KU: 21 / 23 TH: 11 / 14 COMM: 3+3 / 3+3 APPS: 14.5 /15

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Date: April 7 2015

91%

UNIT 3 TEST: Applications of Derivatives

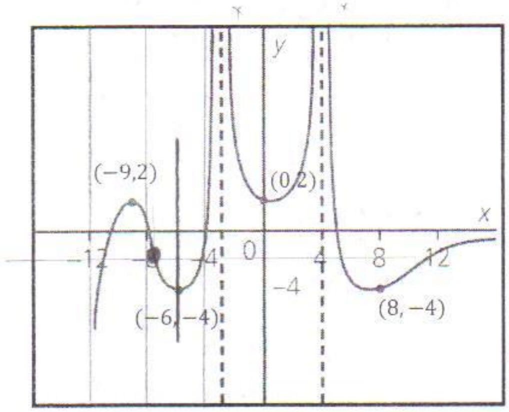
Instructions:
1. Read the instructions carefully.
2. Attempt all problems and don't spend all your time on one problem. Other problems will feel left out ☹
3. Show all work in the space provided. Three (3) marks will be given to the overall mathematical form on this test.
4. ENJOY ☺

PART A: KNOWLEDGE AND UNDERSTANDING

Short Answers

Write the correct answer in the space provided. Full marks will be given for the correct answer.

1. A function $y = f(x)$ is defined in the following graph. The critical points have been located for you. [6]



a) State the intervals where the function is increasing.	$x<-9$, $-6<x<-3$, $0<x<4$, $x>8$ ✓
b) State the intervals where $f'(x) < 0$.	$-9<x<-6$, $-3<x<0$, $4<x<8$ ✓
c) Write the equations for any vertical asymptotes.	VA $x=-3$, $x=4$ ✓
d) What is the value of $f''(x)$ on the interval $-3 < x < 3$?	positive ✓
e) If $x \geq -6$, state the intervals where $f'(x) < 0$ and $f''(x) > 0$.	$-3<x<0$ $4<x<8$ ✓
f) Identify a point of inflection and state the approximate ordered pair for the point.	$(-7.5, -2)$ ✓

6/6

Complete solutions must be shown for full marks.

2. Find and classify the nature of all critical points of the function $f(x) = 3x^5 - 25x^3 + 60x$. [5]

$f'(x) = 15x^4 - 75x^2 + 60$

$\emptyset = (x+2)(15x^3 - 30x^2 - 15x + 30)$

$= (x+2)(x-2)(x+1)(x-1)$

$x=-2 \quad x=2 \quad x=-1 \quad x=1$

$f(-2) = -16 \quad f(2) = 16 \quad f(-1) = -38 \quad f(1) = 38$

$(-2,-16) \quad (2,16) \quad (-1,-38) \quad (1,38)$

5/5

f'(x)

−2	−1	1	2
+ −	− +	+ −	− +
inc dec	dec inc	inc dec	dec inc
∴ (−2, −16) is a local max open down	∴ (−1, −38) a local min open up	∴ (1, 38) is a local ~~max~~ open down	∴ (2, 16) is a local min open up

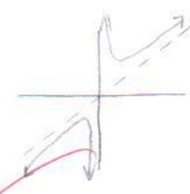
3. Find the interval(s) in which the function $f(x) = \frac{x^2+1}{x}$ is increasing. [4]

$= \frac{x^2}{x} + \frac{1}{x}$

$= \frac{1}{x} + x$

OA $y = x$

VA $x = 0$



$f'(x) = \frac{-1}{x^2} + 1$

$0 = \frac{-1}{x^2} + 1$

$1 = \frac{1}{x^2}$

$x^2 = 1$

$x = \pm\sqrt{1}$

4/4

f'(x)

−1	+1
+ −	− +
increasing decreasing	decreasing increasing

nice!

∴ the function is increasing at $x < -1$, $x > 1$

9 :)

4. Given $f(x) = \frac{x}{(x+2)^2}$, $f'(x) = \frac{2-x}{(x+2)^3}$, $f''(x) = \frac{2x-8}{(x+2)^4}$ [8]

$2x=8$, $x=\frac{8}{2}$, $=4$

Find, showing work

a) x-intercept(s)	b) y-intercept(s)	c) vertical asymptote(s)
$x=\emptyset$ (0,0)	$y=\emptyset$ (0,0)	VA $x=-2$
d) horizontal asymptote(s)	e) nature and coordinates of extremum/extrema max or min? -1	f) coordinates of point(s) of inflection
HA $y=0$	$(2, \frac{1}{8})$ opens up	$(4, \frac{1}{9})$ -1

$0=(x+2)^2$ → $x=-2$

$f(x) = \frac{0}{(0+2)^2}$

$f'(x) = \frac{2-x}{(x+2)^3}$ → $x=2$

$f(2) = \frac{2}{(2+2)^2} = \frac{2}{16} = \frac{1}{8}$

$f(4) = \frac{4}{(4+2)^2} = \frac{4}{36} = \frac{1}{9}$

6/8.

-1 need to check concavity change

	$x<2$	$x>2$
$f'(x)$	−	+
	incres	decrease

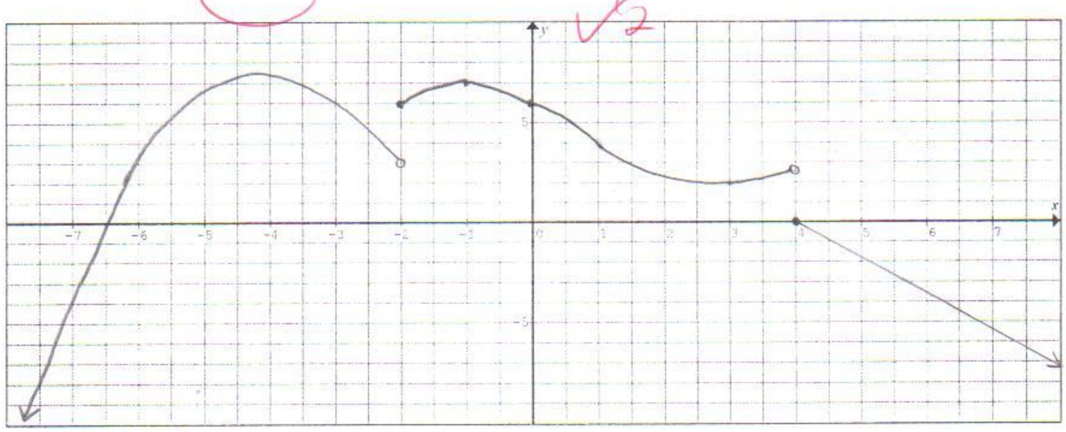
min

− min +

PART B: APPLICATIONS

5. Sketch the graph of a function with the following properties: [4]
   - There are local extrema (but not global extrema) at (-1,7) and (3,2).
   - There is a point of inflection at (1,4).
   - The graph is concave down only when $x < 1$.
   - The x-intercept is (−4,0) and the y-intercept is (0,6).

½



3.5/4

/8+3.5

6. Sketch the function $y = x^4 - 8x^2 + 7$ indicating key features as discussed in class. (3 marks will be given towards communication for clearly showing your steps) [11] A [3] C

y-int = 7

$y' = 4x^3 - 16x$

$y'' = 12x^2 - 16$

$16 = 12x^2$

$\frac{4}{3} = x^2$

$\pm\sqrt{\frac{4}{3}} = x$

POI's

$y' = 4x^3 - 16x$

$\varnothing = 4x^3 - 16x$

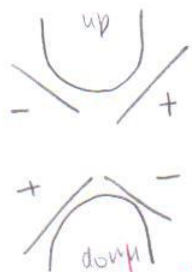
$= 4x(x^2 - 4)$

$x = 0$ $x = 2$ $x = -2$

$y(0) = 7$ $y(2) = -9$ $y(-2) = -9$

crit points

$(0,7)$ $(2,-9)$ $(-2,-9)$



y': 0: + increasing, − decreasing — open down

−2: − decr, + incr. — open up

2: − decr., + incr. — open up

$y'(+\sqrt{\frac{4}{3}}) = -1.\overline{888}$

$y'(-\sqrt{\frac{4}{3}}) = -1.\overline{888}$

$y = x^4 - 8x^2 + 7$

$= (x-1)(x^3 + x^2 - 7x - 7)$

$= (x-1)(x+1)(x^2 - 7)$

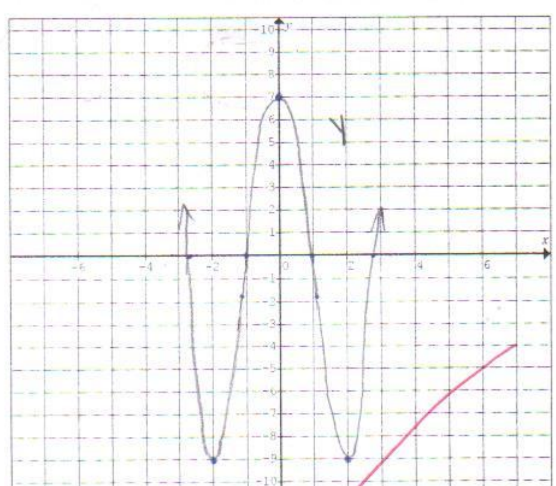
$$\begin{array}{c|ccccc} 1 & 1 & 0 & -8 & 0 & 7 \\ + & \downarrow & 1 & 1 & -7 & -7 \\ \hline & 1 & 1 & -7 & -7 & \varnothing \end{array}$$

$$\begin{array}{c|cccc} -1 & 1 & 1 & -7 & -7 \\ + & \downarrow & -1 & 0 & 7 \\ \hline & 1 & \varnothing & -7 & \varnothing \end{array}$$

$x = 1$ $x = -1$ $x = \pm\sqrt{7}$

$= \pm 2.64$

$\boxed{\begin{array}{l} \text{x-int} \\ x = 1 \\ x = -1 \\ x = 2.64 \\ x = -2.64 \end{array}}$



11 + 3 (14)

PART C: THINKING

7. a) Create an equation for a polynomial function $f(x)$ such that $f(x)$ is above the x-axis when $x > 3$ and below the x-axis when $x < 3$. [6]

$$f(x) = x - 3$$

b) Create an equation for a polynomial function $g(x)$ such that $g(x)$ is an increasing function when $x > -1$ and is a decreasing function when $x < -1$.

$$g(x) = |(x+1)|$$

6/6.

c) Create an equation for a polynomial function $h(x)$ such that $h(x)$ concaves up when $x > 5$ and concaves down when $x < 5$.

$$h(x) = (x-5)^3$$

8. Determine values for a, b, c and d that guarantee that the function $f(x) = ax^3 + bx^2 + cx + d$ will have a local maximum at $(1, -7)$ and a point of inflection at $(2, -11)$. [8]

sub in $(1,-7)$

$f(x) = ax^3 + bx^2 + cx + d$

$-7 = a(1)^3 + b(1)^2 + c(1) + d$

$-7 = a + b + c + d$ ④

sub in $(2,-11)$

$f(x) = ax^3 + bx^2 + cx + d$

$-11 = a(2)^3 + b(2)^2 + c(2) + d$

$-11 = 8a + 4b + 2c + d$ ③

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

∵ local max at $(1,-7)$, point on $f'(x)$ will be $(1,0)$

sub in $(1,0)$

$f'(x) = 3ax^2 + 2bx + c$

$0 = 3a(1)^2 + 2b(1) + c$

$\emptyset = 3a + 2b + c$ ②

$-3a - c = 2b$ ⑥

$c = 2b - 3a$ ⑨

∵ POI at $(2,-11)$, point of $f''(x)$ will be $(2,0)$.

sub in $(2,0)$

$f''(x) = 6ax + 2b$

$0 = 6a(2) + 2b$

$\boxed{\emptyset = 12a + 2b}$ ok.

$6b = a$ ①

this is CORRECT

5/8

sub ① into ②

$0 = 3a + 2b + c$

$= 3(6b) \cancel{- 2b} + c$

$= 18b + 2b + c$

$= 20b + c$

$c = -20b$ ⑤

ok idea

⑦ into ⑤

$-20b = 18 + 7a + 3b$

$-23b = 18 + 7a$

$b = -\frac{18+7a}{23}$ ⑪

④ - ③

$-7 = a + b + c + d$

$- \quad -11 = 8a + 4b + 2c + d$

$18 = -7a - 3b - c$

$c = 18 + 7a + 3b$ ⑦

-3

⑪ into ⑩

$\frac{-18+7a}{23} = -\frac{9a}{6}$

$9a(23) = 6(18+7a)$

$207a = 108 + 42a$

☺ The end ☺

$165a = 108$

$a = 0.6545$

i dont know ¨∩

⑨ into ⑧

$2b - 3a = -12a - 4b$

$6b = -9a$

$b = -\frac{9}{6}a$ ⑩

this isn't

$f'(x) = 3ax^2 + 2bx + c$

$0 = 3a(2)^2 + 2b(2) + c$

$\emptyset = 12a + 4b + c$

$-12a - 4b = c$ ⑧

11