

Question 3

We begin the problem by constructing a graph of nodes $C_1, C_2 \dots C_N$ with links from any computer C_i to C_j denoting a direct link from those two computers as given in our network.

Our task is to achieve a min-cut such that the virus sent from Computer, our source node, will never be able to reach Computer N, our sink node.

The max-flow min-cut theorem states that in any flow network, the minimum cut (what we need to find) is equal to the maximum flow.

Additionally, in order to find the edges (links between computers) that create that minimum cut (that we must disconnect), we use a residual graph.

So, once we have our initial graph, we run the Ford-Fulkerson algorithm which runs in $O(E * f)$; E being the number of edges and f being the maximum flow.

Once this is done, we will be left with a residual graph. Now, we find all vertices that are reachable from our source node.

Any edge that exists from any vertex in our collection to any non-reachable vertex, are the edges which are part of the minimum cut. These edges are the links between any two computers in our network that must be removed so that we minimise our cost.

So, our minimum total cost to disconnect the computers is equal to our result in the Ford-Fulkerson Algorithm, and the edges (links) that must be removed are the edges that we found as described in the residual graph.