MCR3U **Quadratic Functions & Equations** Name Uni Lee

Show work of good form for full marks. **Use exact solutions only (NO decimals).** Good luck! ☺

Part A: Knowledge and Understanding **(26 marks)**

Multiple choice - pick the best answer and write your answer in **CAPITAL** letters on the line.

1. Solve $x^2 - 3x - 28 = 0$ and leave your answer in simplest form.

   A. $x = -4, 7$ B. $x = -7, 4$ C. $x = -28, 0$ D. $x = -24, 4$

   B A

2. Find the optimum value of the function $f(x) = -8x^2 - 11$ and classify it.

   A. $y_{max} = -11$ B. $y_{min} = -8$ C. $x = 0$ D. $y_{min} = 11$

   A ✓

3. Find where the optimum value of the function $c(x) = 2.1x^2 - 12.7x + 167.4$ occurs.

   A. $x = 2$ B. $x = 3$ C. $x = 4$ D. $y_{max} = 5$

   B ✓

4. Determine the **EXACT** roots of the following equation $x^2 - 12x + 8 = 0$

   A. $x = 12 \pm 2\sqrt{7}$ B. $x = 6 \pm 4\sqrt{7}$
   C. $x = 12 \pm \sqrt{112}$ D. $x = 6 \pm 2\sqrt{7}$

   D ✓

5. How many times does the graph of the function $f(x) = 5x^2 - 6x + 1$ intersect the x-axis?

   A. *None* B. *Once* C. *Twice* D. *More than twice*

   ✓ C

6. Which of the following is true about the parabola for the function $f(x) = -9(x + 6)^2 - 8$?

   A. The vertex is (6, -8). B. The y-intercept is 8.
   C. The axis of symmetry is $x = -6$ D. The parabola opens up.

   C ✓

7. The path of a diver under water after a dive is in the shape of a parabola that opens upward. Using the water surface as the x-axis, the x-intercepts that the diver makes are 1 m and 5 m from the edge of the pool. The maximum depth, d, in metres, of the diver under water is 2 m. Which quadratic equation models the path of the diver?

   A. $d = -\frac{1}{2}(x - 1)(x - 5)$ B. $d = \frac{1}{2}(x + 1)(x - 5)$
   C. $d = \frac{1}{2}(x - 1)(x - 5)$ D. $d = -\frac{1}{2}(x + 1)(x + 5)$

   C ✓

16

8. An electronics store sells 15 MP3 players per month at $90 each. For every $10 decrease in cost, the store would sell 3 more MP3 players. If x represents the number of decreases in cost, which function best describes the revenue, R?

D ✓

A. $R = (15 - 3x)(90 + 10x)$
B. $R = (15 + x)(90 - x)$
C. $R = (15 - 10x)(90 + 3x)$
D. $R = (15 + 3x)(90 - 10x)$

9. A highway overpass has a shape that can be modelled by the equation of a parabola. If the edge of the highway is the origin and the highway is 15 m wide, what is the equation of the parabola if the height of the overpass 4 m from the edge of the highway is 18 m?

PICK Correct A value

D ✓

A. $f(x) = -\frac{2}{9}x^2 + \frac{10}{3}x$
B. $f(x) = 15(x - 4)^2 + 18$
C. $f(x) = -\frac{9}{38}x(x - 15)$
D. $f(x) = -\frac{9}{22}x^2 + \frac{10}{3}x$

10. Kelly has 16 m of fencing to enclose her flower garden. The function $f(x) = -(x - 4)^2 + 16$ can be used to model the area of the garden, where x is the length of fencing in metres. What is the domain for the measure of fencing Kelly can use for the length?

A B

A. $\{0 < x < 16\}$
B. $\{0 < x < 8\}$
C. $\{-4 \leq x \leq 4\}$
D. $\{1 \leq x \leq 4\}$

Write full solutions to the following questions. Show work of good form for FULL marks.

11. Find the max/min value of $y = -2x^2 + 3x - 3$ and where it occurs using partial factoring. (5)

FIND X-COORDINATE

$= -2\left(x^2 - \frac{3}{2}x\right) - 3$

$= -2\left(x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right) - 3$

$= -2\left(x - \frac{3}{4}\right)^2 - \frac{15}{8}$

max value $y = -\frac{15}{8}$ 1

at $x = \frac{3}{4}$

/3

12. **Solve** using the **most efficient** method and leave your answers as **exact values** in simplified form.

a) $6(x+3)^2 - 48 = 0$ (3)

$6(x+3)^2 = 48$

$(x+3)^2 = \frac{48}{6}$

$x+3 = \pm\sqrt{8}$

$x = \pm\sqrt{8} - 3$

simplify

$x = -3 \pm 2\sqrt{2}$

2.5

b) $3x^2 - 7x = 20$ (3)

$3x^2 - 7x - 20 = 0$

$(x-4)(x+\frac{5}{3}) = 0$

$x = 4, -\frac{5}{3}$ ✓

13. Determine the equation of the quadratic function that passes through (1, -2) if the roots are $3 \pm \sqrt{5}$. (5)

$y = a(x-r)(x-s)$

sub $(3\pm\sqrt{5})$ and $(1,-2)$

$-2 = a(1-(3-\sqrt{5}))(1-(3+\sqrt{5}))$

$-2 = a(1-3+\sqrt{5})(1-3-\sqrt{5})$

$-2 = a(-2+\sqrt{5})(-2-\sqrt{5})$

$-2 = a(4+2\sqrt{5}-2\sqrt{5}-5)$

$-2 = a(-1)$

$a = 2$

simplify

$y = 2(x-(3-\sqrt{5}))(x-(3+\sqrt{5}))$

$= 2(x-3-\sqrt{5})(x-3+\sqrt{5})$

4.5

10

Part B: Application - Show work of good form for FULL marks. (14 marks)

14. Determine the value(s) of k so that the quadratic $y = 4x^2 - kx + 25$ has two distinct real roots. (4)

$d = b^2 - 4ac$ $\quad a=4 \quad b=-k \quad c=25$

$= (-k)^2 - 4(4)(25)$

$= (-k)^2 - 400$

2 real roots

$d > 0$

2 same roots $d=0$

$0 = (-k)^2 - 400$

$\pm\sqrt{400} = \pm\sqrt{(-k)^2}$

$k = \pm 20$

2 real roots

$d = 500^2 - 400$

$= 249600$

no reall roots

$d = 0^2 - 400$

$= -400$

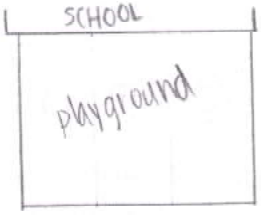
$\therefore k$ has to be $-20 > k > 20$

bf

write separately

4

15. A playground is to be built against the side of a school. The playground is to be rectangular and fenced in on three sides (the school serves as the fourth side of the rectangle). The builders have 60m of fencing. Find the dimensions of the largest playground that can be built. (5)



$A = l \times w$

$P = 60m$

$60m = l + l + w + w$

$l = \frac{60 - 2w}{2}$

$l = 30 - w$

$A_{max} = lw$

$= (30-w)w$

$= 30w - w^2$

$= -w^2 + 30w$

$= -(w^2 - 30w)$

$= -(w^2 - 30w + 15^2 - 15^2)$

$= -(w-15)^2 + 225$

max area $= 225\ m^2$

4

$w = 15\ m$

$l = 30 - (15)$

$= 15\ m$

1.

∴ dimensions 15m × 15m

16. Solve the linear-quadratic system algebraically. Leave your answers as exact numbers and simplified form. (5)

$y = 2x^2 - 3x + 4$ ①

$4x = -y + 7$ ②

$y = -4x + 7$ ③

sub ③ into ①

$2x^2 - 3x + 4 = -4x + 7$

$2x^2 + x - 3 = 0$

$2x^2 - 2x + 3x - 3 = 0$

$2x(x-1) + 3(x-1) = 0$

$(2x+3)(x-1) = 0$

$x = -\frac{3}{2}$ $x = +1$

sub $x = -\frac{3}{2}$ into ③

$y = -\frac{4}{1}(-\frac{3}{2}) + 7$

$= \frac{12}{2} + 7$

$= 13$

$(-\frac{3}{2}, 13)$

sub $x = +1$ into ③

$y = -4(1) + 7$

$= 3$

$(1, 3)$

∴ the POI's are $(-\frac{3}{2}, 13)$ and $(1, 3)$ ✓

Part C: Communication (7 marks) - Proper form

17. Describe three ways to determine the zeros of the quadratic function $f(x) = -3x^2 + 27x - 42$. Which method would be the fastest? (4 marks)

This would be fastest

- less writing and factoring when a value is 1 is easy

- Common factor then factor, then solve for x ✓
- complete the square then solve for x ✓
- quadratic formula ✓

2.5

- Equal signs used appropriately, concluding statements

K/U	APP	COMM
19 /26	13 /14	6.5 /7

Nice!