

Test # 2  
Analytical Geometry

K / U 12/12

A 12.5/13

TIP 12/12

C 4+ Level

## Knowledge and Understanding

1. How far is the point
- $(-10, 24)$
- from the origin?

a)  $\sqrt{26}$       b) 10      c) 26      d) 34      e) 44

2. One end of a line segment has the coordinates
- $(4, -8)$
- . If the middle point is
- $(4, 1)$
- , then what are the coordinates of the other endpoint?

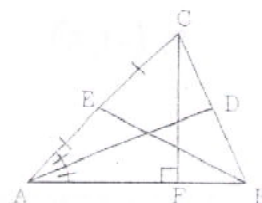
a)  $(8, 4)$       b)  $(4, 10)$       c)  $(10, 4)$       d)  $(4, 8)$       e)  $(4, -17)$ 

3. One end of a line segment has the coordinates
- $(m, n)$
- . If the other end has the coordinates
- $(p, q)$
- , then what are the coordinates of the midpoint?

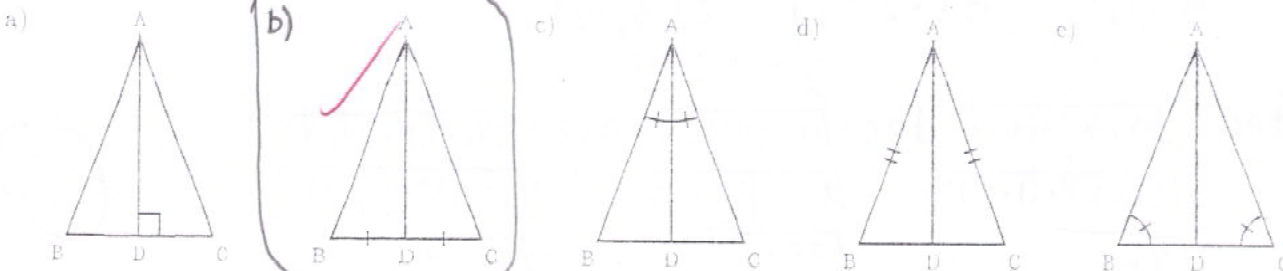
a)  $\left(\frac{m-p}{2}, \frac{n-q}{2}\right)$       b)  $\left(\frac{m+p}{2}, \frac{n+q}{2}\right)$       c)  $(2(m+p), 2(n+q))$   
d)  $\left(\frac{m+q}{2}, \frac{n+p}{2}\right)$       e)  $\left(\frac{p-m}{2}, \frac{q-n}{2}\right)$ 

4. Name a segment that is a median of the given figure.

a) BE      b) CF      c) AD      d) AF      e) BD



5. Which of the following diagrams illustrates segment AD as a segment bisector?



6. A right triangle has vertices
- $A(6, 4)$
- ,
- $B(6, -8)$
- ,
- $C(0, 4)$
- . What are the coordinates of the midpoint of the hypotenuse?

a)  $(3, -6)$       b)  $(3, -2)$       c)  $(3, 4)$       d)  $(6, -2)$       e)  $(6, 4)$ 

7. A parallelogram has vertices
- $A(-4, 1)$
- ,
- $B(1, 2)$
- ,
- $C(2, 6)$
- , and
- $D(-3, 5)$
- . At what point do the diagonals intersect?

a)  $(0, 3\frac{1}{2})$       b)  $(0, 4\frac{1}{2})$       c)  $(-3\frac{1}{2}, 4)$       d)  $(-1, 3\frac{1}{2})$       e)  $(-1, 4\frac{1}{2})$

[3] 8. For the circle  $x^2 + y^2 = 169$  state: the radius:  $r = 13$

the centre:  $(0, 0)$

the intercepts  $(0, 13)$   $(0, -13)$   $(13, 0)$   $(-13, 0)$

3

[2] 9. Write the equation of the circle, centre  $O(0, 0)$ , that passes through point  $(-3, -8)$ .

$$\begin{aligned} r^2 &= x^2 + y^2 \\ r^2 &= (-3)^2 + (-8)^2 \\ \sqrt{r^2} &= \sqrt{9 + 64} \\ r &= \sqrt{73} \end{aligned}$$

$\therefore$  the equation of the circle is

$$\boxed{x^2 + y^2 = 73}$$

2

### Applications

[3] 10.  $\triangle ABC$  has vertices  $A(3, 4)$ ,  $B(-5, 2)$ , and  $C(1, -4)$ .

Determine an equation for  $CD$ , the median from  $C$  to  $AB$ .

$A(3, 4)$   $B(-5, 2)$   $C(1, -4)$

$$\begin{aligned} M_{AB} &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left( \frac{3 + (-5)}{2}, \frac{4 + 2}{2} \right) \\ &= (-1, 3) \end{aligned}$$

$D(-1, 3)$

$$\begin{aligned} M_{CD} &= \left( \frac{y_2 - y_1}{x_2 - x_1} \right) \\ &= \frac{3 + 4}{-1 - 1} \\ &= -\frac{7}{2} \\ y &= -\frac{7}{2}x + b \end{aligned}$$

$$\begin{aligned} \text{sub } C(1, -4) \text{ into } y &= -\frac{7}{2}x + b \\ y &= -\frac{7}{2}x + b \\ -4 &= -\frac{7}{2}(1) + b \\ b &= -0.5 \end{aligned}$$

$\therefore$  the equation for  $\overline{CD}$  is  $y = -\frac{7}{2}x - 0.5$

2.5

[3] 11. A triangle has vertices at  $A(1, 1)$ ,  $B(-2, -1)$ , and  $C(3, -2)$ . Determine the type of triangle  $ABC$ .

$A(1, 1)$   $B(-2, -1)$   $C(3, -2)$

$$\begin{aligned} d_{AB} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} & d_{BC} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} & d_{CA} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1 + 2)^2 + (1 + 1)^2} & &= \sqrt{(-2 - 3)^2 + (-1 + 2)^2} & &= \sqrt{(3 - 1)^2 + (-2 - 1)^2} \\ &= \sqrt{9 + 4} & &= \sqrt{25 + 1} & &= \sqrt{4 + 9} \\ &= \sqrt{13} \text{ units} & &= \sqrt{26} & &= \sqrt{13} \end{aligned}$$

$d_{AB} = d_{CA} \therefore$  this triangle is isosceles.

pyth theorem

$$a^2 + b^2 = c^2$$

LS	RS
$a^2 + b^2$	$c^2$
$13 + 13$	26
26	26

LS = RS

LS = RS

$\therefore$  this is a right angle isosceles triangle.

3

5.5

[4] 12. Show that the quadrilateral with vertices J(-1, 1), K(3, 4), L(8, 4) and M(4, 1) is a rhombus.

J(-1, 1) K(3, 4) L(8, 4) M(4, 1)

$$\begin{aligned} d_{JK} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1 - 3)^2 + (1 - 4)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \end{aligned}$$

$$\begin{aligned} d_{KL} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - 8)^2 + (4 - 4)^2} \\ &= \sqrt{25 + 0} \\ &= \sqrt{25} \end{aligned}$$

$$\begin{aligned} d_{LM} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(8 - 4)^2 + (4 - 1)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \end{aligned}$$

$$\begin{aligned} d_{MJ} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - 1)^2 + (1 - 1)^2} \\ &= \sqrt{9 + 0} \\ &= \sqrt{9} \end{aligned}$$

all sides have the same distance.

∴ It's either square or rhombus.

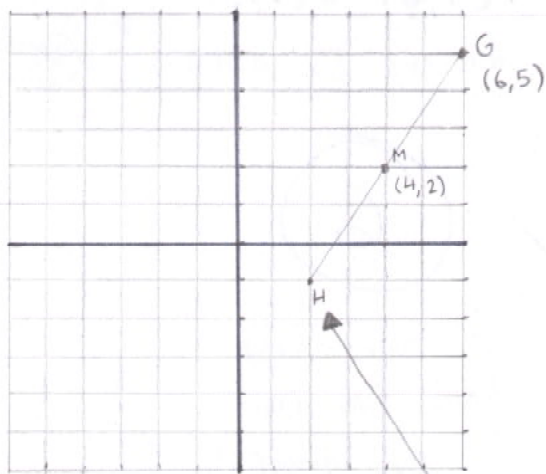
$$\begin{aligned} m_{KL} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 1}{3 - 8} \\ &= \frac{3}{-5} \\ &= -\frac{3}{5} \end{aligned}$$

$$\begin{aligned} m_{ML} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 1}{8 - 4} \\ &= \frac{3}{4} \end{aligned}$$

slope of  $\overline{KL}$  and  $\overline{ML}$   
are not perpendicular

∴ this Quadrilateral  
is a rhombus

[3] 13. For a line segment GH, one endpoint is G(6, 5) and the midpoint is M(4, 2). Find the coordinates of endpoint H.



$$\begin{aligned} d_{MG} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6 - 4)^2 + (5 - 2)^2} \\ &= \sqrt{(2)^2 + (3)^2} \\ &= \sqrt{4 + 9} \\ &= \sqrt{13} \end{aligned}$$

$$\begin{aligned} m_{MG} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 2}{6 - 4} \\ &= \frac{3}{2} \end{aligned}$$

M(4, 2)

$$\begin{aligned} &- (2, 3) \\ &= (2, -1) \end{aligned}$$

∴ the coordinates of H  
is (2, -1)

LS	RS
$d_{MH} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{13}$	$= \sqrt{(4 - 2)^2 + (2 - (-1))^2}$
$= \sqrt{13}$	$= \sqrt{(2)^2 + (3)^2}$
$= \sqrt{13}$	$= \sqrt{13}$

∴ LS = RS

# Thinking / Inquiry / Problem Solving

[6] 14.  $\Delta ABC$  has vertices  $A(6,3)$ ,  $B(2,5)$ , and  $C(0,1)$ . Determine the coordinates of the circumcentre.

**A(6,3) B(2,5) C(0,1)**

$$M_{AB} = \left( \frac{x_2 + x_1}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left( \frac{6+2}{2}, \frac{3+5}{2} \right)$$

$$= (4, 4) \text{ (D)}$$

$$M_{BC} = \left( \frac{x_2 + x_1}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left( \frac{2+0}{2}, \frac{5+1}{2} \right)$$

$$= (1, 3) \text{ (E)}$$

$$M_{CA} = \left( \frac{x_2 + x_1}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left( \frac{0+6}{2}, \frac{1+3}{2} \right)$$

$$= (3, 2) \text{ (F)}$$

$$M_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3-5}{6-2}$$

$$= \frac{-2}{4}$$

$$= -\frac{1}{2}$$

$$M_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5-1}{2-0}$$

$$= \frac{4}{2}$$

$$= 2$$

$$M_{CA} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-3}{0-6}$$

$$= \frac{-2}{-6}$$

$$= \frac{1}{3}$$

Perp eq. of AB

$$y = 2x + b$$

sub (D) into

$$y = 2x + b$$

$$(4) = 2(4) + b$$

$$b = -4$$

Perp eq. of BC

$$y = -\frac{1}{2}x + b$$

sub (E) into

$$y = -\frac{1}{2}x + b$$

$$(3) = -\frac{1}{2}(1) + b$$

$$b = 3.5$$

$$y = 2x - 4 \text{ (1)}$$

$$y = -0.5x + 3.5 \text{ (2)}$$

$$(y = 2x - 4)(-0.5)$$

$$-0.5y = -1x + 2 \text{ (3)}$$

$$(y = -0.5x + 3.5)(2)$$

$$2y = -1x + 7$$

$$2y = -1x + 7$$

$$-0.5y = -1x + 2$$

$$2.5y = 5$$

$$y = 2$$

sub  $y = 2$  into (1)

$$y = 2x - 4$$

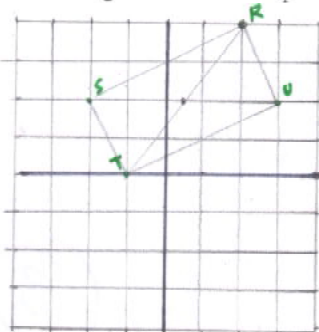
$$2 = 2x - 4$$

$$2x = 6$$

$$x = 3$$

$\therefore$  the coordinates of the circumcentre is  $(3, 2)$

[6] 15. Quadrilateral RSTU has vertices  $R(2, 4)$ ,  $S(-2, 2)$ ,  $T(-1, 0)$  and  $U(3, 2)$ . Verify that the diagonals of the quadrilateral bisect each other and are equal in length.



**R(2,4) S(-2,2) T(-1,0) U(3,2)**

$$M_{RT} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left( \frac{2-1}{2}, \frac{4+0}{2} \right)$$

$$= (0.5, 2)$$

$$M_{SU} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left( \frac{-2+3}{2}, \frac{2+2}{2} \right)$$

$$= (0.5, 2)$$

POI of  $\overline{RT}$  and  $\overline{SU}$  is  $(0.5, 2)$  (A)

$$D_{AT} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0.5 - 1)^2 + (2 - 0)^2}$$

$$= \sqrt{(1.5)^2 + (2)^2}$$

$$= \sqrt{6.25}$$

$$D_{AS} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0.5 - 2)^2 + (2 - 2)^2}$$

$$= \sqrt{(2.5)^2 + (0)^2}$$

$$= \sqrt{6.25}$$

$$D_{AT} = D_{AR}$$

$$\text{and } D_{AS} = D_{AU}$$

$\therefore$  the diagonals bisect each other

$$D_{AR} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0.5 - 2)^2 + (2 - 4)^2}$$

$$= \sqrt{(-1.5)^2 + (-2)^2}$$

$$= \sqrt{6.25}$$

$$D_{AU} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0.5 - 3)^2 + (2 - 2)^2}$$

$$= \sqrt{(-2.5)^2 + (0)^2}$$

$$= \sqrt{6.25}$$

$$\therefore \overline{AT} = \overline{AR}$$