(ANSWERS)

Date: ____________________ Name: ____________________

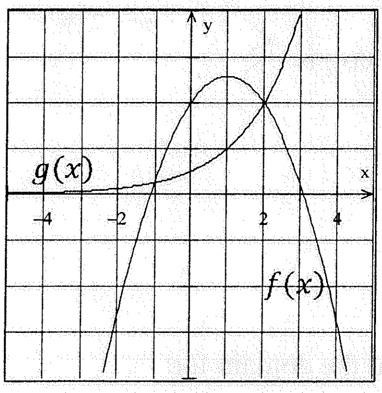
PIERRE ELLIOTT TRUDEAU H.S.

MHF4U: Unit 8 Summative Assessment

Part A: Knowledge and Understanding. [16 marks]

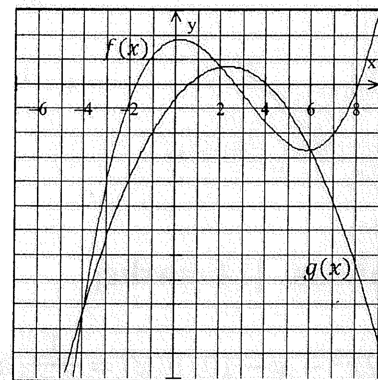
1. Given the diagrams below, solve for the inequalities indicated. [4 marks]

a) $f(x) < g(x)$



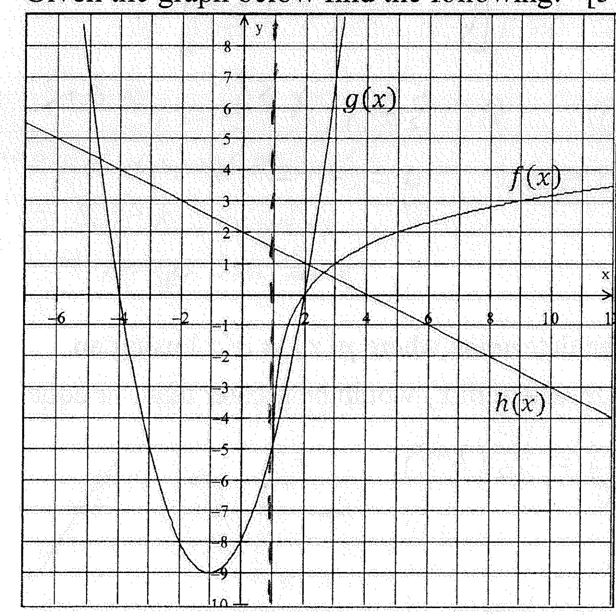
$x < -1$ or $x > 2$, $x \in R$

b) $f(x) \geq g(x)$



$-4 \leq x \leq 2$ or $x \geq 6$, $x \in R$.

2. Given the graph below find the following: [5 marks]



a) $f(9) + g(-3)$ $\quad$ -2

$3 + (-5)$

b) $f(0) - g(0)$ $\quad$ undefined

undefined $-(-8)$

c) $h(f(2))$ $\quad$ 2

0 $\quad h(0) =$

d) $g(3) \times h(10)$ $\quad$ -21

7×-3

e) $(g \circ g \circ f)(5)$ $\quad$ -8

$g(2)$

$g(0)$

3. Given $f(x) = 2x^2 + x - 17$ and $g(x) = x + 3$. Find the equation of the oblique asymptote to the function $y = \frac{f(x)}{g(x)}$. Rewrite this function as a sum of two other functions. [3 marks]

$$
\begin{array}{r}
2x - 5 \\
x+3 \,\big)\, 2x^2 + x - 17 \\
\underline{2x^2 + 6x} \\
-5x - 17 \\
\underline{-5x - 15} \\
-2
\end{array}
$$

The equation of the oblique asymptote is $y = 2x - 5$.

$\therefore y = 2x - 5 + \frac{-2}{x+3}$

4. Given $f(x) = x^2 - 4$ and $g(x) = \sqrt{x+3}$, find a simplified equation for $y = (f \circ g)(x)$. State the domain and range for the composite function, then draw a sketch to represent it. [4 marks]

$y = (f \circ g)(x)$

$= (\sqrt{x+3})^2 - 4$

$= x + 3 - 4$

$y = x - 1$

$\because g(x) = \sqrt{x+3}$

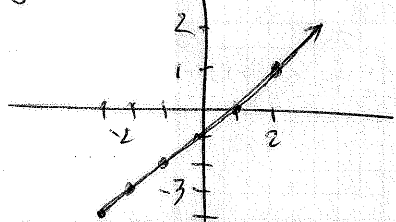
$\therefore D = \{x \mid x \geq -3, x \in \mathbb{R}\}$

$\because$ minimum value for $g(x)$ is 0

$\therefore$ minimum value for $f(g(x)) = 0^2 - 4$

$= -4$

$\therefore R = \{y \mid y \geq -4, y \in \mathbb{R}\}$



Part B: Applications. [14 marks]

5. Given $f(x) = 2x^2 - x - 10$, $g(x) = \frac{1}{x+4}$, and $h(x) = \log(x)$, find the domain for:

a) $(g \circ f)(x)$ [3 marks]

$= \frac{1}{(2x^2 - x - 10) + 4}$

$= \frac{1}{2x^2 - x - 6}$

$= \frac{1}{(2x+3)(x-2)}$

$D: \quad x \neq -\frac{3}{2}, \ x \neq 2, \ x \in \mathbb{R}$

b) $h(f(x))$ [3 marks]

$= \log(2x^2 - x - 10)$

$= \log[(2x-5)(x+2)]$

$D: \ x < -2, \ x > \frac{5}{2}, \ x \in \mathbb{R}.$

$\rightarrow$ since $y = (2x-5)(x+2)$ is positive when $x < -2$ or $x > \frac{5}{2}$ (and not equal to 0)

6. Given $p(x) = 2\sin\left[\frac{\pi}{12}(x-1)\right]$, and $q(x) = 1$, then determine where $p(x) = q(x)$ using an algebraic method. Using this information, generalize where $p(x)$ would be greater than, or equal to $q(x)$. [4 marks]

$1 = 2\sin\left[\frac{\pi}{12}(x-1)\right]$ Let $\theta = \frac{\pi}{12}(x-1)$

$\frac{1}{2} = \sin\theta$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$ or $\theta = \pi - \sin^{-1}\left(\frac{1}{2}\right)$

$\theta = \frac{\pi}{6}$ or $\theta = \frac{5\pi}{6}$

$\therefore \frac{\pi}{12}(x-1) = \frac{\pi}{6}$

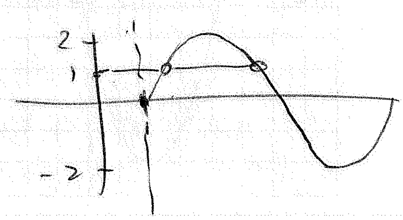
$x - 1 = 2$

$x = 3$

$\therefore \frac{\pi}{12}(x-1) = \frac{5\pi}{6}$

$x - 1 = 10$

$x = 11$



$p(x) \geq q(x)$ when

$3 \leq x \leq 11$

$\because$ This is periodic

$\therefore 3 + pn \leq x \leq 11 + pn, \ n \in \mathbb{Z}$

$\therefore 3 + 24n \leq x \leq 11 + 24n, \ n \in \mathbb{Z}$

$p = 2\pi \div \frac{\pi}{12}$

$p = 24$

7. Using the periodic nature of a sinusoidal function, produce a **reasonably accurate** sketch of the function $\boldsymbol{y = x \sin x}$. Be sure to show the intervals at which the maximum, minimum, and zero values occur by drawing and labelling these points (two cycles to the left, and to the right, of the vertical axis). [~~4~~ marks]

~~2~~ 4

- - symmetry (even).
- - 2 cycles
- - radian measure
- - zeros