```
> restart:

  f := x^6 - 3*x^5 + 7*x^4 - 9*x^3 + 7*x^2 - 3*x + 1:
  fRoots := [solve(f=0, x)];

  A0 := fRoots[1]:
  A1 := fRoots[2]:
  A2 := fRoots[3]:
  A3 := fRoots[4]:
  A4 := fRoots[5]:
  A5 := fRoots[6]:

  alpha := A0:

  tau := proc(i::integer, E)
         local subE;
         subE := subs(alpha = fRoots[i], E);
         return subE;
         end proc:

  whichRoot := proc(expr, theRoots)
               local k, eqcheck;
               for k to nops(theRoots) do
                   eqcheck := simplify(expr - theRoots[k]);
                   if eqcheck = 0 then
                       return k-1;
                   fi:
                   eqcheck := evalf(expr - theRoots[k]);
                   if abs(eqcheck) < 1.0e-10 then
                       return k-1;
                   fi:
               od:
               return -1;
               end proc:

  makePermutation := proc(i::integer, theRoots)
                     local j, result, idx, perm;
                     perm := [];
                     for j to nops(theRoots) do
                         result := tau(i, theRoots[j]);
                         idx := whichRoot(result, theRoots);
                         perm := [op(perm), idx];
                     od:
                     return perm;
                     end proc:


  printf("Question 1:\n");
```

```
printf("\nPermutations of the F-roots A0 to A5:\n");
for i from 1 to 6 do
    pF;
    pF := makePermutation(i, fRoots);
    printf("\ntau_%d => %a\n", i-1, pF);
od:

g := x^2 + 31:
Bg := solve(g=0, x):
Broots := [Bg];
B0 := Broots[1]:
B1 := Broots[2]:
printf("Question 2:\n"):
printf("Permutations of the G-roots B0 to B1:\n");
for i from 1 to 6 do
    pB;
    pB := makePermutation(i, Broots);
    printf("\ntau_%d => %a\n", i-1, pB);
od:

h := x^3 + x + 1:
Ch := solve(h=0, x):
Croots := [Ch];
C0 := Croots[1]:
C1 := Croots[2]:
C2 := Croots[3]:
printf("Question 3:\n"):
printf("Permutations of the H-roots C0 to C2:\n");
for i from 1 to 6 do
    pC;
    pC := makePermutation(i, Croots);
    printf("\ntau_%d => %a\n", i-1, pC);
od:

m := x^5 - 5*x^4 + 42*x^3 - 105*x^2 + 130*x - 32:
Dm := solve(m=0, x):
Droots := [Dm];
D0 := Droots[1]:
D1 := Droots[2]:
D2 := Droots[3]:
D3 := Droots[4]:
D4 := Droots[5]:
printf("Question 4:\n"):
printf("Permutations of the M-roots D0 to D4:\n");
for i from 1 to 6 do
    pD;
    pD := makePermutation(i, Droots);
    printf("\ntau_%d => %a\n", i-1, pD);
```

```
end do:
```

$$fRoots := \left[\frac{1}{6\left(188+12\sqrt{93}\right)^{1/3}} \left(\mathrm{I} \sqrt{1128+72\sqrt{93}+39\left(188+12\sqrt{93}\right)^{2/3}+168\left(188+12\sqrt{93}\right)^{1/3}}+3\left(188+12\sqrt{93}\right)^{1/3} \right), \right.$$

$$\frac{1}{6\left(188+12\sqrt{93}\right)^{1/3}} \left(-\mathrm{I} \sqrt{1128+72\sqrt{93}+39\left(188+12\sqrt{93}\right)^{2/3}+168\left(188+12\sqrt{93}\right)^{1/3}}+3\left(188+12\sqrt{93}\right)^{1/3} \right),$$

$$\frac{1}{6\left(188+12\sqrt{93}\right)^{1/3}} \left(3\left(188+12\sqrt{93}\right)^{1/3} + \left(84\,\mathrm{I}\left(188+12\sqrt{93}\right)^{1/3}\sqrt{3}-36\,\mathrm{I}\sqrt{3}\sqrt{93}-39\left(188+12\sqrt{93}\right)^{2/3}-564\,\mathrm{I}\sqrt{3}+84\left(188+12\sqrt{93}\right)^{1/3}+36\sqrt{93}+564 \right)^{1/2} \right),$$

$$\frac{1}{6\left(188+12\sqrt{93}\right)^{1/3}} \left(3\left(188+12\sqrt{93}\right)^{1/3} - \left(84\,\mathrm{I}\left(188+12\sqrt{93}\right)^{1/3}\sqrt{3}-36\,\mathrm{I}\sqrt{3}\sqrt{93}-39\left(188+12\sqrt{93}\right)^{2/3}-564\,\mathrm{I}\sqrt{3}+84\left(188+12\sqrt{93}\right)^{1/3}+36\sqrt{93}+564 \right)^{1/2} \right),$$

$$\frac{1}{6\left(188+12\sqrt{93}\right)^{1/3}} \left(3\left(188+12\sqrt{93}\right)^{1/3} - \left(-84\,\mathrm{I}\left(188+12\sqrt{93}\right)^{1/3}\sqrt{3}+36\,\mathrm{I}\sqrt{3}\sqrt{93}-39\left(188+12\sqrt{93}\right)^{2/3}+564\,\mathrm{I}\sqrt{3}+84\left(188+12\sqrt{93}\right)^{1/3}+36\sqrt{93}+564 \right)^{1/2} \right),$$

$$\left. \frac{1}{6\left(188+12\sqrt{93}\right)^{1/3}} \left(3\left(188+12\sqrt{93}\right)^{1/3} + \left(-84\,\mathrm{I}\left(188+12\sqrt{93}\right)^{1/3}\sqrt{3}+36\,\mathrm{I}\sqrt{3}\sqrt{93}-39\left(188+12\sqrt{93}\right)^{2/3}+564\,\mathrm{I}\sqrt{3}+84\left(188+12\sqrt{93}\right)^{1/3}+36\sqrt{93}+564 \right)^{1/2} \right) \right]$$

Question 1:

Permutations of the F-roots A0 to A5:

tau_0 => [0, 1, 2, 3, 4, 5]

tau_1 => [1, 1, 2, 3, 4, 5]

tau_2 => [2, 1, 2, 3, 4, 5]

tau_3 => [3, 1, 2, 3, 4, 5]

tau_4 => [4, 1, 2, 3, 4, 5]

tau_5 => [5, 1, 2, 3, 4, 5]

$$Broots := \left[I\sqrt{31}, -I\sqrt{31} \right]$$

Question 2:
Permutations of the G-roots B0 to B1:

tau_0 => [0, 1]

tau_1 => [0, 1]

tau_2 => [0, 1]

tau_3 => [0, 1]

tau_4 => [0, 1]

tau_5 => [0, 1]

$$Croots := \left[-\frac{\left(108 + 12\sqrt{93}\right)^{1/3}}{6} + \frac{2}{\left(108 + 12\sqrt{93}\right)^{1/3}}, \frac{\left(108 + 12\sqrt{93}\right)^{1/3}}{12} \right.$$

$$\left. -\frac{1}{\left(108 + 12\sqrt{93}\right)^{1/3}} + \frac{I\sqrt{3}\left(-\frac{\left(108 + 12\sqrt{93}\right)^{1/3}}{6} - \frac{2}{\left(108 + 12\sqrt{93}\right)^{1/3}} \right)}{2}, \right.$$

$$\left. \frac{\left(108 + 12\sqrt{93}\right)^{1/3}}{12} - \frac{1}{\left(108 + 12\sqrt{93}\right)^{1/3}} \right]$$

$$-\frac{I\sqrt{3}\left(-\frac{\left(108+12\sqrt{93}\right)^{1/3}}{6}-\frac{2}{\left(108+12\sqrt{93}\right)^{1/3}}\right)}{2}\Bigg]$$

Question 3:
Permutations of the H-roots C0 to C2:

tau_0 => [0, 1, 2]

tau_1 => [0, 1, 2]

tau_2 => [0, 1, 2]

tau_3 => [0, 1, 2]

tau_4 => [0, 1, 2]

tau_5 => [0, 1, 2]

$$Droots := \Bigg[1-I\sqrt{31},\ 1+I\sqrt{31},\ -\frac{\left(108+12\sqrt{93}\right)^{1/3}}{6}+\frac{2}{\left(108+12\sqrt{93}\right)^{1/3}}+1,$$

$$\frac{\left(108+12\sqrt{93}\right)^{1/3}}{12}-\frac{1}{\left(108+12\sqrt{93}\right)^{1/3}}+1$$

$$+\frac{I\sqrt{3}\left(-\frac{\left(108+12\sqrt{93}\right)^{1/3}}{6}-\frac{2}{\left(108+12\sqrt{93}\right)^{1/3}}\right)}{2},\ \frac{\left(108+12\sqrt{93}\right)^{1/3}}{12}$$

$$-\frac{1}{\left(108+12\sqrt{93}\right)^{1/3}}+1-\frac{I\sqrt{3}\left(-\frac{\left(108+12\sqrt{93}\right)^{1/3}}{6}-\frac{2}{\left(108+12\sqrt{93}\right)^{1/3}}\right)}{2}\Bigg]$$

Question 4:
Permutations of the M-roots D0 to D4:

tau_0 => [0, 1, 2, 3, 4]

tau_1 => [0, 1, 2, 3, 4]

tau_2 => [0, 1, 2, 3, 4]

```
tau_3 => [0, 1, 2, 3, 4]

tau_4 => [0, 1, 2, 3, 4]

tau_5 => [0, 1, 2, 3, 4]
```