ENV-541 Sensor Orientation
Lab 1 - Stochastic Processes

Michael Spieler

October 5, 2018

1 Noise realizations plots

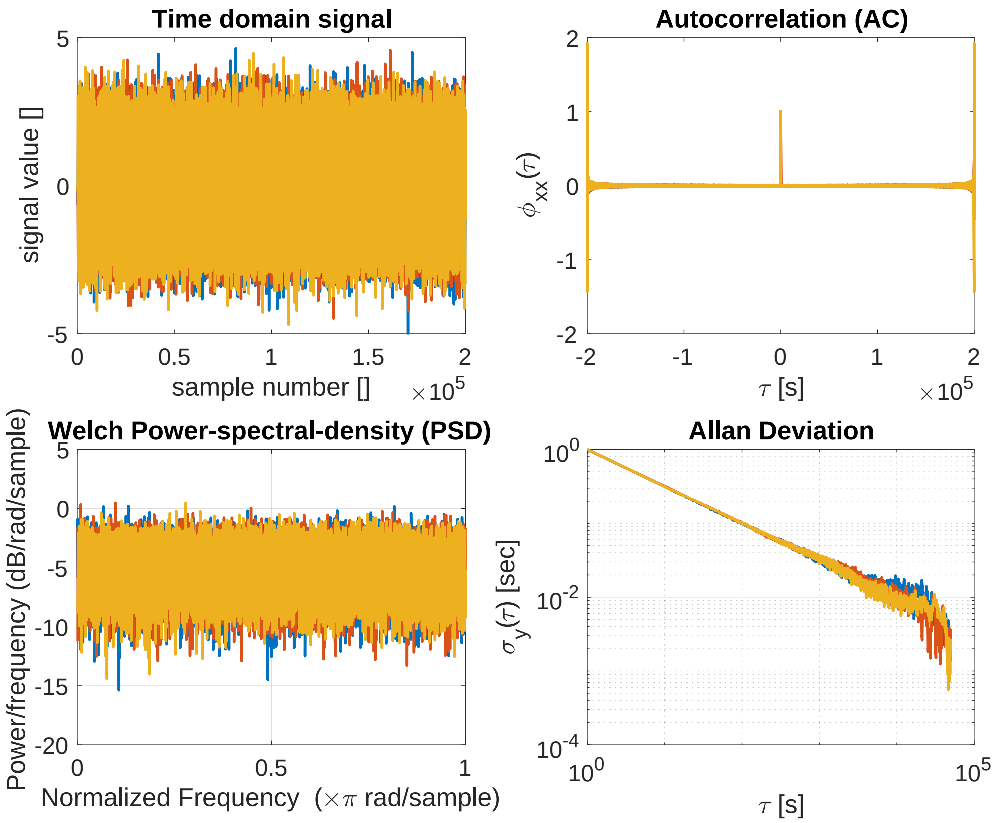

Figure 1: White Noise.

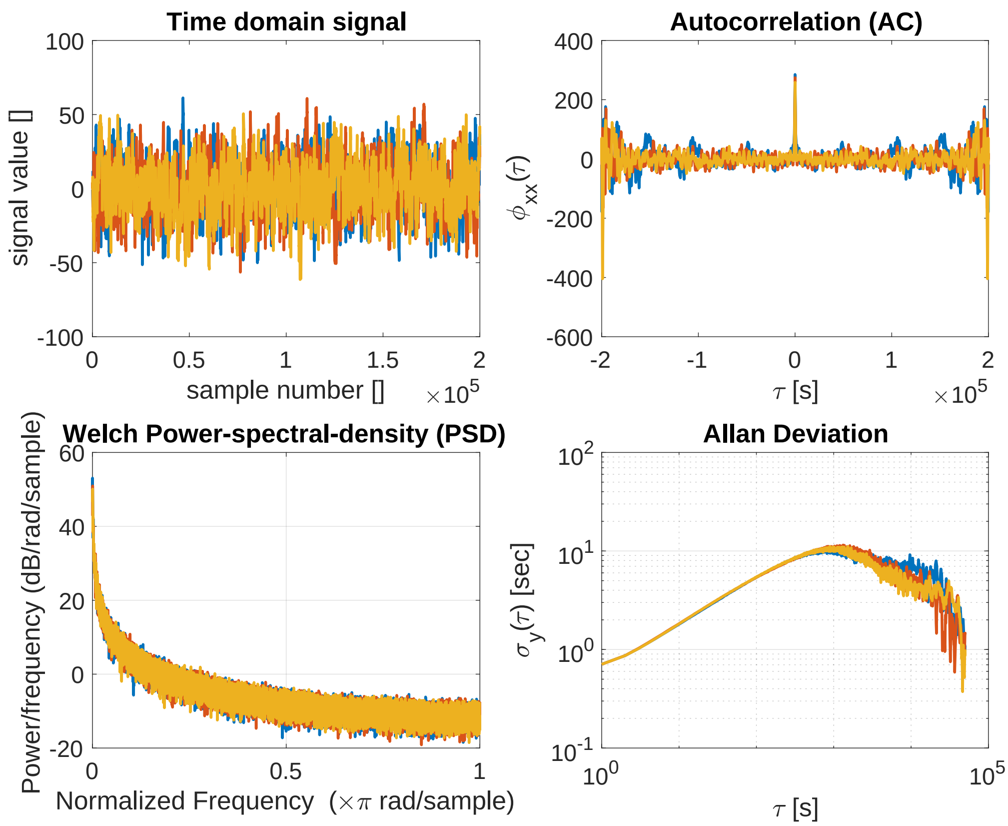

Figure 2: first order Gauss-Markov process with correlation time T=500.

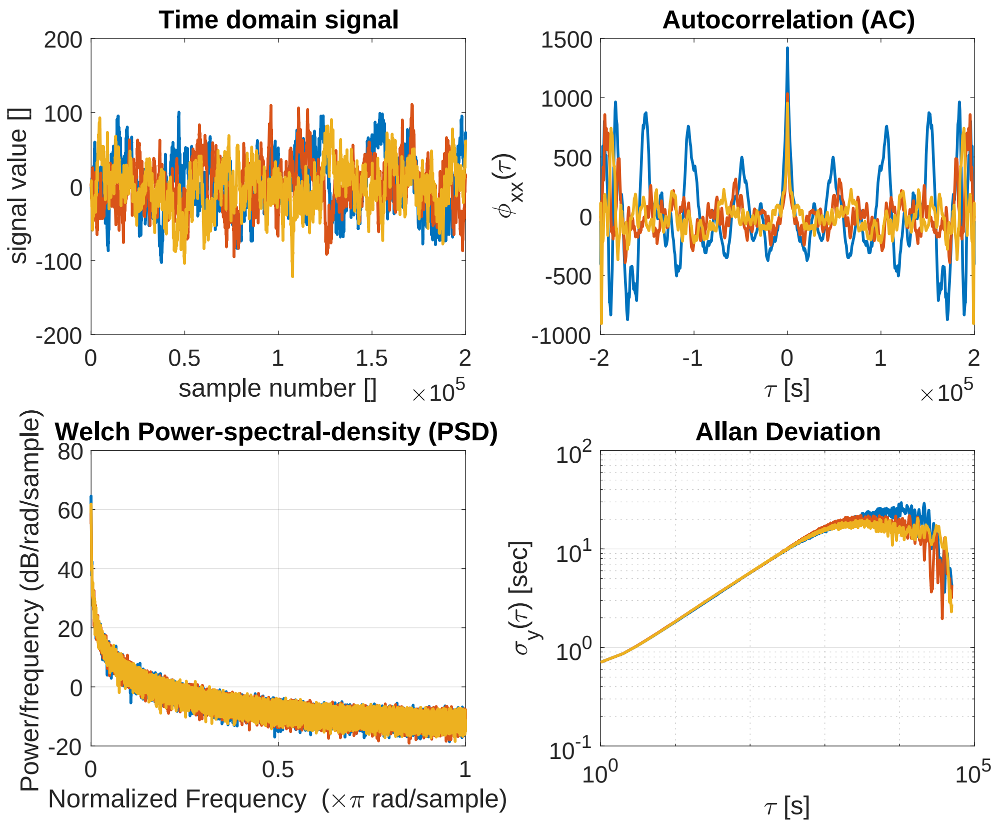

Figure 3: first order Gauss-Markov process with correlation time T=2000.

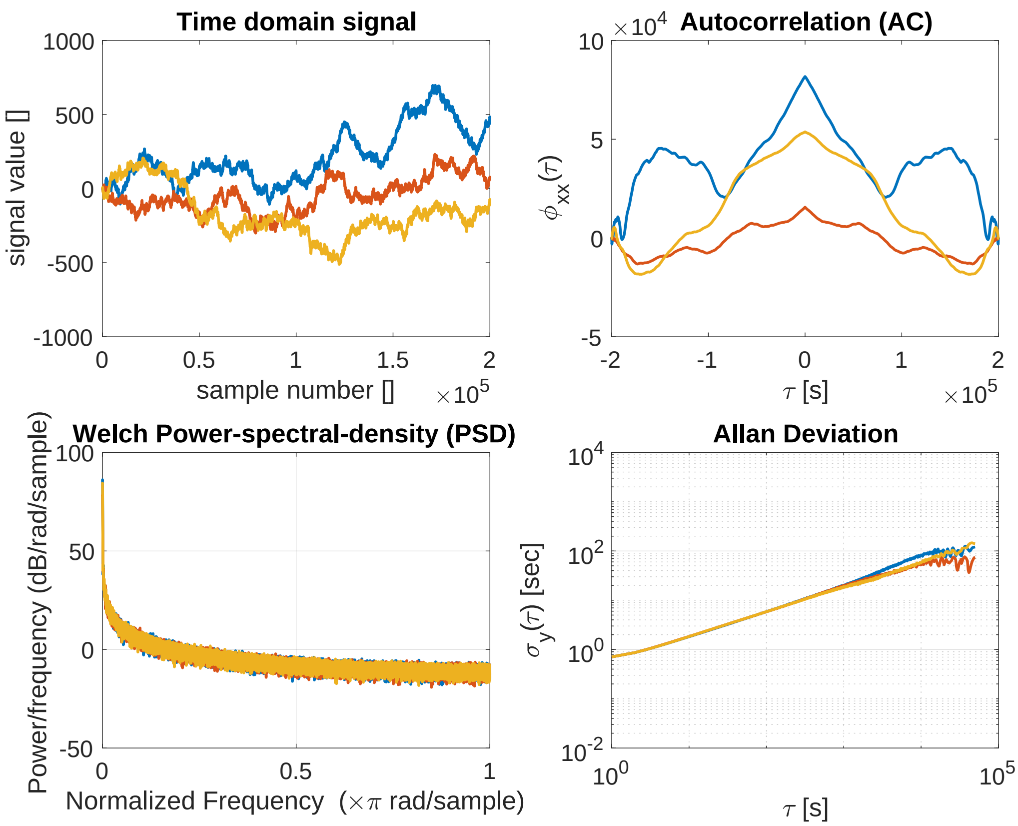

Figure 4: Random Walk.

2 Analysis

2.a Autocorrelation function shape

White Noise The autocorrelation is a dirac.

1st order Gauss-Markov Near to the center ($\tau = 0$), the autocorrelation resembles an exponentail decay in $\mid \tau \mid$, which is expected. There are lobes further away from $\tau = 0$ which does not correspond to the theory but can be explained by the fact that the unbiased version of `xcorr()` was used.

Random Walk The autocorrelation should look like a Gauss-Markov with quasi-infinite correlation time (exponential decay in τ), which it does with some imagination.

Note: In both the Gauss-Markov and the random walk sequences the autocorrelation shows weird lobes in the first realization (blue). Maybe this is due to a badly initialized random number generator since `rng(1)` is used for reproducible plots?

2.b Empirically determined values for standard deviation and correlation length

Parameter β is obtained by reading on the autocorrelation plot the width T of the central peak at σ^2/e. $\beta = 1/T$

sequence nb.	1 (blue)	2 (red)	3 (orange)
σ^2	1421	1034	955.6
T	6662	3282	1838

Table 1: GMP $T = 2000$

Tables 2 and 1 show the measured values for σ^2 and T. The theoretical values for σ^2 are $\sigma^2_{GM} = 1/(2\beta)$ with $q = 1$ and $\beta = 1/T$ which are in this case 1000 and 250 respectively. The variance σ^2 is estimated is quite close to the theoreticl value. However, the values for the correlation length T are off by up to a factor of 3. The reason for the deviation is unclear.

sequence nb.	1 (blue)	2 (red)	3 (orange)
σ^2	285.1	274.6	258.8
T	1136	1042	942

Table 2: GMP $T = 500$

2.c

The correlation time T characterizes best the underlying process because it depends only on parameter β. It tells how close the signal is to white noise. The smaller T, the closer to white noise, the bigger T, the closer to a random walk.

2.d

sequence nb.	1	2	3
σ^2	1.182931e+03	9.042480e+02	9.889570e+02
β	4.229967e-04	5.559729e-04	5.066038e-04
$T = 1/\beta$	2364.1	1798.6	1973.9

Table 3: GMP $T = 2000$

sequence nb.	1	2	3
σ^2	2.664254e+02	2.710082e+02	2.610238e+02
β	1.880129e-03	1.855449e-03	1.921333e-03
$T = 1/\beta$	531.9	539.0	520.5

Table 4: GMP $T = 500$

The GMWM estimates well the both parameters T and σ^2 (maximal error is less than 20%). The estimated correlation time T via GMWM is much closer to the true value than the results from graphical estimation.

Code

```
close all;

rng(1);
wn = randn(200000, 3);
rw = cumsum(wn);
% Gauss-Markov process (GM)
dt = 1;
beta = 1/500;
gm500 = GMP(wn, dt, beta);
beta = 1/2000;
gm2000 = GMP(wn, dt, beta);

%% Export data
dlmwrite('01_white_noise.txt',wn,'precision','%.8f')
dlmwrite('01_random_walk.txt',rw,'precision','%.8f')
dlmwrite('01_gm500.txt',gm500,'precision','%.8f')
dlmwrite('01_gm2000.txt',gm2000,'precision','%.8f')

%% Plots
set(groot,'DefaultAxesFontSize',17)
set(groot,'DefaultLineLineWidth',2)

plot_noise_characteristics(wn, 'White Noise')
plot_noise_characteristics(rw, 'Random Walk')
plot_noise_characteristics(gm500, 'Gauss-Markov, T=500')
plot_noise_characteristics(gm2000, 'Gauss-Markov, T=2000')

%% functions
function x = GMP(w, dt, beta)
    x = zeros(size(w));
    xk = 0;
    for i = 1:length(w)
       xk = exp(-beta*dt)*xk + w(i,:);
       x(i,:) = xk;
    end
end

function [] = plot_noise_characteristics(x, name)
    figure
    subplot(2,2,1);
    plot(x)
    xlabel('sample number []')
    ylabel('signal value []')
    title('Time domain signal')

    subplot(2,2,2);
    tau=-length(x)+1:length(x)-1;
    plot(tau,xcorr(x(:,1), 'unbiased')); hold on
    plot(tau,xcorr(x(:,2), 'unbiased'))
    plot(tau,xcorr(x(:,3), 'unbiased'))
    title('Autocorrelation (AC)')
    xlabel('\tau [s]')
    ylabel('\phi_{xx}(\tau)')

    subplot(2,2,3);
    pwelch(x)
    title('Welch Power-spectral-density (PSD)')

    subplot(2,2,4);
    loglog(allandev(x(:,1), '')); hold on; grid on
    loglog(allandev(x(:,2), ''))
    loglog(allandev(x(:,3), ''))
    title('Allan Deviation')
    xlabel('\tau [s]')
    ylabel('\sigma_y(\tau) [sec]')
    %suptitle(name)
end
```