MCV4U

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KU: 13.5 /18 TH: 5 /6 APP: 6 /8

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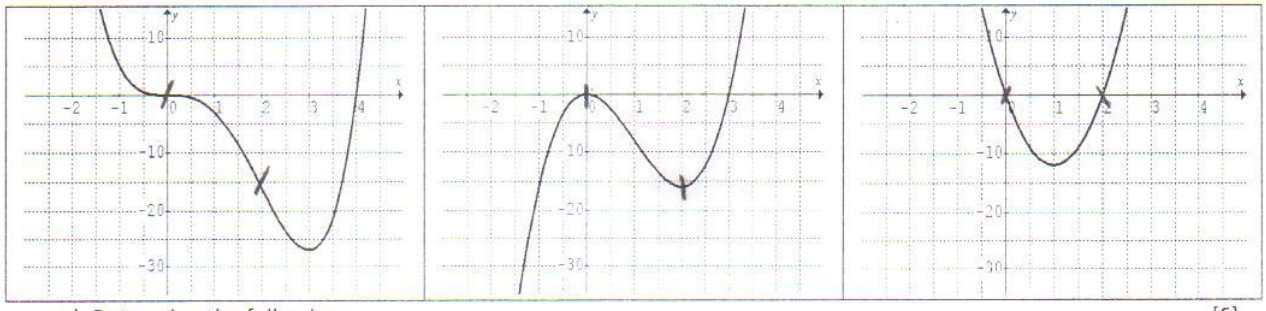
Unit 3 Quiz: Applications of Derivatives

1. The graphs of the function $f(x) = x^4 - 4x^3$, its first derivative $f'(x)$, and its second derivative $f''(x)$ are shown below.

f(x) f'(x) f''(x)



a) Determine the following: [6] K

i) interval(s) of increase	$x > 3$ ✓
ii) interval(s) of decrease	$x < 0$, $0 < x < 3$ ✓
iii) location(s) of any extrema and coordinates	$x = 3$ ✓
iv) intervals where $f(x)$ concaves up	$x < 0$, $x > 2$ ✓
v) intervals where $f(x)$ concaves down	$0 < x < 2$ ✓
vi) location of any point(s) of inflection	$x = \emptyset$ & $x = 2$. ✓½

5.5/6.

b) Confirm your answers above algebraically, using techniques as discussed in class. [12] K

8/12.

$f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

iii) $f'(x) = 4x^3 - 12x^2$

$0 = 4x^2(x-3)$

$x = 0$ $x = 3$

$f(3) = -27$

POI

x	x=-1	x=1
f'(x)	+5	-3

∴ POI

x	x=2	x=3.5
f(x)	-16	-21.43

∴ local extremum min.

-3

i/ii) $f'(x) = 4x^3 - 12x^2$ ✓

$= 4x^2(x-3)$

$x = 0$ $x = 3$

x	x<0	0	0<x<3	3	x>3
f(x)	+		−		+

∴ larger than zero, decreases from x<0

decrease 0<x<3

increase x>3 ∵ larger than x=3

Should sub these into f(x) & find when f'(x)>0 & f'(x)<0. (-1)

iv/v) $f''(x) = 12x^2 - 24x$

$0 = 12x^2 - 24x$

$= 12x(x-2)$

$x = 0$ $x = 2$

		0		2	
	+		−		+

concave up, down, concave up

iv) $f''(x) = 12x^2 - 24x$

$0 = 12x(x-2)$ ✓

$x = 0$ $x = 2$ ✓

x	x=-1	0	POI x=0.5	2	x=1.5	x=3
f''(x)	+		−		−	+

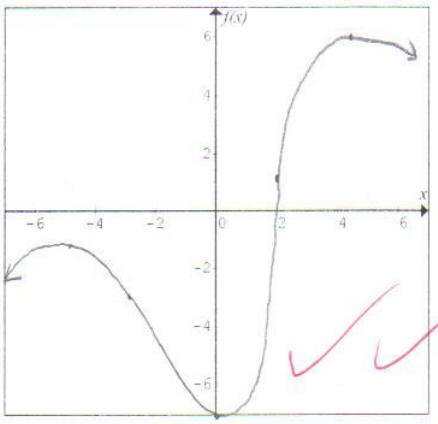
∵ changes concavity, ∴ POI

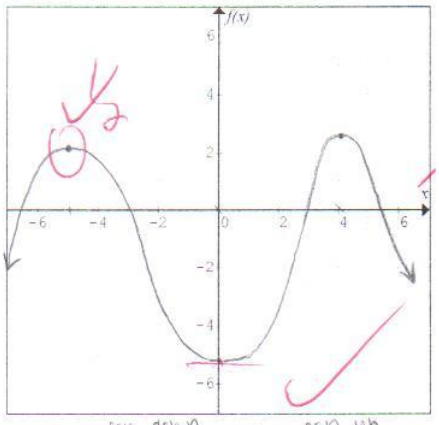
13.5

2. Sketch a **continuous** graph that satisfies each set of conditions. [8]
A

a) $f(-5) = -1, f(-3) = -3, f(0) = -7,$
$f(2) = 1, f(4) = 6$

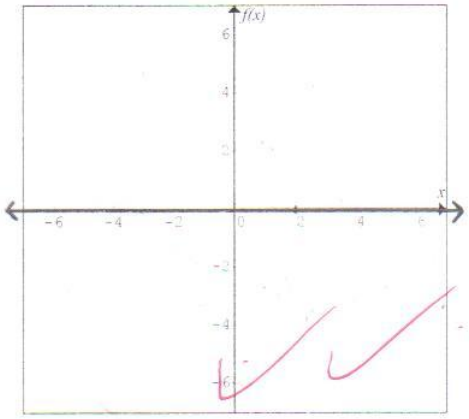
b) $f'(5) = f'(0) = f'(4) = 0$

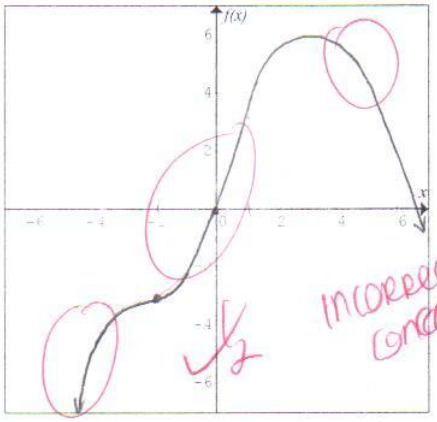




c) $f''(-3) = f'(2) = 0$

d) $f''(x) < 0$ for $-2 < x < 1$, $f''(x) > 0$ for $x < -2$ and $x > 1$, $f(-2) = -3, f(0) = 0$

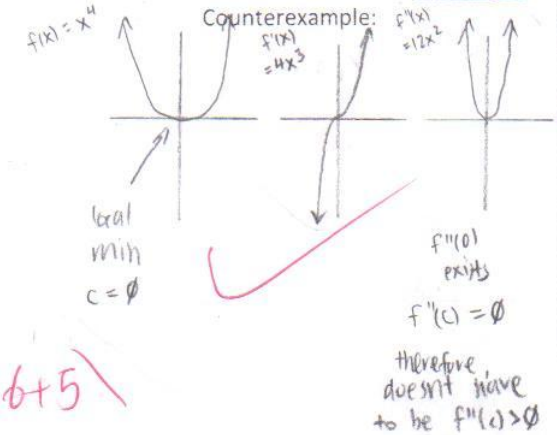




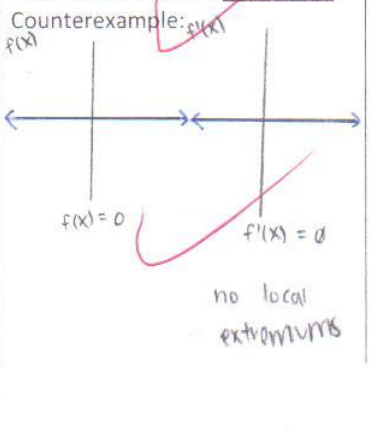
3. Below are 6 statements, 3 of which are FALSE. Identify which of the following statements are false, then provide a COUNTEREXAMPLE (algebraic, graphic or in words) to prove that it is false. [6]
T

A) If a function, f, has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.
B) If $f'(c)$ exists and $f'(c) = 0$ for a function f, then there must be a local extremum.
C) If a function, f, has a local minimum at c and $f''(c)$ exists, then $f''(c) > 0$.
D) If $f'(c) = 0$ and $f''(c) > 0$, then $(c, f(c))$ must be a local minimum.
E) A graph, $y = f(x)$, is concave upward on the interval $a < x < b$, if $f''(x) > 0$ for $a < x < b$.
F) If a function, f, has a point of inflection at c, then $f''(c) = 0$, and concavity must change at c.

False Statement #1: C
Counterexample:



False Statement #2: B
Counterexample:



False Statement #3: F
Counterexample: