

Test # 2  
Analytical GeometryK / U 12 / 12A 12.5 / 13TIP 12 / 12C 4t / Level

## Knowledge and Understanding

1. How far is the point  $(-10, 24)$  from the origin?
- a)  $\sqrt{26}$       b) 10      c) 26      d) 34      e) 44
2. One end of a line segment has the coordinates  $(4, -8)$ . If the middle point is  $(4, 1)$ , then what are the coordinates of the other endpoint?
- a)  $(8, 4)$       b)  $(4, 10)$       c)  $(10, 4)$       d)  $(4, 8)$       e)  $(4, -17)$
3. One end of a line segment has the coordinates  $(m, n)$ . If the other end has the coordinates  $(p, q)$ , then what are the coordinates of the midpoint?

a)  $\left(\frac{m-p}{2}, \frac{n-q}{2}\right)$

b)  $\left(\frac{m+p}{2}, \frac{n+q}{2}\right)$

c)  $(2(m+p), 2(n+q))$

d)  $\left(\frac{m+q}{2}, \frac{n+p}{2}\right)$

e)  $\left(\frac{p-m}{2}, \frac{q-n}{2}\right)$

4. Name a segment that is a median of the given figure.

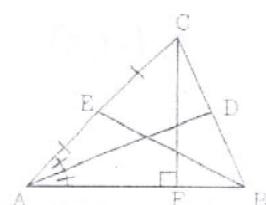
a) BE

b) CF

c) AD

d) AF

e) BD

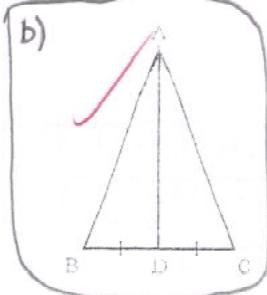


5. Which of the following diagrams illustrates segment AD as a segment bisector?

a)



b)



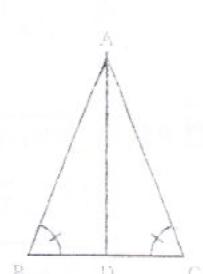
c)



d)



e)



6. A right triangle has vertices A(6, 4), B(6, -8), C(0, 4). What are the coordinates of the midpoint of the hypotenuse?

a)  $(3, -6)$

b)  $(3, -2)$

c)  $(3, 4)$

d)  $(6, -2)$

e)  $(6, 4)$

7. A parallelogram has vertices A(-4, 1), B(1, 2), C(2, 6), and D(-3, 5). At what point do the diagonals intersect?

a)  $(0, 3\frac{1}{2})$

b)  $(0, 4\frac{1}{2})$

c)  $(-3\frac{1}{2}, 4)$

d)  $(-1, 3\frac{1}{2})$

e)  $(-1, 4\frac{1}{2})$

/7

[3] 8. For the circle  $x^2 + y^2 = 169$  state: the radius:  $r = 13$  ✓

the centre:  $(0, 0)$  ✓

the intercepts  $(0, 13)$   $(0, -13)$   $(13, 0)$   $(-13, 0)$  ✓

③

[2] 9. Write the equation of the circle, centre  $O(0,0)$ , that passes through point  $(-3, -8)$ .

$$r^2 = x^2 + y^2$$

$$r^2 = (-3)^2 + (-8)^2$$

$$\sqrt{r^2} = \sqrt{9 + 64}$$

$$r = \sqrt{73}$$

∴ the equation of the circle is

$$x^2 + y^2 = 73$$

②

### Applications

[3] 10.  $\triangle ABC$  has vertices  $A(3, 4)$ ,  $B(-5, 2)$ , and  $C(1, -4)$ .

Determine an equation for  $CD$ , the median from  $C$  to  $AB$ .

$$A(3, 4) \quad B(-5, 2) \quad C(1, -4)$$

$$M_{AB} = \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

$$= \left( \frac{3-5}{2}, \frac{4+2}{2} \right)$$

$$= (-1, 3)$$

$$D(-1, 3)$$

$$M_{CD} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{3+4}{-1-1}$$

$$= -\frac{7}{2}$$

$$y = -\frac{7}{2}x + b$$

sub  $C(1, -4)$  into  $y = -\frac{7}{2}x + b$

$$-4 = -\frac{7}{2}(1) + b$$

$$-4 = -\frac{7}{2} + b$$

$$b = 0.5$$

∴ the equation for  $CD$   
is  $y = -\frac{7}{2}x + 0.5$

2.5

[3] 11. A triangle has vertices at  $A(1, 1)$ ,  $B(-2, -1)$ , and  $C(3, -2)$ . Determine the type of triangle  $ABC$ .

$$A(1, 1) \quad B(-2, -1) \quad C(3, -2)$$

$$\begin{aligned} d_{AB} &= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\ &= \sqrt{(1+2)^2 + (1+1)^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13} \text{ units} \end{aligned}$$

$$\begin{aligned} d_{BC} &= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\ &= \sqrt{(-2-3)^2 + (-1+2)^2} \\ &= \sqrt{25+1} \\ &= \sqrt{26} \end{aligned}$$

$$\begin{aligned} d_{CA} &= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\ &= \sqrt{(3-1)^2 + (-2-1)^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13} \end{aligned}$$

③

$d_{AB} = d_{CA}$  ∴ this triangle is isosceles.

pyth theorem

LS

RS

$$LS = RS$$

$$a^2 + b^2 = c^2$$

$$a^2 + b^2$$

$$c^2$$

$$13 + 13$$

$$26$$

$$26$$

$$26$$

∴ this is a right angle  
isosceles triangle.

55

- [4] 12. Show that the quadrilateral with vertices J(-1, 1), K(3, 4), L(8, 4) and M(4, 1) is a rhombus.

J(-1, 1)    K(3, 4)    L(8, 4)    M(4, 1)

$$d_{JK} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-1-3)^2 + (1-4)^2}$$

$$= \sqrt{16+9}$$

$$= \sqrt{25}$$

$$d_{KL} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3-8)^2 + (4-4)^2}$$

$$= \sqrt{25+0}$$

$$= \sqrt{25}$$

$$d_{LM} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(8-4)^2 + (4-1)^2}$$

$$= \sqrt{16+9}$$

$$= \sqrt{25}$$

$$d_{MJ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4+1)^2 + (1-1)^2}$$

$$= \sqrt{25+0}$$

$$= \sqrt{25}$$

all sides have the same distance.  
∴ It's either square or rhombus.



$$m_{KL} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-4}{3-8}$$

$$= \frac{0}{-5}$$

$$= 0$$

$$m_{ML} = \frac{y_2 - y_1}{x_2 - x_1}$$

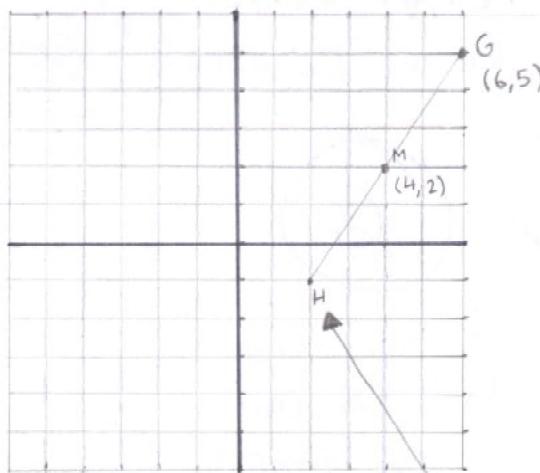
$$= \frac{4-1}{8-4}$$

$$= \frac{3}{4}$$

slope of  $\overline{KL}$  and  $\overline{ML}$   
are not perpendicular

∴ this Quadrilateral  
is a rhombus

- [3] 13. For a line segment GH, one endpoint is G(6, 5) and the midpoint is M(4, 2). Find the coordinates of endpoint H.



$$d_{MG} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(6-4)^2 + (5-2)^2}$$

$$= \sqrt{4+9}$$

$$= \sqrt{13}$$

$$M_{MG} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5-2}{6-4}$$

$$= \frac{3}{2}$$

$$\begin{array}{rcl} M(4, 2) \\ - (2, 3) \\ \hline (2, -1) \end{array}$$

$$H(2, -1)$$

∴ the coordinates of H  
is (2, -1)

$$d_{MH} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{13}$$

$$\therefore \sqrt{13}$$

$$= \sqrt{13}$$

LS

RS

$$\begin{array}{rcl} \sqrt{13} & & \sqrt{13} \\ = \sqrt{(4-2)^2 + (2+1)^2} & & = \sqrt{2^2 + 3^2} \\ = \sqrt{13} & & = \sqrt{13} \end{array}$$

∴ LS = RS

## Thinking / Inquiry/ Problem Solving

[6] 14.  $\triangle ABC$  has vertices A(6,3), B(2,5), and C(0,1). Determine the coordinates of the circumcentre.

$$A(6,3)$$

$$M_{AB} = \left( \frac{x_2+x_1}{2}, \frac{y_2+y_1}{2} \right) \\ = \left( \frac{6+2}{2}, \frac{3+5}{2} \right)$$

$$= (4, 4) \quad \textcircled{D}$$

$$M_{BC} = \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right) \\ = \left( \frac{2+0}{2}, \frac{5+1}{2} \right)$$

$$= (1, 3) \quad \textcircled{E}$$

$$M_{CA} = \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right) \\ = \left( \frac{0+6}{2}, \frac{1+3}{2} \right)$$

$$= (3, 2) \quad \textcircled{F}$$

$$B(2,5)$$

$$M_{AB} = \frac{y_2-y_1}{x_2-x_1} \\ = \frac{3-5}{6-2}$$

$$= -\frac{2}{4}$$

$$= -\frac{1}{2}$$

$$M_{BC} = \frac{y_2-y_1}{x_2-x_1} \\ = \frac{5-1}{2-0}$$

$$= \frac{4}{2}$$

$$= 2$$

$$M_{CA} = \frac{y_2-y_1}{x_2-x_1} \\ = \frac{1-3}{0-6}$$

$$= \frac{-2}{-6}$$

$$= \frac{1}{3}$$

Perp eq. of AB

$$y = 2x + b$$

sub \textcircled{D} into

$$y = 2x + b$$

$$(4) = 2(4) + b$$

$$b = -4$$

Perp eq. of BC

$$y = -\frac{1}{2}x + b$$

sub \textcircled{E} into

$$y = -\frac{1}{2}x + b$$

$$(3) = -\frac{1}{2}(1) + b$$

$$b = 3.5$$

$$\rightarrow y = 2x - 4 \quad \textcircled{1}$$

$$\rightarrow y = -0.5x + 3.5 \quad \textcircled{2}$$

$$(y = 2x - 4)(-0.5)$$

$$-0.5y = -1x + 2 \quad \textcircled{3}$$

$$(y = -0.5x + 3.5)(2)$$

$$2y = -1x + 7$$

$$2y = -1x + 7$$

$$-0.5y = -1x + 2$$

$$2.5y = 5$$

$$y = 2$$

sub  $y = 2$  into \textcircled{1}

$$y = 2x - 4$$

$$2 = 2x - 4$$

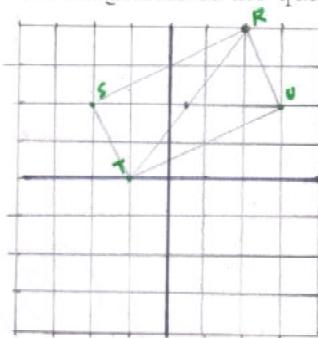
$$2x = 6$$

$$x = 3$$

$\therefore$  the coordinates  
of the circumcentre  
is  $(3, 2)$

6

[6] 15. Quadrilateral RSTU has vertices R(2, 4), S(-2, 2), T(-1, 0) and U(3, 2). Verify that the diagonals of the quadrilateral bisect each other and are equal in length.



$$R(2,4) \quad S(-2,2) \quad T(-1,0) \quad U(3,2)$$

$$M_{RT} = \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

$$= \left( \frac{2-1}{2}, \frac{4+0}{2} \right)$$

$$= (0.5, 2)$$

$$M_{SU} = \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

$$= \left( \frac{-2+3}{2}, \frac{2+2}{2} \right)$$

$$= (0.5, 2)$$

POI of  $\overline{RT}$  and  $\overline{SU}$  is  $(0.5, 2)$  \textcircled{A}

6

$$DAT = DAR$$

$$\text{and } DAS = DAU$$

$\therefore$  the diagonals

bisect each other

$$DAT = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\ = \sqrt{(0.5+1)^2 + (2-0)^2} \\ = \sqrt{(1.5)^2 + (2)^2} \\ = \sqrt{6.25}$$

$$DAS = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\ = \sqrt{(0.5+2)^2 + (2-2)^2} \\ = \sqrt{(2.5)^2 + (0)^2} \\ = \sqrt{6.25}$$

$$DAR = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\ = \sqrt{(0.5-2)^2 + (2-4)^2} \\ = \sqrt{(-1.5)^2 + (-2)^2} \\ = \sqrt{6.25}$$

$$DAU = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\ = \sqrt{(0.5-3)^2 + (2-2)^2} \\ = \sqrt{(-2.5)^2 + (0)^2} \\ = \sqrt{6.25}$$

$$\therefore DAT = DAR$$