Discretization of the ODE

Discretize time interval:

$$[0, T] \to \{0 = t_0, t_1, \dots, t_{m-1}, t_m = T\}$$

Discretize state and control:

$$x_n = x(t_n)$$
$$u_n = u(t_n)$$

Discretization of the ODE

Explicit Euler for every time step $h_n = t_{n+1} - t_n$:

$$x_{n+1} \approx \tilde{x}_{n+1} = x_n + h_n f(x_n, u_n, p) =: \Psi(x_n, u_n, p)$$

Discrete constraint:

$$0 = x_{n+1} - \Psi(x_n, u_n, p) =: \Phi_n(x, u, p) \quad \forall n = 0, \dots, m-1$$

Problem Formulation

Natural approach: Optimal Control Problem

$$\begin{aligned} \min_{x,p} \quad & \frac{1}{2}\|\bar{x} - x\|^2 \\ \text{s. t.} \quad & \Phi(x, u, p) = 0 \\ & p \geq 0 \end{aligned}$$

state	$x = (s, \theta, \dot{s}, \dot{\theta})^T$
control	$u = (\tau_1, \tau_2)^T$
parameters	$p = (p_1, ..., p_k)^T$
desired motion	$\bar{x}$

Problem Formulation

Input:

- control u
- desired motion $\bar{x}$ related to u

Output:

- parameters p of the excavator
- x, but not of interest

Idea:

- get rid of variable x
- set $x := \bar{x}$
- solve a relaxed problem

Problem Formulation

Original Problem

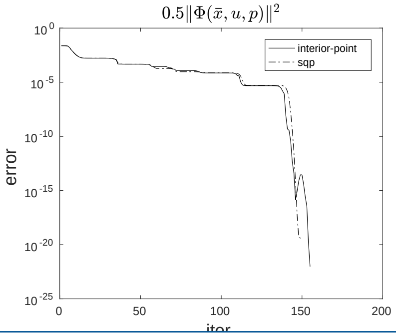
$$\begin{aligned} \min_{x,p} \quad & \frac{1}{2}\|\bar{x} - x\|^2 \\ \text{s.t.} \quad & \Phi(x, u, p) = 0 \\ & p \geq 0 \end{aligned}$$

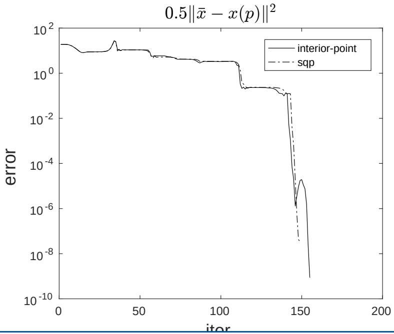
Reinterpreted Problem

$$\begin{aligned} \min_{p} \quad & \frac{1}{2}\|\Phi(\bar{x}, u, p)\|^2 \\ \text{s.t.} \quad & p \geq 0 \end{aligned}$$

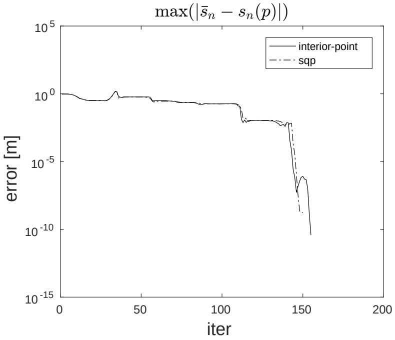
Results

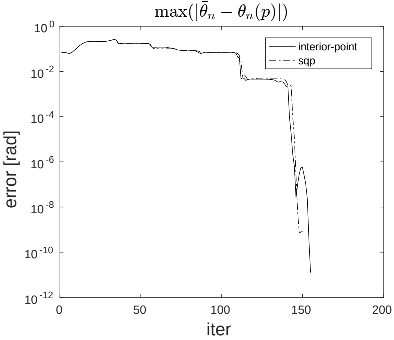
Without approximation error
x(p) solution of ODE using parameters p
Explicit Euler





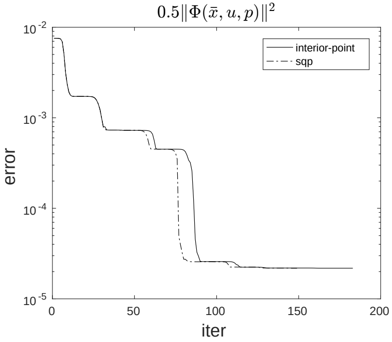
Results

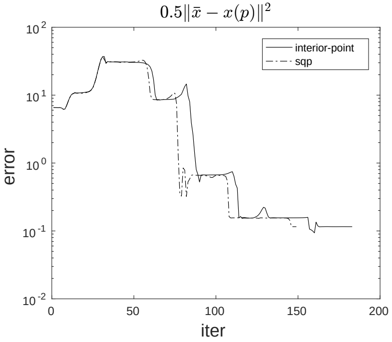

$\max(|\bar{s}_n - s_n(p)|)$
interior-point
sqp
error [m]
iter
10^{5}
10^{0}
10^{-5}
10^{-10}
10^{-15}
0
50
100
150
200


$\max(|\bar{\theta}_n - \theta_n(p)|)$
interior-point
sqp
error [rad]
iter
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
0
50
100
150
200

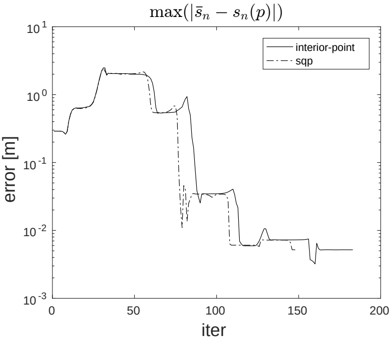
Results

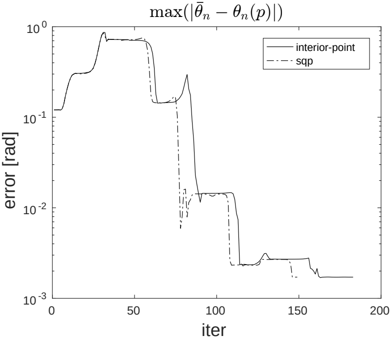
With approximation error





Results





Exact up to 1cm

Test Run (1)

14 different cases:

- Starting values: $N(\mu, \frac{\mu}{2}), N(\mu, \frac{\mu}{20}), N(\mu, \frac{\mu}{200}), N(\frac{\mu}{10}, \frac{\mu}{20}), N(10\mu, 5\mu)$
- Swarm sizes: $4, 10, 40$
- Objective function: torques+positions, only positions
- 4 solvers, each 5 loops per case

Results:

- Better values for "only position" (Particle Swarm: equal)
- Genetic Algorithm / Simulated Annealing better for good starting values, but much worse than Particle Swarm / Pattern Search
- Swarm Size changes have no effect
- Funccount independent from starting values, independent from objective function
- Time behaves as funccount

Test Run (2)

5 different cases:

- Starting values as before
- Solvers: Particle Swarm (torques + positions, only positions), Pattern Search (only positions)
- each 10 loops per case

Results:

- Starting values too big $\Rightarrow$ unrealistic results
- Similar results
- Particle Swarm is much faster
- Friction still hard to determine

Results

Test Run (1)

	value	evaluations	time	dev_{max}	dev_{mean}
Particle Swarm	10^{-12}	2200	6 min	$10^{-1.0}$	$10^{-3.8}$
Pattern Search	10^{-11}	6500	17 min	$10^{-1.3}$	$10^{-3.7}$
Genetic Algorithm	10^{-4}	5500	14 min	$10^{-0.5}$	$10^{-1.0}$
Simulated Annealing	10^{-3}	4200	11 min	$10^{-0.3}$	$10^{-0.6}$

Test Run (2)

	value	evaluations	time	dev_{max}	dev_{mean}
Particle Swarm (t+p)	10^{-10}	2900	8 min	$10^{-1.2}$	$10^{-4.0}$
Particle Swarm (p)	10^{-12}	2000	5 min	$10^{-1.0}$	$10^{-3.9}$
Pattern Search (p)	10^{-12}	6200	17 min	$10^{-1.5}$	$10^{-3.8}$