} - \underline{j})$ respectively.

Find the vector AB.

If $\underline{l}$ and $\underline{m}$ are the position vectors of the mid-point of AB and AC respectively, find $\underline{l}$ and $\underline{m}$.

Hence, find the vector LM. 9

Relative to the origin O, the position vector of two points A and B are $\vec{a} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ respectively.

If the unit vector $\overline{u} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, find a, b and c.

If the third point C lies on the line AB and divides it in the ratio AC : CB = 2:1, find the cosine of angle COB. 11

2003

P and Q are the mid-point of AB and CB respectively where A = (1, 1, 0), B = (7, 2, 1) and C = (3, -2, 1). Show that $\overrightarrow{PQ}$ is parallel to $\overrightarrow{AC}$. 10

Relative to the origin O, vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ have position vectors $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ respectively. Find:

the vector equation of the line joining A and C.

the angle between $\overrightarrow{AC}$ and $\overrightarrow{BC}$. 12

Let $\underline{r} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ be a vector equation of a straight line. A line perpendicular to $\underline{r}$ passes through a point with position vector $\begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$ on the line $\underline{r}$ and through the point with a position vector $\begin{pmatrix} -2 \\ -1 \\ d \end{pmatrix}$ not on $\underline{r}$. Calculate the value of d. 7

2004

The position vectors of points R and T relative the the origin O are $\overline{OR} = \underline{i} - 2\underline{j} + 7\underline{k}$ and $\overline{OT} = 5\underline{i} + \underline{j} - 5\underline{k}$. Find

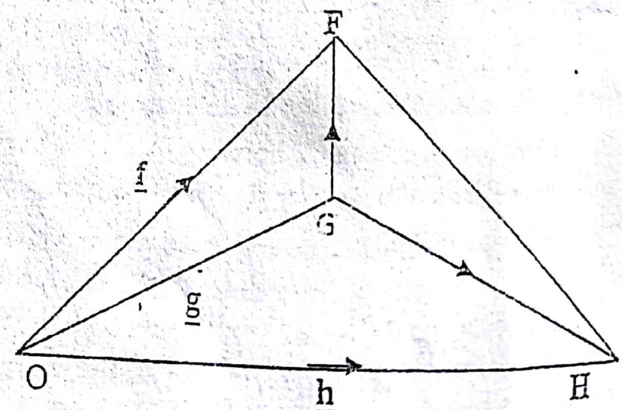
the distance between the origin and point R. 3

vector RT. 3

the position vector of the mid-point of $\overline{OR}$. 3

The position vectors of points P and N are $3\underline{i} - 4\underline{j} + 5\underline{k}$ and $\underline{i} - \underline{k}$ respectively. If N is between P and Q such that PN : NQ = 2 : 3, calculate the position vector of point Q. 6

Figure below shows points F, G and H whose position vectors are $4\underline{i}-\underline{k}$, $\underline{i}+2\underline{j}-\underline{k}$ and $3\underline{i}+3\underline{j}-6\underline{k}$ respectively.



Show that vectors $\overline{GF}$, and $\overline{GH}$ are perpendicular to each other.

Calculate the value of angle GHO. 10

A straight line passes through the point with position vector $2\underline{i} - \underline{j}$ and is parallel to the vector $\underline{i} - 3\underline{j}$. Using vector approach, find the point where the line intersects the line $y = x$. 8

2005

Find the vector equation of a straight line passing through the points A and B with position vectors $5\underline{i} + 2\underline{j} - 2\underline{k}$ and $\underline{i} + 8\underline{j} + 2\underline{k}$, respectively. 4

Given that $|a\underline{i} + b\underline{j}| = 5$ and $a\underline{i} + b\underline{j}$ is **perpendicular** to $8\underline{i} - 6\underline{j}$, calculate the values of a and b. 10

Points A and B have position vectors $4\underline{i} - 3\underline{j}$ and $\underline{i} + t\underline{j}$, respectively, with respect to the origin, O. If cosine of angle AOB is $\frac{2}{\sqrt{5}}$, calculate the value of t. 10

Given that $\underline{a} - \underline{b} = \begin{pmatrix} -9 \\ -12 \\ -10 \end{pmatrix}$ and $2\underline{a} + \underline{b} = \begin{pmatrix} 24 \\ 30 \\ 16 \end{pmatrix}$, find the vectors $\underline{a}$ and $\underline{b}$. 6

2006

The position vectors of points A, B and C are $3\underline{i} + 5\underline{j}$, $4\underline{i} + 7\underline{j}$ and $6\underline{i} + 11\underline{j}$ respectively. Show that the points are *collinear*. 5

The position vector of points P and Q relative to the origin O are $\binom{5}{2}$ and $\binom{1}{-4}$, respectively. If R is a point such that $\overline{OP} = 3\overline{OQ} + 2\overline{OR}$, find the position vectors of R. 5

Triangle OQR has points X and Y on OR and QR, respectively.
If RX : XO = RY : RQ = 1 : 3, $\overline{RQ} = \underline{p}$ and $\overrightarrow{OR} = \underline{r}$, show that vector **XY** is **parallel** to vector OQ. 5

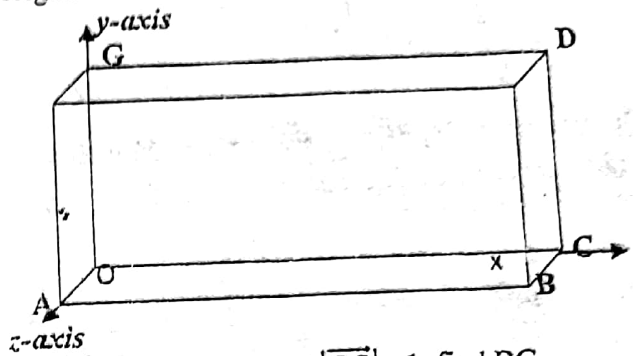
Points A and B are (1, 1, 1) and (13, 4, 5) respectively.
find $\overrightarrow{AB}$, 3
calculate length of $\left|\frac{1}{13}\overline{AB}\right|$. 4

2007

Given that $\underline{a} = 2\underline{i} + 3\underline{j}$ and $\underline{b} = 6\underline{i} + 12\underline{j}$.
Find
i. $\underline{a}.\underline{b}$
ii. ab 7

Calculate the **unit vector** in the **direction** of $\underline{b}$. 3

Figure below shows a cuboid with one of its sides ABCO lying on XZ plane where O is the origin.



If $|\overline{OA}|=3$, $|OC|= 2$ and $|\overline{OG}| =1$, find BG.
Find the **equation** of $\overline{AB}$. 11

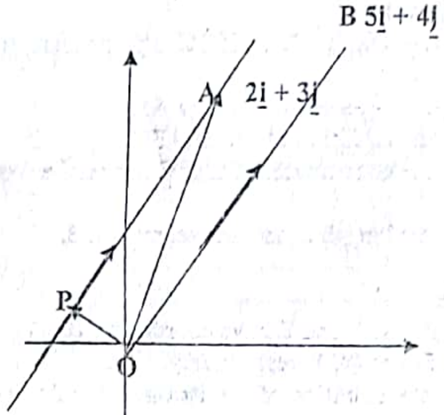
Relative to the origin **O**, the position vectors of points A and B are $\binom{1}{3}$ and $\binom{5}{5}$ respectively. If a straight line passing through A and B intersects the line $\binom{0}{5} + \beta\binom{6}{-2}$, find the **position vector** of the **point of intersection**. 12

2008

Given that $\overrightarrow{AB} = \binom{2}{3}$ and $\overrightarrow{PQ} = \binom{-3}{2}$, calculate $|3\overrightarrow{AB} - 2\overrightarrow{PQ}|$. 6

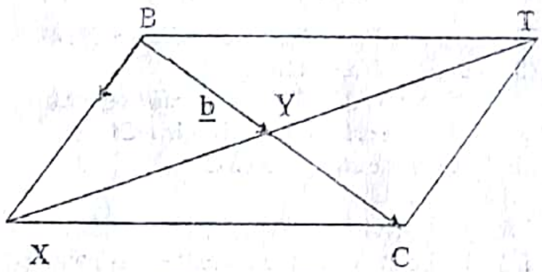
Given that $\overline{PQ} = \binom{4}{3}$ and $\overline{QR} = \binom{8}{-6}$, calculate the angle between the two vectors. 9

In figure below, the position vector of point A is 2i+3j and point B is 5i+4j.



Given that point P is such that PA is parallel to OB and OP is perpendicular to PA, find the position vectors of point P. 11

In figure below XBTC is a quadrilateral such that XY = YT and BY = YC.



Given that $\overrightarrow{BX} = \underline{a}$ and $\overrightarrow{BY} = \underline{b}$, use vector method to prove that XBTC is a parallelogram. 10

2009

Given that $\underline{a} = \begin{pmatrix} 3 \\ 5 \\ -6 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} -6 \\ 6 \\ -4 \end{pmatrix}$. Find

the value of $|2\underline{a} + \underline{b}|$. 5

the angle between $\underline{a}$ and $\underline{b}$. 6

Find the equation of a straight line through $\binom{3}{5}$ perpendicular to line $\underline{r} = \binom{3}{5} + \lambda\binom{5}{-2}$. 4

Given that points A, B and C have position vectors $\underline{a} = \binom{2}{1}$, $\underline{b} = \binom{3}{4}$ and $\underline{c} = \binom{5}{0}$, show that triangle ABC is isosceles. 6

The position vectors of points A, B and C are $\underline{i} + 3\underline{j} + 4\underline{k}$, $-5\underline{i} + 3\underline{j} + \underline{k}$ and $3\underline{i} + 6\underline{j} + 6\underline{k}$ respectively. Find the area of triangle ABC. 10

2010

Given the points P (8, 14, 12) and Q (2, 6, 12), calculate the **unit vector** parallel to $\overline{PQ}$ in its simplest form. 7

Given that the position vectors of A and B are $(4\underline{i} + 8\underline{j} - \underline{k})$ and $(-12\underline{i} + 4\underline{j} - \underline{k})$ respectively, find

the position vector of the point N where $\overline{AN} = \frac{3}{5}\overline{NB}$. 7

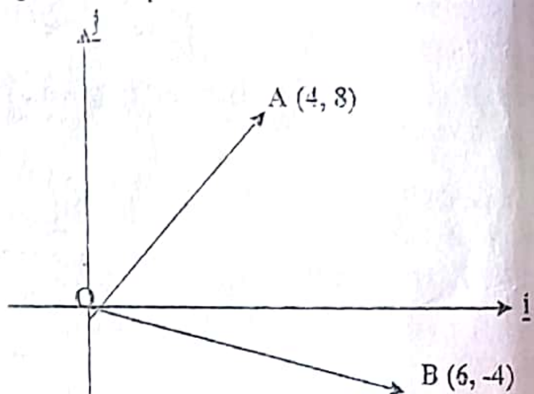
the vector equation of the line passing through B parallel to $-\underline{i} + 2\underline{j} + \underline{k}$. 2

Given that $\underline{a} = (4\underline{i} + 4\underline{j})$, $\underline{b} = (4\underline{i} + 4\underline{j} + 4q\underline{k})$ and angle between $\underline{a}$ and $\underline{b}$ is $\frac{\pi}{4}$, find the values of q. 9

Given that position vectors of points A, B and C are $(\underline{i} + 2\underline{j} - 2\underline{k})$, $(3\underline{i} + \underline{j} + \underline{k})$ and $(2\underline{i} + \underline{j} - 3\underline{k})$ respectively, show that $\overline{BA}$ is perpendicular to $\overline{AC}$. 3

2011

Figure shows position vectors $\overline{OA}$ and $\overline{OB}$.



a. Express the vectors $\overline{OA}$ and $\overline{AB}$ in terms of $\underline{i}$ and $\underline{j}$. 4
b. Find the length of vector $\overline{AB}$. 3

Given the position vectors A $\binom{1}{2}$, B $\binom{3}{-1}$, C $\binom{-1}{-1}$, D $\binom{k}{1}$ and that $\overline{AB}$ is perpendicular to $\overrightarrow{CD}$, calculate the value of k. 9

CARTESIAN GEOMETRY

Distance formula, gradient and equation of a straight line.

1999 (sample paper)
A is a point (3, -6), B is a point (-5, 8) and P is the point (h, k),
a. Write down the gradients of AP and BP.
b. Given that AP is perpendicular to BP, find the equation which must be satisfied by h and k.
10

1991
a. Write down the equation of the line:
  i. which passes through (2, 1) and (-7, -3).
  ii. which has a gradient -2 and passes through (6, 1) 7
b. Find the distance between the points (6, -4) and (2, -2). 2

1992
Find the equation of the line through (-2, 4) which is
  i. parallel to the line given by the equation $3x+y-2 = 0$
  ii. perpendicular to the line given by the equation $3x+y-2 = 0$. 7

1995
PQRS is a parallelogram in which QR is perpendicular to PR, and QS is parallel to the line $y + 4x = 2$, P and R are points (-1, 4) and (4, 1) respectively.
a. Find the coordinates of the mid-point of PR. 1
b. Find the equation of QS. 5
c. Find the equation of QR. 3
d. Calculate;
  i. the coordinates of Q
  ii. the coordinates of S
  iii. the area of PQRS. 11

1996
A right angled triangle ABC has two of its vertices as
A (1, 6) and B (-1, 2). If BC is perpendicular to AB, find;
a. the equation of the line AB. 3
b. the equation of the line BC. 3
c. the coordinates of C if C lies on the x-axis.
1
d. the length of the line segment AB. 2

1997
Points X, Y and Z have coordinates (0, -1), (2, 5) and (4, 1) respectively. Find:
a. the equation of the straight line through X and Y. 2
b. the slope of the line through Z perpendicular to XY, and hence its equation. 3
c. the point C where the perpendicular from Z to XY meets XY. 3
d. the distance from Z to XY. 2

The points A, B and C have coordinates (2, 4), (8, 5) and (6, 1) respectively.
  i. Show that OABC is a parallelogram.
  ii. Find the size of the triangle AOC
  iii. Find the area of OABC 10

Triangle OAB has vertices O (0, 0), A (1, 2) and B (4, 0). Points C, D and E are the mid-points of OB, AB and OA respectively. Show that AC, OD and BE intersect in one point and find the coordinates of the point. 11

1998
A line through A (1, 5) is drawn parallel to the x-axis and meets BP whose equation is
$3y = 2x - 5$ at B. If points P and Q on PB lie on the y and x-axis respectively. Find
  i. the length of PB.
  ii. the equation of the perpendicular form from A to PQ. 9

1999
A line passes through points A (-2, 3) and B (7, 2). If the perpendicular bisector of AB cuts the x-axis at C, and the y-axis at D, calculate the area of triangle OCD where O is the origin.8

2000
Quadrilateral ABCD has vertices A (1, 7), B (7, 5), C (6, 2) and D (0, 4).

a. Show that the quadrilateral is a rectangle. 5

b. Find the coordinates of the point of intersection of the diagonals of the rectangle. 7

2002
Let P be a point where line $x - y - 2 = 0$ and $2x + y + 3 = 0$ intersect. Find the equation of line, through the point of intersection perpendicular to the line $4x - 3y = 5$. 8

Triangle ABC is such that point A = (2, -1), B = (3, 2) and C = (4, 3). If D is the mid-point of BC, show that $AB^2 + AC^2 = 2AD^2 + 2BD^2$. 10

The line $2x - 6 = -3y$ meets the y-axis at A and the x-axis at B. C is the point such that AB = BC. CD is drawn perpendicular to AC to meet the line through A parallel to $5x = 7 - y$ at D.

a. Find the coordinates of A, B and C. 7

b. State the equation of CD and AD. Hence find the coordinates of D. 9

c. Calculate the area of triangle ACD. 4

2003

a. Find the equation of a line passing through the points A (4, 3) and B (2, 1). 4

b. Find the equation of the straight line through the point C (1, 1) perpendicular to the line AB in part (a). 3

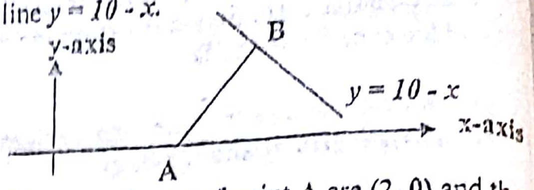
2004
Show that the lines joining the points A (1, 6), B (1, 4) and C (2, 1) form a right angled triangle. 8

Given the points A (2, 3), B (2, 4) and C (5, -1) find;

i. the equation of the line which passes through A and is perpendicular to BC. 5

ii. the equation of the line passing through B to the mid-point of AC. 5

2005
Figure shows a line AB that intersects another line $y = 10 - x$.



If the coordinates of point A are (2, 0) and the length of AB is $\sqrt{32}$, find the coordinates of point B. 7

Lines $y = 2x$ and $y + x - 3 = 0$ intersect at point A. Find the equation of a straight line which passes through the point A and is parallel to the line $x + 4y - 1 = 0$. 8

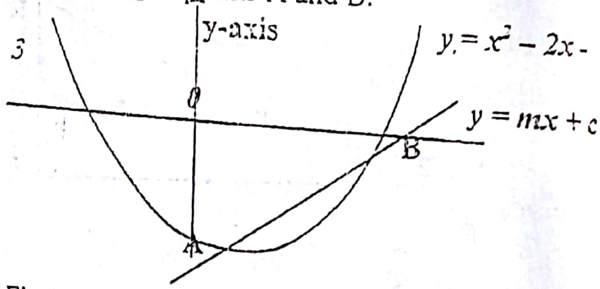
2006
Points A, B and C lie on a straight line such that B is the mid-point of AC. If points A and B have coordinates (-1 6) and (2, 4) respectively, find the coordinates of C. 7

2007
Points X and Y are (4, -2) and (-8, 6) respectively and Z is the mid-point of XY.

a. Find;
   i. the distance from X and Y.
   ii. the coordinates of Z. 6

b. Another point Q (3, m) is such that angle $QXY = 90^0$, find m. 6

Figure below shows a sketch of a quadratic graph and a straight line graph which are intersecting at points A and B.

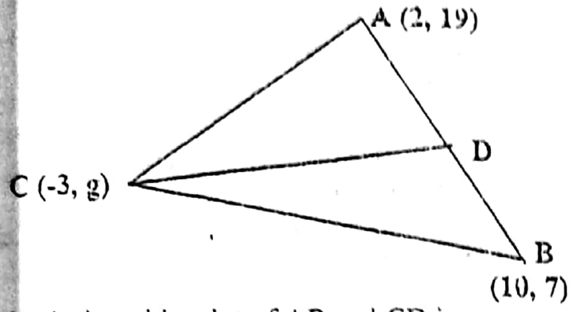


Find the equation of the straight line passing through points A and B. 9

2008

Given that line **AB** is perpendicular to **BD**, point **A** (3, 1), point **B** (0, 6) and **D** lies on the x-axis, find the coordinates of D. 9

Figure below shows triangle with vertices **A** (2, 19), **B** (10, 7) and **C** (-3, g).



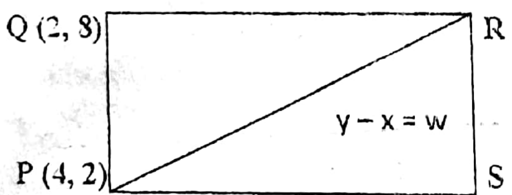
If **D** is the mid-point of **AB** and **CD** is perpendicular to **AB**, find the value of g. 10

2009

The vertices of a quadrilateral ABCD are **A** (2, 4), **B** (-1, 3), **C** (1, 2) and **D** (4, 3). X and **Y** are the mid-points of AD and BC respectively. **Show** that XY is parallel to AB. 7

A straight line Q passes through (1.5, 1) and its gradient is -2. Another line **L** whose equation is 3y - 2x = 5 meets the line Q at P. Find the coordinates of the point P. 9

Figure below shows a rectangle PQRS such that P and Q are points (4, 2) and (2, 8) respectively. PR is the diagonal whose equation is y - x = w.



Find;

i. the **equation** of QR in the form ax + **by** = c.

ii. the **coordinates** of R.

11

2010

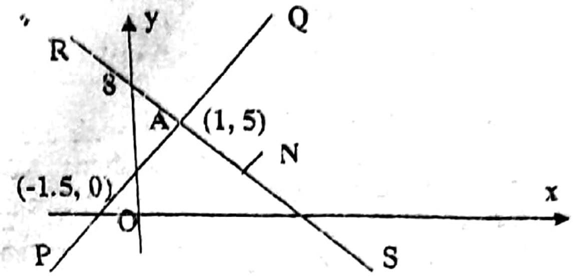
The coordinates of points **P**, Q and **R** are (2, 4), (8, -4) and (14, 8) respectively. C is the point where the perpendicular bisectors of **PQ** and **QR** meet. Given that the equation of the perpendicular bisector of **PR** is y = - 3x + 30 and the coordinates **of the mid-points** of **PQ** and **PR** are **(5, 0) and (8, 6)** respectively, **show** that **CP** = $\sqrt{50}$. 12

2011

A straight line passes **through points X (-3, 4)** and Y (7, 8). **Show that the point (6, -4)** lies on the perpendicular **bisector of XY**. 10

2012

The figure below **shows two lines PQ and RS intersecting at right angles at A. If the y-**coordinate **for N on RS is *a*, find the x-**coordinate **for N in terms of *a*.**



2013

The line y + 3x + 1 = 0 meets the curve $y = x^2 - 2x - 3$ at points A and B. Find the coordinates of the midpoint of **AB**. 11

2014

The perpendicular distance from the point (1, c) to the line 6x + 9 = 8y is 5.5. Calculate the possible values of c. 8

Four points have coordinates **A** (-*a*, 1), **B** (2, 7), **P** (-1, 2*a*) and **Q** (5, 1). If the line **AB** is perpendicular to **PQ**, find the value of ***a***. 6

2015

The coordinates of points **P**, **Q** and **N** are (1, 5), (5,-9) and (10, 9) respectively. Find the **distance** from N to the **midpoint** of **PQ**. 5

The equation of a straight line $y = \frac{1}{2}x + 8$ crosses the y-axis at **P**. Another straight line **AB**, perpendicular to the line $y = \frac{1}{2}x + 8$, passes through **P**.

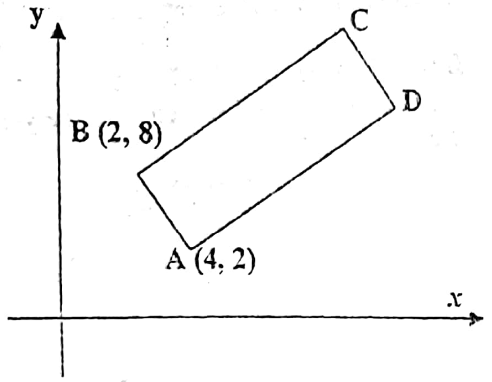
Find the **equation** of the line **AB**. 4

If the line **AB** crosses *x*-axis at **Q**, find the **coordinates** of **Q**. 3

P (0, 0), Q (1, 2) and R (4, 0) are three points on a Cartesian plane. Points C, D and E are mid points of PR, QR and PQ respectively. Given that QC, PD and RE intersect at point B, find the coordinates of B. 12

2017

Figure below shows a rectangle ABCD in which A = (4, 2) and B = (2, 8).



i. Find the equation of the line BC.

ii. If the equation of the line AC is $y = x - 2$, calculate the coordinates of point C. 9

2018

If points A $(-3, -2)$, B $(-1, a-2)$ and C$(a, 7)$ are collinear, calculate the values of a. 8

Find the equation of the perpendicular bisector of the line joining the points: P (- 3, 3) and Q (1, - 5). 8

Centre, radius and equation of a circle, tangent and normal to a circle:

1989

A is point (9, -7), B is (5, 5) and C is (-1, 3).

a. Show that P (4, -2) is the centre of the circumcircle of triangle ABC.
b. Find the equation of this circle
c. Calculate the equation of the tangent to this circle at A and also the equation of the tangent at the other point whose x coordinate is 9. 18

1990(sample paper)

A triangle ABC has vertices at the points A (-2, 8), B (1, 1) and C (5, 1).

a. Calculate the equation of the perpendicular bisector of the sides AB and BC.
b. Given that these perpendiculars meet at the point D,
   i. find the coordinates of D,
   ii. verify that D is equidistant from A, B and C.
   iii. calculate the equation of the circle through A, B and C. 20

1991

a. Find the equation of the circle centre (4, -5) passing through (7, -3). 5
b. Find the centre and radius of the circle $x^2 + y^2 - 6x - 4y + 4 = 0$. Sketch the circle and calculate where it cuts the x-axis.

1992 15

If the *x-axis* is a tangent to a circle with centre (4, 2), find the equation of the circle. 3

Show that the line $y = 2x+9$ is tangent to the circle $x^2 + y^2 - 2x - 4y = 0$.

a. Find
   i. the point of contact of the line and the circle.
   ii. the equation of the diameter to the circle through the point of contact.
   iii. the points where the circle cuts the x-axis.

20

1995

Find the equation of the circle which crosses the x-axis at (1, 0) and (4, 0) to which the y-axis is tangent. 7

1996

a. Write the equation of the circle centre (2, -3) and radius 5.
Calculate the length of the tangent to the circle from the point Q (7, 6).
Show that the line $y = \frac{3x}{4} + 1\frac{3}{4}$ is a tangent to the circle. 12

b. Show that the point Z (4, 2) lies on the circle $(x - 1)^2 + (y + 2)^2 = 25$.
Find the equation of the tangent at Z.
8

1997

Find;
the equation of the circle centre (2, 1) which
i. has the x-axis as one of its tangents,
ii. where the line $y = \frac{4x}{3}$ meets the circle in (i) above and name this line. 9

Points A, B and C have coordinates (1, 0), (3, 0) and (2, 4) respectively.

a. Find the mid-point D of AC. 2
b. Find the equation of the line through D perpendicular to AC. 4
c. Given that x = 2 is a perpendicular bisector of the line AB, find the centre and radius of the circle through A, B and C hence its equation. 7

1998

Find the centre and the radius of the circle whose equation is $x^2 + y^2 - 6x + 4y + 4 = 0$.
7

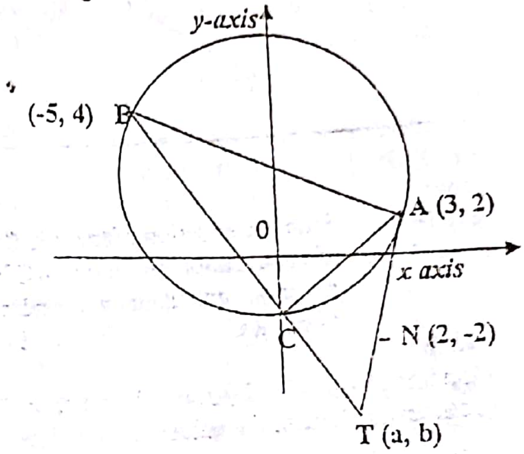
A tangent is drawn from a point T (6, 8) to the circle $(x+2)^2 + (y-3)^2 = 25$.

a) Find the length of the tangent.
b) Show that A (-2, 8) is one place where the tangent from T touches the circle.
c) Let B be the other point of contact where the tangent from T touches the circle, and C be the centre of the circle. Find the area of the quadrilateral ATBC. 12

The points (8, 4) and (2, 2) are the ends of the diameter of a circle.
Find:
the equation of the circle
the equation of the tangent at (8,4). 8

1999

In figure below (not drawn to scale), AB is a diameter of a circle ABC with A (3, 2) and B (-5, 4). A point N (2, -2) is on the line TA and C is a point where the circle cuts the y-axis. 3T is a straight line.



a. Find the equation of the circle. 4
b. Prove that TA is a tangent to the circle.
3
c. Calculate the values of a and b. 9
d. Find the area of triangle ACT. 4

2000

Triangle ABC has its vertices at points A (-2, 8), B (1, 1) and C (1, 5).

a. Find the equations of the perpendicular bisectors of the sides AB and BC. 5
b. If these perpendiculars in (a) meet at the point D, find the coordinates of the point D.
8
c. Find
the equation of the circle through A, B and C.
the equation of a tangent to the circle through C. 7

A circle passes through the points A (0, 1), B (6, 5) and O (0, 0). Find;

- the length of chord AB. 2
- the coordinates of the centre of the circle.
7

2001

The centre of a circle is the point C (3, 7). If A (5, 3) is a point on the circumference of the circle,

a) Calculate the radius of the circle.
b) Find the equation of the line through A perpendicular to AC. 4
c) Is the point (1, 1) on the circumference of the circle? 9

The point H (2, 5) is a centre of a circle of radius 3 units. The equation of the line L is $x + y = 2$.

a) Find the distance of h from L.
b) Determine whether or not, L is a tangent to the circle. 11

2003

The coordinates of points A, B and T are (a, 0), (- a, 0) and (5, 4) respectively. If the distance between A and T is half the distance between B and T.

a. Calculate the possible values of a.
b. Find the equation of a circle with centre A touching TB using the smaller value of a. 20

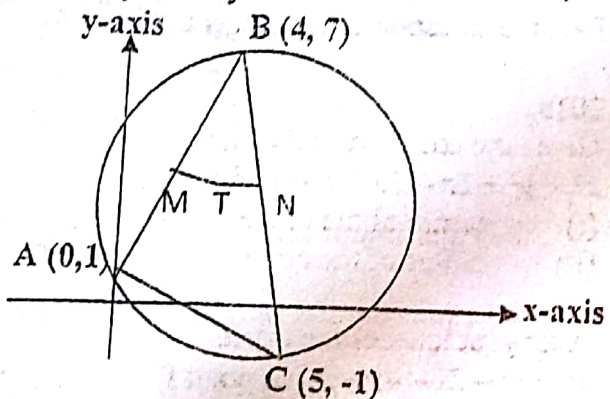
2004

Find the equation of the tangent to the circle $x^2 + y^2 - 4x + 10y - 8 = 0$ at the point (3, 1). 10

2005

Given that $36x^2 + 36y^2 - 24x + 36y - 23 = 0$ is the equation of a circle, find the coordinates of the centre of the circle. 6

Figure show a circle ABC with centre T such that points A, B and C are (0, 1), (4, 7) and (5, -1) respectively.



If M and N are the mid-point of AB and CB respectively, find the coordinates of the centre of the circle. 13

2006

Given the equation of circle $x^2 + y^2 - 2x - 2y = 7$ and that of a line $3x + 4y = 7$, find the;

a. centre of the circle 4
b. equation of the line L, passing through the centre perpendicular to the line $3x + 4y = 7$. 5
c. coordinates of the points where line L intersecting the circle. 11

2007

The equation of a circle is $x^2 + y^2 - 6x + 4y + 2 = 0$. Find the centre and the radius of the circle. 6

Show that the line $4x + 3y = 25$ is a tangent to the circle $x^2 + y^2 = 25$. 10

2008

Find the equation of the circle centre (5, -7) passing through (11, 4). 5

A point P (4, 3) lies on the circumference of a circle centre (0, 0). Find the equation of the tangent to the circle at P in the form $y = mx+c$. 6

2009

Find the radius of the circle whose equation is given by $y^2 - 2x^2 + 3x = \frac{18}{16}$. 5

2010

A point (3, k) lies on the circle $x^2 + y^2 - 3x - 2y + 1 = 0$. Calculate the possible values of k. 5

Find the equation of the circle whose diameter AB has the coordinates A (-3, 4) and B (2, 1). 6

A tangent touches a circle whose equation is $x^2 - 4x + y^2 - 10y - 71 = 0$ at the point (8, -3). Find the equation of the tangent. 10

2011

A line PT is a tangent to a circle centre (5, 7) at T. Given that the gradient is 2 and P has coordinates (-12, 3), find the equation of the circle. 13

2012
Show that $2x^2 + 2y^2 - 6x + 10y = 1$ is an equation of a circle. 6

Show that the line $5y = 12x - 33$ is a **tangent** to the circle $x^2 + y^2 + 2x - 8y = 8$. 10

2013
Given the equation of the circle $2x^2 + 2y^2 - 8x + 4y - 15 = 0$, find the;

i. **radius** of the circle
ii. **coordinates** of the centre of the circle. 7

Find the **equation of the normal** to the circle $x^2 + y^2 - 3x - y - 2 = 0$ at the point (3, 2). 8

2014
A triangle PQR has vertices P (2, 14), Q (-6, 2) and R (12, -10). If angle PQR is a right angle, find the equation of the circle that passes through the three points P, Q and R in the form $ax^2 + by^2 + cx + dy + e = 0$. 7

The tangent to the circle $x^2 + y^2 - 4x + 6y - 77 = 0$ at the point (5, 6) meets x and y axes at points A and B respectively. Find the coordinates of A and B. 11

2016
Find the coordinates of the point where the line $x + y = 1$ meets the circle $x^2 + y^2 - 2x - 4y + 1 = 0$. 8

Find the *equation of a tangent* at a point (8, 4) on the circumference of a circle whose diameter has end points (8, 4) and (2, 2). 8

The vertices of triangle PQR are P(-4, 2), Q(4, 2) and R(2, 6).
a) Show that angle PQR = 90°.
b) Find the *equation of the circle* passing through points P, Q and R. 12

2017
The centres of the circles A and B are 7 cm apart. The equation of circle A is $x^2 + y^2 = 4x - 6y + 9 = 0$. If the two circles touch each other externally, calculate the *radius* of circle B. 7

Find the **equation of the normal** to the circle $x^2 + y^2 + 6x - 4y - 24 = 0$ at the point *W (3, 1)* on the circumference. 8

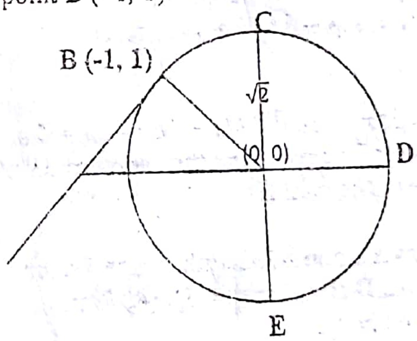
2018
A circle P passes through a point (2, 6) and another circle Q has the equation is $x^2 + y^2 - 2x - 10y + 18 = 0$. If the two circles have the same centre, find the *equation* of the circle P. 7

Given that the line $3y = 4x + c$ is a tangent to the circle $x^2 + y^2 + 4x - 10y - 7 = 0$, calculate the value of c. 11

Sample Paper
Calculate the *length of the tangent* from the point (9, 8) to the circle $x^2 + y^2 - 2x - 4y = 3$. 7

A circle of radius *2 cm* touches the axes of the *xy*-plane in the first quadrant. Find the *equation* of the circle. 6

The figure below shows a tangent AB at B to a circle BCDE centre (0, 0) with radius $\sqrt{2}$ at a point B (- *1, 1*).



Find the *equation* of the tangent. 4

2019
Given the equation of the circle $x^2 + y^2 + 6x - 2y + 6 = 0$, find the:
(i) centre of the circle
(ii) radius of the circle 6

A tangent touches the circle $x^2 + y^2 - 2x - 4y = 31$ at point P. Given that point X (9, 8) is on the tangent, calculate the length of PX. 10

Compiled by K.A. ADINI

ALGEBRA

FUNCTIONS

1991:

Let $f(x) = \frac{x^2-3x}{3x-2}$

a. Find:
   i. f(2) 2
   ii. $f\left(\frac{2}{x}\right)$ and simplify your answer completely. 4

b. Solve $f(x) = \frac{x}{3x-2}$ 5

1992:

If $f(x) = 3x^2 - 1$ and $g(x) = 2 - x$, find

a. f(2) 1
b. g(3) 1
c. f(g(x)) 2
d. f(g(1-t)) 3

1996:

Let $f(x) = 2x+1$ and $g(x) = \frac{1}{1+x^2}$. Find

a. f(2) 2
b. g(-3) 2
c. g(f(0)) 2
d. f(g(0)) - g(f(0)) 2

1997:

The functions f and g, where $0 \le x \le \pi$ are defined by $f: x \rightarrow \cos x$ and $g: x \rightarrow x - \frac{\pi}{2}$. Find

a. $f(\pi)$ 1
b. $g(\pi)$ 1
c. $fg(\frac{\pi}{2})$ and 2
d. the value of x if $fg(x) = \sin^2 x$ 3

1998:

Functions f and g are defined for real values of x by $f: x \rightarrow 3x + 2$ and $g: x \rightarrow mx + 3$.

a. Find
   i. g(-3) 2
   ii. the value of m for which fg(x) = gf(x) 4
   iii. When m takes this value, for what value of x does $f(x) = g(x-1)$? 3

1999:

Given that $f(x) = 10^{2x}$, and $g(x) = \log x$, find:

a. f(3) 1
b. fg(100) 3
c. fg(1) 3
d. gf(10) 3

2000:

Functions f and g are defined by $f: x \rightarrow 15 - 7x - x^2$ and $g: x \rightarrow (3 - x)^2$. Find:

a. f(3) 2
b. fg(-1) 3
c. the value of x for which $f(x) = g(x)$ 4

2001:

Given that $f(x) = \frac{2x-1}{x^2 - x + 5}$ calculate

i. f(2) 3
ii. x if f(x) = 1 3

2003:

Given that $f(\theta) = \sin^2\theta - \cos^2\theta$,

a. find $f(\frac{\pi}{3})$.
b. If $f(\theta) = 0$, find θ for $0 \le \theta \le \pi$. 11

2004:

Given that $f(x) = x^2$, simplify $\frac{f(a+h) - f(a)}{h}$. 5

2005:

Given than $E(x) = 8^x$.

a. Find $E(\frac{2}{3})$ 3
b. If E(x) = 16, calculate the value of x. 5

Given that $f: x \rightarrow \frac{20}{x}$ and $g: x \rightarrow x - 1$, calculate the value of x for which g(f(x)) = 0. 3

2006:

Let $f: x \rightarrow \frac{1}{2+x}$

a. Find f (f(1)). 4
b. Find the range if the domain is the set $\{x: 1 \le x \le 5\}$ 3

2007:

Given that $f(x) = \frac{x-2}{3}$ and $g(x) = 3x + 2$, find $gf(x)$ in the simplest form. 4

2008:

$$f(x) = \begin{cases} 2x^2 - 1, 0 < x < 3 \\ 0, x = 3 \\ x^2 + 1, x > 3 \end{cases} \text{ find } ff(2)$$ 5

A function $g(x)$ is given by $g(x) = ax^2 + bx - 3$ where a and b are constants. Given that $g(1) = 4$ and $g(2) = 15$, calculate the values of a and b.

2010: 7

Give that $f(x) = 2x + 1$ and $g(x) = \frac{1}{2}(x-1)$, show that $fg(x) = gf(x)$.

7

2011:

Given that $h(y) = (\frac{y}{3})^2$ and $g(y) = 3y - 12$. Find $h(g(y+4))$.

4

Given that a function $f(y) = \log_a y$ and that $f(4x-a) = 2f(x) + f(3)$. Find all possible values of x in terms of a.

11

2012:

Given that $h(x) = \cos x$ and $g(x) = \frac{2x-\pi}{2}$ find $hg(\frac{\pi}{2})$.

5

Given that $f(x) = \frac{x}{x+2}$ and $g(x) = \frac{1+2x}{1-x}$, show that $gf(x) = \frac{1+2x}{3}$

7

2013:

Given that $f(x) = 2x+3$ and $g(x) = \frac{5}{x-2}$ for $x \neq 2$ find x for which $ff(x) = gf(2)$. 7

2014

Given that $f(x) = x^2 - x$ and $g(x) = 2x - 3$, find the values of x when $f(x) + g(-x) = 1$. 6

2015

Given that $f: x \to 9x^2 - 4$ and $g: x \to \sqrt{x+1}$ for $x \geq 0$, find $fg(x)$.

5

2016

Given that $g(x) = 2x + 3$ and $f(x) = x^2 + 4$, find $gf(x)$.

4

2017

The function g is defined by $g(x) = \frac{3}{x+1}$ for $x \neq -1$. Find the expression of $g(g(x))$ in its simplest form.

4

2018

Given that $f: x \to x^2 - 9$ and $g: x \to |x + 2|$, calculate the value of $gf(2)$.

4

Given that $fg(x) = gf(x)$ where $f(x) = ax + b$ and $g(x) = 3x + 5$, where a and b are constants, find a in terms of b.

6

Sample Paper

Given that $f(x) = 1 + 2x$ and $g(x) = \frac{1}{x-1}$ where $x \neq 1$, find $fg(x)$ in its simplest form. 5

Let $h(y) = \frac{3y^2+5}{\log_{10} y+4}$, calculate $\lim_{y \to 1} h(y)$.

4

Show that the function $y = x^2 + 5$ is a *one-to-one* function but not *onto*.

6

2019:

Given that $f(x) = \frac{\pi}{x-\pi}$, where $x \neq \pi$ and $g(x) = 2x$, evaluate $fg(5\pi)$.

5

Quadratic functions

1992:
By completing the square, find the *maximum value* of $f(x) = -x^2 - 2x - 2$. Sketch the curve. 6

1996:
By completing the square, find the *minimum value* of $y = x^2 + 4x - 1$ and find coordinates of the point where it occurs. 3

1997:
The expression $p + qx - x^2$ has its greatest value of 5 when $x = 2$.

i. By completing the square or otherwise, find the numerical values of p and q. 4

ii. Sketch the curve $y = p + qx - x^2$ with these values p and q. 2

If $y = 2 - 2x - x^2$, find by completing the square

i. the value of x when $y = 0$,

ii. coordinates of the *maximum point* of the curve. 7

2004
Give the function $f(x) = x^2 + px + p - 1$, has the minimum value of 0. Find, by completing the square;

i. the value of x in terms of p at the minimum point.

ii. the value of p. 11

2006
By completing the square, find the *minimum value* of $y = x^2 + 4x - 1$ and the coordinates of the point where it occurs. 4

2010
By completing the square, calculate the *maximum value* of y for the function
$y = \frac{7}{8} - x - 2x^2$. 5

2012
A graph of the form $y = ax^2 + bx + c$ crosses the y-axis at (0, 3) and has turning point at (1, 2). Find the values of a, b and c. 9

2017 Sketch the graph of $y = -1(x-4)(x-12)$

2016
The curve $f(x) = x^2 - mx + n$ has a turning point at $x = 2$. Given that the **maximum value** of this function at this point is -1, by **completing a square**, find the values of m and n. 10

2019
Given that $y = 1 - 4x - 2x^2$, by completing the square, find the greatest value of y. 5

Exponential functions

1991
Solve the equation $10 = 2^{x+1}$ for the giving your answer correct to 3 significant figures. 4

Plot the graph of $y = 2^{x+1}$ for the values of x between -1 and 4. From the graph, find the value of y when x = 25. 8

1996
Solve for x in these equations;

a. $2^{x^2} = 16$ 3

b. $2^{2x} - 5(2^x) + 4 = 0$ 4

1997
Solve the equation $2^{2x+1} - 15(2^x) = 8$. 3

Draw the graph of $y = 2^x$ for the values of x from -4 to 4. (Take the scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis). From the graph, estimate the solution of the equation $2^{x-1} = 4x$. 7

2006
Plot the graph of $y = 2^{x+1}$ for the values of x where $-1 \leq x \leq 4$. 6
Use your graph to find the value of x when $y = 22.5$ and show your solution on the graph. 1

Solve the equation $3^{2x} - 12(3^x) + 27 = 0$. 8

2009
Solve the equation $2^{2a+3} - 2^{a+3} - 2^a + 1 = 0$. 9

2010
Given that $f(x) = 3^x$, draw, on the graph paper provided, the graph of $f(x) = 3^x$ for $-2 \leq x \leq 2$.
Indicate, on your graph the value of $f(x) = 3^{1.2}$
From your graph, state the approximate value of $f(x) = 3^{1.2}$ 8

2015
Solve the equation $8^{2x-5} = 5^{x+1}$. 8

2017
Using a scale of 2 cm to represent 1 unit on the horizontal axis and **2 cm** to represent 5 units on the vertical axis, draw a graph of $y = 3^{x+1}$ for **$-3 \leq x \leq 2$.**
Use the graph to solve the equation $3^x = 7$. 10

Rational functions

1998

Sketch the curve $y = \dfrac{-2}{x+1}$. 3

2002

The curves $y = \dfrac{4}{x^2}$ and $y = -x^2 + 5$ intersect at A and B. Find the point of intersection.
Sketch the curves. 7

Inequalities

1998
a. $x(x-1) < 2$ 2
b. $x < \dfrac{2}{x-1}$ 4

1991
i. $x^2+5 > 0$ 2
ii. $(x+4)(x+5) < 0$ 4

1992
$x + 1 < \dfrac{6}{x}$ 10

1996
a. $x^2 + x < 2$
b. $\dfrac{1}{x-1} < 3$ 7

1997
a. $|x-3| < 2$ 2
b. $\dfrac{x+2}{x+1} < 3$ 4

1998
a. $2x^2 + 5x - 12 > 0$ 4
b. $1 + x < \dfrac{6}{x}$ 7

1999
a. $\dfrac{x}{x-1} < 5$ 5
b. $|3x+1| < 4$ 4

2000
a. $(3x+1)(x-5) < 0$ 4
b. $\dfrac{x^2+12}{x} > 7$ 7

2001
a. $\dfrac{2x-4}{x-1} < 1$ 5
b. $4x - x^2 + 6 > 0$ 8

2002
Given that $(a-b)^2 \geq 0$, show that $a^2 + b^2 \geq 2ab$ 2

Solve the inequality $x^2 - 3x + 2 > 0$. 7

2003
Copy and complete the table of values for $y = x^2 - 1$ and $y = \frac{1}{x}$

x	-2	-1.5	-1.0	-0.5	0.5	1	1.5	2
x^2-1	3	1.3		-0.8	-0.8	0	1.3	3
$1/x$	-0.5	-0.7	-1	-2	-2		0.7	0.5

Using a scale of 2 cm to represent 1 unit on both axes, draw the graphs of $y = x^2 - 1$ and $y = \frac{1}{x}$.

Use the graph to solve the inequality $x^2 - 1 < \frac{1}{x}$
14

Solve the inequality $\left|2x - \frac{3}{2}\right| > \frac{1}{2}$. 6

2004
For what values of x does the inequality $\frac{3x}{x-1} > 3$ hold? 4

a. Solve the inequality $6 - x - x^2 \leq 0$. 6
b. Sketch the graph of $y = 6 - x - x^2$ and illustrate the solution of $6 - x - x^2 \leq 0$ on the graph. 3

2005
Solve the inequality $\frac{12}{x-3} > x + 1$. 9

Sketch the graph of $y = |x + 2|$. 3
Use the graph to find the range of values of x for which $|x + 2| > 2$. 4

2006
Solve the inequality $\frac{6x-1}{2x+1} < 2$. 7
A girl is four years older than a boy. The product of their ages is more than 21
Use this information to form an appropriate inequality.
Solve the inequality to find the age range of the boy. 12

2007
$|6x + 5| \leq 4x - 1$ 7

Sketch on the same axes, the graphs of $y = x+1$ and $y = \frac{12}{x+3}$. Use your graph to solve the inequality $\frac{12}{x-3} < x + 1$. 13

2008
Solve the inequality $\frac{(x-1)(x+2)}{3x+2} < 0$. 9

Sketch the graph of $y = x^2 - 2x + 1$ and illustrate the solutions of the inequality $x^2 - 2x + 1 > 4$ on the graph. 12

2009
Find the range of values of x for which
$\frac{x^2+8}{x} > 6$. 8
$|x^2 - 5x - 10| \geq 4$ 11

2010
$1 > 8x^2 + 2x$ 7

$\left|\frac{x+1}{x-2}\right| < 2$ 12

2011
$|4 - 2x| \leq 8$ 6

$\frac{1}{x+1} > \frac{2}{x-3}$ 11

2012
$\frac{(x-1)(x+2)}{3x+2} \geq 0$ 7

A man is 22 years older than his child. In 5 years time, the product of their ages will be at least 240 years.
Formulate an inequality in x to represent the information.
Solve the inequality to find the age range of the child. 11

2013
$\frac{x+1}{3x-5} - 3 \geq 0$ 8
Find the range of values of x for which $\frac{1}{x-2} \leq 3$ 12

2014
Solve, by sketching the graph, the inequality $(x-1)(2x-3)(3-x) < 0$. 7
Find the range of values of x for which $\frac{(x-1)^2}{x+5} < 1$. 11

2015
Find the range of values of x for which
$(3x-1)^2 < 3x^2 + 13$. 7

- Find the range of values of x for which
$x + 3 > \frac{4}{x}$. 10

2016
By sketching the graph, solve the inequality
$4p^2 - 9p \geq 9$. 8

Solve the inequality $|9 - 3x| \leq 27$. 6

2017
On the same axes, sketch the graphs of
$g(x) = x - 2$ and $f(x) = (x-2)(x-4)$
Use the graphs to solve the inequality
$f(x) < g(x)$ 10

2018
Solve the inequality $\frac{1}{|x-3|} < 5$. 7
By sketching a graph, solve the inequality
$\frac{10}{x} \leq 5$. 8

Sketch the graph of $y = x^2 - 2x - 15$ and shade the region in which $x^2 - 2x - 15 > 0$. 5

Sample paper
Find the range of values of x for which
$(3x-1)^2 < 3x^2 + 13$. 8

Solve by sketching a graph, the inequality
$\frac{x-1}{3x-5} - 3 \geq 0$. 10

2019:
Solve the inequality $|2x + 1| \leq 3$. 6

Solve the inequality $x^2 - 2x + 1 > 7 - x$. 8

Binomial theorem

1991
Find the **coefficient** of the 5^{th} term in the expansion of
$(3 + x^2)^8$. 6

1992
a. Write the expansion of $(1 + x)^6$. 3
b. Use the binomial theorem to evaluate $(0.98)^6$, correct to 3 decimal places. 4

1996
Write down the expansion of $(1 + 2x)^4$. Hence, showing all your working, evaluate $(1.02)^4$, correct to 3dp. 4

1997
a. Using the binomial theorem, write down the coefficient of x^4 and x^5 in the expansion of $(2 + x)^7$. 3
b. Calculate the value of x if the 5^{th} and the 6^{th} terms in the expansion of $(2 + x)^7$ in ascending powers of x are equal. 3

1998
Use the binomial theorem to expand $(1 - \frac{x}{2})^6$, hence evaluate $(1.04)^6$, correct to 4 decimal places. (Show all your working). 5

1999
Find the values of a and n if the sum of the first three terms in the expansion of $(1 + ax)^n$ is $1 + 16x + 96x^2$. 8

2000
Use the binomial expansion of $(1+2x)^9$ to evaluate $(1.04)^9$, correct to 3 decimal places. 5

Find the term in x^3 for the product of
$(1-x)^{20}(x + \frac{1}{2})^7$. 11

2001
Given the coefficient of x^2 is equal to the coefficient of x^4 in the expansion $(1+ax^2)^6$, find the value of a. 4

2002
Determine the coefficient of w in the expansion of $(w - \frac{1}{w})^3$. Use the binomial theorem. 5

2003
Find the non-zero value of b if the coefficient of x^3 in the expansion of $(b+x)^8$ is equal to the coefficient of x^8 in the expansion of $(2+bx)^8$. Express your answer in surd form. 7

2004
Find the coefficient of the term with x^{10} in the expansion of $(2x+y)^{14}$. 4

2005
In the expansion of $(x+b)^n$, the coefficients of x^{n-1} and x^{n-2} are -8 and 30, respectively. Find the values of b and n. 10

Given that $(1-\frac{1}{x})^5 = 1 - \frac{5}{x} + \frac{10}{x^2} - \frac{10}{x^3} + \frac{5}{x^4} - \frac{1}{x^5}$, estimate the value of $(\frac{999}{1000})^5$ to 3 significant figures. 4

2006
Use the binomial theorem to expand $(2a-1)^4$ and simplify the coefficients. 4

Calculate the constant term of the expansion of $\left(\frac{1}{3x} - \frac{3x^2}{2}\right)^9$ 8

2007
In the expansion of $(2p-q)^4$, calculate the coefficient of p^3q. 4

2008
The coefficient of x^3 in the expansion of $(1+kx)^4$ is 32. Find the value of k. 6

The term with x^5 in the expansion of $(1+3x)^{10}$ is equal to the term with x^8 in the expansion of $(252+x)^9$. Find the value of x. 10

2009
Use the binomial theorem to expand $\left(2x-\frac{1}{x}\right)^8$ and find the constant term. 4

2010
Use the binomial theorem to find the greatest coefficient in the expansion of $(x+\frac{1}{x})^6$. 4

2011
Use the binomial theorem to evaluate $(1.02)^4$. Correct to 4 decimal places. 6

2012
The first two terms in the expansion of $(2-ax)^6$ are $b+12x$. Find the values of a and b. 6

2013
Use the binomial theorem to find the term in the form $\frac{A}{x}$ where A is a constant, in the expansion of $\left(x^2+\frac{2}{x}\right)^7$. 5

2014
Find the coefficient of x^2 in the expansion of $\left(x-\frac{1}{2x}\right)^{10}$. 5

2015
Given that $10x$ is the second term in the expansion of $(1+2x)^n$, use the binomial theorem to find the value of n. 4

2016
Use the binomial theorem to find the constant term in the expansion of $\left(x+\frac{1}{3x}\right)^6$. 5

2017
The second term in the expansion of $(1+2y)^n$ is $40y$. Calculate the value of n. 5

2018
Calculate the ratio of the coefficient of the second term to the coefficient of the fifth term in the expansion of $(1+3x)^4$. 5

Sample Paper
Find the term independent of x in the expansion of $\left\{x^2-\frac{2}{x}\right\}^{12}$. 4

Given that $(z+1)^{10} = \sum_{r=0}^{10} 10C_r Z^r 1^{10-r}$, use the formula to find the coefficient of z^4. 2

2019
Find the coefficient of the term with x^7 in the expansion of $(2+x)^{10}$. 5

Graphical problems

1989

In an experiment, corresponding value of two variables p and v are obtained as shown in the following table:

V	10	15	20	25	30	100
P	40	22.5	15	11	8.5	1.6

It is suspected that the variables satisfy law of the form $p=kv^n$. ie $\log k + n \log v$. By plotting values of log p against log v, verify that the variables do satisfy a law of this form, and find the values of k and n. 13

1991

The variables w and p are known to satisfy the relationship $w = kp^n$ where k and n are both constants. The data below was obtained from an experiment involving two inequalities.

p	1	2	3	4	5	6
w	5	40	135	320	625	1080

By plotting a straight line of the appropriate variables, find the values of k and n and state the law connecting the two variables. 20

1996

The table below gives some experimental value of x and y.

x	2	3	4	5	6
y	1.09	0.74	0.55	0.44	0.37

It is known that x and y are related by an equation of the form $y = \frac{k}{x}$. By plotting a suitable graph, estimate the value of k. 9

1997

The table below shows corresponding values of x and y obtained in an experiment. It is known that x and y satisfy a relationship of the form $y = ax^b$ where a and b are constants.

x	1.1	1.7	2.1	3.2	3.8
y	12.85	11.47	33.56	86.86	150.5

a. Plot a suitable graph to verify the relationship between x and y. 10

b. Use your graph to

- Show which value of y in the table is not correct and give the correct value of this y. 3
- Estimate the values of a and b. 4
- From (ii) above state the law connecting the two variables x and y. 1
- Estimate the value of x when y = 100. 2

1998

A law of the form $y = ax^n$ connects the variables x and y from a set of readings. $Log\, y$ is then plotted against $log\, x$ to give a line best fit of gradient 4 and intercept 2.3. Deduce the appropriate values of a and n. 5

2000

The table below shows the results of an experiment done in a laboratory.

f	1	1.25	1.75	2	3	4	5
p	25	16	8	6	2.8	1.6	1

It is known that f and p are connected by the equation $f = c/p^k$ where c and k are constants.

a. Draw a suitable graph to show the relationship and estimate the value of c and k. 14

b. Use your graph to estimate
   i. p when f =1.2
   ii. f when p =2.5. 6

2001

The table below shows the appropriate values of the two quantities p and q. The quantities are connected by a relation of the $p \alpha q^n$.

q	3	16	25	34	47
p	45	1282	3126	5780	11042

Draw a suitable graph and determine the proportionality constant. 12

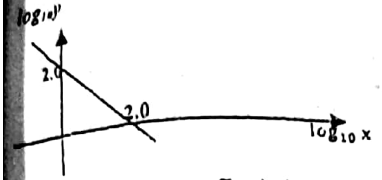
2002

The table below gives experimental values obtained for two values x and y.

x	0	1	2	3	4
y	10.10	19.99	40.10	80.01	159.99

Show that y and x are related by the law in the form $y = ka^x$ and determine the constants k and a. 11

2003
The figure below shows a linear graph of $\log_{10}y$ against $\log_{10}x$ for the equation $y = \frac{A}{x}$



Use the graph to find the values of A.

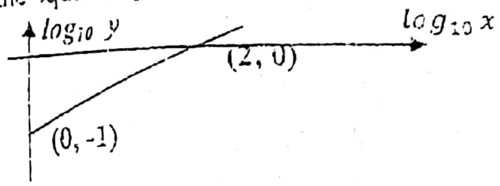
7

2004
The table below shows value of x and y obtained from an experiment.

x	0.2	0.5	1	2	5
y	100	16	4	1	0.16

If it is known that the relationship between x and y is $y = ax^b$ where a and b are constants, draw a suitable graph to determine the values of the constants a and b. 13

2005
Figure shows a graph of $\log_{10}y$ against $\log_{10}x$ for the equation $y = ax^b$ where a and b are constants.



Use the graph to calculate the values of a and b.

10

Table below shows the values of y calculated from the formula $y = ax^2 + bx$, where a and b are constants. However one of the values of y was wrongly calculated $y = ax^2 + bx$

x	4	7	10	13
y	96	133	160	117

What type of graph is expected from the plot of $\frac{y}{x}$ against x?

On the graph paper provided, draw the graph of $\frac{y}{x}$ against x.

Find which values were wrongly calculated.

Estimate the numerical values of a and b.

12

Table shows the results of an experiment in which some liquid was left to cool and its temperature measured at different intervals.

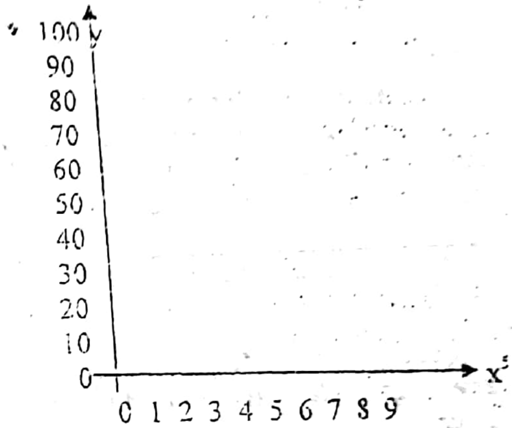
Cooling temperatures

Time (minutes)	0	1	2	3	4	5
Temperature(^{0}c)	100	60	40	30	25	23

Using the scale of 2 cm to represent 1 minute on the x-axis and 2 cm to represent 10^{0}C on the y- axis, draw a graph of these results. 5

Estimate the rate of decrease of the temperature at 3 minutes. 5

2006
The figure below shows plots of points of the relation between y and x^5.



- Copy the figure on the graph paper provided. 1
- Draw a straight line that best fit the points. 2
- Find x when y = 320. 3

2007
The values of x and y are connected such that $y = pq^x$ where p and q are constants. If **log y** is plotted against x. Straight line graph of gradient 0.6 and the y intercept of - 0.2 obtained. Calculate the values of p and q. 7

The table below shows results of an experiment relating x and y.

x	-2	-1	1	2	3
y	-15	-1	3	17	55

It is known that the relationship between x and y is of the form $y = ax^3 + b$ where a and b are constants. Using a scale of 2 cm to represent 5 units on the x-axis and 2 cm to represent 10 units on the y - axis:

Draw a suitable graph to determine the values of the constants a and b.
Use the graph to find the values of the constant a and b. 10

2008

A graph of log y against log x for the equation *log y = d log x+ log c* passes through (*1.3, 3.4*) and (*1.7, 4.0*).

i. Draw a graph of log y against log x.
ii. Use your graph to find the values of c and d.
iii. Express y in terms of x. 15

2009

A straight line graph of log y =b log x+ log a where a and b are constants is drawn. The graph crosses the y-axis at 2.3 and the x-axis at 1.
Calculate the values of a and b. 4
Find the law connecting x and y. 3
Estimate the value of y when x=18. 2

Copy and complete the table of values of log x and log y from the table of values of x and y. 2

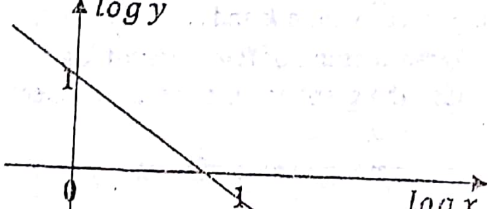
X	20	30	40	50	60
y	0.22	0.18	0.16	0.15	0.13

log x	1.3		1.6		1.8
log y					-0.89

Given the values that satisfy the law $y = kx^p$, draw the graph of *log x* against *log y*. 5
Use your graph to find the values of the constants *k* and *p*. 3

2010

Figure shows a graph of log y against log x



Using the graph, express *y* in terms of x. 8

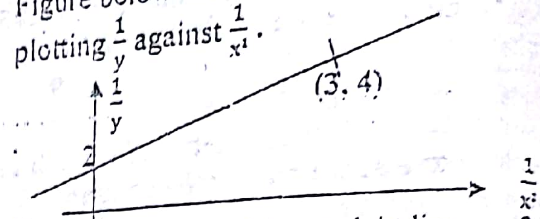
The population P of fish in a lake was estimated every year. The data for 5 years was summarized in the table below.

t (years)	1	2	3	4	5
P (thousand)	12.1	18.4	28.3	42.5	64.6

✓ Using a suitable scale, plot the graph of log p against t.
✓ If the relationship connecting p and t is $p = ab^t$, find values of the constants a and b 16

2011

Figure below is a straight line obtained by plotting $\frac{1}{y}$ against $\frac{1}{x^2}$.

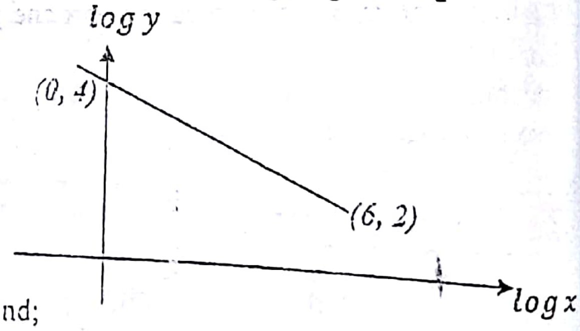


Find the gradient of the straight line. 2
Hence express y in terms of x. 4

2012

Two variables x and y are connected by the equation $\frac{x^2}{p^2} + \frac{2y^2}{q^2} = 1$ where p and q are constants. Given that the straight line graph drawn from this relationship intersects the y - axis at $4\frac{1}{2}$ and its gradient is $-\frac{9}{50}$, find the values of p and q. 8

The figure below shows part of a linear graph obtained by plotting log y againstlog x.



Find;

i. log y in terms of log x
ii. y in terms of x

9

2013

Variables x and y are related in such a way that when $\frac{y}{\sqrt{x}}$ is plotted against x^2, a straight line is produced. If the straight line passes through points A (4, 3) and B (1, $1\frac{1}{2}$), express y in terms of x. 7

The table below shows estimated costs for printing copies of a book.

Number of copies	50	100	200	500
Cost in thousands (MK)	11.5	12.5	14.5	20.5

Draw a graph of the form $y = mx + c$ that connects the cost (y) in thousands and number of copies (x).
Use your graph to find the values of m and c.

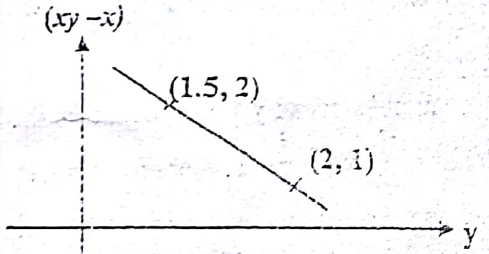
10

2014

A set of values of p and q are connected by the law $p = ab^q$ where a and b are constants. When $\log p$ is plotted against q, the graph is a straight line with gradient 0.56 and y – intercept - 0.95. Without drawing the graph, find the values of a and b.

9

The figure below shows a sketch of a graph of $(xy - x)$ plotted against y, for the equation $y = \frac{x+b}{x+d}$.

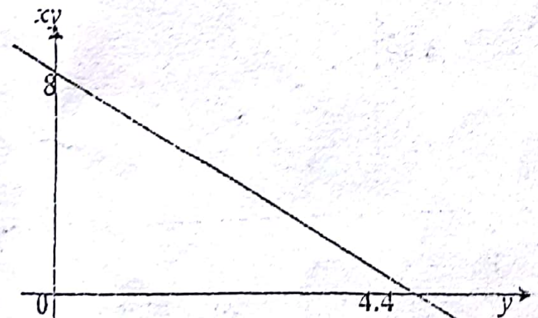


Find the values of b and d in the equation $y = \frac{x+b}{x+d}$.

10

2015

Figure below is a graph of xy against y of the experimental values of two variables x and y.



If x and y are connected by the equation $y = \frac{a}{x+b}$ where a and b are constants, find the value of a and the value of b.

7

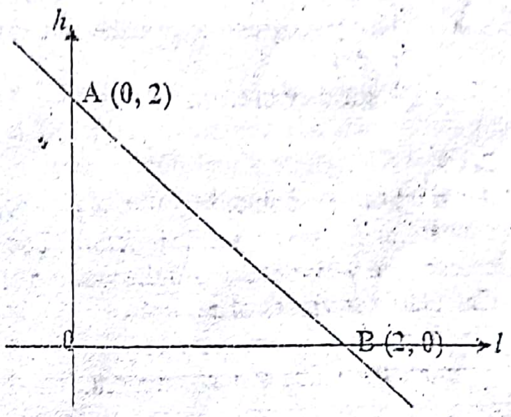
2016

Variables x and y are related by the equation $y = kn^x$. If a straight line of gradient 0.58 drawn from the equation, passes through the points (5, 41) and (6, m), find the value of m to 1 decimal place.

9

2017

Figure below shows a straight line graph drawn from a set of numbers which were obtained from an experiment. The numbers were found to be related in the form $h = dl^c$, where d and c are constants.



Use the graph to find the values of c and d.

8

Write the experimental equation connecting h and l.

2

2018

The graph of a linear function of $cy = ax^2 + x$ where a and c are constants, has the y-intercept at 0.5 and gradient of 2.3. Calculate the value of a and the value of c.

6

Table below shows results of x and y obtained in an experiment.

x	1.1	1.8	2.1	2.7	3.8
y	2.85	14.47	33.56	91.00	150.5

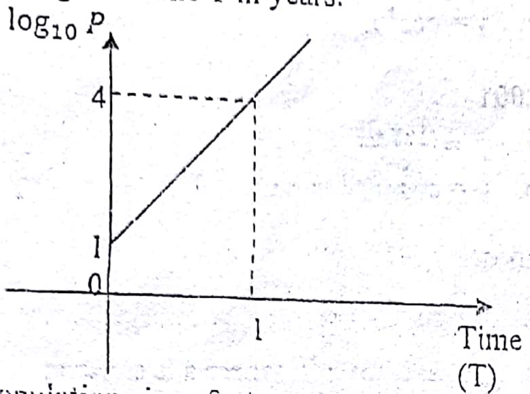
It is known that x and y are connected by the formula $y = kx^n$ where k and n are constants.

i. Draw a graph of $\log y$ against $\log x$.
ii. Use the graph to find the law connecting x and y.
iii. Estimate the value of x when $y = 50$.

15

Sample Paper

The figure below shows a straight line graph of $\log_{10} P$ against time T in years.



If a population size of a community at time T is given by $P = KB^T$ where K and B are constants, calculate the value of K and the value of B. 9

The table below shows data for the law $Y = A \times B^n$, where A and B are constants.

n	0	2	4	6	8
Y	5.00	11.25	25.31	56.96	128.14

i. Draw a graph of $\log_{10} Y$ against n.

ii. Use the graph to estimate the values of A and B. 16

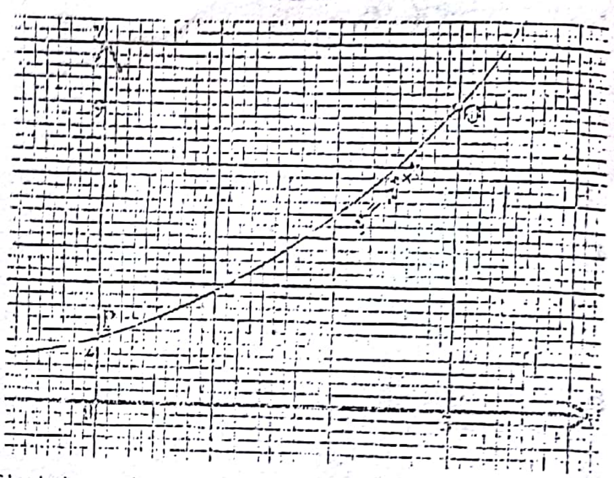
2019

The variables x and y are connected by the equation $y = rx^t$ where r and t are constants. When $\log y$ is plotted against $\log x$, a straight line that cuts the x axis at 2 and y axis at -1 is obtained. Calculate the:

(i) gradient of the line

(ii) value of r 7

The figure below shows a sketch of a graph of $y = a^x + b$ passing through points P and Q, where a and b are constants.



Find the values of a and b. 8

CALCULUS

Differentiation

1989

a. $y = (x^2+2)(\sqrt{(x-3)})$ 3

b. $y = 3e^{3x+2}$ 3

c. $y = \dfrac{\sin x}{1+\tan x}$ 3

Given that $y = x^2 \cos x$, x

a. find: i. $\dfrac{dy}{dx}$

ii. $\dfrac{d^2y}{dx^2}$ 4

b. prove that $x^2\dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} + (6 + x^2)y = 0.$ 5

1991

a. $y = x^3 \sin 2x$ 5

b. $y = (3x^2-2)^5$ 5

1992

a. $y = x \sin 2x$ 3

b. $y = \dfrac{3x}{x+4}$ 3

c. $y = (\cos(5x+2))^4$ 6

1996

a. $y = x^2 \cos x$

b. $y = (3x^2+1)^5$

c. $y = \dfrac{x}{2x+3}$ 8

1997

a. $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$ 2

b. $y = \dfrac{\cos x}{x}$ 3

1998

a. $y = (1-2x^3)^5$ 3

b. $y = \sin x \cos 2x$ 3

c. $y = x^3 \operatorname{cosec} 2x$ 4

1999

a. $y = \dfrac{1}{1+\cos x}$ 3

b. $y = x(\sqrt{x+1})$ 4

2000

a. $y = (x^2-3)(x^2+3)$ 3

b. $y = \dfrac{1-x^2}{x^3+3x^2}$ 4

c. $y = \cos^3 x$ 4

2001

a. $y = \dfrac{(x+1)^2}{x-1}$ 3

b. $y = \cos x(x^2+\sin x)$ 3

2002

a. $y = 8x^{3/2} - \dfrac{2}{\sqrt[3]{x}}$ 4

b. $y = a(1-\cos x)^{1/2}$ where a is constant. 3

c. $y = \dfrac{\sin x}{1+\cos x}$ 3

2003

a. $y = (x^2+2)(2x+3)^{1/2}$ 5

b. $y = \dfrac{\sin x}{1+\tan x}$ 5

2004

$y = \sqrt{x^2+1}$ 5

Show that $\dfrac{d}{dx}(\sin x \cos x) = 2\cos^2 x - 1.$ 5

2005

$y = x\cos(x^2+4)$ 4

2006

$y = \dfrac{\sin x}{2x+1}$ 4

2007

$f(x) = 3x(2x-1)^3$ 4

2008

Show that $\dfrac{d}{dx}\dfrac{x}{(2-x)^2} = \dfrac{x+2}{(2-x)^3}$ 8

2009

Given that $y = (x^2-7)^9$, find $\dfrac{d^2y}{dx^2}$. 6

2010

a. $(3x^3-5)\cos 4x$ 4

b. $(x-6)\sqrt{x-1}$ 6

2011

a. $(3x^2-1)\cos 3x$ 4

b. $(\sin(4x^2-5))^3$ 6

2013

Solve the equation $\frac{d}{dx}(x + \sin 2x) = 2$ for $0 < x < 2\pi$. 5

2014

$y = (x-1)^{5/3}(x+1)^{3/2}$, in its simplest form.
Given that $y = A\cos 2x + B\sin 2x$ where A and B are constants, show that $\frac{d^2y}{dx^2} + 4y = 0$. 6

2017

$y = \sin\sqrt[3]{x^4+1}$ 5

Sample Paper

Differentiate $(x+1)^3(2x-5)^2$ with respect to x. 7

2019:

Evaluate the $\lim_{x\to -2} 2x^3 + 3x^2 + 7$. 4

Integration

1989

a. $\int \sin(2\theta + \frac{\pi}{3})\,d\theta$ 3

b. $\int_0^9 \frac{dx}{2x+2}$ 3

c. $\int_1^4 e^{4-x}\,dx$ 3

1991

a. $\int_{-1}^{2}(3x+2)dx$ 3

b. $\int_0^{\frac{\pi}{3}}\cos(3x+\frac{\pi}{2})\,dx$ 5

1992

a. $\int_{-1}^{1}(3x+1)(3x+2)dx$ 5

b. $\int_0^{\frac{\pi}{3}}\sin(\frac{x}{2}+\frac{\pi}{3})dx$ 6

1996

a. $\int_1^2(x^2-2x+3)dx$. 3

b. $\int \cos(3x+5)dx$ 2

c. $\int_0^1 x^2(x+4)\,dx$ 4

Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ and hence evaluate $\int \sin^3\theta\,d\theta$. 10

1997

a. $\int_1^3(4+3x-x^3)dx$ 3

b. $\int_0^{\frac{\pi}{2}}(\sin 2x - \cos x)dx$ 4

c. $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 4\sin^2\theta\,d\theta$. 8

1998

a. $\int_0^{\frac{\pi}{4}}(1+\sin x)^2\,dx$ 6

b. $\int_1^2(3+\frac{1}{t^4})\,dt$ 5

By expressing $\sin^3 x$ in terms of sin x and sin 3x, show that $\int \sin^3 x\,dx = \frac{1}{3}\cos^3 x - \cos x + C$. (You may use the fact that $\cos 3x = 4\cos^3 x - 3\cos x$). 11

1999

a. $\int_0^{\frac{\pi}{4}}(\cos 2x - \sin 2x)dx$ 4

b. $\int_1^4\left(\frac{7x^3+3x}{\sqrt{x}}\right)dx$ 4

2000

a. $\int(1-\sqrt{x}+x^2)dx$ 2

b. $\int_{-2}^{1}\frac{dx}{2x^5}$

c. $\int_{\frac{\pi}{2}}^{2\pi}\sin x\,dx$ 6

2001

a. $\int_{-1}^{0}(2+3x)^6\,dx$ 5

b. $\int_0^{\frac{\pi}{2}}2\sin\left(2x-\frac{\pi}{2}\right)dx$ 5

c. $\int_0^{\frac{\pi}{4}}\frac{1+\sin\theta}{\cos^2\theta}\,d\theta$ 8

2002

Differentiate $\frac{\sin x}{1+\cos x}$ with x and hence evaluate

$\int_0^{\frac{\pi}{2}}\frac{dx}{1+\cos x}$ 6

$\int_4^9(\sqrt{x}+2)^2\,dx$ 3

2003

a. $\int_4^9\frac{1}{x\sqrt{x}}\,dx$

b. $\int_0^{\pi}\sin x\,(1+\cos x)^5\,dx$ 13

2004

a. $\int_1^2(x^4-\sqrt[3]{x^2})\,dx$ 6

b. $\int_0^{\frac{\pi}{3}}(2-2\sin t)\,dt.$ 5

Given that $\frac{d}{d\theta}(\theta\sin\theta)=\sin\theta+\theta\cos\theta$,

evaluate $\int_0^{\frac{\pi}{2}}\theta\cos\theta\,d\theta$. 7

2005

a. $\int(\cos 2x+5)\,dx.$ 4

b. $\int_0^1(4x^3-6x^2+1)\,dx.$ 3

2006

a. $\int(\sqrt[5]{x}\,dx.$ 4

b. $\int_0^{2\pi}\sin\left(\frac{x}{2}-\frac{3\pi}{4}\right)dx$ 6

2007

a. $\int_0^{\frac{\pi}{2}}(2\cos 2\theta+5\theta)d\theta$ 5

b. $\int\cos 2x\,(\sin 2x+3)^2\,dx$ 5

2008

Use the formula

$\sin\theta\cos\phi=\frac{1}{2}\sin(\theta+\phi)+\frac{1}{2}\sin(\theta-\phi)$ to

evaluate $\int_0^{\frac{\pi}{4}}\sin 3x\cos x\,dx.$ 9

2009

a. $\int x^3(2x^4-1)^5\,dx$ 5

b. $\int\frac{1-\cos 2x}{2}\,dx$ 3

$\int_0^{\frac{2\pi}{8}}(1+\sin 2x)^2\,dx$, leave your answer in terms π. 9

2010

$\int(\sin^2\frac{\theta}{2}+\cos\theta)\,d\theta.$ 7

2012

$\int\cos 2\theta\sqrt{5-\sin 2\theta}\,d\theta$ 5

2013

$\int\frac{\cos 2x}{(4+\sin 2x)^2}\,dx$ 6

2014

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{6}}\sin 3x\,dx$ 5

2015

A function $y=f(x)$ is such that $\frac{d^2y}{dx^2}=6x-2$.

When $x=2$, $y=11$ and $\frac{dy}{dx}=10$. Find $f(x)$.

8

2016

A curve passes through a point (0, 2). If the gradient of the curve is given by $\frac{dy}{dx}=2x-3$, find the equation of the curve. 6

2018

If $\sin 3\theta=3\sin\theta-4\sin^3\theta$, integrate $\sin^3\theta$,

6

Solve the equation $\int_0^{2x}\sin\frac{\theta}{2}\,d\theta=0$, for $0\le x\le\pi$, 6

Sample Paper

Integrate the following

a. $2\sin(t)+5t^3+20$ 5

b. $(x^3+7x-20)^6(3x^2+7)$ 6

Given that $\frac{d}{dx}\left\{\frac{x}{\sin(x)}\right\} = \frac{\sin(x)-x\cos(x)}{\sin^2(x)}$,

evaluate

$\int \sin(x)\frac{\left[1-x\frac{\cos(x)}{\sin(x)}\right]}{\sin^2(x0} dx.$ 4

Application of differentiation
Small increments

1989

The radius of a spherical balloon is r cm, and it decreases by small amount a δr.

a. Find an expression in terms of r and δr for the approximate change δv in the volume.

b. Find the decrease in volume when the radius decreases from 5 cm by 0.1 cm. Give your answer, correct to the nearest cm^3. 9

1997

A close right circular cylinder has height 12 cm and radius r cm. The total surface area is A cm^2.

Show that $\frac{dA}{dr} = 4\pi(r+6)$

Use the results in (a) to approximate the percentage increase in the area when the radius increases from 4 cm to 4.02 cm, the height remaining constant. 7

2000

The radius of a circle increases by 0.5 %. How much percentage change is made to the area of the circle? 5

2004

If $V = t^2 + 5$, approximate the change in V when t changes from 10 to 9.99, using calculus technique. 5

2006

The time T oscillation of a pendulum of length l metres is given by $T = cl^{1/2}$ where c is constant. Estimate the percentage change in T if l increases from 0.60 m to 0.62 m. 7

2008

A 4% error is detected after calculating surface area of a hemisphere due to wrong measurement of its radius. Find the percentage error in the radius. (Surface area of a hemisphere = $3\pi r^2$) 8

2009

The radius of a sphere is increased by 1%. Calculate the approximate percentage change in the volume of the sphere.
(Volume of a sphere = $\frac{4}{3}\pi r^3$). 7

2013

If 0.5 % error was made when measuring the radius of a spherical ball, calculate the percentage error in the volume of the ball.
(Volume of a sphere = $\frac{4\pi}{3} r^3$). 8

2014

If the area of a circle increases by 10%, calculate the approximate percentage change in the radius of the circle. 9

2016

A square has a side x cm. Find the increase in the area of the square when the side increases by 0.01 cm. 6

2017

Quantities x and y are related by the equation $y = 2x^2 - 3x + 1$. Calculate the increase in x when y increases from 3 to 3.015. 6

2018

Given that $A = \pi r^2$, calculate an approximate change in A when r increases from 3 to 3.01, leaving your answer in terms of π. 4

Rates of change

1999

Boyle's law for confined gases states that if the temperature is constant, $PV = C$, where P is the pressure, V is volume, and C is a constant. Initially, volume is 30 m^3 and the pressure is 20 N/m^2. The pressure is decreasing at the rate of 4 N/m^2 every second.

a. Calculate the value of the constant C.

b. At what rate is the volume changing initially? 8

The volume of water in a reservoir, V m^3 is given by the formula, $V=1000\,x^2$ where x is the depth of water. If water is drained out from the reservoir at a steady rate of $50m^3/s$. Calculate that rate at which the level is falling when the depth is 25 m. 6

2001

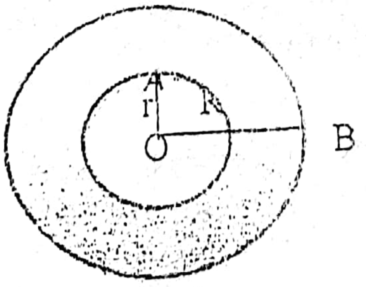
A spherical balloon is being blown up so that its ~~volume~~ increases at a constant rate of 2 cm^3/s. If the volume of the ball is 96 cm^3, find the rate of increase of the radius. 10

2003

Gas is escaping from a spherical balloon at the rate of 0.01 m^3/s. If the radius is 0.5 m, at what rate is it decreasing? Leave your answer in term of π. 6

2007

Figure below shows two concentric circles which are changing such that the radius OB of the larger circle R is three times that of the smaller circle r.



If r is increasing at the rate of 0.3 cm/s, find the rate at which that shaded area is increasing when R is 8 cm. Give your answer in terms of π. 10

2011

Water is emptying out of a tank with uniform cross section at the rate of 600 ml/s. If the cross section is a square base with side 45 cm, calculate:

a. the rate at which the height is falling in the tank,

b. How long will it take for the height to drop by 20 cm. 9

2012

If the volume of a sphere of radius 10 cm increases at the rate of $100\pi\ cm^3\ s^{-1}$, calculate the rate at which the radius of the sphere will change.

(Volume of a sphere $= \frac{4}{3}\pi r^3$) 7

2019

A metal of side x cm was heated and the length of each side increased at the rate of 0.02 cm/s. If the initial length of the edge of the cube was 7 cm, find the rate of increase of the volume of the cube. 6

Kinematics

2016

The distance, s metres, moved by a particle along a straight line OA is given by $s = 2t^3 - 2t^2 + 1$. Find the acceleration of the particle in terms of t. 4

2017

The equation of the motion of a ball thrown vertically upwards is $s = 22t - 5t^2$ where s is distance in metres and t is time in seconds. Calculate the maximum height of the ball. 7

Tangents, normals, minima, maxima, second derivatives graph area below curves, etc

1989

Find the minimum value of $4x^2 + \frac{27}{x}$ where x is a positive number. 9

Given the equation of a curve
$y = x^3 - 4x^2 + 5x + 2$,
a. Find the **minimum point** on the curve.
b. Find also the **equation of the tangent** at the point where $x = 2$.
c. Calculate the **area** enclosed by the curve, the two axes and the line $x = 1$. 18

Given the equation of a curve $y = x(x-2)(x-4)$,
a. sketch the curve.
b. show the tangent at the point A whose x-coordinate is 1 on your sketch,
c. find the **equation of the tangent** at A and calculate the point where it meets the curve again. 9

Given the equation of a curve $y = 4x + x^2$,
a. calculate the **points** where the curve meets the x-axis. 4
b. calculate the **area** enclosed by the curve and the x-axis. 6
c. calculate the **volume** formed when this area is rotated once about the x-axis. 8

1991

Given that $y = \frac{4x^3}{3} + \frac{7x^2}{2} + 3x + 17$, find the values of x at the **maximum** and **minimum** points and distinguish between them. 10

A man wishes to enclose a rectangular plot in his garden using 200 m length of wire, and a straight wall as one side of the enclosure. What is the **largest area** he can enclose? 10

1992

The gradient $\frac{dy}{dx}$ of a curve is $12x^2 - 3$ and the curve passes through (1, 3). Find the **equation of the curve**. 5

The curve $y = -x^2 + 4x - 3$ meets the x-axis at A and B. The tangents to this curve A and B meet at P. Find
a. the **equation of the tangents** at A and B, 10
b. the **area** of triangle **ABP**, 3
c. the **area** above the x-axis enclosed by the curve and the tangents. 7

A man decides to fence in a rectangular section of his garden. One edge is against the side of his house so no fence is needed there. Materials for the fence costs K5 per metre for the side opposite to the house and K4 per metre for the two ends. He spends K500 but wants as **large an area** fenced off as possible. What are the **dimensions** of the largest fenced area? 10

1996

The gradient of the tangent to a curve at any point (x, y) is given $6x^2 + 1$. The curve passes through the point
(1, 5). Find
i. The **equation of the curve**
ii. The **equation of the tangent** to the curve at the point where $x = 0$ 5

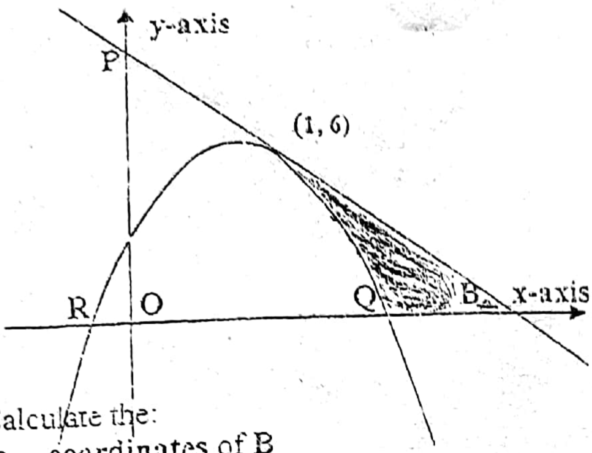
Let $f(x) = x^3 - 3x^2 - x + 3$,
a. solve the equation $f(x) = 0$. 3
b. Find which turning points are the **maxima** and which **minima**, giving the x-values only. 5
c. Sketch the graph of $y = f(x)$. 6
d. find the **finite area** enclosed by the graph and the x-axis. 6

A cylindrical tin, without a top, is to have a volume of 27π **cm^3**. Find the **radius** and the **height** of the tin if its surface area is to be **minimum**. 10

A curve is given by the equation $y = \frac{2x-1}{x-1}$.
Find
i. The **equation of the tangent** to the curve at the point where $x = 2$.
ii. The **equation of the normal** to the curve at the point where $x = 2$
iii. Also the **point** where the normal cuts the curve again. 10

1997

Figure below shows a tangent AB drawn to the curve $y = 6 + x - x^2$ touching the curve at A and cutting the x-axis at B.



Calculate the:

a. coordinates of B
b. coordinates of R and Q,
c. the area of the shaded region. 13

1998

A farmer has 200 m of fencing wire with which to form a rectangular enclosure. What dimensions will give him the maximum possible enclosed area? 4

Find the turning points of the curve $y = 2x^3 + 3x^2 - 12x + 7$ and classify them. 3

Find the equation of the normal to the curve $y = x^2(x-3)$ at the point where it cuts the x-axis. 9

A straight line through the origin cuts the curve $y = 4 - x^2$ at the point $x = 3$.

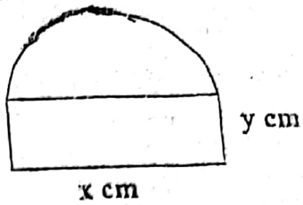
a. Find the equation of the line.
b. Sketch the curve and the line.
c. Calculate the area bounded by the curve and the straight line. 15

1999

Find the turning points on the curve $y = x^3 + x^2 - x + 1$ and classify them. 5

Calculate the area enclosed by the curve and the straight line. 4

A window has the shape of a rectangle surrounded by a semi-circle as shown in the figure below. The perimeter of the window is 40 cm.



- Show that the area of the window is $20x - (\frac{\pi+4}{8})x^2$ cm^2.
- Show that this area has a maximum and find it. 12

Show that the curves $y^2 = 4x$ and $x^2 = 4y$ meet at (4, 4). Sketch the graphs of the two curves and calculate the area enclosed by them. 10

A curve is given by the equation $y = x^3 - 3x^2 + 2x$.

- Find the equation of the tangent to the curve at the point where $x = 3$
- Sketch the curve.
- Find the ordinate of the minimum point.
- Find the area bounded by the curve and x-axis. 16

A solid right circular cylinder has to have a volume of 16π. Calculate its minimum surface area. 9

2001

Given that $y = x + \cos 2x$, find the coordinates of the stationary points for $0 \leq x \leq \pi$. 10

2002:

A cylindrical container closed at one end, has height h cm and base radius r cm. Write down in terms h and r the expressions for:

- the total surface area of the container S cm^2
- the volume of the container V cm^3.
- Hence find the value of r and the corresponding value h which make V maximum. 12

2003:
An open rectangular tank of height h metres with a square base of side x m is to be constructed with a capacity of 500 m^3.

- Show that the surface area of the four walls and the base is $(\frac{2000}{x} + x^2)\ m^2$.
- Find the value of x for which this area is a minimum and hence calculate the minimum area of the rectangular box.

8

The equation of the curve is $y = px + \frac{q}{x}$, where p and q are the constants. Given that the curve passes through the points A(1, 11) and B(4, $21\frac{1}{2}$)

- Evaluate p and q.
- Find the equation of the tangent to the curve at the point where x = 2.
- Show that this tangent is parallel to AB.

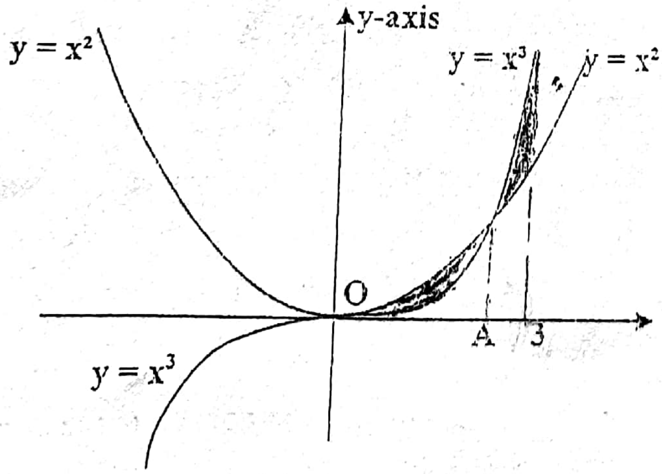
14

Find the point on the curve $y = 2 - 3x^2$ at which the tangent is parallel to the line $y = 2x$.
Given $y = 2x^3 - 3x^2 - 36x$ is the equation of a curve, find

- The turning points and state whether maximum or minimum. 10
- Sketch the curve; show the turning points and where the curve cuts the x-axis.

10

2004
Figure shows two curves $y = x^2$ and $y = x^3$ intersecting at x = 0 and at point x = A.



i. Find the value of A.
ii. Calculate the area of the shaded region between x = 0 and x = 3. 9

Given that $y = \frac{2}{x^2}$ and $T = x - 2y$, find the value of x if T is the maximum and hence find the maximum value of T 9

2005

a. The curves for which $\frac{dy}{dx} = 3x^2 + bx - 9$ where b is a constant has the turning point (-3, 32). Calculate the value of b. 4

b. A function f(x) passes through the point (0, 1). Given that its derivative is $2x^2 + 3x + 5$, find the function. 6

2006
Let $y = \frac{x^4}{4} + x^3 + x^2$.

a. Find $\frac{dy}{dx}$.
2

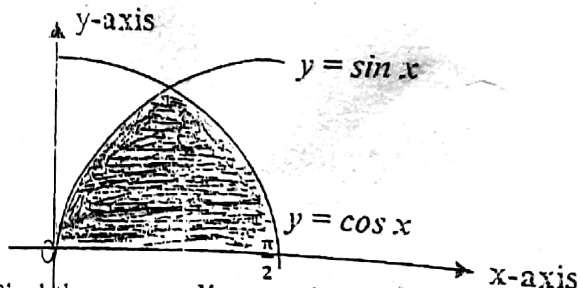
b. Find the values of x where the function $y = \frac{x^4}{4} + x^3 + x^2$.has the turning points and determine whether maximum or minimum.
13

c. Sketch the graph of $y = \frac{x^4}{4} + x^3 + x^2$.
5

2007
Given that $f(x) = \sin^2 x - \sin x + 5$ for $0^0 \leq x \leq 90^0$, find by completing the square,

i. the minimum value of f(x).
ii. the value of x for which f(x) is minimum. 7

2007
Figure shows part of a graph of $y = \sin x$ and $y = \cos x$.

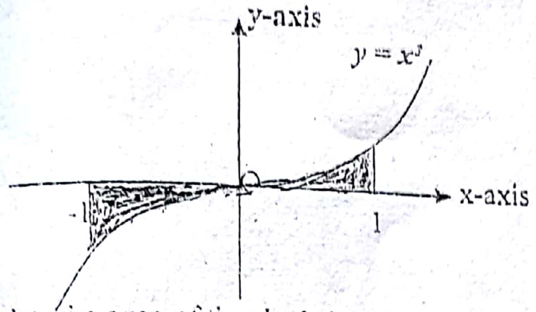


a. Find the x-coordinate of the intersection point of the two curves.
6

b. Calculate the area of the shaded part.
6

2008

Figure below shows a graph of $y = x^3$.



Calculate the area of the shaded region. 5

2009

By completing the square, find the value of θ on the turning point for $\cos^2\theta - \cos\theta - 6 = 0$ for the $0° \leq \theta \leq 90°$. 5

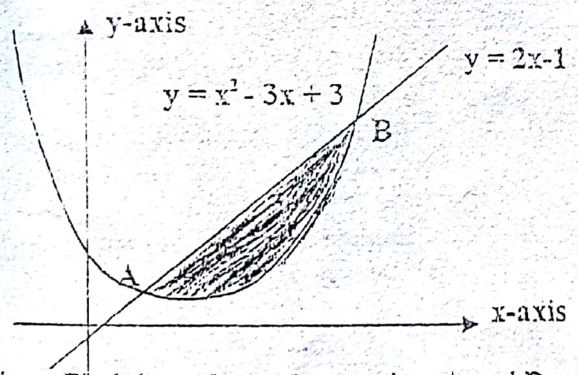
The normal to a curve $y = 3 - x^2$ at the point A (1, 2) meets the curve again at B. Find:

- The equation of the normal at A.
- The coordinates of B. 11

2010

Given that $y = x + 2\cos x$ for $0 \leq x \leq 2\pi$, calculate the coordinates of the stationary points. 10

Figure below shows the curves $y = x^2 - 3x + 3$ and the line $y = 2x - 1$ intersecting at point A and B.



i. Find the values of x at points A and B.

ii. Calculate the area bounded by the curve and the line. 12

2011

Find the equation of the tangents to the curve $y = \frac{5}{1+x^2}$ at $x = 2$, in the form $y = mx + c$. 9

An open cuboid box is formed by cutting squares of side x cm from each of the four corners of a 36 cm by 36 cm cardboard. The remaining flaps are folded to make the vertical sides.

- Write down the **volume** of the cuboid in terms of x.
- Find the value of x that corresponds to the maximum volume of the cuboid.
- Calculate the **maximum volume.** 16

2012

Find the coordinates of two points on the curve $y = x(x^2 - 9)$ at which the tangent is parallel to the line $y = 3x + 5$. 8

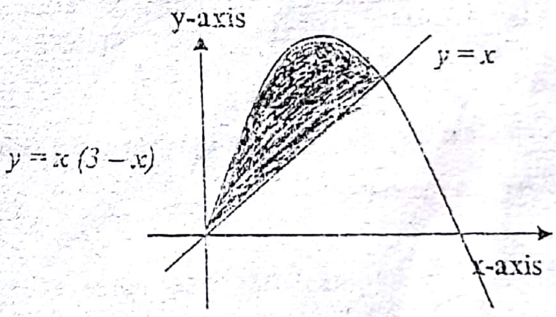
The line $y = x + 3$ intersects the curve $y = x^2 - 4x + 3$ at A and B.

- Find the coordinates of points A and B.
- Sketch the curve $y = x^2 - 4x + 3$ and calculate the area of the region bounded by the curve and the line $y = x + 3$. 13

2013

A tangent to the curve $y = x^3 - px + q$ at the point (2, -8) is parallel the x-axis. Find the values of p and q. 8

Figure below shows graphs of $y = x(3 - x)$ and $y = x$.



Find the area of the shaded part. 7

2014

Find the equation of the normal to the curve $y = 3x^2 + \frac{1}{x^3}$ at the point P (1, 4). 7

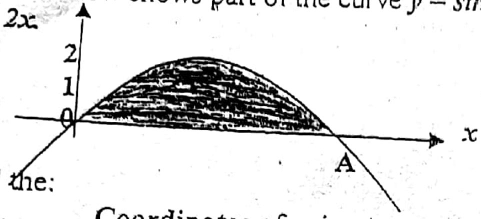
Find the coordinates of the stationary point on the curve $y = \frac{2-3x}{(2x-1)^2}$. 9

2015

Given that $\theta = \frac{400 - x - x^2}{20}$, find the rate of change of θ when $x = 5$. 5

- Find the smallest value of x for which the curve $y = 2x - 3\sin x$ has a gradient of $\frac{1}{2}$. 6

Figure below shows part of the curve $y = \sin x + \sin 2x$



Find the:

i. Coordinates of point A

ii. Area of the shaded region.

10

A tangent is drawn at the point $x = 3$ on the curve $y = \frac{6}{x}$. Calculate the coordinates of the point where the tangent meets the x-axis.

13

Given that $f(x) = \cos 2x - \sin x$ for $180° \leq x \leq 270°$, find by **completing the square**, the coordinates of the **turning point** of $f(x)$.

12

2016

Find the coordinates of the points on the curve $y = 3x^3 + 2x^2 + x - 1$ at which the normal has the equation $y = 2 - x$. 10

Calculate the **gradient of a tangent** at the point $\theta = \frac{\pi}{3}$ on the curve $y = \frac{\sin 2\theta}{\cos 3\theta}$. 7

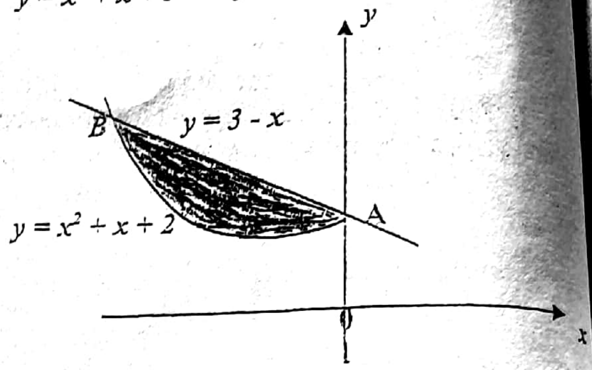
- Find the turning points on the curve $y = 2x^3 - 9x^2 + 12x - 4$.
- Sketch the curve $y = 2x^3 - 9x^2 + 12x - 4$.

14

2017

Find the coordinates of the points on the curve $y = x^3 - \frac{3x^2}{2} - 36x + 12$ at which the tangent is parallel to the x – axis. 10

Figure below shows part of the sketch of the graphs of $y = x^2 + x + 3$ and $y = 3 - x$.



i. Calculate the coordinates of A and B.

ii. Find the area of the shaded region.

10

2018

Given the curve for $y = 3x - 2\cos x$ and the line $y = 4x$, calculate the coordinates of a point on the curve where the tangent is parallel to the given line for $0 \leq x \leq \pi$, giving your answer in terms of π. 8

Given the equation of the function $y = 2x^3 + 3x^2 - 36x + 4$,

i. calculate the coordinates of the stationary points and distinguish between them.

ii. sketch the curve. 14

Sample Paper

A manufacturer wishes to make cylindrical cans with a capacity of $500\ cm^3$ using the smallest quantity of metal possible.

i. Find the expression for the total surface area of a cylindrical can in terms of its radius ***r cm*** only.

ii. Calculate the values of r and the height of the can, h.

10

2019

Find the equation of the tangent to the curve $y = 3x^2 - 2x - 1$ at the point (- 2, 15) 6

The gradient of a curve is $3x^2 - \frac{1}{x^2}$. If the curve passes through the point (- 1, 0), find the equation of the curve.

6

The normal at the point (- 1, 2) on the curve $y = 3 - x^2$ meets the curve at B. Find the coordinates of **B**. 11

Given that $f(x) = x^3 - x^2 - 5x + 6$,

(i) Find the turning points of $f(x)$

(ii) Determine the nature of the turning points of $f(x)$ 12

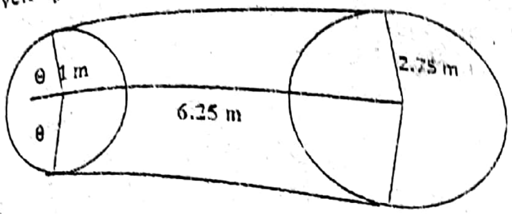
Compiled by K.A. ADINI

TRIGONOMETRY

Radian measure

1991

Figure below shows two wheels of radii **2.75 m** and **1 m** with centres **6.25 m** apart which are driven by continuous belt.

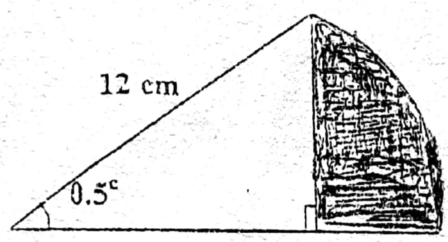


Assuming that the belt is tight, find its total length. (Hint: Find **θ** in **radians** and the **length** of the common tangents). 20

1992

The total **perimeter** of a sector of a circle is 12 cm, and its area is **8 cm^2**. Find **all** possible values of the **radius** of the sector and the corresponding **angles** of the centre. 10

1997



The above figure shows a sector AOB of a circle centre O of radius **12 cm**. **AP** is perpendicular to OB, OP is **10.5 cm** and OAP is a right angled triangle in which angle **AOP** is **0.5 radians**. Calculate

a. **AP** 2

b. the **area of the sector OAB**, 2

c. the **area of the shaded part APB**. 2

1998

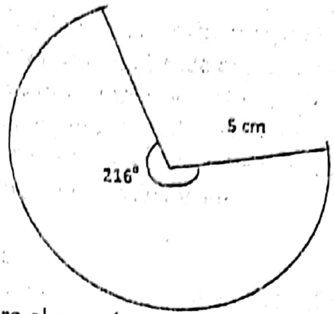


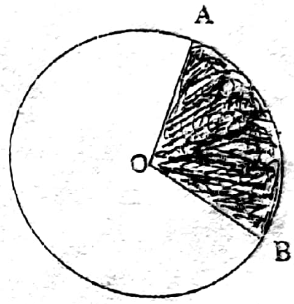
Figure above shows a sector of radius **5 cm** and has an angle of **216^0**. If the sector is bent to form a cone, find

a. the **radius** of the base of the cone.

b. the **vertical angle** of the cone. 5

1999

In figure below, O is the centre of the circle. The area of the sector AOB is directly proportional to the angle AOB, where the angle is measured in degrees.



a. Find the value of **the proportionality constant** and hence express the area of the sector as function of the angle AOB.

b. Using the formula derived in a. (i), find the **area of the sector** when the radius is **10 cm** and angle AOB is **60°** 8

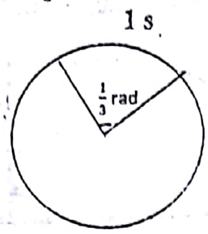
2001

The area of a sector of a circle of diameter **10.62 cm** is **29.2 cm^2**. Calculate the **angle**, in **degrees**, subtended at the centre of the circle. 5

An elastic belt is placed round the rim of a pulley of radius of **5 cm**. One point of the belt is pulled directly away from the centre, **O**, of the pulley until it is at A, **10 cm** from O, find the **length of the belt** that is in contact with the rim of the pulley. 7

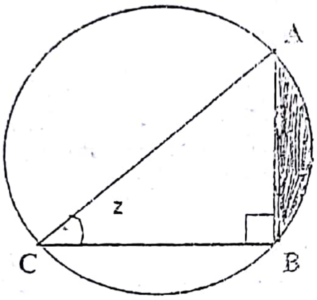
2002

Figure below shows student riding a bicycle along a circular path radius 15 m at constant angular speed of $\frac{1}{3}$ radians per second.



Determine the distance, in metres, that the student will travel in 1 minute. 3

Figure below shows a circle touching the corners of a right-angled triangle.

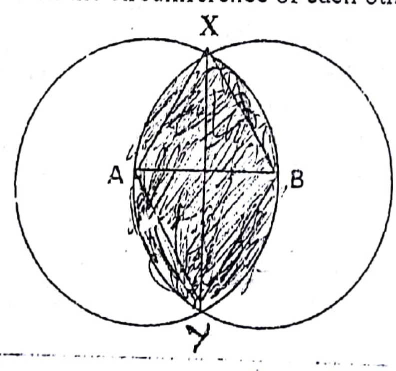


Given that $\tan(z) = \sqrt{3}$, AB is 1 m long and that AC is a diameter, calculate the area of the shaded region without using the area of triangle ABC. Leave your answer in terms of π. (Hint; use radians. Label the centre of the circle O and join OB) 13

2004

In a circle of radius 20 cm, a chord 12 cm long is drawn. Calculate, in radians, the angle the chord subtends at the centre of the circle. 6

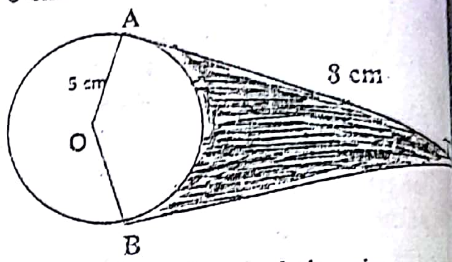
Figure below shows two equal circles intersecting at X and Y such that their centres A and B are on the circumference of each other.



If the radius of the circle is 6 cm, calculate the area of overlap. 11

2005

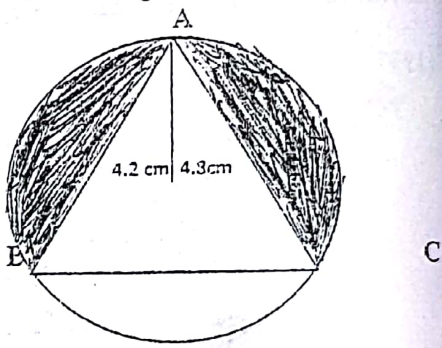
Figure below is a circle with centre O and radius 5 cm. AT and BT are tangents to the circle at the points A and B respectively, such that AT = 8 cm.



Calculate the area of the shaded region. 9

2007

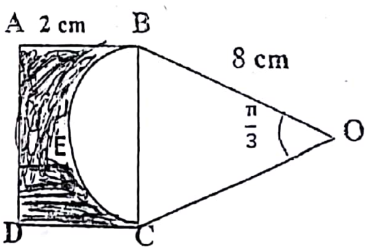
In figure, ABC is a circle with center O, angle ACB = 60^0 and angle ABC = 40^0.



If OA = 2.5 cm, AB = 4.2 cm and AC = 4.8 cm calculate the total area of the shaded segments. 11

2003

In figure below, ABCD is a rectangle, BEC is an arc of radius 8 cm with centre O, angle BOC is $\frac{\pi}{3}$ radians and AB = 2 cm.

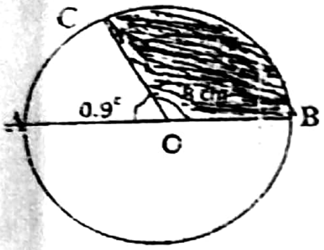


Calculate the perimeter of the shaded part. 7

2009
A piece of wire 40 cm long is bent into the shape of a sector of radius r cm and area 100 cm^2. Find

a. an expression for the sector angle θ in terms of radius r.
b. the value of r and θ. 11

2011
Figure shows a circle centre O of radius 8 cm. Angle AOC = 0.9 radians.

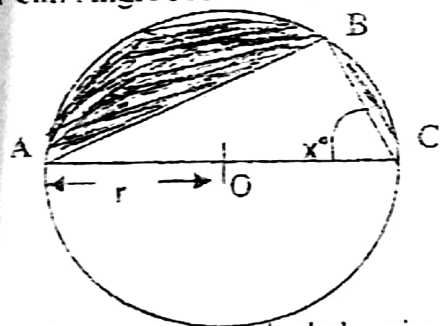


Find the

a. length of arc of the minor segment BC.
b. area of the shaded part. 7

2012
A sector of a circle is formed from a piece of wire 10 cm long. If r is the radius of the circle, show that the area of the sector is $r(5 - r)$ cm^2.
5

2013
Figure below shows a semicircle centre O, of radius r cm. Angle ACB = x°.

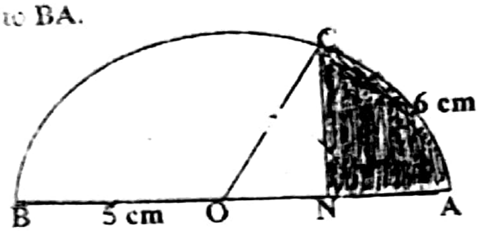


Show that the area of the shaded region is $r^2\left(\frac{\pi}{2} - \sin 2x\right)$ cm^2. 8

2015
The area of a sector of radius 12 cm is y cm^2. Find the length of an arc of the sector in terms of y. 7

2016
A sector of a circle has an area of 20 cm^2. If th angle of the sector is x radians, calculate the radius of the circle in terms of x. 6

2017
Figure below shows a semi-circle BCA with centre O and radius 5 cm. CN is perpendicular to BA.



Given that the length of the arc AC is 6 cm, calculate:

i. angle AOC in radians
ii. the area of the shaded region.

10

2018
The perimeter of a sector of a circle is *48.96* cm. If the arc of the sector of the circle subtends an angle of *2.08* radians at the centre of the circle, calculate the radius of the circle. 5

Sample Paper
A piece of wire *40 cm* long is bent into the shape of a sector. If the area of the sector is *100 cm^2*. calculate the;

i. radius of the sector
ii. angle of the sector 10

2019
The area of a sector of a circle is 300 cm^2. If the arc of the circle subtends an angle of 5.16 radians at the centre of the circle, calculate the radius of the circle. 5

Trigonometric identities

1989

Given that A is an obtuse angle with $\sin A = \frac{4}{5}$, and B is an acute angle with $\cos B = \frac{5}{13}$, calculate, leaving tour answer as a fraction

a. sin 2A

b. cos 2B 4

c. sin(A+B) 3

2

1992

$2\sin^2 \theta = 1 - \cos\theta$, giving solutions for $0° \le \theta \le 360°$.

7

A and B are angles of a triangle in which $\sin A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$. Without using tables and calculator, find the value of $\operatorname{cosec}(A+B)$.

10

1996

$\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta$

8

If A and B are acute angles such that $\sin A = \alpha$ and $\cos B = \beta$. Find in terms of α and β

a. sin B

b. cos A

c. sin 2B

d. tan (A+B) 10

1998

If $\cos 2x = 2\cos^2 x - 1$, show that

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$. 4

1999

$\frac{\cos 2x - \cos x}{\sin x + \sin 2x} \equiv \frac{\cos x - 1}{\sin x}$. For what values of x, $0^0 \le x \le 360^0$, is the identity above not valid?

6

2000

$$\frac{1+\cos 2\theta}{\sin 2\theta} \equiv \frac{\sin 2\theta}{1-\cos 2\theta}$$ 4

2001

Express $4\sin^2\theta + 1$ in terms of $\cos 2\theta$.

5

2002

Given that $\sin\theta = \cos\theta$, simplify $\sin 2\theta$ to an expression without θ. 3

2005

$\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta$. 8

2006

$\frac{1+\cot^2 x}{1+\tan^2 x} \equiv \cot^2 x$. 5

2007

$\operatorname{cosec}^2 A \equiv \frac{\sec A}{\sec A - \cos A}$ 6

2008

Given that $\sin\theta = 0.800$, $\tan\theta = 1.330$, $\cos\emptyset = 0.530$ and $\tan\emptyset = 2.400$, use trigonometric identities to calculate $\sin(\theta - \emptyset)$ leaving your answer to 2 decimal place.

10

2009

$\frac{1}{\sqrt{a^2\operatorname{cosec}^2\theta - a^2}} \equiv \frac{1}{a}\tan\theta$ 5

2010

$\frac{\sin 4\theta}{\sin\theta} \equiv 8\cos^3\theta - 4\cos\theta$. 6

$\frac{\sin 2\theta}{1+\cos 2\theta} \equiv \tan\theta$ 8

2011

$\sec x + \operatorname{cosec} x \cot x \equiv \sec x \operatorname{cosec}^2 x$. 8

$\cos^4 y - \sin^4 y \equiv \cos 2y$. 4

2012

$\frac{1-\cos 2\theta}{\sin 2\theta} \equiv \tan\theta$ 7

2013

$\sin 3x \equiv 3\sin x - 4\sin^3 x$. 7

2014

$\frac{1+\cos 2B}{1-\cos 2B} \equiv \cot^2 B$. 6

2015

If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, evaluate $\tan(A+B)$. 6

Prove the identity $(1 - \sin^2 x)(1 + \tan^2 x) \equiv 1$. 5

2016

Prove the identity $\frac{1-5\sin x}{\cos x} \equiv \sec x - 5\tan x$. 4

2017

Given that $\sin(A-B) = \cos(A+B)$, show that $\tan A = 1$. 8

Trigonometric equations

1989

Solve the following equations for the values of x between 0 and 2π, giving your answer in terms of π

a. $\cot x - 1 = 0$ 3

b. $\cos x + \sqrt{3}\sin x = 0$. 3

c. $\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4} = 1$ 3

1991

$5\sec^2 x = 2\sec x + 3$ for $0^\circ \leq x \leq 180^\circ$. 7

1996

a. $1 + \sin x = \cos^2 x$, for $0^\circ \leq x \leq 360^\circ$. 4

b. $2\sin^2\theta - \sin\theta - 1 = 0$, $0^\circ \leq \theta \leq 360^\circ$,

c. $\tan^5 3\theta - 1 = 0$, $0^\circ \leq \theta \leq 360^\circ$. 10

1997

$2\tan^2 x - 3\tan x + 1 = 0$, $0^\circ \leq x \leq 360^\circ$. 4

1998

a. $\cos 2x + 3\sin x = 2$ 5

b. $2\tan^2 x + \sec x = 1$, $0^\circ \leq x \leq 360^\circ$. 5

1999

a. $\tan^4 x + 7 = 4\sec^2 x$. 5

b. $\sin 2x + \sin x = 0$, $0^\circ \leq x \leq 180^\circ$. 4

2000

a. $\sin x \cos x = 0$ 4

b. $5\sec x + 1 = \tan^2 x + 2\sec^2 x$, for $0 \leq x \leq 2\pi$. 6

2001

a. $\cos\frac{\theta}{2} = -\frac{\sqrt{2}}{2}$ 4

b. $\sin 2\theta = \cos\theta$, for $-360^\circ \leq \theta \leq 360^\circ$. 6

Simplify $\frac{1}{1-\sin\theta} + \frac{1}{\sin\theta+1}$, hence solve the equation $\frac{1}{1-\sin\theta} + \frac{1}{\sin\theta+1} = 4$, for values of $0 \leq \theta \leq 2\pi$. 7

2002

If $2\cos x \sin x + \cos x = 2\sin x + 1$, find all values of x (*general solution*) and leave your answer in degrees. 10

If $\cos\theta = 0.3$, find $\tan^2\theta$. 3

2003

a. $\sin(2x - 40)^\circ = 0.5$. 5

b. $\sin x = \cos 2x$, for $0^\circ \leq x \leq 90^\circ$. 6

2004

$6\cos^2 B - \sin B = 5$ for the values of B between 90° and 270°. 6

2005

$3\sec^2\theta - 5\tan\theta - 5 = 0$, for $0^\circ \leq \theta \leq 180^\circ$. 7

2006

Given that $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $\cos\frac{\pi}{3} = \frac{1}{2}$, without using a calculator or four figure table, evaluate $\frac{\operatorname{cosec}\frac{5\pi}{3}}{\sec\frac{5\pi}{3}}$. 7

$2\cot^2\theta + 8 = \frac{7}{\sin\theta}$, for $0^\circ \leq \theta \leq 360^\circ$. 8

2008

$3\cos^2 x - 2\sin x - 2 = 0$, for $0^\circ \leq \theta \leq 180^\circ$. 7

2009

$\sqrt{3}\sin\theta = -\cos\theta$ for $0^\circ \leq \theta \leq 360^\circ$. 5

$\cos 4x + \cos 6x + \cos 2x = 0$, for $0^\circ \leq x \leq 90^\circ$. 9

2014

$\sec^2\theta = 4 + 2\tan\theta$, for $0^\circ \leq \theta \leq 360^\circ$. 10

2015

$\cos^2 x = 1 + \sin x$, for $0° \leq x \leq 180°$. 7

2016

Given that $\sin x = \frac{-9}{41}$, $\cos^2\left(\frac{x}{2}\right) = \frac{1+\cos x}{2}$ and x is between 270° and 360° calculate the value of $\cos\left(\frac{x}{2}\right)$. 8

2018

Given that $\sin\frac{\theta}{2} = \frac{5}{13}$ and $0° \leq \theta \leq 90°$, calculate the value $\sin \theta$. 7

2019

If $\cos x = -\frac{2}{3}$, evaluate $\cos 2x$. 5

Solve the equation $3\tan^2 x + 2\sec x = 5$ for $0 \leq x \leq \pi$, where x is in radians. 9

Graphs of trigonometric functions

1989

a. Using the same scale and axes, draw the graphs of $y = 3\cos 2x$ and $y = \sin x$ for values of x from 0° to 180°. Take a scale of 2 cm to represent 20° on the x-axis and 2 cm to represent 1 unit on the y-axis. 9

b. For values of x between 0° and 180°, use your graph to find:
- the range of values of x for which $3\cos 2x < -1$
- the solutions of the equation $3\cos 2x - \sin x = 0$
- the solutions of the equation $3\cos 2x - \sin x = 2$. 9

1991

Draw the graph of $y = \sin(2\theta + 40°)$ for $0° \leq \theta \leq 180°$, using a table of values with intervals for θ of 15°. Use the graph to find the values of θ for which

a. $y = \pm 0.6$
b. $y > 0$. 20

1992

Sketch on the same axes the curves:

a. $y = 2\sin x$
b. $y = \tan x$
c. $y = 2 - \frac{4x}{\pi}$, for values of x between 0 and $\frac{\pi}{2}$

On your sketch shade the area satisfied by all the inequalities: $y < \sin x$, $y < \tan x$ and $y > 2 - \frac{4x}{\pi}$. 10

1997

a. Taking the scale of 2 cm to represent 15° on the x- axis and 2 cm to represent 1 unit on the y-axis, draw, on that same axes, the graphs of: $y = 2\sin 2x$ and $y = 2\cos 3x$, plotting the values for which x=0°, 15°, 30°, 45°, 60°, 75°, 90°, 105°, 120°, 135°, 150°, 165°, 180°. 12

b. Using your graphs, estimate:
   i. the solutions to the equation $2\sin 2x = 2\cos 3x$; in the range from 0° to 180°,
   ii. the range of values of x between 0° and 180° for which $\sin 2x \leq \cos 3x$. 8

1998

Given that $f: x \to \sin\frac{3x}{2}$ and $g: x \to -2\cos x$.

a. Find $f(-\pi)$
b. Taking a scale of 2 cm to represent 20° on the horizontal axis and 2 cm to represent 1 unit on the vertical axis, draw the graphs of f(x) and g(x) on the same axes for $0° \leq x \leq 180°$. 12
c. Use your graph to;
   - determine the values of x for which $f(x) > 0$
   - solve $f(x) = g(x)$
   - find the values of x for which $g(x) - \frac{3}{2} < 0$. 6

1999

a. Using the same axes, draw graphs of $y = 1 + \cos x$ and $y = \sin (x - 60)^\circ$ for $0^\circ \leq x \leq 360^\circ$. On the same x-axis take the values of x at 30° interval. 14

b. Use the graphs to;

i. solve the equation $\sin(x - 60)^\circ - \cos x = 1$

ii. estimate the range of values of x for which $1 + \cos x > \sin (x - 60)^\circ$. 6

2000

a. Using the same axes, draw the following graphs for the range of values of θ between 0° and 90° inclusive:

i. $y = \cos^2 2\theta$

ii. $y = \tan 3\theta$. 13

b. From the graphs, estimate the values of θ for which:

i. $\tan 3\theta = \cos^2 2\theta$

ii. $\tan 3\theta \div \cos^2 2\theta > 1$. 7

2001

a. Use appropriate scale to draw the graphs of $y = \cos 2\theta + 2\sin \frac{\theta}{2}$ for values of θ in the range $0^\circ \leq \theta \leq 180^\circ$. 11

b. Use your graph to solve the equations:

i. $2 \sin \frac{\theta}{2} \div \cos 2\theta = 1.5$

ii. $\frac{1}{4} - \frac{1}{2} \cos 2\theta = \sin \frac{\theta}{2}$. 9

x	-90°	-60°	-45°	-30°	0°	30°	45°	60°	90°
y	0		≈ 0.7		1		≈ 0.7		0

2002

Draw the graph of $y = \cos x$ and $y = x$ on the same axes and scale for $0^\circ \leq x \leq 90^\circ$. 6

Approximate the solution of $\cos x = x$ to 1 decimal place. 1

Copy and complete the table below and draw the graph of $y = \cos x$ for x between -90° and 90°. (Use 1 decimal place) 7

2003

a. Using a suitable scale, draw the graph of $y = 2 \sin 2\theta$ for $0^\circ \leq \theta \leq 90^\circ$. 4

b. Using the graph, find the value of θ when $y = 2$. 1

2004

The table below contains information for the function $y = 2 \sin (x - 30)^\circ$

X	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	210	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Y	[illegible]		[illegible]		[illegible]	1.7	[illegible]	0	[illegible]	-1.7		-1.7	[illegible]

a. Calculate the missing values in the table. 4

b. Using a scale of 2 cm to represent 1 unit on the vertical axis, and 2 cm to represent 60° on the horizontal axis, plot a graph of $y = 2 \sin (x - 30)^\circ$. 6

c. Use your graph to

- find the range of values of x in which $2 \sin (x - 30)^\circ > 0$. 2
- solve the equation $\sin (x - 30)^\circ + \frac{1}{4} = 0$. 4

2005

a. Using a scale of 2 cm to represent 20° on the x-axis and 4 cm to represent 1 unit on the y-axis, draw on the same axes, graphs of $y = 2\cos 2x$ and $y = 2\sin x - 2$ for values of $x = 40^\circ, 60^\circ, 80^\circ, 100^\circ, 120^\circ$ and 140°. 11

b. Using your graph, find

- the solution of the equation $\cos 2x = -\frac{1}{2}$
- the values of x for which $\cos 2x + 1 \leq \sin x$. 9

2006

- Using the scale of 2 cm to represent 90° on the horizontal axis and 4 cm to represent 1 unit on the vertical axis, draw on the same axes, the graphs $y = 1 + \sin \theta$ and $y = |\sin \theta|$. 10
- Use the graph to solve the equation $1 + \sin \theta = |\sin \theta|$. 2

Q7

2007

Using an interval of 30°, draw on the same axes the graphs of: $y = \frac{1}{2}\sin\theta$ and $y = \cos 3\theta$ for $0° \le \theta \le 180°$.

- from the graphs, find the values of θ when $\frac{2\cos 3\theta}{\sin\theta} = 1$. 15

2008

- Using a scale of 2 cm to represent 30° on the x-axis and 2 cm to represent 1 unit on the y-axis, draw on the same axes, the graphs of $y = |\cos x|$ and $y = \sin 2x$ for $0° \le x \le 270°$.
- From the graphs, find the values of x for which $\sin 2x > |\cos x|$. 13

2009

a. Using a scale of 2 cm to represent 15° on the horizontal axis and 2 cm to represent 1 unit on the vertical axis, on the same axes, draw the graphs of $y = 2\sin 2\theta$ and $y = \tan\theta$.

b. Use your graph to solve the inequality $\sin 2\theta \le \frac{1}{2}\tan\theta$. 13

2010

a. Using a scale of 2 cm to represent 30° on the horizontal axis and 2 cm to represent 1 unit on the vertical axis, draw on the same axes, the graphs of $y = 1 + \cos x$ and $y = \sin(x - 60)°$ for $0° \le x \le 360°$.

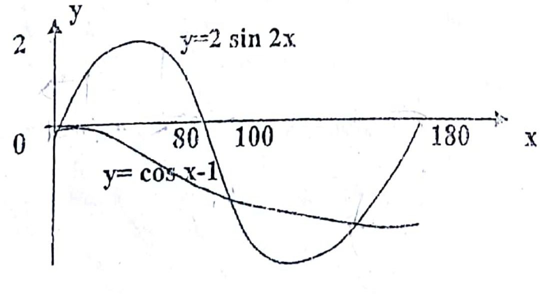
b. Using your graph, find the values of x for which $\sin(x-60)° - \cos x - 1 = 0$ 14

2011

Figure below shows graph of $y = 2\sin 2x$ and $y = \cos x - 1$. From the graphs, state the values of x for which;

a. $2\sin 2x > 1$ 3

b. $\frac{2\sin 2x}{\cos x - 1} = 1$ 4



2012

Using a scale of 2 cm to represent 30° on the horizontal axis and 4 cm to represent 1 unit on the vertical axis, on the same axes, draw the graphs of $y = \cos 3x$ and $y = \frac{1}{2}\sin x$, $0° \le x \le 180°$

Use your graphs to solve the inequality $2\cos 3x > \sin x$. 13

2013

Using a scale of 2 cm to represent 30° on the horizontal axis and 4 cm to represent 1 unit on the vertical axis, draw on the same axes, graphs of $y = \sin 2x$ and $y = \sin x$ for $0° \le x \le 270°$.

Use your graph to solve the inequality $\sin x \ge \sin 2x$ 12

2014

Using a scale of 2 cm to represent 30° on the horizontal axis and 4 cm to represent 1 unit on the vertical axis, draw on the same axes, the graphs of $y = |2\sin x|$ and $y = \left|\frac{x}{\pi} - 1\right|$ for $0° \le x \le 270°$.

Use your graphs to solve the equation $|2\pi \sin x| = |x - \pi|$. 14

2016

Using a scale of 2 cm to represent 60° on the horizontal axis and 4 cm to represent 1 unit on the vertical axis, draw on the same axes, the graphs of $y = 2\sin A$ and $y = \tan A$ for $0° \le A \le 360°$.

Use the graphs to solve the equation $2\sin A = \tan A$. 13

2017

Using a scale of 2 cm to represent 45° on the horizontal axis and 4 cm to represent 1 unit on the vertical axis,

Draw a graph of $y = \cos 2x$ for $0° \le x \le 180°$.

Use the graph to solve the equation $\cos 2x = 0.5$. 10

2018

Using a scale of 2 *cm* to represent 30° on the horizontal axis and 2 *cm* to represent 1 *unit* on the vertical axis, draw on the same scale and axes, graphs of $y = |2\sin x|$ and $y = \frac{x}{\pi}$, for $0° \le x \le 270°$.

Use the graphs to solve the equation $\pi|2\sin x| = x$. 14

Using a scale of 2 *cm* to represent $30°$ on the horizontal axis and 4 *cm* to represent 1 *unit* on the vertical axis, draw on the same axes, the graphs of $y = \cos 2x$ and $y = |\sin 2x|$ for $0° \leq x \leq 270°$.
Use the graphs to solve the equation $\cos 2x = |\sin 2x|$. 14

2019

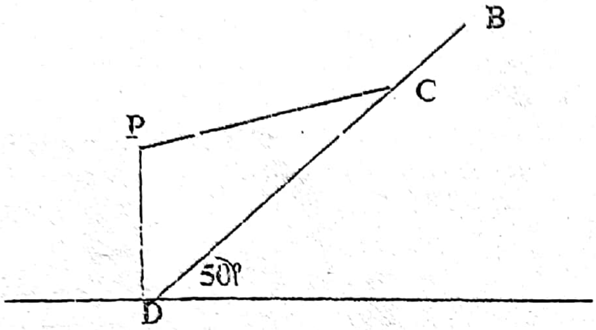
(i) Using a scale of 2 *cm* to represent $45°$ on the horizontal axis and 2 *cm* to represent 1 unit on the vertical axis, draw on the same axes, the graphs of $y = 2 \sin x$ and $y = \cos x + 1$, for $0° \leq x \leq 360°$

(ii) Use the graphs to solve the inequality $\cos x + 1 \geq 2 \sin x$. 12

MECHANICS
Resolution of vectors

1989

In figure below **BD** is a pole of length **14 m**, which is inclined at **50°** to the horizontal with the end **D** on horizontal ground. A peg **P**, is **6 m** vertically above the point **D**. The pole is held in this position by a rope which is attached to a point **C** on the pole, passes over the peg and is fastened to the ground at **D**. **DC** is **10 m** long.



Calculate:

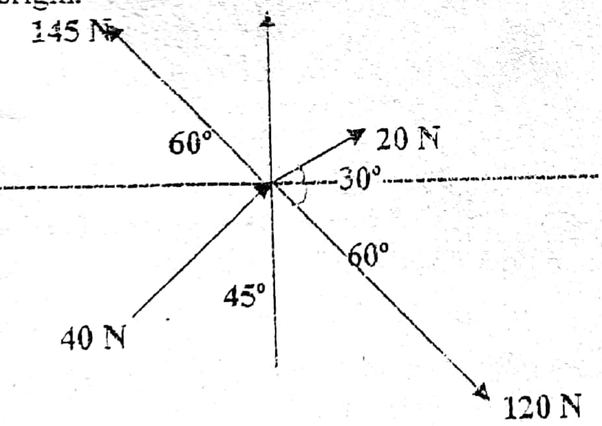
the length of B above the ground. 4

the length of PC 6

the angle DPC 8

1991

Figure below shows a system of forces acting at the origin.

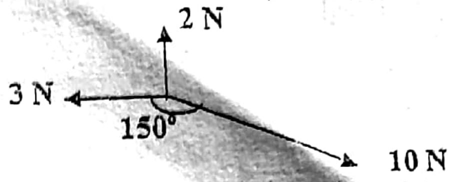


a. Find the **sum of the components** of the above forces acting along the x and y-axes. 12

b. Find the **magnitude and the direction** of the resultant of the system of forces. Give your answer to 3 significant figures 8

1992

Figure shows 3 **forces** at a point O.

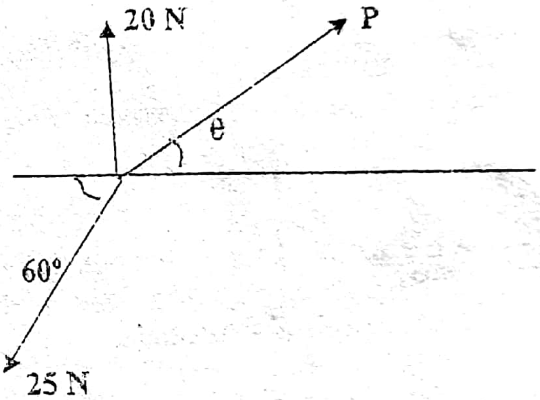


Find the **magnitude and direction** of the resultant. 11

Three forces $\underline{F} = 2i + 3j + 2k$, $\underline{F} = 2j - 3k$, **and** $\underline{F} = 4i - k$ act at a point. Find the **resultant force** and calculate the **magnitude** of the resultant. 4

1995

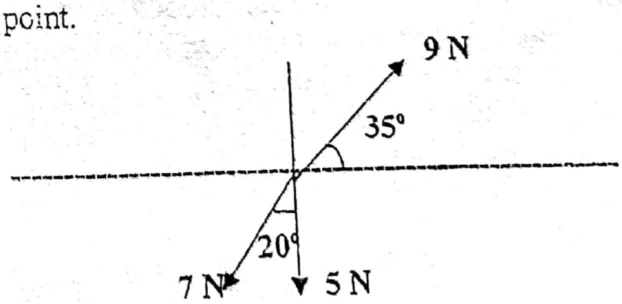
Figure below shows three forces which are in equilibrium.



Find the **magnitude and direction** of force P. 12

1996

Figure below a system of forces acting at a point.



Find:

a. the **sum of the components** of the above system of forces along the x and y axes. 4

b. the **magnitude and direction** of the resultant force of the system force. 3

1997

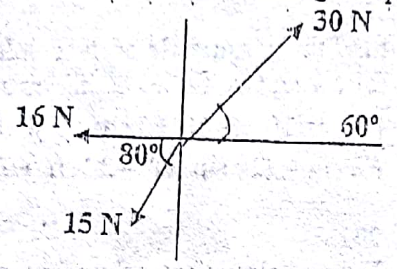
A small boat travels through water at 8 km/h. On a certain day it is being steered **due east**, but there is a current running south at 2 km/h and the wind is blowing in the direction 150° at 4 km/h.

Find the **magnitude of the resultant velocity** of the boat.

If the pilot wishes to move due east, in what **direction** should he steer while maintaining the same resultant velocity? 10

1998

Figure a system of three forces acting at a point.



Find:

the **resultant force**. 4

the **magnitude** and **direction** of the resultant force. 6

1999

If a mass of 3 kg is acted upon by force of 5 N, 10 N, and 8 N in the directions **north**, east and 250° respectively, calculate:

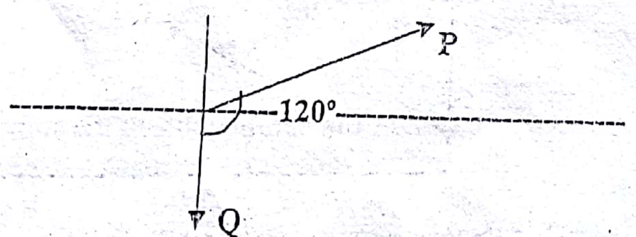
a. the **resultant force**

b. the **acceleration**

c. the **direction** of the acceleration. 10

2000

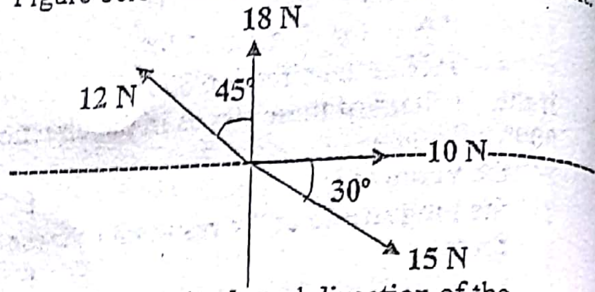
Figure below shows two forces P and Q. The angle between the two forces is 120° and the magnitude of the resultant force is $\sqrt{19}$ N If the direction of P changes so that the angle between the forces is 60°, the magnitude of the resultant would be 7 N.



Calculate forces P and Q. 10

2001

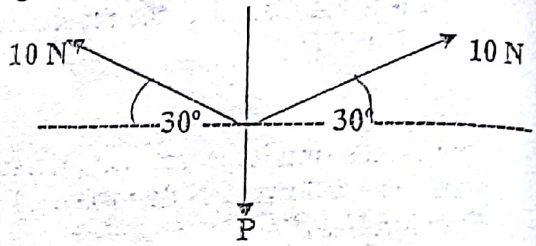
Figure below shows four forces acting at a point.



Find the **magnitude** and **direction** of the resultant force. 10

2002

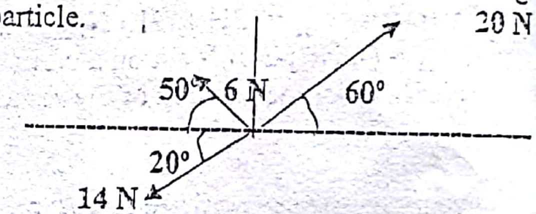
The figure below shows three co-planar forces.



If the resultant force is 0, calculate the **magnitude** of P. 4

2003

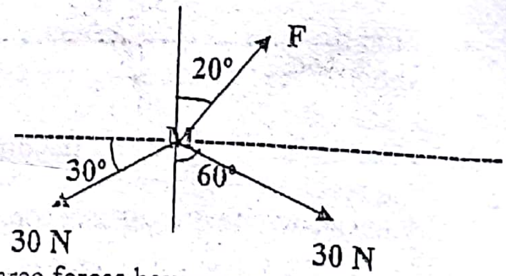
Figure below shoes three co-planar forces acting on a particle.



Find the **magnitude** and **direction** of the resultant force. 14

2004

Figure below shows three forces acting on a body of mass M.



If the three forces have a zero resultant force in the y-axis, calculate the **value of F**. 7

2005

Three co-planar act on a point:

- 6 N in the direction **045°**;
- 8 N in the direction **180°**;
- P N in the direction 330°.

If the resultant of these forces in the direction 000°, calculate:

a. the **value of P**; 6

b. the **magnitude** of the resultant of these forces. 4

A force P of magnitude **28 N** acts on a point A which has position vector **2j + k** in the direction of the vector $3\underline{i} - 2\underline{j} + 6\underline{k}$. A second force Q of magnitude 27 N acts on A in the direction of the vector **i-4j-8k**. Calculate the **resultant** of the forces P and Q. 7

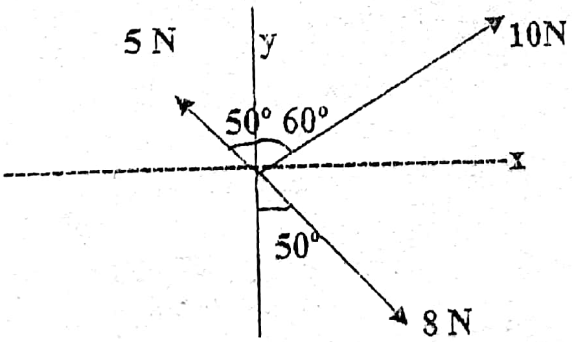
2006

Four forces simultaneously act on a stationary particle as follows: **2 N due west**, **1** N in the direction N 61°W,
7 N in the direction N 46°E and 1 N due south.
Calculate the **magnitude** and direction of the resultant force. 15

2008

Three forces $\underline{P} = (3i+4j-3k)$ N, $\underline{Q} = (2i-5j-2k)$ N and $\underline{x} = (-3i+2j-3k)$ N act on a particle.
Calculate the **magnitude** of the resultant of the three forces. 6

Figure below shows three forces: 5 N in the direction N 50^0W, 10 N in the direction N 60^0E and 8 N the direction S 50^0E acting on a point O.

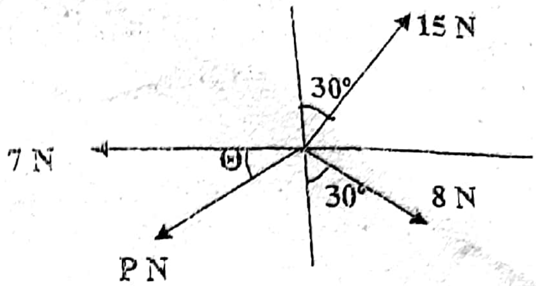


i. Resolve the forces into components in directions $\overrightarrow{OX}$ and $\overrightarrow{OY}$.

ii. Find the **resultant** force. 11

2009

Figure below shows forces which are in equilibrium.



Find the **value of P**. 12

2011

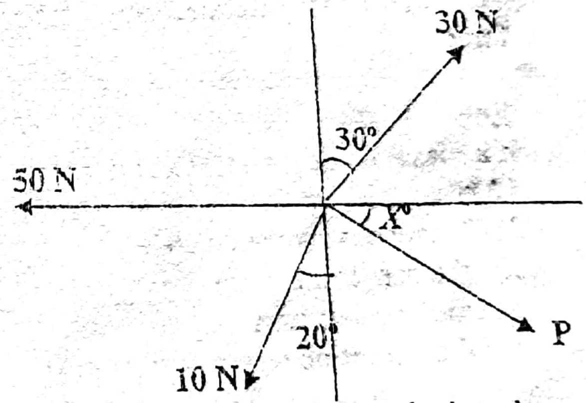
Find the **magnitude** of the resultant of the following two forces acting on a particle:

$\overline{F1}$ **=(10 cos 30°+8 cos 60°)i +0 j newtons**

$\overline{F2}$**=0i + (20 sin 30° – 20 sin 60°) j** newtons. 5

2013

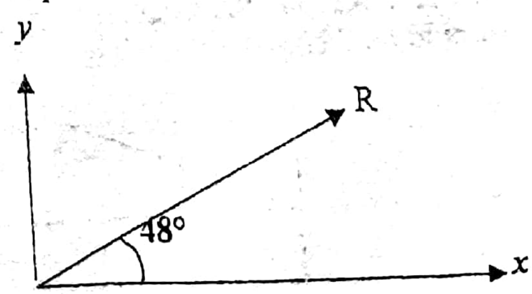
Figure below shows four forces acting on a particle.



If the forces are at equilibrium, calculate the values of force P and **angle x°**. 10

2015

Figure below shows a force **R** drawn on a Cartesian plane.



If **R** = **12 N** and the angle between the force and the *x*-axis is **48°**, resolve **R** along the *x* and *y*-axis. 6

2016
Given that forces
$F_1 = \{(5\cos 60° + 3\cos 30°)\underline{i} + 2\underline{j}\}$ N and
$F_2 = \{3\underline{i} + (15\sin 60° - 4\sin 30°)\underline{j}\}$ N act on an object, calculate the **modulus** of the resultant force. 6

2018
Three forces: 6 N in the direction 045°, F N in the direction 180° and 8 N in the direction 330° act on a particle. Given that the resultant of the forces is in the y-axis, calculate the;
i. the value of F
ii. *magnitude* of the resultant force.
10

Sample Paper
Forces of 10 N, 8 N, and 6 N, act along the sides BC, CA and AB respectively of an equilateral triangle ABC. Calculate:
i. the *resultant force*
ii. the *direction* of the resultant force.
8

2019
Figure below shows two forces of 90 N and 60 N acting on a 10 kg mass at 40° to the resultant force (R).

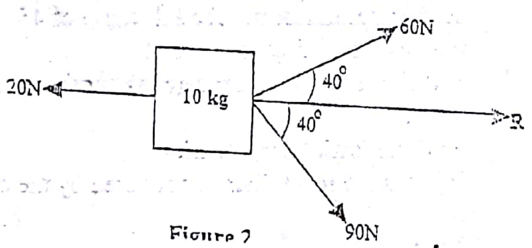


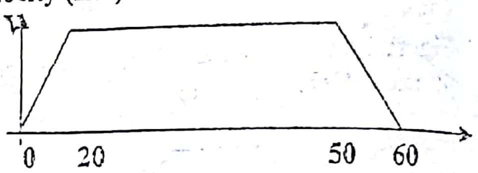
Figure 2

If a frictional force of 20 N acts against the direction of the motion, calculate the:
(i) resultant force
(ii) acceleration of the mass 1

Travel graphs

1990 Sample paper
A car starts from rest and accelerates uniformly for 8 seconds reaching a velocity of 18 m/s. It travels at this steady speed for 1 minute and then decelerates uniformly to stop in a distance of 45 m. Draw a sketch of the **velocity-time graph** of the car and calculate:
a. the **initial acceleration**. 6
b. the **time taken** when deceleration. 6
c. the **total distance** travelled. 6

1991
Figure below shows the velocity-time graph of a body moving in a straight line.
Velocity (m/s)



Time (s)

a. If the body accelerates uniformly for 20 seconds covering 50 m, find the **maximum** velocity attained in this tine. 6
b. Find the **distance** covered between t = 20 seconds and t = 40 seconds. 3
c. Find the **retardation** in the last 10 seconds. 5
d. Find the total distance covered. 6

1992
A coach-line travelling from Blantyre to Lilongwe starts from rest in Blantyre and accelerates uniformly to a speed of 80 km/h. It maintains this speed before retarding uniformly to come to rest in Lilongwe. Taking the distance between the two cities as 300 km and the total time taken as 4 **hours** and given that the **retardation is twice the acceleration** in magnitude,
i. Sketch a **velocity- time graph** to illustrate the coach's journey.
Hence or otherwise find:
ii. the **time** for which the coach is travelling at constant speed.
iii. the **acceleration**.
13

1995

At time zero seconds, car A is travelling at a constant speed of 20 m/s and is passing a stationary car B which immediately accelerates uniformly at 3 ms^{-2} Once car B reaches a speed of 30 m/s both car A and car B decelerate uniformly until they are at rest. However, car B decelerates twice as fast as car A. If the total distance travelled by car A is 300 m.

Draw a v-t graph to illustrate the above information,

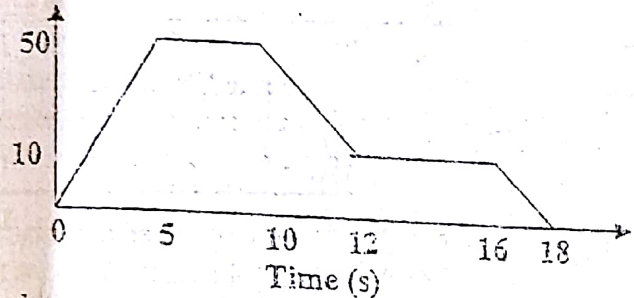
Find:

1) the time taken for car B to reach a speed of 30 m/s
2) the deceleration of car A
3) the total distance travelled by car B.

12

1996

Figure below shows a velocity time graph for a journey. Velocity (m/s)



Calculate

i. the acceleration in the first 5 seconds
ii. the total distance travelled in the first 12 seconds.

6

A car takes 60 s to travel between two sets of traffic lights. Starting from rest at one end coming to rest again at the second. It accelerates uniformly to 12 m/s and uniformly deceleration to rest. How far apart are the two sets? 6

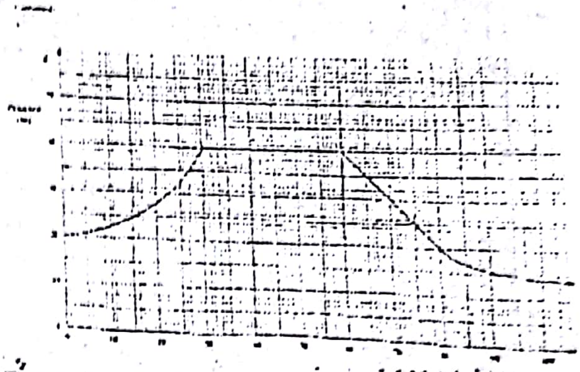
1997

A car starts from rest and accelerates uniformly to a speed of 10m/s in 3 seconds. It travels at a constant speed for another 3 seconds and finally comes to rest in a further 4 seconds under constant retardation. By sketching the velocity-time graph, calculate the distance travelled.

4

1999

Figure below shows a distance-time grap for a journey of a particle



From the graph, estimate the speed when t =20 seconds.

When is the particle moving with constant speed?

When t=80 seconds, how far is the particle from the origin?

When t=80 seconds, what is the total distance travelled by the particle?

10

2000

A car starts from rest and accelerates uniformly for 8 seconds reaching a velocity of 18 m/s. It travels at this speed for 1 minute and then decelerates uniformly for a distance of 45 metres where it comes to rest.

Sketch the velocity-time graph for the motion of the car.

Find the initial acceleration

What is the total distance covered by the car?

10

2004

A particle of mass 2 kg initially at rest is accelerated by a 6 N force for 5 seconds. The particle is then brought to a rest in a further 8 seconds.

Calculate the acceleration during the first 5 seconds. 2

Calculate the highest speed attained. 2

Sketch the speed time graph for the particle.

4

Calculate the total distance covered during this motion.

2

2005

A car accelerates uniformly from rest to a speed of V m/s in 5 seconds. It travels at this speed for 20 seconds and then decelerates for t seconds to come to rest.

Sketch the velocity -time graph for the motion

If the distance travelled while decelerating is $\frac{4}{5}$ of the distance travelled while accelerating, calculate the value of t.

Given that the total distance travelled by the car is 1274 m, calculate the value of V. 14

2010

An object accelerates uniformly for a certain time, then moves at a constant speed for 1 minute and finally decelerates uniformly.

Time (min)	0	1	2	3	4	5	6
Velocity (m/s)	0	20	40	60	60	30	0

Using a scale of 2 cm to represent 1 unit on the horizontal axis and 2 cm to represent 10 units on the vertical axis, draw, on the graph paper provided a velocity-time graph.

Using your graph, calculate the average velocity of the object. 12

2011

A train starts from rest at station A and travels to station B in 50 seconds. Its velocity (V m/s) after t seconds is given by:

$$V = \begin{cases} 0.4t, & t < 10 \\ 4, & 10 \leq t < 40 \\ 20 - 0.4t, & 40 \leq t \leq 50 \end{cases}$$

Sketch a velocity-time graph to represent the motion of the train.

Hence find the total distance travelled by the train. 7

2012

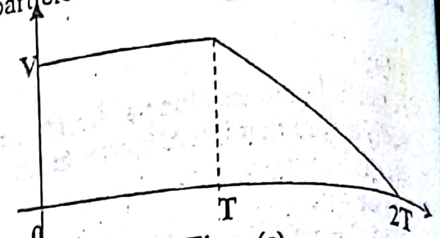
A car accelerates uniformly from rest to reach a certain velocity in 10 minutes. It then continues at this velocity for another 10 minutes and decelerates to rest in a further 5 minutes.

Sketch a velocity-time graph. 4

If the distance covered is 17.5 km, calculate the maximum velocity reached. 4

2017

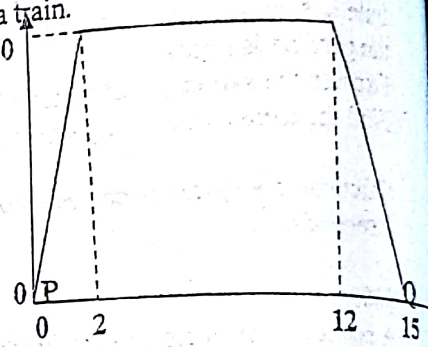
Figure below shows a velocity-time graph of a moving particle.



Given that the average velocity for the whole journey is *37.5 m/s* and that the deceleration between T and 2T is 2.5 m/s^2, calculate the values of V and T. 10

2018

The figure below shows a velocity-time graph PABQ of a train.



Calculate the;

a. *distance* from P to Q in km. 4

b. *average speed* of the train in km/h. 3

Application of calculus to problems involving distance, velocity and acceleration.

1998

A particle moves along a straight line OA in such a way that it is S metres from O after t seconds, where

$s = 6t - t^2$.

Find the velocity of the particle after 2 seconds.
Find the acceleration after 2 seconds.
What is the value of s when the particle changes its direction? 10

2000

The acceleration of a particle moving in a straight line is given by $a = t - \frac{5}{2}$ where t is the time in seconds. Given that the initial velocity of the particle is 3 m/s.
Express the velocity v in terms of t. 4
Find the time t when the particle is at rest. 2
Calculate the distance covered by the particle in 2 seconds. 3

2001

A particle moves along a straight line so that after t seconds, its distance from a fixed point O on the straight line is $s = t^3 - 3t^2 + 2t$.
When is the particle at O?
Find the velocity of the particle at t = 1.
Find the acceleration when t = 2. 13

2002

A particle moves in a straight line and the distance from a fixed point O on the line is given by $s = 2t^3 - 15t^2 + 36t + 4$
Sketch the velocity -time graph
Calculate the times at which the particle is at rest.
Calculate the velocity at t = 1
Calculate the acceleration at t = 1. 16

2006

A particle initially at point O moves in a straight line. Its displacement, in metres, from O after t seconds s given by $s = 2t^3 - 21t^2 + 60t$. Calculate the
times the particle is momentary at rest. 6

initial velocity of the particle 4
acceleration of the particle after 3 seconds 4

2007

The distance time equation of a particle from a fixed point O is given by $S = 3 \sin 3t + 5 \cos 3t$. Find the distance that the particle is from the origin when its velocity is zero. 10

2008

A particle is moving with a velocity (m/s), $v(t) = (47 - 9.8t)\underline{i} + t^2\underline{k}$. Find the acceleration of the particle at t = 10 seconds. 4

2012

A stone is thrown vertically upwards and its height is given by $s = 10t - 5t^2$.

i. Calculate the time the stone will take to reach the maximum height.
ii. Find the maximum height. 7

2013

The displacement of a particle from a fixed point is given by $s = 2t^3 - 15t^2 + 36t + 4$. Calculate the:

a. time at which the particle is at rest. 5
b. acceleration at t = 1. 4

2014

A small body moves along x – axis such that its distance x metres from the origin at time t seconds is given by $x = 2t^3 - 15t^2 + 24t + 20$.
Find the:

i. initial velocity
ii. times when the body is at instantaneous rest. 7

2017

The initial velocity of an aeroplane on the ground is $\frac{9}{2}$ m/s. If its acceleration after t seconds is given by $a = 2t + \frac{1}{5}t^2$, calculate the total distance it travelled on the ground in the first 10 seconds. 6

Uniformly accelerated linear motions and kinematics:

1992

A bullet is projected vertically upwards with an initial velocity of u m/s. Between 4^{th} and 5^{th} second after leaving the ground, it rises 20 m. Calculate the value of u. 7

1995

Two particles A and B are moving on the same horizontal line towards each other. Particle A passes a point P on the line at a speed of 4 m/s and a constant retardation of $\frac{1}{2}$ ms^{-2}. Simultaneously, particle B passes a point Q on the line at a speed of 2 m/s and a constant acceleration of 1 ms^{-2}. Given that the distance PQ is 20 m, calculate the distance from P to the point of collision. 10

1996

A stone of mass 2 kg is thrown up with an initial velocity of 40 m/s.

a. How high does it rise? 3
b. How long does it take to reach the highest point? 2

1997

A particle with an initial velocity of 5 m/s takes 4 seconds to travel 100 metres under uniform acceleration. Find:
the acceleration of the particle
the final velocity of the particle. 6

1998

A particle changes its velocity from $6i-2j$ to $9i+8j$ m/s in 5 s.
What is the acceleration of the body, assumed uniform?
What will be its velocity after a further 2 s?
5

1999

A car starts from rest at point X and travels on a straight road with uniform acceleration of 2 ms^{-2}. Two seconds later, a motorbike passes through X travelling on the same road towards the car at a uniform speed of 9 m/s. Calculate:
the distance from X to the point where the motorbike overtakes the car. 8
the distance from X to where the two have the same velocity. 2

A stone is thrown directly upwards with an initial velocity of 20 m/s, and its height (in metres) above the ground is given by s. The acceleration due to gravity is approximately 10 ms^{-2}. Find;
the acceleration, velocity and distance above the ground, after 5 seconds.
maximum height
How long the flight of the stone took. 9

2000

A student is initially at a point with position vector $\begin{pmatrix}0\\-3\end{pmatrix}$ walking with velocity $\begin{pmatrix}3\\4\end{pmatrix}$ m/s.
How far is she after 3 seconds? 5

2001

A rocket is fired vertically upwards from the ground. After one second, it reaches a height of 15 metres. Calculate:

a. the initial velocity of the rocket. 2
b. the time taken for the rocket to reach its greatest height. 2
c. the greatest height reached. 2

2002

An orange is thrown upwards with an initial velocity of 42 m/s from the edge of a building of height 120 m. Calculate:
the time when the orange hits the ground,
the speed when the orange hits the ground.
6

2007

A stone is thrown vertically upwards with an initial speed of 250 m/s from a fixed point O. Calculate the maximum height reached by the stone from O.
Calculate the time the stone takes to return to O.
The stone falls past point O and lands on the ground 45 m below it. Calculate the time taken by the stone to fall to the ground. 14

2008

A body is decelerating at $0.8\ ms^{-2}$ passes through point A which a speed of 30 m/s. Calculate:

the velocity of the body after 10 seconds after passing the point A.

the distance covered in the 10 seconds.

the distance covered from point A to rest.

10

2009

A stone is thrown vertically upwards from the foot of a building with an initial velocity of 20 m/s. At the same time, a bag is released from the top of the building above the stone. Given that the stone and the bag collided after 2 seconds, calculate the height of the building. 9

2011

Particles P and Q are travelling in the same direction. At time t = 0, particle P, travelling at 2 m/s overtakes Q travelling at 8 m/s. Given that the acceleration of P and Q are $3.2\ ms^{-2}$ and $4\ ms^{-2}$ respectively, calculate the distance covered by the time Q overtakes P. 12

2012

A particle covers a distance of 160 m accelerating from a velocity of 10 m/s to a velocity of 30 m/s. Calculate:

a. the acceleration

b. the time taken. 10

2014

Two particles X and Y are moving in the same direction on parallel horizontal lines. X moves with a velocity of 4 m/s and a constant acceleration of $0.5\ m/s^2$ while Y moves with a velocity of 6 m/s and a constant acceleration of $0.3\ m/s^2$. If the velocity of X is equal to the velocity of Y after t seconds, calculate the

i. value of t

ii. distance between the two particles. 14

2015

Two particles X and Y are moving on the same horizontal line towards each other. Particle X passes point A on the line with a velocity of 8 m/s and acceleration of $\frac{3}{4}\ m/s^2$. Simultaneously particle Y passes point B on the line with a velocity of -4 m/s and acceleration of $-\frac{5}{4}\ m/s^2$. If the distance AB is 64 m, calculate the time for the particles to collide. 10

2016

A particle whose initial velocity is 12 m/s moves for 6 seconds in a horizontal straight line from point P to point Q with a constant acceleration of $8\ m/s^2$. It then continues in the straight line with a constant retardation of $4\ m/s^2$ to come to rest a point R. Calculate the distance PR.

11

2017

A bullet is fired vertically upwards with an initial speed of 100 m/s. Ignoring the effect of air resistance, find the time taken for the bullet to reach a height of 500 m. 7

Sample Paper

A stone is thrown 125 m away with a velocity of 35 m/s. Calculate the angle of projection of the stone. 8

2019

The height, (h metres) of a ball from the ground, t seconds after it was thrown vertically upwards, is given by $h = (2\underline{i} + 5\underline{j}) + (3\underline{i} - 4\underline{j})t - (4\underline{i} + 3\underline{j})t^2$. Calculate the initial velocity of the ball. 4

A particle passes a certain point with a speed of 5 m/s and accelerates at $3\ m/s^2$. Find the time taken for the particle to travel a distance of 44 m. 7

Forces and motion, momentum and impulse:

1990 (specimen papers)

A force of **10 N** acts on a mass of **5 kg** initially at rest. Calculate:

the **acceleration** of the mass

the **time taken** for the mass to reach a speed of **25 m/s**

the **distance** travelled in attaining this speed. 12

1991

A loaded truck having a mass of **10 tonnes** moves at speed of **20 m/s.**

Calculate its **momentum.** 2

What constant **force** is necessary to reduce its speed to **10 m/s** in **20 seconds**? 5

How far will the truck move while this reduction in speed is taking place? 3

1992

A force of 200 N acting on a body for 5 s increases the velocity of the body from 10 m/s to 25 m/s in the directions of the force.

Calculate:

acceleration of the body. 3

the **change in momentum** of the body. 5

a. the **distance** travelled during the 5 s. 3

1995

A ball of mass **0.2 kg** is thrown vertically upwards with a velocity of **25 m/s** from the top of a building which is **70 m** above the ground. The ball hits the ground and remains in contact with it for **0.25 s** before rebounding. While in contact with the ground, it experiences an average forces of **64 N**. Calculate:

i. the **maximum** height the ball reaches,

ii. the **velocity** on the first rebound.

8

1996

A force $\underline{\mathbf{F}} = 14\mathbf{i} - 20\,\mathbf{j}$ N acts for **6 seconds** on a body of mass **8 kg**. Calculate:

i. the **impulse** the body receives

ii. the **velocity** of the body at the instant the forces ceases if its initial velocity was $-3i+2j$ m/s. 7

A particle of mass **10 kg** moves in a straight line from point **A** to point **B** then to point **C**. The distance from **A** to **B** is **80 m** and from **B** to **C** is **60 m**. It leaves **A** acted on by a constant force $\mathbf{F_1}$ at **4 m/s** and reaches **B** travelling at **V m/s**. It leaves **B** at the same speed **V m/s** and comes to rest at **C** acted on by force $\mathbf{F_2}$. The whole journey takes **20 s**.

Find:

i. the value of **V**

ii. the values of F_1 and F_2. 14

1997

A lorry of mass **10 000 kg** is travelling at **30 m/s** when its brakes are applied for a period of **2 s.** If the braking force has constant value of 20000 N, find the final velocity. 9

After the 2 s a child runs into the road **100 metres** in front of the truck. What is the **minimum braking force** that must be applied in order to ensure that the truck does not hit the child? 11

1998

A truck of mass **15 kg** is pulled by horizontal force of 40 N and covers a distance of 3 **m** in 2 seconds starting from rest. Find the magnitude of the friction force (assumed constant) which opposes the motion. 6

A tennis ball of mass **150 g** strikes the racket with velocity **25i-j m/s** and leaves the racket with velocity **-23i+15j m/s.**

What is the **momentum** of the ball before the impact?

Find the **magnitude of the impulse** the ball receives.

Find the **magnitude of the impulse** between the racket and the ball.

If the racket and the ball are in contact for **0.05 seconds**, find the force on the ball.

10

1999

A loaded truck having a mass of **10 tonnes** is moving at a speed of **20 m/s.**

Calculate its **momentum.**

What **constant force** is necessary to reduce its speed to **10 m/s** in **20 seconds?**

What was the **acceleration** of the truck when the force was being applied on it? 7

2000

A force of 10 N acts on a mass of 5 kg initially at rest. Calculate:

the **acceleration** of the mass

the **time taken** for mass to reach a speed of 25 m/s

the **distance** travelled in attaining this speed. 8

2002

A particle P of mass 2 kg is at rest at origin O. The forces $\underline{F} = (4i+3j)$ and $\underline{g} = (2i+j)$ act simultaneously on P. Calculate the distance the particle travelled after 4 seconds. 4

2003

A ball of mass 60 g is dropped from a height of 20 m. Calculate:

the **time taken** to cover the distance

the **velocity** of the ball when it is just about to hit the ground.

the **momentum** of the ball when it first hits the ground. 11

A mass of 200 g is initially at rest. If the mass is accelerated to velocity of $\overline{V} = 6i + 8j + 5k$ in 2 **seconds**, calculate the force acting on the mass expressed in vector form. 6

2004

A body of mass 10 kg an initial velocity of $(i+4j)$ m/s. When a constant force has acted on the body for 5 **seconds** the velocity is $(5i+2j)$ m/s. Calculate the value of the force. 6

A particle of mass 5 kg is pulled along a smooth horizontal plane using a rope. The rope is inclined at 60^0 to the horizontal plane. If the acceleration is 4 m/s^2, calculate the **tension** in the rope. 6

2005

A truck of mass 10 **tonnes** moves at a velocity of 20 m/s. Calculate the **constant** force that is required to reduce its speed to 10 m/s in 20 **seconds**. 6

2006

A particle P of mass 3 kg is initially at rest at point D whose position relative to origin O $\overline{OD} = (i+2j)$ metres. If force $\underline{F} = (9j+3j)$ N acts P, calculate the displacement of the particle after 2 seconds. 6

2007

Three forces P(3i+5j-6k), Q(-6i+3j-4k) and R (5i+5k), act on a moving body of mass 4 kg for 10 seconds. Calculate;

a. the **increase in momentum** 4
b. the **magnitude of momentum**. 3
c. the **acceleration** of the body. 2

2008

A bag of mass 50 kg is moved vertically by a rope. Find the **tension** in the rope when the mass

(i) moves with a constant velocity of 2 m/s.
(iii) accelerates downwards at 5 ms^{-2}

7

2009

A ball of mass 0.5 kg is dropped from the roof 8 m above the floor. The ball hits the floor and rebounds to a height of 2.6 m.

Calculate the **impulse** of the floor on the ball. 7

Given that the average force on the ball on rebound is 64 N, calculate the **time** of contact between the ball and the ground. 3

Three forces P(3i+4j-k), Q(2i-j+6k) and R(8i+2j+4k) are applied on a 10 kg mass for 5 **seconds**. Given that the initial velocity is (5i-j) m/s, calculate the **final velocity**. 9

2010A force acted on a body of mass 12 kg and the speed of the body changed uniformly from (i-j-k) to (5i –j-3k) m/s in 10 **seconds**.

Calculate:

i. the **change in momentum**
ii. the **acceleration** of the body. 8

A load of 400 kg is lifted through a vertical distance of 36 m. The load moves upwards from rest with uniform acceleration of 0.5 ms^{-2}.

Calculate:

i. the **tension** in the cable during acceleration.
ii. the **velocity** of the load attained after it had travelled 36 m. 11

2012

A constant force, F, acts on a lorry of mass 1500 kg. The lorry accelerates from rest to a velocity of 1.2 m/s in 3 seconds. Find the value of F. 4

2013

A stone of mass 80 kg is tied to a rope. Calculate the tension in the rope if the stone moves:

a. upwards with an acceleration of $0.5\ m/s^2$. 4

b. downwards with an acceleration of $0.4\ m/s^2$. 4

2014

A car of mass 2400 kg travelling at 20 m/s is brought to rest in 4 seconds. Find the average:

i. deceleration
ii. braking force 7

2015

A toy of mass 2 kg is given a push of 20 N for 10 seconds. If it started with 5 m/s, find the final velocity. 5

A body of mass 20 kg is acted upon by a constant force for 5 seconds with an initial velocity of $3\underline{i} + 4\underline{j} - \underline{k}$ and a final velocity of $5\underline{i} + 7\underline{j} - 2\underline{k}$. Calculate the magnitude of the force acting on the body. 8

2016

A ball of mass 1 kg travelling at a constant velocity of 20 m/s hits a wall and rebounds with a velocity of 15 m/s. Calculate the impulse of the ball. 6

2017

A force of 250 N acting on a body for 8 seconds increases its velocity from 5 m/s to 45 m/s. Calculate the

i. change in momentum
ii. mass of the body. 11

2018

A force of $15\underline{i} + 20\underline{j}$ N acts on a particle of mass 5 kg. If its velocity is $24\underline{i} + 20\underline{j}$ m/s after 8 seconds, calculate its *initial velocity*. 6

Three forces $A = (2\underline{i} + 3\underline{j})$ N, $B = (4\underline{i} - \underline{j})$N and $C = (-\underline{i} + \underline{j})$N are applied to a body of mass 10 kg for 5 seconds. Calculate the magnitude of the *impulse*. 7

2019

The forces $F = (4\underline{i} + 2\underline{j} - 3\underline{k})$, $G = (2\underline{i} - 3\underline{j} + \underline{k})$ and $H = (-3\underline{i} - \underline{j} - 2\underline{k})$ are applied to an object for 6 seconds. Find the change in momentum in vector form. 5

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