

Two kinds of information processing in cognition

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What is the relationship between information and representation? Dating back at least to Dretske (1981), an influential answer has been that information is a rung on a ladder that gets one to representation. Representation is information, or representation is information plus some other ingredient. In this paper, I argue that this approach oversimplifies the relationship between information and representation. If one takes current probabilistic models of cognition seriously, information is connected to representation in a new way. It enters as a property of the representational content as well as a property of the vehicles of that content. This offers a new, logically independent way in which information and representation are intertwined in cognitive processing.

1 Introduction

There is a new way in which cognition could be information processing. Philosophers have tended to understand cognition's relationship to Shannon information in just one way. This suited an approach that treated cognition as an inference over representations of single outcomes (*there is a face here, there is a line there, there is a house here*). Recent work conceives of cognition differently. Cognition does not involve an inference over representations of single outcomes but an inference over *probabilistic representations* – representations whose content includes multiple outcomes and their estimated probabilities.

My claim in this paper is that these recent probabilistic models of cognition open up new conceptual and empirical territory for saying that cognition is information

processing. Empirical work is already exploring this territory and researchers are drawing tentative connections between the two kinds of Shannon information in the brain. In this paper, my goal is not to propose a specific relationship between these two quantities of information in the brain, although some possible connections are sketched in Section 6. My goal is to convince you that there *are* two logically and conceptually distinct kinds of Shannon information whose relationship should be studied.

Before we proceed, some assumptions. My focus in this paper is only on Shannon information and its mathematical cognates. I do not consider other ways in which the brain could be said to process information.¹ Second, I assume a representationalist theory of cognition. I take this to mean that cognitive scientists find it *useful* to describe at least some aspects of cognition as involving representation. I focus on the role of Shannon information within two different kinds of representationalist model: categorical models and probabilistic models. My claim is that *if* one accepts a probabilistic model of cognition, then there are two ways in which cognition involves Shannon information. I do not attempt to defend representationalist theories of cognition in general.²

Here is a brief preview of my argument. Under probabilistic models of cognition there are two kinds of probability distribution associated with cognition. First, there is the ‘traditional’ kind: probability distributions associated with a specific neural state occurring in conjunction with an environmental state (for example, the probability of a specific neural state occurring when a subject is presented with a line at 45 degrees in a certain portion of her visual field). Second, there is the new kind associated with probabilistic neural representation: probability distributions that are *represented by* neural states. These probability distributions are the brain’s probabilistic guesses about the possible range of environmental outcomes (say, that the line is at 0, 35, 45, or 90 degrees).³ The two kinds of probability distribution – one associated with *a neural/environmental state occurring* and the other associated with *the neural system’s estimate of certain environmental state occurring* – are conceptually and logically distinct. They have different outcomes, different probability values, and different types of probability (objective and subjective) associated with them.

¹See Floridi (2011).

²Note that I define representationalist theories in terms of their *utility* for describing cognitive processes, not in terms of their *truth*. Some deny truth but accept utility: they endorse some form of instrumentalism about representationalist models in cognitive science (for example, Egan, 2010; Colombo and Seriès, 2012; Sprevak, 2013). On my view, this still falls within the representationalist paradigm. To the extent it is legitimate, even if only on pragmatic grounds, to use a representationalist model of cognition, it is legitimate to say that cognition involves two kinds of information processing.

³This distinction is not that between ‘encoding’ and ‘decoding’ probability distributions (Elia-smith, 2005a). Encoding and decoding distributions are discussed in Section 3.

They generate two separate measures of Shannon information in the brain. The algorithms that underlie cognition can be described as processing *either* or *both* of these Shannon information quantities.

2 Shannon information

Before attributing two kinds of Shannon information to the brain, one first needs to know what justifies attributing *any* kind of Shannon information. Below, I briefly review the definitions of Shannon information in order to identify sufficient conditions for a physical system to be ascribed Shannon information. The rest of the paper shows that these conditions are satisfied in two separate ways by the brain. Definitions in this section are taken from MacKay (2003), although similar points can be made with other formalisms.

In order to define Shannon information, one first needs to define a *probabilistic ensemble*:

- Probabilistic ensemble X is a triple (x, A_X, P_X) , where the outcome x is the value of a random variable, which takes on one of a set of possible values, $A_X = \{a_1, a_2, \dots, a_i, \dots, a_I\}$, having probabilities $P_X = \{p_1, p_2, \dots, p_I\}$, with $P(x = a_i) = p_i$, $p_i \geq 0$, and $\sum_{a_i \in A_X} P(x = a_i) = 1$

A necessary and sufficient condition for being a probabilistic ensemble is the existence of a random variable with *multiple possible outcomes* and an associated *probability distribution*.⁴ If the random variable has a finite number of outcomes, this probability distribution takes the form of a probability mass function, assigning a value, p_i , to each possible outcome. If the random variable has an infinite number of outcomes, the probability distribution takes the form a probability density function, assigning a value, p_i , to the possible outcome falling within a certain range. In either case, multiple possible outcomes and a probability distribution over those outcomes is necessary and sufficient to satisfy the definition of a probabilistic ensemble.⁵

If a physical system has multiple possible outcomes and a probability distribution associated with those outcomes, then that physical system can be described as a probabilistic ensemble. If a neuron has multiple possible outcomes (e.g. firing or not), and a probability distribution over those outcomes (reflecting the chances of it firing), then the neuron can be described as a probabilistic ensemble.

⁴The term ‘outcome’ here is not meant to imply that this is the output of a causal process.

⁵In principle, an ensemble might have only one outcome (necessarily, with probability of 1). As we will see, this corresponds to an ensemble and its single outcome having 0 bits of Shannon information. For information processing to be non-trivial, we need more than one possible outcome.

Shannon information is a continuous scalar quantity measured in bits. Shannon information is predicated of at least three different types of subject: *ensembles*, *outcomes*, and *ordered pairs of ensembles*. The definitions differ, so let us consider each in turn.

The Shannon information, $H(X)$, of an *ensemble* is defined as:

$$H(X) = \sum_i p_i \log_2 \frac{1}{p_i}$$

The only independent variables in the definition of $H(X)$ are the possible outcomes of the ensemble (the *is*) and their probabilities (the p_i s). The Shannon information of an ensemble is thus a mathematical function of, and only of, the essential properties of a probabilistic ensemble. Merely being an ensemble – having multiple possible outcomes and a probability distribution over those outcomes – is enough to define a $H(X)$ measure and bestow a quantity of Shannon information. Therefore, any physical system that is described as a probabilistic ensemble *ipso facto* has an associated measure of Shannon information. If a neuron is described as an ensemble (because it has multiple possible outcomes and a probability distribution over those outcomes), then it has, by that very fact, a quantity of Shannon information attached.

The Shannon information, $h(x)$, of an *outcome* is defined as:

$$h(x = a_i) = \log_2 \frac{1}{p_i}$$

$H(X)$ is the expected value of $h(x)$ across all possible outcomes of ensemble X . The single independent variable in the definition of $h(x)$ is the probability of the outcome in question (p_i). This means that again, the existence of an ensemble is a sufficient condition for satisfying the definition of $h(x)$. If an ensemble exists, then *ipso facto* each of its outcomes has a measure of Shannon information. No further conditions would need to be met. If a neuron is described as an ensemble, each of its possible outcomes (e.g. firing or not firing) has a probability value, and hence each possible outcome has a quantity of Shannon information attached.

There are many Shannon measures of information that are defined for *pairs of ensembles*.⁶ Common ones include:

Joint information:

⁶One member of the pair is usually called the ‘sender’ and the other the ‘receiver’.

$$H(X, Y) = \sum_{x, y \in A_X A_Y} P(x, y) \log_2 \frac{1}{P(x, y)}$$

Conditional information:

$$H(X | Y) = \sum_{y \in A_Y} P(y) \sum_{x \in A_X} P(x | y) \log_2 \frac{1}{P(x | y)}$$

Mutual information:

$$I(X; Y) = H(X) - H(X | Y)$$

These measures differ from each other in important ways, but again a sufficient condition for satisfying any one of them is that a physical system has multiple possible outcomes and a probability distribution over those outcomes. Taken separately, two ensembles X and Y have respective outcomes and probability distributions over those outcomes. The Shannon measures above assume that there is also a (joint) probability distribution, $P(X, Y)$, which describes the probability of a pair of outcomes from the ensembles, x, y , occurring.⁷ No more than this is required to define the pairwise measures above. If ensembles X and Y exist, and if pairs of their respective outcomes have probabilities (even if some of these probabilities are 0), then the Shannon measures of joint information, conditional information, and mutual information are defined. Consequently, if two neurons are described as two ensembles, and if there is a joint probability distribution over their respective possible outcomes, then those neurons have associated measures of joint information, conditional information, and mutual information.

A sufficient condition for a physical system to be ascribed Shannon information is that it *has multiple possible outcomes and a probability distribution over those outcomes*. The Shannon information of an ensemble, a single outcome, or a pair of ensembles, is a function of, and only of, the possible outcomes and probability distribution associated with that ensemble, single outcome, or pair. If a physical system is described as an ensemble (or a pair of ensembles whose joint outcomes have a probability distribution), it *ipso facto* has Shannon information attached.

If a physical system changes the probability associated with its possible outcomes over time, its associated Shannon measures are likely to change too. Such a system may be described as ‘processing’ Shannon information. This change could happen in at least two distinct ways. If a physical system modifies the probabilities associated

⁷ $P(X, Y)$ also defines any conditional probability measures, such as $P(X | Y)$.

with its *physical states* occurring (e.g. a neuron makes certain physical states such as firing more or less likely), it can be described as processing Shannon information.⁸ Alternatively, if the firing activity of the neuron *represents* a probability distribution over possible outcomes, and that represented probability distribution changes over time – perhaps as a result of learning or inference – then the neuron’s associated Shannon measures will change too. In both cases, probability distributions and Shannon information change. But distinct probability distributions and distinct measures of Shannon information change in each case. The remainder of this paper will unpack the distinction between the two.

3 The traditional kind of Shannon information

Traditionally, an important role of Shannon information has been to function as a building block in the project of *naturalising representation*. Many versions of information-theoretic semantics try to explain semantic content in terms of Shannon information. These accounts aim to explain how representation, and in particular mental representation, arises from Shannon information-theoretic relations. Such theories often claim that Shannon information is a source of naturalistic, objective facts about representation. Dretske formulated one of the earliest such theories.⁹ Dretske’s (1981) theory aimed to reduce facts about representational content entirely to facts about Shannon information. More recently, other accounts – including Dretske’s later (1988; 1995) views – propose that an information-theoretic condition is only one part of a larger condition on representational content. Additional conditions include variously demands regarding teleology, instrumental (reward-guided) learning, structural isomorphism, and/or appropriate use.¹⁰ In what follows, I will focus solely on the information-theoretic part of such a semantic theory.

Information-theoretic semantics attempts to explain representation in terms of one physical state ‘carrying information’ about another physical state. The relationship

⁸One way in which this could occur is during learning and other kinds of plasticity that potentiate or inhibit certain possible neural firings over the long term. But similar changes also occur during short-term processes. When a neuron fires, it makes a specific outcome – *firing* – certain. That will almost certainly affect the probabilities associated with other neurons in the brain (making their respective outcomes of firing more or less probable), and hence change their associated Shannon measures. Neuroscientists can track how these Shannon measures change as a specific outcome propagates in the brain during cognitive processing. Thanks to Nick Shea for this point.

⁹Prior to Dretske’s work, Shannon information had been linked to semantic content, although not always in reductive fashion (Bar-Hillel and Carnap, 1964; Wiener, 1961).

¹⁰See Millikan (1984); Papineau (1987); Dretske (1988); Shea (2007); Shea (2014a); Skyrms (2010); W. M. Ramsey (2016) for a range of such proposals. Note that some of these authors argue that mental representations sometimes gain their content solely on the basis of non-Shannon factors. Thanks to an anonymous referee for pointing this out.

of ‘carrying information’ is assumed to be a precursor to, or a precondition for, certain varieties of representation. In the context of the brain, such a theory says:

- (R) Neural state, n (from N), represents an environmental state, s (from S) only if n ‘carries information’ about s .

Implicit in R is the idea that neural state, n , and environmental state, s , come from a set of possible alternatives. According to R, neural state n represents s only if n bears the ‘carrying information’ relation to s and not to other outcomes. Different neural states could occur in the brain (e.g. different neurons in a population might fire). Different environmental states could occur (e.g. a face or a house could be present). Crudely, the reason why certain neural firings represent a face and not a house is that those firings, and only those firings, bear the ‘carrying information’ relationship to *face* outcomes, and the relevant firings do not bear that relationship to *house* outcomes. R assumes that we are dealing with multiple possible outcomes: multiple possible representational vehicles (N) and multiple possible environmental states (S). It tries to identify a special relationship between individual outcomes that gives rise to representation. Representation occurs only when n from N bears an ‘carrying information’ relation to s from S .

The primary task for an information-theoretic semantics is to explain what this ‘carrying information’ relation is. Different versions of information-theoretic semantics do this differently.¹¹ Theories can be divided roughly into two camps: those that are ‘correlational’ and those that invoke ‘mutual information’.

The starting point of ‘correlational’ theories is that one physical state ‘carries information’ about another just in case there is a statistical correlation between the two that satisfies some probabilistic condition. This still leaves plenty of questions unanswered: What kind of statistical correlation (Pearson, Spearman, Kendall, mutual information, or something else?)¹² How should the physical states be typed such that a correlation can be measured? How much correlation is enough to set up an information-carrying relation? Does it matter if the correlation is accidental or underwritten by a law or disposition? Rival information-theoretic semantics take different views. Consider the following three proposals:

¹¹The relation of ‘carrying information’ is also sometimes described as one physical state ‘having natural information’ about another; see Stegmann (2015).

¹²Millikan (2001) suggests that one should look at the probabilistic relations that are ‘learnable’ for an agent: A is correlated with B, and hence carries information, if B is learnable (or inferable) from A. However, any degree of probabilistic dependence between A and B (no matter how slight) could, in principle, allow an agent to learn, or infer, one from the other. With suitable rewards, even the mildest degree of probabilistic dependence *could* be a target of learning as an agent could be arbitrarily incentivised to do so. The notion of a ‘learnable’ relation – if it is not merely a synonym for *not probabilistically independent* – is as much in need of explication as the notion of ‘correlation’.

1. $P(S = s \mid N = n) = 1$
2. $P(S = s \mid N = n)$ is ‘high’
3. $P(S = s \mid N = n) > P(S = s \mid N \neq n)$

Dretske (1981) endorses (1): given that the neural state occurs, the probability of the environmental state occurring should be 1. Millikan (2000; 2004) endorses (2): the conditional probability of the environmental state, given the neural state, need only be ‘high’, where what counts as ‘high’ is a complex matter involving the correlation having influenced past agential use via genetic selection or learning.¹³ Shea (2007) and Scarantino and Piccinini (2010) propose that the correlation should be understood in terms of probability raising, (3): the neural state should make the occurrence of the environmental state more probable than it would have been otherwise.

At first glance, there may seem nothing particularly Shannon-like about proposals (1)–(3). Probability theory alone is sufficient to express the conditions necessary for representation without enrichment from Shannon’s concepts. These semantic theories are perhaps better termed ‘probabilistic’ semantics than ‘information-theoretic’ semantics.¹⁴ Nevertheless, there is a legitimate way in which these accounts do show that cognition is Shannon information processing. According to (1)–(3), ‘carrying information’ is a relation between particular outcomes and those outcomes must come from ensembles that have probability distributions. Remember that a sufficient condition for a system to have Shannon information is that it *has multiple possible outcomes and a probability distribution over those outcomes*. (1)–(3) assure us that this condition is true of a cognitive system that contains representations. According to (1)–(3), the representational content of a neural state arises when that neural state is an outcome from an ensemble with other possible outcomes (other possible neural states) that could occur with certain probabilities (and probabilities conditional on various possible environmental outcomes). If cognition involves representation, and those representations gain their content by any of (1)–(3), then cognition *ipso facto* involves Shannon information. Shannon information attaches to representations because of the probabilistic nature of their vehicles. According to (1)–(3), that probabilistic nature is essential to their representational status. Consequently, to the extent that cognition can be described as processing representations, and to the extent that we accept one of these versions of information-theoretic semantics, cognition can be described as processing states with a probabilistic nature, and by that fact, processing states with Shannon information.

‘Mutual information’ versions of information-theoretic semantics unpack ‘carrying

¹³See Stegmann (2015), pp. 873–874 for helpful analysis of Millikan’s view.

¹⁴Timpson (2013), pp. 41–42 makes a similar point with regard to Dretske’s (1981) theory, and related criticisms are raised by commentators in Dretske (1983).

information' differently. They invoke the Shannon concept of mutual information – or rather, pointwise mutual information, the analogue of mutual information for pairs of single outcomes from two ensembles. Mutual information $I(X; Y)$ is the expected value of pointwise mutual information $PMI(x, y)$ over all joint outcomes of a pair of ensembles.¹⁵ Pointwise mutual information for a pair of single outcomes, x, y , from two ensembles is defined as:

$$PMI(x, y) = \log_2 \frac{P(x, y)}{P(x)P(y)} = \log_2 \frac{P(x | y)}{P(x)} = \log_2 \frac{P(y | x)}{P(y)}$$

Skyrms (2010) and Isaac (2017) propose that the information carried by a physical state, n , (its 'informational content') is a vector consisting of the $PMI(n, s)$ value of every possible environmental state, s_i , from S , for that specific n from N : $\langle PMI(n, s_1), \dots, PMI(n, s_n) \rangle$. Isaac identifies the meaning or representational content of n with this PMI -vector. Skyrms says that the meaning/content is likely to be a more traditional semantic object, such as a set of possible worlds. That set is derived from the PMI -vector by finding the environmental states that generate high-value elements in the vector. The representational content of n is the set of possible worlds in which high- PMI -value environmental states occur.

Like Skyrms and Isaac, Usher (2001) and Eliasmith (2005b) appeal to pointwise mutual information to define 'carrying information'. Unlike Skyrms and Isaac, they define 'carrying information' as a relation between a single neural state, n , and a single environmental state, s . They say that n carries information about s just in case s is the environmental state for which $PMI(n, s)$ has its maximum value given that neural state n . Neural state, n , carries information about the s that produces the peak-value element in its PMI -vector. Usher and Eliasmith connect this quantity to what is measured in 'encoding' experiments in neuroscience. In an encoding experiment, many environmental states are presented to a brain and researchers look for the environmental state that best predicts a specific neural response – that maximises $PMI(n, s)$ as one varies s for some fixed n . Usher and Eliasmith also offer a second, complementary definition of 'carrying information'. This is based around what is measured in 'decoding' experiments. In a decoding experiment, researchers examine many neural states and classify them based on which one best predicts an environmental state – i.e. which neural state n maximises $PMI(n, s)$ for a given s . Here, instead of looking for the highest $PMI(n, s)$ value as one varies s and keeps n fixed, one looks for the highest $PMI(n, s)$ value as one varies n and keeps s fixed. There is no reason why the results of encoding and decoding experiments should coincide: they might pick out two entirely different sets of information carrying relations between the brain and its environment. Usher and Eliasmith argue that

¹⁵ $I(X; Y) = \sum_{x, y \in A_X, A_Y} P(x, y) PMI(x, y)$

they provide different, complementary, and equally valid, Shannon information-theoretic accounts of representational content.

On each of these semantic theories, Shannon information arises in a cognitive system because of the probabilistic properties of neural states *qua representational vehicles*. It is because a given neural state is an outcome from a set of possible alternative states, combined with the probability of various environmental outcomes, that the cognitive system has the Shannon information-theoretic properties relevant to representation and hence to cognition. In the next section, I describe a different way in which Shannon information enters cognition. Here, the relevant Shannon quantity arises, not from the probabilistic nature of the physical vehicles and environmental states, but from the *representational content*. ‘Probabilistic’ models of cognition claim that the representational content of a neural state is probabilistic. This means that Shannon information is associated with a cognitive system in a new way: via its content rather than via the probabilistic nature of its neural vehicles.

4 The new kind of Shannon information

Probabilistic models of cognition, like the representationalist accounts discussed in the previous section, attribute representations to the brain. Unlike those accounts however, these models do not aim to naturalise representational content. They *help themselves* to representational content. Their claim is that neural representations have a particular kind of content. They are largely silent about *how* these representations get this content. In principle, probabilistic models of cognition are compatible with a variety of semantic theories, including certain versions of information-theoretic semantics.¹⁶

The central claim of any probabilistic model of cognition is that neural representations have probabilistic content. In contrast, ‘categorical’ approaches to representation assume that neural representations have *single outcomes* as their representational content. A neural state, *n*, represents a single environmental outcome (or a single set of outcomes). Thinking about neural representation in categorical terms has prompted description of neural states early in V1 as *edge detectors*: their activity represents the presence (or absence) of an edge at a particular angle in a portion of the visual field. The represented content is a particular outcome (*edge at ~45 degrees*). Similarly, neurons in the inferior temporal (IT) cortex are described as *hand detectors*: their activity represents the presence (or absence) of a hand. The represented content is a single outcome (*hand present*). Similarly, neurons in the fusiform face area (FFA) are described as *face detectors*: their activity represents the

¹⁶I discuss how information-theoretic semantics might interact with probabilistic models of cognition in Section 6.

presence (or absence) of a face. The represented content is a single outcome (*face present*) (for example, see Gross, 2007; Logothetis and Sheinberg, 1996; Kanwisher, McDermott and Chun, 1997).

There is increasing suspicion that representation in the brain is not like this. Represented content is rarely a single environmental state (e.g. *hand present*); rather, what is represented is a probability distribution over many possible states. The brain represents many outcomes simultaneously to ‘hedge its bets’ during cognitive processing. This allows the brain to store, and make use of, information about multiple rival outcomes if it is uncertain which is the true outcome. Uncertainty may come from unreliability in the perceptual hardware, or from the brain’s epistemic situation that even with perfectly functioning hardware it only has incomplete access to its environment.

Ascribing probabilistic representations to a cognitive agent is not in itself a new idea (de Finetti, 1990; F. P. Ramsey, 1990). However, there is an important difference between past approaches and new probabilistic models. In the past, probabilistic representations were treated as *personal-level* states of a cognitive agent – ‘credences’, ‘degrees of belief’, or ‘personal probabilities’. In new probabilistic models of cognition, probabilistic representations are treated as states of subpersonal *parts* of the agent – of neural populations, or single neurons. The claim is that, regardless of whichever personal-level states are attributed to an agent, various parts of that agent token diverse (and perhaps even conflicting) probabilistic representations. Thinking in these terms has prompted redescription of neural states early in V1 as probabilistically nuanced ‘hypotheses’, ‘guesses’, or ‘expectations’ about edges. Their neural activity does not represent a single state of affairs (*edge at ~45 degrees*) but a probability distribution over multiple edge orientations (Alink et al., 2010). The represented content is a probability distribution over how the environment stands with respect to edges. Similarly, neural activity in the IT cortex does not represent a single state of affairs (*hand present*) but a probability distribution over multiple possible outcomes regarding hands. The represented content is a probability distribution over how the environment stands with respect to hands. Similarly, neural activity in FFA does not represent a single state of affairs (*face present*) but a probability distribution over multiple possible outcomes regarding faces. The represented content is a probability distribution over how the environment stands with respect to faces (Egner, Monti and Summerfield, 2010).

Traditional models of cognition tend to describe cognitive processing as a computational inference over specific outcomes – *if there is an edge here, then that is an object boundary*. Probabilistic models of cognition describe cognitive processing as a computational inference over probability distributions – *if the probability distribution of edge orientations is this, then the probability distribution of object boundaries is*

that. Cognitive processing is a series of computational steps that use one probability distribution to condition, or update, another probability distribution.¹⁷ Neural representations may maintain their probabilistic character right until the moment that the brain is forced to plump for a specific outcome in action. At that point, the brain may select the most probable outcome from its current represented probability distribution conditioned on all its available evidence (or some other point estimate that is easier to compute).

Modelling cognition as probabilistic inference does not mean modelling cognition as non-deterministic or chancy. The physical hardware and algorithms underlying the probabilistic inference may be entirely deterministic. Consider that when your electronic PC filters spam messages from incoming emails, it performs a probabilistic inference, but both the PC's physical hardware and the algorithm that the PC follows are entirely deterministic. A probabilistic inference takes representations of probability distributions as input, yields representations of probability distributions as output, and transforms input to output based on rules of valid (or pragmatically efficacious) probabilistic inference. The physical mechanism and the algorithm for processing representations may be entirely deterministic. What makes the process probabilistic is not the chancy nature of physical vehicles or abstract rules but that probabilities feature in the represented content that is being manipulated.

Perhaps the best known current example of a probabilistic model of cognition is the 'Bayesian brain' hypothesis. This says that brains use probabilistic representations and process them according to rules of Bayesian or approximately Bayesian inference (Knill and Pouget, 2004). Predictive coding provides one proposal about how the Bayesian brain hypothesis could be neurally implemented (Clark, 2013; Friston, 2009). It is worth stressing that the motivation for ascribing probabilistic representations, and for probabilistic models of cognition in general, is much broader than that for the Bayesian brain hypothesis (or for predictive coding). The brain's inferential rules could, in principle, depart very far from Bayesianism and still produce adaptive behaviour under many circumstances. It remains an open question to what extent humans are Bayesian (or approximately Bayesian) reasoners. Probabilistic techniques developed in AI, such as deep learning, reinforcement learning, and generative adversarial models, can produce impressively adaptive behaviour despite having complex and qualified relationships to Bayesian inference. The idea that cognition is a form of probabilistic inference is a much more general idea than that cognition is Bayesian. A researcher in cognitive science may subscribe to probabilistic representation even if they take a dim view of the Bayesian brain

¹⁷Here, conditional probabilities tell the cognitive system how it to update its estimate of an unknown variable based on its knowledge about known variables.

hypothesis.¹⁸

The essential difference between a categorical representation and a probabilistic one lies in their content. Categorical representations aim to represent a single state of affairs. In Section 3, we saw that schema R treats representation as a relation between a neural state, n , and an environmental outcome, s . The representational content is specified by a truth, accuracy, or satisfaction condition. Meeting that condition is assumed to be largely an all-or-nothing matter. A categorical representation effectively ‘bets all its money’ that a certain outcome occurs. An edge detector says *there is an edge*. Multiple states of affairs may sometimes feature inside the representational content (for example, *there is an edge between ~43–47 degrees*), but those states of affairs are grouped together into a single outcome that is represented as true. There is no probabilistic nuance, or apportioning of different degrees of uncertainty, to different outcomes.

In contrast, probabilistic representations aim to represent a probability distribution over multiple outcomes. The probability distribution is a measure of how much the system ‘expects’ that the relevant outcomes are true. Unlike with categorical representations, the represented content does not partition the possible environmental states into only two classes (*true* and *false*). Representation is not an all-or-nothing matter; it involves assigning a weighted probability to various possible outcomes. As we will see, these outcomes need not even coincide with the possible outcomes of S . Whereas representational content is specified by a truth, accuracy, or satisfaction condition for a categorical representation, the content of a probabilistic representation is specified by a probability mass or density function over a set of possible outcomes.

Probabilistic representations could, in principle, use any physical or formal vehicle to carry their content. There is nothing about the physical make-up of a representational vehicle that determines whether it is categorical or probabilistic. Either type could also, in principle, use any number of different representational formats as the formal structure over which its algorithms operate. Possible formal structures include being a setting of weights in a neural network, a symbolic expression, a directed graph, a ring, a tree, a region in continuous space, or an entry in a relational database (Griffiths, Chater et al., 2010; Tenenbaum et al., 2011). Certain physical vehicles and certain formal structures are more apt to serve certain computations rather than others. The choice of physical vehicle and representational format affects how easy it is to mechanise an inference with computation (Marr, 1982). But in principle, there is nothing about the physical make-up or formal structure that determines whether a representation is categorical or probabilistic. That is determined solely by its represented content.

¹⁸See Ma (2012).

The preceding discussion should not be taken as suggesting that a model of cognition can only employ one type of representation (categorical or probabilistic). There is no reason why both types of representation cannot appear in a model of cognition, assuming there are appropriate rules to take the cognitive system between the two. The preceding discussion should also not be taken as suggesting that one type of representation cannot be reduced to the other. A wide range of such reductions may be possible. For example, a cognitive system might use suitably structured complexes of traditional representations to express the probability calculus and thereby express probabilistically nuanced content (maybe this is what we do with the public language of mathematical probability theory). Conversely, a cognitive system might use suitably structured complexes of probabilistic representations to express all-or-nothing-like truth conditions. Feldman (2012) describes a cognitive system in which categorical representations are approximated by probabilistic ones that represent strongly modal (sharply peaked) probability distributions.¹⁹ Categorical representations may reduce to probabilistic representations that assign all their probability mass to a single outcome. Categorical and probabilistic representations may mix in cognition, and perhaps one sometimes, under the right conditions, gives rise to the other.²⁰

5 Two kinds of information processing

In Section 1, we assumed that cognition is profitably described by saying it involves representations. In Section 2, we saw that having multiple outcomes and a probability distribution over those outcomes is sufficient to have an associated measure of Shannon information. We have now seen, in Sections 3 and 4, two ways in which representations in cognition have multiple outcomes and probability distributions associated with them. Consequently, Shannon information attaches to those representations in two distinct ways. What characterises the Shannon information described in Section 3 is that it is associated with probability of the neural *vehicle* occurring (conditional on various environmental outcomes). What characterises the Shannon information described in Section 4 is that it is associated with the probabilities that appear inside represented *content*.

The degree to which these two quantities of Shannon information differ depends on the degree to which the two sets of outcomes and probability distributions differ. In this section, I argue that they typically involve different sets of *outcomes*, different numerical *probability values*, and they must involve different *kinds of probability*.

¹⁹Feldman calls these ‘symbolic representations’, but his claim is about their content, not about the representational format of their vehicles.

²⁰Also see Rahnev (2017) for models of cognition that are ‘intermediate’ between categorical and probabilistic representation.

Different outcomes. In Section 3, the set of possible outcomes is the set of *possible neural and environmental states*. The outcomes are the objective possibilities – neural and environmental – that could occur. What interests Dretske, Millikan, Shea, Skyrms, and others is to know whether a particular neural state from a set of alternatives (N) occurs conditional on a particular environmental state from a set of alternatives (S).²¹ In contrast, in Section 4, the outcomes are the *represented possible states of the environment*. These are the ways that the brain represents the environment could be. The set of represented possibilities need not be the same as what is objectively possible. A cognitive system might make a mistake about what is possible just as it might make a mistake about what is actual: it might represent an environmental outcome that is impossible (e.g. winning a lottery it never entered) or it might fail to represent an environmental state that is possible (e.g. that it is a brain in a vat). Unless the cognitive system represents all and only the genuinely possible outcomes, there is no reason to think that its set of represented outcomes will be the same as the set of objectively possible outcomes. Hence, the set of outcomes represented by a neural state need not be the same as the set of outcomes S . Moreover, for the two sets of outcomes to be the same, the brain would need to represent not only the possible environmental states (S) but also its possible neural states (N). Only in the special case of a cognitive system that (a) represents all and only the possible environmental states and (b) represents all and only the possible neural states, would the respective sets of outcomes coincide.

Different probability values. Suppose that a cognitive system, perhaps due to some quirk of its design, does represent all and only the environmental and neural possibilities. In such a case, the numerical probability values associated with outcomes are still likely to differ. In the context of the naturalising projects of Section 3, these values measure the objective chances, frequencies, propensities, or some other measure of a neural state occurring conditional on a possible environmental state. What interests Millikan, Shea, and others are probabilistic relations between neural states and environment states. In contrast, in the context of the modelling projects of Section 4, the probability values are the cognitive system's *estimation* of how likely each outcome is, not the actual objective probabilities. Brains are described as having 'priors' – probabilistic representations of various outcomes – and a 'likelihood function' or 'probabilistic generative model' – a probabilistic representation of the relationships between outcomes. Psychologists are interested in how the brain uses its priors and generative model to make inferences about unknown events, or in how it updates its priors in light of new evidence. All the aforementioned probabilities are subjective probabilities: the brain's guesses about the possible outcomes and relationships between them. Only a God-like cognitive agent – one who knows

²¹Or whether an environmental state occurs conditional on some neural state occurring. Each can be exchanged for the other via Bayes' theorem.

the truth about the objective probabilities of events and their relations – would assign the right probability values to the various outcomes and relationships. Such a system would have a *veridical* (and *complete*) probabilistic representation of its environment, its own neural states, and the relationships between them. This may be a goal to which a cognitive system might aspire, but it is surely a position that few of us achieve. There is no reason to think that a typical cognitive system would assign numerical probability values to outcomes with the same weight as their corresponding objective probabilities.

Different kinds of probability. Assume, for the sake of argument, that we are dealing with a God-like cognitive agent like the one described above. Even for this unusual agent, there are still two distinct types of Shannon information. The reason is that the respective $P(\cdot)$ values, even if they numerically agree, measure different *kinds* of probability. The $P(\cdot)$ s mean different things in each case. In the context of the naturalising projects of Section 3, the $P(\cdot)$ values measure objective probabilities. These may be chances, frequencies, propensities, or whatever else that corresponds to the objective probability of the relevant outcome occurring.²² In the context of the modelling projects of Section 4, the $P(\cdot)$ values measure subjective probabilities. These are the system's representation of how likely it thinks are the relevant outcomes. Chances, frequencies, propensities, or similar are not the same as a system's representation of how likely an event is to occur. Even for a God-like cognitive agent – for whom the two are equal in terms of their numerical values – the measures are nevertheless distinct. One concerns one type of probability, the other concerns another. Subjective probabilities might agree in terms of numerical value with objective probabilities, but subjective probabilities do not become objective probabilities merely because they happen to accurately reflect them. No than a picture of a Komodo dragon becomes a living, breathing Komodo dragon if that picture happens to be accurate. One is a representation, the other is a state of the world. In the case of our God-like agent, one is a distribution of objective probabilities and the other is the system's (veridical) representation of possible outcomes and estimation of their respective likelihood. Well-known normative principles connect subjective and objective probabilities. However, no matter which normative principles one believes in, and regardless of whether a God-like agent satisfies them, the two kinds of probability are distinct.²³

²²Different theorists in Section 4 take different views about the nature of these objective probabilities. Shea (2007) says the probabilities are chances (although he does not say what chances are); Millikan (2000) focuses on the idea that they are frequencies and she considers the consequent reference class problem. No one entertains the hypothesis that they are subjective probabilities.

²³Skyrms agrees: 'objective and subjective information' may be carried by a neural state (2010, pp. 44–45). Skyrms' concern is with the objective probabilities that are associated with neural states and environmental states. However, he agrees that subjective probabilities (and hence, subjective information) may be carried by a neural state *qua* content.

Two kinds of probability distribution feature in cognition. Each generates an associated measure of Shannon information. The two corresponding Shannon quantities are distinct: they may involve different outcomes, different probability values, and will involve different kinds of probability. This allows us to make sense of two kinds of Shannon information being processed in cognition: two kinds of probability distribution change under probabilistic models of cognition. Cognitive processing involves changes in a system's representational vehicles and changes in a system's probabilistic represented content. Information-processing algorithms that govern cognition can be defined over either or both of these Shannon quantities.²⁴

6 Relationship between the two kinds of information

My claim in the previous section was that the two kinds of Shannon information are *conceptually* and *logically* distinct. I wish to stress that this does not rule out all manner of interesting connections between them. Even if they are different quantities, that does not mean that they must vary independently of each other. This section highlights some possible connections between them.

6.1 Connections via semantic theory

One is likely to be persuaded of connections between the two kinds of Shannon information if one endorses some form of information-theoretic semantics for probabilistic representations. The probabilistic models described in Section 4 are silent about how their representations get their content. In principle, these probabilistic models are compatible with a range of semantic proposals, including some version of the information-theoretic semantics described in Section 3.

Skyrms' or Isaac's theory looks the most promising approach to adapt for an information-theoretic account of probabilistic content. Both their theories attribute as content multiple environmental outcomes plus a graded probabilistic response for each outcome. However, it is not immediately obvious how to connect their probabilistic measures to the notion of subjective probability. The probability distribution represented by n cannot simply be assumed to be the probability distribution of S . As we saw in Section 5, a probabilistic representation may misrepresent the objective possibilities and their probability values. A further

²⁴One might object that there are not two *kinds* of Shannon information, but only two 'applications' of a single kind of Shannon information measure to the brain. However, the same remark could be made about objective and subjective probabilities: both are applications of a single kind of mathematical probability measure (to measure objective chances and agents' uncertainties). To the extent that we are willing to say that there are two 'kinds' of probability here (objective and subjective), we should do the same for Shannon information.

consideration is that the represented probabilities appear to depend, not only on the probabilistic relations between a representational vehicle and its corresponding environmental outcomes, but also on what else the system ‘believes’. The probability that a system assigns to *there is a face* should not be independent of the probability that it assigns to *there is a person*, even if the two outcomes are represented by different neural vehicles. A noteworthy feature of the information-theoretic accounts of Section 3 is that they disregard relationships of probabilistic coherence between representations in assigning representational content. They assign content piecemeal, without considering how the contents may cohere. How to address these two issues and create an information-theoretic semantics for probabilistic representations is presently unclear.²⁵

If an information-theoretic semantics for probabilistic neural representations *could* be developed, it would provide a bridge between the two kinds of Shannon information. One kind of information (associated with the represented probabilities) could not vary independently of the other (associated with the objective probabilities). The two would correlate at least for the cases to which this semantic theory applied. Moreover, if the semantic theory held as a matter of conceptual or logical truth, then the correlation between the two Shannon quantities would hold with a similar strength. Such an information-theoretic account of probabilistic neural representation would illuminate deep connections between the two types of Shannon information in the brain. In the absence of such a semantic theory, however, it is hard to speculate on exactly what those connections are likely to be.

If one is sceptical about the prospects of an information-theoretic semantics for probabilistic neural representation, then one may be less inclined to see deep conceptual or logical connections between the two kinds of Shannon information. If one endorses Grice’s (1957) theory of *non-natural* meaning, for example, then the two Shannon measures will look conceptually and logically independent. Grice said that in cases of non-natural meaning, representational content depends on human intentions and not, for example, on the objective probabilities of a physical vehicle occurring in conjunction with environmental outcomes. There is nothing to stop any physical vehicle representing any content, provided it is underwritten by the right intentions. I might say that the proximity of Saturn to the Sun (appropriately normalised) represents the probability that Donald Trump will be impeached. Provided this is underwritten by the right intentions, probabilistic representation occurs. Representation is, in this sense, an arbitrary connection between a vehicle and a content: it can be set up or destroyed at will, without regard

²⁵See Shea (2014b); Shea (2018) for a promising approach.

for the probabilities of the underlying events.²⁶ If one endorses Grice's theory of non-natural meaning, there need be no conceptual or logical connections between the probabilities of neural and environmental states and what those states represent. One Shannon measure could vary independently of the other. This is not to say that the two measures would not be correlated in the brain; it is just to say that if they correlate, that correlation does not flow from the semantic theory.

6.2 Connections via empirical correlations

Regardless of connections that may flow from one's semantic theory, there are likely to be other reasons why the two measures of Shannon information will correlate in the brain. The nature of these connections will depend on the strategy the brain uses to 'code' its probabilistic representations. This coding scheme describes how probabilistic content – which may consist of probability values, overall shape of the probability distribution, or summary statistics like the mean or variance – maps onto physical activity in the brain or onto physical relations between the brain and environment. The specific scheme that the brain uses to code its probabilistic content is currently unknown and the subject of much speculation. Suggested proposals include that the rate of firing of a neuron, the number of neurons firing in a population, the chance of neurons firing in population, or the spatial distribution of neurons firing in a population, is a monotonic function of characteristic features of the represented probability distribution (see, for example, Barlow, 1969; Averbeck, Latham and Pouget, 2006; Ma et al., 2006; Deneve, 2008; Fiser et al., 2010; Griffiths, Vul and Sanborn, 2012; Pouget et al., 2013). According to these schemes, the probability of various neural states occurring varies in some regular way with their represented probability distributions. This relationship may be straightforward and simple or it may be extremely complicated and vary in different parts of the brain. The same applies to the relationship between the two Shannon quantities. If an experimentalist were to know the brain's coding scheme, she would likely be able to infer one Shannon measure from the other. But even granted this is possible, the two kinds of Shannon information would remain conceptually and logically distinct, for the reasons given in Section 5.

Cognitive processing is sometimes defined over the information-theoretic properties of neural vehicles. Saxe, Calderone and Morale (2018) describe how brain entropy during resting state, as measured by fMRI, correlates with general intelligence. Chang et al. (2018) describe how drinking coffee increases the brain's entropy during resting state. Carhart-Harris et al. (2014) describe the relationship between consciousness and brain entropy, and how this changes after taking the psychedelic

²⁶Skyrms (2010) argues against this that 'all meaning is natural meaning' (p. 1). All meaning depends on the physical probabilities that connect vehicles and their content.

drug, psilocybin. Rieke et al. (1999) advocate an entire research programme that examines information-theoretic properties of neural vehicles (spike trains) and their relationships to possible environmental outcomes. They argue that Shannon information-theoretic properties of the neural vehicles and environmental outcomes allow us to infer possible and likely computations that the brain uses and the efficiency of the brain's coding scheme. In each of these cases, the Shannon measures are defined over possible neural vehicles and environmental states, not over their represented content (although several of the authors suggest that since the two are correlated by the brain's coding scheme, we can use one to draw conclusions about the other).

In contrast, Feldman (2000) looks at algorithms defined over the information-theoretic properties of the represented content. He argues that the difficulty of learning a new Boolean concept correlates with the information-theoretic complexity of the represented Boolean condition. Kemp (2012) and Piantadosi, Tenenbaum and Goodman (2016) extend this idea to general concept learning. They propose that concept learning is a form of probabilistic inference that seeks to find the concept that maximises the probability of the represented data classification. This cognitive process is described as the agent seeking the concept that offers the optimal Shannon compression scheme over its represented perceptual data. Gallistel and Wilkes (2016) describe associative learning as a probabilistic inference about the likely causes of an unconditioned stimulus given the observations. They describe this in terms of Shannon information processing: the cognitive system starts with priors over hypotheses about causes that have maximum entropy (their probability distributions are as 'noisy' as possible consistent with the data); the cognitive system then aims to find the hypotheses that provide optimal compression (that maximise Shannon information) of the represented hypothesis and observed data. In general, modellers move smoothly between probabilistic formulations and information-theoretic formulations of a probabilistic inference when describing a cognitive process. In each of the cases described above, the Shannon information is associated not with the probabilities of certain neural vehicles occurring, but with the represented probability distributions over which inference is performed (although again, one might think that the two are likely to be related via the brain's coding scheme).

6.3 Two versions of the free-energy principle

Friston (2010) claims that his free-energy principle provides a unified theory of how all cognitive and living creatures work. However, he invokes two kinds of Shannon information processing and he effectively describes two separate versions of his 'free-energy principle'.

First, Friston says that the free-energy principle is a claim about the probabilistic inference performed by a cognitive system. He claims that the brain aims to predict upcoming sensory activation and it forms probabilistic hypotheses about the world that are updated in light of its errors in making this prediction. Shannon information attaches to the represented probability distributions over which the inference is performed. Friston says that the brain aims to minimise the ‘surprisal’ of – the Shannon information associated with – new sensory evidence. When the brain is engaged in probabilistic inference, however, Friston says that it does not represent the full posterior probability distributions as a perfect Bayesian reasoner would do. Instead, the brain approximates them with simpler probability distributions, assumed to be Gaussian. Provided the brain minimises the Shannon-information quantity ‘variational free energy’ it will bring these simpler probability distributions into approximate correspondence with the true posterior distributions that a perfect Bayesian would have (Friston, 2009; Friston, 2010). Variational free energy is an information-theoretic quantity, predicated of the agent’s represented probability distributions, that measures how far those subjective probability distributions depart from the optimal guesses of a perfect Bayesian observer. According to Friston, the brain minimises ‘free energy’ and so approximates an ideal Bayesian reasoner.

Friston makes a second, conceptually distinct, claim about cognition (and life in general) aiming to minimise free energy. In this context, his goal is to explain how cognitive (and living) systems maintain their physical integrity and homeostatic balance in the face of a changing physical environment. Cognitive (and living) systems face the problem that their physical entropy tends to increase over time: they generally become more disordered and the chance increases that they will undergo a fatal physical phase transition. Friston says that when living creatures resist this tendency, they minimise free energy (Friston, 2013; Friston and Stephan, 2007). However, the free energy minimised here is not the same as that which attaches to the represented, probabilistic guesses of some agent. Instead, it attaches to the objective probabilities of various possible (fatal) physical states of the agent occurring in response to environmental changes. Minimising free energy involves the system trying to arrange its internal physical states so as to avoid being overly changed by probable environmental transitions. The system strives to maintain its physical nature in equipoise with likely environmental changes. The information-theoretic free-energy minimised here is defined over the objective distributions of possible physical states that could occur, not over the probability distributions represented by an agent’s hypotheses.

Minimising one free-energy measure may help an agent to minimise the other: a good, Bayesian reasoner is plausibly more likely to survive in a changing physical environment than an irrational agent. But they are not the same quantity. Moreover, any correlation between them could conceivably come unstuck. An irrational

agent could depart far from Bayesian ideals but be lucky enough to live in an hospitable environment that maintains its physical integrity and homoeostasis no matter how badly the agent updates its beliefs. Alternatively, an agent may be a perfectly rational Bayesian and update its beliefs perfectly, but its physical environment may change so rapidly and catastrophically that it cannot survive or maintain its homoeostasis. Understanding how Friston's two formulations of the free-energy principle interact – that pertaining to represented subjective probabilities and that pertaining to objective probabilities – is ongoing work.²⁷

7 Conclusion

Traditionally, philosophers have invoked Shannon information as a rung on a ladder that takes one to naturalised representation. In this context, Shannon information is associated with the outcomes and probability distributions of physical states and environmental states. This project, however, obscures a novel way in which Shannon information enters into cognition. Probabilistic models of cognition treat cognition as an inference over representations of probability distributions. Therefore, probabilities enter into cognition in two distinct ways: as the objective probabilities of neural vehicles and/or environmental states occurring and as the subjective probabilities that describe the agent's expectations. Two types of Shannon information are associated with cognition respectively: Shannon information that pertains to the probability of the neural vehicle occurring and Shannon information that pertains to the represented probabilistic content. The former is conceptually and logically distinct from the latter, just as representational vehicles are conceptually and logically distinct from their content. Various (conceptual, logical, empirical) relations may connect the two kinds of Shannon information in the brain, just as various such connections have been mooted for more traditional vehicles and their content. Care should be taken, however, not to conflate the two. For, as we know with the distinction between ordinary representational vehicles and content, much trouble lies that way.

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²⁷Colombo and Wright (2018) draw a similar contrast between the two formulations of the free-energy principle. They describe different versions of the free-energy principle as involving 'epistemic' and 'physical' probabilities.

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