```
> (* Mantej Sokhi *)
```

QUESTION 4A:

```
> restart:
  with(Groebner):
> F := [x+y+z-3,x^2+y^2+z^2-5,x^3+y^3+z^3-7]:
  G := Basis(F,plex(x,y,z)):
  G;
```

$$\left[3z^3 - 9z^2 + 6z + 2,\ y^2 + yz + z^2 - 3y - 3z + 2,\ x + y + z - 3\right] \tag{1}$$

```
> NF := NormalForm(x^4+y^4+z^4-9,G,plex(x,y,z)):
  NF;
```

$$0 \tag{2}$$

```
> NF2 := NormalForm(x^5+y^5+z^5,G,plex(x,y,z)):
  NF2;
```

$$\frac{29}{3} \tag{3}$$

QUESTION 4B:

```
> restart:
  with(Groebner):
> f := x^3+(2*x*y*z)-z^2:
  g := x^2+y^2+z^2-1:
  L := expand(f-lambda*(g)):
  L;
```

$$-\lambda x^2 - \lambda y^2 - \lambda z^2 + x^3 + 2xyz - z^2 + \lambda \tag{4}$$

```
> LX := diff(L,x):
  LY := diff(L,y):
  LZ := diff(L,z):
> LX;
  LY;
  LZ;
```

$$-2\lambda x + 3x^2 + 2yz$$

$$-2\lambda y + 2xz$$

$$-2\lambda z + 2xy - 2z \tag{5}$$

```
> IDE := [LX,LY,LZ,g]:
  GB := Basis(IDE,plex(lambda,x,y,z)):
  GB;
```

$$\left[1152z^7 - 1763z^5 + 655z^3 - 44z,\ -1152z^6 + 118yz^3 + 1605z^4 - 118yz - 453z^2,\right. \tag{6}$$
$$\left. -6912z^5 + 3835y^2z + 10751z^3 - 3839z,\ -9216z^5 + 3835y^3 + 3835yz^2\right.$$

$$+ 11778\,z^3 - 3835\,y - 2562\,z,\ -1152\,z^5 + 3835\,y\,z^2 - 1404\,z^3 + 3835\,x\,z + 2556\,z,$$
$$-19584\,z^5 + 25987\,z^3 + 3835\,x\,y - 6403\,z,\ x^2 + y^2 + z^2 - 1,\ -335232\,z^6 + 477321\,z^4$$
$$- 11505\,y\,z - 134419\,z^2 + 7670\,\lambda - 11505\,x\Big]$$

```
> SYSSOLVE := proc(GB)

  local zRoots,yRoots,xRoots,zi,yi,xi,pts,tol,constCheck,fVal,res,
  i,xLoc,yLoc,zLoc,maxVal:

  zRoots := [fsolve(GB[1])]:
  pts := []:
  fVal := []:
  tol := 1e8:

  for zi in zRoots do:
     if zi=0 then:
        pts := [op(pts),([0,-1,0],[0,1,0],[1,0,0],[-1,0,0])]:
      fi:
     yRoots := [fsolve(subs(z=zi,GB[2]))]:
     if nops(yRoots)=0 then:
        yRoots := [fsolve(subs(z=zi,GB[3]))]:
      fi:
     for yi in yRoots do:
        xRoots := [fsolve(subs(z=zi,y=yi,GB[5]))]:
        for xi in xRoots do:
           constCheck := xi^2+yi^2+zi^2:
           if abs(constCheck) <= tol then
                pts := [op(pts),[xi,yi,zi]]:
              fi:
         od:
     od:
  od:

  res := convert(pts,set):
  res := convert(res,list):
  for i from 1 to nops(res) do:
     xLoc := res[i][1]:
     yLoc := res[i][2]:
     zLoc := res[i][3]:

    fVal := [op(fVal),xLoc^3+2*(xLoc*yLoc*zLoc)-zLoc^2]:
  od:

  maxVal := max(fVal):
```

```
  return maxVal,res:
  end proc:
> maxVal,solList := SYSSOLVE(GB):
  maxVal;
  solList;
```

$$1$$

$$[[-1, 0, 0], [0, -1, 0], [0, 1, 0], [1, 0, 0], [-0.6666666668, -0.3333333337, -0.6666666667], [-0.6666666668, 0.3333333337, 0.6666666667], [-0.3750000003, -0.8794529551, 0.2931509850], [-0.3750000003, 0.8794529551, -0.2931509850], [0., 0., -1.], [0., 0., 1.]] \quad (7)$$

QUESTION 4C:

```
> restart:
  with(Groebner):
> r := m/2:
  f1 := (x[1]-x[2])^2+(y[1]-y[2])^2-m^2:
  f2 := (x[1]-x[3])^2+(y[1]-y[3])^2-m^2:
  f3 := (x[2]-x[3])^2+(y[2]-y[3])^2-m^2:
  f4 := y[1]+r-1:
  f5 := y[2]-r:
  f6 := x[2]-r:
  f7 := x[3]+r-1:
> y[11] := 1-r:
  y[22] := r:
  x[22] := r:
  x[33] := 1-r:
> f1 := simplify(subs(y[1]=y[11],y[2]=y[22],x[2]=x[22],x[3]=x[33],
  f1)):
  f2 := simplify(subs(y[1]=y[11],y[2]=y[22],x[2]=x[22],x[3]=x[33],
  f2)):
  f3 := simplify(subs(y[1]=y[11],y[2]=y[22],x[2]=x[22],x[3]=x[33],
  f3)):
  f4 := m*z-1:
> f1;
  f2;
  f3;
  f4;
```

$$1 + \frac{m^2}{4} + \left(-2 - x_1\right) m + x_1^2$$

$$2 - \frac{m^2}{2} + \left(x_1 + y_3 - 2\right) m + x_1^2 + y_3^2 - 2\,x_1 - 2\,y_3$$

$$1 + \frac{m^2}{4} + \left(-2 - y_3\right) m + y_3^2$$

$$m\,z - 1 \qquad (8)$$

```
> F := [f1,f2,f3,f4]:
  GB := Basis(F,plex(z,x[1],y[3],m)):
  GB;
```

$$\left[m^4 - 32\,m^3 + 80\,m^2 - 64\,m + 16,\ -m^3 + 31\,m^2 - 50\,m + 4\,y_3 + 16,\ -m^3 + 31\,m^2 - 50\,m + 4\,x_1 + 16,\ m^3 - 32\,m^2 + 80\,m + 16\,z - 64\right] \qquad (9)$$

```
> p1 := factor(GB[1]):
  p1;
```

$$m^4 - 32\,m^3 + 80\,m^2 - 64\,m + 16 \qquad (10)$$

```
> rootP := [fsolve(p1,m)]:
  rootP;
```

$$[0.5086661901,\ 0.7943953532,\ 1.349198186,\ 29.34774027] \qquad (11)$$

```
> m := rootP[1]:
  m;
```

$$0.5086661901 \qquad (12)$$