Name: uni

Date:

KU: 8 / 12 TH: 10 / 12 COMM: 3 / 3 APPS: 14 /15

84%

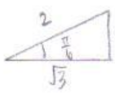
UNIT 4 QUEST: Sinusoidal Functions
This is one **TRIGGY** quest!

Instructions:
1. Read the instructions carefully.
2. Attempt all problems and don't spend all your time on one problem. Other problems will feel left out ☹
3. Show all work in the space provided. Three (3) marks will be given to the overall mathematical form on this test.
4. ENJOY ☺

PART A: KNOWLEDGE AND UNDERSTANDING

Complete solutions must be shown for full marks.

1. a) Find the slope of the tangent at the given x-value for each of the following functions. [6]
Give answers in exact value.



i) $f(x) = \sin^3 x$ at $x = \frac{\pi}{6}$

$f'(x) = 3\sin^2 x(\cos x)$ ✓ ✓

at $x = \frac{\pi}{6}$

$= 3\sin^2\left(\frac{\pi}{6}\right)\left(\cos\frac{\pi}{6}\right)$

$= 3\left(\frac{1}{2}\right)^2\left(\frac{\sqrt{3}}{2}\right)$

$= 3\left(\frac{1}{4}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{3}{4}\cdot\frac{\sqrt{3}}{2}$

$\boxed{= \frac{3\sqrt{3}}{8}}$ ✓

ii) $g(x) = \frac{\cos 2x}{x}$ at $x = \frac{\pi}{2}$

$g'(x) = \frac{x(-\sin 2x)(2) - \cos 2x}{x^2}$ ✓

$= \frac{2\left(\frac{\pi}{2}\right)(-\sin 2\frac{\pi}{2}) - \cos 2\frac{\pi}{2}}{\left(\frac{\pi}{2}\right)^2}$

$= \frac{\pi(-\sin\pi) - \cos\pi}{\left(\frac{\pi}{2}\right)^2}$

$= \frac{\pi(0) - 1 - (-1)}{\left(\frac{\pi}{2}\right)^2}$ ✓

$= -1 \div \left(\frac{\pi}{2}\right)^2$

$= -1 \times \frac{4}{\pi^2}$

$\boxed{= \frac{-4}{\pi^2}}$ X

5/6.

b) Determine the convacity at the given x-value for each of the functions above. [6]

3/6

i) $f(x) = \sin^3 x$ at $x = \frac{\pi}{6}$

$f'(x) = 3\sin^2 x(\cos x)$

$f''(x) = 6\sin x(\cos x)(-\sin x)$ X

product Rule!!

(-2)

at $x = \frac{\pi}{6}$

$= 6\sin\frac{\pi}{6}\left(\cos\frac{\pi}{6}\right)\left(-\sin\frac{\pi}{6}\right)$

$= 6\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right)$

$= 3\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right)$

$= \frac{3\sqrt{3}}{2}\left(-\frac{1}{2}\right)$

$\boxed{= -\frac{3\sqrt{3}}{4}}$

$\because f''(x) < 0$

$\therefore$ concaves down

✓ ok..

ii) $g(x) = \frac{\cos 2x}{x}$ at $x = \frac{\pi}{2}$

$g'(x) = \frac{2x(-\sin 2x) - \cos 2x}{x^2}$

$g''(x) = \frac{(x^2)[(2x)(-\cos 2x)(2) + (-\sin 2x)(2) - 2\sin 2x] - [2x(-\sin 2x) - \cos 2x]2x}{x^4}$

$= \frac{\left(\frac{\pi}{2}\right)^2\left[\left(2\frac{\pi}{2}\right)(-\cos 2\frac{\pi}{2})(2) + (-\sin 2\frac{\pi}{2})(2) - 2\sin 2\frac{\pi}{2}\right] - \left[2\frac{\pi}{2}(-\sin 2\frac{\pi}{2}) - \cos 2\frac{\pi}{2}\right]2\left(\frac{\pi}{2}\right)}{\left(\frac{\pi}{2}\right)^4}$

$= \frac{\left(\frac{\pi^2}{4}\right)[(\pi)(-\cos\pi)(2) + (-\sin\pi)(2) - 2\sin\pi] - [\pi(-\sin\pi) - \cos\pi]\pi}{\frac{\pi^4}{16}}$

$= \frac{\frac{\pi^2}{4}[\pi(0)(2) + (-1)(2) - 2(1)] - [\pi(-1) - 0]\pi}{\frac{\pi^4}{16}}$

(-1)

$= \left(\frac{\pi^2}{4}[-4] - [-\pi]\pi\right)\times\frac{16}{\pi^4}$

$= \left(-\frac{4\pi^2}{4} + \pi^2\right)\times\frac{16}{\pi^4}$

$= (-\pi^2 + \pi^2)\times\frac{16}{\pi^4}$

$= \emptyset \times \frac{16}{\pi^4}$ X

$\boxed{= \emptyset}$

$\because g''(x) = \emptyset$, $\therefore$ it's a POI

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PART B: APPLICATIONS

2. Find the equation of the tangent to the function of $f(x) = 4\sin\left(\frac{1}{2}x\right)$ at $x = \frac{\pi}{2}$. [4]

$f'(x) = 4\cos(\frac{1}{2}x)(\frac{1}{2})$

$= 2\cos(\frac{1}{2}x)$

at $x = \frac{\pi}{2}$

$= 2\cos(\frac{1}{2}\cdot\frac{\pi}{2})$

$= 2\cos(\frac{\pi}{4})$

$= 2(\frac{1}{\sqrt{2}})$

$= \cancel{2}(\frac{\sqrt{2}}{\cancel{2}})$

$= \sqrt{2}$

(x, y)

$(\frac{\pi}{2}, 4\sin(\frac{1}{2}(\frac{\pi}{2})))$

$(\frac{\pi}{2}, 2\sqrt{2})$

$\therefore$ the equation is

$y = \sqrt{2}\cdot x + 2\sqrt{2} - 4$

$y = mx + b$

$2\sqrt{2} = \sqrt{2}(2\sqrt{2}) + b$

$2\sqrt{2} = 4 + b$

$2\sqrt{2} - 4 = b$

$\frac{\pi}{2}$

X

3/4

3. At what points on the curve $y = \cos(x) + \sin(x)$ is the tangent line horizontal for $x \in \mathbb{R}$? Give answers in exact value. [6]

$y' = -\sin x + \cos x$ $\qquad p = 2\pi$

$0 = \cos x - \sin x$

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$\tan x = 1$

$\boxed{x_1 = \frac{\pi}{4}} + k2\pi, \ k \in \mathbb{Z}$

$x_3 = \pi + \frac{\pi}{4}$

$= \frac{4\pi}{4} + \frac{\pi}{4}$

$\boxed{x_3 = \frac{5\pi}{4}} + k2\pi, \ k \in \mathbb{Z}$

Therefore, the points are $(\frac{\pi}{4} + k2\pi, \sqrt{2})$ and $(\frac{5\pi}{4} + k2\pi, -\sqrt{2})$ where $k \in \mathbb{Z}$

y at $x_1 = \frac{\pi}{4}$

$y = \cos(\frac{\pi}{4}) + \sin(\frac{\pi}{4})$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$= \frac{2\sqrt{2}}{2}$

$(\frac{\pi}{4} + k2\pi, \sqrt{2})$

y at $x_3 = \frac{5\pi}{4}$

$y = \cos\frac{5\pi}{4} + \sin\frac{5\pi}{4}$

$= -\frac{\sqrt{2}}{2} + -\frac{\sqrt{2}}{2}$

$= -\frac{2\sqrt{2}}{2}$

$(\frac{5\pi}{4} + k2\pi, -\sqrt{2})$

6/6

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4. An AC-DC coupled circuit has a voltage (in volts) at time t (in seconds) that is given by the function:

$$V(t) = 3\sin(t) + 15$$

a) Determine the maximum and minimum voltage. At what times do these occur? [4]

$V'(t) = 3\cos t$

$0 = 3\cos t$

$0 = \cos t$

$t = \cos^{-1} 0$

$\boxed{t_1 = \frac{\pi}{2}}$

$t_4 = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$

S | A
T | C

$\because p = 2\pi$

$\therefore + k2\pi,\ k \in \mathbb{Z}$

$V(t)$ at $\frac{\pi}{2}$

$V(t) = 3\sin(\frac{\pi}{2}) + 15$

$= 3(1) + 15$

$= 18\text{ v}$

$V(t)$ at $\frac{3\pi}{2}$

$V(t) = 3\sin(\frac{3\pi}{2}) + 15$

$= 12\text{ v}$

$\therefore$ max voltage is 18 v and min is 12 v. Max occurs every $\frac{\pi}{2} + k2\pi,\ k \in \mathbb{Z}$. Min occurs every $\frac{3\pi}{2} + k2\pi,\ k \in \mathbb{Z}$

5/5

b) Determine the amplitude of the voltage. [1]

$\frac{18\text{v} + 12\text{v}}{2} = 3$

$\therefore$ amplitude is 3

PART C: THINKING

5. Determine whether or not the following statements are true or false. [5]
If the statement is true, prove by finding its derivative.
If the statement is false, explain using a counterexample (i.e. proving without finding its derivative).

Statement 1: The derivative of the function $y = \csc x$ is $\frac{dy}{dx} = -\csc x \cot x$.

Statement 2: The derivative of the function $y = \sec x$ is $\frac{dy}{dx} = \csc x \cot x$

$\csc x = \frac{1}{\sin x}$

$\frac{d}{dx}\left(\frac{1}{\sin x}\right)$

$= \frac{\sin x(0) - 1(\cos x)}{\sin^2 x}$

$= \frac{-\cos x}{\sin^2 x}$

$\therefore$ statement 1 is correct

$-\csc x \cot x$

$= -\frac{1}{\sin x} \cdot \frac{1}{\tan x}$

$= -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$

$= \frac{-\cos x}{\sin^2 x}$

3/5

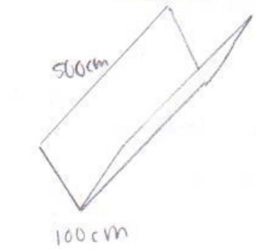
$-\csc x \cot x$ is the same as $\csc x \cot x$ except its negative. the negative wont change the derivative or original equation. $\therefore$ statement 2 is false

2

need counter example!

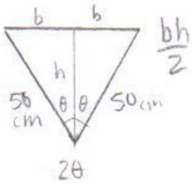
/5 + 3

6. A V-shaped trough 5m long is made from a rectangular sheet of aluminum 100 cm wide by bending it down the middle. What angle between the sides of the trough will maximize its capacity? [7]
To get full marks, you must take the derivative with respect to the angle.



$V = \frac{bh}{2} \times l$

$= \frac{bh}{2} \times 500cm$



$\frac{bh}{2}$

$A = \frac{bh}{2}$

$= \frac{50\sin\theta(50\cos\theta)}{2}$

$\sin\theta = \frac{b}{50}$

$\cos\theta = \frac{h}{50}$

$A' = \frac{2[50\sin\theta(50\sin\theta) + (50\cos\theta)(50\cos\theta)] - 0(50\sin\theta)(50\cos\theta)}{4}$

$= \frac{2[50\sin\theta(-50\sin\theta) + (50\cos\theta)(50\cos\theta)]}{4}$

$= \frac{50\sin\theta(-50\sin\theta) + (50\cos\theta)(50\cos\theta)}{2}$

$(50\sin\theta)^2 = (50\cos\theta)^2$

$\frac{2500\sin^2\theta}{2500\cos^2\theta} = \frac{2500\cos^2\theta}{2500\cos^2\theta}$

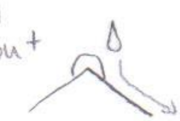
$\sqrt{\tan^2\theta} = \pm\sqrt{1}$

$\tan\theta = -1$ $\tan\theta = 1$

$\theta_2 = \frac{3\pi}{4}$ $\boxed{\theta_1 = \frac{\pi}{4}}$

$\theta_4 = \frac{7\pi}{4}$ $\theta_3 = \frac{5\pi}{4}$

the rest are rejected. ☺ The end ☺
because $2 \times \theta_2$ is $> \pi$ so water will fall out



$2\theta_1$
$= 2(\frac{\pi}{4})$
$\boxed{= \frac{\pi}{2}}$

∴ the angle should be $\frac{\pi}{2}$ rad to max its capacity

7/7

7 (11)