<u>An Empirical Comparison of the Dependence of Conditional Volatility Parameter for Short-term Interest Rate Models</u>

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Forecasting short-term interest rates is of critical concern to policymakers, economists, and investors because there is a massive amount of notional value tied to those rates. In this research, we use monthly observations of 90-day constant maturity treasury bill rates and AA commercial paper rates to examine which of two models is a better characterization of interest rate dynamics: one that uses the elasticity of volatility with respect to interest rate level as shown in Chan et al (1992), or one that uses the volatility clustering effect to predict changes. Volatility clustering is when large changes tended to group together over time series data and was developed by Mandelbrot (1963). At the heart of this problem is determining whether interest rates become more volatile as interest rates rise, or if periods of stress are interspersed amongst periods of calm. The two different approaches to modeling short-term interest rates can show us a very important feature of interest rates. Throughout this paper, we will answer several other questions such as, how has monetary policy may have affected overall volatility, what are the parameter estimates for slightly riskier assets, and what conclusions can we draw from the parameter estimates of the two models? The empirical estimates show that both dynamics can be used to model interest rates, but that using a model which incorporates GARCH effects might be more appropriate. This suggests that commercial paper rates and treasury bill rates experience volatility clustering.

Forecasting short-term interest rates is of critical concern to policymakers, economists, and investors because there is nearly $2 trillion in outstanding commercial paper and the amount outstanding for Treasury bills is larger. In the last 20 years, we have seen significant disruption in the financial markets. Moreover, the valuation of risk has always been a central cornerstone to

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all financial institutions, creditors, and corporations. Therefore, forecasting short-term rates can provide important information to various market participants. For individual investors and firms, these forecasts can help reduce interest rate risk, and help them better manage their borrowing costs.

Models for the nominal interest rate have been well studied and several methods and models have been proposed. This research will determine which of these models is the best characterization of interest rates. Indeed, Anke (2012) shows that this is no small task as there are a dizzying amount of different methods to choose from and interest rates tend to be difficult to predict. There are two important characteristics of interest rate dynamics which we will use as the theoretical foundation for the models. The elasticity of volatility with respect to interest rate levels, which was shown by Chan et al. (1992), is the relationship between volatility and its level. For example, if interest rates are high, volatility would be high as well. The Generalized Method of Moments (GMM) approach is used to estimate 4 parameters; one of which will measure the elasticity of volatility with respect to interest rate levels. The motivations for doing so will be discussed later in the paper. The other dynamic of interest rates is the volatility clustering effect. That is, large changes tend to be followed by large changes, and small changes tend to be followed by small changes. Or, in other words, periods of stress are interspersed amongst periods of relative calm. The ARMA-GARCH method will be used to show the volatility clustering effect. The purpose of this research is to propose an appropriate model for short-term commercial paper rates and determine if these characteristics of interest rates can reasonably be used to produce accurate forecasts.

For this research, we will look at monthly data for two rates: 90-day constant maturity Treasury Bill rates and 90-day constant maturity AA Commercial Paper Rate. The Board of Governors of the Federal Reserve System maintains the historical data and is released daily as part of the H.15 release.

The remainder of the paper is organized as follows: Section II will provide context to this research by reviewing the literature, Section III will describe the different theoretical approaches to modeling the data, Section IV will describe the data and the methodology, Section V will present the empirical results, section VI will discuss the performance the models, and Section VII will conclude and discuss the implications of the results. For each section, we will first discuss the GMM approach and then discuss the ARMA-GARCH approach.

Section II: Literature Review

Chan et al (1992) sheds light on an important feature of nominal interest rate modeling: the dependency of volatility on the level of interest rates. Kharmov (2013), following Chan et al. (1992), used GMM to estimate models of the real interest rate. The choice of GMM was deliberate because “GMM estimators are consistent even if the errors are conditionally heteroskedastic, which is important in our case, as the variance of the interest rate process depends on the level of interest rates” (Kharmov 2013). The GMM approach was put forward by Hansen (1982). In his paper, he showed the model was more flexible and can work with correlated and heteroskedastic data which we will show is present within our data. Both Chan et al. (1992) and Kharmov (2013) compare several interest rate models put forward by academic researchers including Merton (1973), Brennan and Schwartz (1977,1979, 1980), Vasicek (1997), Dothan (1978), and Cox, Ingersoll, and Ross (1980, 1985). Despite this extensive research, it was previously difficult to compare these different models primarily because there was not a common framework in which different models could be nested and their performance compared. The primary goal of Chan et al. (1992) was to determine which model was able to capture an important volatility structure parameter which “is the elasticity of volatility with respect to the level of interest rates”. Whereas the primary goal of Kharmov (2013) was to apply this same approach to a real interest rate model. Importantly, this parameter is the only parameter that makes up the non-linear component of the model.

For this research, we estimate the parameters of 8 models proposed by these economists using the GMM approach. Each model contains a parameter restriction, which we use to evaluate the significance of the parameter value and show that certain parameters are or are not different from zero.

The ARMA-GARCH approach is a hybrid model. The ARMA model is used to model time series data and it results in an auto-regressive linear model. If the error terms of this model can be graphed by a function, or experience GARCH effects, by using the ARMA-GARCH model, we can study the heteroskedasticity of the changes. These models are commonly employed in modeling financial time series that exhibit time-varying volatility clustering, i.e. periods of swings interspersed with periods of relative calm. The GARCH method was introduced by Bollerslev (1986); he showed that “while conventional time series and econometric models operate under an assumption of constant variance, the Autoregressive Conditional Heteroskedastic (GARCH) process allows the conditional variance to change over time as a function of past squared errors".

ARMA-GARCH has traditionally been used to show the volatility clustering effect in stock market returns. Including Chen (1997), who studied S&P 500 index volatility. Wherever there exists some volatility clustering effect, the ARMA-GARCH approach should be considered.

The key distinction between these two methods of modeling is that the GMM approach uses a parameter to show the dependency of volatility on the level of interest rates. Whereas, the ARMA-GARCH approach uses residuals of forecasted changes minus actual changes to show the dependency of changes on their past changes. While this distinction might appear innocuous at first, the implications are significant for interest rate dynamics. Fundamentally, does the volatility of interest rates respond to their levels or the magnitude and direction of previous changes? When the underlying security is less creditworthy and therefore creditors will require more compensation to invest, interest rates climb (price of bond falls). This decreased creditworthiness will increase the volatility of the security as investors respond to the new interest rate level. This is the key economic argument for utilizing the GMM approach. This argument is countered by the argument that volatility responds to stress in the markets. There might be periods of intense volatility which might be caused by economic conditions, followed by relative calm as more level heads prevail or the result of monetary policy pushing volatility down.

These two competing theories of interest rate dynamics might both be true to some degree. But, given the importance and potential applications of forecasting interest rates, we will provide a structured approach to comparing the two models.

Section III: Econometric Approach

In this section, we will discuss the econometric approach to estimating the parameters of the two models. Our first approach is the Generalized Method of Moments developed by Hansen (1982). When modeling interest rate dynamics, we will consider a continuous-time diffusion process defined by:

$$dr = \alpha + \beta r_t + \sigma r^{\gamma} dz \qquad \text{(I)}$$

Where α represents drift, β represents the mean-reversion parameter, σ is the variance level, and γ is the "dependence of volatility on the interest rate level, or the elasticity of the interest rate volatility", and dZ represents some Brownian motion. This is the unrestricted process; however, the same approach can then be used to nest models developed by other economists by imposing the appropriate parameter restrictions. For our purposes, we also need the discrete-time analog of this continuous-time process, which is:

$$r_{t+1} - r_t = \alpha + \beta r_t + \epsilon_t \tag{II}$$

$$E(\varepsilon_{t+1}) = 0,\ E(\epsilon_{t+1}^2) = \sigma^2 r^{2\gamma} \tag{III}$$

As you can see, the discrete-time equations are simply a combination of a linear part plus the error term, which contains an exponential variable to be estimated. The GMM approach chooses the parameters to equate the sample moments and their theoretical values under the model. The GMM approach will estimate the parameter values which will minimize the residuals of the model to zero. It is important to note that the discrete-time process is only an approximation of the continuous-time analog. As shown in Grossman, Melino, and Shiller (1987) and Campbell (1986), the amount of error can be reduced if changes in the rate are measured over short periods of time.

There are eight parameter restrictions that we consider. The table below shows the degrees of freedom and the parameters that are restricted in these models.

Models	Model Name	Parameters				Deg. Of Freedom
		A	β	σ	γ	
Model 1	Unrestricted	-	-	-	-	0
Model 2	CEV	**0**	-	-	-	1
Model 3	Merton	-	**0**	-	**0**	2
Model 4	Vasicek	-	-	-	**0**	1
Model 5	GBM	**0**	-	-	**1**	2
Model 6	CIR-SR	-	-	-	**1**	1
Model 7	Dothan	**0**	**0**	-	**1**	3
Model 8	Brennan-Schwartz	-	-	-	**1**	1
Model 9	CIR-VR	**0**	**0**	-	**1.5**	3

The unrestricted model was previously estimated by Chen et al (1992). The constant elasticity of variance (CEV) process restricts the drift parameter to zero which was developed by

Cox (1975) for treasury bonds who found that there does not exist any drift in interest rate dynamics. Merton (1973), published in the Journal of Finance, developed a model for discount bond prices which is slightly different than yields and is used to price almost any type of financial instrument using data that is highly observable. The Vasicek model was introduced in 1977; one of its key advantages is that it is theoretically possible for the interest rate to become negative, which has become a desirable condition in the post-crisis markets. The Geometric Brownian motion (GBM) has been shown to model stock prices well but can be applied to several financial and time-series data as well. The CIR-SR model, which avoids the possibility of negative interest rates, and, consequently, when interest rates become close to zero, the equation becomes dominated by the drift parameter. Dothan (1978) and Brennan-Schwartz (1980) used discount bonds and convertible bonds, respectively, to estimate the parameters of this model. Finally, Cox, Ingersoll, and Ross (1980) used this model to study variable-rate securities. For all of these models, we will use the p-value to reject or accept these models as compared to the unrestricted model.

The ARMA-GARCH approach uses a similar discrete-time equation to estimate the parameters of the model. In this case, there are 5 parameters. The ARMA-GARCH is a hybrid model, which combines the Autoregressive Moving Average (ARMA) mean equation with the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) variance equation. The ARMA model has been proposed in literature for modeling time series; however, in this model, the variance is constant. For this reason, we use the GARCH model to capture the conditional variance. Below is the discrete-time equation for the ARMA-GARCH process that we will use:

$$r_{t+1} - r_t = \alpha + \beta(r_t - r_{t-1}) + \varepsilon_{t+1} \qquad \text{(IV)}$$

$$\varepsilon_{t+1} = z_t \sigma_t \qquad \text{(V)}$$

$$\sigma_t^2 = \omega + \xi(\varepsilon_{t-1}^2) + \gamma(\sigma_{t-1}^2) \qquad \text{(VI)}$$

The 5 parameters that will be estimated: the mean parameter, α, the linear coefficient for the relationship of current changes to previous changes, β, the conditional heteroskedasticity of residuals, ξ *(pronounced: Xi)*, which will show the dependence of this periods changes to the magnitude of the error in the last period, *ω (pronounced: omega)*, which is some constant added to the nonlinear function for the variance, and lastly, the conditional variance term, γ.

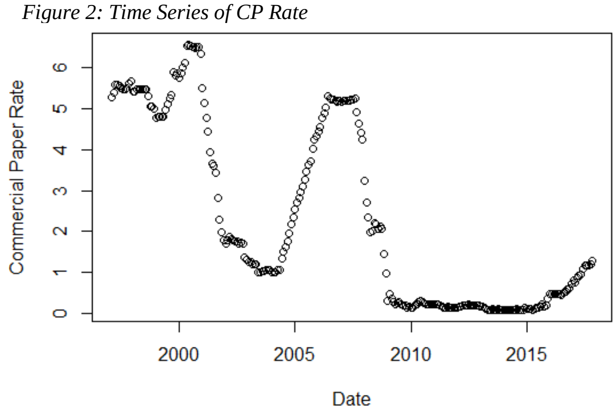
When there is a period of higher volatility, and large changes in period t, the model will produce large changes in period t+1. γ shows the clustering effect, or the tendency for periods of large changes to be followed by large changes. It represents the conditional variance. We use the ξ parameter to show the tendency of large errors to be group together. The focus of the research is to compare the γ values for the two models.

Section IV: Data and Methodology

Our data begins in 1997. I use 3-month Treasury-Bill interest rate and 3-month AA Commercial Paper Rate. For context, interest rates in the early 1980's were relatively high. This was the result of Fed Chairman Paul Volker tightening monetary conditions to rein in runaway inflation. After this period, interest rates started to fall gradually through the late 1980s through the late 1990s and monetary policy normalized. Investors started to sell these positions when the tech bubble burst in the early 2000s. Under the direction of Alan Greenspan, the Fed pushed rates down to stimulate growth. This started to heat the economy in the years just prior to the financial crisis of 2007-09. This financial crisis severely affected credit conditions, which caused the cost of borrowing for companies to sky-rocket sometimes overnight. In response, the Fed, under Chairman Bernanke, lowered interest rates to near zero for years to stimulate economic growth

Figure 1: Time Series of T-bill Rate

Figure 2: Time Series of CP Rate



and inflation, but also to incentivize borrowing. Interest rates have been on quite a rollercoaster ride over the past 40 years which represents a challenge when modeling these rates.

Table 1: Descriptive Statistics

	T-bill Rate	CP Rate	SPREAD
No. Of Obs	251	251	251
Mean	4.9100	5.3000	0.2700
SD	2.0681	2.2161	0.2551
Median	1.1700	1.3100	0.1500

Commercial paper rates (CP rates) have largely followed the same trends, which can be seen in Figure 2. By way of summary statistics, for the period of 1997-2017, T-bill rates had an average of just below 5%. CP rates had a slightly higher average of 5.3%, which is consistent with our understanding that CP rates have an implied risk premium. This difference between the CP rate and the T-bill rate is called the spread. It averaged about 27 basis points. The 95% confidence interval on the spread over this period is from 24 to 27 basis points.

We show serial correlation between the data and its lags. There exist significant correlations between the rate and it's 1-month and 12-month lags. The correlation between T-bill rate and its one-month lag was 0.9952, and 0.8125 with its 12-month lag. Commercial paper rates were similarly correlated with its lags. In addition, you can see that there exists a correlation between changes and their lags. CP rate changes correlation with its lagged values was 0.5908 and was 0.45 for T-bill rates.

Table 2: Correlogram of Lags

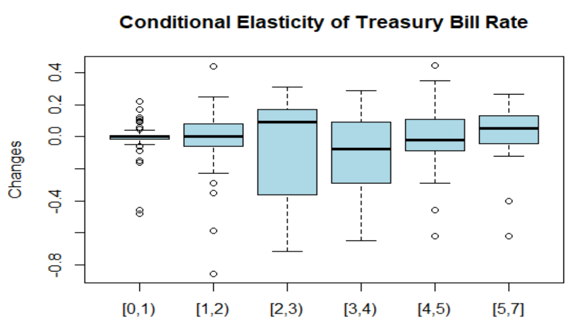
	T-Bill Rate	LAG T-bill Rate	CP Rate	LAG CP Rate
T-Bill Rate	1.0000			
LAG T-Bill Rate	0.9952	1.0000		
CP Rate	0.9939	0.9946	1.0000	
LAG CP Rate	0.9877	0.9939	0.9962	1.0000

	T-bill Rate	12mo LAG T-bill	CP Rate	12mo LAG CP Rate
T-Bill Rate	1			
12mo LAG T-Bill	0.8125	1		
CP Rate	0.9948	0.8294	1	
12mo LAG CP Ra	0.8004	0.9952	0.8201	1

	CP Changes	*Lag CP*	*T-bill Changes*	*Lag T-Bill*
CP Changes	1			
Lag CP	0.5908	1		
T-bill Changes	0.6541	0.4287	1	
Lag T-Bill	0.6348	0.6537	0.44912	1

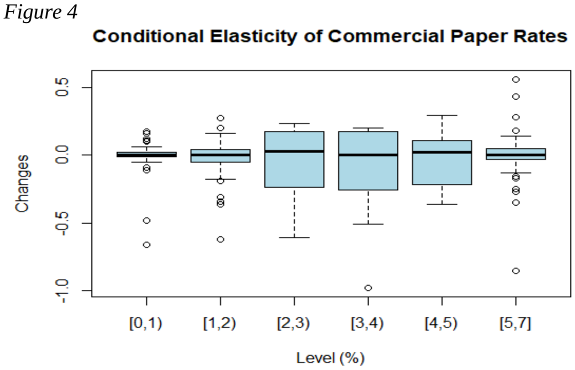
We use a histogram to examine to see if the elasticity of volatility dynamic exists within our data. To do this, we binned interest rates into 5 bins and looked at their changes. By looking at the boxplot, we see that the standard deviation is smaller near zero, but as interest rates are higher, the changes are larger and tend to be more volatile. This suggests that the GMM approach is appropriate. This is the relationship between changes and level that the value of γ will predict. Figures 3 and 4 are the boxplots for the elasticity of volatility for T-bill rates and CP rates, respectively.

Figure 3



Frequency					
[0,1)	[1,2)	[2,3)	[3,4)	[4,5)	[5,7]
116	37	12	16	41	28

Figure 4



Frequency					
[0,1)	[1,2)	[2,3)	[3,4)	[4,5)	[5,7]
102	44	16	10	19	59

One thing of note, in the [2,3) and [3,4) bins, there is a large degree of variance, as you can see by the boxplots, but the frequency of the observations is relatively low. Additionally, there is smaller variance in the 2 highest bins.

Section V: Performance Tests

Testing for serial correlation of errors is important, because if the errors are correlated then the model is misspecified. The first such test is the Autocorrelation Function (ACF) test which is shown in Figure 6. This is used to see if there is serial correlation, and to what degree there is. As you can see, the ACF of residuals clearly shows that there is no correlation between the residuals, which suggests that this model is appropriate for modeling T-bill and CP rates. The ACF of standardized residuals showed that there was volatility clustering within the data. From the ACF plots in Figure 5 and 6, we can see that most values are within the 95% confidence interval for Gaussian white noise, which are the dotted horizontal lines in the graphs. If the lags rise above the bars, then this model isn't appropriate to use. Moreover, the variance is non-stationary and suggests that the GARCH process may be appropriate for this data.

Figure 5: ACF of Standardized Residuals for T-Bill rate

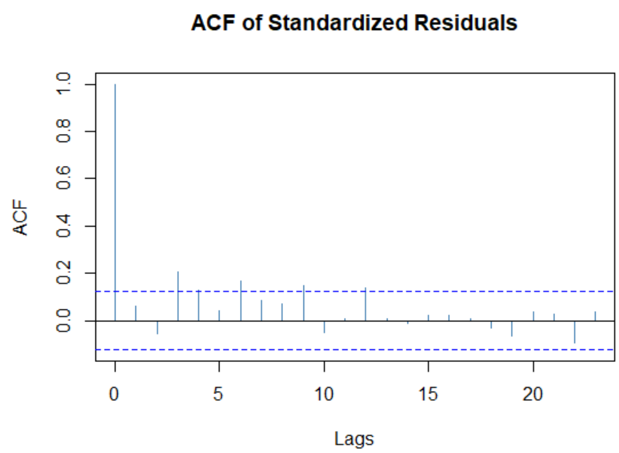
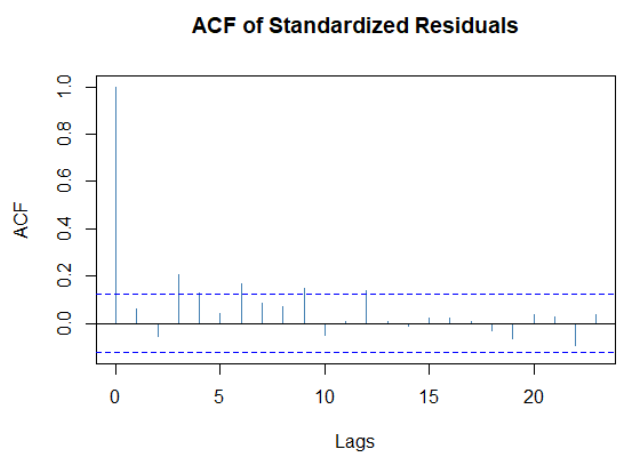


Figure 6: ACF of Standardized Residuals for CP rate



Because the models were not nested, to compare the models, we used to compare predicted values vs the actual values. We show summary statistics for the error terms of each model. We look at averages, standard deviations, skewness, and the largest and smallest errors which will tell us small insights into the performance of our models.

For Treasury-Bill rates, the average error was small for both the ARMA-GARCH and GMM model. The standard deviation for the ARMA-GARCH model was .1802 which is compared with 0.1797 for the GMM model. From this we can see that there wasn't a large amount of error and its distribution had little variance. Both are good signs for the performance of the models. The skewness is less for the ARMA-GARCH model was smaller than the GMM

Table 3: ARMA-GARCH Model Errors

ARMA-GARCH	*T-Bill Rate Model*	*CP Rate Model*
Mean	-0.009	-0.010
Standard Error	0.011	0.009
Median	0.002	0.000
Mode		-0.002
Standard Deviation	0.180	0.139
Sample Variance	0.032	0.019
Kurtosis	5.632	12.788
Skewness	-1.226	-2.015
Range	1.561	1.414
Minimum	-0.801	-0.890
Maximum	0.760	0.524
Sum	-2.361	-2.472
Count	250	250
Confidence Level(95.0%)	0.0225	0.0173

model. The skewness generally measures the length of the tails, so it can tell us what the distribution looks like. Specifically, skewness for the ARMA-GARCH model was -1.23 and -1.88 for the GMM model, which indicates that the errors were not distributed normally. Excess kurtosis was near 5.5 for both models which shows bushiness at the tails of the distribution. Generally indicating that errors were very spread out. The graph of the error terms tends to indicate that there were larger errors near the beginning of the period for both models. Then during the financial crisis, the error terms spiked before falling. From the statistics we derive 3 conclusions: 1) that the model tended to perform well, with small average, median and standard distributions, 2) the model tended to forecast well when interest rates were low and loss volatile, 3) the ARMA-GARCH approach was slightly better at predicting treasury bill rates than the GMM approach. This last insight demonstrates that Treasury bills respond to stress. Their movements tend to experience volatility clustering.

Table 4: GMM Model Errors

GMM	*Tbill Rate*	CP Rate
Mean	0.000	0.001
Standard Error	0.012	0.010
Median	0.000	0.000
Mode	0.000	0.000
Standard Deviation	0.188	0.157
Sample Variance	0.035	0.025
Kurtosis	8.415	9.117
Skewness	0.489	-0.410
Range	1.928	1.613
Minimum	-0.830	-0.782
Maximum	1.099	0.831
Sum	0.039	0.199
Count	250.000	250.000
Confidence Level(95.0%)	0.023	0.020

Now we will consider commercial paper rates. For the most part, all the same conclusions hold. There are some interesting differences between the values. On average, the errors were very small, as was the standard deviation. The average error for the GMM approach was significantly larger than that of the ARMA-GARCH model. The average error for ARMA-GARCH was -0.010 and for GMM it was -0.0003. The standard deviations are similarly smaller. This suggests that the ARMA-GARCH approach might be the more appropriate model because it has smaller errors. The error terms weren't distributed normally, and the tails were significantly wider for commercial paper rates, which follows our intuition. Indicating that commercial paper rates tend to experience higher volatility than the model might predict in times of stress. So again, we can see an advantage for using the ARMA-GARCH model.

Lastly, we compare the correlations between the predicted values vs actual values. This is the best method we have to compare the models' performances with each other. Shown in the table below, the ARMA-GARCH and the GMM approach produced values that were moderately correlated with its actual values. The closeness of their correlations makes sense given how similar the equations are for both methods.

	T-bill Changes	*CP Changes*
ARMA-GARCH Predicted	0.4466	0.5907
GMM Predicted	0.4484	0.5912

Section VI: Empirical Results

First, I ran the results for the Unrestricted model on 90-day T-bill rates. The results suggest that alpha and beta are not statistically different with zero, which is consistent with prior research. Shown in Table 5, the value of sigma is 0.168 which is larger than expected and is most likely due to the continued near-zero interest rate environment which shows more variance. Gamma is smaller than expected, suggesting that perhaps high volatility and high interest rates of the 1980s increased the parameter estimate. In 1997-2017, monetary policy played an important role in setting rates, and perhaps lowering overall volatility.

The parameter estimates for all the models can be seen in Table 2. Many of the other models were not statistically significant. In fact, only the unrestricted model and the CEV model were statistically significant. Both parameter estimates for sigma and gamma were significant at the 95% level. Because the CEV model was significant, we have shown that there are not any drift dynamics in interest rate changes.

Mathematically, gamma represents an exponential parameter on the variable $r_{t.}$ Because gamma = 0.218, the graph of conditional volatility will look the graph of y = sqrt(x). As interest rates are low, so too is volatility. As the level of interest rates get higher, the conditional volatility gets larger quickly before slowing the pace and increasing at a gradual rate.

Next, we estimated the parameters for commercial paper rates, shown in Table 3. Many of the same conclusions are drawn regarding their dynamics which is expected given how correlated the two rates are. The unrestricted model for both T-bill and CP rates demonstrated that interest rates are dependent on the elasticity of volatility. The model of T-bill rate fit the data well, but the model was only significant at the 90% level for CP rates. The implied risk premium for owning the riskier commercial paper vs T-bills is seen by the larger value of gamma for commercial paper.

To go back to the research question, the GMM approach is clearly an appropriate model to use when modeling T-bill rates. There exists a significant dependence on elasticity of volatility within interest rates, and moreover, we have estimated a parameter of this relationship. We derived two main conclusions from the results of this parameter estimation: (1) the estimate is lower than previous research which signals falling volatility; and (2) there exists a larger degree of nonlinearity in conditional volatility for commercial paper rates. This can be seen in both the parameter estimates and the overall fit of the two models compared to each other. It was slightly more difficult for the model to forecast commercial paper rates than T-bill rates.

Now, we will discuss the results for the ARMA-GARCH model, and then we will compare the GMM and ARMA-GARCH approach to each other. Again, we began by estimating the five parameters. From this we see 3 things: 1) there is a strong relationship between current changes and past changes. Lambda is a linear parameter that shows this relationship and is equal to 0.358. This suggests that regressing with respect to changes as opposed to levels might be more appropriate. 2) the estimate for xi, which shows the stickiness of changes with respect to past residuals, is 0.574. More specifically, this parameter shows the relationship between the residuals to their prior values. If the linear model produces large errors last period, they're likely associated with large errors in this period. 3) The conditional volatility parameter is 0.617. Clearly showing the tendency for large changes to be clustered together. Thus, we conclude that there exists the volatility clustering effect for interest rates. All the parameter values are presented in Table 5 and 6.

When I ran this model on commercial paper rates, I found interesting differences between the parameter values for the two rates. While many of the same conclusions can be drawn there is a noticeable difference in lambda suggesting that there is an even greater relationship between current and past changes for commercial paper rates. While lambda was larger for CP rate, xi was much smaller. Meaning that there might be less tendency for the residuals to be serially correlated. However, rho is also larger. For commercial paper rate, rho = 0.732. There is a higher conditional volatility for commercial paper. This is indicative of higher volatility for slightly riskier assets. All the parameter values for the ARMA-GARCH model are shown in Table 6.

We conclude that the characteristics described at the beginning of this paper are true. Interest rate changes will be more volatile the higher that interest rate levels become, and they will be volatility clustering when previous errors and changes are large.

Table 5: Parameter Estimates for GMM Model

T-bill Rate							
		α	β	σ	γ	p-value	Degrees of Freedom
Model 1	Unrestricted	0.000	-0.008	0.168	0.219	*******	0
	s.e.	0.017	0.008	0.002	0.076		
	t-stat	0.019	-0.980	7.192	2.896		
Model 2	CEV	**0.000**	-0.007	0.169	0.218	0.985	1
	s.e.		0.007	0.017	0.053		
	t-stat		-1.050	9.671	4.115		
Model 3	Merton	-0.015	**0.000**	0.168	**0.000**	0.001	2
	s.e.	0.016		0.021			
	t-stat	-0.938		7.975			
Model 4	Vasicek	-0.022	0.009	0.168	**0.000**	0.001	1
	s.e.	0.016	0.005	0.022			
	t-stat	-1.364	1.800	7.723			
Model 5	GBM	**0.000**	0.001	-0.037	**1.000**	0.001	2
	s.e.		0.007		0.007		
	t-stat		0.095		-5.235		
Model 6	CIR-SR	0.026	-0.012	0.105	**0.500**	0.034	1
	s.e.	0.016	0.007	0.013			
	t-stat	2.238	-1.554	7.826			
Model 7	Dothan	**0.000**	**0.000**	-0.368	**1.000**	0.001	3
	s.e.			0.005			
	t-stat			-7.961			
Model 8	Brennan-Schwartz	0.031	-0.009	0.043	**1.000**	0.005	1
	s.e.	0.012	0.008	0.006			
	t-stat	2.516	-1.133	6.625			
Model 9	CIR-VR	**0.000**	**0.000**	-0.026	**1.500**	0.001	3
	s.e.			0.002			
	t-stat			-7.111			

Commericial Paper Rate							
		α	β	σ	γ	p-value	Degrees of Freedom
Model 1	Unrestricted	-0.002	-0.006	0.146	0.270	*******	0
	s.e.	0.015	0.008	0.025	0.102		
	t-stat	-0.126	-0.808	5.474	2.635		
Model 2	CEV	**0.000**	-0.006	0.144	0.278	0.900	1
	s.e.		0.007	0.019	0.084		
	t-stat		-0.873	7.719	3.325		
Model 3	Merton	-0.009	**0.000**	0.145	**0.000**	0.021	2
	s.e.	0.014		0.024			
	t-stat	-0.588		6.150			
Model 4	Vasicek	-0.010	0.005	0.146	**0.000**	0.008	1
	s.e.	0.015	0.005	0.024			
	t-stat	-0.694	0.971	6.054			
Model 6	GBM	**0.000**	0.000	-0.036	**1.000**	0.007	2
	s.e.		0.007	0.008			
	t-stat		-0.065	-4.774			
Model 7	CIR-SR	0.014	-0.009	0.100	**0.500**	0.115	1
	s.e.	0.011	0.007	0.015			
	t-stat	1.329	-1.203	6.493			
Model 8	Dothan	**0.000**	**0.000**	-0.035	**1.000**	0.017	3
	s.e.			0.006			
	t-stat			-6.345			
Model 9	Brennan-Schwa	0.021	-0.006	-0.038	**1.000**	0.150	1
	s.e.	0.011	0.008	0.007			
	t-stat	1.977	-8.342	-5.293			
Model 10	CIR-VR	**0.000**	**0.000**	-0.014	**1.500**	0.011	3
	s.e.			0.003			
	t-stat			-5.629			

Table 6: Parameter Estimates for ARMA-GARCH Model

Treasury bill Rate				
	est.	*s.e.*	*t-stat*	*sig.*
α	0.000	0.002	-0.204	
β	0.358	0.070	5.142	***
ω	0.000	0.000	1.771	
ξ	0.574	0.177	3.245	**
γ	0.617	0.069	8.912	***
* Signficance	0 ***	0.001**	0.01*	

Commercial Paper Rate				
	est.	*s.e.*	*t-stat*	*sig.*
α	0.002	0.002	0.793	
β	0.487	0.065	7.450	***
ω	0.000	0.000	2.046	*
ξ	0.230	0.072	3.177	**
γ	0.732	0.049	14.934	***
* Signficance	0 ***	0.001**	0.01*	

Section VI: Conclusion

For investors and firms, it’s important to hedge interest rate risk so you can take advantage of borrowing cost fluctuations. In doing so, it is necessary to understand their dynamics. Knowing this information can be useful for a corporation trying to hedge against sudden rises in corporate borrowing costs.

We considered two models; however, further research might attempt to nest the GMM with the ARMA-GARCH process if possible. If we could consider a model which can test both dynamics of interest rates at the same time, it may be able to provide the best approximation and forecast ability.

We said from the beginning that modeling forecasts can serve as an assessment for policy changes. Over the last 20 years, the Fed has pursued lower interest rates than ever before. This monetary policy has reduced overall volatility for interest rates. Intuitively, this makes sense because investors have more policy certainty when the Fed is active and communicating with markets. However, I offer an alternative explanation. Because of the financial crisis, central banks started to buy ultra-safe government securities and regulators increased capital requirements on all the banks. Corporations and governments began buying many of the safest assets they could hold such as treasury bills and commercial paper, which has lowered volatility in these markets because of the immense demand and general economic conditions decreased periods of stress in the short-term market. The data at the time were key financial variables and were highly considered the risk-free rate or very close to it. We show a slight credit premium associated with conditional volatility for commercial paper rates which is consistent with our understanding of these two rates.

The two models show that the dynamics for commercial paper rates are different. When the model depended on the elasticity of volatility, they approximated treasury bills well. Which shows that treasury bills are less volatile overall. However, commercial paper rates respond to stress in the markets, i.e. periods of sudden high volatility. The question then becomes, why do two different approaches model one rate better than the other even though both rates are extremely correlated? We propose 2 alternative explanations for as to why this might be the case: 1) AA Commercial paper is always issued by public companies, and commercial paper rates may respond to stock market movements; whereas, treasury bills are a more defensive asset; and 2) the direct interference by the Fed into the government securities market lowered conditional volatility, but they have been less active in the commercial paper market.

References

Bollerslev, Tim. "Generalized Autoregressive Conditional Heteroskedasticity." Journal of Econometrics 31, no. 3 (February 1986): 307-27.

Brennan, Michael J., and Eduardo S. Schwartz. "A Continuous Time Approach to the Pricing of Bonds." Journal of Banking & Finance 3, no. 2 (1979): 133-55.

Campbell, John. "Stock Returns and the Term Structure." Journal of Financial Economics18 18, no. 2 (1985): 373-99.

Chan, K. C., G. Andrew Karolyi, Francis A. Longstaff, and Anthony B. Sanders. "An Empirical Comparison of Alternative Models of the Short-Term Interest Rate." The Journal of Finance 47, no. 3 (January 3, 1992): 1209-227.

Chen, Cathy W. S., and Jack C. Lee. "On Selecting a Power Transformation in Time-Series Analysis." Journal of Forecasting 16, no. 5 (1997): 343-54.

Cox, John C., Jonathan E. Ingersoll, and Stephen A. Ross. "A Theory of the Term Structure of Interest Rates." Econometrica 53, no. 2 (March 1985): 385.

Dothan, L.uri. "On the Term Structure of Interest Rates." Journal of Financial Economics 6, no. 1 (1978): 59-69.

Hansen, Lars Peter. "Large Sample Properties of Generalized Method of Moments Estimators." Econometrica 50, no. 4 (July 1982): 1029.

J., Grossman Sanford, Melino Angelo, and Shiller Robert J. "Estimating the Continuous Time Consumption Based Asset Pricing Model." Journal of Business and Economic Statistics 5 (1985): 315-27.

Khramov, Vadim. "Estimating Parameters of Short-Term Real Interest Rate Models." IMF Working Papers 13, no. 212 (October 2013): 1.

Merton, Robert C. "An Intertemporal Capital Asset Pricing Model." Econometrica 41, no. 5 (September 1973): 867.

Vasicek, Oldrich. "An Equilibrium Characterization of the Term Structure." Journal of Financial Economics 5, no. 2 (August 1977): 177-88.