MCR 3U **Test 2 - Polynomials & Rational Expressions** Name Uni Lee

Calculators are allowed. Be sure to show work of good form for full marks. Good luck! ☺

Part A: Knowledge and Understanding **(22 marks)**

Multiple Choice - Write the **CAPITAL** letter of the best answer on the line. (1 mark each)

1. Expand and simplify $6x(9x - 3x^2 - 10)$ B ✓

   a. $-18x^2 + 54x - 60$ c. $51x^2 - 10$

   b. $-18x^3 + 54x^2 - 60x$ d. -24

2. Which of the following are factors for the polynomial $4s^2 - 16s + 16 - t^2$? D ✓

   a. $(2s - 4 - t)^2$ c. $(2s + 4 - t)(2s - 4 + t)$

   b. $4(s - 2 - t)^2$ d. $(2s - 4 - t)(2s - 4 + t)$

3. Simplify. $\frac{4x-5}{6} \div \frac{8x-10}{12}$ B ✓

   a. $48x - 60$ c. $\frac{(4x-5)(4x-5)}{24}$

   b. 1 d. $8x - 10$

4. Referring to question 3, what is/are the restrictions? A ✓

   a. $\frac{5}{4}$ c. 10

   b. 12 d. 0

5. Simplify: $\frac{m-n}{n-m}$ C ✓

   a. 0 c. -1

   b. 1 d. $\frac{-m+n}{m+n}$

6. Simplify $-2\sqrt{45} - 3\sqrt{20} - 2\sqrt{6}$ C ✓

   a. $12\sqrt{5} - 2\sqrt{6}$ c. $-12\sqrt{5} - 2\sqrt{6}$

   b. $5\sqrt{10} - \sqrt{2}$ d. $12\sqrt{10} - \sqrt{2}$

6

7. State the restrictions for the following rational expressions, if they exist. (3)

(a) $\frac{2x-1}{4x}$ (b) $\frac{4x^2+5}{x^2-4}$ (c) $\frac{5}{x^2-4x+5}$

2

$x \neq 0$ ✓ $x \neq \pm 2$ ✓ no restrictions ✗

8. Factor fully.

a) $n^3 - 2n^2 + 4n - 8$ (2)

$= n^2(n-2) + 4(n-2)$

$= (n^2+4)(n-2)$ ✓

b) $y^4 - 5y^2 - 36$ (2)

integers no fractions

$(y^2 - 2.23y)(y^2 + 2.23y) \cdot 36$

$= (y - \sqrt{5}\,y + 6)(y^2 + \sqrt{5}\,y - 6)$ 1

$= (y-3)(y+3)(y+2)^2$

5

9. Simplify fully. **Do not state restrictions.**

a) $\frac{b^2+2b-80}{2b^3-24b^2+64b}$ (3)

$= \frac{(b+10)(b-8)}{2b(b^2-12b+32)}$

$= \frac{(b+10)\cancel{(b-8)}}{2b\cancel{(b-8)}(b-4)}$

$= \frac{(b+10)}{2b(b-4)}$ ✓

b) $(2\sqrt{3}+5\sqrt{2})(\sqrt{2}-2\sqrt{2})$ (3)

$= 2\sqrt{6} - 4\sqrt{6} + 5\sqrt{4} - 10\sqrt{4}$

$= -2\sqrt{6} - 5\sqrt{4}$ ✓

$= -2\sqrt{6} - 10$

6

c) $\frac{5\sqrt{2}}{\sqrt{7}-3\sqrt{2}}$ (3)

$= \frac{5\sqrt{2}}{\sqrt{7}} - \frac{5\sqrt{2}}{3\sqrt{2}}$ can't separate like this

$= \frac{5\sqrt{2}}{\sqrt{7}} - \frac{5}{3}$ $= \frac{-30-5\sqrt{14}}{11}$

11

Part B: Application – Show work of good form for FULL marks. **(14 marks)**

10. The length of a rectangle is $\frac{w+3}{w^2+w-12}$ and the width is $\frac{w^2+7w+12}{w^2-9}$. What is the area of the rectangle in simplest form? (4)

l

w $\boxed{A}$

$A = l \times w$

$l = \frac{w+3}{w^2+w-12}$ $w = \frac{w^2+7w+12}{w^2-9}$

$= \left(\frac{w+3}{w^2+w-12}\right) \times \left(\frac{w^2+7w+12}{w^2-9}\right)$

$= \frac{\cancel{(w+3)}}{\cancel{(w+4)}(w-3)} \times \frac{(w+3)\cancel{(x+4)}}{\cancel{(w+3)}(w-3)}$

$\boxed{= \frac{(w+3)}{(w-3)^2} \text{ units}^2}$ ✓

$w \neq \pm 3, -4$

4

. Simplify and state any restrictions on the variable. Remember the order of operations.

$$\frac{1}{x^2-1} - \frac{2}{x^2+3x+2} \times \frac{x+1}{4x^2}$$ (5)

$$= \frac{1}{(x+1)(x-1)} - \frac{2}{\cancel{(x+1)}(x+2)} \times \frac{\cancel{(x+1)}}{4x^2}$$

$$= \frac{(+1)}{(x+1)(x-1)} + \frac{(-2)}{(x+2)(4x^2)}$$

$$= \frac{1(x+2)(4x^2) + (-2)(x+1)(x-1)}{(x+1)(x-1)(x+2)(4x^2)}$$

$$= \frac{(x+2)(4x^2)+(-2x-2)(x-1)}{(x+1)(x-1)(x+2)(4x^2)}$$

$$= \frac{4x^3+8x^2 + (-2x^2+2x-2x+2)}{(x+1)(x-1)(x+2)(4x^2)}$$

$$\boxed{= \frac{4x^3+6x^2+2}{(x+1)(x-1)(x+2)(4x^2)}}$$ ✓

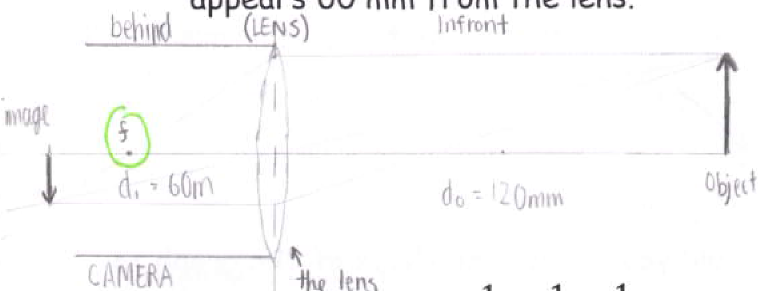
$$\boxed{x \neq \pm 1, -2, 0}$$ ✓

4.5

12. The formula for the focal length f in a camera is: $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$ where p represents the distance from the object to the lens and q represents the distance from the lens to the image.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

(a) Calculate the focal length if the object is 120 mm in front of a lens and the image appears 60 mm from the lens. (3)



$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{f} = \frac{1}{120mm} + \frac{1}{60mm}$$

$$\frac{1}{f} = \frac{3}{120mm}$$ ✓

$3f = 120$ mm

$\boxed{f = 40mm \text{ behind the lens}}$

(b) Rewrite the formula $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$ so that it is in the form $f =$ (2)

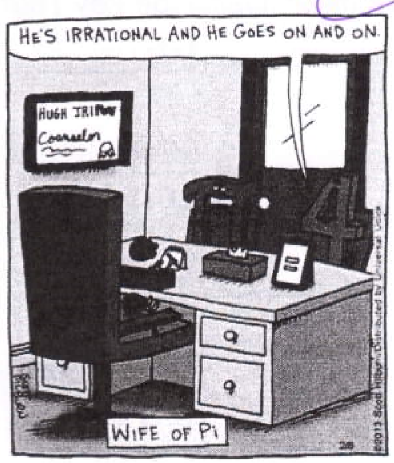
$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{f} = \frac{q}{pq} + \frac{p}{pq}$$

$$\frac{1}{f} = \frac{q+p}{pq}$$

$$f(q+p) = pq$$

$$\boxed{f = \frac{pq}{q+p}}$$ ✓



9.5

Part C: Communication (5 marks + 3 marks for overall good form on the test) **(8 marks)**

(Equal signs properly used, proper order of expressions)

12. Explain 2 different ways to determine if two expressions are equivalent. Then use one of the methods to determine if they are equivalent. (5)

① simplify and see if they're the same

② sub in 3 diff numbers and see if answers are the same

$f(x) = 3x^3 + 9x + 13,\ x \neq 0$ and $g(x) = \dfrac{9x^4 + 27x^2 + 39x}{3x},\ x \neq 0$

sub 10 into x

$f(x) = 3x^3 + 9x + 13$
$= 3(10)^2 + 9(10) + 13$
$= 403$

$g(x) = \dfrac{9x^4 + 27x^2 + 39x}{3x}$
$= \dfrac{9(10)^4 + 27(10)^2 + 39(10)}{3(10)}$
$= 3103$

$\therefore f(x) \neq g(x)$

sub 5000 into x

$f(x) = 3x^2 + 9x + 13$
$= 3(5000)^2 + 9(5000) + 13$
$= 75045013$

$g(x) = \dfrac{9x^4 + 27x^2 + 39x}{3x}$
$= \dfrac{9(5000)^4 + 27(5000)^2 + 39(5000)}{3(5000)}$
$= 375000045000$

$\therefore f(x) \neq g(x)$

sub 2 into x

$f(x) = 3x^2 + 9x + 13$
$= 3(2)^2 + 9(2) + 13$
$= 43$

$g(x) = \dfrac{9x^4 + 27x^2 + 39x}{3x}$
$= \dfrac{9(2)^4 + 27(2)^2 + 39(2)}{3(2)}$
$= 55$

$\therefore f(x) \neq g(x)$

therefore, the 2 expressions are not equivalent

Part D: Thinking – Show work of good form for FULL marks. **(8 marks)**

13. Solve for x if the reciprocal of $\left(\dfrac{1}{x} - 1\right)$ is - 2. (4)

$\frac{1}{x} - 1 = -\frac{1}{2}$

reciprical of $\frac{-2}{1} = -\frac{1}{2}$

$\frac{1}{x} = \frac{1}{2}$

$x = 2$

$\boxed{x = 2}$

14. If you were to graph $y = \dfrac{12x^3 - 5x^2 - 2x}{3x^2 - 2x}$, what would you expect the shape of the graph to look like? Describe key features but DO NOT GRAPH. Show work to support your answer. (4)

$y = \dfrac{12x^3 - 5x^2 - 2x}{3x^2 - 2x}$

$= \dfrac{\cancel{(x)}\cancel{(3x-2)}(4x+1)}{\cancel{(x)}\cancel{(3x-2)}}$

$\boxed{y = (4x+1)}$ $\boxed{x \neq 0, \tfrac{2}{3}}$

Linear

it would look like a line going ↗ with a hole at $x = 0$ and $x = \frac{2}{3}$

Knowledge	Thinking	Communication	Application
17 / 22	8 / ~~9~~ 8	8 / 8	13.5 / 14

Well Done!