Date: Dec 17 2014 Name: Uni Lee

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MHF4U Test #4: Trigonometry

Parent Signature: ____________

K & U: 15.5 /18 /21 **APP:** 12 /12 **Comm:** 6 /9 **TIPS:** /8

Part A: Knowledge and Understanding. [18 marks]

Short Answers. [13 marks]

1. Three related angles, between 0 and 2π, to $\frac{\pi}{4}$ are $\frac{3\pi}{4}$ $\frac{5\pi}{4}$ $\frac{7\pi}{4}$ 1.5 1½✓
2. The next co-terminal, positive angle to $\frac{3\pi}{11}$ is $\frac{14\pi}{11}$ ½✓
3. Four co-related angles, between 0 and 2π, to $\frac{2\pi}{5}$ are $\frac{\pi}{10}$ $\frac{9\pi}{10}$ $\frac{11\pi}{10}$ $\frac{19\pi}{10}$ 2 ✓✓
4. The equivalent angle measure to 80° in radians is $\frac{4\pi}{9}$ or ~~1.396~~ ½✓ 1
5. The equivalent angle measure to $\frac{7\pi}{15}$ rad in degrees is 84° ½✓
6. What are the two different trigonometric relationships you can write about complementary acute angles? $\cos x = \sin\theta$ $\tan\theta = \cot x$ of 2 ✓✓
7. Angles, measured from standard position, that look identical are co-terminal angles ✓ 1
8. An equivalent trig. expression to $\cos\left(\frac{5\pi}{8}\right)$, using a co-related acute angle is $-\sin\frac{\pi}{8}$ ✓ ½
9. $P\left(\cos\frac{5\pi}{12}, \sin\frac{5\pi}{12}\right)$ can be rewritten, using a co-related acute angle, as $\left(\sin\frac{\pi}{12}, \cos\frac{\pi}{12}\right)$ ✓
10. Determine the exact values for each of the following: ✓✓✓

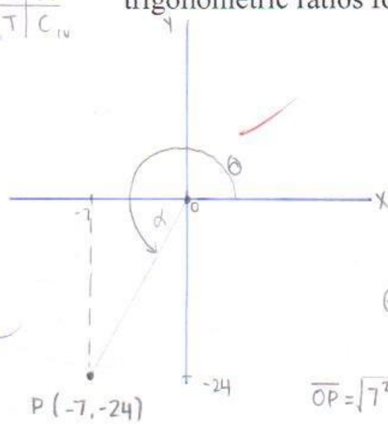
a) $\sin\left(\frac{5\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

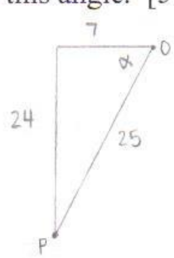
b) $\sec\left(\frac{41\pi}{6}\right) = \sec\left(\frac{5\pi}{6}\right) = -\frac{2}{\sqrt{3}}$

c) $\cot\left(\frac{-23\pi}{2}\right) = \cot\left(\frac{\pi}{2}\right)$ = undefined 2

11. The point $P(-7, -24)$ is on the terminal arm of an angle measured from standard position. Draw a diagram to model this information, then determine the exact values for the primary and secondary trigonometric ratios for this angle. [5 marks]

"S | A"
"T | C"





$\theta = \frac{\pi}{2} + \alpha$

$\overline{OP} = \sqrt{7^2 + 24^2}$

$= 25$ units

$\alpha = 73.73°$

$\alpha = 1.287$ rad

$-\sin\alpha = \frac{24}{25}$

$-\cos\alpha = \frac{7}{25}$

$\tan\alpha = \frac{24}{7}$

$\sin\left(\frac{\pi}{2}+\alpha\right) = \frac{24}{25}$

$\sin\theta = -\frac{24}{25}$

$\cos\left(\frac{\pi}{2}+\alpha\right) = -\frac{7}{25}$

$\cos\theta = \frac{7}{25}$

$\tan\left(\frac{\pi}{2}+\alpha\right) = \frac{24}{7}$

$\tan\theta = \frac{24}{7}$

3

14.5

$\cos(2x) = \cos^2 x - \sin^2 x$

$\sin 2x = 2\sin x \cos x$

12. Express as a single trigonometric ratio. [3 marks]

a) $2 - 4\sin^2\left(\frac{5\theta}{2}\right)$

$= 2 - 2\sin^2\left(\frac{5\theta}{2}\right) - 2\sin^2\left(\frac{5\theta}{2}\right)$

$= 2\sin^2\left(\frac{5\theta}{2}\right) + 2\sin^2\left(\frac{5\theta}{2}\right) - 1 - 1$

$= \cos\left[2\left(\frac{5\theta}{2}\right)\right] + \cos\left[2\left(\frac{5\theta}{2}\right)\right]$

$= 2\cos(5\theta)$

1

b) $2\cos^2\left(\frac{x}{2} + \frac{\pi}{4}\right) - 1$

$= 2\left(\cos\frac{x}{2}\cos\frac{\pi}{4} - \sin\frac{x}{2}\sin\frac{\pi}{4}\right) - 1$

$= 2\left(\cos^2\frac{x\pi}{8} - \sin^2\frac{x\pi}{8}\right) - 1$

$= 2\left(\cos 2\left(\frac{x\pi}{8}\right)\right) - 1$

? C

c) $3\sec x \csc x$

$= 3\,\frac{1}{\cos x \sin x}$

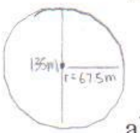
$= \frac{3}{\cos x \sin x}$

$=$

2

1

Part B: Applications. [12 marks]



13. The London Eye is a large Ferris wheel located on the banks of the Thames River in London, England. Each sealed and air-conditioned passenger capsule holds about 25 passengers. The diameter of the wheel is 135 m, and the wheel takes about half an hour to complete one revolution. Determine the following: [4 marks]

a) How far, along the circumference of the wheel, does a passenger travel in 5 minutes?

1 rev = 30 mins
$= 30\,\text{min} \times \frac{60\,s}{1\,min}$
$= 1800\,s$

$C = \pi d$
$= \pi(135m)$
$\doteq 424.115\,m$

$5\,min = 5\,mins \times \frac{60\,s}{1\,min}$
$= 300\,s$

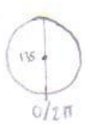
distance = speed · Δt
$= 0.2356\,m/s \cdot (300\,s)$
$\doteq 70.68\,m$

capsule speed $= \frac{424.115m}{1800\,s}$
$\doteq 0.2356\,m/s$

2.5

Therefore, a passenger travels about 70.68 m in 5 minutes

b) What is the angular velocity of a passenger, in radians per second?



speed $\doteq 0.2356\,m/s$

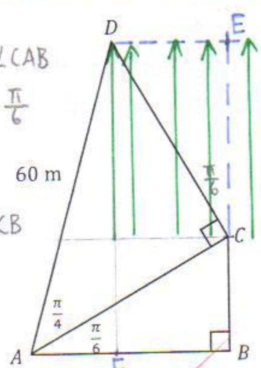
1 rev = 2π

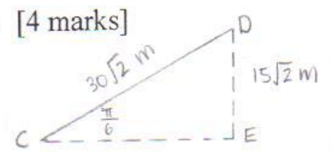
ang velocity $= \frac{0.2356\,m/s}{424.115\,m} \times 2\pi = 0.00349$ rad/s

1.5

∴, the angular velocity is 0.00349 rad/s

14. Find the vertical distance D is above C, as an exact value, given the diagram below. [4 marks]





$\angle ACB = \pi - \frac{\pi}{2} - \angle CAB$
$= \pi - \frac{\pi}{2} - \frac{\pi}{6}$
$= \frac{\pi}{3}$ rad

$\angle DCE = \pi - \frac{\pi}{2} - \angle ACB$
$= \pi - \frac{\pi}{2} - \frac{\pi}{3}$
$= \frac{\pi}{6}$ rad

$\cos\frac{\pi}{4} = \frac{\overline{CD}}{60}$

$\frac{\sqrt{2}}{2} = \frac{\overline{CD}}{60}$

$2\overline{CD} = 60\sqrt{2}$

$\overline{CD} = 30\sqrt{2}\,m$

$\frac{\overline{AF}}{60} = \cos(\angle DAF)$

$= \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \left(\frac{\sqrt{6}}{4}\right) - \left(\frac{\sqrt{2}}{4}\right)$

$\frac{\overline{AF}}{60} = \frac{\sqrt{6} - \sqrt{2}}{4}$

$4\overline{AF} = 60(\sqrt{6} - \sqrt{2})$

$\overline{AF} = \frac{60\sqrt{6} - 60\sqrt{2}}{4}$

$\overline{AF} = 15\sqrt{6} - 15\sqrt{2}\,m$

4

$\cos\frac{\pi}{4} = \frac{\overline{AC}}{60}$

$\frac{\sqrt{2}}{2} = \frac{\overline{AC}}{60}$

$2\overline{AC} = 60\sqrt{2}$

$\overline{AC} = 30\sqrt{2}\,m$

$\cos\frac{\pi}{6} = \frac{\overline{AB}}{\overline{AC}}$

$\frac{\sqrt{3}}{2} = \frac{\overline{AB}}{30\sqrt{2}}$

$2\overline{AB} = \sqrt{3}(30\sqrt{2})$

$2\overline{AB} = 30\sqrt{6}$

$\overline{AB} = 15\sqrt{6}\,m$

$\overline{DE} = \overline{AB} - \overline{AF}$
$= 15\sqrt{6} - (15\sqrt{6} - 15\sqrt{2})$
$= 15\sqrt{6} - 15\sqrt{6} + 15\sqrt{2}$

$\overline{DE} = 15\sqrt{2}\,m$

$\cos\frac{\pi}{6} = \frac{\overline{CE}}{\overline{CD}}$

$\frac{\sqrt{3}}{2} = \frac{\overline{CE}}{30\sqrt{2}}$

$2\overline{CE} = \sqrt{3}(30\sqrt{2})$

$2\overline{CE} = 30\sqrt{6}$

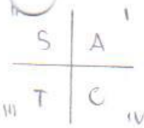
$\overline{CE} = 15\sqrt{6}\,m$

3

Therefore, the vertical distance D is above C is $15\sqrt{6}$ m

15. Find an exact value for each of the following. Express in simplest form. [4 marks]

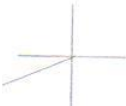
a) $\cos\left(\frac{8\pi}{9}\right)\cos\left(\frac{5\pi}{18}\right) - \sin\left(\frac{8\pi}{9}\right)\sin\left(\frac{5\pi}{18}\right)$



$= \cos\left(\frac{8\pi}{9} + \frac{5\pi}{18}\right)$

$= \cos\left(\frac{16\pi}{18} + \frac{5\pi}{18}\right)$

$= \cos\frac{21\pi}{18}$



$= \cos\frac{7\pi}{6}$

$= -\cos\frac{\pi}{6}$

$\boxed{= -\frac{\sqrt{3}}{2}}$

2

b) $\sin 105°$

~~$=\sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$~~ degrees - -

$= \cos\frac{\pi}{4}\sin\frac{\pi}{3} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$ acc

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

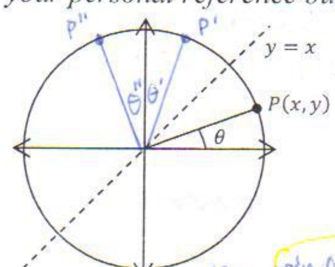
$\boxed{= \frac{\sqrt{6}+\sqrt{2}}{4}}$

2

4

Part C: Communication. [9 marks]

16. Derive a co-function formula for sine and cosine, relating θ (an acute angle) to $\frac{\pi}{2} + \theta$. Provide a detailed explanation, **using words only**, on how you obtained the relationship. (*The diagram is for your personal reference but will not be looked at as part of your explanation*). [4 marks]



- we have the point $P(x,y)$ which is $P(\cos\theta, \sin\theta)$. When we reflect this over the $y=x$ line, point P' is created with coordinates of (y,x) which is $(\sin\theta, \cos\theta)$. Then it is reflected over the y axis which makes point P'' with the coordinates (y,x) which is $(-\sin\theta, \cos\theta)$. GIE

$(-y, x)$? ? ?

so, $\sin\theta = y \rightarrow \sin\theta' = x \rightarrow \sin\theta'' = x \rightarrow \sin\left(\frac{\pi}{2}+\theta\right) = x \rightarrow \boxed{\sin\left(\frac{\pi}{2}+\theta\right) = \sin\theta}$

so, $\cos\theta = x \rightarrow \cos\theta' = y \rightarrow \cos\theta'' = -y \rightarrow \cos\left(\frac{\pi}{2}+\theta\right) = -y \rightarrow \boxed{\cos\left(\frac{\pi}{2}+\theta\right) = -\cos\theta}$

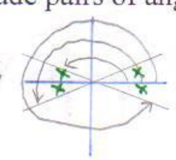
because $\theta'' = \frac{\pi}{2} + \theta$

sub in yellow

2.5

17. Explain, in detail, what is meant by "related angles", "co-related angles", and co-terminal angles. Be sure to include pairs of angles that satisfy each type. [5 marks]

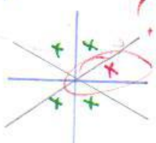
related angles



such as: $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

- related angles are measured from standard position and in each quadrant, when they add, or subtract the acute from the x axis, those angles are related.

co-related angles

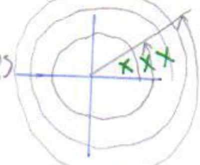


such as $\frac{\pi}{6}$ and $\frac{\pi}{3}$

- the red angle and green angle are co-related. they add up to $\frac{\pi}{2}$. used in trig relation ships. when $\sin X$ will be equal to $\cos X$. Also $\tan X = \tan X$.

3.5

co-terminal angles



such as $\frac{\pi}{6}$ and $\frac{13\pi}{6}$

$+2\pi$

- co-terminals look the same when measured from standard position. Its just they add or minus extra revolutions. Infinite amount of co-terminal angles because you ~~just~~ +/- $2\pi n$ on your angle

6

Part D: Thinking, Inquiry and Problem Solving. [8 marks]

19. Prove the following identities. [8 marks]

a) $\dfrac{\sec(\pi - x)}{\sin(\pi + x)} = \tan(\pi + x) + \cot(x)$

✓✓✓✓

$LS = \dfrac{\sec(\pi - x)}{\sin(\pi + x)}$

$= \dfrac{\left(\frac{1}{\cos(\pi - x)}\right)}{\cos\pi\sin x + \cos x\sin\pi}$

$= \dfrac{\left(\frac{1}{\cos\pi\cos x + \sin\pi\sin x}\right)}{\cos\pi\sin x + \cos x\sin\pi}$

$= \dfrac{1}{\cos\pi\cos x + \sin\pi\sin x} \times \dfrac{1}{\cos\pi\sin x + \cos x\sin\pi}$

$\because LS = RS$

$\therefore$ It is an identity

How?

I tried...

$RS = \tan(\pi + x) + \cot(x)$

$= \dfrac{\tan\pi + \tan x}{1 - \tan\pi\tan x} + \dfrac{1}{\tan x}$

$= \dfrac{\left(\frac{\sin\pi}{\cos\pi} + \frac{\sin x}{\cos x}\right)}{1 - \left(\frac{\sin\pi}{\cos\pi} \cdot \frac{\sin x}{\cos x}\right)} + \dfrac{1}{\left(\frac{\sin x}{\cos x}\right)}$

$= \dfrac{\left(\frac{\sin\pi\cos x + \sin x\cos\pi}{\cos\pi\cos x}\right)}{1 - \left(\frac{\sin\pi\sin x}{\cos\pi\cos x}\right)} + \dfrac{1}{\left(\frac{\sin x}{\cos x}\right)}$?

$= \dfrac{\left(\frac{\sin\pi + x}{\cos\pi\cos x}\right)}{1 - \frac{\sin\pi\sin x}{\cos\pi\cos x}} + \dfrac{1}{\left(\frac{\sin x}{\cos x}\right)}$? ?

b) $\sin 3A\csc A - \cos 3A\sec A = 2$

✓✓✓✓

$LS = \sin(3A)\csc(A) - \cos(3A)\sec(A)$

$= \sin(3A)\cdot\frac{1}{\sin A} - \cos 3A \cdot \frac{1}{\cos A}$

$= \dfrac{\sin 3A}{\sin A} - \dfrac{\cos 3A}{\cos A}$

$= \sin 2A - \cos 2A$ X

$= 2\sin A\cos A - \cos^2 X - \sin^2 X$?

$= -2\sin A\cos A + \cos^2 X + \sin^2 X$

$= -2\sin A\cos A + 1$

$= 1 + 1$

$= 2$

$RS = 2$

$\because LS = RS$

$\therefore$ It is an identity

I tried...

Just write a random therefore statement for part marks