CHAPTER 1

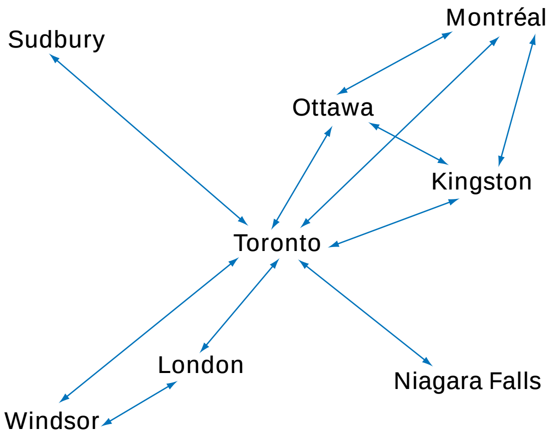
Tools for Data Management

Specific Expectations	Section
Locate data to answer questions of significance or personal interest, by searching well-organized databases.	1.3
Use the Internet effectively as a source for databases.	1.3
Create database or spreadsheet templates that facilitate the manipulation and retrieval of data from large bodies of information that have a variety of characteristics.	1.2, 1.3, 1.4
Represent simple iterative processes, using diagrams that involve branches and loops.	1.1
Represent complex tasks or issues, using diagrams.	1.1, 1.5
Solve network problems, using introductory graph theory.	1.5
Represent numerical data, using matrices, and demonstrate an understanding of terminology and notation related to matrices.	1.6, 1.7
Demonstrate proficiency in matrix operations, including addition, scalar multiplication, matrix multiplication, the calculation of row sums, and the calculation of column sums, as necessary to solve problems, with and without the aid of technology.	1.6, 1.7
Solve problems drawn from a variety of applications, using matrix methods.	1.6, 1.7

Chapter Problem

VIA Rail Routes

When travelling by bus, train, or airplane, you usually want to reach your destination without any stops or transfers. However, it is not always possible to reach your destination by a non-stop route. The following map shows the VIA Rail routes for eight major cities. The arrows represent routes on which you do not have to change trains.



1. **a)** List several routes you have travelled where you were able to reach your destination directly.
   b) List a route where you had to change vehicles exactly once before reaching your destination.
2. **a)** List all the possible routes from Montréal to Toronto by VIA Rail.
   b) Which route would you take to get from Montréal to Toronto in the least amount of time? Explain your reasoning.
3. **a)** List all the possible routes from Kingston to London.
   b) Give a possible reason why VIA Rail chooses not to have a direct train from Kingston to London.

This chapter introduces graph theory, matrices, and technology that you can use to model networks like the one shown. You will learn techniques for determining the number of direct and indirect routes from one city to another. The chapter also discusses useful data-management tools including iterative processes, databases, software, and simulations.

Review of Prerequisite Skills

If you need help with any of the skills listed in purple below, refer to Appendix A.

1. **Order of operations** Evaluate each expression.
   a) $(-4)(5) + (2)(-3)$
   b) $(-2)(3) + (5)(-3) + (8)(7)$
   c) $(1)(0) + (1)(1) + (0)(0) + (0)(1)$
   d) $(2)(4) + \frac{12}{3} - (3)^2$

2. **Substituting into equations** Given $f(x) = 3x^2 - 5x + 2$ and $g(x) = 2x - 1$, evaluate each expression.
   a) $f(2)$
   b) $g(2)$
   c) $f(g(-1))$
   d) $f(g(1))$
   e) $f(f(2))$
   f) $g(f(2))$

3. **Solving equations** Solve for x.
   a) $2x - 3 = 7$
   b) $5x + 2 = -8$
   c) $\frac{x}{2} - 5 = 5$
   d) $4x - 3 = 2x - 1$
   e) $x^2 = 25$
   f) $x^3 = 125$
   g) $3(x + 1) = 2(x - 1)$
   h) $\frac{2x - 5}{2} = \frac{3x - 1}{4}$

4. **Graphing data** In a sample of 1000 Canadians, 46% have type O blood, 43% have type A, 8% have type B, and 3% have type AB. Represent these data with a fully-labelled circle graph.

5. **Graphing data** Organize the following set of data using a fully-labelled double-bar graph.

City	Snowfall (cm)	Total Precipitation (cm)
St. John's	322.1	148.2
Charlottetown	338.7	120.1
Halifax	261.4	147.4
Fredericton	294.5	113.1
Québec City	337.0	120.8
Montréal	214.2	94.0
Ottawa	221.5	91.1
Toronto	135.0	81.9
Winnipeg	114.8	50.4
Regina	107.4	36.4
Edmonton	129.6	46.1
Calgary	135.4	39.9
Vancouver	54.9	116.7
Victoria	46.9	85.8
Whitehorse	145.2	26.9
Yellowknife	143.9	26.7

6. **Graphing data** The following table lists the average annual full-time earnings for males and females. Illustrate these data using a fully-labelled double-line graph.

Year	Women ($)	Men ($)
1989	28 219	42 767
1990	29 050	42 913
1991	29 654	42 575
1992	30 903	42 984
1993	30 466	42 161
1994	30 274	43 362
1995	30 959	42 338
1996	30 606	41 897
1997	30 484	43 804
1998	32 553	45 070

7. **Using spreadsheets** Refer to the spreadsheet section of Appendix B, if necessary.

a) Describe how to refer to a specific cell.

b) Describe how to refer to a range of cells in the same row.

c) Describe how to copy data into another cell.

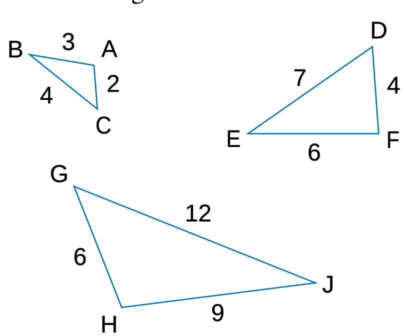
d) Describe how to move data from one column to another.

e) Describe how to expand the width of a column.

f) Describe how to add another column.

g) What symbol must precede a mathematical expression?

8. **Similar triangles** Determine which of the following triangles are similar. Explain your reasoning.



9. **Number patterns** Describe each of the following patterns. Show the next three terms.

a) 65, 62, 59, …

b) 100, 50, 25, …

c) $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$

d) a, b, aa, bb, aaa, bbbb, aaaa, bbbbbbbb, …

10. **Ratios of areas** Draw two squares on a sheet of grid paper, making the dimensions of the second square half those of the first.

a) Use algebra to calculate the ratio of the areas of the two squares.

b) Confirm this ratio by counting the number of grid units contained in each square.

c) If you have access to *The Geometer's Sketchpad*® or similar software, confirm the area ratio by drawing a square, dilating it by a factor of 0.5, and measuring the areas of the two squares. Refer to the help menu in the software, if necessary.

11. **Simplifying expressions** Expand and simplify each expression.

a) $(x - 1)^2$

b) $(2x + 1)(x - 4)$

c) $-5x(x - 2y)$

d) $3x(x - y)^2$

e) $(x - y)(3x)^2$

f) $(a + b)(c - d)$

12. **Fractions, percents, decimals** Express as a decimal.

a) $\frac{5}{20}$ b) $\frac{23}{50}$ c) $\frac{2}{3}$

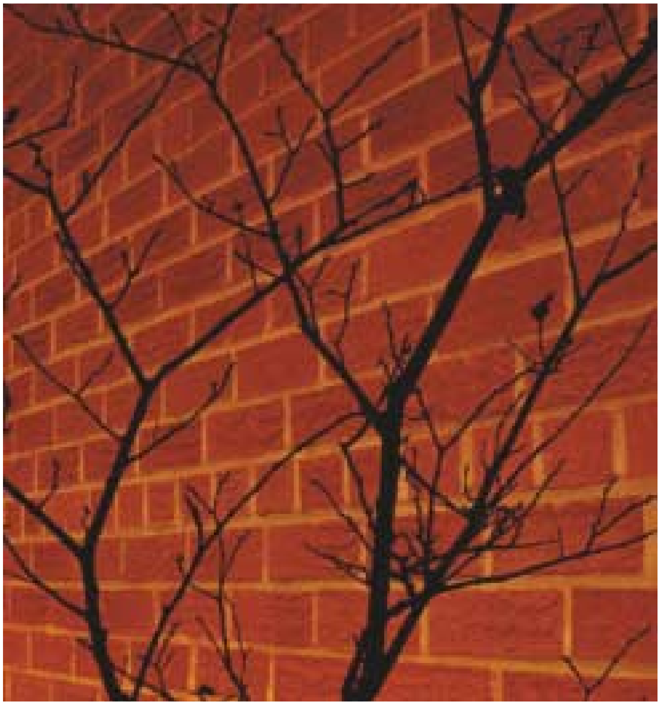
d) $\frac{138}{12}$ e) $\frac{6}{7}$ f) 73%

13. **Fractions, percents, decimals** Express as a percent.

a) 0.46 b) $\frac{4}{5}$ c) $\frac{1}{30}$

d) 2.25 e) $\frac{11}{8}$

1.1 The Iterative Process



If you look carefully at the branches of a tree, you can see the same pattern repeated over and over, but getting smaller toward the end of each branch. A nautilus shell repeats the same shape on a larger and larger scale from its tip to its opening. You yourself repeat many activities each day. These three examples all involve an iterative process. **Iteration** is a process of repeating the same procedure over and over. The following activities demonstrate this process.

INVESTIGATE & INQUIRE: Developing a Sort Algorithm

Often you need to sort data using one or more criteria, such as alphabetical or numerical order. Work with a partner to develop an algorithm to sort the members of your class in order of their birthdays. An **algorithm** is a procedure or set of rules for solving a problem.

1. Select two people and compare their birthdays.
2. Rank the person with the later birthday second.
3. Now, compare the next person's birthday with the last ranked birthday. Rank the later birthday of those two last.
4. Describe the continuing process you will use to find the classmate with the latest birthday.
5. Describe the process you would use to find the person with the second latest birthday. With whom do you stop comparing?
6. Describe a process to rank all the remaining members of your class by their birthdays.
7. Illustrate your process with a diagram.

The process you described is an iterative process because it involves repeating the same set of steps throughout the algorithm. Computers can easily be programmed to sort data using this process.

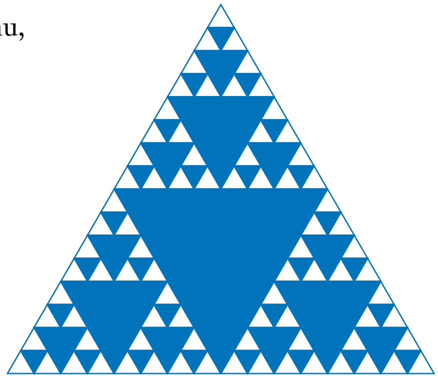
INVESTIGATE & INQUIRE: The Sierpinski Triangle

Method 1: Pencil and Paper

1. Using isometric dot paper, draw a large equilateral triangle with side lengths of 32 units.
2. Divide this equilateral triangle into four smaller equilateral triangles.
3. Shade the middle triangle. What fraction of the original triangle is shaded?
4. For each of the unshaded triangles, repeat this process. What fraction of the original triangle is shaded?
5. For each of the unshaded triangles, repeat this process again. What fraction of the original triangle is shaded now?
6. Predict the fraction of the original triangle that would be shaded for the fourth and fifth steps in this iterative process.
7. Predict the fraction of the original triangle that would be shaded if this iterative process continued indefinitely.

Method 2: *The Geometer's Sketchpad®*

1. Open a new sketch and a new script.
2. Position both windows side by side.
3. Click on REC in the script window.
4. In the sketch window, construct a triangle. Shift-click on each side of the triangle. Under the Construct menu, choose Point at Midpoint and then Polygon Interior of the midpoints.
5. Shift-click on one vertex and the two adjacent midpoints. Choose Loop in your script.
6. Repeat step 5 for the other two vertices.
7. Shift-click on the three midpoints. From the Display menu, choose Hide Midpoints.
8. Stop your script.
9. Open a new sketch. Construct a new triangle. Mark the three vertices. Play your script at a recursion depth of at least 3. You may increase the speed by clicking on Fast.



10. a) What fraction of the original triangle is shaded
    i) after one recursion?
    ii) after two recursions?
    iii) after three recursions?

    b) Predict what fraction would be shaded after four and five recursions.

    c) Predict the fraction of the original triangle that would be shaded if this iterative (recursion) process continued indefinitely.
11. Experiment with recursion scripts to design patterns with repeating shapes.

The Sierpinski triangle is named after the Polish mathematician, Waclaw Sierpinski (1882–1924). It is an example of a **fractal**, a geometric figure that is generally created using an iterative process. One part of the process is that fractals are made of **self-similar** shapes. As the shapes become smaller and smaller, they keep the same geometrical characteristics as the original larger shape. Fractal geometry is a very rich area of study. Fractals can be used to model plants, trees, economies, or the honeycomb pattern in human bones.

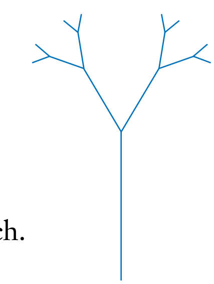
WEB CONNECTION

www.mcgrawhill.ca/links/MDM12

Visit the above web site and follow the links to learn more about the Sierpinski triangle and fractals. Choose an interesting fractal and describe how it is self-similar.

Example 1 Modelling With a Fractal

Fractals can model the branching of a tree. Describe the algorithm used to model the tree shown.



Solution

Begin with a 1-unit segment. Branch off at 60° with two segments, each one half the length of the previous branch. Repeat this process for a total of three iterations.

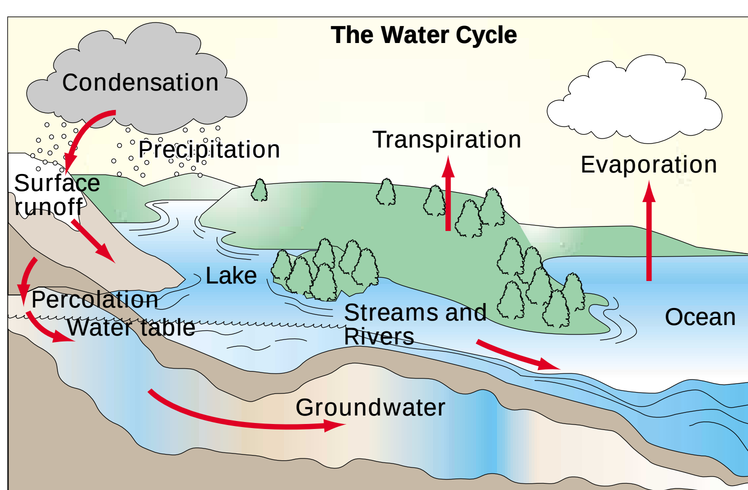
Arrow diagrams can illustrate iterations. Such diagrams show the sequence of steps in the process.

Example 2 The Water Cycle

Illustrate the water cycle using an arrow diagram.

Solution

The water, or hydrologic, cycle is an iterative process. Although the timing of the precipitation can vary, the cycle will repeat itself indefinitely.



Example 3 Tree Diagram

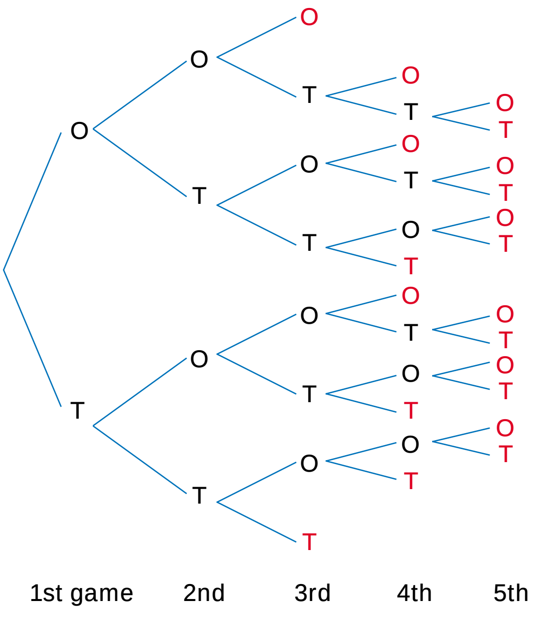
a) Illustrate the results of a best-of-five hockey playoff series between Ottawa and Toronto using a tree diagram.

b) How many different outcomes of the series are possible?

Solution

a) For each game, the tree diagram has two branches, one representing a win by Ottawa (O) and the other a win by Toronto (T). Each set of branches represents a new game in the playoff round. As soon as one team wins three games, the playoff round ends, so the branch representing that sequence also stops.

b) By counting the endpoints of the branches, you can determine that there are 20 possible outcomes for this series.



Example 4 Recursive Formula

The **recursive formula** $t_n = 3t_{n-1} - t_{n-2}$ defines a sequence of numbers. Find the next five terms in the sequence given that the **initial** or **seed values** are $t_1 = 1$ and $t_2 = 3$.

Solution

$$\begin{aligned} t_3 &= 3t_2 - t_1 \\ &= 3(3) - 1 \\ &= 8 \end{aligned} \qquad \begin{aligned} t_4 &= 3t_3 - t_2 \\ &= 3(8) - 3 \\ &= 21 \end{aligned} \qquad \begin{aligned} t_5 &= 3t_4 - t_3 \\ &= 3(21) - 8 \\ &= 55 \end{aligned}$$

$$\begin{aligned} t_6 &= 3t_5 - t_4 \\ &= 3(55) - 21 \\ &= 144 \end{aligned} \qquad \begin{aligned} t_7 &= 3t_6 - t_5 \\ &= 3(144) - 55 \\ &= 377 \end{aligned}$$

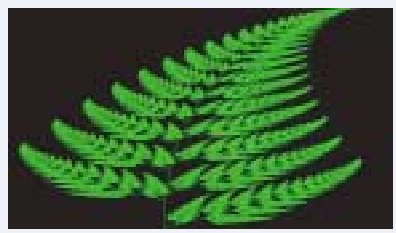
The next five terms are 8, 21, 55, 144, and 377.

Key Concepts

- Iteration occurs in many natural and mathematical processes. Iterative processes can create fractals.
- A process that repeats itself can be illustrated using arrows and loops.
- A tree diagram can illustrate all the possible outcomes of a repeated process involving two or more choices at each step.
- For recursive functions, the first step is calculated using initial or seed values, then each successive term is calculated using the result of the preceding step.

Communicate Your Understanding

1. Describe how fractals have been used to model the fern leaf shown on the right.



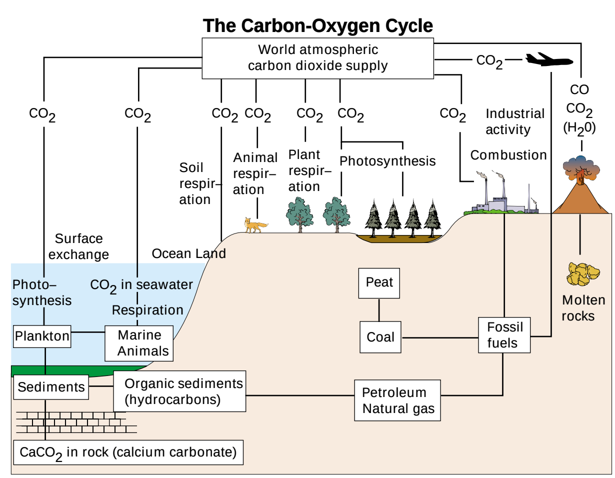
2. Describe your daily routine as an iterative process.

Practise

A

1. Which of the following involve an iterative process?
   a) the cycle of a washing machine
   b) your reflections in two mirrors that face each other
   c) the placement of the dials on an automobile dashboard
   d) a chart of sunrise and sunset times
   e) substituting a value for the variable in a quadratic equation
   f) a tessellating pattern, such as paving bricks that fit together without gaps

2. The diagram below illustrates the carbon-oxygen cycle. Draw arrows to show the gains and losses of carbon dioxide.



3. Draw a tree diagram representing the playoffs of eight players in a singles tennis tournament. The tree diagram should show the winner of each game continuing to the next round until a champion is decided.

Apply, Solve, Communicate

B

4. Draw a diagram to represent the food chain.

5. **Communication** Describe how the tracing of heartbeats on a cardiac monitor or electrocardiogram is iterative. Illustrate your description with a sketch.

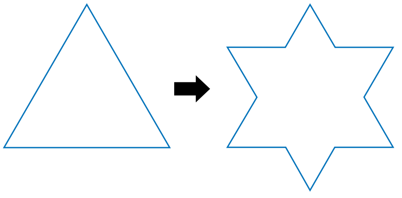
6. In the first investigation, on page 6, you developed a sort algorithm in which new data were compared to the lowest ranked birthday until the latest birthday was found. Then, the second latest, third latest, and so on were found in the same manner.

a) Write a sort algorithm in which this process is reversed so that the highest ranked item is found instead of the lowest.

b) Write a sort algorithm in which you compare the first two data, then the second and third, then the third and fourth, and so on, interchanging the order of the data in any pair where the second item is ranked higher.

7. **Application** Sierpinski's carpet is similar to Sierpinski's triangle, except that it begins with a square. This square is divided into nine smaller squares and the middle one is shaded. Use paper and pencil or a drawing program to construct Sierpinski's carpet to at least three stages. Predict what fraction of the original square will be shaded after n stages.

8. **Application** In 1904 the Swedish mathematician Helge von Koch (1870–1924) developed a fractal based on an equilateral triangle. Using either paper and pencil or a drawing program, such as *The Geometer's Sketchpad*®, draw a large equilateral triangle and trisect each side. Replace each middle segment with two segments the same length as the middle segment, forming an equilateral triangle with the base removed, as shown below.



Repeat the process of trisection and replacement on each of the 12 smaller segments. If you are using a computer program, continue this process for at least two more iterations.

a) How many segments are there after three iterations?

b) How many segments are there after four iterations?

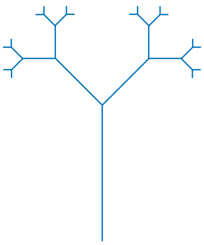
c) What pattern can you use to predict the number of segments after n iterations?

9. The first two terms of a sequence are given as $t_1 = 2$ and $t_2 = 4$. The recursion formula is $t_n = (t_{n-1})^2 - 3t_{n-2}$. Determine the next four terms in the sequence.

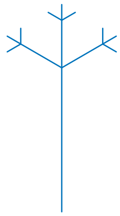
10. Each of the following fractal trees has a different algorithm. Assume that each tree begins with a segment 1 unit long.

a) Illustrate or describe the algorithm for each fractal tree.

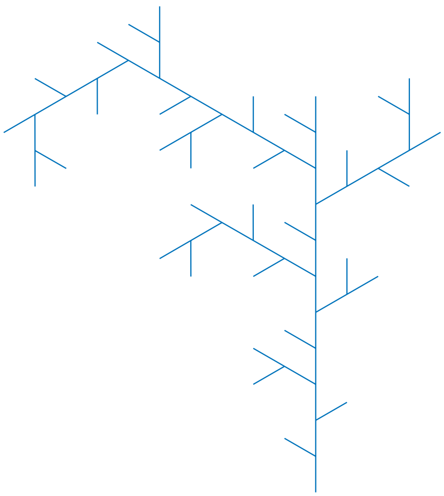
i)



ii)



iii)



b) What is the total length of the branches in each tree?

c) An interesting shape on a fractal tree is a spiral, which you can trace by tracing a branch to its extremity. Are all spirals within a tree self-similar?

d) Write your own set of rules for a fractal tree. Draw the tree using paper and pencil or a drawing program.

11. Inquiry/Problem Solving Related to fractals is the mathematical study of chaos, in which no accurate prediction of an outcome can be made. A random walk can illustrate such "chaotic" outcomes.

a) Select a starting point near the centre of a sheet of grid paper. Assign the numbers 1 to 4 to the directions north, south, east, or west in any order. Now, generate random whole numbers between 1 and 4 using a die, coin, or graphing calculator. Draw successive line segments one unit long in the directions corresponding to the random numbers until you reach an edge of the paper.

b) How would a random walk be affected if it were self-avoiding, that is, not allowed to intersect itself? Repeat part a) using this extra rule.

c) Design your own random walk with a different set of rules. After completing the walk, trade drawings with a classmate and see if you can deduce the rules for each other's walk.

WEB CONNECTION

www.mcgrawhill.ca/links/MDM12

To learn more about chaos theory, visit the above web site and follow the links. Describe an aspect of chaos theory that interests you.

12. Use the given values for t_1 to find the successive terms of the following recursive formulas. Continue until a pattern appears. Describe the pattern and make a prediction for the value of the nth term.

a) $t_n = 2^{-t_{n-1}}$; $t_1 = 0$

b) $t_n = \sqrt{t_{n-1}}$; $t_1 = 256$

c) $t_n = \dfrac{1}{t_{n-1}}$; $t_1 = 2$

ACHIEVEMENT CHECK

Knowledge / Understanding	Thinking / Inquiry / Problem Solving	Communication	Application

13. a) Given $t_1 = 1$, list the next five terms for the recursion formula $t_n = n \times t_{n-1}$.

b) In this sequence, t_k is a factorial number, often written as $k!$ Show that

$$t_k = k! = k(k-1)(k-2)\ldots(2)(1).$$

c) Explain what 8! means. Evaluate 8!

d) Explain why factorial numbers can be considered an iterative process.

e) Note that

$$\begin{aligned}&(2^5)(5!)\\ &= (2 \times 2 \times 2 \times 2 \times 2)(5 \times 4 \times 3 \times 2 \times 1)\\ &= (2 \times 5)(2 \times 4)(2 \times 3)(2 \times 2)(2 \times 1)\\ &= 10 \times 8 \times 6 \times 4 \times 2\end{aligned}$$

which is the product of the first five even positive integers. Write a formula for the product of the first n even positive integers. Explain why your formula is correct.

f) Write $\dfrac{10!}{(2^5)(5!)}$ as a product of consecutive odd integers.

g) Write a factorial formula for the product of

i) the first six odd positive integers

ii) the first ten odd positive integers

iii) the first n odd positive integers

C

14. **Inquiry / Problem Solving** Recycling can be considered an iterative process. Research the recycling process for a material such as newspaper, aluminum, or glass and illustrate the process with an arrow diagram.

15. **Inquiry / Problem Solving** The infinite series $S = \cos\theta + \cos^2\theta + \cos^3\theta + \ldots$ can be illustrated by drawing a circle centred at the origin, with radius of 1. Draw an angle θ and, on the x-axis, label the point $(\cos\theta, 0)$ as P_1. Draw a new circle, centred at P_1, with radius of $\cos\theta$. Continue this iterative process. Predict the length of the line segment defined by the infinite series $S = \cos\theta + \cos^2\theta + \cos^3\theta + \ldots$.

16. **Communication** Music can be written using fractal patterns. Look up this type of music in a library or on the Internet. What characteristics does fractal music have?

17. Computers use binary (base 2) code to represent numbers as a series of ones and zeros.

Base 10	Binary
0	0
1	1
2	10
3	11
4	100
⋮	⋮

a) Describe an algorithm for converting integers from base 10 to binary.

b) Write each of the following numbers in binary.

i) 16 **ii)** 21
iii) 37 **iv)** 130

c) Convert the following binary numbers to base 10.

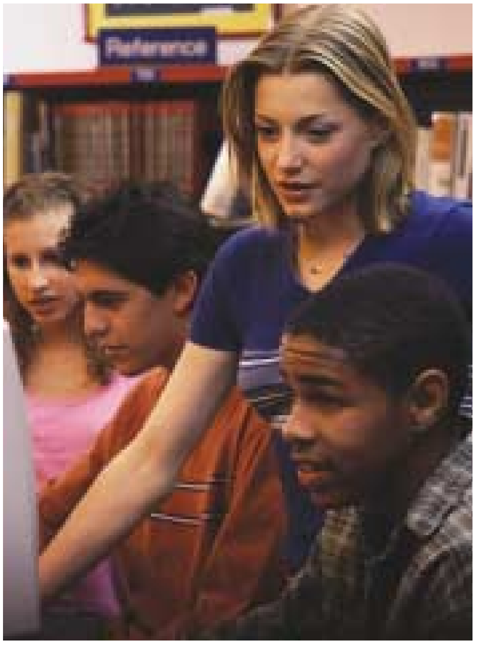
i) 1010 **ii)** 100000
iii) 111010 **iv)** 111111111

1.2 Data Management Software

INVESTIGATE & INQUIRE: Software Tools

1. List every computer program you can think of that can be used to manage data.
2. Sort the programs into categories, such as word-processors and spreadsheets.
3. Indicate the types of data each category of software would be best suited to handle.
4. List the advantages and disadvantages of each category of software.
5. Decide which of the programs on your list would be best for storing and accessing the lists you have just made.

Most office and business software manage data of some kind. Schedulers and organizers manage lists of appointments and contacts. E-mail programs allow you to store, access, and sort your messages. Word-processors help you manage your documents and often have sort and outline functions for organizing data within a document. Although designed primarily for managing financial information, spreadsheets can perform calculations related to the management and analysis of a wide variety of data. Most of these programs can easily transfer data to other applications.



Database programs, such as Microsoft® Access and Corel® Paradox®, are powerful tools for handling large numbers of records. These programs produce **relational databases**, ones in which different sets of records can be linked and sorted in complex ways based on the data contained in the records. For example, many organizations use a relational database to generate a monthly mailing of reminder letters to people whose memberships are about to expire. However, these complex relational database programs are difficult to learn and can be frustrating to use until you are thoroughly familiar with how they work. Partly for this reason, there are thousands of simpler database programs designed for specific types of data, such as book indexes or family trees.

Of particular interest for this course are programs that can do statistical analysis of data. Such programs range from modest but useful freeware to major data-analysis packages costing thousands of dollars. The more commonly used programs include MINITAB™, SAS, and SST (Statistical

Software Tools). To demonstrate statistical software, some examples in this book have alternative solutions that use Fathom™, a statistical software package specifically designed for use in schools.

Data management programs can perform complex calculations and link, search, sort, and graph data. The examples in this section use a spreadsheet to illustrate these operations. A spreadsheet is software that arranges data in rows and columns. For basic spreadsheet instructions, please refer to the spreadsheet section of Appendix B. If you are not already familiar with spreadsheets, you may find it helpful to try each of the examples yourself before answering the Practise questions at the end of the section. The two most commonly used spreadsheets are Corel® Quattro® Pro and Microsoft® Excel.

Formulas and Functions

A formula entered in a spreadsheet cell can perform calculations based on values or formulas contained in other cells. Formulas retrieve data from other cells by using **cell references** to indicate the rows and columns where the data are located. In the formulas C2*0.05 and D5+E5, each reference is to an individual cell. In both Microsoft® Excel and Corel® Quattro® Pro, it is good practice to begin a formula with an equals sign. Although not always necessary, the equals sign ensures that a formula is calculated rather than being interpreted as text.

Built-in formulas are called **functions**. Many functions, such as the **SUM function** or **MAX function** use references to a range of cells. In Corel® Quattro® Pro, precede a function with an @ symbol. For example, to find the total of the values in cells A2 through A6, you would enter

Corel® Quattro® Pro: @SUM(A2..A6)   Microsoft® Excel: SUM(A2:A6)

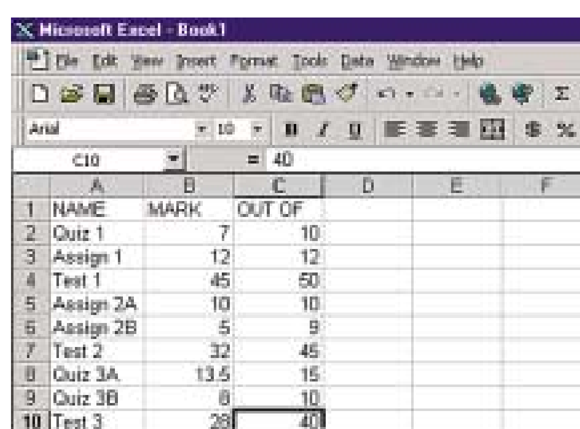
Similarly, to find the total for a block of cells from A2 through B6, enter

Corel® Quattro® Pro: @SUM(A2..B6)   Microsoft® Excel: SUM(A2:B6)

A list of formulas is available in the Insert menu by selecting Function…. You may select from a list of functions in categories such as Financial, Math & Trig, and Database.

Example 1   Using Formulas and Functions

The first three columns of the spreadsheet on the right list a student's marks on tests and assignments for the first half of a course. Determine the percent mark for each test or assignment and calculate an overall midterm mark.

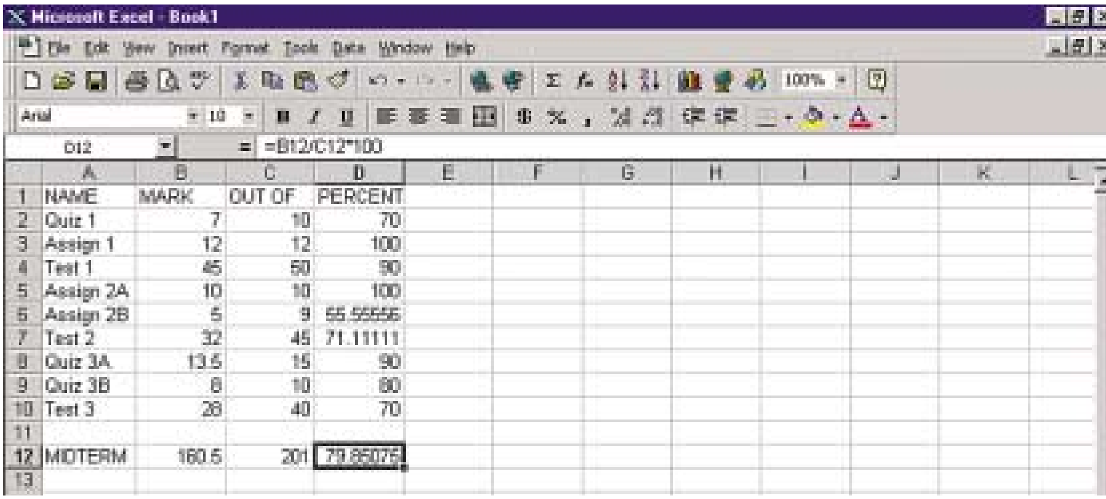


	A	B	C	D	E	F
1	NAME	MARK	OUT OF			
2	Quiz 1	7	10			
3	Assign 1	12	12			
4	Test 1	45	50			
5	Assign 2A	10	10			
6	Assign 2B	5	9			
7	Test 2	32	45			
8	Quiz 3A	13.5	15			
9	Quiz 3B	8	10			
10	Test 3	28	40			

Solution

In column D, enter formulas with cell referencing to find the percent for each individual mark. For example, in cell D2, you could use the formula B2/C2*100.

Use the **SUM function** to find totals for columns B and C, and then convert to percent in cell D12 to find the midterm mark.


Microsoft Excel - Book1

D12 = =B12/C12*100

	A	B	C	D
1	NAME	MARK	OUT OF	PERCENT
2	Quiz 1	7	10	70
3	Assign 1	12	12	100
4	Test 1	45	50	90
5	Assign 2A	10	10	100
6	Assign 2B	5	9	55.55556
7	Test 2	32	45	71.11111
8	Quiz 3A	13.5	15	90
9	Quiz 3B	8	10	80
10	Test 3	28	40	70
11				
12	MIDTERM	160.5	201	79.85075
13				

Relative and Absolute Cell References

Spreadsheets automatically adjust cell references whenever cells are copied, moved, or sorted. For example, if you copy a **SUM function**, used to calculate the sum of cells A3 to E3, from cell F3 to cell F4, the spreadsheet will change the cell references in the copy to A4 and E4. Thus, the value in cell F4 will be the sum of those in cells A4 to E4, rather than being the same as the value in F3.

Because the cell references are relative to a location, this automatic adjustment is known as **relative cell referencing**. If the formula references need to be kept exactly as written, use **absolute cell referencing**. Enter dollar signs before the row and column references to block automatic adjustment of the references.

Fill and Series Features

When a formula or function is to be copied to several adjoining cells, as for the percent calculations in Example 1, you can use the **Fill feature** instead of Copy. Click once on the cell to be copied, then click and drag across or down through the range of cells into which the formula is to be copied.

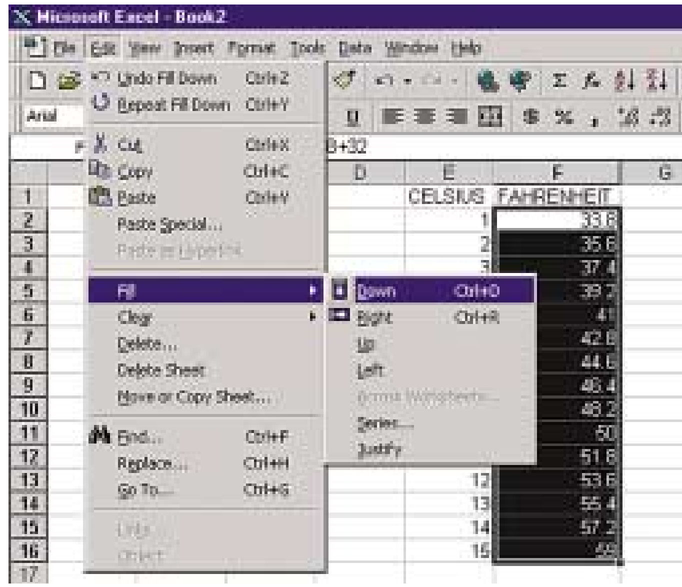
To create a sequence of numbers, enter the first two values in adjoining cells, then select Edit/Fill/Series to continue the sequence.

Example 2 Using the Fill Feature

The relationship between Celsius and Fahrenheit temperatures is given by the formula Fahrenheit = 1.8 × Celsius + 32. Use a spreadsheet to produce a conversion table for temperatures from 1°C to 15°C.

Solution

Enter 1 into cell E2 and 2 into cell E3. Use the **Fill feature** to put the numbers 3 through 15 into cells E4 to E16. Enter the conversion formula E2*1.8+32 into cell F2. Then, use the **Fill feature** to copy the formula into cells F3 through F16. Note that the values in these cells show that the cell references in the formulas did change when copied. These changes are an example of relative cell referencing.



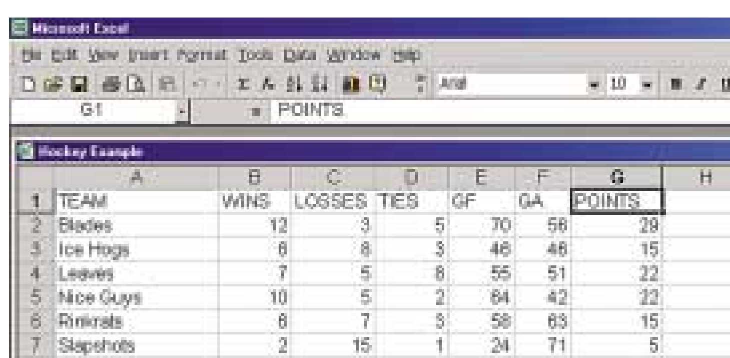
Charting

Another important feature of spreadsheets is the ability to display numerical data in the form of charts or graphs, thereby making the data easier to understand. The first step is to select the range of cells to be graphed. For non-adjoining fields, hold down the Ctrl key while highlighting the cells. Then, use the **Chart feature** to specify how you want the chart to appear.

You can produce legends and a title for your graph as well as labels for the axes. Various two- and three-dimensional versions of bar, line, and circle graphs are available in the menus.

Example 3 Charting

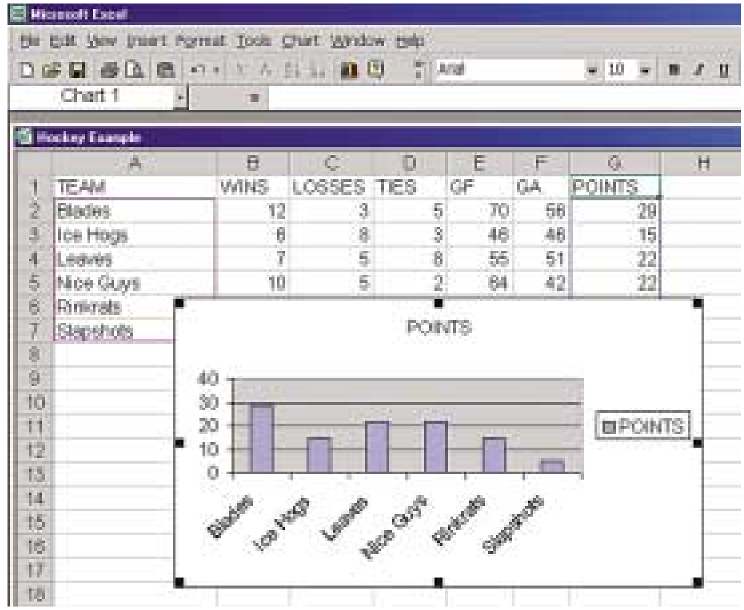
The results and standings of a hockey league are listed in this spreadsheet. Produce a two-dimensional bar chart using the TEAM and POINTS columns.



TEAM	WINS	LOSSES	TIES	GF	GA	POINTS
Blades	12	3	5	70	58	29
Ice Hogs	6	8	3	46	48	15
Leaves	7	5	8	55	51	22
Nice Guys	10	5	2	64	42	22
Rinkrats	6	7	3	58	63	15
Slapshots	2	15	1	24	71	5

Solution

Holding down the Ctrl key, highlight cells A1 to A7 and then G1 to G7. Use the **Chart feature** and follow the on-screen instructions to customize your graph. You will see a version of the bar graph as shown here.



Sorting

Spreadsheets have the capability to sort data alphabetically, numerically, by date, and so on. The sort can use multiple criteria in sequence. Cell references will adjust to the new locations of the sorted data. To sort, select the range of cells to be sorted. Then, use the **Sort feature**.

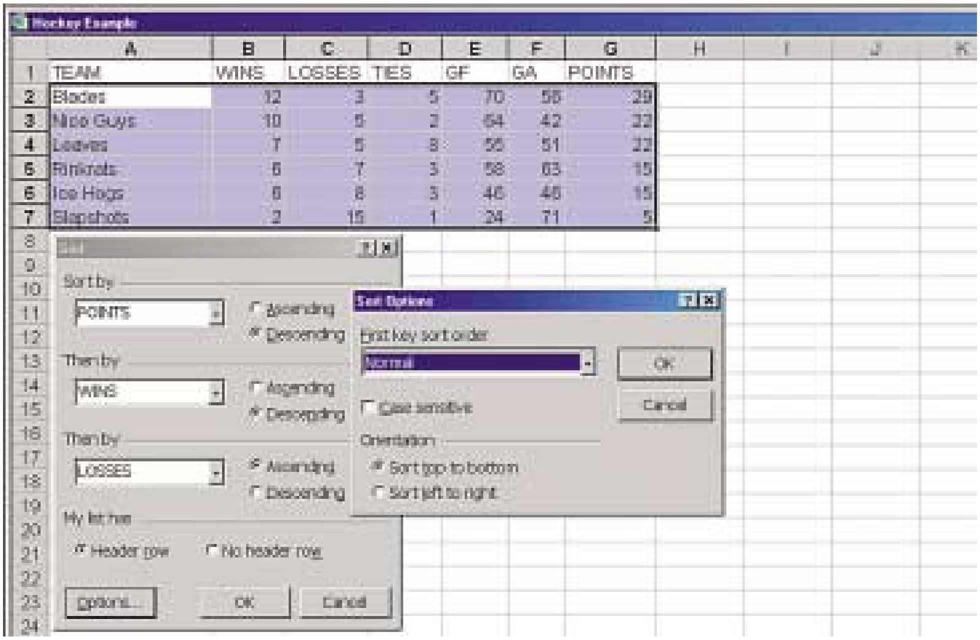
Select the criteria under which the data are to be sorted. A sort may be made in ascending or descending order based on the data in any given column. A sort with multiple criteria can include a primary sort, a secondary sort within it, and a tertiary sort within the secondary sort.

Example 4 Sorting

Rank the hockey teams in Example 3, counting points first (in descending order), then wins (in descending order), and finally losses (in ascending order).

Solution

When you select the **Sort feature**, the pop-up window asks if there is a header row. Confirming that there is a header row excludes the column headings from the sort so that they are left in place. Next, set up a three-stage sort:
- a primary sort in descending order, using the points column
- then, a secondary sort in descending order, using the wins column
- finally, a tertiary sort in ascending order, using the losses column



Searching

To **search** for data in individual cells, select Find and Replace.

Then, in the dialogue box, enter the data and the criteria under which you are searching. You have the option to search or to search and replace.

A **filtered search** allows you to search for rows containing the data for which you are searching.

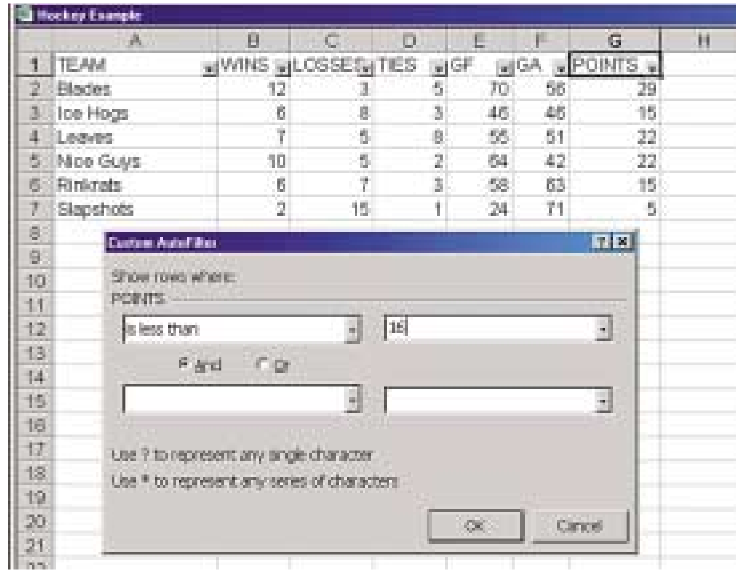
Arrows will appear at the top of each column containing data. Clicking on an arrow opens a pull-down menu where you can select the data you wish to find. The filter will then display only the rows containing these data. You can filter for a specific value or select custom… to use criteria such as greater than, begins with, and does not contain. To specify multiple criteria, click the And or Or options. You can set different filter criteria for each column.

Example 5 Filtered Search

In the hockey-league spreadsheet from Example 3, use a **filtered search** to list only those teams with fewer than 16 points.

Solution

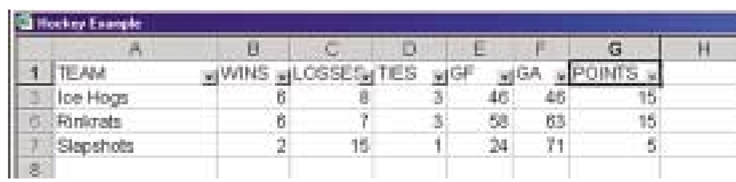
In Microsoft® Excel, select Data / Filter / Autofilter to begin the filter process. Click on the arrow in the POINTS column and select custom… In the dialogue window, select is less than and key in 16.



	A	B	C	D	E	F	G	H
1	TEAM	WINS	LOSSES	TIES	GF	GA	POINTS	
2	Blades	12	3	5	70	56	29	
3	Ice Hogs	6	8	3	46	46	15	
4	Leaves	7	5	8	55	51	22	
5	Nice Guys	10	5	2	64	42	22	
6	Rinkrats	6	7	3	58	63	15	
7	Slapshots	2	15	1	24	71	5	

In Corel® Quattro® Pro, you use Tools / Quickfilter / custom….

Now, the filter shows only the rows representing teams with fewer than 16 points.



	A	B	C	D	E	F	G	H
1	TEAM	WINS	LOSSES	TIES	GF	GA	POINTS	
3	Ice Hogs	6	8	3	46	46	15	
6	Rinkrats	6	7	3	58	63	15	
7	Slapshots	2	15	1	24	71	5	

Adding and Referencing Worksheets

To **add worksheets** within your spreadsheet file, click on one of the sheet tabs at the bottom of the data area. You can enter data onto the additional worksheet using any of the methods described above or you can copy and paste data from the first worksheet or from another file.

To **reference data from cells in another worksheet**, preface the cell reference with the worksheet number for the cells.

Such references allow data entered in sheet A or sheet 1 to be manipulated in another sheet without changing the values or order of the original data. Data edited in the original sheet will be automatically updated in the other sheets that refer to it. Any sort performed in the original sheet will carry through to any references in other sheets, but any other data in the secondary sheets will not be affected. Therefore, it is usually best to either reference all the data in the secondary sheets or to sort the data only in the secondary sheets.

Project Prep

The calculation, sorting, and charting capabilities of spreadsheets could be particularly useful for your tools for data management project.

Example 6 Sheet Referencing

Reference the goals for (GF) and goals against (GA) for the hockey teams in Example 3 on a separate sheet and rank the teams by their goals scored.

Solution

Sheet 2 needs only the data in the columns titled GF and GA in sheet 1. Notice that cell C2 contains a cell reference to sheet 1. This reference ensures the data in cell F2 of sheet 1 will carry through to cell C2 of sheet 2 even if the data in sheet 1 is edited. Although the referenced and sorted data on sheet 2 appear as shown, the order of the teams on sheet 1 is unchanged.


C2 =Sheet1!F2

Hockey Example

	A	B	C	D
1	TEAM	GF	GA	
2	Blades	70	56	
3	Nice Guys	64	42	
4	Rinkrats	58	63	
5	Leaves	55	51	
6	Ice Hogs	46	46	
7	Slapshots	24	71	

Key Concepts

- Thousands of computer programs are available for managing data. These programs range from general-purpose software, such as word-processors and spreadsheets, to highly specialized applications for specific types of data.
- A spreadsheet is a software application that is used to enter, display, and manipulate data in rows and columns. Spreadsheet formulas perform calculations based on values or formulas contained in other cells.
- Spreadsheets normally use relative cell referencing, which automatically adjusts cell references whenever cells are copied, moved, or sorted. Absolute cell referencing keeps formula references exactly as written.
- Spreadsheets can produce a wide variety of charts and perform sophisticated sorts and searches of data.
- You can add additional worksheets to a file and reference these sheets to cells in another sheet.

Communicate Your Understanding

1. Explain how you could use a word-processor as a data management tool.
2. Describe the advantages and drawbacks of relational database programs.
3. Explain what software you would choose if you wanted to determine whether there was a relationship between class size and subject in your school. Would you choose different software if you were going to look at class sizes in all the schools in Ontario?
4. Give an example of a situation requiring relative cell referencing and one requiring absolute cell referencing.
5. Briefly describe three advantages that spreadsheets have over hand-written tables for storing and manipulating data.

Practise

A

1. **Application** Set up a spreadsheet page in which you have entered the following lists of data. For the appropriate functions, look under the Statistical category in the Function list.

   Student marks:

   65, 88, 56, 76, 74, 99, 43, 56, 72, 81, 80, 30, 92

   Dentist appointment times in minutes:

   45, 30, 40, 32, 60, 38, 41, 45, 40, 45

   a) Sort each set of data from smallest to greatest.

   b) Calculate the mean (average) value for each set of data.

   c) Determine the median (middle) value for each set of data.

   d) Determine the mode (most frequent) value for each set of data.

2. Using the formula features of the spreadsheet available in your school, write a formula for each of the following:

   a) the sum of the numbers stored in cells A1 to A9

   b) the largest number stored in cells F3 to K3

   c) the smallest number in the block from A1 to K4

   d) the sum of the cells A2, B5, C7, and D9

   e) the mean, median, and mode of the numbers stored in the cells F5 to M5

   f) the square root of the number in cell A3

   g) the cube of the number in cell B6

   h) the number in cell D2 rounded off to four decimal places

   i) the number of cells between cells D3 and M9 that contain data

   j) the product of the values in cells A1, B3, and C5 to C10

   k) the value of π

Apply, Solve, Communicate

B

3. Set up a spreadsheet page that converts angles in degrees to radians using the formula Radians = π×Degrees/180, for angles from 0° to 360° in steps of 5°. Use the series capabilities to build the data in the Degrees column. Use π as defined by the spreadsheet. Calculations should be rounded to the nearest hundredth.

4. The first set of data below represents the number of sales of three brands of CD players at two branches of Mad Dog Music. Enter the data into a spreadsheet using two rows and three columns.

Branch	Brand A	Brand B	Brand C
Store P	12	4	8
Store Q	9	15	6

   The second set of data represents the prices for these CD players. Enter the data using one column into a second sheet of the same spreadsheet workbook. Set up a third sheet of the spreadsheet workbook to reference the first two sets of data and calculate the total revenue from CD player sales at each Mad Dog Music store.

Brand	Price
A	$102
B	$89
C	$145

5. **Application** In section 1.1, question 12, you predicted the value of the *n*th term of the recursion formulas listed below. Verify your predictions by using a spreadsheet to calculate the first ten terms for each formula.

   a) $t_n = 2^{-t_{n-1}}$; $t_1 = 0$

   b) $t_n = \sqrt{t_{n-1}}$; $t_1 = 256$

   c) $t_n = \dfrac{1}{t_{n-1}}$; $t_1 = 2$

6. a) Enter the data shown in the table below into a spreadsheet and set up a second sheet with relative cell references to the Name, Fat, and Fibre cells in the original sheet.

Nutritional Content of 14 Breakfast Cereals (amounts in grams)

Name	Protein	Fat	Sugars	Starch	Fibre	Other	TOTALS
Alphabits	2.4	1.1	12.0	12.0	0.9	1.6	
Bran Flakes	4.4	1.2	6.3	4.7	11.0	2.4	
Cheerios	4.0	2.3	0.8	18.7	2.2	2.0	
Crispix	2.2	0.3	3.2	22.0	0.5	1.8	
Froot Loops	1.3	0.8	14.0	12.0	0.5	1.4	
Frosted Flakes	1.4	0.2	12.0	15.0	0.5	0.9	
Just Right	2.2	0.8	6.6	17.0	1.4	2.0	
Lucky Charms	2.1	1.0	13.0	11.0	1.4	1.5	
Nuts 'n Crunch	2.3	1.6	7.1	16.5	0.7	1.8	
Rice Krispies	2.1	0.4	2.9	22.0	0.3	2.3	
Shreddies	2.9	0.6	5.0	16.0	3.5	2.0	
Special K	5.1	0.4	2.5	20.0	0.4	1.6	
Sugar Crisp	2.0	0.7	14.0	11.0	1.1	1.2	
Trix	0.9	1.6	13.0	12.0	1.1	1.4	
AVERAGES							
MAXIMUM							
MINIMUM							

b) On the first sheet, calculate the values for the TOTALS column and AVERAGES row.

c) Determine the maximum and minimum values in each column.

d) Rank the cereals using fibre content in decreasing order as a primary criterion, protein content in decreasing order as a secondary criterion, and sugar content in increasing order as a tertiary criterion.

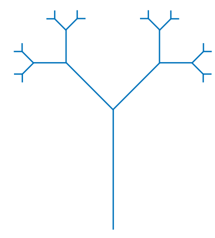
e) Make three circle graphs or pie charts: one for the averages row in part b), one for the cereal at the top of the list in part d), and one for the cereal at the bottom of the list in part d).

f) Perform a search in the second sheet to find the cereals containing less than 1 g of fat and more than 1.5 g of fibre. Make a three-dimensional bar graph of the results.

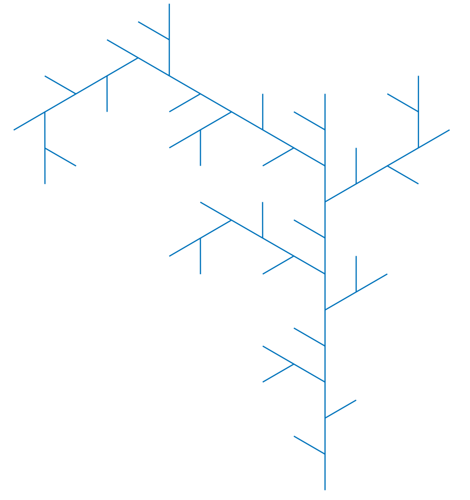
C

7. In section 1.1, question 10, you described the algorithm used to draw each fractal tree below. Assuming the initial segment is 4 cm in each tree, use a spreadsheet to determine the total length of a spiral in each tree, calculated to 12 iterative stages.

a)



b)



8. **Communication** Describe how to lock column and row headings in your spreadsheet software so that they remain visible when you scroll through a spreadsheet.

9. **Inquiry/Problem Solving** Outline a spreadsheet algorithm to calculate $n \times (n - 1) \times (n - 2) \ldots 3 \times 2 \times 1$ for any natural number n without using the built-in factorial function.

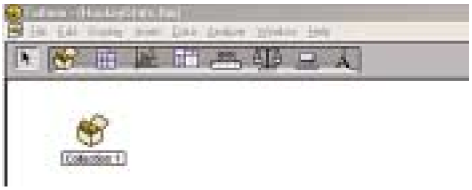
Introduction to Fathom™

Fathom™ is a statistics software package that offers a variety of powerful data-analysis tools in an easy-to-use format. This section introduces the most basic features of Fathom™: entering, displaying, sorting, and filtering data. A complete guide is available on the Fathom™ CD. The real power of this software will be demonstrated in later chapters with examples that apply its sophisticated tools to statistical analysis and simulations.

Appendix B includes details on all the Fathom™ functions used in this text.

When you enter data into Fathom™, it creates a **collection**, an object that contains the data. Fathom™ can then use the data from the collection to produce other objects, such as a graph, table, or statistical test. These secondary objects display and analyse the data from the collection, but they do not actually contain the data themselves. If you delete a graph, table, or statistical test, the data still remains in the collection.

Fathom™ displays a collection as a rectangular window with gold balls in it. The gold balls of the collection represent the original or "raw" data. Each of the gold balls represents a **case**. Each case in a collection can have a number of **attributes**. For example the cases in a collection of medical records could have attributes such as the patient's name, age, sex, height, weight, blood pressure, and so on. There are two basic types of attributes, **categorical** (such as male/female) and **continuous** (such as height or mass). The **case table** feature displays the cases in a collection in a format similar to a spreadsheet, with a row for each case and a column for each attribute. You can add, modify, and delete cases using a case table.



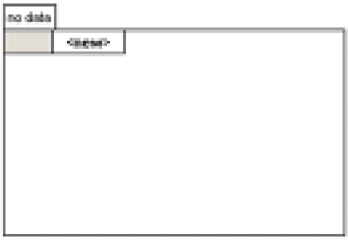
Example 1 Tables and Graphs

a) Set up a collection for the hockey league standings from section 1.2, Example 3 on page 17.

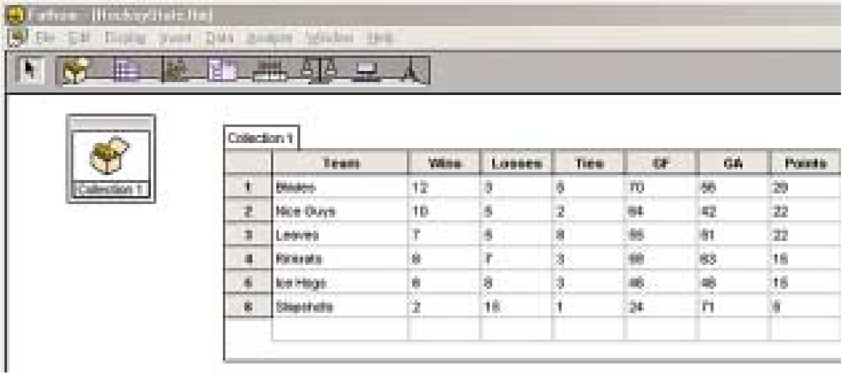
b) Graph the Team and Points attributes.

Solution

a) To enter the data, start Fathom™ and drag the **case table** icon from the menu bar down onto the work area.



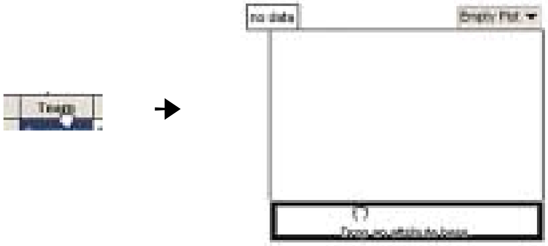
Click on the attribute <new>, type the heading Team, and press Enter. Fathom™ will automatically create a blank cell for data under the heading and start a new column to the right of the first. Enter the heading Wins at the top of the new column, and continue this process to enter the rest of the headings. You can type entries into the cells under the headings in much the same way as you would enter data into the cells of a spreadsheet.



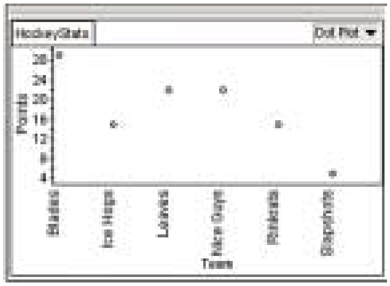
	Team	Wins	Losses	Ties	GF	GA	Points
1	Blades	12	3	5	70	55	29
2	Nice Guys	10	5	2	64	42	22
3	Leaves	7	5	8	65	51	22
4	Rinkrats	6	7	3	59	63	15
5	Ice Hogs	6	8	3	46	46	15
6	Slapshots	2	16	1	24	71	5

Note that Fathom™ has stored your data as Collection 1, which will remain intact even if you delete the **case table** used to enter the data. To give the collection a more descriptive name, double-click on Collection 1 and type in HockeyStats.

b) Drag the **graph icon**  onto the work area. Now, drag the Team attribute from the **case table** to the *x*-axis of the graph and the Points attribute to the *y*-axis of the graph.



Your graph should look like this:



Fathom™ can easily sort or filter data using the various attributes.

Example 2 Sorting and Filtering

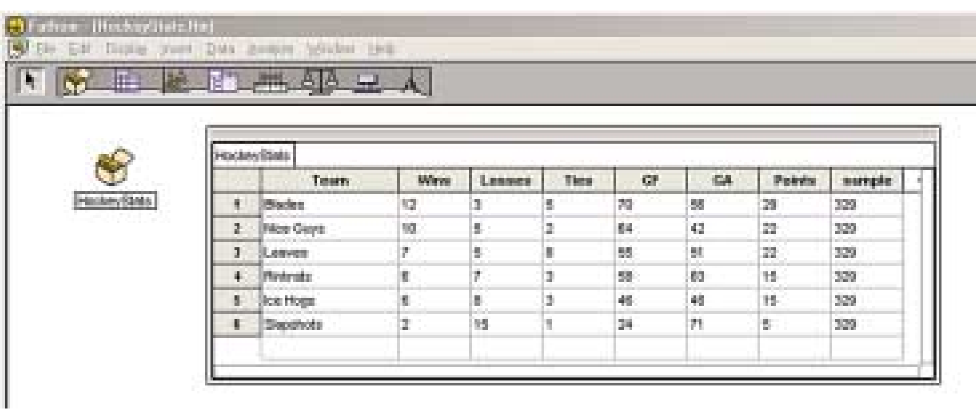
a) Rank the hockey teams in Example 1 by points first, then by wins if two teams have the same number of points, and finally by losses if two teams have the same number of points and wins.

b) List only those teams with fewer than 16 points.

c) Set up a separate table showing only the goals for (GF) and goals against (GA) data for the teams and rank the teams by their goals scored.

Solution

a) To **Sort** the data, right-click on the Points attribute and choose Sort Descending. Fathom™ will list the team with the most points first, with the others following in descending order by their point totals. To set the secondary sort, right-click on the Wins attribute and choose Sort Descending. Similarly, right-click on the Losses attribute and choose Sort Ascending for the final sort, giving the result below.

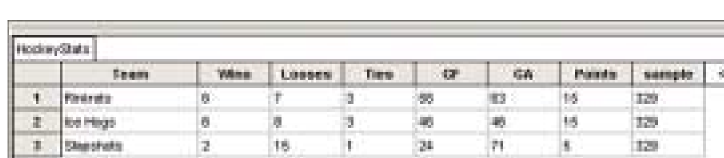


	Team	Wins	Losses	Ties	GF	GA	Points	sample
1	Blades	12	3	5	70	58	29	329
2	Nice Guys	10	5	2	64	42	22	329
3	Leaves	7	5	8	55	51	22	329
4	Rinkrats	6	7	3	58	63	15	329
5	Ice Hogs	6	8	3	46	48	15	329
6	Slapshots	2	15	1	24	71	5	329

b) To **Filter** the data, from the Data menu, choose Add Filter. Click on the plus sign beside Attributes.

Now, double-click on the Points attribute, choose the less-than button, and type 16. Click the Apply button and then OK.

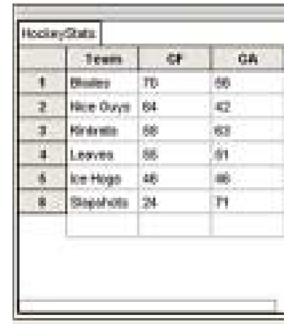
The results should look like this:



	Team	Wins	Losses	Ties	GF	GA	Points	sample
1	Rinkrats	6	7	3	58	63	15	329
2	Ice Hogs	6	8	3	46	48	15	329
3	Slapshots	2	15	1	24	71	5	329

The **Filter** is listed at the bottom as Points < 16.

c) Click on HockeyStats, and then drag a new table onto the work area. Click on the Wins attribute. From the Display menu, choose Hide Attribute. Use the same method to hide the Losses, Ties, and Points attributes. Right-click the GF attribute and use Sort Descending to rank the teams.



For details on functions in Fathom™, see the Fathom™ section of Appendix B or consult the Fathom™ Help screen or manual.

1. Enter the data from Example 1 into Fathom™. Use the built-in functions in Fathom™ to find the following.

a) the mean of goals against (GA)

b) the largest value of goals for (GF)

c) the smallest value of GF

d) the sum of GA

e) the sum of GA and GF for each case

2. a) Set up a new collection with the following student marks:
65, 88, 56, 76, 74, 99, 43, 56, 72, 81, 80, 30, 92

b) Sort the marks from lowest to highest.

c) Calculate the mean mark.

d) Determine the median (middle) mark.

3. Explain how you would create a graph of class size versus subjects in your school using Fathom™.

4. Briefly compare the advantages and disadvantages of using Fathom™ and spreadsheets for storing and manipulating data.

WEB CONNECTION

www.mcgrawhill.ca/links/MDM12

For more examples, data, and information on how to use Fathom™, visit the above web site and follow the links.

1.3 Databases

A **database** is an organized store of records. Databases may contain information about almost any subject—incomes, shopping habits, demographics, features of cars, and so on.

INVESTIGATE & INQUIRE: Databases in a Library

In your school or local public library, log on to the library catalogue.

1. Describe the types of fields under which a search can be conducted (e.g., subject).
2. Conduct a search for a specific topic of your choice.
3. Describe the results of your search. How is the information presented to the user?

INVESTIGATE & INQUIRE: The E-STAT Database

1. Connect to the Statistics Canada web site and go to the E-STAT database. Your school may have a direct link to this database. If not, you can follow the Web Connection links shown here. You may need to get a password from your teacher to log in.
2. Locate the database showing the educational attainment data for Canada by following these steps:
   a) Click on Data.
   b) Under the heading *People*, click on Education.
   c) Click on Educational Attainment, then under the heading *Census databases*, select Educational Attainment again.
   d) Select Education, Mobility and Migration for the latest census.

WEB CONNECTION

www.mcgrawhill.ca/links/MDM12

To connect to E-STAT visit the above web site and follow the links.

3. Scroll down to the heading *University, pop. 15 years and over by highest level of schooling*, hold down the Ctrl key, and select all four subcategories under this heading. View the data in each of the following formats:
   a) table b) bar graph c) map
4. Describe how the data are presented in each instance. What are the advantages and disadvantages of each format? Which format do you think is the most effective for displaying this data? Explain why.
5. Compare the data for the different provinces and territories. What conclusions could you draw from this data?

A database **record** is a set of data that is treated as a unit. A record is usually divided into **fields** that are reserved for specific types of information. For example, the record for each person in a telephone book has four fields: last name, first name or initial, address, and telephone number. This database is sorted in alphabetical order using the data in the first two fields. You search this database by finding the page with the initial letters of a person's name and then simply reading down the list.

A music store will likely keep its inventory records on a computerized database. The record for each different CD could have fields for information, such as title, artist, publisher, music type, price, number in stock, and a product code (for example, the bar code number). The computer can search such databases for matches in any of the data fields. The staff of the music store would be able to quickly check if a particular CD was in stock and tell the customer the price and whether the store had any other CDs by the same artist.

Databases in a Library

A library catalogue is a database. In the past, library databases were accessed through a card catalogue. Most libraries are now computerized, with books listed by title, author, publisher, subject, a Dewey Decimal or Library of Congress catalogue number, and an international standard book number (ISBN). Records can be sorted and searched using the information in any of the fields.

Such catalogues are examples of a well-organized database because they are easy to access using keywords and searches in multiple fields, many of which are cross-referenced. Often, school libraries are linked to other libraries. Students have access to a variety of print and online databases in the library. One powerful online database is Electric Library Canada, a database of books, newspapers, magazines, and television and radio transcripts. Your school probably has access to it or a similar library database. Your local public library may also have online access to Electric Library Canada.

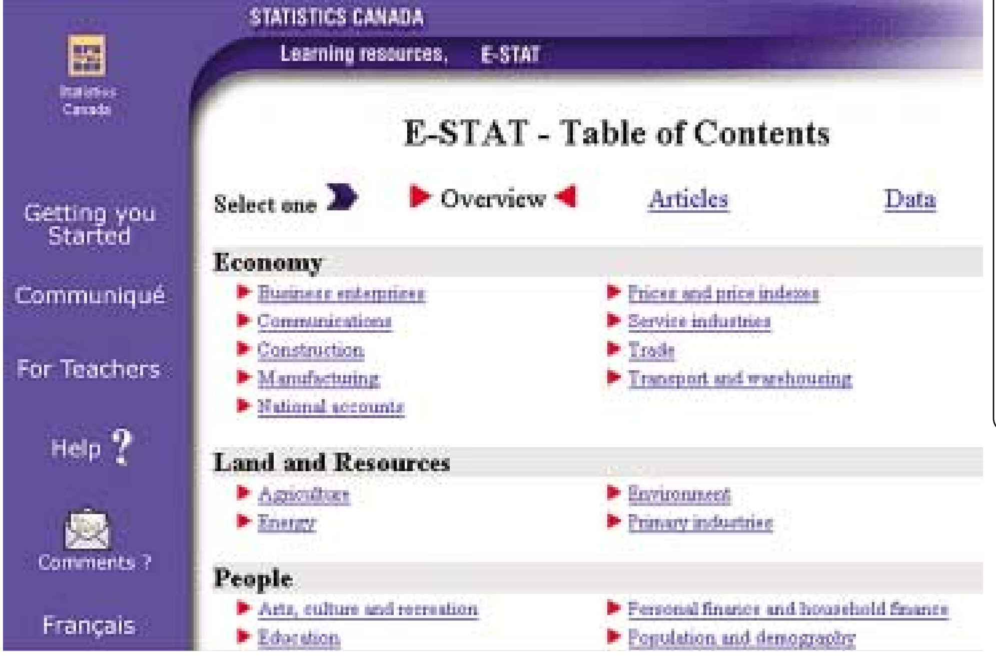
Project Prep

Skills in researching library and on-line databases will help you find the information needed for your tools for data management project.

Statistics Canada

Statistics Canada is the federal government department responsible for collecting, summarizing, analysing, and storing data relevant to Canadian demographics, education, health, and so on. Statistics Canada maintains a number of large databases using data collected from a variety of sources including its own research and a nation-wide census. One such database is **CANSIM II** (the updated version of the Canadian Socio-economic Information Management System), which profiles the Canadian people, economy, and industries. Although Statistics Canada charges a fee for access to some of its data, a variety of CANSIM II data is available to the public for free on Statistics Canada's web site.

Statistics Canada also has a free educational database, called **E-STAT**. It gives access to many of Statistics Canada's extensive, well-organized databases, including CANSIM II. E-STAT can display data in a variety of formats and allows students to download data into a spreadsheet or statistical software program.



Data in Action

By law, Statistics Canada is required to conduct a census of Canada's population and agriculture every five years. For the 2001 census, Statistics Canada needed about 37 000 people to distribute the questionnaires. Entering the data from the approximately 13.2 million questionnaires will take about 5 billion keystrokes.

Key Concepts

- A database is an organized store of records. A well-organized database can be easily accessed through searches in multiple fields that are cross-referenced.
- Although most databases are computerized, many are available in print form.

Communicate Your Understanding

1. For a typical textbook, describe how the table of contents and the index are sorted. Why are they sorted differently?
2. Describe the steps you need to take in order to access the 1860–61 census results through E-STAT.

Practise

A

1. Which of the following would be considered databases? Explain your reasoning.
   - a) a dictionary
   - b) stock-market listings
   - c) a catalogue of automobile specifications and prices
   - d) credit card records of customers' spending habits
   - e) an essay on Shakespeare's *Macbeth*
   - f) a teacher's mark book
   - g) the *Guinness World Records* book
   - h) a list of books on your bookshelf

Apply, Solve, Communicate

B

2. Describe each field you would include in a database of
   - a) a person's CD collection
   - b) a computer store's software inventory
   - c) a school's textbook inventory
   - d) the backgrounds of the students in a school
   - e) a business's employee records
3. a) Describe how you would locate a database showing the ethnic makeup of your municipality. List several possible sources.

   b) If you have Internet access, log onto E-STAT and go to the data on ethnic origins of people in large urban centres:
   - i) Select Data on the Table of Contents page.
   - ii) Select Population and Demography.
   - iii) Under *Census*, select Ethnic Origin.
   - iv) Select Ethnic Origin and Visible Minorities for the latest census in large urban centres.
   - v) Enter a postal code for an urban area and select two or more ethnic origins while holding down the Ctrl key.
   - vi) View table, bar graph, and map in turn and describe how the data are presented in each instance.

   c) Compare these results with the data you get if you leave the postal code section line blank. What conclusions could you draw from the two sets of data?

4. **Application**

a) Describe how you could find data to compare employment for males and females. List several possible sources.

b) If you have Internet access, log onto E-STAT and go to the data on employment and work activity:

i) Under the *People* heading, select Labour.

ii) Under the *Census databases* heading, select Salaries and Wages.

iii) Select Sources of Income (Latest census, Provinces, Census Divisions, Municipalities).

iv) While holding down the Ctrl key, click on All persons with employment income by work activity, Males with employment income by work activity, and Females with employment income by work activity.

v) Download this data as a spreadsheet file. Record the path and file name for the downloaded data.

c) Open the data file with a spreadsheet. You may have to convert the format to match your spreadsheet software. Use your spreadsheet to

i) calculate the percentage difference between male and female employment

ii) display all fields as a bar graph

5. **Communication** Go to the reference area of your school or local library and find a published database in print form.

a) Briefly describe how the database is organized.

b) Describe how to search the database.

c) Make a list of five books that are set up as databases. Explain why they would be considered databases.

6. **Application** The Internet is a link between many databases. Search engines, such as Yahoo Canada, Lycos, Google, and Canoe, are large databases of web sites. Each search engine organizes its database differently.

a) Use three different search engines to conduct a search using the keyword *automobile*. Describe how each search engine presents its data.

b) Compare the results of searches with three different search engines using the following keywords:

i) computer monitors

ii) computer+monitors

iii) computer or monitors

iv) "computer monitors"

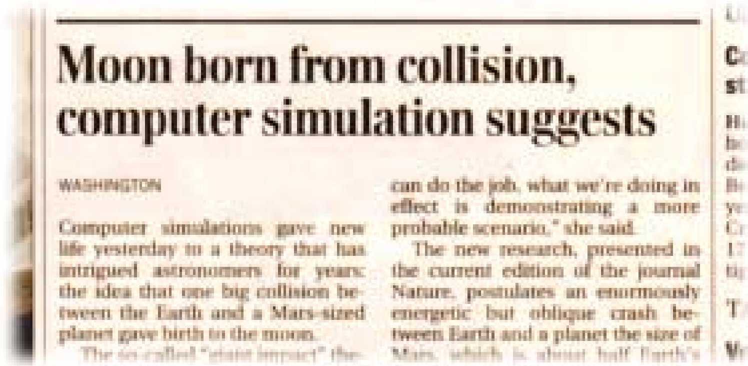
Chapter Problem

7. Use the Internet to check whether the map of VIA Rail routes at the start of this chapter is up-to-date. Are there still no trains that go from Montréal or Kingston right through to Windsor?

8. **Communication** Log on to the Electric Library Canada web site or a similar database available in your school library. Enter your school's username and password. Perform a search for magazine articles, newspaper articles, and radio transcripts about the "brain drain" or another issue of interest to you. Describe the results of your search. How many articles are listed? How are the articles described? What other information is provided?

1.4 Simulations

A **simulation** is an experiment, model, or activity that imitates real or hypothetical conditions. The newspaper article shown here describes how astrophysicists used computers to simulate a collision between Earth and a planet the size of Mars, an event that would be impossible to measure directly. The simulation showed that such a collision could have caused both the formation of the moon and the rotation of Earth, strengthening an astronomical theory put forward in the 1970s.



Moon born from collision, computer simulation suggests

WASHINGTON

Computer simulations gave new life yesterday to a theory that has intrigued astronomers for years: the idea that one big collision between the Earth and a Mars-sized planet gave birth to the moon.

The so-called "giant impact" the-

can do the job, what we're doing in effect is demonstrating a more probable scenario," she said.

The new research, presented in the current edition of the journal Nature, postulates an enormously energetic but oblique crash between Earth and a planet the size of Mars, which is about half Earth's

INVESTIGATE & INQUIRE: Simulations

For each of the following, describe what is being simulated, the advantages of using a simulation, and any drawbacks.

a) crash test dummies
b) aircraft simulators
c) wind tunnels
d) zero-gravity simulator
e) 3-D movies
f) paint-ball games
g) movie stunt actors
h) grow lights
i) architectural scale models

In some situations, especially those with many variables, it can be difficult to calculate an exact value for a quantity. In such cases, simulations often can provide a good estimate. Simulations can also help verify theoretical calculations.

Example 1 Simulating a Multiple-Choice Test

When writing a multiple-choice test, you may have wondered "What are my chances of passing just by guessing?" Suppose that you make random guesses on a test with 20 questions, each having a choice of 5 answers. Intuitively, you would assume that your mark will be somewhere around 4 out of 20 since there is a 1 in 5 chance of guessing right on each question. However, it is possible that you could get any number of the questions right—anywhere from zero to a perfect score.

a) Devise a simulation for making guesses on the multiple-choice test.

b) Run the simulation 100 times and use the results to estimate the mark you are likely to get, on average.

c) Would it be practical to run your simulation 1000 times or more?

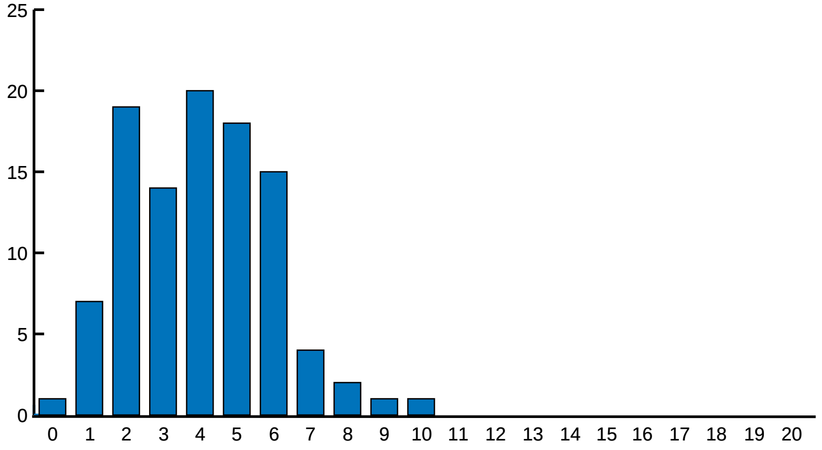
Solution 1 Using Pencil and Paper

a) Select any five cards from a deck of cards. Designate one of these cards to represent guessing the correct answer on a question. Shuffle the five cards and choose one at random. If it is the designated card, then you got the first question right. If one of the other four cards is chosen, then you got the question wrong.

Put the chosen card back with the others and repeat the process 19 times to simulate answering the rest of the questions on the test. Keep track of the number of right answers you obtained.

b) You could run 100 simulations by repeating the process in part a) over and over. However, you would have to choose a card 2000 times, which would be quite tedious. Instead, form a group with some of your classmates and pool your results, so that each student has to run only 10 to 20 repetitions of the simulation.

Make a table of the scores on the 100 simulated tests and calculate the mean score. You will *usually* find that this average is fairly close to the intuitive estimate of a score around 4 out of 20. However, a mean does not tell the whole story. Tally up the number of times each score appears in your table. Now, construct a bar graph showing the frequency for each score. Your graph will look something like the one shown.



This graph gives a much more detailed picture of the results you could expect. Although 4 is the most likely score, there is also a good chance of getting 2, 3, 5, or 6, but the chance of guessing all 20 questions correctly is quite small.

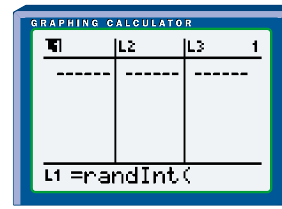
c) Running the simulation 1000 times would require shuffling the five cards and picking one 20 000 times—possible, but not practical.

Solution 2 Using a Graphing Calculator

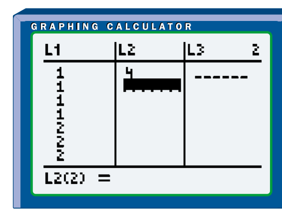
See Appendix B for more details on how to use the graphing calculator and software functions in Solutions 2 to 4.

a) You can use random numbers as the basis for a simulation. If you generate random integers from 1 to 5, you can have 1 correspond to a correct guess and 2 through 5 correspond to wrong answers.

Use the STAT EDIT menu to view lists L1 and L2. Make sure both lists are empty. Scroll to the top of L1 and enter the **randInt function** from the MATH PRB menu. This function produces random integers.



Enter 1 for the lower limit, 5 for the upper limit, and 20 for the number of trials. L1 will now contain 20 random integers between 1 and 5. Next, sort the list with the **SortA function** on the LIST OPS menu. Press 2nd 1 to enter L1 into the sort function. When you return to L1, the numbers in it will appear in ascending order. Now, you can easily scroll down the list to determine how many correct answers there were in this simulation.



b) The simplest way to simulate 100 tests is to repeat the procedure in part a) and keep track of the results by entering the number of correct answers in L2. Again, you may want to pool results with your classmates to reduce the number of times you have to enter the same formula over and over. If you know how to program your calculator, you can set it to re-enter the formulas for you automatically. However, unless you are experienced in programming the calculator, it will probably be faster for you to just re-key the formulas.

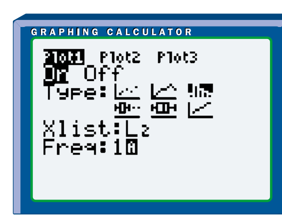
Once you have the scores from 100 simulations in L2, calculate the average using the **mean function** on the LIST MATH menu. To see which scores occur most frequently, plot L2 using **STAT PLOT**.

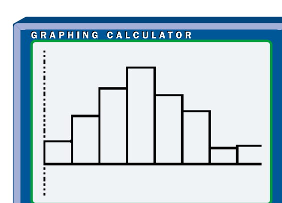
i) Turn off all plots except Plot1.

ii) For Type, choose the bar-graph icon and enter L2 for Xlist. Freq should be 1, the default value.

iii) Use ZOOM/ZoomStat to set the window for the data. Press WINDOW to check the **window settings**. Set Xscl to 1 so that the bars correspond to integers.

iv) Press GRAPH to display the bar graph.





c) It is possible to program the calculator to run a large number of simulations automatically. However, the maximum list length on the TI-83 Plus is 999, so you would have to use at least two lists to run the simulation a 1000 times or more.

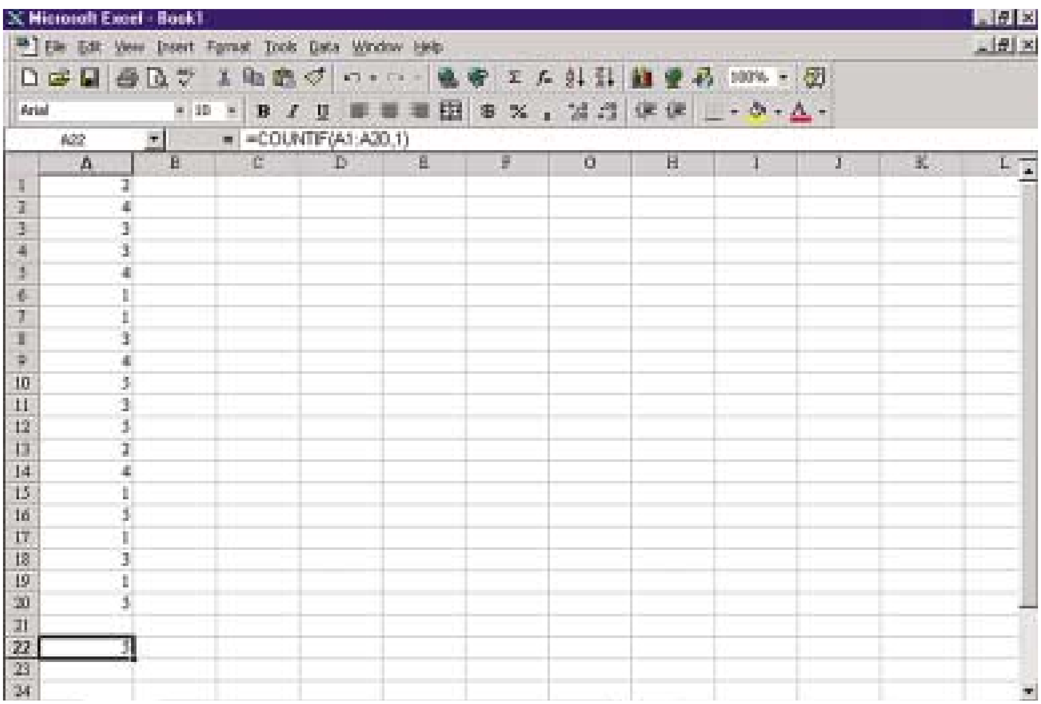
Solution 3 Using a Spreadsheet

a) Spreadsheets have built-in functions that you can use to generate and *count* the random numbers for the simulation.

The **RAND() function** produces a random real number that is equal to or greater than zero and less than one. The **INT function** rounds a real number down to the nearest integer. Combine these functions to generate a random integer between 1 and 5.

In Microsoft® Excel, you can use RANDBETWEEN only if you have the Analysis Toolpak installed.

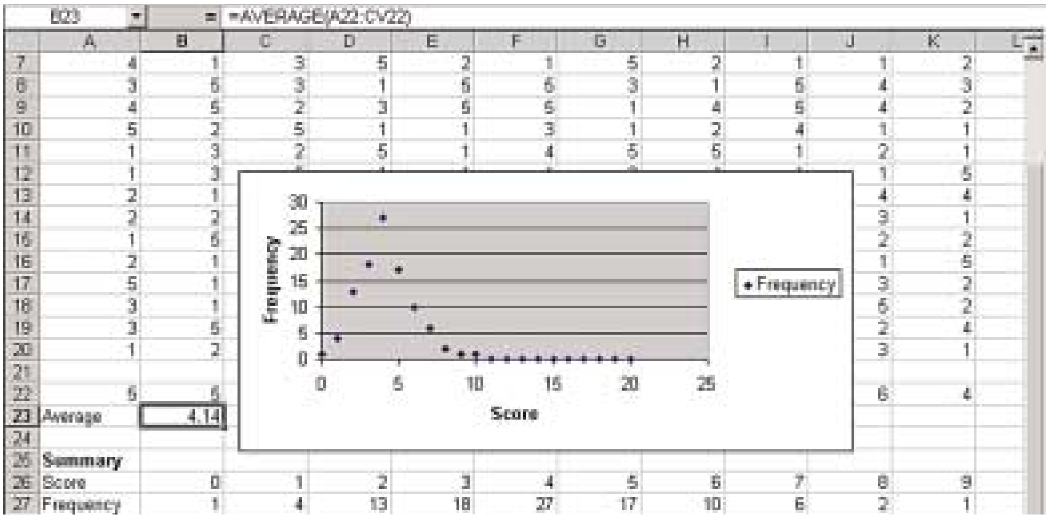
Enter the formula INT(RAND( )*5)+1 or RANDBETWEEN(1,5) in A1 and copy it down to A20. Next, use the **COUNTIF function** to count the number of *1*s in column A. Record this score in cell A22.



b) To run 100 simulations, copy A1:A22 into columns B through CV using the **Fill feature**. Then, use the **average function** to find the **mean** score for the 100 simulated tests. Record this average in cell B23.

Next, use the **COUNTIF function** to find the number of times each possible score occurs in cells A22 to CV22. Enter the headings SUMMARY, Score, and Frequency in cells A25, A26, and A27, respectively. Then, enter 0 in cell B26 and highlight cells B26 through V26. Use the **Fill feature** to enter the integers 0 through 20 in cells B26 through V26. In B27, enter the formula for the number of zero scores; in C27, the number of *1*s; in D27, the number of *2*s; and so on, finishing with V27 having the number of perfect scores. Note that by using **absolute cell referencing** you can simply copy the **COUNTIF function** from B27 to the other 20 cells.

Finally, use the **Chart feature** to plot frequency versus score. Highlight cells A26 through V27, then select Insert/Chart/XY.



c) The method in part b) can easily handle 1000 simulations or more.

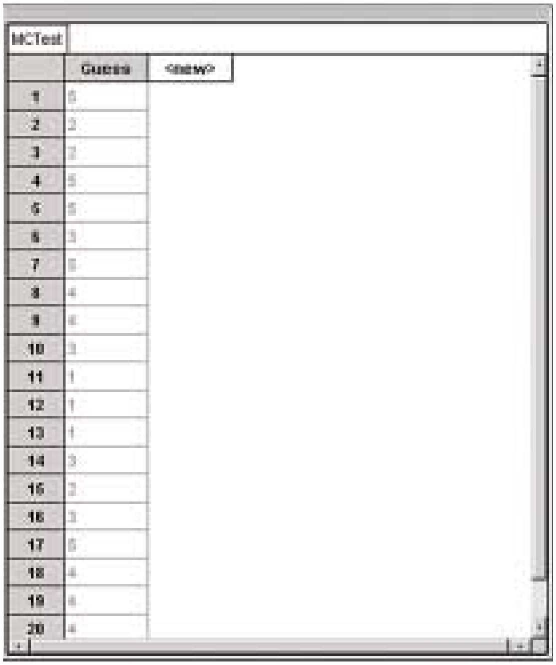
Solution 4 Using Fathom™

a) Fathom™ also has built-in functions to generate random numbers and count the scores in the simulations.

Launch Fathom™ and open a new document if necessary. Drag a new **collection** box to the document and rename it MCTest. Right-click on the box and create 20 new cases.

Drag a **case table** to the work area. You should see your 20 cases listed. Expand the table if you cannot see them all on the screen.

Rename the <new> column Guess. Right-click on Guess and select Edit Formula, Expand Functions, then Random Numbers. Enter 1,5 into the **randomInteger() function** and click OK to fill the Guess column with random integers between 1 and 5. Scroll down the column to see how many correct guesses there are in this simulation.



b) You can run a new simulation just by pressing Ctrl-Y, which will fill the Guess column with a new set of random numbers. Better still, you can set Fathom™ to automatically repeat the simulation 100 times automatically and keep track of the number of correct guesses.

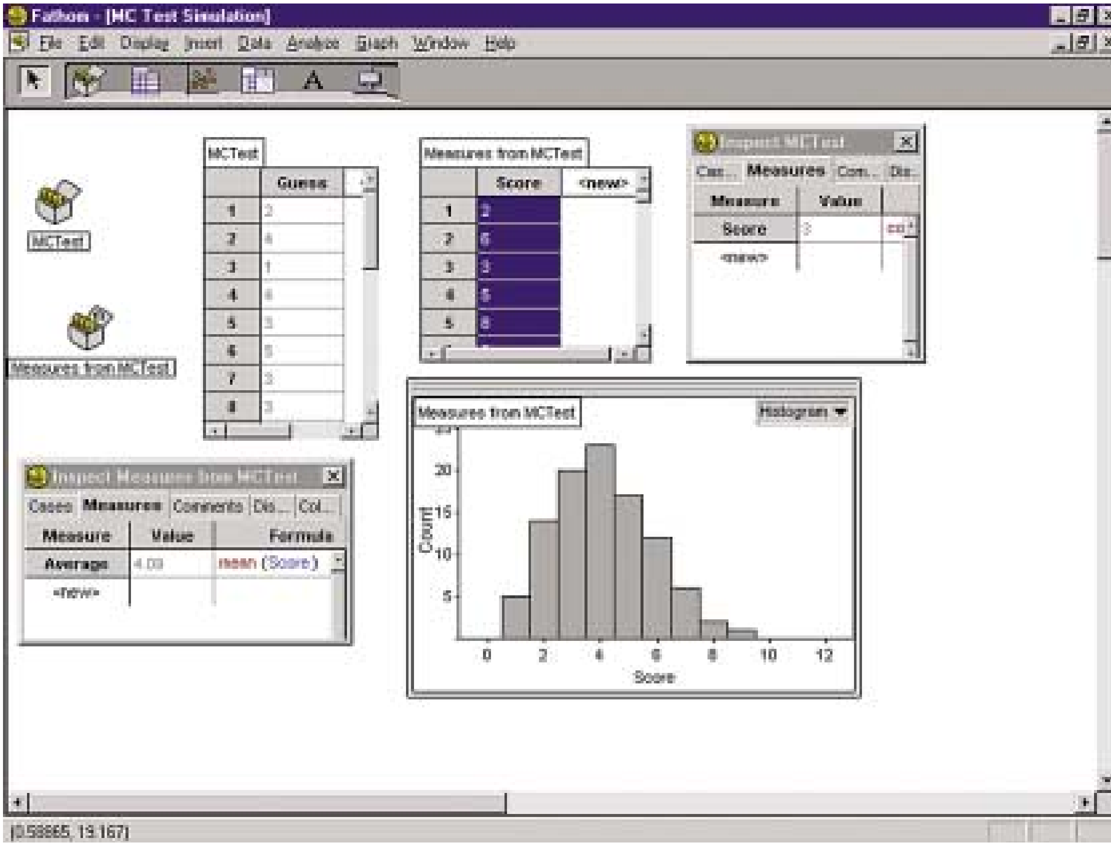
First, set up the count function. Right–click on the **collection** box and select Inspect Collection. Select the Measures tab and rename the <new> column Score. Then, right-click the column below Formula and select Edit Formula, Functions, Statistical, then One Attribute. Select count, enter Guess = 1 between the brackets, and click OK to count the number of correct guesses in your case table.

Click on the MCTest **collection** box. Now, select Analyse, Collect Measures from the main menu bar, which creates a new collection box called Measures from MCTest. Click on this box and drag a new **case table** to the document. Fathom™ will automatically run five simulations of the multiple-choice test and show the results in this **case table** .

To simulate 100 tests, right-click on the Measures from MCTest **collection** box and select Inspect Collection. Turn off the animation in order to speed up the simulation. Change the number of measures to 100. Then, click on the Collect More Measures button. You should now have 100 measures in the **case table** for Measures from MCTest.

Next, use the **mean function** to find the average score for these simulations. Go back to the Inspect Measures from MCTest **collection** box and change the column heading <new> to Average. Right-click on Formula and select Edit Formula, Functions, Statistical, then One Attribute. Select mean, enter Score between the brackets, and select OK to display the mean mark on the 100 tests.

Finally, plot a histogram of the scores from the simulations. Drag the **graph icon** onto the work area. Then, drag the Score column from the Measures from MCTest **case table** to the horizontal axis of the graph. Fathom™ then automatically produces a dot plot of your data. To display a histogram instead, simply click the menu in the upper right hand corner of the graph and choose Histogram.



c) Fathom™ can easily run this simulation 1000 times or more. All you have to do is change the number of measures.

Key Concepts

- Simulations can be useful tools for estimating quantities that are difficult to calculate and for verifying theoretical calculations.
- A variety of simulation methods are available, ranging from simple manual models to advanced technology that makes large-scale simulations feasible.

Communicate Your Understanding

1. Make a table summarizing the pros and cons of the four simulation methods used in Example 1.
2. A manufacturer of electric motors has a failure rate of 0.2% on one of its products. A quality-control inspector needs to know the range of the number of failures likely to occur in a batch of 1000 of these motors. Which tool would you use to simulate this situation? Give reasons for your choice.

Practise

A

1. Write a graphing calculator formula for

a) generating 100 random integers between 1 and 25

b) generating 24 random integers between –20 and 20

2. Write a spreadsheet formula for

a) generating 100 random numbers between 1 and 25

b) generating 100 random integers between 1 and 25

c) generating 16 random integers between –40 and 40

d) counting the number of entries that equal 42.5 in the range C10 to V40

Apply, Solve, Communicate

B

3. Communication Identify two simulations you use in everyday life and list the advantages of using each simulation.

4. Describe three other manual methods you could use to simulate the multiple-choice test in Example 1.

5. Communication

a) Describe a calculation or mechanical process you could use to produce random integers.

b) Could you use a telephone book to generate random numbers? Explain why or why not.

6. Application A brother and sister each tell the truth two thirds of the time. The brother stated that he owned the car he was driving. The sister said he was telling the truth. Develop a simulation to show whether you should believe them.

7. Inquiry/Problem Solving Consider a random walk in which a coin toss determines the direction of each step. On the odd-numbered tosses, walk one step north for heads and one step south for tails. On even-numbered tosses, walk one step east for heads and one step west for tails.

a) Beginning at position (0, 0) on a Cartesian graph, simulate this random walk for 100 steps. Note the coordinates where you finish.

b) Repeat your simulation 10 times and record the results.

c) Use these results to formulate a hypothesis about the endpoints for this random walk.

d) Change the rules of the random walk and investigate the effect on the end points.

ACHIEVEMENT CHECK

Knowledge/Understanding	Thinking/Inquiry/Problem Solving	Communication	Application

8. a) Use technology to simulate rolling two dice 100 times and record the sum of the two dice each time. Make a histogram of the sums.

b) Which sum occurs most often? Explain why this sum is likely to occur more often than the other sums.

c) Which sum or sums occur least often? Explain this result.

d) Suppose three dice are rolled 100 times and the sums are recorded. What sums would you expect to be the most frequent and least frequent? Give reasons for your answers.

C

9. Communication Describe a quantity that would be difficult to calculate or to measure in real life. Outline a simulation procedure you could use to determine this quantity.

1.5 Graph Theory

Graph theory is a branch of mathematics in which graphs or networks are used to solve problems in many fields. Graph theory has many applications, such as

- setting examination timetables
- colouring maps
- modelling chemical compounds
- designing circuit boards
- building computer, communication, or transportation networks
- determining optimal paths

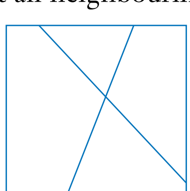
In graph theory, a graph is unlike the traditional Cartesian graph used for graphing functions and relations. A **graph** (also known as a **network**) is a collection of line segments and **nodes**. Mathematicians usually call the nodes **vertices** and the line segments **edges**. Networks can illustrate the relationships among a great variety of objects or sets.

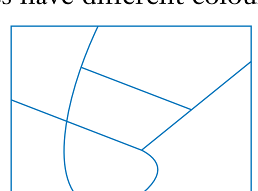
This network is an illustration of the subway system in Toronto. In order to show the connections between subway stations, this map is not to scale. In fact, networks are rarely drawn to scale.

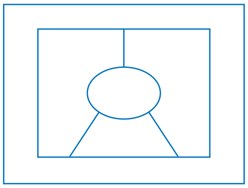
INVESTIGATE & INQUIRE: Map Colouring

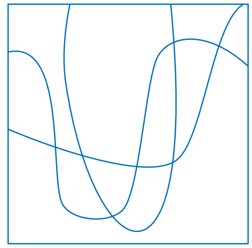
In each of the following diagrams the lines represent borders between countries. Countries joined by a line segment are considered **neighbours**, but countries joining at only a single point are not.

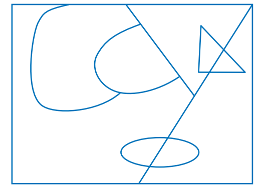
1. Determine the smallest number of colours needed for each map such that all neighbouring countries have different colours.

a) 

b) 

c) 

d) 

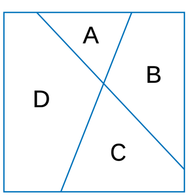
e) 

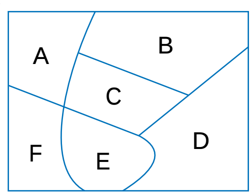
2. Make a conjecture regarding the maximum number of colours needed to colour a map. Why do you think your conjecture is correct?

Although the above activity is based on maps, it is very mathematical. It is about solving problems involving **connectivity**. Each country could be represented as a node or **vertex**. Each border could be represented by a segment or **edge**.

Example 1 Representing Maps With Networks

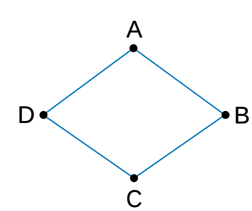
Represent each of the following maps with a network.

a) 

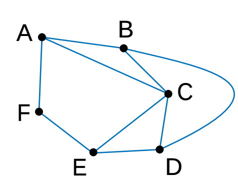
b) 

Solution

a) Let A, B, C, and D be vertices representing countries A, B, C, and D, respectively. A shares a border with both B and D but not with C, so A should be connected by edges to B and D only. Similarly, B is connected to only A and C; C, to only B and D; and D, to only A and C.

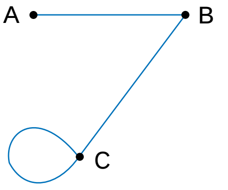


b) Let A, B, C, D, E, and F be vertices representing countries A, B, C, D, E, and F, respectively. Note that the positions of the vertices are not important, but their interconnections are. A shares borders with B, C, and F, but not with D or E. Connect A with edges to B, C, and F only. Use the same process to draw the rest of the edges.



As components of networks, edges could represent connections such as roads, wires, pipes, or air lanes, while vertices could represent cities, switches, airports, computers, or pumping stations. The networks could be used to carry vehicles, power, messages, fluid, airplanes, and so on.

If two vertices are connected by an edge, they are considered to be **adjacent**. In the network on the right, A and B are adjacent, as are B and C. A and C are not adjacent.



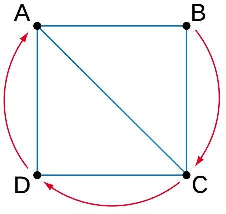
The number of edges that begin or end at a vertex is called the **degree** of the vertex. In the network, A has degree 1, B has degree 2, and C has degree 3. The loop counts as both an edge beginning at C and an edge ending at C.

Any connected sequence of vertices is called a **path**. If the path begins and ends at the same vertex, the path is called a **circuit**. A circuit is independent of the starting point. Instead, the circuit depends on the route taken.

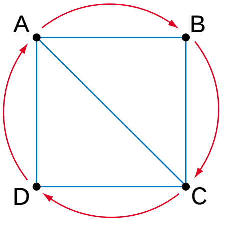
Example 2 Circuits

Determine if each path is a circuit.

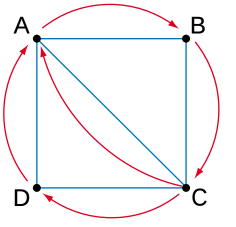
a)



b)



c)



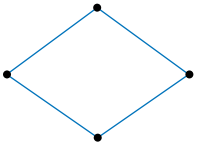
Solution

a) Path: BC to CD to DA
Since this path begins at B and ends at A, it is not a circuit.

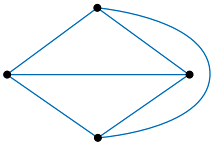
b) Path: BC to CD to DA to AB
This path begins at B and ends at B, so it is a circuit.

c) Path: CA to AB to BC to CD to DA
Since this path begins at C and ends at A, it is not a circuit.

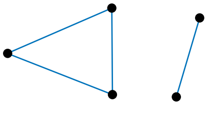
A network is **connected** if and only if there is at least one path connecting each pair of vertices. A **complete** network is a network with an edge between every pair of vertices.



Connected but not complete: Not all vertices are joined directly.

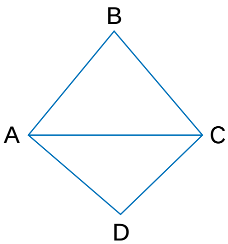


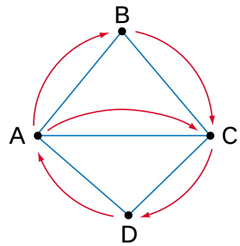
Connected and complete: All vertices are joined to each other by edges.



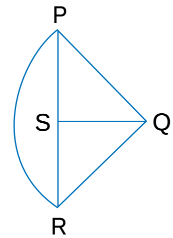
Neither connected nor complete: Not all vertices are joined.

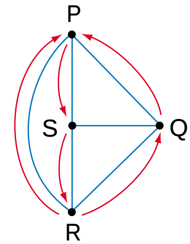
In a **traceable** network all the vertices are connected to at least one other vertex and all the edges can be travelled exactly once in a continuous path.





Traceable: All vertices are connected to at least one other vertex, and the path from A to B to C to D to A to C includes all the edges without repeating any of them.

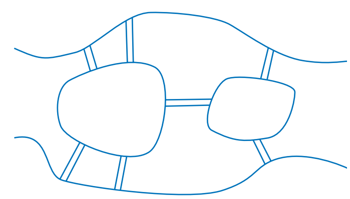




Non-traceable: No continuous path can travel all the edges only once.

Example 3 The Seven Bridges of Koenigsberg

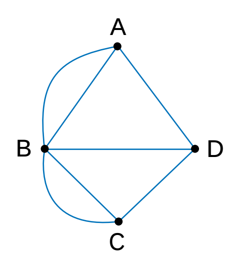
The eighteenth-century German town of Koenigsberg (now the Russian city of Kaliningrad) was situated on two islands and the banks of the Pregel River. Koenigsberg had seven bridges as shown in the map.



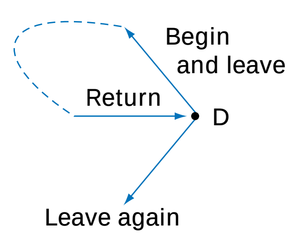
People of the town believed—but could not prove—that it was impossible to tour the town, crossing each bridge exactly once, regardless of where the tour started or finished. Were they right?

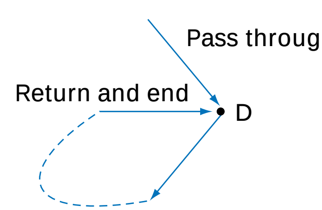
Solution

Reduce the map to a simple network of vertices and edges. Let vertices A and C represent the mainland, with B and D representing the islands. Each edge represents a bridge joining two parts of the town.



If, for example, you begin at vertex D, you will leave and eventually return but, because D has a degree of 3, you will have to leave again.





Conversely, if you begin elsewhere, you will pass through vertex D at some point, entering by one edge and leaving by another. But, because D has degree 3, you must return in order to trace the third edge and, therefore, must end at D. So, your path must either begin or end at vertex D. Because all the vertices are of odd degree, the same argument applies to all the other vertices. Since you cannot begin or end at more than two vertices, the network is non-traceable. Therefore, it is indeed impossible to traverse all the town's bridges without crossing one twice.

Leonhard Euler developed this proof of Example 3 in 1735. He laid the foundations for the branch of mathematics now called graph theory. Among other discoveries, Euler found the following general conditions about the traceability of networks.

- A network is traceable if it has only vertices of even degree (even vertices) or exactly two vertices of odd degree (odd vertices).
- If the network has two vertices of odd degree, the tracing path must begin at one vertex of odd degree and end at the other vertex of odd degree.

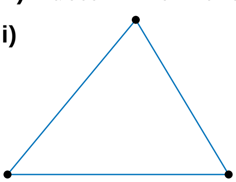
Example 4 Traceability and Degree

For each of the following networks,

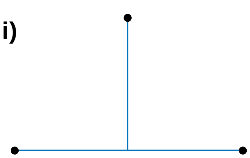
a) list the number of vertices with odd degree and with even degree

b) determine if the network is traceable

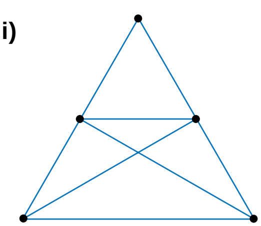
i)



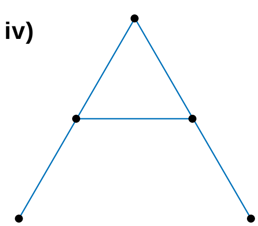
ii)



iii)



iv)



Solution

i) **a)** 3 even vertices
0 odd vertices
b) traceable

ii) **a)** 0 even vertices
4 odd vertices
b) non-traceable

iii) **a)** 3 even vertices
2 odd vertices
b) traceable

iv) **a)** 1 even vertex
4 odd vertices
b) non-traceable

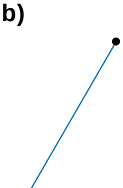
If it is possible for a network to be drawn on a two-dimensional surface so that the edges do not cross anywhere except at vertices, it is **planar.**

Example 5 Planar Networks

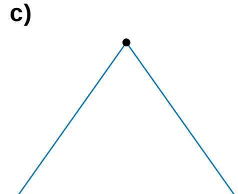
Determine whether each of the following networks is planar.

a)

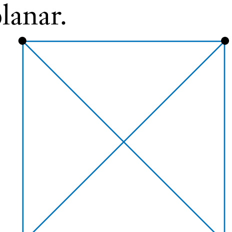
b)



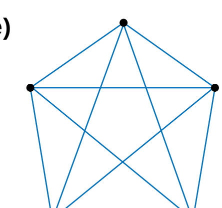
c)



d)



e)

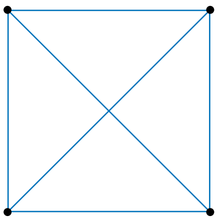
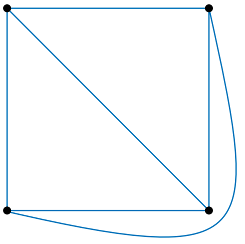


Solution

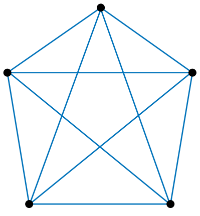
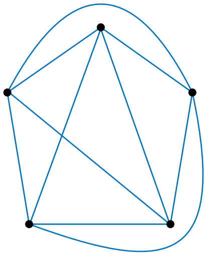
a) Planar

b) Planar

c) Planar

d)  can be redrawn as 

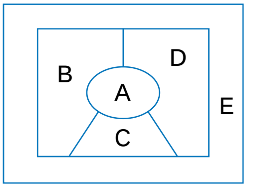
Therefore, the network is planar.

e)  cannot be redrawn as a planar network: 

Therefore, the network is non-planar.

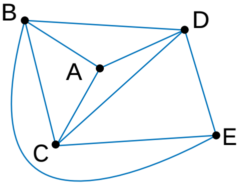
Example 6 Map Colouring (The Four-Colour Problem)

A graphic designer is working on a logo representing the different tourist regions in Ontario. What is the minimum number of colours required for the design shown on the right to have all adjacent areas coloured differently?



Solution

Because the logo is two-dimensional, you can redraw it as a planar network as shown on the right. This network diagram can help you see the relationships between the regions. The vertices represent the regions and the edges show which regions are adjacent. Vertices A and E both connect to the three other vertices but not to each other. Therefore, A and E can have the same colour, but it must be different from the colours for B, C, and D. Vertices B, C, and D all connect to each other, so they require three different colours. Thus, a minimum of four colours is necessary for the logo.



This example is a specific case of a famous problem in graph theory called the four-colour problem. As you probably conjectured in the investigation at the start of this section, the maximum number of colours required in *any* planar map is four. This fact had been suspected for centuries but was not proven until 1976. The proof by Wolfgang Haken and Kenneth Appel at the University of Illinois required a supercomputer to break the proof down into cases and many years of verification by other mathematicians. Non-planar maps can require more colours.

WEB CONNECTION

www.mcgrawhill.ca/links/MDM12

Visit the above web site and follow the links to find out more about the four-colour problem. Write a short report on the history of the four-colour problem.

Example 7 Scheduling

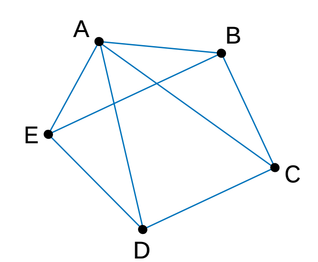
The mathematics department has five committees. Each of these committees meets once a month. Membership on these committees is as follows:

Committee A: Szczachor, Large, Ellis
Committee B: Ellis, Wegrynowski, Ho, Khan
Committee C: Wegrynowski, Large
Committee D: Andrew, Large, Szczachor
Committee E: Bates, Card, Khan, Szczachor

What are the minimum number of time slots needed to schedule the committee meetings with no conflicts?

Solution

Draw the schedule as a network, with each vertex representing a different committee and each edge representing a potential conflict between committees (a person on two or more committees). Analyse the network as if you were colouring a map.



The network can be drawn as a planar graph. Therefore, a maximum of four time slots is necessary to "colour" this graph. Because Committee A is connected to the four other committees (degree 4), at least two time slots are necessary: one for committee A and at least one for all the other committees. Because each of the other nodes has degree 3, at least one more time slot is necessary. In fact, three time slots are sufficient since B is not connected to D and C is not connected to E.

Time Slot	Committees
1	A
2	B, D
3	C, E

Project Prep

Graph theory provides problem-solving techniques that will be useful in your tools for data management project.

Key Concepts

- In graph theory, a graph is also known as a network and is a collection of line segments (edges) and nodes (vertices).
- If two vertices are connected by an edge, they are adjacent. The degree of a vertex is equal to the number of edges that begin or end at the vertex.
- A path is a connected sequence of vertices. A path is a circuit if it begins and ends at the same vertex.
- A connected network has at least one path connecting each pair of vertices. A complete network has an edge connecting every pair of vertices.
- A connected network is traceable if it has only vertices of even degree (even vertices) or exactly two vertices of odd degree (odd vertices). If the network has two vertices of odd degree, the tracing must begin at one of the odd vertices and end at the other.
- A network is planar if its edges do not cross anywhere except at the vertices.
- The maximum number of colours required to colour any planar map is four.

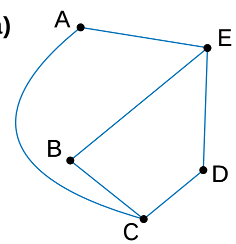
Communicate Your Understanding

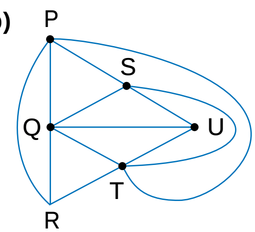
1. Describe how to convert a map into a network. Use an example to aid in your description.
2. A network has five vertices of even degree and three vertices of odd degree. Using a diagram, show why this graph cannot be traceable.
3. A modern zoo contains natural habitats for its animals. However, many of the animals are natural enemies and cannot be placed in the same habitat. Describe how to use graph theory to determine the number of different habitats required.

Practise

A

1. For each network,
   i) find the degree of each vertex
   ii) state whether the network is traceable

a) 

b) 

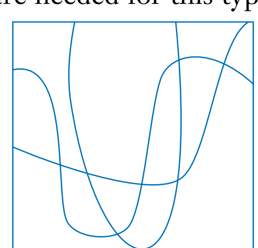
2. Draw a network diagram representing the maps in questions 1d) and 1e) of the investigation on pages 41 and 42.

3. a) Look at a map of Canada. How many colours are needed to colour the ten provinces and three territories of Canada?
   b) How many colours are needed if the map includes the U.S.A. coloured with a single colour?

Apply, Solve, Communicate

B

4. The following map is made up of curved lines that cross each other and stop only at the boundary of the map. Draw three other maps using similar lines. Investigate the four maps and make a conjecture of how many colours are needed for this type of map.



5. Is it possible to add one bridge to the Koenigsberg map to make it traceable? Provide evidence for your answer.

6. **Inquiry/Problem Solving** The following chart indicates the subjects studied by five students.

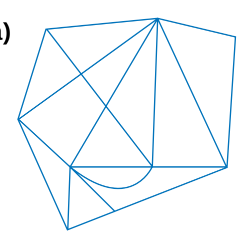
C. Powell	B. Bates	G. Farouk
English	Calculus	Calculus
French	French	French
History	Geometry	Geography
Music	Physics	Music

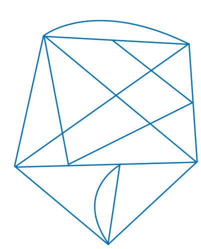
E. Ho	N. Khan
Calculus	English
English	Geography
Geometry	Mathematics of Data Management
Mathematics of Data Management	Physics

   a) Draw a network to illustrate the overlap of subjects these students study.
   b) Use your network to design an examination timetable without conflicts.

   (*Hint*: Consider each subject to be one vertex of a network.)

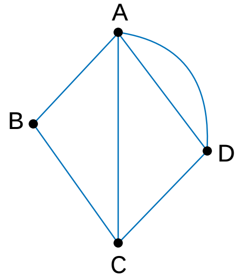
7. A highway inspector wants to travel each road shown once and only once to inspect for winter damage. Determine whether it is possible to do so for each map shown below.

a) 

b) 

8. **Inquiry/Problem Solving**

a) Find the degree of each vertex in the network shown.

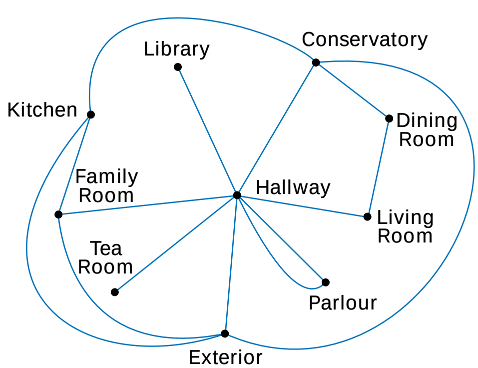


b) Find the sum of the degrees of the vertices.

c) Compare this sum with the number of edges in the network. Investigate other networks and determine the sum of the degrees of their vertices.

d) Make a conjecture from your observations.

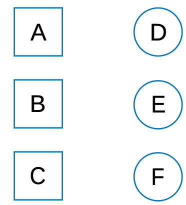
9. **a)** The following network diagram of the main floor of a large house uses vertices to represent rooms and edges to represent doorways. The exterior of the house can be treated as one room. Sketch a floor plan based on this network.



b) Draw a floor plan and a network diagram for your own home.

10. **Application**

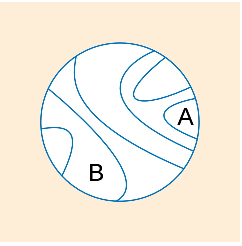
a) Three houses are located at positions A, B, and C, respectively. Water, gas, and electrical utilities are located at positions D, E, and F, respectively. Determine whether the houses can each be connected to all three utilities without any of the connections crossing. Provide evidence for your decision. Is it necessary to reposition any of the utilities? Explain.



b) Show that a network representing two houses attached to n utilities is planar.

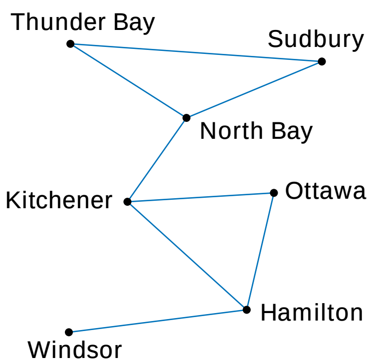
11. The four Anderson sisters live near each other and have connected their houses by a network of paths such that each house has a path leading directly to each of the other three houses. None of these paths intersect. Can their brother Warren add paths from his house to each of his sisters' houses without crossing any of the existing paths?

12. In the diagram below, a sheet of paper with a circular hole cut out partially covers a drawing of a closed figure. Given that point A is inside the closed figure, determine whether point B is inside or outside. Provide reasons for your answer.

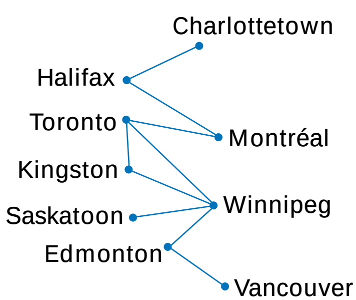


13. Application A communications network between offices of a company needs to provide a back-up link in case one part of a path breaks down. For each network below, determine which links need to be backed up. Describe how to back up the links.

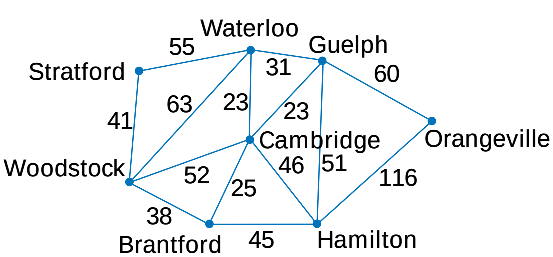
a)



b)



14. During an election campaign, a politician will visit each of the cities on the map below.

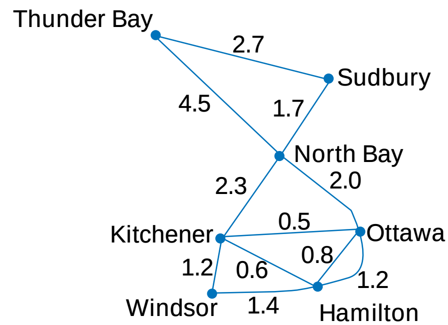


a) Is it possible to visit each city only once?

b) Is it possible to begin and end in the same city?

c) Find the shortest route for visiting all the cities. (*Hint*: You can usually find the shortest paths by considering the shortest edge at each vertex.)

15. In a communications network, the **optimal path** is the one that provides the fastest link. In the network shown, all link times are in seconds.



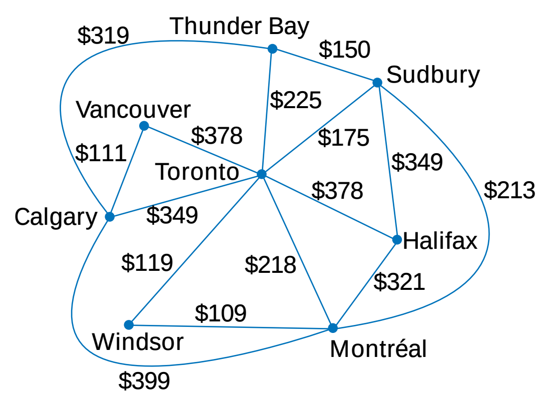
Determine the optimal path from

a) Thunder Bay to Windsor

b) Hamilton to Sudbury

c) Describe the method you used to estimate the optimal path.

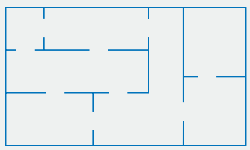
16. A salesperson must travel by air to all of the cities shown in the diagram below. The diagram shows the cheapest one-way fare for flights between the cities. Determine the least expensive travel route beginning and ending in Toronto.



ACHIEVEMENT CHECK

Knowledge / Understanding	Thinking / Inquiry / Problem Solving	Communication	Application

17. The diagram below shows the floor plan of a house.



a) Find a route that passes through each doorway of this house exactly once.

b) Use graph theory to explain why such a route is possible.

c) Where could you place two exterior doors so that it is possible to start outside the house, pass through each doorway exactly once, and end up on the exterior again? Explain your reasoning.

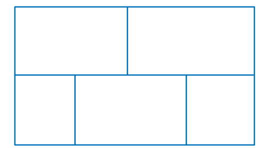
d) Is a similar route possible if you add three exterior doors instead of two? Explain your answer.

C

18. a) Six people at a party are seated at a table. No three people at the table know each other. For example, if Aaron knows Carmen and Carmen knows Allison, then Aaron and Allison do not know each other. Show that at least three of the six people seated at the table must be strangers to each other. (*Hint*: Model this situation using a network with six vertices.)

b) Show that, among five people, it is possible that no three all know each other and that no three are all strangers.

19. **Inquiry / Problem Solving** Use graph theory to determine if it is possible to draw the diagram below using only three strokes of a pencil.

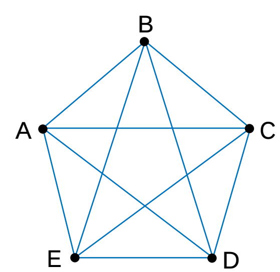


20. **Communication**

a) Can a connected graph of six vertices be planar? Explain your answer.

b) Can a complete graph of six vertices be planar? Explain.

21. Can the graph below represent a map in two dimensions. Explain.



22. Can a network have exactly one vertex with an odd degree? Provide evidence to support your answer.

23. **Communication** A graph is regular if all its vertices have the same degree. Consider graphs that do not have either loops connecting a vertex back to itself or multiple edges connecting any pair of vertices.

a) Draw the four regular planar graphs that have four vertices.

b) How many regular planar graphs with five vertices are there?

c) Explain the difference between your results in parts a) and b).

1.6 Modelling With Matrices

A **matrix** is a rectangular array of numbers used to manage and organize data, somewhat like a table or a page in a spreadsheet. Matrices are made up of horizontal rows and vertical columns and are usually enclosed in square brackets. Each number appearing in the matrix is called an **entry**. For instance, $A = \begin{bmatrix} 5 & -2 & 3 \\ 2 & 1 & 0 \end{bmatrix}$ is a matrix with two rows and three columns, with entries 5, –2, and 3 in the first row and entries 2, 1, and 0 in the second row. The **dimensions** of this matrix are 2×3. A matrix with m rows and n columns has dimensions of $m \times n$.

INVESTIGATE & INQUIRE: Olympic Medal Winners

At the 1998 Winter Olympic games in Nagano, Japan, Germany won 12 gold, 9 silver, and 8 bronze medals; Norway won 10 gold, 10 silver, and 5 bronze medals; Russia won 9 gold, 6 silver, and 3 bronze medals; Austria won 3 gold, 5 silver, and 9 bronze medals; Canada won 6 gold, 5 silver, and 4 bronze medals; and the United States won 6 gold, 3 silver, and 4 bronze medals.

1. Organize the data using a matrix with a row for each type of medal and a column for each country.
2. State the dimensions of the matrix.
3. a) What is the meaning of the entry in row 3, column 1?
   b) What is the meaning of the entry in row 2, column 4?
4. Find the sum of all the entries in the first row of the matrix. What is the significance of this **row sum**? What would the **column sum** represent?
5. Use your matrix to estimate the number of medals each country would win if the number of Olympic events were to be increased by 20%.
6. a) Interchange the rows and columns in your matrix by “reflecting” the matrix in the diagonal line beginning at row 1, column 1.
   b) Does this **transpose matrix** provide the same information? What are its dimensions?
7. State one advantage of using matrices to represent data.

In general, use a capital letter as the symbol for a matrix and represent each entry using the corresponding lowercase letter with two indices. For example,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} \qquad C = \begin{bmatrix} c_{11} & c_{12} & c_{13} & \cdots & c_{1n} \\ c_{21} & c_{22} & c_{23} & \cdots & c_{2n} \\ c_{31} & c_{32} & c_{33} & \cdots & c_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{m1} & c_{m2} & c_{m3} & \cdots & c_{mn} \end{bmatrix}$$

Here, a_{ij}, b_{ij}, and c_{ij} represent the entries in row i and column j of these matrices.

The transpose of a matrix is indicated by a superscript t, so the transpose of A is shown as A^t. A matrix with only one row is called a **row matrix**, and a matrix with only one column is a **column matrix**. A matrix with the same number of rows as columns is called a **square matrix**.

$$\begin{bmatrix} 1 & -2 & 5 & -9 \end{bmatrix} \qquad \begin{bmatrix} -3 \\ 0 \\ 5 \end{bmatrix} \qquad \begin{bmatrix} 3 & 4 & 9 \\ -1 & 0 & 2 \\ 5 & -10 & -3 \end{bmatrix}$$

a row matrix   a column matrix   a square matrix

Example 1 Representing Data With a Matrix

The number of seats in the House of Commons won by each party in the federal election in 1988 were Bloc Québécois (BQ), 0; Progressive Conservative Party (PC), 169; Liberal Party (LP), 83; New Democratic Party (NDP), 43; Reform Party (RP), 0; Other, 0. In 1993, the number of seats won were BQ, 54; PC, 2; LP, 177; NDP, 9; RP, 52; Other, 1. In 1997, the number of seats won were BQ, 44; PC, 20; LP, 155; NDP, 21; RP, 60; Other, 1.

a) Organize the data using a matrix S with a row for each political party.

b) What are the dimensions of your matrix?

c) What does the entry s_{43} represent?

d) What entry has the value 52?

e) Write the transpose matrix for S. Does S^t provide the same information as S?

f) The results from the year 2000 federal election were Bloc Québécois, 38; Progressive Conservative, 12; Liberal, 172; New Democratic Party, 13; Canadian Alliance (formerly Reform Party), 66; Other, 0. Update your matrix to include the results from the 2000 federal election.

Solution

a)

$$S = \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{ccc} 1988 & 1993 & 1997 \end{array} \\ \begin{bmatrix} 0 & 54 & 44 \\ 169 & 2 & 20 \\ 83 & 177 & 155 \\ 43 & 9 & 21 \\ 0 & 52 & 60 \\ 0 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{BQ} \\ \text{PC} \\ \text{LP} \\ \text{NDP} \\ \text{RP} \\ \text{Other} \end{array}$$

Labelling the rows and columns in large matrices can help you keep track of what the entries represent.

b) The dimensions of the matrix are 6×3.

c) The entry s_{43} shows that the NDP won 21 seats in 1997.

d) The entry s_{52} has the value 52.

e) The transpose matrix is

$$S^t = \begin{array}{c} \begin{array}{cccccc} \text{BQ} & \text{PC} & \text{LP} & \text{NDP} & \text{RP} & \text{Other} \end{array} \\ \begin{bmatrix} 0 & 169 & 83 & 43 & 0 & 0 \\ 54 & 2 & 177 & 9 & 52 & 1 \\ 44 & 20 & 155 & 21 & 60 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ 1988 \\ 1993 \\ 1997 \end{array}$$

Comparing the entries in the two matrices shows that they do contain exactly the same information.

f)

$$\begin{array}{c} \begin{array}{cccc} 1988 & 1993 & 1997 & 2000 \end{array} \\ \begin{bmatrix} 0 & 54 & 44 & 38 \\ 169 & 2 & 20 & 12 \\ 83 & 177 & 155 & 172 \\ 43 & 9 & 21 & 13 \\ 0 & 52 & 60 & 66 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{BQ} \\ \text{PC} \\ \text{LP} \\ \text{NDP} \\ \text{CA (RP)} \\ \text{Other} \end{array}$$

Two matrices are equal only if each entry in one matrix is equal to the corresponding entry in the other.

For example, $\begin{bmatrix} \frac{3}{2} & \sqrt{16} & (-2)^3 \\ 5^{-1} & -4 & -(-2) \end{bmatrix}$ and $\begin{bmatrix} 1.5 & 4 & -8 \\ \frac{1}{5} & -4 & 2 \end{bmatrix}$ are equal matrices.

Two or more matrices can be added or subtracted, provided that their dimensions are the same. To add or subtract matrices, add or subtract the corresponding entries of each matrix. For example,

$$\begin{bmatrix} 2 & -1 & 5 \\ 0 & 7 & -8 \end{bmatrix} + \begin{bmatrix} 0 & 5 & -3 \\ -2 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2 \\ -2 & 11 & -9 \end{bmatrix}$$

Matrices can be multiplied by a **scalar** or constant. To multiply a matrix by a scalar, multiply each entry of the matrix by the scalar. For example,

$$-3\begin{bmatrix} 4 & 5 \\ -6 & 0 \\ 3 & -8 \end{bmatrix} = \begin{bmatrix} -12 & -15 \\ 18 & 0 \\ -9 & 24 \end{bmatrix}$$

Example 2 Inventory Problem

The owner of Lou's 'Lectronics Limited has two stores. The manager takes inventory of their top-selling items at the end of the week and notes that at the eastern store, there are 5 video camcorders, 7 digital cameras, 4 CD players, 10 televisions, 3 VCRs, 2 stereo systems, 7 MP3 players, 4 clock radios, and 1 DVD player in stock. At the western store, there are 8 video camcorders, 9 digital cameras, 3 CD players, 8 televisions, 1 VCR, 3 stereo systems, 5 MP3 players, 10 clock radios, and 2 DVD players in stock. During the next week, the eastern store sells 3 video camcorders, 2 digital cameras, 4 CD players, 3 televisions, 3 VCRs, 1 stereo system, 4 MP3 players, 1 clock radio, and no DVD players. During the same week, the western store sells 5 video camcorders, 3 digital cameras, 3 CD players, 8 televisions, no VCRs, 1 stereo system, 2 MP3 players, 7 clock radios, and 1 DVD player. The warehouse then sends each store 4 video camcorders, 3 digital cameras, 4 CD players, 4 televisions, 5 VCRs, 2 stereo systems, 2 MP3 players, 3 clock radios, and 1 DVD player.

a) Use matrices to determine how many of each item is in stock at the stores after receiving the new stock from the warehouse.

b) Immediately after receiving the new stock, the manager phones the head office and requests an additional 25% of the items presently in stock in anticipation of an upcoming one-day sale. How many of each item will be in stock at each store?

Solution 1 Using Pencil and Paper

a) Let matrix A represent the initial inventory, matrix B represent the number of items sold, and matrix C represent the items in the first shipment of new stock.

$$A = \begin{array}{c} \begin{array}{cc} \text{E} & \text{W} \end{array} \\ \begin{bmatrix} 5 & 8 \\ 7 & 9 \\ 4 & 3 \\ 10 & 8 \\ 3 & 1 \\ 2 & 3 \\ 7 & 5 \\ 4 & 10 \\ 1 & 2 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{camcorders} \\ \text{cameras} \\ \text{CD players} \\ \text{TVs} \\ \text{VCRs} \\ \text{stereos} \\ \text{MP3 players} \\ \text{clock radios} \\ \text{DVD players} \end{array} \qquad B = \begin{bmatrix} 3 & 5 \\ 2 & 3 \\ 4 & 3 \\ 3 & 8 \\ 3 & 0 \\ 1 & 1 \\ 4 & 2 \\ 1 & 7 \\ 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 4 & 4 \\ 3 & 3 \\ 4 & 4 \\ 4 & 4 \\ 5 & 5 \\ 2 & 2 \\ 2 & 2 \\ 3 & 3 \\ 1 & 1 \end{bmatrix}$$

Since the dimensions of matrices A, B, and C are the same, matrix addition and subtraction can be performed. Then, the stock on hand before the extra shipment is

$$D = A - B + C = \begin{bmatrix} 5 & 8 \\ 7 & 9 \\ 4 & 3 \\ 10 & 8 \\ 3 & 1 \\ 2 & 3 \\ 7 & 5 \\ 4 & 10 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 5 \\ 2 & 3 \\ 4 & 3 \\ 3 & 8 \\ 3 & 0 \\ 1 & 1 \\ 4 & 2 \\ 1 & 7 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 4 \\ 3 & 3 \\ 4 & 4 \\ 4 & 4 \\ 5 & 5 \\ 2 & 2 \\ 2 & 2 \\ 3 & 3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 8 & 9 \\ 4 & 4 \\ 11 & 4 \\ 5 & 6 \\ 3 & 4 \\ 5 & 5 \\ 6 & 6 \\ 2 & 2 \end{bmatrix}$$

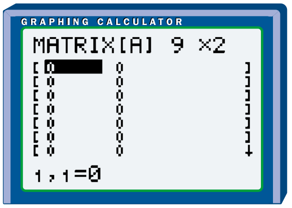
Let E represent the stock in the stores after the extra shipment from the warehouse.

$$E = 125\% \times D = 1.25 \begin{bmatrix} 6 & 7 \\ 8 & 9 \\ 4 & 4 \\ 11 & 4 \\ 5 & 6 \\ 3 & 4 \\ 5 & 5 \\ 6 & 6 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 7.5 & 8.75 \\ 10 & 11.25 \\ 5 & 5 \\ 13.75 & 5 \\ 6.25 & 7.5 \\ 3.75 & 5 \\ 6.25 & 6.25 \\ 7.5 & 7.5 \\ 2.5 & 2.5 \end{bmatrix}$$

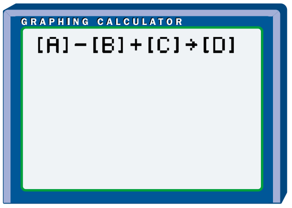
Assuming the manager rounds to the nearest whole number, the stock at the eastern store will be 8 video camcorders, 10 digital cameras, 5 CD players, 14 televisions, 6 VCRs, 4 stereo systems, 6 MP3 players, 8 clock radios, and 3 DVD players in stock. At the western store, there will be 9 video camcorders, 11 digital cameras, 5 CD players, 5 televisions, 8 VCRs, 5 stereo systems, 6 MP3 players, 8 clock radios, and 3 DVD players in stock.

Solution 2 Using a Graphing Calculator

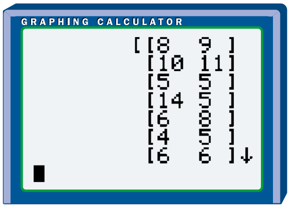
a) As in the pencil-and-paper solution, let matrix A represent the initial inventory, matrix B the items sold, and matrix C the first shipment of new stock. Use the MATRX EDIT menu to **store matrices**. Press ENTER to select a matrix name, then key in the dimensions and the entries. The calculator will store the matrix until it is cleared or overwritten. Matrix names and entries appear in square brackets on the calculator screen.



Use the MATRX NAMES menu to copy the matrices into the expression for D, the matrix representing the stock on hand before the extra shipment. Just move the cursor to the matrix you need and press ENTER.



b) To find the stock on hand after the extra shipment for the one-day sale, multiply matrix D by 1.25 and store the result in matrix E. Then, you can use the **round function** in the MATH NUM menu to display the closest whole numbers for the entries in matrix E.



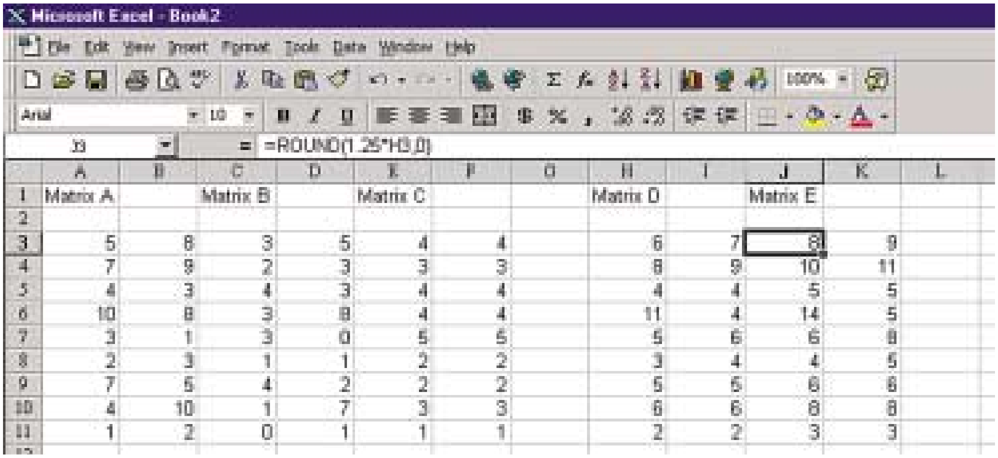
Solution 3 Using a Spreadsheet

a) You can easily perform **matrix operations** using a spreadsheet. It is also easy to add headings and row labels to keep track of what the entries represent. Enter each matrix using two adjacent columns: matrix A (initial stock) in columns A and B, matrix B (sales) in columns C and D, and matrix C (new stock) in columns E and F.

To find the amount of stock on hand after the first shipment from the warehouse, enter the formula A3–C3+E3 in cell H3.

Then, use the **Fill feature** to copy this formula for the rest of the entries in columns H and I.

b) Use the **Fill feature** in a similar way to copy the formula for the entries in matrix E, the stock on hand after the extra shipment from the warehouse. You can use the **ROUND function** to find the nearest whole number automatically. The formula for cell J3, the first entry, is ROUND(1.25*H3,0).



	A	B	C	D	E	F	G	H	I	J	K	L
1	Matrix A		Matrix B		Matrix C			Matrix D		Matrix E		
2												
3	5	8	3	5	4	4		6	7	8	9	
4	7	9	2	3	3	3		8	9	10	11	
5	4	3	4	3	4	4		4	4	5	5	
6	10	8	3	8	4	4		11	4	14	5	
7	3	1	3	0	5	5		5	6	6	8	
8	2	3	1	1	2	2		3	4	4	5	
9	7	5	4	2	2	2		5	5	6	6	
10	4	10	1	7	3	3		6	6	8	8	
11	1	2	0	1	1	1		2	2	3	3	

Key Concepts

- A matrix is used to manage and organize data.
- A matrix made up of m rows and n columns has dimensions $m \times n$.
- Two matrices are equal if they have the same dimensions and all corresponding entries are equal.
- The transpose matrix is found by interchanging rows with the corresponding columns.
- To add or subtract matrices, add or subtract the corresponding entries of each matrix. The dimensions of the matrices must be the same.
- To multiply a matrix by a scalar, multiply each entry of the matrix by the scalar.

Communicate Your Understanding

1. Describe how to determine the dimensions of any matrix.

2. Describe how you know whether two matrices are equal. Use an example to illustrate your answer.

3. Can transpose matrices ever be equal? Explain.

4. a) Describe how you would add two matrices. Give an example.

b) Explain why the dimensions of the two matrices need to be the same to add or subtract them.

5. Describe how you would perform scalar multiplication on a matrix. Give an example.

Practise

A

1. State the dimensions of each matrix.

a) $\begin{bmatrix} 4 & 5 & -1 \\ -2 & 3 & 8 \end{bmatrix}$ **b)** $[1 \;\; 0 \;\; -7]$

c) $\begin{bmatrix} 3 & -9 & -6 \\ 5 & 4 & 7 \\ 1 & 0 & 8 \\ 8 & -1 & 2 \end{bmatrix}$

2. For the matrix $A = \begin{bmatrix} -5 & 3 & 2 \\ 6 & 0 & -1 \\ 4 & 8 & -3 \\ 7 & 1 & -4 \end{bmatrix}$,

a) state the value in entry

i) a_{21} **ii)** a_{43} **iii)** a_{13}

b) state the entry with value

i) 4 **ii)** −3 **iii)** 1

3. Let $A = \begin{bmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & o \\ p & q & r & s & t \end{bmatrix}$

and $B = \begin{bmatrix} u & v \\ w & x \\ y & z \end{bmatrix}$.

For each of the following, replace a_{ij} or b_{ij} with its corresponding entry in the above matrices to reveal a secret message.

a) $a_{33}a_{11}a_{45}a_{43}a_{24}a_{13}a_{15}a_{44}$
$a_{11}a_{43}a_{15}$ $\quad a_{21}b_{11}a_{34}$

b) a_{24} $\quad a_{32}a_{35}b_{12}a_{15}$ $\quad a_{33}a_{11}a_{45}a_{23}$

c) $b_{21}a_{35}b_{21}$ $\quad a_{45}a_{23}a_{24}a_{44}$
$a_{24}a_{44}$ $\quad a_{21}b_{11}a_{34}$

4. a) Give two examples of row matrices and two examples of column matrices.

b) State the dimensions of each matrix in part a).

5. a) Give two examples of square matrices.

b) State the dimensions of each matrix in part a).

6. a) Write a 3×4 matrix, A, with the property that entry $a_{ij} = i + j$.

b) Write a 4×4 matrix, B, with the property that entry $b_{ij} = \begin{cases} 3 \text{ if } i = j \\ i \times j \text{ if } i \neq j \end{cases}$

7. Solve for w, x, y, and z.

a) $\begin{bmatrix} x & 4 \\ -2 & 4z-2 \end{bmatrix} = \begin{bmatrix} 3 & y-1 \\ w & 6 \end{bmatrix}$

b) $\begin{bmatrix} w^3 & x^2 \\ 2y & 3z \end{bmatrix} = \begin{bmatrix} 8 & 9 \\ 8-2y & 2z-5 \end{bmatrix}$

8. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 9 \\ 5 & 0 \\ -4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ -6 & 1 \\ 8 & 2 \\ -1 & -5 \end{bmatrix}$,

and $C = \begin{bmatrix} 3 & -2 & 6 & 5 \\ 1 & 4 & 0 & -8 \end{bmatrix}$.

Calculate, if possible,

a) $A + B$ **b)** $B + A$ **c)** $B - C$

d) $3A$ **e)** $-\frac{1}{2}B$ **f)** $2(B - A)$

g) $3A - 2B$

9. Let $A = \begin{bmatrix} 8 & -6 \\ 1 & -2 \\ -4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 2 & 4 \\ 9 & -3 \end{bmatrix}$,

and $C = \begin{bmatrix} 2 & 3 \\ 8 & -6 \\ 4 & 1 \end{bmatrix}$.

Show that

a) $A + B = B + A$
(commutative property)

b) $(A + B) + C = A + (B + C)$
(associative property)

c) $5(A + B) = 5A + 5B$
(distributive property)

10. Find the values of w, x, y, and z if

$$\begin{bmatrix} 5 & -1 & 2 \\ 4 & x & -8 \\ 7 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 6 & y & 5 \\ -3 & 2 & 1 \\ 2 & -3 & z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 34 & 10 & 24 \\ -4 & 24 & -12 \\ 2w & -12 & 42 \end{bmatrix}$$

11. Solve each equation.

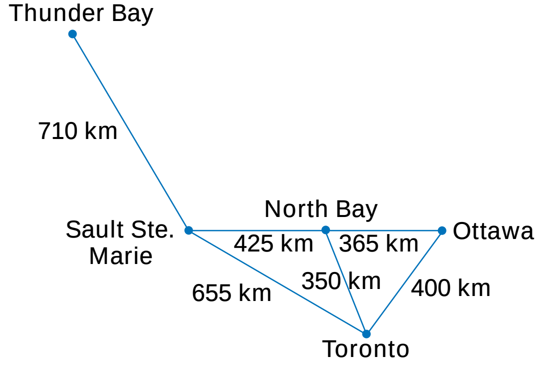
a) $\begin{bmatrix} 3 & 2 & -5 \\ 2 & 0 & 8 \end{bmatrix} + A = \begin{bmatrix} 7 & 0 & 1 \\ -4 & 3 & -2 \end{bmatrix}$

b) $\begin{bmatrix} 5 & 7 \\ 4 & 0 \\ -1 & -3 \end{bmatrix} + y\begin{bmatrix} 1 & 6 \\ 0 & -4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 19 \\ 4 & -8 \\ 3 & 7 \end{bmatrix}$

Apply, Solve, Communicate

B

12. **Application** The map below shows driving distances between five cities in Ontario.



a) Represent the driving distances between each pair of cities with a matrix, A.

b) Find the transpose matrix, A^t.

c) Explain how entry a_{23} in matrix A and entry a_{32} in matrix A^t are related.

13. Nobel prizes are awarded for physics, chemistry, physiology/medicine, literature, peace, and economic sciences. The top five Nobel prize-winning countries are U.S.A. with 67 Nobel prizes in physics, 43 in chemistry, 78 in physiology/medicine, 10 in literature, 18 in peace, and 25 in economic sciences; U.K. with 21 Nobel prizes in physics, 25 in chemistry, 24 in physiology/medicine, 8 in literature, 13 in peace, and 7 in economic sciences; Germany with 20 Nobel prizes in physics, 27 in chemistry, 16 in physiology/medicine, 7 in literature, 4 in peace, and 1 in economic sciences; France with 12 Nobel prizes in physics, 7 in chemistry, 7 in physiology/medicine, 12 in literature, 9 in peace, and 1 in economic sciences; and Sweden with 4 Nobel prizes in physics, 4 in chemistry, 7 in physiology/medicine, 7 in literature, 5 in peace, and 2 in economic sciences.

a) Represent this data as a matrix, N. What are the dimensions of N?

b) Use row or column sums to calculate how many Nobel prizes have been awarded to citizens of each country.

14. The numbers of university qualifications (degrees, certificates, and diplomas) granted in Canada for 1997 are as follows: social sciences, 28 421 males and 38 244 females; education, 8036 males and 19 771 females; humanities, 8034 males and 13 339 females; health professions and occupations, 3460 males and 9613 females; engineering and applied sciences, 10 125 males and 2643 females; agriculture and biological sciences, 4780 males and 6995 females; mathematics and physical sciences, 6749 males and 2989 females; fine and applied arts, 1706 males and 3500 females; arts and sciences, 1730 males and 3802 females.

The numbers for 1998 are as follows: social sciences, 27 993 males and 39 026 females; education, 7565 males and 18 391 females; humanities, 7589 males and 13 227 females; health professions and occupations, 3514 males and 9144 females; engineering and applied sciences, 10 121 males and 2709 females; agriculture and biological sciences, 4779 males and 7430 females;

mathematics and physical sciences, 6876 males and 3116 females; fine and applied arts, 1735 males and 3521 females; arts and sciences, 1777 males and 3563 females.

a) Enter two matrices in a graphing calculator or spreadsheet—one two-column matrix for males and females receiving degrees in 1997 and a second two-column matrix for the number of males and females receiving degrees in 1998.

b) How many degrees were granted to males in 1997 and 1998 for each field of study?

c) How many degrees were granted to females in 1997 and 1998 for each field of study?

d) What is the average number of degrees granted to females in 1997 and 1998 for each field of study?

15. **Application** The table below shows the population of Canada by age and gender in the year 2000.

Age Group	Number of Males	Number of Females
0–4	911 028	866 302
5–9	1 048 247	996 171
10–14	1 051 525	997 615
15–19	1 063 983	1 007 631
20–24	1 063 620	1 017 566
25–29	1 067 870	1 041 900
30–34	1 154 071	1 129 095
35–39	1 359 796	1 335 765
40–44	1 306 705	1 304 538
45–49	1 157 288	1 162 560
50–54	1 019 061	1 026 032
55–59	769 591	785 657
60–64	614 659	641 914
65–69	546 454	590 435
70–74	454 269	544 008
75–79	333 670	470 694
80–84	184 658	309 748
85–89	91 455	190 960
90+	34 959	98 587

a) Create two matrices using the above data, one for males and another for females.

b) What is the total population for each age group?

c) Suppose that Canada's population grows by 1.5% in all age groups. Calculate the anticipated totals for each age group.

16. **a)** Prepare a matrix showing the connections for the VIA Rail routes shown on page 3. Use a 1 to indicate a direct connection from one city to another city. Use a 0 to indicate no direct connection from one city to another city. Also, use a 0 to indicate no direct connection from a city to itself.

Chapter Problem

b) What does the entry in row 4, column 3 represent?

c) What does the entry in row 3, column 4 represent?

d) Explain the significance of the relationship between your answers in parts b) and c).

e) Describe what the sum of the entries in the first row represents.

f) Describe what the sum of the entries in the first column represents.

g) Explain why your answers in parts e) and f) are the same.

C

17. **Inquiry / Problem Solving** Show that for any $m \times n$ matrices, A and B

a) $(A^t)^t = A$ **b)** $(A + B)^t = A^t + B^t$

18. **Communication** Make a table to compare matrix calculations with graphing calculators and with spreadsheets. What are the advantages, disadvantages, and limitations of these technologies?

19. **Inquiry / Problem Solving** Search the newspaper for data that could be organized in a matrix. What calculations could you perform with these data in matrix form? Is there any advantage to using matrices for these calculations?

1.7 Problem Solving With Matrices

The previous section demonstrated how to use matrices to model, organize, and manipulate data. With multiplication techniques, matrices become a powerful tool in a wide variety of applications.

INVESTIGATE & INQUIRE: Matrix Multiplication

The National Hockey League standings on March 9, 2001 in the Northeast Division are shown below along with the league's point system for a win, loss, tie, or overtime loss (OTL).

Team	Win	Loss	Tie	OTL
Ottawa	39	17	8	3
Buffalo	36	25	5	1
Toronto	31	23	10	5
Boston	28	27	6	7
Montréal	23	36	5	4

Score	Points
Win	2
Loss	0
Tie	1
OTL	1

1. Calculate the number of points for each team in the Northeast Division using the above tables. Explain your method.
2. a) Represent the team standings as a 5×4 matrix, A.

   b) Represent the points system as a column matrix, B.
3. Describe a procedure for determining the total points for Ottawa using the entries in row 1 of matrix A and column 1 of matrix B.
4. How could you apply this procedure to find the points totals for the other four teams?
5. Represent the total points for each team as a column matrix, C. How are the dimensions of C related to those of A and B?
6. Would it make sense to define matrix multiplication using a procedure such that $A \times B = C$? Explain your reasoning.

In the above investigation, matrix A has dimensions 5×4 and matrix B has dimensions 4×1. Two matrices can be multiplied when their inner dimensions are equal. The outer dimensions are the dimensions of the resultant matrix when matrices A and B are multiplied.

$$A_{5\times4} \times B_{4\times1} = C_{5\times1}$$

same dimensions

outer dimensions give dimensions of resultant matrix

Example 1 Multiplying Matrices

Matrix A represents the proportion of students at a high school who have part-time jobs on Saturdays and the length of their shifts. Matrix B represents the number of students at each grade level.

$$A = \begin{array}{c} \begin{array}{cccc} \text{Gr 9} & \text{Gr 10} & \text{Gr 11} & \text{Gr 12} \end{array} \\ \left[\begin{array}{cccc} 0.20 & 0.10 & 0.20 & 0.15 \\ 0.25 & 0.30 & 0.25 & 0.45 \\ 0.05 & 0.25 & 0.15 & 0.10 \end{array}\right] \end{array} \begin{array}{l} \\ \le 4\text{ h} \\ 4.1 - 6\text{ h} \\ > 6\text{ h} \end{array} \qquad B = \begin{array}{c} \begin{array}{cc} \text{M} & \text{F} \end{array} \\ \left[\begin{array}{cc} 120 & 130 \\ 137 & 155 \\ 103 & 110 \\ 95 & 92 \end{array}\right] \end{array} \begin{array}{l} \\ \text{Gr 9} \\ \text{Gr 10} \\ \text{Gr 11} \\ \text{Gr 12} \end{array}$$

a) Calculate AB. Interpret what each entry represents.

b) Calculate BA, if possible.

Solution

a) A and B have the same inner dimensions, so multiplication is possible and their product will be a 3×2 matrix: $A_{3\times4} \times B_{4\times2} = C_{3\times2}$

$$AB = \begin{bmatrix} 0.20 & 0.10 & 0.20 & 0.15 \\ 0.25 & 0.30 & 0.25 & 0.45 \\ 0.05 & 0.25 & 0.15 & 0.10 \end{bmatrix} \begin{bmatrix} 120 & 130 \\ 137 & 155 \\ 103 & 110 \\ 95 & 92 \end{bmatrix}$$

$$= \begin{bmatrix} (0.20)(120) + (0.10)(137) + (0.20)(103) + (0.15)(95) & (0.20)(130) + (0.10)(155) + (0.20)(110) + (0.15)(92) \\ (0.25)(120) + (0.30)(137) + (0.25)(103) + (0.45)(95) & (0.25)(130) + (0.30)(155) + (0.25)(110) + (0.45)(92) \\ (0.05)(120) + (0.25)(137) + (0.15)(103) + (0.10)(95) & (0.05)(130) + (0.25)(155) + (0.15)(110) + (0.10)(92) \end{bmatrix}$$

$$\doteq \begin{bmatrix} 73 & 77 \\ 140 & 148 \\ 65 & 71 \end{bmatrix}$$

Approximately 73 males and 77 females work up to 4 h; 140 males and 148 females work 4–6 h, and 65 males and 71 females work more than 6 h on Saturdays.

b) For $B_{4\times2} \times A_{3\times4}$, the inner dimensions are not the same, so BA cannot be calculated.

Technology is an invaluable tool for solving problems that involve large amounts of data.

Example 2 Using Technology to Multiply Matrices

The following table shows the number and gender of full-time students enrolled at each university in Ontario one year.

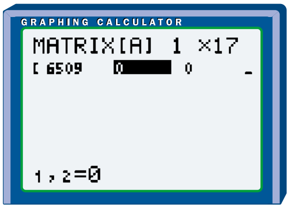
University	Full-Time Students	Males (%)	Females (%)
Brock	6509	43	57
Carleton	12 376	55	45
Guelph	11 773	38	62
Lakehead	5308	48	52
Laurentian	3999	43	57
McMaster	13 797	46	54
Nipissing	1763	34	66
Ottawa	16 825	42	58
Queen's	13 433	44	56
Ryerson	10 266	47	53
Toronto	40 420	44	56
Trent	3764	36	64
Waterloo	17 568	55	45
Western	21 778	46	54
Wilfred Laurier	6520	45	55
Windsor	9987	46	54
York	27 835	39	61

a) Set up two matrices, one listing the numbers of full-time students at each university and the other the percents of males and females.

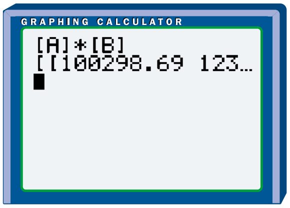
b) Determine the total number of full-time male students and the total number of full-time female students enrolled in Ontario universities.

Solution 1 Using a Graphing Calculator

a) Use the MATRX EDIT menu to store matrices for a 1×17 matrix for the numbers of full-time students and a 17×2 matrix for the percents of males and females.



b) To multiply matrices, use the MATRX NAMES menu. Copy the matrices into an expression such as [A]*[B] or [A][B].



There are 100 299 males and 123 622 females enrolled in Ontario universities.

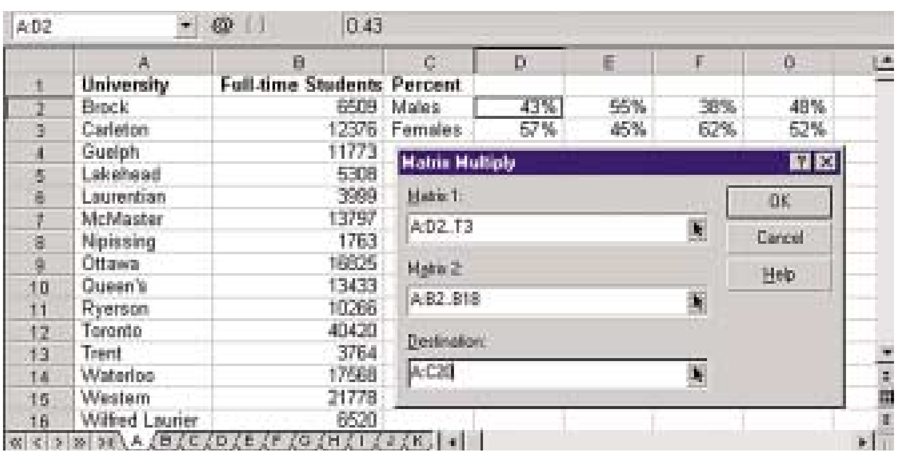
You can also enter matrices directly into an expression by using the square brackets keys. This method is simpler for small matrices, but does not store the matrix in the MATRX NAMES menu.

Solution 2 Using a Spreadsheet

Enter the number of full-time students at each university as a 17×1 matrix in cells B2 to B18. This placement leaves you the option of putting labels in the first row and column. Enter the proportion of male and female students as a 2×17 matrix in cells D2 to T3.

Both Corel® Quattro® Pro and Microsoft® Excel have built-in functions for **multiplying matrices**, although the procedures in the two programs differ somewhat.

Corel® Quattro® Pro:
On the Tools menu, select Numeric Tools/Multiply. In the pop-up window, enter the cell ranges for the two matrices you want to multiply and the cell where you want the resulting matrix to start. Note that you must list the 2×17 matrix first.



Project Prep

You can apply these techniques for matrix multiplication to the calculations for your tools for data management project.

Microsoft® Excel:
The MMULT(matrix1,matrix2) function will calculate the product of the two matrices but displays only the first entry of the resulting matrix. Use the INDEX function to retrieve the entry for a specific row and column of the matrix.

D21 = =INDEX(MMULT(D2:T3,B2:B18),2,1)

	A	B	C	D	E	F	G	H	I	J
1	**University**	**Full-time Students**	**Percent**							
2	Brock	6509	Males	43%	55%	38%	48%	43%	46%	34%
3	Carleton	12376	Females	57%	45%	62%	52%	57%	54%	66%
4	Guelph	11773								
5	Lakehead	5308								
6	Laurentian	3989								
7	McMaster	13797								
8	Nipissing	1763								
9	Ottawa	16825								
10	Queen's	13433								
11	Ryerson	10266								
12	Toronto	40420								
13	Trent	3764								
14	Waterloo	17568								
15	Western	21778								
16	Wilfred Laurier	6520								
17	Windsor	9987								
18	York	27835								
19										
20	**Totals**	Males		100298.7						
21		Females		123622.3						
22										

Identity matrices have the form $I = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \end{bmatrix}$ with entries of 1 along the main diagonal and zeros for all other entries. The identity matrix with dimensions $n \times n$ is represented by I_n. It can easily be shown that $A_{m\times n}I_n = A_{m\times n}$ for any $m \times n$ matrix A.

For most square matrices, there exists an **inverse matrix** A^{-1} with the property that $AA^{-1} = A^{-1}A = I$. Note that $A^{-1} \neq \frac{1}{A}$.

For 2×2 matrices, $AA^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Multiplying the matrices gives four simultaneous equations for w, x, y, and z. Solving these equations yields $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. You can confirm that $A^{-1}A = I$, also. If $ad = bc$, then A^{-1} does not exist since it would require dividing by zero.

The formulas for the inverses of larger matrices can be determined in the same way as for 2×2 matrices, but the calculations become much more involved. However, it is relatively easy to find the inverses of larger matrices with graphing calculators since they have the necessary formulas built in.

Example 3 Calculating the Inverse Matrix

Calculate, if possible, the inverse of

a) $A = \begin{bmatrix} 3 & 7 \\ 4 & -2 \end{bmatrix}$ **b)** $B = \begin{bmatrix} 6 & 8 \\ 3 & 4 \end{bmatrix}$

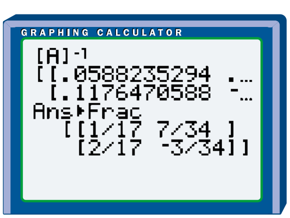
Solution 1 Using Pencil and Paper

a)
$$\begin{aligned} A^{-1} &= \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \\ &= \frac{1}{(3)(-2)-(7)(4)}\begin{bmatrix} -2 & -7 \\ -4 & 3 \end{bmatrix} \\ &= -\frac{1}{34}\begin{bmatrix} -2 & -7 \\ -4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{17} & \frac{7}{34} \\ \frac{2}{17} & -\frac{3}{34} \end{bmatrix} \end{aligned}$$

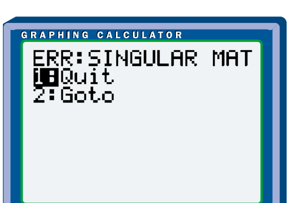
b) For B, $ad - bc = (6)(4) - (8)(3) = 0$, so B^{-1} does not exist.

Solution 2 Using a Graphing Calculator

a) Use the MATRX EDIT menu to **store** the 2×2 **matrix**. Retrieve it with the MATRX NAMES menu, then use x^{-1} to find the inverse. To verify that the decimal numbers shown are equal to the fractions in the pencil-and-paper solution, use the **►Frac function** from the MATH NUM menu.



b) For B, the calculator shows that the inverse cannot be calculated.



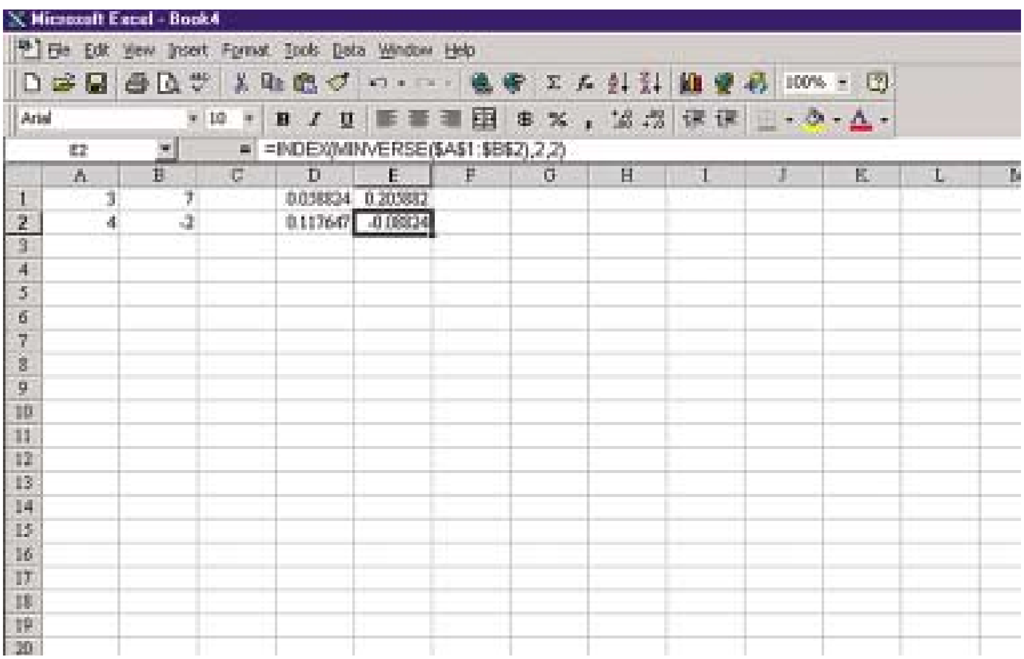
Solution 3 Using a Spreadsheet

The spreadsheet functions for **inverse matrices** are similar to those for matrix multiplication.

a) Enter the matrix in cells A1 to B2.

In Corel® Quattro® Pro, use Tools/Numeric Tools/Invert… to enter the range of cells for the matrix and the cell where you want the inverse matrix to start. Use the **Fraction feature** to display the entries as fractions rather than decimal numbers.

In Microsoft® Excel, use the MINVERSE function to produce the inverse matrix and the INDEX function to access the entries in it. If you put absolute cell references in the MINVERSE function for the first entry, you can use the **Fill feature** to generate the formulas for the other entries.
Use the **Fraction feature** to display the entries as fractions rather than decimal numbers.



During the 1930s, Lester Hill, an American mathematician, developed methods for using matrices to decode messages. The following example illustrates a simplified version of Hill's technique.

Example 4 Coding a Message Using Matrices

a) Encode the message PHONE ME TONIGHT using 2×2 matrices.

b) Determine the matrix key required to decode the message.

Solution

a) Write the message using 2×2 matrices. Fill in any missing entries with the letter Z.

$$\begin{bmatrix} P & H \\ O & N \end{bmatrix}, \begin{bmatrix} E & M \\ E & T \end{bmatrix}, \begin{bmatrix} O & N \\ I & G \end{bmatrix}, \begin{bmatrix} H & T \\ Z & Z \end{bmatrix}$$

Replace each letter with its corresponding number in the alphabet.

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

$$\begin{bmatrix} 16 & 8 \\ 15 & 14 \end{bmatrix}, \begin{bmatrix} 5 & 13 \\ 5 & 20 \end{bmatrix}, \begin{bmatrix} 15 & 14 \\ 9 & 7 \end{bmatrix}, \begin{bmatrix} 8 & 20 \\ 26 & 26 \end{bmatrix}$$

Now, encode the message by multiplying with a **coding matrix** that only the sender and receiver know. Suppose that you chose $C = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ as your coding matrix.

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 16 & 8 \\ 15 & 14 \end{bmatrix} = \begin{bmatrix} 63 & 38 \\ 110 & 68 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 5 & 13 \\ 5 & 20 \end{bmatrix} = \begin{bmatrix} 20 & 59 \\ 35 & 105 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 15 & 14 \\ 9 & 7 \end{bmatrix} = \begin{bmatrix} 54 & 49 \\ 93 & 84 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 8 & 20 \\ 26 & 26 \end{bmatrix} = \begin{bmatrix} 50 & 86 \\ 92 & 152 \end{bmatrix}$$

You would send the message as 63, 38, 110, 68, 20, 59, 35, 105, 54, 49, 93, 84, 50, 86, 92, 152.

b) First, rewrite the coded message as 2×2 matrices.

$$\begin{bmatrix} 63 & 38 \\ 110 & 68 \end{bmatrix}, \begin{bmatrix} 20 & 59 \\ 35 & 105 \end{bmatrix}, \begin{bmatrix} 54 & 49 \\ 93 & 84 \end{bmatrix}, \begin{bmatrix} 50 & 86 \\ 92 & 152 \end{bmatrix}$$

You can decode the message with the inverse matrix for the coding matrix.

$$C^{-1} \times CM = C^{-1}C \times M = IM = M$$

where M is the message matrix and C is the coding matrix.
Thus, the **decoding matrix,** or key, is the inverse matrix of the coding matrix. For the coding matrix used in part a), the key is

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}^{-1} = \frac{1}{(3)(2) - (5)(1)} \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

Multiplying the coded message by this key gives

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 63 & 38 \\ 110 & 68 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 15 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 20 & 59 \\ 35 & 105 \end{bmatrix} = \begin{bmatrix} 5 & 13 \\ 5 & 20 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 54 & 49 \\ 93 & 84 \end{bmatrix} = \begin{bmatrix} 15 & 14 \\ 9 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 50 & 86 \\ 92 & 152 \end{bmatrix} = \begin{bmatrix} 8 & 20 \\ 26 & 26 \end{bmatrix}$$

The decoded message is 16, 8, 15, 14, 5, 13, 5, 20, 15, 14, 9, 7, 8, 20, 26, 26. Replacing each number with its corresponding letter in the alphabet gives PHONEMETONIGHTZZ, the original message with the two Zs as fillers.

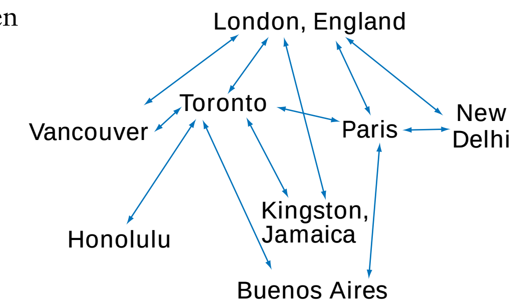
Matrix multiplication and inverse matrices are the basis for many computerized encryption systems like those used for electronic transactions between banks and income tax returns filed over the Internet.

Transportation and communication networks can be represented using matrices, called **network matrices**. Such matrices provide information on the number of direct links between two **vertices** or points (such as people or places). The advantage of depicting networks using matrices is that information on indirect routes can be found by performing calculations with the network matrix.

To construct a network matrix, let each vertex (point) be represented as a row and as a column in the matrix. Use 1 to represent a direct link and 0 to represent no direct link. A vertex may be linked to another vertex in one direction or in both directions. Assume that a vertex does not link with itself, so each entry in the main diagonal is 0. Note that the network matrix provides information only on direct links.

Example 5 Using Matrices to Model a Network

Matrixville Airlines offers flights between eight cities as shown on the right.



a) Represent the network using a matrix, A. Organize the matrix so the cities are placed in alphabetical order.

b) Calculate A^2. What information does it contain?

c) How many indirect routes with exactly one change of planes are there from London to Buenos Aires?

d) Calculate $A + A^2$. What information does it contain?

e) Explain what the entry from Vancouver to Paris in $A + A^2$ represents.

f) Calculate A^3. Compare this calculation with the one for A^2.

g) Explain the significance of any entry in matrix A^3.

Solution

a)

$$A = \begin{array}{c} \begin{array}{cccccccc} B & H & K & L & N & P & T & V \end{array} \\ \left[\begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \begin{array}{c} \\ B \\ H \\ K \\ L \\ N \\ P \\ T \\ V \end{array}$$

b) Since the dimensions of matrix A are 8×8, you may prefer to use a calculator or software for this calculation.

$$A^2 = \left[\begin{array}{cccccccc} 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 & 1 & 2 & 1 & 2 \\ 2 & 1 & 1 & 5 & 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 & 1 \\ 1 & 1 & 2 & 2 & 1 & 4 & 2 & 2 \\ 1 & 0 & 1 & 3 & 2 & 2 & 6 & 2 \\ 1 & 1 & 2 & 1 & 1 & 2 & 1 & 2 \end{array}\right]$$

The entries in A^2 show the number of indirect routes with exactly one change of planes. A^2 does not contain any information on direct routes.

c) There are two indirect routes with exactly one change of planes from London to Buenos Aires.

London → Paris → Buenos Aires
London → Toronto → Buenos Aires

d) $$A + A^2 = \begin{bmatrix} 2 & 1 & 1 & 2 & 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 & 1 & 2 & 2 & 2 \\ 2 & 1 & 2 & 5 & 2 & 3 & 4 & 2 \\ 1 & 0 & 1 & 2 & 2 & 2 & 2 & 1 \\ 2 & 1 & 2 & 3 & 2 & 4 & 3 & 2 \\ 2 & 1 & 2 & 4 & 2 & 3 & 6 & 2 \\ 1 & 1 & 2 & 2 & 1 & 2 & 2 & 2 \end{bmatrix}$$

Since A shows the number of direct routes and A^2 shows the number of routes with one change of planes, $A + A^2$ shows the number of routes with at most one change of planes.

e) The entry in row 8, column 6 of $A + A^2$ shows that there are two routes with a maximum of one change of planes from Vancouver to Paris.

Vancouver → Toronto → Paris
Vancouver → London → Paris

f) $$A^3 = \begin{bmatrix} 2 & 1 & 3 & 5 & 3 & 6 & 8 & 3 \\ 1 & 0 & 1 & 3 & 2 & 2 & 6 & 1 \\ 3 & 1 & 2 & 8 & 3 & 4 & 9 & 2 \\ 5 & 3 & 8 & 8 & 7 & 11 & 12 & 8 \\ 3 & 2 & 3 & 7 & 2 & 6 & 5 & 3 \\ 6 & 2 & 4 & 11 & 6 & 6 & 12 & 4 \\ 8 & 6 & 9 & 12 & 5 & 12 & 8 & 9 \\ 3 & 1 & 2 & 8 & 3 & 4 & 9 & 2 \end{bmatrix}$$

The calculation of $A^3 = A^2 \times A$ is more laborious than that for $A^2 = A \times A$ since A^2 has substantially fewer zero entries than A does. A calculator or spreadsheet could be useful.

g) The entries in A^3 tell you the number of indirect routes with exactly two changes of planes between each pair of cities.

Key Concepts

- To multiply two matrices, their inner dimensions must be the same. The outer dimensions give the dimensions of the resultant matrix: $A_{m \times n} \times B_{n \times p} = C_{m \times p}$. To find the entry with row i and column j of matrix AB, multiply the entries of row i of matrix A with the corresponding entries of column j of matrix B, and then add the resulting products together.
- The inverse of the 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ provided that $ad \neq bc$. Larger inverse matrices can be found using a graphing calculator or a spreadsheet.
- To represent a network as a matrix, use a 1 to indicate a direct link and a 0 to indicate no direct link. Calculations with the square of a network matrix and its higher powers give information on the various direct and indirect routings possible.

Communicate Your Understanding

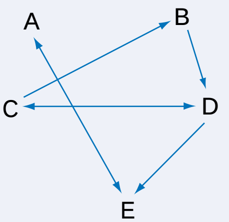
1. Explain how multiplying matrices is different from scalar multiplication of matrices.

2. Describe the steps you would take to multiply $\begin{bmatrix} 4 & -2 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 3 & 6 & 2 & -1 \\ 0 & 4 & 5 & 7 \end{bmatrix}$.

3. Is it possible to find an inverse for a matrix that is not square? Why or why not?

4. Explain why a network matrix must be square.

5. Describe how you would represent the following network as a matrix. How would you find the number of routes with up to three changeovers?



Practise

A

1. Let $A = \begin{bmatrix} 4 & 7 & 0 \\ -3 & -5 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 9 \\ -7 & 0 \end{bmatrix}$,

$C = \begin{bmatrix} 1 & 5 & 8 \\ 2 & 0 & -4 \\ -3 & -2 & 8 \end{bmatrix}$, $D = \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix}$, $E = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$.

Calculate, if possible,

a) BD **b)** DB **c)** B^2 **d)** EA

e) AC **f)** CE **g)** DA

2. Given $A = \begin{bmatrix} 4 & 2 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 \\ -2 & 0 \end{bmatrix}$, show that $A^2 + 2B^3 = \begin{bmatrix} 16 & -30 \\ 24 & 1 \end{bmatrix}$.

3. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $A^4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, the 2×2 **zero matrix**.

4. Let $A = \begin{bmatrix} 5 & 0 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ -2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -3 \\ 0 & 7 \end{bmatrix}$.

Show that

a) $A(B + C) = AB + AC$ (distributive property)

b) $(AB)C = A(BC)$ (associative property)

c) $AB \neq BA$ (not commutative)

5. Find the inverse matrix, if it exists.

a) $\begin{bmatrix} 0 & -1 \\ 2 & 4 \end{bmatrix}$ **b)** $\begin{bmatrix} 4 & -6 \\ -2 & 3 \end{bmatrix}$ **c)** $\begin{bmatrix} 3 & 0 \\ -6 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}$ **e)** $\begin{bmatrix} 10 & 5 \\ 4 & 2 \end{bmatrix}$

6. Use a graphing calculator or a spreadsheet to calculate the inverse matrix, if it exists.

a) $A = \begin{bmatrix} 1 & -3 & 1 \\ -2 & 1 & 3 \\ 0 & -1 & 0 \end{bmatrix}$

b) $B = \begin{bmatrix} -2 & 0 & 5 \\ 2 & -1 & -1 \\ 3 & 4 & 0 \end{bmatrix}$

c) $C = \begin{bmatrix} 2 & -1 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ -2 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{bmatrix}$

Apply, Solve, Communicate

B

7. For $A = \begin{bmatrix} 2 & -4 \\ -2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix}$, show that

a) $(A^{-1})^{-1} = A$

b) $(AB)^{-1} = B^{-1}A^{-1}$

c) $(A^{t})^{-1} = (A^{-1})^{t}$

8. Application Calculators Galore has three stores in Matrixville. The downtown store sold 12 business calculators, 40 scientific calculators, and 30 graphing calculators during the past week. The northern store sold 8 business calculators, 30 scientific calculators, and 21 graphing calculators during the same week, and the southern store sold 10 business calculators, 25 scientific calculators, and 23 graphing calculators. What were the total weekly sales for each store if the average price of a business calculator is \$40, a scientific calculator is \$30, and a graphing calculator is \$150?

9. Application The manager at Sue's Restaurant prepares the following schedule for the next week.

Employee	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.	Wage Per Hour
Chris	–	8	–	8	8	–	–	\$7.00
Lee	4	4	–	–	6.5	4	4	\$6.75
Jagjeet	–	4	4	4	4	8	8	\$7.75
Pierre	–	3	3	3	3	8	–	\$6.75
Ming	8	8	8	8	–	–	–	\$11.00
Bobby	–	–	3	5	5	8	–	\$8.00
Nicole	3	3	3	3	3	–	–	\$7.00
Louis	8	8	8	8	8	–	–	\$12.00
Glenda	8	–	–	8	8	8	8	\$13.00
Imran	3	4.5	4	3	5	–	–	\$7.75

a) Create matrix A to represent the number of hours worked per day for each employee.

b) Create matrix B to represent the hourly wage earned by each employee.

c) Use a graphing calculator or spreadsheet to calculate the earnings of each employee for the coming week.

d) What is the restaurant's total payroll for these employees?

10. According to a 1998 general social survey conducted by Statistics Canada, the ten most popular sports for people at least 15 years old are as follows:

Sport	Total (%)	Male (%)	Female (%)
Golf	7.4	11.1	3.9
Ice Hockey	6.2	12.0	0.5
Baseball	5.5	8.0	3.1
Swimming	4.6	3.6	5.6
Basketball	3.2	4.6	1.9
Volleyball	3.1	3.3	2.8
Soccer	3.0	4.6	1.5
Tennis	2.7	3.6	1.8
Downhill/Alpine Skiing	2.7	2.9	2.6
Cycling	2.5	3.0	2.0

In 1998, about 11 937 000 males and 12 323 000 females in Canada were at least 15 years old. Determine how many males and how many females declared each of the above sports as their favourite. Describe how you used matrices to solve this problem.

11. **Application** A company manufacturing designer T-shirts produces five sizes: extra-small, small, medium, large, and extra-large. The material and labour needed to produce a box of 100 shirts depends on the size of the shirts.

Size	Cloth per shirt (m^2)	Labour per 100 shirts (h)
Extra-small	0.8	8
Small	0.9	8.5
Medium	1.2	9
Large	1.5	10
Extra-large	2.0	11

a) How much cloth and labour are required to fill an order for 1200 small, 1500 medium, 2500 large, and 2000 extra-large T-shirts?

b) If the company pays $6.30 per square metre for fabric and $10.70 per hour for labour, find the cost per box for each size of T-shirt.

c) What is the total cost of cloth and labour for filling the order in part a)?

12. Use the coding matrix $\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}$ to encode each message.

a) BIRTHDAY PARTY FRIDAY

b) SEE YOU SATURDAY NIGHT

13. **Application** Use the decoding matrix $\begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$ to decode each message.

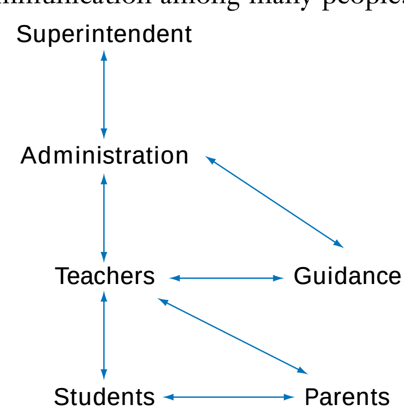
a) 64, 69, 38, 45, 54, 68, 31, 44, 5, 115, 3, 70, 40, 83, 25, 49

b) 70, 47, 39, 31, 104, 45, 61, 25, 93, 68, 57, 44, 55, 127, 28, 76

14. a) Create a secret message about 16 to 24 letters long using the coding matrix $\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$.

b) Trade messages with a classmate and decode each other's messages.

15. Quality education at a school requires open communication among many people.



a) Represent this network as a matrix, A.

b) Explain the meaning of any entry, a_{ij}, of matrix A.

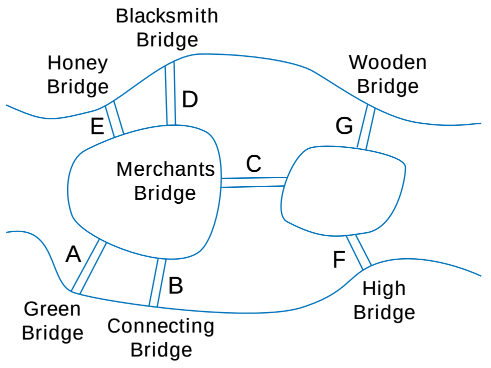
c) Describe what the sum of the entries in the third column represents.

d) Calculate A^2.

e) How many indirect links exist with exactly one intermediary between the principal and parents? List these links.

f) Calculate $A + A^2$. Explain what information this matrix provides.

16. Network matrices provide another approach to the Koenigsberg bridges example on page 44.



Use network matrices to answer the following questions.

a) How many ways can you get from Honey Bridge to Connecting Bridge by crossing only one of the other bridges? List these routes.

b) How many ways can you get from Blacksmith Bridge to Connecting Bridge without crossing more than one of the other bridges?

c) Is it possible to travel from Wooden Bridge to Green Bridge without crossing at least two other bridges?

17. Use network matrices to find the number of VIA Rail routes from

a) Toronto to Montréal with up to two change-overs

b) Kingston to London with up to three change-overs

C

18. **Inquiry/Problem Solving** Create your own network problem, then exchange problems with a classmate. Solve both problems and compare your solutions with those of your classmate. Can you suggest any improvements for either set of solutions?

19. Show how you could use inverse matrices to solve any system of equations in two variables whose matrix of coefficients has an inverse.

20. **Communication** Research encryption techniques on the Internet. What is meant by 128-bit encryption? How does the system of private and public code keys work?

21. **Inquiry/Problem Solving**

a) Suppose you receive a coded message like the one in Example 4, but you do not know the coding matrix or its inverse. Describe how you could use a computer to break the code and decipher the message.

b) Describe three methods you could use to make a matrix code harder to break.

22. a) Show that, for any $m \times n$ matrix A and any $n \times p$ matrix B, $(AB)^t = B^tA^t$.

b) Show that, if a square matrix C has an inverse C^{-1}, then C^t also has an inverse, and $(C^t)^{-1} = (C^{-1})^t$.

Review of Key Concepts

1.1 The Iterative Process

Refer to the Key Concepts on page 10.

1. a) Draw a tree diagram showing your direct ancestors going back four generations.

b) How many direct ancestors do you have in four generations?

2. a) Describe the algorithm used to build the iteration shown.

b) Continue the iteration for eight more rows.

c) Describe the resulting iteration.

```
            MATH
          MATHMATH
      MATH        MATH
    MATHMATHMATHMATH
  MATH                MATH
MATHMATH            MATHMATH
```

3. a) Construct a Pythagoras fractal tree using the following algorithm.

Step 1: Construct a square.

Step 2: Construct an isosceles right triangle with the hypotenuse on one side of the square.

Step 3: Construct a square on each of the other sides of the triangle.

Repeat this process, with the newly drawn squares to a total of four iterations.

b) If the edges in the first square are 4 cm, determine the total area of all the squares in the fourth iteration.

c) Determine the total area of all the squares in the diagram.

4. Design an iterative process using the percent reduction capabilities of a photocopier.

1.2 Data Management Software

Refer to the Key Concepts on page 21.

5. List three types of software that can be used for data management, giving an example of the data analysis you could do with each type.

6. Evaluate each spreadsheet expression.

a) F2+G7–A12
where F2=5, G7=–9, and A12=F2+G7

b) PROD(D3,F9)
where D3=6 and F9=5

c) SQRT(B1)
where B1=144

7. Describe how to reference cells A3 to A10 in one sheet of a spreadsheet into cells B2 to B9 in another sheet.

8. Use a spreadsheet to convert temperatures between –30° C and 30° C to the Fahrenheit scale, using the formula Fahrenheit = 1.8 × Celsius + 32. Describe how you would list temperatures at two-degree intervals in the Celsius column.

1.3 Databases

Refer to the Key Concepts on page 31.

9. Describe the characteristics of a well-organized database.

10. Outline a design for a database of a shoe store's customer list.

11. a) Describe the types of data that are available from Statistics Canada's E-STAT database.

b) What can you do with the data once you have accessed them?

12. What phrase would you enter into a search engine to find

a) the top-selling cookbook in Canada?

b) the first winner of the Fields medal?

c) a list of movies in which bagpipes are played?

1.4 Simulations

Refer to the Key Concepts on page 39.

13. List three commonly used simulations and a reason why each is used.

14. Write out the function to generate a random integer between 18 and 65 using

a) a graphing calculator

b) a spreadsheet

15. A chocolate bar manufacturer prints one of a repeating sequence of 50 brainteasers on the inside of the wrapper for each of its chocolate bars. Describe a manual simulation you could use to estimate the chances of getting two chocolate bars with the same brainteaser if you treat yourself to one of the bars every Friday for five weeks.

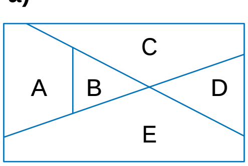
16. Outline how you would use technology to run a simulation 500 times for the scenario in question 15.

1.5 Graph Theory

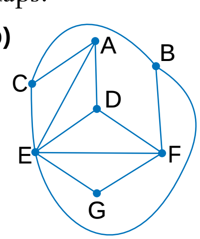
Refer to the Key Concepts on page 48.

17. How many colours are needed to colour each of the following maps?

a)



b)



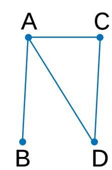
18. State whether each network is

i) connected

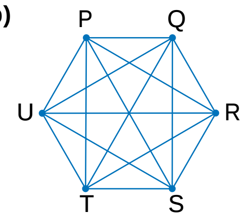
ii) traceable

iii) planar

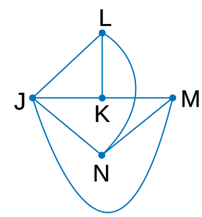
a)



b)



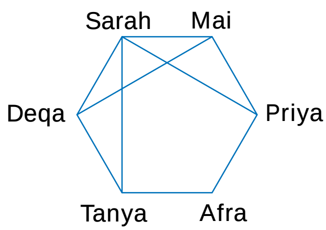
c)



19. For each network in question 18, verify that $V - E + R = 2$, where V is the number of vertices, E is the number of edges, and R is the number of regions in a graph.

20. The following is a listing of viewing requests submitted by patrons of a classic film festival. Use graph theory to set up the shortest viewing schedule that has no conflicts for any of these patrons.
Person A: *Gone With the Wind*, *Curse of The Mummy*, *Citizen Kane*
Person B: *Gone With the Wind*, *Jane Eyre*
Person C: *The Amazon Queen*, *West Side Story*, *Citizen Kane*
Person D: *Jane Eyre*, *Gone With the Wind*, *West Side Story*
Person E: *The Amazon Queen*, *Ben Hur*

21. Below is a network showing the relationships among a group of children. The vertices are adjacent if the children are friends.



a) Rewrite the network in table form.

b) Are these children all friends with each other?

c) Who has the most friends?

d) Who has the fewest friends?

1.6 Modelling With Matrices

Refer to the Key Concepts on page 59.

22. For the matrix $A = \begin{bmatrix} 2 & -1 & 5 \\ 0 & 4 & 3 \\ 7 & -8 & -6 \\ -2 & 9 & 1 \end{bmatrix}$,

a) state the dimensions

b) state the value of entry

i) a_{32} ii) a_{13} iii) a_{41}

c) list the entry with value

i) 3 ii) 9 iii) −1

23. Write a 4×3 matrix, A, with the property that $a_{ij} = i \times j$ for all entries.

24. Given $A = \begin{bmatrix} 3 & 2 & -1 \\ -7 & 0 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 8 & -2 \\ 3 & 4 \\ 2 & 5 \end{bmatrix}$,

$C = \begin{bmatrix} 6 & 1 & -4 \\ -5 & 9 & 0 \end{bmatrix}$, and $D = \begin{bmatrix} 4 & 3 \\ -1 & 7 \\ 6 & 2 \end{bmatrix}$.

Calculate, if possible,

a) $A + C$ b) $C - B$

c) $A + B$ d) $3D$

e) $-\frac{1}{2}C$ f) $3(B + D)$

g) $A^t + B$ h) $B^t + C^t$

25. The manager of a sporting goods store takes inventory at the end of the month and finds 15 basketballs, 17 volleyballs, 4 footballs, 15 baseballs, 8 soccer balls, 12 packs of tennis balls, and 10 packs of golf balls. The manager orders and receives a shipment of 10 basketballs, 3 volleyballs, 15 footballs, 20 baseballs, 12 soccer balls, 5 packs of tennis balls, and 15 packs of golf balls. During the next month, the store sells 17 basketballs, 13 volleyballs, 17 footballs, 12 baseballs, 12 soccer balls, 16 packs of tennis balls, and 23 packs of golf balls.

a) Represent the store's stock using three matrices, one each for the inventory, new stock received, and items sold.

b) How many of each item is in stock at the end of the month?

c) At the beginning of the next month, the manager is asked to send 20% of the store's stock to a new branch that is about to open. How many of each item will be left at the manager's store?

26. Outline the procedure you would use to subtract one matrix from another

a) manually

b) using a graphing calculator

c) using a spreadsheet

1.7 Problem Solving With Matrices

Refer to the Key Concepts on page 74.

27. Let $A = \begin{bmatrix} 4 & 5 \\ -6 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ -5 & 7 \end{bmatrix}$,

$C = \begin{bmatrix} 3 & 6 & -1 \\ 2 & 0 & 4 \\ -5 & -2 & 8 \end{bmatrix}$, and $D = \begin{bmatrix} 5 \\ 4 \\ -3 \end{bmatrix}$.

Calculate, if possible,

a) AB **b)** BA **c)** A^2

d) DC **e)** C^2

28. a) Write the transpose of matrices $A = \begin{bmatrix} 1 & 5 \\ 8 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ 6 & -1 \end{bmatrix}$.

b) Show whether $(AB)^t = B^tA^t$.

29. A small accounting firm charges $50 per hour for preparing payrolls, $60 per hour for corporate tax returns, and $75 per hour for audited annual statements. The firm did the following work for three of its clients:

XYZ Limited, payrolls 120 hours, tax returns 10 hours, auditing 10 hours

YZX Limited, payrolls 60 hours, tax returns 8 hours, auditing 8 hours

ZXY Limited, payrolls 200 hours, tax returns 15 hours, auditing 20 hours

a) Use matrices to determine how much the accounting firm should bill each client.

b) How can you determine the total billed to the three clients?

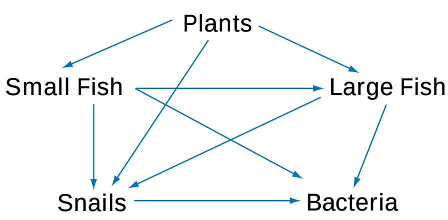
30. Suppose you were to encode a message by writing it in matrix form and multiplying by a coding matrix. Would your message be more secure if you then multiplied the resulting matrices by another coding matrix with the same dimensions as the first one? Explain why or why not.

31. a) Write an equation to show the relationship between a matrix and its inverse.

b) Show that $\begin{bmatrix} 1.5 & 0 & -1 \\ 20 & -1.5 & -13 \\ -7.5 & 0.5 & 5 \end{bmatrix}$ is the inverse of $\begin{bmatrix} 4 & 2 & 6 \\ 10 & 0 & 2 \\ 5 & 3 & 9 \end{bmatrix}$.

c) Find the inverse of $\begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}$.

32. The following diagram illustrates the food chains in a pond.



a) Represent these food chains as a network matrix, A.

b) Calculate A^2.

c) How many indirect links with exactly one intermediate step are there from plants to snails?

d) Calculate $A + A^2$. Explain the meaning of any entry in the resulting matrix.

e) Calculate A^3.

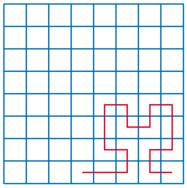
f) List all the links with two intermediate steps from plants to bacteria.

Chapter Test

ACHIEVEMENT CHART				
Category	Knowledge / Understanding	Thinking / Inquiry / Problem Solving	Communication	Application
Questions	All	8, 9, 14	1, 2, 5, 6, 7, 8, 9, 14	9, 10, 13, 14

1. a) Describe an iterative process you could use to draw the red path.

b) Complete the path.



2. Find the first few terms of the recursion formula $t_n = \frac{1}{t_{n-1} + 2}$, given $t_1 = 0$.

Is there a pattern to these terms? If so, describe the pattern.

3. A "fan-out" calling system is frequently used to spread news quickly to a large number of people such as volunteers for disaster relief. The first person calls three people. Each of those people calls an additional three people; each of whom calls an additional three people, and so on.

a) Use a tree diagram to illustrate a fan-out calling system with sufficient levels to call 50 people.

b) How many levels would be sufficient to call 500 people?

4. Rewrite each of the following expressions as spreadsheet functions.

a) C1+C2+C3+C4+C5+C6+C7+C8

b) The smallest value between cells A5 and G5

c) $\frac{5 - \sqrt{6}}{10 + 15}$

5. Suppose that, on January 10, you borrowed \$1000 at 6% per year compounded monthly (0.5% per month). You will be expected to repay \$88.88 a month for 1 year. However, the final payment will be less than \$88.88. You set up a spreadsheet with the following column headings: MONTH, BALANCE, PAYMENT, INTEREST, PRINCIPAL, NEW BALANCE

The first row of entries would be:
MONTH: February
BALANCE: 1000.00
PAYMENT: 88.88
INTEREST: 5.00
PRINCIPAL: 83.88
NEW BALANCE: 916.12

Describe how you would

a) use the cell referencing formulas and the Fill feature to complete the table

b) determine the size of the final payment on January 10 of the following year

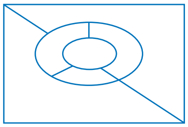
c) construct a line graph showing the declining balance

6. Describe how you would design a database of the daily travel logs for a company's salespersons.

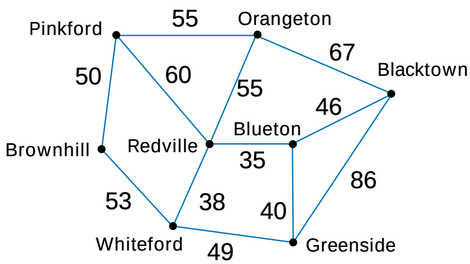
7. Describe three different ways to generate random integers between 1 and 50.

8. a) Redraw this map as a network.

b) How many colours are needed to colour the map? Explain your reasoning.



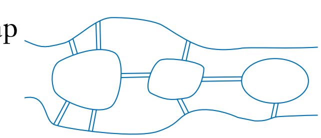
9. A salesperson must visit each of the towns on the following map.



a) Is there a route that goes through each town only once? Explain.

b) Find the shortest route that begins and ends in Pinkford and goes through all the towns. Show that it is the shortest route.

10. The following map shows the bridges of Uniontown, situated on the banks of a river and on three islands. Use graph theory to determine if a continuous path could traverse all the bridges once each.



11. Let $A = \begin{bmatrix} 4 & -2 & 6 \\ -8 & 5 & 9 \\ 0 & 1 & -1 \\ 3 & -7 & -3 \end{bmatrix}$.

a) State the dimensions of matrix A.

b) What is the value of entry a_{23}?

c) Identify the entry of matrix A with value -2.

d) Is it possible to calculate A^2? Explain.

12. Let $A = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$, $B = [7 \quad 5 \quad 0]$, $C = \begin{bmatrix} 4 & 8 \\ 5 & -3 \end{bmatrix}$,

$D = \begin{bmatrix} 2 & -7 \\ 9 & 1 \end{bmatrix}$, and $E = \begin{bmatrix} 8 & -2 \\ 5 & 0 \\ -4 & 1 \end{bmatrix}$.

Calculate, if possible,

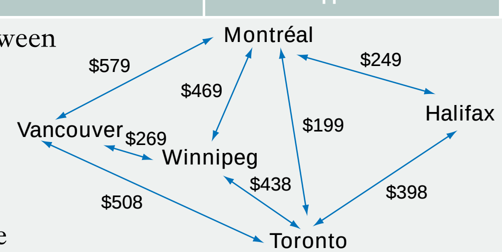
a) $2C + D$ **b)** $A + B$ **c)** AD **d)** EC **e)** E^t

13. A local drama club staged a variety show for four evenings. The admission for adults was \$7.00, for students \$4.00, and for children 13 years of age and under \$2.00. On Wednesday, 52 adult tickets, 127 student tickets, and 100 child tickets were sold; on Thursday, 67 adult tickets, 139 student tickets, and 115 child tickets were sold; on Friday, 46 adult tickets, 115 student tickets, and 102 child tickets were sold; and on Saturday, 40 adult tickets, 101 student tickets, and 89 child tickets were sold. Use matrices to calculate how much money was collected from admissions.

ACHIEVEMENT CHECK

Knowledge / Understanding	Thinking / Inquiry / Problem Solving	Communication	Application

14. The network diagram below gives the cost of flights between five Canadian cities.



a) Construct a network matrix A for these routes.

b) Calculate A^2 and A^3.

c) How many ways can a person travel from Halifax to Vancouver by changing planes exactly twice? Describe each route. Which route is most economical?

Wrap-Up

Implementing Your Action Plan

1. With your whole class or a small group, brainstorm criteria for ranking universities and community colleges. List the three universities or colleges that you think will most likely be the best choices for you.
2. Have a class discussion on weighting systems.
3. Look up the *Maclean's* university and community college rankings in a library or on the Internet. Note the criteria that *Maclean's* uses.
4. Determine your own set of criteria. These may include those that *Maclean's* uses or others, such as travelling distances, programs offered, size of the city or town where you would be living, and opportunities for part-time work.
5. Choose the ten criteria you consider most important. Research any data you need to rate universities and colleges with these criteria.
6. Assign a weighting factor to each of the ten criteria. For example, living close to home may be worth a weighting of 5 and tuition cost may be worth a weighting of 7.
7. Use a spreadsheet and matrix methods to determine an overall score for each university or community college in Ontario. Then, rank the universities or community colleges on the spreadsheet. Compare your rankings with those in *Maclean's* magazine. Explain the similarities or differences.
8. From your rankings, select the top five universities or community colleges. Draw a diagram of the distances from each university or college to the four others and to your home. Then, use graph theory to determine the most efficient way to visit each of the five universities or community colleges during a five-day period, such as a March break vacation.
9. Based on your project, select your top three choices. Comment on how this selection compares with your original list of top choices.

Suggested Resources

- *Maclean's* magazine rankings of universities and community colleges
- Other publications ranking universities and community colleges
- University and community college calendars
- Guidance counsellors
- Map of Ontario
- Spreadsheets

Refer to section 9.3 for information on implementing an action plan and Appendix C for information on research techniques.

WEB CONNECTION

www.mcgrawhill.ca/links/MDM12

For details of the *Maclean's* rankings of universities and colleges, visit the above web site and follow the links.

Evaluating Your Project

1. Reflect on your weighting formula and whether you believe it fairly ranks the universities and community colleges in Ontario.

2. Compare your rating system to that used by one of your classmates. Can you suggest improvements to either system?

3 What went well in this project?

4. If you were to do the project over again, what would you change? Why?

5. If you had more time, how would you extend this project?

6. What factors could change between now and when you make your final decision about which university or college to attend?

Presentation

Prepare a written report on your findings. Include

- the raw data
- a rationale for your choice of criteria
- a rationale for your weightings
- a printout of your spreadsheet
- a diagram showing the distances between your five highest-ranked universities or community colleges and the route you would use to visit them
- a summary of your findings

Preparing for the Culminating Project

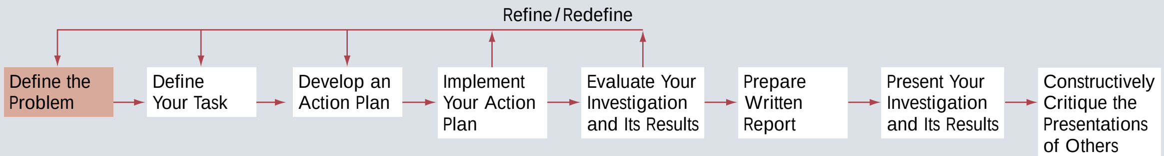
Applying Project Skills

Consider how the data management tools you used on this project could be applied to the culminating project in Chapter 9 to

- access resources
- carry out research
- carry out an action plan
- evaluate your project
- summarize your findings in a written report

Keeping on Track

Now is a good time to draw up a schedule for your culminating project and to investigate methods for selecting a topic. Refer to Chapter 9 for an overview of how to prepare a major project. Section 9.1 suggests methods for choosing a topic. Also, consider how to find the information you will need in order to choose your topic.



Career Connection

Cryptographer

In this digital era, information is sent with blinding speed around the world. These transmissions need to be both secure and accurate. Although best known for their work on secret military codes, cryptographers also design and test computerized encryption systems that protect a huge range of sensitive data including telephone conversations among world leaders, business negotiations, data sent by credit-card readers in retail stores, and financial transactions on the Internet. Encrypted passwords protect hackers from reading or disrupting critical databases. Even many everyday devices, such as garage-door openers and TV remote controls, use codes.

Cryptographers also develop error-correcting codes. Adding these special codes to a signal allows a computer receiving it to detect and correct errors that have occurred during transmission. Such codes have numerous applications including CD players, automotive computers, cable TV networks, and pictures sent back to Earth by interplanetary spacecraft.

Modern cryptography is a marriage of mathematics and computers. A cryptographer must have a background in logic, matrices, combinatorics, and computer programming as well as fractal, chaos, number, and graph theory. Cryptographers work for a wide variety of organizations including banks, government offices, the military, software developers, and universities.

WEB CONNECTION

www.mcgrawhill.ca/links/MDM12

Visit the above web site and follow the links for more information about a career as a cryptographer and about other careers related to mathematics.

Life Expectancies

Background

Do women live longer than men? Do people live longer in warmer climates? Are people living longer today than 50 years ago? Do factors such as education and income affect life expectancy? In this project, you will answer such questions by applying the statistical techniques described in the next two chapters.

Your Task

Research and analyse current data on life expectancies in Canada, and perhaps in other countries. You will use statistical analysis to compare and contrast the data, detect trends, predict future life expectancies, and identify factors that may affect life expectancies.

Developing an Action Plan

You will need to find sources of data on life expectancies and to choose the kinds of comparisons you want to make. You will also have to decide on a method for handling the data and appropriate techniques for analysing them.