Workforce Scheduling at United Airlines

- United Airlines employs 5,000 reservation and customer service agents.
- Some part-time (2-8 hour shifts), some full-time (8-10 hour shifts).
- Workload varies greatly over day.

Modeled problem as LP:

- Decision variables: how many employees of each shift length should begin at each potential start time (half-hour intervals).
- Constraints: minimum required employees for each half-hour.
- Objective: minimize cost.

United Airlines saved about $6 million annually, improved customer service, the model is still in use today.

For more details, see Jan-Feb 1986 Interfaces article "United Airlines Station Manpower Planning System" (available in the course webpage)

Optimizing Prototype Vehicle Testing at Ford Motor Company

Go Further

The Problem

- The prototype vehicles that Ford Motor Company uses to verify new designs are a major annual investment.
- But the cost of building a prototype routinely exceeds **$250,000**, while complex vehicle programs commonly require over 100 full-vehicle prototypes and sometimes require over 200 in the course of product development.
- The company wants to identify the most efficient use of the prototypes, thus cutting costs while still meeting demands for high quality.

The Analytics Solution

- A team of engineering/operations research noted that prototypes sit idle much of the time waiting for various tests, so increasing their usage was a clear path to cost savings. The barrier to sharing these idle prototypes among design groups lay in determining an optimal set of vehicles that could be used to satisfy all the testing needs. The team developed an optimization model to reduce the number of prototype vehicles Ford needed to verify the designs of its vehicles and perform necessary tests.

The Value

- **Ford reduced annual prototype costs by more than $250 million**. The model dramatically shortened the planning process, established global procedures, and created a common structure for dialogue between budgeting and engineering.
- The model became an integral part of Ford's tactical and strategic planning of product development.

Cereals, Ltd

Cereals, Ltd is a company specialized in preparing and packing wheat and corn for several retailing stores.There are no limits concerning the supply of these cereals and demand can be considered unlimited (in other words, the whole production is sold). The production is processed in three phases: Pre-Processing (I), Processing (II) and Packing (III).

	I Pre-Processing	II Processing	III Packing	
Weekly production capacity (h)	120	100	150	
Time (h) needed to prepare 1 ton				Profit(€/ton)
Wheat	6 h	1 h	5 h	4
Corn	2 h	4 h	5 h	3

Currently, the company produces 18 tons of wheat and 6 tons of corn per week.
Is this the best option?

Cereals, Ltd - Formulation

Decision variables

x = tons of wheat to produce weekly

y = tons of corn to produce weekly

Constraints

$$6x + 2y \leq 120$$

$$x + 4y \leq 100$$

$$5x + 5y \leq 150$$

$$x \geq 0, y \geq 0$$

Objective function: to maximize the profit $\max 4x + 3y$

$$\max 4x+3y$$

$$\text{s.a} \quad 6x+2y \le 120$$
$$x+4y \le 100$$
$$5x+5y \le 150$$
$$x \ge 0, y \ge 0$$

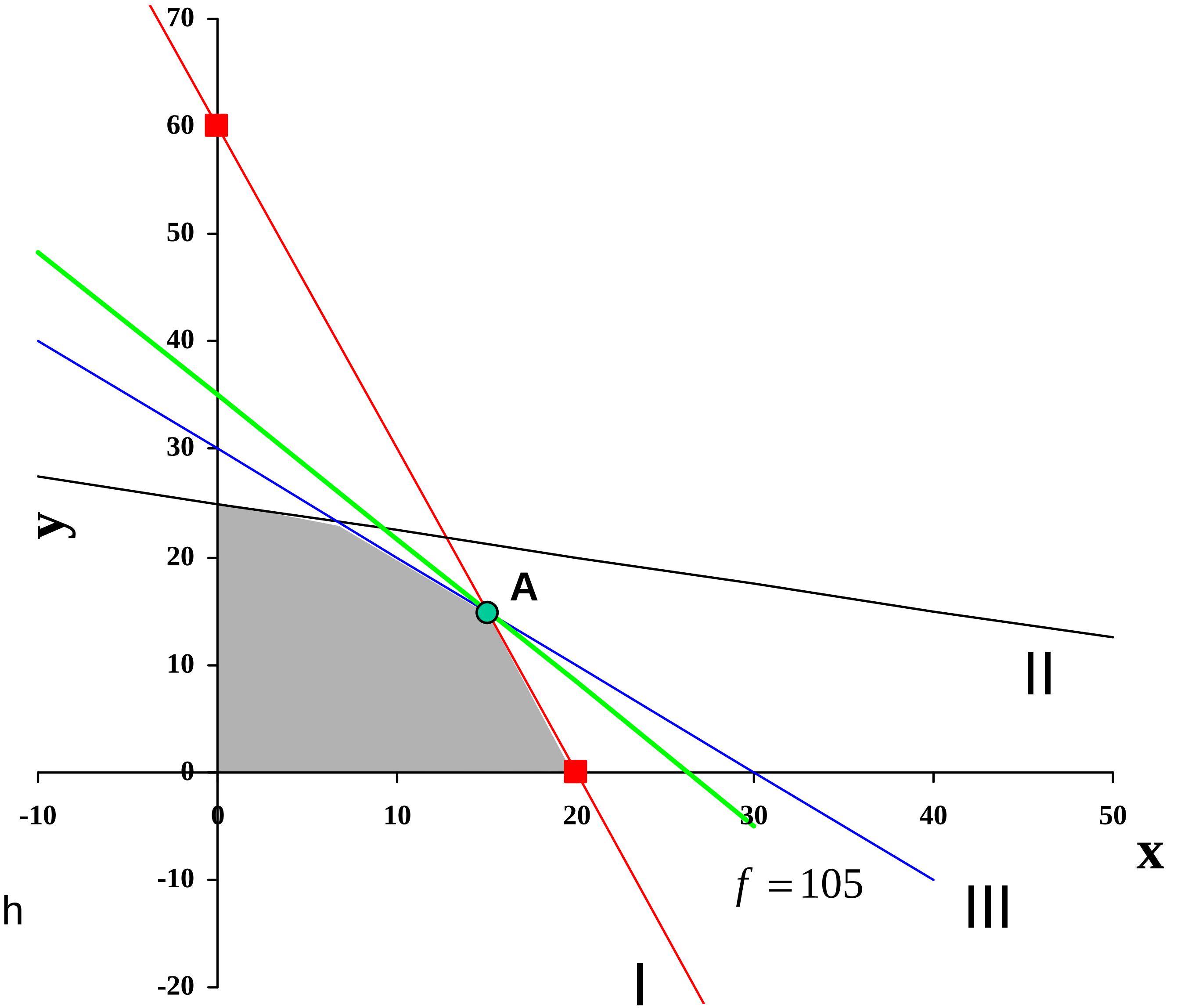
Consider again the example of Cereals, Ltd

Optimal solution A = (15,15)

Profit: $4x+3y=105$

What happens if it would be possible to increase the profit of each ton of wheat to 4,35 €?

And what if the production capacity in section III (packing) is reduced to 126h per week?



Once the optimal solution of a linear problem is obtained, what should we do if changes in the parameters occur?

Sensitivity Analysis

Analyses the effect of (small) changes on the parameter values in the optimal solution.

Case 1: changes in the coefficients of the objective function (c_j)

Example: what is the possible variation for unitary profit of wheat (x) and corn (y) without changing the optimal solution (x=15 and y=15)?

Caso 2: changes in the right side of constraints alterações (b_i)

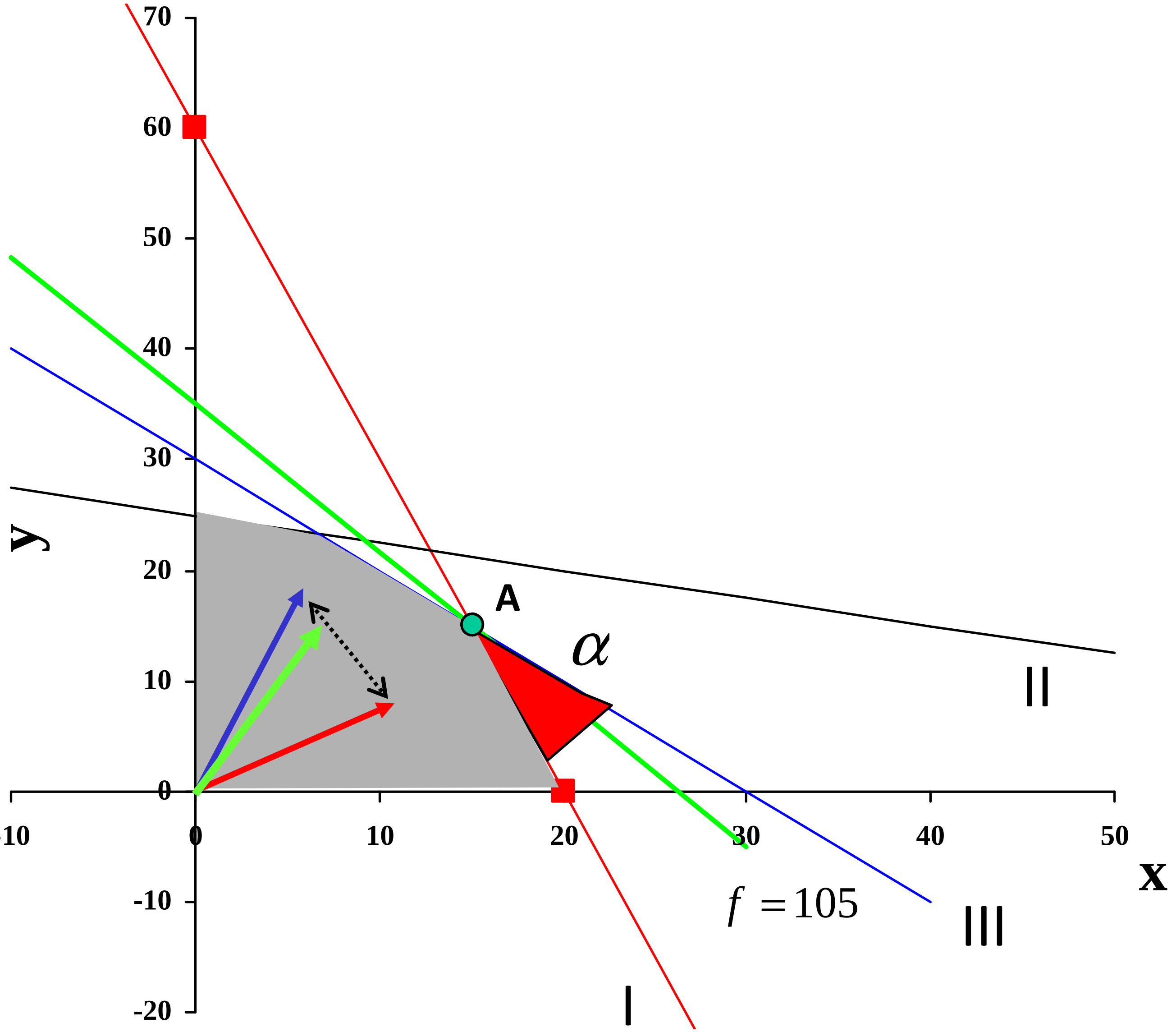
Example: what is the effect of changing the production capacity in each section (I, II and III)?

Case 1 - changes in the coefficients of the objective function (c_j)

Lines **I** and **III** make an angle **a** at point **A**.

If **f** rotates inside angle **a**, the optimal solution is maintained.

Rotating **f** inside angle **a** means that the slope of f varies between the slopes of **I** and **III**.



Case 1 - changes in the coefficients of the objective function (c_j)

Slope of I = **- 3**

$$6x+2y=120 \Leftrightarrow y=-3x+60$$

Slope of III = **-1**

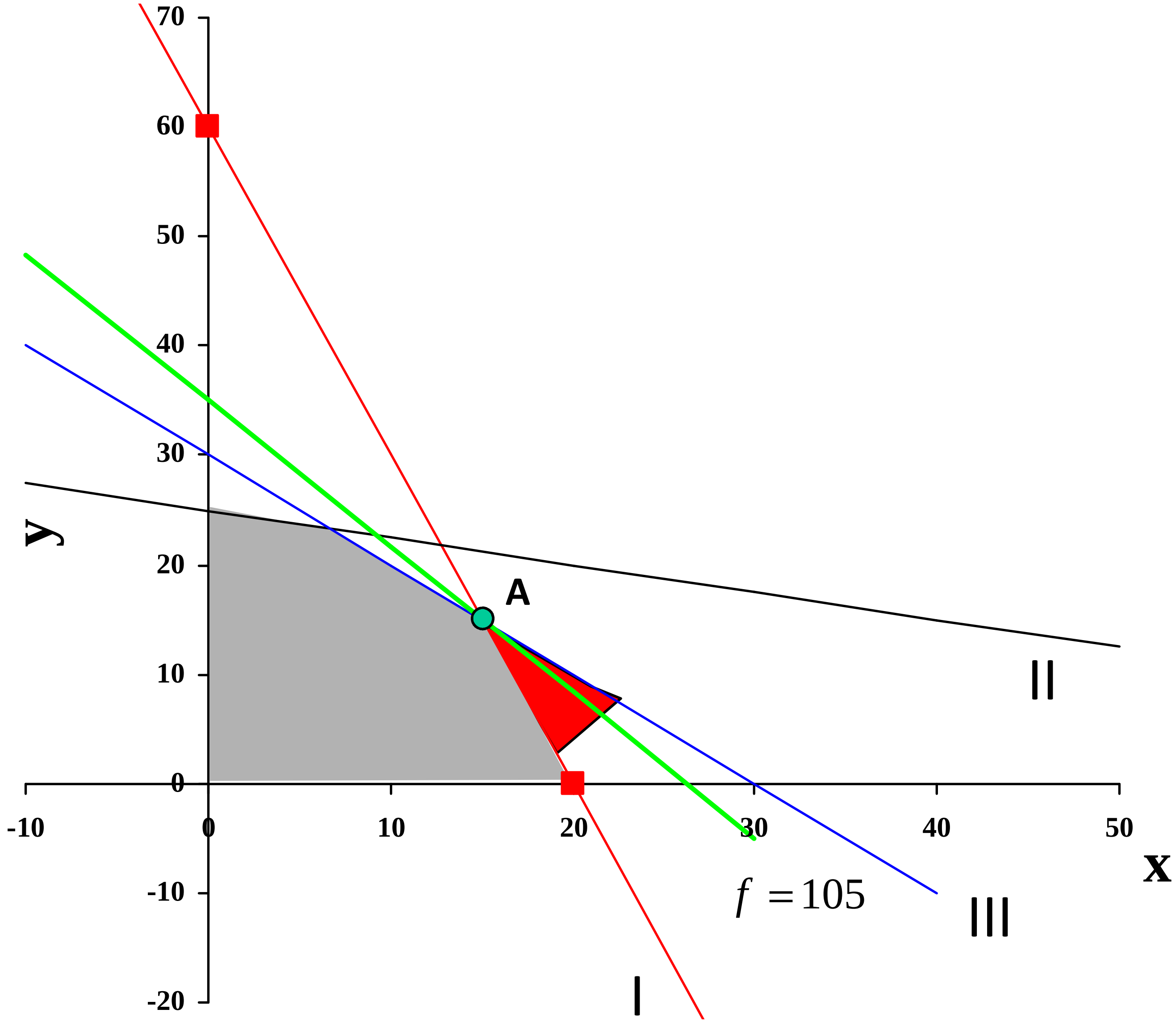
$$5x+5y=150 \Leftrightarrow y=-x+30$$

Slope of f = **- 4 / 3**

$$4x+3y=k \Leftrightarrow y=-\frac{4}{3}x+\frac{k}{3}$$

In fact,

$$-3 \le -\frac{4}{3} \le -1$$



Case 1 - changes in the coefficients of the objective function (c_j)

Let a, b be the coefficients of the objective function $f(x,y) = ax + by = k \Leftrightarrow y = -\frac{a}{b}x + \frac{k}{b}$

The optimal solution (i.e, the values of x and y) remains unchanged if $-3 \leq -\frac{a}{b} \leq -1$

although the **f** value (in this example, the profit) may vary.

- In particular, if we modify the value of **a**, keeping **b = 3**:

$$-3 \leq -\frac{a}{3} \leq -1 \Leftrightarrow -9 \leq -a \leq -3 \Leftrightarrow 3 \leq a \leq 9$$

- If we change the value of **b** instead, keeping **a = 4**:

$$-3 \leq -\frac{4}{b} \leq -1 \Leftrightarrow 1 \leq -\frac{4}{b} \leq 3 \Leftrightarrow \frac{4}{3} \leq b \leq 4$$

Case 1 - changes in the coefficients of the objective function (c_j)

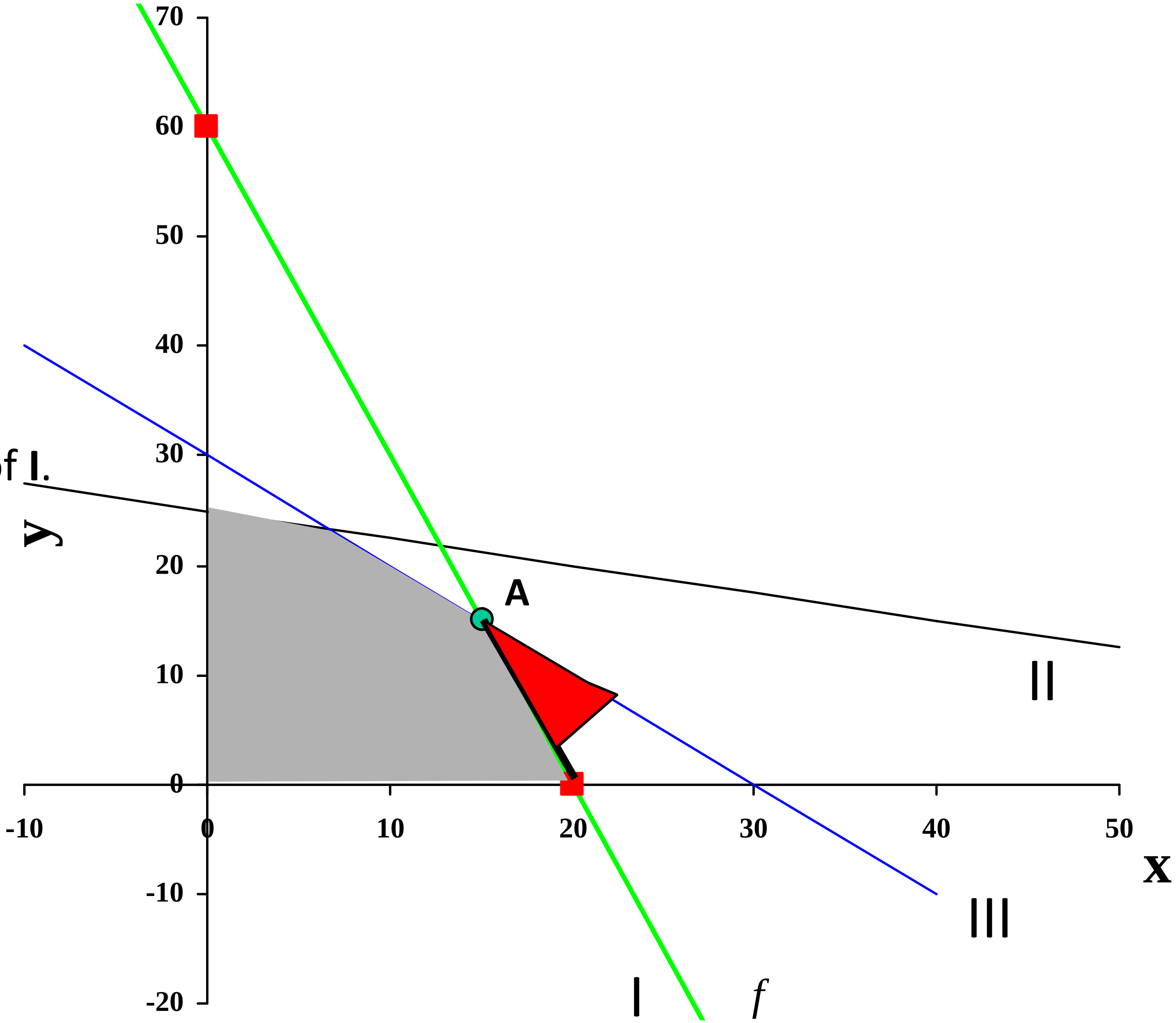
- If $-3 < -\frac{a}{b} < -1$, the optimal solution is unique (point A).

- If $-\frac{a}{b} = -3$

(*e.g, a = 4, b=4/3 or a=9, b=3*),

the slope of **f** is equal to the slope of **I**.

In this case, we will have an infinite number of optimal solutions.

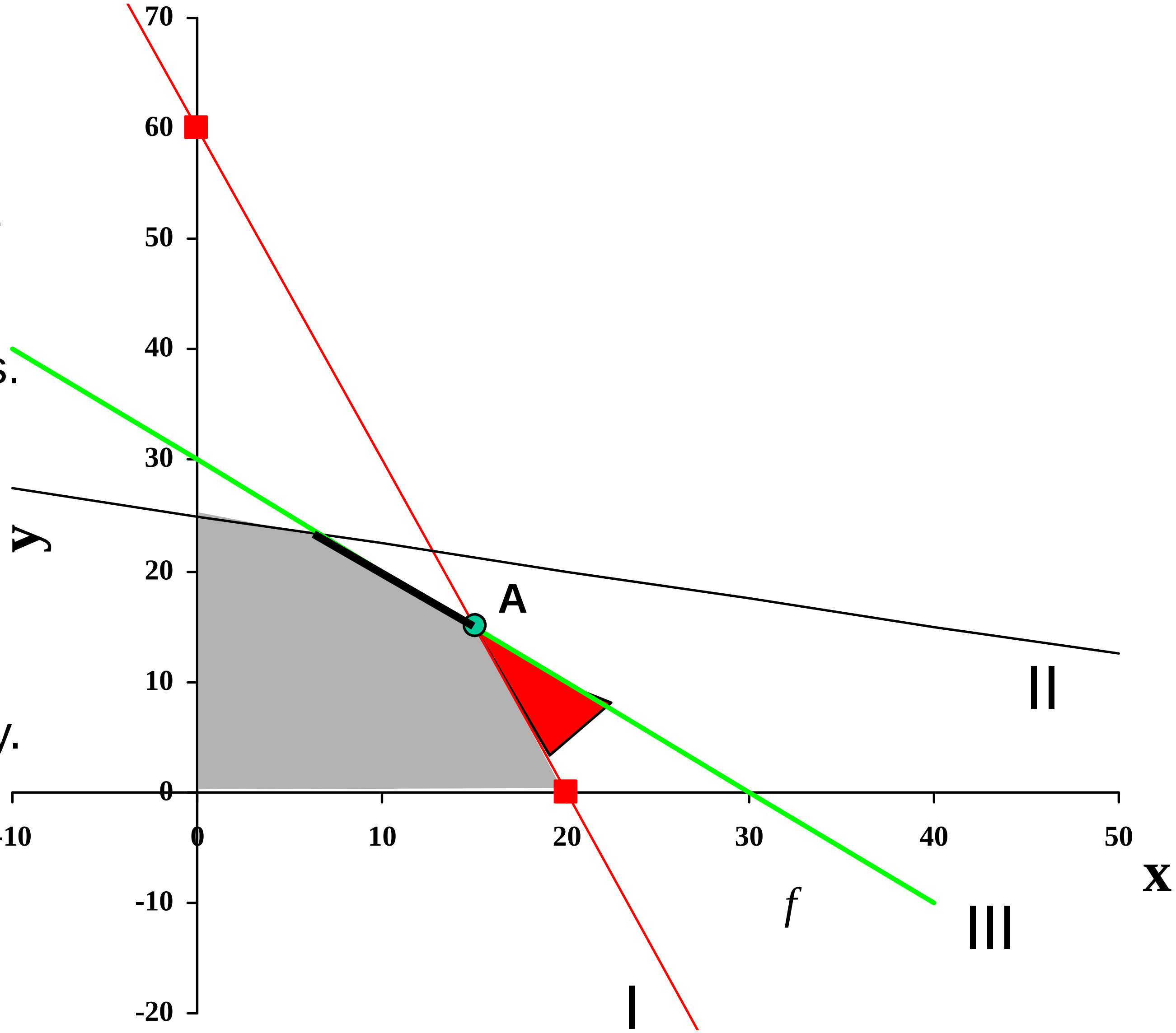


Case 1 - changes in the coefficients of the objective function (c_j)

- If $-\frac{a}{b} = -1$

(*e.g, a = 4, b=4 or a=3, b=3*),

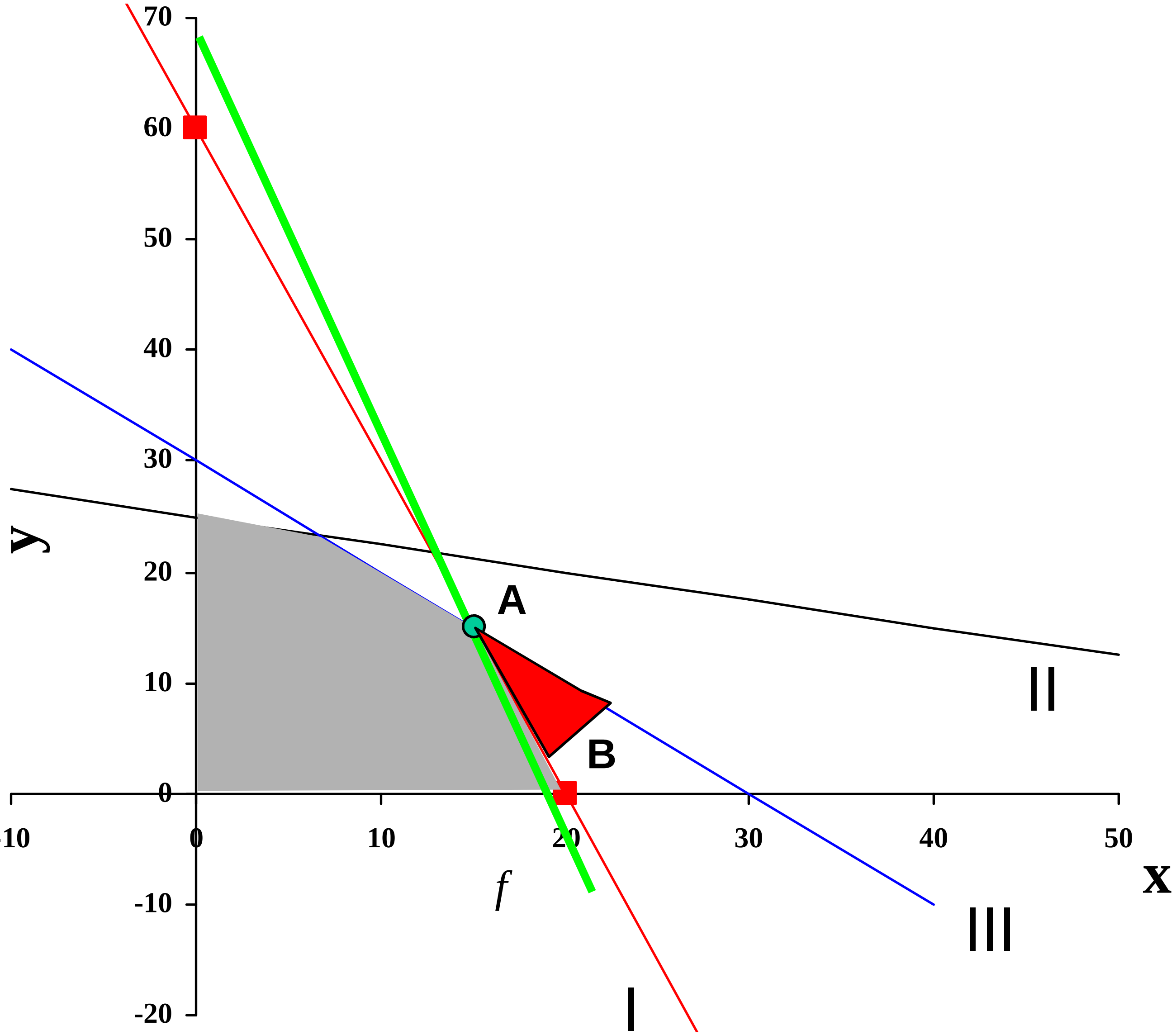
The slope of **f** is equal to the slope of **III** and we will also have an infinite number of optimal solutions.



Note that, for the same production plan (x=15, y=15), the profit is different if we have 4x+4y or 3x+3y.

Case 1 - changes in the coefficients of the objective function (c_j)

What happens if **f** rotates beyond angle α?



- If $-\frac{a}{b} < -3$, we can see graphically that the new optimal solution is **B**.

- But, in the general case, we can only say that the optimal solution will change and it is necessary to solve the new problem.

Case 2: changes in the right side of constraints (b_i)

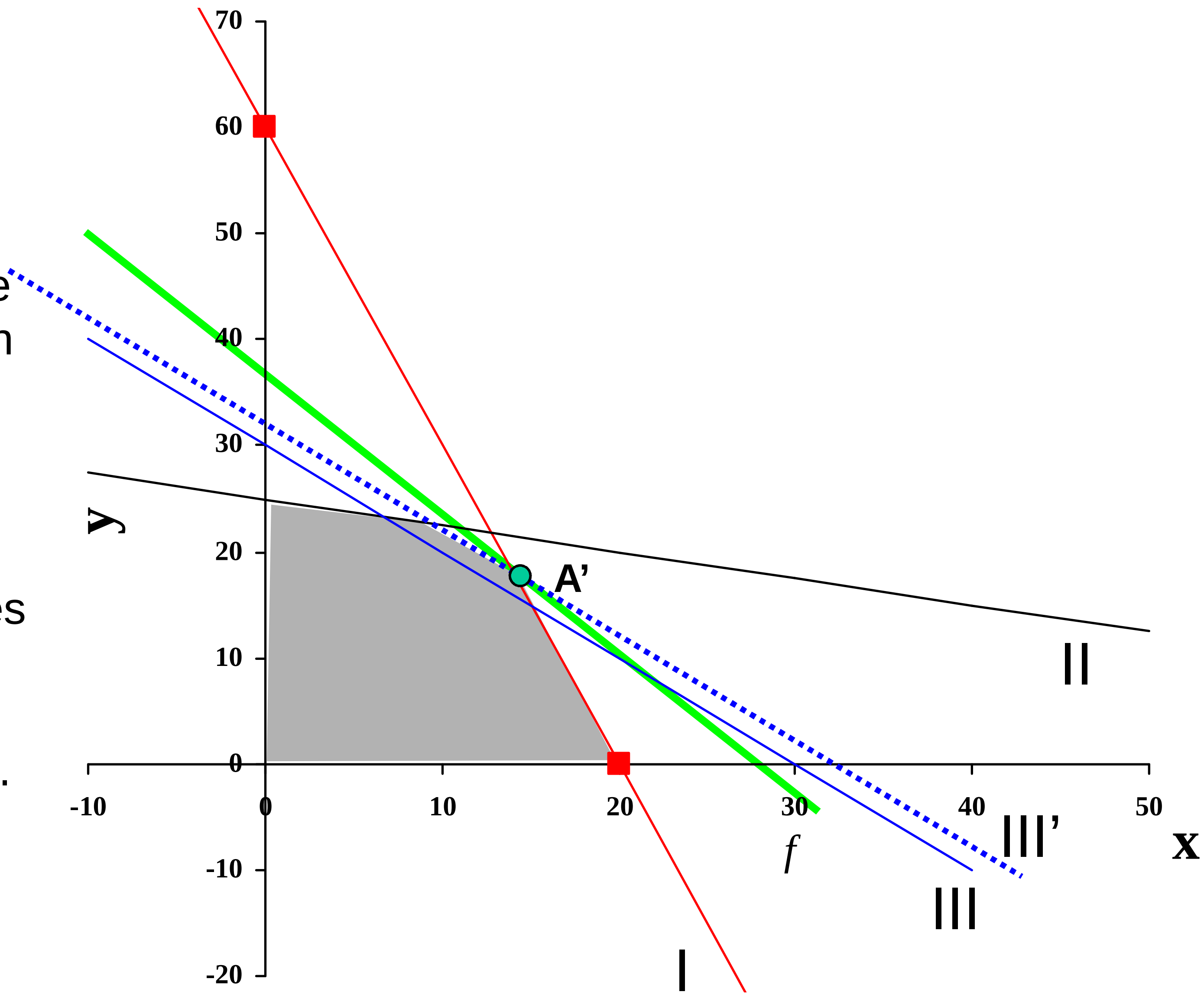
Consider now constraint **III**:

$$5x + 5y \le 150$$

What happens if we increase the production capacity (k) in section **III**?

Let $5x + 5y \le k$

As **k** increases, we will have lines (like **III'**) parallel to **III**, and the optimal solution is in the intersection point of **I** and **III'** (A').



Case 2: changes in the right side of constraints (b_i)

- If we increase one unit to resource **III**: **k = 151**.
- The new optimal solution is the intersection point of:

$$\begin{cases} 5x+5y=151 \\ 6x+2y=120 \end{cases} \Leftrightarrow \begin{cases} x=14{,}9 \\ y=15{,}3 \end{cases}$$

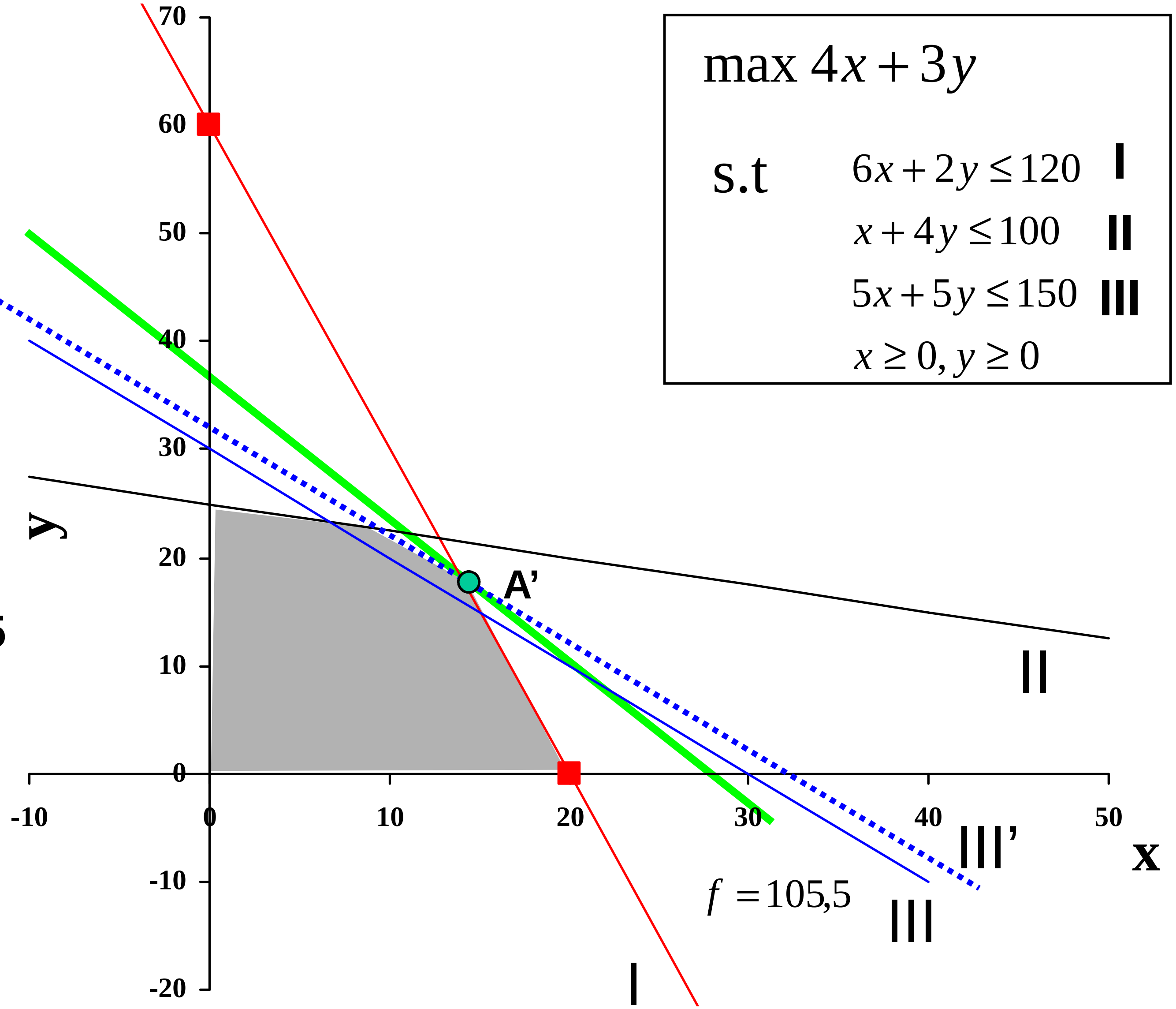
The profit changed from **f* =105** to:

$$4x+3y=4\times 14{,}9+3\times 15{,}3$$
$$=105{,}5$$

$$\max 4x+3y$$

s.t

$$6x+2y\le 120 \quad \mathbf{I}$$
$$x+4y\le 100 \quad \mathbf{II}$$
$$5x+5y\le 150 \quad \mathbf{III}$$
$$x\ge 0,\ y\ge 0$$



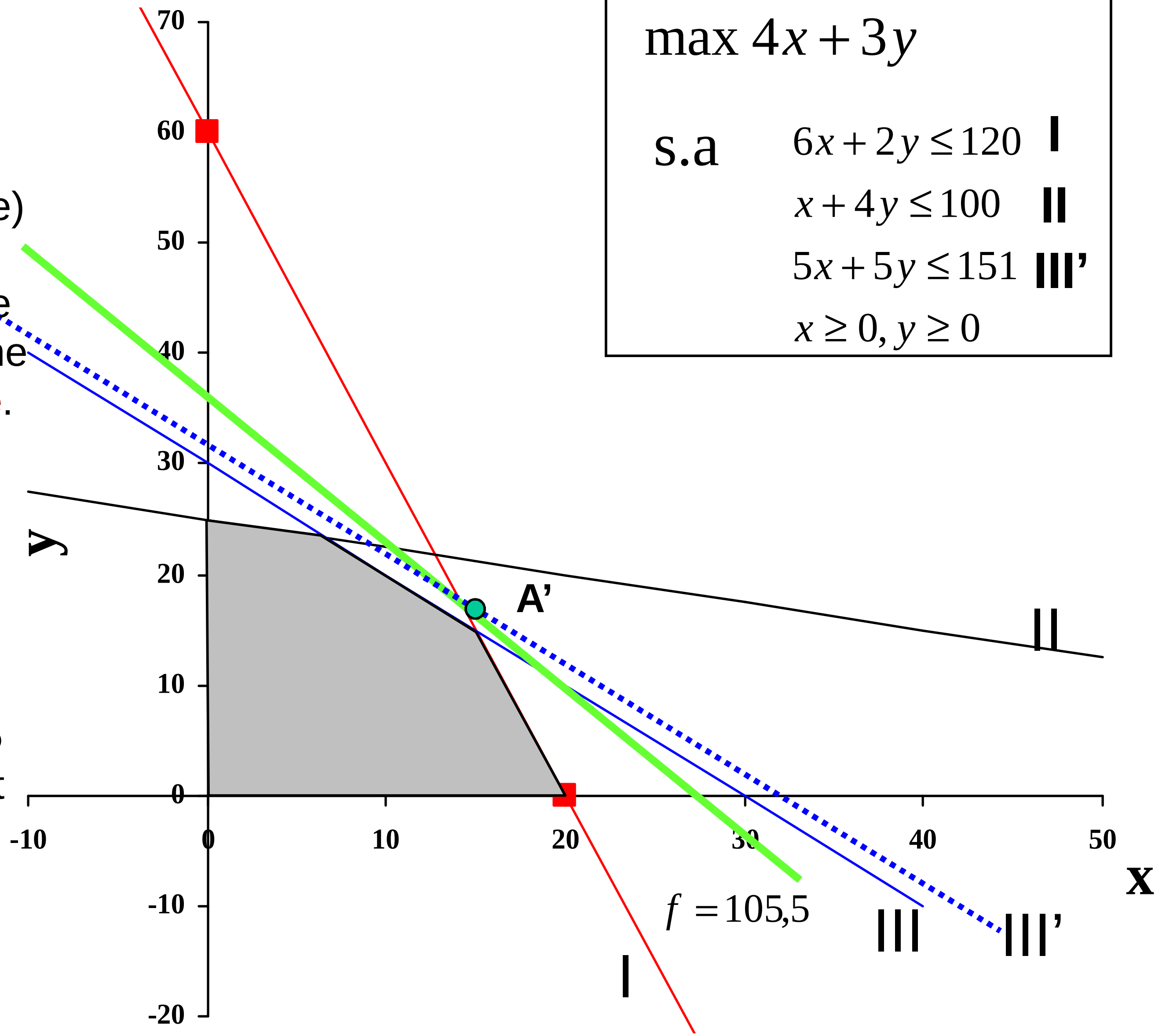
Shadow price

- The amount added to profit (in this case) as a result of the additional unit of resources is seen as the marginal value of the resources and is referred to as the **opportunity cost** or the **shadow price**.

- In this example, the shadow price of resource **III** is the marginal profit obtained when we have an additional hour in the packing section.
- Since the profit has increased from **105** to **105,5**, the shadow price of constraint **III** is **0,5 €**.

$$\max\ 4x + 3y$$

$$\text{s.a}\quad \begin{aligned} 6x + 2y &\le 120 & \text{I}\\ x + 4y &\le 100 & \text{II}\\ 5x + 5y &\le 151 & \text{III'}\\ x \ge 0, y &\ge 0 \end{aligned}$$

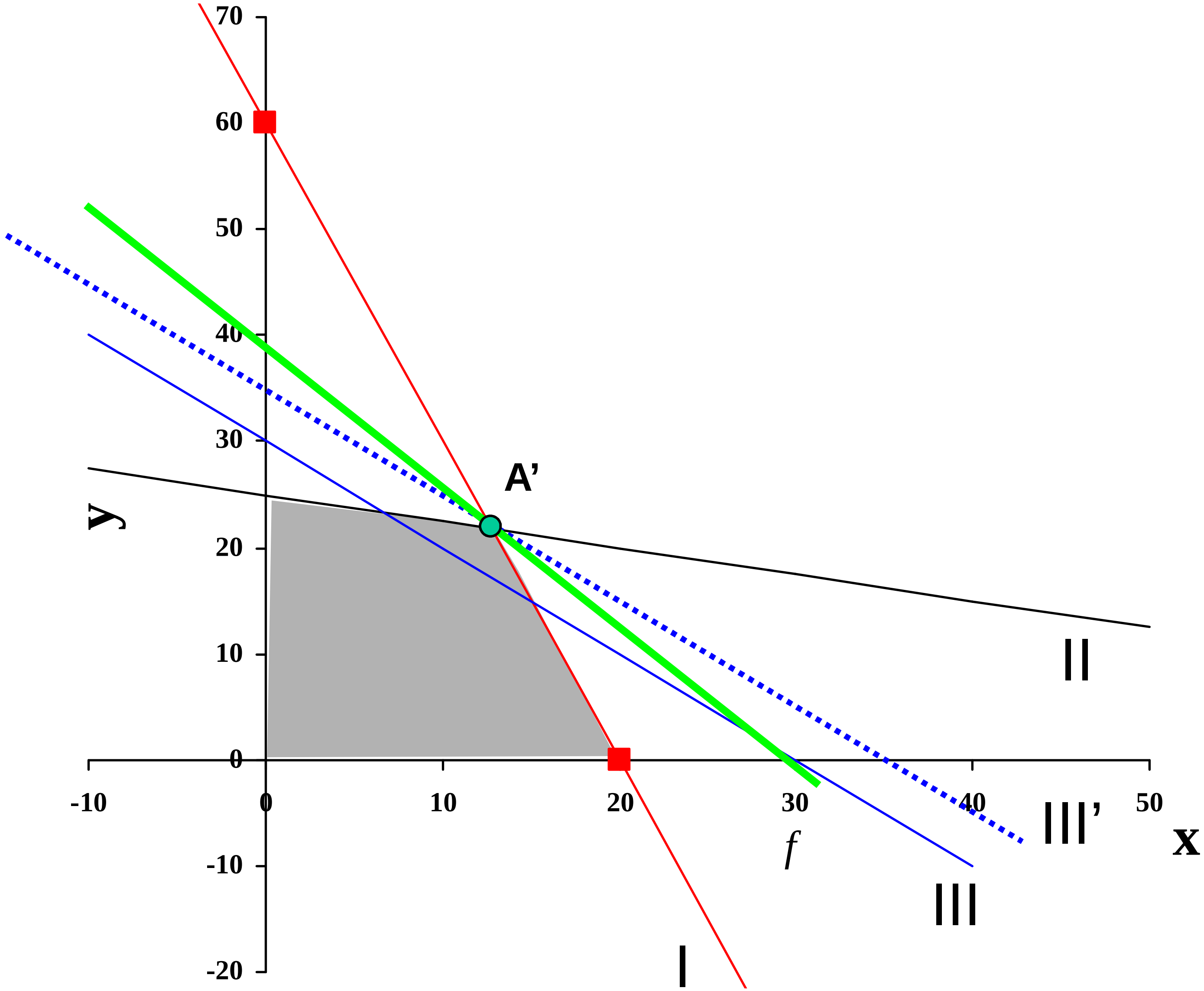


Case 2: changes in the right side of constraints (b_i)

- When **k = 172,727** (why?) the optimal solution lies in the intersection point of **I**, **II** and **III'**.
- What happens if **k > 172,727** ?

For example, **k=175**

$$5x + 5y \leq 175$$



Case 2: changes in the right side of constraints (b_i)

- If $5x + 5y \leq 175$ (**III'**)
  the optimal solution **A'** lies in the intersection point of **I** and **II** and constraint **III'** becomes redundant.

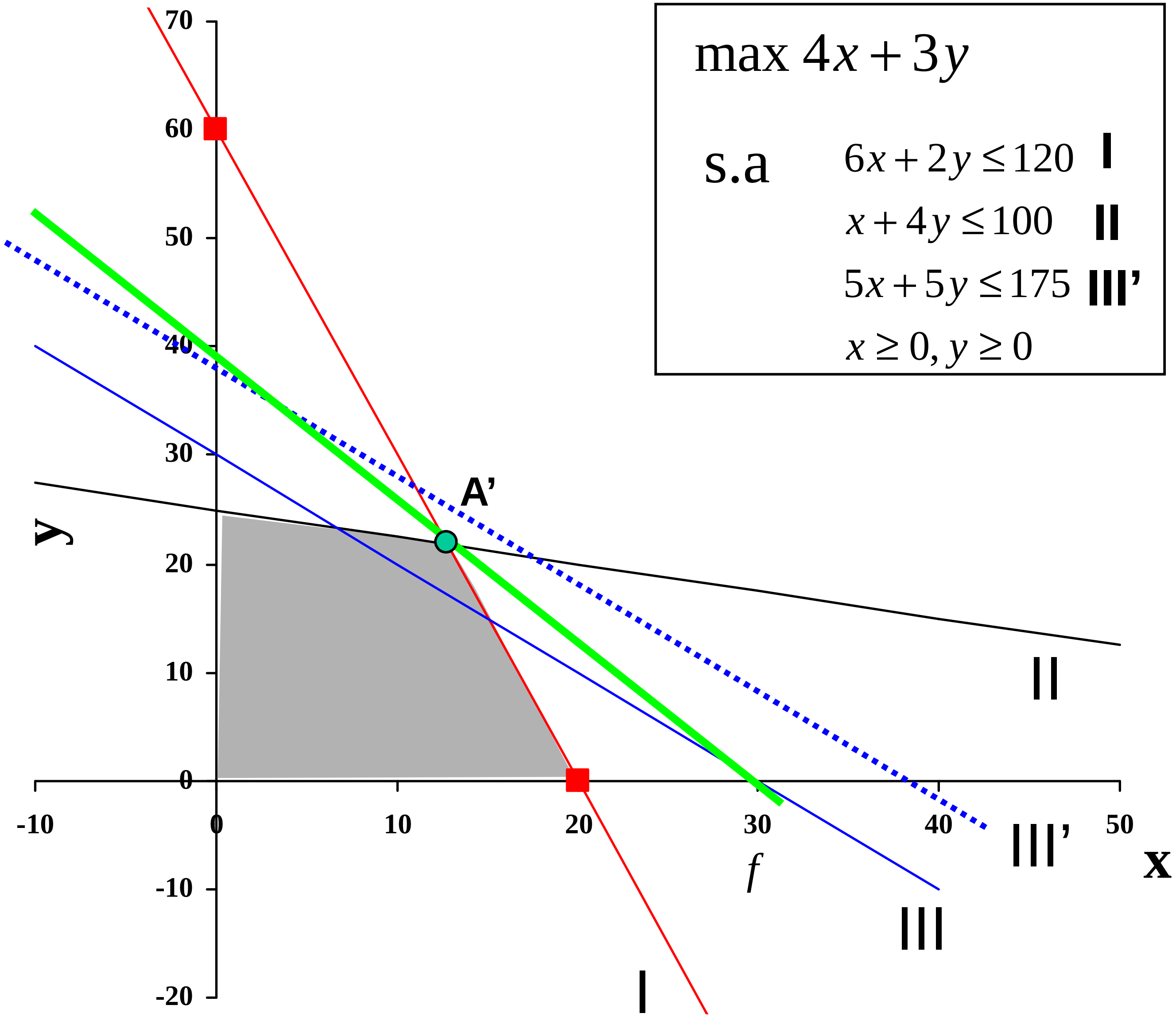
  In this case the shadow price meaning no longer applies, since the increase of a unit in resource III does no longer corresponds to a variation in the objective function

  We will have to solve the new problem.

$$\max\ 4x + 3y$$

s.a

$$6x + 2y \leq 120 \quad \textbf{I}$$
$$x + 4y \leq 100 \quad \textbf{II}$$
$$5x + 5y \leq 175 \quad \textbf{III'}$$
$$x \geq 0,\ y \geq 0$$



Case 2: changes in the right side of constraints (b_i)

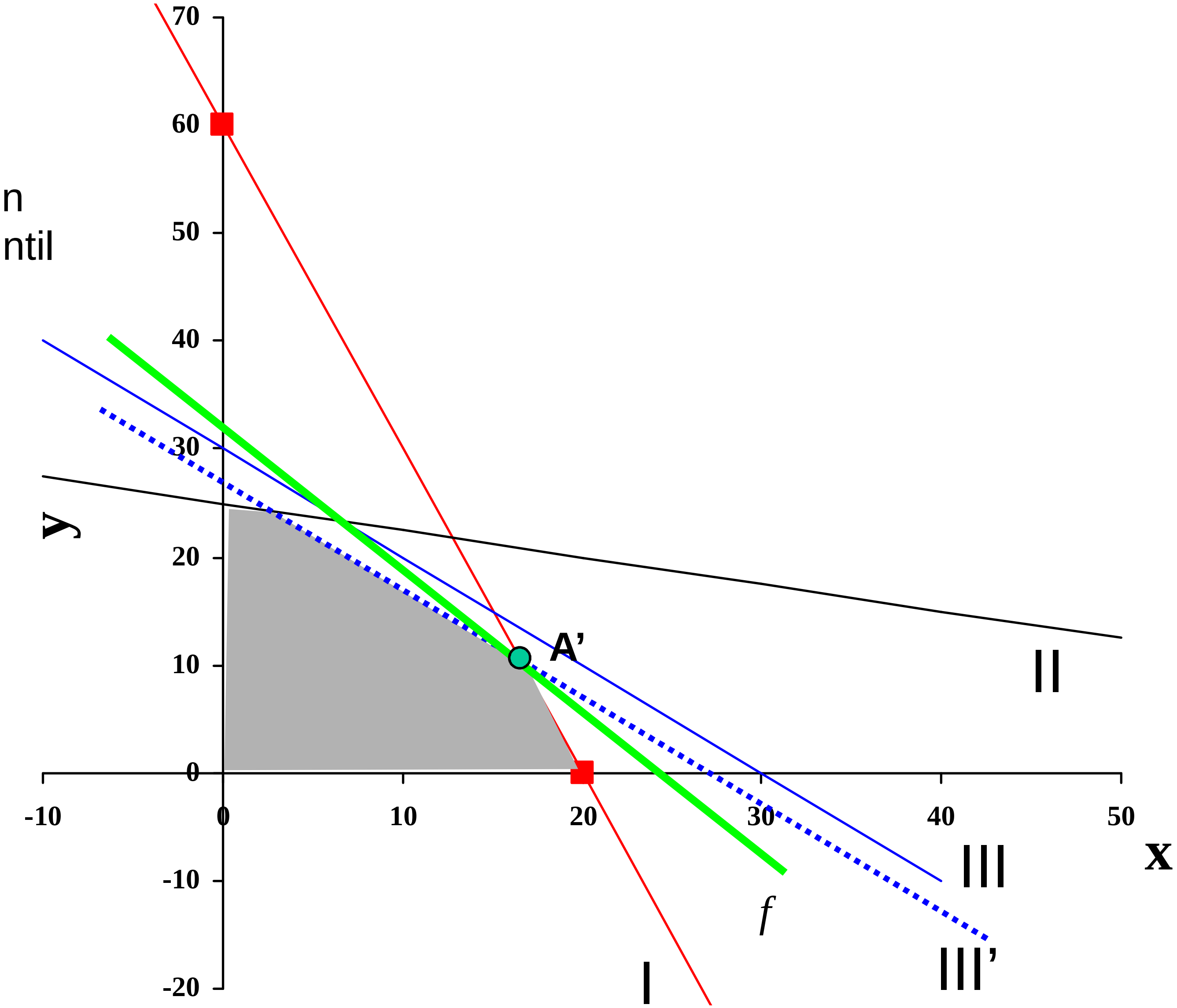
- What happens when **k < 150**?

  As k decreases, the optimal solution lies in the intersection of **I** and **III'** until **k=100** (why?).

- And when **k < 100** ?

  For example, if k=90

$$5x + 5y \leq 90$$

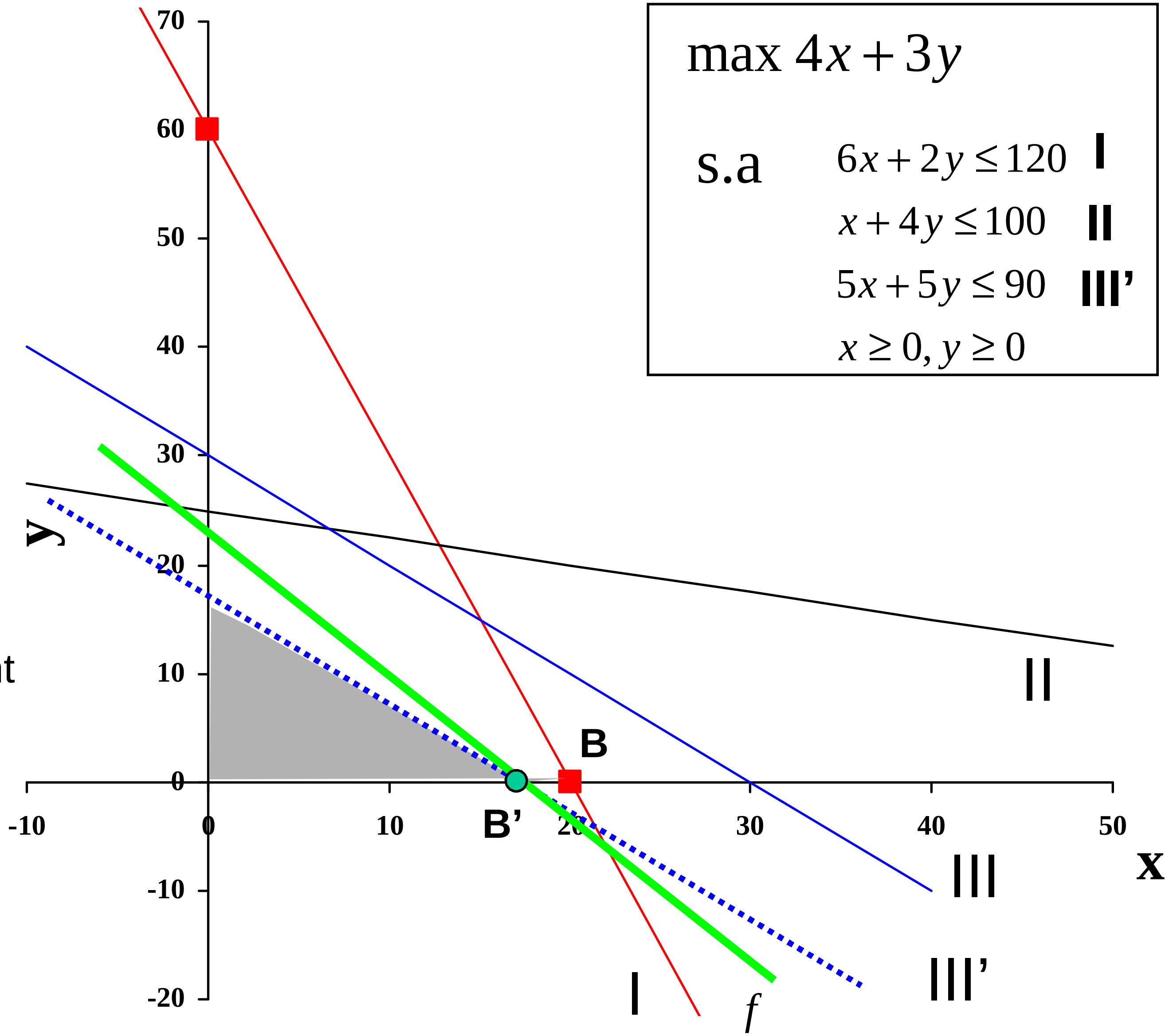


Case 2: changes in the right side of constraints (b_i)

- If $5x + 5y \leq 90$ the optimal solution no longer lies in the intersection of **I** with **III'**. Now it lies in the intersection of **III'** with **y=0** (we say that the *basis has changed*).

- Now we have a slack in constraint I. The value and the meaning of the shadow price associated to constraint **III** no longer apply.

- We will have to solve the new problem.



$$\max\ 4x + 3y$$

s.a

$$6x + 2y \leq 120 \quad \textbf{I}$$
$$x + 4y \leq 100 \quad \textbf{II}$$
$$5x + 5y \leq 90 \quad \textbf{III'}$$
$$x \geq 0, y \geq 0$$

Reduced cost

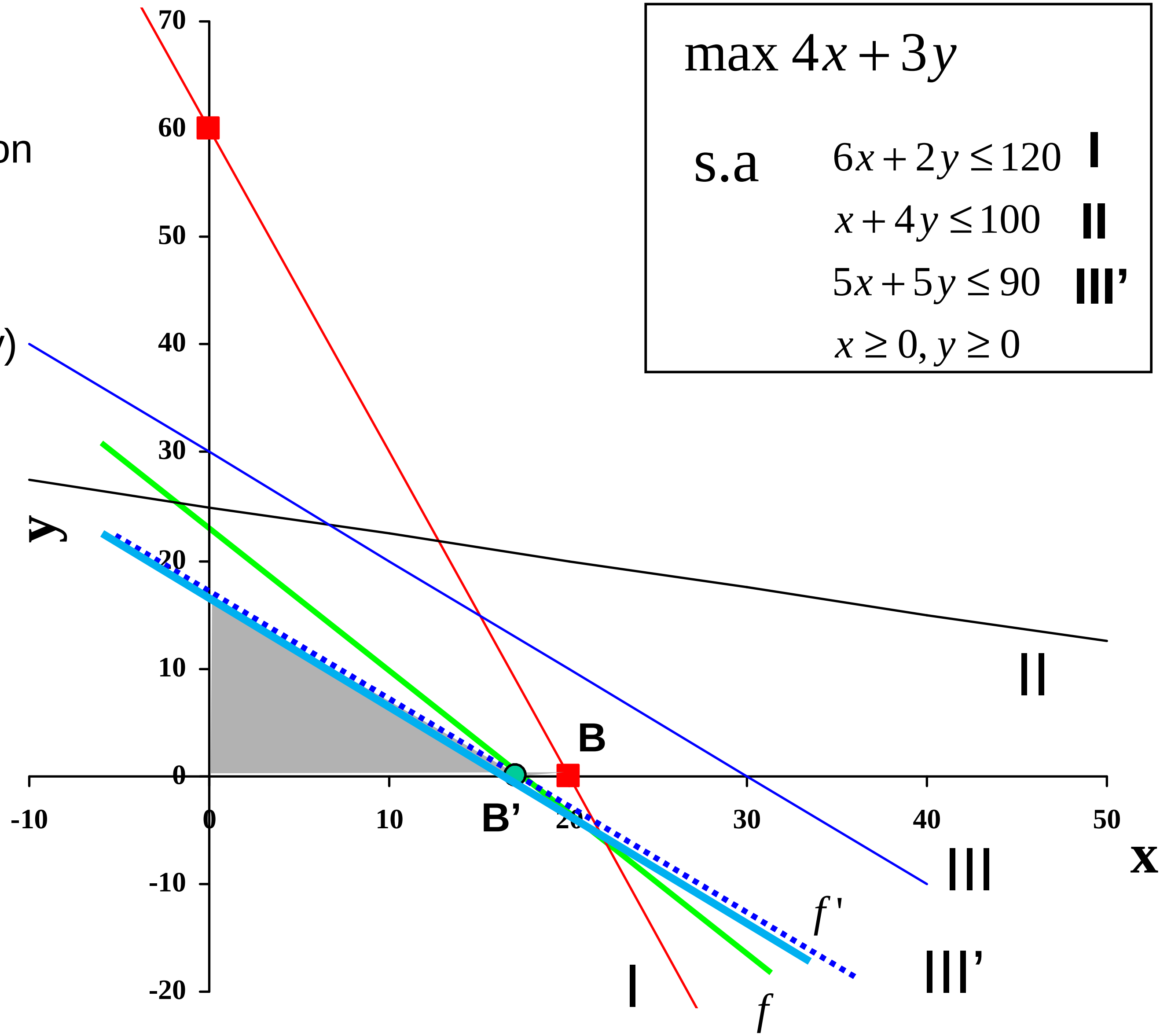
- In this case, the optimal solution indicates that the optimal production plan should not include the production of corn (**y=0**).
- The **reduced cost** of this product shows the increment that the corn (y) unitary profit should have in order to include it in the optimal production plan.
- In this example, if the corn unitary profit is 4 €/ton (f'), it may be considered in the production plan.

Hence, since the increment is of 1€/ton, the **reduced cost of y is 1**.

$$\max\ 4x+3y$$

s.a

$$6x+2y \le 120 \quad \text{I}$$
$$x+4y \le 100 \quad \text{II}$$
$$5x+5y \le 90 \quad \text{III'}$$
$$x \ge 0,\ y \ge 0$$



Case 2: changes in the right side of constraints (b_i)

Consider now constraint **II**

$$x + 4y \leq 100$$

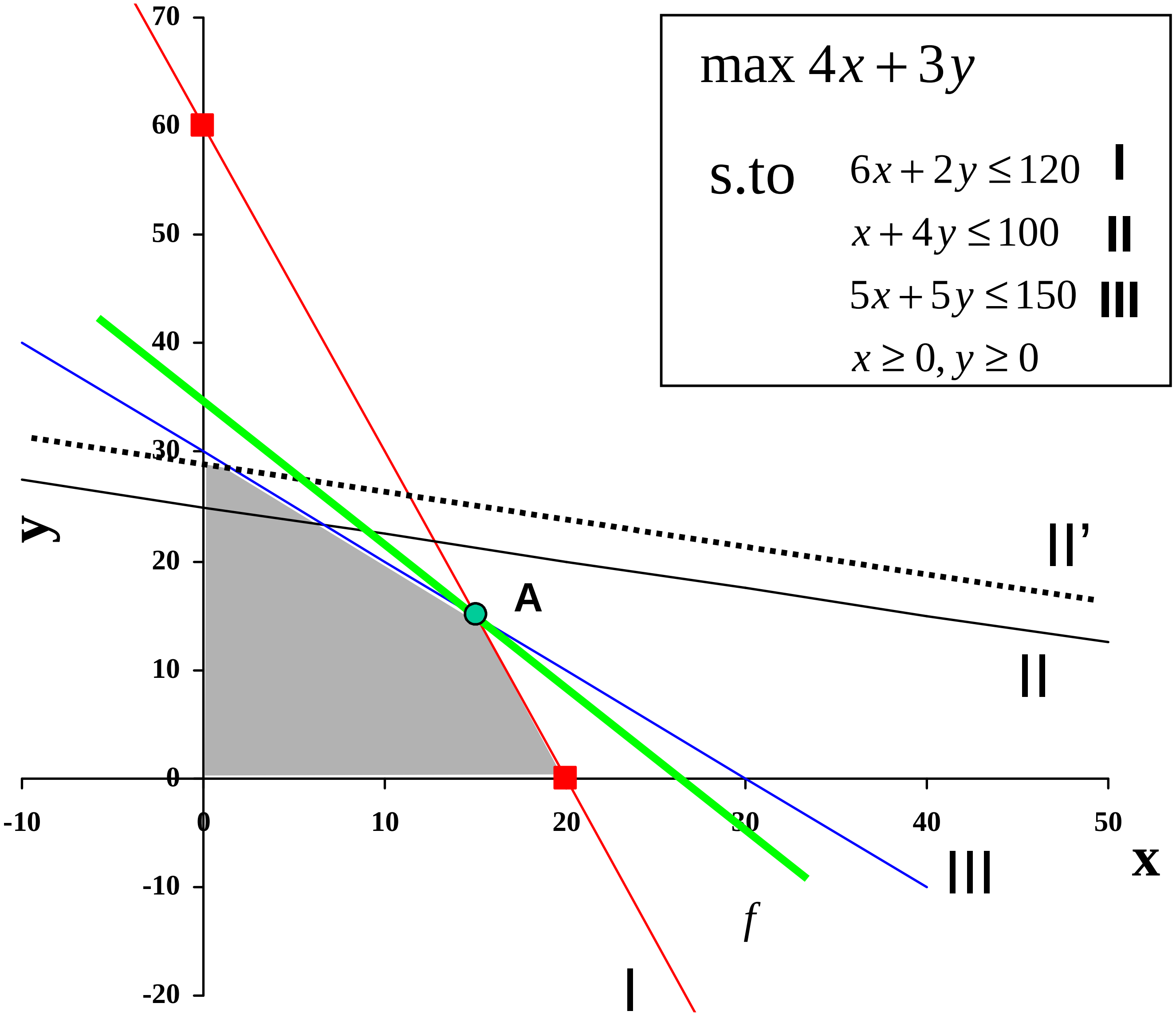
What happens if we increase the production capacity (k) in section **II**?

$$x + 4y \leq k$$

- For k > 100, the optimal solution does not change.
- And when k < 100? (for example, if k = 75?)
- And if k < 75?

$$\max 4x + 3y$$

$$\text{s.to} \quad \begin{aligned} 6x + 2y &\leq 120 & \text{I} \\ x + 4y &\leq 100 & \text{II} \\ 5x + 5y &\leq 150 & \text{III} \\ x \geq 0, y &\geq 0 \end{aligned}$$



In short, for a maximization problem

Case 1 - changes in the coefficients of the objective function (c_j)

For a particular optimal solution, and for each coefficient of the objective function (cj), it is possible to determine an interval of variation that will keep the optimal solution unchanged (note that the value of the objective function may change).

If, in the optimal solution, the value of a decision variable is zero (xi = 0), its reduced cost is the increment that the corresponding coefficient in the objective function should have in order to include that variable in the optimal solution (xi > 0).

Case 2: changes in the right side of constraints (b_i)

(i) If a constraint is active (there is no slack or surplus) then increasing or decreasing the amount of the resource associated to that constraint could lead to a change in the value of the objective function in the optimal solution.
The shadow price of a resource is the increment in the objective function generated by an additional unit of that resource.

.

(ii) If there is a slack in a constraint (the constraint is inactive), the value of the objective function in the optimal solution does not alter if we increase the amount of available resource. However, if we decrease the amount of available resource, the value of the objective function in the optimal solution may change.

Solution of Cereals, Ltd obtained by Lindo software (http://www.lindo.com)

LP OPTIMUM FOUND AT STEP 1

OBJECTIVE FUNCTION VALUE

1) 105.0000

VARIABLE	VALUE	REDUCED COST
X	15.000000	0.000000
Y	15.000000	0.000000

ROW	SLACK OR SURPLUS	**DUAL PRICES**
2)	0.000000	0.250000
3)	25.000000	0.000000
4)	0.000000	0.500000

SENSITIVITY ANALYSIS

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VAR	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X	4.000000	5.000000	1.000000
Y	3.000000	1.000000	1.666667

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	120.000000	60.000000	33.333332
3	100.000000	INFINITY	25.000000
4	150.000000	22.727272	49.999996

Exercise
Sensitivity Analysis using the optimal solution obtained by Lindo

Consider again the example of Cereals, Ltd.

Due to new market challenges, it was decided to produce a new cereal, barley (‘cevada’, in Portuguese).
The marginal profits now are of 4 € per ton of wheat, 1 € per ton of corn and 3 € per ton of barley.
In each section (I, II and III) , the production times for each ton of barley are 3.5, 6 and 4 hours, respectively. Also, the company is committed to produce at least 12 tons of wheat and 10 tons of barley each week.

Problem formulation:

$$
\begin{aligned}
\max\ & 4x + y + 3z \\
\text{s.to}\ & 6x + 2y + 3.5z \le 120 \\
& x + 4y + 6z \le 100 \\
& 5x + 5y + 4z \le 150 \\
& x \ge 12 \\
& z \ge 10 \\
& x, y, z \ge 0
\end{aligned}
$$

The new problem has been solved using Lindo, and we obtained the following tableaux for the optimal solution

OBJECTIVE FUNCTION VALUE

1) 89.14286

VARIABLE	VALUE	REDUCED COST
X	12.000000	0.000000
Y	0.000000	0.714286
Z	13.714286	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	0.857143
3)	5.714286	0.000000
4)	35.142857	0.000000
5)	0.000000	-1.142857
6)	3.714286	0.000000

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X	4.000000	1.142857	INFINITY
Y	1.000000	0.714286	INFINITY
Z	3.000000	INFINITY	0.666667

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	120.000000	3.333333	12.999999
3	100.000000	INFINITY	5.714286
4	150.000000	INFINITY	35.142857
5	12.000000	2.166667	0.615385
6	10.000000	3.714286	INFINITY

Questions:

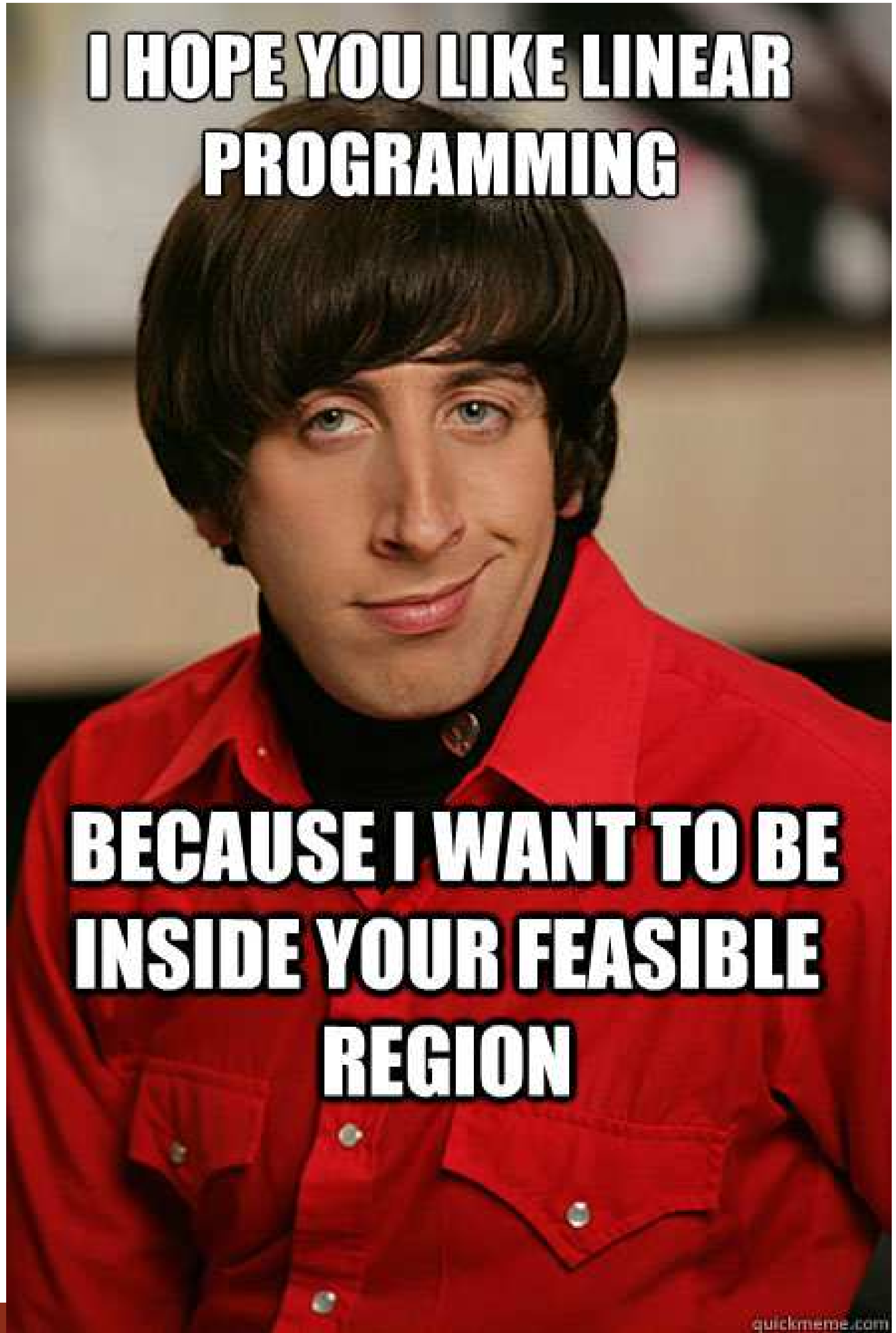
1. What is the profit in the optimal solution? What is the optimal production plan?
2. Suppose that the wheat (x) profit has an increment of 0.75 €/ton. Which is the impact of this change on the optimal production plan and on the profit? And if the increment is of 1.5 €/ton? And if the wheat profit decreases 0.05 €/ton? And if it decreases 2 €/ton?
3. What should be done to make corn production (y) profitable?
4. If the company was forced to produce some corn, which would be the impact of that decision on the company profit?
5. Suppose that there is an additional hour available in section I. What is the impact on profit? And if there are 2 additional hours?

4. Suppose that the number of available hours in section I diminishes to 119 hours. What is the impact on profit? And if we only dispose of 105 hours in this section?

6) Comment the importance of hiring multifunctional employees that can work in different sections.
7) What happens if the minimum amount of wheat to produce increases of 1.5 tons? And if it decreases of 0,29 tons?
8) What happens if the minimum amount of corn to produce increases of 0.5 tons? And if it decreases of 7 tons?

Linear Programming

Simplex Method


I HOPE YOU LIKE LINEAR PROGRAMMING
BECAUSE I WANT TO BE INSIDE YOUR FEASIBLE REGION
quickmeme.com

Simplex Method

Motivation:

The graphical method cannot be applied to problems with more than 2 variables.

Basic Idea:

The Simplex method is based in the fact that any LP optimal solution lies on a vertex of the feasible region.

Basic Method:

- Start by calculating the objective function value for any vertex of the domain.
- Jump to an adjacent vertex corresponding to a better objective function value.
- Continue with this process until it is no possible to improve the objective function.

Cereals, Ltd

Cereals, Ltd is a company specialized in preparing and packing wheat and corn for several retailing stores.There are no limits concerning the supply of these cereals and demand can be considered unlimited (in other words, the whole the production is sold).
The production is processed in three phases: Pre-Processing (I), Processing (II) and Packing (III)

	I Pre-Processing	II Processing	III Packing	
Weekly production capacity (h)	120	100	150	
Time (h) needed to prepare 1 ton				Profit(€/ton)
Wheat	6 h	1 h	5 h	4
Corn	2 h	4 h	5 h	3

Currently, the company produces 18 tons of wheat and 6 tons of corn per week.
Is this the best option?

Cereals, Ltd - Formulation

Decision variables

x = tons of wheat to produce weekly

y = tons of corn to produce weekly

Constraints

$$6x + 2y \leq 120$$

$$x + 4y \leq 100$$

$$5x + 5y \leq 150$$

$$x \geq 0, y \geq 0$$

Objective function: to maximize the profit $\max 4x + 3y$

Graphical method

$$\max \text{ f } = 4x_1 + 3x_2$$

$$\text{s.to} \quad 6x_1 + 2x_2 \le 120$$
$$x_1 + 4x_2 \le 100$$
$$5x_1 + 5x_2 \le 150$$
$$x_1 \ge 0, x_2 \ge 0$$

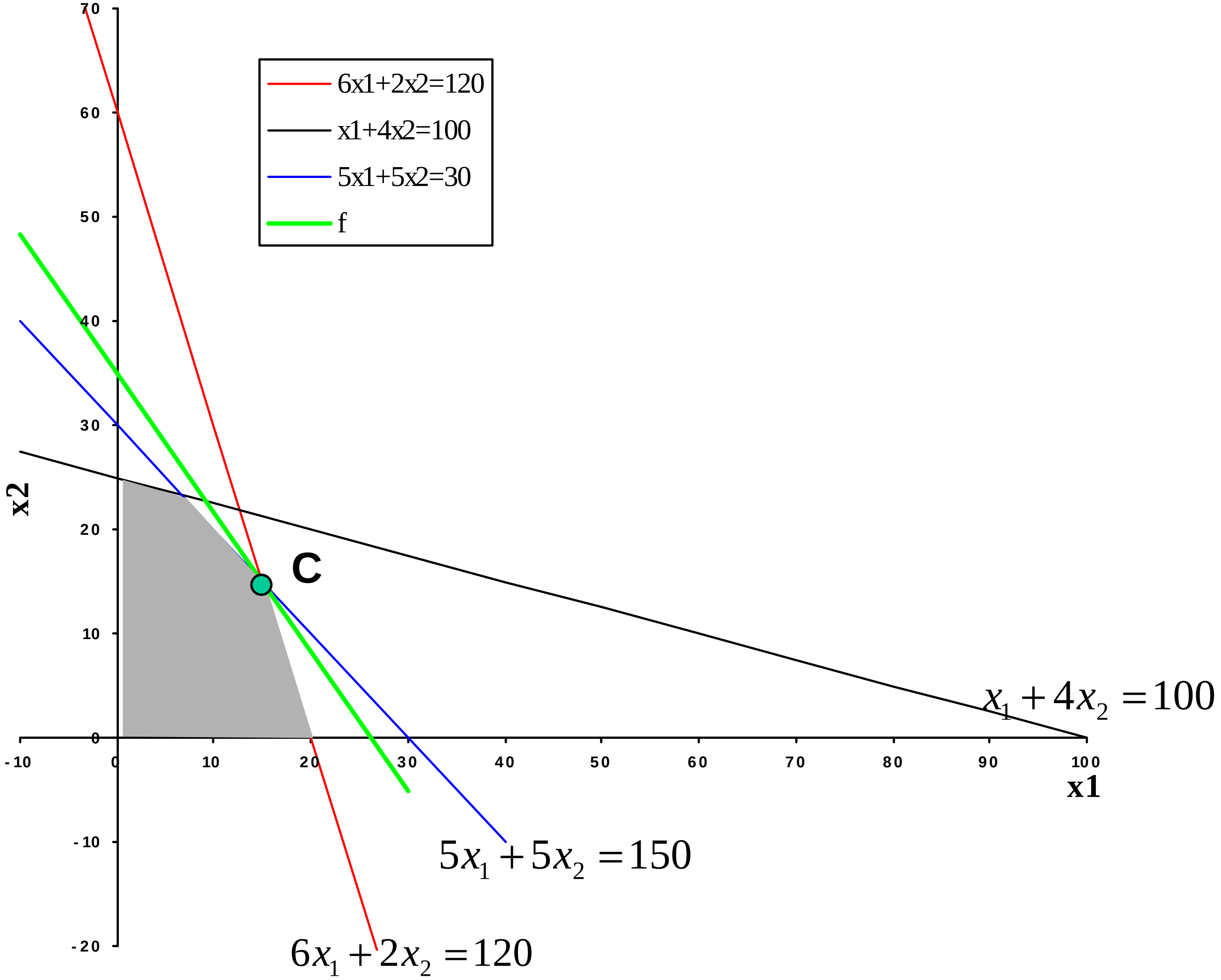
In order to obtain the optimal solution we have to calculate the intersection point (A) of the straight lines:

$$6x_1 + 2x_2 = 120$$
$$5x_1 + 5x_2 = 150$$

C = (15,15)

This production plan yields a profit of 105 €

$$4x_1 + 3x_2 = 105$$



Simplex Method

Step 1: Writing the problem in the canonical form

Consider that all the inequalities (constraints) are of <= type with positive values in the right side. The LP is in canonical form when the inequalities are changed to equalities by adding a **slack variable** to each constraint.

Maximizar (ou Minimizar) $f = c_1 \cdot x_1 + c_2 \cdot x_2 + \dots + c_n \cdot x_n$

sujeito a

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + \dots + a_{1n} \cdot x_n = b_1$$
$$a_{21} \cdot x_1 + a_{22} \cdot x_2 + \dots + a_{2n} \cdot x_n = b_2$$
$$\dots\dots\dots$$
$$a_{m1} \cdot x_1 + a_{m2} \cdot x_2 + \dots + a_{mn} \cdot x_n = b_m$$

Com: $m < n$ restrições

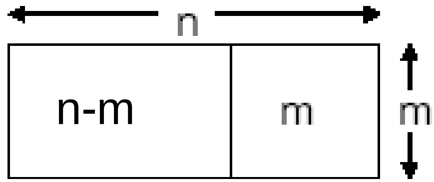
$$b_1, b_2, \dots, b_m \geq 0$$
$$x_1, x_2, \dots, x_n \geq 0$$

Exemplo:

$$\max f = 4x_1 + 3x_2$$
$$6x_1 + 2x_2 + s_1 = 120$$
$$x_1 + 4x_2 + s_2 = 100$$
$$5x_1 + 5x_2 + s_3 = 150$$
$$x_1 \geq 0, x_2 \geq 0$$

Simplex Method (Step 2)

Step 2: Find an initial feasible basic solution



Consider a LP in the canonical form with

- n variables
- m constraints (equalities), with m<n

A **basic solution** is obtained by assigning n-m variables to zero and solving the constraints (equations) for the remaining variables (m).

The n-m null variables are referred to as **non-basic variables**.
The others are the **basic variables**.

The basic solutions can be:

feasible basic solutions: when all the basic variables are non-negative.
infeasible basic solutions: when at least one basic variable is negative.

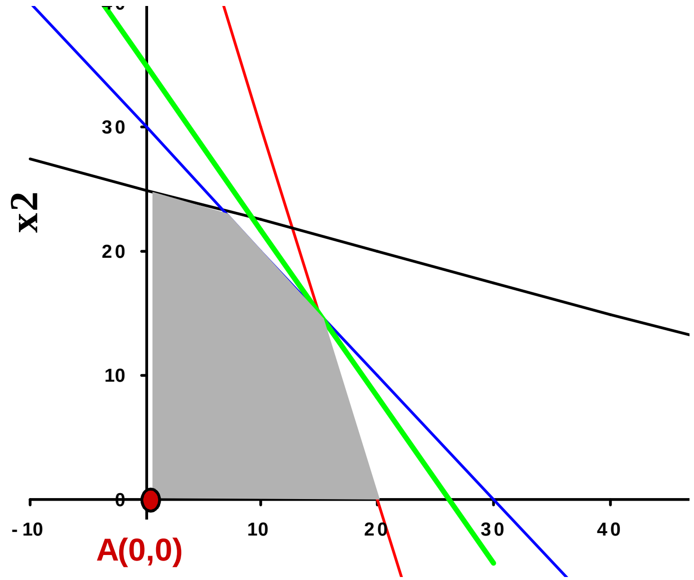
$$\max f = 4x_1 + 3x_2$$

$$6x_1 + 2x_2 + s_1 = 120$$

$$x_1 + 4x_2 + s_2 = 100$$

$$5x_1 + 5x_2 + s_3 = 150$$

$$x_1 \geq 0, x_2 \geq 0$$



Tabular form

basis	X1	X2	S1	S2	S3	value
S1	6	2	1	0	0	120
S2	1	4	0	1	0	100
S3	5	5	0	0	1	150
f	4	3	0	0	0	0

- The coefficient matrix of the basic variables is an identity matrix (or is convertible into one by row or column swaps)
- The coefficients of the basic variables in the objective function are null.

Basic variables: S1 = 120, S2 = 100, S3 = 150

Non-basic variables: X1 = 0, X2 = 0

Point A

Simplex Method (step 3)

Step 3: Verify if the basic solution found is optimal:

For a maximization problem, if all the coefficients in the objective function are non-positive (<=0), then the problem is solved.

For a minimization problem, if all the coefficients in the objective function are non-negative (>=0), then the problem is solved.

basis	X1	X2	S1	S2	S3	value
S1	6	2	1	0	0	120
S2	1	4	0	1	0	100
S3	5	5	0	0	1	150
f	4	3	0	0	0	0

- At this moment, the value of x1 and x2 is zero (and also the o.f. value).
- As both coefficients in the objective function (o.f.) are positive numbers, if any of these variables becomes positive, the value of the o.f. would increase

  (f = 4 x1 + 3 x2).

Which of these two variables should be chosen to enter the basis?

Simplex Method (Step 4)

Step 4: Find a new basic solution that improves the objective function

To find a new basic solution, we will choose a **non-basic variable to enter the basis** and a **basic variable to leave the basis.**

Step 4.1: **Choose a non-basic variable to enter the basis** . The column corresponding to this variable is called *pivot column*.
In a maximization problem choose, amongst the variables with positive coefficients, the one with the highest positive value. In a minimization problem choose, amongst the variables with negative coefficients, the one with the highest negative value

Step 4.2 Choose a basic variable to leave the basis, The row corresponding to this variable is called *pivot row*.
Calculate the ratio between the right side members of the equations and the corresponding members in the pivot column. From the set of non-negative ratios, the row with the lowest ratio will be the pivot row.

	basis	X1	X2	S1	S2	S3	value		
L1	S1	6	2	1	0	0	120	120/6 =	20
L2	S2	1	4	0	1	0	100	100/1 =	100
L3	S3	5	5	0	0	1	150	150/5 =	30
f	f	4	3	0	0	0	0		

Pivot row

*Choose to leave the basis the variable with the lowest **non-negative** ratio.*

Pivot column

In a maximization problem, choose to enter the basis the variable with the highest positive value in the o.f.

-f = 0

This value is the simmetric of the o.f. value for the current solution.

x1 was chosen to enter the basis, keeping x2 = 0 (non-basic).
X1 should take the highest possible value while satisfying the constraints.

$$s_1 = 120 - 6x_1 \quad \text{Since } s_1 \geq 0, 120 - 6x_1 \geq 0 \Leftrightarrow \boxed{x_1 \leq \frac{120}{6} = 20}$$

Choose the most restrictive condition

$$s_2 = 100 - x_1 \quad \text{Since } s_2 \geq 0, 100 - x_1 \geq 0 \Leftrightarrow x_1 \leq 100$$

$$s_3 = 150 - 5x_1 \quad \text{Since } s_3 \geq 0, 150 - 5x_1 \geq 0 \Leftrightarrow x_1 \leq \frac{150}{5} = 30$$

When x1 = 20, S1 is null (leaves the basis)

	basis	X1	X2	S1	S2	S3	value		
L1	S1	6	2	1	0	0	120	120/6 =	20
L2	S2	1	4	0	1	0	100	100/1 =	100
L3	S3	5	5	0	0	1	150	150/5 =	30
f	f	4	3	0	0	0	0		

Pivot row

Choose to leave the basis the variable with the lowest non-negative ratio.

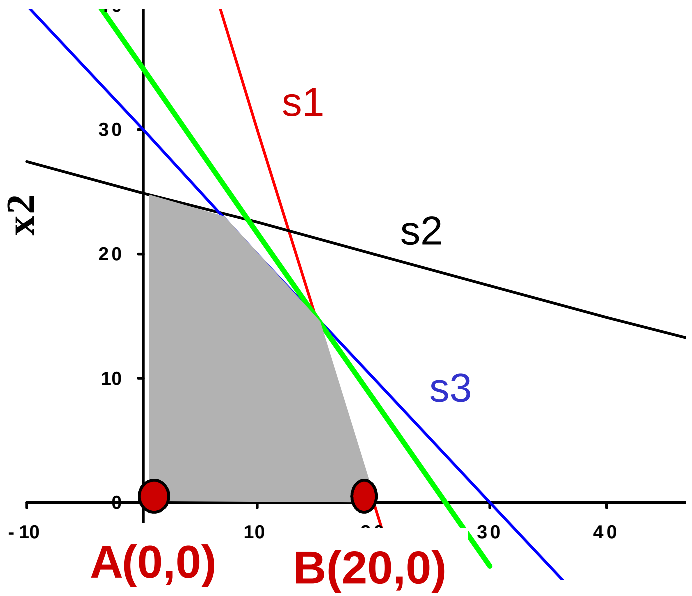
Pivot column

-f = 0

In a maximization problem, choose to enter the basis the variable with the highest positive value in the o.f.

X1 enters the basis, taking a non-negative value
S1 leaves the basis , taking the null value

Going from point A to Point B (20,0)



Simplex Method (step 5)

Step 5: Update Simplex tableaux to identify the new basic solution

The procedure is based in algebraic operations performed on the rows of the Simplex tableaux in order to build a new identity matrix with the rows and columns of the basic variables.

We perform algebraic operations in order to set the value 1 to the intersection of the pivot row and the pivot column and zero values in all the other coefficients of the pivot column (including the o.f.).

After identifying the new basic solution, go to step 3 to verify if the new solution is optimal.

Iteration 0 (point A)

	basis	X1	X2	S1	S2	S3	value		
L1	S1	6	2	1	0	0	120	120/6 =	20
L2	S2	1	4	0	1	0	100	100/1 =	100
L3	S3	5	5	0	0	1	150	150/5 =	30
f	f	4	3	0	0	0	0		

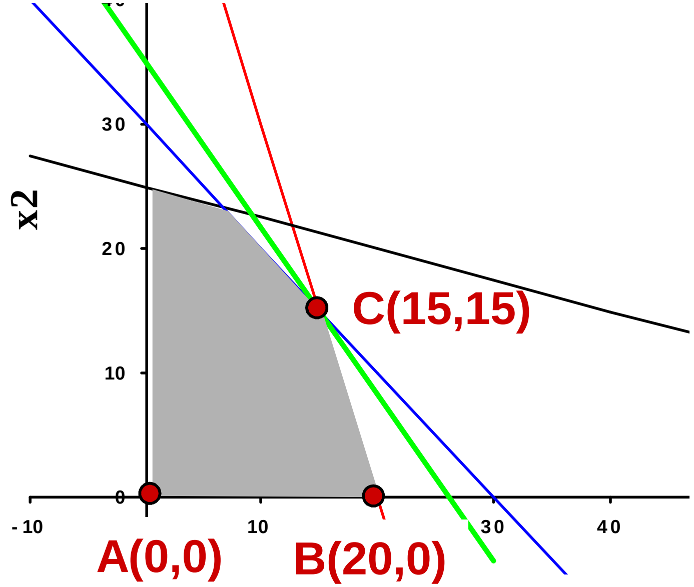
Divide by 6 all the values in this line

Iteration 1 (point B)

	basis	X1	X2	S1	S2	S3	value		
L'1 = L1/6	X1	1	0,3333	0,1667	0	0	20	20/0,33=	60
L'2 = L2 - L'1	S2	0	3,6667	-0,167	1	0	80	80/3,66=	21,82
L'3 = L3 - 5.L'1	S3	0	3,3333	-0,833	0	1	50	50/3,33=	15
f' = f' - 4.L'1	f	0	1,6667	-0,667	0	0	-80		

X2 enters the basis with a non-negative value
S3 leaves the basis, with null value

Going from point B to Point C (15,15)




BRACE YOURSELVES
(ANOTHER) ITERATION IS COMING

Iteration 1 (point B)

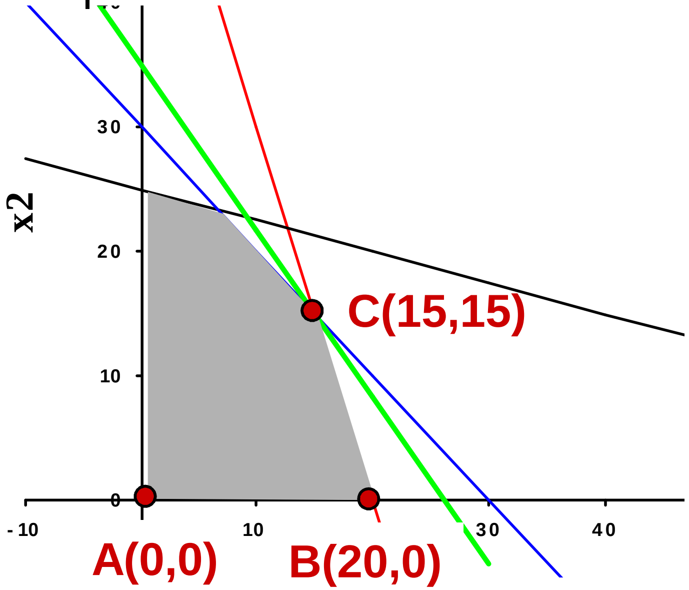
basis	X1	X2	S1	S2	S3	value
X1	1	0,3333	0,1667	0	0	20
S2	0	3,6667	-0,167	1	0	80
S3	0	3,3333	-0,833	0	1	50
f	0	1,6667	-0,667	0	0	-80

Iteration 2 (point C)

	basis	X1	X2	S1	S2	S3	value
L"1 = L'1 - 0,33.L"3	X1	1	0	0,25	0	-0,1	15
L"2 = L'2 - 3,67.L"3	S2	0	0	0,75	1	-1,1	25
L"3 = L'3 / 3,33	X2	0	1	-0,25	0	0,3	15
f" = f' - 1,67.L"3	f	0	0	-0,25	0	-0,5	-105

***Point C is the optimal solution**, since none of the coefficients in the o.f is positive (we are solving a maximization problem)*

f = 4*X1+3*X2 = 4*15+3*15 = 60+45=**105**



Exercise 1

$$\max f = -x_1 + 2x_2 + x_3$$
$$2x_1 + x_2 - x_3 \leq 2$$
$$2x_1 - x_2 + 5x_3 \leq 6$$
$$4x_1 + x_2 + x_3 \leq 6$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

Iteration 0 : Point (0,0,0)

	basis	X1	X2	X3	S1	S2	S3	value		
L1	S1	2	1	-1	1	0	0	2	2/1 =	2
L2	S2	2	-1	5	0	1	0	6		
L3	S3	4	1	1	0	0	1	6	6/1 =	6
f	f	-1	2	1	0	0	0	0		

Variable X2 enters the basis and variable S1 leaves the basis

Iteration 1 : Point (0,2,0)

	basis	X1	X2	X3	S1	S2	S3	value		
L'1 = L1	X2	2	1	-1	1	0	0	2		
L'2 = L2+L'1	S2	4	0	4	1	1	0	8	8/4 =	2
L'3 =L3-L'1	S3	2	0	2	-1	0	1	4	4/2 =	2
f' = f-2*L'1	f	-5	0	3	-2	0	0	-4		

Variable X3 enters the basis and variable S2 leaves the basis

Iteration 1 : Point (0,2,0)

	basis	X1	X2	X3	S1	S2	S3	value		
L'1 = L1	X2	2	1	-1	1	0	0	2		
L'2 = L2+L'1	S2	4	0	4	1	1	0	8	8/4 =	2
L'3 =L3-L'1	S3	2	0	2	-1	0	1	4	4/2 =	2
f' = f-2*L'1	f	-5	0	3	-2	0	0	-4		

Variable X3 enters the basis and variable S2 leaves the basis

Iteration 2 : Point (0,4,2)

	basis	X1	X2	X3	S1	S2	S3	value
L"1 =L'1+L"2	X2	3,00	1	0	1,25	0,25	0	4,00
L"2 = L'2/4	X3	1,00	0	1,00	0,25	0,25	0	2,00
L"3 =L'3-2L"2	S3	0	0	0	-1,5	-0,5	1	0
f" = f' - 3*L"2	f	-8	0	0	-2,75	-0,75	0	-10

Optimal solution: X1 = 0
X2 = 4
X3 = 2

Optimal value of o.f. = 10