MCR3U **TEST: EXPONENTIAL FUNCTIONS** Name: Uni Lee

No Graphing Calculators allowed. Show all work of good form for full marks. Good luck!

**Part A: Knowledge** ( /18)

Multiple Choice: Select the best answer. Write your answer in CAPITAL letters on the line. (6)

1. Evaluate $\frac{4^{-2} \times 5^{-3}}{4^{-3} \times 5^{-1}}$. A ✓

A) $\frac{4}{25}$ B) $-\frac{25}{4}$ C) $\frac{1}{20}$ D) $-\frac{81}{1600}$

2. Evaluate $9^{-\frac{1}{2}} + 25^{-\frac{1}{2}} + 32^{\frac{3}{5}}$ B ✓

A) 0 B) $8\frac{8}{15}$ C) $8\frac{1}{4}$ D) 16

3. Which of the following is equivalent to the expression $\left(a^3b^4\right)^{-2}\left(a^{-3}b^{-5}\right)^{-4}$? A ✓

A) $a^6b^{12}$ B) $\frac{1}{a^6b^{12}}$ C) $a^{12}b^{15}$ D) $\frac{1}{a^6b^7}$

4. Which of the following is equivalent to the expression $\left(64f^6g^{-3}\right)^{\frac{1}{2}}\left(81f^{-4}g^8\right)^{\frac{1}{4}}$? B ✓

A) $\frac{36}{f^4g^4}$ B) $36g^3$ C) $\frac{72g^3}{f}$ D) $863.87g^3$

5. Which of the following is equivalent to the expression $\left(\frac{\left(3j^{-3}k^6\right)^4}{j^{-2}k^{-8}}\right)^{\frac{1}{2}}$? C ✓

A) $\frac{j^{20}}{162k^{64}}$ B) $\frac{9k^8}{j^5}$ C) $\frac{9k^{16}}{j^5}$ D) $\frac{40.5k^{16}}{j^5}$

6. Which of the following is equivalent to the expression $\left(\frac{\sqrt[3]{z^7}}{\sqrt{z^5}}\right)^4$? D ✓

A) $\frac{1}{\sqrt{z^3}}$ B) $\sqrt[35]{z^4}$ C) $z$ D) $\frac{1}{\sqrt[3]{z^2}}$

7. Complete the following table. (8)

| Function | Equation of the asymptote | Range (formal set notation) | y-intercept | Increasing / decreasing |
|---|---|---|---|---|
| $g(x) = -(2)^{\frac{1}{2}x} - 1$ | HA $y = -1$ ✓ | $\{y \in \mathbb{R} \mid y < -1\}$ ✓ | $y = -2$ ✓ | decreasing ✓ |
| $h(x) = -\frac{1}{2}\left(\frac{1}{3}\right)^{-x+2} + 4$ | HA $y = 4$ ✓ | $\{y \in \mathbb{R} \mid y < 4\}$ ✓ | $y = \frac{71}{18}$ ✓ | increasing |

7

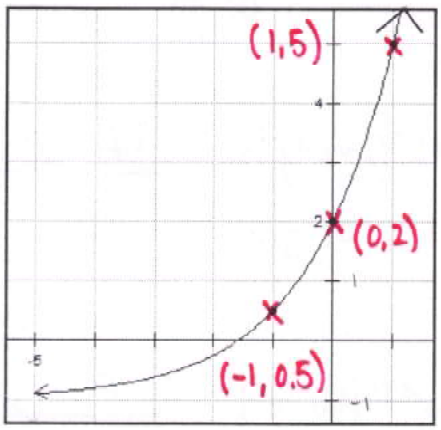
8. Given the graph of $f(x) = ab^x + c$, (4)

a) State the equation of the horizontal asymptote. (1)

HA $y = +c$ $-1$

0.5

b) Determine the equation of $f(x)$. Show all your work. (3)




HA $y = -1$

**Work** use point (0,2)

$y = ab^x + c$

$2 = ab^0 - 1$

$2 = a(1) - 1$

$3 = a(1)$

$a = 3$

sub $a = 3$ into $y = ab^x + c$

use point (1,5)

$y = ab^x + c$

$5 = 3b^1 - 1$

$6 = 3b^1$

$\frac{6}{3} = b^1$

$b = 2$

Equation of $f(x)$= $3(2)^x - 1$ ✓

3

0.5

**Part B: Application** ( /31)

9. Solve for $x$.

a) $-208 = -x^3 + 8$ (3)

$-216 = -x^3$

$x^3 = 216$

$(x^3)^{\frac{1}{3}} = 216^{\frac{1}{3}}$

$x = (6^3)^{\frac{1}{3}}$

$x = 6$ ✓

b) $5^{4x-1} = 125^{x+3}$ (3)

$5^{4x-1} = (5^3)^{x+3}$

$5^{4x-1} = 5^{3x+9}$

$4x - 1 = 3x + 9$

$x = 10$

c) $3^{3x-5} = (9^{x+2})\left(\frac{1}{3}\right)^x$ (3)

$3^{3x-5} = [(3^2)^{x+2}][(3^{-1})^x]$

$3^{3x-5} = (3^{2x+4})(3^{-x})$

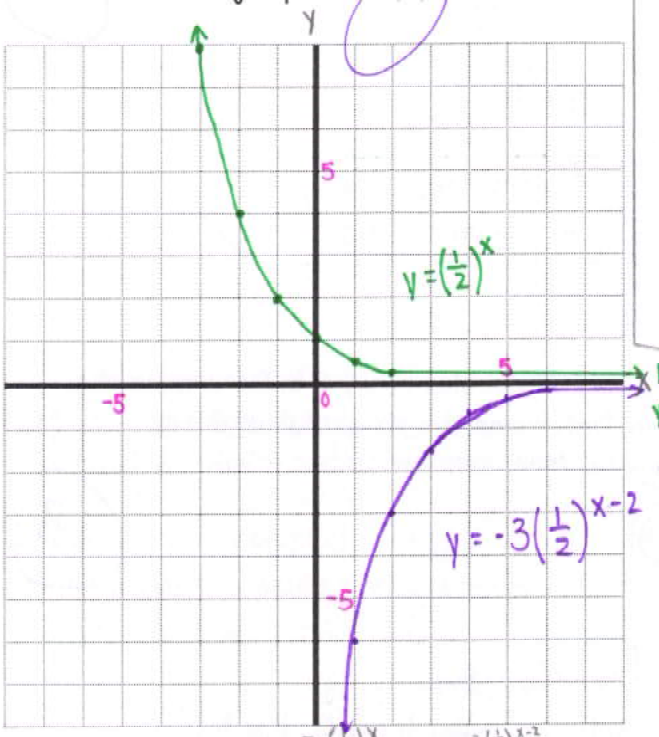
$3^{3x-5} = 3^{x+4}$

$3x - 5 = x + 4$

$x = 4.5$

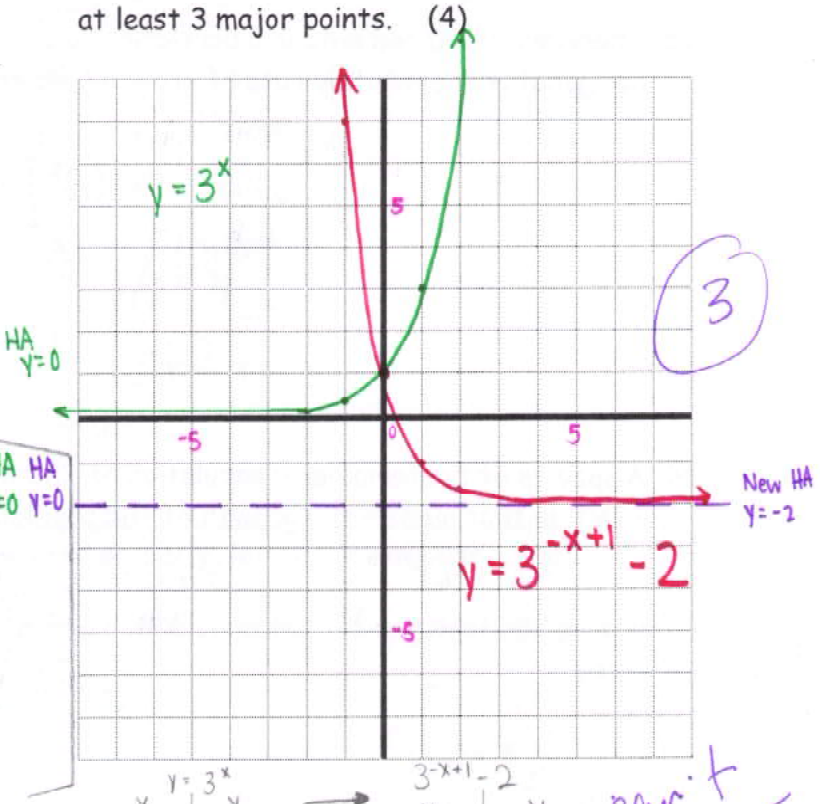
3

10. Graph the function $f(x) = -3\left(\frac{1}{2}\right)^{x-2}$. Label at least 3 major points. (4)




| $y = \left(\frac{1}{2}\right)^x$ | | $-3\left(\frac{1}{2}\right)^{x-2}$ | |
|---|---|---|---|
| x | y | x+2 | -3y |
| -1 | 2 | 1 | -6 |
| 0 | 1 | 2 | -3 |
| 1 | $\frac{1}{2}$ | 3 | -1.5 |
| 2 | $\frac{1}{4}$ | 4 | -0.75 |

11. Graph the function $g(x) = 3^{-x+1} - 2$. Label at least 3 major points. (4)



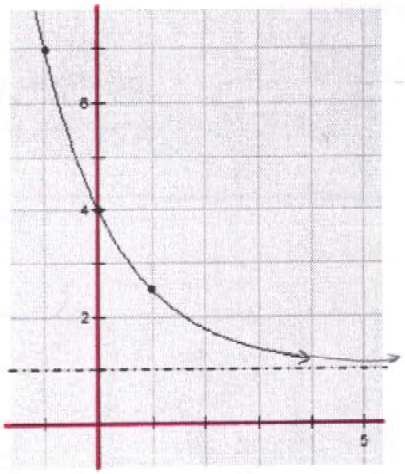

| $y = 3^x$ | | $3^{-x+1} - 2$ | |
|---|---|---|---|
| x | y | x | y |
| -1 | $\frac{1}{3}$ | -1 | 7 |
| 0 | 1 | 0 | 1 |
| 1 | 3 | 1 | -1 |
| 2 | 9 | 2 | $-1\frac{2}{3}$ |

Can't use a T.O.V.

12. Match the graph to the function. (Both graphs have a scale of 1.) (2)

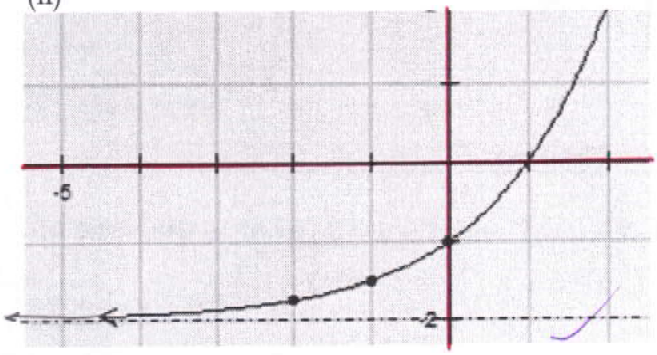
(a) $y = 2^{\frac{1}{2}x+2}$ (b) $y = 3\left(\frac{1}{2}\right)^x + 1$ (c) $y = \frac{1}{2}(2)^{x+1} - 2$ (d) $y = 3(2)^{-x-1} + 1$

(i)




B

(ii)




C



/9

***For the following questions, do not forget to introduce your variables using let statements.***

13. There are 5400 red ants in a particular colony. If there were 1200 ants in the colony 8 months ago, what is the monthly rate of growth? Round to one decimal place. (4)

$A(n) = A_0 r^n$

$5400 = (1200)(r)^8$

$4.5 = r^8$

$4.5^{\frac{1}{8}} = (r^8)^{\frac{1}{8}}$

$r = 1.2$

$20\%$

therefore, the rate of growth is 1.2 ✓

14. A species of bacteria has a population of 250 at 6 a.m. It doubles every 8 h. Determine the function that models the growth of the population. Determine the population at 9 p.m. (4)

$A(n) = A_0 r^n$

$500 = 250(r)^8$

$2 = r^8$

$2^{\frac{1}{8}} = (r^8)^{\frac{1}{8}}$

$r = 1.09$

sub $r = 1.09$

$A(15) = 250(1.09)^{15}$

time between 6am - 9pm = 15h

$= 910$

2

3

Therefore, there will be about 910 bacterias at 9 p.m.

15. An 80 g sample of plutonium-238 has a half-life of 88 years. How long will it take for this sample to decay to 10 g? (4)

n rep years

$A(n) = A_0 r^n$

$40 = 80(r)^{88}$

$\frac{40}{80} = r^{88}$

$0.5^{\frac{1}{88}} = (r^{88})^{\frac{1}{88}}$

$0.992 = r$

$10 = 80(0.992)^n$

$\frac{1}{8} = 0.992^n$

$\log \frac{1}{8} = n \log 0.992$

$-0.90308 = -0.003488\,(n)$

$\frac{-0.90308}{-0.003488} = n$

$n = 264$ years

Therefore, I will take 264 years to decay to 10g

11

**Part C: Thinking** ( /7)

16. If $x + y = 1$ and $x - y = 3$, what is the value of $2^{(y^2 - x^2)}$? (3)

$y + x = 1$ $\quad y - x = -3$

$2^{(y^2 - x^2)}$

$= 2^{(y+x)(y-x)}$

$= 2^{(1)(-3)}$

$= 2^{-3}$

$= \frac{1}{2^3}$

$= \frac{1}{8}$ ✓

3

17. If $g(x) = (-4)^x$ and $g(x+1) - g(x+2) + g(x+3) = kg(x)$, what is the value of $k$? (4)

$$(-4)^{(x+1)} + (-4)^{(x+2)} + (-4)^{(x+3)} = K(-4)^x$$

$$-4^x\left((-4)^1 + (-4)^2 + (-4)^3\right) = K(-4)^x$$

$$-4^x\left[-4 + 16 - 64\right] = K(-4)^x$$

$$-4^x(-52) = K(-4)^x$$

$$K = -52$$

3.5

**Part D: Communication** ( /8)

( /3 graphs labelled, equal signs used properly, let statements used, proper form)

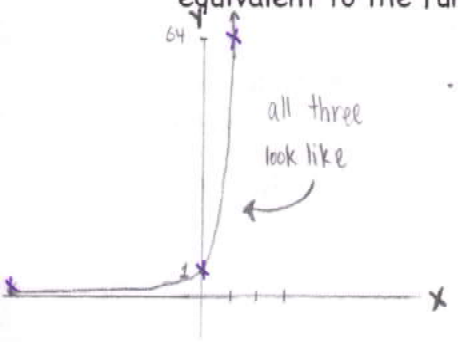
18. For the function $f(x) = -4\left(\frac{1}{3}\right)^{\frac{1}{2}x-1} + 2$, state the base graph and the transformations on it, in the order you would perform them. (3)

base graph $f(x) = \left(\frac{1}{3}\right)^x$

- reflection in x-axis ✓
- vertical stretch by factor of 4 ✓
- horizontal stretch by factor of 2 ✓
- horizontal translation right 1 unit.
- vertical translation up 2 units ✓

2.5

19. Name two different parent functions and their corresponding transformations that would be equivalent to the function $f(x) = 64^x$. Justify your answer with a brief explanation. (2)

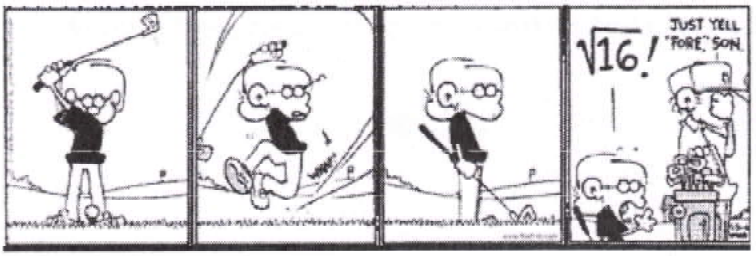



$f(x) = 8^{2x}$ · HC by factor of $\frac{1}{2}$

$f(x) = 2^{6x}$ · HC by factor of $\frac{1}{6}$

$64 = 8^2 = 2^6$

after you fill in the "x" the y is all the same




| K/U | APP | THK | COMM |
|---|---|---|---|
| 16.5/18 | 23 /31 | 6.5 /7 | 7.5 /8 |

25