

(ANSWERS)

Date: _____

Name: _____

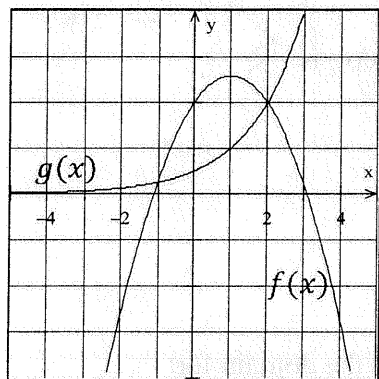
PIERRE ELLIOTT TRUDEAU H.S.

MHF4U: Unit 8 Summative Assessment

Part A: Knowledge and Understanding. [16 marks]

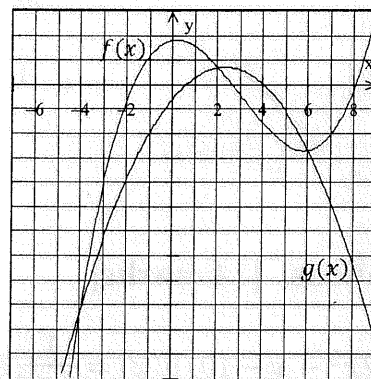
1. Given the diagrams below, solve for the inequalities indicated. [4 marks]

a) $f(x) < g(x)$



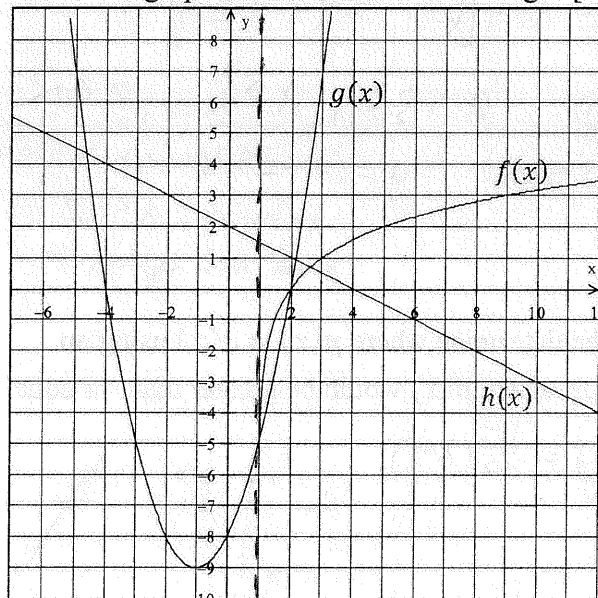
$x < -1$ or $x > 2, x \in \mathbb{R}$

b) $f(x) \geq g(x)$



$-4 \leq x \leq 2$ or $x \geq 6, x \in \mathbb{R}$

2. Given the graph below find the following: [5 marks]



a) $f(9) + g(-3)$
 $3 + (-5)$

-2

b) $f(0) - g(0)$
 $\text{undefined} - (-8)$

undefined

c) $h(f(2))$
 $h(0)$

2

d) $g(3) \times h(10)$
 7×-3

-21

e) $(g \circ g \circ f)(5)$
 $g(g(f(5)))$

-8

3. Given $f(x) = 2x^2 + x - 17$ and $g(x) = x + 3$. Find the equation of the oblique asymptote to the function $y = \frac{f(x)}{g(x)}$. Rewrite this function as a sum of two other functions. [3 marks]

$$\begin{array}{r} 2x - 5 \\ x+3 \overline{) 2x^2 + x - 17} \\ \underline{2x^2 + 6x} \\ -5x - 17 \\ \underline{-5x + 15} \\ -2 \end{array}$$

The equation of the oblique asymptote is $y = 2x - 5$.

$\therefore y = 2x - 5 + \frac{-2}{x+3}$

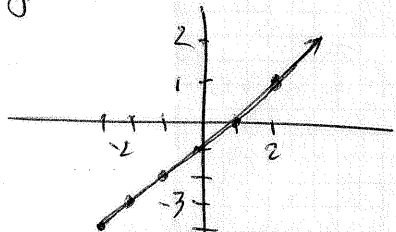
4. Given $f(x) = x^2 - 4$ and $g(x) = \sqrt{x+3}$, find a simplified equation for $y = (f \circ g)(x)$. State the domain and range for the composite function, then draw a sketch to represent it. [4 marks]

$$y = (f \circ g)(x)$$

$$= (\sqrt{x+3})^2 - 4$$

$$= x+3-4$$

$$y = x-1$$



$$\therefore g(x) = \sqrt{x+3}$$

$$\therefore D = \{x | x \geq -3, x \in \mathbb{R}\}$$

\therefore minimum value for $g(x)$ is 0

$$\therefore \text{minimum value for } f(g(x)) = 0^2 - 4 = -4$$

$$\therefore R = \{y | y \geq -4, y \in \mathbb{R}\}$$

Part B: Applications. [14 marks]

5. Given $f(x) = 2x^2 - x - 10$, $g(x) = \frac{1}{x+4}$, and $h(x) = \log(x)$, find the domain for:

a) $(g \circ f)(x)$

[3 marks]

$$= \frac{1}{(2x^2 - x - 10) + 4}$$

$$= \frac{1}{2x^2 - x - 6}$$

$$= \frac{1}{(2x+3)(x-2)}$$

$$D: x \neq -\frac{3}{2}, x \neq 2, x \in \mathbb{R}$$

b) $h(f(x))$

[3 marks]

$$= \log(2x^2 - x - 10)$$

$$= \log[(2x-5)(x+2)]$$

$$D: x < -2, x > \frac{5}{2}, x \in \mathbb{R}$$

\rightarrow since $y = (2x-5)(x+2)$ is positive when $x < -2$ or $x > \frac{5}{2}$ (and not equal to 0)

6. Given $p(x) = 2 \sin\left[\frac{\pi}{12}(x-1)\right]$, and $q(x) = 1$, then determine where $p(x) = q(x)$ using an algebraic method. Using this information, generalize where $p(x)$ would be greater than, or equal to $q(x)$. [4 marks]

$$1 = 2 \sin\left[\frac{\pi}{12}(x-1)\right] \quad \text{Let } \theta = \frac{\pi}{12}(x-1)$$

$$\frac{1}{2} = \sin \theta$$

$$\theta = \sin^{-1}\left(\frac{1}{2}\right) \quad \text{or } \theta = \pi - \sin^{-1}\left(\frac{1}{2}\right)$$

$$\theta = \frac{\pi}{6}$$

$$\text{or } \theta = \frac{5\pi}{6}$$

$$\therefore \frac{\pi}{12}(x-1) = \frac{\pi}{6}$$

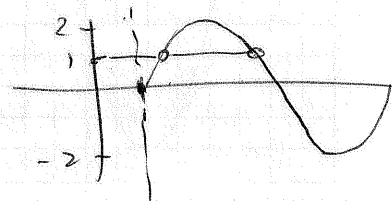
$$x-1 = 2$$

$$x = 3$$

$$\therefore \frac{\pi}{12}(x-1) = \frac{5\pi}{6}$$

$$x-1 = 10$$

$$x = 11$$



$p(x) \geq q(x)$ when

$$3 \leq x \leq 11$$

\therefore This is periodic

$$\therefore 3 + pn \leq x \leq 11 + pn, n \in \mathbb{Z}$$

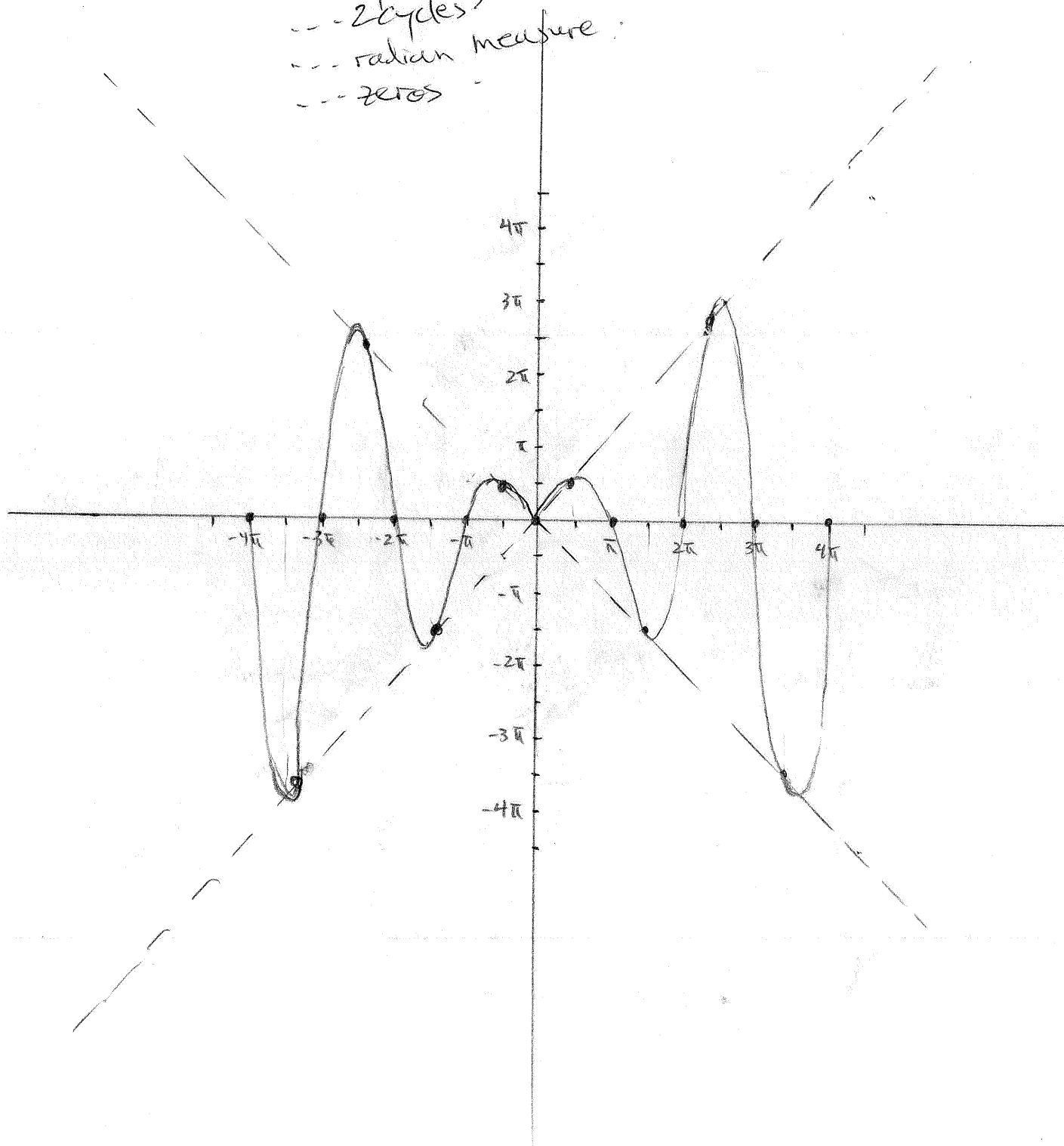
$$\therefore 3 + 24n \leq x \leq 11 + 24n, n \in \mathbb{Z}$$

$$p = 2\pi \div \frac{\pi}{12}$$

$$p = 24$$

7. Using the periodic nature of a sinusoidal function, produce a **reasonably accurate** sketch of the function $y = x \sin x$. Be sure to show the intervals at which the maximum, minimum, and zero values occur by drawing and labelling these points (two cycles to the left, and to the right, of the vertical axis). [4 marks]

~~2~~ 4 --- symmetry (even).
 --- 2 cycles
 --- radian measure.
 --- zeros



During the previous course of a number of months, I have been engaged in the study of the history of the United States, and have been particularly interested in the life of George Washington. I have been particularly interested in the life of George Washington, and have been particularly interested in the life of George Washington.

Washington was born on February 22, 1732, in the town of Stratford, Maryland. He was the second of five children of Augustine Washington, a planter and a member of the House of Burgesses. His mother was Mary Ball Washington.

He was educated at the College of William and Mary in Virginia, and at the University of Pennsylvania in Philadelphia.

Washington was a member of the Continental Congress, and was elected President of the United States in 1789.

He died on December 14, 1799, at Mount Vernon, Virginia.

His death was a great loss to the young nation.