

#### Question 4

First, we order the jobs based off its profit in descending order.

So,

$$p_1 \geq p_2 \geq p_3 \geq \dots \geq p_n$$

We earn this profit  $p_i$ , given that its deadline  $d_i \geq 1$  is less than the current time,  $t$ . Each job takes one unit of  $t$  to complete. We must find the subset of jobs that maximises profit.

The solution to this problem is to use a **greedy algorithm**.

So, our approach is that we schedule each job,  $p_i$  in order (starting from  $p_1$ ) at the latest possible timeslot available (or the next best), given the condition that  $d_i \leq t$ .

If such a timeslot is unavailable, then we do not schedule the job.

For example, if we have 4 jobs, already sorted with profits of \$20, \$15, \$12 and \$10, with deadlines  $t = 4, 2, 2$  and  $1$ . Each job takes 1 unit of time.

We schedule the slots as shown.

T	1	2	3	4
P	\$12	\$15	n/a	\$20

We can see here, that despite the \$12 having a deadline of  $t = 2$ , we placed it into the next best position in order to attain its profit.

Unfortunately, there is no space for the job that provides a profit of \$10 as there is not a suitable slot that fits its deadline. Nevertheless, this is the most optimal solution.

Since we are placing the jobs at its latest possible deadline, we iterate through in  $O(n)$  and each job assignment is similarly in  $O(n)$ , so the algorithm is run in  $O(n^2)$  time.