

Case Studies Nonlinear Optimization

# Open Cast Mining

Final Presentation

July 09, 2016

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# Table of Contents

- 1 Project Overview
- 2 Physical Model
- 3 Parameter Identification: Physical Model
- 4 Parameter Identification: Black Box Model
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originally posted to Flickr by FAndrey at <http://flickr.com/photos/43301444@N06/4141786255>

- Goal: **Optimization of model parameters**
- Models of technical system = Physical properties + Control properties

# Problem Setting

# Procedure

## Physical Model

- Rope properties
- Lagrange Formalism



## Parameter Identification

## Discretization + Optimization

# Procedure

## Physical Model

- Rope properties
- Lagrange Formalism

## Blackbox Model

- Realistic model
- Confidential information

and

## Parameter Identification

Discretization + Optimization



## Parameter Identification

Derivative-free optimization

# Physical Modeling

## Why?

Building an  
accurate model



Good description of  
the effects of control  
on motion

# Physical Modeling

## How?

To consider:

- Friction in cable reels
- Potential/Kinetic energy
- etc

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Lagrange Formalism  
(ODE)

# Parameter Identification

## What are parameters?

- Friction coefficients
- Mass
- Inertia

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- Mass
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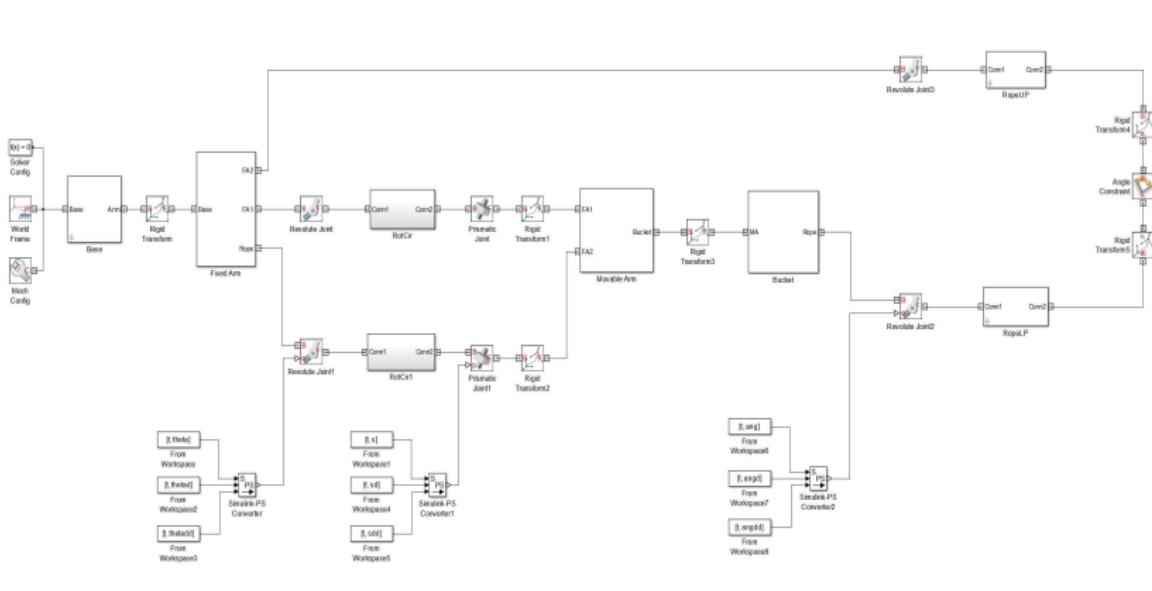
## Why?

Accurate and  
realistic parameters



Better prediction  
and planning of  
motion

# Visualization: Simulink



## Visualization Example

Same parameters except different load weights

**vs**

Light load

Heavy load

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# Lagrange Formalism

Method to describe dynamics of an accelerated system

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r    points of actions of forces F

q    free variables

Q    generalized forces

# Lagrange Formalism

Method to describe dynamics of an accelerated system

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i$$

$$Q = \left( \frac{\partial r}{\partial q} \right)^T F$$

T kinetic energy

V potentials

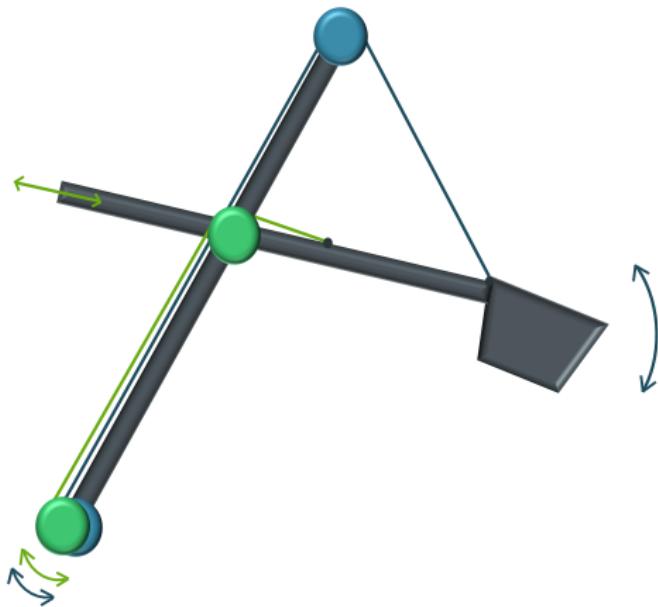
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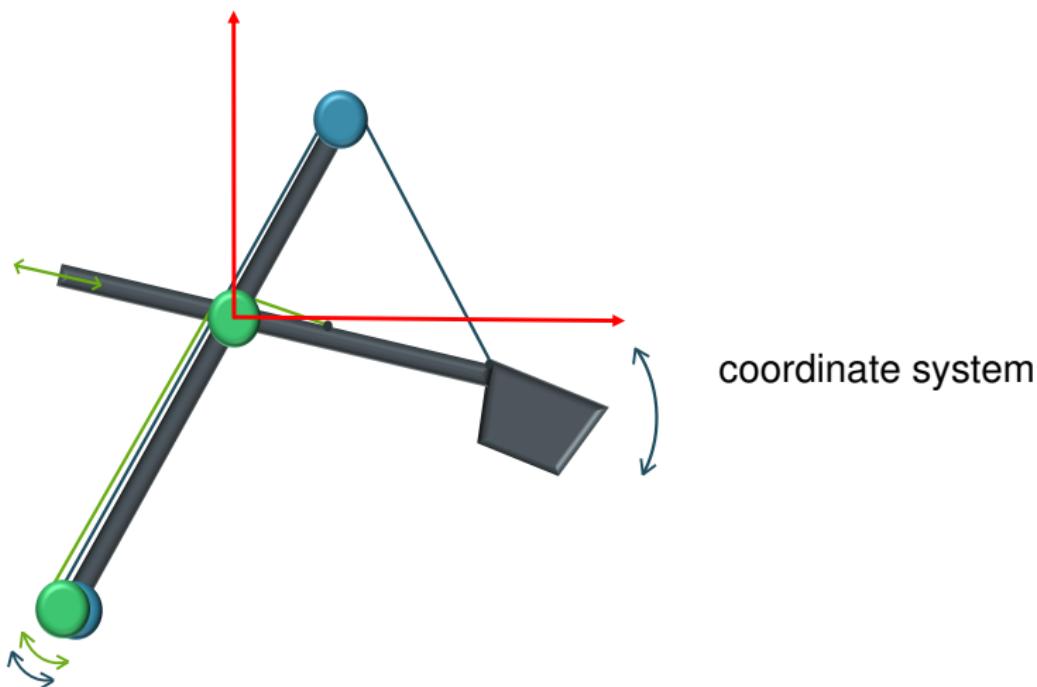
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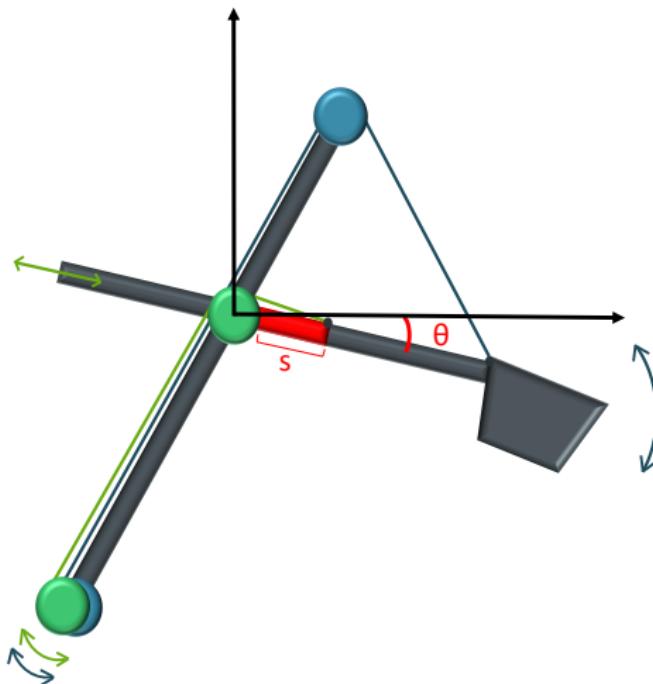
# Physical Model of Excavator



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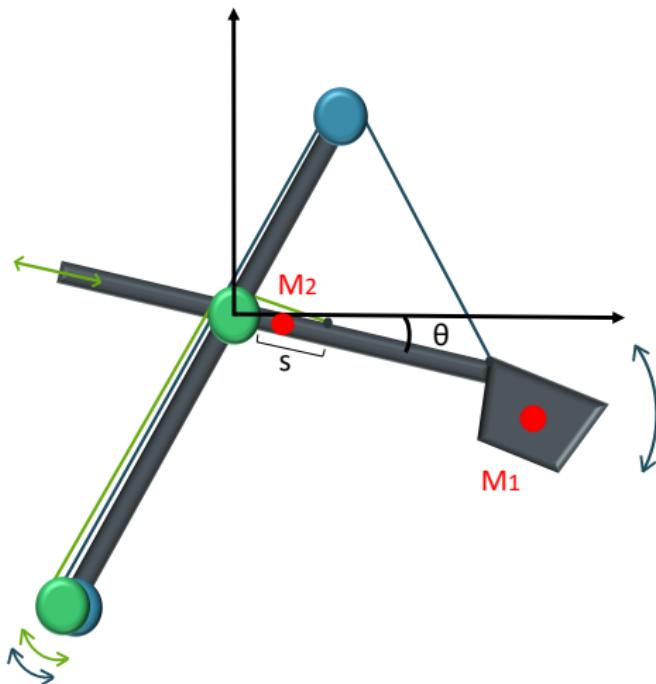
# Physical Model of Excavator



degrees of freedom

- length  $s$
- tilt angle  $\theta$

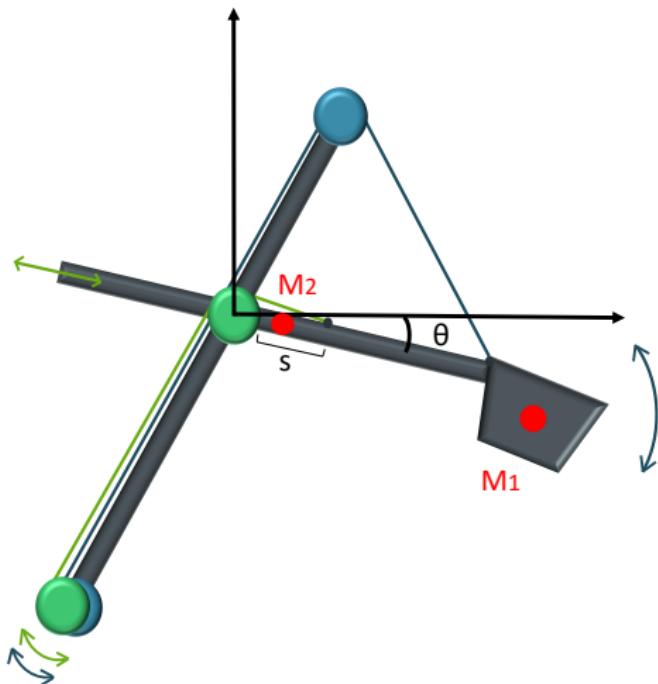
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movable centers of gravity of

- shovel  $M_1$
- arm  $M_2$

# Physical Model of Excavator



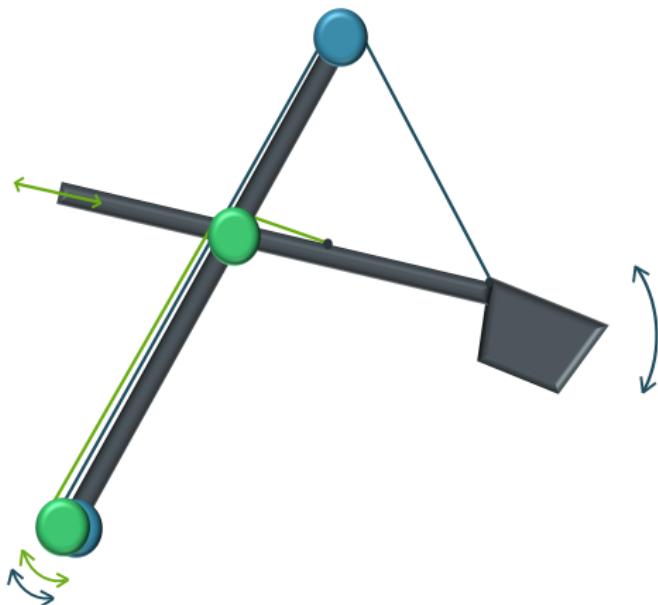
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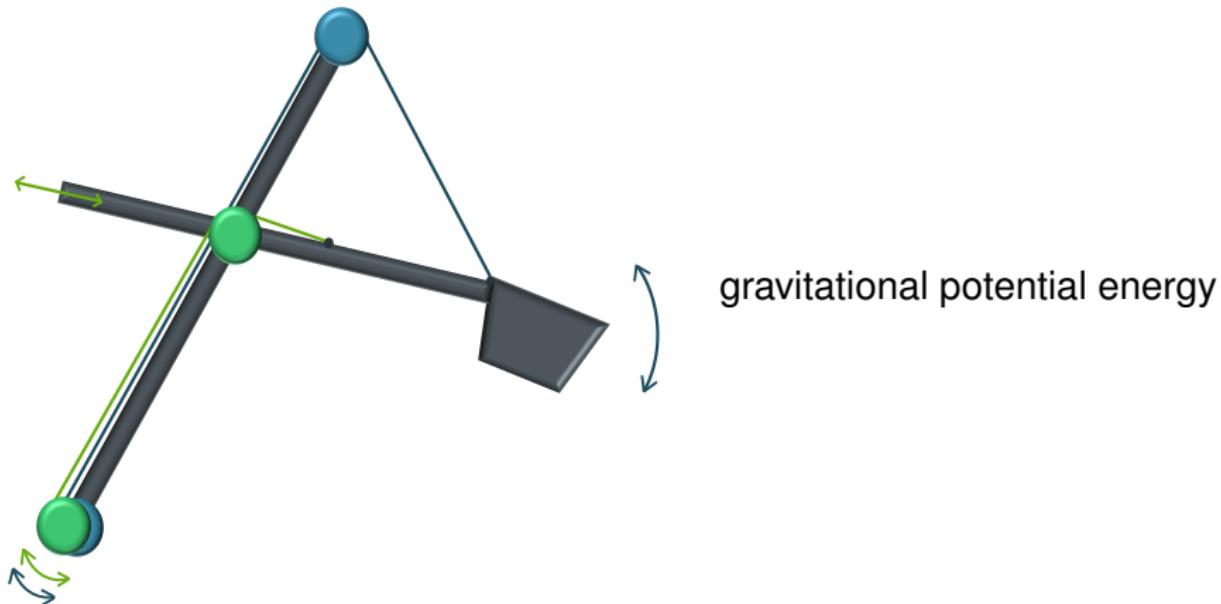
$$Q = \left( \frac{\partial r}{\partial q} \right)^T F$$

# Kinetic Energy

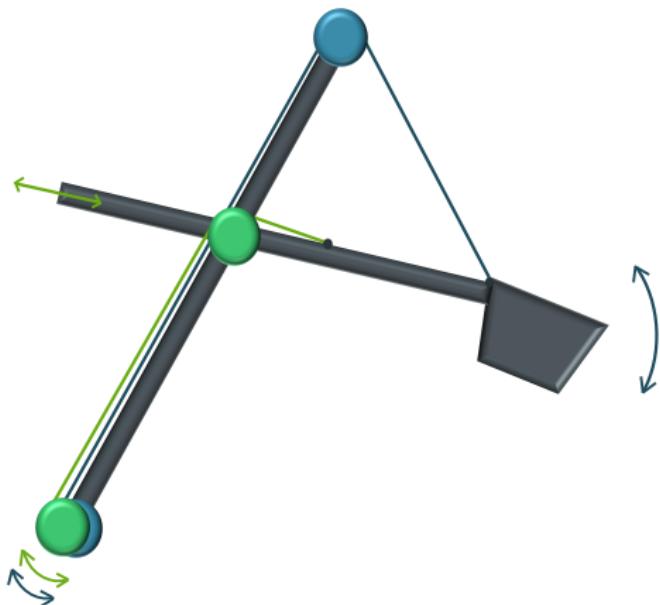


- movement of mass
- rotation of cable reel

# Potential Energy



# Generalized Forces



- torque on cable reels
- friction of cable reels

# Physical Model of Excavator

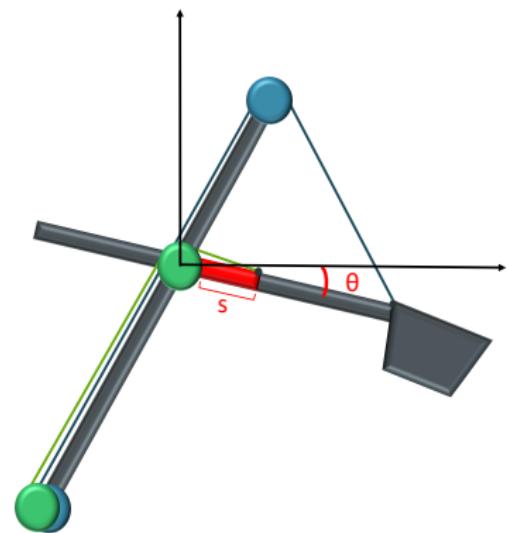
Assumptions to the model:

- no mass for the ropes
- shovel as point mass
- no slack / friction between ropes and cable reels

# Lagrange Formalism

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{s}} \right) - \frac{\partial T}{\partial s} + \frac{\partial V}{\partial s} = Q_s$$

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\theta}} \right) - \frac{\partial T}{\partial \theta} + \frac{\partial V}{\partial \theta} = Q_\theta$$



# Resulting ODE

$$A(x, p) \begin{pmatrix} \ddot{s} \\ \ddot{\theta} \end{pmatrix} = b(x, u, p)$$

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→ Transformation into 1st order ODE

$$\frac{d}{dt} \begin{pmatrix} s \\ \theta \\ \dot{s} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \dot{s} \\ \dot{\theta} \\ A^{-1}(x, p)b(x, u, p) \end{pmatrix} = f(x, u, p)$$

state	$x = (s, \theta, \dot{s}, \dot{\theta})^T$
control	$u = (\tau_1, \tau_2)^T$
parameters	$p = (p_1, \dots, p_k)^T$

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## Discretization of the ODE

Discretize time interval:

$$[0, T] \rightarrow \{0 = t_0, t_1, \dots, t_{m-1}, t_m = T\}$$

Discretize state and control:

$$x_n = x(t_n)$$

$$u_n = u(t_n)$$

## Discretization of the ODE

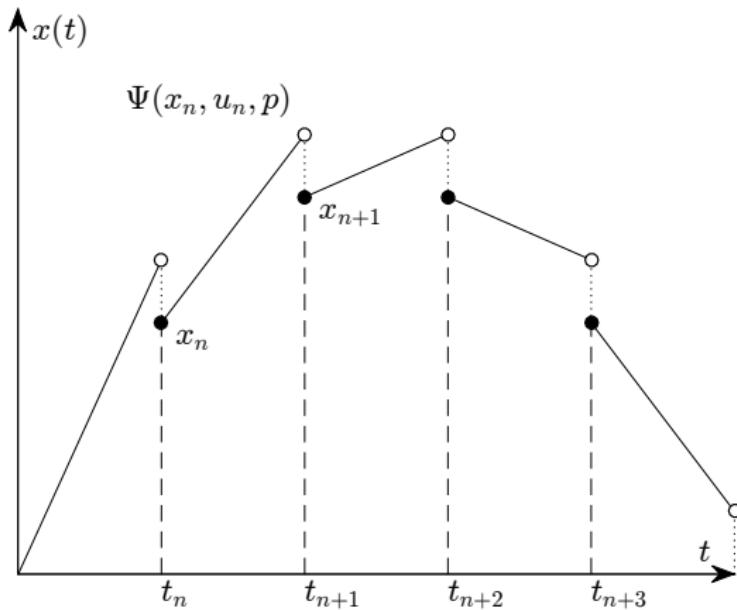
Explicit Euler for every time step  $h_n = t_{n+1} - t_n$ :

$$\Psi(x_n, u_n, p) = x_n + h_n f(x_n, u_n, p)$$

Discrete constraint:

$$0 = x_{n+1} - \Psi(x_n, u_n, p) =: \Phi_n(x, u, p) \quad \forall n = 0, \dots, m-1$$

# Discretization of the ODE



# Problem Setting

Given:

- control  $\bar{u}$
- motion  $\bar{x}$  related to  $\bar{u}$  and  $\bar{p}$

Unknown:

- parameters  $\bar{p}$  of the excavator

Output:

- parameters  $p$

# Problem Formulation

Natural approach: Optimal Control Problem

$$\begin{aligned} \min_{x,p} \quad & \frac{1}{2} \|\bar{x} - x\|^2 \\ \text{s. t.} \quad & \Phi(x, \bar{u}, p) = 0 \\ & p \geq 0 \end{aligned}$$

state	$x = (s, \theta, \dot{s}, \dot{\theta})^T$
parameters	$p = (p_1, \dots, p_k)^T$
control	$\bar{u} = (\tau_1, \tau_2)^T$
desired motion	$\bar{x}$

# Problem Formulation

## Original Problem

$$\begin{aligned} \min_{x,p} \quad & \frac{1}{2} \|\bar{x} - x\|^2 \\ \text{s. t.} \quad & \Phi(x, \bar{u}, p) = 0 \\ & p \geq 0 \end{aligned}$$

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Set  $x \leftarrow \bar{x}$

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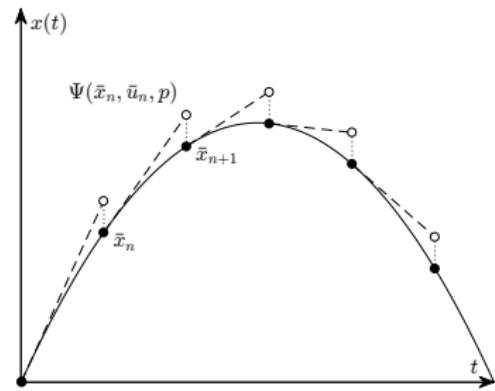
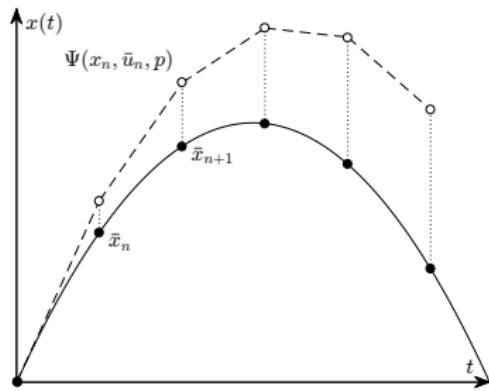
## Reinterpreted Problem

$$\begin{aligned} \min_p \quad & \frac{1}{2} \|\Phi(\bar{x}, \bar{u}, p)\|^2 \\ \text{s. t.} \quad & p \geq 0 \end{aligned}$$

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- $\bar{x}$  solves ODE for  $\bar{p}$
- $\Phi(\bar{x}, \bar{u}, \bar{p}) \rightarrow 0$  for discretization  $m \rightarrow \infty$
- number of parameters fix

# Comparison of the Approaches

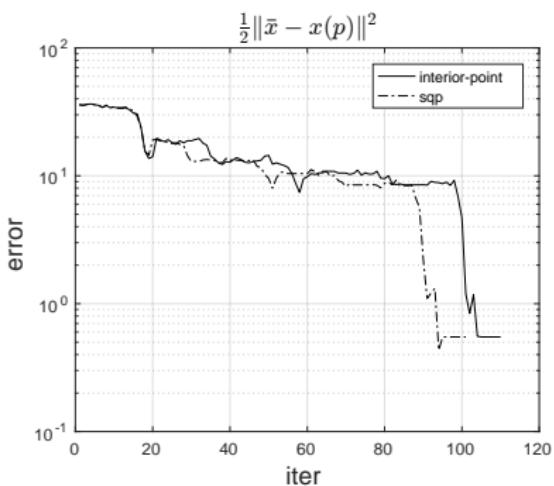
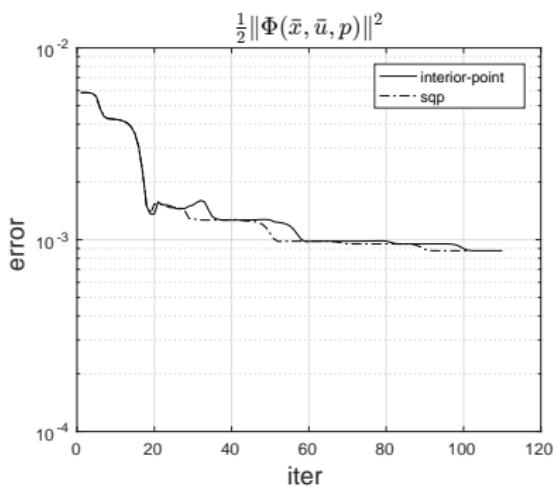


continuous vs. stepwise Approximation

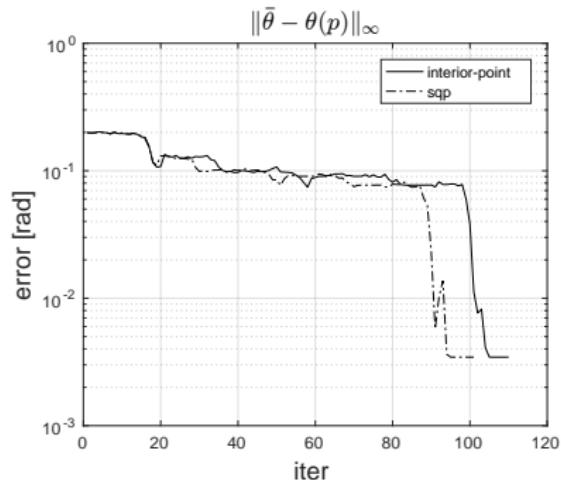
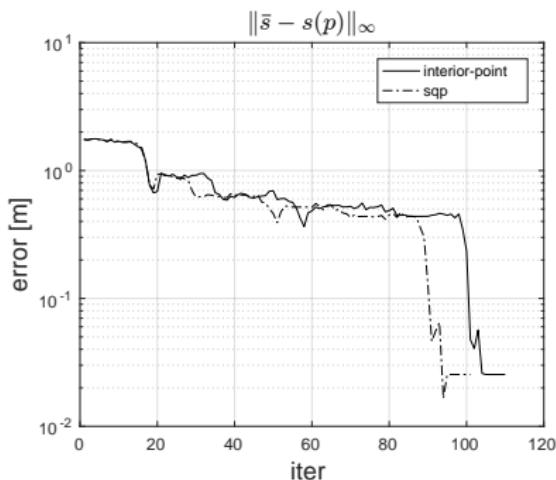
## Example Instance

- $[0, T] = [0, 14s]$
- 1500 time steps
- $p_0 \in [0.8\bar{p}, 1.2\bar{p}]$
- $x(p)$  solution of ODE for given  $p$
- internally 5 trajectories in parallel

# Results

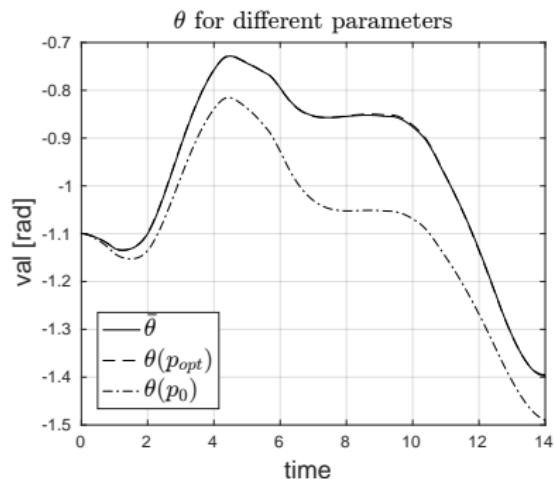
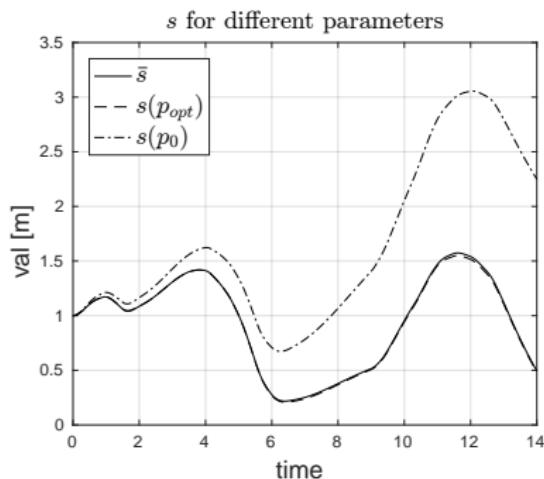


# Results



Exact up to 3cm

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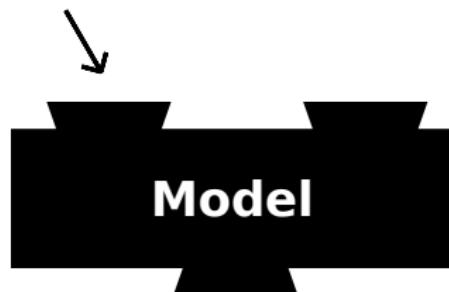
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**Control**



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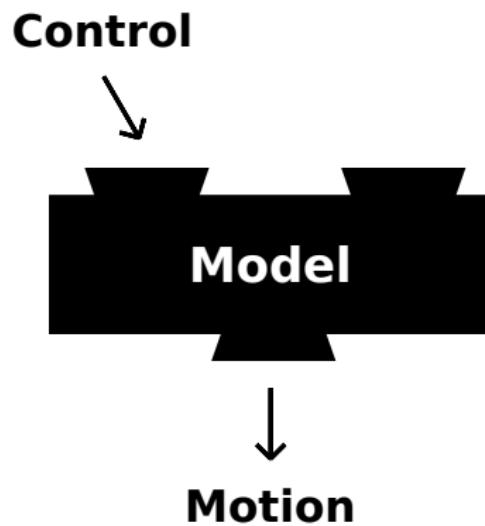
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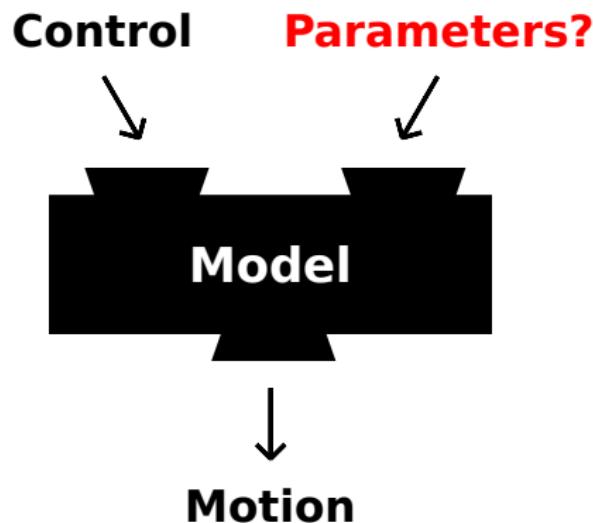
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# Trajectories

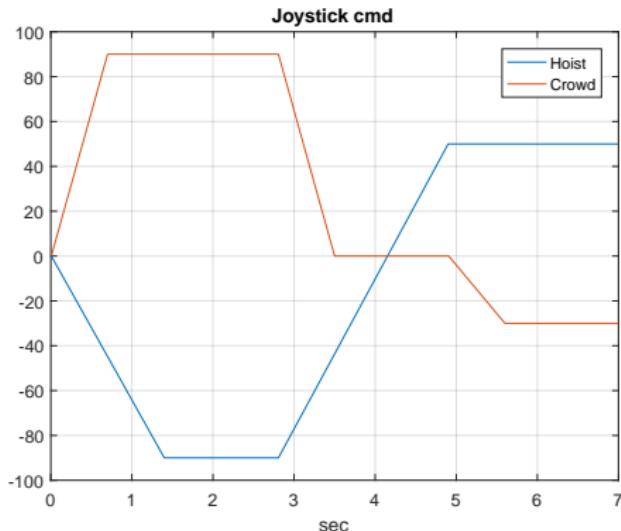
**Input:** Joystick commands for Up/Down and Back/Forth

**Output:** Position of the shovel

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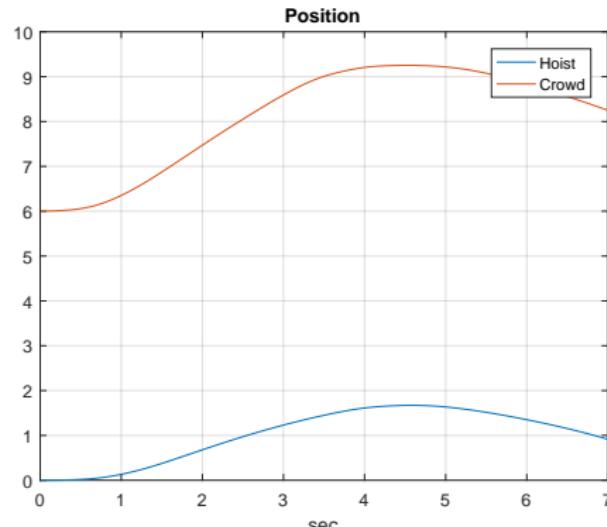
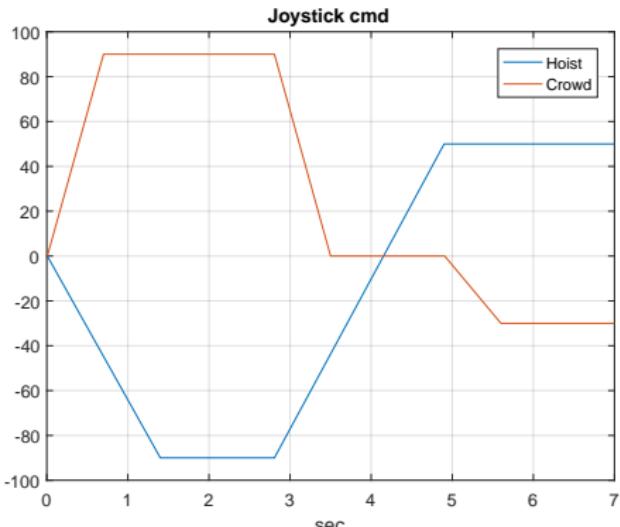
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# Objective Function

## Optimized Parameters:

- Inertia (Engine)
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$$\|\bar{X}_i - X_i(p)\|^2$$

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## Optimized Parameters:

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$$\frac{\|\bar{X}_i - X_i(p)\|^2}{\|\bar{X}_i\|^2} + \frac{\|\bar{Y}_i - Y_i(p)\|^2}{\|\bar{Y}_i\|^2}$$

$\bar{X}_i, \bar{Y}_i$  reference trajectories

# Objective Function

## Optimized Parameters:

- Inertia (Engine)
- Inertia (Arm)
- Friction
- Mass

## Penalty Term:

$$\frac{1}{n} \cdot \sum_{i=1}^n \left( \frac{\|\bar{X}_i - X_i(p)\|^2}{\|\bar{X}_i\|^2} + \frac{\|\bar{Y}_i - Y_i(p)\|^2}{\|\bar{Y}_i\|^2} \right)$$

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# Objective Function

## Optimized Parameters:

- Inertia (Engine)
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## Penalty Term:

$$\min_{p \in \mathbb{R}^4} f(p) = \frac{1}{n} \cdot \sum_{i=1}^n \left( \frac{\|\bar{X}_i - X_i(p)\|^2}{\|\bar{X}_i\|^2} + \frac{\|\bar{Y}_i - Y_i(p)\|^2}{\|\bar{Y}_i\|^2} \right)$$

s. t.

$$p_j \geq 0$$

$\bar{X}_i, \bar{Y}_i$  reference trajectories

# Influence of the Parameters

10% parameter deviation:

- Inertia (Engine):  $1 \cdot 10^{-3}$
- Inertia (Arm):  $3 \cdot 10^{-3}$
- Friction:  $8 \cdot 10^{-11}$
- Mass:  $5 \cdot 10^{-2}$

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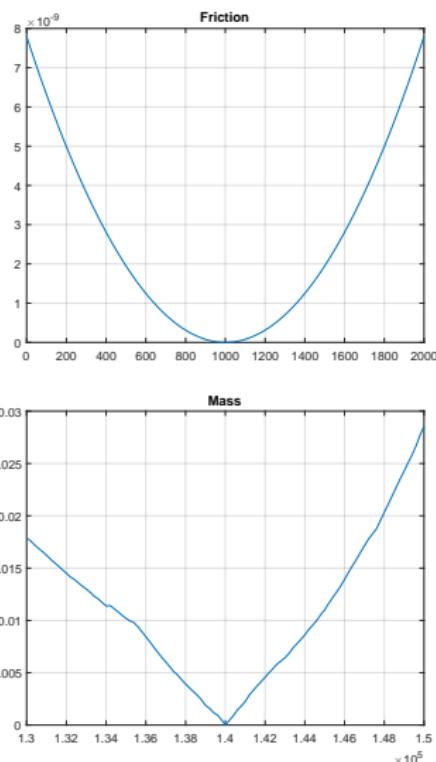
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Pattern Search	$10^{-11}$	6500	17 min	$10^{-1.3}$	$10^{-3.7}$
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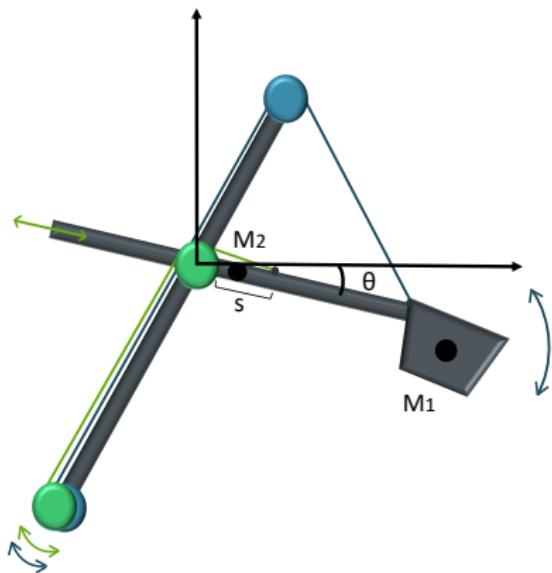
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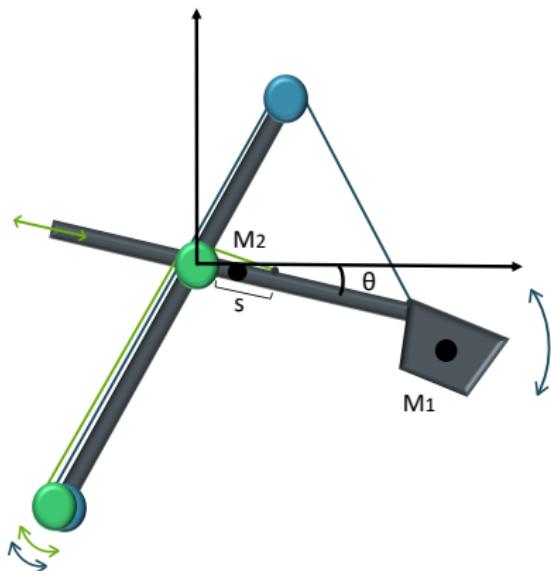
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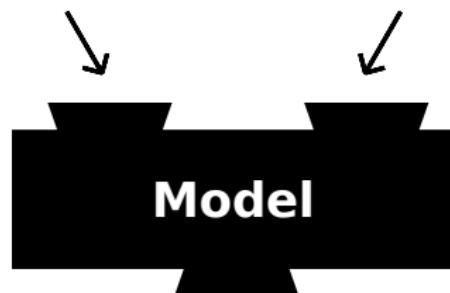


# Summary



**Control**

**Parameters?**



**Motion**