

Question 5A:

```
> restart:  
with(LinearAlgebra):  
> vec_b := Vector[column](datatype=integer[4],[1,1,0,1,1]);
```

$$\text{vec_b} := \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

(1)

```
> mat_A := Matrix(5,5,readonly=false,datatype=integer[4],[[0,1,0,0,  
1],[1,0,1,0,0],[0,1,0,1,0],[0,0,1,0,1],[1,0,0,0,1]]);
```

$$\text{mat_A} := \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(2)

```
> det_mat_A := Determinant(mat_A,method=float):  
printf("\n The determinant of the matrix A is: %5.5f.\n",  
det_mat_A);
```

The determinant of the matrix A is: 2.00000.

```

> mat_B := MatrixInverse(mat_A,method='LU');
> compute_and_check := proc(a::Matrix,b::Matrix)
  local var_A, var_B, compute_mat_product;
  var_A := a;
  var_B := b;
  compute_mat_product := MatrixMatrixMultiply(var_A, var_B);
  print(compute_mat_product);
  if IsSimilar(compute_mat_product,IdentityMatrix(5)) = true then
    return printf("\n Yes. The matrix AB = I.\n");
  else
    return "No. The matrix product AB does not equal I.";
  end if;
end proc;
> compute_and_check(mat_A,mat_B);

```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Yes. The matrix AB = I.

```

> e_val_mat_A := Eigenvalues(mat_A,output='Vector[column]');

```

$e_val_mat_A :=$

$$\begin{bmatrix} 1 \\ 2 \\ -1 \\ \frac{1}{2}\sqrt{5} - \frac{1}{2} \\ -\frac{1}{2}\sqrt{5} - \frac{1}{2} \end{bmatrix}$$

(3)

> solve_sys := LinearSolve(mat_A,vec_b,method=QR,conjugate=true);

(4)

solve_sys :=

$$\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

(4)

> x := 'x':

> char_mat_A := CharacteristicMatrix(mat_A, x);

(5)

$$char_mat_A := \begin{bmatrix} x & -1 & 0 & 0 & -1 \\ -1 & x & -1 & 0 & 0 \\ 0 & -1 & x & -1 & 0 \\ 0 & 0 & -1 & x & -1 \\ -1 & 0 & 0 & 0 & x-1 \end{bmatrix} \quad (5)$$

> det_char_mat_A := Determinant(char_mat_A);

$$det_char_mat_A := x^5 - x^4 - 4x^3 + 3x^2 + 3x - 2 \quad (6)$$

> solve_for_char := solve(factor(det_char_mat_A)=0);

$$solve_for_char := -1, 1, 2, \frac{1}{2}\sqrt{5} - \frac{1}{2}, -\frac{1}{2}\sqrt{5} - \frac{1}{2} \quad (7)$$