```
> (* Mantej Sokhi *)
```

QUESTION 3A:

```
> restart:
  with(Groebner):
> IDE[1] := [(x*y)-z,(x*z)-y]:
  IDE[2] := [x+1,(x*y)+1,y-1]:
> SP[1] := SPolynomial(IDE[1][1],IDE[1][2],plex(x,y,z)):
  SP[1];
```

$$y^2 - z^2 \tag{1}$$

```
> (* Using THM. 6 since r != 0 we conclude that G=IDE[1] is not a
  GB for IDE[1] *)

  NF[1] := NormalForm(SP[1],IDE[1],plex(x,y,z)):
  NF[1];
```

$$y^2 - z^2 \tag{2}$$

```
> G[2] := IDE[2]:
  for i from 1 to nops(G[2]) do:
     for j from (i+1) to nops(G[2]) do:
        r := NormalForm(SPolynomial(G[2][i],G[2][j],plex(x,y,z)),
  G[2],plex(x,y,z)):
        printf("S(f[%d],f[%d]) mod G[2] = %a\n",i,j,r);
     od:
  od:

S(f[1],f[2]) mod G[2] = 0
S(f[1],f[3]) mod G[2] = 0
S(f[2],f[3]) mod G[2] = 0
```

```
> (* This is not minimal as LT(f1) divides LT(f2). It is also not
  reduced. *)
  GB := G[2]:
  GB;
```

$$[x+1, xy+1, y-1] \tag{3}$$

QUESTION 3B:

```
> restart:
  with(Groebner):
> (* For Monomial Ordering: GRLEX> where x>y>z *)
  IDE := [x-z^2,y-z^3]:
  G[1] := IDE:
> SP[1] := SPolynomial(IDE[1],IDE[2],grlex(x,y,z)):
  SP[1];
```

$$-xz+y \tag{4}$$

```
> NF[1] := NormalForm(SP[1],G[1],grlex(x,y,z)):
  NF[1];
```

$$-xz+y \tag{5}$$

```
> G[2] := [op(G[1]),NF[1]]:
  G[2];
```

$$\left[-z^2+x,\ -z^3+y,\ -xz+y\right] \tag{6}$$

```
> for i from 1 to nops(G[2]) do:
      for j from (i+1) to nops(G[2]) do:
         r := NormalForm(SPolynomial(G[2][i],G[2][j],grlex(x,y,z)
  ),G[2],grlex(x,y,z)):
         printf("S(f[%d],f[%d]) mod G[2] = %a\n",i,j,r);
      od:
  od:

S(f[1],f[2]) mod G[2] = 0
S(f[1],f[3]) mod G[2] = -x^2+y*z
S(f[2],f[3]) mod G[2] = 0
```

```
> G[3] := [op(G[2]),-x^2+y*z]:
  G[3];
```

$$\left[-z^2+x,\ -z^3+y,\ -xz+y,\ -x^2+yz\right] \tag{7}$$

```
> for i from 1 to nops(G[3]) do:
      for j from (i+1) to nops(G[3]) do:
         r := NormalForm(SPolynomial(G[3][i],G[3][j],grlex(x,y,z)
  ),G[3],grlex(x,y,z)):
         printf("S(f[%d],f[%d]) mod G[3] = %a\n",i,j,r);
      od:
  od:

  G[3];

S(f[1],f[2]) mod G[3] = 0
S(f[1],f[3]) mod G[3] = 0
S(f[1],f[4]) mod G[3] = 0
S(f[2],f[3]) mod G[3] = 0
S(f[2],f[4]) mod G[3] = 0
S(f[3],f[4]) mod G[3] = 0
```

$$\left[-z^2+x,\ -z^3+y,\ -xz+y,\ -x^2+yz\right] \tag{8}$$

```
> subsop(1=NULL,G[3]);
```

$$\left[-z^3+y,\ -xz+y,\ -x^2+yz\right] \tag{9}$$

```
> for i to nops(G[3]) do:
    G[3][i] := NormalForm(G[3][i],subsop(i=NULL,G[3]),grlex(x,y,
  z));
    print(G[3][i]);
  od:
  G[3];
```

$$-z^2 + x$$

$$0$$

$$-x z + y$$

$$-x^2 + y z$$

$$\left[-z^2 + x,\ 0,\ -x z + y,\ -x^2 + y z\right] \quad \textbf{(10)}$$

```
> (* REDUCED GB *)
  G[3] := -1*G[3]:
  G[3] := [G[3][1],G[3][3],G[3][4]]:
  G[3];
```

$$\left[z^2 - x,\ x z - y,\ x^2 - y z\right] \quad \textbf{(11)}$$

```
> (* CHECKING WITH MAPLE *)
  GB := Basis(IDE,grlex(x,y,z)):
  GB;
```

$$\left[z^2 - x,\ x z - y,\ x^2 - y z\right] \quad \textbf{(12)}$$

```
> restart:
  with(Groebner):
> (* For Monomial Ordering: LEX> where x>y>z *)
  IDE := [x-z^2,y-z^3]:
  G[1] := IDE:
> SP[1] := SPolynomial(IDE[1],IDE[2],plex(x,y,z)):
  SP[1];
```

$$x z^3 - y z^2 \quad \textbf{(13)}$$

```
> NF[1] := NormalForm(SP[1],G[1],plex(x,y,z)):
  NF[1];
```

$$0 \quad \textbf{(14)}$$

```
> for i to nops(G[1]) do:
    G[1][i] := NormalForm(G[1][i],subsop(i=NULL,G[1]),plex(x,y,z)
  ):
  od:
  G[1];
```

$$\left[-z^2 + x,\ -z^3 + y\right] \quad \textbf{(15)}$$

```
> (* CHECKING WITH MAPLE *)
  GB := Basis(IDE,plex(x,y,z)):
```

```
GB;
```

$$\left[-z^3+y,\ -z^2+x\right] \quad (16)$$

QUESTION 3C:

```
> restart:
  with(Groebner):
> IDE := [x*y-1,x*z-1,y*z-1]:
> G[1] := IDE:
> for i from 1 to nops(G[1]) do:
     for j from (i+1) to nops(G[1]) do:
        r := NormalForm(SPolynomial(G[1][i],G[1][j],grlex(x,y,z)
  ),G[1],grlex(x,y,z)):
        printf("S(f[%d],f[%d]) mod G[1] = %a\n",i,j,r);
      od:
  od:

  G[1];
S(f[1],f[2]) mod G[1] = y-z
S(f[1],f[3]) mod G[1] = x-z
S(f[2],f[3]) mod G[1] = x-y
```

$$\left[x\,y-1,\ x\,z-1,\ y\,z-1\right] \quad (17)$$

```
> G[2] := [op(G[1]),(y-z,x-z,x-y)]:
  G[2];
```

$$\left[x\,y-1,\ x\,z-1,\ y\,z-1,\ y-z,\ x-z,\ x-y\right] \quad (18)$$

```
> for i from 1 to nops(G[2]) do:
     for j from (i+1) to nops(G[2]) do:
        r := NormalForm(SPolynomial(G[2][i],G[2][j],grlex(x,y,z)
  ),G[2],grlex(x,y,z)):
        printf("S(f[%d],f[%d]) mod G[2] = %a\n",i,j,r);
      od:
  od:
S(f[1],f[2]) mod G[2] = 0
S(f[1],f[3]) mod G[2] = 0
S(f[1],f[4]) mod G[2] = 0
S(f[1],f[5]) mod G[2] = 0
S(f[1],f[6]) mod G[2] = 0
S(f[2],f[3]) mod G[2] = 0
S(f[2],f[4]) mod G[2] = 0
S(f[2],f[5]) mod G[2] = z^2-1
S(f[2],f[6]) mod G[2] = 0
S(f[3],f[4]) mod G[2] = z^2-1
S(f[3],f[5]) mod G[2] = 0
```

```
S(f[3],f[6]) mod G[2] = 0
S(f[4],f[5]) mod G[2] = 0
S(f[4],f[6]) mod G[2] = 0
S(f[5],f[6]) mod G[2] = 0
```

```
> G[3] := [op(G[2]),z^2-1]:
  G[3];
```

$$\left[x\,y-1,\,x\,z-1,\,y\,z-1,\,y-z,\,x-z,\,x-y,\,z^2-1\right] \tag{19}$$

```
> for i from 1 to nops(G[3]) do:
      for j from (i+1) to nops(G[3]) do:
         r := NormalForm(SPolynomial(G[3][i],G[3][j],grlex(x,y,z)
  ),G[3],grlex(x,y,z)):
         printf("S(f[%d],f[%d]) mod G[3] = %a\n",i,j,r);
      od:
  od:
S(f[1],f[2]) mod G[3] = 0
S(f[1],f[3]) mod G[3] = 0
S(f[1],f[4]) mod G[3] = 0
S(f[1],f[5]) mod G[3] = 0
S(f[1],f[6]) mod G[3] = 0
S(f[1],f[7]) mod G[3] = 0
S(f[2],f[3]) mod G[3] = 0
S(f[2],f[4]) mod G[3] = 0
S(f[2],f[5]) mod G[3] = 0
S(f[2],f[6]) mod G[3] = 0
S(f[2],f[7]) mod G[3] = 0
S(f[3],f[4]) mod G[3] = 0
S(f[3],f[5]) mod G[3] = 0
S(f[3],f[6]) mod G[3] = 0
S(f[3],f[7]) mod G[3] = 0
S(f[4],f[5]) mod G[3] = 0
S(f[4],f[6]) mod G[3] = 0
S(f[4],f[7]) mod G[3] = 0
S(f[5],f[6]) mod G[3] = 0
S(f[5],f[7]) mod G[3] = 0
S(f[6],f[7]) mod G[3] = 0
```

```
> G[3];
```

$$\left[x\,y-1,\,x\,z-1,\,y\,z-1,\,y-z,\,x-z,\,x-y,\,z^2-1\right] \tag{20}$$

```
> for i to nops(G[3]) do:
     G[3][i] := NormalForm(G[3][i],subsop(i=NULL,G[3]),grlex(x,y,
  z)):
  od:
```

```
G[3];
```

$$\left[0, 0, 0, y-z, 0, x-z, z^2-1\right] \quad (21)$$

```
> G[3] := [y-z,x-z,z^2-1]:
  G[3];
```

$$\left[y-z, x-z, z^2-1\right] \quad (22)$$

```
> (* MAPLE CHECK *)
  GB := Basis(IDE,grlex(x,y,z)):
  GB;
```

$$\left[y-z, x-z, z^2-1\right] \quad (23)$$

```
> (* <LT(I)> = <y,x,z^2> *)
```