# Discretization of the ODE

Discretize time interval:

$$[0, T] \to \{0 = t_0, t_1, \ldots, t_{m-1}, t_m = T\}$$

Discretize state and control:

$$x_n = x(t_n)$$
$$u_n = u(t_n)$$

# Discretization of the ODE

Explicit Euler for every time step $h_n = t_{n+1} - t_n$:

$$x_{n+1} \approx \tilde{x}_{n+1} = x_n + h_n f(x_n, u_n, p) =: \Psi(x_n, u_n, p)$$

Discrete constraint:

$$0 = x_{n+1} - \Psi(x_n, u_n, p) =: \Phi_n(x, u, p) \quad \forall n = 0, \ldots, m-1$$

# Problem Formulation

Natural approach: Optimal Control Problem

$$\begin{aligned} \min_{x,p} \quad & \frac{1}{2}\|\bar{x} - x\|^2 \\ \text{s. t.} \quad & \Phi(x, u, p) = 0 \\ & p \geq 0 \end{aligned}$$

| | |
|---|---|
| state | $x = (s, \theta, \dot{s}, \dot{\theta})^T$ |
| control | $u = (\tau_1, \tau_2)^T$ |
| parameters | $p = (p_1, ..., p_k)^T$ |
| desired motion | $\bar{x}$ |

# Problem Formulation

Input:

- control $u$
- desired motion $\bar{x}$ related to $u$

Output:

- parameters $p$ of the excavator
- x, but not of interest

Idea:

- get rid of variable $x$
- set $x := \bar{x}$
- solve a relaxed problem

# Problem Formulation

## Original Problem

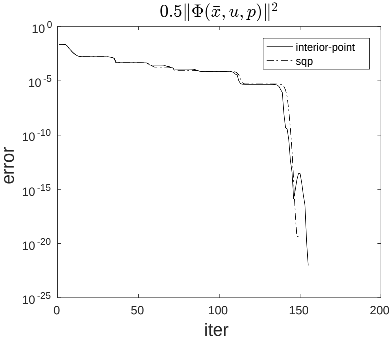
$$
\begin{aligned}
\min_{x,p} \quad & \frac{1}{2}\|\bar{x} - x\|^2 \\
\text{s.t.} \quad & \Phi(x, u, p) = 0 \\
& p \geq 0
\end{aligned}
$$

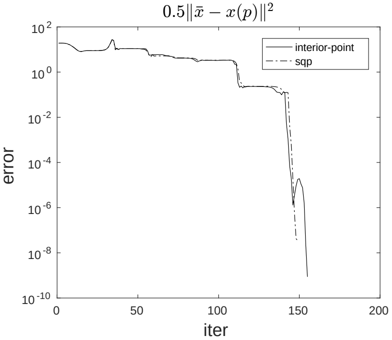
## Reinterpreted Problem

$$
\begin{aligned}
\min_{p} \quad & \frac{1}{2}\|\Phi(\bar{x}, u, p)\|^2 \\
\text{s.t.} \quad & p \geq 0
\end{aligned}
$$

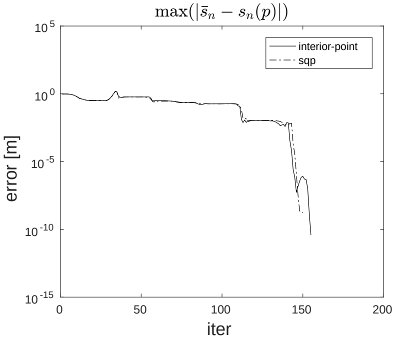
# Results

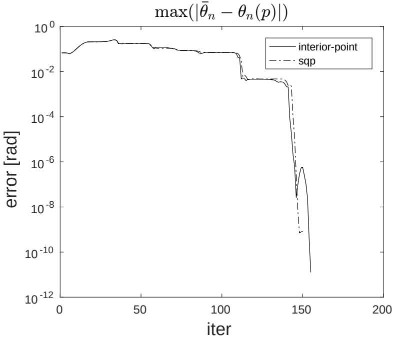
Without approximation error
x(p) solution of ODE using parameters p



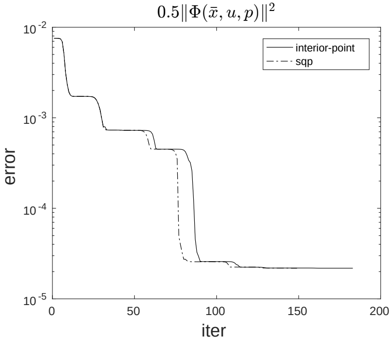
# Results

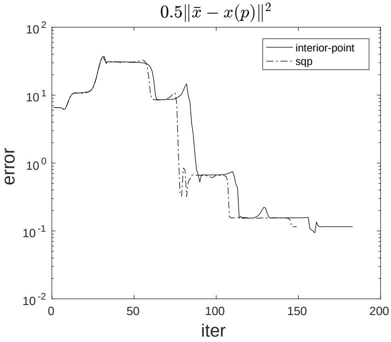

$\max(|\bar{s}_n - s_n(p)|)$
error [m]
iter
interior-point
sqp
10^5
10^0
10^-5
10^-10
10^-15
0
50
100
150
200


$\max(|\bar{\theta}_n - \theta_n(p)|)$
error [rad]
iter
interior-point
sqp
10^0
10^-2
10^-4
10^-6
10^-8
10^-10
10^-12
0
50
100
150
200

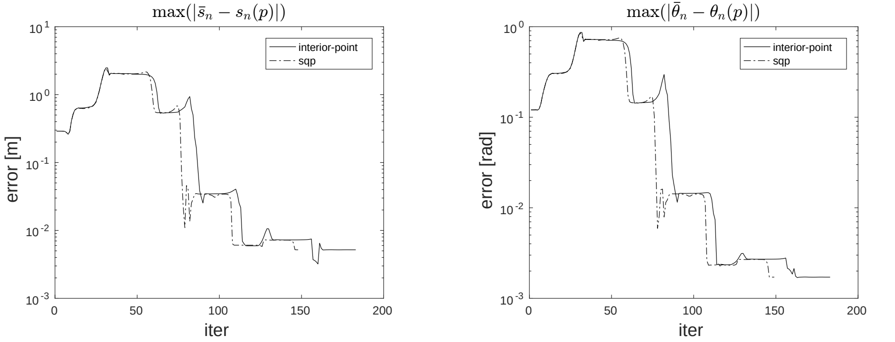
# Results

## With approximation error



# Results




Exact up to 1cm