Master Thesis

Optimal Integration of Autonomous Vehicles in Car Sharing: Development of a Heuristic considering Multimodal Transport and Integration in an Optimal Framework

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1 Problem Formulation

1.1 Problem Description and Notation

This formulation models the problem of optimal integration of autonomous vehicles in car sharing, considering multimodal transport.

Notation

We are given a set of vehicles $\mathcal{V}$ and a set of costumers $\mathcal{C}$. For public transport, we have a set of available stations $\mathcal{S}$ and a set of public transport rides $\mathcal{P}$. A ride $p \in \mathcal{P}$ is a finite sequence of stations at time points $p = ((s_1, z_1), \ldots, (s_k, z_k))$ with $s_i \in \mathcal{S}$ and z_i a time point for $i \in [k]$.

We are further given a set of trips $\mathcal{T}$; each trip $t \in \mathcal{T}$ is either a car trip or a public transport trip and has a start and end location $p_t^{\text{start}}, p_t^{\text{end}}$ and a start and end time $z_t^{\text{start}}, z_t^{\text{end}}$. Accordingly, we define $\mathcal{T} = \mathcal{T}_{\text{car}} \uplus \mathcal{T}_{\text{public}}$. A public transport trip $t \in \mathcal{T}_{\text{public}}$ is a connected subsequence of a public transport ride $p \in \mathcal{P}$ and it holds

$$p_t^{\text{start}} = s_i^p \qquad p_t^{\text{end}} = s_j^p \qquad z_t^{\text{start}} = z_i^p \qquad z_t^{\text{end}} = z_j^p$$

for some $i < j$.

The start position and the starting time of a vehicle $v \in \mathcal{V}$ is p_v and z_v.

Additionally, we have a set of refuel stations $\mathcal{R}$. A refuel station $r \in \mathcal{R}$ has a location p_r. In this model, a car is allowed to refuel at most once between two trips. We define $f_{s,t}^{\text{d}}$ for $s \in \mathcal{V} \cup \mathcal{T} \cup \mathcal{R}, t \in \mathcal{T} \cup \mathcal{R}$ as the amount, the fuel level decreases along the deadhead trip. f_t^{t} for $t \in \mathcal{T} \cup \mathcal{R}$ is the amount of fuel, the car needs for a trip. For $r \in \mathcal{R}$ holds $f_r^{\text{t}} \leq 0$. f_v^0 for $v \in \mathcal{V}$ is the initial fuel state of a car. The fuel of a car is in the interval $[0, 1]$ describing the relative fuel state.

We define the time, a car needs to get from position p_1 to p_2, as t_{p_1, p_2}. We define

$$t_{s,t} = \begin{cases} t_{p_s^{\text{end}}, p_t^{\text{start}}} & \text{if } s, t \in \mathcal{T}_{\text{car}} \\ t_{p_s, p_t^{\text{start}}} & \text{if } s \in \mathcal{V} \cup \mathcal{R}, t \in \mathcal{T}_{\text{car}} \\ t_{p_s^{\text{end}}, p_t} & \text{if } s \in \mathcal{T}_{\text{car}}, t \in \mathcal{R} \\ t_{p_s, p_t} & \text{if } s \in \mathcal{V}, t \in \mathcal{R} \end{cases}$$

as the time a car needs from one trip to another.

We are given a set of multimodal routes $\mathcal{M}$. A route $m = (t_1, \ldots, t_k)$ is a sequence of trips with the following properties:

$$p_{t_i}^{\text{end}} = p_{t_{i+1}}^{\text{start}} \qquad z_{t_1}^{\text{end}} \leq z_{t_{i+1}}^{\text{start}} \qquad t_i \in \mathcal{T}_{\text{car}} \Rightarrow t_{i+1} \in \mathcal{T}_{public} \qquad \text{for all } i \in [k-1].$$

We define the route start and end locations and times for $m \in \mathcal{M}$

$$p_m^{\text{start}} := p_{t_1}^{\text{start}} \qquad p_m^{\text{end}} := p_{t_k}^{\text{end}} \qquad z_m^{\text{start}} := z_{t_1}^{\text{start}} \qquad z_m^{\text{end}} := z_{t_k}^{\text{end}}.$$

Each costumer $c \in \mathcal{C}$ has a finite set of alternative routes. The mapping $C : \mathcal{M} \to \mathcal{C}$ shows which route belongs to which costumer. For each route of the same costumer $m \in C^{-1}(c)$, the start and end positions are the same, the start and end times may differ. We define the costumer start and end times for $c \in \mathcal{C}$

$$z_c^{\text{start}} := \min_{m \in C^{-1}(c)} z_m^{\text{start}} \qquad\qquad z_c^{\text{end}} := \max_{m \in C^{-1}(c)} z_m^{\text{end}}. \qquad (1)$$

Problem Description

The problem is the following: Find a schedule of trips for every vehicle including refueling stops and a sequence of trips for every costumer. Therefore, the car trips are fulfilled by the scheduled car and the public transport trips by public transport according to its timetable. For this, we have the following conditions:

- Each car is able to serve its scheduled trips, considering time and location.

- The fuel state of each car is always in a feasible range.

- Each costumer is able to complete his trip, considering time and location.

- For each costumer, exactly one trip is chosen.

The goal is to find a cost-minimal feasible schedule considering all these constraints.

Costs

We have the following types of costs:

- Vehicles costs c^{v}: unit costs for each used car

- Deadhead costs $c_{s,t}^{\text{d}}$ for $s \in \mathcal{V} \cup \mathcal{T} \cup \mathcal{R}, t \in \mathcal{T} \cup \mathcal{R}$: costs, if a car drives to a trip or a refuel station without a costumer using it

- Trip costs c_t^{t} for $t \in \mathcal{T}_{\text{car}}$: costs for fulfilling a trip

For public transport, we define either trip costs for each public transport trip or fixed costs for each costumer using public transport. Finally, we define costs to consider the costumer preferences.

- Trip costs c_t^{t} for $t \in \mathcal{T}_{\mathrm{public}}$: costs for using public transport

- Route-dependent costs c_m^{r} for $m \in \mathcal{M}$: costs for costumer preferences and unit costs for using public transport

Since the trip costs for public transport are connected with the choice of the route, we easily add these costs $\sum_{t \in m \cap \mathcal{T}_{\mathrm{public}}} c_t^{\mathrm{t}}$ to the trip costs.

The route costs additionally include costumer preferences. This can be the total travel time, the number of changes or the costs for the costumers. Typically, a pure car trip is faster but more expensive. Further, a late departure time or an early arrival time can be criteria for this cost function.

Partial Order of the Trips

In order to decide whether a car can fulfill two trips in a row, we define a partial ordering of the car set and the set of car trips. The set of public transport trips is left out in this definition.

Definition 1 (Partial order of trips)**.** The binary relation $\prec$ on $\mathcal{V} \times \mathcal{T}_{\mathrm{car}}$ is defined as follows:

$$s \prec t \qquad :\Leftrightarrow \qquad \left(z_s^{\mathrm{end}} + t_{s,t} \leq z_t^{\mathrm{start}} \right) \wedge \left(C(s) \neq C(t) \vee \exists m \in \mathcal{M} : s, t \in m \right)$$
$$\text{for all } s \in \mathcal{V} \uplus \mathcal{T}_{\mathrm{car}}, t \in \mathcal{T}_{\mathrm{car}}$$
$$s \not\prec t \qquad\qquad\qquad\qquad\qquad \text{for all } s \in \mathcal{V} \uplus \mathcal{T}_{\mathrm{car}}, t \in \mathcal{V}$$

The binary relation $\preceq$ on $\mathcal{V} \times \mathcal{T}_{\mathrm{car}}$ we define as:

$$s \preceq t \qquad :\Leftrightarrow \qquad s = t \wedge s \prec t \qquad \text{for all } s, t \in \mathcal{V} \uplus \mathcal{T}_{\mathrm{car}}$$

The expression $s \prec t$ means, that one car is able to fulfill both trips, first s and then t. A car must not cover two trips of the same costumer, except they belong to the same route. This results from the problem setting, that for each costumer exactly one trip is fulfilled.

1.2 Route Creation

We are not given the set of routes $\mathcal{M}$ in advance. For each costumer $c \in \mathcal{C}$, we have start and end location $p_c^{\mathrm{start}}, p_c^{\mathrm{end}}$ and a start and end time $\hat{z}_c^{\mathrm{start}}, \hat{z}_c^{\mathrm{end}}$. All the trips of the costumer lie in this interval, i.e.

$$\hat{z}_c^{\mathrm{start}} \leq z_m^{\mathrm{start}} \qquad\qquad z_m^{\mathrm{end}} \leq \hat{z}_c^{\mathrm{end}} \qquad\qquad \text{for all } m \in C^{-1}(c).$$

Basic Restrictions

To simplify the creation of the routes, we make some assumptions. For every route $m \in \mathcal{M}$ holds:

- There are not two car trips in a row.

- There is no car trip between two public transport trips.

- The number of public transport trips is restricted. Usually, one can reach every station with at most two changes.

- We define a walking distance d^{walk}. If the distance between the start position and the first station or between the last station and the end position, there is no car trip necessary.

We assume that we have some oracle that provides the set of feasible public transport routes for costumer $c \in \mathcal{C}$:

$$M_c = \quad \Big\{ (s_1, z_1, s_2, z_2) \, | \, s_1, s_2 \in \mathcal{S}, \hat{z}_c^{\text{start}} \le t_1 < t_2 \le \hat{z}_c^{\text{end}}, \text{ there is a public}$$
$$\text{transport route from } s_1 \text{ to } s_2 \text{ with start time } z_1 \text{ and end time } z_2 \Big\}$$

The fact, whether the costumer changes during his usage of public transport, has no effect on the model. Thus, we can consider each element in M_c as a public transport trip.

Route Creation

We create the set of multimodal routes $\mathcal{M}$. For this, we set a car trip before and after each public transport trip in order to bring the costumer from his start to his destination, except when it is possible to walk the distance. We also have to consider the given time restrictions. Further, we create the pure car trips. How the set $\mathcal{M}$ is created in detail, is described in algorithm 1.

Until now, we do not consider any changing times between a car trip and a public transport trip.

Further, we assume that the given costumer start and and times are feasible, i.e. $\hat{z}_c^{\text{start}} + t_{p_c^{\text{start}}, p_c^{\text{end}}} \le \hat{z}_c^{\text{end}}$ for all $c \in \mathcal{C}$.

Algorithm 1: Creation of the routes

Input: costumer $\mathcal{C}$; $M_c, p_c^{\text{start}}, p_c^{\text{end}}, \hat{z}_c^{\text{start}}, \hat{z}_c^{\text{end}}$ for all $c \in \mathcal{C}$
Output: set of routes $\mathcal{M}$, set of trips $\mathcal{T}_{\text{car}}, \mathcal{T}_{\text{public}}$

1 $\mathcal{T}_{\text{car}} \leftarrow \emptyset$;
2 $\mathcal{T}_{\text{public}} \leftarrow \emptyset$;
3 $\mathcal{M} \leftarrow \emptyset$;
4 **foreach** $c \in \mathcal{C}$ **do**
5 $\quad$ determine M_c;
6 $\quad$ **foreach** $(s_1, z_1, s_2, z_2) \in M_c$ **do**
7 $\quad\quad$ create public transport trip t;
8 $\quad\quad$ $p_t^{\text{start}} \leftarrow s_1, p_t^{\text{end}} \leftarrow s_2, z_t^{\text{start}} \leftarrow z_1, z_t^{\text{end}} \leftarrow z_2$;
9 $\quad\quad$ create car trips t_1, t_2;
10 $\quad\quad$ $p_{t_1}^{\text{start}} \leftarrow p_c^{\text{start}}, p_{t_1}^{\text{end}} \leftarrow s_1, z_{t_1}^{\text{start}} \leftarrow z_1 - t_{p_c^{\text{start}}, s_1}, z_{t_1}^{\text{end}} \leftarrow z_1$;
11 $\quad\quad$ $p_{t_2}^{\text{start}} \leftarrow s_2, p_{t_2}^{\text{end}} \leftarrow p_c^{\text{end}}, z_{t_2}^{\text{start}} \leftarrow z_2, z_{t_2}^{\text{end}} \leftarrow z_2 + t_{s_2, p_c^{\text{end}}}$;
12 $\quad\quad$ **if** $\hat{z}_c^{\text{start}} \leq z_{t_1}^{\text{start}} \wedge z_{t_2}^{\text{end}} \leq \hat{z}_c^{\text{end}}$ **then**
13 $\quad\quad\quad$ create multimodal route m;
14 $\quad\quad\quad$ $\mathcal{T}_{\text{public}} \leftarrow \mathcal{T}_{\text{public}} \cup t$;
15 $\quad\quad\quad$ **if** $d_{p_c^{\text{start}}, s_1} \geq d^{\text{walk}}$ **then** $m \leftarrow (t_1, t)$; $\mathcal{T}_{\text{car}} \leftarrow \mathcal{T}_{\text{car}} \cup \{t_1\}$;
16 $\quad\quad\quad$ **else** $m \leftarrow (t)$;
17 $\quad\quad\quad$ **if** $d_{s_2, p_c^{\text{end}}} \geq d^{\text{walk}}$ **then** append t_2 to m; $\mathcal{T}_{\text{car}} \leftarrow \mathcal{T}_{\text{car}} \cup \{t_2\}$;
18 $\quad\quad\quad$ $C(m) \leftarrow c$;
19 $\quad\quad\quad$ $\mathcal{M} \leftarrow \mathcal{M} \cup \{m\}$;
20 $\quad\quad$ **end**
21 $\quad$ **end**
22 $\quad$ create car trips t_1, t_2;
23 $\quad$ $p_{t_1}^{\text{start}} \leftarrow p_c^{\text{start}}, p_{t_1}^{\text{end}} \leftarrow p_c^{\text{end}}, z_{t_1}^{\text{start}} \leftarrow \hat{z}_c^{\text{start}}, z_{t_1}^{\text{end}} \leftarrow \hat{z}_c^{\text{start}} + t_{p_c^{\text{start}}, p_c^{\text{end}}}$;
24 $\quad$ $p_{t_2}^{\text{start}} \leftarrow p_c^{\text{start}}, p_{t_2}^{\text{end}} \leftarrow p_c^{\text{end}}, z_{t_2}^{\text{start}} \leftarrow \hat{z}_c^{\text{end}} - t_{p_c^{\text{start}}, p_c^{\text{end}}}, z_{t_2}^{\text{end}} \leftarrow \hat{z}_c^{\text{end}}$;
25 $\quad$ create multimodal routes m_1, m_2;
26 $\quad$ $m_1 \leftarrow (t_1), m_2 \leftarrow (t_2)$;
27 $\quad$ $\mathcal{T}_{\text{car}} \leftarrow \mathcal{T}_{\text{car}} \cup \{t_1, t_2\}, \mathcal{M} \leftarrow \mathcal{M} \cup \{m_1, m_2\}$;
28 **end**
29 **return** $\mathcal{M}, \mathcal{T}_{\text{car}}, \mathcal{T}_{\text{public}}$

Further Restrictions

If the routes are created as described in algorithm 1, there are routes using every available station as long as it is feasible. Most of these routes are obviously bad for the costumer since they cause a big detour. What is more, a large number of routes enlarge the problem size and leads to a bad performance for solving it. Therefore, we try to restrict the set of alternatives to a reasonable size.

Example 1. Let $\mathcal{S} = \{s_1, \ldots, s_n\}$ with a single public transport ride serving all stations. Let $\mathcal{C} = \{c_1, c_2\}$ with $p_{c_1}^{\text{end}} = s_n$ and $p_{c_2}^{\text{start}} = s_k$ for a certain $k \in [n-1]$. The alternative routes are

$$\mathcal{M} = \underbrace{\left\{ \left(\left(p_{c_1}^{\text{start}}, s_i \right), (s_i, s_n) \right) | i \in [n-1] \right\}}_{\text{for } c_1} \cup \underbrace{\left\{ \left(s_k, p_{c_2}^{\text{end}} \right) \right\}}_{\text{for } c_2}$$

with $\left(p_{c_1}^{\text{start}}, s_k \right) \prec \left(s_k, p_{c_2}^{\text{end}} \right)$ and $\left(p_{c_1}^{\text{start}}, s_i \right) \not\prec \left(s_k, p_{c_2}^{\text{end}} \right)$ for all $i \in [n] \backslash \{k\}$.

We get the only solution, where only one car is needed, when c_1 drives to s_k, wherever the station s_k is. Every route of c_1 can be the optimal route, considering the other costumers. Therefore, an exact reduction of $\mathcal{M}$ is not possible without the the risk of pruning the optimal solution.

It is not practicable to consider all possible multimodal routes due to computation reasons. But it is also not possible to reduce the number of routes without risking to lose the optimal solution. Hence, we try to make reasonable restrictions which keep the problem size small.

Pareto Optimality

The idea is to choose only Pareto optimal multimodal routes (cf. Kaiser/Knoll, cap. 3.2.2) in order to determine good routes.

Definition 2 (Pareto optimality). Let V in $\mathbb{R}^n$.

1. The partial order $\leq$ on $\mathbb{R}^n$ is given by

$$v \leq w \qquad :\Leftrightarrow \qquad v_i \leq w_i \qquad \forall i \in [n] \qquad \text{for all } v, w \in \mathbb{R}^n$$

2. An element $w \in V$ is Pareto optimal in V if it is minimal with respect to $\leq$ in V, i.e.

$$v \leq w \qquad \Rightarrow \qquad v = w \qquad \text{for all } v \in V$$

3. The Pareto frontier of V with respect to $\leq$ is the set of Pareto optimal elements in V, i.e.

$$min_{\leq}V := \{w \in V | \forall v \in V : v \leq w \Rightarrow v = w\}$$

Let $m \in \mathcal{M}$ be a multimodal route. We define

$$\varphi : \mathcal{M} \to \mathbb{R}^5 \qquad\qquad m \mapsto \begin{pmatrix} c^{\mathrm{r}} + \sum_{t \in m} c_t^{\mathrm{t}} \\ c^{\mathrm{r}} \\ |\mathcal{T}_{\mathrm{car}} \cap \{t \in m\}| \\ \sum_{t \in m \cap \mathcal{T}_{\mathrm{car}}} z_t^{\mathrm{end}} - z_t^{\mathrm{start}} \\ \sum_{t \in m \cap \mathcal{T}_{\mathrm{car}}} f_t^{\mathrm{t}} \end{pmatrix}$$

The function φ grades a route to their costs, their route costs, the number of cars needed, the time of a car needed and the fuel consumption.
From now on, we will use the Pareto frontier of $\varphi(\mathcal{M})$ as a restricted route set:

$$\hat{\mathcal{M}} := min_{\leq}\varphi(\mathcal{M}) \tag{2}$$

2 Mathematical Models

We introduce the mathematical model, with which the previously described problem is solved.

2.1 Task Graph

For tackling the problem, we introduce a task graph, on which the model is based. The graph is basically the same as in (Kaiser, Knoll, cap. 3.1) with the restriction, that only car trips $t \in \mathcal{T}_{\text{car}}$ are considered.

Definition 3 (Task graph)**.** Let $d^{\text{s}}, d^{\text{e}}$ be special vertices describing the source and sink of the vehicle flow. We define the task graph as $\hat{G} = (\hat{V}, \hat{A})$, where

$$\hat{V} := \{d^{\text{s}}, d^{\text{e}}\} \cup \mathcal{V} \cup \mathcal{T}_{\text{car}}$$

is the vertex set consisting of the source, the sink, the vehicle set $\mathcal{V}$ and the set of car trips $\mathcal{T}_{\text{car}}$. The arc set is

$$\hat{A} := (\{d^{\text{s}}\} \times \mathcal{V}) \cup \left\{(s,t) \in (\mathcal{V} \cup \mathcal{T}_{\text{car}})^2 \, | \, s \prec t\right\} \cup ((\mathcal{V} \cup \mathcal{T}_{\text{car}}) \times \{d^{\text{e}}\}).$$

A vertex $s \in \mathcal{V}$ represents the initial state of a vehicle s where is becomes available for the first time. Each $d^{\text{s}} - d^{\text{e}}$-path in $\hat{G}$ is the duty of one vehicle, i.e. this vehicle fulfills the trips in the order given by the path. Hence, two trips are connected only if it is possible that one car fulfills both trips, i.e. the relation $\prec$ holds.
To consider the refuel stations, we introduce an extended task graph.

Definition 4 (Extended task graph)**.** For every $s, t \in \mathcal{V} \cup \mathcal{T}_{\text{car}}$ with $s \prec t$ we create a copy of $\left\{r \in \mathcal{R} \, | \, z_s^{\text{end}} + t_{s,r} + t_{r,t} \leq z_t^{\text{start}}\right\}$ denoted by $\mathcal{R}_{s,t}$. This means, various copied sets are pairwise disjoint. We define the extended task graph $G = (V, A)$ with vertex set

$$V := \hat{V} \cup \bigcup_{\substack{s,t \in \mathcal{V} \cup \mathcal{T}_{\text{car}} \\ s \prec t}} \mathcal{R}_{s,t}$$

and arc set

$$A := \hat{A} \cup \{(s,r) | s, t \in \mathcal{V} \cup \mathcal{T}_{\text{car}}, s \prec t, r \in \mathcal{R}_{s,t}\} \cup \{(r,t) | s, t \in \mathcal{V} \cup \mathcal{T}_{\text{car}}, s \prec t, r \in \mathcal{R}_{s,t}\}.$$

In this graph, each feasible refuel station is considered for each feasible pair of trips. There is a method to reduce the size of $\mathcal{R}_{s,t}$ significantly without losing the optimal solution. This method is described in (Kaiser, Knoll, cap. 3.2.2). From now on, we will use $G = (V, A)$ with restricted $\mathcal{R}_{s,t}$.

2.2 Arc Flow Formulation

We develop a model for solving the problem via a flow of the cars.

The multimodal routes are given in advance. The car trips are adjusted in such a way, that they fit to the public transport routes (in location and time). It is not possible to model each route as a trip because then the car availabilities are not considered.

We model a flow of the cars. The public transport trips work only as constraints for this flow. Constraints are the fulfilling of one multimodal route per costumer and the fuel constraints.

Basic Model

We model the arc flow as an ILP. The formulation is basically built on (MILP), described in (Kaiser, Knoll, cap. 3.2). We use the following decision variables:

- $x_{s,t} \in \{0,1\}$ for $(s,t) \in A$: indicates, whether trip $t \in \mathcal{T}_{\text{car}}$ is fulfilled after $s \in \mathcal{V} \uplus \mathcal{T}_{\text{car}}$

- $z_{s,r,t} \in \{0,1\}$ for $t \in \mathcal{T}_{\text{car}}, s \in \text{N}_G^-(t), r\mathcal{R}_{s,t}$: indicates, whether refuel station $r \in \mathcal{R}$ is visited between s and t

- $e_s \in [0,1]$ for $s \in \mathcal{V} \backslash \{d^{\text{s}}, d^{\text{e}}\}$: states the fuel of the respective car after fulfilling trip $s \in \mathcal{T}_{\text{car}}$

If $s \in \mathcal{V}$, then $x_{s,t}$ determines, whether trip t is the first trip fulfilled by s and e_s is the initial fuel state f_s^0 of vehicle s.

Additionally to (MILP) we introduce decision variables to determine the fulfilling of routes:

- $u_m \in \{0,1\}$ for $m \in \mathcal{M}$: indicates whether multimodal route m is fulfilled

The basic constraints are developed in detail in (Kaiser, Knoll, cap. 3.2) and not treated in detail here. The basic constraints are the flow conservation constraint, the constraint for considering every car and the constraints guaranteeing feasible fuel states all the time.

Costumer and Route Constraints

In (MILP), each costumer has a set of alternative trips and from this set, exactly one trips has to be fulfilled. This is modeled as follows:

$$\sum_{t\in C^{-1}(c)} \sum_{s\in \text{N}_G^-(t)} x_{s,t} = 1 \qquad \text{for all } c \in \mathcal{C} \qquad (3.17)$$

In contrast to (MILP), each costumer has a set of alternative routes consisting of trips and from this set, exactly one route has to be fulfilled. Therefore, we replace (3.17) by the following formulation:

$$\sum_{m \in C^{-1}(c)} u_m = 1 \qquad \qquad \text{for all } c \in \mathcal{C} \qquad \qquad (3)$$

$$\sum_{s \in \mathrm{N}_G^-(t)} x_{s,t} = u_m \qquad \qquad \text{for all } m \in \mathcal{M}, t \in m \qquad \qquad (4)$$

The constraint (3) says, that for every costumer exactly one route is fulfilled. The constraint (4) says, if a route is fulfilled then every trip of this route must be fulfilled.

Objective Function

The objective function in (MILP) is given by

$$\sum_{s \in \mathcal{V}} \sum_{t \in \mathrm{N}_G^+(s) \setminus \{d^e\}} x_{s,t} c^{\mathrm{v}} + \sum_{t \in \mathcal{T}} \sum_{s \in \mathrm{N}_G^-(t)} \left[x_{s,t} \left(c_{s,t}^{\mathrm{d}} + c_t^{\mathrm{t}} \right) + \sum_{r \in \mathcal{R}_{s,t}} z_{s,r,t} \left(c_{s,r}^{\mathrm{d}} + c_{r,t}^{\mathrm{d}} - c_{s,t}^{\mathrm{d}} \right) \right]$$

considering the vehicle costs c^{v}, the trip costs c_t^{t} for $t \in \mathcal{T}_{\mathrm{car}}$ and the deadhead costs c^{d}. What is missing, are the route-dependent costs c^{r} and the public transport trip costs c_t^{t} for $t \in \mathcal{T}_{\mathrm{public}}$. Since the route is fixed in advance, we can easily add the public transport costs to the route costs. Thus, we define

$$\hat{c}_m^{\mathrm{r}} := c^{\mathrm{r}} + \sum_{t \in m \cap \mathcal{T}_{\mathrm{public}}} c_t^{\mathrm{t}} \qquad \qquad \text{for all } m \in \mathcal{M}$$

and add

$$\sum_{m \in \mathcal{M}} u_m \hat{c}^{\mathrm{r}}$$

to the objective function.

LP Formulation

Putting all this together, we get the following formulation, called (MMILP):

$$\min \sum_{s \in \mathcal{V}} \sum_{t \in \mathrm{N}_G^+(s) \setminus \{d^e\}} x_{s,t} c^v + \sum_{m \in \mathcal{M}} u_m \hat{c}^r$$

$$+ \sum_{t \in \mathcal{T}} \sum_{s \in \mathrm{N}_G^-(t)} \left[x_{s,t} \left(c_{s,t}^d + c_t^t \right) + \sum_{r \in \mathcal{R}_{s,t}} z_{s,r,t} \left(c_{s,r}^d + c_{r,t}^d - c_{s,t}^d \right) \right] \quad \text{(MMILP)}$$

$$\text{s.t.} \quad \sum_{t \in \mathrm{N}_G^-(s)} x_{t,s} = \sum_{t \in \mathrm{N}_G^+(s)} x_{s,t} \qquad \text{for all } s \in V \setminus \{d^s, d^e\} \tag{3.15}$$

$$\sum_{s \in \mathrm{N}_G^-(t)} x_{s,t} = 1 \qquad \text{for all } t \in \mathcal{V} \tag{3.16}$$

$$\sum_{m \in C^{-1}(c)} u_m = 1 \qquad \text{for all } c \in \mathcal{C} \tag{3}$$

$$\sum_{s \in \mathrm{N}_G^-(t)} x_{s,t} = u_m \qquad \text{for all } m \in \mathcal{M}, t \in m \tag{4}$$

$$\sum_{r \in \mathcal{R}_{s,t}} z_{s,r,t} \leq x_{s,t} \qquad \text{for all } t \in \mathcal{T}, s \in \mathrm{N}_G^-(t) \tag{3.18}$$

$$e_s \leq f_s^0 \qquad \text{for all } s \in \mathcal{V} \tag{3.19}$$

$$0 \leq e_s - \sum_{r \in \mathcal{R}_{s,t}} z_{s,r,t} f_{s,r}^d \qquad \text{for all } t \in \mathcal{T}, s \in \mathrm{N}_G^-(t) \tag{3.12}$$

$$e_t \leq 1 - f_t^t - \sum_{r \in \mathcal{R}_{s,t}} z_{s,r,t} f_{r,t}^d \qquad \text{for all } t \in \mathcal{T}, s \in \mathrm{N}_G^-(t) \tag{3.13}$$

$$e_t \leq e_s - x_{s,t} \left(f_{s,t}^d + f_t^t \right) - \sum_{r \in \mathcal{R}_{s,t}} z_{s,r,t} \left(f_{s,r}^d + f_r^t + f_{r,t}^d - f_{s,t}^d \right) + (1 - x_{s,t})$$

$$\qquad \qquad \text{for all } t \in \mathcal{T}, s \in \mathrm{N}_G^-(t) \tag{3.14}$$

$$x_{s,t} \in \{0,1\} \qquad \text{for all } (s,t) \in A \tag{3.20}$$

$$z_{s,r,t} \in \{0,1\} \qquad \text{for all } t \in \mathcal{T}, s \in \mathrm{N}_G^-(t), r \in \mathcal{R}_{s,t} \tag{3.21}$$

$$e_s \in [0,1] \qquad \text{for all } s \in V \setminus \{d^s, d^e\} \tag{3.22}$$

3 Heuristics

There is already a heuristic for solving an easier version of the problem (Knoll, cap. 10). This heuristic only handles the case without costumers. This means, there is a trip set $\mathcal{T}$ and each of these trips has to be fulfilled. This is even a simplification to (MILP). We try to extend this heuristic such that it can tackle the problem considering multimodal transport.

3.1 Splitting the Problem

We define the splitting of the task graph similarly to (Knoll, cap. 8) with the difference, that the splittings can be defined generally here.

Definition 5 (Splitting). Let $n \in \mathbb{N}$ and let

$$\mathcal{T}_{\text{car}} = \bigcup_{i=1}^{n+1} \mathcal{T}_i \qquad\qquad \mathcal{V} = \bigcup_{i=1}^{n+1} \mathcal{V}_i$$

be partitions of the set of car trips, respectively vehicles. Then we call $\{\mathcal{T}_i | i \in [n+1]\}$ and $\{\mathcal{V}_i | i \in [n+1]\}$ splitting of $\mathcal{T}$ and $\mathcal{V}$ and $\mathcal{T}_i$ and $\mathcal{V}_i$ partial trip respectively vehicle set.

Definition 6 (Transformed Task Graph). Let $\{\mathcal{T}_1, \ldots \mathcal{T}_n\}$ be a splitting of $\mathcal{T}_{\text{car}}$ according to Definition 5. Then we define:

1. Split Point: Let $s \in \mathcal{T}_i$ for $i \in [n] \backslash \{1\}$. For $j \in [i-1]$, we define the split point $\text{SP}_j(s)$ with $p_{\text{SP}_j(s)}^{\text{start}} = p_{\text{SP}_j(s)}^{\text{end}} =: p_s^{\text{start}}, z_{\text{SP}_j(s)}^{\text{start}} = z_{\text{SP}_j(s)}^{\text{end}} =: z_s^{\text{start}}$ and $f_{\text{SP}_j(s)}^{\text{t}} =: 0$.

2. For $i \in [n] \backslash \{1\}$ and $j \in [i-1]$, we define $P_{j,i} := \{\text{SP}_j(s) | s \in \mathcal{T}_i\}$.

3. Partial Split Point Set: For $j \in [n-1]$, we define the partial split point set $P_j := \bigcup_{i=j+1}^{n} P_{j,i}$.

4. Split Point Set: We define the split point set $P := \bigcup_{j=1}^{n-1} P_j$.

Let $G = (V, A)$ the task graph, $\{\mathcal{V}_1, \ldots, \mathcal{V}_n\}$ be a splitting of $\mathcal{V}$.

1. Transformed Task Graph: We define the transformed task graph $\overline{G} = \left(\overline{V}, \overline{A} \right)$ with vertex set

$$\overline{V} := V \cup P = V \cup \{\text{SP}_i(s) | i \in [n-1], j \in [n] \backslash [i], s \in \mathcal{T}_j\}$$

and arc set

$$\overline{A} := (d^{\mathrm{s}} \times \mathcal{V}) \cup \bigcup_{i=1}^{n} \left\{ (s,t) \in (\mathcal{V}_i \cup \mathcal{T}_i) \times (\mathcal{T}_i \cup P_i) \,|\, s \prec t \right\}$$

$$\cup \bigcup_{i=1}^{n} \left\{ (s,t) \in \left(\left(\bigcup_{j=1}^{i-1} P_{j,i} \right) \times \mathcal{T}_i \right) \,\middle|\, s = \mathrm{SP}_i(t) \right\} \cup (\mathcal{V} \times \{d^{\mathrm{e}}\}) \cup (\mathcal{T}_{\mathrm{car}} \times \{d^{\mathrm{e}}\})$$

3.2 Costumer-dependent Splitting

In contrast to the splitting performed in (Knoll, cap. 8), the trips are not split according to their start times but according to their costumers' start times. This means, that all trips of a route and all routes of a costumer are in the same splitting. For each spitting, we apply (EMILP) to receive an optimal partial solution and connect the partial solutions to a feasible overall solution.

Splitting

Given points in time c_i for $i \in [n]$ with $c_i < c_{i+1}$ for $i \in [n-1]$. We first define a splitting of the costumers $\mathcal{C} = \bigcup_{i=1}^{n+1} \mathcal{C}_i$ as

$$\mathcal{C}_i := \begin{cases} \{c \in \mathcal{C} \,|\, z_c^{\mathrm{start}} \leq c_1\} & \text{for } i = 1 \\ \{c \in \mathcal{C} \,|\, c_{i-1} < z_c^{\mathrm{start}} \leq c_i\} & \text{for } i \in [n]\backslash\{1\} \\ \{c \in \mathcal{C} \,|\, c_n < z_c^{\mathrm{start}}\} & \text{for } i = n+1. \end{cases}$$

Based on the costumer splitting, we define the splittings of $\mathcal{T}_{\mathrm{car}}$ and $\mathcal{V}$ as

$$\mathcal{T}_i := \{t \in \mathcal{T}_{\mathrm{car}} \,|\, (M \circ C)(t) \in \mathcal{C}_i\} \qquad \text{for } i \in [n+1]$$

and

$$\mathcal{V}_i := \begin{cases} \{v \in \mathcal{V} \,|\, z_v \leq c_1\} & \text{for } i = 1 \\ \{v \in \mathcal{V} \,|\, c_{i-1} < z_v \leq c_i\} & \text{for } i \in [n]\backslash\{1\} \\ \{v \in \mathcal{V} \,|\, c_n < z_v\} & \text{for } i = n+1. \end{cases}$$

Solving of the Partial Instances

Since for costumer $c \in \mathcal{C}_i$ all his trips are in splitting $\mathcal{T}_i$, costumer c has to be satisfied only in the partial instance i. For solving the partial instances, we modify the (PLMILP_i) from (Knoll, cap. 10) as follows: The constraint

$$\sum_{s \in \mathrm{N}^-_{\overline{G}_i}(t)} x_{s,t} = 1 \qquad \text{for all } t \in \hat{\mathcal{V}}_i \cup \mathcal{T}_i \cup \hat{P}_i \qquad (10.1)$$

ensures that each trip in this partial instance is fulfilled. This constraint is replaced by

$$\sum_{m \in C^{-1}(c)} u_m = 1 \qquad \text{for all } c \in \mathcal{C}_i \qquad (5)$$

$$\sum_{s \in \mathrm{N}^-_{\overline{G}_i}(t)} x_{s,t} = u_m \qquad \text{for all } m \in C^{-1}(\mathcal{C}_i), t \in m \qquad (6)$$

$$\sum_{s \in \mathrm{N}^-_{\overline{G}_i}(t)} x_{s,t} = 1 \qquad \text{for all } t \in \hat{\mathcal{V}}_i \cup \hat{P}_i \qquad (7)$$

where (7) ensures that all vehicles and split points are considered. (5) and (6) guarantee that for every costumer in this partial instance exactly one route is chosen.

The further procedure is similar to (Knoll, Cap. 10). For each solved partial instance i, the partial split point set $\hat{P}_i$ is created. Therefore, only the chosen trips are considered. All trips that were not chosen in the partial instance, are neglected. The partial solutions are feasibly connected to a feasible overall solution according to (Knoll, cap. 10.2).

Model Equivalence This heuristic formulation is not equivalent to the original formulation (MMILP). This is shown by the following example.

Example 2. Let t_1, t_2, t_3 with $t_1 \prec t_2 \prec t_3$ be trips with the following properties:

Trip	Start	End	Route	Costumer
t_1	8:00	8:15	m_1	C_1
t_2	8:30	8:45	m_2	C_2
t_3	9:00	9:15	m_1	C_1

Table 1: Trips

In this case, costumer C_1 uses public transport between 8:15 and 9:00. The duty (t_1, t_2, t_3) is a feasible result of the (MMILP).

If there is a split point at 8:15 then the splittings are $\mathcal{T}_1 = \{t_1, t_3\}, \mathcal{T}_2 = \{t_2\}$. Hence, there is one split point $\mathrm{SP}_1(t_2)$ with $z^{\mathrm{start}}_{\mathrm{SP}_1(t_2)} = 8{:}30$. The partial solution of instance 1 is (t_1, t_3) and $t_3 \not\prec \mathrm{SP}_1(t_2)$. Thus, the partial solutions cannot be feasibly connected to the solution (t_1, t_2, t_3).

With this example we have seen, that the formulations (EMILP) and (MMILP) are not equivalent. It is even possible, that an optimal solution of (MMILP) is not feasible in (EMILP).

Although the formulations are not equivalent, we can give an estimation on the objective value when we make some restrictions.

Definition 7. Consider a costumer set $\mathcal{C}$ and split points c_i for $i \in [n]$ with $c_i < c_{i+1}$. We define the following values:

- Costumer Extension for $c \in \mathcal{C}$: $L_{\mathrm{C}}(c) := \max\limits_{t \in (M \circ C)^{-1}(c)} z_t^{\mathrm{start}} - \min\limits_{t \in (M \circ C)^{-1}(c)} z_t^{\mathrm{start}}$

- Costumer Extension: $L_{\mathrm{C}} := \max\limits_{c \in \mathcal{C}} L_{\mathrm{C}}(c)$

- Splitting Length: $L_{\mathrm{S}} := \min\limits_{i \in [n-1]} c_{i+1} - c_i$

Theorem 1. *Consider the problem with costumer set $\mathcal{C}$ and split points c_i for $i \in [n]$ with $c_i < c_{i+1}$. Let*

$$L_{\mathrm{C}} \leq L_{\mathrm{S}} \tag{8}$$

Let $d := (t_1, \ldots, t_k)$ be the duty of a vehicle of a feasible solution of the (MMILP)*. Then, there are duties $d_1 \uplus d_2 = d$, where d_1, d_2 are part of a feasible solution of (EMILP). Moreover,*

$$\mathrm{cost}\,(d_1) + \mathrm{cost}\,(d_2) \leq 2 \cdot \mathrm{cost}\,(d)\,. \tag{9}$$

Proof. We consider the vehicles duty $d = (t_1, \ldots t_k)$. We write $s \prec t$ according to Definition 1, i.e. (s, t) is feasible in (MMILP). We write $s \to t$ iff (s, t) is feasible in (EMILP).

Feasibility For arbitrary $i \in [k-2]$ holds: $t_i \prec t_{i+1} \prec t_{i+2}$, therefore also $t_i \prec t_{i+2}$. Assume $t_2 \not\to t_3$. Then holds $t_3 \in \mathcal{T}_l, t_2 \in \mathcal{T}_{l+1}$ for some $l \in [k]$. Since $t_1 \prec t_3$, $t_1 \in \bigcup_{j=1}^{l+1} \mathcal{T}_|$. Therefore $t_1 \to t_2$ or $t_1 \to t_3$.

$\square$

A Previous Formulations

(MILP)

$$\min \sum_{s \in \mathcal{V}} \sum_{t \in \mathrm{N}_G^+(s) \setminus \{d^{\mathrm{e}}\}} x_{s,t} c^{\mathrm{v}}$$

$$+ \sum_{t \in \mathcal{T}} \sum_{s \in \mathrm{N}_G^-(t)} \left[x_{s,t} \left(c_{s,t}^{\mathrm{d}} + c_t^{\mathrm{t}} \right) + \sum_{r \in \mathcal{R}_{s,t}} z_{s,r,t} \left(c_{s,r}^{\mathrm{d}} + c_{r,t}^{\mathrm{d}} - c_{s,t}^{\mathrm{d}} \right) \right] \qquad \text{(MILP)}$$

$$\text{s.t.} \sum_{t \in \mathrm{N}_G^-(s)} x_{t,s} = \sum_{t \in \mathrm{N}_G^+(s)} x_{s,t} \qquad \text{for all } s \in V \setminus \{d^{\mathrm{s}}, d^{\mathrm{e}}\} \qquad (3.15)$$

$$\sum_{s \in \mathrm{N}_G^-(t)} x_{s,t} = 1 \qquad \text{for all } t \in \mathcal{V} \qquad (3.16)$$

$$\sum_{t \in C^{-1}(c)} \sum_{s \in \mathrm{N}_G^-(t)} x_{s,t} = 1 \qquad \text{for all } c \in \mathcal{C} \qquad (3.17)$$

$$\sum_{r \in \mathcal{R}_{s,t}} z_{s,r,t} \leq x_{s,t} \qquad \text{for all } t \in \mathcal{T}, s \in \mathrm{N}_G^-(t) \qquad (3.18)$$

$$e_s \leq f_s^0 \qquad \text{for all } s \in \mathcal{V} \qquad (3.19)$$

$$0 \leq e_s - \sum_{r \in \mathcal{R}_{s,t}} z_{s,r,t} f_{s,r}^{\mathrm{d}} \qquad \text{for all } t \in \mathcal{T}, s \in \mathrm{N}_G^-(t) \qquad (3.12)$$

$$e_t \leq 1 - f_t^{\mathrm{t}} - \sum_{r \in \mathcal{R}_{s,t}} z_{s,r,t} f_{r,t}^{\mathrm{d}} \qquad \text{for all } t \in \mathcal{T}, s \in \mathrm{N}_G^-(t) \qquad (3.13)$$

$$e_t \leq e_s - x_{s,t} \left(f_{s,t}^{\mathrm{d}} + f_t^{\mathrm{t}} \right) - \sum_{r \in \mathcal{R}_{s,t}} z_{s,r,t} \left(f_{s,r}^{\mathrm{d}} + f_r^{\mathrm{t}} + f_{r,t}^{\mathrm{d}} - f_{s,t}^{\mathrm{d}} \right) + (1 - x_{s,t})$$

$$\text{for all } t \in \mathcal{T}, s \in \mathrm{N}_G^-(t) \qquad (3.14)$$

$$x_{s,t} \in \{0, 1\} \qquad \text{for all } (s, t) \in A \qquad (3.20)$$

$$z_{s,r,t} \in \{0, 1\} \qquad \text{for all } t \in \mathcal{T}, s \in \mathrm{N}_G^-(t), r \in \mathcal{R}_{s,t} \qquad (3.21)$$

$$e_s \in [0, 1] \qquad \text{for all } s \in V \setminus \{d^{\mathrm{s}}, d^{\mathrm{e}}\} \qquad (3.22)$$

(AMILP)

$$\min \sum_{s\in\mathcal{V}} \sum_{t\in\mathrm{N}_G^+(s)\setminus\{d^{\mathrm{e}}\}} x_{s,t} c^{\mathrm{v}}$$

$$+ \sum_{t\in\mathcal{T}} \sum_{s\in\mathrm{N}_G^-(t)} \left[x_{s,t}\left(c_{s,t}^{\mathrm{d}} + c_t^{\mathrm{t}}\right) + \sum_{r\in\mathcal{R}_{s,t}} z_{s,r,t}\left(c_{s,r}^{\mathrm{d}} + c_{r,t}^{\mathrm{d}} - c_{s,t}^{\mathrm{d}}\right) \right] \qquad \text{(AMILP)}$$

$$\text{s.t.} \quad \sum_{t\in\mathrm{N}_G^-(s)} x_{t,s} = \sum_{t\in\mathrm{N}_G^+(s)} x_{s,t} \qquad \text{for all } s \in V\setminus\{d^{\mathrm{s}}, d^{\mathrm{e}}\} \qquad (3.15)$$

$$\sum_{s\in\mathrm{N}_G^-(t)} x_{s,t} = 1 \qquad \text{for all } t \in \mathcal{T}\cup\mathcal{V} \qquad (6.2)$$

$$\sum_{r\in\mathcal{R}_{s,t}} z_{s,r,t} \le x_{s,t} \qquad \text{for all } t \in \mathcal{T}, s \in \mathrm{N}_G^-(t) \qquad (3.18)$$

$$e_s \le f_s^0 \qquad \text{for all } s \in \mathcal{V} \qquad (3.19)$$

$$0 \le e_s - \sum_{r\in\mathcal{R}_{s,t}} z_{s,r,t} f_{s,r}^{\mathrm{d}} \qquad \text{for all } t \in \mathcal{T}, s \in \mathrm{N}_G^-(t) \qquad (3.12)$$

$$e_t \le 1 - f_t^{\mathrm{t}} - \sum_{r\in\mathcal{R}_{s,t}} z_{s,r,t} f_{r,t}^{\mathrm{d}} \qquad \text{for all } t \in \mathcal{T}, s \in \mathrm{N}_G^-(t) \qquad (3.13)$$

$$e_t \le e_s - x_{s,t}\left(f_{s,t}^{\mathrm{d}} + f_t^{\mathrm{t}}\right) - \sum_{r\in\mathcal{R}_{s,t}} z_{s,r,t}\left(f_{s,r}^{\mathrm{d}} + f_r^{\mathrm{t}} + f_{r,t}^{\mathrm{d}} - f_{s,t}^{\mathrm{d}}\right) + (1 - x_{s,t})$$

$$\text{for all } t \in \mathcal{T}, s \in \mathrm{N}_G^-(t) \qquad (3.14)$$

$$x_{s,t} \in \{0,1\} \qquad \text{for all } (s,t) \in A \qquad (3.20)$$

$$z_{s,r,t} \in \{0,1\} \qquad \text{for all } t \in \mathcal{T}, s \in \mathrm{N}_G^-(t), r \in \mathcal{R}_{s,t} \qquad (3.21)$$

$$e_s \in [0,1] \qquad \text{for all } s \in V\setminus\{d^{\mathrm{s}}, d^{\mathrm{e}}\} \qquad (3.22)$$

(LMILP)

$$\min \sum_{s\in\mathcal{V}} \sum_{t\in N_{\overline{G}}^+(s)\setminus\{d^e\}} x_{s,t}c^v + \sum_{s\in P} \sum_{t\in N_{\overline{G}}^-(s)} x_{s,t}c_t^t$$

$$+ \sum_{t\in\mathcal{T}\cup P} \sum_{s\in N_{\overline{G}}^-(t)} \left[x_{s,t}\left(c_{s,t}^d + c_t^t\right) + \sum_{r\in\mathcal{R}_{s,t}} z_{s,r,t}\left(c_{s,r}^d + c_{r,t}^d - c_{s,t}^d\right) \right] \quad \text{(LMILP)}$$

$$\text{s.t.} \sum_{t\in N_{\overline{G}}^-(s)} x_{t,s} = \sum_{t\in N_{\overline{G}}^+(s)} x_{s,t} \qquad \text{for all } s\in\overline{V}\setminus\{d^s,d^e\} \tag{8.1}$$

$$\sum_{s\in N_{\overline{G}}^-(t)} x_{s,t} = 1 \qquad \text{for all } t\in\mathcal{T}\cup\mathcal{V} \tag{8.2}$$

$$\sum_{r\in\mathcal{R}_{s,t}} z_{s,r,t} \le x_{s,t} \qquad \text{for all } t\in\mathcal{T}\cup P, s\in N_{\overline{G}}^-(t)\setminus P \tag{8.3}$$

$$e_s \le f_s^0 \qquad \text{for all } s\in\mathcal{V} \tag{8.4}$$

$$0 \le e_s - \sum_{r\in\mathcal{R}_{s,t}} z_{s,r,t}f_{s,r}^d \qquad \text{for all } t\in\mathcal{T}\cup P, s\in N_{\overline{G}}^-(t)\setminus P \tag{8.5}$$

$$e_t \le 1 - f_t^t - \sum_{r\in\mathcal{R}_{s,t}} z_{s,r,t}f_{r,t}^d \qquad \text{for all } t\in\mathcal{T}\cup P, s\in N_{\overline{G}}^-(t)\setminus P \tag{8.6}$$

$$e_t \le e_s - x_{s,t}\left(f_{s,t}^d + f_t^t\right) - \sum_{r\in\mathcal{R}_{s,t}} z_{s,r,t}\left(f_{s,r}^d + f_r^t + f_{r,t}^d - f_{s,t}^d\right) + (1 - x_{s,t})$$

$$\text{for all } t\in\mathcal{T}\cup P, s\in N_{\overline{G}}^-(t)\setminus P \tag{8.7}$$

$$x_{s,t} \in \{0,1\} \qquad \text{for all } (s,t)\in\overline{A} \tag{8.9}$$

$$z_{s,r,t} \in \{0,1\} \qquad \text{for all } t\in\mathcal{T}\cup P, s\in N_{\overline{G}}^-(t)\setminus P, r\in\mathcal{R}_{s,t} \tag{8.10}$$

$$e_s \in [0,1] \qquad \text{for all } s\in\overline{V}\setminus\{d^s,d^e\} \tag{8.11}$$