THE BRITISH TABULATING MACHINE COMPANY LTD

# an introduction to binary arithmetic

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- BINARY TO DENARY
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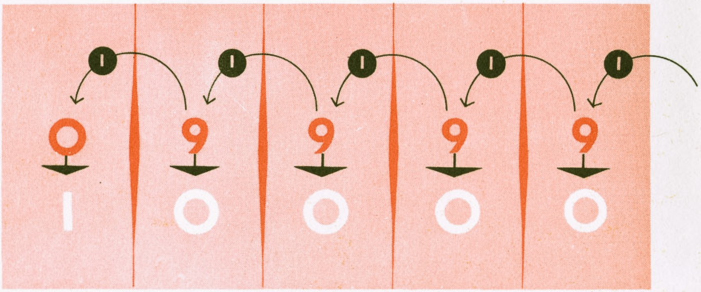
*. . . scale of ten*

The most commonly used method of numerical notation is, of course, the decimal, or denary, system. The "scale of ten" makes use of the ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The value of each digit is governed by its position and we have no difficulty in understanding that the number 47,283, for example, is made up of four ten-thousands (40,000), seven thousands (7,000), two hundreds (200), eight tens (80) and three units (3).

In mechanical counter assemblies we know that the addition of an impulse to the "units" counter wheel, if standing at "9", automatically results in the "carry over" of "1" to the adjacent "tens" counter wheel, and the "units" wheel is restored to zero. Each of the counter wheels is similarly affected whenever, having reached "9", a further impulse is added. Numbers of any size, limited only by the number of counter wheels available, can thus be contained in counter mechanisms of this type.




But the denary scale is not the only one with which we are familiar. In sterling currency and in the tables of Imperial weights and measures there is, as every schoolboy knows, an almost bewildering variety of different notational scales, e.g.,

| | | |
|---|---|---|
| Four farthings | = | One penny (Quaternary) |
| Twelve pennies | = | One shilling (Duodecimal) |
| Twenty shillings | = | One pound (Vigesimal) |

There are also scales of "sixteen" (ounces to pounds), "fourteen" (pounds to stones), "eight" (furlongs to miles), "three" (feet to yards) etc.

There is nothing very remarkable therefore about the "scale of two," or the scale of binary notation. In fact we use it every day when we convert amounts of ten shillings into pounds sterling, and pints into quarts.



# BINARY EXPLAINED

## 1) INTEGERS

We have said that the value of each digit in any decimal number is governed by its position. Reading from right to left, each digit represents in ascending order a higher power of *ten*, i.e.,

$$10^n \;\dots\;\dots\quad 10^4 \quad 10^3 \quad 10^2 \quad 10^1 \quad 10^0$$

Similarly, in binary arithmetic each symbol represents a successively higher power of *two*, i.e.,

$$2^n \;\dots\;\dots\quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$$

which, in non-mathematical language, would be simply:

| | | | | | | |
|---|---|---|---|---|---|---|
| | $2^n$ ... ... | $2\times2\times2\times2$ | $2\times2\times2$ | $2\times2$ | 2 | 1* |
| so that the number | ... | 1 | 1 | 1 | 0 | 1 |

expresses in binary form the decimal equivalent of

$$16 + 8 + 4 + 0 + 1 = 29$$

It will consequently be appreciated that numbers of any magnitude can be expressed precisely in binary notation as a series of "0's" and "1's". Thus, for example:

11111111111111111111111111111111

is the binary equivalent of 4,294,967,295 (denary).

## 2) FRACTIONS

Fractions can also conveniently be recorded in binary notation. It is necessary only to remember that the binary symbols "0" and "1" to the right of the binary point are in a *descending* scale of value, each successively in denomination being half the value of its predecessor.

Thus, binary "·1" is the same as decimal "·5", and binary "·01" is equivalent to decimal "·25" and so on.

For example, binary fraction "·11011" becomes decimal fraction "·84375" by addition of the decimal equivalents of each binary digit†, as shown below:—

Binary:
·1 1 0 1 1

Decimal:
·5 + ·25 + ·0 + ·0625 + ·03125 = ·84375

** By mathematical convention, $2^0$ is taken to be equal to one, in fact any number to the power of nought is regarded as equal to one.*

*† Strictly speaking, the word "digit" in this connexion is a misnomer, a digit being a numeral from 0 to 9. In electronic computer language the difficulty is surmounted by describing a single binary numeral as a "bit."*

*. . . the higher power of two*

*** BINI-TEN NOTATION** Denary digits are expressed individually in binary, with each occupying a four-bit array.

| 5 | 6 | 9 |
|---|---|---|
| 0101 | 0110 | 1001 |

Denary number 569 is 0101,0110,1001 in Bini-ten.

**BINI-TEN STERLING NOTATION**

| | £ | | s. | | d. | |
|---|---|---|---|---|---|---|
| | 2 | 7 | 1 | 6 | 1 | 1 |
| | 0010 | 0111 | 0001 | 0110 | 0001 | 0001 |
| or, more usually | 0010 | 0111 | 0001 | 0110 | 1011 | |

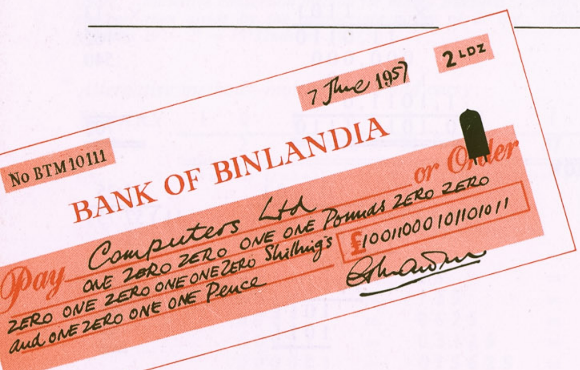
is £27 16s. 11d. in Bini-ten sterling notation.

**BINI-TEN GROSS NOTATION**

| Gross | | Dozens | Singles |
|---|---|---|---|
| 1 | 0 | 11 | 11 |
| 0001 | 0000 | 1011 | 1011 |

is 10 gross, 11 dozen and 11 singles in Bini-ten gross notation.

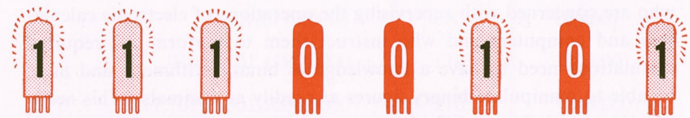
* *Alternatively described as Binary Coded Decimal Notation.*








The two-symbol binary scale of notation is particularly suited to the requirements of the latest electronic calculating and computing machines. The machines are equipped with batteries of valves, or other two-state devices such as transistors or magnetic cores, with which they do their arithmetic. For the sake of simplicity, each may be considered at any moment as being in an "on" or "off" state. A series of valves therefore might momentarily be in the condition:—

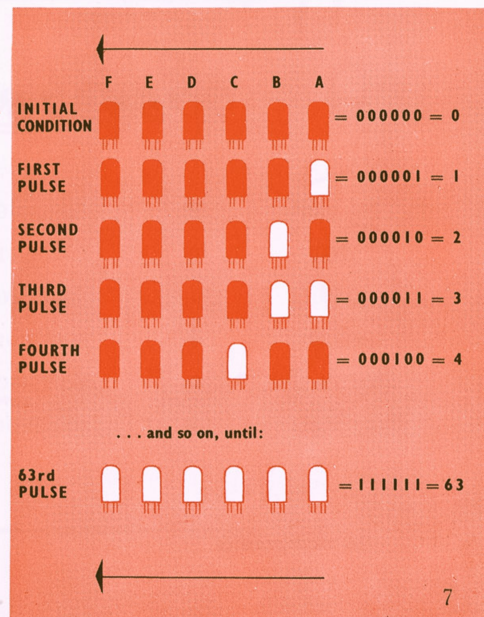



It is possible to stop the machine's operations and examine the state of the valve register. In this case the operator would at once know that the binary number held in the register was 11100101 (or denary 229).

Electronic computers are able to calculate at lightning speed because, unlike mechanical calculators, they are not hampered by the inertia of counter wheels and other such moving parts. The fact that

**COUNTING IN BINARY —THE FLIP-FLOP**

*Two valves linked together form a flip-flop. With, for example, six valves as shown, any number up to 63 can be accumulated. When valve (A) is OFF a pulse will switch it to the ON state; when it is ON a further pulse will switch it to the OFF state and make it emit a pulse which will cause the next valve (B) to assume the ON condition; and so on for each valve in the series. By the addition of more flip-flops larger numbers, of course, can be accommodated.*



## 3 CONVERSION TABLES

| | BINARY | | | | | | | BINARY | | | | | | | | BINARY | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Dec. | 32 | 16 | 8 | 4 | 2 | 1 | Dec. | 64 | 32 | 16 | 8 | 4 | 2 | 1 | Dec. | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 51 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 101 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 2 | 0 | 0 | 0 | 0 | 1 | 0 | 52 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 102 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 3 | 0 | 0 | 0 | 0 | 1 | 1 | 53 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 103 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| 4 | 0 | 0 | 0 | 1 | 0 | 0 | 54 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 104 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 1 | 0 | 1 | 55 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 105 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 6 | 0 | 0 | 0 | 1 | 1 | 0 | 56 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 106 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 7 | 0 | 0 | 0 | 1 | 1 | 1 | 57 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 107 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| 8 | 0 | 0 | 1 | 0 | 0 | 0 | 58 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 108 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 9 | 0 | 0 | 1 | 0 | 0 | 1 | 59 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 109 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 10 | 0 | 0 | 1 | 0 | 1 | 0 | 60 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 110 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| 11 | 0 | 0 | 1 | 0 | 1 | 1 | 61 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 111 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 12 | 0 | 0 | 1 | 1 | 0 | 0 | 62 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 112 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 1 | 1 | 0 | 1 | 63 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 113 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| 14 | 0 | 0 | 1 | 1 | 1 | 0 | 64 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 114 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 15 | 0 | 0 | 1 | 1 | 1 | 1 | 65 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 115 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 16 | 0 | 1 | 0 | 0 | 0 | 0 | 66 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 116 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| 17 | 0 | 1 | 0 | 0 | 0 | 1 | 67 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 117 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 18 | 0 | 1 | 0 | 0 | 1 | 0 | 68 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 118 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 |
| 19 | 0 | 1 | 0 | 0 | 1 | 1 | 69 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 119 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 20 | 0 | 1 | 0 | 1 | 0 | 0 | 70 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 120 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 21 | 0 | 1 | 0 | 1 | 0 | 1 | 71 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 121 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 22 | 0 | 1 | 0 | 1 | 1 | 0 | 72 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 122 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| 23 | 0 | 1 | 0 | 1 | 1 | 1 | 73 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 123 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 24 | 0 | 1 | 1 | 0 | 0 | 0 | 74 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 124 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 25 | 0 | 1 | 1 | 0 | 0 | 1 | 75 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 125 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| 26 | 0 | 1 | 1 | 0 | 1 | 0 | 76 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 126 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 27 | 0 | 1 | 1 | 0 | 1 | 1 | 77 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 127 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 28 | 0 | 1 | 1 | 1 | 0 | 0 | 78 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 128 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 29 | 0 | 1 | 1 | 1 | 0 | 1 | 79 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 129 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 30 | 0 | 1 | 1 | 1 | 1 | 0 | 80 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 130 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 31 | 0 | 1 | 1 | 1 | 1 | 1 | 81 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 131 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 32 | 1 | 0 | 0 | 0 | 0 | 0 | 82 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 132 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 33 | 1 | 0 | 0 | 0 | 0 | 1 | 83 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 133 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 34 | 1 | 0 | 0 | 0 | 1 | 0 | 84 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 134 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 35 | 1 | 0 | 0 | 0 | 1 | 1 | 85 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 135 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 36 | 1 | 0 | 0 | 1 | 0 | 0 | 86 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 136 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 37 | 1 | 0 | 0 | 1 | 0 | 1 | 87 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 137 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 38 | 1 | 0 | 0 | 1 | 1 | 0 | 88 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 138 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| 39 | 1 | 0 | 0 | 1 | 1 | 1 | 89 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 139 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| 40 | 1 | 0 | 1 | 0 | 0 | 0 | 90 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 140 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 41 | 1 | 0 | 1 | 0 | 0 | 1 | 91 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 141 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| 42 | 1 | 0 | 1 | 0 | 1 | 0 | 92 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 142 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 43 | 1 | 0 | 1 | 0 | 1 | 1 | 93 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 143 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 44 | 1 | 0 | 1 | 1 | 0 | 0 | 94 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 144 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 45 | 1 | 0 | 1 | 1 | 0 | 1 | 95 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 145 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 46 | 1 | 0 | 1 | 1 | 1 | 0 | 96 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 146 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 47 | 1 | 0 | 1 | 1 | 1 | 1 | 97 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 147 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 48 | 1 | 1 | 0 | 0 | 0 | 0 | 98 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 148 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 49 | 1 | 1 | 0 | 0 | 0 | 1 | 99 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 149 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| 50 | 1 | 1 | 0 | 0 | 1 | 0 | 100 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 150 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 |

| | | | |
|---|---|---|---|
| 200 ... ... 1100,1000 | FRACTIONS | | 400 ... ... 1,1001,0000 |
| 250 ... ... 1111,1010 | Dec. | Bin. | 450 ... ... 1,1100,0010 |
| | ·5 ... ... | ·1 | |
| 300 ... ... 1,0010,1100 | ·25 ... ... | ·01 | 500 ... ... 1,1111,0100 |
| | ·125 ... ... | ·001 | |
| 350 ... ... 1,0101,1110 | ·0625 ... ... | ·0001 | 1000 ... ... 11,1110,1000 |





## 4 DENARY VALUES AND RELATIVE POSITIONS OF THE FIRST THIRTY-TWO BINARY NUMERALS

| Group Position | 8 | | | | 7 | | | |
|---|---|---|---|---|---|---|---|---|
| Position in Group | 4 | 3 | 2 | 1 | 4 | 3 | 2 | 1 |
| DENARY | 2,147,483,648 | 1,073,741,824 | 536,870,912 | 268,435,456 | 134,217,728 | 67,108,864 | 33,554,432 | 16,777,216 |
| Position in Series | 32 | 31 | 30 | 29 | 28 | 27 | 26 | 25 |
| BINARY | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| | 6 | | | | 5 | | | |
|---|---|---|---|---|---|---|---|---|
| | 4 | 3 | 2 | 1 | 4 | 3 | 2 | 1 |
| | 8,388,608 | 4,194,304 | 2,097,152 | 1,048,576 | 524,288 | 262,144 | 131,072 | 65,536 |
| | 24 | 23 | 22 | 21 | 20 | 19 | 18 | 17 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| | 4 | | | | 3 | | | |
|---|---|---|---|---|---|---|---|---|
| | 4 | 3 | 2 | 1 | 4 | 3 | 2 | 1 |
| | 32,768 | 16,384 | 8,192 | 4,096 | 2,048 | 1,024 | 512 | 256 |
| | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| | 2 | | | | 1 | | | |
|---|---|---|---|---|---|---|---|---|
| | 4 | 3 | 2 | 1 | 4 | 3 | 2 | 1 |
| | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |
| | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

**BINARY NUMBER**

1111,1111,1111,1111,1111,1111,1111,1111,

is the equivalent of DENARY

**4,294,967,295**

HOLLERITH REGD

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| | |
|---|---|
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