Grain of Salt

An Automated Way to Test Stream Ciphers through SAT Solvers

Presentation at Tools for Cryptanalysis

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Motivations and goals

Motivations

- Stream ciphers may be broken using SAT solving tools
- Analysis of stream ciphers is possible using SAT solvers

Goals

- Describe different methods to translate shift register-based stream ciphers to SAT problems
- Demonstrate a tool that can help in this process

What is a SAT solver

Solves a problem in CNF

CNF is an "`and` of `or`-s"

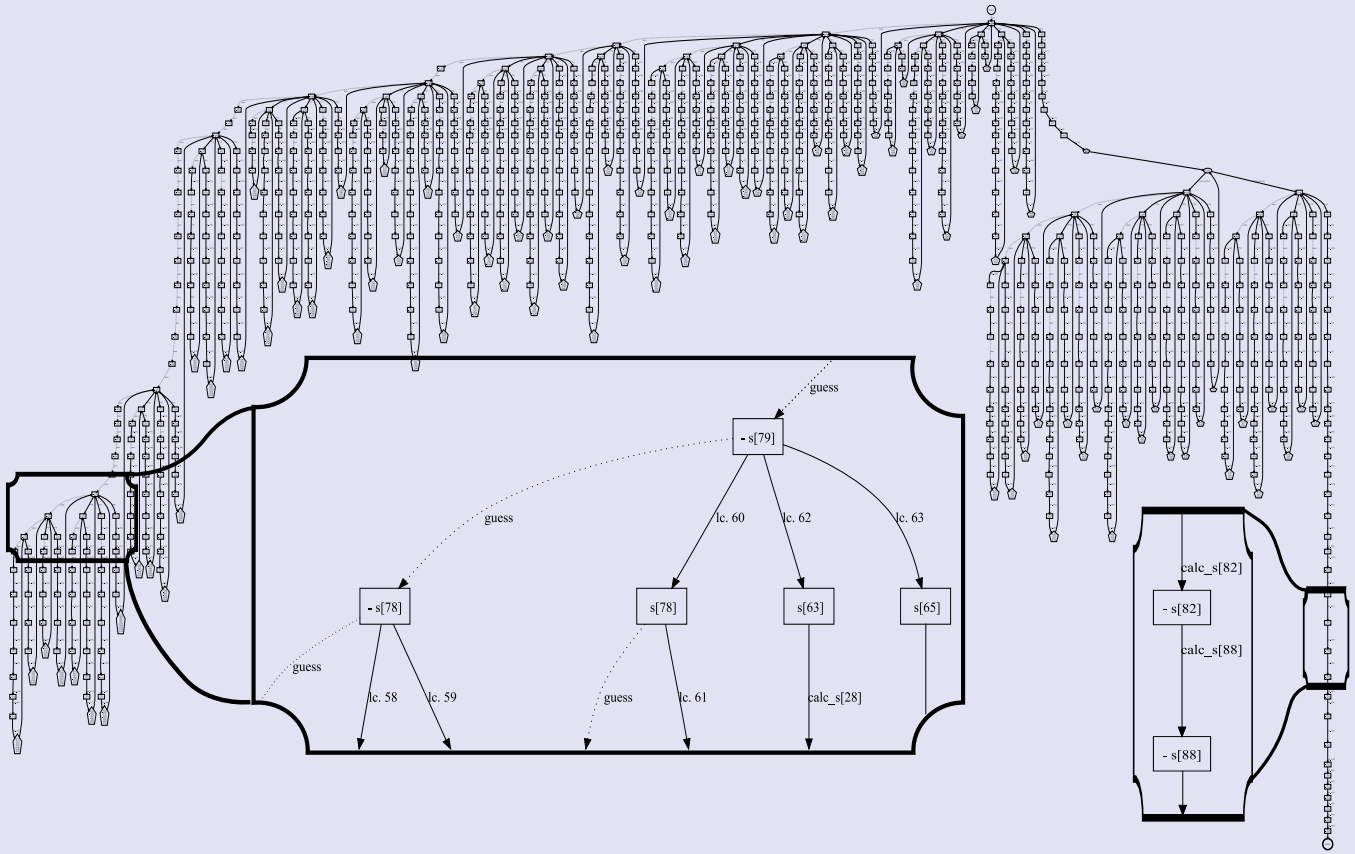
$$(x_1 \vee \neg x_3) \quad \wedge \quad (\neg x_2 \vee x_3) \quad \wedge \quad (x_1 \vee x_2)$$

Uses DPLL(φ) algorithm

1. If formula φ is trivial, return SAT/UNSAT
2. ret = DPLL(φ with $v \leftarrow$ `true`)
3. if ret == SAT, return SAT
4. ret = DPLL(φ with $v \leftarrow$ `false`)
5. if ret == SAT, return SAT
6. return UNSAT

Search tree

Example search tree



Visualised

- Guesses
- Propagations
- Generated learnt clauses
- Clause group causing the propagation

Calculated stats

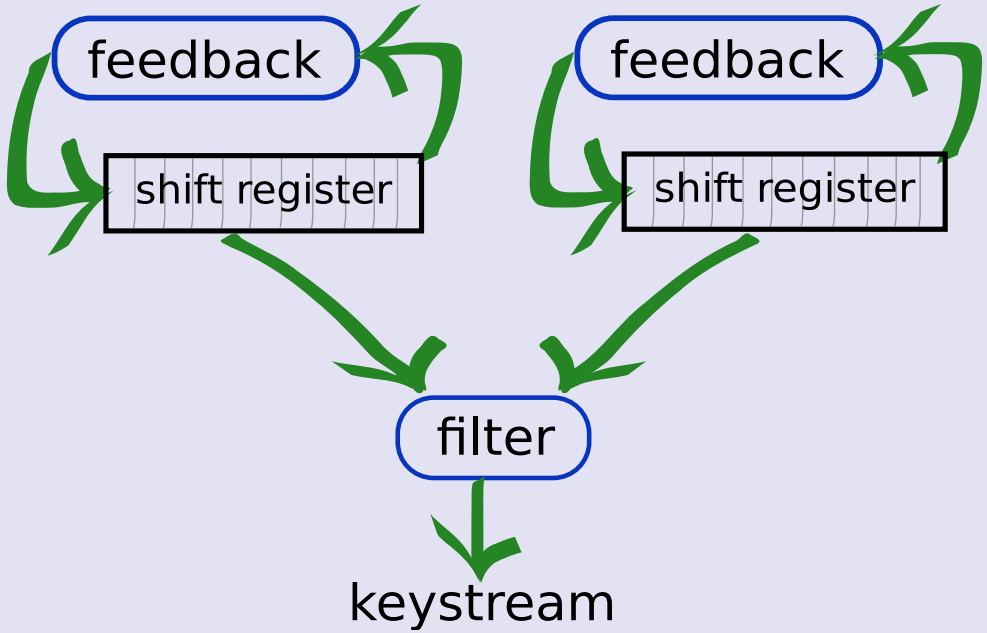
- Average depth
- Most conflicted clauses
- No. of guess/branch
- Most guessed vars
- Most propagated vars

Stream ciphers

Shift register-based stream ciphers

- Use a set of *shift registers*
- Shift registers' *feedback function* is either linear or non-linear
- Uses a *filter function* to generate 1 secret bit from the state
- Working: clock-filter-clock-filter. . . feedback-filter-feedback-filter. . .

Example cipher



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Translating shift register-based stream ciphers to CNF

Unwinding time

- SAT solvers don't understand the notion of "time"
- We must present the problem as if no timing was involved
- To make unwinding simple, regular clocking is needed

Describe feedback and filter functions

- Translate ANF (Algebraic Normal Form) to CNF
- Either through XORs and Monomials
- Or through direct translation using Karnaugh maps

Give help bits to aid solving

- Select and fix shift register states: guess-and-determine
- Select shift register states randomly, fix randomly, and solve for random problems

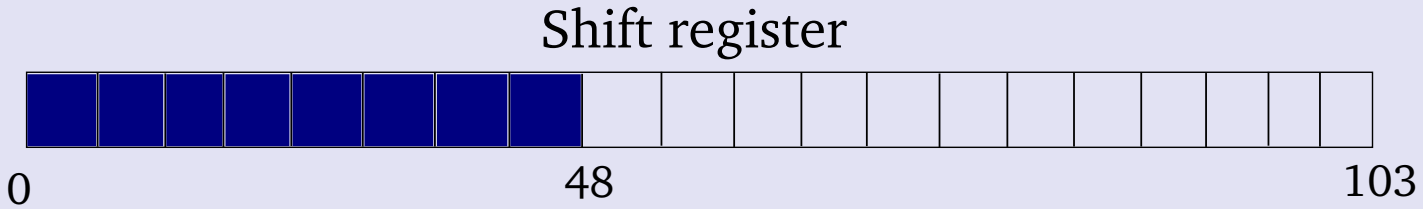
Unwinding time

Why unwind?

- CNF cannot handle the notion of "time"
- Must present all shift register states in the CNF at once
- No irregular clockings

Example

- Crypto1 state: 48 bits
- We have observed 56 bits of output
- We need to clock 56 times



Unwinding the time

Base-shifting

- If the feedback functions are reversible, we can start in the middle
- Usually best to shift to the middle:

- The total depth of the search is then halved

Grain of Salt

- `./grainofsalt --outputs 56 --crypto crypto1 --base-shift 32`
- `./grainofsalt --outputs 160 --crypto grain --base-shift 80,80`

Describing feedback and filter functions

ANF→CNF

- ANF (Algebraic Normal Form): $a = bc \oplus b \oplus c$
- Must be converted to CNF: $x \vee y = \texttt{true}$

1. Assign each higher degree monomial an internal variable
2. Describe the XOR as CNF

Extended monomials

- CNF does not limit terms to be non-negated as ANF does
- $bc \oplus b \oplus c = b(c+1) \oplus c = (b+1)(c+1) \oplus 1 = \neg b \neg c \oplus 1$
- Default in `grainofsalt`, can be disabled with `--noextmonomials`

Karnaugh maps

- Uses Karnaugh maps to map functions to CNF
- Does not need internal variables
- May generate substantially smaller function descriptions

Describing feedback and filter functions

Function descriptions in `grainofsalt`

File `grain/functions/sr0/feedback.txt`

```
sr1-62
sr1-51
sr1-38
sr1-23
sr1-13
sr1-0
```

Defines feedback function: $s_{i+80} = s_{i+62} + s_{i+51} + s_{i+38} + s_{i+23} + s_{i+13} + s_i$

Arbitrary functions

- Arbitrary functions can be defined and used in other functions
- Dependency graph is built from output bits, only used functions are generated in CNF
- Allows to efficiently map ciphers built from functional blocks

Describing feedback and filter functions

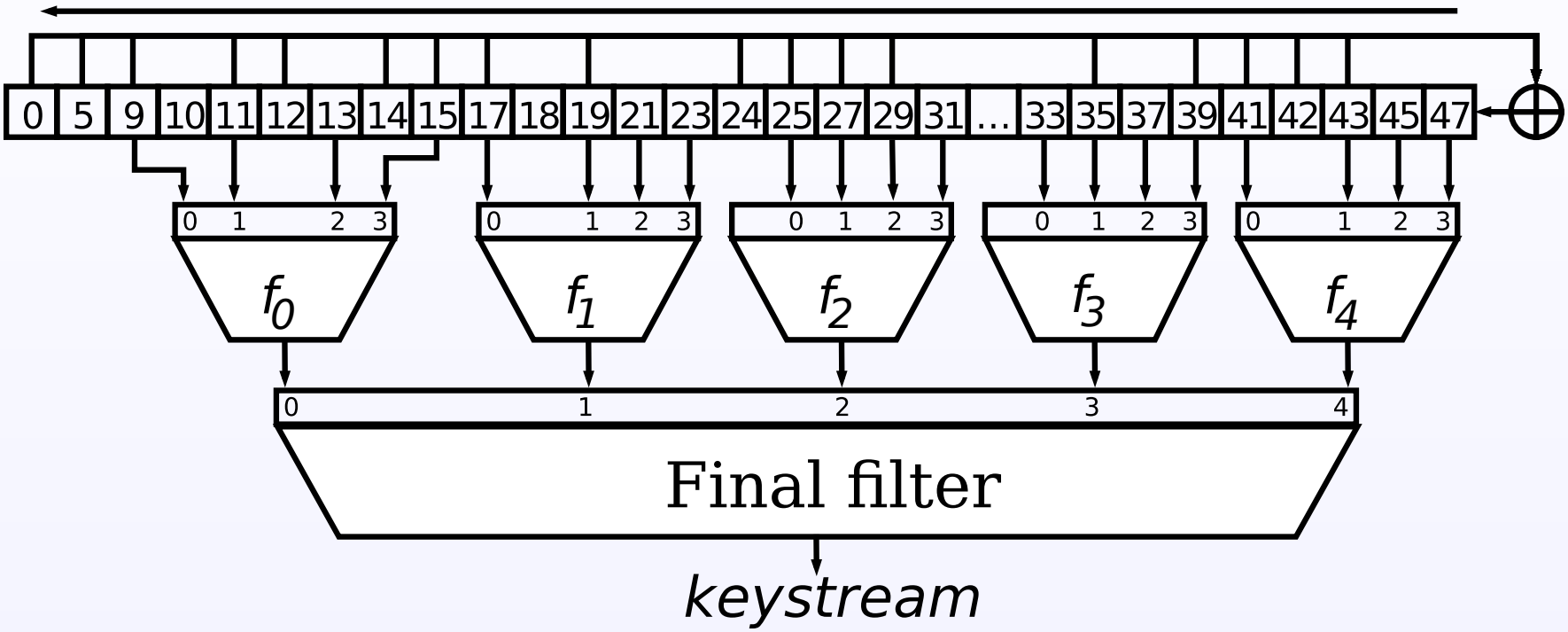


Figure: Crypto-1 cipher filter function diagram

Help bits

Why?

- Usually the generated problem is too slow to solve
- Except when breaking Crypto-1 (London travel card) — 40sec. approx

1. Give bits at fixed positions, multiply
2. Give bits at random positions, extrapolate

Given information use

- Recursively propagated at the ANF level
- Monomials are replaced with their constant equivalents
- Monomials are replaced with their monomial equivalents
- To disable: `--nopropagate`

Help bits

Fixed help bits

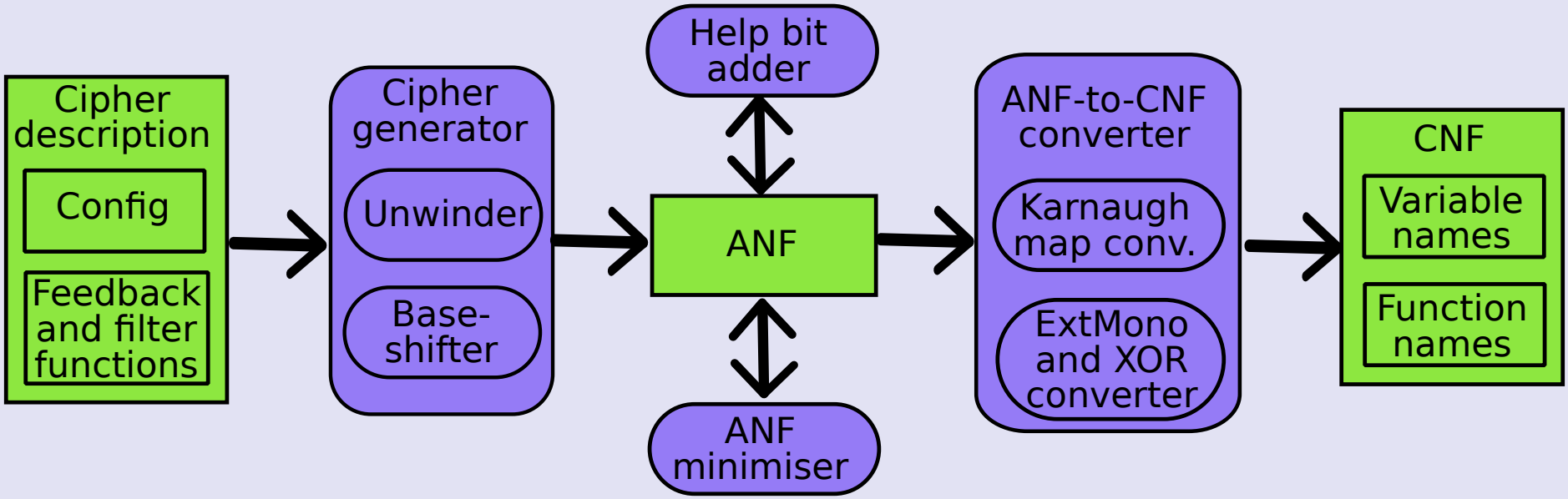
- Difficult to know which combination of variables will be fastest
- Automated, randomised (Monte Carlo) greedy algorithm
- Sets all bits, tests difficulty based on generated CNF size
- Average smallest size wins
- To generate: `--genDeterBits N` To use: `--deterBits N`

Probabilistic help bits

- Fix n randomly picked vars randomly, measure time
- Do above step (*everything* random) many times and average
- Do above steps for $n-1, n-2 \ldots$ until possible
- Plot graph — if done well, it is perfectly exponential
- But exponential factor $\ll 2$
- To use: `--probBits N`

Overview

Flow diagram



Example usage

- `./grainofsalt --crypto grain --outputs 100 --genDeterBits 60`
- `./grainofsalt --crypto grain --outputs 100 --stats --cnfDir generatedCNFs --num 100 --deterBits 55`

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Conclusions

Concluding remarks

- `grainofsalt` gives an integrated platform to conduct experiments with SAT solvers on shift register-based stream ciphers
- It is GPL, with open GIT repository, well-documented, and extensible
- `http://gitorious.com/grainofsalt/`

Future work

- Add more ciphers — they are very simple to describe
- Add more functionality — non-shift register based ciphers
- Could be integrated into sage partially/fully

Thank you for your time

Any questions?