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MHF4U Test #2: Polynomial Functions

Parent Signature: ____________________

K & U: 14.5 /17 **APP:** 13 /15 **Comm:** 4.5 /7 **TIPS:** 8.5 /9

Part A: Knowledge and Understanding. [17 marks]

1. Fill in the blanks. [11 marks]

a) Determine the nature of the roots for the following equations:

i) $(x-5)(2x^2+5x+4)=0$ 1 real distinct root, and 2 complex roots 1

ii) $(x+1)^3(x^2-4)=0$ 2 real distinct roots, and 3 real equal roots 1

iii) $(x^2+6x+5)(3x^2-7x+4)=0$ 4 real distinct roots 1

iv) $x^3-27=0$ 1 real distinct root and 2 complex roots 1

b) The solution(s) to the inequality $(x^2+6x)(x-1)^3>0$ is/are: (✓✓)

$x \in \mathbb{R}$ $-6<x<0$ and $x \in \mathbb{R}$ $x>1$ 1.5

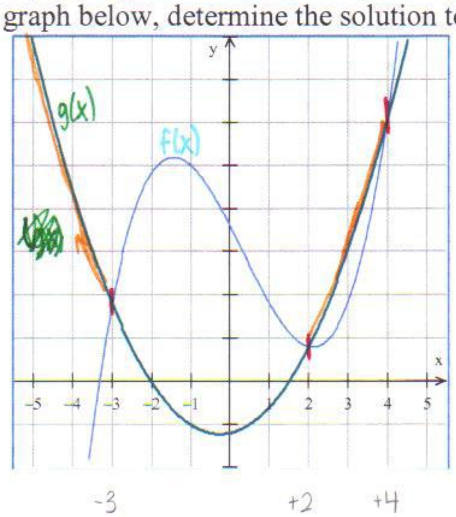
cannot be and.

c) The remainder when $2x^3+5x^2-7x-3$ is divided by $x-2$ is 19 1

d) One factor of the polynomial x^3+4x^2+x-6 is $x+3$. Determine the other two binomial, linear factors. (✓✓)

other factors are $(x+2)$ and $(x-1)$ 2

e) If $f(x)$ represents the cubic function, and $g(x)$ represents the quadratic function, then from the graph below, determine the solution to $f(x)<g(x)$. (✓✓)



the function f(x) is less than the function g(x) when $x \in \mathbb{R}$ $x<-3$ or $x \in \mathbb{R}$ $2<x<4$.

2

10.5

2. Find the equation for the family of polynomial functions with roots of -3, $4 \pm 2\sqrt{3}$. [4 marks]

$x=-3 \quad x=4+2\sqrt{3} \quad x=4-2\sqrt{3}$

$(x+3) \quad (x-4-2\sqrt{3}) \quad (x-4+2\sqrt{3})$

factors: $(x-6-\sqrt{3}) \quad (x-2+\sqrt{3})$

2

Therefore, the equation for family is $f(x) = a(x+3)(x-6-\sqrt{3})(x-2+\sqrt{3})$

expand and simplify

3. Given $f(x) = 2x^3 + bx^2 + 5x - 7$, and $f(4) = 45$, find the value of 'b'. [2 marks]

$45 = 2(4)^3 + b(4)^2 + 5(4) - 7$

$\emptyset = 128 + 16b + 20 - 7 - 45$

$= 16b + 96$

$-96 = 16b$

$-\frac{96}{16} = b$

$-6 = b$

2

Therefore, the value of b is -6

4

Part B: Application. [15 marks]

4. Factor completely the polynomial $x^4 + 2x^3 - 4x^2 - 5x + 6$ (with integer numbers). [4 marks]

$P(1) = (1)^4 + 2(1)^3 - 4(1)^2 - 5(1) + 6$

$= 1 + 2 - 4 - 5 + 6$

$= \emptyset$

$\therefore (x-1)$ is a factor.

$$\begin{array}{r|rrrrr} 1 & 1 & 2 & -4 & -5 & 6 \\ + & \downarrow & 1 & 3 & -1 & -6 \\ \hline & 1 & 3 & -1 & -6 & \emptyset \end{array}$$

$x^3 + 3x^2 - x - 6$

$P(-2) = (-2)^3 + 3(-2)^2 - (-2) - 6$

$= -8 + 12 + 2 - 6$

$= \emptyset$

$\therefore (x+2)$ is a factor

$$\begin{array}{r|rrrr} -2 & 1 & 3 & -1 & -6 \\ + & \downarrow & -2 & -2 & 6 \\ \hline & 1 & 1 & -3 & \emptyset \end{array}$$

$x^4 + 2x^3 - 4x^2 - 5x + 6 = (x-1)(x+2)(x^2 + x - 3)$

4.

5. Solve the following inequality $x^3 + 3x^2 - 10x + 18 \geq 2x^2 + 7x + 3$, showing a logical progression on how you determined your solution. Use an interval table. [6 marks]

$x^3 + 3x^2 - 10x + 18 \geq 2x^2 + 7x + 3$

$x^3 + 3x^2 - 10x + 18 - 2x^2 - 7x - 3 \geq \emptyset$

$x^3 + x^2 - 17x + 15 \geq \emptyset$

$P(1) = (1)^3 + (1)^2 - 17(1) + 15$

$= 1 + 1 - 17 + 15$

$= \emptyset$

$\therefore (x-1)$ is a factor

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -17 & 15 \\ + & \downarrow & 1 & 2 & -15 \\ \hline & 1 & 2 & -15 & \emptyset \end{array}$$

$x^2 + 2x - 15$

$= (x+5)(x-3)$

$x^3 + x^2 - 17x + 15 = (x+1)(x+5)(x-3)$

$x=-1 \quad x=-5 \quad x=3$

	$x<-5$	$x=-5$	$-5<x<-1$	$x=-1$	$-1<x<3$	$x=3$	$3<x$
$(x+1)$	−		−	0	+		+
$(x+5)$	−	0	+		+		+
$(x-3)$	−		−		−	0	+
$f(x)$	−	0	+	0	−	0	+

5.5

Therefore, $x^3 + 3x^2 - 10x + 18$ is equal or greater than $2x^2 + 7x + 3$ when $x \in \mathbb{R}$ $-5 \leq x \leq 1$ or $x \in \mathbb{R}$ $3 \leq x$ not backwards

follow conventions.

9.5

6. Sketch the polynomial function $P(x) = 2x^3 - 11x^2 + 2x + 15$. Show calculations to support the appearance of your function. [5 marks]

y int is +15

$P(-1) = 2(-1)^3 - 11(-1)^2 + 2(-1) + 15$

$= -2 - 11 - 2 + 15$

$= 0$

$\therefore (x+1)$ is a factor.

$$\begin{array}{r|rrrr} -1 & 2 & -11 & 2 & 15 \\ + & \downarrow & -2 & 13 & -15 \\ \hline & 2 & -13 & 15 & 0 \end{array}$$

$0 = (x+1)(2x^2 - 13x + 15)$?

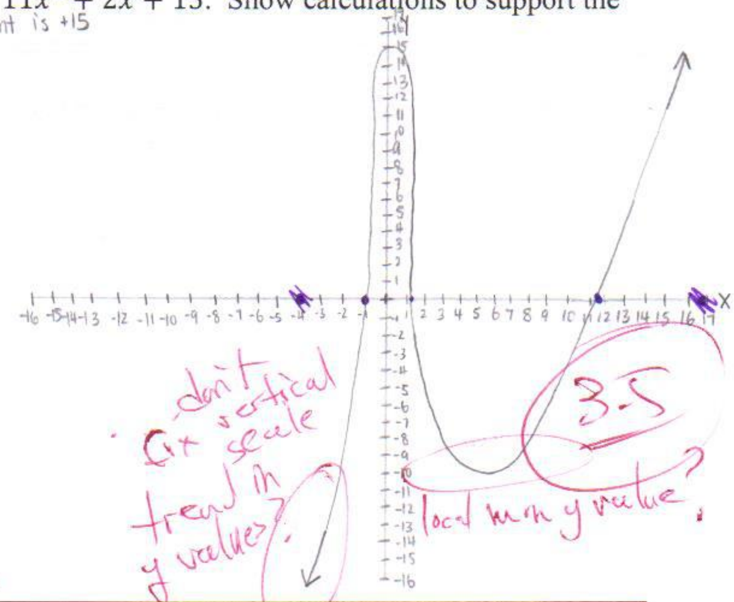
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-13) \pm \sqrt{(13)^2 - 4(1)(15)}}{2(1)}$

$= \frac{13 \pm \sqrt{109}}{2}$

$x = \frac{13 + \sqrt{109}}{2} = 11.7\ldots$

$x = \frac{13 - \sqrt{109}}{2} = 1.2798\ldots$



don't fix vertical scale

trend in y values?

local min y value?

3.5

Part C: Communication. [7 marks]

7. If given the equation $x^3 + 4x^2 - 2x - 14 = 2x^2 + 3x - 8$, then provide an interpretation of what the roots could represent. When would we say that the roots will be the same as finding the zeros, or x-intercepts? Explain. [3 marks]

When given that equation, you are looking for the Points of Intersection. of what? When you move them all to one side, like $x^3 + 2x^2 - 5x + 22 = 0$, it is a new function with what? where the zeros are the points of intersection? to find what x can equal to. You only find zeros when one side is equal to zero because those are x-ints. Roots are when you solve for x, but the other side may or may not be zero.

→ value that satisfies condition.

2

8. Divide $x^4 + 2x^3 - 9x - 11$ by $x^2 - 4$, using all the correct procedures in long division. Include a division statement. [4 marks]

(-1 place holders

$x^2 + 0x - 4$

$$\begin{array}{r} x^2 + 2x + 4 \\ x^2 - 4 \overline{) x^4 + 2x^3 + 0x^2 - 9x - 11} \\ x^4 \qquad - 4x^2 \qquad\qquad \\ \hline 2x^3 + 4x^2 - 9x - 11 \\ 2x^3 \qquad - 8x \qquad \\ \hline 4x^2 - x - 11 \\ 4x^2 \qquad - 16 \\ \hline -x + 5 \end{array}$$

reqd

$\therefore x^4 + 2x^3 - 9x - 11 = (x^2 - 4)(x^2 + 2x + 4) - x + 5$

2.5

4.5

Part D: Thinking, Inquiry and Problem Solving. [9 marks]

9. The polynomial $2x^3 + bx^2 + 7x + d$, when divided by $(x - 2)$ gave a remainder of 9. When divided by $(x + 1)$ the polynomial gave a remainder of -18. Solve for the unknowns. [4 marks]

$9 = P(2)$

$9 = 2(2)^3 + b(2)^2 + 7(2) + d$

$0 = 16 + 4b + 14 + d - 9$

$0 = 4b + d + 21$ maintain equation

$-d = 4b + 21$

① $d = -4b - 21$

$-18 = P(-1)$

$-18 = 2(-1)^3 + b(-1)^2 + 7(-1) + d$

$0 = -2 + b - 7 + d + 18$

$-d = b + 9$

② $d = -b - 9$

sub 1 into 2

$-4b - 21 = -b - 9$

$-12 = 3b$

③ $-4 = b$

sub 3 into 2

$d = -b - 9$

$d = -(-4) - 9$

$d = 4 - 9$

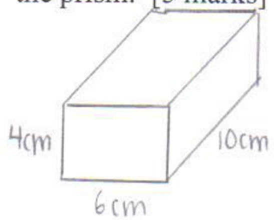
$d = -5$

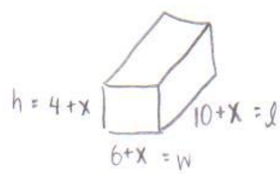
Therefore, $d = -5$ and $b = -4$

3.5

10. A rectangular prism has dimensions 10 cm by 6 cm by 4 cm. The dimensions of the prism are to be increased by the same amount so that the volume it can hold is at least 576 cm^3. Develop an equation to model this relationship. Solve the equation to determine the minimum dimensions of the prism. [5 marks]

Let...





$V = lwh$

$576 = (10+x)(6+x)(4+x)$

$0 = x^3 + 20x^2 + 124x + 240 - 576$

$= x^3 + 20x^2 + 124x - 336$

$P(2) = (2)^3 + 20(2)^2 + 124(2) - 336$

$= 8 + 80 + 248 - 336$

$= 0$

$\therefore (x-2)$ is a factor

$$\begin{array}{r|rrrr} 2 & 1 & 20 & 124 & -336 \\ + & \downarrow & 2 & 44 & 336 \\ \hline & 1 & 22 & 168 & 0 \end{array}$$

5

$V = (x-2)(x^2 + 22x + 168)$

$x = 2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-22 \pm \sqrt{22^2 - 4(1)(168)}}{2(1)}$

$= \frac{-22 \pm \sqrt{-188}}{2}$

$= \frac{-22 \pm \sqrt{4 \cdot -47}}{2}$

$= \frac{-22 \pm 2\sqrt{-47}}{2}$

$x = \frac{-22 + 2\sqrt{-47}}{2}$ $\quad x = \frac{-22 - 2\sqrt{-47}}{2}$ } Can't be a dimension for prism

sub $x=2$ into $h = 4+x$

$h = 4 + (2)$

$= 6$ cm

sub $x=2$ into $w = 6+x$

$w = 6 + (2)$

$= 8$ cm

sub $x=2$ into $l = 10+x$

$l = 10 + (2)$

$= 12$ cm

8.5

Therefore, the minimum dimensions are 6 cm × 8 cm × 12 cm