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KU: 15 / 19 TH: 10.5 / 11 COMM: 4+3 / 7 APPS: 16/16 Date: May 13 2015

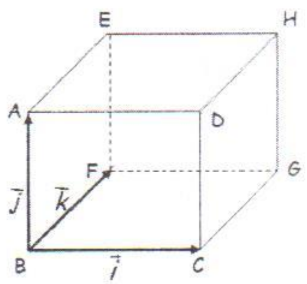
92%

UNIT 5 TEST: Geometric Vectors

Instructions:
1. Read the instructions carefully.
2. Attempt all problems and don't spend all your time on one problem. Other problems will feel left out ☹
3. Show all work in the space provided. Three (3) marks will be given to the overall mathematical form on this test.
4. **Round all magnitudes to two (2) decimal places when necessary and round all angles to the nearest angle.**
5. ENJOY ☺

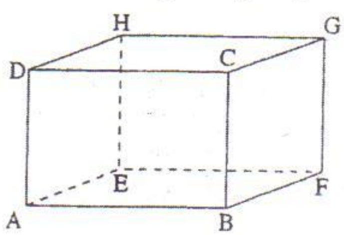
PART A: KNOWLEDGE AND UNDERSTANDING

1. Given the following rectangular prism, identify each of the following as a single vector (use the letters of the vertices to name them). [4]



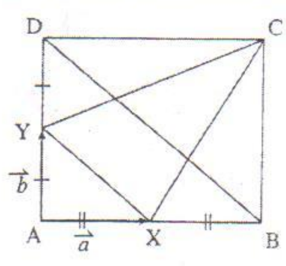
a) $\vec{i}+\vec{j}+\vec{k}$	$\overrightarrow{BH}$ ✓
b) $\vec{k}-\vec{i}+\vec{j}$	$\overrightarrow{CE}$ ✓
c) $\vec{k}-(\vec{i}+\vec{j})$	$\overrightarrow{DF}$ ✓
d) $(\vec{i}+\vec{j})-\vec{k}$	$\overrightarrow{FD}$ ✓

2. Given the following rectangular prism, express the following as a single vector. [3]



a) $\overrightarrow{AC}+\overrightarrow{CF}+\overrightarrow{FA}$	$\vec{0}$ ✓
b) $\overrightarrow{FE}-\overrightarrow{AE}+\overrightarrow{BC}$	$\overrightarrow{FD}$ ✓
c) $\overrightarrow{AF}+\overrightarrow{EH}-\overrightarrow{CH}$	not possible ✓

3. In rectangle $ABCD$, X and Y are midpoints of AB and AD, respectively. If $\overrightarrow{AX}=\vec{a}$ and $\overrightarrow{AY}=\vec{b}$, express each vector as a linear combination of $\vec{a}$ and $\vec{b}$. [3]



a) $\overrightarrow{AC}$	$2\vec{a}+2\vec{b}$ ✓
b) $\overrightarrow{CX}$	$-2\vec{b}-\vec{a}$ ✓
c) $3\overrightarrow{XY}$	$3(\vec{b}-\vec{a})$ ✗

/12(11)

4. Simplify $3(4\vec{u} + \vec{v}) - 2\vec{u} - 3(\vec{u} - \vec{v})$. [2]

$= 12\vec{u} + 3\vec{v} - 2u - 3\vec{u} + 3\vec{v}$ ✓

$= 6\vec{u} + 6\vec{v}$ ✗

1/2

5. If $|\vec{u}|=10$, $|\vec{v}|=8$ and the angle between $\vec{u}$ and $\vec{v}$ is 40°,

a) find $|3\vec{u} + \vec{v}|$. *Include a diagram in your solution.* [3]

$|\vec{R}|^2 = |\vec{v}|^2 + |3\vec{u}|^2 - 2|\vec{v}||3\vec{u}| \cos 140°$

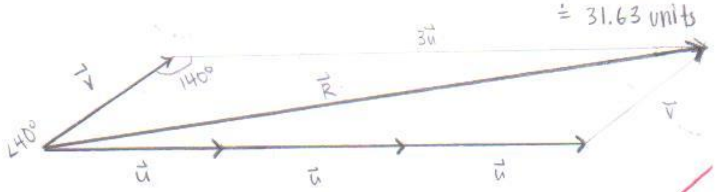
$= 8^2 + 30^2 - 2(8)(3)\cos 140°$ ✗ 30

$= 1000.770...$

$R = 31.6349$

$\doteq 31.63$ units ✓ ok

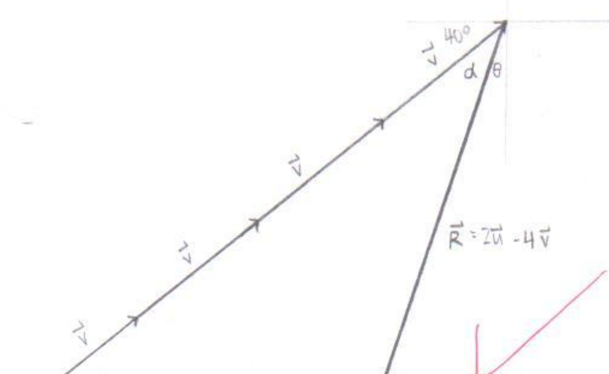
2/3



therefore, $|3\vec{u} + \vec{v}|$ is 31.63 units

✓

b) find $2\vec{u} - 4\vec{v}$. *Include a diagram in your solution.* [4]



$|\vec{R}|^2 = |4\vec{v}|^2 + |2\vec{u}|^2 - 2|4\vec{v}||2\vec{u}| \cos 40°$

$= 32^2 + 20^2 - 2(32)(20)\cos 40°$

$= 443.4631128...$ ✓

$R = 21.05856388...$

$\doteq 21.06$ units ✓

3/4

✓

therefore, $2\vec{u} - 4\vec{v}$ is about 21.06 units [37.62° CCW of $4\vec{v}$]

✗

tail to tail!

6

$\frac{\sin 40}{21.06} = \frac{\sin \alpha}{20}$

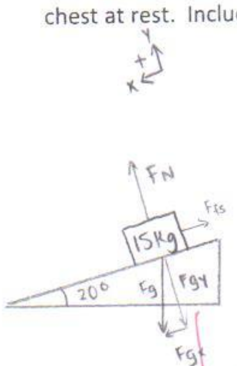
$\sin \alpha \; 21.06 = 12.85575...$

$\alpha = 37.62093236...°$

$= 37.62°$

PART B: APPLICATIONS

6. A 15 kg treasure chest, filled with golden protractors, is resting on a ramp, with an incline of 20°. Determine the force of friction parallel to the ramp, and the force perpendicular to the ramp which keep the treasure chest at rest. Include a sketch. [4]



$m\vec{a} = \Sigma \vec{F}_x$

$\emptyset = \vec{F}_{gx} + \vec{F}_{fs}$

$-\vec{F}_{fs} = \vec{F}_g(\sin 20°)$

$= m\vec{a}\ (\sin 20°)$

$= 15\text{ kg}\ (9.81\text{ m/s}^2)(\sin 20°)$

$= -50.32826\ldots\text{N}$

$\vec{F}_{fs} \doteq +50.33\text{ N}$

$\doteq 50.33\text{N}$ [up the ramp]

$\vec{F}_{gy} = \vec{F}_g(\cos 20°)$

$= m\vec{a}\ (\cos 20°)$

$= (15\text{ kg})(-9.81\text{ m/s}^2)(\cos 20°)$

$= -138.2757191\ldots\text{ N}$

$\doteq -138.28\text{ N}$

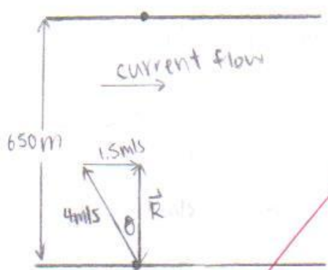
therefore, the force of friction is 50.33 N, and the force perpendicular is 138.28 N

4/4.

7. A ferry boat crosses a river and arrives at a point on the opposite bank directly across from its starting point. The boat can travel at 4 m/s and the current is 1.5 m/s. If the river is 650 m wide at the crossing point, [4]

a) in what direction must the boat steer and

b) how long will it take to cross?



$4^2 = 1.5^2 + |\vec{R}|^2$

$R = 3.708099\ldots\text{ m/s}$

$\doteq 3.71\text{ m/s}$

$\sin\theta = \frac{1.5\text{ m/s}}{4\text{ m/s}}$

$\theta = \sin^{-1}(0.375)$

$= 22.02°$

a) therefore, the boat should steer 22.02° towards the current

b) $\Delta\vec{d} = \vec{R}(\Delta t)$

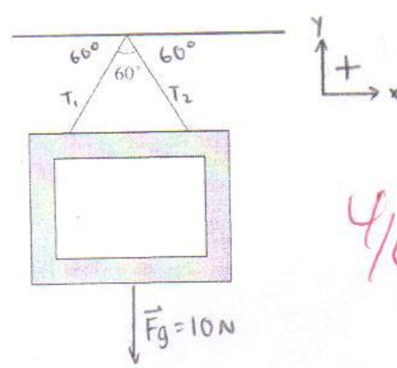
$+650\text{m} = +3.71\text{ m/s}\ (\Delta t)$

$\Delta t = 175.202\ldots\text{ s}$

→ therefore, it will take about 175.20 s to cross

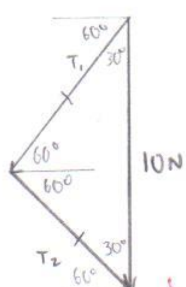
4/4

8. A picture of weight 10 N [down] hangs from two wires as shown in the diagram. Determine the magnitude of the tension in each wire assuming that the picture is hung symmetrically on the wires. Include a vector diagram in your solution. [4]



$\Sigma\vec{F} = m\vec{a}$ (rest)

$= \emptyset$



$\frac{\sin 120°}{10\text{ N}} = \frac{\sin 30°}{T_2}$

$\sin 120°\ T_2 = 5\text{N}$

$T_2 = 5.7735\ldots\text{N}$

$\frac{\sin 120°}{10\text{N}} = \frac{\sin 30°}{T_1}$

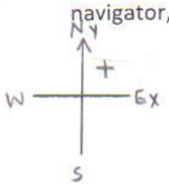
$T_1 = 5.7735\ldots\text{ N}$

Therefore, the magnitude of the tension in each wire is 5.77 N

4/4

/12(11)

9. Victor Vector, ace pilot for Pakho Airlines needs to fly to a destination on a bearing of S30˚W. There is a 60 km/h wind **from** N70˚W. Victor plans on flying his plane at 600 km/h. Determine the heading that his navigator, Scott Scalar, should give him in order to accomplish this. [4]

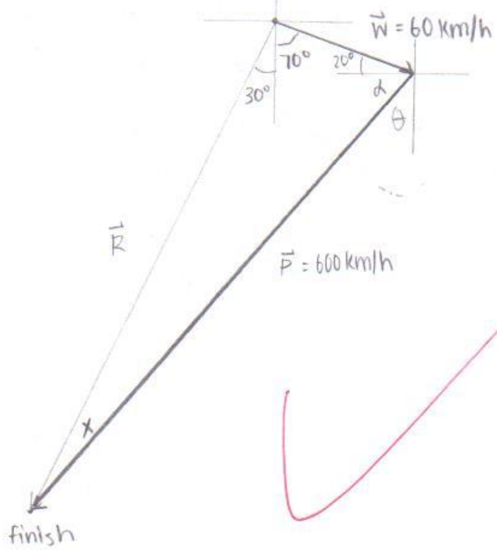


$\vec{P}$ 600 km/h [?]
$\vec{W}$ 60 km/h [N70° W]
$\vec{R}$? [S 30° W]

$\vec{R} = \vec{W} + \vec{P}$

$\frac{\sin 100°}{600 \text{ km/h}} = \frac{\sin x}{60 \text{ km/h}}$

$x = 5.65°$



4/4

triangle rule

$\alpha = 180° - x - 70° - 30° - 20°$

$= 180° - 5.65° - 70° - 30° - 20°$

$= 54.35°$

∴, they should fly the plane 600 km/h [W 54.35° S]

[S 35.65° W]

PART C: COMMUNICATION

10. If $a\vec{u} + b\vec{v} = \vec{0}$ where a and b are positive, comment on the following:

a) If $a \neq 0$, and $b \neq 0$, what is the relationship between vectors $\vec{u}$ and $\vec{v}$? Explain briefly. [2]

$\vec{u}$ and $\vec{v}$ must be opposites so that they could add up to zero. 180° rotated in direction since a and b are not the negative ones

4/4

b) If $a = -b$, what is the relationship between vectors $\vec{u}$ and $\vec{v}$? Explain briefly. [2]

sub in $a = -b$

$-b\vec{u} + b\vec{v} = 0$

$\cancel{b}\vec{v} = \cancel{b}\vec{u}$

$\vec{v} = \vec{u}$

it means $\vec{u}$ and $\vec{v}$ are identical

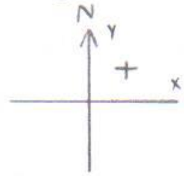
equivalent vectors

4+4(11)

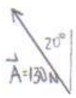
PART D: THINKING

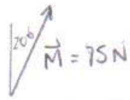
11. Ash, Misty and Brock are trying to move a boulder at the same time, but they weren't very cooperating very well. Each of them applied the following force on the boulder:

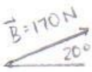
    Ash: 130 N [N20°W]
    Misty: 75 N at 020° (true bearing)
    Brock: 170 N 20° in standard position.



a) What is the resultant force? [6]





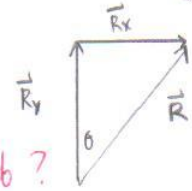


	y-dir	x-dir
$\vec{A}$	$\vec{A}_y = 130\cos 20°$ $= +122.16N$	$\vec{A}_x = 130\sin 20°$ $= -41.04N$
$\vec{M}$	$\vec{M}_y = 75\cos 20°$ $= +70.48N$	$\vec{M}_y = 75\sin 20°$ $= +25.65N$
$\vec{B}$	$\vec{B}_y = 170\sin 20°$ $= +58.14N$	$\vec{B}_x = 170\cos 20°$ $= +159.75N$
$\vec{R}$	$\vec{R}_y = +250.78N$	$\vec{R}_x = +144.36N$

-44.46 ?

½

ok.



$|\vec{R}|^2 = |\vec{R}_y|^2 + |\vec{R}_x|^2$

$R = 289.362...\ N$

$\tan\theta = \frac{R_x}{R_y}$

$\theta \doteq 29.93°$

ok

5.5/6

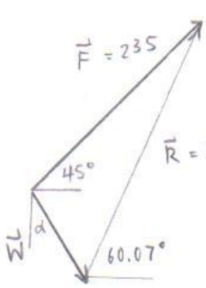
∴ the resultant force is 289.36 N [N 29.93° E] ok

b) What will the equilibrant force be? [1]

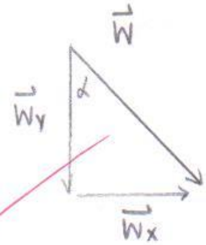
289.36 N [S 29.93° W] ok.

1/1

c) Mr. Wong came along and pushed the boulder as well and the resultant force became 235 N [N45°E], determine the force that was applied by Mr. Wong. [4]



	y-dir	x-dir
$\vec{F}$	$\vec{F}_y = 235\sin 45°$ $= +166.577N$	$\vec{F}_x = 235\cos 45$ $= +166.871N$
$\vec{R}$	$\vec{R}_y = +250.78N$	$\vec{R}_x = +144.36N$
$\vec{W}$	$\vec{W}_y = -83.903N$	$\vec{W}_y = +22.517$



$|\vec{W}|^2 = |\vec{W}_y|^2 + |\vec{W}_x|^2$

$W = 86.87\ N$

$\tan\alpha = \frac{W_x}{W_y}$

$\alpha = 15.02°$

4/4

∴ mr Wong applied 86.87N [S 15.02° E]

☺ The end ☺

ok.

/10.5