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Question 5

Let $x \ = \ \langle \alpha_0, \alpha_1, \alpha_2, \dots, \alpha_n \rangle$

And so,

$$x \ = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \cdots + \alpha_n x^n$$

We know that $x * \langle 1, 1, -1 \rangle = \ \langle 1, 0, -1, 2, -1 \rangle$, so

$$(\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \cdots + \alpha_n x^n) * (1 + x - x^2) = 1 - x^2 + 2x^3 - x^4$$

Expanding,

$$(\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \cdots + \alpha_n x^n) + (\alpha_0 x + \alpha_1 x^2 + \alpha_2 x^3 + \cdots + \alpha_n x^{n+1}) - (\alpha_0 x^2 + \alpha_1 x^3 + \alpha_2 x^4 + \cdots + \alpha_n x^{n+2}) = 1 - x^2 + 2x^3 - x^4$$

We can compute $\alpha_0, \alpha_1, \alpha_2, \dots, \alpha_n$ by comparing coefficients in the above equation

$\alpha_0 = 1$
$\alpha_1 + \alpha_0 = 0 \rightarrow \ \alpha_1 = -1$
$\alpha_2 + \alpha_1 - \alpha_0 = -1 \rightarrow \alpha_2 = 1$
$\alpha_3 + \alpha_2 - \alpha_1 = 2 \rightarrow \alpha_3 = 0$
$\alpha_4 + \alpha_3 - \alpha_2 = -1 \rightarrow \alpha_4 = 0$

And $\alpha_n = 0$ for $n \geq 3$

And so,

$$x = \ \langle 1, -1, 1 \rangle$$