Hoken

Date: Dec. 16/14 Name: Uni.

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MHF4U Test #5: Trigonometric Functions

K & U: 13 **/16** **APP:** 9 **/11** **Comm:** 11 **/11** **TIPS:** 11.5 **/12**

Part A: Knowledge and Understanding. [16 marks]

1. Fill in the blanks. [12 marks]

a) A basic cosine function has a minimum value that occurs at intervals of $\pi + 2\pi n,\ n \in \mathbb{Z}$ 1

b) The interval at which asymptotes occur in a cosecant function is $\frac{\pi}{2} + \pi n,\ n \in \mathbb{Z}$ 0.5

c) The function(s) with zeros at intervals of $\pi n, n \in \mathbb{Z}$ is/are $y = \sin x$ $y = \tan x$ 1

d) The function with a period of 2π and asymptotes at $\frac{\pi}{2} + \pi n, n \in \mathbb{Z}$ is $y = \sec x$ 1

e) Vertical translations for trig. functions should be referred to as vertical displacements 1

f) If a tangent ratio is negative it would mean that the terminal arm would lie in Q2 or Q4, and the measures for these two angles will be between $\frac{\pi}{2} < x < \pi$ or $\frac{3\pi}{2} < x < 2\pi$ (in radians). 2

S A
T C

0.5

g) The trigonometric function that is increasing throughout is the tangent function ½✓

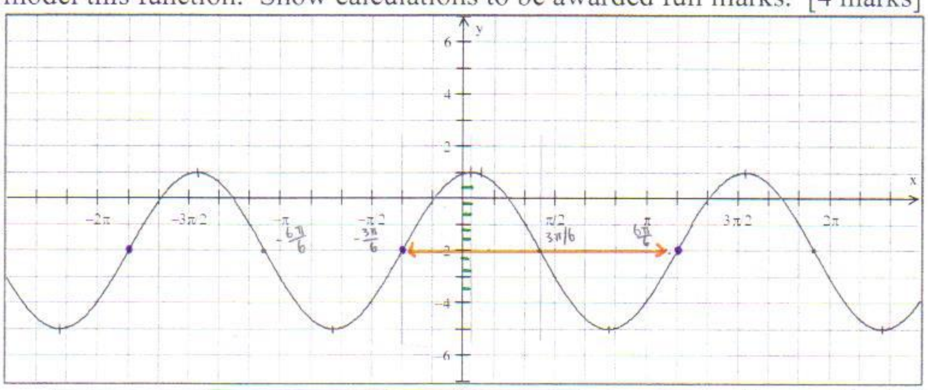
h) The instantaneous rate of change at $x = \frac{\pi}{2}$ for the function $y = \sin x$ is 0 ½✓ 0.5

i) The value(s) of x that satisfy the equation $\cos x = \frac{\sqrt{2+\sqrt{2}}}{2}$, $0 \le x \le 2\pi$, is/are $\frac{\pi}{12}$ or $\frac{23\pi}{12}$ 0

j) What is the equation of a cosecant function that will be identical to $y = \sec x$? $y = \csc\left(x + \frac{\pi}{2}\right)$ 1

k) The mapping rule for $y = 3\cos\left[\frac{3}{5}\left(x + \frac{\pi}{12}\right)\right] - 5$ is $(x, y) \rightarrow \left(\frac{5}{3}x - \frac{\pi}{12},\ 3y - 5\right)$ 1

2. A sinusoidal function is given in the graph below. Determine a sine and a cosine equation to model this function. Show calculations to be awarded full marks. [4 marks]



Therefore, the sine function is
$y = 3\sin\left[\frac{4}{3}\left(x + \frac{\pi}{3}\right)\right] - 2$
and the cosine function is
$y = 3\cos\left[\frac{4}{3}\left(x - \frac{\pi}{24}\right)\right] - 2$

3.5

$p = x_2 - x_1$
$= \frac{7\pi}{6} - \left(-\frac{2\pi}{6}\right)$
$= \frac{9\pi}{6}$

$|a| = \frac{y_{max} - y_{min}}{2}$
$= \frac{(+1) - (-5)}{2}$
$= \frac{6}{2}$
$= 3$

$c = \frac{y_{max} + y_{min}}{2}$
$= \frac{(+1) + (-5)}{2}$
$= -2$

For Sine Function:
$d = +\frac{2\pi}{6}$
$= +\frac{\pi}{3}$

For Cosine Function:
$d = -\frac{\pi}{24}$

$K = \frac{2\pi}{p}$
$= 2\pi \times \frac{6}{9\pi}$
$= \frac{12\pi}{9\pi}$
$= \frac{4}{3}$

q.i. $= \frac{p}{4}$
$= \frac{9\pi}{6} \times \frac{1}{4}$
$= \frac{9\pi}{24}$

Green
$= \frac{9\pi}{24} - \frac{8\pi}{24}$
$= \frac{\pi}{24}$

opposite

13

Part B: Applications. [11 marks]

3. The function $y = 4\cos^2\left(\frac{x}{3}\right) - 4\sin^2\left(\frac{x}{3}\right) + 3$ is equivalent to what sinusoidal function? Show calculations to support your answer. What is the period of this function? [3 marks]

$y = 4\cos^2\left(\frac{x}{3}\right) - 4\sin^2\left(\frac{x}{3}\right) + 3$

$= 4\left(1 - \sin^2\left(\frac{x}{3}\right)\right) - 4\sin^2\left(\frac{x}{3}\right) + 3$

$= 4 - 4\sin^2\left(\frac{x}{3}\right) - 4\sin^2\left(\frac{x}{3}\right) + 3$

$= 7 - 8\sin^2\left(\frac{x}{3}\right)$?

$= -8\sin^2\left(\frac{x}{3}\right) + 7$

$= -8\sin^2\left(\frac{1}{3}x\right) + 7$?

$K = \frac{2\pi}{p}$

$\frac{1}{3} = \frac{2\pi}{p}$

$p = 6\pi$

$\frac{6\pi}{2}$

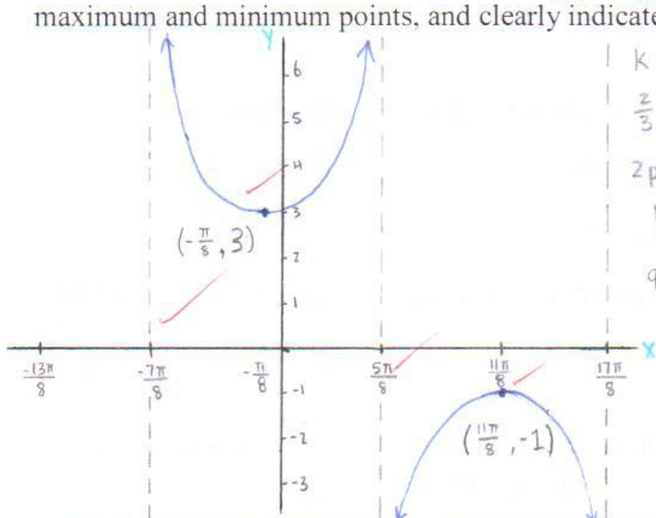
$\boxed{\text{Therefore, the period of this function is } 3\pi}$ OK.

- squared means all on the bottom ∴ p is halved

∴ Its equivalent to a sine square function with transformations

2.5

4. Sketch one cycle of the function $y = 2\sec\left[\frac{2}{3}\left(x + \frac{\pi}{8}\right)\right] + 1$. State the coordinates of the local maximum and minimum points, and clearly indicate all asymptotes. [4 marks]



$K = \frac{2\pi}{p}$

$\frac{2}{3} = \frac{2\pi}{p}$

$2p = 6\pi$

$p = 3\pi$

$q.i. = \frac{3\pi}{4}$

$= \frac{6\pi}{8}$

4

vertical asymptots $\frac{5\pi}{8} + \frac{3\pi}{2}n, n \in \mathbb{Z}$

5. Solve the equation given below. Express your answers as exact values. [4 marks]

$\cos x \cos\frac{3\pi}{8} - \sin x \sin\frac{3\pi}{8} = -\frac{\sqrt{3}}{2}$, $0 \le x \le 2\pi$

$\cos\left(x + \frac{3\pi}{8}\right) = -\frac{\sqrt{3}}{2}$ not mult.

$x + \frac{3\pi}{8} = (\cos^{-1})\left(-\frac{\sqrt{3}}{2}\right)$ inconsistent.

$x + \frac{3\pi}{8} = \frac{\pi}{6}$

$x_1 = \frac{\pi}{6} - \frac{3\pi}{8}$

$= \frac{8\pi}{48} - \frac{18\pi}{48}$

$= \frac{10\pi}{48}$ Q1 all pos

$= \frac{5\pi}{24}$ can't do this first

II S | A I
III T | C IV

$x_2 = \pi - \left(\frac{5\pi}{24}\right)$

$= \frac{24\pi}{24} - \frac{5\pi}{24}$

$= \frac{19\pi}{24}$

$x_3 = \pi + \frac{5\pi}{24}$

$= \frac{24\pi}{24} + \frac{5\pi}{24}$

$= \frac{29\pi}{24}$

2.5

Therefore, the angles that satisfy are $\frac{19\pi}{24}$ rad or $\frac{29\pi}{24}$ rad

9

Part C: Communication. [11 marks]

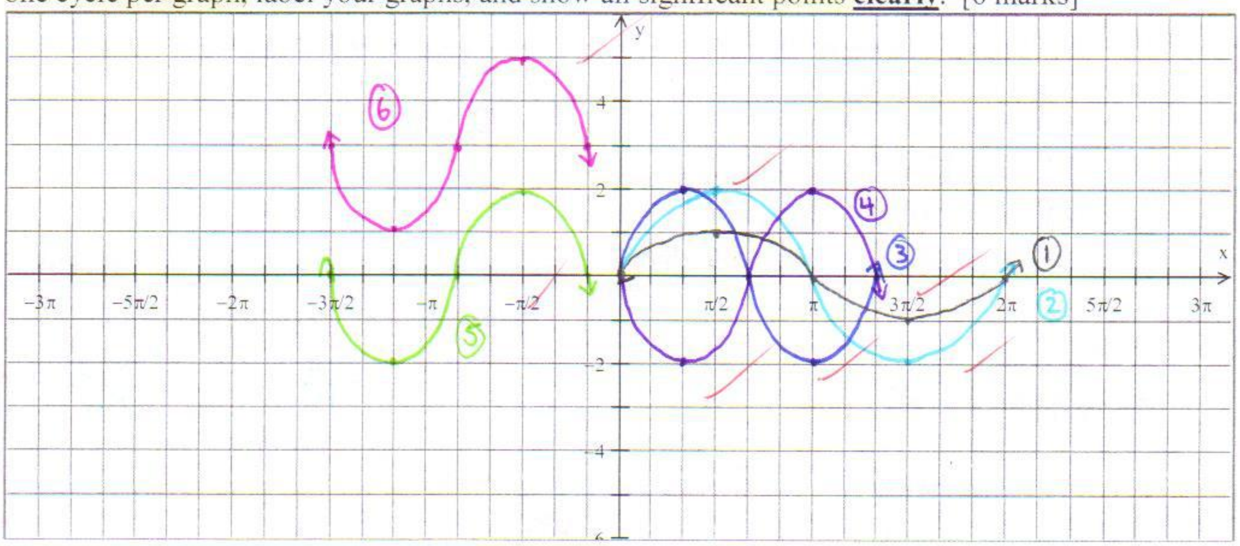
$\left[\frac{3}{4}\left(x + \frac{\pi}{9}\right)\right]$

6. Describe the transformations on the function $y = -3\cos\left(\frac{3}{4}x + \frac{\pi}{12}\right) + 7$. [5 marks]

The cosine function is vertically stretched by a factor of 3, then horizontally stetched by a factor of $\frac{4}{3}$, then reflected in the x axis. Then phase shifted $\frac{\pi}{9}$ rad to the left. Finally, theres a vertical displacement of 7 units upwards

5

7. Graph the function $y = -2\sin\left[\frac{3}{2}\left(x + \frac{3\pi}{2}\right)\right] + 3$, starting with the original, non-transformed function, then showing transformations in each successive graph, in the appropriate order. Draw one cycle per graph, label your graphs, and show all significant points **clearly**. [6 marks]



① $y = \sin x$

② $y = 2\sin x$

③ $y = 2\sin\left(\frac{3}{2}x\right)$

④ $y = -2\sin\left(\frac{3}{2}x\right)$

⑤ $y = -2\sin\left[\frac{3}{2}\left(x + \frac{3\pi}{2}\right)\right]$

⑥ $y = -2\sin\left[\frac{3}{2}\left(x + \frac{3\pi}{2}\right)\right] + 3$

$d = +\frac{3\pi}{2}$

$= +\frac{9\pi}{6}$

$=$ left $\frac{9\pi}{6}$ rad

$k = \frac{2\pi}{p}$

$\frac{3}{2} = \frac{2\pi}{p}$

$3p = 4\pi$

$p = \frac{4}{3}\pi$

q.i. $= p \div 4$

$= \frac{4\pi}{3} \times \frac{1}{4}$

$= \frac{4\pi}{12}$

$= \frac{2\pi}{6}$

6

11

Part D: Thinking, Inquiry and Problem Solving. [12 marks]

8. Solve the following trigonometric equation where $0 \le x \le 2\pi$. [5 marks]
$3\tan x = \tan 2x$

$$3\tan x = \frac{2\tan x}{1-\tan^2 x}$$

$$2\tan x = 3\tan x(1-\tan^2 x)$$

$$2\tan x = 3\tan x - 3\tan^3 x$$

$$-\tan x = -3\tan^3 x$$

$$0 = 3\tan^3 x - \tan x$$

$$0 = \tan x(3\tan^2 x - 1)$$

$$0 = \tan x(3\tan^2 x - 1)$$

$\tan x = 0$

$x_1 = \tan^{-1}(0)$
$= 0$ rad

$x_3 = \pi + 0$
$= \pi$

0 rad $= 2\pi$

$3\tan^2 x - 1 = 0$

$\tan^2 x = \frac{1}{3}$

$\sqrt{\tan^2 x} = \pm\sqrt{\frac{1}{3}}$

$\tan x = \pm\frac{1}{\sqrt{3}}$

$\tan x = -\frac{1}{\sqrt{3}}$

$x_1 = \frac{\pi}{6}$

$x_2 = \pi - \frac{\pi}{6}$
$= \frac{5\pi}{6}$

$x_4 = 2\pi - \frac{\pi}{6}$
$= \frac{11\pi}{6}$

$\tan x = \frac{1}{\sqrt{3}}$

$x_1 = \frac{\pi}{6}$

$x_3 = \pi + \frac{\pi}{6}$
$= \frac{7\pi}{6}$

S | A
T | C

5

Therefore, the angles that satisfy are $0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$ rad.

9. Solve the equation $8\cos^3 x - 14\cos^2 x - 7\cos x + 6 = 0$, $0 \le x \le 2\pi$. [7 marks]

$8\cos^3 x - 14\cos^2 x - 7\cos x + 6 = 0$

Let x represent cos x

$8x^3 - 14x^2 - 7x + 6 = 0$

$p(2) = 8(2)^3 - 14(2)^2 - 7(2) + 6$
$= 64 - 56 - 14 + 6$
$= 0$

$\therefore$ x-2 is a factor

-2	8	-14	-7	6
-	↓	-16	-4	6
	8	2	-3	0

$0 = (x-2)(8x^2+2x-3)$

Quad Formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{4-4(8)(-3)}}{2(8)}$$
$$= \frac{-2 \pm 10}{16}$$

$x = \frac{-2+10}{16}$ or $x = \frac{-2-10}{16}$

$= \frac{1}{2}$ $\quad$ $= -\frac{3}{4}$

$0 = (x-2)(x-\frac{1}{2})(x+\frac{3}{4})$

$0 = (x-2)(x-\frac{1}{2})(x+\frac{3}{4})$ ← sub in x = cos x

$0 = (\cos x - 2)(\cos x - \frac{1}{2})(\cos x + \frac{3}{4})$

$\cos x - 2 = 0$
$\cos x = 2$
not possible

$\cos x - \frac{1}{2} = 0$
$\cos x = \frac{1}{2}$

$x_1 = \frac{\pi}{3}$
$x_4 = 2\pi - \frac{\pi}{3}$
$= \frac{5\pi}{3}$

$\cos x + \frac{3}{4} = 0$
$\cos x = -\frac{3}{4}$

$x_1 = 0.7227...$
$x_2 = \pi - 0.7227...$
$= 2.41889...$
$x_4 = \pi + 0.7227$
$= 3.8643...$

6.5

Therefore, the angles that satisfy are $\frac{\pi}{3}$, $\frac{5\pi}{3}$, 2.418, or 3.8643

11.5