MCR 3U

TEST: TRIGONOMETRIC EQUATIONS

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Show all work using good form for full marks.
Good luck!

K/U	THK	COMM	APP
17 /21 (18)	5.5 /8	5 /6	19 /19

Part A: Knowledge and Understanding /21

Multiple Choice - Write the answer on the line provided. Each question is worth one mark.

1. B ✓ What is the exact value of $\tan 30°$?

a) $\sqrt{3}$ b) $\frac{\sqrt{3}}{3}$ c) $\frac{\sqrt{3}}{2}$ d) 1

2. B ✓ What is the exact value of $\sin -225°$?

a) $\frac{2}{\sqrt{3}}$ b) $\frac{\sqrt{2}}{2}$ c) -1 d) $-\frac{\sqrt{2}}{2}$

3. C ✓ What is the exact value of $\cot 450°$?

a) 1 b) -1 c) 0 d) undefined

4. D ✓ Find the value(s) of θ for $\tan\theta = \sqrt{3}$, for $0° \le \theta \le 360°$.

a) $-60°$ and $120°$ b) $-60°$ and $150°$ c) $120°$ and $300°$ d) $60°$ and $240°$

5. B ✓ Find the value(s) of θ for $\sec\theta = -\frac{2\sqrt{3}}{3}$, for $0° \le \theta \le 360°$.

a) $30°$ and $150°$ b) $150°$ and $210°$ c) $30°$ and $210°$ d) $210°$ and $330°$

6. A ✓ Find the value(s) of θ for $\csc\theta = 1.2208$, for $0° \le \theta \le 360°$.

a) $55°$ and $125°$ b) $235°$ and $305°$ c) $55°$ and $305°$ d) $55°$ and $235°$

7. D ✓ $P(2,-3)$ lies on the terminal arm of an angle in standard position on the unit circle. What is the value of the principal angle θ to the nearest degree?

a) $56°$ b) $146°$ c) $236°$ d) $304°$

8. A ✓ Given $\cos 170°$, determine which of the following is an equivalent expression.

a) $\cos 190°$ b) $\cos 10°$ c) $\cos 350°$ d) $\cos 280°$

/8

9. D ✓ In ΔABC we know $\angle A = 64°$, $\angle C = 90°$ and $c = 9cm$. What is the length of side b?

a) 20.53cm b) 10.01cm c) 4.39cm d) 3.95cm

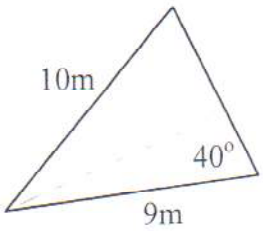
10. C ✓ Which of the following is true?

a) The ambiguous case cannot occur in a side-side-angle triangle.
b) The reference angle is the smallest angle from the terminal arm to the x- or y-axis.
c) In ΔABC with $\angle C = 90°$, $\sin \angle A = \cos \angle B$.
d) A trigonometric equation is true for all defined values of the variable.

2

11. Determine if the following triangles have no solution, one solution, or two solutions. (3)

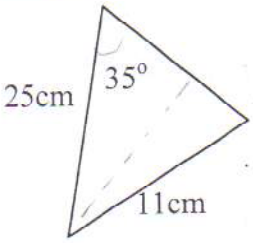
a)



2

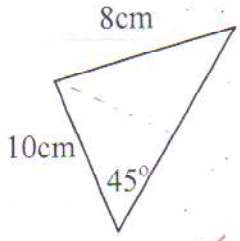
two ✗

b)



none ✓

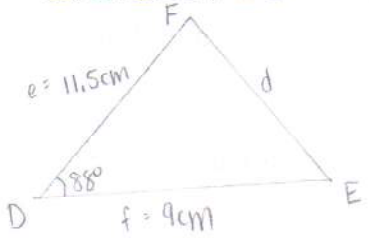
c)



two ✓

Round side lengths to one decimal place and angles to the nearest whole number. For #12 and #13 it is recommended that you include a diagram in your solutions.

12. In ΔDEF, $\angle D = 88°$, $e = 11.5$ cm and $f = 9$ cm. Solve for the unknown length and angles. (5)



$d^2 = e^2 + f^2 - 2ef \cos 88°$

$\sqrt{d^2} = \sqrt{11.5^2 + 9^2 - 2(11.5)(9)\cos 88}$

$d = 14.353599$ cm

$\frac{\sin 88}{14.353599} = \frac{\sin F}{9}$

$\sin F (14.353599) = 8.994517443$

$\sin F = 0.626638478$

$\angle F = 38.8$

$\angle F = 38.8°$

$\angle E = 180° - 38.8° - 88°$

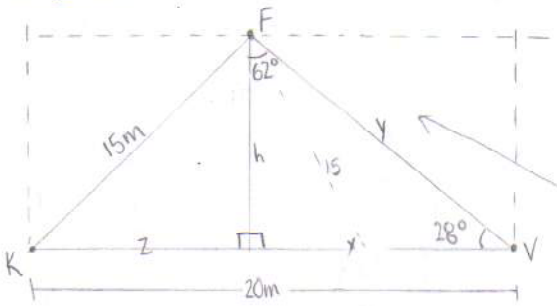
$= 53.2°$

$\angle D = 88°$
$\angle E = 53°$
$\angle F = 39°$
$d = 14.4$ cm
$e = 11.5$ cm
$f = 9$ cm ✓

9

Part B: Application /19

13. Vivian has just observed a firework go off in the sky at an angle of 28° with the ground. Khadija was 15 m away from the firework and was standing 20 m from Vivian. How far was Vivian from the firework when it went off in the sky? Round to the closest metre. **Include a well-labelled diagram.** (6)



$x = 20 - z$

$15^2 = 20^2 + y^2 - 2(20)(y)\cos 28°$

$225 = 400 + y^2 - 40y\cos 28°$

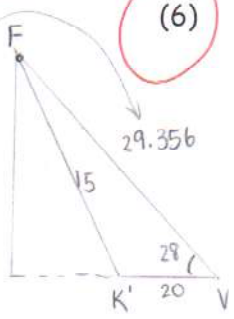
$225 = 400 + y^2 - 35.3179y$

$y^2 - 35.3179y + 175$

$y = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{35.3179 \pm \sqrt{(35.3179)^2-4(1)(175)}}{2(1)}$

$= \frac{35.3179 \pm \sqrt{547.3540604}}{2}$



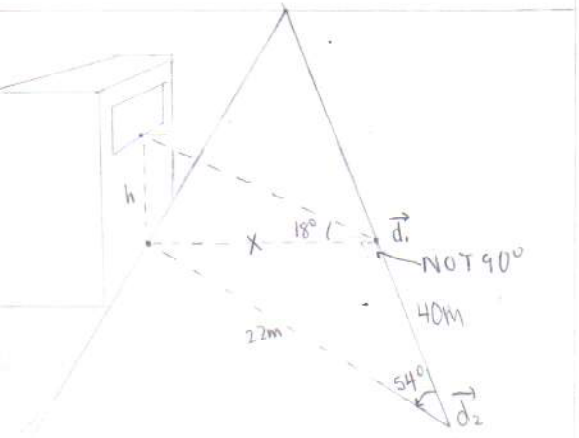
$y = 29.356$ m OR $y = 5.96115$

Depends on where vivian is standing, she can be 29m or 6m away.

Sine Law is easier

Interesting method (11)

14. Sarah spots One Direction on a hotel balcony. She needs to figure out how high they are so she can buy the proper ladder. She stands directly across from the group and notices the angle of elevation to the group is 18°. Sarah then walks 40 m parallel to the road and observes the angle between the base of the hotel and Sarah's previous spot is 54°. At her second spot, Sarah notices (while crying) that she is only 22 m away from the base of the hotel. How far up is the group, to the nearest tenth of a metre? **Include a well-labelled diagram.** (5)



$x^2 = 22^2 + 40^2 - 2(22)(40)\cos 54°$

$x = 32.395955$ m

$\tan 18° = \frac{h}{x}$

$0.3249x = h$

$h = 0.3249(32.395955)$

$= 10.52608$ m ✓

Therefore, the group is 10.5 m off the ground

11

15. Prove each identity showing work of good form. (8)

a) $\sin^2 x\cos^2 x + \cos^4 x = (1 - \sin x)(1 + \sin x)$ (4)

Let θ represent x

LS = $\sin^2\theta \cos^2\theta + \cos^4\theta$

$= \sin^2\theta \cos^2\theta + \cos^2\theta \cos^2\theta$

$= \cos^2\theta (\sin^2\theta + \cos^2\theta)$

$= \cos^2\theta (1)$

$= \cos^2\theta$

$= (1 - \sin^2\theta)$

RS = $(1 - \sin\theta)(1 + \sin\theta)$

$= (1 - \sin^2\theta)$

$\therefore$ LS = RS

$\therefore$ it is an identity

b) $\sin x \tan x + \frac{\sin x}{\tan x} = \frac{1}{\cos x}$ (4)

Let θ represent x

LS = $\sin\theta \tan\theta + \frac{\sin\theta}{\tan\theta}$

$= \sin\theta\left(\frac{\sin\theta}{\cos\theta}\right) + \frac{\sin\theta}{\tan\theta}$

$= \frac{\sin^2\theta}{\cos\theta} + \frac{\sin\theta}{\tan\theta}$

$= \frac{\sin^2\theta}{\cos\theta} + \sin\theta \times \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta}{\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta \sin\theta}{\cos\theta \sin\theta} + \frac{\sin\theta \cos^2\theta}{\cos\theta \sin\theta}$

$= \frac{\sin^2\theta \sin\theta + \sin\theta \cos^2\theta}{\cos\theta \sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$

$= \frac{1}{\cos\theta}$

RS = $\frac{1}{\cos\theta}$

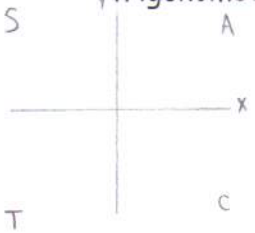
$\therefore$ LS = RS

$\therefore$ its an identity

Part C: Communication /6

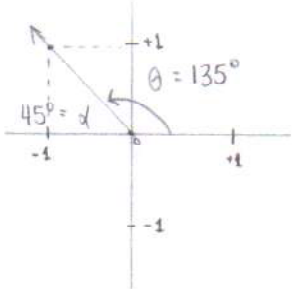
(/3 marks for equal signs, triangles labeled, units, L.S./R.S. for proofs)

16. Explain the roles of the reference angle α and the CAST rule in finding the exact value of trigonometric ratios for angles greater than 90°. Use an example to help you explain. (3)



α is the angle from the terminal arm to X-axis.

CAST shows where it is positive.



$\sin 135°$

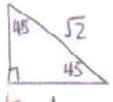
$\sin\theta = \sin\alpha$

$\sin 135° = \sin 45°$

because C A S T, 135° is positive

$\sin 45° = \frac{\sqrt{2}}{2}$ ← you use because it is easier.

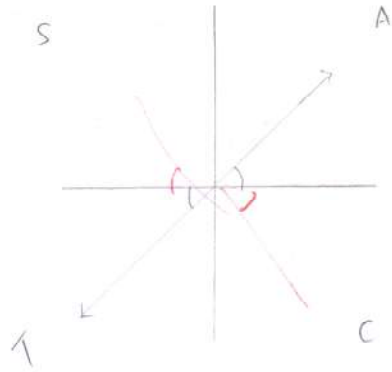
$\sin 135° = \frac{\sqrt{2}}{2}$



Why is it equivalent?

Part D: Thinking /8

17. Solve for θ in the equation $2\cot^2\theta - 5 = -3$, $0° \le \theta \le 360°$. (4)



Let x rep cot θ

$2x^2 - 5 = -3$

$2x^2 - 2 = 0$

$2x^2 + 2x - 2x - 2 = 0$

$2x(x+1) - 2(x+1) = 0$

$(2x-2)(x+1) = 0$

$2(x-1)(x+1) = 0$

$x = 1$ $\quad x = -1$

$\cot\theta = 1$ $\quad \cot\theta = -1$

$\theta = 45°$ $\quad \theta = 225°$

therefore,
$\theta = 45°, 225°$

3

18. Choose EITHER a) OR b) to answer. **Clearly indicate if you are solving a) or b)** (4)

a) Prove that

$$\frac{\csc^2 x + \sec^2 x}{\csc x \sec x} = \tan x + \cot x$$

Let θ represent x

LS: $\dfrac{\csc^2\theta + \sec^2\theta}{\csc\theta \sec\theta}$

$= \dfrac{\left(\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}\right)}{\left(\frac{1}{\sin\theta} \times \frac{1}{\cos\theta}\right)}$

$= \dfrac{\frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta \cos^2\theta}}{\frac{1}{\sin^2\theta \cos^2\theta}}$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\sin^2\theta \cos^2\theta} \times \dfrac{\sin^2\theta \cos^2\theta}{1}$

$= \dfrac{\cos^2\theta + \sin^2\theta}{1}$

$= 1$

RS: $\tan\theta + \cot\theta$

$= \tan\theta + \dfrac{1}{\tan\theta}$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\frac{\sin\theta}{\cos\theta}}$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{1} \times \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta}{\cos\theta \sin\theta} + \dfrac{\cos^2\theta}{\cos\theta \sin\theta}$

$= \dfrac{1}{\cos\theta \sin\theta}$

$= 1$

∵ LS = RS

∴ it is an identity

OR b) θ and β are angles in standard position. θ has its terminal arm in the forth quadrant and β has its terminal arm in the first quadrant. If $\sin\theta = -\frac{3}{5}$ and $\cos\beta = \frac{1}{3}$, what is the exact value of $3\cot\theta + \csc^2\beta - \cos\theta$?

2.5

Colour me?