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MHF4U1 Test #1: Graphs of Polynomials

K & U: 21.5/24 **APP:** 13 /15 **Comm:** 8 /9 **TIPS:** 4.5/9

Part A: Knowledge and Understanding. [24 marks]

1. Fill in the blanks. [8 marks]

a) What is the constant finite difference for $g(x) = -\frac{1}{2}x^4 + 3x^2 - 9x^2 + 3$? (✓) -12

b) The $f(x) = 3x^5 - 2x^3 + 7x$ can be classified as an odd function since it shows symmetry around the origin (0,0) (✓)

c) Express the domain and range for $y = -3\left(\frac{1}{2}x + 4\right)^6 - 15$ in interval/bracket notation. (✓✓✓)

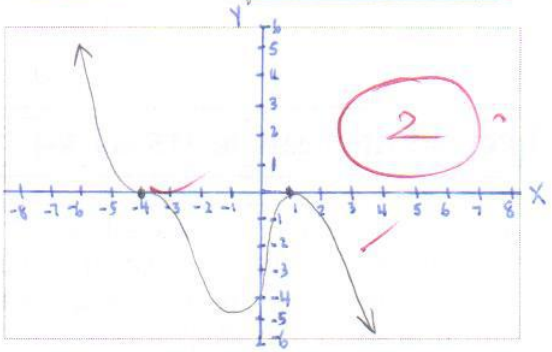
DOMAIN: $x \in (-\infty, \infty)$ RANGE: $y \in (-\infty, -15]$ specify.

d) State the relationship between the degree and the number of "turns" in a polynomial function. (✓)

the number of degree -1 is how many ∧ hills/valleys there are. max

e) The end behaviour of even degree power functions will extend from ... (✓) quadrant 2 → Quadrant 1

f) Sketch an odd degree polynomial function with a lead coefficient that is negative, and roots of −4 (order 3) and 1 (order 2). (✓✓)
... label and scale the appropriate axis!



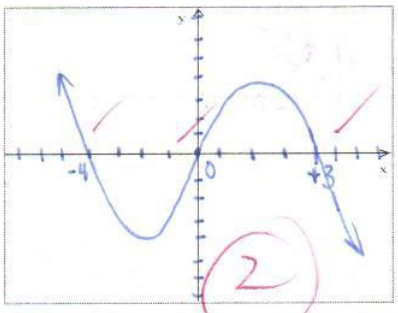
2. Given the equations below, state the indicated characteristics. [6 marks]

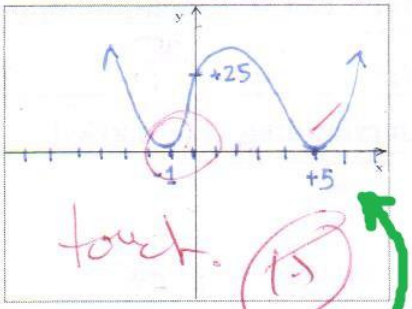
	$y = -3(x+6)^2(1-x)^3(5-x)$	$y = -\frac{1}{4}\left[\frac{1}{2}(x+4)\right]^4 + 13$
a) the name of the polynomial (✓✓)	sextic function	quartic function
b) the vertex (✓)		(-4, 13)
c) the end behaviour (✓)	extends from Q3 to Q4	
d) the x-intercept(s) (✓)	$x =$ -6 (order 2) +1 (order 3) and +5	
e) the y-intercept (✓)		y = -311

13

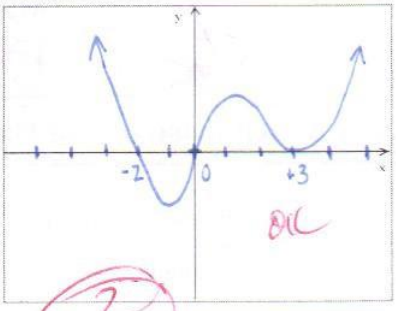
3. Sketch the graphs of the following polynomial functions. [6 marks]

a) $y = -x(x+4)(x-3)$ b) $y = (x+1)^2(x-5)^2$ c) $y = x(2+x)(3-x)^2$





touch.



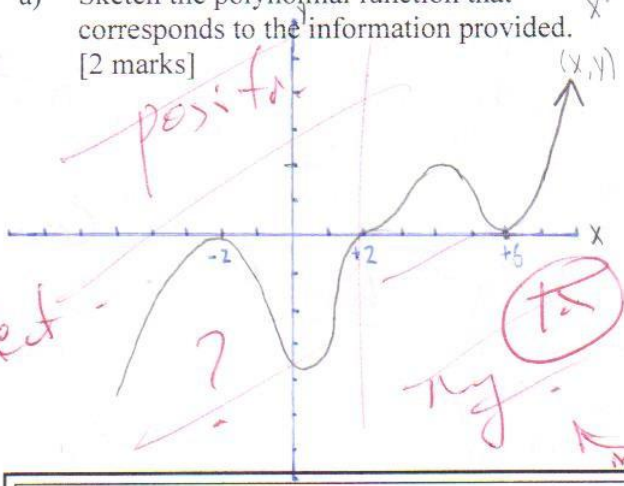
BAC

2

He took off 0.5 bc I didnt touch the line...

4. Given the following conditions: $y > 0$ when $x \in (-\infty, 2)$ and $y < 0$ when $x \in (2, \infty)$; the zeros are at -2, 2 and 6, with one of these being of order 3, the others of order 2. Do the following:

a) Sketch the polynomial function that corresponds to the information provided. [2 marks]



positive

reflect.

7

Try.

b) What is the equation for this family of polynomials, and what is the sign on 'a'? [2 marks]

$y = a(x+2)^2(x-2)^3(x-6)^2$

sub $(0,-4)$ cont.

$-4 = a(0+2)^2(0-2)^3(0-6)^2$

$= a(4)(-8)(36)$

$a = -\frac{1}{8}$

'a' cannot be determined.

$\therefore$ equation is $y = -\frac{1}{8}(x+2)^2(x-2)^3(x-6)^2$

inconsistent

Part B: Application. [15 marks]

5. The roots of a function are -4, 3, & 5 (all of order one) and 1 (order two). If the function passes through the point $(2, -12)$, determine its equation. [3 marks]

$y = a(x+4)(x-3)(x-5)(x-1)^2$

sub in $(2,-12)$

$-12 = a(2+4)(2-3)(2-5)(2-1)^2$

$-12 = 18a$

$a = -\frac{12}{18}$

$= -\frac{2}{3}$

3

therefore, the equation is $y = -\frac{2}{3}(x+4)(x-3)(x-5)(x-1)^2$

6. The table below shows the height of a ball in metres, thrown in the air, at various times, in seconds. Estimate what the instantaneous velocity is at 5.5 s. [3 marks]

t	$h(t)$
4	125
4.5	123.75
5	120
5.5	113.75
6	105
6.5	93.75

(5.5s, 113.75m)

(5.50001s, 113.750206 8m)

How?

You don't have the defining equation

$\vec{V}_{in@5.5s} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{113.750206\,8\text{m} - 113.75\text{m}}{5.50001\text{s} - 5.5\text{s}}$

$= 20.68\ldots$ m/s

$\doteq 20.7$ m/s ?

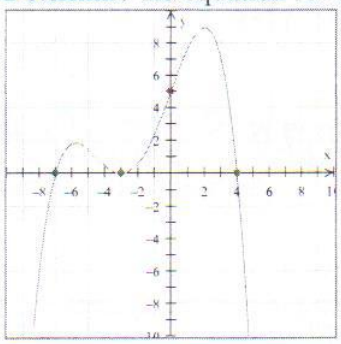
1

Therefore, the instantaneous velocity of the ball at 5.5s is about 20.7 m/s

4

8.5

7. Determine the equation for the function given by the graph below: [4 marks]



x-ints are $-7, -3$ (order 2), $+4$

y-int is $+5 \rightarrow (0, +5)$

$y = a(x+7)(x+3)^2(x-4)$

sub in $(0, +5)$

$5 = a(0+7)(0+3)^2(0-4)$

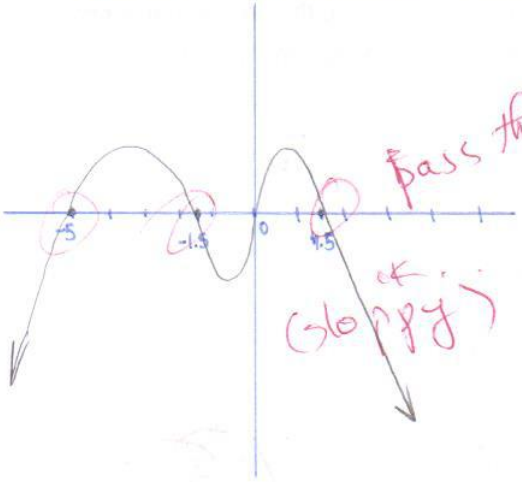
$5 = -252a$

$a = -\frac{5}{252}$ sub back into

Therefore, the equation of this function is $y = -\frac{5}{252}(x+7)(x+3)^2(x-4)$

4

8. Sketch in the space below, the function $y = -4x^4 - 20x^3 + 9x^2 + 45x$. Show calculations to support how you determined the various aspects of the graph. [5 marks]



$y = -4x^4 - 20x^3 + 9x^2 + 45x$

$= -4x^3(x+5) + 9x(x+5)$

$= (-4x^3 + 9x)(x+5)$

$= -x(4x^2 - 9)(x+5)$

$= -x(2x+3)(2x-3)(x+5)$

x-ints are $0, -1.5, +1.5, -5$

find y-int

$y = -(0)(2(0)+3)(2(0)-3)((0)+5)$

$= 0$

- 4th degree function
- quartic function
- leading coef (-4)
- end behavior Q3 → Q4

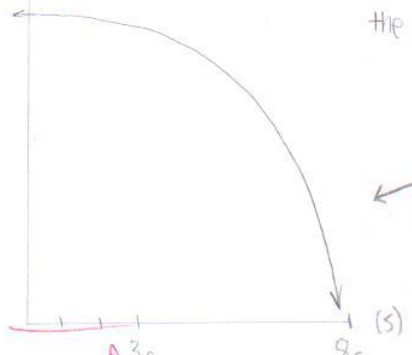
3

9

Part C: Communication. [9 marks]

9. On a given position-time graph modelled by a polynomial function, the slopes of tangents were taken within an interval from $t = 3\ s$ to $t = 8\ s$. These values were $-\frac{1}{3}, -\frac{3}{5}, -\frac{5}{2}, -6, -10$ and -38, measured from left to right. What would the curve within this function look like? Describe, using terminology from this unit to explain how you would know. [4 marks]

(m)



the slopes are $-\frac{1}{3}$ $-\frac{3}{5}$ $-\frac{5}{2}$ -6 -10 between 3s to 8s. the slopes are getting more and more steep. looks like a parabola upside down

rate of change is increasing decreasing but it is pointed downwards because of negative sign

the object is speeding up towards the starting point.

→ tangent lines are getting steeper ... slope values are decreasing

(sign and trend?)

3

10. Name the function, then describe its transformations, if given $y = 2\left[-\frac{3}{2}x + 6\right]^5 - 11$. [5 marks]

$= 2\left[-\frac{3}{2}(x-4)\right]^5 - 11$

- vertically stretch the quintic function by factor of 2
- horizontally compress it by factor of $\frac{2}{3}$
- reflect in y axis
- horizontally translate it 4 units right
- vertically translate it 11 units downwards

Complete sentences not point form.

5

Part D: Thinking, Inquiry and Problem Solving. [9 marks]

11. The function $f(x) = (x-3)(x+4)(x+1) - 6$ undergoes the following successive transformations: a translation the 2 units to the left, a reflection in the x-axis, then a translation by 3 units upward, and followed by a vertical compression by a factor of $\frac{2}{3}$. Determine a simplified equation to represent this transformed function. [4 marks]

$f(x) = (x-1)(x+6)(x+3) - 6$ (2 units left) changes as well

$f(x) = -(x-1)(x+6)(x+3) - 6$ (reflection in x axis)

$f(x) = -(x-1)(x+6)(x+3) - 3$ (3 units upward) OK.

$f(x) = -\frac{2}{3}(x-1)(x+6)(x+3) - 3$ (Vertical compression by factor of $\frac{2}{3}$) X

Therefore, the equation to represent this transformed function is $f(x) = -\frac{2}{3}(x-1)(x+6)(x+3) - 3$.

2.5

12. Determine the equation of the polynomial function given some of its points in the table below. [5 marks]

x	$f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
−1	−17			
0	−4	>13	> −10	>12
1	−1	>3	> 2	>12
2	4	>5	> 14	>12
3	23	>19	> 26	
4	68	>45		

3rd dif

$\Delta^4 f(x) = a(4!)$?

$12 = a(24)$ X

$a = \frac{1}{2}$ X

y-int $(0, -4)$ ✓

Therefore, the equation of the function is $y = \frac{1}{2}(x^4) - 4$ X

1.5

4.5