MPM2D1

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TEST #5
Quadratic Equations.

K/U 12/12 A 15/15 TIP 12.5/13 C 4+/Level

Knowledge and Understanding

1. Solve: $(x-2)^2 = 14$

   a) ± 4 b) $-2 \pm \sqrt{14}$ c) $2 \pm \sqrt{14}$ d) -4 ± 14 e) 4 ± 14

2. The trinomial $x^2 - 10kx + R$ is a perfect square. Find the value of R.

   a) 25 b) $5k^2$ c) $25k^2$ d) $100k^2$ e) $25k^{2x^2}$

3. Solve $x^2 - 8x + 2 = 0$ using the Quadratic Formula.

   a) $x = -8 \pm \sqrt{14}$ b) $x = -4 \pm \sqrt{14}$ c) $x = 4 \pm \sqrt{2}$ d) $x = 4 \pm \sqrt{14}$ e) $x = 8 \pm \sqrt{14}$

4. Which one of the following quadratic equations will have *two equal roots*?

   a) $b^2 - 16 = 0$ b) $9y^2 - 30y + 25 = 0$ c) $3x^2 + 14x + 49 = 0$
   d) $-4c^2 + 12c + 9 = 0$ e) $n^2 + 12n - 36 = 0$

5. Consider solving $x^2 + 14x + 3 = 0$ by *completing the square*. At which of the following equations will you arrive?

   a) $(x+7)^2 = 52$ b) $(x+7)^2 = 46$ c) $(x-7)^2 = 52$ d) $(x-7)^2 = 46$ e) $(x+14)^2 = 193$

6. A rectangle's length is 8 m longer than its width, and its perimeter is 128 m^2. Which of the following equations would be suitable to find the dimensions of the rectangle?

   a) $2w + 2(w+8) = 128$ b) $w(w+8) = 128$ c) $w + (w-8) = 128$
   d) $2w(2w-16) = 128$ e) $w + 8 = 128$

7. The parabola $y = 3x^2 + 18x + 19$ will have

   a) a maximum at $(3, -8)$ b) a minimum at $(-9, -62)$ c) a maximum at $(-3, 10)$
   d) a minimum at $(-3, -8)$ e) a maximum at $(-3, -8)$

8. Write the equation of the parabola that opens down, has a vertex $V(-2, 51)$, and is congruent to $y = -4x^2$. Answer in the form $y = a(x-h)^2 + k$.

   a) $y = 4(x-2)^2 - 51$ b) $y = -4(x-2)^2 - 51$ c) $y = -4(x+2)^2 - 51$
   d) $y = 4(x+2)^2 + 51$ e) $y = -4(x+2)^2 + 51$

8

[4] 12. A rectangle is 3 cm longer than it is wide. The area is 40 cm^2. Find the dimensions of the rectangle.

let w be the width
let l be the length

$l = w + 3$ ①

$lw = 40$ ②

sub ① into ②

$lw = 40$

$(w+3)w = 40$

$w^2 + 3w - 40 = 0$

$w = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-3 \pm \sqrt{3^2-4(1)(-40)}}{2}$

$= \frac{-3 \pm \sqrt{169}}{2}$

$w_1 = \frac{-3+\sqrt{169}}{2}$ $\quad$ $w_2 = \frac{-3-\sqrt{169}}{2}$

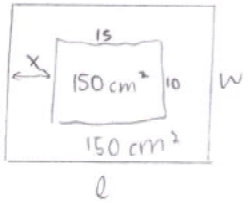
$= 5$ cm $\quad$ $= -8$

$l_1 = 8$ cm $\quad$ $l_2 = -5$ rejected.

∴, the dimensions are 5cm × 8cm

4

[4] 13. Tom wants to frame a picture. He wants to make a frame the same area as a picture itself. If the picture measures 10 cm x 15 cm how wide should the frame be?



let w be the width $\quad$ let x be the wide
let l be the length.

$lw = 300$ ①

$l = 2x + 15$ ②

$w = 2x + 10$ ③

sub ②&③ into ①

$lw = 300$

$(2x+15)(2x+10) = 300$

$4x^2 + 50x + 150 = 300$

$4x^2 + 50x - 150 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-50 \pm \sqrt{50^2-4(4)(-150)}}{8}$

$= \frac{-50 \pm \sqrt{4900}}{8}$

$x_1 = \frac{-50+\sqrt{4900}}{8}$ $\quad$ $x_2 = \frac{-50-\sqrt{4900}}{8}$

$= 2.5$ $\quad$ $= -15$

therefore, the wide should be 2.5 cm. $\quad$ rejected.

4

A: 8

Thinking. Inquiry. Problem Solving.

[2] 14. Give an example of quadratic equation that has no real roots. Use the vertex form.

$-4x^2 - 2x - 1000$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{2 \pm \sqrt{4-4(-4)(-1000)}}{-8}$

$= \frac{2 \pm \sqrt{-15996}}{-8}$

No real roots

$-4x^2 - 2x - 1000$

$= -4(x^2 + 0.5x) - 1000$

$= -4(x^2 + 0.5x + 0.25^2 - 0.25^2) - 1000$

$= -4(x+0.25)^2 - 999.75$

2

TIP 2

[4] 9. Solve by factoring and check your solution.

$0 = 2x^2 - 7x + 6$

$= 2x^2 - 4x - 3x + 6$

$= 2x(x-2) - 3(x-2)$

$= (2x-3)(x-2)$

$x = 1.5$ $x = 2$

Check $x = \frac{3}{2}$

LS	RS
0	$2x^2 - 7x + 6$
0	$= 2(1.5)^2 - 7(1.5) + 6$
0	$= 4.5 - 10.5 + 6$
0	$= 0$

$\therefore$ LS = RS

Check $x = 2$

LS	RS
0	$2x^2 - 7x + 6$
0	$= 2(2)^2 - 7(2) + 6$
0	$= 8 - 14 + 6$
0	$= 0$

$\therefore$ LS = RS

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Applications.

[3] 10. Solve by completing a square. Give your answer as an exact root.

$-2x^2 - 8x + 10 = 0$

$-2(x^2 + 4x) + 10 = 0$

$-2(x^2 + 4x + 4 - 4) + 10 = 0$

$-2(x+2)^2 + 18 = 0$

$\sqrt{(x+2)^2} = \sqrt{9}$

$x = \pm 3 - 2$

$x_1 = -5$

$x_2 = 1$

3

[4] 11. If a ball was thrown on Mars, its height, h, in meters might be modelled as

$h = -1.9t^2 + 18t + 1$

a) What would be the maximum height of the ball on Mars.

$h = -1.9t^2 + 18t + 1$

$= -1.9(t^2 - 9.4736t) + 1$

$= -1.9(t^2 - 9.4736t + 4.7368^2 - 4.7368^2) + 1$

$= -1.9(t - 4.7368)^2 + 43.63082$

The maximum height is 43.6 m.

4

b) When does it happen?.

$t = 4.7368$

it happens at 4.7 seconds

A: 7

[4]. 15. A bus company has 4000 passengers daily, each paying a fare of $2. For each $ 0.15 increase, the company estimates that it will lose 40 passengers. If the company needs to take in $ 10 450 to stay in business, what fare should be charged?

3.0

let $0.15n$ be amount of increase

let $4000 - 40n$ be amount of passengers

max profit = passengers × fare

$= (4000 - 40n)(0.15n + 2)$

$= -6n^2 + 520n + 8000$

$10450 = -6n^2 + 520n + 8000$

$0 = -6n^2 + 520n - 2450$

$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-520 \pm \sqrt{520^2 - 4(-6)(-2450)}}{-12}$

$= \frac{-520 \pm \sqrt{211600}}{-12}$ ✓

$n_1 = \frac{-520 + \sqrt{211600}}{-12}$

$= 5$

$n_2 = \frac{-520 - \sqrt{211600}}{-12}$

$= 81.666$ (rejected) why?

Fare $= 0.15n + 2$

$= 0.15(5) + 2$

$= 2.75$

∴ the should charge $2.75

[4] 16. The sum of squares of two consecutive odd integers is 90. Find the numbers.

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let x and y be the numbers

$x^2 + y^2 = 90$ ①

$y = x + 2$ ②

sub ② into ①

$x^2 + y^2 = 90$

$x^2 + (x+2)^2 = 90$

$x^2 + x^2 + 4x + 4 - 90 = 0$

$2x^2 + 4x - 86 = 0$ ✓

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{4^2 - 4(2)(-86)}}{4}$

$= \frac{-4 \pm \sqrt{704}}{4}$

$x_1 = \frac{-4 + \sqrt{704}}{4}$ $\quad x_2 = \frac{-4 - \sqrt{704}}{4}$

$= 5.6332$ $\quad = -7.6332$

$y_1 = 7.6332$ $\quad = -5.6332$

There fore, the numbers can be 7.6332 and 5.6332 or -7.6332 and -5.6332.

[3] 17. Solve using quadratic formula. Give your answer as approximate roots.

$x^2 = \frac{7x + 6}{5}$

$5x^2 = 7x + 6$

$x = -5 + 7x + 6$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-7 \pm \sqrt{49 - 4(-5)(6)}}{-10}$

$= \frac{-7 \pm \sqrt{169}}{-10}$

3

$x_1 = \frac{-7 + \sqrt{169}}{-10}$ $\quad x_2 = \frac{-7 - \sqrt{169}}{-10}$

$x_1 = -0.6$ $\quad x_2 = 2$

10.5