H_∞ Loop Shaping Control for Quadruple Tank System

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***Abstract*—In this paper, the control system of quadruple tank is designed which is based on the structure of experimental facility of quadruple tank and the method of H_∞ loop shaping control. Firstly, The system model of the quadruple tank are acquired with system identification. Then the pre- and post-compensation are selected to make the singular values of the nominal controlled system become the desired open-loop gain. Finally the H_∞ loop shaping controller is designed based on the robust stabilization theory. The controller we have obtained will be reduced to the PID controller which is easy to implement. The final controller is firstly used in the simulink environment to get the simulation result. Then the result is verified on the experimental facility of quadruple tank, and the experimental waveforms indicates that the controller have good control performance and robust performance.**

Keywords-component; H_∞ loop shaping; quadruple tank; robust performance; PID controller

I. INTRODUCTION

The tank is the ideal controlled object in the development process of the process control theory all the time. It's structure is continually improved form one tank to quadruple tank. The schematic diagram of quadruple tank model which is put forward in [1] is clear, as is shown in Fig. 1. The quadruple tank system is a typically multivariable system and it has the nature of nonlinearity, strong coupling and long time-delay, so it is the representative object in the process control research. Many scholars do many theory research and design plenty of physical models that are related to the quadruple tank in recent years. For example, predictive control scheme was proposed in [2], PID controller is designed for the modified quadruple tank process using root locus in [3], and so on.

The laboratory independently designs and develops the quadruple tank control system experimental facility [4] on the basic of achievement of predecessor. Many advanced control methods such as the fuzzy control, the model predictive control, the ADRC and the adaptive control have acquired better effect in the laboratory. Some of these methods are based on the precise mathematic model and the industrial field has many uncertain factors, so these higher-order controllers of these methods are difficult to achieve and they can not be applied to industrial field [5].

H_∞ loop shaping is put forward by McFarlane and Glover, and it is an effective control method for evaluating four closed-loop properties simultaneously. It is the classical approach which combines the classical control theory with modern robust optimal control. The open-loop plant is augmented by pre and post-compensators to give a desired shape to the singular values of the open-loop frequency response. In order to apply this method to the design of quadruple tank control system, the design strategy of H_∞ loop shaping is described as follow.

First of all, we should establish the mathematical model according to the experiment method of step response. Then we design the control system by the use of the above design method. Finally, the performance of the controller will be verified on the experimental platform.

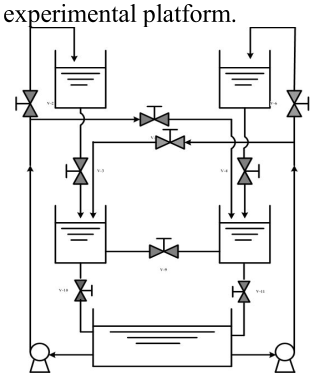

Figure 1. The structural schematic diagram of quadruple tank

II. THE ESTABLISHMENT OF THE MATHEMATICAL MODEL

A. The introduction of quadruple tank

The quadruple tank we design is shown in Fig. 2. The hardware mainly consist of two dc speed regulating pumps, four connected tanks, four ultrasonic sensors, nine valves, a PLC of siemens 300, a switched-mode power supply and a power of siemens. The left pump can inject water to tank4 and tank2 through the the valve Fa1 and Fa3, so the right pump can inject water to tank3 and tank1 through the the valve Fa2 and Fa4. The water of tank4 flow into tank1 through the valve Fa5. Similarly, the water of tank3 flow into tank3 through the valve Fa6. We can set up various mathematical models to conduct all kinds of experiment through adjusting the opening of the valve.

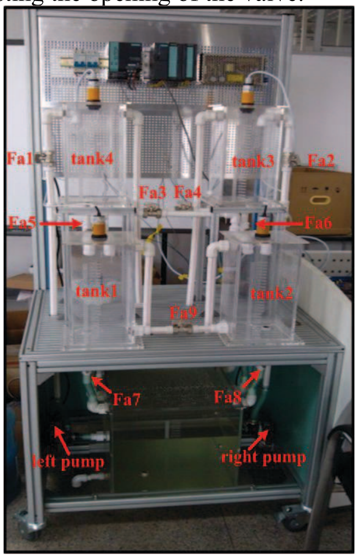

Figure 2. The physical diagram of quadruple tank

Software part mainly includes programming software STEP7, monitoring software WINCC and mathematics application software MATLAB. WINCC communicates with MATLAB in real time by OPC, therefore it can realize that the water level is displayed and the fixed variable of pump is given without delay. The experimental facility of quadruple tank not only combines the advanced control theory with industrial object compactly, but also provides a good solution for solving the problem of complex industrial scene. In the laboratory environment, teachers and students can rely on the platform to do theory teaching and applied research, and some professor will develop some advanced optimization control methods.

B. The identification of system model

The mathematical model of quadruple tank is builded by making use of experiment method of step response. We can construct a system of double input and double output on the experiment platform through opening and closing the valves. Obviously, the left pump respectively constitutes the two order system and one order system with tank1 and tank2. In the same way, the right pump respectively constitutes the one order system and two order system with tank1 and water tank2. Because the system coupling is high and four water tanks are connected, we must ensure that the object is in a stable state before the experiment in order to getting reliable test results. Firstly, we close the valves Fa3, Fa4, Fa7 and Fa8, simultaneously we give quantitative pump the same fixed variable and adjust valves Fa1 and Fa2 to ensure that the rising speed and height of tanks3, 4 is same. Then we need open and regulate valves Fa3, Fa4, Fa7 and Fa8 for the purpose that the level of tank3,4 is stable. In the end, the levels of four tanks are stable and the whole system state is constant through opening and regulating valves Fa3, Fa4, Fa7, Fa8. We should give left pump a step from 30 to 40 when the whole system state is invariable. After that we can get and save the step response data of level 1 and 2. And the operation of right pump is the same as the left pump. The two groups of data curve is shown in Fig. 3.

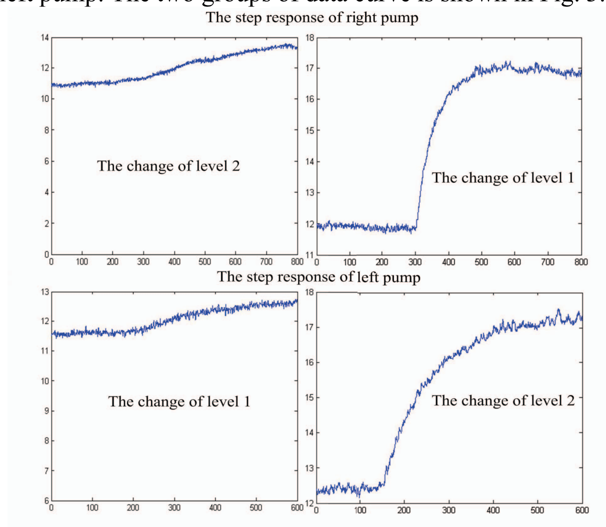

Figure 3. The change curve of level

Then we need process the two sets of data with the identification toolbox of MATLAB and select the form of transfer function according to the structure of quadruple tank. So we work out two transfer functions of first-order inertia link with pure time delay and the other two transfer functions of second-order inertia link with pure time delay [2]. Because the final controller of this article is the closed-loop PID controller and it do not require quite accurate mathematical model of the object, system mathematical model can be approximatively concluded as follows:

$$G(S)=\begin{bmatrix} \dfrac{0.43385}{205.22s+1} & \dfrac{0.4259}{(1538.9s+1)(35.637s+1)} \\ \\ \dfrac{0.38056}{(2312.5s+1)(4.4552s+1)} & \dfrac{0.43192}{213.89s+1} \end{bmatrix}$$

III. Design Control System Based on H_∞ Loop Shaping

A. Design loop shaping

The pre- and post-compensation are selected to make the singular values of the nominal controlled system become the desired open-loop gain. The step of loop shaping is as follow Fig. 4. The controlled object P and shaping function W_1, W_2 are merged as the controlled object P_S after shaping. $P_S = W_2 P W_1$, there is no implicit modal in W_1 and W_2 which makes P_S unstable.

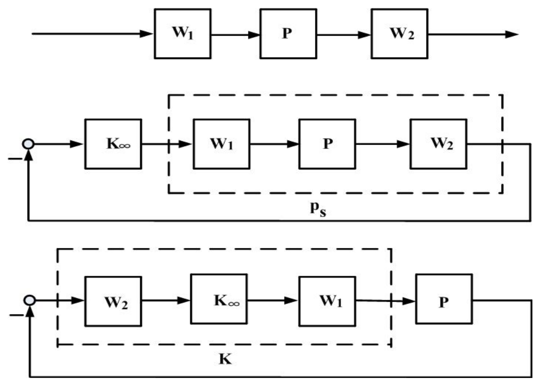

Figure 4. The design steps of loop shaping

B. The robust stabilization

a) Calculate:

$$\varepsilon_{\max} = \left(\inf_{K\text{镇定}} \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I + P_S K)^{-1} \tilde{M}_S^{-1} \right\| \right)^{-1} = \sqrt{1 - \left\| \begin{bmatrix} \tilde{N}_S & \tilde{M}_S \end{bmatrix} \right\|_H^2} < 1$$

$\tilde{M}_S$ and $\tilde{N}_S$ are regularization co-prime decomposition of P_S as follow:

$$P_S = \tilde{M}_S^{-1} \tilde{N}, \tilde{M}_S \tilde{M}_S^{\sim} + \tilde{N}_S \tilde{N}_S^{\sim} = I$$

In consideration of the system model and the loop shaping method, we can acquire design indicator $\varepsilon_{\max}$ after a few tests of W_1 and W_2. Finally

$$W_1 = \begin{bmatrix} \frac{0.1S+0.1}{S} & 0 \\ 0 & \frac{0.1S+0.05}{S} \end{bmatrix}, W_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\varepsilon_{\max} = 0.4539$$

b) We chose the parameters and make it satisfy $\varepsilon \le \varepsilon_{\max}$ *,then design the stabilization controller* K_∞*:*

$$\left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + P_S K_\infty)^{-1} \tilde{M}_S^{-1} \right\|_\infty \le \varepsilon^{-1}$$

C. Design the feedback controller

It's necessary to combine the H_∞ controller K_∞ with shaping function W_1 and W_2 for forming the the feedback controller K

$$K = W_1 K_\infty W_2$$

The closed-loop characteristic of system is directly related to the shape of the open loop. It can improve the rapidity and stability by changing the spectrum characteristics of open loop system, and enhance the anti-interference ability by reducing the decibel value at high frequencies. As the stability margin of the controlled object after shaping($\varepsilon \le \varepsilon_{\max}$), it is not only a guarantee of the robust stability of closed-loop system, but also a indicator of the success of loop shaping. If it is small, the specified loop shape is incompatible with robust stability requirements, so you should adjust accordingly and redesign.

D. Optimization and improvement of controller

The controller is not easy to realize and adjust online on account of that the order of the above controller is too high, so reducing the order [3] of the controller is necessary. The characteristics of the low frequency of the controller and the characteristics of *PID* controller are very close, simultaneously intermediate frequency and high frequency characteristics are relatively less important, so we can reject them and obtain the controller that the open loop characteristic of the system can satisfy the requirement of closed loop control system. The main steps of order reduction are as follows:

a) Select the nonsingular matrix T and eigenvalue decomposition of A_K

$$T A_K T^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & a_2 \end{bmatrix}$$

Among it a_2 do not contain zero eigenvalues. It makes the zero eigenvalue separate from other characteristic values.

b) The conversion of C_k *and* B_k *as follow:*

$$C_k T = [C_1 \ C_2], T^{-1} B_K = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

c) As a result, high order controller is similar to $K_P + K_I / S + K_D S$,

$$K_P = D_K - c_2 a_2^{-1} b_2,\ K_I = c_1 b_1,\ K_D = -c_2 a_2^{-2} b_2$$

The above method is that you need find the coefficient of Maclaurin series 3 of the controller on the basis of frequency domain variable s.

$$C_K (sI - A_K)^{-1} B_K + D_K = [c_1 \ c_2] \left(sI - \begin{bmatrix} 0 & 0 \\ 0 & a_2 \end{bmatrix} \right)^{-1} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + D_K =$$

$$\frac{c_1 b_1}{s} + (D_K - c_2 a_2^{-1} b_2) - c_2 a_2^{-2} b_2 s + O(|s|)$$

So high order controller is reduced to the *PID* controller which is easy to implement. It can draw the following parameters on the basis of the above design method.

$$K_P = \begin{bmatrix} 4.2015 & 0.4604 \\ 0.1391 & 2.3780 \end{bmatrix} K_I = \begin{bmatrix} 0.0507 & 0.0023 \\ 0.0002 & 0.0254 \end{bmatrix}$$

We can set up control system as Fig. 5 according to the design method of controller and the model structure of quadruple tank.

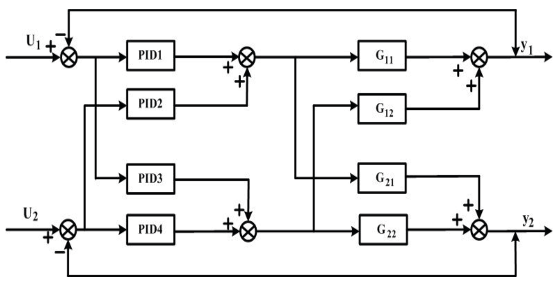

Figure 5. The block diagram of control system

IV. THE NUMERICAL SIMULATION AND THE ACTUAL VALIDATION

We can appropriately change model parameters in order to verifying the control performance and the robust performance of the controller. Firstly, a step from 0 to 16 is given to the setpoint of level1 and level2. The changed trend is shown in Fig. 6, and you will find that the system reach steady state quickly after good volatility in the simulation environment. However, the verification of simulation results should be carried out on the experimental platform. You should give two water levels a setpoint of 15 and 12 when level1 and level2 is respectively stable in a certain value. Two water levels can reach set point fast and are stable as shown in Fig. 7. Because data acquisition of sensor and data processing of model identification are all not very accurate, the mathematical model we obtain is not quite match with the actual system. Yet the controller on the basis of the mathematical model can well control the two water levels, it indicates that the controller have good control performance and robust performance.

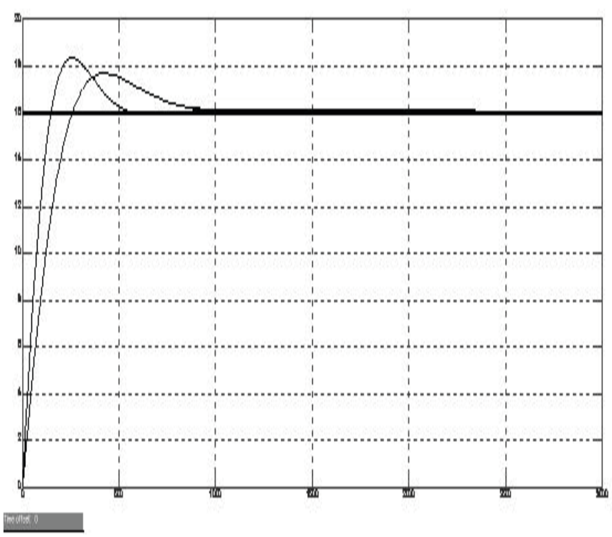

Figure 6. The control effect of simulation

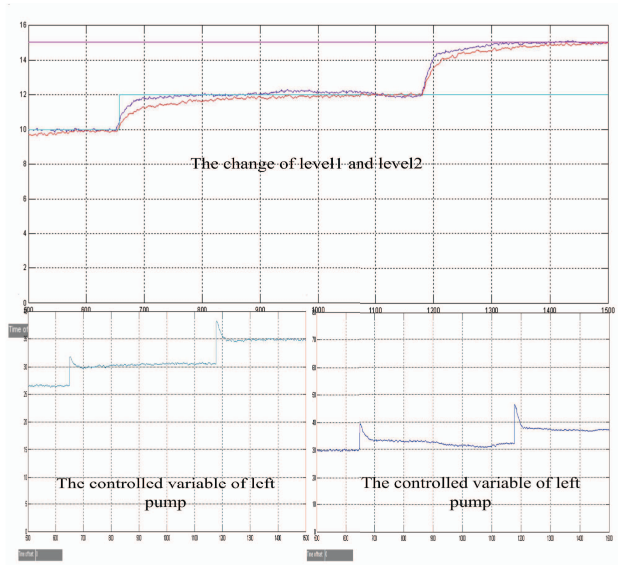

Figure 7. The actual control effect

V. THE CONCLUSION

The results of simulation and practice shows that H_∞ loop shaping is a good method which solves the problem of tradeoff between robustness and stability of multivariable control system. In the meanwhile controller order reduction method can achieve that the advanced control algorithm is easily applied to the actual industrial process. The controller in this paper realizes decoupling between dual channel of quadruple tank and achieves good control effect. In addition, it has certain robustness for the parameter perturbation system.

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