Case Studies Nonlinear Optimization

Open Cast Mining

Final Presentation

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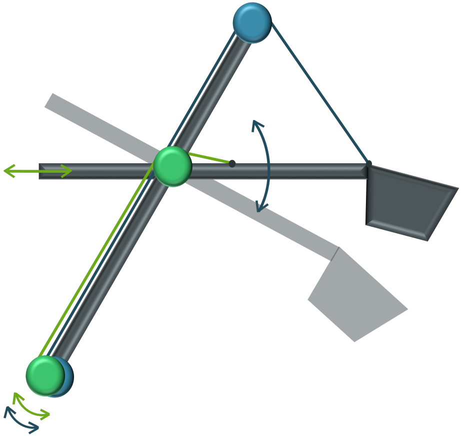
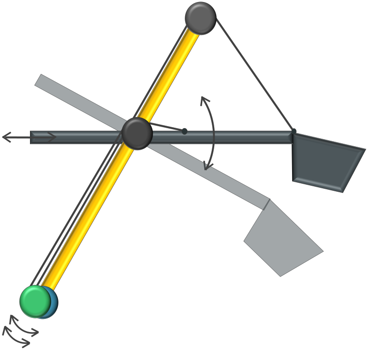
originally posted to Flickr by FAndrey at http://flickr.com/photos/43301444@N06/4141786255

- Goal: **optimization of model parameters**
- Models of technical system = physical properties + control properties

Problem Setting

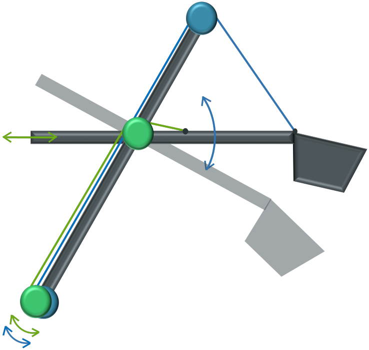


Problem Setting



- arm element fixed to base
- cannot be moved w.r.t. the base

Problem Setting



- green shovel motion **back** and **forth**
- blue shovel motion **up** and **down**

Main Problems

1. Physical Modelling
   - Modelling rope properties
   - Determining information needed for calibration of model

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1. Physical Modelling
   - Modelling rope properties
   - Determining information needed for calibration of model

2. Parameter Optimization
   - Optimizing parameters for a complex, unknown model (black box)

Physical Modelling

Why?

Building an accurate model $\Rightarrow$ Good description of the effects of control and motion

Physical Modelling

Why?

Building an accurate model

$\Rightarrow$

Good description of the effects of control and motion

To consider:

- Friction in cable reels
- Deformation of ropes
- etc.

Which color do we want to use?

TUMblue

TUMblue1

TUMblue2

TUMblue3

TUMblue4

TUMblue5

Parameter Optimization

What are parameters?

- Friction coefficients
- Mass
- Inertia

Parameter Optimization

What are parameters?

- Friction coefficients
- Mass
- Inertia

Why?

Accurate and realistic parameters $\Rightarrow$ Better prediction and planning of motion

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Lagrange Formalism

Method to describe dynamics of an accelerated system

T	kinetic energy
V	potentials
F	non-conservative external forces
r	points of actions of forces F
q	free variables
Q	generalized forces

Lagrange Formalism

Method to describe dynamics of an accelerated system

T	kinetic energy
V	potentials
F	non-conservative external forces
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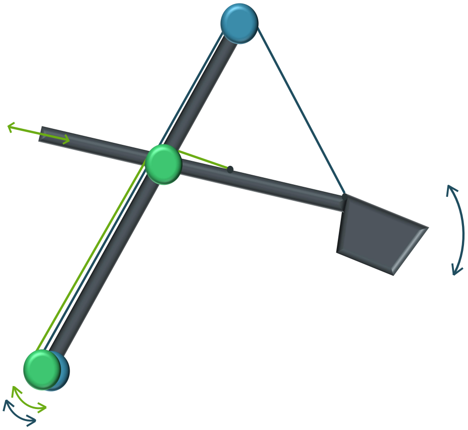
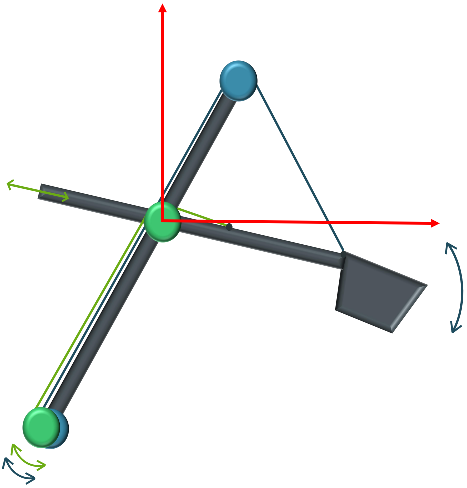
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i$$

$$Q = \left(\frac{\partial r}{\partial q}\right)^T F$$

Physical Model of Excavator

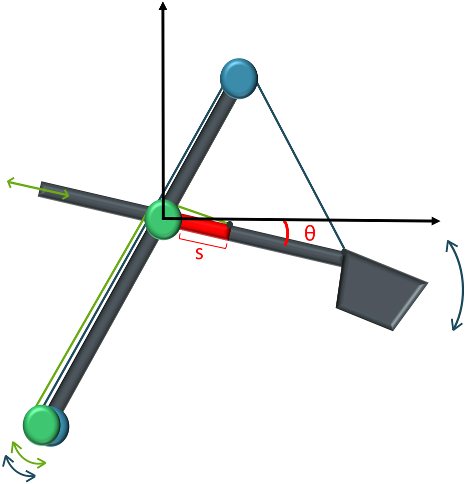


Physical Model of Excavator



coordinate system

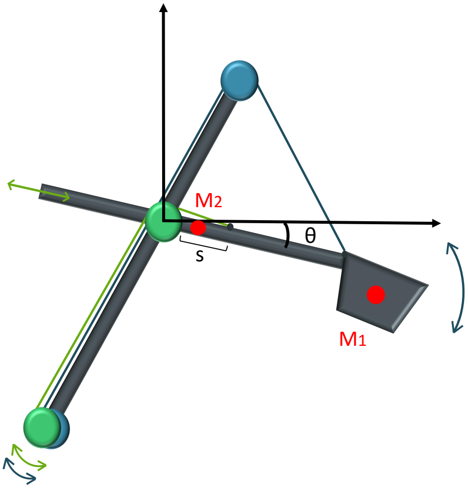
Physical Model of Excavator



degrees of freedom

- length s
- tilt angle θ

Physical Model of Excavator



movable centers of gravity of

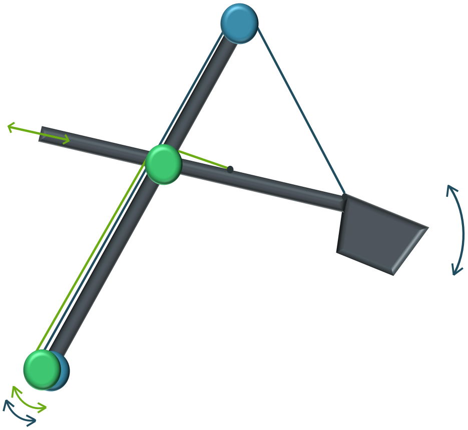
- shovel M_1
- arm M_2

Physical Model of Excavator

Assumptions to the model:

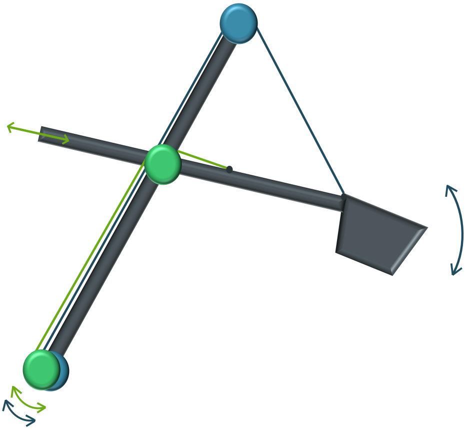
- no mass for the ropes
- shovel as point mass
- no slack/friction between ropes and cable reels

Kinetic Energy



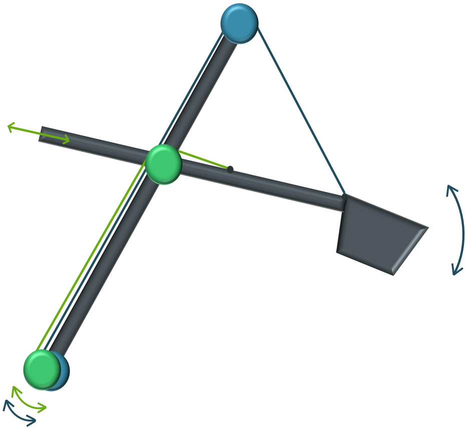
- movement of mass
- rotation of cable reel

Potential Energy



gravitational potential energy

Generalized Forces



- torque on cable reel
- friction of cable reel

Lagrange Formalism

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{s}}\right) - \frac{\partial T}{\partial s} + \frac{\partial V}{\partial s} = Q_s$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} + \frac{\partial V}{\partial \theta} = Q_\theta$$

Resulting ODE

Second order ODE from Lagrange Formalism:

$$A(x,p)\begin{pmatrix}\ddot{s}\\ \ddot{\theta}\end{pmatrix} = b(x,u,p)$$

Transformed into first order ODE:

$$\frac{d}{dt}\begin{pmatrix}s\\ \theta\\ \dot{s}\\ \dot{\theta}\end{pmatrix} = \begin{pmatrix}\dot{s}\\ \dot{\theta}\\ A^{-1}(x,p)b(x,u,p)\end{pmatrix} = f(x,u,p)$$

state	$x = (s, \theta, \dot{s}, \dot{\theta})^T$
control	$u = (\tau_1, \tau_2)^T$
parameters	$p = (p_1, ..., p_k)^T$

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Discretization of the ODE

Discretize time interval:

$$[0, T] \to \{0 = t_0, t_1, \ldots, t_{m-1}, t_m = T\}$$

Discretize state and control:

$$x_n = x(t_n)$$
$$u_n = u(t_n)$$

Discretization of the ODE

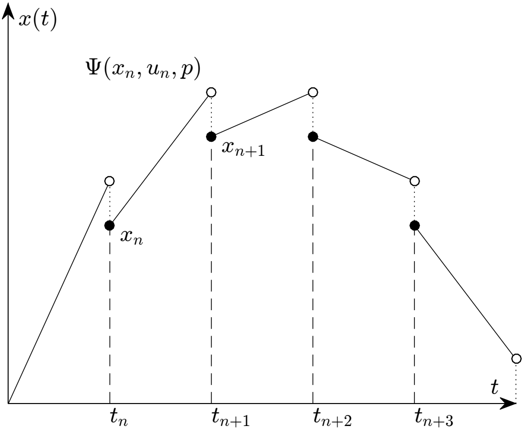
Explicit Euler for every time step $h_n = t_{n+1} - t_n$:

$$\Psi(x_n, u_n, p) = x_n + h_n f(x_n, u_n, p)$$

Discrete constraint:

$$0 = x_{n+1} - \Psi(x_n, u_n, p) =: \Phi_n(x, u, p) \quad \forall n = 0, \ldots, m-1$$

Discretization of the ODE



Problem Formulation

Natural approach: Optimal Control Problem

$$
\begin{aligned}
\min_{x,p} \quad & \frac{1}{2}\|\bar{x} - x\|^2 \\
\text{s. t.} \quad & \Phi(x, \bar{u}, p) = 0 \\
& p \geq 0
\end{aligned}
$$

state	$x = (s, \theta, \dot{s}, \dot{\theta})^T$
parameters	$p = (p_1, ..., p_k)^T$
control	$\bar{u} = (\tau_1, \tau_2)^T$
desired motion	$\bar{x}$

Problem Formulation

Original Problem

$$
\begin{aligned}
\min_{x,p} \quad & \frac{1}{2}\|\bar{x} - x\|^2 \\
\text{s.t.} \quad & \Phi(x, \bar{u}, p) = 0 \\
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\end{aligned}
$$

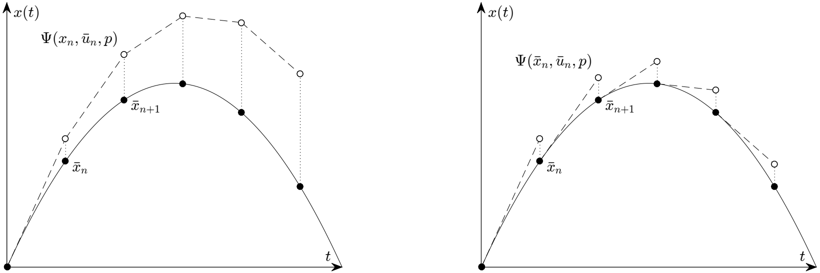
Reinterpreted Problem

$$
\begin{aligned}
\min_{p} \quad & \frac{1}{2}\|\Phi(\bar{x}, \bar{u}, p)\|^2 \\
\text{s.t.} \quad & p \geq 0
\end{aligned}
$$

$$\begin{aligned} \min_{p} \quad & \frac{1}{2}\|\Phi(\bar{x}, \bar{u}, p)\|^2 \\ \text{s.t.} \quad & p \geq 0 \end{aligned}$$

- $\bar{x}$ solves ODE
- $\Phi(\bar{x}, \bar{u}, \bar{p}) \to 0$ for discretization $m \to \infty$
- Problem dimension fixed for $m \to \infty$

Comparison of the Approaches

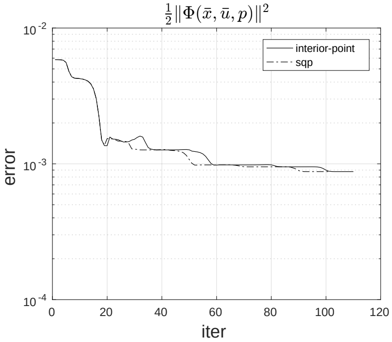


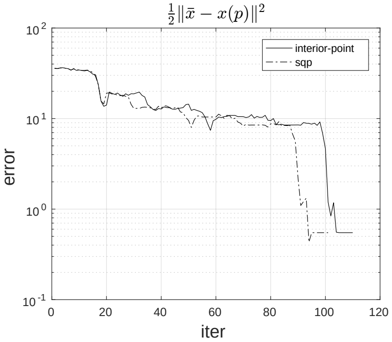
continuous vs. stepwise Approximation

Example Instance

- $[0, T] = [0, 14s]$
- 1500 time steps
- $p_0 \in [0.8\bar{p}, 1.2\bar{p}]$
- $x(p)$ solution of ODE for given p

Results


$\frac{1}{2}\|\Phi(\bar{x}, \bar{u}, p)\|^2$
interior-point
sqp
error
iter
10^{-2}
10^{-3}
10^{-4}
0
20
40
60
80
100
120


$\frac{1}{2}\|\bar{x} - x(p)\|^2$
interior-point
sqp
error
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10^{2}
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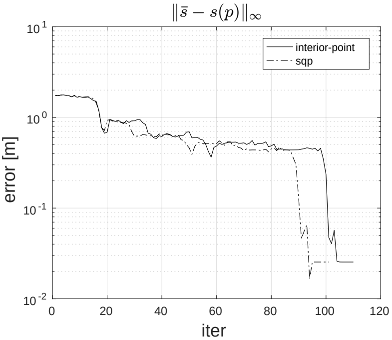
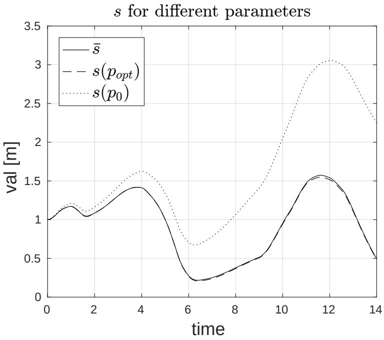


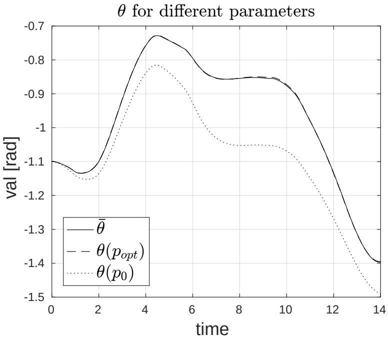
Exact up to 3cm

Results

movies: reference and optimizes trajectory

Results


s for different parameters
$\bar{s}$
$s(p_{opt})$
$s(p_0)$
val [m]
time
0
0.5
1
1.5
2
2.5
3
3.5
0
2
4
6
8
10
12
14


θ for different parameters
$\bar{\theta}$
$\theta(p_{opt})$
$\theta(p_0)$
val [rad]
time
-0.7
-0.8
-0.9
-1
-1.1
-1.2
-1.3
-1.4
-1.5
0
2
4
6
8
10
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14

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Motivation

Examples

- Friction coefficients
- Masses
- Inertia

Black box model

- Realistic model from Siemens
- Confidential information

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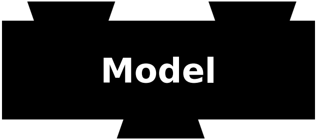

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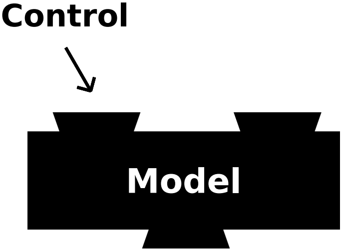

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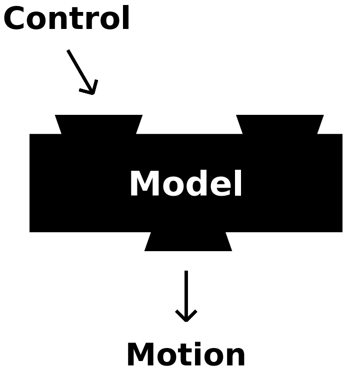

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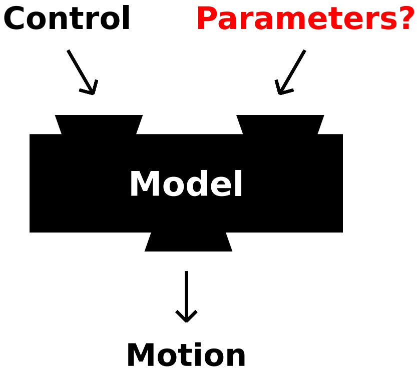
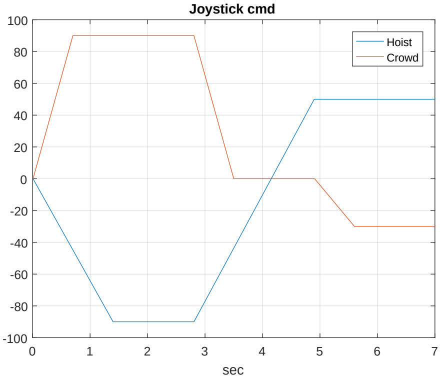
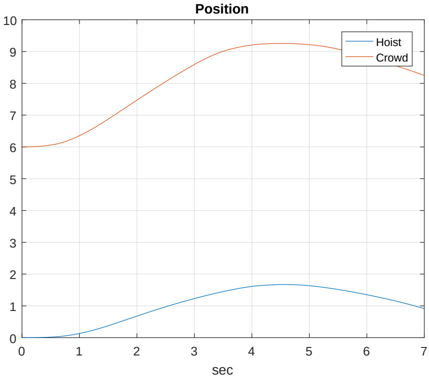


Trajectories

Input: Joystick commands for Up/Down and Forth/Back

Output: Position of the shovel





Objective Function

Optimized Parameters:

- Inertia (Engine)
- Inertia (Arm)
- Friction
- Mass

Penalty Term:

$$\min_{p \in \mathbb{R}^4} f(p) = \frac{1}{n} \cdot \sum_{i=1}^{n} \left(\frac{\| \overline{X}_i - X_i(p) \|^2}{\| \overline{X}_i \|^2} + \frac{\| \overline{Y}_i - Y_i(p) \|^2}{\| \overline{Y}_i \|^2} \right)$$

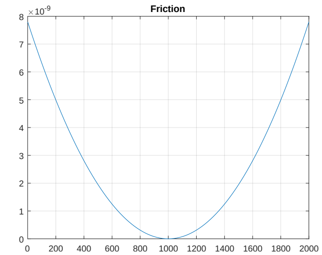
$$\text{s.t.} \quad p_j \geq 0$$

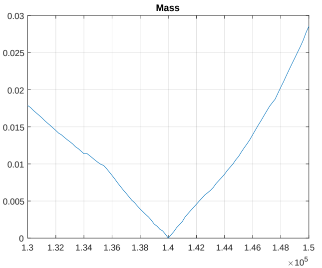
$\overline{X}_i$ reference trajectory

Influence of the Parameters

10% parameter deviation:

- Inertia (Engine): $1 \cdot 10^{-3}$
- Inertia (Arm): $3 \cdot 10^{-3}$
- Friction: $8 \cdot 10^{-11}$
- Mass: $5 \cdot 10^{-2}$





Solvers

- Derivative free optimization methods
- Deterministic or stochastic approaches
- Decrease function value by evaluating systematically

	value	evaluations	time	dev_{max}	dev_{mean}
Particle Swarm	10^{-12}	2200	6 min	$10^{-1.0}$	$10^{-3.8}$
Pattern Search	10^{-11}	6500	17 min	$10^{-1.3}$	$10^{-3.7}$
Genetic Algorithm	10^{-4}	5500	14 min	$10^{-0.5}$	$10^{-1.0}$
Simulated Annealing	10^{-3}	4200	11 min	$10^{-0.3}$	$10^{-0.6}$

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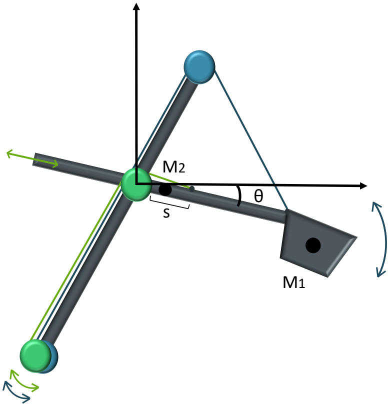
Summary



Summary