

Question 2

We are given

$$P(x) = A_0 + A_1x^{100} + A_2x^{200}$$

Let $y = x^{100}$

$$P(x) = A_0 + A_1y + A_2y^2$$

$$P(x)^2 = A_0^2 + 2A_0A_1y + 2A_0A_2y^2 + A_1^2y^2 + 2A_1A_2y^3 + A_2^2y^4$$

The 'large integer multiplications' that we need to calculate are:

$$M_0 = A_0^2$$

$$M_1 = 2A_0A_1$$

$$M_2 = 2A_0A_2 + A_1^2$$

$$M_3 = 2A_1A_2$$

$$M_4 = A_2^2$$

We can use the fact that

$$(A_0 + A_1 + A_2)^2 = A_0^2 + A_1^2 + A_2^2 + 2A_0A_1 + 2A_0A_2 + 2A_1A_2$$

Noticing that the highlighted figures are M_2 , we can shrewdly rewrite M_2 as

$$M_2 = (A_0 + A_1 + A_2)^2 - M_0 - M_1 - M_3 - M_4$$

M_0, M_1, M_3 and M_4 are all each one 'large integer multiplication', the co-efficient of 2 is not considered large and so is not counted.

For M_2 , only one multiplication of $(A_0 + A_1 + A_2)^2$ is required, the remaining addition and subtraction is not counted.

And so, via this method, only 5 'large multiplications' are required.