```
> (* Mantej Sokhi *)
```

QUESTION 1B:

```
> restart:
  with(LinearAlgebra):
> alpha := 1+sqrt(2)+sqrt(3):
  basisA := [1,sqrt(2),sqrt(3),sqrt(6)]:

> expand(alpha^0);
  expand(alpha^1);
  expand(alpha^2);
  expand(alpha^3);
  expand(alpha^4);
```

$$1$$

$$1+\sqrt{2}+\sqrt{3}$$

$$6+2\sqrt{2}+2\sqrt{3}+2\sqrt{2}\,\sqrt{3}$$

$$16+14\sqrt{2}+12\sqrt{3}+6\sqrt{2}\,\sqrt{3}$$

$$80+48\sqrt{2}+40\sqrt{3}+32\sqrt{2}\,\sqrt{3} \qquad \textbf{(1)}$$

```
> cord1 := <1,0,0,0>:
  cord2 := <1,1,1,0>:
  cord3 := <6,2,2,2>:
  cord4 := <16,14,12,6>:
  cord5 := <80,48,40,32>:
  matA := <cord1|cord2|cord3|cord4|cord5>:
  matA;
```

$$\begin{bmatrix} 1 & 1 & 6 & 16 & 80 \\ 0 & 1 & 2 & 14 & 48 \\ 0 & 1 & 2 & 12 & 40 \\ 0 & 0 & 2 & 6 & 32 \end{bmatrix} \qquad \textbf{(2)}$$

```
> minP := NullSpace(matA):
  minP;
```

(3)

$$\left\{\begin{bmatrix} -8 \\ 16 \\ -4 \\ -4 \\ 1 \end{bmatrix}\right\} \tag{3}$$

```
> (* We can just read off the minP now *)
  (* z^4-4*z^3-4*z^2+16*z-8 *)
```

```
> restart:
  with(Groebner):
```

```
> ID := [z-1-s-t,s^2-2,t^2-3]:
```

```
> GB := Basis(ID,plex(s,t,z)):
```

```
> GB;
```

$$\left[z^4 - 4z^3 - 4z^2 + 16z - 8,\ z^3 - 3z^2 + 2t - 8z + 10,\ -z^3 + 3z^2 + 2s + 6z - 8\right] \tag{4}$$

```
> res := GB[1]:
  res;
```

$$z^4 - 4z^3 - 4z^2 + 16z - 8 \tag{5}$$

```
> temp := gcdex(z,res,z,'s','t'):
  s;
```

$$\frac{1}{8}z^3 - \frac{1}{2}z^2 - \frac{1}{2}z + 2 \tag{6}$$

```
> check := rem(s*z,res,z):
  check;
```

$$1 \tag{7}$$