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MHF4U Test #6: Exponential and Logarithmic Functions

K & U: 12.5 **/18** **APP:** 9 **/16** **Comm:** 6 **/8** **TIPS:** 7 **/11**

Part A: Knowledge and Understanding. [18 marks]

1. Fill in the blanks. [13 marks]

a) Simplify the logarithm $\log_a \sqrt[n]{a}$ ½✓ $\frac{1}{n}$ ✗

b) Simplify the logarithm $2^{\log_2(3x+4)}$ ½✓ $3x+4$ ✓ 0.5

c) The restriction on $\log_2(x+2) + \log_2(x-2)$ is $\{x \in \mathbb{R} \mid x > 2\}$ ✓ 1

$x=-2$ $x=2$

d) Express $3\log C - \log P - h\log O$ *(... letter O, not 'zero' ...)* $\log \frac{C^3 \cdot O^h}{P}$ ✗ 0.5

e) Simplify $\log_a 27 + \log_a 4 - \log_a 12$ (*exact value, no decimals*) $\log_a 9$ ✓ 1

$2\log_a 3$

f) The population of a colony of ants, after '*t*' weeks, is modelled by $P(t) = 1000(1.28)^t$. What is the doubling time, expressed to two decimal places?

$2000 = 1000(1.28)^t$

$2 = 1.28^t$

$t = 2.81$

1½✓ 2.81 weeks ✓ 1.5

g) Given $2^x = 3^{x+1}$, solve for '*x*'. Express your answer as an exact value!

✓✓ $x = -\frac{\log 3}{\log 3 - \log 2}$ ✓ 2

h) Simplify $\log_3 x + \log_9 x + \log_{27} x$. Express as an exact value. 1½✓ ✗

i) Evaluate the following by first rewriting the logarithm with an appropriate base change (not base 10 or *e*), then simplifying. Express your answer as an exact value, in fraction form. ✓✓

$\log_{243} 81 = \log_{3^5} 3^4$?

$\boxed{= \frac{4}{5}}$ 1

j) Rewrite $\log_4 8x^3$ in terms of $\log_2 x$ ✓✓ $\log_2 x^3 - 1$

$\log_{2^2} 8x^3$

$= \log_{2^2}(2^4)(x^3)$ ✓

$\log_2 \frac{1}{2}x^3$? ✗

0.5

8

2. Given $\log_a 3 = 0.6131$ and $\log_a 8 = 1.1606$, rewrite the following in terms of $\log_a 3$ and $\log_a 8$, then evaluate the following. [5 marks]

a) $\log_a 72$

$= \log_a(3\cdot3\cdot8)$

$= \log_a 3 + \log_a 3 + \log_a 8$

$= 0.6131 + 0.6131 + 1.1606$

$\boxed{= 2.3868}$

2.5

b) $\log_a 3.375$

$= \log_a\left(\frac{3\cdot3\cdot3}{8}\right)$

$= \log_a 3 + \log_a 3 + \log_a 3 - \log_a 8$

$= 0.6131 + 0.6131 + 0.6131 - 1.1606$

$\boxed{= 0.233}$? ✗

2

4.5

Part B: Application. [16 marks]

3. Use the laws of logarithms to express the following as a single logarithm. Show a logical progression to be awarded full marks. [6 marks]

a) $\log_a 36 - \left(2\left(\frac{1}{\log_9 a}\right) - 3\log_a x\right)$

$= \log_a 36 - \left(\frac{2}{\log_9 a} - 3\log_a x\right)$

$= \log_a 36 - \frac{2}{\log_9 a} + \log_a x^3$

$=$

1

b) $\left(2\log_2 10a\right) - \left(\frac{1}{2}\log_2 25b\right) - \left(3\frac{\log c}{\log 2}\right)$

$= \log_2(10a)^2 - \log_2(25b)^{\frac{1}{2}} - 3\log C - 3\log 2$ ✗

$= \log_2(100a^2) - \log_2(5b^{\frac{1}{2}}) - \log C^3 - \log 2^3$?

$= \log_2\frac{100a^2}{5\sqrt{b}} - \log C^3 - \log 2^3$?

$=$

1.5

4. A \$2000 investment earns 8% interest, compounded monthly. Write an equation for the value of the investment as a function of time, in years, then determine how long it would take for the investment to triple in value. [3 marks]

$\frac{0.08}{12} = 0.00\overline{66}$

$A(t) = 2000\left(1 + \frac{0.08}{12}\right)^{\frac{t}{12}}$ ✗

$6000 = 2000\left(1.00\overline{66}\right)^{\frac{t}{12}}$?

$3 = \left(1.00\overline{66}\right)^{\frac{t}{12}}$

$\frac{\log 3}{\log 1.0066} = \frac{t}{12}$

$167.005 = \frac{t}{12}$

$t = 167$

∴ It will take 167 years to triple in value

2.5

5

5. Solve the following equations. Express your answer as exact values. [7 marks]

a) $3^{x+1} + 3^x = 96$ ✓✓✓

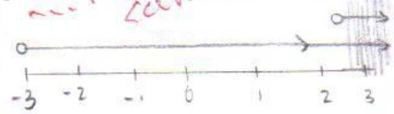
=

0

b) $\log_9(2x-5) + \log_9(x-3) = \frac{1}{2}$ ✓✓✓✓

$\log_9 (2x-5)(x-3) = 9^{\frac{1}{2}}$ ... not on the same line

overlap



$2x - 5 > 0$??

$2x^2 - 11x + 15 = 3$

$2x^2 - 11x + 12 = 0$

$(x-4)(2x-3) = 0$ must be $> \frac{5}{2}$

4

$x - 4 = 0$ $2x - 3 = 0$

$x = 4$ $x = \frac{3}{2}$ cant because restriction

$\therefore$ the values are $x = 4$

4

Part C: Communication. [8 marks]

6. Use the laws of logarithms to rewrite the function $y = \log_2 8x^5$ in terms of $\log_2 x$. Use this new expression to describe the transformations on this new logarithmic function. [5 marks]

$y = \log_2 8x^5$

$= \log_2 8 + \log_2 x^5$

$= \log_2 8 + 5 \log_2 x$

$= 3 + 5 \log_2 x$

$= 5 \log_2 x + 3$

stretch the logarithmic function (base of 2) vertical by a factor of 5. Then translate it 3 units up

4.5

7. Does the equation below have an extraneous root? Explain what an extraneous root is, and why not all of the answers found would satisfy. [3 marks]

$\log_2(x-4) + \log_2(x+3) = 3$

$\log_2 (x-4)(x+3) = 3$

$(x-4)(x+4) = 2^3$? ✗

$x^2 - 16 = 8$ ✗

$x^2 = 24$

$x = \pm\sqrt{24}$ ← cant have negative number because you cant have negative number into $\log_{10}(x)$

$+\sqrt{24}$ ~~$-\sqrt{24}$~~

extraneous root is a root for the equation, but it cant be used. logs cant have a negative number ?

1.5

Therefore, only $+\sqrt{24}$ will satisfy. $-\sqrt{24}$ would not.

6

Part D: Thinking, Inquiry and Problem Solving. [11 marks]

8. If $\log_3 b = x$ and $\log_a 3 = n$, express the following in terms of 'n' and 'x'. Express your answer as a single rational expression, showing a logical progression to be awarded full marks. [4 marks]
$\log_{b^2}(27a^5)$

$= \log_{b^2}(27) + \log_{b^2}(a^5)$

$= \log_{b^2}(27) + 5\log_{b^2} a$
sub ② and ③ into

$= \log_{3^{2x}}(27) + 5\log_{3^{2x}}(3^n)$

$\boxed{= \log_{9^x}(27 \times 243^n)}$

$\log_3 b = x$ $\quad$ $b = 3^x$ ②

$\log_a 3 = n$ $\quad$ $3 = a^n$ ①

$\frac{\log 3}{\log a} = n$?

$\frac{0.47712}{\log a} = n$

$\log a = 0.47712n$

$a = 10^{0.47712n}$

$a = 3^n$ ③

2

9. Solve the following. [5 marks]
$\frac{1}{5-\log x} + \frac{2}{1+\log x} = 1$

Let k rep $\log x$

$\frac{1}{5-k} + \frac{2}{1+k} = 1$

$\frac{1+k+2(5+k)}{(5-k)(1+k)} = 1$

$\frac{1+k+10+2k}{5+4k-k^2} = 1$

$\frac{3k+11}{5+4k-k^2} = 1$

$3k+11 = -k^2+4k+5$ ok

$k^2-k+6 = 0$

$(k-3)(x+2) = 0$

can't

sub log x into k
$((\log x)-3)((\log x)+2) = 0$

$\log x = 3$ $\quad$ $\log x = -2$

$x = 10^3$ $\quad$ $x = 10^{-2}$

$= 1000$ $\quad$ $= 0.01$

Therefore, the values that satisfy are $x = 1000$ or $x = 0.01$

3.5

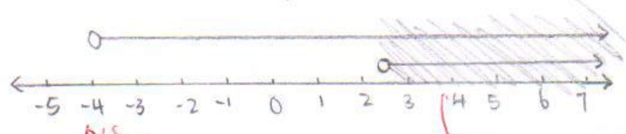
10. Determine the restrictions on the expression $\log(x+4) + \log(-2x+5)$. Show work to justify your answer. [2 marks]

$x+4=0$
~~$x=-4$~~
can't be negative

$-2x+5=0$
$-2x=-5$
$x=\frac{5}{2}$

?

1.5



$x \in \mathbb{R} \mid x > \frac{5}{2}$

?

/70
Exam
20 MC
16 s/a Fill
34 Full

-9
-5 polynomial equalities synth eq.