

Two kinds of information processing in cognition

Mark Sprevak
University of Edinburgh

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What is the relationship between information and representation? Dating back at least to Dretske (1981), an influential answer has been that information is a rung on a ladder that gets one to representation. Representation is information, or representation is information plus some other ingredient. In this paper, I argue that this approach oversimplifies the relationship between information and representation. If one takes current probabilistic models of cognition seriously, information is connected to representation in a new way. It enters as a property of the representational content as well as a property of the vehicles of that content. This offers a new, logically independent, way in which information and representation are intertwined in cognitive processing.

1 Introduction

There is a new way in which cognition could be information processing. Philosophers have tended to conceptualise cognition's relationship to Shannon information in just one way. This suited an approach that treated cognition as inference over representations of single outcomes (*there is a face here, there is a line there, there is a house here*). Recent work conceives of cognition differently. Cognition does not involve an inference over representations of single outcomes but an inference over *probabilistic representations* – representations whose content includes multiple outcomes and their estimated probabilities.

My claim in this paper is that these recent probabilistic models of cognition open up new conceptual and empirical territory for saying that cognition is information

processing. Empirical work is already exploring this territory and researchers are drawing tentative connections between the two kinds of information in the brain. In this paper, my goal is not to propose a specific relationship between these two quantities of information in the brain, although some possible connections are sketched in Section 6. My goal is to convince you that there *are* two logically and conceptually distinct kinds of information whose relationship should be studied.

Before we proceed, some assumptions. My focus in this paper is only on Shannon information and its mathematical cognates. I do not consider other ways in which the brain could be said to process information.¹ Second, I assume a representationalist theory of cognition. I take this to mean that cognitive scientists find it *useful* to describe at least some aspects of cognition as processing representations. I focus on the role of Shannon information within two different kinds of representationalist model: categorical models and probabilistic models. My claim is that *if* one accepts a probabilistic model, then there are two ways in which cognition involves Shannon information processing. I do not attempt to defend representationalist theories of cognition in general.²

Here is a brief preview of my argument. Under probabilistic models of cognition there are two kinds of probability distribution associated with cognition. First, there is the ‘traditional’ kind: probability distributions associated with a specific neural state occurring in conjunction with an environmental state (for example, the probability of a specific neural state occurring when a subject is presented with a line at 45 degrees in a certain portion of her visual field). Second, there is the new kind associated with probabilistic neural representations: probability distributions that are *represented by* neural states. These probability distributions are the brain’s probabilistically-weighted guesses about various environmental outcomes (say, that the line is at 0, 35, 45, or 90 degrees).³ The two kinds of probability distribution – one associated with *a neural/environmental state occurring* and the other associated with *the neural system’s estimate of certain environmental states occurring* – are conceptually and logically distinct. They have different outcomes, different probability values, and different types of probability (objective and subjective) associated with them. They generate two separate measures of Shannon information in the brain.

¹See Floridi (2011).

²Note that I define representationalist theories in terms of their *utility* for describing cognitive processes, not in terms of their *truth*. Some deny truth but accept utility: they endorse some form of instrumentalism about representationalist models in cognitive science (for example, Egan, 2010; Colombo and Seriès, 2012; Sprevak, 2013). On my view, this still falls within the representationalist paradigm. To the extent it is legitimate, even if only on pragmatic grounds, to use a representationalist model of cognition, it is legitimate to say that cognition involves two kinds of information processing.

³This distinction is not that between ‘encoding’ and ‘decoding’ probability distributions (Elia-smith, 2005a). Encoding and decoding distributions are discussed in Section 3.

The algorithms that underlie cognitive processing can be described as processing *either* or *both* of these Shannon information quantities.

2 Shannon information

Before attributing two kinds of Shannon information to the brain, one first needs to know what justifies attributing *any* kind of Shannon information to the brain. Below, I briefly review the definitions of Shannon information in order to identify the conditions sufficient for a physical system to be ascribed Shannon information. The rest of the paper shows that these conditions are satisfied in two separate ways by the brain. Definitions in this section are taken from MacKay (2003), although similar points can be made with other formalisms.

In order to define Shannon information, one first needs to define the notion of a *probabilistic ensemble*:

- Probabilistic ensemble X is a triple (x, A_X, P_X) , where the outcome x is the value of a random variable, which takes on one of a set of possible values, $A_X = \{a_1, a_2, \dots, a_i, \dots, a_I\}$, having probabilities $P_X = \{p_1, p_2, \dots, p_I\}$, with $P(x = a_i) = p_i$, $p_i \geq 0$, and $\sum_{a_i \in A_X} P(x = a_i) = 1$

A necessary and sufficient condition for satisfying the definition of a probabilistic ensemble is the existence of a random variable with *multiple possible outcomes* and an associated *probability distribution*.⁴ If the random variable has a finite number of possible outcomes, this probability distribution takes the form of a probability mass function, assigning a value, p_i , to each outcome. If the random variable has an infinite number of possible outcomes, the probability distribution takes the form a probability density function, assigning a value, p_i , to the outcome falling within a certain range. In either case, multiple outcomes and a probability distribution over those outcomes is necessary and sufficient for a probabilistic ensemble.⁵

If a physical system has multiple possible outcomes and a probability distribution associated with those outcomes, then that physical system can be described as a probabilistic ensemble. If a neuron has multiple possible outcomes (e.g. firing or not), and a probability distribution over those outcomes (reflecting the chances of the neuron firing), then the neuron's activity can be described as a probabilistic ensemble.

Shannon information is a continuous scalar quantity measured in bits. Shannon

⁴The term 'outcome' here is not meant to imply that this is the output of a causal process.

⁵In principle, an ensemble might have only one outcome (necessarily, with probability of 1). As we will see, this corresponds to an ensemble and its single outcome having 0 bits of Shannon information. For information processing to be non-trivial, we need more than one possible outcome.

information is predicated of at least three different types of subject: *ensembles*, *outcomes*, and *ordered pairs of ensembles*. The definitions differ, so let us consider each in turn.

The Shannon information, $H(X)$, of an *ensemble* is defined as:

$$H(X) = \sum_i p_i \log_2 \frac{1}{p_i}$$

The only independent variables in $H(X)$ are the possible outcomes of the ensemble (the *is*) and their probabilities (the p_i s). The Shannon information of an ensemble is thus a mathematical function of, and only of, the defining properties of a probabilistic ensemble. Merely being an ensemble – having multiple possible outcomes and a probability distribution over those outcomes – is enough to define a $H(X)$ measure and bestow a quantity of Shannon information on the ensemble. Therefore, any physical system that is described as a probabilistic ensemble *ipso facto* has an associated measure of Shannon information. If a neuron is described as an ensemble (because it has multiple possible outcomes and a probability distribution over those outcomes), then it has, by that very fact, a quantity of Shannon information attached.

The Shannon information, $h(x)$, of a *single outcome* is defined as:

$$h(x = a_i) = \log_2 \frac{1}{p_i}$$

$H(X)$ is the expected value of $h(x)$ across all possible outcomes of the ensemble. The only independent variable in $h(x)$ is the probability of the outcome in question. So again, the mere existence of an ensemble is a sufficient condition for satisfying the definition of $h(x)$. If an ensemble exists, then each of its outcomes has a measure of Shannon information. No further conditions need to be met. If a neuron is described as an ensemble, then each of its possible outcomes (firing or not firing) has a probability, and hence has a quantity of Shannon information attached.

There are many Shannon measures of information that are defined for *pairs of ensembles*.⁶ Common ones include:

Joint information:

$$H(X, Y) = \sum_{x,y \in A_X A_Y} P(x, y) \log_2 \frac{1}{P(x, y)}$$

⁶One member of the pair is usually called the ‘sender’ and the other the ‘receiver’.

Conditional information:

$$H(X | Y) = \sum_{y \in A_Y} P(y) \sum_{x \in A_X} P(x | y) \log_2 \frac{1}{P(x | y)}$$

Mutual information:

$$I(X; Y) = H(X) - H(X | Y)$$

These measures differ from each other in important ways, but again a sufficient condition for satisfying any of them is the presence of multiple possible outcomes and a probability distribution over those outcomes. Ensembles X and Y have individual outcomes and individual probability distributions over those outcomes. The pairwise Shannon measures above assume that conjunctions of outcomes from the ensembles also have an associated (joint) probability distribution, $P(X, Y)$, which describes the probability of a pair (x, y) of outcomes occurring and defines any conditional probability measures, such as $P(X | Y)$. No more than this is required to define the Shannon measures above. If X and Y exist, and if combinations of their respective outcomes have probabilities (even if some of these probabilities are 0), then the Shannon measures of joint information, conditional information, and mutual information are defined. Consequently, if two neurons are described as two ensembles and have a joint probability distribution, then that pair has associated measures of joint information, conditional information, and mutual information.

A sufficient condition for a physical system to be attributed Shannon information is that it *has multiple possible outcomes and a probability distribution over those outcomes*. Shannon information of an ensemble, a single outcome, or a pair of ensembles, is a function of, and only of, the possible outcomes and probability distribution associated with that ensemble, single outcome, or pair of ensembles. If a physical system is described as an ensemble (or a pair of ensembles), it *ipso facto* has associated Shannon information.

If a physical system changes the probabilities associated with its possible outcomes over time, its associated Shannon measures are likely to change too. Such a system may be described as ‘processing’ Shannon information. This change could happen in at least two ways. If a physical system modifies the probabilities associated with its *physical states* occurring (e.g. a neuron makes certain physical states such as

firing more or less likely), it can be described as processing Shannon information.⁷ Alternatively, if the firing activity of the neuron *represents* a probability distribution over possible outcomes, and that represented probability distribution changes over time – perhaps as a result of inference or learning – then its associated Shannon measures will change too. In both cases, probability distributions and Shannon information change. But distinct probability distributions and distinct measures of Shannon information are changed in each case. The remainder of this paper will unpack the distinction between them.

3 The traditional kind of Shannon information

An important role for Shannon information has been as a building block in the project of *naturalising representation*. Versions of information-theoretic semantics try to explain semantic content in terms of Shannon information. These accounts aim to explain how representation, and in particular mental representation, arises from Shannon information-theoretic relations. Such theories often claim that Shannon information is a source of naturalistic, objective facts about representation. Dretske formulated one of the earliest such theories.⁸ Dretske's (1981) theory aimed to reduce facts about representational content entirely to facts about Shannon information. More recently, other accounts – including Dretske's later (1988; 1995) views – propose that an information-theoretic condition is only one part of a larger condition for representational content. Additional conditions include variously conditions concerning teleology, instrumental (reward-guided) learning, structural isomorphism, and/or appropriate use.⁹ In what follows, I focus solely on the information-theoretic part of such a semantic theory.

Information-theoretic semantics attempts to explain representation in terms of one physical state 'carrying information' about another state. The relationship of 'carrying information' is assumed to be a precursor to, or a precondition for, certain

⁷One example would be neural changes that occur during learning and other kinds of long-term plasticity. But similar changes also occur during short-term processing. When a neuron fires, it makes a specific outcome – its *firing* – certain. This change will almost certainly affect the probabilities associated with other neurons in the brain (making their respective outcomes more or less probable), and hence change their associated Shannon measures. Neuroscientists can track how these Shannon measures change as a specific outcome propagates in the brain during cognitive processing. Thanks to Nick Shea for this point.

⁸Prior to Dretske's work, Shannon information had been linked to semantic content, although not always in reductive fashion (Bar-Hillel and Carnap, 1964; Wiener, 1961).

⁹See Millikan (1984); Papineau (1987); Dretske (1988); Shea (2007); Shea (2014a); Skyrms (2010); W. M. Ramsey (2016) for a range of such proposals. Note that some of these authors argue that representations sometimes gain their content solely on the basis of these other non-Shannon factors. Thanks to an anonymous referee for pointing this out.

kinds of representation. In the context of the brain, such a theory says:

- (R) Neural state, n (from N), represents an environmental state, s (from S) only if n ‘carries information’ about s .

Implicit in R is the idea that neural state, n , and environmental state, s , come from a set of possible alternatives. According to R, neural state n represents s only if n bears the ‘carrying information’ relationship to this s and not to other outcomes. Different neural states could occur in the brain (e.g. different neurons in a population might fire). Different environmental states could occur (e.g. a face or a house could be present). Crudely, the reason why certain neural firings represent a face and not a house is that those firings, and only those firings, bear the ‘carrying information’ relation to *face* outcomes, and the relevant firings do not bear that relation to *house* outcomes. R assumes that we are dealing with multiple possible outcomes: multiple possible representational vehicles (N) and multiple possible environmental states (S). It tries to identify a special relationship between individual outcomes that gives rise to representation. Representation occurs only when n from N bears an ‘carrying information’ relationship to s from S .

The primary task for an information-theoretic semantics is to explain what this ‘carrying information’ relation is. Different versions of information-theoretic semantics do this differently.¹⁰ Theories can be divided roughly into two camps: those that are ‘correlational’ and those that invoke ‘mutual information’.

The starting point for ‘correlational’ theories is that one physical state ‘carries information’ about another just in case there is a correlation between the two that satisfies some probabilistic condition. This still leaves plenty of questions unanswered: What kind of correlation (Pearson, Spearman, Kendall, mutual information, something else?)¹¹ How should physical states be typed into sets of outcomes between which a correlation is measured? How much correlation is enough to set up an information-carrying relation? Does it matter if the correlation is accidental or underwritten by a law or disposition? Rival information-theoretic semantics take different views on these questions. Consider the following three proposals:

¹⁰The relation of ‘carrying information’ is also sometimes described as one physical state ‘having natural information’ about another; see Stegmann (2015).

¹¹Millikan (2001) suggests that the relevant correlational relations are those that are ‘learnable’ for an agent: A is correlated with B, and hence carries information about B, if B is learnable (or inferable) from A. However, any degree of probabilistic dependence between A and B (no matter how slight) could, in principle, allow an agent to learn, or infer, one from the other. With suitable rewards on offer, even the mildest degree of probabilistic dependence could be a target of learning as an agent could reap arbitrarily large rewards for doing so. The notion of a ‘learnable’ relation – if it is not merely to be a synonym for *not probabilistically independent* – is as much in need of explication as the notion of ‘correlation’.

1. $P(S = s \mid N = n) = 1$
2. $P(S = s \mid N = n)$ is ‘high’
3. $P(S = s \mid N = n) > P(S = s \mid N \neq n)$

Dretske (1981) endorses (1): a neural state carries information about an environmental state just in case an agent, given the neural state, could infer with certainty that the environmental state occurs (and this could not have been inferred using the agent’s background knowledge alone). Millikan (2000; 2004) endorses (2): the conditional probability of the environmental state need only be ‘high’, where what counts as ‘high’ is a complex matter involving the correlation relation having influenced past use via genetic selection or learning.¹² Shea (2007) and Scarantino and Piccinini (2010) propose that correlation should be understood in terms of probability raising, (3): the neural state should make the occurrence of the environmental state more probable than it would have been otherwise.

At first glance, there may seem nothing particularly Shannon-like about proposals (1)–(3). Probability theory alone is sufficient to express the conditions necessary for representation without enrichment from Shannon’s concepts. These semantic theories are perhaps better termed ‘probabilistic’ semantics than ‘information-theoretic’ semantics.¹³ Nevertheless, there is a legitimate way in which these accounts show that cognition is Shannon information processing. According to (1)–(3), ‘carrying information’ is a relationship between particular outcomes and those outcomes come from ensembles that have probability distributions. A sufficient condition for a system to have Shannon information is that it *has multiple possible outcomes and a probability distribution over those outcomes*. (1)–(3) assure us that this condition is met by a cognitive system that contains representations. According to (1)–(3), the representational content of a neural state arises only if that neural state is an outcome from an ensemble with other possible outcomes (other possible neural states) that could occur with certain probabilities (and probabilities conditional on various environmental outcomes). If cognition involves representation, and representations gain their content by any of (1)–(3), then cognition *ipso facto* involves Shannon information. Shannon information attaches to representations because of the probabilistic nature of their vehicles. That probabilistic nature is, according to (1)–(3), essential to their representational status. Consequently, to the extent that cognition can be described as processing representations, and to the extent that we accept one of these versions of information-theoretic semantics, cognition can be described as processing states with a probabilistic nature, and *ipso facto* can be described as processing states with Shannon information.

¹²See Stegmann (2015), pp. 873–874 for helpful analysis of Millikan’s view.

¹³Timpson (2013), pp. 41–42 makes a similar point with regard to Dretske’s (1981) theory, and related criticisms are raised by commentators in Dretske (1983).

‘Mutual information’ versions of information-theoretic semantics unpack ‘carrying information’ differently. They invoke the Shannon concept of mutual information, or rather, pointwise mutual information, the analogue of Shannon mutual information for pairs of single outcomes. Mutual information is the expected value of pointwise mutual information over all joint outcomes of a pair of ensembles.¹⁴ Pointwise mutual information for a pair of single outcomes, x, y , from two ensembles, X, Y , is defined as:

$$PMI(x, y) = \log_2 \frac{P(x, y)}{P(x)P(y)} = \log_2 \frac{P(x | y)}{P(x)} = \log_2 \frac{P(y | x)}{P(y)}$$

Skyrms (2010) and Isaac (2017) propose that the information carried by a physical state, n , (its ‘informational content’) is a vector consisting of the $PMI(n, s)$ value of every possible environmental state, s_i , from S , for that specific n from N : $\langle PMI(n, s_1), \dots, PMI(n, s_n) \rangle$. Isaac identifies the meaning or semantic content of n with this PMI -vector. Skyrms says that the meaning/semantic content can be considered a more traditional semantic object, such as a set of possible worlds, which is derived from the PMI -vector by finding the environmental states that generate high-value elements of the PMI -vector. The meaning/semantic content is the set of possible worlds in which these high- PMI -value environmental states obtain.

Like Skyrms and Isaac, Usher (2001) and Eliasmith (2005b) appeal to pointwise mutual information to define ‘carrying information’. Unlike Skyrms and Isaac, they define ‘carrying information’ as a relationship between a single neural state, n , and a single environmental state, s . They say that n carries information about s just in case s is the state for which $PMI(n, s)$ has its maximum value for that n . Neural state, n , carries information about the s that produces the peak value in its PMI -vector. Usher and Eliasmith connect this quantity to what is measured in ‘encoding’ experiments in neuroscience. In an encoding experiment, many environmental states are presented to a brain and researchers look for the environmental state that best predicts a neural response – that maximises $PMI(n, s)$ as one varies s for a given n . Usher and Eliasmith also offer a second, complementary definition of ‘carrying information’. This is based around what is measured in ‘decoding’ experiments. In a decoding experiment, researchers examine many neural states and classify them based on which one best predicts an environmental state – i.e. which n maximises $PMI(n, s)$ for a given s . Here, instead of looking for the highest $PMI(n, s)$ value as one varies s and keeps n fixed, one looks for the highest $PMI(n, s)$ value as one varies n and keeps s fixed. There is no reason why the results of encoding and decoding experiments should coincide: they might pick out two entirely different sets of information carrying relations between brain and world. Usher and Eliasmith argue

¹⁴ $I(X; Y) = \sum_{x, y \in A_X, A_Y} P(x, y) PMI(x, y)$

that they provide different, complementary, and equally valid, Shannon information-theoretic accounts of representational content. In both cases, representational content is categorical: the content is the single environmental state that produces the peak *PMI* value (across variations in n or s).

On each of these semantic theories, Shannon information arises in a cognitive system because of the probabilistic properties of neural states *qua vehicles*. It is because a given neural state is an outcome from a range of possible alternatives combined with the probability of various environmental outcomes, that the cognitive system has the Shannon information-theoretic properties relevant to representation and hence to cognition. In the next section, I describe a different way in which Shannon information enters into cognition. Here, the Shannon quantity arises, not from the probabilistic nature of the physical vehicles and environmental states, but from the *represented content*. ‘Probabilistic’ models of cognition claim that the represented content of a neural state is probabilistic. This means that Shannon information is associated with a cognitive system in a new way: via its content rather than via the probabilistic nature of its neural vehicles.

4 The new kind of Shannon information

Probabilistic models of cognition, like the representationalist accounts discussed in the previous section, attribute representations to the brain. Unlike those accounts however, these accounts do not aim to naturalise representational content. They *help themselves* to representational content. Their claim is that neural representations have a particular kind of content. Probabilistic models of cognition are largely uncommitted about *how* these representations get this content. In principle, they are compatible with a variety of semantic theories, including certain versions of information-theoretic semantics.¹⁵

The central claim of any probabilistic model of cognition is that neural representations have probabilistic content and those neural representations they enter into probabilistic inferences. In contrast, categorical approaches to representation assume that *single outcomes* are the represented content of representational vehicles. A neural state, n , represents a single environmental outcome (or a single set of outcomes). Thinking about neural representation in these terms has prompted description of neural states early in V_1 as *edge detectors*: their activity represents the presence (or absence) of an edge at a particular angle in a portion of the visual field. The represented content is a particular outcome (*edge at ~45 degrees*). Similarly, neurons in the inferior temporal (IT) cortex are described as *hand detectors*: their

¹⁵I discuss how information-theoretic semantics might interact with probabilistic models of cognition in Section 6.

activity represents the presence (or absence) of a hand. The represented content is a single outcome (*hand present*). Similarly, neurons in the fusiform face area (FFA) are described as *face detectors*: their activity represents the presence (or absence) of a face. The represented content is a single outcome (*face present*) (for example, see Gross, 2007; Logothetis and Sheinberg, 1996; Kanwisher, McDermott and Chun, 1997).

There is increasing suspicion that representation in the brain is not like this. Representational content is rarely a single environmental state (e.g. *hand present*); rather, it is a probability distribution over multiple possible states. The brain represents many outcomes simultaneously to ‘hedge its bets’ during processing. This allows the brain to store, and make use of, information about different possible outcomes when it is uncertain about which is the true outcome. The brain’s uncertainty may come from the unreliability of the perceptual hardware, or from its epistemic situation that, even with perfectly reliable hardware, it only has incomplete access to its environment. Ascribing probabilistic content to a cognitive agent is not in itself new (de Finetti, 1990; F. P. Ramsey, 1990). However, there is an important difference between past approaches and current probabilistic models. In the past, probabilistic representations were treated as *personal-level* states of an agent – ‘credences’, ‘degrees of belief’, or ‘personal probabilities’. In new probabilistic models, probabilistic representations are treated as states of subpersonal *parts* of the agent – neural populations, or single neurons. The claim is that, irrespective of the personal-level states that are attributed to the agent, various parts of the agent token diverse (and perhaps even conflicting) probabilistic representations. Thinking in these terms has prompted redescription of neural states early in V1 as probabilistically nuanced ‘hypotheses’, ‘guesses’, or ‘expectations’ about edges. Their neural activity does not represent a single state of affairs (*edge at ~45 degrees*) but a probability distribution over multiple possible edge orientations (Alink et al., 2010). Their representational content contains multiple possible outcomes regarding edges along with a probability distribution over those outcomes. Similarly, neural activity in the IT cortex does not represent a single state of affairs (*hand present*) but a probability distribution over multiple possible outcomes regarding hands. The represented content contains multiple possible outcomes regarding hands along with a probability distribution over those outcomes. Similarly, neural activity in FFA does not represent a single state of affairs (*face present*) but a probability distribution over multiple possible outcomes regarding faces. The represented content contains multiple possible outcomes regarding faces along with a probability distribution over those outcomes (Egner, Monti and Summerfield, 2010).

Traditional models of cognitive processing tend to describe cognition as a computational process that implements an inference over specific outcomes – *if there is an edge here, then that is an object boundary*. Probabilistic models of processing

describe the processing as a computational process that implements an inference over probability distributions – *if the probability distribution of edge orientations is this, then the probability distribution of possible object boundaries is that*. Cognitive processing is a series of computational steps that uses representations of one probability distribution to condition, or update, representations of another probability distribution.¹⁶ Neural representation may remain probabilistic right until the moment when the brain is forced to plump for a specific outcome in action. At that point, the brain may select the most probable outcome from its current represented probability distribution conditioned on all its available evidence (or some other point estimate that is easier to compute).

Modelling cognition as a form of probabilistic inference does not mean modelling cognition as non-deterministic or chancy. The physical hardware and algorithms underlying the probabilistic inference may be entirely deterministic. Consider that when your electronic PC filters spam messages from incoming emails, it performs a probabilistic inference, but both the PC's physical hardware and the abstract algorithm that the PC follows are entirely deterministic. A probabilistic inference takes representations of probability distributions as input, yields representations of probability distributions as output, and transforms input into output based on rules of valid (or pragmatically efficacious) probabilistic inference. The physical mechanism and the formal rules for processing representations may be entirely deterministic. What makes the process probabilistic is not the chancy nature of the physical transitions or formal rules but that probabilities feature in the represented content that is being manipulated.

Perhaps the best known current example of a probabilistic model of cognition is the 'Bayesian brain' hypothesis. This model says that brains represent their environment probabilistically and process those representations according to rules of Bayesian or approximately Bayesian inference (Knill and Pouget, 2004). Predictive processing provides a proposal about how this Bayesian inference could be neurally implemented (Clark, 2013; Friston, 2009). It is worth stressing that the motivation for positing probabilistic representations in the brain, and for probabilistic models of cognition in general, is much broader than that for the Bayesian brain hypothesis (or for predictive processing). The brain's inferential methods could, in principle, depart very far from Bayesianism and still produce adaptive behaviour under many circumstances. It remains an open question to what extent humans are Bayesian (or approximately Bayesian) reasoners. Probabilistic techniques adapted from AI, such as deep learning, reinforcement learning, and generative adversarial models,

¹⁶This is not Bayesian updating but simply using conditional probabilities to guide steps in an inference: conditional probabilities tell the cognitive system how to make inferences about the distribution of values of an unknown variable based on its knowledge about known variables.

can produce impressively rational behaviour despite having complex and qualified relationships to Bayesian inference. The idea that cognition is a form of probabilistic inference is a much more general idea than the idea that cognition is Bayesian. A researcher in cognitive science may subscribe to probabilistic representation even if they take a dim view of the Bayesian brain hypothesis.¹⁷

The essential difference between a categorical representation and a probabilistic representation lies in their content. Categorical representations aim to represent a single state of affairs. In Section 3, we saw that schema R treats representation as a relation between a neural state, n , and an environmental outcome, s . The represented content is specified by a truth, accuracy, or satisfaction condition. Meeting this truth, accuracy, or satisfaction condition is assumed to be largely an all-or-nothing matter. A categorical representation effectively ‘bets all its money’ that a certain outcome occurs. An edge detector says that *there is an edge*. Multiple states of affairs may sometimes feature in the represented content (for example, *there is an edge between ~43–47 degrees*), but those states of affairs are grouped together into a single outcome that is represented as true. There is no probabilistic nuance, or apportioning of different degrees of uncertainty, to different outcomes.

In contrast, probabilistic representations aim to represent multiple outcomes and a probability distribution over those outcomes. The probability distribution is a graded measure of how much the system ‘expects’ that the relevant outcome is the true one. Unlike categorical representations, represented content does not partition environmental states into two classes (*true* and *false*). Representation is not an all-or-nothing matter; it involves assigning a weighted probability value to various possible outcomes. The probabilistic notion of representation pairs a neural state, n , with multiple outcomes and a probability distribution over those outcomes. As we will see, these represented outcomes need not even coincide with the possible outcomes of S . Whereas the content of a categorical representation is specified by a truth, accuracy, or satisfaction condition, the content of a probabilistic representation is specified by a probability mass or density function over a set of possible outcomes.

Probabilistic representations could, in principle, use any physical vehicle to represent their content. There is nothing about the physical make-up of a representational vehicle that determines that the representation has categorical rather than probabilistic content. Structured complexes of probabilistic representations could use any number of different formal formats as their syntactic structure. Possible formal formats include being a setting of weights in a neural network, a symbolic expression, a directed graph, a ring, a tree, a region in continuous space, or an entry in a relational database (Griffiths, Chater et al., 2010; Tenenbaum et al., 2011). Certain physical vehicles and formal formats are more apt to serve in certain computational

¹⁷See Ma (2012).

processes than they are in others. The choice of physical vehicle and formal format affect how easy it is to implement an inference over representations with computation (Marr, 1982). But in principle there is nothing about the physical make-up or formal format of a representation that determines whether its content is categorical or probabilistic. That is determined solely by its represented content.

The preceding discussion should not be taken as suggesting that a model of cognition may only employ one type of representation (categorical or probabilistic) exclusively. A model could mix the two types of representation, assuming it has appropriate rules to take the system between the two in inference. The preceding discussion should also not be taken as suggesting that there are no possible reductive connections between the two types of representation. A wide range of such connections may exist. For example, a cognitive system might use structured complexes of categorical representations to represent the mathematical probability calculus and thereby represent probabilistically nuanced content (this is maybe what we do with our notation for mathematical probability theory). Conversely, a cognitive system might use structured complexes of probabilistic representations to represent categorical content. Feldman (2012) describes a cognitive model in which categorical content is approximated by probabilistic representations that have strongly modal (sharply peaked) probability distributions – probabilistic representations that effectively ‘bet all their money’ on a single outcome.¹⁸ Categorical and probabilistic representations may co-exist in cognition, and it is conceivable that one type of representation might reduce, in various ways, to the other.¹⁹

5 Two kinds of information processing

We assumed at the start that cognition is profitably described by saying it involves representations. In Section 2, we saw that having multiple possible outcomes and a probability distribution over those outcomes is enough to have an associated measure of Shannon information. In both the naturalising projects of Section 3 and the probabilistic modelling projects of Section 4, representations have multiple possible outcomes and probability distributions associated with them. Consequently, in the context of each project – naturalising representational content and ascribing probabilistic content – representations have associated measures of Shannon information. What characterises the Shannon information described in Section 3 is that it is associated with probability of the neural *vehicle* occurring conditional on various environmental outcomes. What characterises the Shannon information

¹⁸Feldman calls these ‘symbolic representations’, but his claim is about their content, not about the syntactic format of their vehicles.

¹⁹Also see Rahnev (2017) for a review of models of cognition that are ‘intermediate’ between categorical representation and representation of full-blown probability distributions.

described in Section 4 is that it is associated with the probabilities represented by the representational *content*.

The degree to which these two ‘types’ of Shannon information differ depends on the degree to which the underlying outcomes and probability distributions differ. In this section, I argue that they typically involve different sets of *outcomes*, different numerical *probability values*, and they must involve different *kinds of probability*.

Different outcomes. In the context of the naturalising project of Section 3, the set of possible outcomes is the set of *possible neural and environmental states*. The outcomes are the objective possibilities – neural and environmental – that could occur. What interests Dretske, Millikan, Shea, Skyrms, and others is to know whether a particular neural state from a set of alternatives (N) occurs conditional on a particular environmental state from a set of alternatives (S).²⁰ In contrast, in the context of the project of Section 4, the outcomes are the *represented possible states of the environment*. These are the ways that the brain represents the environment could be. This set of represented possibilities need not be identical with what is objectively possible. A cognitive system might make be mistaken about what is possible just as it might be mistaken about what is actual: it might represent an environmental outcome that is impossible (e.g., winning a lottery one never entered) or it might fail to represent an environmental state that is possible (e.g., that it is a brain in a vat). Unless the cognitive system represents all and only the objectively possible environmental outcomes, there is no reason to think that its set of represented outcomes will be the same as the set of objectively possible environmental outcomes. Hence, the set of outcomes represented by a neural state need not be the same as the set of outcomes S . Moreover, for the respective sets to coincide, the brain would need not only to represent the objectively possible environmental states but also to represent the objectively possible outcomes regarding its own neural states. Only in the special case of a cognitive system that (a) represents all and only the possible environmental states and (b) represents all and only its possible neural states, would the two sets of outcomes over which probabilities are defined coincide.

Different probability values. Suppose that a cognitive system, perhaps through a quirk of fate, *does* represent all and only the real environmental and neural possibilities. It is worth nothing that even in such a case, the numerical probability *values* associated with each outcome might differ. The probability values that pertain to the naturalising project are the values of the objective chances, frequencies, propensities, or some other measure of a neural state occurring conditional on a possible environmental state. What interests Millikan, Shea, and others are these objective probability values. In contrast, the probability values that pertain to probabilistic

²⁰Or whether an environmental state occurs conditional on some neural state occurring. Each can be exchanged for the other via Bayes’ theorem.

representations are the system's probabilistic guesses about different represented outcomes. These are its *estimation* of how likely each outcome is, not the actual objective probabilities. Brains are ascribed 'priors' – representations of the unconditional probabilities of various environmental outcomes – and a 'likelihood function' or 'generative model' – a probabilistic representation of the relationships between possible outcomes. Psychologists are interested in how the brain uses its priors and likelihood function to make a judgement about unknown events (or in how it modifies those priors in light of new certain knowledge). The aforementioned probabilities are the agent's guesses concerning outcomes and relations between them. Only a God-like cognitive agent – one who knows the truth about the objective unconditional probabilities of events and their probabilistic relations – would assign the right probability values to the various outcomes and relations. Such a system would have a *veridical* (and *complete*) probabilistic representation of its environment, its own neural states, and the relationships between them. This may be a goal to which a rational agent may aspire, but it is surely a position that few of us achieve. There is no reason to think that an ordinary cognitive agent assigns probability values to its represented outcomes with the same numerical value as their corresponding objective probabilities.

Different kinds of probability. Even for a God-like cognitive agent, we would still have two conceptually and logically distinct kinds of probability. For a God-like cognitive agent, we are assuming (for the sake of argument) that the numerical $P(\cdot)$ values and the sets of possible outcomes are identical. But the respective $P(\cdot)$ values still measure different *kinds* of probability. $P(\cdot)$ refers to something different in the two cases. In the case of the naturalising project of Section 3, the $P(\cdot)$ values measure objective probabilities. These may be chances, frequencies, propensities, or whatever else that corresponds to the objective probability of the relevant outcome occurring.²¹ In the case of ascribing probabilistic representation, the $P(\cdot)$ values measure subjective probabilities. These are the cognitive system's representation of how likely the relevant outcomes are to occur. Chances, frequencies, propensities, or similar are not the same as a system's representation of – its guess concerning – how likely an event is to occur. For our God-like cognitive agent, the two may agree in terms of their numerical value and set of possible outcomes, but the two are nonetheless distinct. One quantifies objective probabilities; the other quantifies subjective probabilities. They may happen to align in terms of numerical value, but subjective probabilities do not become objective probabilities merely because they happen to accurately represent them. No than a picture of a Komodo dragon would

²¹Different theorists in Section 4 take different views about the nature of these objective probabilities. Shea (2007) says the probabilities are chances (although he does not say what chances are); Millikan (2000) focuses on the idea that they are frequencies and she considers the consequent reference class problem. No one entertains the hypothesis that they are subjective probabilities.

become a living Komodo dragon if that picture happens to accurately represent one. One is a representation and the other is a state of the world. In the case of our God-like agent, one is a distribution of objective probabilities and the other is the system's (veridical) representation of possible outcomes and its associated estimate of their probabilities. Well-known normative principles connect subjective and objective probabilities. However, no matter which normative principles one believes in, and regardless of whether a God-like agent satisfies them, the two probability distributions are distinct.²²

Two kinds of probability distribution feature in cognition. Each generates an associated measure of Shannon information. The two corresponding Shannon quantities are similarly distinct: they may involve different outcomes, different probability values, and will involve different kinds of probability. This allows us to make sense of two kinds of Shannon information being processed in cognition: two kinds of probability distribution change under probabilistic models of cognition. Cognitive processing involves changes in a system's representational vehicles and changes in a system's probabilistic represented content. Information-processing algorithms that explain cognitive processing can be defined over either or both of these Shannon quantities.²³

6 Relationship between the two kinds of information

My claim is that the two forms of Shannon information are *conceptually* and *logically* distinct. This does not rule out all manner of interesting connections between them. They might measure different quantities, but that does not mean the two quantities must always vary independently of each other. This section highlights some possible connections between them.

6.1 Connections via semantic theory

One is likely to be persuaded of deep connections between the two kinds of Shannon information if one endorses some form of information-theoretic semantics for

²²Skyrms agrees: 'objective and subjective information' may be carried by a neural state (2010, pp. 44–45). Skyrms' concern is with the objective probabilities that pertain to neural states and environmental states. However, he agrees that subjective probabilities (and hence, subjective information) may be carried by a neural state *qua* content.

²³Rather than saying that there are two 'kinds' of Shannon information, one might say that there are two 'applications' of a single kind of Shannon measure to the brain. Thanks to an anonymous referee for this comment. However, a similar comment could be made about objective and subjective probabilities: they are two 'applications' of a single kind of probability measure. It seems reasonable to say that, to the extent that we are willing to grant that there are two 'kinds' of probability here, we should do the same for Shannon information.

probabilistic representations. The probabilistic models of cognition described in Section 4 are silent about how probabilistic representations get their content. In principle, they are compatible with a range of semantic proposals, including some modified version of the information-theoretic semantics described in Section 3.

Skyrms' or Isaac's theory looks the most promising current theory to adapt for an information-theoretic account of probabilistic content. Both their approaches attribute multiple environmental outcomes plus a graded probabilistic response for each outcome. However, it is not immediately obvious how to connect their probabilistic measures to the notion of subjective probability. The probability distribution represented by n cannot merely be assumed to be the probability distribution of S . As we saw in Section 5, a probabilistic representation may misrepresent the true possibilities and their probability values. A further consideration is that the represented probabilities appear to depend not only on the probabilistic relationship between a single representational vehicle and possible environmental outcomes, but also on what else the system 'believes'. The probability that a system assigns to *there is a face* should not be independent of the probability it assigns to *there is a person*, even if the two outcomes are represented by different neural vehicles. A noteworthy feature of the information-theoretic accounts of Section 3 is that they disregard relationships of probabilistic coherence between representations in assigning representational content. They assign content piecemeal, without considering how the contents may cohere. How to address these two issues and create an information-theoretic semantics for probabilistic representations is presently unclear.²⁴

If an information-theoretic semantics for probabilistic neural representations could be developed, it would provide a bridge between the two Shannon measures. One kind of information (associated with represented subjective probabilities) could not vary independently of the other (associated with the objective probabilities). The two would correlate in the cases to which the semantic theory applied. Moreover, if the semantic theory held as a matter of conceptual or logical necessity, then the correlation between the two Shannon quantities would hold with a similar strength. An information-theoretic account of probabilistic neural representation is therefore likely to illuminate conceptual or logical connections between the two types of Shannon information in the brain. In the absence of such a semantic theory, however, it is hard to speculate on exactly what those connections are likely to be.

If one is sceptical about the prospects of an information-theoretic semantics for probabilistic neural representation, one may be less inclined to see conceptual or logical connections between the two kinds of Shannon information. If one endorses Grice's (1957) theory of non-natural meaning, for example, then the two Shannon

²⁴See Shea (2014b); Shea (2018) for a promising response to these problems.

measures will look conceptually and logically independent. Grice said that in the case of non-natural meaning, representational content depends exclusively on intentions and not, for example, the objective probabilities of a physical vehicle occurring in conjunction with environmental outcomes. On this semantic theory, there is nothing to stop any physical vehicle representing any content, provided it is underwritten by the right intentions. I might say that the proximity of Saturn to the Sun (appropriately normalised) represents the probability that Donald Trump will be impeached. Provided this is underwritten by the right intentions, probabilistic representation occurs. Representation is, in this sense, an arbitrary connection between a vehicle and a content: it can be set up or destroyed at will, without regard for probabilities of the underlying events.²⁵ If one endorses Grice's theory of non-natural meaning, there need be no conceptual or logical connections between the probabilities of neural and environmental states and what those states represent. One Shannon measure could vary independently of the other. This is not to say that the Shannon measures would be uncorrelated in the brain. It is just to say that if they correlate, that correlation does not flow from the semantic theory.

6.2 Connections via empirical correlations

Regardless of connections that may flow from one's semantic theory, there are likely to be other reasons why the two measures will correlate in the brain. The nature of some of these correlations will depend on which strategy the brain uses to 'code' its probabilistic representations. This coding scheme describes how the probabilistic content – probability values, overall shape of the probability distribution, or summary statistics like mean or variance – map onto physical activity of the brain or onto physical relations between the brain and environment. The coding scheme that the brain uses for coding its probabilistic content is currently unknown and the subject of much speculation. Proposals include that the rate of firing of a neuron, the number of neurons firing in a population, the chance of neurons firing in population, or the spatial distribution of neurons firing in a population, is a function of some defining feature of the represented probability distribution (see, for example, Barlow, 1969; Averbeck, Latham and Pouget, 2006; Ma et al., 2006; Deneve, 2008; Fiser et al., 2010; Griffiths, Vul and Sanborn, 2012; Pouget et al., 2013). According to these coding schemes, the probability of various neural states occurring varies in some regular way with their represented probability distribution. This relationship may be straightforward and simple or it may be extremely complex and vary in different parts of the brain. The same qualification applies to the relationship between the two Shannon measures. If an experimentalist were to know the coding scheme of the brain, she would be able to reliably infer one measure from the other. But

²⁵Skyrms argues against this that all meaning is natural meaning. All meaning depends on the physical probabilities that connect vehicles and their content.

irrespective of their empirical connections, the two kinds of Shannon information would remain conceptually and logically distinct, for the reasons given in Section 5.

Cognitive processing in the brain is sometimes defined over the traditional, physical Shannon measure. Saxe, Calderone and Morale (2018) describe how brain entropy during resting state, as measured by fMRI, correlates with general intelligence. Chang et al. (2018) describe how drinking coffee increases brain entropy during resting state. Carhart-Harris et al. (2014) describe the relationship between consciousness and brain entropy, and how this changes after taking the psychedelic psilocybin. Rieke et al. (1999) advocate an entire research programme that examines information-theoretic properties of neural vehicles (spike trains) and their relations to possible environmental outcomes. They argue that Shannon information-theoretic properties of the physical vehicles and environmental outcomes allow us to make inferences about possible and likely computations that the brain uses and the efficiency of its coding scheme. In each of these cases, the Shannon measures are defined over possible neural vehicles and environmental states, not over their represented content (although several of the authors suggest that since the two are correlated via the brain's coding scheme, we can use one to draw conclusions about the other).

In contrast, Feldman (2000) looks at algorithms defined over the information-theoretic properties of the represented content. He argues that the difficulty of learning a new Boolean concept correlates with the information-theoretic complexity of the represented Boolean condition. Kemp (2012) and Piantadosi, Tenenbaum and Goodman (2016) extend the idea to general concept learning. They propose that concept learning is a form of probabilistic inference that seeks to find the represented concept that maximises the probability of the represented data classification. This cognitive process is described as the agent seeking the concept that offers the optimal Shannon compression scheme over the represented data. Gallistel and Wilkes (2016) describe associative learning as a probabilistic inference about the most likely cause of an unconditioned stimulus given the observations. They describe this in terms of Shannon information processing: the cognitive system starts with priors over hypotheses about causes that have maximum entropy (their probability distributions are as 'noisy' as possible consistent with the data); the cognitive system then aims to find the hypotheses that provide optimal compression (that maximise Shannon information) of the represented hypothesis and observed data. In general, theorists move smoothly between probabilistic-inference formulations and information-theoretic formulations when describing a cognitive process. In each of these cases, the Shannon information is associated not with the probabilities of certain neural vehicles occurring, but with the represented probability distributions over which the probabilistic inference is performed (although again, one might think that the two are likely to be related).

6.3 Two versions of the free-energy principle

Friston (2010) claims that the free-energy principle provides a unified theory about how all cognitive and living creatures work, and that various computational proposals, such as those used in predictive coding, flow from the free-energy principle (Clark, 2013). He invokes two kinds of Shannon information processing in two separate versions of his ‘free-energy principle’.

First, Friston says that the free-energy principle is a claim about the probabilistic inference performed by a cognitive system. He claims that the brain aims to predict upcoming sensory activation and it forms probabilistic hypotheses about the world that are updated in light of errors it makes in making this prediction. Shannon information attaches to the represented probability distributions over which the inference is performed. Friston says that the brain aims to minimise the surprisal of – the Shannon information associated with – new sensory evidence. When the brain is engaged in probabilistic inference, however, Friston says that its neural activity does not represent the full posterior probability distributions. Instead, the brain approximates them with simpler probability distributions, assumed to be Gaussian. Provided the brain minimises the Shannon-information quantity ‘variational free energy’ it will bring these simpler probability distributions into approximate correspondence with the true posterior distributions that a perfect Bayesian reasoner would have (Friston, 2009; Friston, 2010). Variational free energy is an information-theoretic quantity, predicated of the agent’s represented probability distributions, that measures how far those subjective probability distributions depart from the optimal guesses of a perfect Bayesian observer. According to Friston, the brain minimises ‘free energy’ and so approximates an ideal Bayesian reasoner.

Friston makes a second, conceptually distinct, claim about cognition (and life generally) aiming to minimise free energy. In this context, the aim is to explain how cognitive (and living) systems maintain their physical integrity and homeostatic balance in the face of a changing physical environment. Cognitive (and living) systems face the problem that their physical entropy tends to increase over time: they generally become more disordered and the chance increases that they will undergo a fatal physical phase transition. Friston says that when living systems resist this tendency, they minimise free energy (Friston, 2013; Friston and Stephan, 2007). However, the free energy minimised is not that which attaches to the represented, probabilistic guesses of some agent. Instead, it attaches to the objective probabilities of various possible (fatal) physical states of the agent occurring in response to environmental changes. Minimising free energy involves the system trying to arrange its internal physical states so as to avoid being overly changed by probable environmental transitions. The system strives to maintain its physical nature in equipoise with likely environmental changes. The information-theoretic free-energy

minimised here is defined over the objective distributions of possible physical states that could occur, not over the probability distributions represented by an agent's hypotheses.

Minimising one free-energy measure may help an agent to minimise the other: a good, Bayesian reasoner is arguably more likely to survive in a changing physical environment than an irrational agent. But they are not the same quantity. Moreover, any correlation between them could conceivably come unstuck. An irrational agent could depart far from Bayesian ideals but be lucky enough to live in an hospitable environment that maintains its physical integrity and homoeostasis no matter how badly the agent updates its beliefs. Alternatively, an agent may be a perfectly rational Bayesian and update its beliefs perfectly, but its physical environment may change so rapidly and catastrophically that it cannot survive or maintain its homoeostasis. Understanding how Friston's two formulations of the free-energy principle interact – that pertaining to represented probabilities and that pertaining to objective probabilities – is ongoing work.²⁶

7 Conclusion

Traditionally, philosophers have invoked Shannon information as a rung on a ladder to naturalise representation. In this context, Shannon information is associated with the outcomes and probability distributions of physical states and environmental states. This project obscures a novel way in which Shannon information enters into cognition. Probabilistic models of cognition treat cognition as an inference over representations of probability distributions. This means that probabilities enter into cognition in two distinct ways: as the objective probabilities of neural vehicles and/or environmental outcomes occurring and as subjective probabilities that describe the agent's expectations. Two types of Shannon information are associated respectively: Shannon information that pertains to the probability of the vehicle occurring and Shannon information that pertains to the represented probabilistic content. The former is conceptually and logically distinct from the latter, just as representational vehicles and their objective probabilities are conceptually and logically distinct from their content and the associated subjective probabilities. Various (conceptual, logical, empirical) relations may connect the two kinds of Shannon information in the brain, just as various relations connect traditional, categorical vehicles and their contents. Care should be taken, however, not to conflate the two. For, as we know for the distinction between categorical vehicles and content, much trouble lies that way.

²⁶Colombo and Wright (2018) draw a similar contrast between the two formulations of the free-energy principle. They describe different versions of the free-energy principle as involving 'epistemic' and 'physical' probabilities.

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Bibliography

- Alink, A., C. M. Schwiedrzik, A. Kohler, W. Singer and L. Muckli (2010). “Stimulus predictability reduces responses in primary visual cortex”. In: *Journal of Neuroscience* 30, pp. 2960–2966.
- Averbeck, B. B., P. E. Latham and A. Pouget (2006). “Neural correlations, population coding and computation”. In: *Nature Reviews Neuroscience* 7, pp. 358–366.
- Bar-Hillel, Y. and R. Carnap (1964). “An outline of a theory of semantic information”. In: *Language and Information*. Reading, MA: Addison-Wesley, pp. 221–274.
- Barlow, H. B. (1969). “Pattern recognition and the responses of sensory neurons”. In: *Annals of the New York Academy of Sciences* 156, pp. 872–881.
- Carhart-Harris, Robin, Robert Leech, Peter Hellyer, Murray Shanahan, Amanda Feilding, Enzo Tagliazucchi, Dante Chialvo and David Nutt (2014). “The entropic brain: A theory of conscious states informed by neuroimaging research with psychedelic drugs”. In: *Frontiers in Human Neuroscience* 8, pp. 1–22.
- Chang, D., D. Song, J. Zhang, Y. Shang, Q. Ge and Z. Wang (2018). “Caffeine caused a widespread increase in brain entropy”. In: *Scientific Reports* 8, p. 2700.
- Clark, A. (2013). “Whatever next? Predictive brains, situated agents, and the future of cognitive science”. In: *Behavioral and Brain Sciences* 36, pp. 181–253.
- Colombo, M. and P. Seriès (2012). “Bayes on the brain—On Bayesian modelling in neuroscience”. In: *The British Journal for the Philosophy of Science* 63, pp. 697–723.
- Colombo, M. and C. Wright (2018). “First principles in the life sciences: The free-energy principle, organicism, and mechanism”. In: *Synthese*. DOI: [10.1007/s11229-018-01932-w](https://doi.org/10.1007/s11229-018-01932-w).
- de Finetti, B. (1990). *Theory of Probability*. Vol. 1. New York, NY: Wiley & Sons.

- Deneve, S. (2008). “Bayesian spiking neurons I: Inference”. In: *Neural Computation* 20, pp. 91–117.
- Dretske, F. (1981). *Knowledge and the Flow of Information*. Cambridge, MA: MIT Press.
- (1983). “Précis of *Knowledge and the Flow of Information*”. In: *Behavioral and Brain Sciences* 6, pp. 55–90.
- (1988). *Explaining Behavior*. Cambridge, MA: MIT Press.
- (1995). *Naturalizing the Mind*. Cambridge, MA: MIT Press.
- Egan, F. (2010). “Computational models: a modest role for content”. In: *Studies in History and Philosophy of Science* 41, pp. 253–259.
- Egner, T., J. M. Monti and C. Summerfield (2010). “Expectation and surprise determine neural population responses in the ventral visual system”. In: *Journal of Neuroscience* 30, pp. 16601–16608.
- Eliasmith, C. (2005a). “A new perspective on representational problems”. In: *Journal of Cognitive Science* 6, pp. 97–123.
- (2005b). “Neurosemantics and categories”. In: *Handbook of Categorization in Cognitive Science*. Ed. by H. Cohen and C. Lefebvre. Amsterdam: Elsevier, pp. 1035–1055.
- Feldman, J. (2000). “Minimization of Boolean complexity in human concept learning”. In: *Nature* 407, pp. 630–633.
- (2012). “Symbolic representation of probabilistic worlds”. In: *Cognition* 123, pp. 61–83.
- Fiser, J., P. Berkes, G. Orbán and M. Lengyel (2010). “Statistically optimal perception and learning: From behavior to neural representations”. In: *Trends in Cognitive Sciences* 14, pp. 119–130.
- Floridi, L. (2011). *The Philosophy of Information*. Oxford: Oxford University Press.
- Friston, K. (2009). “The free-energy principle: a rough guide to the brain?” In: *Trends in Cognitive Sciences* 13, pp. 293–301.
- (2010). “The free-energy principle: A unified brain theory?” In: *Nature Reviews Neuroscience* 11, pp. 127–138.
- (2013). “Life as we know it”. In: *Journal of the Royal Society Interface* 10, p. 20130475.
- Friston, K. and K. E. Stephan (2007). “Free-energy and the brain”. In: *Synthese* 159, pp. 417–458.

- Gallistel, C. R. and J. T. Wilkes (2016). “Minimum description length model selection in associative learning”. In: *Current Opinion in Behavioral Sciences* 11, pp. 8–13.
- Grice, P. (1957). “Meaning”. In: *Philosophical Review* 66, pp. 377–388.
- Griffiths, T. L., N. Chater, C. Kemp, A. Perfors and J. B. Tenenbaum (2010). “Probabilistic models of cognition: exploring representations and inductive biases”. In: *Trends in Cognitive Sciences* 14, pp. 357–364.
- Griffiths, T. L., E. Vul and A. N. Sanborn (2012). “Bridging levels of analysis for probabilistic models of cognition”. In: *Current Directions in Psychological Science* 21, pp. 263–268.
- Gross, C. G. (2007). “Single neuron studies of inferior temporal cortex”. In: *Neuropsychologia* 46, pp. 841–852.
- Isaac, A. M. C. (2017). “The semantics latent in shannon information”. In: *The British Journal for the Philosophy of Science*. DOI: [10.1093/bjps/axx029](https://doi.org/10.1093/bjps/axx029).
- Kanwisher, N., J. McDermott and M. M. Chun (1997). “The fusiform face area: A module in human extrastriate cortex specialized for face perception”. In: *Journal of Neuroscience* 17, pp. 4302–4311.
- Kemp, C. (2012). “Exploring the conceptual universe”. In: *Psychological Review* 119, pp. 685–722.
- Knill, D. C. and A. Pouget (2004). “The Bayesian brain: the role of uncertainty in neural coding and computation”. In: *Trends in Neurosciences* 27, pp. 712–719.
- Logothetis, N. K. and D. L. Sheinberg (1996). “Visual object recognition”. In: *Annual Review of Neuroscience* 19, pp. 577–621.
- Ma, W. J. (2012). “Organizing probabilistic models of perception”. In: *Trends in Cognitive Sciences* 16, pp. 511–518.
- Ma, W. J., J. M. Beck, P. E. Latham and A. Pouget (2006). “Bayesian inference with probabilistic population codes”. In: *Nature Neuroscience* 9, pp. 1432–1438.
- MacKay, D. J. C. (2003). *Information Theory, Inference, and Learning Algorithms*. Cambridge: Cambridge University Press.
- Marr, D. (1982). *Vision*. San Francisco, CA: W. H. Freeman.
- Millikan, R. G. (1984). *Language, Thought and Other Biological Categories*. Cambridge, MA: MIT Press.
- (2000). *On Clear and Confused Ideas*. Cambridge: Cambridge University Press.

- Millikan, R. G. (2001). "What has natural information to do with intentional representation?" In: *Naturalism, Evolution and Mind*. Ed. by D. Walsh. Cambridge: Cambridge University Press, pp. 105–125.
- (2004). *The Varieties of Meaning*. Cambridge, MA: MIT Press.
- Papineau, D. (1987). *Reality and Representation*. Oxford: Blackwell.
- Piantadosi, S. T., J. B. Tenenbaum and N. D. Goodman (2016). "The Logical Primitives of Thought: Empirical Foundations for Compositional Cognitive Models". In: *Psychological Review* 123, pp. 392–424.
- Pouget, A., J. M. Beck, W. J. Ma and P. E. Latham (2013). "Probabilistic brains: Knows and unknowns". In: *Nature Neuroscience* 16, pp. 1170–1178.
- Rahnev, D. (2017). "The case against full probability distributions in perceptual decision making". In: *bioRxiv*. DOI: [10.1101/108944](https://doi.org/10.1101/108944).
- Ramsey, F. P. (1990). *Philosophical Papers*. Ed. by D. H. Mellor. Cambridge: Cambridge University Press.
- Ramsey, W. M. (2016). "Untangling two questions about mental representation". In: *New Ideas in Psychology* 40, pp. 3–12.
- Rieke, F., D. Warland, R. R. van Steveninck and W. Bialek (1999). *Spikes*. Cambridge, MA: MIT Press.
- Saxe, G. N., D. Calderone and L. J. Morale (2018). "Brain entropy and human intelligence: A resting-state fMRI study". In: *PLoS ONE* 13, e0191582.
- Scarantino, A. and G. Piccinini (2010). "Information without truth". In: *Metaphilosophy* 41, pp. 313–330.
- Shea, N. (2007). "Consumers need information: Supplementing teleosemantics with an input condition". In: *Philosophy and Phenomenological Research* 75, pp. 404–435.
- (2014a). "Exploitable isomorphism and structural representation". In: *Proceedings of the Aristotelian Society* 114, pp. 123–144.
- (2014b). "Neural signaling of probabilistic vectors". In: *Philosophy of Science* 81, pp. 902–913.
- (2018). *Representation in Cognitive Science*. Oxford: Oxford University Press.
- Skyrms, B. (2010). *Signals*. Oxford: Oxford University Press.
- Sprevak, M. (2013). "Fictionalism about neural representations". In: *The Monist* 96, pp. 539–560.

- Stegmann, U. E. (2015). "Prospects for probabilistic theories of natural information". In: *Erkenntnis* 80, pp. 869–893.
- Tenenbaum, J. B., C. Kemp, T. L. Griffiths and N. D. Goodman (2011). "How to Grow a Mind: Statistics, Structure, and Abstraction". In: *Science* 331, pp. 1279–1285.
- Timpson, C. G. (2013). *Quantum Information Theory and the Foundations of Quantum Mechanics*. Oxford: Oxford University Press.
- Usher, M. (2001). "A statistical referential theory of content: Using information theory to account for misrepresentation". In: *Mind and Language* 16, pp. 311–334.
- Wiener, N. (1961). *Cybernetics*. 2nd ed. New York, NY: Wiley & Sons.