

```
> (* Mantej Sokhi *)
```

QUESTION 5A:

```
> restart:
```

```
> minP := z^2+1:  
alias(omega=RootOf(minP)):
```

```
> N := proc(f)  
local res:  
  
res := resultant(minP,subs(omega=z,f),z):  
return res:  
end proc:
```

```
> f1 := x^4+x^2+2*x+1:  
f1 := unapply(f1,x):  
normF1 := N(f1(x)):  
normF1;
```

$$(x^4 + x^2 + 2x + 1)^2 \quad (1)$$

```
> r := N(f1(x-4*omega)):  
r ;
```

$$x^8 + 66x^6 + 4x^5 + 1571x^4 - 124x^3 + 15718x^2 - 3004x + 58145 \quad (2)$$

```
> sqFree := gcd(r,diff(r,x)):  
sqFree;
```

$$1 \quad (3)$$

```
> rFactorQ := factor(r):  
rFactorQ;
```

$$(x^4 + 25x^2 - 14x + 145) (x^4 + 41x^2 + 18x + 401) \quad (4)$$

```
> b1,b2 := op(%):
```

```
> fact1 := gcd(f1(x-4*omega),b1,'q1'):  
fact1;  
q1;
```

$$\begin{aligned} -7x\omega + x^2 + \omega - 12 \\ -9x\omega + x^2 - \omega - 20 \end{aligned} \quad (5)$$

```
> fact2 := gcd(q1,b2,'q2'):  
fact2;  
q2;
```

$$\begin{aligned} -9x\omega + x^2 - \omega - 20 \\ 1 \end{aligned} \quad (6)$$

```
> fact1 := subs(x=x+4*omega,fact1):
```

```

fact2 := subs(x=x+4*omega,fact2):
fact1;
fact2;

```

$$\begin{aligned}
& -7(x+4\omega)\omega + (x+4\omega)^2 + \omega - 12 \\
& -9(x+4\omega)\omega + (x+4\omega)^2 - \omega - 20
\end{aligned}$$

(7)

```

> f(x)=fact1*fact2:
> mapFact := factor(f1(x),omega):
mapFact;

```

$$-(x\omega + x^2 + \omega)(x\omega - x^2 + \omega)$$

(8)

```

> evalRes := evala(Expand(fact1*fact2)):
evalRes;

```

$$x^4 + x^2 + 2x + 1$$

(9)

```

> compF1 := evala(fact1):
compF2 := evala(fact2):
compF1;
compF2;

```

$$\begin{aligned}
& x\omega + x^2 + \omega \\
& -x\omega + x^2 - \omega
\end{aligned}$$

(10)

```

> restart:
> minP := z^2+1:
alias(omega=RootOf(minP)):

```

```

> N := proc(f)
local res:

```

```

res := resultant(minP,subs(omega=z,f),z):
return res:
end proc:

```

```

> f2 := x^4+(2*omega*x^3)-x^2+1:
f2 := unapply(f2,x):
normF2 := N(f2(x)):
normF2;

```

$$x^8 + 2x^6 + 3x^4 - 2x^2 + 1$$

(11)

```

> r := normF2:
r;

```

$$x^8 + 2x^6 + 3x^4 - 2x^2 + 1$$

(12)

```

> sqFree := gcd(r,diff(r,x)):
sqFree;

```

(13)

```
> rFactorQ := factor(r):
rFactorQ;
```

$$(x^4 + x^2 + 2x + 1) (x^4 + x^2 - 2x + 1)$$

(14)

```
> b1,b2 := op(%):
> fact1 := gcd(f2(x),b1,'q1'):
fact1;
q1;
```

$$\omega x + x^2 + \omega$$

$$\omega x + x^2 - \omega$$

(15)

```
> fact2 := gcd(q1,b2,'q2'):
fact2;
q2;
```

$$\omega x + x^2 - \omega$$

$$1$$

(16)

```
> f(x)=fact1*fact2:
> mapFact := factor(f2(x),omega):
mapFact;
```

$$(\omega x + x^2 + \omega) (\omega x + x^2 - \omega)$$

(17)

```
> evalRes := evala(Expand(fact1*fact2)):
evalRes;
```

$$2\omega x^3 + x^4 - x^2 + 1$$

(18)

```
> compF1 := evala(fact1):
compF2 := evala(fact2):
compF1;
compF2;
```

$$\omega x + x^2 + \omega$$

$$\omega x + x^2 - \omega$$

(19)

QUESTION 5B:

```
> restart:
> minP := z^2+1:
alias(omega=RootOf(minP)):
> N := proc(f)
local res:

res := resultant(minP,subs(omega=z,f),z):
return res:
end proc:
```

```

> BADS := proc(paramA::polynom,omega,minP)
  local f,s,shiftF,resShiftF,diffRes,res,newRest:

  f := paramA:
  f := unapply(f,x):
  s := 's':
  shiftF := expand(f(x-s*omega)):
  resShiftF := resultant(minP,subs(omega=z,shiftF),z):
  diffRes := diff(resShiftF,x):
  newRest := resultant(resShiftF,diffRes,x):
  res := [solve(newRest=0,s)]:

  return res:
end proc:

```

```

> f1 := x^4+x^2+(2*x)+1:
f2 := x^4+(2*omega*x^3)-x^2+1:

```

```

> res1 := BADS(f1,omega,minP):
res2 := BADS(f2,omega,minP):
res1;
res2;

```

$$\begin{aligned}
& \left[0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2} \sqrt{1-4I}, -\frac{1}{2} \sqrt{1-4I}, \frac{1}{2} \sqrt{1-4I}, -\frac{1}{2} \sqrt{1-4I}, \right. \\
& \quad \left. \frac{1}{2} \sqrt{1+4I}, -\frac{1}{2} \sqrt{1+4I}, \frac{1}{2} \sqrt{1+4I}, -\frac{1}{2} \sqrt{1+4I}, \right. \\
& \quad \sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=1)}, \\
& \quad -\sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=1)}, \\
& \quad \sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=2)}, \\
& \quad -\sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=2)}, \\
& \quad \sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=3)}, \\
& \quad -\sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=3)}, \\
& \quad \sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=4)}, \\
& \quad -\sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=4)}, \\
& \quad \sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=1)}, \\
& \quad \left. -\sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=1)}, \right]
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=2)}, \\
& -\sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=2)}, \\
& \sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=3)}, \\
& -\sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=3)}, \\
& \sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=4)}, \\
& -\sqrt{\text{RootOf}(16_Z^4 - 24_Z^3 + _Z^2 - 11_Z + 1, \text{index}=4)} \Big]
\end{aligned}$$

$$\begin{aligned}
& \left[\frac{1}{2} - \frac{1}{2} \sqrt{1+4I}, \frac{1}{2} + \frac{1}{2} \sqrt{1+4I}, \frac{1}{2} - \frac{1}{2} \sqrt{1+4I}, \frac{1}{2} + \frac{1}{2} \sqrt{1+4I}, \frac{1}{2} \right. \\
& - \frac{1}{2} \sqrt{1-4I}, \frac{1}{2} + \frac{1}{2} \sqrt{1-4I}, \frac{1}{2} - \frac{1}{2} \sqrt{1-4I}, \frac{1}{2} + \frac{1}{2} \sqrt{1-4I}, \frac{1}{2} \\
& - \frac{1}{4} \sqrt{2+2\sqrt{17}}, \frac{1}{2} + \frac{1}{4} \sqrt{2+2\sqrt{17}}, \frac{1}{2} - \frac{1}{4} I \sqrt{-2+2\sqrt{17}}, \frac{1}{2} \\
& + \frac{1}{4} I \sqrt{-2+2\sqrt{17}}, \frac{1}{2} - \frac{1}{4} \sqrt{2+2\sqrt{17}}, \frac{1}{2} + \frac{1}{4} \sqrt{2+2\sqrt{17}}, \frac{1}{2} \\
& - \frac{1}{4} I \sqrt{-2+2\sqrt{17}}, \frac{1}{2} + \frac{1}{4} I \sqrt{-2+2\sqrt{17}}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\
& - \frac{1}{4} \sqrt{2+2\sqrt{17}}, \frac{1}{2} + \frac{1}{4} \sqrt{2+2\sqrt{17}}, \frac{1}{2} - \frac{1}{4} I \sqrt{-2+2\sqrt{17}}, \frac{1}{2} \\
& + \frac{1}{4} I \sqrt{-2+2\sqrt{17}}, \frac{1}{2} - \frac{1}{4} \sqrt{2+2\sqrt{17}}, \frac{1}{2} + \frac{1}{4} \sqrt{2+2\sqrt{17}}, \frac{1}{2} \\
& \left. - \frac{1}{4} I \sqrt{-2+2\sqrt{17}}, \frac{1}{2} + \frac{1}{4} I \sqrt{-2+2\sqrt{17}}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right] \quad (20)
\end{aligned}$$