MCR 3U

# TEST: SINUSOIDAL FUNCTIONS

Name: Uni Lee

No Graphing Calculators are allowed. Show all work of good form for full marks. Good luck!

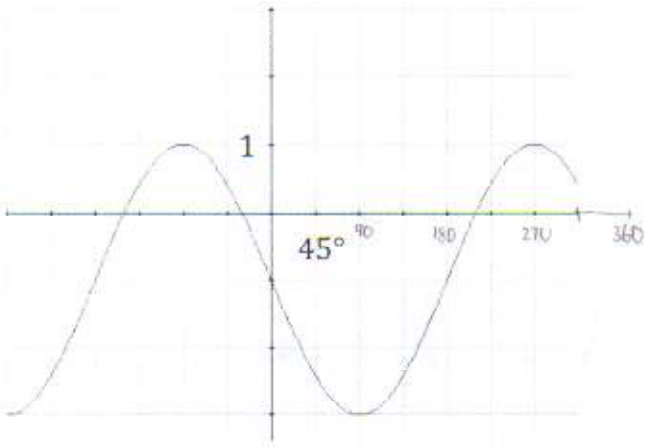
| K/U | THK | COMM | APP |
|---|---|---|---|
| ~~19~~ 16 /19 | ~~5~~ 4 /8 | 10 /10 | 27 /28 |

## Part A: Knowledge and Understanding /19

1. Complete the following table. (10)

| Function | Period | Amplitude | Phase Shift | Vertical Displacement | Max Value | Min Value |
|---|---|---|---|---|---|---|
| $y = 2\cos(2\theta + 180°) - 1$ | 180° ✓ | 2 ✓ | left 90° ✓ | down 1 unit ✓ | 1 ✓ | -3 ✓ |
| $y = \frac{1}{2}\sin\left[\frac{1}{3}(\theta - 60°)\right]$ ✓ | 1080° | $\frac{1}{2}$ | 60° right | none | $\frac{1}{2}$ ✓ | $-\frac{1}{2}$ ✓ |

2. State the amplitude, period, phase shift and vertical displacement. Label these features on the graph, then determine the equation of the **cosine** function. (9) ✓




Amplitude: 2

Period: 360° ✓

Phase shift: left 90° ✓

Vertical displacement: down 1 unit ✓

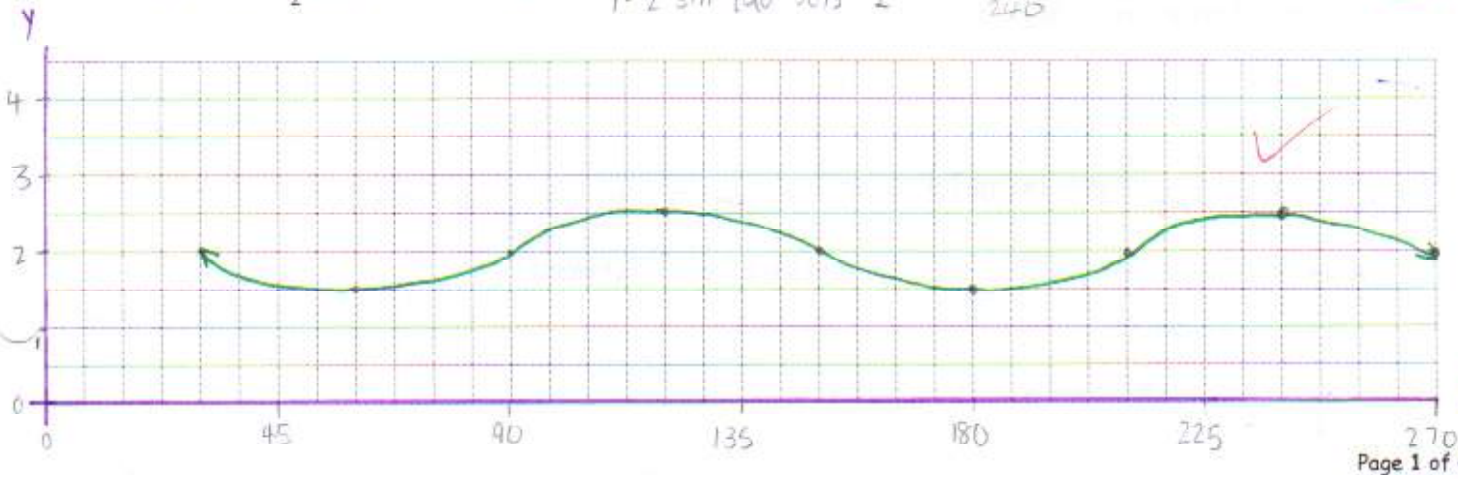
Equation of the **cosine** function:

$y = 2\cos(\theta + 90°) - 1$ ✓

## Part B: Application /28

3. Graph the following functions. **Show <u>two</u> complete cycles.** (5)

a) $y = -\frac{1}{2}\sin(3\theta - 90°) + 2$ $y = -\frac{1}{2}\sin[3(\theta - 30)] + 2$ 120 240

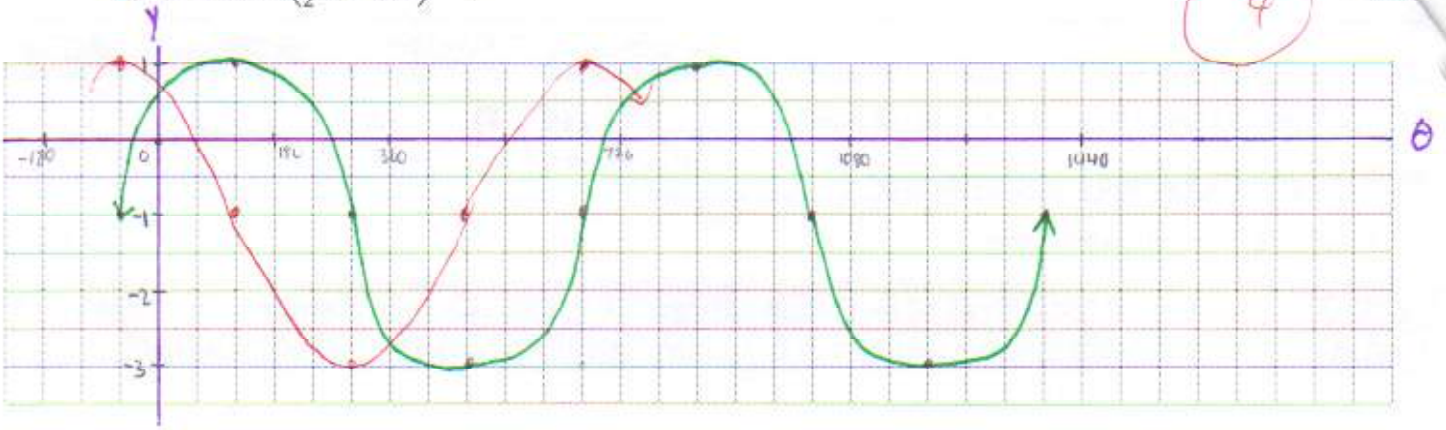





/5

b) $y = 2\cos\left(\frac{1}{2}\theta + 30°\right) - 1$ (5)

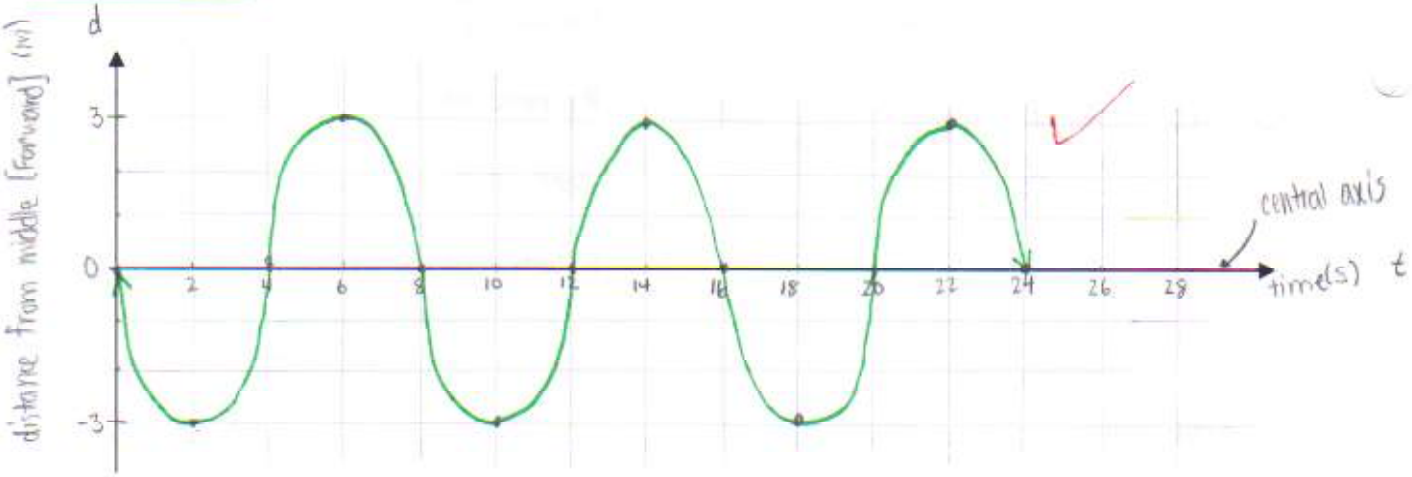
4




4. Jacqueline is on a swing, she starts in the middle, swings back a horizontal distance of 3 m, then back to the middle, then forward a horizontal distance of 3 m, then back to the middle and so forth. Jacqueline swings at a rate of 2 cycles every 16 seconds. (9)

a) Sketch the graph of Jacqueline's **horizontal distance** (in metres) from the **middle** verses **time** (in seconds) for 24 seconds.

Be sure to show the 5 major points of each period, the central axis, and the appropriate labels. (4)




b) Write the equation of the height that represents the horizontal distance, *d*, from the middle at time, *t*, as a cosine **and** sine function. (4)

SINE EQUATION: $d = -3\sin[45t]$ COSINE EQUATION: $d = \cos[45(t-6)]$

c) At what time(s) will Jacqueline's horizontal distance be 3m forward? (1)

at 6s, 14s, 22s

$t_n = 6 + (8n-8)$ second

every 8s after the first 6s

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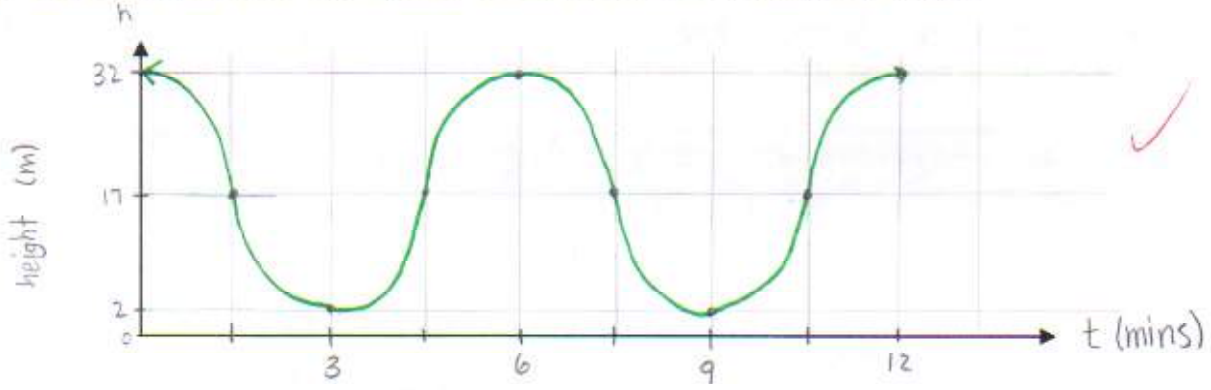
A Ferris wheel has a radius of 15m and its centre is 17m above the ground. It completes one rotation every 6 minutes. Uni rides the Ferris wheel as Arman records her height above the ground. He starts recording when Uni is at the top of the wheel. (9)

a) Sketch a graph for two cycles. Be sure to properly label your graph. (4)




b) Write the equation of the Uni's height, $h(t)$ metres, as a function of the time, $t$ minutes, as both a **sine** and **cosine** equation. (4)

SINE EQUATION: $h(t) = -15\sin[60(t-1.5)]+17$ COSINE EQUATION: $h(t) = 15\cos[60t]+17$

c) At what time(s) will Uni be at the same height as the axel of the wheel? (1)

at 1.5 mins, 4.5 mins, 7.5 mins, 10.5 mins

every 3 mins after 1.5 mins $t_n = 1.5 + (3n-3)$

## Part C: Thinking /8

6. A Ferris wheel at the local carnival has a diameter of 24 metres and its centre is 15 metres above the ground. The wheel completes one revolution every 20 seconds. (3)

(a) What would happen to the graph if only the diameter of the Ferris wheel was shorter?

the amplitude would be smaller.

it stretches vertically less

period, time and centre is the same

(b) What would happen to the graph if only the centre of the wheel was higher above the ground?

vertical displacement is moved up.

the wave is moved up

max and min is increased

time and period, amp is the same

(c) What would happen to the graph if each revolution took only 15 seconds?

20s 

15s 

the period will be smaller, more dense horizontal compression

7. The diameter of a car's tire is 60 cm. While the car is being driven, the tire picks up a nail. How high above the ground is the nail after the car has travelled1 km? (3)

$2\pi r = 188.5$ m

car travels 188.5 m every rotation


$h = -30\cos(1.9d) + 30$

1 km = 1000 m = ? cm

$h = -30\cos(1.9(1000)) + 30$

$= +35.2$ cm

Therefore, the nail would be 35.2cm above the ground at 1 km

8. State the domain and range of the function $= \frac{1}{\cos\theta}$. (2)

undefined when $\cos\theta = 0$

$D: \{\theta \in \mathbb{R}\}$

$R: \{y \in \mathbb{R} \mid -1 \le y \le 1\}$

$y \ge 1$ or $y \le -1$

**Part D: Communication** /10

( /3 Graphs labeled, function notation, units, equal signs)

9. Describe the transformations, in the correct order, applied to the graph of $y = \sin\theta$ to produce $y = -4\sin(2\theta) - 5$. (4)

- reflection in x-axis
- horizontal compression by factor of $\frac{1}{2}$
- vertical stretch by factor of 4
- vertical translation down 5 units

10. Explain how you would determine the **maximum** and **minimum** for the function $y = 2\cos(\theta + 135°) - \frac{5}{2}$ using proper mathematical terminology. (3)

$-\frac{5}{2}$ is the central axis and "2" is the amp, which is the amount it goes up and down from the central axis

take the vertical translation, and $\pm$ the vertical stretch amount, and that will be max and min of the function

max: $-\frac{1}{2}$

min: $+4\frac{1}{2}$