```
> (* Mantej Sokhi *)
```

QUESTION 2A:

```
> restart:
  with(NumberTheory):
  with(PolynomialTools):
> m := z^4+z^3+z^2+z+1:
  alias(omega=RootOf(m,z)):
> m1 := MinimalPolynomial(omega,z):
  m2 := MinimalPolynomial(omega^2,z):
  m1;
  m2;
```

$$z^4 + z^3 + z^2 + z + 1$$

$$z^4 + z^3 + z^2 + z + 1 \tag{1}$$

QUESTION 2B:

```
> inv := gcdex(z^4+3*z^3,m,z,'s'):
> inv;
  s;
```

$$1$$

$$-\frac{9}{61} z^3 + \frac{18}{61} z^2 - \frac{2}{61} z - \frac{3}{61} \tag{2}$$

```
> x := subs(z=omega,rem(s*(z^3+1),m,z)):
  x;
```

$$-\frac{10}{61} \omega^3 + \frac{20}{61} \omega^2 - \frac{9}{61} \omega + \frac{17}{61} \tag{3}$$

```
> evala(1/(omega^4+3*omega^3));
```

$$-\frac{9}{61} \omega^3 + \frac{18}{61} \omega^2 - \frac{2}{61} \omega - \frac{3}{61} \tag{4}$$

```
> x := 'x':
  res := solve({(omega+4)*x+omega*y=1,omega^3*x+omega^4*y=-1},{x,y}
  ) :
> res;
```

$$\left\{ x = -\frac{10}{61} \omega^3 + \frac{20}{61} \omega^2 - \frac{9}{61} \omega + \frac{17}{61}, y = \frac{17}{61} \omega^3 + \frac{27}{61} \omega^2 - \frac{64}{61} \omega + \frac{26}{61} \right\} \tag{5}$$

```
> res := convert(omega,radical):
  res;
```

$$\frac{1}{4} \sqrt{5} - \frac{1}{4} + \frac{1}{4} I \sqrt{2} \sqrt{5 + \sqrt{5}} \tag{6}$$

```
> res := evalf(omega):
  res;
```
$$0.3090169944 + 0.9510565163\,I \qquad\qquad (7)$$

QUESTION 2C:

```
> a := x^2+(omega^3+omega^2+2*omega)*x-omega:
  b := x^2+(-omega^2+omega)*x+omega^2:
> gcd(a,b);
```
$$\omega^3 + \omega + x \qquad\qquad (8)$$

```
> aFact := factor(a):
  bFact := factor(b):
  aFact;
  bFact;
```
$$\left(\omega^3 + \omega + x\right)\left(\omega^2 + \omega + x\right)$$
$$-\left(\omega^3 + \omega^2 - x\right)\left(\omega^3 + \omega + x\right) \qquad\qquad (9)$$