Calcul numérique des solides et structures non linéaires

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First course

Introduction to non-linearities in mechanics

Two types of non-linearities

- Material non-linearities
  - Stress is no longer proportional to strain
- Geometric non-linearities
  - The hypothesis of small disturbances around a natural state is no longer valid

Purpose of the Course: To calculate with finite element methods in these cases

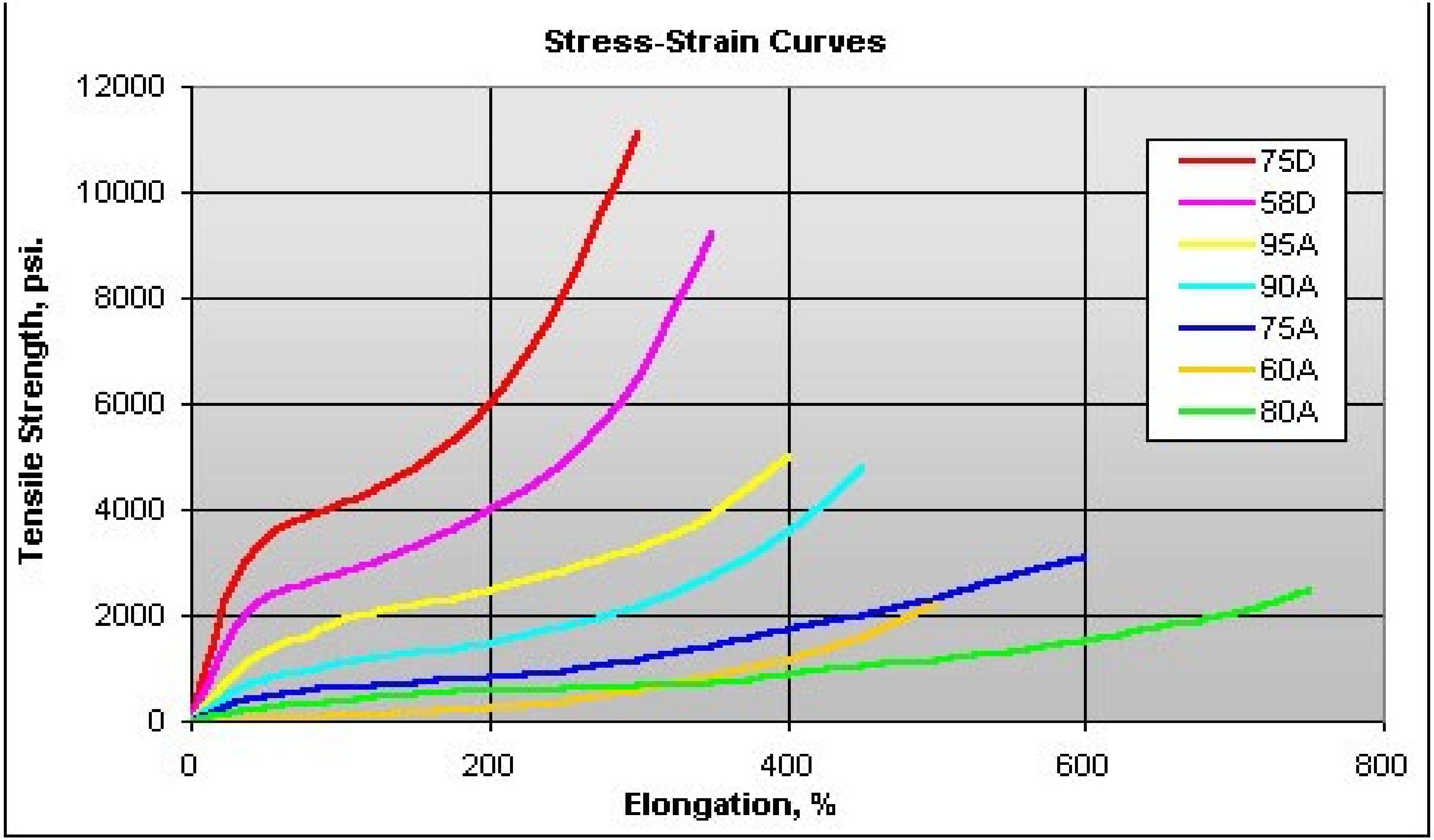
Material Nonlinearities

- Non-linear elasticity
- Plasticity
- (Viscoelasticity and) viscoplasticity
- Damage
- Fatigue
- Dependence of materials / environment

Non-linear elasticity

- Elastomer, rubber, wood
- Same path for loading and unloading
- No dissipation
- Existence of potential
- Deformation energy

Nonlinear stress-strain relationships



Strain energy for the one-dimensional linear case

$$\psi = \frac{1}{2} E e^2$$

$$\sigma = \frac{\partial \psi}{\partial e} = E e$$

For the three-dimensional non-linear case, something like

$$" \sigma_{ij} = \frac{\partial \psi}{\partial e_{ij}} "$$

Hyperelastic material with different models: Neo-Hook, Mooney Rivlin, ...

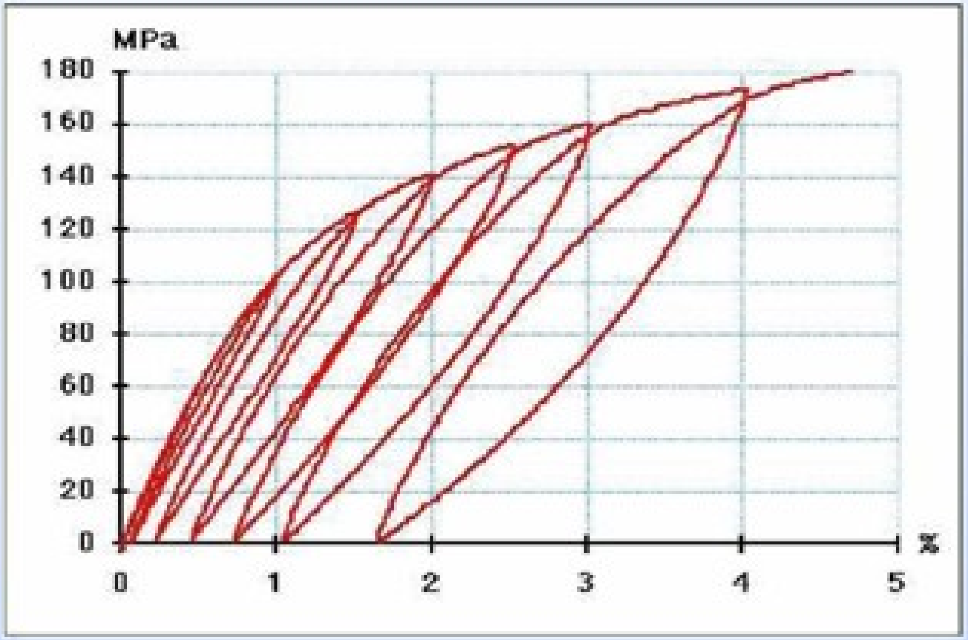
Can be incompressible (rubber)

Plasticity

- Ferrous metals
- Elastic limit
- Plasticity criterion
  - Von Mises 2nd invariant of stress deviator
  - Tresca max (σ_i - σ_k)
- Plastic flow law

Plastic behaviour

Essai



Traction à 45^0 : **Comportement plastique**

Example of plastic behaviour

* When $\sigma < \sigma_0$

$\sigma = E\epsilon$ Reversible linear elastic behaviour

* When $\sigma > \sigma_0$

$$\epsilon = \epsilon^e + \epsilon^p$$

ϵ^e Reversible elastic deformation

ϵ^p Plastic deformation

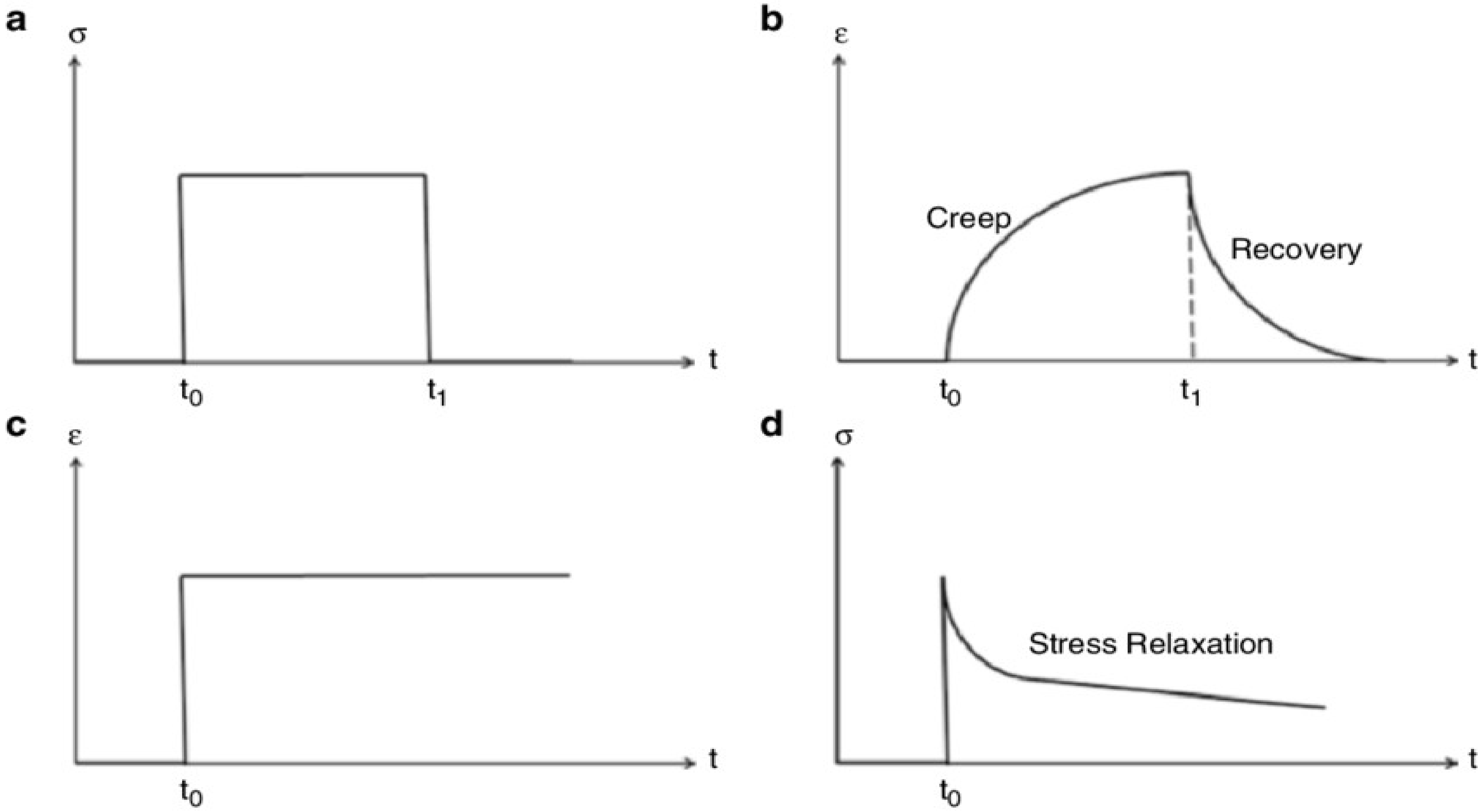
Flow law

$$\dot{\epsilon}^p = f(\sigma)$$

Viscoelasticity - Viscoplasticity

- State of the material that evolves with time. Stress and strain.
- Creep. Constant stress, deformation increases.
- Relaxation. Constant strain, stress decreases.
- Maxwell model
- Kelvin model
- Often coupled with plasticity

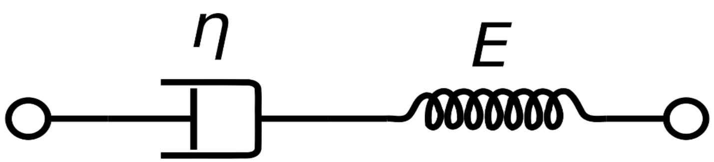
Stress strain relationship



Viscoelasticity models

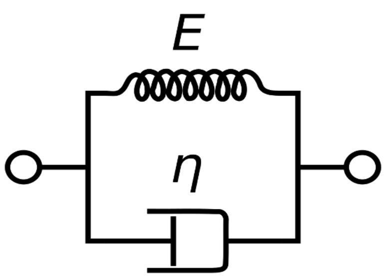
Maxwell model

$$\sigma+\frac{\eta}{E}\dot{\sigma}=\eta\,\dot{\epsilon}$$



Kelvin model

$$\sigma=E\,\epsilon+\eta\,\dot{\epsilon}$$



Damage

- Metals - composites - ceramics
- Plasticity + change of elastic modulus.
- Growth of cavities in the material associated (or not) with plasticity
- Ultimate state ($\varepsilon > 10\%$)

Damage

Cracking

Crack opening and crack growth when stresses exceed a threshold

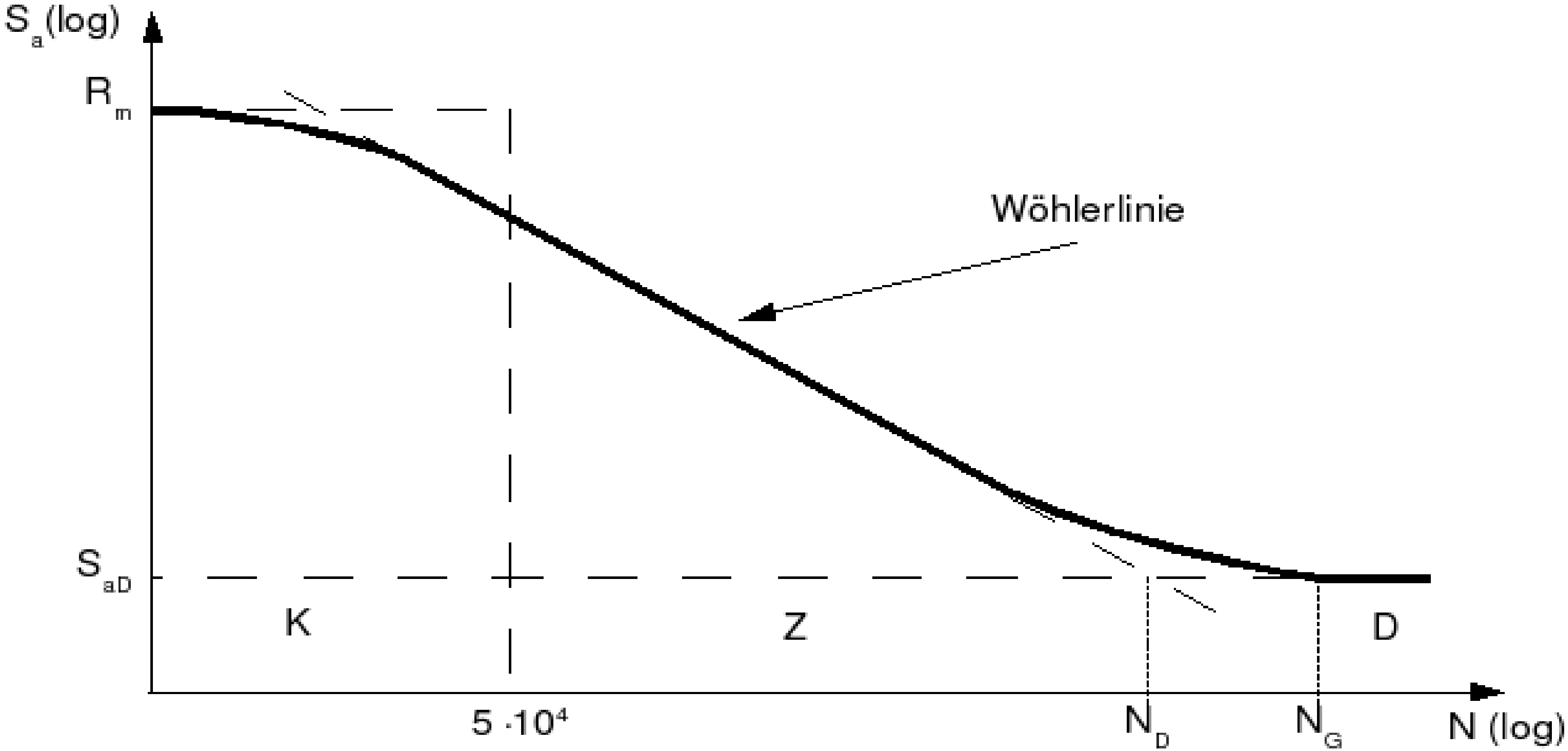
- Slow growth or
- Sudden rupture

Fatigue

- Light alloys, steel
- Aging of the material according to the number and intensity of cycles
- Effect: decreases the capacity of deformation then rupture
- Method: modification of the properties of the material established from tests

Fatigue fracture

S-N Curves



Dependence of materials / environment

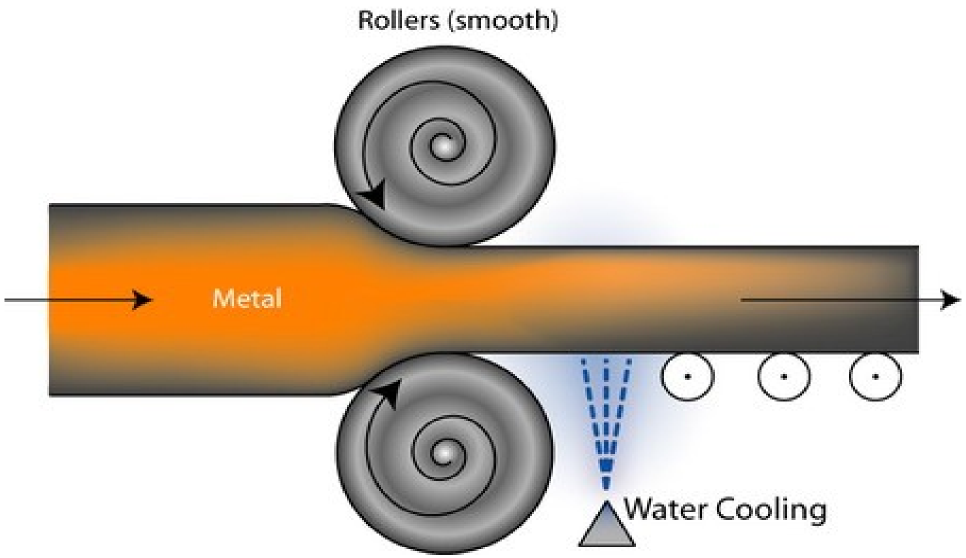
Properties function of

- Temperature
- Strain state
- Chemistry, phase change
- Aging
- Hygrometry
- ...

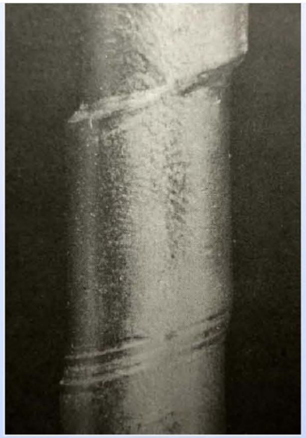
Geometric non-linearities

- Large strain
- Prestressed load
- Buckling
- Contact, boundary conditions
- Loading function of the geometry
- Mechanical properties function of the temperature

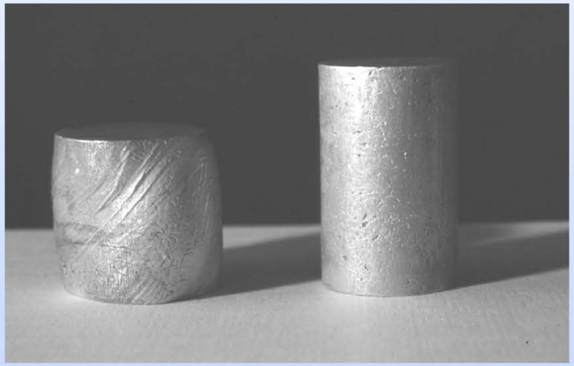
Large strain



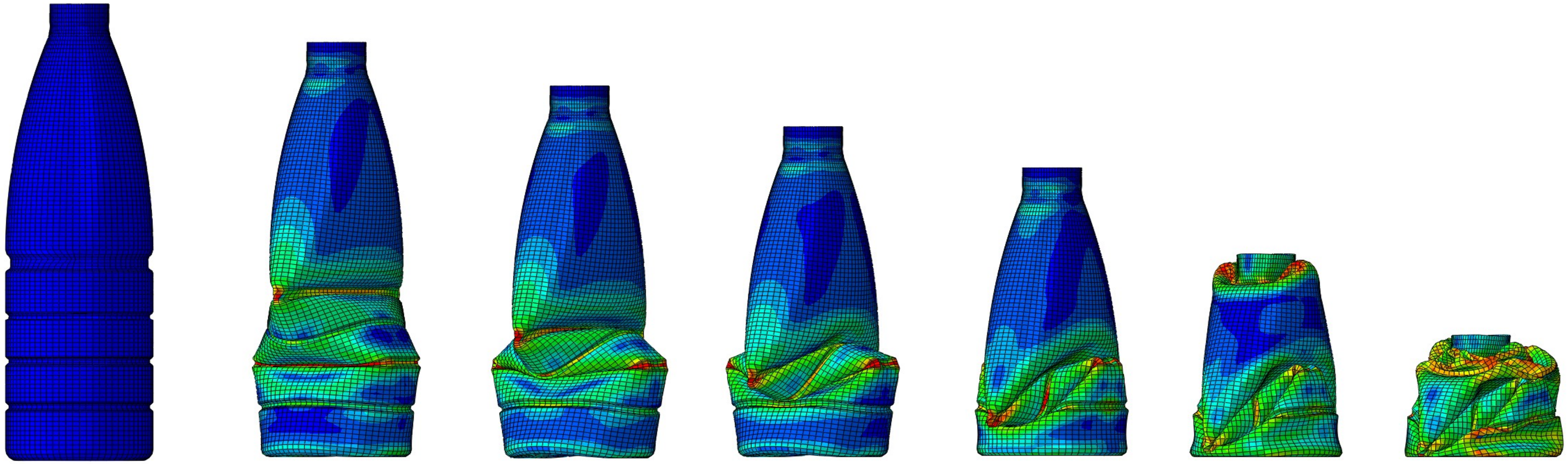
Rolling



Traction sur monocristal de Zinc.



compression sur monocristal d'Aluminium.



crash

Large strain

Use the strain tensor

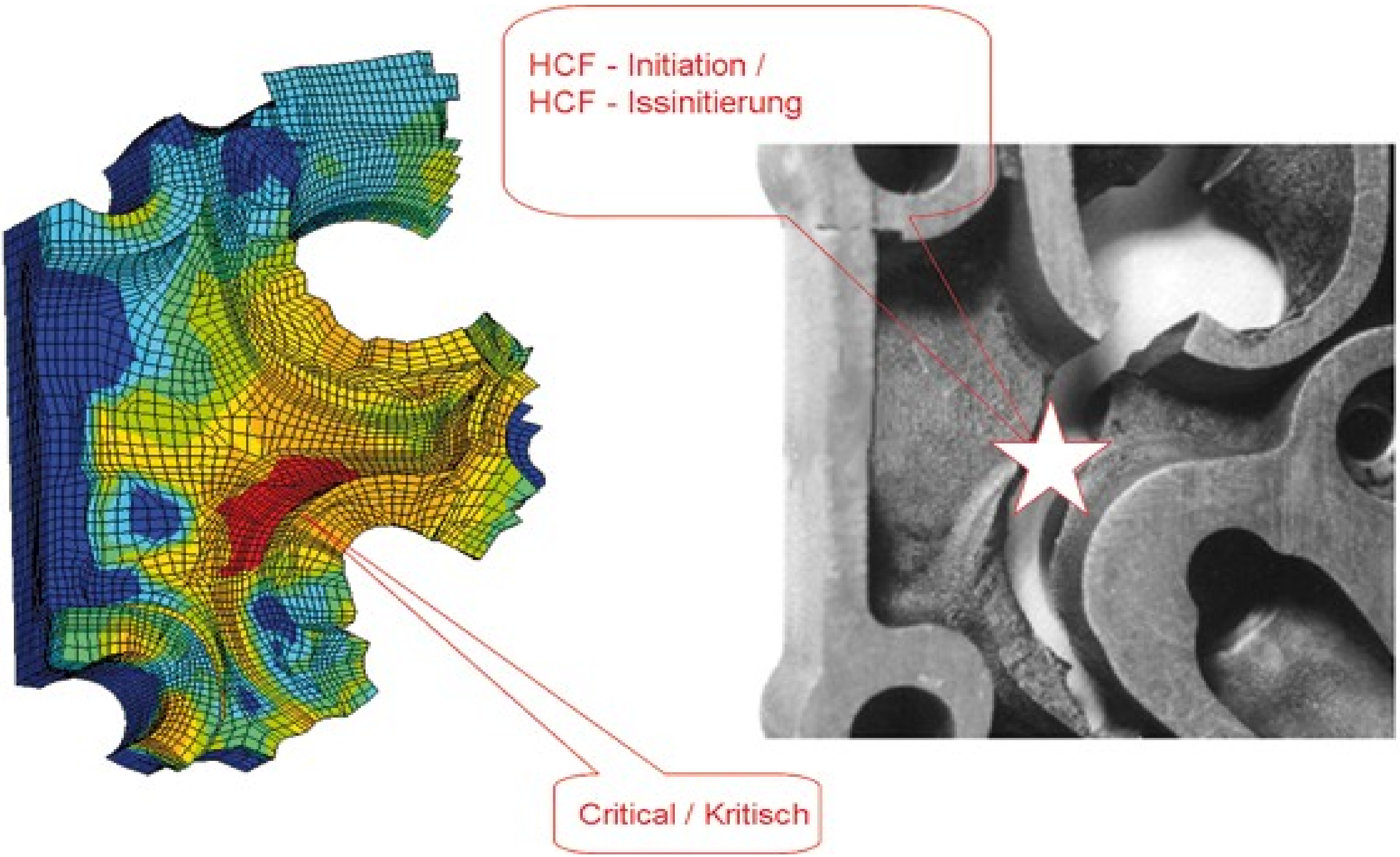
$$e = \frac{1}{2}\left({}^{t}\nabla \phi \, \nabla \phi - I\right)$$

Use the correct constitutive law of the material giving
the correct stress tensor (Piola, Cauchy) from the strain tensor *e*

Set the equilibrium equation on the deformed
or reference configurations

Prestressed load

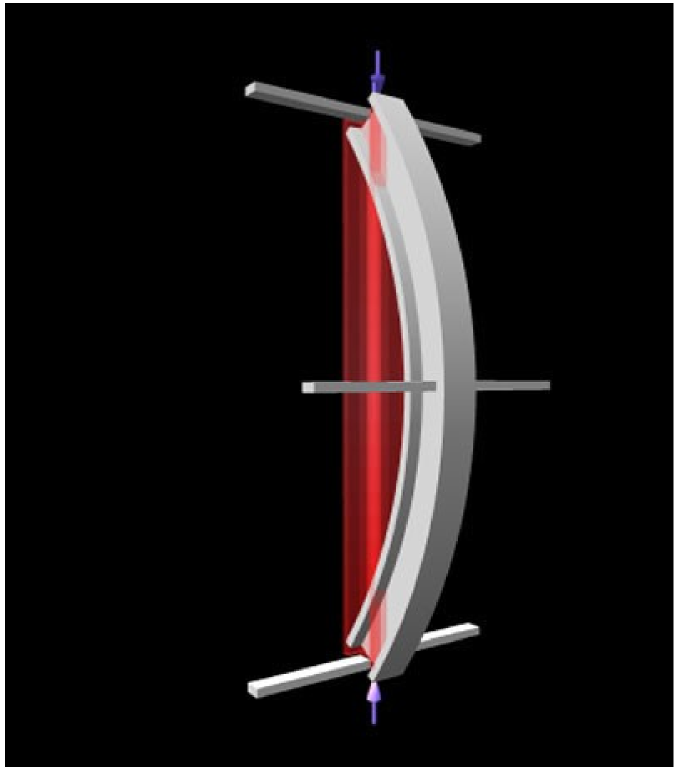
Residual stress

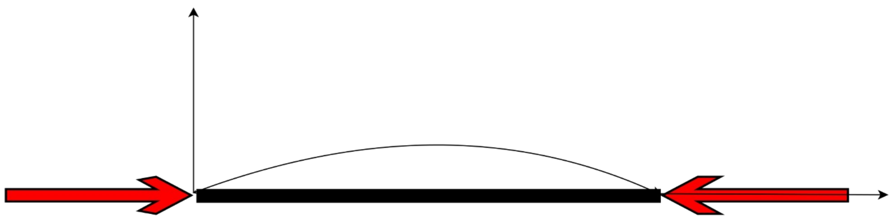


Prestressed load

- The reference state is not a natural state
- This state is at equilibrium without external load
- A new load is applied in addition to the prestressed load
- Other example: a guitar string has a static preload and vibrations are applied in addition to the prestressed load

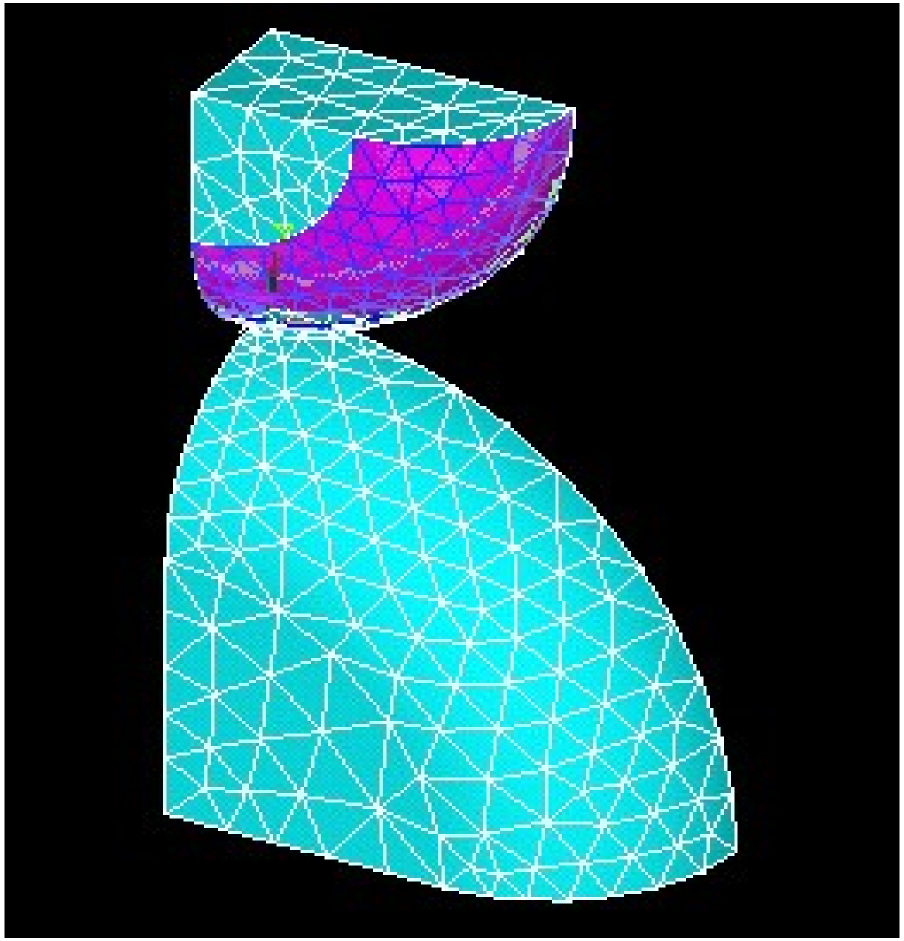
Buckling

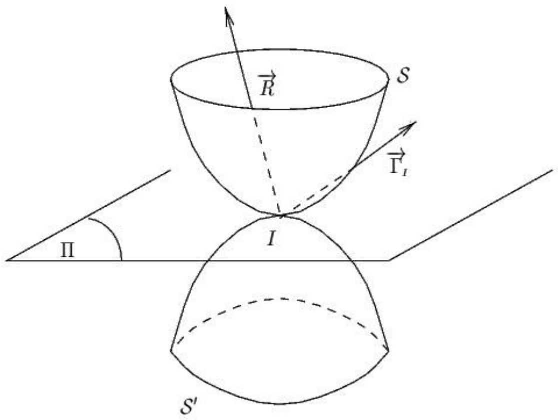




$$When \quad F > \pi^2 \frac{EI}{L^2}$$

Contact, friction





No interpenetration, only detachment is possible

$$(\boldsymbol{u_2} - \boldsymbol{u_1}) . \boldsymbol{n} \geq 0$$

Unilateral contact, only compression is possible

$$R_{n_1} = -R_{n_2} \leq 0$$

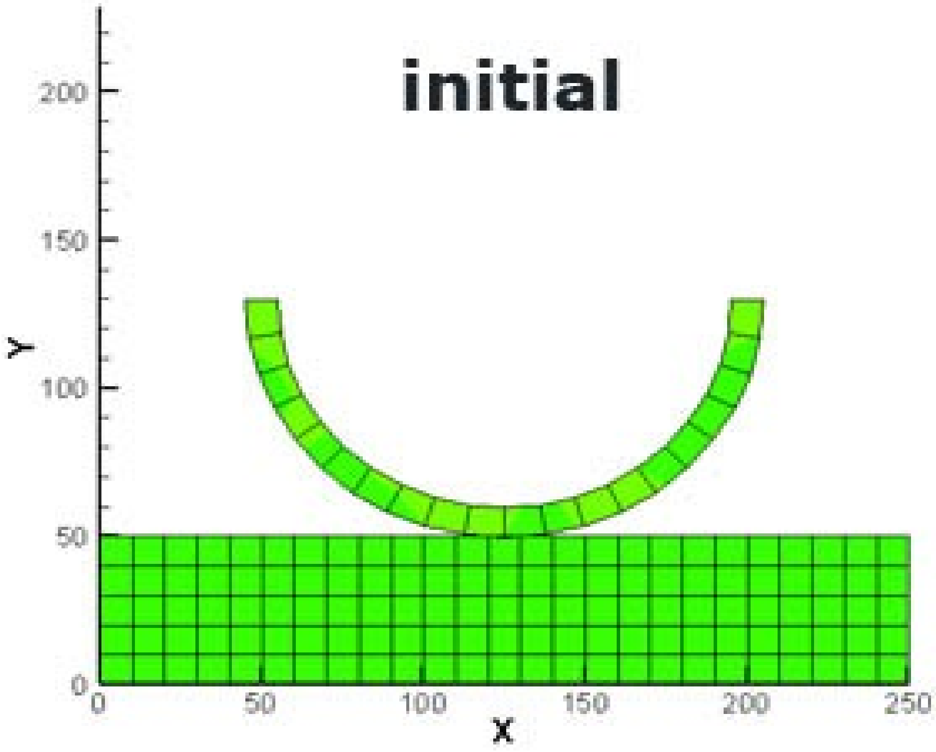
No friction, change this relation if there is friction

$$R_{t_1} = -R_{t_2} = 0$$

Either contact or detachment

$$((\boldsymbol{u_2} - \boldsymbol{u_1}) . \boldsymbol{n}) R_n = 0$$

Finding the contact zone is a part of the problem


initial
deformed
Y
X
0
50
100
150
200
250

Non-linear boundary condition

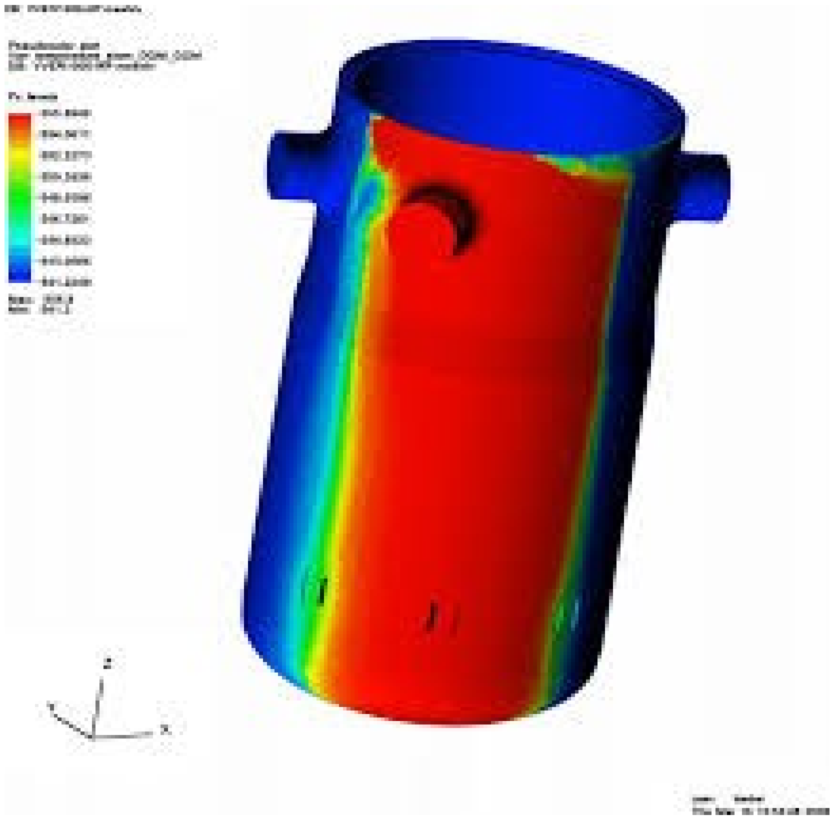
Follower pressure in piping, tank, dawn

F = P N

Required for stability analysis

Thermics

Position-dependent temperature, change of elastic properties



Steps of a non linear computation

A non linear computation is an iterative process

Non linear dynamics

Static or dynamic

Conclusion

- Understand the mechanics and the equations

- Be able to solve with FeniCS
  Other possibilities:
  - ✓ Abaqus, Ansys, Nastran
  - ✓ FreeFem++
  - ✓ Write the program with python, C++, matlab

- Have a critical look at the results

- Postprocessing

THE END