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Question 4

a)

We have the sequence $\langle 1, \underbrace{0, 0, \dots, 0}_{k}, 1\rangle$

Equate this to a suitable polynomial, $P(x)$

$$P(x) = 1x^0 + \cdots + 1x^{n-1}$$

$$= 1 + \cdots + x^{n-1}$$

To calculate the convolution of $\langle 1, \underbrace{0, 0, \dots, 0}_{k}, 1\rangle * \langle 1, \underbrace{0, 0, \dots, 0}_{k}, 1\rangle$, we compute the two sequences multiplied, i.e. $P(x)^2$.

$$P(x)^2 = x^0 + \cdots + 2x^{n-1} + \cdots + x^{2n-2}$$

However, we must compute the number of $0's$ which we ignored in our initial multiplication.

These zeroes exist in between x^0 and $2x^{n-1}$, and in between $2x^{n-1}$ and x^{2n-2}, i.e. the coefficients for x^1 up to x^{n-2}, and x^n to x^{2n-3}.

We know that two sequences of $P(x)$'s type with α and β elements, produce a sequence of length $\alpha + \beta - 1$.

Using the fact that the polynomial $P(x)$ has $k + 2$ elements, we can use the point above to calculate the number of elements in $P(x)^2$.

$$2(k + 2) - 1$$
$$= 2k + 3$$

Since $P(x)^2$ has 3 non-zero coefficients, subtract this from the total amount of elements to find the number of 0's.

$$2k + 3 - 3 = 2k$$

There are $2k$ many 0 coefficients in the convolution, so our solution is

$$\langle 1, \underbrace{0, 0, \dots, 0}_{k}, 2, \underbrace{0, \dots 0, 0}_{k}, 1\rangle$$

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b)

Our sequence is $A = \langle 1, \underbrace{0, 0, \dots, 0}_{k}, 1 \rangle$

We can form the corresponding polynomial,

$$P_A(x) = \sum\nolimits_{k=0}^{n-1} A_k x^k$$

And extend it to

$$P_A(x) = 1 + \sum\nolimits_{k=1}^{n-2} A_k x^k + x^{n-1}$$

Now, we evaluate the above for its complex roots of unity, i.e.

$$\langle P_A(1), P_A(\omega_n), P_A({\omega_n}^2), \dots, P_A({\omega_n}^{n-1}) \rangle$$

We know that $n = k + 2$, so

$$\hat{A} = \langle 1 + {\omega_{k+2}}^{0\times(k+1)}, 1 + {\omega_{k+2}}^{1\times(k+1)}, \dots, 1 + {\omega_{k+2}}^{(k+1)\times(k+1)} \rangle$$

$$\hat{A} = \langle 2, 1 + {\omega_{k+2}}^{k+1}, \dots, 1 + {\omega_{k+2}}^{(k+1)^2} \rangle$$