MCV4U

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KU: 9 /8 TH: 6 /6 APP: 7.5 /8

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Unit 2 Quiz: Derivatives

1. Find the first derivative to the following functions. Express your answers in unsimplified form. [8] K

a) $f(x) = 5x^3 - 3x^2 + 2x + 5$

$f'(x) = 15x^2 - 6x + 2$ ✓ ✓

b) $g(x) = \sqrt{x}(x^2 - 1)$

$g'(x) = (\sqrt{x})(2x) + (x^2-1)\left(\frac{1}{2\sqrt{x}}\right)$ ✓ ✓

8/8

c) $h(x) = (x-1)^2(x^2+2)^3$

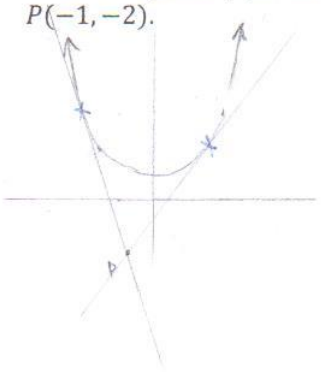
$h'(x) = (x-1)^2(3)(x^2+2)^2(2x) + (x^2+2)^3(2)(x-1)$ ✓ ✓

d) $i(x) = \frac{-2-3x}{x+x^2+2x^3}$

$= (-2-3x)(x+x^2+2x^3)^{-1}$

$i'(x) = (-2-3x)(-1)(x+x^2+2x^3)^{-2}(1+2x+6x^2) + (x+x^2+2x^3)(-3)$ ✓ ✓

2. Find the equation(s) of the tangent line(s) to the curve $f(x) = x^2 + 1$ passing through the point $P(-1, -2)$. [5] A



$f'(x) = 2x$

point on parabola (x, x^2+1)

5/5

$m_{tan} = \frac{y_2 - y_1}{x_2 - x_1}$

$2x = \frac{x^2+1-(-2)}{x-(-1)}$

$2x = \frac{x^2+1+2}{x+1}$

$2x(x+1) = x^2+1+2$

$2x^2+2x = x^2+1+2$

$x^2+2x-3 = 0$

$(x+3)(x-1) = 0$

$x = -3$ or $x = 1$

sub $x=-3$ into $f(x)$: $f(x) = (-3)^2+1 = 10$, point $(-3, 10)$

sub $x=1$ into $f(x)$: $f(x) = (1)^2+1 = 2$, point $(1, 2)$

m_{tan} at point $(-3, 10)$: $m_{tan} = 2x = 2(-3) = -6$

$y = mx+b$

$10 = -6(-3)+b$

$10 = 18+b$

$-8 = b$

$y = -6x-8$ ✓

m_{tan} at point $(1, 2)$: $m_{tan} = 2x = 2(1) = 2$

$y = mx+b$

$2 = 2(1)+b$

$2 = 2+b$

$0 = b$

$y = 2x$ ✓

∴, the equations of the tangent lines are $y = -6x-8$ and $y = 2x$

8+5=13

3. Given the parabola $y = -x^2 + 10x - 16$, find the equation of the normal at the point $(4,8)$. [3] A

$\frac{dy}{dx} = -2x + 10$

m_{tan} at point $(4,8)$

$m_{tan} = -2(4) + 10$

$= -8 + 10$

$= -2$ careful!!

$m_{tan\perp} = \frac{1}{2}$ ok

sub in $m_{tan\perp}$ and point $(4,8)$

$y = mx + b$

$8 = \frac{1}{2}(4) + b$

$8 = 2 + b$

$b = 6$

$y = \frac{1}{2}x + 6$ ok

$\therefore$ the equation of the normal is $y = \frac{1}{2}x + 6$

2.5/3

4. Mr. Wong was doing the solutions on the board; however, he has made many mistakes. [6] T

i) Find which step did he **FIRST** makes his mistake on.
ii) Explain what mistake he made.
iii) Correct his mistake.

a) Differentiate $y = \frac{5x^2}{2x^5 - 3x^3}$

Solution:

$y = \frac{5x^2}{2x^5 - 3x^3}$

Step 1: $y' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

Step 2: $y' = \frac{5x^2(10x^4 - 9x^2) - 10x(2x^5 - 3x^3)}{(2x^5 - 3x^3)^2}$

Step 3: $y' = \frac{50x^6 - 45x^4 - 20x^6 - 30x^4}{4x^{10} - 9x^6}$

Step 4: $y' = \frac{30x^6 - 75x^4}{4x^{10} - 9x^6}$

i) Which step was the first mistake?

Step 2 ✓

ii) Why was it wrong?

$g(x) \neq 5x^2$ $f'(x) \neq (10x^4 - 9x^2)$

he switched $f(x)$ and $g(x)$, which will affect the answer ✓

iii) What should that step look like?

Step 2:

$y' = \frac{10x(2x^5 - 3x^3) - 5x^2(10x^4 - 9x^2)}{(2x^5 - 3x^3)^2}$ ✓

6/6

b) Differentiate $y = x(5x^3 - 7)^3$

Solution:

$y = x(5x^3 - 7)^3$

Step 1: $y' = x(3)(5x^3 - 7)^2 + (1)(5x^3 - 7)^3$ (missing)

Step 2: $y' = 3x(25x^6 - 49) + (125x^9 - 243)$

Step 3: $y' = 75x^7 - 147x + 125x^9 - 243$

Step 4: $y' = 1125x^8 + 525x^6 - 147$

i) Which step was the first mistake?

Step 1 ✓

ii) Why was it wrong?

did not derive $(5x^3 - 7)^3$ properly. didn't put in derivative of the inner function $5x^3 - 7$ ✓

iii) What should that step look like?

Step 1:

$y' = x(3)(5x^3 - 7)^2(15x^2) + (1)(5x^2 - 7)^3$ ✓

2.5+6