MCV4U

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Date: April 17 2015

Formative Quiz: Derivatives of Sinusoidal Functions

1. Find $\frac{dy}{dx}$ for each of the following functions. Do not need to simplify. [12]

a) $y = \sin^2 x + \cos\left(\frac{3x}{2}\right)$

$= \sin^2 x + \cos(1.5x)$

$y' = 2\sin x\cos x + (-\sin(1.5x)(1.5))$

$= 2\sin x\cos x - 1.5\sin(1.5x)$ ✓

b) $y = \cos(\sin(2x))$

$y' = -\sin(\sin(2x))(\cos(2x)(2))$ ✓

c) $y = \frac{1+\cos x}{\sin x}$

$y' = \frac{\sin x(-\sin x) - (1+\cos x)(\cos x)}{\sin^2 x}$ ✓

d) $y = \sin^4\left(2x - \frac{\pi}{2}\right)$

~~$y' = 4\cos^3(2x - \frac{\pi}{2})(2)$~~

$y = [\sin(2x - \frac{\pi}{2})]^4$

$y' = 4[\sin(2x - \frac{\pi}{2})]^3[\cos(2x\frac{\pi}{2})](2)$

$= 8\sin^3(2x - \frac{\pi}{2})\cos(2x\frac{\pi}{2})$

e) $y = \tan^5(x^2 - 4x + 1)$

~~$y' = 5\sec^4(x^2 - 4x + 1)(2x - 4)$~~

$y = [\tan(x^2 - 4x + 1)]^5$

$y' = 5[\tan(x^2 - 4x + 1)]^4[\sec(x^2 - 4x + 1)](2x - 4)$

f) $y = \frac{\sin x}{1-\cos x}$

$y' = \frac{(1-\cos x)(\cos x) - (\sin x)(\sin x)}{(1-\cos x)^2}$ ✓

2. Find the value of the slope of the tangent line to the curve $y = x\cos 2x$ at $x = \frac{\pi}{2}$ [2]

$y' = 1\cos 2x + x(-\sin(2x)) \cdot 2$

$= \cos 2x - 2x\sin(2x)$

@ $x = \frac{\pi}{2}$

$= \cos \cancel{2}(\frac{\pi}{\cancel{2}}) - \cancel{2}(\frac{\pi}{\cancel{2}})\sin(\cancel{2}(\frac{\pi}{2}))$

$= \cos\pi - \pi\sin\pi$

$= -1 - \pi(0)$

$= -1$

~~$\frac{dy}{dx} = -x\sin 2x(2)$~~

~~$= -2x\sin(2x)$~~

~~$M = -2(\frac{\pi}{2})\sin(2(\frac{\pi}{2}))$~~

~~$= -\frac{2\pi}{2}\sin\frac{2\pi}{2}$~~

~~$= -\pi\sin\pi$~~

~~$= -\pi(0)$~~

~~$= 0$~~

3. Find an equation for the tangent to the curve $y = \sin\left(2x + \frac{\pi}{3}\right)$ at $x = \frac{\pi}{6}$ [4]

$(x, y)\ \left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$ ✓

$y' = \cos\left(2x+\frac{\pi}{3}\right)(2)$

$= 2\cos\left(2x+\frac{\pi}{3}\right)$

$M = 2\cos\left(2\left(\frac{\pi}{6}\right)+\frac{\pi}{3}\right)$

$= 2\cos\left(\frac{\pi}{3}+\frac{\pi}{3}\right)$

$= 2\cos\left(\frac{2\pi}{3}\right)$

$= -1$ ✓

$y = mx + b$

$y = -1x + b$

$\frac{\sqrt{3}}{2} = -1\left(\frac{\pi}{6}\right) + b$

$\frac{\sqrt{3}}{2} = -\frac{\pi}{6} + b$

$\boxed{\cancel{\frac{\sqrt{3}}{2} \times \frac{-6}{\pi} = b} \quad \cancel{\frac{-6\sqrt{3}}{2\pi} = b}}$

$b = \frac{\sqrt{3}}{2} + \frac{\pi}{6}$

$= \frac{3\sqrt{3}+\pi}{6}$

$\therefore$ equation is

$\cancel{y = -1x - \frac{6\sqrt{3}}{2\pi}}$

$\cancel{y = -1x\ \frac{3\sqrt{3}+\pi}{6}}$

$= -x + \frac{3\sqrt{3}+\pi}{6}$

4. At what points on the curve $y = \cos x - \sin x$, $0 \le x \le 2\pi$ is the tangent line horizontal? [5]

$y' = -\sin x - \cos x$

$0 = \sin x - \cos x$

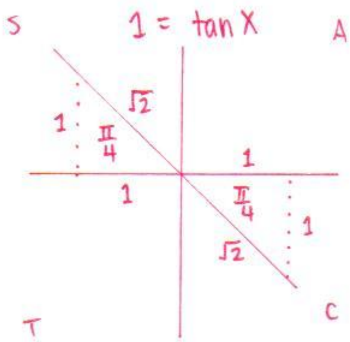
$\cos x = \sin x$

$\cancel{\frac{\cos x}{\sin} = x}$

$\cancel{\cot x = x}$

$\frac{\cos x}{\cos x} = \frac{\sin x}{\cos x}$

$1 = \tan x$



$\therefore x = \frac{3\pi}{4}, \frac{7\pi}{4}$

for $x = \frac{3\pi}{4}$

$y = \cos\left(\frac{3\pi}{4}\right) - \sin\left(\frac{3\pi}{4}\right)$

$= -\frac{2}{\sqrt{2}}$

$= -\sqrt{2}$

for $x = \frac{7\pi}{4}$

$y = \cos\left(\frac{7\pi}{4}\right) - \sin\left(\frac{7\pi}{4}\right)$

$= \frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)$

$= \frac{2}{\sqrt{2}}$

$= \sqrt{2}$

$\therefore \left(\frac{3\pi}{4}, -\sqrt{2}\right)\ \&\ \left(\frac{7\pi}{4}, \sqrt{2}\right)$