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UNIT 7 QUEST: Lines and Planes

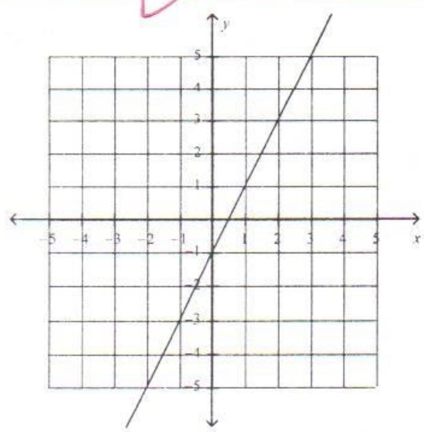
Instructions:
1. Read the instructions carefully.
2. Attempt all problems and don't spend all your time on one problem. Other problems will feel left out ☹
3. Show all work in the space provided. Three (3) marks will be given to the overall mathematical form on this test.
4. ENJOY ☺

PART A: KNOWLEDGE AND UNDERSTANDING

Multiple Choice. Write the answer to the Multiple Choice section in the chart below. Please use CAPITAL LETTERS.

1. B	2. D B	3. A	4. B C	5. C
6. B D	7. D	8. C	9. C	

1. Which vector equation represents the line **perpendicular** to the line shown at right? [1]



A. $\vec{r} = (0,-1) + s(1,2)$, $s \in \mathbb{R}$

B. $\vec{r} = (0,-1) + s(-2,1)$, $s \in \mathbb{R}$

C. $\vec{r} = (0,-1) + s(2,1)$, $s \in \mathbb{R}$

D. None of these

2. Which of the following is a direction vector for the line with scalar equation $3x + 2y - 1 = 0$? $y = \frac{3}{2}x + \frac{1}{2}$ [1]

A. $\vec{m} = (3,2)$ B. $\vec{m} = (-2,3)$ C. $\vec{m} = (3,-2)$ D. None of these

3. Which of the following is the parametric equation of the line with symmetric equation $\frac{x+3}{3} = \frac{y+2}{2} = \frac{z-1}{2}$? [1]

A. $x = 3t - 3, y = 2t - 2, z = 2t + 1, t \in \mathbb{R}$

B. $x = 3t + 3, y = 2t + 2, z = 2t - 1, t \in \mathbb{R}$

C. $x = 3t - 3, y = 2t - 2, z = t + 2, t \in \mathbb{R}$

D. None of these

4. Which of the following is **not** an equation for the line passing through the points $P(1,4,-3)$ and $Q(3,2,1)$? [1]

A. $\vec{r} = (1,4,-3) + s(2,-2,4)$, $s \in \mathbb{R}$

B. $x = -t + 3, y = t + 2, z = -2t + 1, t \in \mathbb{R}$

C. $\frac{x-3}{2} = \frac{y-2}{-2} = \frac{z+1}{4}$

D. $\vec{r} = (3,2,1) + s(1,-1,2)$, $s \in \mathbb{R}$

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5. Which of the following is **not** a plane? [1]

A. $\vec{r} = (1,3,4) + s(2,-1,2) + t(1,1,1)$, $s,t \in \mathbb{R}$

B. $\vec{r} = (2,4,2) + s(1,-2,3) + t(3,2,2)$, $s,t \in \mathbb{R}$

C. $\vec{r} = (3,2,3) + s(4,-4,2) + t(-2,2,-1)$, $s,t \in \mathbb{R}$

D. $\vec{r} = (-2,1,4) + s(2,2,-1) + t(2,2,1)$, $s,t \in \mathbb{R}$

6. A plane is defined by the equation $3x - 2z = 4y + 1$. Which of the following is the normal vector of this plane? [1]

A. $\vec{n} = (3,-2,4)$ B. $\vec{n} = (3,4,-2)$ C. $\vec{n} = (3,2,4)$ D. $\vec{n} = (3,-4,-2)$

7. Which plane goes through the origin and is perpendicular to the line $\vec{r} = (2,-2,1) + s(2,3,-4)$, $s \in \mathbb{R}$? [1]

A. $2x - 2y + z = 0$ B. $2x + 3y - 4z = 0$ C. $2x + 3y + z - 4 = 0$ D. None of these

8. The two planes $3x - 2y - 4z + 1 = 0$ and $2x - 5y - 3 = 0$ are: [1]

A. Perpendicular B. Coincident C. Parallel and Distinct D. None of these

9. Which of the following is not a point on the plane with Cartesian equation $3x + 2y - 2z - 4 = 0$? [1]

A. $P(2,0,1)$ B. $Q(2,3,4)$ C. $R(1,1,1)$ D. $S(0,3,1)$

Short Answer. Write your answer in the space provided.

10. State the symmetric equations of the line that passes through $(0,5)$ and is parallel to $\vec{r} = (3,-2) + t(4,1), t \in \mathbb{R}$ [1]

$\vec{p} = (0,5) + t(1,4)$

$x = 0 + t \quad y = 5 + 4t$

$x = t \quad \frac{y-5}{4} = t$

$\boxed{x = \frac{y-5}{4}}$

11. State the vector equation of the line perpendicular to $5x - 2y = 3$ and through $P(7,1)$. [1]

$-2y = -5x + 3$

$y = \frac{5}{2}x - \frac{3}{2}$

$(5,-2)$

$\vec{r} = (7,1) + t(2,5)$, $t \in \mathbb{R}$

12. State the vector equation of a line through $(1,4,8)$ and parallel to the x-axis. [1]

$\vec{r} = (1,4,8) + t(1,0,0)$, $t \in \mathbb{R}$

13. State the symmetric equations of the line that passes through $(1,2,3)$ and is parallel to $x = 4 - t, y = 5t$, $z = 1 + 2t, t \in \mathbb{R}$. [1]

$\vec{m} = (-1,5,2)$

$\vec{m}_{\perp} = (5,1,2)$

$\vec{r} = (1,2,3) + t(5,1,2)$

$x = 1 + 5t \quad y = 2 + t \quad z = 3 + 2t$

$\frac{x-1}{5} = t \quad y - 2 = t \quad \frac{z-3}{2} = t$

$\frac{x-1}{5} = y - 2 = \frac{z-3}{2}$

$\vec{r} = (1,2,3) + t(-1,5,2)$

$\therefore \frac{x-1}{-1} = \frac{y-2}{5} = \frac{z-3}{2}$

1

14. Find the vector equation of the line that is parallel to the line $\frac{x+1}{3} = \frac{2-y}{5} = z + 4$ and has an x-intercept of 5. [2]

(5,0,0)

1

$\vec{r} = (5,0,0) + t(3,-5,-4), t \in \mathbb{R}$

$t = \frac{x+1}{3}$ $t = \frac{2-y}{5}$ $t = z+4$

$3t = x+1$ $5t = 2-y$ $t-4 = z$

$3t-1 = x$ $y = 2-5t$ $z = -4+t$

$x = -1+3t$

coefficient of t is 1 !! (not 4)

1/2.

15. A line passes through $P(2,5,-1)$ and $Q(9,-5,-7)$.

a) State the vector equation of the line. [1]

$\vec{m}(7,-10,-6)$

$\vec{r} = (2,5,-1) + t(7,-10,-6), t \in \mathbb{R}$

b) State the corresponding parametric equations [1]

$\left.\begin{array}{l} x = 2+7t \\ y = 5-10t \\ z = -1-6t \end{array}\right\} t \in \mathbb{R}$

2/2.

16. Determine the scalar equation for the plane that passes through the point $(1,2,-4)$ and has normal $\vec{n} = (3,2,4)$ [2]

x y z

A B C

~~$\vec{m} = (2,-3,4)$~~

$Ax + By + Cz + D$

$= 3(1) + 2(2) + 4(-4) + D$

$D = 9$

$\therefore$ eq $= 3x + 2y + 4z + 9$

$[x,y,z] = (1,2,-4) + s(2,-3,4) + t(3,2,4)$

this is vector equation

Ø.

PART B: APPLICATION

For full marks, show work of good form in the space provided.

17. Determine the value of k that makes the lines $\frac{x+2}{4} = \frac{y+1}{5} = \frac{z-3}{3}$ and $\vec{r} = (1,3,6) + t(-2k, 2, k), t \in \mathbb{R}$ perpendicular. [3]

$t = \frac{x+2}{4}$ $t = \frac{y+1}{5}$ $t = \frac{z-3}{3}$

$4t = x+2$ $5t = y+1$ $3t = z-3$

$-2+4t =$ $y = 1-5t$ $z = 3+3t$

$\vec{s} = (-2,1,3) + t(4,-5,3)$

$\vec{r} = (1,3,6) + t(-2k,2,k)$

$\vec{m}_\perp = (5,4,3)$

$\frac{4}{2}$

$\frac{5}{2} = -2k$

$\frac{3}{2} = k$

$-4k = 5$

$0 = (4,5,3) \cdot (-2k,2,k)$

$= -8k + 10 + 3k$

$= 5k - 10$

$10 = 5k$

$k = 2$

$\boxed{\therefore k \text{ value should be } k = \frac{3}{2}}$

Ø.

18. A line that passes through the origin intersects a plane at the point $P(2,3,-5)$. If the line is perpendicular to the plane, determine the scalar equation of the plane. [3]

$\vec{l} = (0,0,0) + t(2,3,-5), t \in \mathbb{R}$ $\vec{m}_\perp = (3,-2,-5)$

$2x + 3y - 5z + D = 0$

$2(2) + 3(3) - 5(5) = 0$

$D + 4 + 9 + 25 = 0$

$D = 38$

$\therefore -2x + 3y + 5z + 38$

$\pi = (2,3,-5) + t(3,-2,-5) + s(2,3,-5)$

Ø

/3 + Ø

19. Determine the scalar equation of the plane having x-, y-, and z-intercepts of 2, 5 and 3 respectively. [3]

$0 = 2x + 5y + 3z$

$=$

20. Given the planes $\begin{cases} \pi_1: 2x - y + kz = 8 \\ \pi_2: 2kx - 6y + 36z = 5 \end{cases}$, determine a value of k if these planes are parallel, if possible. [3]

$2 : 2k$
$-y : -6y$
$k : 36$

not possible.
cant have
k value that
satifys at the
same time

21. Find the intersection of the following pairs of lines, if any. [4]

$$l_1 : \begin{cases} x = -1 + 3t \\ y = 1 + 4t \\ z = -2t \end{cases} \quad \text{and} \quad l_2 : \begin{cases} x = -1 + 2s \\ y = 3s \\ z = -7 + s \end{cases}$$

① $-1 + 3t = -1 + 2s$
② $1 + 4t = 3s$
③ $-2t = -7 + s$

$\vec{m}_1 = (3, 4, -2)$
$\vec{m}_2 = (2, 3, 1)$
∵ not scalar multiple
∴ not coincidental

② $1 + 4t = 3s$
$+2$ ③ $-4t = -14 + 2s$

$1 = -14 + 5s$
$15 = 5s$
$s = 3$

$1 + 4t = 3(3)$
$4t = 8$
$t = 2$

$s = 3$ & $t = 2$ into ①

LS	RS
$-1 + 3(t)$	$-1 + 2s$
$= -1 + 3(2)$	$= -1 + 2(3)$
$= 5$	$= 5$

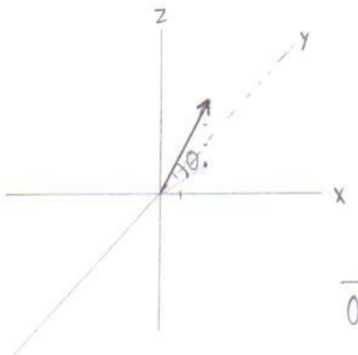
∵ LS = RS
∴ one POI

$x = -1 + 3(2)$ $= 5$
$y = 1 + 4(2)$ $= 9$
$z = -2(2)$ $= -4$

∴ POI is at point $(5, 9, -4)$

PART D: THINKING

22. Determine the distance from the point $A(1,2,3)$ to the plane π: $9x + y + z + 7 = 0$. [4]



$\sqrt{1^2+2^2}$

$=\sqrt{5}$

$\sqrt{\sqrt{5}^2+3^2}$

$\overrightarrow{OA} = \sqrt{14}$

$y = -9x - z - 7$

$2 = -9(1) - (3) - 7$

$0 = -9 - 10 - 2$

$= -21$

$\boxed{\therefore \text{ 21 units away}}$

$\cos\theta = \frac{\sqrt{5}}{\sqrt{14}}$

$\theta = 53.3°$

$D = \frac{|Ax + By + Cz + D|}{\sqrt{A^2+B^2+C^2}}$

$= \frac{|9(1) + 1(2) + 1(3) + 7|}{\sqrt{9^2+1^2+1^2}}$

$= \frac{21}{\sqrt{83}}$ units

0

0

23. Show why the planes $2x + ky + 2z - 3 = 0$ and $5x + ky + 3kz + 2 = 0$ are never perpendicular. [4]

in order for them to be perp

$\vec{n}_1 \cdot \vec{n}_2 = 0$

$2x + ky + 2z - 3 = 0$
$5x + ky + 3kz + 2 = 0$

$\vec{n}_1 = (2, k, 2)$
$\vec{n}_2 = (5, k, 3k)$

$(2, k, 2) \cdot (5, k, 3k) = 0$

$10 + k^2 + 6k =$

$k^2 + 6k + 10 =$

Quad Form

$k = \frac{-6 + \sqrt{-4}}{2}$ $k = \frac{-6 - \sqrt{-4}}{2}$

Imaginary number impossible

because coeffiicents before x y z are all positive. perpendicular must be negative slope if other is positive

X

0

☺ The end ☺

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