# Calcul numérique des solides et structures non linéaires

D. Duhamel, C. Lestringan,
C. Maurini, S. Neukirch

# First course

# Introduction to non-linearities in mechanics

# Two types of non-linearities

- Material non-linearities
  - Stress is no longer proportional to strain

- Geometric non-linearities
  - The hypothesis of small disturbances around a natural state is no longer valid

Purpose of the Course: To calculate with finite element methods in these cases

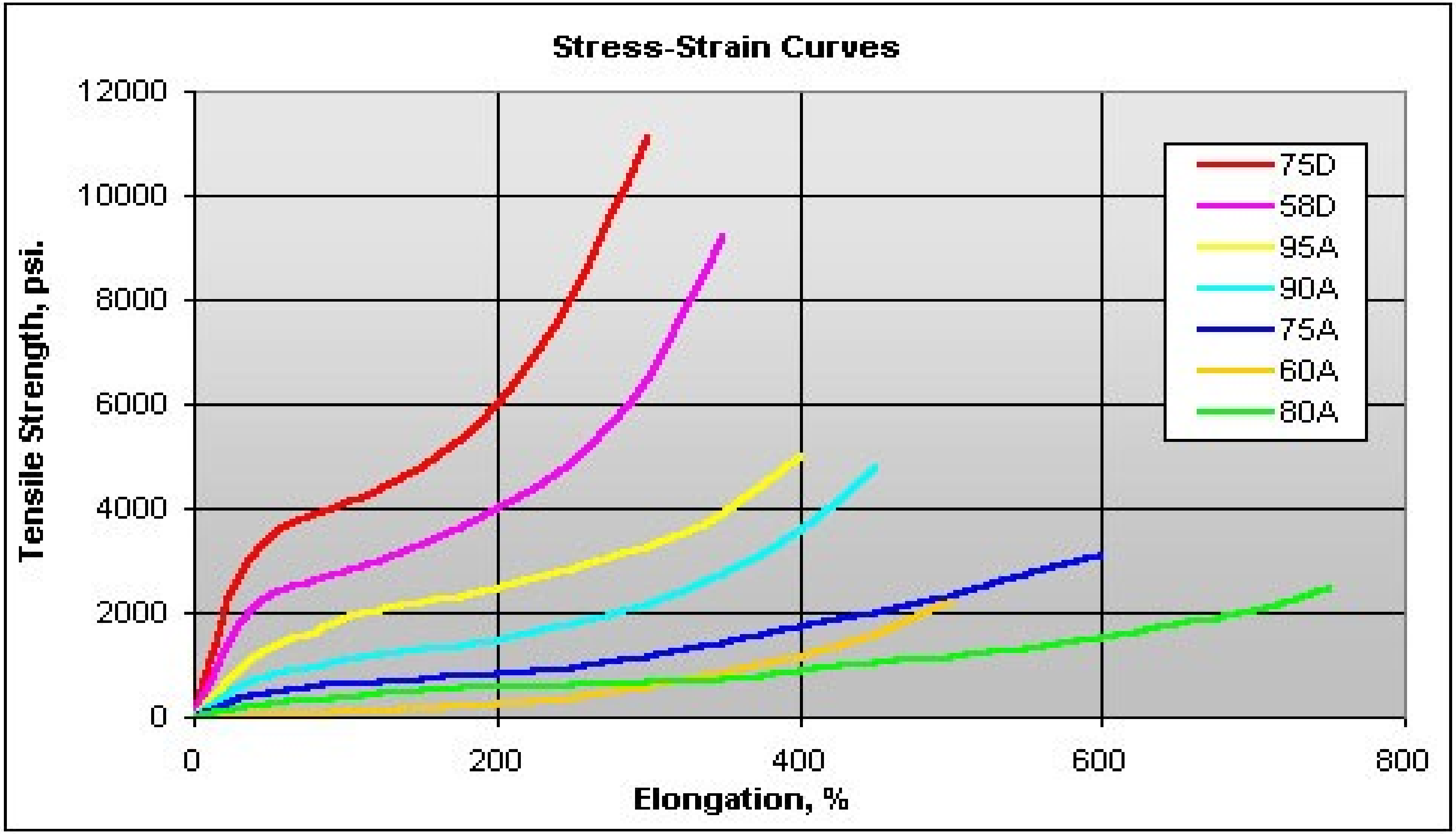
# Material Nonlinearities

- Non-linear elasticity
- Plasticity
- (Viscoelasticity and) viscoplasticity
- Damage
- Fatigue
- Dependence of materials / environment

# Non-linear elasticity

- Elastomer, rubber, wood
- Same path for loading and unloading
- No dissipation
- Existence of potential
- Deformation energy

# Nonlinear stress-strain relationships



Strain energy for the one-dimensional linear case

$$\psi = \frac{1}{2} E e^2$$

$$\sigma = \frac{\partial \psi}{\partial e} = E e$$

For the three-dimensional non-linear case, something like

$$" \sigma_{ij} = \frac{\partial \psi}{\partial e_{ij}} "$$

Hyperelastic material with different models: Neo-Hook, Mooney Rivlin, ...

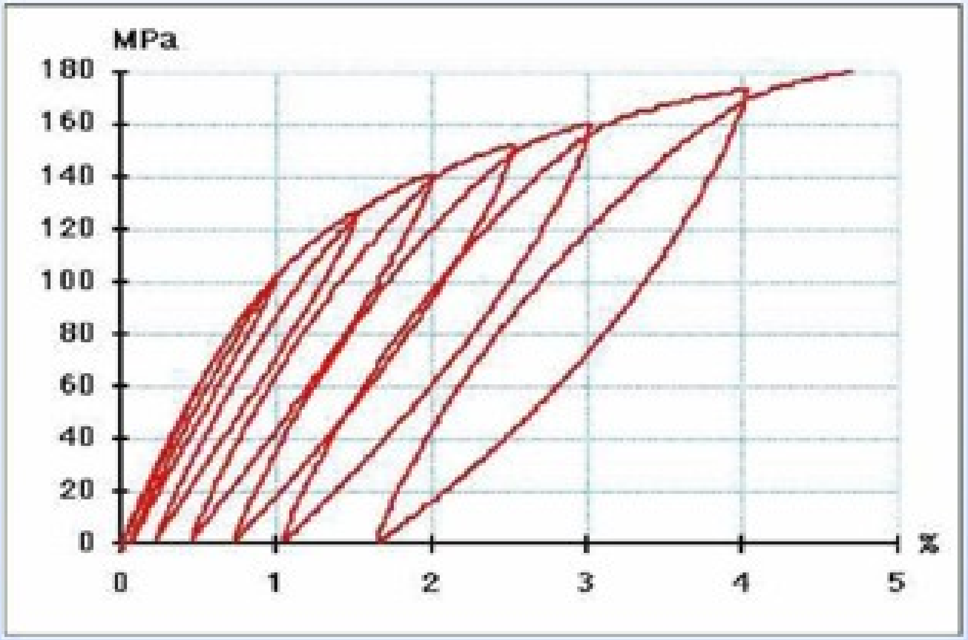
Can be incompressible (rubber)

# Plasticity

- Ferrous metals
- Elastic limit
- Plasticity criterion
  - Von Mises 2nd invariant of stress deviator
  - Tresca max ($\sigma_i$ - $\sigma_k$)
- Plastic flow law

# Plastic behaviour

Essai




Traction à $45^0$ : **Comportement plastique**

# Example of plastic behaviour

* When $\sigma < \sigma_0$

$$\sigma = E\epsilon$$ Reversible linear elastic behaviour

* When $\sigma > \sigma_0$

$$\epsilon = \epsilon^e + \epsilon^p$$

$\epsilon^e$ Reversible elastic deformation

$\epsilon^p$ Plastic deformation

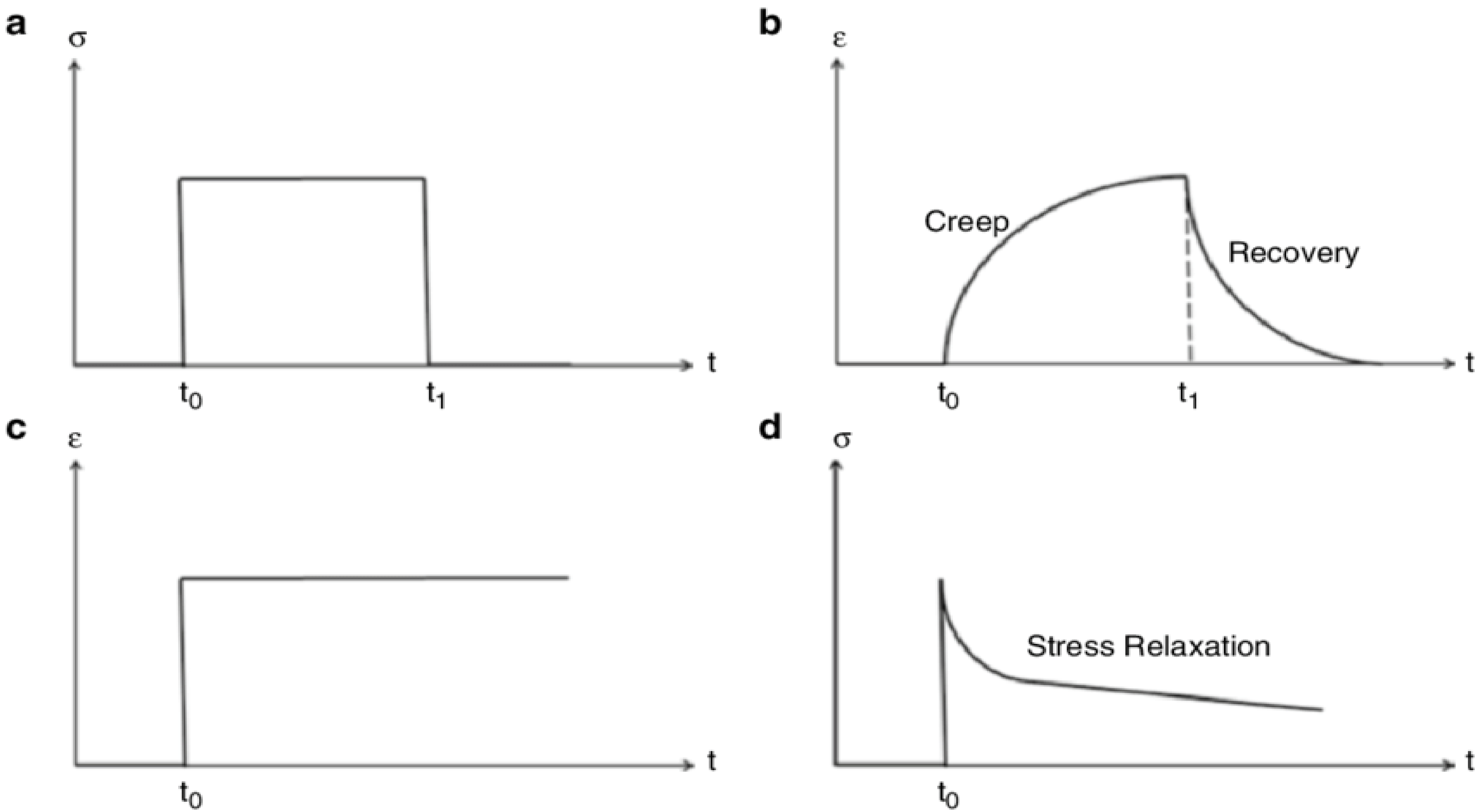
Flow law

$$\dot{\epsilon}^p = f(\sigma)$$

# Viscoelasticity - Viscoplasticity

- State of the material that evolves with time. Stress and strain.
- Creep. Constant stress, deformation increases.
- Relaxation. Constant strain, stress decreases.
- Maxwell model
- Kelvin model
- Often coupled with plasticity

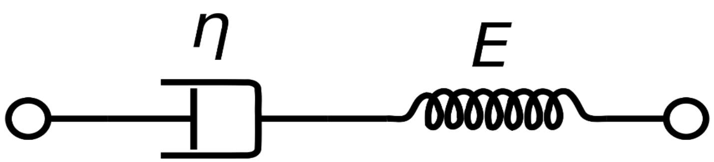
# Stress strain relationship



# Viscoelasticity models

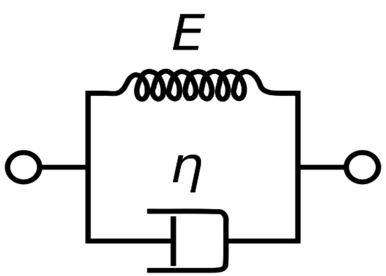
## Maxwell model

$$\sigma + \frac{\eta}{E}\dot{\sigma} = \eta\,\dot{\epsilon}$$




## Kelvin model

$$\sigma = E\,\epsilon + \eta\,\dot{\epsilon}$$



# Damage

- Metals - composites - ceramics
- Plasticity + change of elastic modulus.
- Growth of cavities in the material associated (or not) with plasticity
- Ultimate state ($\varepsilon$ > 10%)

# Damage

# Cracking

Crack opening and crack growth when stresses exceed a threshold

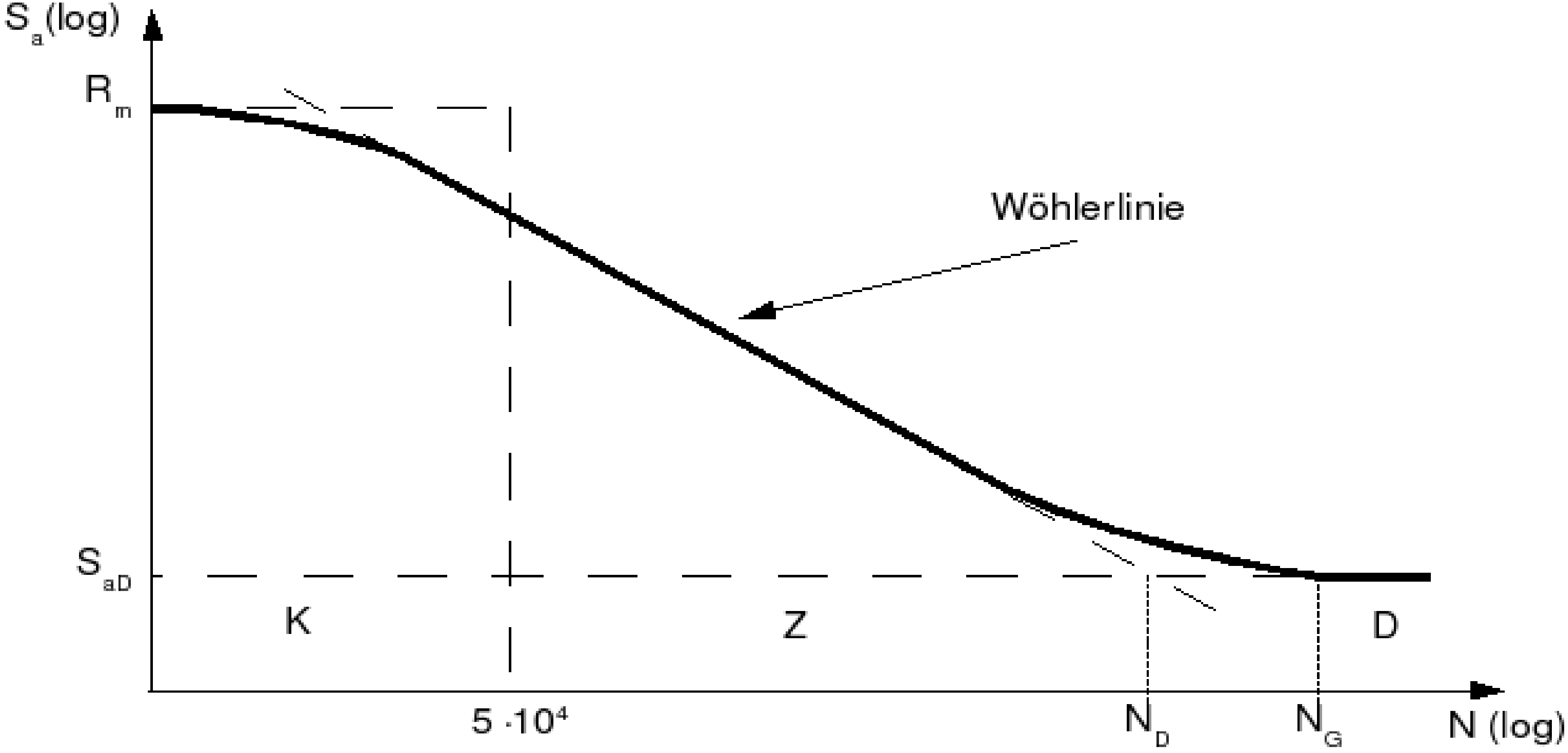
- Slow growth or
- Sudden rupture

# Fatigue

- Light alloys, steel
- Aging of the material according to the number and intensity of cycles
- Effect: decreases the capacity of deformation then rupture
- Method: modification of the properties of the material established from tests

# Fatigue fracture

# S-N Curves



# Dependence of materials / environment

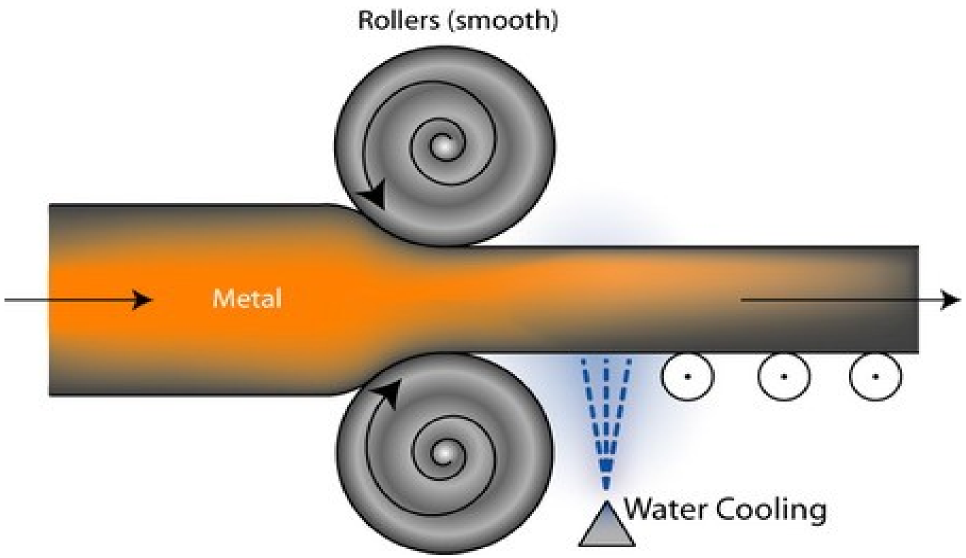
Properties function of

- Temperature
- Strain state
- Chemistry, phase change
- Aging
- Hygrometry
- ...

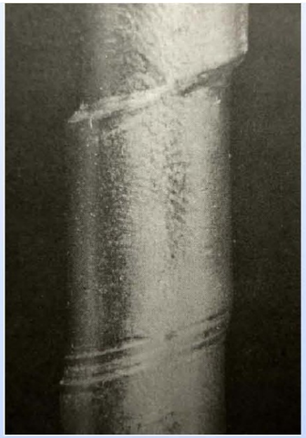
# Geometric non-linearities

- Large strain
- Prestressed load
- Buckling
- Contact, boundary conditions
- Loading function of the geometry
- Mechanical properties function of the temperature

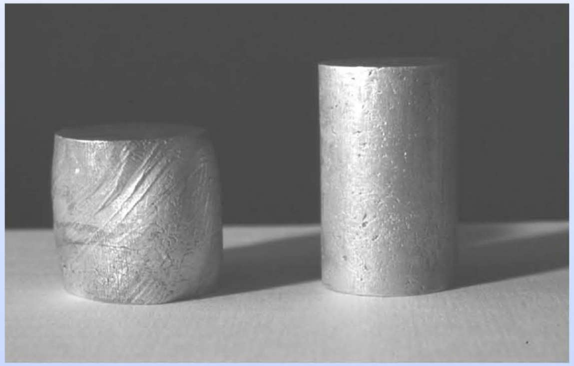
# Large strain



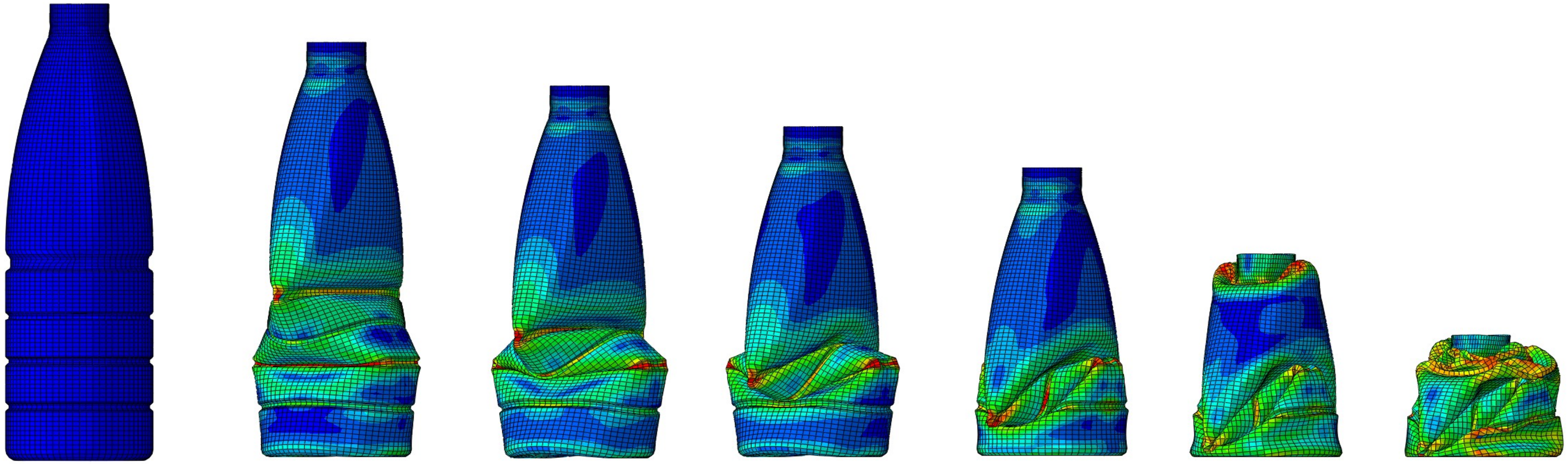

Rolling



Traction sur monocristal de Zinc.



compression sur monocristal d'Aluminium.




crash

# Large strain

Use the strain tensor

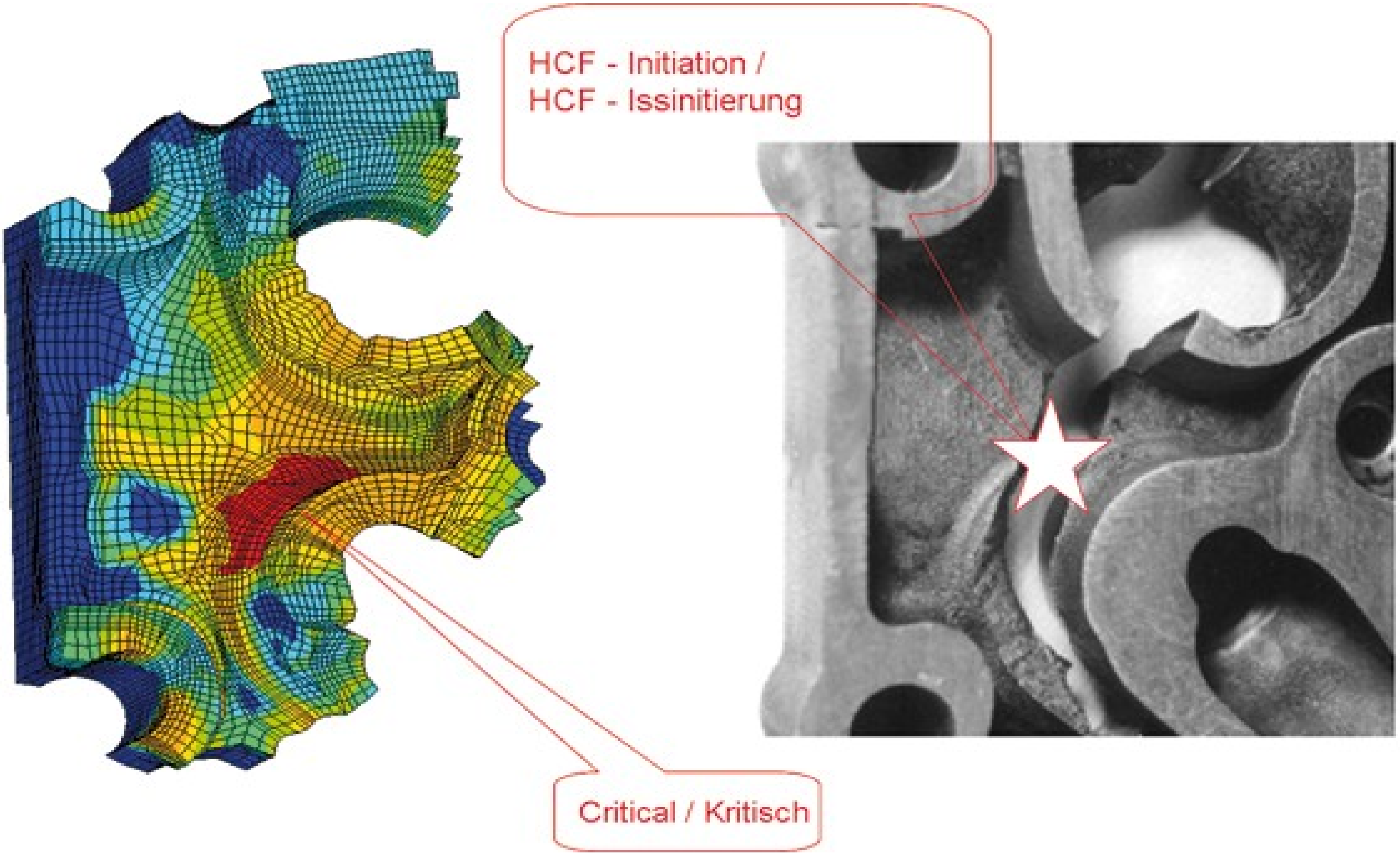
$$e = \frac{1}{2}\left({}^t\nabla\phi\,\nabla\phi - I\right)$$

Use the correct constitutive law of the material giving
the correct stress tensor (Piola, Cauchy) from the strain tensor *e*

Set the equilibrium equation on the deformed
or reference configurations

# Prestressed load

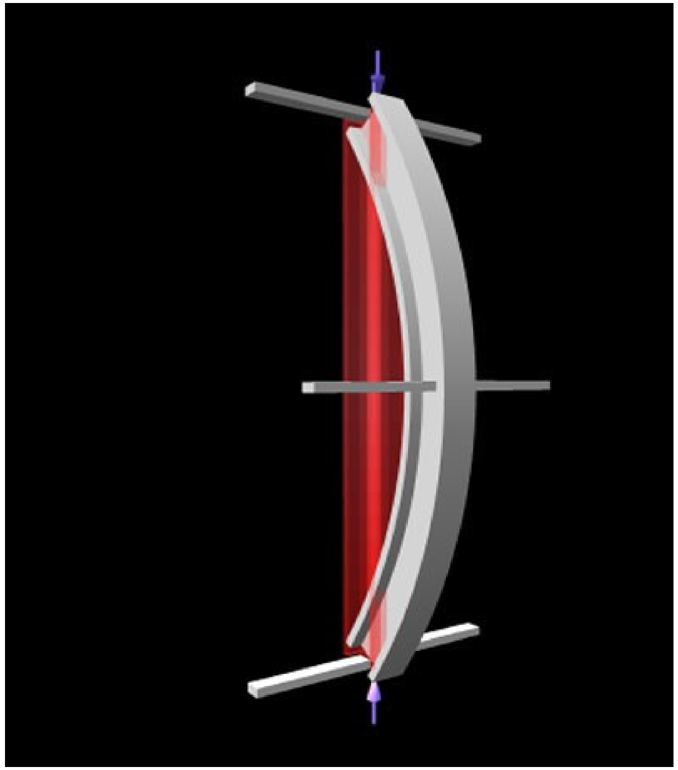
# Residual stress

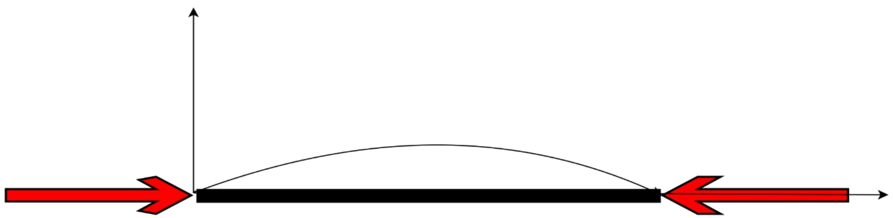


# Prestressed load

- The reference state is not a natural state
- This state is at equilibrium without external load
- A new load is applied in addition to the prestressed load
- Other example: a guitar string has a static preload and vibrations are applied in addition to the prestressed load

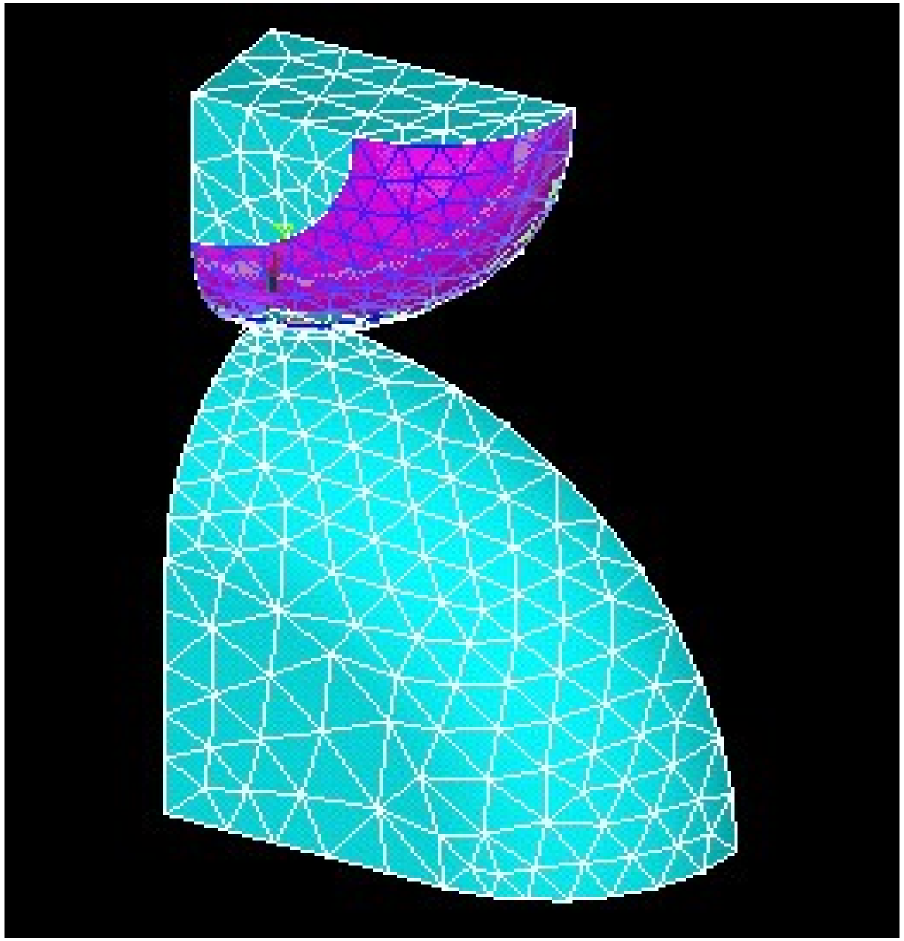
# Buckling

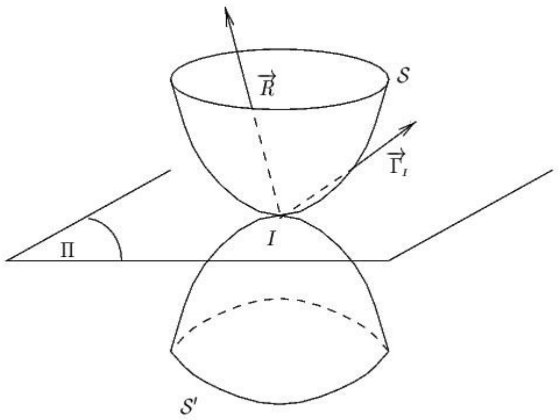




$$\text{When} \quad F > \pi^2 \frac{EI}{L^2}$$

# Contact, friction






No interpenetration, only detachment is possible

$$(\boldsymbol{u_2}-\boldsymbol{u_1}).\boldsymbol{n}\geq 0$$

Unilateral contact, only compression is possible

$$R_{n_1}=-R_{n_2}\leq 0$$

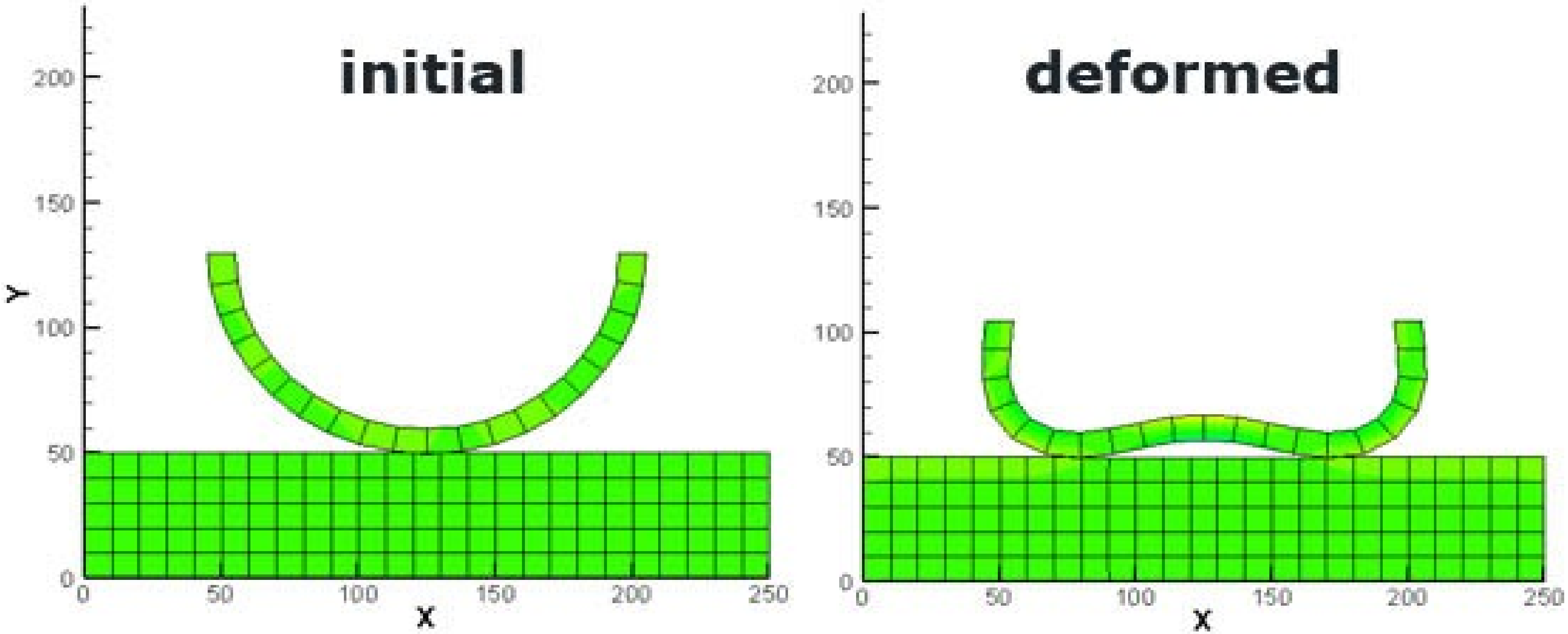
No friction, change this relation if there is friction

$$R_{t_1}=-R_{t_2}=0$$

Either contact or detachment

$$((\boldsymbol{u_2}-\boldsymbol{u_1}).\boldsymbol{n})R_n=0$$

Finding the contact zone is a part of the problem


initial
deformed
Y
X
X
0
50
100
150
200
250

# Non-linear boundary condition

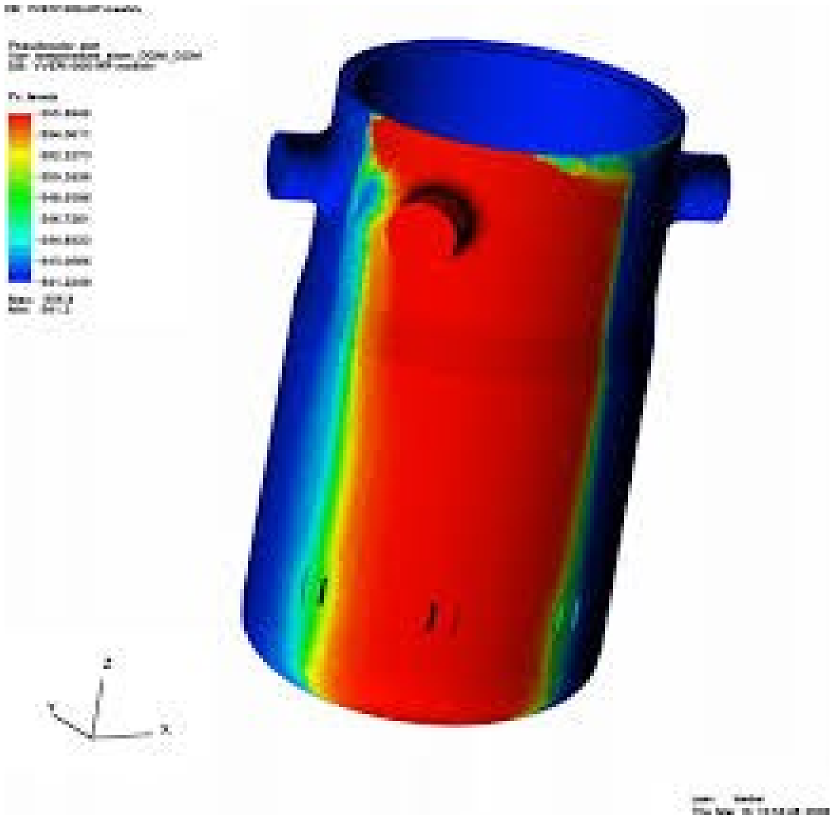
Follower pressure in piping, tank, dawn

F = P N

Required for stability analysis

Thermics

Position-dependent temperature, change of elastic properties



# Steps of a non linear computation

A non linear computation is an iterative process

# Non linear dynamics

Static or dynamic

# Conclusion

- Understand the mechanics and the equations

- Be able to solve with FeniCS
  Other possibilities:
  - ✓ Abaqus, Ansys, Nastran
  - ✓ FreeFem++
  - ✓ Write the program with python, C++, matlab

- Have a critical look at the results

- Postprocessing

THE END