Chapter 1 Introduction to Probability

Prerequisite Skills, pages 4–5

1. **a)** 0.25, 25% **b)** 0.833…, $83.\overline{3}\%$
c) 0.666…, $66.\overline{6}\%$ **d)** 0.65, 65%

2. **a)** $\frac{3}{4}$ **b)** $\frac{1}{4}$ **c)** $\frac{22}{35}$ **d)** $\frac{4}{9}$

3. **a)** $\frac{1}{2}$, 0.5, 50% **b)** $\frac{11}{12}$, $0.91\overline{6}$, $91.\overline{6}\%$
c) $\frac{5}{12}$, $0.41\overline{6}$, $41.\overline{6}\%$ **d)** $\frac{3}{4}$, 0.75, 75%

4. **a)** $\frac{1}{12}$, $0.08\overline{3}$, $8.\overline{3}\%$ **b)** $\frac{1}{6}$, $0.1\overline{6}$, $16.\overline{6}\%$
c) $\frac{5}{9}$, $0.\overline{5}$, $55.\overline{5}\%$ **d)** $\frac{1}{8}$, 0.125, 12.5%

5. **a)** 3:10
b) blue:total = 2:10 = 1:5; yellow:total = 5:10 = 1:2
c) red: 30%, blue: 20%, yellow: 50%

6. **a)** $\frac{\text{hits}}{\text{times at bat}} = \frac{10}{35}$ **b)** $\frac{10}{35} \approx 0.286$
c) about 114 hits

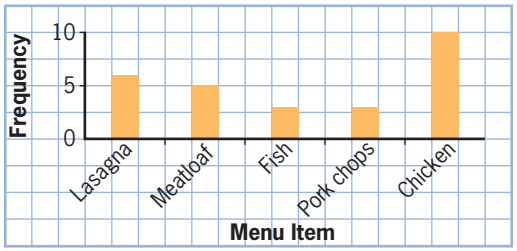
7. **a)** random: The coin is equally likely to land on heads or tails.
b) non-random: The intersection is not entered unless it is safe.
c) non-random: You select exactly what you want.
d) random: Without looking, each candy is equally likely to be chosen.

8. Answers may vary.
a) rolling a die
b) selecting your favourite pair of jeans to wear

9. **a)** $\frac{1}{4}$ of the deck **b)** about 11.5% of the deck

10. **a)** 1 way: 1 and 1
b) 6 ways: 6 and 1, 5 and 2, 4 and 3, 3 and 4, 2 and 5, 1 and 6
c) 0 ways
d) 6 ways: 1 and 1, 2 and 2, 3 and 3, 4 and 4, 5 and 5, 6 and 6
e) 7 ways: 1 and 3, 2 and 2, 3 and 1; 3 and 6, 4 and 5, 5 and 4, 6 and 3

11.



12 **a)** chicken; about 37% **b)** $\frac{21}{27}$ or $\frac{7}{9}$
c) Answers may vary. What percent of the class chose fish? $\frac{3}{27}$, or $\frac{1}{9}$ of the class chose fish.

1.1 Simple Probabilities, pages 6–15

Example 1 Your Turn

a) orange: $\frac{1}{4}$, 0.25, 25%; red: $\frac{1}{8}$, 0.125, 12.5%; purple: $\frac{1}{4}$, 0.25, 25%; green: $\frac{3}{8}$, 0.375, 37.5%
b) Answers may vary. The spinner could be one-quarter orange, one-eighth red, one-quarter purple, and three-eighths green.
c) It is possible that there is a fifth colour, but that the arrow did not land on it in any of these 32 spins.

Example 2 Your Turn

a) cable: about 42%, satellite: about 37%, Internet: about 13%, antenna: about 3%, none: about 5%
b) Answers may vary. The cable company would be interested for marketing. They can claim to be the most popular.
c) Answers may vary. Internet TV service will increase and cable/satellite will decrease. Access to Internet TV will increase with sales of smart TVs and as networks make more programming available.

Example 3 Your Turn

Answers may vary.
a) 0.01; In Ontario, July is a summer month so the probability of a snow day in July is highly unlikely.
b) 1; The sun always sets in the west so the probability of the sun setting in the west tonight is guaranteed.
c) 0.5; The next person to enter the school cafeteria will be male or female.

Reflect

R1. **a)** Experimental probability is probability based on experimental trials. It is calculated as the number of times an outcome happens divided by total number of trials.
b) Experimental probability is a useful tool for making predictions. It can tell you what might happen and how likely it is to occur based on what has been observed.
c) While experimental probability can be close enough to help with decision making, it is based on an experiment. Conditions or circumstances may change and the next experiment could result in different experimental probabilities.

R2. **a)** The event is impossible.
b) The event is certain to happen.
c) This is the range representing impossible to certain.

R3. **a)** #probability is an estimate of how likely something will occur based on intuition
b) Answers may vary. I think there is a 0.95 probability that Canada will win the gold medal in women's hockey.

Practise

1. C

2. **a)** 0.4 **b)** 0.6

Apply

3. **a)** 0.7 **b)** 0.65
c) Since the two experimental probabilities of her scoring were both less than 80% (70% and 65%), her statement is not accurate. She has over-estimated her scoring ability.

4. **a)** yellow: $\frac{1}{2}$, green: $\frac{1}{6}$, purple: $\frac{1}{4}$, blue: $\frac{1}{12}$
b) Answers may vary. It could be one-half yellow, one-sixth green, one-quarter purple, and one-twelfth blue.
c) Yes, because it is based on experimental probability.

5. Answers may vary.

a) The table shows the results for a mystery spinner.

Colour	Favourable Outcomes, $n(A)$
Green	8
Orange	4

b)

6. a) vanilla: about 6%, chocolate: about 14%, raspberry ripple: about 28%, pralines and cream: about 52%

b) The stand sells about the same number of pralines and cream cones as all the other flavours together. The owner can use this information to ensure that there is enough pralines and cream.

7. 21 rainy days

8. a) fast ball: about 91%, curve ball: about 9%, knuckle ball: 0%

b) Answers may vary. The batter would know that the pitch will most likely be a fast ball.

9. a) straight down the line: 7.5%, middle: 30%, outside: 62.5%

b) Answers may vary. Sandeep's opponent would know that the serve will most likely end up in the outside region.

10. Answers may vary.

a) 0.9; I study a lot and do all the homework assignments.

b) 0.9; I study a lot, do all the homework assignments, and have passed all the exams.

c) 0.7; We are currently in the middle of a record snowfall period.

d) 0.99; My favourite song is currently number one on the charts, and I listen to the radio every day for several hours.

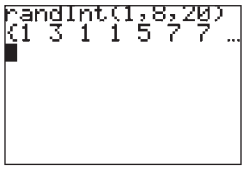
11. Answers may vary. 0.9; They have won more than any other team since the competition first started, and this year they are the host country.

12. 2 male puppies

14. 3 times

15. Answers may vary.

a)



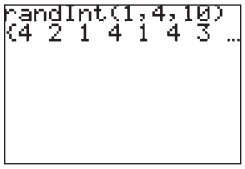
b)

Random Number	Tally
1	4
2	0
3	2
4	2
5	3
6	2
7	4
8	3

c) rolling a 1: 20%, rolling a 2: 0, rolling a 3: 10%, rolling a 4: 10%, rolling a 5: 15%, rolling a 6: 10%, rolling a 7: 20%, rolling an 8: 15%

d) The values are not equal. I think they should be equal with repeated experiments. Each number should have the same likelihood of being generated.

16. Answers may vary.



rolling a 2: 10%

Extend

17. Answers may vary.

18. Answers may vary. Staffing is appropriate for the morning, understaffed for the afternoon, and overstaffed for the evening. The afternoon should increase to 11 open cash registers, and the evening should be reduced to 3 open cash registers.

1.2 Theoretical Probability, pages 16–25

Example 1 Your Turn

about 33%

Example 2 Your Turn

about 67%

Example 3 Your Turn

a) 3:2 **b)** 4:1

Reflect

R1. a) The set of all possible outcomes is the sample space. An event is a set of outcomes in the sample space that have a common characteristic. Then, the probability of an event A happening is the number of outcomes in that subset divided by the total number of outcomes in the sample space. Diagrams may vary.

b) Answers may vary. There are 3 red, 2 blue, and 1 yellow marbles in a bag. What is the theoretical probability of randomly picking a red one? Sample space: R, R, R, B, B, Y; Event of picking red: R, R, R; Theoretical probability of picking red: 0.5.

R2. a) The complement of an event is the set of possible outcomes not included in an event.

b) Answers may vary. There are 3 red, 2 blue, and 1 yellow marbles in a bag. What is the theoretical probability of randomly picking a red one? The event, A, is picking red. The complement, A', is not picking red. The theoretical probability of the complement, not picking a red marble, is $1 - 0.5$, or 0.5.

R3. a) A ratio of the probability that an event will happen to the probability that it will not happen.

b) A ratio of the probability that an event will not happen to the probability that it will happen.

c) The concepts are similar in that they are a ratio of the same probabilities. The difference is the order in which they are presented.

Practise

1. D **2.** A **3.** C

Apply

4. a) $\frac{1}{12}$ **b)** $\frac{1}{6}$ **c)** $\frac{1}{2}$ **d)** $\frac{31}{36}$ **e)** $\frac{29}{36}$

5. a) $S = \{B_1B_2, B_1B_3, B_1W, B_1G_1, B_1G_2, B_2B_1, B_2B_3, B_2W, B_2G_1, B_2G_2, B_3B_1, B_3B_2, B_3W, B_3G_1, B_3G_2, WB_1, WB_2, WB_3, WG_1, WG_2, G_1B_1, G_1B_2, G_1B_3, G_1W, G_1G_2, G_2B_1, G_2B_2, G_2B_3, G_2W, G_2G_1\}$

b) $A = \{B_1B_2, B_1B_3, B_2B_1, B_2B_3, B_3B_1, B_3B_2, G_1G_2, G_2G_1\}$

c) $\frac{4}{15}$ **d)** 4:11

6. a) 0.75 **b)** about 0.66

7. a) about 0.033 **b)** about 0.967

c) Answers may vary. The monkey has associated red with no reward.

8. a) subjective probability **b)** $\frac{1}{5}$

9. a) 4:1 **b)** 3:1

11. Answers may vary.

12. Answers may vary.

a) $\frac{1}{9}$ **b)** 8 times

13. If there are k possible outcomes to a certain probability experiment, all equally likely, then the theoretical probability of any one outcome is $\frac{1}{k}$. Then, the sum of theoretical probabilities of the outcomes is

$$\begin{aligned}\text{Sum} &= \underbrace{\frac{1}{k}+\frac{1}{k}+\frac{1}{k}+\cdots+\frac{1}{k}}_{k \text{ times}} \\ &= k\left(\frac{1}{k}\right) \\ &= 1\end{aligned}$$

14. $P(\text{Canadiens winning}) = \frac{1}{1+8} \approx 0.111$

$P(\text{Canucks winning}) = \frac{2}{2+17} \approx 0.105$

So, the Canadiens are more likely to win.

15. a) The reporter has expressed the odds against incorrectly as $n(S):n(A)$ instead of $P(A'):P(A)$.

b) The odds against the Ottawa Senators winning against the Vancouver Canucks are 2:1 because they have won only one of their three meetings so far this year.

Extend

16. a) $P(n \text{ heads}) = \left(\frac{1}{2}\right)^n$ **b)** $\left(\frac{1}{2}\right)^{10} \approx 9.766 \times 10^{-4}$

17. a) The odds in favour of A are equal to the reciprocal of the odds against A.

b) Answers may vary. The odds in favour of the Canadian women's hockey team winning the gold medal at the next Winter Olympics are 3:1. Then, the odds against the Canadian women's hockey team winning the gold medal at the next Winter Olympics are 1:3.

c) $$\begin{aligned}\text{odds in favour of } A &= \frac{P(A)}{P(A')} \\ &= \frac{1}{\frac{P(A')}{P(A)}} \\ &= \frac{1}{\text{odds against } A}\end{aligned}$$

1.3 Compare Experimental and Theoretical Probabilities, pages 26–33

Reflect

R1. Theoretical probability cannot predict the actual outcome of a probability experiment, but it can give you an idea of what is likely to happen. Experimental probability is not a perfect predictor of the outcome of a probability experiment because results of experiments can change.

R2. If the number of times an outcome occurs is observed over a very large number or trials, you can be more certain of the likelihood of its occurrence.

Practise

1. A **2.** B

Apply

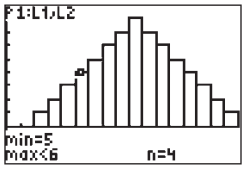
3. Answers may vary.

4. Answers may vary.

5. a)

Sum, x	Frequency	Theoretical Probability
2	1	$\frac{1}{64}$
3	2	$\frac{1}{32}$
4	3	$\frac{3}{64}$
5	4	$\frac{1}{16}$
6	5	$\frac{5}{64}$
7	6	$\frac{3}{32}$
8	7	$\frac{7}{64}$
9	8	$\frac{1}{8}$
10	7	$\frac{7}{64}$
11	6	$\frac{3}{32}$
12	5	$\frac{5}{64}$
13	4	$\frac{1}{16}$
14	3	$\frac{3}{64}$
15	2	$\frac{1}{32}$
16	1	$\frac{1}{64}$

b)



c) In a few trials, the statistical and theoretical probabilities vary greatly. After a very large number of trials, the statistical probabilities are much closer to the theoretical probabilities.

d) Answers may vary.

7. Answers may vary.

a) Two 4-sided dice are rolled and the sum is recorded.

Sum, x	Frequency	Theoretical Probability
2	1	$\frac{1}{16}$
3	2	$\frac{1}{8}$
4	3	$\frac{3}{16}$
5	4	$\frac{1}{4}$
6	3	$\frac{3}{16}$
7	2	$\frac{1}{8}$
8	1	$\frac{1}{16}$

c) As the number of trials increases, the statistical probability values approach the theoretical probability values.

8. a) No. There are 36 possible outcomes for the sum of two dice. With 20 trials, it is mathematically impossible for the statistical probabilities to equal the corresponding theoretical probabilities.

b) In theory, the minimum number of trials necessary would be 36. However, it is almost impossible for the statistical and theoretical probabilities to agree in this case.

Extend

9. Answers may vary.

a) I chose 5. Draw Cards. You can choose from 1 to 3 decks, with or without replacement, and a 52-card deck or a 32-card deck. Each card is shown in a table along with number and suit.

b) What is the theoretical probability of drawing a heart from a deck of cards, with replacement? Conduct a large number of trials. How does the experimental probability of drawing a heart from a deck of cards, with replacement, compare?
The theoretical probability of drawing a heart from a deck of cards, with replacement, is $\frac{1}{4}$.

For the experimental probability, run repeated trials, save the data, check for the number of hearts (= 1) in the list and divide by the number of trials.

10. Answers may vary.

1.4 Mutually Exclusive and Non-Mutually Exclusive Events, pages 34–43

Example 1 Your Turn
60%

Example 2 Your Turn
50%

Example 3 Your Turn
about 67%

Example 4 Your Turn
50%

Reflect

R1.

Definition cannot occur simultaneously	**Characteristics** events that have different attributes
MUTUALLY EXCLUSIVE EVENTS	
Examples Coin: either heads or tails, not both. Card: heart, diamond, club or spade, not a combination	**Non-examples** Card: red card and face card

R2. a) If A and B are non-mutually exclusive events, then the total number of favourable outcomes is: $n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$.

b) When events are non-mutually exclusive. It ensures that items are not counted twice.

R3. Answers may vary. Rolling doubles or a sum 6.

Practise

1. C **2.** D

Apply

3. a) pink shirt or purple shirt: $\frac{3}{5}$,
pink shirt or a short-sleeved shirt: $\frac{4}{5}$

b) first scenario: mutually exclusive events. The shirt cannot be pink and purple.
second scenario: non-mutually exclusive events. One shirt is pink and short-sleeved.

4. about 36%

5. about 78%

6. a) 37.5% **b)** 50% **c)** 62.5%

7. a) 50%

b) I used the principle of inclusion and exclusion.

8. a) 20% **b)** 40%

c) There are five possible outcomes to this game: 8 same, 7 same, 6 same, 5 same, or 4 same. If there are 3 the same, then there are 5 of the other colour, and so on. So, the probability of 8 buttons the same colour is 20%. Scoring at least 4 points means 7 or 8 buttons the same colour. So, the probability of 7 or 8 buttons the same colour is 40%.

10. Answers may vary.

a) Since there are 3 blue marbles and the probability of green or yellow is 50%, there must be at least 3 marbles that belong to the mutually exclusive event of "green or yellow." This could mean that there is 1 green and 2 yellow marbles along with the 3 blue marbles.

b) Two green and 1 yellow marble along with the 3 blue marbles.

11. 11:7

12. Answer may vary. What is the probability of rolling either doubles or a sum of 5 with a standard pair of dice? $\frac{5}{18}$

Extend

13. From the principle of inclusion and exclusion, $n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$.

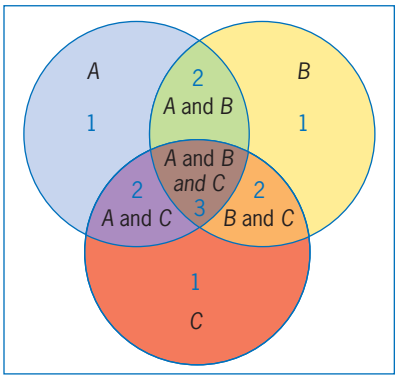
$$\begin{aligned} P(A \text{ or } B) &= \frac{n(A \text{ or } B)}{n(S)} \\ &= \frac{n(A) + n(B) - n(A \text{ and } B)}{n(S)} \\ &= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \text{ and } B)}{n(S)} \\ &= P(A) + P(B) - P(A \text{ and } B) \end{aligned}$$

14. a) $\frac{1}{3}$

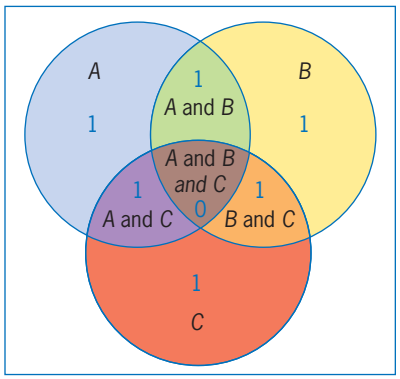
b) Answers may vary. I solved this using a tree diagram with four time periods and looked for outcomes that included B and C in periods one and two or B and C in periods three and four.

c) Answers may vary. I assumed that any of Renzo's classes could be in any time period.

15. a) Starting with $n(A) + n(B) + n(C)$, regions A and B, B and C, and A and C will be counted twice, while region A and B and C is counted three times.



Subtract the regions
$n(A \text{ and } B) + n(B \text{ and } C) + n(A \text{ and } C)$:
$n(A) + n(B) + n(C) - n(A \text{ and } B) - n(B \text{ and } C)$
$- n(A \text{ and } C)$



This results in excluding the count for region A and B and C altogether. Add the region A and B and C.
$n(A \text{ or } B \text{ or } C) = n(A) + n(B) + n(C) - n(A \text{ and } B)$
$- n(B \text{ and } C) - n(A \text{ and } C) + n(A \text{ and } B \text{ and } C)$
Then, divide both sides by $n(S)$.
$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$
$- P(A \text{ and } B) - P(B \text{ and } C) - P(A \text{ and } C)$
$+ P(A \text{ and } B \text{ and } C)$

b) Answers may vary. What is the probability of rolling a sum of 6 or doubles or an even sum?
In this case, $n(A) = 5$, $n(B) = 6$, $n(C) = 18$, $n(A \text{ and } B) = 1$, $n(B \text{ and } C) = 6$, $n(A \text{ and } C) = 5$, $n(A \text{ and } B \text{ and } C) = 1$, and $n(S) = 36$.
$P(A \text{ or } B \text{ or } C)$
$= P(A) + P(B) + P(C) - P(A \text{ and } B)$
$- P(B \text{ and } C) - P(A \text{ and } C) + P(A \text{ and } B \text{ and } C)$
$= \frac{5}{36} + \frac{6}{36} + \frac{18}{36} - \frac{1}{36} - \frac{6}{36} - \frac{5}{36} + \frac{1}{36}$
$= \frac{18}{36}$
$= 0.5$
The probability of rolling a sum of 6 or doubles or an even sum is 50%.

1.5 Independent and Dependent Events, pages 44–55

Example 1 Your Turn
6.25%

Example 2 Your Turn
24%

Example 3 Your Turn
25%

Example 4 Your Turn
10%

Example 5 Your Turn
15 sales

Reflect

R1. a) Independent events have no influence on each other's probability of occurring, while dependent events do influence the probability of the other event occurring.

b) Answers may vary. Drawing two cards from a deck with replacement versus drawing two cards from a deck without replacement.

R2. The second scenario is more likely, because the first scenario involves multiple events while the second only involves a single event.

R3. a) Conditional probability is the probability of a second event occurring, given that a first event occurred.

b) Answers may vary. Three green marbles and two yellow marbles are placed into a bag. What is the probability of randomly drawing a second green marble given that a green marble was already chosen?
What is the probability of randomly drawing a green marble followed by a green marble, assuming that the first marble is replaced before the second marble is drawn?

R4. Answers may vary. A probability tree diagram makes it easier to see the event branch of interest and aids in the calculation of probabilities.

Practise

1. C **2.** B **3.** D

Apply

4. a) 12.5% **b)** about 16.7%
c) Part a) involves independent events, while part b) involves dependent events.

5. a) Yes; both players have the same probability of winning a point on a given trial (about 4.2%).
b) Answers may vary. Player A wins a point if the result is Red-1. Player B wins a point if the result is Green or Blue-4. Then, player A has about a 4.2% probability of winning and player B has a $\frac{1}{16}$, or 6.25% of winning.

6. 4 sales

7. a) 6.25%
b) first path decision is correct: 12.5%, first two path decision are correct: 25%

8. a) about 9.1% **b)** about 0.43%
c) 0%

10. a) about 0.14, assume that the crowd's experimental probability of 85% is accurate
b) about 9 times

Extend

11. a) 1:7 **b)** 11:5

12. Answers may vary. In part a), the superior team would have a higher probability of winning. In part b), the probability of playing seven games would decrease.

13. No. In general, $P(A|B)$ will not equal $P(B|A)$.

14. Answers may vary.

Chapter 1 Review, pages 56–57

1. a) blue: $\frac{24}{149} \approx 16.11\%$, green: $\frac{48}{149} \approx 32.21\%$, yellow: $\frac{51}{149} \approx 34.23\%$, purple: $\frac{26}{149} \approx 17.45\%$

b) Answers may vary. blue sector: about 58°, green sector: about 116°, yellow sector: about 123°, purple sector: about 63°.

c) Yes; it is based on experimental probability.

2. a) 0.6 **b)** 168 throws

3. a) 0.9; Since Canada has won at least one medal since 1900, the probability is high that we will win at least one medal in the next Olympics.

b) 0.1; About 10% of the population is left-handed.

c) 0.25; There are typically four grades in a high school.

4. a) $\frac{1}{36}$ **b)** $\frac{1}{9}$ **c)** $\frac{5}{6}$ **d)** $\frac{29}{36}$

5. a) 25% **b)** $\frac{1}{13}$ **c)** $\frac{3}{13}$

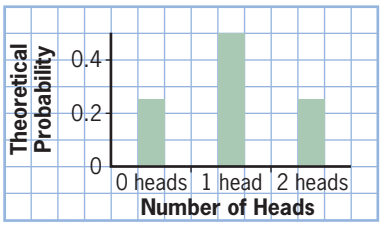
6. 3:1

7. a) 25% **b)** $\frac{1}{6}$

c) Experimental probability is not a perfect predictor of the outcome of a probability experiment because results of experiments can change. Experimental probability approaches theoretical probability as a very large number of trials are conducted.

8. a) outcomes: (H, H), (H, T), (T, H), (T, T)

b)



c) The probability of 1 head is twice that of 0 or 2 heads.

9. a) 25% **b)** 62.5% **c)** 62.5% **d)** 62.5%

10. a) 62.5% **b)** 50% **c)** 50% **d)** 75%

11. a) The first scenario of an even value and a heart will be drawn is more likely to occur.

b) $P(\text{even number}) = \frac{1}{2}$, $P(\text{heart}) = \frac{1}{4}$.

So, $P(\text{even number and heart}) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$.

$P(\text{composite number}) = \frac{1}{3}$, $P(\text{face card}) = \frac{3}{13}$.

So, $P(\text{composite number and face card})$

$= \frac{1}{3} \times \frac{3}{13} = \frac{1}{13}$

12. a) 25% **b)** 20%

c) Part a) involves independent events, while part b) involves dependent events.

13. 6 times

Chapter 1 Test Yourself, pages 58–59

1. C **2.** C **3.** B **4.** 25%

5. a) Since this is Marlis's opinion, it is subjective probability. **b)** 4:1

6. a) about 8.7% **b)** about 45.6%

7. a) 11:7

b) These are mutually exclusive events. There are 6 ways to roll seven, 2 ways to roll 11, and 6 ways to roll doubles. 36 outcomes are possible.

So, $P(A) = \frac{6}{36} + \frac{2}{36} + \frac{6}{36}$, or $\frac{14}{36}$.

Then, $P(A') = 1 - \frac{14}{36}$, or $\frac{22}{36}$.

8. a) 25% **b)** 87.5% **c)** 62.5% **d)** 37.5%

9. a) $\frac{1}{12}$ **b)** $\frac{1}{3}$

10. a) $\frac{1}{3}$ **b)** $\frac{1}{20}$ **c)** $\frac{1}{15}$

Chapter 2 Permutations

Prerequisite Skills, pages 62–63

1. a) 0.039, 0.24, 0.5, 0.718 **b)** 3.0078, 3.078, 3.78

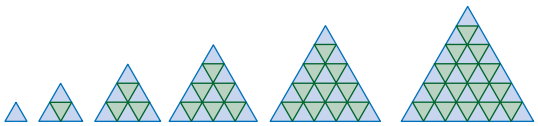
c) $\frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$ **d)** $\frac{7}{12}, \frac{5}{8}, \frac{3}{4}, \frac{5}{6}$

2. a) 27.5% **b)** 490% **c)** 12 562%

d) 40% **e)** 475%

3. $\frac{57}{36}$

4. a) Starting with one triangle, add an increasing number of odd triangles to form a larger triangle.



b) Starting with 12, subtract 3 continuously.

12, 9, 6, 3, $3 - 3 = 0$, $0 - 3 = -3$, $-3 - 3 = -6$, …

c) Starting with the expression $n - 2$, subtract 1 continuously.

$n - 2$, $n - 3$, $n - 4$, $n - 4 - 1 = n - 5$, $n - 5 - 1 = n - 6$, $n - 6 - 1 = n - 7$, …

d) Starting with $\frac{1}{2}$, multiply the denominator by 2 continuously.

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8 \times 2} = \frac{1}{16}, \frac{1}{16 \times 2} = \frac{1}{32}, \frac{1}{32 \times 2} = \frac{1}{64}$, …

5. Answers may vary.

a) If you view the diagrams as stairs, start with 2 steps then add 1 step continuously to get the next diagram in the pattern. 2, 3, 4, …

Starting with a perimeter comprised of 6 line segments, add 2 line segments continuously to get the next diagram in the pattern. 6, 8, 10, …

b) 2, 3, 4, 5, 6, …; 6, 8, 10, 12, 14, …

6. a) 816 **b)** 30 **c)** 15 120

d) 2850 **e)** $\frac{65}{81}$ **f)** $\frac{1}{256}$

7. a) 360 **b)** 1716

c) 35 **d)** approximately 0.005 530

8. a) 60 **b)** 63 **c)** 15 **d)** 722

9. a) $x^3 - 3x^2 + 2x$ **b)** $2x^2 + 4$

c) $x + 5$ **d)** $x^2 - 5x + 6$

10. a) 336 **b)** 5040 **c)** 6 **d)** 2772

11. a) $\frac{1}{6}$ **b)** $\frac{5}{36}$ **c)** $\frac{1}{2}$ **d)** $\frac{1}{4}$ **e)** $\frac{1}{6}$

12. a) independent, the outcome of flipping a coin does not affect the outcome of rolling a die

b) dependent, the outcome of dealing a first card affects the second card dealt

c) independent, the outcome of first die does not affect the outcome of the second die

d) independent, the outcome of randomly selecting a date does not affect the outcome of randomly selecting someone's name

13. a) $\frac{1}{20}$ **b)** $\frac{1}{20}$ **c)** $\frac{2}{20}$ or $\frac{1}{10}$

d) $\frac{10}{20}$ or $\frac{1}{2}$ **e)** $\frac{8}{20}$ or $\frac{2}{5}$ **f)** $\frac{14}{20}$ or $\frac{7}{10}$

14. a) mutually exclusive

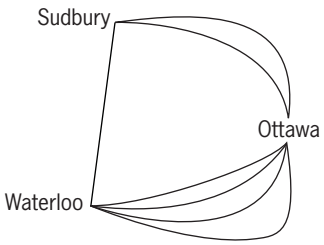
b) non-mutually exclusive

c) mutually exclusive

d) non-mutually exclusive

e) non-mutually exclusive

15.



Tree diagram outcomes: (WOS), (WOS), (WOS), (WOS), (WOS),(WOS), (WOS), (WOS), (WS)

16.

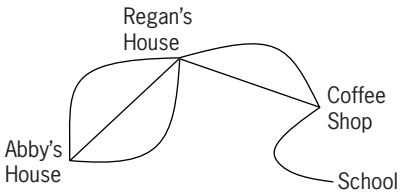
First Die / Second Die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

36 possible outcomes

2.1 Organized Counting, pages 64–69

Example 1 Your Turn

a)



Tree diagram outcomes: (A, R, C, S), (A, R, C, S), (A, R, C, S), (A, R, C, S), (A, R, C, S), (A, R, C, S)

b) six routes

Example 2 Your Turn

a) 24 possible outcomes

b) (Z, B, R, D)

c) 24

Reflect

R1. Answers may vary. While a tree diagram does show all possible outcomes, the actual drawing gets more complicated with more stages. For three stages, a tree diagram is an efficient way to illustrate the outcomes of three spins.

R2. Answers may vary.

a) A table of values is faster to create.

b) When there are three or fewer stages.

Practise

1. 32

T, T, T, T, T T, F, F, T, T T, T, F, F, F T, F, F, T, F
T, T, T, T, F F, F, T, T, T T, F, F, F, T T, F, T, F, F
T, T, T, F, T F, T, F, T, T F, F, F, T, T F, F, F, F, T
T, T, F, T, T F, T, T, F, T F, F, T, F, T F, F, F, T, F
T, F, T, T, T F, T, T, T, F F, F, T, T, F F, F, T, F, F
F, T, T, T, T T, F, T, F, T F, T, F, F, T F, T, F, F, F
T, T, T, F, F T, F, T, T, F F, T, F, T, F T, F, F, F, F
T, T, F, F, T T, T, F, T, F F, T, T, F, F F, F, F, F, F

2. a) 9 outcomes: (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)

b) 27 outcomes: (1, 1, 1), (1, 1, 2), (1, 1, 3), (1, 2, 1), (1, 2, 2), (1, 2, 3), (1, 3, 1), (1, 3, 2), (1, 3, 3), (2, 1, 1), (2, 1, 2), (2, 1, 3), (2, 2, 1), (2, 2, 2), (2, 2, 3), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 1, 1), (3, 1, 2), (3, 1, 3), (3, 2, 1), (3, 2, 2), (3, 2, 3), (3, 3, 1), (3, 3, 2), (3, 3, 3)

3. B **4.** C

Apply

5. Answers may vary.

6. a) 20 possible outcomes: (A, A, A), (A, A, B, A), (A, A, B, B, A), (A, A, B, B, B), (A, B, A, A), (A, B, A, B, A), (A, B, A, B, B), (A, B, B, A, A), (A, B, B, A, B), (A, B, B, B), (B, A, A, A), (B, A, A, B, A), (B, A, A, B, B), (B, A, B, A, A), (B, A, B, A, B), (B, A, B, B), (B, B, A, A, A), (B, B, A, A, B), (B, B, A, B), (B, B, B)

b)

AAA	ABAA	ABBAB	BAABB	BBAAA
AABA	ABABA	ABBB	BABAA	BBAAB
AABBA	ABABB	BAAA	BABAB	BBAB
AABBB	ABBAA	BAABA	BABB	BBB

c) 20

7. a)

Member 1	Member 2
A	B or C or D or E or F
B	A or C or D or E or F
C	A or B or D or E or F
D	A or B or C or E or F
E	A or B or C or D or F
F	A or B or C or D or E

b) 30

c) 15 outcomes

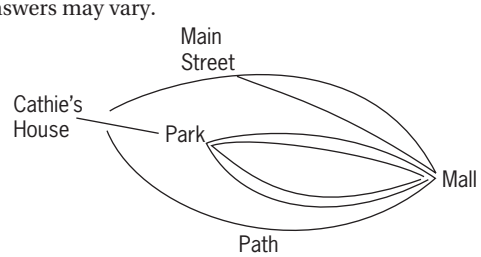
Member 1	Member 2
A	B or C or D or E or F
B	C or D or E or F
C	D or E or F
D	E or F
E	F

8. No. Tree diagram outcomes for die, then coin: (1, H), (1, T), (2, H), (2, T), (3, H), (3, T), (4, H), (4, T), (5, H), (5, T), (6, H), (6, T)
Tree diagram outcomes for coin, then die: (H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 5), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)

9. Answers may vary. Assume that the possible test results are letter grades A, B, and C and that a student stops testing once she/he receives an A grade.
Tree diagram outcomes: (A), (B, A), (B, B, A), (B, B, B), (B, B, C), (B, C, A), (B, C, B), (B, C, C), (C, A), (C, B, A), (C, B, B), (C, B, C), (C, C, A), (C, C, B), (C, C, C)
15 sets of results are possible.

10. Answers may vary.

a)



Tree diagram outcomes: (Cathie, Main, Mall), (Cathie, Main, Mall), (Cathie, Park, Mall), (Cathie, Park, Mall), (Cathie, Park, Mall), (Cathie, Park, Mall), (Cathie, Path, Mall)

b) 7

12. a) 45 configurations

b) Adding a siding colour results in an increase of 15 choices. An additional trim colour results in only 9 more choices.

13. a) 2 **b)** 19 **c)** 4

Extend

14. There are 1 190 000 different local phone numbers Sarah can call.

15. Assume rows on checkerboard are numbered 0 to 7. The portion of the tree diagram that starts with a move diagonally left has 49 possible paths to the opposite side. Similarly, the portion of the tree diagram that starts with a move diagonally right has 54 possible paths to the opposite side. The total number of possible paths to the opposite side is 49 + 54, or 103.

16. a) 8 **b)** 16 **c)** 2^n

2.2 The Fundamental Counting Principle, pages 70–75

Example 1 Your Turn

60

Example 2 Your Turn

a) 308 915 776 **b)** 19 770 609 664

Example 3 Your Turn

13 800

Reflect

R1. Johnny is wrong. He should apply the fundamental counting principle: $4 \times 8 \times 3 = 96$.

R2. Answers may vary. The fundamental counting principle is the product of the number of ways multiple events can occur. For example, there are 3 flavours of ice cream and 6 choices of toppings to create a sundae. Event one, choose ice cream flavour, can happen in 3 ways. Event two, choosing a topping, can happen in 6 ways. The result in 3×6, or 18 different 1-topping ice cream sundaes.

Practise

1. a) 4 **b)** 8 **c)** 16 **d)** 2^n

2. a) 210 **b)** 2730

3. 240

4. a) 3 **b)** appetizers: 4, main course: 5, dessert: 3

c) 60

5. C

6. B

7. a) 25 **b)** 20

Apply

8. a) 16 **b)** 64 **c)** 64 **d)** 4096

e) 144 **f)** 248 832 **g)** n^k

9. 19 000

10. 90

11. 7776

12. a) 12 960 000 **b)** 11 703 240

13. a) 216 000 **b)** 205 320

15. a) 456 976 000 **b)** 17 576 000 **c)** 1 000 000

16. An Alberta licence plate will have much fewer choices than an Ontario licence plate. There are 26 choices for a letter, while there are only 10 choices for a digit.

17. 24

18. The same. In event one and two, the colour of the dice does not affect the choices for a die, and rolling three dice once has the same results as rolling one die three times.

19. Answers may vary.

a) My security code is 325. I pressed ENTER 145 times before I saw my code. The actual number of possible outcomes for a three-digit security is 1000.

b) It might take 100 times longer to break a five-digit code versus a three-digit code. The actual number of possible outcomes for a five-digit security is 100 000. Using a graphing calculator to randomly generate a five-digit code could possibly take more than 10 000 presses of ENTER because of duplicates or your code may never be generated.

20. Answers may vary. To find the number of choices for each of the three toppings, factor 4080: $2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 17$. Using all the factors, create three values. For example, there could be $(2 \times 2 \times 2)$ choices for sauce, $(2 \times 3 \times 5)$ choices for actual topping ingredient, and 17 choices for cheese. Another possibility is that this includes four different sizes (S, M, L, XL). Then, there are actually 1020 $(2 \times 2 \times 3 \times 5 \times 17)$ topping options.

21. a) 1 048 576 **b)** 9 756 625

Extend

22. 52 400 215 plates

23. a) 60 **b)** 52

24. 8223

25. 1 275 120

2.3 Permutations and Factorials, pages 76–81

Example 1 Your Turn

a) 24 **b)** 720

c) 7920 **d)** 144

Example 2 Your Turn

40 320

Example 3 Your Turn

59 280

Example 4 Your Turn

240

Reflect

R1. Arranging r people from a group of n people with regard to order will have more possibilities. For example, ABC can be arranged in 6 ways: ABC, ACB, BAC, BCA, CAB, CBA. Without regard for order these arrangements are the same.

R2. Answers may vary. Using a calculator, 0! has a value of 1. Look at the formula for permutations. Suppose three people are awarded first, second, and third prize. So, $r = n = 3$ and the formula becomes $\frac{3!}{0!}$. In order for this to make sense, choose 0! to be defined as 1.

Practise

1. a) 362 880 **b)** 3 991 680 **c)** 5040 **d)** 6720

2. a) $\frac{6!}{2!}$ **b)** $\frac{15!}{9!}$ **c)** 7! **d)** $\frac{8!}{4!}$

e) $\frac{n!}{(n-4)!}$ **f)** $(n+1)!$

3. a) ${}_6P_6$ **b)** ${}_{91}P_6$ **c)** ${}_{18}P_6$

4. C **5.** B **6.** 2730 **7.** 73 440

Apply

8. ${}_{22}P_9$, or 180 503 769 600

9. a) 10! **b)** 99! **c)** 10!

d) $n!$ **e)** $(n+2)!$

10. **a)** 15!, or 1 307 674 368 000 **b)** 32 760
c) 13 650
11. **a)** 3 628 800 **b)** 604 800
12. 3 720 087 **14.** 3 628 800 **15.** 207 3660 000

Extend

16. **a)** $n = 11$ **b)** $n = 6$
17. 371 589 120
18. 23!, or 25 852 016 738 884 976 640 000
19. **a)** $\frac{9!}{8!!}$ **b)** $\frac{(2k+1)!}{2k!!}$ **c)** $2^n n!$
20. 6

2.4 The Rule of Sum, pages 82–87

Example 1 Your Turn

a) 408 240
b) 46 080

Example 2 Your Turn

a) ${}_{13}P_3$
b) ${}_4P_3$
c) 1739

Example 3 Your Turn

480

Reflect

R1. It is simpler to calculate the number of executives without any males or females than all the possibilities for at least one male and one female.
R2. Use the fundamental counting principle when the events are independent. For example, rolling a die twice. The outcome of the first event does not affect the second. Use the rule of sum when events are mutually exclusive. For example, rolling a 1 or a 2. Both events cannot happen at the same time.

Practise

1. 2016
2. **a)** 8 **b)** 18
3. A **4.** D

Apply

5. **a)** 130 **b)** 78
6. 182 520 000 **7.** 173 659 200
8. **a)** 120 100 **b)** 980 200
c) When the answers in parts a) and b) are expanded into factorial form, all three expressions in part b) are at least 2 times as big as those in part a). So, the result is more than $2^3 = 8$ times the answer in part a).
9. **a)** 60
b) Answers may vary. Question: Five speakers, P, Q, R, S, and T, are available to address a meeting. The organizer must decide whether to have four or five speakers. How many options would the organizer have for the meeting? Answer: $5! + 4! = 144$. There are 144 options.
10. 61 328 **11.** 3 628 799
12. **a)** 48 **b)** 126
13. Answers may vary. For each roll of two dice, there are six ways to get doubles. There are $6 + 6 + 6$, or 18 ways to get doubles in one or two or three rolls.
14. Morse code is used to represent 26 letters, 10 digits, and 8 punctuation symbols, or a total of 44 symbols. Since each character has two options (dot or dash), a maximum of six characters is needed: $2^6 = 64$.

Extend

16. 82
17. **a)** 9 **b)** 44
18. **a)** 265 **b)** 455 **c)** 1

2.5 Probability Problems Using Permutations, pages 88–95

Example 1 Your Turn

a) $P(\text{all same}) = \frac{1}{10\,000\,000\,000}$
b) $P(\text{all 6s}) = \frac{1}{7776}$
For independent trials,
P(all the same) = (P(a success))$^{\#\text{ trials}}$.

Example 2 Your Turn

$P(\text{in grade order}) = \frac{1}{24}$

Example 3 Your Turn

a) $P(\text{ace, ace, ace, jack, jack}) = \frac{1}{1\,082\,900}$
b) $P(\text{heart, heart, club, club, club}) = \frac{143}{166\,600}$

Example 4 Your Turn

a) approximately 0.7164 **b)** approximately 0.2836

Reflect

R1. No. The probability that at least two people have the same birthday is approximately 0.6269.
R2. Answers may vary. If the trials are dependent, permutations can be used. Look for restrictions such as, "without replacement" or "alphabetical order."
R3. Answers may vary. The first represents 3 of 12 objects being arranged. The second is 3 times 1 of 12 objects being arranged.

Practise

1. $P(\text{king, queen, jack}) = \frac{8}{16\,575}$
2. $\frac{1}{15}$ **3.** A **4.** C

Apply

5. $\frac{1\,307\,674\,367\,999}{1\,307\,674\,368\,000} : \frac{1}{1\,307\,674\,368\,000}$
6. **a)** approximately 0.000 505
b) $\frac{{}_{30}P_3}{{}_{365}P_3} \approx 0.000\,505$
7. **a)** $P(\text{doubles}) = \frac{1}{6}$ **b)** $P(\text{doubles twice}) = \frac{1}{36}$
c) They are the same.
8. **a)** $P(3 \text{ boys}) = \frac{1}{8}$ **b)** $P(4 \text{ boys}) = \frac{1}{16}$
c) $P(5 \text{ boys}) = \frac{1}{32}$ **d)** $P(n \text{ boys}) = \frac{1}{2^n}$
9. **a)** $P(\text{MATH}) = \frac{1}{3024}$ **b)** $P(\text{M,A,T,H}) = \frac{1}{126}$
c) $P = \frac{4}{9}$
10. **a)** $P(\text{ascending order}) \approx 4.1697 \times 10^{-5}$
b) $P(\text{no same denomination}) \approx 0.2102$
11. $P(\text{at least two the same}) \approx 0.4114$
12. 23
13. **a)** $P(\text{songs in order}) = \frac{1}{3\,628\,800}$
b) $P = \frac{1}{45}$
14. **a)** 0.8203 : 0.1797 **b)** 0.4160 : 0.5840
15. Answers may vary. Example: 7

16. a) i) $\frac{1}{38\,955\,840}$ ii) $\frac{1}{78\,960\,960}$ iii) $\frac{1}{146\,611\,080}$

b) The probability of cracking the safe decreases as the five different numbers are chosen from a greater range of number.

18. The probability that at least two people have the same birthday as you is approximately 0.5687.

19. a) not throwing a sum of 7 on consecutive rolls

b) three different letters being arranged in alphabetical order

c) two out of five friends having the same birth month

Extend

20. 3.1664×10^{-7}

21. Answers may vary. Any scenario that has $n(A) = 1$ and $n(S) = {}_{15}P_7$. For example, winning first prize similar to question 20.

22. a) approximately 0.0947

b) approximately 6.9613×10^{-5}

23. a) approximately 0.0188

b) approximately 0.1004

24. a) approximately 2.2355×10^{-6}

b) approximately 0.0026

Chapter 2 Review, pages 96–97

1. 27 possible outcomes

2.

First Die / Second Die	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

The sum of 9 occurs eight times. There is only one occurrence of the sum 2 and sum 16.

3. a) 60 possible outcomes **b)** (Q, K, A) **c)** 60

4. a) 100 000 **b)** 800 000 s, or about 9.3 days

5. a) 360

b) Ryan has 432 choices to configure his computer. Increasing the number of choices for any option will increase the total number of possible configurations.

6. 150

7. 60

8. a) and **b)**

1
2 2
3 6 6
4 12 24 24
5 20 60 120 120
6 30 120 360 720 720

The first term in row n is n. To obtain the remaining terms in row n, multiply all the terms in the row above by n.

c) Answers may vary. The last term in row n equals $n!$. The last two terms in each row are equal.

9. 87 091 200

10. a) 144 **b)** 576 **c)** 5040

11. 576

12. 60

13. a) approximately 2.7557×10^{-7}

b) $1 - 2.7557 \times 10^{-7}$

14. a) $\frac{1}{30}$ **b)** $\frac{2}{15}$ **c)** $\frac{29}{30}$

15. a) approximately 8.4165×10^{-8}

b) $1 - 8.4165 \times 10^{-8}$

Chapter 2 Test Yourself, pages 98–99

1. C **2.** D **3.** A

4. ${}_9P_{10}$ is not defined, $n < r$.

$${}_9P_{10} = \frac{9!}{(9-10)!}$$
$$= \frac{9!}{(-1)!}$$

5. a) 24 possible outcomes **b)** 6

6. 1152 **7.** 95 040 **8.** $\frac{1}{56}$

9. a) 40 320 **b)** 25 200

10. 32 659 200

11. a) 3 575 880 **b)** 3 156 000 **c)** 1 806 000

12. a) $\frac{1}{456\,976}$ **b)** $\frac{1}{358\,800}$

13. approximately 0.9345

14. a) 311 875 200 **b)** 158 146 560

c) approximately 3.6938×10^{-6}

d) approximately 3.5013×10^{-5}

Chapter 3 Combinations

Prerequisite Skills, pages 102–103

1. a) 40 320 **b)** 60 480 **c)** 144 **d)** 151 200

e) 1320 **f)** 35 **g)** 330 **h)** 504 504

2. a) $n!$ is a product of sequential natural numbers with the form $n! = n(n-1)(n-2) \times \ldots \times 2 \times 1$.

b) The number of permutations of r items from a collection of n items is written as ${}_nP_r$ or $P(n, r)$.

${}_nP_r = \frac{n!}{(n-r)!}$, $n \geq r$

3. a) $\frac{7!}{4!}$ **b)** $\frac{100!}{8!}$ **c)** $\frac{n!}{(n-6)!}$ **d)** $\frac{15!}{(15-r)!}$

4. a) 40 320 **b)** 6720 **c)** 1716

5. a) 39 916 800 **b)** 86 400

6. a) 40 320 **b)** 336

7. a) The first and last terms are 1. The remaining terms are the sum of the two adjacent terms in the row above.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

b) Answers may vary. Consider the top of the triangle row 0. Then, the sum of entries in row n equals 2^n. The second diagonal contains the counting numbers 1, 2, 3, 4, 5, ….

8. a) $\frac{1}{8}$ **b)** $\frac{1}{8}$ **c)** $\frac{1}{8}$ **d)** $\frac{3}{8}$

9. a) approximately 0.0060 **b)** approximately 0.2549

c) approximately 0.3077

10. a) $\frac{1}{18}$ **b)** $\frac{1}{18}$ **c)** $\frac{1}{9}$

d) approximately 1.5619×10^{-16}

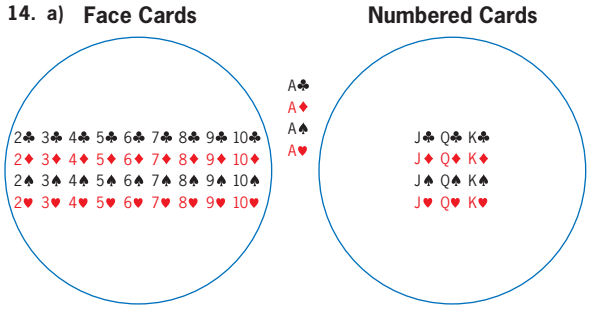
11. a) It could have six faces, two of each colour.

b) 18 **c)** $\frac{1}{18}$ **d)** $\frac{2}{3}$

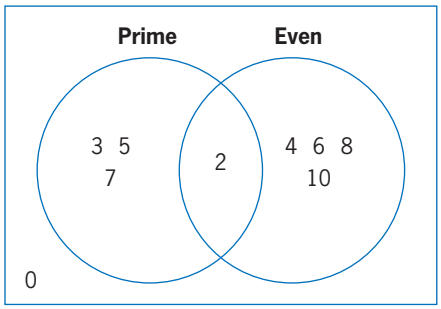
12. The events A and B are not mutually exclusive, since the overlap shows there are common elements. If A and B are non-mutually exclusive events, then the total number of favourable outcomes is: $n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$.

13. 7

14. a)



b)



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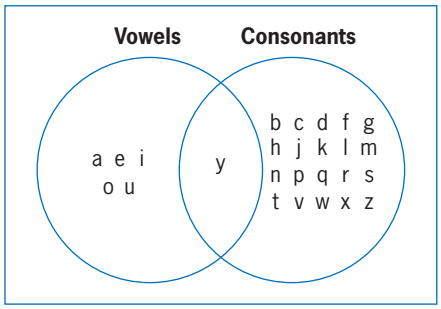
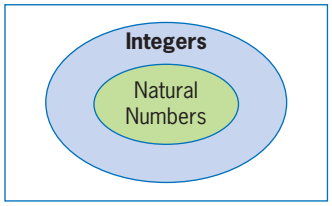


d)



15. a) x^6 **b)** $4a^2$ **c)** $25m^6$ **d)** $81k^{12}$

16. a) $x^2 + 2xy + y^2$ **b)** $a^3 + 3a^2b + 3ab^2 + b^3$

c) $4p^2 + 4pq + q^2$

17. a) $(n-1)(n-2)$ **b)** $n(n-1)$ **c)** n

3.1 Permutations With Non-Ordered Elements, pages 104–109

Example 1 Your Turn

a) In each case, there are 3! permutations.

b) In each case, there are 2!2! permutations.

Example 2 Your Turn

I would expect the number of orders of the second team to be higher.

Example 3 Your Turn

840

Reflect

R1. There are four identical 2s.

R2. No. The number of permutations of three girls and four boys, or seven people, is 7!
The number of permutations of three red balls and four green balls is $\frac{7!}{3!4!}$. All red balls are identical and all green balls are identical.

R3. Answers may vary. It is much quicker to use the formula. Drawing a tree diagram or chart may not be practical and takes longer.

Practise

1. a) 2520 **b)** 1680
c) 420 **d)** 1 905 780 240

2. B

3. D

4. a) 20 160 **b)** 420 **c)** 415 800 **d)** 180

5. a) 60 **b)** 20 **c)** 30 **d)** 5

Apply

6. a) 369 600 **b)** 34 650 **c)** 924

7. 70 **8.** 2520 **9.** 9 459 450

10. Since it is not possible to have 0 or a fractional number of ways to do something, the number of permutations involving identical objects will always be a natural number. The denominator must be a factor of the numerator.

11. 462; Assume that the streets are laid out in a grid pattern, and that all of the streets are continuous between his school and his home.

12. a) 369 600 **b)** 7 484 400

13. 42

14. 24

16. a) 10 764 000 **b)** 43 056 000 **c)** 1 794 000
d) The number of licence plates in part c) is about 0.4% of the total licence plates without restrictions.

17. Answers may vary. How many arrangements are there of 12 flags in a row if two are red, three are green, four are blue, and three are yellow?

Extend

18. 1320 **19.** 60 **20.** 185 794 560

3.2 Combinations, pages 110–115

Example 1 Your Turn

a) 210 **b)** 252 **c)** 210

Example 2 Your Turn

525

Example 3 Your Turn

35

Reflect

R1. Answers may vary.

a) For permutations, order matters. For example, select five out of eight for five different offices of the committee.

b) For combinations, order does not matter. For example, select five out of eight for a committee.

R2. Answers may vary. Examples: selecting groceries, selecting toppings for a sandwich

R3. A situation in which order matters (permutations) will have more possibilities. ${}_nC_r = \frac{{}_nP_r}{r!}$

For each combination of r items there are $r!$ permutations. So, the number of combinations is $r!$ times smaller than the number of permutations.

Practise

1. a) 126 **b)** 70 **c)** 220
d) 462 **e)** 420 **f)** 27 772 222 500

2. B **3.** B **4.** 210 **5.** 330 **6.** 1

7. a) 1 **b)** 1 **c)** 1 **d)** 1 **e)** 1

Apply

8. 168

9. a) 65 780 **b)** 792 **c)** 575 757
d) 845 000 **e)** 1 096 680

10. a) 5 586 853 480 **b)** 3 838 380
c) There is a larger number of ways to choose a 12-person jury than a 6-person jury. The denominator in part a) (28!12!) is smaller than that in part b) (34!6!).

11. a) 330 **b)** 150 **c)** 60 **d)** 5 **e)** 15
f) Parts b) to e) are subsets of part a). The combinations add to 230. With the inclusion of a three truck and one car option, the total is 330.

12. a) This is a combination situation, since the order does not matter.
b) ${}_{14}C_2 = 91$

13. a) 210 **b)** 252
c) ${}_{10}C_n$, $3 \le n \le 10$

14. a) **i)** ${}_7C_2 = 21$, ${}_7C_5 = 21$
ii) ${}_4C_3 = 4$, ${}_4C_1 = 4$
iii) ${}_{12}C_4 = 495$, ${}_{12}C_8 = 495$
The values in each pair are the same.
b) ${}_nC_r = {}_nC_{n-r}$. The only difference is the order of the terms in the denominator. The number of combinations of n items taken r at a time is equivalent to the number of combinations of n items taken $n - r$ at a time.
c)
$$\begin{aligned} {}_nC_r &= \frac{n!}{(n-r)!r!} \\ &= \frac{n!}{r!(n-r)!} \\ &= \frac{n!}{(n-(n-r))!(n-r)!} \\ &= {}_nC_{n-r} \end{aligned}$$

15. a) 6 126 120 **b)** 6 126 120
c) The results for parts a) and b) are the same. The order in which the jobs are assigned is irrelevant.

16. a) 20
b) The general formula for the number of diagonals in a polygon with n sides is $\frac{n(n-3)}{2}$. Using combinations, select two points from the n vertices: ${}_nC_2$. However, this also includes consecutive vertices that form a side of the polygon. So, subtract n, the number of sides. There are ${}_nC_2 - n$ diagonals in an n-sided convex polygon.

18. 756 756

19. 2 375 880 867 360 000

20. The techniques from the two sections result in the same answer.

21. There are ${}_{30}C_5 \times {}_{25}C_5 \times {}_{20}C_5 \times {}_{15}C_5 \times {}_{10}C_5 \times {}_5C_5$ ways to divide a class of 30 students into six teams of five members.
The number of ways to arrange a total of 30 balls with six different colours is also ${}_{30}C_5 \times {}_{25}C_5 \times {}_{20}C_5 \times {}_{15}C_5 \times {}_{10}C_5 \times {}_5C_5$.

22. For $r > 0$, there will always be more r-permutations of n items than r-combinations or n items.
${}_nC_r = \frac{{}_nP_r}{r!}$
In permutations order matters, not in combinations. For each combination of r items there are $r!$ permutations. So, the number of combinations is $r!$ times smaller than the number of permutations.

23. 42

Extend

24. a) Let the three consecutive numbers be represented by n, $n-1$, and $n-2$.
$$\begin{aligned} \frac{n(n-1)(n-2)}{3!} &= \frac{n(n-1)(n-2)(n-3)!}{(n-3)!3!} \\ &= {}_nC_3 \end{aligned}$$
b) Let the r consecutive numbers be represented by $n, n-1, n-2, \ldots, (n-r+1)$.
$$\begin{aligned} &\frac{n(n-1)(n-2)\cdots(n-r+1)}{r!} \\ &= \frac{n(n-1)(n-2)\cdots(n-r+1)(n-r)!}{(n-r)!r!} \\ &= {}_nC_r \end{aligned}$$

25. $n = 5$

26. 816

3.3 Problem Solving With Combinations, pages 116–121

Example 1 Your Turn

Combinations:
$$\begin{aligned} &{}_8C_1 + {}_8C_2 + {}_8C_3 + {}_8C_4 + {}_8C_5 + {}_8C_6 + {}_8C_7 + {}_8C_8 \\ &= \frac{8!}{(8-1)!1!} + \frac{8!}{(8-2)!2!} + \frac{8!}{(8-3)!3!} + \frac{8!}{(8-4)!4!} \\ &\quad + \frac{8!}{(8-5)!5!} + \frac{8!}{(8-6)!6!} + \frac{8!}{(8-7)!7!} + \frac{8!}{(8-8)!8!} \\ &= 255 \end{aligned}$$
Indirect Method:
$$\begin{aligned} 2^8 - 1 &= 256 - 1 \\ &= 255 \end{aligned}$$

Example 2 Your Turn

a) 576 050 767 488 **b)** 2 672 060
c) 4 306 559 400 **d)** 181 823 183 256
e) 158 362 127 352

Example 3 Your Turn

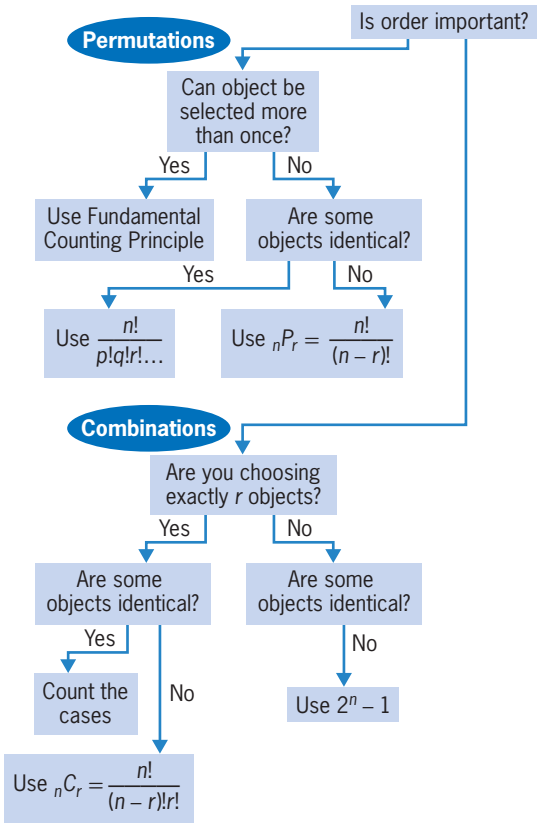
a) 2 052 000 **b)** 307 800

Reflect

R1. When determining the total number of subsets of a set, you add the number of possibilities in each case because the events are mutually exclusive.

R2. When using cases to determine the number of ways of selecting objects from different sets, you add because the events are mutually exclusive.

R3. Answers may vary.



Practise

1. 15 **2.** B **3.** C

4. a) 6720 **b)** 5880 **c)** 2016 **d)** 14 826

Apply

5. a) combinations; The order in which the 5 members are chosen does not matter.

b) permutations; Order matters, since each position holds an office.

c) both; The order in which the members of the team are chosen does not matter. When arranging for a photo, order matters.

d) permutations; Order matters because there are 3 different prizes.

6. 32 767 **7.** 92 **8.** 63 **9.** 15 **10.** 14 400

11. a) 1 237 792 **b)** 6 799 260 **c)** 3 219 112

12. a) 5 326 270 **b)** 6 864 396 000

13. 600 **14.** 2 041 200 000 **15.** 160

Extend

17. 968 **18.** 2 560 481 280

3.4 Combinations and Pascal's Triangle, pages 122–127

Example 1 Your Turn

$1 + 7 + 28 + 84 + 210 = 330$.

Comparing the terms in Pascal's triangle to combinations gives ${}_6C_6 + {}_7C_6 + {}_8C_6 + {}_9C_6 + {}_{10}C_6 = {}_{11}C_7$.

Example 2 Your Turn

Pascal's Method:

School

210	84	28	7		1
126	56	21	6		1
70	35	15	5		1
35	20	10	4		1
15	10	6	3		1
5	4	3	2		1
1	1	1	1		Home

Bill can take 210 different routes to school.

Combinations: ${}_{10}C_4 \times {}_6C_6 = 210$

Example 3 Your Turn

a) $(a+b)^4 = {}_4C_0a^4b^0 + {}_4C_1a^3b^1 + {}_4C_2a^2b^2 + {}_4C_3a^1b^3 + {}_4C_4a^0b^4$
$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

The terms ${}_4C_r$, where $r = 0$ to 4 correspond to row 4 in Pascal's triangle. The degree of each term is 4.

b) $(p+q)^5 = {}_5C_0p^5q^0 + {}_5C_1p^4q^1 + {}_5C_2p^3q^2 + {}_5C_3p^2q^3 + {}_5C_4p^1q^4 + {}_5C_5p^0q^5$
$= p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$

The terms ${}_5C_r$, where $r = 0$ to 5 correspond to row 5 in Pascal's triangle. The degree of each term is 5.

Reflect

R1. Answers may vary. The term labels begin with $t_{0,0}$. This maintains the pattern of first and last terms in each row both being 1, since there is only one term.

R2. The terms in row n of Pascal's triangle correspond to the combinations $t_{n,r} = {}_nC_r$. Each row in Pascal's triangle represents the combinations of choosing 0 items, 1 item, 2 items, and so on, out of n items.

R3. Yes. Finding the number of arrangements of n items with p of one type identical and q of another type identical is a valid solution. The result is the same $\frac{9!}{4!5!}$.

Practise

1. a) ${}_9C_0$ ${}_9C_1$ ${}_9C_2$ ${}_9C_3$ ${}_9C_4$ ${}_9C_5$ ${}_9C_6$ ${}_9C_7$ ${}_9C_8$ ${}_9C_9$

b) ${}_4C_4$ ${}_5C_4$ ${}_6C_4$ ${}_7C_4$ ${}_8C_4$

2. $a = 286 + 78$ $= 364$ $b = 1001 - 286$ $= 715$ $c = a + 1001$ $= 364 + 1001$ $= 1365$

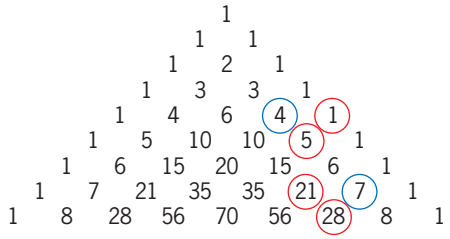
3. D

4. B

5.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

a) ${}_6C_3 = 20$ **b)** ${}_8C_4 = 70$



c) ${}_4C_3 = 4$ **d)** ${}_7C_6 = 7$

Apply

6. a) **i)** 4 **ii)** 9 **iii)** 16

b) They are perfect squares.

c) These occur in diagonal 2.

d) Each perfect square greater than 1 is equal to the sum of a pair of adjacent terms on diagonal 2 of Pascal's triangle: $n^2 = {}_nC_2 + {}_{n+1}C_2$, $n > 1$

7. a) 165; ${}_7C_7 + {}_8C_7 + {}_9C_7 + {}_{10}C_7 = {}_{11}C_8$

b) ${}_rC_r + {}_{r+1}C_r + {}_{r+2}C_r + \ldots + {}_{r+k-1}C_r = {}_{r+k}C_{r+1}$

8. 35 **9.** 27 **10.** 180

11. a) 32; Since this is a triangular array, combinations can be used to solve this question.

b) 20; Since this is not a triangular array, combinations cannot be used.

12. a) diagonal 2

b) $1 + 2 = {}_3C_2$ $1 + 2 + 3 = {}_4C_2$ $1 + 2 + 3 + 4 = {}_5C_2$
The sum of the first n natural numbers is ${}_{n+1}C_2$.

c) Example 1 involved sums of terms in diagonal 2. This question involves sums of terms in diagonal 1.

13. a) $x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8xy^7 + y^8$

b) $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

c) $16a^4 + 32a^3b + 24a^2b^2 + 8ab^3 + b^4$

d) $x^6 - 6x^4 + 12x^2 - 8$

14. a)

Number of Lines	Number of Regions	Rewrite the Number of Regions
0	1	1
1	2	1 + 1
2	4	1 + (1 + 2)
3	7	1 + (1 + 2 + 3)
4	11	1 + (1 + 2 + 3 + 4)
⋮		⋮
n		1 + (1 + 2 + 3 + … + n)

The values being added represent the triangular numbers, whose sum is $\frac{n(n+1)}{2}$. The formula for the number of regions is $R(n) = 1 + \frac{n(n+1)}{2}$.
The sum of the first n natural numbers is ${}_{n+1}C_2$.
So, $R(n) = 1 + {}_{n+1}C_2$.

b) 211

15. a)

n	Sum of Squares $1^2 + 2^2 + \ldots + n^2$	$t_{n+1,3} + t_{n+2,3}$
1	$1^2 = 1$	
2	$1^2 + 2^2 = 5$	$t_{3,3} + t_{4,3} = 1 + 4 = 5$
3	$1^2 + 2^2 + 3^2 = 14$	$t_{4,3} + t_{5,3} = 4 + 10 = 14$
4	$1^2 + 2^2 + 3^2 + 4^2 = 30$	$t_{5,3} + t_{6,3} = 10 + 20 = 30$
5	$1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55$	$t_{6,3} + t_{7,3} = 20 + 35 = 55$
6	$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 91$	$t_{7,3} + t_{8,3} = 35 + 56 = 91$

b) The values in columns two and three are the same.

c) The sum of the first n squares is ${}_{n+1}C_3 + {}_{n+2}C_3$, $n > 1$.

d) 42 925

16. a)

Layer, n	Total Number of Oranges	$t_{n+1,2} + t_{n+1,3}$
1	1	
2	1 + 3 = 4	$t_{3,2} + t_{3,3} = 3 + 1 = 4$
3	1 + 3 + 6 = 10	$t_{4,2} + t_{4,3} = 6 + 4 = 10$
4	1 + 3 + 6 + 10 = 20	$t_{5,2} + t_{5,3} = 10 + 10 = 20$

b) The total number of oranges needed for a stack of n layers can be found in diagonal 3 of Pascal's triangle.

c) ${}_{n+1}C_2 + {}_{n+1}C_3$, $n > 1$ **d)** 220

18. $(h + t)^5 = {}_5C_0h^5t^0 + {}_5C_1h^4t^1 + {}_5C_2h^3t^2 + {}_5C_3h^2t^3 + {}_5C_4h^1t^4 + {}_5C_5h^0t^5$
$= 1h^5t^0 + 5h^4t^1 + 10h^3t^2 + 10h^2t^3 + 5h^1t^4 + 1h^0t^5$
If a coin is tossed five times, there is
1 way to get 5 heads and 0 tails
5 ways to get 4 heads and 1 tail
10 ways to get 3 heads and 2 tails
10 ways to get 2 heads and 3 tails
5 ways to get 1 head and 4 tails
1 way to get 0 heads and 5 tails

Extend

19. a) $p^5 - 5p^3 + 10p - \frac{10}{p} + \frac{5}{p^3} - \frac{1}{p^5}$

b) $81m^8 + 216m^4 + 216 + \frac{96}{m^4} + \frac{16}{m^8}$

3.5 Probabilities Using Combinations, pages 128–133

Example 1 Your Turn

a) approximately a 0.000 113% chance

b) approximately a 0.015 765% chance

c) approximately 0.999 841 22

d) Answers may vary. It is extremely unlikely that anyone will win the lottery prizes.

Example 2 Your Turn

a) approximately 0.36 **b)** approximately 0.28

c) It is more likely that there will be equal numbers of male and female students than more female than male students.

Example 3 Your Turn

Slots A and F: 0; Slots B and E: $\frac{1}{8}$; Slots C and D: $\frac{3}{8}$

Reflect

R1. Answers may vary. A student selects three cards in order, without replacement, from a standard deck. What is the probability that the student selects a king, then two queens? What is the probability that a hand of three cards contains only face cards?

R2. If you interpret the language to mean Jake is first and Hamid is second, order matters. So, the probability that two are the top two finishers is $\frac{{}_2P_2}{{}_8P_2}$.
If you interpret the language to mean Jake and Hamid are top two with no assigned place (first or second), order does not matter. So, the probability that two are the top two finishers is $\frac{{}_2C_2}{{}_8C_2}$. Both expressions result in the same probability of $\frac{1}{28}$.

Practise

1. **a)** approximately 0.000 495
   b) approximately 0.025
   c) approximately a 0.000 305
2. $\frac{1}{3}$ **3.** B **4.** C

Apply

5. **a)** approximately 0.006 **b)** approximately 0.076
   c) approximately 0.002 **d)** approximately 0.7
6. approximately 1.575×10^{-12}
7. approximately 0.303
8. approximately 0.145 : 0.855
9. approximately 0.167
10. **a)** space D at about 0.38
    b) If the checker begins in a different location, the number of possible paths ending at each destination will be different.
11. approximately 0.908
12. Answers may vary. The probability of the disc landing in each slot at the bottom of the board depends on its starting slot. Dropping the ball from one of two centre slots (3 or 4) will give the most paths, so that is the best strategy. Starting in slot 3, the probability of the ball landing in A or F: $\frac{1}{32}$; B or E: $\frac{5}{32}$; C or D: $\frac{5}{16}$; G: 0.
13. approximately 0.54
14. approximately 0.476

Extend

16. $\frac{7}{9}$
17. **a)** approximately 0.952 **b)** approximately 0.952
    c) approximately 0.548
18. 0.225

Chapter 3 Review, pages 134–135

1. 840
2. **a)** 840 **b)** 3 326 400 **c)** 277 200
3. 1001
4. **a)** **i)** $r = 4$ **ii)** $r = 5$
   iii) $r = 3$ or $r = 4$ **iv)** $r = 7$ or $r = 8$
   b) The greatest number of combinations when n is even occurs at $r = \frac{n}{2}$. The greatest number of combinations when n is odd occurs at $r = \frac{n}{2} \pm 0.5$.
5. **a)** 352 800 **b)** 210
6. **a)** 210
   b) Answers may vary. A committee has 10 people. In how many ways could a president and vice president be chosen?
   c) Answers may vary. From a committee of 10 people, there are ${}_{10}P_3$, or 90 ways to choose a president and vice president.
7. 300
8. 31
9. **a)** 30 257 175 **b)** 22 120 065 **c)** 22 116 900
10. $a = 792$, $b = 462$
11. **a)** **i)** 1 **ii)** 5 **iii)** 15
    b) They are entries in diagonal 4 of Pascal's triangle.
    c) They are represented by ${}_nC_4$.
    d) 495
12. **a)** row 9 **b)** row 12
13. Pascal's Method:

Home	1	1	1	1
1	2	3	4	5
1	3	6	10	15
1	4	10	20	35
1	5	15	35	70
1	6	21	56	126
1	7	28	84	210
1	8	36	120	330
				School

Stephen can take 330 different routes to school.
Combinations: ${}_{11}C_2 \times {}_7C_7 = 330$

14. **a)** $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$
    b) $16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$
15. **a)** approximately 0.000 285
    b) approximately 0.000 495
16. **a)** approximately 0.005 **b)** approximately 0.587
17. **a)** approximately 0.004 **b)** approximately 0.496
    c) 0.504

Chapter 3 Test Yourself, pages 136–137

1. C **2.** B **3.** B **4.** A
5. 1001 **6.** 35 **7.** 70 **8.** 15 120
9. **a)** ${}_8C_3 = \frac{{}_8P_3}{3!}$
   b) Both ${}_8C_3$ and ${}_8P_3$ represent the number of arrangements of 3 items from 8. However, combinations have no regard for order, while permutations do. Combination: A committee of three people can be chosen from a list of 8 people in ${}_8C_3$, or 56 ways. Permutation: From a committee of 8 people, there are ${}_8P_3$, or 336 ways to choose a president, vice president, and secretary.
10. approximately 0.396
11. 210
12. Permutations With Like Objects:
    $\frac{18!}{3!3!3!3!3!3!} = 137\,225\,088\,000$
    Combinations:
    ${}_{18}C_3 \times {}_{15}C_3 \times {}_{12}C_3 \times {}_9C_3 \times {}_6C_3 \times {}_3C_3 = 137\,225\,088\,000$
13. **a)** Each row in Pascal's triangle represents the combinations of choosing 0 items, 1 item, 2 items, and so on, out of n items.
    b) The terms of Pascal's triangle are generated by adding two adjacent terms and placing the result immediately below them in the next row.
    $t_{n,r} + t_{n,r+1} = t_{n+1,r+1}$
    Using combinations, ${}_nC_r + {}_nC_{r+1} = {}_{n+1}C_{r+1}$.
14. **a)** Alternately subtracting and adding successive terms in a row of Pascal's triangle results in 0.
    b) For $n > 0$, ${}_nC_0 - {}_nC_1 + {}_nC_2 - \ldots {}_nC_n$.
15. **a)** 169
    b) It would be greater, since there are more chances to get olives or mushrooms. ${}_{15}C_4 - {}_{13}C_4 = 650 > 169$
16. **a)** approximately 0.81 **b)** approximately 0.008
    c) approximately 0.184
17. **a)** approximately 0.0002 **b)** approximately 0.043
    c) approximately 0.381 **d)** approximately 0.624
    e) approximately 0.351

Chapters 1 to 3 Cumulative Review, pages 138–139

1. a) $\frac{2}{3}$ **b)** $\frac{1}{3}$

2. a) 1:4 **b)** 1:1

3. a) Experimental probability is based on experimental trials, while theoretical probability is based on the analysis of all outcomes.

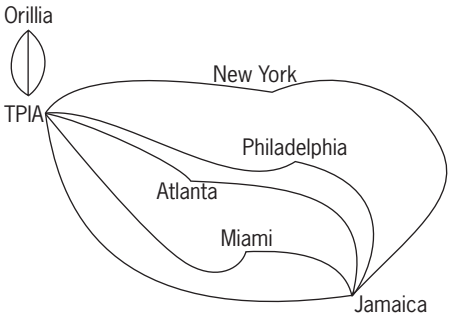
b) Experimental probability approaches theoretical probability as a very large number of trials are conducted.

4. $\frac{2}{9}$ **5.** $\frac{7}{13}$

6. a) $\frac{9}{25}$ **b)** $\frac{3}{10}$

c) The answers to parts a) and b) are different because one deals with replacement and the other does not.

7. Map:



Tree diagram outcomes: (O, D, NY, J), (O, D, M, J), (O, D, A, J), (O, D, P, J), (O, D, J), (O, B, NY, J), (O, B, M, J), (O, B, A, J), (O, B, P, J), (O, B, J), (O, T, NY, J), (O, T, M, J), (O, T, A, J), (O, T, P, J), (O, T, J)

List:

OT_cNJ	OT_cPJ	OT_cAJ	OT_cMJ	OT_cJ
OT_bNJ	OT_bPJ	OT_bAJ	OT_bMJ	OT_bJ
OT_tNJ	OT_tPJ	OT_tAJ	OT_tMJ	OT_tJ

8. a) 216 **b)** 1296 **c)** 64 **d)** 1728

9. a) 1680

b) Adjacent countries share boundaries. These boundaries are more visible if the countries are different colours. With only 8 colours available, there could be many countries that are coloured the same colour, but adjacent countries should not be.

10. 124 251 000 **11.** 1680 **12.** 399 168 000

13. a) $\frac{1}{120}$

b) Winning would be less probable if the digits could be repeated, because there would be more possible outcomes.

14. 63 **15.** 1 646 400

16. a) 210

b) Answers may vary. ${}_{10}C_6 = 210$. Pascal's method will arrive at the same result by adding the number of paths to the adjacent grid points to determine the number of paths to the given point.

17. a)

n	${}_nC_2 \div {}_nC_1$	Result
2	1 ÷ 2	0.5
3	3 ÷ 3	1
4	6 ÷ 4	1.5
5	10 ÷ 5	2
6	15 ÷ 6	2.5
7	21 ÷ 7	3
8	28 ÷ 8	3.5
9	36 ÷ 9	4

b) When n is odd, ${}_nC_2$ is divisible by ${}_nC_1$.

c) When n is odd, ${}_nC_2$ is divisible by ${}_nC_1$. These rows have an even number of terms.

d) Yes, ${}_{15}C_2$ is divisible by ${}_{15}C_1$, because n is odd.

18. 56

19. a) 31 **b)** 31

20. a) 6

b) Yes. Let the three directions the spider can move be right, left, and down. The spider needs to travel 3 edges to its destination. Select any one of these three edges to travel, say right. From two remaining edges, select another direction, say left. Then, the last edge travel down.

$${}_3C_1 \times {}_2C_1 \times {}_1C_1 = \frac{3!}{2!1!} \times \frac{2!}{1!1!} \times \frac{1!}{0!1!}$$
$$= 6$$

21. a) approximately 0.190 **b)** approximately 0.9903

Chapter 4 Probability Distributions for Discrete Variables

Prerequisite Skills, pages 142–143

1. a) $\frac{4}{52}$ or $\frac{1}{13}$ **b)** $\frac{26}{52}$ or $\frac{1}{2}$ **c)** $\frac{13}{52}$ or $\frac{1}{4}$

2. a)

Sum	Possible Groupings	Number of Outcomes	Probability
3	(1,1,1)	1	$\frac{1}{216}$
4	(1,2,1)	3	$\frac{3}{216}$
5	(1,3,1), (1,2,2)	6	$\frac{6}{216}$
6	(1,4,1), (1,3,2), (2,2,2)	10	$\frac{10}{216}$
7	(1,4,2), (1,3,3), (5,1,1), (3,2,2)	15	$\frac{15}{216}$
8	(1,4,3), (1,2,5), (1,1,6), (4,2,2), (3,3,2)	21	$\frac{21}{216}$
9	(6,2,1), (5,3,1), (5,2,2), (4,4,1), (4,3,2), (3,3,3)	25	$\frac{25}{216}$
10	(6,3,1), (6,2,2), (5,3,2), (5,4,1), (4,4,2), (4,3,3)	27	$\frac{27}{216}$
11	(6,4,1), (6,3,2), (5,5,1), (5,4,2), (5,3,3), (4,4,3)	27	$\frac{27}{216}$
12	(6,5,1), (6,4,2), (6,3,3), (5,5,2), (5,4,3), (4,4,4)	25	$\frac{25}{216}$
13	(6,6,1), (6,5,2), (6,4,3), (5,5,3), (5,4,4)	21	$\frac{21}{216}$
14	(6,4,4), (6,5,3), (5,5,4), (6,6,2)	15	$\frac{15}{216}$
15	(6,6,3), (6,4,5), (5,5,5)	10	$\frac{10}{216}$
16	(6,6,4), (6,5,5)	6	$\frac{6}{216}$
17	(6,6,5)	3	$\frac{3}{216}$
18	(6,6,6)	1	$\frac{1}{216}$

b) The sum of the probabilities is 1.

3. a) $\frac{3}{8}$ **b)** $\frac{1}{8}$

4. a) $\frac{1}{2}$ **b)** $\frac{2}{7}$

5. a) $\frac{8}{12}$ or $\frac{2}{3}$ **b)** $\frac{7}{12}$ **c)** 0

6. a) independent **b)** dependent
c) independent **d)** dependent

7. a) Tree diagram outcomes: (H, H, H), (H, H, T), (H, T, H), (H, T, T), (T, H, H), (T, H, T), (T, T, H), (T, T, T)

b) independent; the outcome of flipping a coin does not affect the outcome of flipping another coin.

8. a) $\frac{1}{156}$ **b)** $\frac{1}{78}$ **c)** $\frac{15}{26}$

9. a) 26 400 **b)** 31 119

10. a) 108 **b)** 252 **c)** 894

11. a) $\frac{15!}{3!3!3!3!3!}$ or ${}_{15}C_3 \times {}_{12}C_3 \times {}_9C_3 \times {}_6C_3 \times {}_3C_3$ or 168 168 000

b) ${}_{15}C_5 \times {}_{10}C_5 \times {}_5C_5$ or 756 756

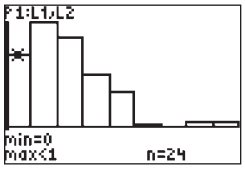
12. a) 44 352 **b)** 0.0707… **c)** 0.3426 …
d) 0.2304 **e)** 0.0823 …

13. a) $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

b) $1024x^5 + 3840x^4y + 5760x^3y^2 + 4320x^2y^3 + 1620xy^4 + 243y^5$

c) 1 **d)** 1

14.

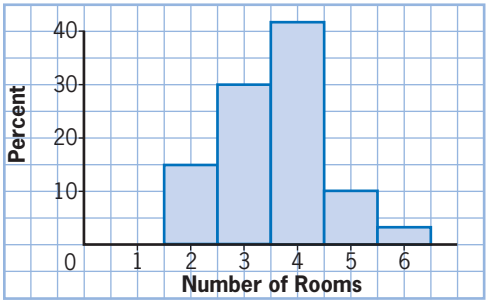


4.1 Probability Distributions, pages 144–153

Example 1 Your Turn

a) the number of rooms in apartments in a particular complex

b)



c) The area of each bar represents its probability, as a percent. The width of each bar is 1, so the probability, as a percent is shown on the vertical axis.

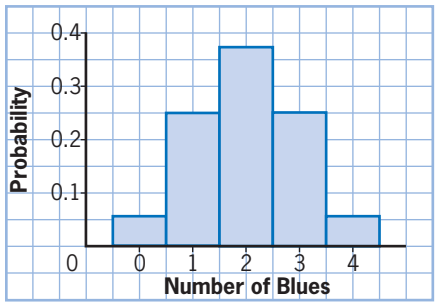
d) Four-room apartments occur most frequently, and the probability decreases as the room value increases or decreases from four.

e) Sum of the probabilities is 1. Yes.

Example 2 Your Turn

a) Tree diagram outcomes:
(R, R, R, R), (R, R, R, B), (R, R, B, R), (R, R, B, B), (R, B, R, R), (R, B, R, B), (R, B, B, R), (R, B, B, B), (B, R, R, R), (B, R, R, B), (B, R, B, R), (B, R, B, B), (B, B, R, R), (B, B, R, B), (B, B, B, R), (B, B, B, B)

b)



c) 2

d) On average, the spinner would land on blue 2 out of the 4 spins.

Example 3 Your Turn

a) −$2.25

b) Answers may vary. No; You lose money if you buy a ticket.

c) Answers may vary. The price could be reduced or more prizes could be given away.

Reflect

R1. While it is impossible to have 1.8 children, expected values are predicted average values and should not be rounded.

R2. Answers may vary. The number of email messages you receive each day of the week and the number of students in each mathematics class at your school. These examples are discrete because each must be a whole number.

R3. Answers may vary. Create a table showing all possible sums of the two 12-sided dice. Determine the frequency of each sum and its probability. Then, construct a histogram to illustrate the probability distribution.

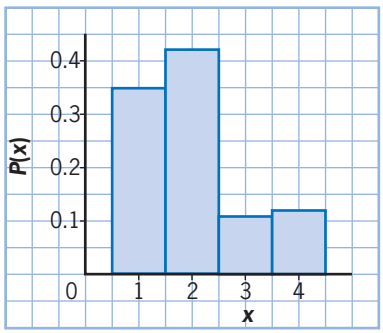
Practise

1. a) discrete **b)** continuous **c)** discrete
d) discrete **e)** continuous

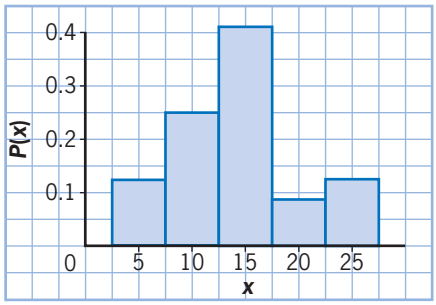
2. A

3. C

4. a)



b)



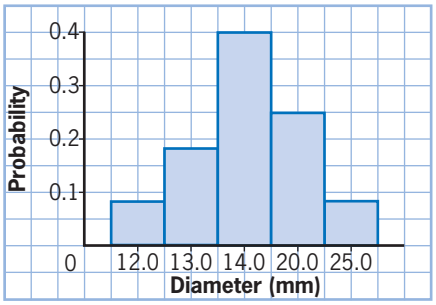
5. a) 2.6 **b)** 3.8

Apply

6. a) the diameter of the marbles in a bag

b) discrete; the number of marbles in a bag is a whole number

c)



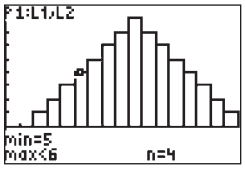
d) The area of each bar represents its probability. The width of each bar is 1, so the probability is shown on the vertical axis.

e) about 16.067; the weighted mean of the outcomes equals the expectation

7. a)

Sum, x	Frequency
2	1
3	2
4	3
5	4
6	5
7	6
8	7
9	8
10	7
11	6
12	5
13	4
14	3
15	2
16	1

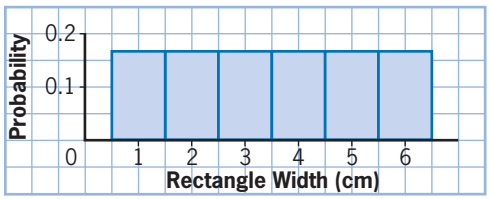
b)



c) 9; On average, the expected sum of two dice is 9.

8. Answers may vary. Perimeter = 24 cm.

Rectangle Width (cm), x	Distribution of Dimensions	Frequency	Probability, $P(x)$
1	1 by 11	1	$\frac{1}{6}$
2	2 by 10	1	$\frac{1}{6}$
3	3 by 9	1	$\frac{1}{6}$
4	4 by 8	1	$\frac{1}{6}$
5	5 by 7	1	$\frac{1}{6}$
6	6 by 6	1	$\frac{1}{6}$



9. a) $\frac{9}{2000}$ **b)** \$0.525 **c)** \$4.475 **d)** \$5.03

11. a) Let D represent rolling doubles and ND represent not rolling doubles. Tree diagram outcomes: (D, D, D), (D, D, ND), (D, ND, D), (D, ND, ND), (ND, D, D), (ND, D, ND), (ND, ND, D), (ND, ND, ND)

b)

Number of Doubles	Distribution of Doubles	Probability
0	(ND,ND,ND)	$\frac{125}{216}$
1	(D,ND,ND) (ND,D,ND) (ND,ND,D)	$\frac{75}{216}$
2	(D,D,ND) (D,ND,D) (ND,D,D)	$\frac{15}{216}$
3	(D,D,D)	$\frac{1}{216}$

c) 0.5

12.

Sum	Possible Groupings	Number of Outcomes	Probability
3	(1,1,1)	1	$\frac{1}{216}$
4	(1,2,1)	3	$\frac{3}{216}$
5	(1,3,1), (1,2,2)	6	$\frac{6}{216}$
6	(1,4,1), (1,3,2), (2,2,2)	10	$\frac{10}{216}$
7	(1,4,2), (1,3,3), (5,1,1), (3,2,2)	15	$\frac{15}{216}$
8	(1,4,3), (1,2,5), (1,1,6), (4,2,2), (3,3,2)	21	$\frac{21}{216}$
9	(6,2,1), (5,3,1), (5,2,2), (4,4,1), (4,3,2), (3,3,3)	25	$\frac{25}{216}$
10	(6,3,1), (6,2,2), (5,3,2), 5,4,1), (4,4,2), (4,3,3)	27	$\frac{27}{216}$
11	(6,4,1), (6,3,2), (5,5,1), (5,4,2), (5,3,3), (4,4,3)	27	$\frac{27}{216}$
12	(6,5,1), (6,4,2), (6,3,3), (5,5,2), (5,4,3), (4,4,4)	25	$\frac{25}{216}$
13	(6,6,1), (6,5,2), (6,4,3), (5,5,3), (5,4,4)	21	$\frac{21}{216}$
14	(6,4,4), (6,5,3), (5,5,4), (6,6,2)	15	$\frac{15}{216}$
15	(6,6,3), (6,4,5), (5,5,5)	10	$\frac{10}{216}$
16	(6,6,4), (6,5,5)	6	$\frac{6}{216}$
17	(6,6,5)	3	$\frac{3}{216}$
18	(6,6,6)	1	$\frac{1}{216}$

13. Answers may vary.

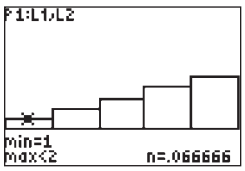
14. a) Answers may vary. There are many more values in these data sets that start with one compared to other digits.

b) 3.441

15. a) $\frac{4}{52}$ or $\frac{1}{13}$ **b)** $\frac{12}{169}$

c) $\frac{144}{2197}$ **d)** $\left(\frac{48}{52}\right)^{n-1} \times \frac{4}{52}$

16.



17. approximately 7.02

4.2 Uniform Distributions, pages 154–159

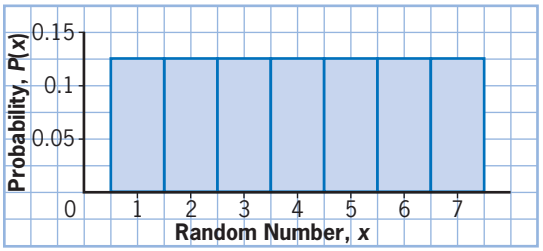
Example 1 Your Turn

a) Yes. Each randomly generated radius is equally likely and there is a single trial.

b)

Random Number, x	$P(x)$	$x \cdot P(x)$
1	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{8}$	$\frac{2}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$
4	$\frac{1}{8}$	$\frac{4}{8}$
5	$\frac{1}{8}$	$\frac{5}{8}$
6	$\frac{1}{8}$	$\frac{6}{8}$
7	$\frac{1}{8}$	$\frac{7}{8}$
8	$\frac{1}{8}$	$\frac{8}{8}$

c)



d) 4.5; On average, the expected radius length will be 4.5.

Example 2 Your Turn

No. A fair game will have an expectation equal to 0. This is not a fair game because the player will win 0.75 point on each turn, on average.

Reflect

R1. Yes, randomly selecting students by their student number is uniform. Each randomly generated student number is equally likely in a single trial.

R2. a)

b)

R3. The expected profit per ticket is \$2 – \$0.75 = \$1.25. This gives an advantage to the school.

Practise

1. a) no; there are different probabilities for different sums in a trial

b) yes; there is an equal probability of selecting each card in a trial

c) yes; there is an equal probability of each song being randomly selected in a trial

d) no; there are different probabilities for different numbers of boys in a family of five

e) yes; there is an equal probability of each person being randomly selected in a trial

2. C

3. D

4. 6 green balls

5. a) 15 **b)** 3.5

Apply

6. a) 6.5

b) No. The expectation is simply the predicted average of all outcomes. Each number between 1 and 12 is equally likely.

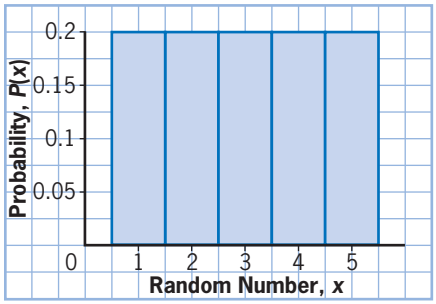
7. a) $\frac{1}{52}$

b) yes; there is an equal probability of selecting any specific card in a trial

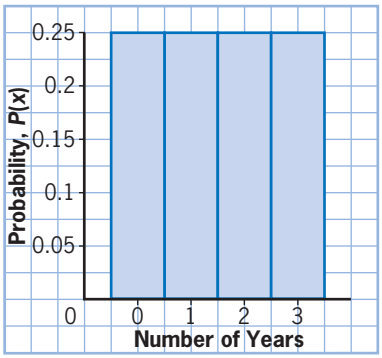
c) $\frac{1}{51}$

d) no; the probability of any specific card changes after a card is removed

8. Let A = 1, B = 2, C = 3, D = 4, and E = 5.



9. a)



b) Answers may vary. The only chance of going free is if prisoner P confesses. Prisoner P should confess. If Q confesses, then P should also confess. If Q denies, then P should confess because he will be set free.

10. a) Answers may vary. 2.5, 3.5, 4.5, 6.5, and 10.5 (the average of the face values for each platonic solid).

b) four faces: 2.5; six faces: 3.5; eight faces: 4.5; twelve faces: 6.5

c) Answers may vary. My findings in part b) confirm my prediction for the icosahedron.

11. \$700.

12. a) The areas of the three regions are not equal so this is not a uniform distribution.

b) Answers may vary. Let A be worth 5 points, B be worth 16 points, and C be worth 20 points.

c) Answers may vary. A similar target with a uniform distribution would have regions of equal area.

13. –\$1.3726 **14.** \$5.55

16. Answers may vary.

Extend

17. The expected outcome is $\frac{n+1}{2}$.

18. Choosing 1 gives the greatest expected outcome at \$70.50.

4.3 Binomial Distribution, pages 160–169

Example 1 Your Turn

a) ${}_{10}C_1\left(\frac{1}{13}\right) \times {}_9C_9\left(\frac{12}{13}\right)^9$ or approximately 0.3743

b) ${}_{10}C_3\left(\frac{1}{13}\right)^3 \times {}_7C_7\left(\frac{12}{13}\right)^7$ or approximately 0.0312

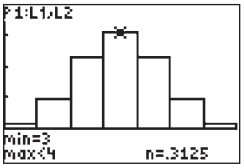
Example 2 Your Turn

a) X = the number of occurrences of a girl

b)

Number of Girls, x	Probability, $P(x)$	$x \bullet P(x)$
0	${}_6C_0(0.5)^0(0.5)^6 = 0.015\,625$	0
1	${}_6C_1(0.5)^1(0.5)^5 = 0.093\,75$	0.093 75
2	${}_6C_2(0.5)^2(0.5)^4 = 0.234\,375$	0.468 75
3	${}_6C_3(0.5)^3(0.5)^3 = 0.3125$	0.9375
4	${}_6C_4(0.5)^4(0.5)^2 = 0.234\,375$	0.9375
5	${}_6C_5(0.5)^5(0.5)^1 = 0.093\,75$	0.468 75
6	${}_6C_6(0.5)^6(0.5)^0 = 0.015\,625$	0.093 75

c) The probabilities sum to 1.

d) 

This probability distribution is more closely bell-shaped with the mode at $x = 3$ girls. It is slightly skewed to the left.

e) 3; On average, you can expect to have three girls in a family of six children.

Example 3 Your Turn

a) about 95.02%

b) 3.2 days

Reflect

R1. ${}_nC_x$ represents the number of ways each of the number of successes can happen.

R2. Answers may vary. In a binomial distribution, p and q represent the probability of success and failure, respectively. The probability of drawing a heart (success) from a deck of cards is 0.25, while the probability of not drawing a heart (failure) is 0.75.

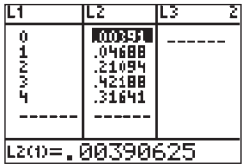
R3. Answers may vary. In a group of 25 people, the expected number of left-handed people is 2.75. However, the probability of one person in 25 being left-handed is ${}_{25}C_1(0.11)^1(0.89)^{24}$, or about 16.78%.

Practise

1. A **2.** B **3.** D

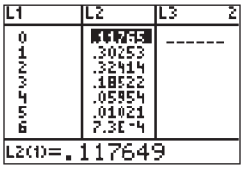
4. a) Tree diagram outcomes:
(1, 1, 1, 1), (1, 1, 1, 2), (1, 1, 2, 1), (1, 1, 2, 2),
(1, 2, 1, 1), (1, 2, 1, 2), (1, 2, 2, 1), (1, 2, 2, 2),
(2, 1, 1, 1), (2, 1, 1, 2), (2, 1, 2, 1), (2, 1, 2, 2),
(2, 2, 1, 1), (2, 2, 1, 2), (2, 2, 2, 1), (2, 2, 2, 2)

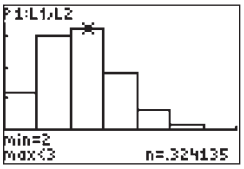
b) In the tree diagram, each roll of 1 has probability of 0.75 and each roll of 2 has probability of 0.25.

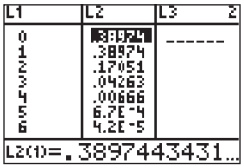
c)


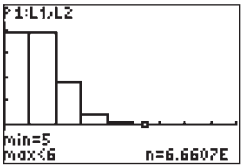
Assume a success is a rolling a 1. Use a graphing calculator to determine all $P(x) = {}_nC_x p^x q^{n-x}$, where $n = 4$, $x = 0, 1, 2, 3, 4$, $p = 0.75$, and $q = 0.25$.

d) The expansion contains the same terms as determined using the binomial distribution formula, but in reverse order.

5. a)




b)




6. about 333.33

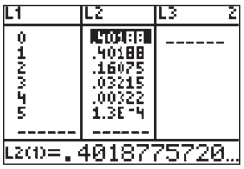
7. a) ${}_5C_2(0.5)^2(0.5)^3$ or 0.3125

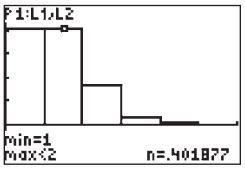
b) ${}_5C_3(0.5)^3(0.5)^2$ or 0.3125.

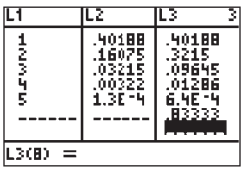
Apply

8. a) 0.0305

b) 0.0328

9. a)


b)


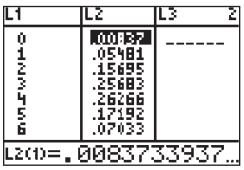
c)


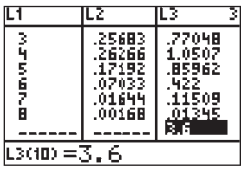
The expected number of sums of 7 in five rolls is about 0.8333.

$E(X) = np$

$= \frac{5}{6}$

≈ 0.8333

10. a)


b)


The expected number of bull's-eyes is 3.6.

c) $P(8) = 0.001\,68$, so it is highly unlikely.

11. a) ${}_5C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$ or about 0.1608

b) $\left(\frac{1}{6} + \frac{5}{6}\right)^5 = 1\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^0 + 5\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^1 + 10\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 + 10\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 + 5\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^4 + 1\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^5$

c) The fourth term: ${}_5C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$.

d) The general term, ${}_nC_x p^x q^{n-x}$, in the binomial expansion represents the probability of x successes in n independent trials, where p is the probability of success on an individual trial and q is the probability of failure on that same individual trial.

12. a) about 0.0168

b) about 2.7901×10^{-7}

c) about 0.018 84

d) Answers may vary.

e) Answers may vary. Yes. The expected number of bulbs that do not meet specification out of 10 bulbs is 10(0.06), or 0.6.

13. Calculate the expectation. Points received indicate a positive value for the random variable, x. Each point represents a car given away.

Number of Cars That Start	Point Value, x	$P(x)$	$x \cdot P(x)$
0	0	${}_5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^5$	0
1	0	${}_5C_1\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^4$	0
2	2	${}_5C_2\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^3$	0.1458
3	3	${}_5C_3\left(\frac{1}{10}\right)^3\left(\frac{9}{10}\right)^2$	0.0243
4	4	${}_5C_4\left(\frac{1}{10}\right)^4\left(\frac{9}{10}\right)^1$	0.0018
5	5	${}_5C_5\left(\frac{1}{10}\right)^5\left(\frac{9}{10}\right)^0$	0.00005
		Sum	0.17195

A fair game will have an expectation equal to 0. This game favours the contestant, on average.

14. a) ${}_{10}C_8(0.8)^8(0.2)^2$ or about 0 30.2%

b) about 67.8%

16. a) The probability of success changes with each trial.

b) Answers may vary. A jar contains 12 red balls and eight green balls. Six balls are removed with replacement. What is the probability that four of the balls are red?

c) 0.311 04

17. a) sample of 7: approximately 0.1998; sample of 100: approximately 7.3783×10^{-4}; sample of 1000: 0

b) The probability of the majority of people approving the transit system decreases with the increase in sample size.

18. $3700

19. Answers may vary.

20. Answers may vary. 19% of the Canadian population live in rural areas. A group of 30 Canadians is selected at Pearson International Airport. What is the probability that more than two of them live in a rural area? What is the expected number of rural citizens?

Extend

21. a) $P(\text{outcome 1, outcome 2, outcome 3}) = \frac{n!}{a!b!c!}p^aq^br^c$ where

P is the probability of outcome 1, outcome 2, and outcome 3

a is the number of times outcome 1 occurs

b is the number of times outcome 2 occurs

c is the number of times outcome 3 occurs

p is the probability of outcome 1

q is the probability of outcome 2

r is the probability of outcome 3

b) about 0.0427

22. $E(X)$

$$= \sum_{i=1}^{n} x_i \cdot P(x_i)$$

$$= \sum_{x=0}^{n} x \cdot {}_nC_x p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x\frac{n!}{(n-x)!x!}p^x(1-p)^{n-x}$$

$$= \sum_{x=1}^{n} x\frac{n!}{(n-x)!x(x-1)!}p^x(1-p)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(n-x)!(x-1)!}p^x(1-p)^{n-x}$$

$$= np\sum_{x=1}^{n} \frac{(n-1)!}{(n-x)!(x-1)!}p^{x-1}(1-p)^{n-x}$$

$$= np\sum_{x=1}^{n} \frac{(n-1)!}{((n-1)-(x-1))!(x-1)!}p^{x-1}(1-p)^{(n-1)-(x-1)}$$

Let $m = n-1$ and $y = x-1$.

$$= np\sum_{y=0}^{m} \frac{m!}{(m-y)!y!}p^y(1-p)^{m-y}$$

This is a form of the binomial theorem $(x+y)^n$ with $x = p$, $y = 1-p$, and $n = m$.

$= np(p + (1-p))^m$

$= np(1)^m$

$= np$

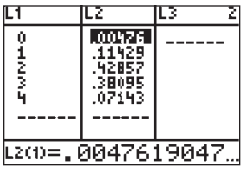
4.4 Hypergeometric Distributions, pages 170–179

Example 1 Your Turn

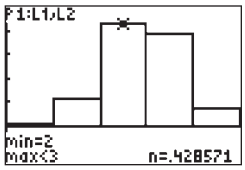
about 39.56%

Example 2 Your Turn

a)



b)



c) The probability is less bell-shaped with the mode at $x = 2$. It is skewed to the left.

d) $P(0) = 0.00476$ or 0.476%, which means that getting no green jelly beans is very unlikely.

e) 2.4; On average, there will be 2.4 green jelly beans in four selections.

Example 3 Your Turn

a) approximately 0.6430

b) about 2.9167

Example 4 Your Turn

about 1176 foxes

Reflect

R1. a) The random variable is the number of red marbles selected. The size of the sample space is 5 marbles. The size of the population is 10 marbles. The range of the random variable is 0 to 5.

b) The random variable is the number of hearts selected. The size of the sample space is 7 cards. The size of the population is 52 cards. The range of the random variable is 0 to 7.

R2. Since the trials are independent, this is not a hypergeometric probability situation.

Practise

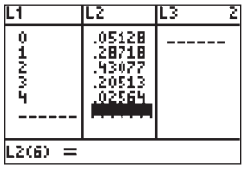
1. D **2.** A

3. a) $n = 6 + 9$, or 15, and $r = 3 + 2$, or 5

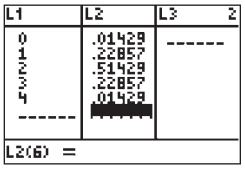
b) $a = 10 - 7$, or 3, and $b = 6 - 5$, or 1

c) $c = 25 - 6$, or 19, and $d = 3 + 2$, or 5

4. a)

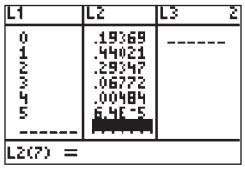


b)



Apply

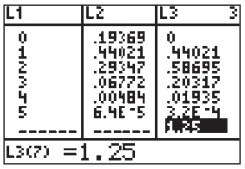
5. a)



b) $E(X) = \frac{ra}{n}$

$= \frac{5(5)}{20}$

$= 1.25$



6. a) about 0.6009 **b)** 0.75

c) The probability that all three light bulbs are defective is about 0.88%, which is very unlikely.

7. a) about 0.0026 **b)** about 0.6962

c) about 2.6084×10^{-6}

8. The probability of fewer than three means $P(0) + P(1) + P(2)$.

The probability of more than seven typically means $P(8) + P(9) + P(10)$. However, there are only 8 mice with a defective mutation. So, the probability of more than seven means $P(8)$. Knowing the shape of the distribution, the ends are highly unlikely.

There is a greater probability of the mice having the genetic mutation in fewer than three.

9. about 1429 foxes

10. a) about 17

b) Yes. The expected number of tagged deer is 17.

11. a) Answers may vary. A five-card hand will have a greater probability of no spades.

b) For a seven-card hand,

$$P(\text{no spades}) = \frac{{}_{13}C_0 \times {}_{39}C_7}{{}_{52}C_7}$$

$$= 0.114\,967\ldots$$

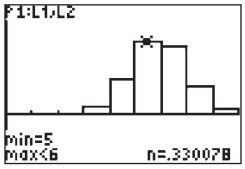
For a five-card hand,

$$P(\text{no spades}) = \frac{{}_{13}C_0 \times {}_{39}C_5}{{}_{52}C_5}$$

$$= 0.221\,533\ldots$$

As the number of cards in a hand increases, so does the possibility that it contains a spade.

13. a)



b) Answers may vary. **c)** Answers may vary.

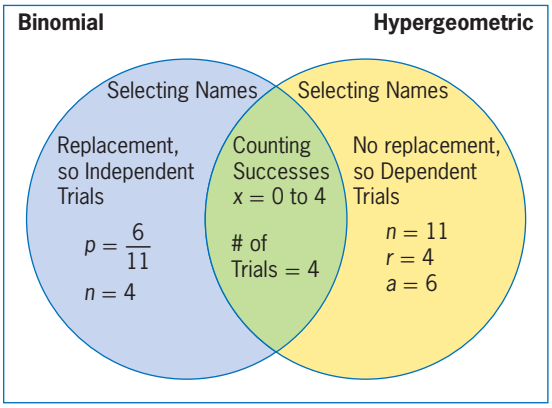
Extend

14. about 0.0263 **15.** about 0.0182

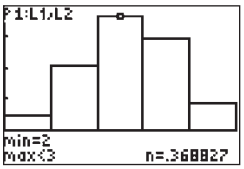
4.5 Comparing and Selecting Discrete Probability Distributions, pages 180–187

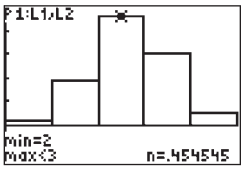
Example Your Turn

a)



b)

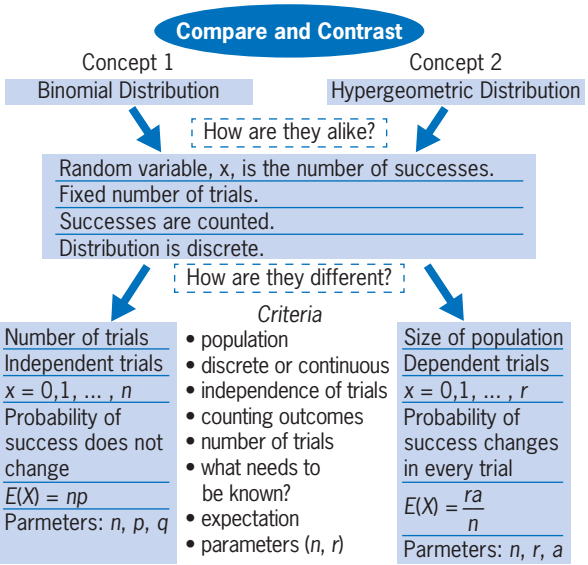




c) The graphs have the same bell-like shape, with the $x = 2$ female names being the most likely outcome. The hypergeometric graph is slightly taller than the binomial graph at $x = 2$ and $x = 3$, and shorter at the other values of x.

Reflect

R1. Answers may vary.



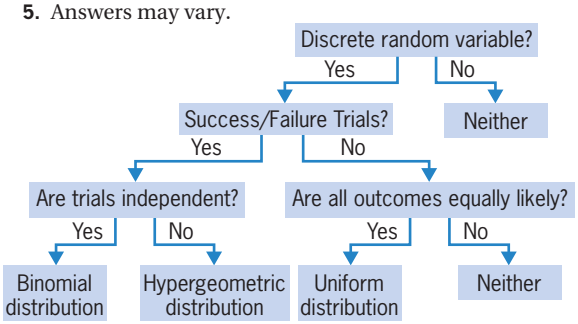
R2. No. For a binomial distribution, the probability of each *success* is the same, but with hypergeometric the probability of success changes with each trial.

Practise

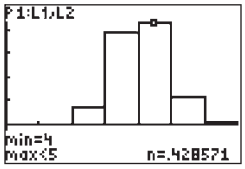
1. **a)** hypergeometric; the trials are dependent
   b) binomial; the trials are independent and given the probability of failure
   c) hypergeometric; the trials are dependent
   d) uniform; the trials are independent and all outcomes are equally likely
2. C
3. C
4. Answers may vary.
   a) the number of cards of a particular suit or denomination, $x = 0$ to 5
   b) the number of grade 11 students or grade 12 students, $x = 0$ to 4
   c) the value of the outcome (a particular suit or denomination)
   d) the number of a particular digit, $x = 0$ to r
   e) the number of defective bottles, $x = 0$ to r
   f) the value of the outcome (winning square)

Apply

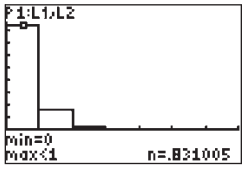
5. Answers may vary.



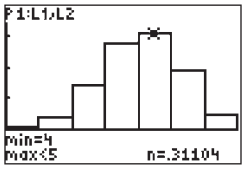
6. **a)** uniform; each outcome is equally likely
   b) Answers may vary. Six people are asked to choose a number between 1 and 15. What is the probability that two people choose the number 13?
7. **a)** binomial; these are independent success/failure trials
   b) Answers may vary. At Bill's Burger Barn, there is a hat with five free hamburger tickets and ten free fries tickets. If three tickets are drawn, what is the probability of winning a free hamburger?
8. **a)** hypergeometric; these are dependent success/failure trials
   b) Answers may vary. For a random draw, 20 slips of paper containing people's names are placed into a bin, 20% of which are her friends. What is the probability that at least one friend's name will be drawn?
   c) In the first scenario, $P(x \geq 1) \approx 0.7183$
   In the second scenario, $P(x \geq 1) \approx 0.6723$
   d) Since there is a higher probability of a friend winning in the first scenario, Barb would be happier with the hypergeometric distribution.
9. Answers may vary.
10. **a)** about 0.3077 **b)** 2 **c)** 1
12. Answers may vary.
   a) Show the probability distribution for the number of successful ring-tosses in eight attempts.
   b) There are 15 bottles, with five green and ten clear. Show the probability distribution for the number of successful ring-tosses onto green bottles in four attempts.
   c) There are eight bottles numbered 1 to 8, and the probability of a successful ring-toss on any one is equally likely. Show the probability distribution for the bottle number.
13. **a)** population of 10:



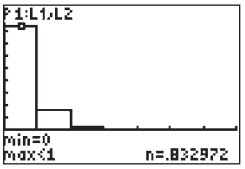
population of 200:



   b) population of 10:



population of 200:



   c) The two distributions for $n = 10$ both have somewhat similar bell shapes, with a mode of $x = 4$. The hypergeometric graph is slightly taller than the binomial graph at $x = 3$ and $x = 4$, and shorter at the other values of x.
   The two distributions for $n = 200$ both have almost identical half-bell shapes, with a mode of $x = 0$. The hypergeometric graph is slightly taller than the binomial graph at $x = 1$, and slightly shorter at the other values of x.
   d) The binomial distribution gives a close approximation of a hypergeometric distribution when r is small in relation to n.

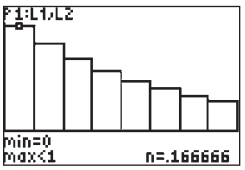
Extend

14. L2 shows $P(x) = \frac{e^{-np}(np)^x}{x!}$, where $n = 2000$, $p = 0.015$, $x = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$.
   L3 shows $P(x) = {}_nC_x p^x q^{n-x}$, where $n = 2000$, $x = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$, $p = 0.015$, and $q = 0.985$.

L1	L2	L3 1
4	3.2E-9	2.7E-9
5	1.9E-8	1.6E-8
6	9.5E-8	8.2E-8
7	4.1E-7	3.6E-7
8	1.5E-6	1.3E-6
9	5.1E-6	4.6E-6
	7.1E-6	6.4E-6
L1(11) =		

   The Poisson distribution gives approximately 7.1×10^{-6}, where as the binomial distribution gives approximately 6.4×10^{-6}.
15. **a)** **i)** $E(X) = \frac{n+1}{2}$ **ii)** $E(X) = a$ **iii)** $E(X) = a$
   b) The expected value for the binomial distribution equals that of the hypergeometric distribution.
16. **a)** **i)** 0.4 **ii)** 0.24 **iii)** 0.144 **iv)** $(0.6)^{n-1}(0.4)$
   b) (probability of failure)^n × (probability of success)
   c) The probability of success after a waiting time of x failures is $P(x) = q^x p$, where p is the probability of success in each single trial and q is the probability of failure.
   d)

L1	L2	L3 2
0	[illegible]	------
1	.13889	
2	.11574	
3	.09645	
4	.08038	
5	.06698	
6	.05582	
L2(1)=.1666666666...		



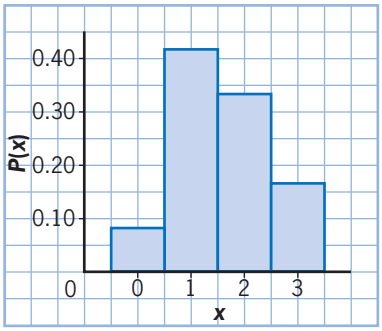
   e) All three distributions have two possible outcomes, success or failure. The geometric and binomial distributions involve independent trials where

the probability of success is the same in every trial. This is in contrast to the hypergeometric distribution, where trials are dependent and the probability of success is not the same in every trial. Binomial and hypergeometric distributions involve a fixed number of trials, whereas geometric distributions involve continued trials until success.

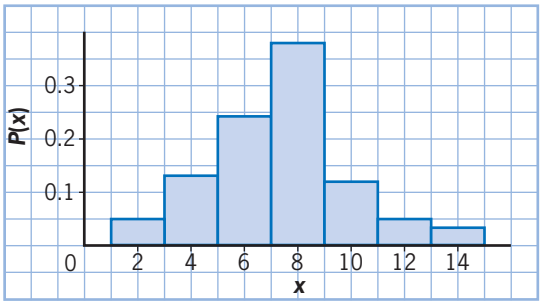
Chapter 4 Review, pages 188–189

1. a) continuous **b)** discrete
c) discrete **d)** continuous

2. a)



b)

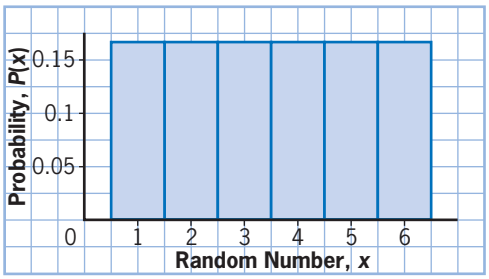


3. a) about 1.58 **b)** 7.32

4. A uniform distribution occurs when, in a single trial, all outcomes are equally likely.

5. a)

Random Number, x	$P(x)$	$x \cdot P(x)$
1	$\frac{1}{6}$	$\frac{1}{6}$
2	$\frac{1}{6}$	$\frac{2}{6}$
3	$\frac{1}{6}$	$\frac{3}{6}$
4	$\frac{1}{6}$	$\frac{4}{6}$
5	$\frac{1}{6}$	$\frac{5}{6}$
6	$\frac{1}{6}$	$\frac{6}{6}$



b) 3.5; the predicted average value of the random number will be 3.5.

6. a) Amounts received indicate a positive value for the random variable, x. Amounts lost indicate a negative value for x.

Number of Green Balls	Amount ($), x	$P(x)$	$x \cdot P(x)$
0	–360	${}_3C_0(0.4)^0(0.6)^3$	–77.76
1	–40	${}_3C_1(0.4)^1(0.6)^2$	–17.28
2	280	${}_3C_2(0.4)^2(0.6)^1$	80.64
3	600	${}_3C_3(0.4)^3(0.6)^0$	38.40
		Sum	24

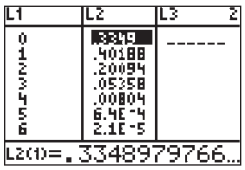
This is not a fair game because the player will win $24 on each turn, on average.

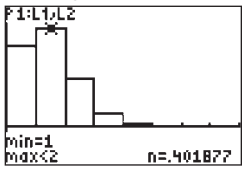
b)

Number of Green Balls	Amount ($), x	$P(x)$	$x \cdot P(x)$
0	–360	${}_3C_0\left(\frac{15}{25}\right)\left(\frac{14}{24}\right)\left(\frac{13}{23}\right)$	–71.22
1	–40	${}_3C_1\left(\frac{10}{25}\right)\left(\frac{15}{24}\right)\left(\frac{14}{23}\right)$	–18.26
2	280	${}_3C_2\left(\frac{10}{25}\right)\left(\frac{9}{24}\right)\left(\frac{15}{23}\right)$	82.17
3	600	${}_3C_3\left(\frac{10}{25}\right)\left(\frac{9}{24}\right)\left(\frac{8}{23}\right)$	31.30
		Sum	23.99

This is not a fair game because the player will win $23.99 on each turn, on average.

7.

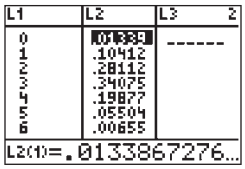


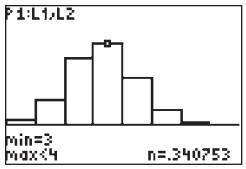


8. a) 55.2; the probability of success (type O) does not change with each trial.
b) Type A: 50.4; Type B: 10.8; Type AB: 3.6

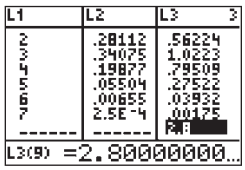
9. a) about 0.3798 **b)** about 0.1087 **c)** 1.2

10. a)





b) $E(X) = \frac{ra}{n}$
$= \frac{7(10)}{25}$
$= 2.8$



11. a) approximately 1.228×10^{-6}
b) about 0.0133 **c)** about 0.9043

12. 1000 seals

13. a) binomial **b)** hypergeometric
c) none **d)** uniform

14. a) $\frac{{}_{12}C_5 \times {}_{40}C_2}{{}_{52}C_7}$, or about 0.0046

b) ${}_7C_5\left(\frac{3}{13}\right)^5\left(\frac{10}{13}\right)^2$, or about 0.0081

c) There is a greater probability of five face cards with replacement because the trials are independent and the probability of a face card does not change.

Chapter 4 Test Yourself, pages 190–191

1. C **2.** D **3.** A **4.** A **5.** B

6. 4

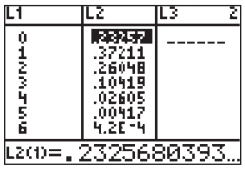
7. about 0.4632

8. a) hypergeometric distribution; the trials are dependent (without replacement)

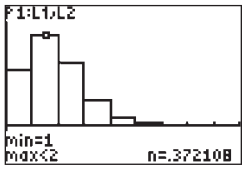
b) make it with replacement

9. 29 beavers

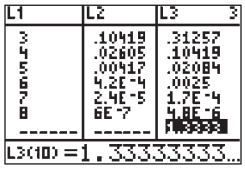
10. a)



b)



c)



The expected number of sums of 7 is about 1.3333.

11. a) about 0.9945 **b)** 14.25

12. $P(0 \text{ women}) = \frac{{}_5C_0 \times {}_{10}C_4}{{}_{15}C_4}$

≈ 0.1538

$P(\text{at least 1 woman}) = 1 - P(0)$

≈ 0.8462

There is a much greater chance that at least one female was promoted, so the committee's claim is unfounded.

13. a) about 0.1556 **b)** 0.72

14. a) about 0.0082 **b)** about 0.0156.

c) about 0.7335

15. Answers may vary. The binomial distribution gives a close approximation of a hypergeometric distribution when r is small in relation to n. If n is very large, non-replacement of successes will not change the ratio of successes to the population that much, and a binomial distribution will be a good approximation to the hypergeometric distribution.

16. a) about 0.5045 **b)** about 0.2003

Chapter 5 Organization of Data for Analysis

Prerequisite Skills, pages 194–195

1. a) line graph **b)** double bar graph

c) scatter plot **d)** circle graph

e) bar graph **f)** histogram

2. a) 10 to 14, 15 to 19, 20 to 24

b) a little over $100 per address

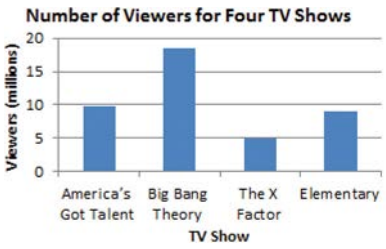
c) Incidents of crime seem to increase with an increase in officers.

d) Xbox 360 **e)** November

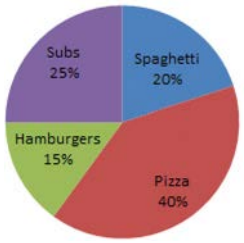
f) Breaking Dawn - Part 2

3. bar graph: shows the frequencies of the values of the variable with disconnected bars, represents non-numeric or unordered data; histogram: uses connected bars, represents a continuous range of values in numeric order

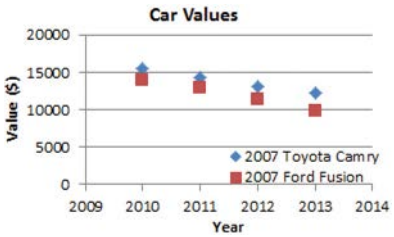
4.



5.



6.



7.

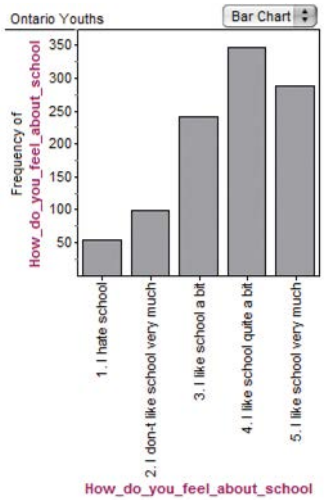
5.1 Data Concepts and Graphical Summaries, pages 196–205

Example 1 Your Turn

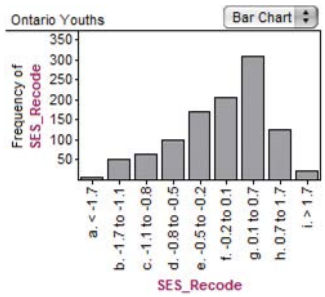
Answers may vary. Climate change: 97% of climate scientists agree that climate-warming trends are very likely due to human activities. Others argue that the warming is attributable to natural causes.

Example 2 Your Turn

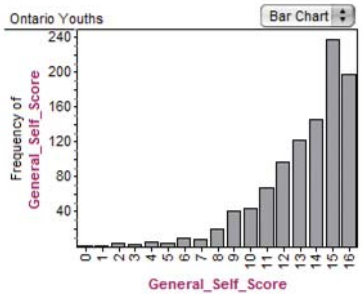
categorical, ordinal data:



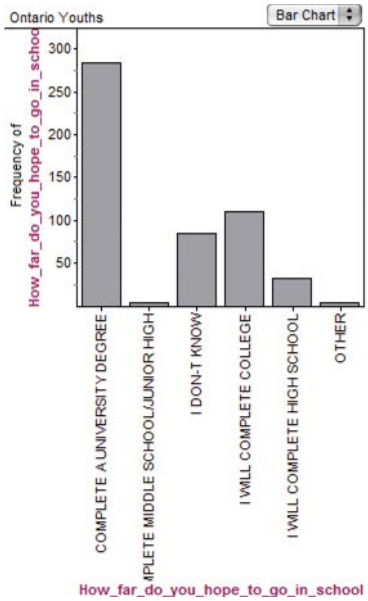
numerical, continuous data:



numerical, discrete data:



categorical, nominal data:



The only data that cannot be represented as a graph is the child ID number. There is simply one per student.

Example 3 Your Turn

Answers may vary.

a) #vacation: tweets go up and down; #math: tweets go up and down; #Junos: tweets are almost non-existent until 4:00 p.m., then there is a peak at 8:00 p.m., dropping to nothing again by 12:00 a.m.

b) #math and #vacation are not that common (low constant volume) and only peak when there is an interest.

c) 8:00 pm, because that hashtag was tweeted less often from then on.

d) No; there were so many more tweets in the #Junos graph. Even at 12:00 p.m., when the number of tweets for #Junos was near its lowest, there were more #Junos tweets than there were at any time for #math.

Reflect

R1. Answers may vary. Predictions made by political experts are not reliable. They are only correct about half the time.

R2. Answers may vary. Positive: Availability, ease of backup, lower risk of loss, and not susceptible to viruses. Negative: Access speed, requires Internet connection, environmental impact with electricity use and carbon emissions of data centres.

Practise

1. Answers may vary.

a) Continuous and discrete data can be numeric, but continuous data can have any value within a given range. Discrete data can have only certain separate values. Heights of students is continuous but the number of students enrolled in each class is discrete.

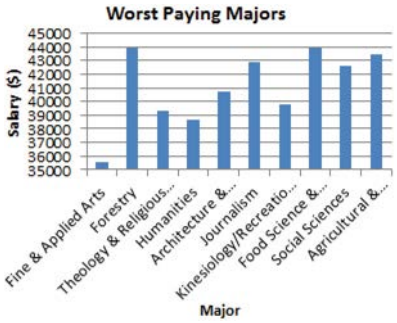
b) Ordinal and nominal data are qualitative, but ordinal data can be ranked and nominal data cannot. A rating scale for a taste test shows ordinal data, while the type of cookie preferred shows nominal data.

c) Numerical data is quantitative and can be continuous or discrete, while categorical data is qualitative. How many movies you see in a year shows numerical data, and the types of movies you like shows categorical data.

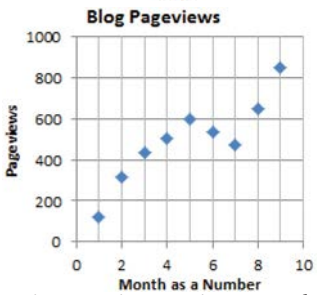
2. a) numerical data: armspan, height; the remaining columns contain categorical nominal data.

b) continuous data: reaction time, travel time; discrete data: number of languages spoken, number of siblings; the remaining columns contain categorical nominal data.

3.



4. a)



b) an increase in pageviews over the first few months

c) School is not in session in June and July.

d) Answers may vary. About 1100.

5. a) There are different scales on the two vertical axes (one for Canada, one for the U.S.), but they both show that auto sales are nearing pre-crisis levels. If there was only one scale used, Canada's auto sales would look like a relatively flat horizontal line.

b) Answers may vary. This graph does not show a dramatic drop in sales as well.

6. Answers may vary.
   a) It reveals underlying trends in data that seasonal movements tend to mask.
   b) To reveal an underlying trend.

Extend

8. a) Answers may vary.
   b) Answers may vary.
   c) Answers may vary. Examples: Roy peaked in the 1890s. Marilyn and Joan both peaked in the 1930s.
   d) Answers may vary. Examples: Madison, Jackson, Avery
9. a) School A appears to be doing the best because the results steadily increased each year.
   b) Answers may vary. School A: 84% maintained standard but 5% dropped from standard. School B: 26% rose to standard but 19% never met standard. School C: 80% maintained standard but 13% dropped from standard. Based on the cohort data, School B appears to have the best results; it has the greatest percent that rose to standard.

5.2 Principles of Data Collection, pages 206–211

Example Your Turn

a) convenience sample
b) stratified sample
c) cluster sample
d) simple random sample
e) systematic sample
f) voluntary response
g) multistage sample

Reflect

R1. Answers may vary. Both divide the population into groups, just differently. In a cluster sample, a number of groups are chosen randomly, and each member of the chosen groups is sampled. In a multistage sample, the group is subdivided further by creating a hierarchy and choosing a random sample at each level.

R2. Answers may vary. If a population is biased one way or another, that will be evident in the smaller sample size.

Practise

1. a) systematic sample
   b) simple random sample
   c) voluntary response
   d) stratified sample
2. A population is all the individuals in a group being studied (all students in a school), while a sample is a group selected from the population (one class in the school).
3. Answers may vary.
   #SimpleRandomSample each member of population has equal chance of being selected.
   #SystematicSample orders population and people are chosen at regular intervals.
   #StratifiedSample has same proportion of members from each group as population does.
   #ClusterSample divides population into groups, randomly selects groups, and samples each member of chosen groups.
   #MultistageSample divides population into a hierarchy and chooses random sample at each level.
   #ConvenienceSample choose members of population who are easy to access.
   #VoluntarySample people choose whether to participate so contains members heavily for or against topic.
4. Answers may vary. Stratified: use grade level or gender. Cluster: use random class sample at each grade level. Voluntary: allow students to choose whether to respond.
5. Answers may vary. Systematic sample: randomly select a time each hour and then test the next 10 batches of candy mix on the factory line.
6. Answers may vary. Audrey Tobias is an 89-year-old peace activist who refused to fill out the 2011 census because of its link to a U.S. military contractor. She faced a criminal charge and her lawyer argued that forcing her to complete the census would violate her freedoms of conscience and free expression. She was aquitted due to reasonable doubt as to her intent at the time of refusal.
7. a) These are both random sampling techniques that divide the population into groups, but differently. Stratified sampling uses the same proportions as the proportions in the entire population (when surveying teachers, use gender groups). Multistage sampling uses sequential levels of sampling by subdividing the group into smaller groups (select from a group of high schools, then departments, then teachers).
   b) These are both non-random sampling techniques that can produce unreliable results because people who respond are not necessarily representative of the entire population. Convenience sampling bases selection on easy access (poll people at a movie theatre about the best movie). Voluntary sampling is comprised of only those who choose to participate (ask people to vote online about the best movie).
8. a) customers of the car dealership
   b) SUV/truck: 57 calls; Minivan: 81 calls; Midsize car: 32 calls, Economy car: 65 calls; Sports car: 15
   c) Answers may vary. They might hope to get more responses by calling, it might be more cost effective.
   d) Answers may vary. Ensure actual customer is surveyed not just a person living at that household.

Extend

10. Answers may vary. Pros: Canadians are not forced to divulge detailed personal information under threat of prosecution. Cons: Since it is not mandatory, data collected on a voluntary basis will not be as accurate and there will most likely be fewer respondents.
11. Aerial surveys estimate the number of moose. The park is divided into 57 plots representing either high-quality or low-quality winter moose habitat. Twenty percent of the 57 blocks are randomly selected and moose are counted to estimate the population. This is similar to cluster sampling. The habitat is divided into areas, a number of areas are randomly chosen, and population is counted in those areas.

5.3 Collecting Data, pages 212–221

Example 1 Your Turn

1. a) there are no controls for the number of colds and the subjects are not randomized
   b) Answers may vary. Add a control group that does not go to the gym.
2. a) control group: plants growing in water with neutral pH; experimental groups: plants grown using water with increasingly acidic pH levels.
   b) Answers may vary. To help eliminate any previously diseased plants.

Example 2 Your Turn

a) Answers may vary. There are only three options.

What is your favourite game system?

b) It is biased. It suggests that leadership training is necessary to run a business.

Do you think that leadership training would help you run a business?
■ Yes ■ No ■ No opinion

c) Asks for an opinion about two things.

How important do you think speed of service is?
■ Very Important ■ Important ■ Somewhat Important
■ Of Little Importance ■ Not important

How important do you think quality of service is?
■ Very Important ■ Important ■ Modestly Important
■ Of Little Importance ■ Not Important

Example 3 Your Turn

Answers may vary.

Reflect

R1. Answers may vary. To get accurate and unbiased information.

R2. Answers may vary. To account for effects from treatment that do not depend on the treatment itself, such as knowledge and expectations of treatment. A placebo group gives researchers a comparison group to determine whether the treatment itself had any effect.

R3. Answers may vary. Informed consent, voluntary participation, avoidance of harm.

Practise

1. C

2. B

3. a) Ask two separate questions.

Do you exercise daily? ■ Yes ■ No
Do you get enough sleep at night? ■ Yes ■ No

b) Rephrase to make it neutral.

Do you think that the Star Wars saga is one of the best science fiction stories of all time?
■ Strongly agree ■ Agree ■ Don't know ■ Disagree
■ Strongly disagree

c) Make it an open question.

What is your favourite type of music?

d) Make more concise.

How do you feel about the following statement: We should increase the number of recycling days in the school.
■ Strongly agree ■ Agree ■ Don't know ■ Disagree
■ Strongly disagree

4. Answers may vary. How often do you walk to school?

5. a) observational study; whether magnetic therapy reduces pain

b) observational study; whether lying in bed for an extended period of time exhibits similar issues to weightlessness in space

c) experimental study; whether handedness affects math grades

Apply

6. a) 410 people

b) They have not used the Internet.

c) Answers may vary. I think that the 0 computer category will either not exist or be very small, and the 1 and 2 computer categories will greatly increase. The no for connected to the Internet will either disappear or be very small, and the yes will greatly increase. Also, all of the red portions should be much smaller, because most people have used the Internet.

d) Answers may vary.

7. Answers may vary.

Which is your favourite sport?
■ Tennis ■ Hockey ■ Basketball

How do you like K-pop music?
■ Pretty Good ■ Good ■ Great ■ Fantastic ■ Awesome

How important do you think exercise and eating healthy are?
■ Very important ■ Important ■ Somewhat important
■ Of little importance ■ Unimportant

8. Answers may vary.

a) A list and description of 43 attributes from Statistic Canada's 2001 Census of Population for Canada.

b) and **c)** Answers may vary.

d) To study cultural trends, health care needs, and other social-economic issues.

9. Answers may vary.

How concerned are you about safety in sports?
■ No opinion ■ Not concerned ■ A little concerned
■ Concerned ■ Very concerned

What is your favourite classification of sports?
■ Endurance sports ■ Team sports ■ Target sports
■ Strength sports ■ Athletics ■ Other

How many hours a week do you watch sports?
■ 0–5 ■ 6–10 ■ 11–15 ■ 16–20 ■ more than 20

What sport do you play?

What is your level of expertise in your sport on a scale of 1 to 5 with 1 being poor and 5 being an expert?

10. Answers may vary.

Do you agree that cell phones should be silenced in movie theatres?
■ Strongly agree ■ Agree ■ Undecided ■ Disagree ■ Strongly disagree

How often do you see a person using a cell phone while he or she is driving?
■ Very frequently ■ Frequently ■ Occasionally ■ Rarely ■ Never

I find my mathematics class interesting.
■ Almost always true ■ Usually true ■ Occasionally true
■ Sometimes true ■ Never true

How well do you like the latest sci-fi movie release?
■ Like a lot ■ Like a little ■ Do not like or dislike
■ Dislike a little ■ Dislike a lot

How important do you think global warming is?
■ Very important ■ Important ■ Modestly important
■ Of little importance ■ Unimportant

11. Answers may vary.

12. a) observational study

b) Answers may vary. A control crop that uses chemical fertilizers is needed, then the treatment crop uses natural fertilizers. The effect being studied is whether natural organic crops yield less than traditional crops.

14. Answers may vary. The subjects cannot be randomly assigned to a treatment group or control group. It is not feasible to assign the "treatment" of academic success.

Extend

15. a) Answers may vary. Studies may seem questionable if the source does not seem reliable, if the sample seems skewed, or if the methods seem biased.

b) Answers may vary.

c) observational study. It found a positive association between coffee consumption and all-cause mortality in men and in men and women younger than 55 years. This group should avoid drinking more than four cups of coffee a day. The study population included a range of ages all of

which completed a medical history and lifestyle questionnaire, were given a baseline examination for mortality, and present findings for all-cause mortality are consistent with those of earlier studies. However, the results of recent studies have been highly variable. The non-coffee-drinking group may have had a higher mortality risk not related to the consumption of coffee.

16. Answers may vary.

5.4 Interpreting and Analysing Data, pages 222–232

Example 1 Your Turn

Answers may vary.

a) The average domestic fare decreased from 2008 to 2009 by around $30.

b) The average domestic fares for 2008 and 2011 by individual city, by province, by cities within a province, or for Canada

Example 2 Your Turn

a) name, artist, composer, album, genre, size of the file, length of the song, year it was released, sampling bit rate, number of times it was played

b) Answers may vary. The most common length for a song is from 210 s to less than 230 s.

c) No.

d) Answers may vary. If you know how long the song is *and* the sampling rate, you can determine the size of the file.

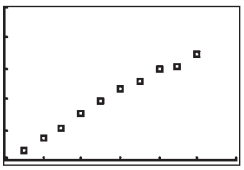
Example 3 Your Turn

a) experiment; the student is controlling the drop height of the ball

b) Drop height of the ball is controlled, and the different bounce heights are recorded.

c) Bounce height appears to be about 70% of the drop height.

d) The relationship appears to be linear.



e) Yes, use a line of best fit.

Reflect

R1. Each provides an organized store of records containing information on players, teams, and games.

R2. Secondary data; we typically are exposed to someone else's use of primary data.

Practise

1. B **2.** C

3. Table 128-0014: Electricity generated from fossil fuels
Table 203-0026 : Survey of household spending (SHS), household spending, by age of reference person.
Table 427-0004: Number of international tourists entering or returning to Canada, by province of entry.
Table 361-0013 : Spectator sports, event promoters, artists and related industries, summary statistics, by North American Industry Classification System (NAICS)

4. **a)** primary data **b)** secondary data
c) primary data **d)** secondary data
e) primary data **f)** secondary data.

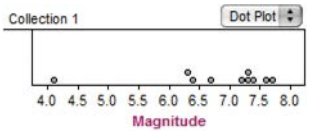
Apply

5. Answers may vary.

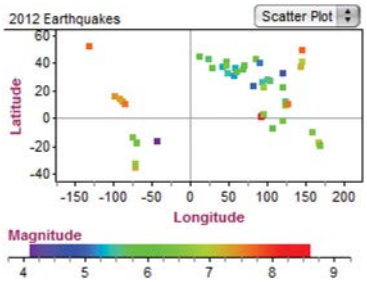
6. **a)** about 21%

b) Queen Charlotte Islands. It was the largest in North or South America in 2012.

c) Dot plot compares magnitudes.

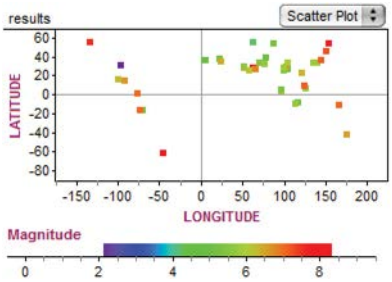


d) Answers may vary. Colour shows the severity of magnitude.



e) South America

f)

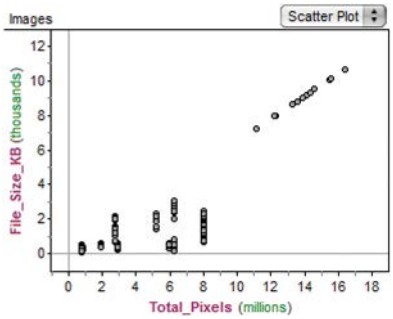


There were more larger magnitude earthquakes in 2013.

7. **a)** date, time, exposure time, F-stop, Exp program, metering mode, whether flash was used, focal length, ISO speed, orientation, dimensions, total pixels, file size in kilobytes

b) Answers may vary.

c)



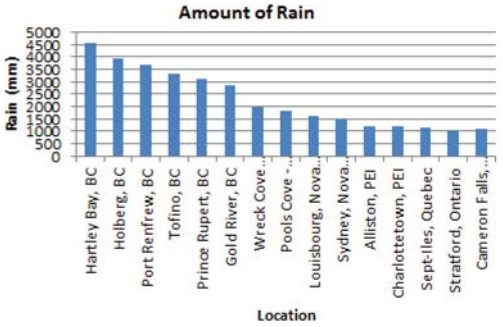
Number of pixels determines the file size, then divide the remaining storage capacity by the file size and estimate how many more photos can be taken at that setting.

8. **a)** The numbers in the table are primary data from weather stations. The table represents secondary data for anyone else who uses it.

b) Stratford: 1064 mm of rain, Hartley Bay: gets over 4 times more rain at 4549 mm per year,

Henderson Lake: gets over 8 times more rain at 9082 mm per year

c) With locations organized by region, it is easy to see that the six highest rainfall locations are all in BC.



Extend

10. Answers may vary.

a) The various sizes and colours of dots begin to change size and location to show that income per person and life expectancy increased over time.

b) size: population of the country, colour: geographic region. The size is in relation to the world population.

c) Each country's story is told in regards to income per person and life expectancy over time. Canada for the most part has continuously increased in both income per person and life expectancy.

d) Answers may vary.

5.5 Bias, pages 233–243

Example 1 Your Turn

a) response bias
b) sampling bias
c) non-response bias
d) measurement bias

Example 2 Your Turn

Answers may vary.

a) no vertical scale, the points suggest a linear relationship, but the number of jobs lost does not change by a constant amount. The horizontal scale is not consistent.

b) The rate did not drop in October. Also, the vertical scale does not start at 0.

c) This information is most likely anecdotal and cannot be verified. In addition, no actual data on the amount of weight lost is given or whether consumers were on a diet and exercise regimen.

Example 3 Your Turn

Answers may vary.

a) The amount spent is about $62.23 per household. For countries that do not celebrate Halloween, this is a lot to spend on candy, costumes, and decorations. However, for many Canadians and Americans this is not a lot of money.

b) Apple's gross profit in 2010 was $26.71 billion and in 2011 it was $45.63 billion, so a $19 billion is a large increase of about 59%.

Example 4 Your Turn

a) The Canadian teams have a white background and the logo is inserted inside the bar.

b) It would have the smallest percent of Canadians and Americans.

c) Answers may vary. There are more Canadian-born players in the NHL and on each team.

d) Answers may vary. The actual percent of American-born players on each team.

e) No. The presentation and information given slants it towards Canadian-born players.

f) Answers may vary.

Reflect

R1. Answers may vary. Yes, by displaying it in a distorted fashion.

R2. Answers may vary. By omitting other information basing the statistics on faulty experiments or studies.

R3. Answers may vary. The consumer is at fault. Just because the product is called *vitamin*water® this in no way should imply that it contains vitamins and consumers must read the fine print labels.

Practise

1. D

2. Answers may vary.

a) response bias; make the response anonymous

b) non-response bias and sampling bias; hold several open public meetings to gather a more accurate opinion.

c) sampling bias; make the poll available in other ways

d) response bias; ask how often the person exercises

e) sampling bias; make the petition available in other ways

3. Answers may vary. Measurement bias. A survey question asks, "Who is the best supercross rider of all time, Ricky Carmichael or James Stewart?

4. Answers may vary.

a) The statement does not explain what the typical number of headaches was, nor does it identify other potential factors (diet, water consumption, weather).

b) The two sections that indicate people like math look significantly larger than the two sections that indicate people do not like math.

c) The break in the vertical scale exaggerates the change in population.

d) It is very difficult to determine actual values from the use of icons.

e) The break in the vertical scale exaggerates the change in units sold, and the horizontal scale does not show consistent monthly increments.

6. a) This number represents about three pieces of mail per dwelling per day, which seems typical. So, the number 9.8 billion is not significant.

b) If each viewer watches about 5.8 h of video each year which does not seem like a lot. So, the number 70 billion is not significant.

c) Since Clayton Kershaw is the highest paid player in Major League history, and he earns more than 15% more than the next highest paid player, this number is significant.

d) Compare this with last year's national debt of $602.4 billion, it is an increase of about 8%. This is a significant number.

Apply

7. a) Answers may vary. The sun-like design and bright colours.

b) 29th. Answers may vary. Environmental conditions, lack of federal spending, public attitudes, and other cheaper forms of generating electricity.

c) The sun-like shape and use of "sun" colours.

d) Answers may vary. They are trying to imply that Canada is far behind in its use of solar energy and needs to improve its standing.

8. Answers may vary. Shocking: Cancer rates up by 25%. Neutral: Cancer rate changes from 1.6% in 2013 to 2% in 2014.

9. Answers may vary. The push poll is a form of telemarketing-based propaganda disguised as a poll that attempts to influence or alter the view of voters.

10. Answers may vary. The Hawthorne effect refers to the fact that people will modify their behavior simply because they are being observed. Example: a typically slow worker who works faster and does a better job when his or her boss is watching.

11. Answers may vary.

a) Confirmation bias is a tendency for people to favour information that confirms their preconceptions regardless of whether the information is true or not. For example, watching certain TV news programs or only visiting websites that express your opinion.

b) Observational selection bias is the effect of suddenly noticing things you did not notice that much before and you wrongly assume that the frequency has increased. For example, when one of your car's headlights is out and suddenly you notice other cars with the same problem.

12. Answers may vary.

a) measurement bias. How important is it for you to use high-tech farming methods?

b) The firm presents the information in this misleading manner to exaggerate their growth and to try to convince potential new clients to "get onboard."

13. Answers may vary.

a) The majority of movies are sci-fi/fantasy, the highest ranked movie has made over $100 million more than the movie ranked at number two.

b) The greatest number of movies were released after 2010. These movies have an unfair advantage over much older movies, which had cheaper ticket prices.

c) This graph is almost the reverse of the other one. Adjusting for inflation negates the effect of higher ticket prices in more recent years.

d) Answers may vary. The top movie on the unadjusted list drops to number three on the adjusted list. Also, the adjusted list contains quite a few different movies and only five movies were released in the 2000s.

e) Answers may vary.

Extend

14. Answers may vary.

a) There is an anti-vaccination movement with celebrity supporters who believe there is a link between vaccines and autism.

b) The possible link between vaccines and autism is in part based on a very small and flawed study published in the UK in 1998. The study was retracted in 2004, and several major health services have all concluded that there is no evidence of a link between the MMR vaccine and autism.

15. Answers may vary. In a double blind study, neither the subjects nor the scientists know who is receiving the medication and who is receiving a placebo. This helps to prevent results from being influenced by the placebo effect or observer bias.

Chapter 5 Review, pages 244–245

1. Answers may vary.

a) numerical, discrete: number of students in a class

b) numerical, continuous: height of students in a class

c) categorical, ordinal: answers to a question that uses a rating scale

d) categorical, nominal: eye colour of students in a class

2. a) The peaks coincide with weekends, since they occur on a regular weekly basis.

b) Answers may vary. Yes, they used about 102 Gb.

c) Using the sum function in a spreadsheet, the family actually used 104.73 Gb.

d) The average daily Internet usage of the family is 3.491 Gb. Answers may vary. It is not useful. They almost always use less than this on weekdays, and significantly more on weekends.

3. a) voluntary response survey

b) convenience sample

4. a) There are two possible responses for age 15, 20, and 35.

How old are you?
■15 and below ■16–19 ■20–34 ■35–60 ■Above 60

b) This question is biased.

How do you feel about a violence rating system for video games?
■Strongly agree ■Agree ■Agree a little ■Don't agree

c) Give a range of responses.

Do you like the new logo for the school?
■Like it a lot ■Like it ■Do not like or dislike
■Dislike a little ■Dislike a lot

5. a) experimental **b)** observational

6. a) The table is primary data for whoever collected it. The table represents secondary data for anyone else who uses it.

b) primary **c)** secondary

7. Answers may vary.

a) Any scenario where a small group that is not representative of the population is surveyed.

b) Any scenario where a limited list of answer choices is given or an opinion is stated before asking the question.

c) Any scenario that influences an answer so as to avoid embarrassment or please the questioner.

d) Any scenario where a response is voluntary.

8. a) The size of the fruit icons is misleading. The rows are basically the same length, but due to the size of the icons used each line actually represents a different number.

b) It is difficult to judge the sizes of each sector and no percent data is given.

9. Answers may vary.

Chapter 5 Test Yourself, pages 246–247

1. A **2.** B **3.** B **4.** B

5. Answers may vary.

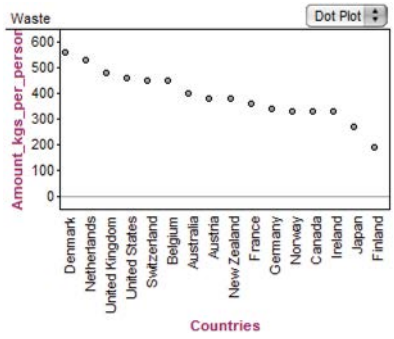
a) Each household varies in is electricity usage. The sales representative cannot guarantee anything

and should instead offer an assessment of what is needed and the payback timeline.

b) Response bias, because a student may respond falsely to avoid embarrassment. The teacher could make the response anonymous by asking students to rate their understanding on a scale of 1 to 5 on a piece a paper, all of which are collected in a bag.

6. Answers may vary.

a)

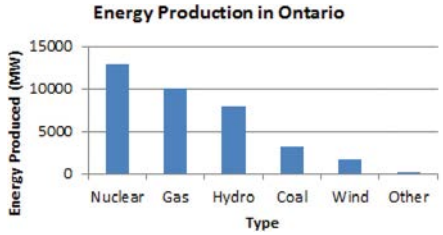


b) The amount of waste in kilograms per person per year by country. A person in Denmark creates the most waste, while a person in Finland creates the least.

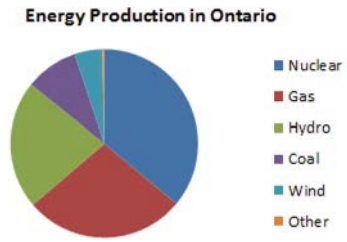
7. Answers may vary.

8. Answers may vary.

a)



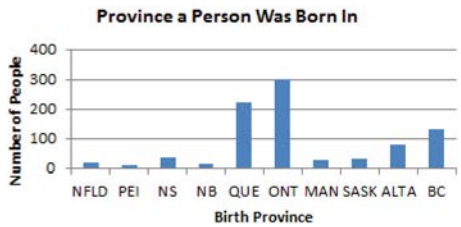
b)



It does not show the percent for each section.

9. Answers may vary.

a)



b) stratified sample; the number of people born in each province approximately matches the percent of Canada's population in each province.

c) The percent of Ontarians travelling is slightly higher than their relative percent of the population. The percent of people from New Brunswick travelling is much greater relative to their percent of the population.

d) Provide the percent of the population that travels rather than just the number of trips.

10. a) Answers may vary.

b) Answers may vary. Highly credible: sample should reflect the characteristics of the population, use a non-biased sampling, and use survey questions free of bias. Not very credible: sample that reflects only some portion of the population, use a biased sampling technique, and use survey questions that contain bias.

Chapter 6 One-Variable Data Analysis

Prerequisite Skills, pages 250–251

1. a) mean: about 45.7, median: 44, mode: 12

b) mean: about 12.7, median: 14, mode: 14

c) mean: about 13.5, median: 12.85, no mode

d) mean: about 147.2, median: 142, mode: 134

2. 12

3. a) categorical **b)** numerical

c) categorical **d)** ordinal data

e) numeric **f)** numeric data

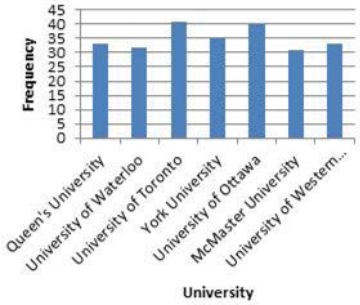
4. a) bar graph **b)** stem and leaf plot

c) pictograph **d)** circle graph

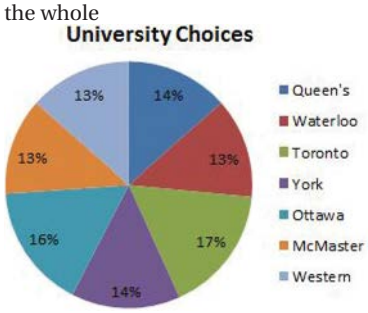
e) histogram **f)** circle graph

g) bar graph

5. the bar graph gives a visual representation of the nominal data



the circle graph allows for easier comparison to the whole



6. a) A stem and leaf plot preserves each data value, whereas a histogram does not with class intervals.

Stem	Leaf
4	3
5	2 5 9
6	1 5 7
7	3 7 8 9
8	0 3
9	9

b) The stems will be the whole number values and the leaves will be the decimal values. This is not practical or useful. Using class intervals of a histogram is a much better choice.

Stem	Leaf
44	5
45	
46	
47	
48	
49	
50	0
51	
52	
53	
54	9

6.1 Measures of Central Tendency, pages 252–265

Example 1 Your Turn

a) mean: about 31.7 °C, median: 29 °C, mode: 29 °C

b) The weather forecast does not match any of the calculated measures of central tendency. However, the mean is affected by the outlier, 45 °C, and the median and mode are the same. Therefore, the weather forecast is not accurate.

c) The outlier of 45 °C causes the mean to be inflated. It does not affect the median or mode.

d) The mean is significantly affected by the outlier. So, the median or mode is more representative of the data.

Example 2 Your Turn

a) estimated mean: about 4 h, median: 3 h, modal interval: 2–4 h

b) Since the data are positively skewed, the modal interval is the least appropriate measure of central tendency. The mean is the greatest of the measures. The median would be an appropriate measure.

Example 3 Your Turn

a) about 84%

b) No, she would have to score 103.17% on the exam. This is not possible, so she cannot receive a final mark of 90% based on the final exam score alone.

Reflect

R1. The mean; an outlier skews the distribution, pushing the mean away from the centre.

R2. Answers may vary.

a) Assignment grade data when you are looking for the average mark.

b) A final course grade that is comprised of a number of categories with varying percents.

c) The results of a survey on how many hours a week students spend on homework.

R3. Answers may vary.

a) Mode: The "model" that occurs most often. Most people have two hands, two eyes, and two legs.

b) Mean: The sum of the travel times divided by the number of trips. The mean time it takes to get to school is 38 min.

c) Median: The middle student in an ordered list. Johnny is an above median student.

Practise

1. a) mean: about 8.7, median: 8, modes: 4, 7, and 15

b) mean: about 13.3, median: 12, mode: 9

c) mean: about 125.9, median: 121, mode: 110

2. a) mean: about 1.45 min, median: about 1.44 min

b) The median best describes Nina's average time. Half of her times are below this and half are above.

3. C

4. D

Apply

5. 80

6. 4.5 cm

7. a) invalid; the mean is not necessarily in the centre of the data. So, the company cannot claim that half the team members sold more than $16 235.

b) invalid; outliers could cause the mean to be inflated and the distribution to be positively skewed. Thus, resulting in 78% of the salaried to be below the mean.

c) valid; if the class sizes are the same, or invalid if the class sizes are different

d) invalid; the median value times 12 does not equal the sum of the monthly expenses for the year. The total expenses for the year is the mean times 12.

8. a) median; The mode could occur anywhere, and not necessarily near the centre of the data. The small size of the sample means that outliers would have a greater effect on the mean than on the median.

b) mode; This would result in the movie chosen most often.

c) mean; By definition, the mean times the number of employees is the necessary budget.

d) median; The mode could occur anywhere, and not necessarily near the centre of the data. Also, any outliers would have a greater effect on the mean than on the median.

9. Answers may vary.

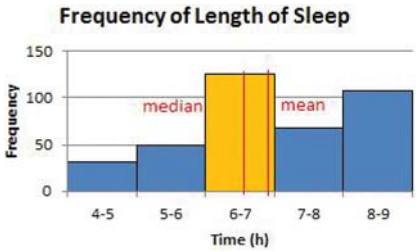
a) Ice time, in minutes, for a hockey player.
15 18 13 15 15 14 16

b) Used car values, in thousands, sold during the first eight days of a month.
25 36 42 8 5 4 7 7

c) The temperatures on March 1st for 10 consecutive years.
16, 17.5, 3.5, 15, 5, −1, 16.5, −2.5, −14.5, 16.5

d) It affects the mean the most by making it too small or too large. The location of the median in a data set is not affected by outliers.

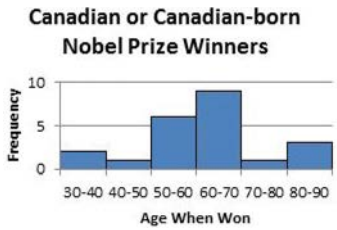
10. a) and **c)**



b) mean: about 6.9 h, median: 6.5 h, modal interval: 6–7 h

d) Since the data are negatively skewed, the modal interval is the least appropriate measure of central tendency. The median and mean are very close together, so either one would be an appropriate measure.

11. a)



b) mean: about 62, median: 61, mode: 61

c) The "average" age of a Canadian Nobel Prize winner is 61 years old. Recognition does not occur until the achievement has been widely accepted, and this sometimes takes decades.

12. a) Russia drops from first to 12th. This moves Norway from 2nd to 1st. Canada drops from 3rd to 10th.

b) Answers may vary. Russia stays 1st. Canada moves from 3rd up to 2nd. Norway moves from 2nd to 3rd.

c) Answers may vary.

13. a) 76% **b)** 92% **c)** at least 59%

d) No. Karen would need an exam score of 102%.

14. Answers may vary. A sample of car owners categorized by age.

16. approximately 1.85 kg

17. Answers may vary.

Extend

18. a) Competitor B

b) Both competitors would have a mark of 8.5.

19. a) $\frac{84}{39}$ **b)** $\frac{84}{39}$ **c)** \$1.31/kg

20. a) about 5.24 **b)** about 2.88%

6.2 Measures of Spread, pages 266–277

Example 1 Your Turn

a) 40th percentile: 16.55 min, 95th percentile: 23.85 min.

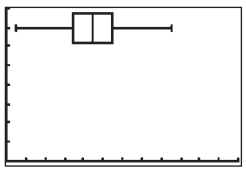
b) **i)** 7th percentile **ii)** 82nd percentile **iii)** 61st percentile

Example 2 Your Turn

a) median: 22.5 m, range: 45 m, Q1: 17.5 m, Q3: 27.5 m, interquartile range: 10 m

b) 25% of the data are contained in each of the intervals 0 to 17.5 m, 17.5 to 22.5 m, 22.5 to 27.5 m, and 27.5 to 45 m.

c) no outliers



Example 3 Your Turn

Answers may vary. The median birth length for boys is 0.5 cm greater than the median birth length for girls. The middle 50% of the birth lengths for boys lie between 48.5 cm and 52 cm, for an IQR of 3.5 cm. The middle 50% of the birth lengths for girls lie between 47.5 cm and 51 cm, for an IQR of 3.5 cm. Both the range and IQR for boys is greater than for girls. So, the birth lengths for boys are more spread out.

Reflect

R1. The median for Data Set 1 is 0.8 less than the median for Data Set 2. The middle 50% for Data Set 1 lie between 60.2 and 43.2, for an IQR of 17. The middle 50% for Data Set 2 lie between 63.2 and 34.5, for an IQR of 28.7. Both the range and IQR for Data Set 2 is greater than for Data Set 1. So, the values for Data Set 2 are more spread out.

R2. The range only gives information about the extreme values, not how closely the data is clustered around its centre.

R3. The interquartile range contains the middle 50% of the data. The smaller this value, the more closely the data is clustered around the centre.

Practise

1.

Mark	Percentile Rank
4.0	about 3rd
5.0	about 13th
6.0	about 32nd
7.0	about 59th
8.0	about 82nd
9.0	about 91st
10.0	about 97th

2. 62

3. a) median: 51, range: 77, Q1: 35.5, Q3: 62, interquartile range: 26.5, outliers: 87, 99

b) median: 233, range: 340, Q1: 214, Q3: 264, interquartile range: 50, outliers: 127, 467

c) median: 4.5, range: 9, Q1: 3, Q3: 6, interquartile range: 3, no outliers exist

d) median: 5968, range: 4559, Q1: 3567, Q3: 7659, interquartile range: 4092, no outliers exist

4. D

5. D

6. a)

Infant Mortality Rate by Province and Territory	2011
Yukon	0.0
New Brunswick	3.5
British Columbia	3.8
Prince Edward Island	4.2
Québec	4.3
Ontario	4.6
Nova Scotia	4.9
Alberta	5.3
Newfoundland and Labrador	6.3
Saskatchewan	6.7
Northwest Territories	7.2
Manitoba	7.7
Nunavut	26.3

b) Answers may vary. New Brunswick: 12th percentile, Prince Edward Island: 27th percentile, Ontario: 42th percentile, Alberta: 58th percentile, and Northwest Territories: 81st percentile.

7. a)

	2007	2008	2009	2010	2011
Median	5.2	5.3	5.8	5	4.9
Interquartile Range	3.2	2.75	3.25	2.2	2.95

b) Answers may vary. The median infant mortality rate increased from 2007 to 2009, and then decreased through 2011. The interquartile range of the infant mortality rates seem to cycle.

c) Answers may vary. Because of outliers.

8. a) Nunavut is an outlier.

b) Answers may vary. This could be due to low birth weight, lack of hospitals, or population prone to respiratory-track infections.

9. Answers may vary.

a)

Population of Canada by Age Group	
Age Group (years)	2009
100 or over	5474
90 to 100	180 409
80 to 90	1 075 522
70 to 80	1 994 853
60 to 70	3 299 618
0 to 10	3 626 272
10 to 20	4 253 528
30 to 40	4 534 301
20 to 30	4 608 623
50 to 60	4 798 598
40 to 50	5 251 373

Population of Canada by Age Group	
Age Group (years)	2013
100 or over	6 911
90 to 100	242 124
80 to 90	1 181 124
70 to 80	2 202 364
0 to 10	3 804 924
60 to 70	3 857 403
10 to 20	4 048 205
30 to 40	4 762 084
20 to 30	4 855 939
40 to 50	4 940 356
50 to 60	5 256 870

2009: 80 to 90: 2nd percentile, 0 to 10: 25th percentile, 50 to 60: 77th percentile
2013: 80 to 90: 2nd percentile, 0 to 10: 16th percentile, 50 to 60: 93rd percentile

b) Age groups 60 to 70 and 0 to 10 swapped positions from 2009 to 2013, as well as age groups 50 to 60 and 40 to 50. In particular, the 0 to 10 age group went from being in the 25th percentile in 2009 to the 16th percentile in 2013, and the 50 to 60 age group went from being in the 77th percentile in 2009 to the 93rd percentile in 2013.

10. a) A quintile divides a data set into five equal groups.

b) Answers may vary. While the percent of total net worth remained the same for the first quintile, all others changed. The second and third quintiles saw declines in the percent of total net worth, while the fourth and fifth quintiles saw increases. Basically, the wealthiest 20% of Canadians increased their share of the total wealth at the expense of the second the third quintiles.

12. December is an outlier. This is most likely caused by a combination of bad weather and increased number of flights due to the holiday season.

Extend

13. Answers may vary.

a) Locate the age of the child along the horizontal axis and follow the line up to the BMI. Interpret the percentile using the following table.

Weight Status Category	Percentile
Underweight	Less than 5th percentile
Healthy weight	5th percentile to less than 85th percentile
Overweight	85th to less than the 95th percentile
Obese	Equal to or greater than the 95th percentile

b)

Percentile	BMI Range by Age		
	2	10	20
$p < 5$th	BMI < 14.7	BMI < 14.2	BMI < 19.1
5th $\le p <$ 10th	14.7 ≤ BMI < 15.1	14.2 ≤ BMI < 14.6	19.1 ≤ BMI < 19.8
10th $\le p <$ 25th	15.1 ≤ BMI < 15.8	14.6 ≤ BMI < 15.5	19.8 ≤ BMI < 21.2
25th $\le p <$ 50th	15.8 ≤ BMI < 16.6	15.5 ≤ BMI < 16.6	21.2 ≤ BMI < 23
50th $\le p <$ 75th	16.6 ≤ BMI < 17.6	16.6 ≤ BMI < 18.2	23 ≤ BMI < 25.4
75th $\le p <$ 85th	17.6 ≤ BMI < 18.2	18.2 ≤ BMI < 19.4	25.4 ≤ BMI < 27
85th $\le p <$ 90th	18.2 ≤ BMI < 18.6	19.4 ≤ BMI < 20.3	27 ≤ BMI < 28.3
90th $\le p <$ 95th	18.6 ≤ BMI < 19.3	20.3 ≤ BMI < 22.1	28.3 ≤ BMI < 30.6
95th $\le p$	19.3 ≤ BMI	22.1 ≤ BMI	30.6 ≤ BMI

14. Answers may vary.

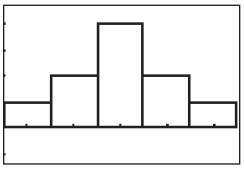
6.3 Standard Deviation and *z*-Scores, pages 278–289

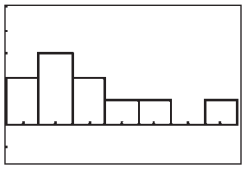
Example 1 Your Turn

Answers may vary.

Set A	
Height (cm)	Frequency
150–155	1
155–160	2
160–165	4
165–170	2
170–175	1
175–180	0
180–185	0

Set B	
Height (cm)	Frequency
150–155	2
155–160	3
160–165	2
165–170	1
170–175	1
175–180	0
180–185	1

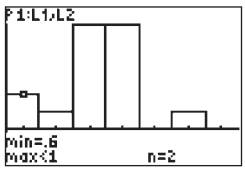




The histogram for Set B will have a greater standard deviation.

Example 2 Your Turn

a) Answers may vary.



b) mean: about 1.8 dB, standard deviation: approximately 0.447 dB

c) The standard deviation would increase because 1.5 dB is farther from the mean and the spread of the data would increase.

d) The standard deviation would decrease because 1.7 dB is closer to the mean and the spread of the data would decrease.

e) Since values decrease by the same amount, the spread will not change. The standard deviation would be unchanged.

f) levels less than 2.3 dB or greater than 1.3 dB

Example 3 Your Turn

a) mean: about 1.53 mm, standard deviation: approximately 0.2196 mm

b) about 0.3188; A gap of 1.6 mm is about 0.3188 standard deviations greater than the mean.

c) about −0.5920; A gap of 1.4 mm is about 0.5920 standard deviations less than the mean. A gap of 1.4 mm is farther from the mean than a gap of 1.6 mm, and is to the left of the means instead of to the right.

d) Any gaps that are less than 1.0908 mm or greater than 1.9692 mm would be rejected. None of the cars in this sample would be rejected.

Reflect

R1. a) A data point is two standard deviations below (less) the mean. In this case, 17.3 has a z-score of –2.

b) A data point is 1.5 standard deviations above (greater) the mean. In this case, 28.15 has a z-score of 1.5.

R2. Answers may vary. Since stock A has a smaller standard deviation, its price is more consistent, or reliable, than stock B. If you are a cautious investor, stock A is less

risky than stock B. However, the potential for greater profit is with stock A.

R3. a) x is greater than the mean
b) x is less than the mean
c) x is equal to the mean

R4. Answers may vary. Use population formulas when all values of a population are included. Use sample formulas when only a sample of the population is taken.

Practise

1. D **2.** A

3. a) about 0.5288; A value of 27.2 is about 0.5288 standard deviations greater than the mean.
b) about –0.3333; A value of 24.1 is about 0.3333 standard deviations less than the mean.
c) about –0.9444; A value of 21.9 is about 0.9444 standard deviations less than the mean.
d) 1.25; A value of 29.8 is 1.25 standard deviations greater than the mean.

4. a) approximately 2.6092 cm
b) approximately 8.5201 home runs
c) approximately a 17.2620 points

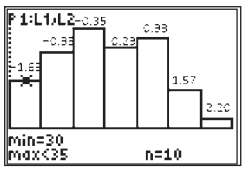
5. a) Sample: Researchers are studying a sample of the population, females ages 35 to 50 years old.
b) Population: This is a national survey for which researchers want to describe the variability in all ages.
c) Population: The teacher wants to summarize the results of all the students in her class.

6. a) years worked that are greater than 6 years and less than 15 years
b) about 97%
c) Answers may vary. The graph visualizes the number of standard deviations an observation is from the mean.

7. a) mean: approximately 45.24 min, standard deviation: approximately 7.831 min
b) Answers may vary. Sample formulas, since 93 customers is likely a sample from one day.
c)

Time Midpoint	z-Score
32.5	about –1.63
37.5	about –0.99
42.5	about –0.35
47.5	about 0.29
52.5	about 0.93
57.5	about 1.57
62.5	about 2.20

d)



8. a) Mississauga-Erindale: about 1.8632, Parkdale-High Park: about –0.0228
b) Answers may vary. They are underrepresented.

9. 92%

10. a) mean: approximately 1.007 L, standard deviation: approximately 0.014 L
b) Sample formulas, since volume is checked using a selection of 102 cartons.
c) Yes, it is within two standard deviations of the mean.
d) Answers may vary. The mean volume of milk increased from 1.007 L on the first day to 1.012 L on the second day. However, the standard deviation decreased from 0.014 L on the first day to 0.009 L on the second day. The data is less spread out from the mean on the second day.

11. a) mean: approximately 7.336 m, standard deviation: approximately 1.570 m
b) Answers may vary. The mean length of logs decreased from 8.44 m on the previous day to 7.336 m on this day. In addition, the standard deviation decreased from 1.836 m on the previous day to 1.570 m on this day. The data is less spread out from the mean on this day.
c) Answers may vary. For quality control purposes. It would help identify any problems in sawing-machine centres, sawing systems, or set repeatability systems.

13. The standard deviation is the square root of the variance. In other words, the standard deviation squared equals the variance. A value between 0 and 1, when squared results in a smaller value. So, the standard deviation will be larger than the variance when it is between 0 and 1. If $s = 0.99$ then $s^2 = 0.9801$.

14. Answers may vary. In the case of a career with a standard deviation of \$15 000, the salary range is more spread out and includes a potential high salary of \$86 000 but also a low salary of \$26 000. Compare this to the case of a career with a standard deviation of \$5000, where the salary range is more clustered around the mean. Here a potential high salary may be only \$66 000 but a low salary of \$46 000 is far above that of the other career.

15. a) Answers may vary. The uncertainty of the investment, or the difference between observed and expected rate of returns.
b) The standard deviation would increase for a risker investment.

16. Answers may vary. Since quartiles divide a set of ordered data into four groups with equal numbers of values, the interquartile range will not change. For a set of five values, Q1 still falls between data values one and two and Q3 still falls between data values four and five. The standard deviation will change because the new value is closer to the mean.

Extend

17. mean: $\bar{x} + a$, standard deviation remains unchanged.

18. mean: $c\bar{x}$, standard deviation: becomes cs.

19.
$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$
$$= \sqrt{\frac{\sum (x_i^2 - 2x_i\mu + \mu^2)}{N}}$$
$$= \sqrt{\frac{\sum x_i^2 - \sum 2x_i\mu + \sum \mu^2}{N}}$$
$$= \sqrt{\frac{\sum x_i^2 - 2\mu \sum x_i + N\mu^2}{N}}$$
$$= \sqrt{\frac{\sum x_i^2}{N} - \frac{2\mu \sum x_i}{N} + \frac{N\mu^2}{N}}$$
$$= \sqrt{\frac{\sum x_i^2}{N} - 2\mu^2 + \mu^2}$$
$$= \sqrt{\frac{\sum x_i^2}{N} - \mu^2}$$
$$= \sqrt{\frac{\sum x_i^2 - N\mu^2}{N}}$$

6.4 Interpreting Statistical Summaries, pages 290–301

Example 1 Your Turn

Answers may vary.

a) the median and mean for a particular Internet service provider, the IQR for the industry

b) For a claim that this ISP is "always" faster, they are using their mean data most of the time. When the mean is greater than the median, the median must be what they are using.

c) The claim is inaccurate. The mean is influenced by outliers, so extremely fast response times will significantly decrease the mean. Since the industry IQR does not include the median, it is not possible to compare this provider's response time to the industry.

d) The vertical scale does not start at 0 and the choice of units is milliseconds. As a result, the differences between the graphs seem to be greater than in reality.

Example 2 Your Turn

a)

b) Answers may vary. The multiple bar graph and the split bar graph both show the breakdown of sales within the regions. In the situations where the sales values are not that different, it is easier to see the difference in the multiple bar graph. However, total sales are different each quarter, so the relative frequency graph needs to be used to compare how well each region did.

Example 3 Your Turn

Answers may vary.

a) While Facebook is a critical tool for communicating, shopping, and listening music, privacy settings are not seen as important.

b) Use of a different colour to circle one statistic for emphasis. Graphics are used to signify each statistic. Various font sizes and font colours are used to catch the eye.

c) How large was the sample? How was the sample chosen and was it a random selection? What questions were asked in the survey?

d) What was the source of the data? Were the data primary or secondary? Who sponsored the survey?

Reflect

R1. Answers may vary. All three types use different colours and length of bars to represent data. However, a multiple bar graph and split bar graph show different quantities, where as a relative split bar graph shows different percents. Only the multiple bar graph displays bars side by side, while the other two types use bars placed one above the other.

R2. Answers may vary.

a) To determine whether you can make valid generalizations. Statistics are often used to represent certain points of view by manipulating graph axes, by citing only one measure of central tendency, or through measurement or sampling bias.

b) How large was the sample? How was the sample chosen and was it a random selection? What questions were asked in the survey? What was the source of the data? Were the data primary or secondary? Who sponsored the survey?

Practise

1. Answers may vary. No. The sample sizes appear to be the same, 100 of each gender. So, the graph shows that a slightly higher percent of grade 12 males have their G2 driver's licence. However, more than 50% of both females and males have their G2 licences.

2. Answers may vary.

a) No. The sample size is too small and was collected during a particular season when it is most likely to be a popular item.

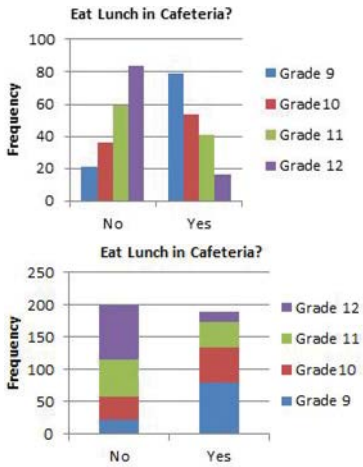
b) increase the sample size and ensure that the sample accurately represents the entire population.

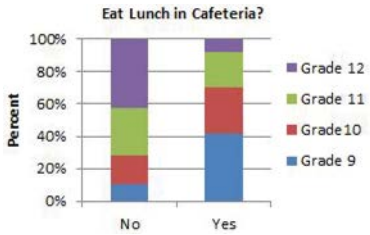
3. D

4. B

5. sample size of trout, the mean length, and the standard deviation for the length

6. a)





b) Since the sample sizes are not the same for each grade level, the relative split bar graph can be used to compare grade levels.

Apply

7. Answers may vary. Using 2013 data, "2% of the world's population has more than half the world's wealth" translates to "142 000 people have more than $115.5 trillion in wealth." In contrast, "half the world's population has only 1% of the world's wealth" translates to "3 550 000 people have only $2.31 trillion in wealth."

8. Answers may vary.

a) That climate change must not be important (or not being taken seriously). The graph shows that more countries have increased their greenhouse gas emissions than reduced them.

b) No. While Russia is the second largest producer of greenhouse emissions in this graph, the country has reduced emissions from 1990 to 2004, while others have increased. Compared to countries not included in this graph, say China and India, Russia is most likely lower on the list.

c) No. While Russia shows the largest decrease in emissions, the graph contains no information on how this was accomplished.

d) The headline implies that climate change must not really be happening since very few countries have attempted to reduce their greenhouse emissions. The graph does show very few countries have decreased their greenhouse gas emissions, but the graph includes only nine countries.

9. Answers may vary.

a) Boys: median: 50%, IQR: 20%, range: 85%
Girls: median: 55%, IQR: 20%, range: 60%
The median for girls is greater than for boys, 55% compared to 50%. While the IQR is the same for both genders, the range for boys is considerable greater than the girls. This means that the data values are more spread out.

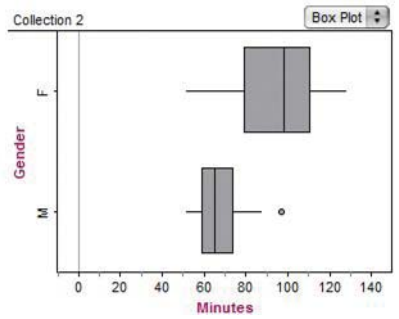
b) The median, Q1, and Q3 values for the girls are greater than the corresponding values for the boys. However, the maximum score for boys is 95% compared to girls at 90%. For this data set, I think the evidence supports that on average girls are better than boys at math.

10. The difference between a z-score of –2 and a z-score of –1 represents one standard deviation. As well, the difference between a z-score of 0 and a z-score of 1 is also one standard deviation. So, the statement means that the mean height is 96 cm with a standard deviation of 3.8 cm.

11. a)

Measure of Central Tendency	Male	Female
Mean	68.1 min	93.4 min
Median	65 min	98 min
Mode	none	none

b)



c) Answers may vary.

12. Answers may vary. While Fox has the heavy coverage of the Western Conference, it has little coverage of the Eastern Conference. NBC has heavy coverage of the Eastern Conference and little coverage of the Western Conference. CBS is the only network with equal coverage of both Conferences.

13. Answers may vary.

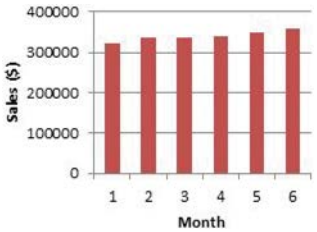
a) Life Sciences is the fastest growing industry.

b) Statistics Canada (Labour Force Survey) and Ontario Ministry of Finance

14. mean depth: approximately 1.23 m,
standard deviation: approximately 1.03 m

15. Answers may vary. Determine the size of the sample, how the sample chosen and if it was a random selection, what was measured, the source of the data and whether it was primary or secondary, if any sponsors were involved.

16. a)



b)

18. Answers may vary.

a) The North American Growth Fund. Returns have been in Q3 for three of the five years and Q2 for the two years. The other two funds showed returns in three or four different quartiles, indicating that they are more volatile.

b) For the long haul, I would recommend the North American Growth Fund because of its consistency, although it does not have high returns. For a short term riskier investment, I would recommend the Canadian Mineral Resource Fund because it appears to cycle to a high every third year.

19. Answers may vary.

Extend

20. a)

Phase of Flight	Percent of Bird Strikes
Descent	2
Enroute	5
Climb	8
Approach	21
Land & Taxi	27
Takeoff	37

b)

Phase of Flight	Percent of Bird Strikes	Percentile
Descent	2	1
Enroute	5	4.5
Climb	8	11
Approach	21	25.5
Land & Taxi	27	49.5
Takeoff	37	81.5

c)

Phase of Flight	Bird Strike Percent / Phase Flight Percent	Risk Percent
Takeoff	37	55.9
Climb	0.533	0.8
Enroute	0.088	0.1
Descent	0.182	0.3
Approach	1.4	2.1
Land & Taxi	27	40.8

Phase of Flight	Risk Percent	Percentile
Enroute	0.1	0.05
Descent	0.3	0.25
Climb	0.8	0.8
Approach	2.1	2.25
Land & Taxi	40.8	23.7
Takeoff	55.9	72.05

6.5 Analysing Data From Statistics Canada, pages 302–307

Reflect

R1. Answers may vary. The data are used by all levels of government, the private sector, and social and community groups.

R2. Answers may vary. Factors to be considered include sample design, questionnaire design, and data collection.

R3. Answers may vary. In order to evaluate the strength of the evidence and draw conclusions on that basis.

Practise

1. B **2.** C

3. a) customize the data according to how data is represented over time

b) customize the data according to which groups and areas are represented

4. population and dwelling counts, age characteristics, marital status, family characteristics, household and dwelling characteristics, detailed mother tongue, knowledge of official languages, first official language spoken, detailed language spoken most often at home, and detailed other language spoken regularly at home

Apply

5. Answers may vary.

6. a) increase: food, shelter, household operations, furnishings and equipment, transportation, recreation, education and reading, and alcoholic beverages and tobacco products
decrease: clothing and footwear and health and personal care

b) increase: food, shelter, household operations, furnishings and equipment, recreation, education and reading, and alcoholic beverages and tobacco products
decrease: clothing and footwear, transportation, and health and personal care

c) 12 months

7. Answers may vary.

a) The pyramid is wider at the base, with the maximum number of males and females at about age 5. In the late 30s to late 70s, there are more males than females. From there, the age population is split between the genders.

b) The pyramid is the widest around age 50. In general, the two genders mirror each other in growth and decline until the mid 60s. From there, the age population is comprised of more females than males.

c) Descriptions may vary. Any birth year in the late 1990s has a similar shape to 2011 but is the widest in the mid 30s.

d) The width of the base goes from about 240 000 people in 1946 to almost double that 480 000 people.

e) male aged 10: 270 000, male aged 50: 180 000, female aged 10: 280 000, female aged 50: 180 000

8. Answers may vary.

9. Answers may vary.

10. Answers may vary.

11. Answers may vary.

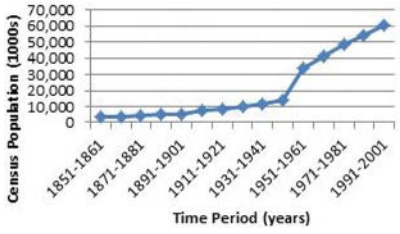
12. Answers may vary.

13. Answers may vary.

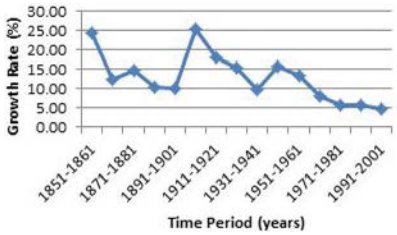
a)

Period	Census Population (1000s)	Growth	Growth Rate (%)
1851–1861	3230	793	24.55
1861–1871	3689	459	12.44
1871–1881	4325	636	14.71
1881–1891	4833	508	10.51
1891–1901	5371	538	10.02
1901–1911	7207	1836	25.48
1911–1921	8788	1581	17.99
1921–1931	10377	1589	15.31
1931–1941	11507	1130	9.82
1941–1951	13648	2141	15.69
1951–1961	34319	4590	13.37
1961–1971	41583	3330	8.01
1971–1981	48270	2859	5.92
1981–1991	54132	3211	5.93
1991–2001	60632	2990	4.93

b) The graph appears to show continued growth from 1851 to 2001. There was a big jump in the population from 1951 to 1961.



c) While the population from 1851 through 2001 was growing steadily, the rate of growth was increasing and decreasing in a fluctuating manner. During almost the entire time period from 1911 to 2011, the rate of growth was declining. During these 100 years, the rate of growth increased once from 1941 to 1951. From 1981, the population is growing at an almost steady rate of about 5%.



Chapter 6 Review, pages 308–310

1. a) mean: the average of a set of data; median: the middle number when the numbers are arranged in numerical order; mode: the number that occurs most often

b) mean: the sum of the data entries divided by the number of entries.
median: for an odd number of data, the median is the middle value of all the data points when the data values are listed in order from least to greatest. For an even number of data, the median is the average between the two middle values.
mode: found by inspection

c) Answers may vary. Mean: the average length of time it takes to get to school. Median: to represent a typical salary among employees. Mode: to represent the most popular song.

2. a) mean: about 286.3, median: 76, modes: 54 and 675

b) mean: 22.5, median: 18, mode: 7

c) mean: 4587.5, median: 3773, modes: 2346 and 9564

3. a) approximately 0.643 **b)** approximately 0.878

c) approximately 0.581

4. a) mean: approximately $57 235, median: $55 000, mode: 50–60 salary interval

b)

5. a) A percentile is the percent of all the data that are less than or equal to the specific data point. Quartiles divide the data set into four equal parts. Q1 is the 25th percentile, Q2 is the median (or 50th) percentile, and Q3 is the 75th percentile.

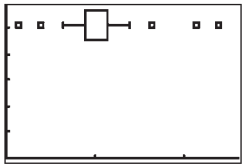
b) Answers may vary. A shoe store may use the IQR to determine the typically sold sizes. Then, use this information to order shoe sizes.

6. a)

Number of Friends	Percentile
0–25	0.8
25–50	6.7
50–75	16.3
75–100	30.6
100–125	57.9
125–150	81.7
150–175	92.1
175–200	96.1
200–225	97.5
225–250	99.4

b) Q1: 87.5 friends, Q3: 112.5 friends, Interquartile range: 25 friends

c)



d) 42 outliers

7. a) standard deviation: approximately 15 895 students, variance: 252 651 025 students^2

b) about 1.5398

c) No universities have a z-score of –2 or less.

8. To have a z-score of 1.5 means that the value is 1.5 standard deviations above the mean.

9. a) 500.14 mL to 502.06 mL

b) Answers may vary. The company may want to overfill the bottles to account for air in the dispensing of the drink liquid.

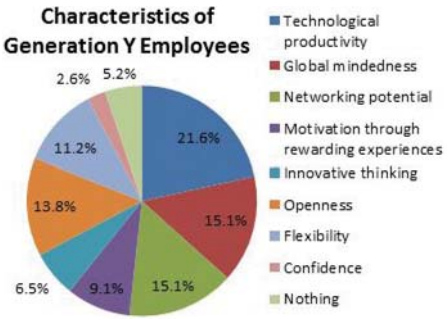
c) The bottles with volumes of 501.0 mL and 500 mL are acceptable.

10. Answers may vary.

a) each region's information is represented by one of the stacked bars, each stacked bar shows which parties are be considered in that region, and each stacked bar shows the percent of the sample of voters that intend to vote those parties

b) No; this is only a sample of 2000 voters from across all of Ontario.

11. Answers may vary. From a manger's perspective, the most important characteristic of a Generation Y employee is his or her technological productivity.



12. Answers may vary.

13. Answers may vary.

a) there was markedly different wage growth across age groups and that there was narrowing of wage differences across education levels during the 2000s

b) and **c)** The article contains two multiple line graphs and one multiple bar graph. Without grid backgrounds, the values are difficult to read. The vertical scale of charts 1 and 3 (line graphs) does not start of 0 and may exaggerate the results.

d) Statistics Canada surveys

e) and **f)** No specifics are given on sample size, though it covers full-time workers aged 17 to 64 from 1981 to 2011.

g) No actual methods are listed. However, it is secondary data.

h) The article gives references for information on previous trends.

i) René Morissette, Garnett Picot, and Yuqian Lu of the Economic Analysis Division at Statistics Canada

j) No. Technically, wages did not *steadily* increase of the past 30 years. Chart 1 shows that while overall hourly wages continued to increase, there were various times over the 30-year period that wages decreased.

Chapter 6 Test Yourself, pages 311–313

1. C **2.** D **3.** C **4.** B **5.** B

6. Yes. The IQR is the spread of the middle 50% of the data. The smaller this range, the smaller the spread of the central half of the data or the more consistent the player. In this case, Joshua is the more consistent player with an IQR of 3 compared to Ron with an IQR of 4.

7. The national census is sent to every household in Canada and completion is mandatory, so there is no concern that any parts of the population will not be represented.

8. −17.4 °C is the closest to the monthly mean

9. a) the source, date published, table number, table title and content, database, and URL

b) the source, date, graph title and content, and URL

10. Answers may vary. Apples: mean: approximately $2.20, standard deviation: approximately $0.40
Plums: mean: approximately $0.60, standard deviation: approximately $0.40
Oranges: mean: approximately $2.80, standard deviation: approximately $1

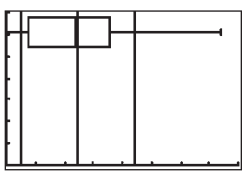
11. a) mean: approximately 12.43 years, standard deviation: approximately 9.38 years, Q1: 4 years, Q3: 18 years, Interquartile range: 14

b)

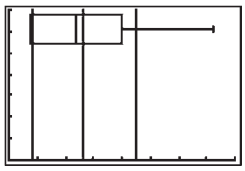
Number of Years Teaching	Frequency
0–8	11
8–16	8
16–24	7
24–32	3
32–40	1

c) mean: approximately 13.33 years, standard deviation: approximately 9.18 years, Q1: 4 years, Q3: 20 years, Interquartile range: 16
The mean and the IQR have increased, but the standard deviation has decreased.

d) vertical lines represent one standard deviation below the mean (3.05), the mean (12.43), and one standard deviation above the mean (21.81):



vertical lines represent one standard deviation below the mean (4.15), the mean (13.33), and one standard deviation above the mean (22.51):



e) 43rd percentile

f) 26 years of teaching

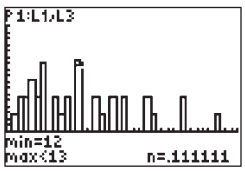
g) no outliers exist in this data set

h) Answers may vary. Ryan's data is not valid because the results do not accurately represent the entire population. Since Ryan sent the survey to the teachers, each chooses whether or not to respond. It is a voluntary response survey. Not knowing how many teachers are at Ryan's school, I assume that the entire population is not represented in the sample.

Chapters 4 to 6 Cumulative Review, pages 314–315

1. a)

Product, x	Frequency	Probability, $P(x)$
1	1	$\frac{1}{36}$
2	2	$\frac{1}{18}$
3	2	$\frac{1}{18}$
4	3	$\frac{1}{12}$
5	2	$\frac{1}{18}$
6	4	$\frac{1}{9}$
8	2	$\frac{1}{18}$
9	1	$\frac{1}{36}$
10	2	$\frac{1}{18}$
12	4	1/9
15	2	$\frac{1}{18}$
16	1	$\frac{1}{36}$
18	2	$\frac{1}{18}$
20	2	$\frac{1}{18}$
24	2	$\frac{1}{18}$
25	1	$\frac{1}{36}$
30	2	$\frac{1}{18}$
36	1	$\frac{1}{36}$



2. 249.5. The predicted average value of the card turned up will be 249.5.

3. a) about 0.1240 **b)** about 0.0027

c) 1×10^{-7}

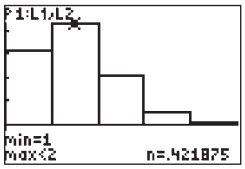
4. about 32.27%

5. a)

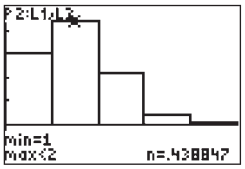
L1	L2	L3 3
0	.31641	.30382
1	.42188	.43885
2	.21094	.21349
3	.04688	.0412
4	.00391	.00264
------	------	▮▮▮▮▮▮

L3(6) =

i)



ii)



b) The graphs have the same bell-like shape, with the $x = 1$ diamond being the most likely outcome. The hypergeometric graph is slightly taller than the binomial graph at $x = 1$ and $x = 2$, and shorter at the other values of x.

c) The expected value for the binomial distribution is 4(0.25), or 1. The expected value for the hypergeometric distribution is $\frac{4(13)}{52}$, or 1. On average, there will be 1 diamond in a four-card selection, with or without replacement.

6. a) Since the blue graph shows the greatest number of searches from August 2012 to Jan 2013, it represents the Google search data related to "Gangnam Style." Since the yellow graph shows a spike in searches about mid-April 2013, it represents the Google search data related to "Gentleman."

b) The scale on the two graphs are very different. In the first five months, "Gangnam Style" had over 900 000 000 reviews, while "Gentleman" had only about 550 000 000 views. So, "Gentleman" is doing worse than "Gangnam Style."

7. Answers may vary.

a) I would use a sample that is representative of the population. So, selection for the sample must be random.

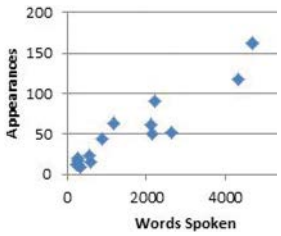
b) I would ask questions that are anonymous, clear, concise, and free of bias.

c) I would collect both continuous (e.g., hours) and discrete (e.g., number of people) numerical data, as well as categorical ordinal (e.g., rating scale) and nominal (e.g., type of sport) data.

d) I will keep the data free of bias by using a collection method that is free from sampling, measurement, response, or non-response bias. I will also display the data in an unbiased fashion.

8. a) Those two characters have the most appearances and largest speaking roles.

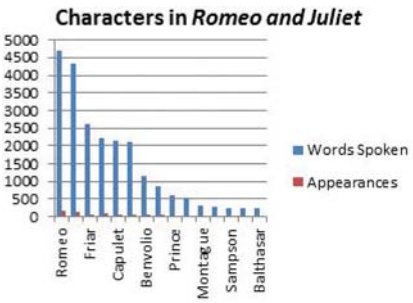
b)



There appears to be an upward trend. As the number of words spoken increases, so does the number of appearances.

c) the Friar

d)



9. a) mean: approximately 71.93 °C, median: 72 °C, mode: 72 °C

b) range: 20 °C, standard deviation: approximately 4.28 °C, variance: 18.3184 °C²

c) Q1: 69 °C, Q2: 72 °C, Q3: 73 °C, Interquartile range: 4

10. a) Two outliers exist in this data set: 81 and 87.

b) mean: approximately 71.1 °C, median: 71.5 °C, mode: 72 °C. The median is in the middle of the data, so it would be the best measure to represent the average temperature of the coffee.

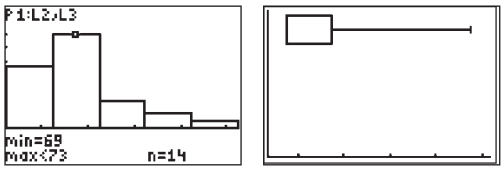
c) The mean is not appropriate because it is affected by the outliers and it is the least of the three measures. The mode is not appropriate because it is now the greatest measure.

11. Answers may vary.

a)

Coffee Temperature (°C)	Frequency
65–69	9
69–73	14
73–77	4
77–81	2
81–85	1

b)



12. a) 2 **b)** 65 °C, 85 °C

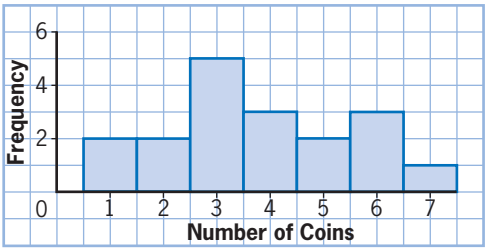
13. Yes. The middle 50% of the coffee temperatures lie between 69 °C and 73 °C.

Chapter 7 Probability Distributions for Continuous Variables

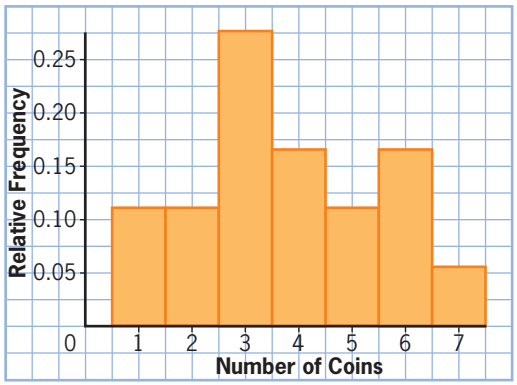
Prerequisite Skills, pages 318–319

1. a) 7.98 m² **b)** 3.33 cm²

2. a)



b)



c) mean: about 3.7778 coins, standard deviation: about 1.7675 coins.

d) about –1.0058

3. a) 78 **b)** No.

4. 32

5. a) 120 **b)** 6 **c)** 90 **d)** 35

6. 6188

7. a) 479 001 600 **b)** 1296

8. $\frac{1}{3}$ **9.** $\frac{1}{720}$ **10.** –$1

11. about 0.0577 **12.** about 0.0993

13. $\frac{1}{14\ 950}$ **14.** about 0.217

15. a) about 0.2149 **b)** about 0.4750

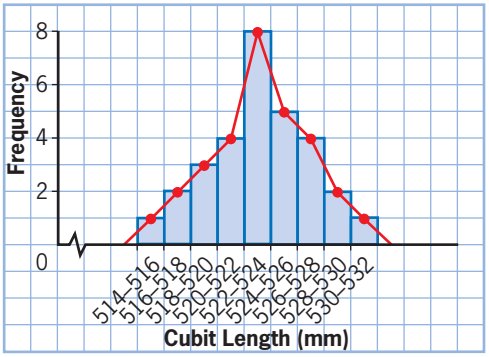
7.1 Continuous Random Variables, pages 320–331

Example 1 Your Turn

a) 0.25 **b)** 0.625 **c)** 0

Example 2 Your Turn

a) and **b)**



c) around 523 mm

d) All outcomes are not equally likely. So, this is not a uniform distribution.

Reflect

R1. The number of guests that occupy the hotel each day is discrete data, while the time a guest waits for an elevator is continuous data. It is possible to list all values of the discrete distribution, since these would be values from 0 up to the maximum capacity of the hotel. It is not possible to list all values of the continuous distribution, since time can be recorded in fractions of a second.

R2. Answers may vary. Maya could be standing differently each time, bad measuring technique, and misreading the measuring device. This is most likely measurement error, not bias.

R3. Answers may vary. The probability that a variable falls within a range of values is equal to the area under the probability density graph for that range of values. The area method cannot be used for single values of a continuous variable, only for a range of values. A continuous random variable can take on an infinite number of values. The probability that it will equal a specific value is always zero.

Practise

1. a) No; mass represents a continuous variable

b) Yes; the value of a card represents a discrete variable

c) No; barometric pressure represents a continuous variable

2. a) No; the number of students with blue eyes represents a discrete variable

b) Yes; weight represents a continuous variable

c) No; the number of cartons of milk represents a discrete variable

d) No; the number of defective tablets represents a discrete variable

3. a) No; each waist size interval is not equally likely

b) 90

c) 36–38 interval

Apply

4. a) It is not obvious whether the distribution is uniform or not.

b) Use a table to determine the frequency for each interval. If all frequencies are equal, then the distribution is uniform.

Volume of Sample (mL)	Frequency
54–55	4
55–56	4
56–57	4
57–58	4
58–59	4
59–60	4
60–61	4
61–62	4
62–63	4
63–64	4
64–65	4

This distribution is uniform.

5. a) Use a table to determine the frequency for each interval. If all frequencies are equal, then the distribution is uniform.

Pressure (psi)	Frequency
2995–2997	8
2997–2999	6
2999–3001	7
3001–3003	7
3003–3005	4

The distribution is not uniform.

b) In general, pressure is a continuous variable. So, the distribution is continuous.

c) Answers may vary. The gauge used to measure the pressure only has a whole-number scale.

6. a) 0.25 **b)** 0.5

c) This type of distribution was used because all times are equally likely.

7. a) 88 keys

b) Answers may vary. No, because I doubt that all keys will be used in a piece of music.

c) Answers may vary. Since each key represents a specific note (frequency), the distribution in part b) is discrete.

8. Answers may vary.

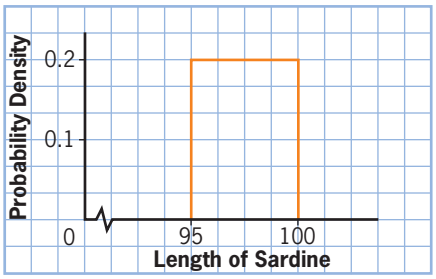
a) If a trombone player plays many notes at random frequencies, I would guess that he is constantly moving the slider to new positions while blowing. Since the trombone can play all frequencies between notes, I would expect the distribution to be continuous.

b) If a trombone player plays a musical composition, he plays specific notes. I would expect the distribution to be discrete.

9. a) There are six possible lengths for the sardines. There are six occurrences of 99 mm. So, it is not possible for the 24 sardine sample to be a uniform distribution.

b) 0.2

c)



d) 0.6

10. The area method cannot be used for single values of a continuous variable. So, Jon is incorrect. The probability that a variable falls within a range of values is equal to the area under the probability density graph for that range of values. The area shaded is from 2.5 to 3.5. So, Sunita is correct.

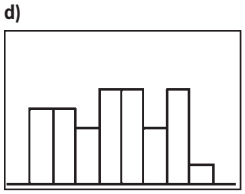
12. a) Speed is a continuous variable, so the data is continuous.

b) The data ranges from 64.6 to 80.2. So, I chose intervals of width two, starting at 64.5.

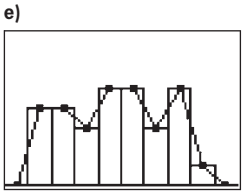
c)

Speed (km/h)	Frequency
64.5–66.5	4
66.5–68.5	4
68.5–70.5	3
70.5–72.5	5
72.5–74.5	5
74.5–76.5	3
76.5–78.5	5
78.5–80.5	1

d)



e)



f) The mean appears to be around 72.5 km/h.

g) Answers may vary. Since only one speed was recorded over 80 km/h, my estimate is $\frac{1}{30}$, or about 0.033.

13. Answers may vary. I think that hair colour should be considered continuous, since there are unlimited numbers of shades between blonde, red, brown, and black.

Extend

14. Answers may vary.

a) Three instruments that can play only discrete frequencies are electric piano, organ, and harp.

b) Three instruments that can play any frequency over a range are bass, cello, violin.

15. Answers may vary.

a) Use the formula for the area of a trapezoid with height 1 to calculate a probability to the left or right of 0. Then, determine the sum if needed.

b) 0.5

7.2 The Normal Distribution and z-Scores, pages 332–345

Example 1 Your Turn

a) 0.175 **b)** 0.205

c) 0.770 **d)** 0

Example 2 Your Turn

a) mean: 35.2 yd, standard deviation: about 9.567 yd

b) about 0.2934

c) about 0.6360

Reflect

R1. Answers may vary. As Kunal practises and gains more skill, I expect the mean and standard deviation to change. As he improves and becomes more consistent, I expect the mean to increase and the standard deviation to decrease.

R2. The z-scores for a normal distribution follow a normal distribution themselves, with a mean of 0 and a standard deviation of 1. From the graph, $P(z < -5)$ is located far in the left tail. So, the probability that far from the central peak is essentially zero.

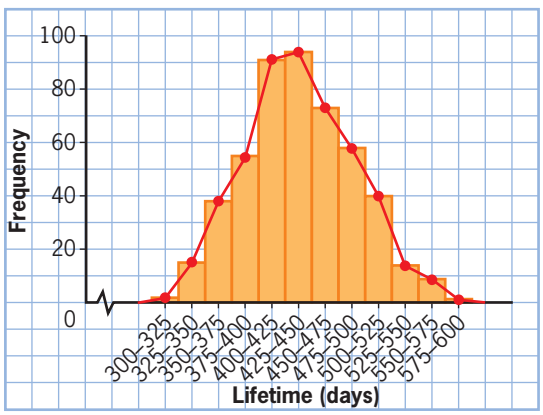
Practise

1. C **2.** A **3.** 0

4. Answers may vary. Data collected one week could be different from another because of the path chosen, traffic lights, or weather condition. Roberta could obtain more reliable values for the mean and standard deviation by combining the data from the two weeks.

Apply

5. a)



b) around 437.5 days

c)

Lifetime (days)	Frequency	Relative Frequency
300–325	2	0.004
325–350	15	0.030
350–375	38	0.076
375–400	55	0.110
400–425	91	0.182
425–450	94	0.188
450–475	73	0.146
475–500	68	0.136
500–525	40	0.080
525–550	14	0.028
550–575	9	0.018
575–600	1	0.002

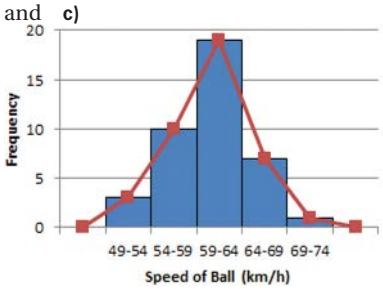
d) 0.220

e) Answers may vary. Replace the light bulbs every 437.5 days, the estimated mean, to be reasonably sure that there would never be a burned out light bulb.

6. a)

Speed of Ball (km/h)	Frequency
49–54	3
54–59	10
59–64	19
64–69	7
69–74	1

b) and c)



d) Answers may vary. While the distribution is centred around a central value, it does not drop off symmetrically to the left and right.

e) around 61.5 days

f)

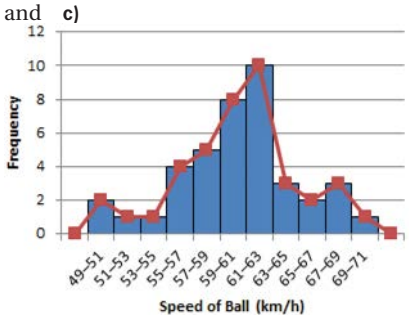
Speed of Ball (km/h)	Frequency	Relative Frequency
49–54	3	0.075
54–59	10	0.250
59–64	19	0.475
64–69	7	0.175
69–74	1	0.025

g) 0.65

7. a)

Speed of Ball (km/h)	Frequency
49–51	2
51–53	1
53–55	1
55–57	4
57–59	5
59–61	8
61–63	10
63–65	3
65–67	2
67–69	3
69–71	1

b) and c)



d) around 61 km/h

e) Answers may vary. I think the smaller interval width made it easier to estimate the mean.

f) Answers may vary. The smaller interval width gives a clearer picture of the actual distribution because it better approximates the shape of the frequency distribution.

8. a)

Horizontal Error (cm)	Frequency
(–10)–(–8)	1
(–8)–(–6)	3
(–6)–(–4)	6
(–4)–(–2)	6
(–2)–0	8
0–2	3
2–4	4
4–6	4
6–8	1

b)

Horizontal Error (cm)	Frequency	Relative Frequency
(–10)–(–8)	1	0.0278
(–8)–(–6)	3	0.0833
(–6)–(–4)	6	0.1667
(–4)–(–2)	6	0.1667
(–2)–0	8	0.2222
0–2	3	0.0833
2–4	4	0.1111
4–6	4	0.1111
6–8	1	0.0278

c) 0.5833

d) Answers may vary. A misadjusted sight with a bias to the left would result in more negative values.

9. a) 35.145 cm b) about 2.3612 cm

c) Answers may vary. If a variable is expected to follow a normal distribution, you can take a representative sample. However, there is not enough data to predict whether the distribution is normal. The data given shows that distribution may be centred, but it is unclear whether it will drop off symmetrically to the left and right.

Tree Height (cm)	Frequency
30–32	3
32–34	2
34–36	7
36–38	7
38–40	1

10. a) Calculate the z-score for each student. Determine which student is performing at a higher number of standard deviations from the respective means.

b) Answers may vary. The university must assume that the test are comparable in material and level of difficulty.

c) z-score of region A ≈ 1.4286; z-score of region B $= 1$. Since $1.429 > 1$, the student from region A performed at a level that is further above average than the student from region B.

11. Answers may vary.

a) marks on an exam, heights of plants, battery life, shoe size, masses of infants. Once data is collected and entered into technology, the remaining parts follow a similar procedure to #12.

13. a) 0.1788 **b)** 0.2676 **c)** 0.3108

Extend

14. Answers may vary.

a) To compare the two distributions, use the z-score. Determine the z-score for this year's class and then use that to determine the equivalent score using the historical mean and standard deviation.

b) 76

c) Marks this year might be lower than expected because the calibre of the students has declined.

15. a) Consider the formula for the mean of grouped data, which is the same as the sum of the product of the midpoint of each interval, m_i, and the relative frequency of each interval, rf_i.

$$\begin{aligned}\bar{x} &= \frac{\sum f_i m_i}{\sum f_i} \\ &= \frac{f_1 m_1 + f_2 m_2 + \cdots f_n m_n}{\sum f_i} \\ &= \frac{f_1 m_1}{\sum f_i} + \frac{f_2 m_2}{\sum f_i} + \cdots \frac{f_n m_n}{\sum f_i} \\ &= m_1 \frac{f_1}{\sum f_i} + m_2 \frac{f_2}{\sum f_i} + \cdots m_n \frac{f_n}{\sum f_i} \\ &= m_1 rf_1 + m_2 rf_2 + \cdots m_n rf_n \\ &= \sum m_i rf_i\end{aligned}$$

b) Determine the midpoint of each interval.

Volume (mL)	Frequency, f	Midpoint, m_i	Relative Frequency, $rf = \frac{f}{200}$
490–492	0	491	0.000
492–494	0	493	0.000
494–496	2	495	0.010
496–498	11	497	0.055
498–500	43	499	0.215
500–502	81	501	0.405
502–504	48	503	0.240
504–506	14	505	0.070
506–508	1	507	0.005
508–510	0	509	0.000

$$\begin{aligned}\bar{x} &= \sum m_i rf_i \\ &= 491(0) + 493(0) + 495(0.10) + \cdots + 509(0) \\ &= 501.08\end{aligned}$$

The mean soft drink volume is 501.08 mL.

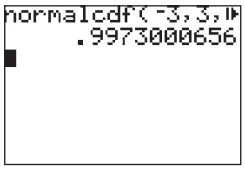
c) The value of the mean makes sense. It occurs in the centre of the bell-shaped frequency polygon.

7.3 Applications of the Normal Distribution, pages 346–351

Example Your Turn

a) about 99.7%

b)



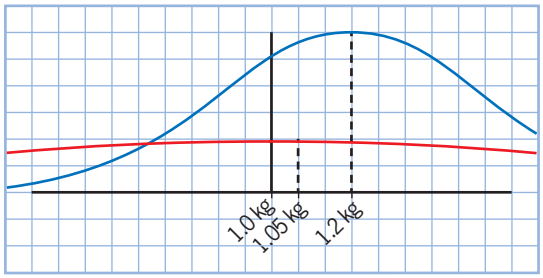
Reflect

R1. Answers may vary. Determine the z-score for a length of 4.7 cm. Use $x = 4.7$, $\bar{x} = 5$, and $s = 0.1$.

$$\begin{aligned}z &= \frac{x - \bar{x}}{s} \\ &= \frac{4.7 - 5.0}{0.1} \\ &= -3\end{aligned}$$

The probability that the length of a bolt is 4.7 cm or less is only about 0.1%, so this would be a surprising value.

R2. Answers may vary. Each machine produces a normal distribution for the mass of honey in a jar. The distributions have different means, one at 1.05 kg and the other at 1.2 kg. Each also produces a "tail" to the left of 1.0 kg. If the standard deviations are correct, the two tail areas will be equal, i.e., 0.001. Under these conditions, the results are possible.



Practise

1. D **2.** A

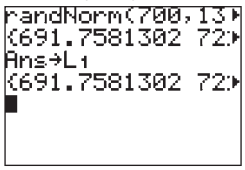
3. The probability that a mass is greater than two standard deviations is about 2.3%. Since this represents approximately 1 cat, this claim makes sense.

4. Answers may vary. A quality control engineer would want to ensure that they will actually fit onto the standard bolts. In particular, the internal diameter of the washer must not be smaller the exterior diameter of the bolt shaft.

Apply

5. Answers may vary.

a)



b) mean: about 701 mm,
standard deviation: about 20.1 mm

c) They are fairly close to the underlying normal distribution.

d)–f)

Sample Size	Mean (mm)	Standard Deviation (mm)
population	$\mu = 700$	$\sigma = 13.2$
10	$\bar{x} = 701$	$s \approx 20.1$
100	$\bar{x} = 700$	$s \approx 14.9$
1000	$\bar{x} = 700$	$s \approx 13.1$

g) The larger the sample, the closer the sample measures are to the underlying normal distribution.

6. a) 0.0122 **b)** 0.121

7. a) 2.5% **b)** $11 400
c) No, 90.82% of the watches fail within 10 years.

8. a) 136 students **b)** 190 students
c) about 5 students

9. 0.000 05 in.

11. a) mean: about 503 g, standard deviation: about 2 g
b) 80%
c) The expected percent of the data within one standard deviation is 68%. So, these values are more clustered around the mean.

Extend

12. a) about 12.1 g
b) To ensure that no more than 0.5% of the sandwiches contain less than 200 g of corned beef, Randy could buy a better slicing machine. New standard deviation: about 7.8 g. Randy could increase the slicing machine mean. New mean: about 231.2 g.
c) Answers may vary. Most likely increasing the slicing machine mean will be more cost effective than buying a new machine.

13. a) For two standard deviations from the mean, $k = 2$. According to Chebyshev's Theorem, no more than $\frac{1}{2^2}$, or 25% of the values lie more than two standard deviations from the mean.
b) Since 95% of the data values lie within two standard deviations of a normal distribution, then 5% of the data lie outside this range. Chebyshev's Theorem does not exactly agree, but could since the 25% value is a maximum.
c) The proportion of values that must lie within k standard deviations of the mean is given by $1 - \frac{1}{k^2}$.

7.4 Confidence Intervals, pages 352–361

Example 1 Your Turn

a) about 11.2% **b)** 63.8% to 86.2%
c) Answers may vary. *Hockey Night in Canada* is watched by 75% of households. This estimate is considered correct within ±11.2%, 99 times out of 100.

Example 2 Your Turn

a) about 2.8% **b)** increase to about 1561 pills

Example 3 Your Turn

a) mean: 1232.75 h, standard deviation: about 121.24 h
b) about 920.44 h to 1545.06 h

Reflect

R1. No. The confidence level is the probability that a particular statistic is within the range indicated by the margin of error. The confidence level's related z-score is used to calculate the margin of error.

R2. The lower end of the range is 4.2%. The upper end of the range is 7%. The 90% confidence interval for the mean defective tablets within one year is 4.2% to 7%. This is the range of possible percents of defective tablets. This would help the manufacturer budget for returns and provide information on the reliability of the manufacturing process.

R3. 90% confidence interval: 9 times out of 10
99% confidence interval: 99 times out of 100

Practise

1. a) about 1.6 **b)** about 2 **c)** about 2.6

2. B **3.** D **4.** 500 km

Apply

5. a) about 19.8% **b)** 62.2% to 101.8%
c) Answers may vary. Students recorded a mean mark of 82% ± 19.8%, 99 times out of 100.

6. Answers may vary. For the original poll, the lower end of the range is 30.6%. The upper end of the range is 37.4%. The 95% confidence interval for the average percent of support is 30.6% to 37.4%. The other two polls are both outside of this range, suggesting that support is not consistent.

7. a) mean: about 48.3 g, standard deviation: about 0.45 g. I assumed that the standard deviation stayed the same as the Single Crème line.
b) about 47.4 g to 49.2 g
c) Answers may vary. No. The company is not justified in claiming the line contains twice the filling. The Double Crème line contains twice the filling when compared to approximately the lower half of the Single Crème line distribution. So, for only less than half of the cookies is the claim valid.

8. a) about 68.9 min to 75.5 min
b) Answers may vary. I would advise the manufacturer that 70 min is a reasonable value for t, since it lies within the 95% confidence interval.

9. about 11 to 13

10. about 2

11. 90.5%

12. approximately 59 patients

13. 15 843.2 h to 16 156.8 h

15. Anywhere from 54% to 60% of students would vote for Adam and anywhere from 48% to 54% of students would vote for Meghan with a 95% confidence level.

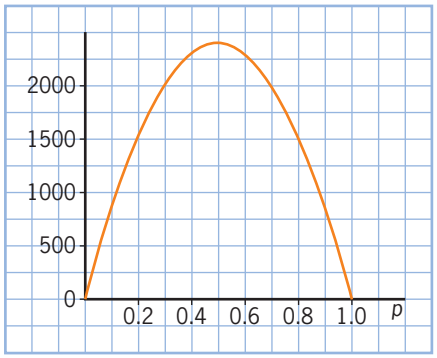
Extend

16. Answers may vary.
a) Since the masses of 20-year-old men most likely follow a normal distribution, I would expect to end up with more men close to the population mean, which is consistent with the bell-shaped curve.
b) This results in the standard deviation of the sample mean being smaller than that of the population.
c) The effect in part b) fits with the formula for the standard deviation of the sample, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$. The standard deviation of the population is divided by $\sqrt{n}$, thus decreasing the value.

17. $z = 1.960$, $E \leq 0.02$

$$E = z\sqrt{\frac{p(1-p)}{n}}$$
$$0.02 \geq 1.960\sqrt{\frac{p(1-p)}{n}}$$
$$\frac{0.02}{1.960} \geq \sqrt{\frac{p(1-p)}{n}}$$
$$\left(\frac{0.02}{1.960}\right)^2 \geq \frac{p(1-p)}{n}$$
$$n \geq p(1-p)\left(\frac{1.960}{0.02}\right)^2$$
$$n \geq 9604p(1-p)$$

The minimum number of voters who must be interviewed is 2401.



7.5 Connections to Discrete Random Variables, pages 362–371

Example 1 Your Turn

a) $np = 6.25$ and $nq = 18.75$. Since both are greater than 5, the normal approximation is reasonable in this case.
b) mean: 6.25, standard deviation: about 2.165
c) about 0.718

Example 2 Your Turn

a) There are 90 sock in the drawer, of which 7 are chosen. The number of trials is less than 10% of the population. The normal approximation is reasonable for this hypergeometric distribution.
b) mean: about 2.333, standard deviation: about 1.204
c) about 0.441

Reflect

R1. Answers may vary. For example, number of heads when flipping a coin or failure rate for quality control.
R2. Answers may vary. For example, choosing certain people to be on a committee or the number of a particular type of card in a five-card hand.
R3. Answers may vary. Using the approximation allows the probabilities of value ranges to be calculated more easily than with the binomial or hypergeometric formulas.
R4. Answers may vary. In the case of the normal approximation for a binomial distribution, any scenario where $np \leq 5$ or $nq \leq 5$. In the case of the normal approximation for a hypergeometric distribution, any scenario where the number of dependent trials is greater than or equal to 10% of the population.

Practise

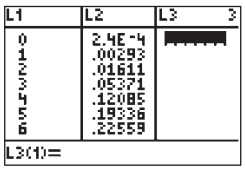
1. A **2.** D **3.** 31 rolls **4.** 5 balls

5. a) Technically, 10% of the population is 0.10(52), or 5.2. So, five cards meets the restriction.
b) 1.25 **c)** about 0.930

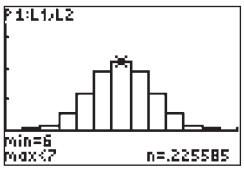
Apply

6. a) about 0.1024 **b)** about 0.1118
c) Answers may vary. Since both np and nq are greater than five, I would expect close agreement between the two methods.

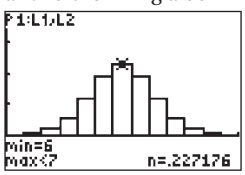
7. a)



b)



c) the probability of a particular number of heads
d) 1
e) Yes; the probability distribution is centred around a value and drops off symmetrically to the right and left forming a bell-like shape.
f)



g) When tossing coins, the mean is $\mu = 6$ and the standard deviation is

$$\sigma = \sqrt{(npq)}$$
$$= \sqrt{12 \times 0.5 \times 0.5}$$
$$\approx 1.732$$

Using normalcdf(–0.5,12.5,6,1.732), the probability is about 0.9998.
h) The areas are the same, 1.

8. a) about 0.0584
b) Since np is less than 5, it is not reasonable to model this distribution using a normal approximation.
c) mean: 2.5, standard deviation: about 1.369
d) about 0.0578.
e) The answers to parts a) and d) are very close. They both round to 5.8%.

10. a) about 0.0586
b) Answers may vary. I chose to use the normal approximation for a hypergeometric distribution because of the number of calculations needed to calculate $P(X \geq 10)$ using the hypergeometric distribution.

11. a) 0.0735
b) No; the population mean and standard deviation are given for the normal distribution.
c) about 505.02 g

Extend

12. a) about 0 **b)** about 0.1016.
13. a) 6 **b)** about 0.2649
c) Answers may vary. No. With a probability of 26.5% that this scenario could happen by chance, there is not enough information to conclude that the program was effective. More data needs to be collected. If the program was effective, I would expect the probability of 4 or fewer drivers being distracted to be higher.

Chapter 7 Review, pages 372–374

1. a) The data are difficult to analyse in this form. It is not obvious whether the distribution is uniform or not.

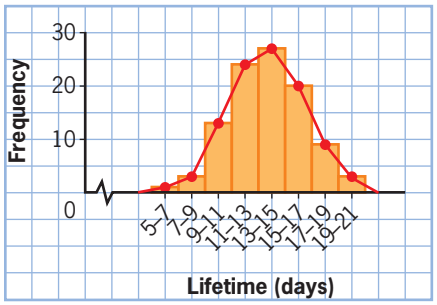
b) Use a table to determine the frequency for each interval. If all frequencies are equal, then the distribution is uniform.

Percent Oxygen	Frequency
31.7–31.8	3
31.8–31.9	3
31.9–32.0	3
32.0–32.1	3
32.1–32.2	3

This distribution is uniform.

2. a) approximately 0.263 **b)** 0.4997 **c)** 0.263

3. a) and **b)**



c) 14 days

4. a)

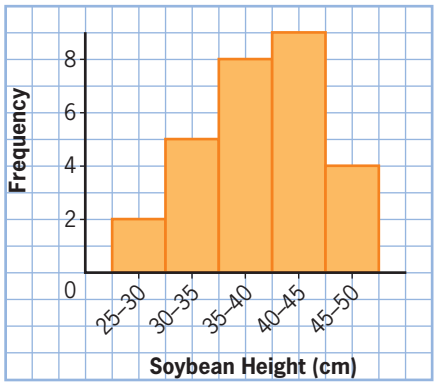
Lifetime (days)	Frequency	Relative Frequency
5–7	1	0.01
7–9	3	0.03
9–11	13	0.13
11–13	24	0.24
13–15	27	0.27
15–17	20	0.20
17–19	9	0.09
19–21	3	0.03

b) 0.01 **c)** 0.71

5. a) mean: 39.225 cm

b) standard deviation: about 5.8260 cm

c)



d) Answers may vary. No. While the distribution is somewhat bell-shaped, it is negatively skewed.

6. 0.1151

7. a) about 1.6% **b)** 0.0894

8. about 0.1052

9. a) about 7.9% **b)** 34.1% to 49.9%

10. a) 1562.75 km

b) 188 437.25 km to 191 562.75 km

11. a) about 0.8941

b) Since nq is less than 5, it is not reasonable to model this distribution using a normal approximation.

c) mean: 61.75, standard deviation: about 1.757.

d) about 0.8834

e) The answer to part a) is slightly higher than that of part d), approximately 89% compared to 88%.

12. a) Answers may vary. No; the class is not a representative sample of the population of the town because it is only comprised of high school students.

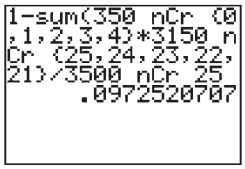
b) The number of trials is less than 10% of the population. The normal approximation is reasonable for this hypergeometric distribution.

c) mean: 2.5, standard deviation: about 1.4948

d) about 0.0905

e) In this situation, $n = 3500$, $r = 25$, and $a = 350$.

$$P(x \geq 5) = 1 - P(0) - P(1) - P(2) - P(3) - P(4)$$
$$\approx 0.6430$$



The probability that at least five students have the flu is about 0.0973.

The normal approximation results in a higher probability, about 9.7% versus 9.0%.

Chapter 7 Test Yourself, pages 375–377

1. C **2.** B **3.** C **4.** B **5.** C **6.** B **7.** 0.2

8. a) 10 students **b)** 0.78

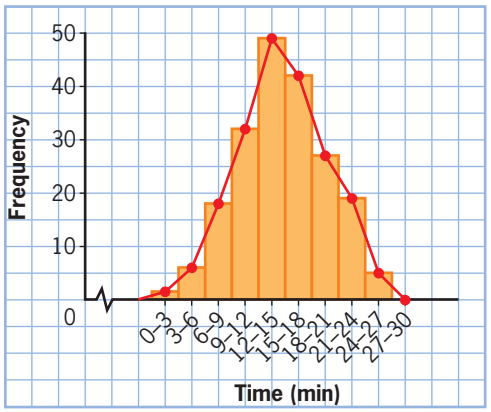
9. 0.0918

10. 492.4 h

11. a) Since np and nq are both greater than 5, it is reasonable to model this distribution using a normal approximation.

b) mean: 750, standard deviation: about 13.693

12. a) and **b)**



c) Yes. Since the probability distribution is centred around a value and drops off mostly symmetrically to the right and left forming a bell-like shape, the data appear to follow a normal distribtion.

13. a) about 3.7% **b)** about 102 dB

14. 79.8% to 88.2%

15. a) hypergeometric probability distribution; there are two outcomes, success and failure, and all trials are dependent.

b) Yes; the number of trials is less than 10% of the population.

c) mean: 29.01, standard deviation: about 0.9463.

d) about 0.9383

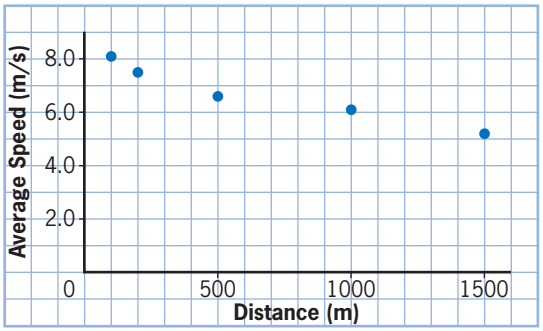
Chapter 8 Two-Variable Data Analysis

Prerequisite Skills, pages 380–381

1. a) upward and positive

b) Answers may vary. As precipitation increases, visibility and speed decrease, so travel time increases.

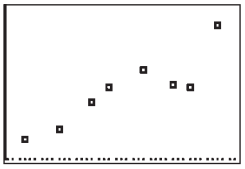
2. a)



b) downward and negative

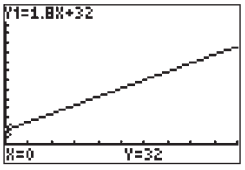
c) Answers may vary. Using a line of best and interpolate: about 6.4 m/s.

3. a)



b) Answers may vary. With the exception of two outliers, the general trend is upward and positive.

4. a)



b) $F = 32$. This is the vertical intercept and means that 0° C equals 32° F.

c) 1.8° F/°C. Every 1° C change results in a 1.8° F.

d) 25° C = 77° F

e) $F = 1.8C + 32$
$= 1.8(25) + 32$
$= 77$

f) Answers may vary.

5. a) 50° F = 10° C **b)** 50° F = 10° C

c) Answers may vary.

6. a) Because only the football team are asked, it is likely the respondents will overwhelmingly want to increase the football program's budget. This is sampling bias.

b) Answers may vary. Survey the entire student body.

7. Usually, the people who call in to radio shows are those with extreme opinions. People who are indifferent or do not think the topic is important may not vote, so a large proportion of listeners choose not to respond. This is non-response bias. Also, since only listeners of the radio show will call in, sampling bias is present, too; the respondents likely would not properly represent the entire population.

8. a) mean: 1.6 m, median: 1.5 m, mode: 1.5 m

b) Outliers have a greater effect on the mean than other measures. In this case, they increase the value of the mean.

c) standard deviation: about 0.1844 m; z-score: about 1.63

9. a) 7 **b)** 7

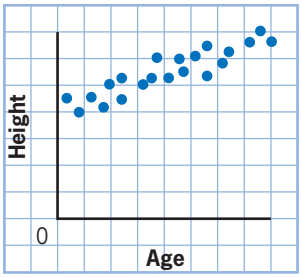
c) 2; it represents the "middle half" of the data.

8.1 Line of Best Fit, pages 382–391

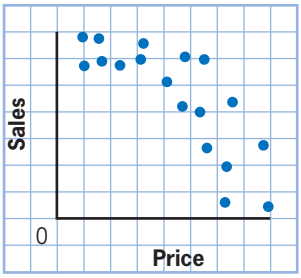
Example 1 Your Turn

Answers may vary.

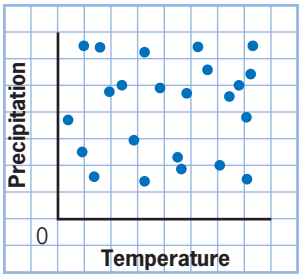
a)



b)

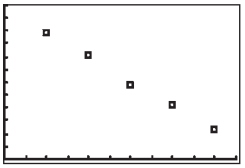


c)



Example 2 Your Turn

a)



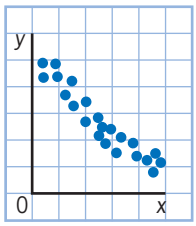
b) $r \approx -0.999$

c) $d = -0.189t + 11.71$; the cyclist started 11.71 km from home and cycled at a rate of 0.189 km/min toward home.

d) Answers may vary. The value of r, which can be between –1 and 1, gives an indication of how closely the data points relate to the line of best fit.

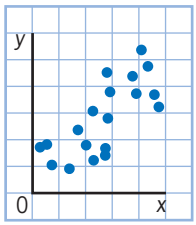
Reflect

R1. a)



b) The correlation coefficient, r, would be somewhere between –0.67 and –1.

R2. a)



b) The correlation coefficient, r, would be somewhere between 0.33 and 0.67.

R3. Answers may vary. $r \approx 0.995$, so there is a near perfect positive linear correlation between distance and time. The equation of the line of best fit is $d = 0.51t + 0.49$. The equation shows that the student started 0.49 m from the motion sensor and walked at a rate of 0.51 m/s away from the sensor.

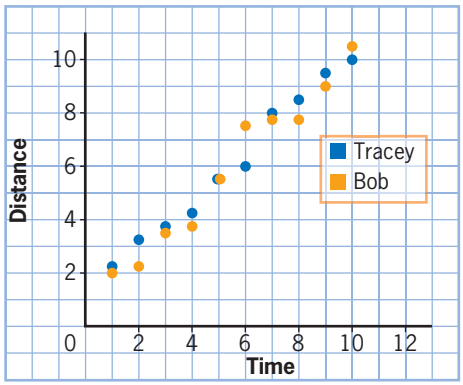
Practise

1. C **2.** B **3.** D

4. Answers may vary.

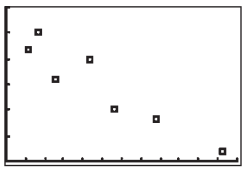
a) same: equation of line of best fit, initial distance, walking away at same rate
different: Tracey's pace is more consistent

b)



Apply

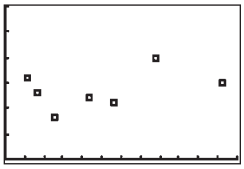
5. a)



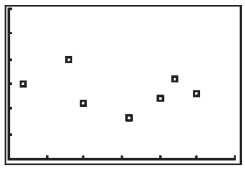
b) strong negative linear correlation

c) $r \approx -0.93$. Yes, r is between –1 and –0.67.

6. a) a weak moderate linear correlation. $r \approx 0.39$



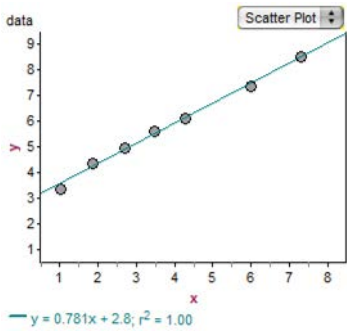
b) a weak linear correlation. $r \approx -0.27$



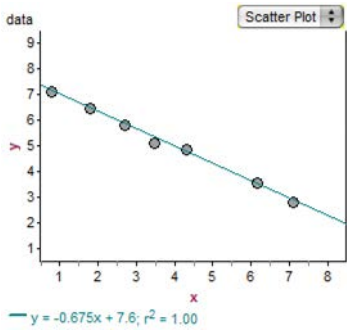
7. Answers may vary.

8. Answers may vary.

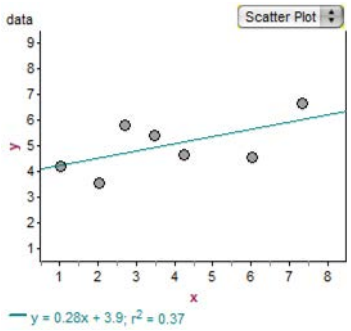
a)



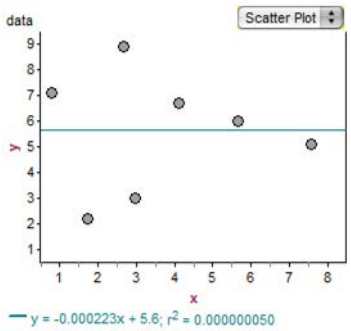
b)



c)

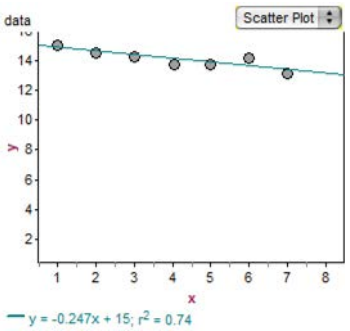


d)



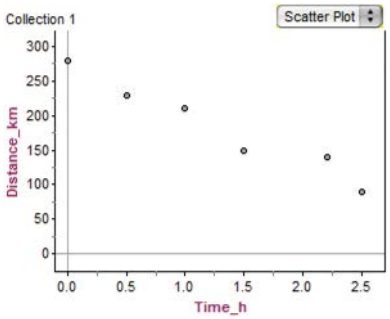
9. Answers may vary.

a)

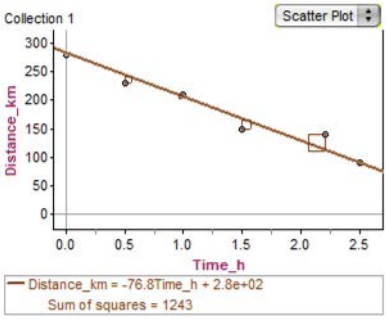


b) The line of best fit starts at the point where $x = 0$ and $y = 15$. The line drops 0.25 on the y-axis for each incremental increase on the x-axis. Because $r^2 = 0.75$, $r \approx -0.866$. There is a strong linear correlation, so the data points are close to the line of best fit.

11. a)

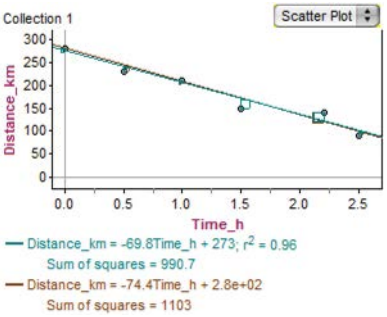


b) and **c)** Several boxes appear. An equation for the line and the sum of squares appear.



d) Estimates may vary. 1103

e) The sum of squares for the line of best fit is 990.7.



Extend

12. Answers may vary. From #5, $r^2 \approx 0.86$. Since the value is fairly close to 1, the choice of linear regression is a pretty good fit. From #6, $r^2 \approx 0.15$. Since the value is much closer to 0 than 1, linear regression is a poor fit.

13. Answers may vary.

a) The coefficient of determination allows you to check the closeness of fit for non-linear regression situations.

b) values from 0 to 1

c) Both coefficients test how well a model fits the data. The correlation coefficient only applies to linear regression, while the coefficient of determination applies to any type of regression curve. The coefficients also take on different values: $-1 \leq r \leq 1$ and $0 \leq r^2 \leq 1$.

8.2 Cause and Effect, pages 392–401

Example 1 Your Turn

a) $r \approx 0.88$, suggesting a strong correlation between number of successful free throws and hours spent practising. Since most coaches agree that long practise hours will result in higher scoring, it is reasonable to characterize this relationship as an example of cause and effect.

b) The value of the correlation coefficient suggests that the line of best fit for these data is relatively good for predicting a team member's free throw performance. The equation relating free throws, n, to hours studied, h, is $n = 1.4h$. The linear model predicts no successful free throws if a team member does not practise at all. The rate of change is 1.4, which means that successful free throws will increase by approximately 1.4 for each additional hour practised.

c) This model suggests that continually increasing practice will result in more than 10 successful attempts out of 10 tries, which is impossible.

Example 2 Your Turn

Answers may vary. It is more likely that a common cause is involved, such as climate change.

Example 3 Your Turn

Answers may vary.

a) cause and effect; Many studies have shown that a patient's stress level can be reduced by exercise.

b) common cause relationship; Both of these variables likely share a positive correlation with strong study habits.

c) accidental relationship; No clear connection exists between pancake sales and the amount of rainfall.

d) presumed relationship; It seems logical that a person from a stable relationship with assumed good communication skills would interview well. It would be difficult to suggest that one causes the other.

Reflect

R1. It does not explain how and why such a correlation exists.

R2. Answers may vary.

Cases of the flu and the amount of severe winter weather most likely have a presumed relationship. Cold weather does not cause the flu.

Cases of the flu and tissue sales most likely have a reverse cause and effect relationship. More tissues are purchased when more people are sick.

The amount of severe winter weather and tissue sales most likely have a common cause effect relationship. More people are sick during the winter and sick people buy more tissues.

R3. Answers may vary.
- **a)** caffeine consumption causes nervousness
- **b)** nervous people are more likely to drink coffee

R4. Answers may vary.
- **a)** The number of females enrolled in undergraduate engineering programs and the number of reality shows on TV both increased for several years.
- **b)** These are likely to be coincidental.

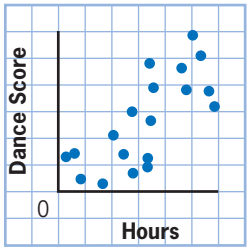
Practise

1. D **2.** B **3.** B

4. a) No. More people at the ski resort most likely results in increased sales of many items, not just hot chocolate.
b) An increase in the number of customers.

Apply

5. a) Answers may vary.



b) Yes; many hours of dedicated practise typically results in better performance.

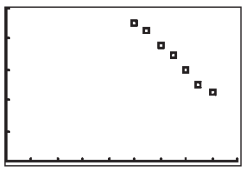
6. Answers may vary.
- **a)** reverse cause and effect: grass growth is directly affected by the amount of rainfall
- **b)** common cause: arm length and leg length are typically related to height
- **c)** accidental: there is no obvious connection between sandwich sales and dog bites
- **d)** presumed: it seems logical that sports would be of interest to fit people but difficult to prove one variable affects the other
- **e)** presumed: it seems logical that cases of diabetes would decrease with healthy eating habits, however this only lowers the risk as other factors also play a role
- **f)** common cause: both heart disease and lung cancer can be tied to smoking

7. a) It is unlikely that eating more fast food would cause a person to sleep less.
b) Answers may vary. The person's overall health.

8. a) No. Ice cream consumption is not likely to cause heat stroke. Heat stroke is generally caused by extremely hot weather.
b) This relation is likely a common cause relationship; incidents of heat stroke and ice cream sales are likely to increase when the weather is hot.

9. a) The deer population sustains the wolf population.
b) Wolf population would decline as its source of food declines.

11. a)

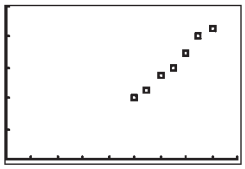


strong negative correlation

b) By definition of demand, the variables are directly related, so it is reasonable to characterize this relationship as likely cause and effect.

c) Because price affects demand, price is the independent variable and demand is the dependent variable.

12. a)

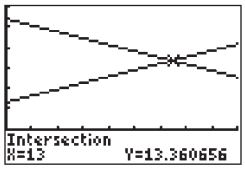


strong positive correlation

b) By definition of supply, the variables are directly related, so it is reasonable to characterize this relationship as likely cause and effect.

c) Because price affects supply, price is the independent variable and supply is the dependent variable.

13. a)

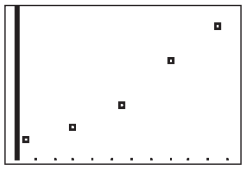


b) (13, 13.36). This shows the quantity for which demand and supply are the same price.

c) Answers may vary. Above $13.36: supply increases but demand drops. Below $13.36: demand increases but supply drops.

Extend

14. a)

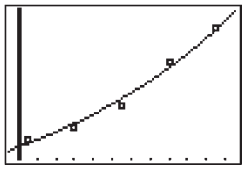


b) Yes. The relationship appears to show a strong positive linear correlation.

c) $r \approx 0.98$, indicating a very good linear fit.

d) The shape of the curve passes very close to the data points.

e) $r^2 \approx 0.99$, indicating that this is a better model for the data.



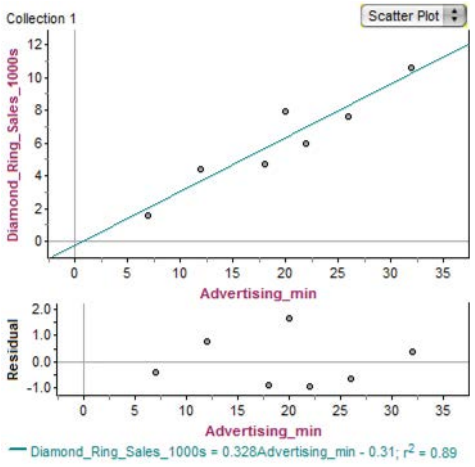
f) As new housing developments are constructed, the population increases at a faster rate.

15. Answers may vary.

8.3 Dynamic Analysis of Two-Variable Data, pages 402–415

Example 1 Your Turn

a) and **c)**



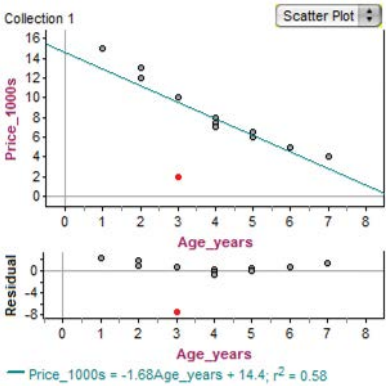
As advertising increases, diamond ring sales increase. $r = \sqrt{0.89} \approx 0.94$ confirms a strong positive correlation. A linear model is appropriate.

b) For (20, 7.9), the residual is 1.65, which means that the actual duration of advertising is 1.65 min greater than what the linear model predicts.
For (18, 4.7), the residual is –0.894, which means that the actual duration of advertising is 0.894 min less than what the linear model predicts.

c) There is no clear pattern in the residual plot.

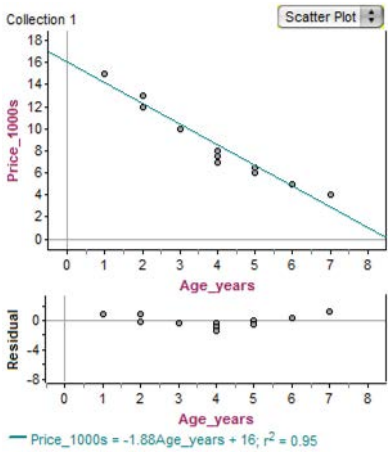
Example 2 Your Turn

a)–c)



This line of best fit is not a good model for the data. The extrapolated vertical intercept suggests around $15 500 for a brand-new motorcycle, which does not sound right if a 1-year bike is $15 000. The correlational coefficient, $r = -\sqrt{0.58} \approx -0.76$, suggests a moderately strong linear correlation, but, except for one point, (3, 2), the data appear to form a much stronger linear trend.
The residual of (3, 2) is much farther from the residual line than the other residuals which appear to have a pattern. This suggests that the outlier has a strong influence on the linear model.

d)



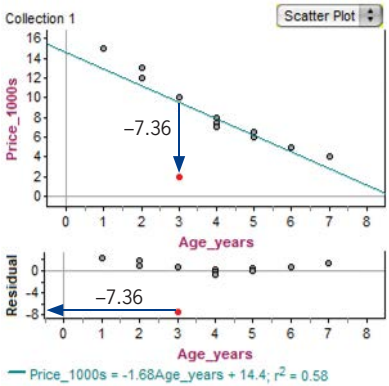
e) The original linear model was influenced by the outlier. After the outlier was removed from the analysis, the linear model appeared to be quite strong.

Example 3 Your Turn

Answers may vary. No. Looking at the data points from 1996 to 2006, only (2005, 6.4) appears to be a possible outlier. If the paternity leave had a significant effect, I would expect the correlation coefficient to be closer to the low end of the moderate range.

Reflect

R1. Answers may vary. A residual plot shows the value of each residual graphically as the vertical distance from a horizontal axis.



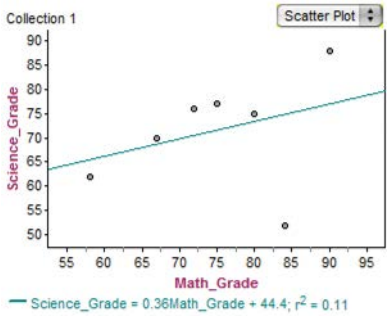
R2. a) A data point that does not fit well in an otherwise linear trend.

b) In a scatter plot, the outlier is relatively far from the line of best fit. In a residual plot, an outlier is either relatively far above or below the horizontal line.

Practise

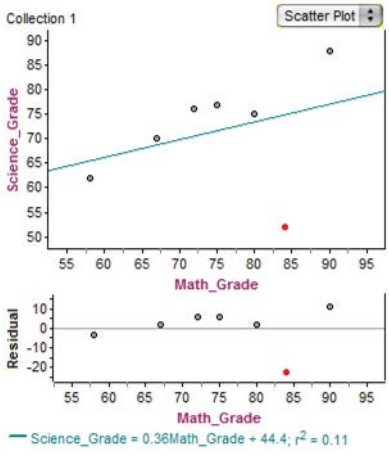
1. C **2.** D **3.** C

4. a)



b) No. $r \approx 0.33$, which suggests weak linear correlation, but, except for one point, (84, 52), the data appear to form a much stronger linear trend.

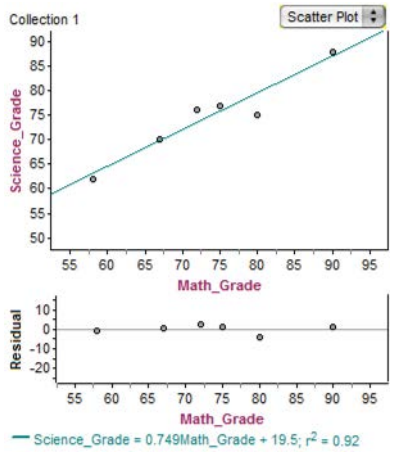
5. a)



b) –22.64.

c) It is much farther from the residual line than the other residuals.

6. a) This line of best fit appears to serve as a very good model for the data. $r \approx 0.96$, which suggests a strong linear correlation.



This appears to be a strong linear model for predicting the science grade based on a math grade.

b) The original linear model was influenced by an outlier. After the outlier was removed from the analysis, the linear model appeared to be quite strong.

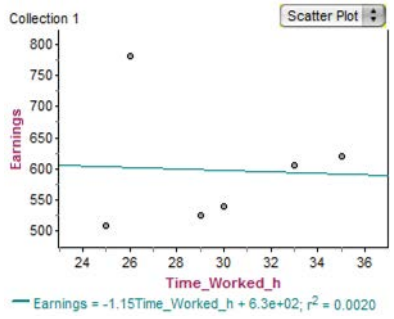
Apply

7. a) 40%; all of Jonathon's other marks were in the 80s.

b) mean: 74%, median: 83%, mode: 83%

c) In this case, removing the outlier only impacted the mean, which is now 83%, the same as the median and the mode.

8. a) and **b)** With the exception of one point, (26, 780), the data appear to have a strong positive linear correlation.

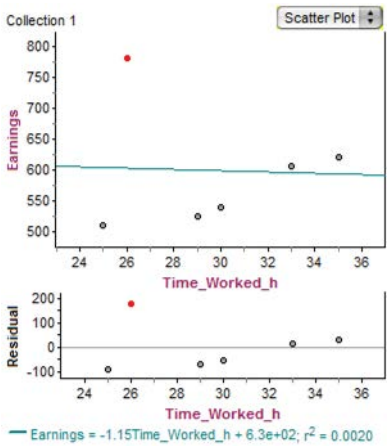


b) $r \approx -0.04$, which suggests a very weak linear correlation.

The equation of the line of best fit is $e = -1.15t + 630$. The equation shows that earnings start at \$630 and decrease at a rate of \$1.15 per hour worked.

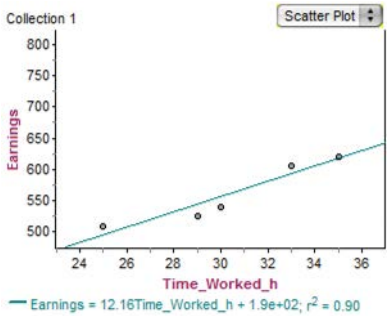
c) No. This model makes no sense and is unrealistic.

9. a)



b) The residual of (26, 780) is much farther from the residual line than the other residuals. This suggests that the outlier has a strong influence on the linear model. This unusual data point could be caused by overly generous tippers.

c) This line of best fit appears to serve as a very good model for the data. $r \approx 0.95$, which suggests a strong linear correlation.

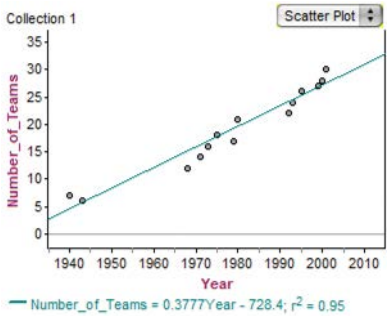


The equation of the line of best fit is $e = 12.16t + 190$. The equation shows that earnings start at \$190 and increase at a rate of \$12.16 per hour worked. This is a useful linear model that is realistic.

10. Answers may vary.

11. a) An overall negative trend over the 50-year period.

b) Answers may vary. There are two distinct trends, between 1940 and 1943 and between 1968 and 2001, with the first being negative and the second positive.



From 1968 on, the league added teams. The expansions of the league could be a hidden variable that might account for the correlation shown in the Montréal Canadiens Stanley Cup Wins graph.

12. Answers may vary. While a lockout cancelled the entire 2004–05 NHL season, it is only one season in the decade and may not invalidate the general trend shown in this study.

13. Answers may vary. Yes. During the 50s and 60s, the Canadiens competed in a league of six teams. However, in the 1970s they competed in a league ranging from 12 to 18 teams and managed to win the greatest number of Stanley Cups during that time.

Extend

15. a) Any points above the line of best fit will have a positive residual and any below the line will have a negative residual.

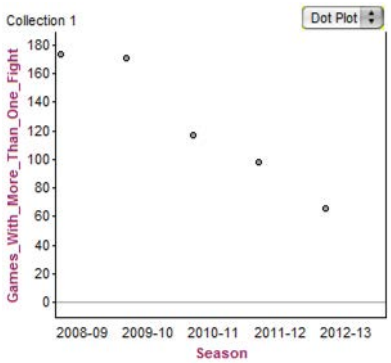
b) 0. By definition, the line of best fit typically has a number of points above and below it, so the positive error will balance out the negative error, resulting in a sum of 0.

16. Answers may vary.

8.4 Uses and Misuses of Data, pages 416–423

Example 1 Your Turn

a)



b) There is an outlier (2012-13, 66) due to a hidden variable. Only 720 games were played during this season due to a labour disruption. It should be removed because it exaggerates the downward trend.

c) Sample size is another source of bias. Only five points were chosen, perhaps to hide that the trend is less downward over a longer period of time.

Example 2 Your Turn

Answers may vary. No. This could be a typical percent of UFO sightings for Ontario, since approximately 40% of the population of Canada resides in Ontario.

Reflect

R1. Answers may vary.

a) choice of scales or titles on a graph, sample size, use of outliers, making inappropriate conclusions

b) to grab the attention of the viewer, for humour, to sway opinion, or to exaggerate a point

R2. Answers may vary.

a) To get more sales.

b) how the data were collected, the sample size, and what exactly "twice as popular" means

R3. Answers may vary.

a) Sensationalism is a type of bias where a piece of work uses tactics such as over-exaggeration to provoke an emotional response.

b) to provoke controversy or discussion or to gain attention

Practise

1. D **2.** A **3.** B

4. a) not reliable

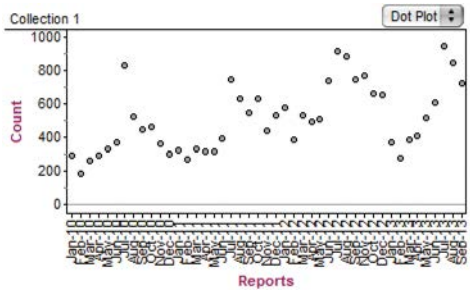
b) the sample size, whether there was any bias in the survey of sampling technique.

c) to sell more tabloid newspapers

Apply

5. a) (Oct-13, 34). It should be excluded from the analysis since it does not include the whole month of October's data.

b)



c) Answers may vary. No, it appears to rise and fall with some regularity each year.

6. a) Sightings reach a maximum in the summer months and a minimum in the winter months.

b) Answers may vary. No. People probably spend more time outside in the evenings during the summer months than in the winter months.

7. Answers may vary.

a) Yes. There may be voluntary response bias as those who claim sightings are more likely to believe in extra-terrestrial beings and therefore more likely to misinterpret a normal event as a UFO sighting.

b) Is the sample representative of the population? What type of sampling technique and data collections methods were used? Are the data verified?

Extend

9. Answers may vary.

a) and **b)** Voter turnout in Australia seems very consistent at just over 80%. This is due to a compulsory voting system in which electors are obliged to vote. Voter turnout for Canada appears to be declining slightly. According to the Conference Board of Canada, this may be due to lower participation of young people. Voter turnout for Switzerland appears to remain at about the 40% mark. The Swiss government is highly decentralized with limited powers, and referendums for important decisions are common, so individual votes for the federal legislature are less likely to have a significant effect on the nation.

10. Answers may vary.

8.5 Advanced Techniques for Data Analysis, pages 424–433

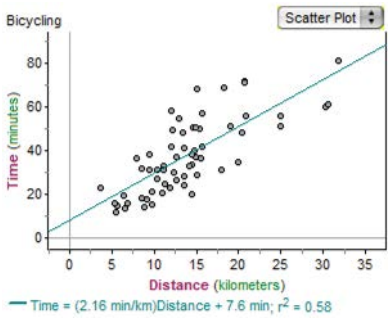
Example 1 Your Turn

Row Labels	Average of Heart Rate 1 Min After Exercise (bpm)	Average of Heart Rate 30 Min After Exercise (bpm)
A	118.6	77.6
B	131.5	91.33333333
Grand Total	125.6363636	85.09090909

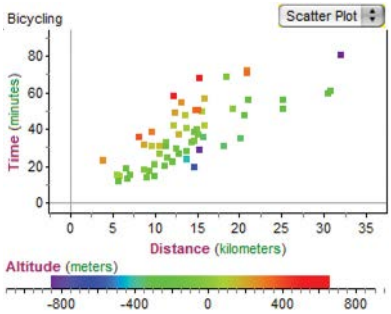
Both the pivot table and chart show that marathon runners had a lower average heart rate 1 min after exercise than non-runners, with an average of 118.6 bpm compared to 131.5 bpm. They also show that marathon runners had a lower average heart rate 30 min after exercise than non-runners with an average of 77.6 bpm compared to 91.3 bpm.

Example 2 Your Turn

a) There appears to be a fairly strong linear correlation between run time and distance. $r \approx 0.76$, which suggests a mildly strong linear correlation.



The altitude attribute scale confirms that altitude is obscuring the linear correlation between run time and distance. The uphill runs tend to have longer run times than the downhill runs.



b) With the exception of (18.2 km, 69 min, –60 m) and (31.8 km, 81 min, –860 m), the results are consistent with those in the example. Perhaps something occurred on those two runs to cause the unusual times, like an accident, construction, or flat tire.

Reflect

R1. a) a table which subdivides data into two or more categories

b) for comparing quantitative data across different categories

R2. a) to compare aggregate resting heart rate and maximum heart rate scores

b) Group A had both a lower resting heart and a lower working heart rate than Group B.

R3. They showed the influence of a third variable by confirming that change in altitude was obscuring the linear correlation between run time and distance.

R4. Answers may vary.

Practise

1. B **2.** C **3.** C

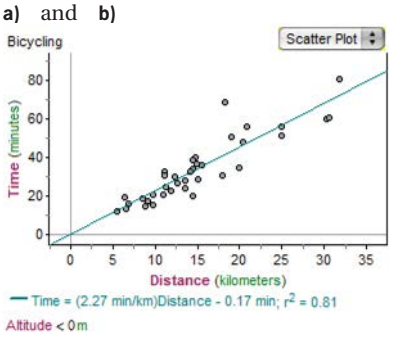
4. males: about 2.15 h, females: about 2.4 h

Apply

5. standard deviation [StdDev()], standard error [StdError()], and the count for the number of cases for which a value of a specified attribute is missing [count(missing())]

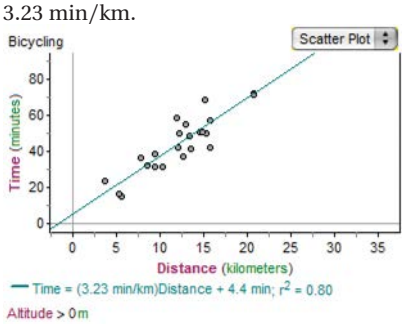
6. minimum value [min()], first quartile [Q1()], median [median()], third quartile [Q3()], and maximum value [max()]

7. a) and **b)**



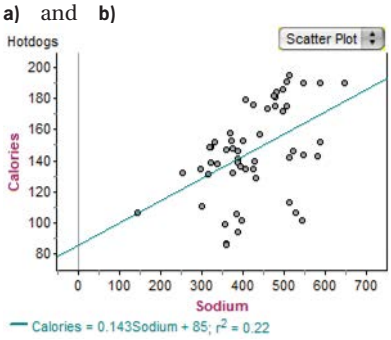
b) $r = 0.9$, which suggests a strong linear correlation. The equation of the line of best fit is $t = 2.27d - 0.17$. The equation shows that time starts at –0.17 min, which makes no sense, and increases at a rate of 2.27 min/km.

8. a) and **b)** $r \approx 0.89$, which suggests a strong linear correlation. The equation of the line of best fit is $t = 3.23d + 4.4$. The equation shows that time starts at 4.4 min and increases at a rate of 3.23 min/km.



b) While the two linear models differ, it makes sense that the rate for the downhill runs (2.27 min/km) would be less than the rate for the uphill runs (3.23 min/km).

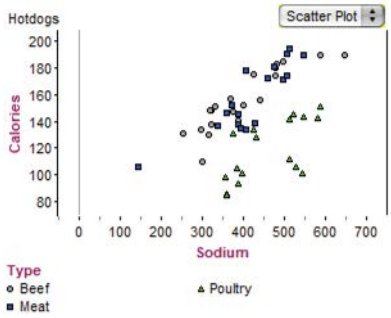
9. a) and **b)**



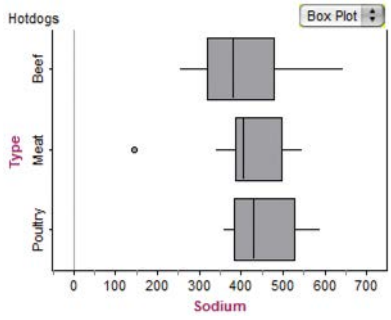
$r \approx 0.47$, which suggests a barely moderate linear correlation.

c) beef: $r \approx 0.89$, which suggests a fairly strong linear correlation.
meat: $r \approx 0.87$, which suggests a fairly strong linear correlation.
poultry: $r \approx 0.66$, which suggests a solid moderate linear correlation.
Each of the individual meat types has an improved linear correlation compared to the original model.

d) Yes. This can be confirmed with the legend attribute.



10. a)



b)

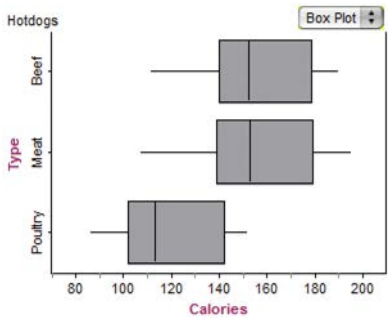
Hotdogs

Type	Sodium
Beef	401.15 20
Meat	418.529 17
Poultry	459 17
Column Summary	424.833 54

S1 = mean ( )
S2 = count ( )

Beef hotdogs have the lowest mean sodium of about 401 mg compared to about 418 mg for meat and 459 mg for poultry.

11. a)



b)

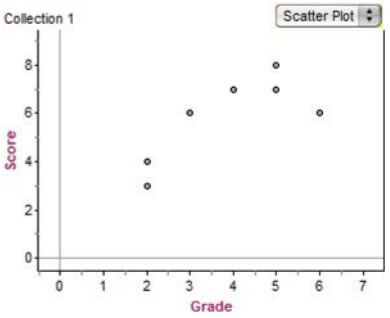
Hotdogs

Type	Calories
Beef	156.85 20
Meat	158.706 17
Poultry	118.765 17
Column Summary	145.444 54

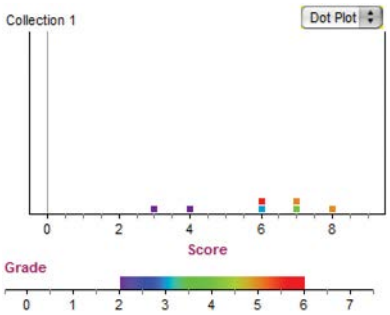
S1 = mean ( )
S2 = count ( )

Poultry hotdogs have the lowest mean calories of about 119 calories per hotdog compared to about 157 calories per beef hotdog and about 159 calories per meat hotdog.

13. a) The correlation appears to be a strong positive one.



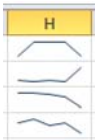
b)



c) The colour trend in the dot plot shows the correlation between grade and score; the purple and blue dots represent younger grades and they tend to have lower scores than the orange and red dots that represent older grades.

Extend

14. a)



b) A tiny line graph representing each row of data appears.

15. a) data trends

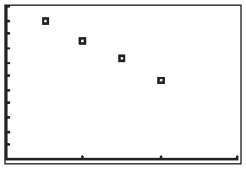
b) Answers may vary. They do not include actual data values, so it is difficult to compare with other data sets.

c) The sparkline for the Green party distorts the data. These mini-graphs are each absolute, not relative.

Chapter 8 Review, pages 434–435

1. graph on the left: –0.97; graph on the right: 0.56

2. a)



b) strong negative correlation between distance and time

c) $r \approx -0.999$, which suggests a strong linear correlation. The equation of the line of best fit is $d = -145t + 572.5$. The equation shows that the initial distance from home was 572.5 km and the distance decreases at a rate of –145 km/h.

3. a) As self-esteem increases, so does the level of achievement.

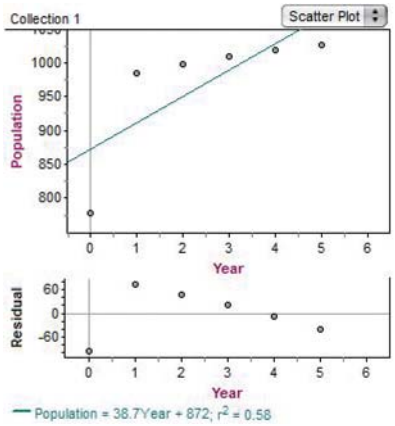
b) As the level of achievement increases, so does self-esteem.

4. a) reverse cause: it is more likely that computer sales drop with increased unemployment

b) accidental: there is no obvious connection between the price of gas and the performance of a football team

c) cause and effect: heart rate is directly affected by running speed

5. a)–c)



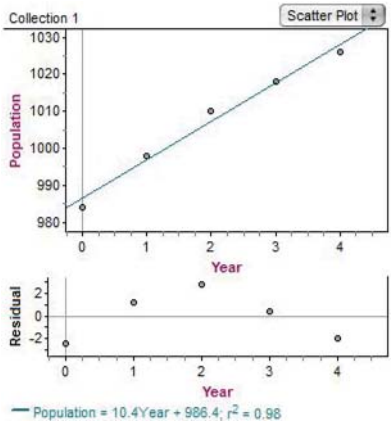
With the exception of (0, 778), the data appear to have a strong positive linear correlation.

b) $r \approx 0.76$, which suggests a mildly strong linear correlation. The equation of the line of best fit is $p = 38.7y - 872$. The equation shows that the population started in year zero at about 872, and increased at a rate of 38.7 students per year. This is not very accurate and is likely due to the outlier at (0, 778).

c) The residual of (0, 778) is much farther from the residual line than the other residuals and the other points form a pattern. This suggests that this is not a good linear model.

6. a) An outlier can have a strong impact on a linear regression model if the number of data points is relatively small. The outlier is due to a hidden variable, only three grade levels. It should be removed because it exaggerates the upward trend.

b) This line of best fit appears to serve as a very good model for the data. $r \approx 0.99$ which suggests strong linear correlation.



The equation of the line of best fit is $p = 10.4y + 986.4$. The equation shows that the population started in year zero at about 986, and increased at a rate of 10.4 students per year. This is an accurate reflection of the school's initial population and rate of growth.

There is no obvious pattern to the residuals and none appear overly far from the residual line. This appears to be a strong linear model for predicting the population based on year.

c) original model: about 1220 students;
corrected model: about 1080 students
The principal should rely on the corrected model because it is more accurate.

7. a) a general downward trend between circulation andyear

b) Yes, there are two distinct trends, from 2005 to 2008 and from 2009 to 2012, which suggests a hidden variable.

c) In 2009, and that is what likely caused the sudden drop in circulation.

d) The price change is what caused the fragmented trend.

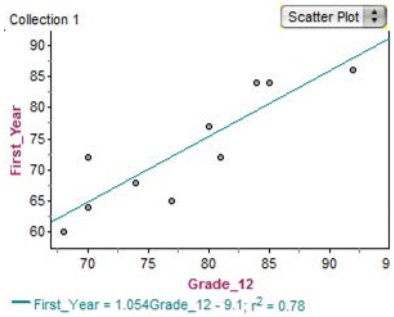
8. Since the vertical scale does not start at 0, it exaggerates the downward trend.

9. Answers may vary.

a) The language is not neutral.

b) "Newspaper circulation declines"

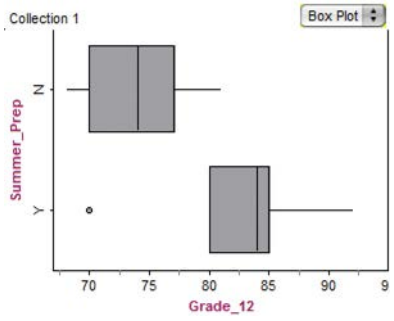
10. a) and **b)**

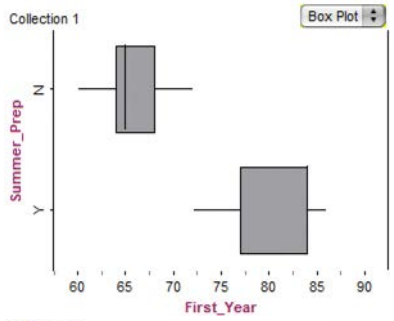


$r \approx 0.88$, which suggests a strong linear correlation.

c) Answers may vary. It is unclear whether the summer prep course was helpful. It is most likely a hidden variable that is obscuring the linear correlation.

11. a)





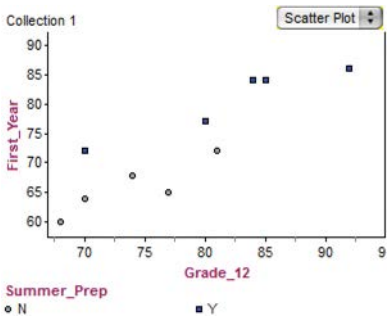
Collection 1

		Grade_12	First_Year
Summer_Prep	N	74 5	65.8 5
	Y	82.2 5	80.6 5
Column Summary		78.1 10	73.2 10

S1 = mean ( )
S2 = count ( )

b) Students who took the summer prep course had higher mean grade 12 calculus marks of about 82 compared to a mean of 74 for students that did not take the summer prep course. Students that took the summer prep course had higher mean first year university marks, with a mean of about 81 compared to a mean of about 66 for students that did not take the summer prep course, so the summer prep course seems to have been helpful.

12. a)



b) The summer prep attribute scale confirms that students who took the summer prep course had higher marks than those who did not. This confirms that the summer prep course is helpful.

Chapter 8 Test Yourself, pages 436–437

1. a) A **b)** D

2. D **3.** D

4. Answers may vary.

a) common cause: snow tire sales and hot chocolate sales are typically related to winter

b) reverse cause and effect: it is more likely that box office sales drop with increased ticket prices

c) presumed: it seems logical that height can play a role in driving safely but difficult to prove one variable affects the other

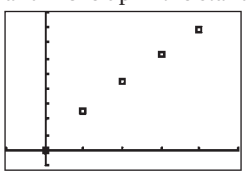
d) accidental: there is no obvious connection between cheeseburger sales and pita sales

5. Answers may vary.

a) As a team's position in the league standings increases, so does attendance at games. People like to support and watch winning teams, so the better the team, the better the support by fans willing to attend games.

b) As attendance at games increases, so does a team's position in the league standings. As the team gets more support and encouragement, they improve and move up in the standings.

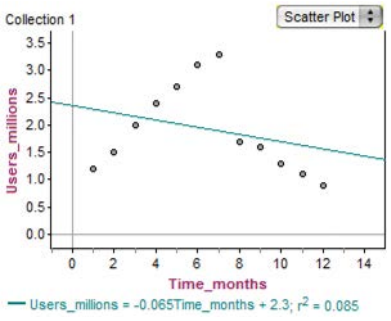
6. a)



a strong positive correlation between speed and time

b) $r \approx 0.996$. The equation of the line of best fit is $s = 3.91t + 0.66$. The equation shows the skydiver's speed started as 0.66 m/s and increased at a rate of 3.91 m/s.

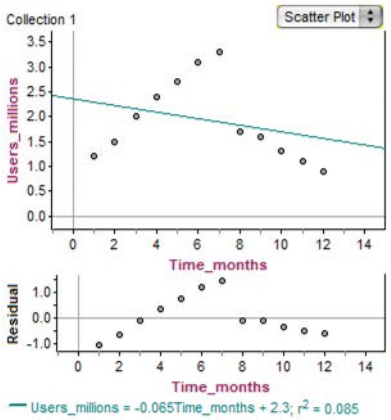
7. a) and **b)**



There are two separate trends, one upward and one downward.

b) $r \approx 0.29$, which suggests a fairly weak linear correlation. The equation of the line of best fit is $u = -0.065t + 2.3$. The equation shows that the number of users started at 2 300 000 and decreased at a rate of –0.065 million users per month.

c)

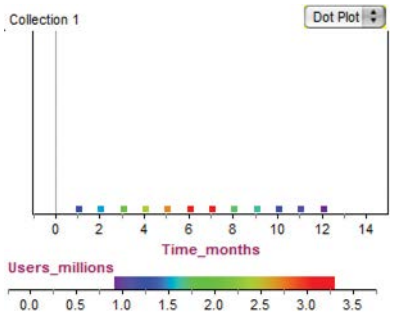


The residuals form two linear patterns which suggests that this is not a good linear model.

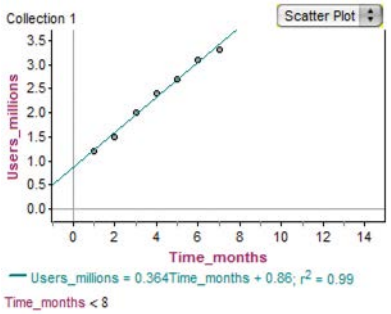
d) The fragmented trend is evidence of a hidden variable. Something happened in month 8.

e) The website started charging a fee in month 8. That is what caused the break in the trend, and changed it from positive to negative.

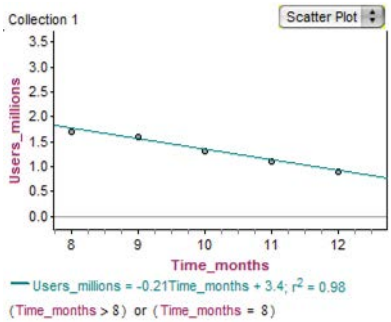
f)



g) The equation of the line of best fit for the positive trend is $u = 0.364t + 0.86$ shows that the number of users started at 860 000 and increased at a rate of 0.86 million users per month. Analyse the sets separately.



The line of best fit for the latter negative trend, $u = -0.21t + 3.4$, indicates that the social networking site is headed towards no users within another five months.



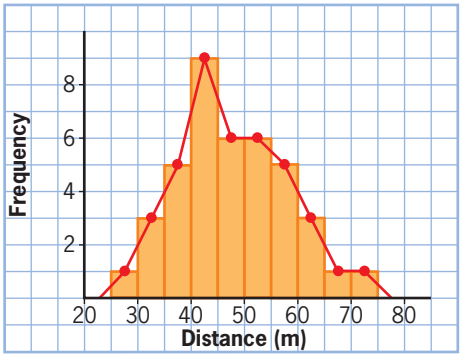
The most common way for social networking sites to make money is by allowing companies to advertise on their site. If the website wants to grow its user base, I think they should drop the fee. Then, the first positive trend can be used to predict the future popularity of this website, assuming that the users return.

Chapters 7 and 8 Cumulative Review, pages 438–439

1. a) Yes. **b)** about 0.1 **c)** 0.5

d) Answers may vary. I expect the distribution to no longer be uniform if the general population was measured due to the wide range of wellness.

2. a) and **b)**



c) around 42.5 m

d)

Distance (m)	Frequency	Relative Frequency
25–30	1	0.025
30–35	3	0.075
35–40	5	0.125
40–45	9	0.225
45–50	6	0.150
50–55	6	0.150
55–60	5	0.125
60–65	3	0.075
65–70	1	0.025
70–75	1	0.025

e) 0.125

f) Answers may vary. The entire distribution would move to the left. This would be due to the reduced skill level in throwing a discus.

3. a) 0.62%

b) 10.56%

c) Answers may vary. I would expect the entire distribution to move to the right. The mean would increase as well as the standard deviation. This

would be due to the reduced skill level in building airplane kits.

4. a) 3.7% to 7.9%

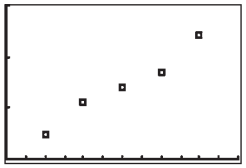
b) Answers may vary. 5.8% of industrial wind turbines will not meet the standard. This estimate is considered correct within ±2.1%, 99 times out of 100.

5. a) Since np and nq are both greater than 5, it is reasonable to model this distribution using a normal approximation.

b) mean: 30, standard deviation: about 5.422

c) about 0.04.

d) Answers may vary. Since the probability is higher, the company is not justified in its new claim.

6. a)  a strong positive correlation

b) $r \approx 0.987$. The equation of the line of best fit is $d = 0.044t + 1$. The equation shows the hiker started 1 km from camp and hiked at a rate of 0.044 km/min away from camp.

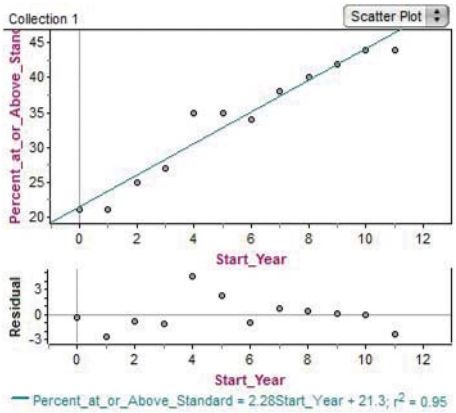
7. Answers may vary.

a) No. It seems unlikely that a higher income level would directly decrease absenteeism. Just because you earn more does not mean you work more days. Both of these variables likely share a negative correlation with an external variable.

b) Possible common causes that explain this relationship are job satisfaction, financial wellness, or family factors.

8. a) accidental: there is no obvious connection between automobile sales and rainfall

b) cause and effect: long practise hours typically will result in fewer musical errors

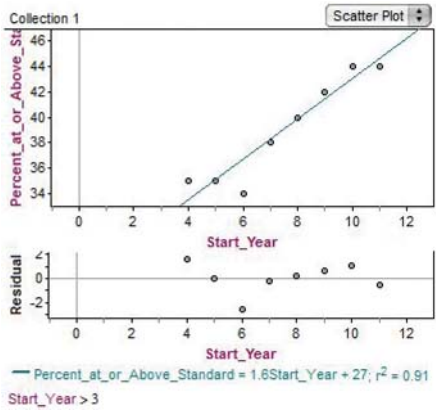
9. a)–c) 

There are two separate trends, both upward, between the percent achieving at or above provincial standard and the year.

b) $r \approx 0.97$ which suggests a strong linear correlation. The equation of the line of best fit is $p = 2.28y - 21.3$. The equation shows that the percent achieving at or above provincial standard started at 21.3 in year zero and increased at a rate of 2.28 percent per year.

c) The residuals in the middle seem to form a linear pattern. This suggests that this is not a good linear model and that there is possible evidence of a hidden variable.

d) Yes. The curriculum perhaps became better aligned with the content of the exam.

e) 

The correlational coefficient, $r \approx 0.95$, still suggests a strong linear correlation. The equation of the line of best fit is $p = 1.6y + 27$. The equation shows that the percent achieving at or above provincial standard started at 27 and increased at a rate of 1.6 percent per year. There is no obvious pattern to the residuals and none appear overly far from the residual line.

f) From the residual plot, this appears to be a stronger linear model for predicting the percent achieving at or above provincial standard based on time, in years.

10. a) There is bias in the title that is not neutral, the sample size is only five seasons, and the vertical scale does not start at 0.

b) Since the graph exaggerates Smyth's skills, it was made by Smyth's agent, most likely in hopes of getting his client a better contract.

c) Bias can be removed by using a neutral title and starting the vertical axis at 0.

11. Answers may vary.