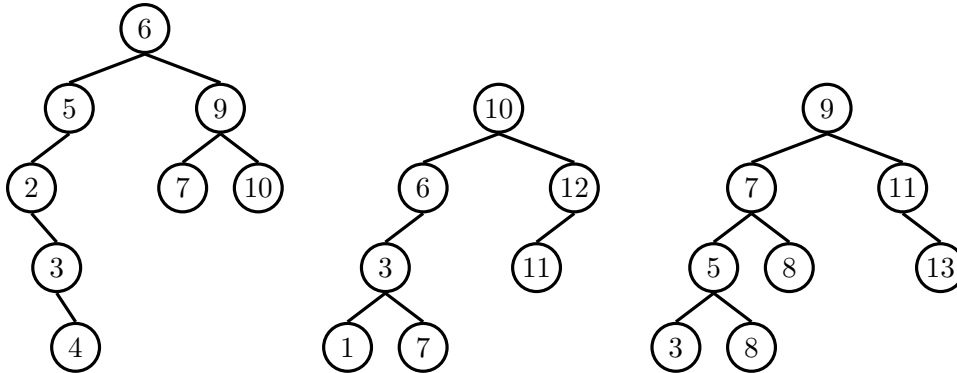


## 1 Unbalanced Trees

- 1.1 For each of the following binary trees, determine if the height of the tree is the same as the height of the optimal tree with the same elements.



- 1.2 Provide tight asymptotic runtime bounds in terms of  $N$ , the number of nodes in the tree, for the following operations and data structures.

Operations	Binary Search	Balanced Search
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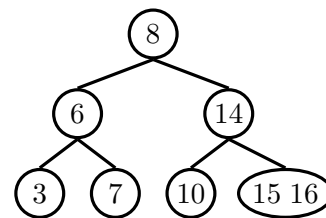
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**boolean** contains(E e);

**boolean** add(E e);

## 2 2-3-Forever

2.1 Draw the 2-3 tree would look like after inserting 18, 12, and 13.



2.2 Now, convert the resulting 2-3 tree to a left-leaning red-black tree.

## 3 New World Order

3.1 Draw the trie that results from inserting "hi", "hello", and "hey".

- 3.2 Given a list of words (possibly repeated), devise a strategy to efficiently return a list of all the words that start with a given prefix.
- 3.3 Given a dictionary of words, describe a procedure for checking if a new word can be created out of the concatenation of two words in the dictionary. For example, if our dictionary contains the words, "news", "paper", "new", and "ape", we should be able to discover the new word, "newspaper".

# 4 Sorting Mechanics *Extra Practice*

- 4.1 Each column below gives the contents of a list at some step during sorting. Match each column with its corresponding algorithm.

· Merge sort · Quicksort · Heap sort · LSD radix sort · MSD radix sort

For quicksort, choose the topmost element as the pivot. Use the recursive (top-down) implementation of merge sort.

	Start	A	B	C	D	E	Sorted
1	4873	1876	1874	1626	9573	2212	1626
2	1874	1874	1626	1874	7121	8917	1874
3	8917	2212	1876	1876	9132	7121	1876
4	1626	1626	1897	4873	6973	1626	1897
5	4982	3492	2212	4982	4982	9132	2212
6	9132	1897	3492	8917	8917	6152	3492
7	9573	4873	4873	9132	6152	4873	4873
8	1876	9573	4982	9573	1876	9573	4982
9	6973	6973	6973	1897	1626	6973	6152
10	1897	9132	6152	3492	1897	1874	6973
11	9587	9587	7121	6973	1874	1876	7121
12	3492	4982	8917	9587	3492	9877	8917
13	9877	9877	9132	2212	4873	4982	9132
14	2212	8917	9573	6152	2212	9587	9573
15	6152	6152	9587	7121	9587	3492	9587
16	7121	7121	9877	9877	9877	1897	9877