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## MHF4U Test #3: Rational Functions

Parent Signature: ______________________

**K & U:** 18 /21 **APP:** 8 /15 **Comm:** 4.5 /7 **TIPS:** 8.5 /12

## Part A: Knowledge and Understanding. [21 marks]

Fill in the blanks (questions 1 – 3). [13 marks] → didn't say horizontal + vert.

1. Determine the equations of all the asymptotes for each of the following rational functions, using the acceptable shortcut techniques outlined in class.

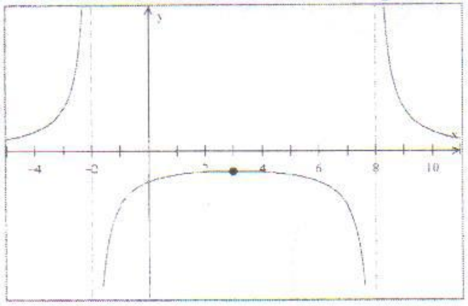
a) n>m $y = \dfrac{2x^2 - 6x}{x^2 - 2x - 3} = \dfrac{2x^2-6x}{(x+1)(x-3)}$ vertical asymptotes are at x=-1 and x=3. no horizontal asymtotes ✓✓ ①

b) n<m $y = \dfrac{x^2 + 9}{x^3 - 6x^2 + 8x} = \dfrac{x^2+9}{x(x-2)(x-4)}$ vertical asymtotes are at x=0, x=2, and x=4. horizontal asymtotes at y=0 ✓✓✓ ③

c) n>m $y = \dfrac{x^3 - 2x^2 - 7x + 3}{x^2 + 2x}$ $= \dfrac{x^3-2x^2-7x+3}{(x+2)x}$ vertical asymptotes are at x=-2 and x=0. no horizontal asymtotes. ✓✓✓ ②

2. State the range for the function $y = \frac{1}{x^2 - 4x}$ ✓✓ R: {y∈R | y ≠ 0} 0.5

3. In the diagram below the point indicated is the local maximum, whose tangent has a slope of 0. Determine where the following occurs:



a) The function values are negative. ✓

the function values are negative where -2<x<8

b) The function values are positive and increasing. ✓

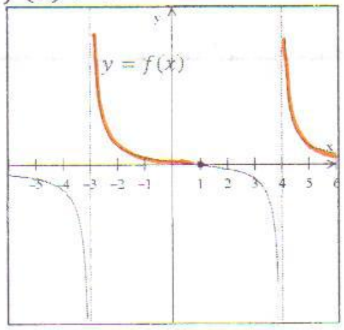
The function values are positive and increasing where x<-2 ③

c) The slope values are negative and decreasing. ✓

The slope values are negative and decreasing where 3<x<8

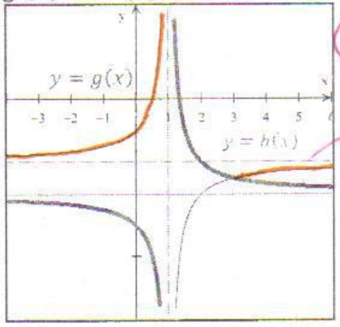
4. Given the diagrams below, and that points of intersection are at integer coordinates of '$x$', find the solution(s) for the functions below: [3 marks]

a) $f(x) > 0$



F(x) is greater than 0 when x∈R| -3<x<1 or x∈R| x>4 ②

b) $g(x) \leq h(x)$



0.5

→ this is where it's greater than

g(x) is less than or equal to h(x) when x∈R| x<1 or x∈R| x≥3

?

⑫

5. State the domain and range for the function given below. [3 marks]

$$y = \frac{x+7}{x+4}$$

x-int at $x=-7$

VA $x=-4$

HA $y=1$

D: $\{x \in \mathbb{R} \mid x \neq -4\}$

R: $\{y \in \mathbb{R} \mid y \neq 1\}$

3

6. Solve the following inequality using a number line. [3 marks]

$$\frac{2x-3}{x+2} \leq 1$$

$$\frac{2x-3}{x+2} - 1 \leq 0$$

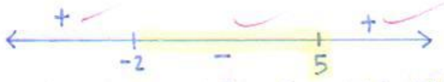
$$\frac{2x-3}{x+2} - \frac{x+2}{x+2} \leq 0$$

$$\frac{(2x-3)-(x+2)}{x+2} \leq 0$$

$$\frac{x-5}{x+2} \leq 0$$

x-int at $x=5$

VA at $x=-2$



| sub in $x=-5$ | sub in $x=0$ | sub in $x=6$ |
|---|---|---|
| $f(x) = 3.\overline{33}$ | $f(x) = -2.5$ | $f(x) = 0.125$ |

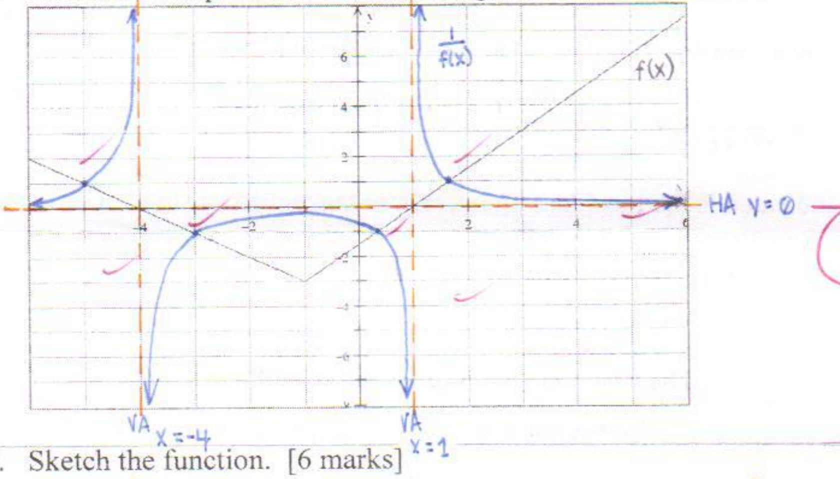
3

Therefore, $\frac{2x-3}{x+2}$ is less than or equal to 1 where $x \in \mathbb{R} \mid -2 < x \leq 5$.

6

## Part B: Application. [15 marks]

7. Sketch the reciprocal of the function given below. [4 marks]



4

8. Sketch the function. [6 marks]

$$y = \frac{x^2-2x-3}{x^3+8x^2+16x}$$

$$= \frac{(x-3)(x+1)}{x(x^2+8x+16)}$$

$$= \frac{(x-3)(x+1)}{x(x+4)(x+4)}$$

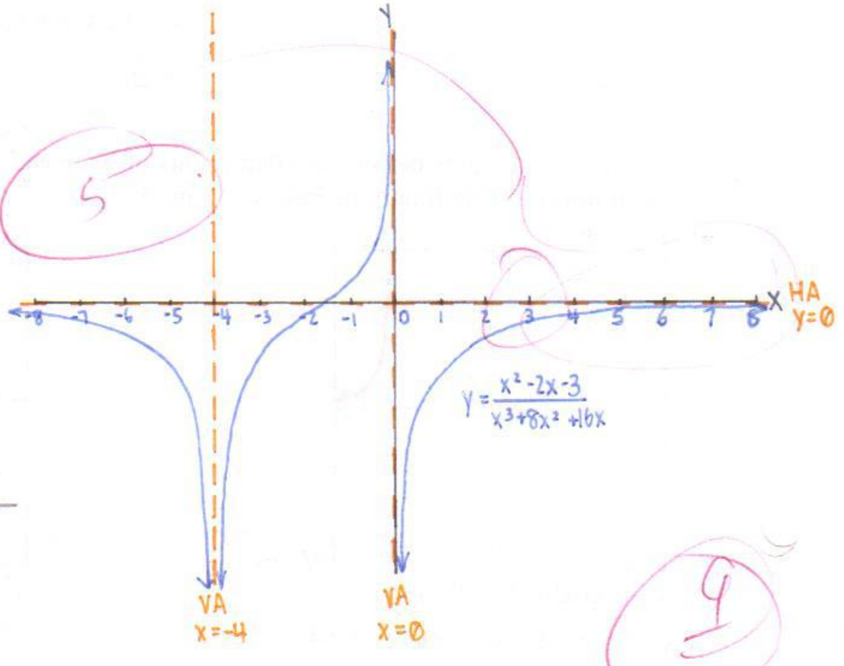
no y-int because vertical asymtote $x=0$

x-intercepts at $x=3$, $x=-1$

vertical asymptotes at $x=0$, $x=-4$

$\because n<m$ $\therefore$ horizontal asymptote is at $y=0$

| as $x \to -4^-$ | as $x \to -4^+$ | as $x \to 0^-$ | as $x \to 0^+$ |
|---|---|---|---|
| $\frac{(x-3)(x+1)}{x(x+4)(x+4)}$ | $\frac{(x-3)(x+1)}{x(x+4)(x+4)}$ | $\frac{(x-3)(x+1)}{x(x+4)(x+4)}$ | $\frac{(x-3)(x+1)}{x(x+4)(x+4)}$ |
| $=\frac{(-)(-)}{(-)(0^-)(0^-)}$ | $=\frac{(-)(-)}{(-)(0^+)(0^+)}$ | $=\frac{(-)(+)}{(0^-)(+)(+)}$ | $=\frac{(-)(+)}{(0^+)(+)(+)}$ |
| $=(-)$ | $=(-)$ | $=(+)$ | $=(-)$ |
| $y \to -\infty$ | $y \to -\infty$ | $y \to +\infty$ | $y \to -\infty$ |



5

9

9. The rational function $y = \frac{2x^3-9x^2-18x+3}{x^2-6x}$ can be looked at as a division of a cubic function by a quadratic function. Express this function as an addition of functions. State each function, and the equation of any asymptotes. [5 marks]

$$\begin{array}{r} 2x+3 \\ x^2-6x \,\overline{)\, 2x^3-9x^2-18x+3} \\ \underline{2x^3-12x^2 \downarrow \downarrow} \\ 0 \quad 3x^2-18x+3 \\ \underline{3x^2-18x \downarrow} \\ 0 \quad 0 \quad +3 \end{array}$$

$$y = \frac{2x^3-9x^2-18x+3}{x^2-6x}$$

$$= \frac{(x^2-6x)(2x+3)+3}{x^2-6x}$$

$$= \frac{\cancel{(x^2-6x)}(2x+3)}{\cancel{(x^2-6x)}} + \frac{3}{x^2-6x}$$

$$= 2x+3 + \frac{3}{x^2-6x}$$

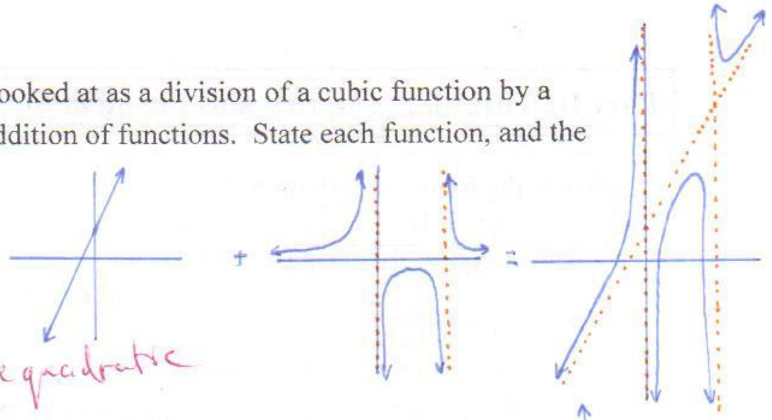
linear function

reciprocal function of a quadratic

$f(x) = 2x+3$

$g(x) = \frac{3}{x^2-6x}$

$= \frac{3}{x(x-6)}$



? oblique asymptote

$y = 2x+3$

req'd.

vertical asymptotes at $x = 0$, $x = 6$

$\because n < m$ $\therefore$ horizontal asymptote is at $y = 0$

X not present

4

## Part C: Communication. [7 marks]

10. Explain the difference between the terms rational expression, rational function and rational equation, using words. [4 marks]

rational expression is a fraction that has polynomials in the numerator and denominator $\frac{\text{polynomial}}{\text{polynomial}}$ ?

rational function is a category of functions ~~that has rational expression~~. $f(x) = \frac{\text{polynomial}}{\text{polynomial}}$

ind./dep. --

rational equation is a rational expression that is equal to a value. you can solve for the variable in the equation

specific condition.

$0 = \frac{\text{polynomial}}{\text{polynomial}}$

$x = ?$ $x = ?$ $x = ?$

3

11. The function given below has a horizontal asymptote of $y = 0$. Verify this end behaviour algebraically. [3 marks]

$$y = \frac{x+4}{x^2+6x+7}$$

$$= \frac{\cancel{x^2}\left(\frac{1}{x}+\frac{4}{x^2}\right)}{\cancel{x^2}\left(1+\frac{6}{x}+\frac{7}{x^2}\right)}$$

$$= \frac{\frac{1}{x}+\frac{4}{x^2}}{1+\frac{6}{x}+\frac{7}{x^2}}$$

$x \to \infty$

as $y \to 0^+$

sub in 0.0001

$= \frac{\text{small number}}{\text{big number}}$

$=$ close to 0 ?

$x \to 0$

$x \to -\infty$

as $y \to 0^-$

sub in -0,0001

$= \frac{\text{small number}}{\text{big number}}$

$=$ close to 0 ?

$x \to 0$

substitution ?

1.5

$\because$ when y approaches zero from both the negative and positive side, x approaches zero.

$\therefore$ the horizontal asymptote is at $y = 0$

4.5

## Part D: Thinking, Inquiry and Problem Solving. [12 marks]

12. Sketch the following. [8 marks]

$$y = \frac{x^2 - 3x - 10}{x^3 + 3x^2 - 10x - 24}$$

$\because n<m$

$\therefore$ HA $y=0$

$= \frac{(x-5)(x+2)}{x^3+3x^2-10x-24}$ ← $P(3) = (3)^3 + 3(3)^2 - 10(3) - 24$

$= 27 + 27 - 30 - 24$

$= 0$

$\therefore x-3$ is a factor.

$= \frac{(x+5)(x+2)}{(x-3)(x^2+6x+8)}$ ✗

$= \frac{(x+5)(x+2)}{(x-3)(x+2)(x+4)}$

```
3 | 1   3  -10  -24
  | ↓   3   18   24
  -----------------
    1   6    8    0
```

x-int at $x=-5$ ✗

Vertical asymptotes are at:

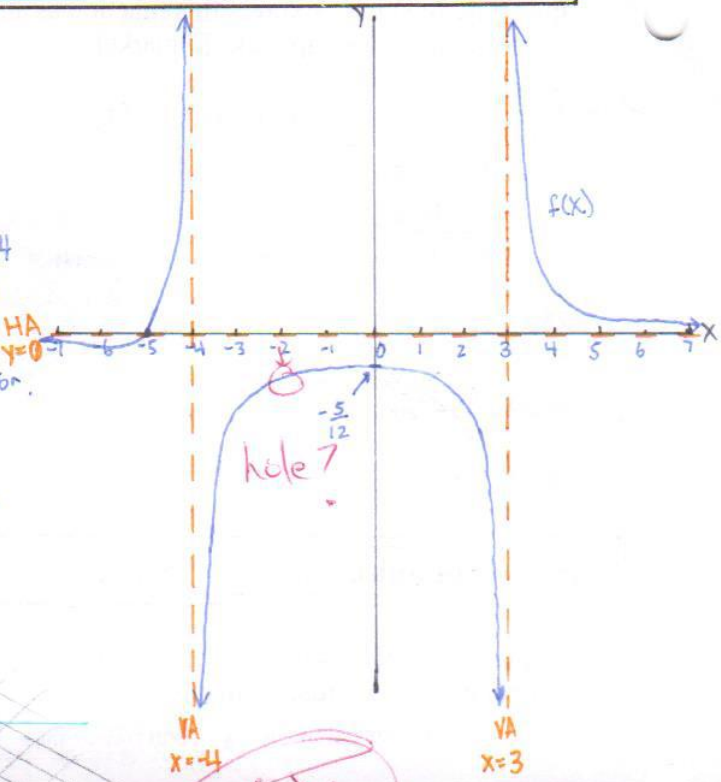
$x=3 \quad x-4$

hole at $x=-2$

y-int at $y=-\frac{5}{12}$

8

| as $x \to -4^-$ | as $x \to -4^+$ | as $x \to 3^-$ | as $x \to 3^+$ |
|---|---|---|---|
| $\frac{(x+5)}{(x-3)(x+4)}$ | $\frac{(x+5)}{(x-3)(x+4)}$ | $\frac{(x+5)}{(x-3)(x+4)}$ | $\frac{(x+5)}{(x-3)(x+4)}$ |
| $=\frac{(+)}{(-)(0^-)}$ | $=\frac{(+)}{(-)(0^+)}$ | $=\frac{(+)}{(0^-)(+)}$ | $=\frac{(+)}{(0^+)(+)}$ |
| $=(+)$ | $=(-)$ | $=(-)$ | $=(+)$ |
| $y \to +\infty$ | $y \to -\infty$ | $y = -\infty$ | $y \to +\infty$ |



6.5

13. Solve the following using an algebraic technique. [4 marks]

$$\frac{x-3}{x+5} > \frac{x-3}{2x+3}$$

can't cross multiply changed the inequality

$(x-3)(x+5) > (x-3)(2x+3)$ ✗

$x^2 + 5x - 3x - 15 > 2x^2 + 3x - 6x - 9$

$x^2 + 2x - 15 > 2x^2 - 3x - 9$

$0 > x^2 - 5x + 6$

$0 > (x-3)(x+2)$

$x=3 \qquad x=-2$

LS > RS

sub $x=-3$ into $(x-3)(x+2)$

$=(-)(-)$

$=(+)$

above 0

sub $x=0$ into $(x-3)(x+2)$

$=(-)(+)$

$=(-)$

below 0

sub $x=4$ into $(x-3)(x+2)$

$=(+)(+)$

$=(+)$

above 0

2

Therefore, $\frac{x-3}{x+5}$ is greater than $\frac{x-3}{2x+3}$ when $x<-2$ or $x>3$ ?

test $x=-3$ $LS = \frac{-6}{2} = -3$ $RS = \frac{-6}{-3} = 2$

LS<RS

85