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Question 5

We can determine the answer by using the asymptotic notation rules:

“Big Oh” Notation: $f(n) = O(g(n))$ is an abbreviation for

There exists positive constants c and n_0 such that

$$0 \leq f(n) \leq c * g(n) \text{ for all } n \geq n_0$$

In this case we say that $g(n)$ is an asymptotic upper bound for $f(n)$

“Omega” Notation: $f(n) = \Omega(g(n))$ is an abbreviation for

There exists positive constants c and n_0 such that

$$0 \leq c * g(n) \leq f(n) \text{ for all } n \geq n_0$$

“Theta Notation: $f(n) = \ominus\left(g(n)\right)$ if and only if $f(n) = O\big(g(n)\big)$ and $f(n) = \Omega\big(g(n)\big)$.
Meaning $f(n)$ and $g(n)$ have the same asymptotic growth rates

a)

We are given:

$$f(n) = (\log_2(n))^{\,2} \quad ; \quad g(n) = \log_2(n^{\log_2 n})^2$$

$$\begin{aligned} g(n) &= 2 * \log_2 n^{\log_2 n} \\ &= 2 * \log_2 n * \log_2 n \\ &= 2 * (\log_2 n)^2 \end{aligned}$$

Looking at Big Oh Notation,

$$0 \leq (\log_2(n))^{\,2} \leq c * 2 * (\log_2 n)^2$$

Which is true for all $c \geq 0.5$

Now, looking at Omega Notation,

$$0 \leq c * 2 * (\log_2 n)^2 \leq (\log_2(n))^{\,2}$$

Which is true for all c such that $0 \leq c \leq 0.5$

Therefore, for large n, $f(n)$ and $g(n)$ have the same asymptotic rates, satisfying the Theta Notation

i.e.: $f(n) = \ominus\left(g(n)\right)$

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b)

Unable to complete as of yet.

c)

We are given,

$$f(n) = n^{1+(-1)^n} \quad ; \quad g(n) = n$$

Consider the two cases whereby n is odd, and whereby n is even

i.e.: $n = 2k + 1$ (odd), $n = 2k$ (even) for some positive integer, k.

Using the Big Oh Notation for even and odd cases

Odd Case:

$f(n) = 1$ *and so*

$$f(n) = O(g(n)) \text{ for } 0 \leq 1 \leq c * n$$

Even Case:

$f(n) = (2k)^{1+(-1)^{2k}}$

$= 2k * (2k)^{(-1)^{2k}}$
$= n^2$ *and so*

$$f(n) = O\big(g(n)\big) \text{ for } 0 \leq n^2 \leq c * n$$

However, as n tends to infinity, $n^2 \geq c * n$ which contradicts our findings and thus rules out the Big Oh Notation.

Using the Omega Notation for even and odd cases

Odd Case:

$$f(n) = \Omega(g(n)) \text{ for } 0 \leq c * n \leq 1$$

Even Case:

$$f(n) = \Omega(g(n)) \text{ for } 0 \leq c * n \leq n^2$$

Again, as n tends to infinity, $c * n$ would grow larger than 1 for our Odd Case. And so we have a contradiction which rules out the Omega Notation.
So, neither the Big Oh nor the Omega Notation fits the functions