

1 Test

$$1.1 \quad \sum_{i=0}^{\log_4 n} \left(\frac{3}{4}\right)^i n \log \frac{n}{4^i}$$

$$\begin{aligned} \sum_{i=0}^{\log_4 n} \left(\frac{3}{4}\right)^i n \log \frac{n}{4^i} &< \sum_{i=0}^{\log_4 n} \left(\frac{3}{4}\right)^i n \log n \\ &\in O(n \log n) \end{aligned}$$

$$\begin{aligned} \sum_{i=0}^{\log_4 n} \left(\frac{3}{4}\right)^i n \log \frac{n}{4^i} &= 1 \cdot n \log \frac{n}{1} + \cdots \\ &> n \log n \\ &\in \Omega(n \log n) \end{aligned}$$

$$O(n \log n), \quad \Omega(n \log n) \implies \Theta(n \log n)$$