BehavioralDeePC.jl

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Behavioral data-driven control

- On the equivalence of subspaces

$$V \subseteq \mathbb{R}^n, \text{ a subspace} \iff V = \{x \in \mathbb{R}^n \mid \exists A, b, \; Ax = b\}$$

- In an analogous way, we can equivalently describe dynamic systems

$$\begin{aligned}
&\mathcal{B} \text{ is an LTI dynamic system} \iff \\
&\mathcal{B} \text{ is described by a state-space model:} \\
&\qquad x_{k+1} = Ax_k + Bu_k, \; y_k = Cx_k, \iff \\
&\mathcal{B} = \Big\{(\{u_k\}_{k\geq 0}, \{y_k\}_{k\geq 0}) \mid \exists A, B, C, x_0 \text{ such that} \\
&\qquad x_{k+1} = Ax_k + Bu_k, \; y_k = Cx_k\Big\}
\end{aligned}$$

Truncation requires Persistence of Excitation

- $\mathcal{B}$ is infinite dimensional
- Can we instead describe $\mathcal{B}$ (equivalently) by a single finite trajectory $(\{u_k\}_{k=0}^{T}, \{y_k\}_{k=0}^{T}),\ T < \infty$?
- The answer is yes, given Persistence of Excitation (PE)
- PE (roughly) means that $\{u_k\}_{k=0}^{T}$ is **rich** enough and **long** enough to excite all the possible modes of the underlying dynamics
- Therefore, towards a finite description of $\mathcal{B}$, $(\{u_k\}_{k=0}^{T}, \{y_k\}_{k=0}^{T})$ has to be of specific nature and of a minimum length[1].

[1] For more detailed discussion visit [2].

Two key ideas

1. **(Initial Conditions)** For minimum (controllable and observable) state-space representation

$$\begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix} = \mathcal{O}_N(A, C)x_0 + \mathcal{C}(A, B, C, D) \begin{pmatrix} u_1 \\ \vdots \\ u_N \end{pmatrix},$$

   $\mathcal{O}_N(A, C)$ observability matrix (left-invertible for minimal systems).

2. **(Dynamic Constraints)** For persistently excited data $(\{u_k\}_{k=0}^T, \{y_k\}_{k=0}^T)$

   range$(\mathcal{H}_t) = \mathcal{B}_t =$ dynamically feasible input/output sequences of length t,

   where $\mathcal{H}_t$ is the Hankel matrix of input/output sequences. That is,

$$\mathcal{H}_t g = \begin{bmatrix} y_1 & \dots & y_{T-t+1} \\ \vdots & & \vdots \\ y_t & \dots & y_T \\ u_1 & \dots & u_{T-t+1} \\ \vdots & & \vdots \\ u_t & \dots & u_T \end{bmatrix} g = \begin{pmatrix} y_1^\star \\ \vdots \\ y_N^\star \\ u_1^\star \\ \vdots \\ u_N^\star \end{pmatrix}, \ \star \text{ denotes new data}$$

DeePC: Data-Enabled Predictive Control[2]

- Solve

$$\min_{g,u,y} \sum_{k=0}^{N-1} y_k^\top Q y_k + u_k^\top R u_k, \tag{1}$$

- subject to

$$\begin{pmatrix} U_p \\ Y_p \\ U_f \\ Y_f \end{pmatrix} g = \begin{pmatrix} u_{ini} \\ y_{ini} \\ u \\ y \end{pmatrix}$$

- and $u_k \in \mathcal{U}$, $y_k \in \mathcal{Y}$, $k = 0, \ldots, N-1$.
- In red: U_p, Y_p are past data (persistently excited), u_{ini}, y_{ini} the previous to current-time input/output data. The rows in red are responsible of deciding g that initiates this trajectory.
- In blue: U_p, Y_p are past data (persistently excited), and u, y are the decision variables over the N future time-steps. These rows are responsible of u, y consistent with the dynamics, and that are continuations of u_{ini}, y_{ini}.

[2] A slightly modified version from that in the paper [1]

DeePC for stochastic/nonlinear

- With the slightest noise/nonlinearity, the condition

$$\begin{pmatrix} U_p \\ Y_p \\ U_f \\ Y_f \end{pmatrix} g = \begin{pmatrix} u_{ini} \\ y_{ini} \\ u \\ y \end{pmatrix}$$

  may no longer be satisfied by any g.

- The solution is using a slack variable
  - Solve

$$\min_{g,u,y,\sigma} \sum_{k=0}^{N-1} y_k^\top Q y_k + u_k^\top R u_k + \lambda_g \|g\|_1 + \lambda_\sigma \|\sigma\|_1, \tag{2}$$

  - subject to

$$\begin{pmatrix} U_p \\ Y_p \\ U_f \\ Y_f \end{pmatrix} g = \begin{pmatrix} u_{ini} \\ y_{ini} + \sigma \\ u \\ y \end{pmatrix}$$

BehavioralDeePC.jl

- BehavioralDeePC.jl is a Julia package that implements the DeePC algorithm, as in the paper [1].
- The repository of this package: https://github.com/msramada/BehavioralDeePC.jl.
- docs/example.ipynb is a notebook with a step-by-step example implementation.

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