Course Review

Chapter 1 Functions

1. Determine the domain and the range for each relation. Sketch a graph of each.

a) $y = \frac{3}{x - 9}$ **b)** $y = \sqrt{2 - x} - 4$

2. Which of the following is NOT true?

A All functions are also relations.

B The vertical line test is used to determine if the graph of a relation is a function.

C All relations are also functions.

D Some relations are also functions.

3. The approximate time for an investment to double can be found using the function $n(r) = \frac{72}{r}$, where n represents the number of years and r represents the annual interest rate, as a percent.

a) How long will it take an investment to double at each rate?

i) 3% **ii)** 6% **iii)** 9%

b) Graph the data to illustrate the function.

c) Determine the domain and range in this context.

4. Determine the vertex of each quadratic function by completing the square. Verify your answer by using partial factoring. State if the vertex is a minimum or a maximum.

a) $f(x) = 3x^2 + 9x + 1$

b) $f(x) = -\frac{1}{2}x^2 + 3x - \frac{5}{2}$

5. A small company manufactures a total of x items per week. The production cost is modelled by the function $C(x) = 50 + 3x$. The revenue is given by the function $R(x) = 6x - \frac{x^2}{100}$. How many items per week should be manufactured to maximize the profit for the company?

Hint: Profit = Revenue − Cost

6. Simplify.

a) $2\sqrt{243} - 5\sqrt{48} + \sqrt{108} - \sqrt{192}$

b) $\frac{2}{3}\sqrt{125} - \frac{1}{3}\sqrt{27} + 2\sqrt{48} - 3\sqrt{80}$

7. Expand. Simplify where possible.

a) $(\sqrt{5} + 2\sqrt{3})(3\sqrt{5} + 4\sqrt{3})$

b) $(4 - \sqrt{6})(1 + \sqrt{6})$

8. Find a simplified expression for the area of the circle.

9. Solve $3x^2 + 9x - 30 = 0$ by

a) completing the square

b) using a graphing calculator

c) factoring

d) using the quadratic formula

10. The length of a rectangle is 5 m more than its width. If the area of the rectangle is 15 m², what are the dimensions of the rectangle, to the nearest tenth of a metre?

11. Find an equation for the quadratic function with the given zeros and containing the given point. Express each function in standard form. Graph each function to check.

a) $2 \pm \sqrt{3}$, point $(4, -6)$

b) 4 and −1, point $(1, -4)$

12. An arch of a highway overpass is in the shape of a parabola. The arch spans a distance of 16 m from one side of the road to the other. At a horizontal distance of 1 m from each side of the arch, its height above the road is 6 m.

a) Sketch the quadratic function if the vertex of the parabola is on the y-axis and the road is along the x-axis.

b) Use this information to determine the equation of the function that models the arch.

c) Find the maximum height of the arch.

13. At a fireworks display, the path of the biggest firework can be modelled using the function $f(x) = -0.015x^2 + 2.24x + 1.75$, where x is the horizontal distance from the launching platform. The profile of a hill, some distance away from the platform, can be modelled with the equation $h(x) = 0.7x - 83$, with all distances in metres. Will the firework reach the hill? Justify your answer.

Chapter 2 Transformations of Functions

14. Test whether the functions in each pair are equivalent by

- **i)** testing three different values of x
- **ii)** simplifying the expressions on the right sides
- **iii)** graphing using graphing technology

a) $f(x) = -2(x + 3)^2 + (5x + 1)$,
$g(x) = -2x^2 - 7x - 17$

b) $f(x) = \dfrac{x^2 - 2x - 15}{x^2 - 9x + 20}$,
$g(x) = \dfrac{x + 3}{x - 4}$

15. Simplify and state the restrictions.

a) $\dfrac{-x + 1}{8x} : \dfrac{2x - 2}{14x^2}$

b) $\dfrac{x^2 + 5x - 36}{x^2 - 2x} \div \dfrac{x^2 + 11x + 18}{8x^2 - 4x^3}$

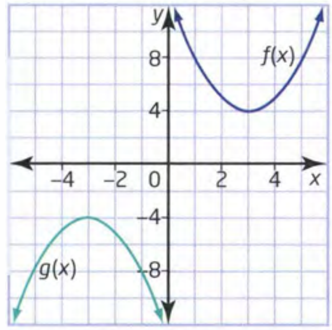
c) $\dfrac{x^2 - 25}{x - 4} \times \dfrac{x^2 - 6x + 8}{3x + 15}$

16. For each function $g(x)$, state the corresponding base function $f(x)$. Describe the transformations that must be applied to the base function using function notation and words. Then, transform the graph of $f(x)$ to sketch the graph of $g(x)$ and state the domain and range of each function.

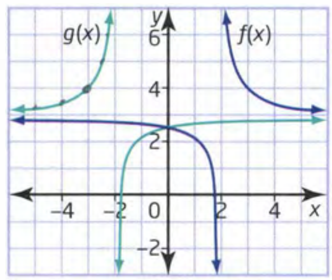
a) $g(x) = \dfrac{1}{x + 5} - 1$

b) $g(x) = \sqrt{x + 7} - 9$

a)



b)



18. For each of the functions $f(x) = x^2$, $f(x) = \sqrt{x}$, and $f(x) = \dfrac{1}{x}$, write an equation to represent $g(x)$ and $h(x)$ and describe the transformations. Then, transform the graph of $f(x)$ to sketch graphs of $g(x)$ and $h(x)$ and state the domain and range of the functions.

a) $g(x) = 4f(-x)$ and $h(x) = \dfrac{1}{4}f(x)$

b) $g(x) = f(4x)$ and $h(x) = -f\left(\dfrac{1}{4}x\right)$

19. A ball is dropped from a height of 32 m. Acceleration due to gravity is $-9.8\ \text{m/s}^2$. The height of the ball is given by $h(t) = -4.9t^2 + 32$.

a) State the domain and range of the function.

b) Write the equation for the height of the object if it is dropped on a planet with acceleration due to gravity of $-11.2\ \text{m/s}^2$.

c) Compare the domain and range of the function in part b) to those of the given function.

20. Describe the combination of transformations that must be applied to the base function $f(x)$ to obtain the transformed function $g(x)$. Then, write the corresponding equation and sketch its graph.

a) $f(x) = x$, $g(x) = -2f[3(x - 4)] - 1$

b) $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{3}f\left[\frac{1}{4}(x - 2)\right] + 3$

21. For each function $f(x)$,

i) determine $f^{-1}(x)$

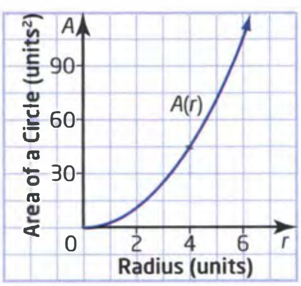
ii) graph $f(x)$ and its inverse

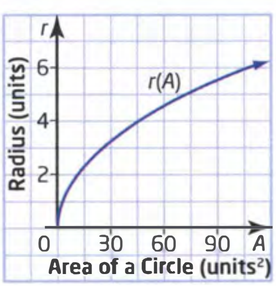
iii) determine whether the inverse of $f(x)$ is a function

a) $f(x) = 4x - 5$

b) $f(x) = 3x^2 - 12x + 3$

22. The relationship between the area of a circle and its radius can be modelled by the function $A(r) = \pi r^2$, where A is the area and r is the radius. The graphs of this function and its inverse are shown.





a) State the domain and range of the function $A(r)$.

b) Determine the equation of the inverse of the function. State its domain and range.

Chapter 3 Exponential Functions

23. A petri dish contains an initial sample of 20 bacteria. After 1 day, the number of bacteria has tripled.

a) Determine the population after each day for 1 week.

b) Write an equation to model this growth.

c) Graph the relation. Is it a function? Explain why or why not.

d) Assuming this trend continues, predict the population after

i) 2 weeks

ii) 3 weeks

e) Describe the pattern of finite differences for this relationship.

24. Tritium is a substance that is present in radioactive waste. It has a half-life of approximately 12 years. How long will it take for a 50-mg sample of tritium to decay to 10% of its original mass?

25. Apply the exponent rules first, if possible, and then evaluate.

a) $(-8)^{-2} + 2^{-6}$

b) $(3^{-3})^{-2} \div 3^{-5}$

c) $\left(\frac{2^3}{3^2}\right)^{-2}$

d) $\frac{(6^6)(6^{-3})}{6^2}$

26. Simplify.

a) $(4n^{-2})(-3n^5)$

b) $\frac{12c^{-3}}{15c^{-5}}$

c) $(3a^2b^{-2})^{-3}$

d) $\left(\frac{-2p^3}{3q^4}\right)^{-5}$

27. Evaluate.

a) $16^{-\frac{3}{4}}$

b) $\left(\frac{4}{9}\right)^{-\frac{1}{2}}$

c) $\left(-\frac{8}{125}\right)^{-\frac{2}{3}}$

28. Simplify. Express your answers using only positive exponents.

a) $\frac{a^{-2}b^3}{a^{\frac{1}{4}}b^{\frac{2}{3}}}$

b) $(u^{-\frac{2}{3}}v^{\frac{1}{4}})^{\frac{3}{5}}$

c) $w^{\frac{7}{8}} \div w^{-\frac{3}{4}}$

Graph each exponential function. Identify the

- domain
- range
- x- and y-intercepts, if they exist
- intervals of increase/decrease
- asymptote

a) $y = 5\left(\frac{1}{3}\right)^x$ **b)** $y = -4^{-x}$

30. A radioactive sample has a half-life of 1 month. The initial sample has a mass of 00 mg.

a) Write a function to relate the amount remaining, in milligrams, to the time, in months.

b) Restrict the domain of the function so the mathematical model fits the situation it is describing.

. Sketch the graph of each function, using the graph of $y = 8^x$ as the base. Describe the effects, if any, on the

- asymptote
- domain
- range

a) $y = 8^{x-4}$ **b)** $y = 8^{x+2} + 1$

32. Write the equation for the function that results from each transformation applied to the base function $y = 11^x$.

a) reflect in the x-axis and stretch vertically by a factor of 4

b) reflect in the y-axis and stretch horizontally by a factor of $\frac{4}{3}$

3. At midnight, one hospital patient contracts an unknown virus. By 1 a.m., three other hospital patients are diagnosed with the same virus. One hour later, nine more patients are found to have the virus, and by 3 a.m., 27 more patients have the virus. The virus continues to spread this way through the hospital.

a) Make a table of values to relate the number of new patients who are diagnosed with the virus to time, in 1-h intervals.

b) Make a scatter plot. Describe the trend.

c) What type of function represents the spread of this virus? Justify your answer.

d) Determine an equation to model this relation. Explain the method you chose to determine the equation.

Chapter 4 Trigonometry

34. a) To find trigonometric ratios for 240° using a unit circle, a reference angle of 60° is used. What reference angle should you use to find the trigonometric ratios for 210°?

b) Use the unit circle to find exact values of the three primary trigonometric ratios for 210° and 240°.

35. A fishing boat is 15 km south of a lighthouse. A yacht is 15 km west of the same lighthouse.

a) Use trigonometry to find an exact expression for the distance between the two boats.

b) Check your answer using another method.

36. Without using a calculator, determine two angles between 0° and 360° that have a sine of $\frac{\sqrt{3}}{2}$.

The point P(-2, 7) is on the terminal arm of ∠A.

a) Determine the primary trigonometric ratios for ∠A and ∠B, such that ∠B has the same sine as ∠A.

b) Use a calculator and a diagram to determine the measures of ∠A and ∠B, to the nearest degree.

38. Consider right $\triangle PQR$ with side lengths $PQ = 5$ cm and $QR = 12$ cm, and $\angle Q = 90°$.

a) Determine the length of side PR.

b) Determine the six trigonometric ratios for $\angle P$.

c) Determine the six trigonometric ratios for $\angle R$.

39. Determine two possible measures between 0° and 360° for each angle, to the nearest degree.

a) $\csc A = \frac{7}{3}$

b) $\sec B = -6$

c) $\cot C = -\frac{9}{4}$

40. An oak tree, a chestnut tree, and a maple tree form the corners of a triangular play area in a neighbourhood park. The oak tree is 35 m from the chestnut tree. The angle between the maple tree and the chestnut tree from the oak tree is 58°. The angle between the oak tree and the chestnut tree from the maple tree is 49°.

a) Sketch a diagram of this situation. Why is the triangle formed by the trees an oblique triangle?

b) Is it necessary to consider the ambiguous case? Justify your answer.

c) Determine the unknown distances, to the nearest tenth of a metre. If there is more than one possible answer, determine both.

41. At noon, two cars travel away from the intersection of two country roads that meet at a 34° angle. Car A travels along one of the roads at 80 km/h and car B travels along the other road at 100 km/h. Two hours later, both cars spot a jet in the air between them. The angle of depression from the jet to car A is 20° and the distance between the jet and the car is 100 km. Determine the distance between the jet and car B.

42. Isra parks her motorcycle in a lot on the corner of Canal and Main streets. She walks 60 m west to Maple Avenue, turns 40° to the left, and follows Maple Avenue for 90 m to the office building where she works. From her office window on the 18th floor, she can see her motorcycle in the lot. Each floor in the building is 5 m in height.

a) Sketch a diagram to represent this problem, labelling all given measurements.

b) How far is Isra from her motorcycle, in a direct line?

43. Prove each identity.

a) $\frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} = \left(\tan \theta + \frac{1}{\tan \theta}\right)^2$

b) $\csc \theta\left(\frac{1}{\cot \theta} + \frac{1}{\sec \theta}\right) = \sec \theta + \cot \theta$

Chapter 5 Trigonometric Functions

44. a) Sketch a periodic function, $f(x)$, with a maximum value of 5, a minimum value of -3, and a period of 4.

b) Select a value a for x, and determine $f(a)$.

c) Determine two other values, b and c, such that $f(a) = f(b) = f(c)$.

45. While visiting a town along the ocean, Bashira notices that the water level at the town dock changes during the day as the tides come in and go out. Markings on one of the piles supporting the dock show a high tide of 4.8 m at 6:30 a.m., a low tide of 0.9 m at 12:40 p.m., and a high tide again at 6:50 p.m.

a) Estimate the period of the fluctuation of the water level at the town dock.

b) Estimate the amplitude of the pattern.

c) Predict when the next low tide will occur.

46. Consider the following functions.

i) $y = 4 \sin \left[\frac{1}{3}(x + 30°)\right] - 1$

ii) $y = -\frac{1}{2} \cos [4(x + 135°)] + 2$

a) What is the amplitude of each function?

b) What is the period of each function?

c) Describe the phase shift of each function.

d) Describe the vertical shift of each function.

e) **Use Technology** Graph each function. Compare the graph to the characteristics expected.

47. A sinusoidal function has an amplitude of 6 units, a period of 150°, and a maximum at (0, 4).

a) Represent the function with an equation using a sine function.

b) Represent the function with an equation using a cosine function.

48. The height, h, in metres, above the ground of a rider on a Ferris wheel after t seconds can be modelled by the sine function $h(t) = 9 \sin [2(t - 30)] + 10$.

a) **Use Technology** Graph the function.

b) Determine

i) the maximum and minimum heights of the rider above the ground

ii) the height of the rider above the ground after 30 s

iii) the time required for the Ferris wheel to complete one revolution

49. Marcia constructs a model alternating current (AC) generator in physics class and cranks it by hand at 4 revolutions per second. She is able to light up a flashlight bulb that is rated for 6 V. The voltage can be modelled by a sine function of the form $y = a \sin [k(x - d)] + c$.

a) What is the period of the AC produced by the generator?

b) Determine the value of k.

c) What is the amplitude of the voltage function?

d) Model the voltage with a suitably transformed sine function.

e) **Use Technology** Graph the voltage function over two cycles. Explain what the scales on the axes represent.

Chapter 6 Discrete Functions

50. Write the ninth term, given the explicit formula for the nth term of the sequence.

a) $t_n = \frac{n^2 - 1}{2n}$

b) $f(n) = (-3)^{n-2}$

51. Write the first five terms of each sequence.

a) $t_1 = 3,\ t_n = \frac{t_{n-1}}{0.2}$

b) $f(1) = \frac{2}{5},\ f(n) = f(n - 1) - 1$

52. A hospital patient, recovering from surgery, receives 400 mg of pain medication every 5 h for 3 days. The half-life of the pain medication is approximately 5 h. This means that after 5 h, about half of the medicine is still in the patient's body.

a) Create a table of values showing the amount of medication remaining in the body after each 5-h period.

b) Write the amount of medication remaining after each 5-h period as a sequence. Write a recursion formula for the sequence.

c) Graph the sequence.

d) Describe what happens to the medicine in the patient's body over time.

Use Pascal's triangle for questions 53 and 54.

53. Expand each power of a binomial.

a) $(x - y)^6$

b) $\left(\frac{x}{3} - 2x\right)^4$

54. Write each as the sum of two terms, each in the form $t_{n,r}$.

a) $t_{5,2}$

b) $t_{10,7}$

55. a) State whether or not each sequence is arithmetic. Justify your answer.

i) 9, 5, 1, -3, ...

ii) $\frac{1}{5}, \frac{3}{5}, 1, \frac{7}{5}, \frac{9}{5}, \ldots$

iii) $-4.2, -3.8, -3.5, -3.3, -3.2, \ldots$

b) For those sequences that are arithmetic, write the formula for the general term.

56. For each geometric sequence, determine the formula for the general term and then write t_{10}.

a) 90, 30, 10, ...

b) $\frac{1}{4}, \frac{1}{6}, \frac{1}{9}, \ldots$

c) $-0.0035, 0.035, -0.35, \ldots$

57. Determine the sum of the first 10 terms of each arithmetic series.

a) $a = 2, d = -3, t_{10} = 56$

b) $a = -5, d = 1.5$

58. Determine the sum of each geometric series.

a) $45 + 15 + 5 + \cdots + \frac{5}{729}$

b) $1 - x + x^2 - x^3 + \cdots - x^{15}$

59. A bouncy ball bounces to $\frac{3}{5}$ of its height when dropped on a hard surface. Suppose the ball is dropped from 45 m.

a) What height will the ball bounce back up to after the seventh bounce?

b) What is the total distance travelled by the ball after 12 bounces?

Chapter 7 Financial Applications

60. Richard deposits $1000 into a guaranteed investment certificate (GIC) that earns 4.5% per year, simple interest.

a) Develop a linear model to relate the amount in the GIC to time. Identify the fixed part and the variable part. Graph the function.

b) How long will it take, to the nearest month, for the investment to double?

c) What annual rate of interest must be earned so that the investment doubles in 6 years?

61. Suppose you have $2500 to invest for 6 years. Two options are available:

- Top Bank: earns 6% per year, compounded quarterly
- Best Credit Union: earns 5.8% per year, compounded weekly

Which investment would you choose and why?

62. Five years ago, money was invested at 6.75% per year, compounded annually. Today the investment is worth $925.

a) How much money was originally invested?

b) How much interest was earned?

63. At the end of every month, Cassie deposits $120 into an account that pays 5.25% per year, compounded monthly. She does this for 5 years.

a) Draw a time line to represent this annuity.

b) Determine the amount in the account after 5 years.

c) How much interest will have been earned?

64. Murray plans to withdraw $700 at the end of every 3 months, for 5 years, from an account that earns 7% interest, compounded quarterly.

a) Draw a time line to represent this annuity.

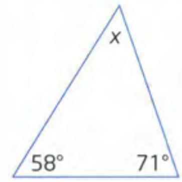
b) Determine the present value of the annuity.

c) How much interest will have been earned?

Prerequisite Skills Appendix

Angle Sum of a Triangle

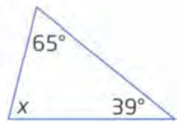
Use the fact that the sum of the interior angles of a triangle is 180° to find the measure of x.



$$x + 58° + 71° = 180°$$
$$x = 180° - 129°$$
$$x = 51°$$

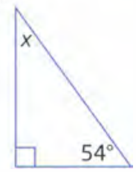
1. Find the measure of each angle x.

a)



b)

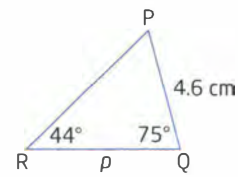
c)



Apply the Sine Law and the Cosine Law

Use the sine law to solve any acute triangle given
- the measures of two angles and any side
- the measures of two sides and the angle opposite one of the given sides

To find the length of side p, first determine the measure of $\angle$P.



Apply the angle sum of a triangle.

$$\angle P = 180° - 44° - 75°$$
$$= 61°$$

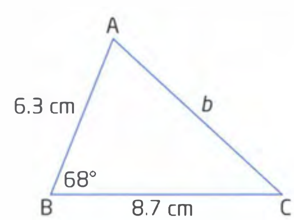
Then, use the sine law.

$$\frac{p}{\sin 61°} = \frac{4.6}{\sin 44°}$$
$$p = \frac{4.6 \sin 61°}{\sin 44°}$$
$$\doteq 5.8$$

The length of side p is approximately 5.8 cm.

Use cosine law to solve any triangle given
- the measures of two sides and the contained angle
- the measures of three sides

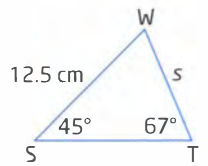
Use the cosine law to find b.

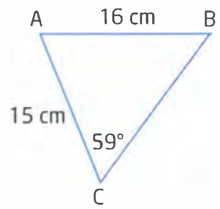


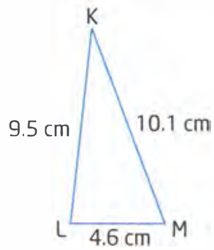
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$b^2 = 8.7^2 + 6.3^2 - 2(8.7)(6.3) \cos 68°$$
$$b^2 \doteq 74.316$$
$$b \doteq 8.6$$

The length of side b is approximately 8.6 cm.

1. Determine the measure of the angle or side indicated.
   a) length of s
   b) measure of $\angle B$
   c) measure of $\angle M$





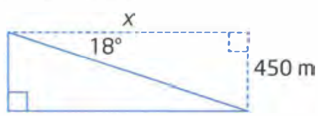


Apply Trigonometric Ratios to Problems

Various problems involving right triangles can be solved using trigonometric ratios.

An airplane that is 450 m above the ground is coming down for a landing at an angle of depression of 18°. How far, horizontally, is the plane from its landing point?

Draw and label a diagram to represent the given information.
Use the tangent ratio and solve for x.



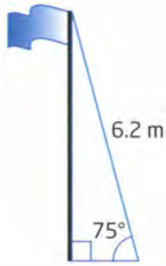
$$\tan 18° = \frac{450}{x}$$

$$x = \frac{450}{\tan 18°}$$

$$\doteq 1385$$

The plane is about 1385 m horizontally from its landing point.

1. A guy wire attached to the ground and the top of a flagpole is 6.2 m long. The wire makes an angle of 75° with the ground. How tall is the flagpole?



2. The ramp from the back of a truck is 4.0 m long. If the back of the truck is 0.6 m above the ground, at what angle is the ramp inclined?

Classify Triangles

Triangles can be classified according to their side lengths or the measures of their angles.

Classification by Side Length

- Equilateral: all three sides equal
- Isosceles: two sides equal
- Scalene: no sides equal

Classification by Angle Measure

- Right triangle: contains a 90° angle
- Acute triangle: has all angles less than 90°
- Obtuse triangle: contains an angle greater than 90°

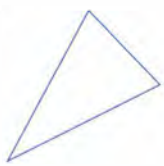
This is a scalene right triangle.

This is an isosceles acute triangle.

This is a scalene obtuse triangle.

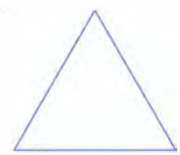
1. Classify each triangle by its sides and angles.

a)



b)

c)



Common Factors

To factor an expression, determine the greatest common factor (GCF) of each term.

To factor $6m^2 - 15m$, write each term in expanded form.
$6m^2 = 2 \times 3 \times m \times m$
$15m = 3 \times 5 \times m$

$3m$ is the GCF.
Find the second factor by dividing the GCF into each term in the original expression.

$$\frac{6m^2}{3m} - \frac{15m}{3m} = 2m - 5$$

Therefore, $6m^2 - 15m = 3m(2m - 5)$.
A common factor can have more than one term. For example, $3(x - 2) + x^2(x - 2) = (x - 2)(3 + x^2)$.

1. Find the GCF of each set of terms.

a) $8x$, $12y$

b) $-12a^3$, $6a^2b$, $9ab$

c) $10xy^3$, $35x^3y^2$

d) $24m^3n^2$, $-72m^2n^4$, $96m^2n^3$

e) $6(a^2 + 3)$, $-5(a^2 + 3)$

f) $3x^2y + 12xy$, $15xy^3 - 6x^2y$

2. Factor fully.

a) $16x^2 + 20x$

b) $5x^2y^2 + 10xy^3$

c) $3a^3 - 9a^2$

d) $4r^5s^2 + 16r^2s$

e) $8a^3b - 10ab + 4a^2b^2$

f) $-6x^3y - 18x^4y^3 - 36x^2y^4$

g) $12p(p + 3q) - q(p + 3q)$

h) $8x(y - 2x^2) + 20xy(y - 2x^2)$

Determine an Angle Given a Trigonometric Ratio

To determine the measure of an acute angle, use the corresponding inverse function on a scientific or graphing calculator.

If $\sin A = 0.3897$, then
$\angle A = \sin^{-1} 0.3897$
$\angle A \doteq 22.9°$

1. Determine the measure of each acute angle, to the nearest degree.

a) $\cos A = 0.2598$

b) $\sin Q = 0.8339$

c) $\tan T = 2.4591$

d) $\cos P = 0.7662$

e) $\sin X = 0.3478$

f) $\tan C = 0.6264$

Direct Variation and Partial Variation

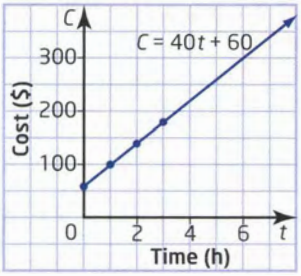
In a direct variation, one variable is a constant multiple of another variable. For example, $y = 4x$ is a direct variation. It is a linear relation that passes through the origin.

A partial variation is a linear relation between two variables that involves a fixed amount plus a variable amount.

A plumber charges \$60, plus \$40/h for labour.
The equation of the relationship is $C = 40t + 60$, where C represents the total cost, in dollars, and t represents the time, in hours.
The fixed part is 60 and the variable part is 40.

Use a table of values to graph the relationship.

Time (h)	Cost (\$)
0	60
1	100
2	140
3	180



The slope of the line is $\frac{100 - 60}{1 - 0} = 40$.

The vertical intercept is 60.

The slope is the coefficient of the variable part and represents the hourly cost of labour. The vertical intercept is the fixed part and represents the fixed or initial cost of labour.

1. For an advertisement in the classified section, a newspaper charges \$25 plus \$12 per day.
   a) Write an equation to relate the total cost, C, in dollars, of the advertisement to the number, d, of days.
   b) Identify the fixed part and the variable part of this relation.
   c) Graph the relation.
   d) Determine the slope and the vertical intercept of the graph.
   e) Explain how your answers in parts b) and d) are related.

2. Irene works part-time at a clothing store. She earns \$200 per week, plus a commission of 5% of her sales.
   a) Write an equation to relate her weekly salary to her sales.
   b) Identify the fixed part and the variable part of this relation.
   c) Graph the relation.
   d) Determine the slope and the vertical intercept of the graph.
   e) Explain how your answers in parts b) and d) are related.

Distributive Property

According to the distributive property, $a(x + y) = ax + ay$.
An algebraic expression in factored form can be expanded by multiplying each term in the brackets by the term outside.

For example, $3(x + 7) = 3x + 21$.

1. Expand.
   a) $2(a + b)$ b) $6(x - 4)$ c) $4(k^2 + 5)$ d) $-3(x - 2)$
   e) $5(x^2 - 2x + 1)$ f) $2x(3x - 4)$ g) $8a(3 + a)$ h) $-2x(x + y - 3)$

Evaluate Expressions

To find the percent of a number, change the percent to a decimal number. Then, multiply by the number.

$$\begin{aligned} 16\% \text{ of } 50 &= 0.16 \times 50 \\ &= 8 \end{aligned}$$

1. Evaluate.
   a) 45% of 120 b) 3% of 64 c) 20% of 95
   d) 5.5% of 2036 e) 4.25% of 600 f) 140% of 230

To add or subtract rational numbers in fraction form, find the least common denominator (LCD), multiply accordingly to get equivalent fractions, and add or subtract the numerators.

To evaluate $\frac{3}{8} - \frac{7}{12}$, use the LCD of 24.

$$\begin{aligned} \frac{3}{8} - \frac{7}{12} &= \frac{9}{24} - \frac{14}{24} \\ &= -\frac{5}{24} \end{aligned}$$

Refer to **Work With Fractions** on page 494.

To multiply rational numbers in fraction form, multiply numerators and denominators. To divide, multiply by the reciprocal of the second fraction. For all operations, convert any mixed numbers to improper fractions first.

$$\begin{aligned} -\frac{5}{6} \div 2\frac{1}{2} &= -\frac{5}{6} \div \frac{5}{2} \\ &= -\frac{5}{6} \times \frac{2}{5} \\ &= -\frac{10}{30} \\ &= -\frac{1}{3} \end{aligned}$$

2. Evaluate.

a) $\frac{3}{4} + \left(-\frac{1}{2}\right)$ **b)** $1\frac{2}{3} - \frac{5}{12}$ **c)** $-\frac{5}{8} + \left(-1\frac{1}{6}\right)$

d) $\frac{7}{9} \times \left(-\frac{3}{4}\right)$ **e)** $3\frac{1}{8} \div \left(-1\frac{1}{4}\right)$ **f)** $-1\frac{1}{5} \div 6$

Exponent Rules

To multiply powers with the same base, add the exponents.

$$\begin{aligned} x^3 \times x^2 &= x^{3+2} \\ &= x^5 \end{aligned}$$

To divide powers with the same base, subtract the exponents.

$$\begin{aligned} x^6 \div x^2 &= x^{6-2} \\ &= x^4 \end{aligned}$$

To raise a power to a power, multiply the exponents.

$$\begin{aligned} (x^2)^3 &= x^{2 \times 3} \\ &= x^6 \end{aligned}$$

1. Simplify, using the exponent rules. Leave answers in exponential form.

a) $2^3 \times 2^4$ **b)** $5^2 \times 5^4$ **c)** $3^5 \div 3^2$

d) $4^8 \div 4^3$ **e)** $(6^4)^2$ **f)** $(9^3)^7$

g) $a^5 \times a^5$ **h)** $z^4 \times z^4$ **i)** $3x^2 \times 2x^3$

j) $y^8 \div y^5$ **k)** $(p^3)^6$ **l)** $n^6 \div n$

m) $(12x^7) \div (-3x^4)$ **n)** $(2t^4)^3$ **o)** $(-4x^2)^4$

2. Evaluate.

a) $8^6 \div 8^4$ **b)** $2^2 \times 2^3 \div 2^4$ **c)** $3^9 \div 3^3$

d) $(2^4)^2$ **e)** $(4^2)^3$ **f)** $\left(\frac{1}{4}\right)^3$

Factor Quadratic Expressions

To factor a quadratic expression:

- check for common factors
- for expressions in the form $x^2 + bx + c$, find two integers, m and n, that have a sum of b and a product of c, and factor as $(x + m)(x + n)$
- for expressions in the form $ax^2 + bx + c$, find two integers, m and n, that have a sum of b and a product of ac, and then rewrite as $ax^2 + mx + nx + c$ and factor by grouping
- look for special products such as differences of squares or perfect squares

To factor $x^2 + 2x - 15$, note that $5 + (-3) = +2$ and $(5)(-3) = -15$.
Therefore, $x^2 + 2x - 15 = (x + 5)(x - 3)$.

In the expression $12x^2 - 60x + 75$, the number 3 is a common factor.
So, $12x^2 - 60x + 75 = 3(4x^2 - 20x + 25)$.
The coefficients of the first term and the last term are perfect squares, so that the trinomial inside the brackets may be a perfect square.
Since the coefficient of the middle term, -20, is twice the product of the square roots of 4 and 25, it is a perfect square trinomial.
Therefore, $12x^2 - 60x + 75 = 3(2x - 5)^2$.

1. Factor fully.

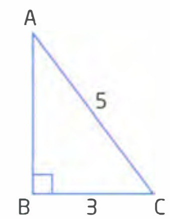
a) $x^2 + 6x + 8$ **b)** $x^2 - 7x + 12$ **c)** $2x^2 + 6x - 36$

d) $3x^2 - 48$ **e)** $3x^2 - 11x + 10$ **f)** $x^2 - 6x + 9$

g) $4x^2 - 100$ **h)** $2x^2 + 3x - 20$ **i)** $4x^2 - 15x + 14$

Find Primary Trigonometric Ratios

To determine the primary trigonometric ratios for ∠A, determine the length of the third side using the Pythagorean theorem.



$c^2 = 5^2 - 3^2$

$c = \sqrt{16}$

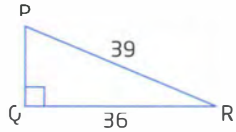
$= 4$

$\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{3}{5}$ $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{4}{5}$ $\tan A = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{3}{4}$

1. Find the primary trigonometric ratios for ∠P in each triangle.

a)

b)



Use a calculator to find the primary trigonometric ratios for any angle. For the angle 25°, sin 25° = 0.4226, cos 25° = 0.0963, and tan 25° = 0.4663, rounded to four decimal places.

2. Use a calculator to find the primary trigonometric ratios for each angle. Round each answer to four decimal places. Be sure your calculator is in degree mode.

a) 58° **b)** 79° **c)** 15°

Finite Differences

Given a table of values where the x-values change in constant steps, it is possible to determine the type of relationship that exists between the variables by calculating finite differences. The first differences are found by subtracting successive y-values. If the first differences are constant, the relationship is linear. If not, calculate the second differences by subtracting successive first differences. If the second differences are constant, the relationship is quadratic. If not, then the relationship is neither linear nor quadratic.

x	y	First Differences
1	−3	3 − (−3) = 6
2	3	13 − 3 = 10
3	13	27 − 13 = 14
4	27	45 − 27 = 18
5	45	

The first differences are not constant. So, the relationship is not linear.

x	y	First Differences	Second Differences
1	−3	3 − (−3) = 6	10 − 6 = 4
2	3	13 − 3 = 10	14 − 10 = 4
3	13	27 − 13 = 14	18 − 14 = 4
4	27	45 − 27 = 18	
5	45		

The second differences are constant. So, the relationship is quadratic.

1. Use finite differences to determine whether each relationship is linear, quadratic, or neither.

a)

x	y
1	5
2	1
3	−3
4	−7
5	−11

b)

x	y
1	−2
2	2
3	18
4	52
5	110

c)

x	y
1	15
2	28
3	39
4	48
5	55

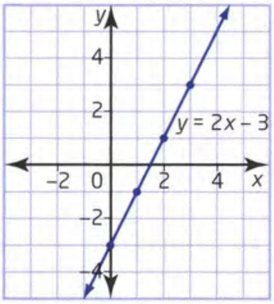
Graphs and Lines

Graph a linear relation by using

- a table of values
- the slope and the y-intercept
- the intercepts

To graph $y = 2x - 3$, make a table of values by choosing simple values for x and substituting to calculate y.

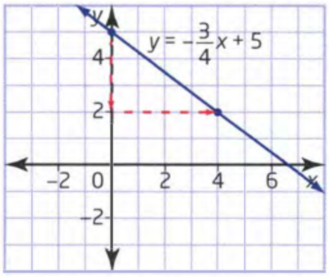
x	y
0	−3
1	−1
2	1
3	3



Plot the four points and draw a line through them.

To graph the line $y = -\frac{3}{4}x + 5$, use the y-intercept, 5, and the slope, or $\frac{\text{rise}}{\text{run}}$, $\frac{-3}{4}$.

Start on the y-axis at (0, 5). Then, use the slope to get to another point on the line. Here, counting down 3 and right 4 leads to (4, 2). Then, draw a line through the two points.

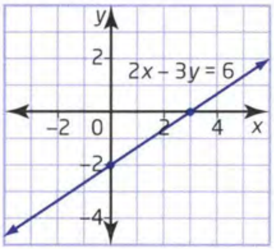


To graph $2x - 3y = 6$, use intercepts.

At the x-intercept, $y = 0$.

$$\begin{aligned} 2x - 3(0) &= 6 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

The x-intercept is 3. A point on the line is (3, 0).



At the y-intercept, $x = 0$.

$$\begin{aligned} 2(0) - 3y &= 6 \\ -3y &= 6 \\ y &= -2 \end{aligned}$$

The y-intercept is −2. A point on the line is (0, −2). Draw a line through the two points.

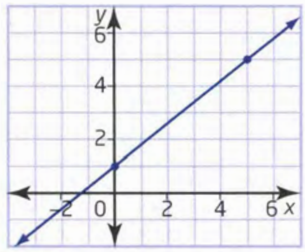
1. Graph each line using a convenient method.

a) $y = \frac{2}{3}x - 4$ **b)** $y = -3x + 6$ **c)** $x + 2y = 8$

d) $5x - 3y = 15$ **e)** $y = -\frac{1}{2}x + 5$ **f)** $y = 4x - 7$

To determine the equation of a line when given a graph, identify the y-intercept and the slope. Then, write the equation in slope y-intercept form, $y = mx + b$.

The y-intercept of this line is 1.
The line passes through the point (5, 5).
Starting from the y-intercept, the rise is 4 and the run is 5.
The slope is $\frac{4}{5}$.
The equation of the line is $y = \frac{4}{5}x + 1$.



If you are given two points that are on the line, determine an equation for the line by finding the slope and the y-intercept.
Find the slope of the line passing through (3, −6) and (15, 2).

$$m = \frac{2 - (-6)}{15 - 3}$$
$$= \frac{8}{12} \text{ or } \frac{2}{3}$$

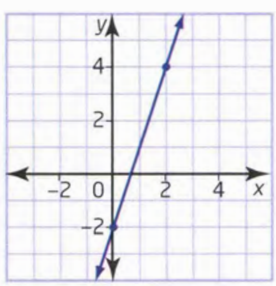
Therefore, $y = \frac{2}{3}x + b$.
Substitute the point (3, −6) to solve for b.

$$-6 = \frac{2}{3}(3) + b$$
$$-6 = 2 + b$$
$$b = -8$$

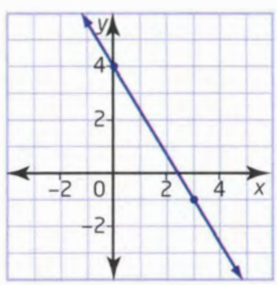
The equation of the line is $y = \frac{2}{3}x - 8$.

2. Determine the equation in the form $y = mx + b$ for each line.

a)



b)



3. Determine the equation in the form $y = mx + b$ for the line that passes through each pair of points.

a) (1, 9) and (3, 13) **b)** (3, 1) and (−12, 8) **c)** (−5, −14) and (10, −5)

Identify Patterns

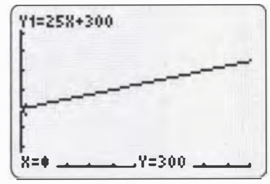
1. Identify the next three terms in each pattern.

a) Z, ZY, ZYX, … **b)** −3, 2, 7, 12, … **c)** 3, 9, 27, …

d) 19, 11, 3, −5, … **e)** $\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \ldots$ **f)** $54x, 18x^2, 6x^3, \ldots$

Linear and Exponential Growth

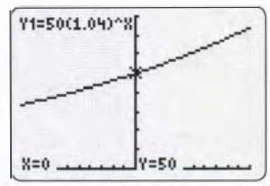
A graph of $y = 25x + 300$ is shown.
The relation is linear.
The y-intercept is 300 and the slope is 25.



A table of values with first differences shows that the first differences are constant.

x	y	First Differences
0	300	
		$325 - 300 = 25$
1	325	
		$350 - 325 = 25$
2	350	
		$375 - 350 = 25$
3	375	

A graph of $y = 50(1.04)^x$ is shown.
The relationship is exponential.
The y-intercept is 50.



A table of values with first differences, second differences, and common ratios is shown.

x	y	First Differences	Second Differences	Common Ratios
0	50			
		$52 - 50 = 2$		$\frac{52}{50} = 1.04$
1	52		$2.08 - 2 = 0.08$	
		$54.08 - 52 = 2.08$		$\frac{54.08}{52} = 1.04$
2	54.08		$2.163 - 2.08 = 0.083$	
		$56.243 - 54.08 = 2.163$		$\frac{56.243}{54.08} = 1.04$
3	56.243			

Successive first and second differences can be obtained by multiplying by 1.04. The common ratios are all equal to 1.04.

1. Graph the relation $y = 30x + 200$.

a) What kind of relation is this?

b) Identify the slope and the y-intercept.

c) Construct a table of values for $x = 0, 1, 2, 3$.

d) Calculate the first differences and describe their pattern.

2. Graph the relation $y = 25(1.1)^x$.

a) What type of relation is this?

b) Identify the y-intercept.

c) Construct a table of values for $x = 0, 1, 2, 3$.

d) Calculate the first and second differences and describe any patterns.

e) Calculate the common ratios of consecutive terms and describe their pattern.

Quadratic Relations

An equation in vertex form $y = a(x - h)^2 + k$ or standard form $y = ax^2 + bx + c$, $a \neq 0$, represents a quadratic relation. The graph of such a relation has a characteristic shape called a parabola.

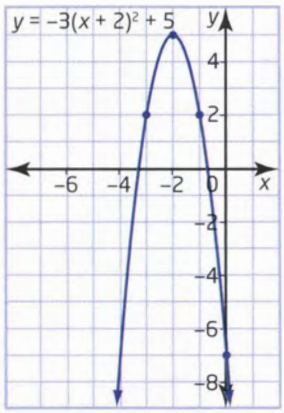
Given an equation in vertex form, (h, k) represents the coordinates of the parabola's vertex, $x = h$ is the equation of the axis of symmetry, and a represents the vertical stretch factor. If a is positive, the parabola opens upward; if a is negative, the parabola opens downward.

For the quadratic relation $y = -3(x + 2)^2 + 5$, the coordinates of the vertex are $(-2, 5)$ and the axis of symmetry is $x = -2$. The vertical stretch factor is 3 and the parabola opens downward because a is negative.

To determine the y-intercept, substitute $x = 0$.

$$\begin{aligned} y &= -3(0 + 2)^2 + 5 \\ &= -3(4) + 5 \\ &= -7 \end{aligned}$$

The y-intercept is -7.



To sketch the graph, plot the vertex $(-2, 5)$. Substitute to find that when $x = -1$, $y = 2$. By symmetry, another point is $(-3, 2)$. Plot the y-intercept. Then, draw a smooth U-shaped curve passing through all four points.

1. For each quadratic relation, state

i) the coordinates of the vertex

ii) the equation of the axis of symmetry

iii) the direction of opening

iv) the y-intercept

Then, sketch the graph of the relation.

a) $y = 2(x - 3)^2 - 8$ **b)** $y = -4(x + 1)^2 + 3$ **c)** $y = 3(x - 5)^2 + 1$

2. Compare the graph of each quadratic function to the graph of $y = x^2$. Identify the direction of opening and state whether the parabola has been vertically stretched or compressed. Justify your answer.

a) $y = 5x^2$ **b)** $y = -\frac{1}{4}(x - 1)^2$ **c)** $y = -3(x + 5)^2 + 2$

To identify the coordinates of the vertex of a quadratic relation using an equation in standard form, change to vertex form by completing the square.

If $a = 1$, then complete the square as described in the second step below.

If a is any number other than 1, then the first step is to factor this from the x-terms.

Write $y = 2x^2 - 12x + 19$ in vertex form as follows.

$y = 2(x^2 - 6x) + 19$	Factor a (i.e., 2) from the first two terms.
$= 2(x^2 - 6x + 9 - 9) + 19$	Divide the coefficient of the x-term by 2 and square the result. This yields 9. Add this number to and subtract it from the expression in brackets.
$= 2(x - 3)^2 + 2(-9) + 19$	Express the perfect square, $x^2 - 6x + 9$, in factored form. Multiply the subtracted value (i.e., 9) by a.
$= 2(x - 3)^2 + 1$	Simplify.

The vertex is (3, 1).

3. Complete the square to express each quadratic relation in the form $y = a(x - h)^2 + k$. Then, state the coordinates of the vertex.

a) $y = x^2 + 4x + 7$ **b)** $y = x^2 - 12x + 3$

c) $y = 3x^2 + 18x - 2$ **d)** $y = -2x^2 + 16x + 9$

Rearrange Formulas

To rearrange a formula to isolate a variable, apply the same steps as for solving an equation.

Solve for a in the formula $I = \frac{50d}{a + b}$.

$$I = \frac{50d}{a + b}$$

$I(a + b) = 50d$	Multiply both sides by $a + b$.
$a + b = \frac{50d}{I}$	Divide both sides by I.
$a = \frac{50d}{I} - b$	Subtract b from both sides.

1. Solve each formula for the variable indicated.

a) $y = -4x + 5$ for x **b)** $P = 2\ell + 2w$ for w **c)** $x^2 + y^2 = r^2$ for y

d) $V = \frac{1}{3}\pi r^2 h$ for h **e)** $s = \frac{2 \quad 10e}{t}$ for e **f)** $A = P(1 + rt)$ for r

Solve Equations

To solve $4(x + 2) = x + 5$, expand to remove brackets and then isolate the variable.

$4(x + 2) = x + 5$	
$4x + 8 = x + 5$	Expand.
$3x + 8 = 5$	Subtract x from both sides.
$3x = -3$	Subtract 8 from both sides.
$x = -1$	Divide both sides by 3.

To check, substitute -1 for x in the original equation.

L.S. $= 4(x + 2)$	R.S. $= x + 5$
$= 4(-1 + 2)$	$= -1 + 5$
$= 4(1)$	$= 4$
$= 4$	

Since L.S. = R.S., the solution is $x = -1$.

To solve $\frac{x+5}{2} - \frac{x}{3} = 1$, multiply both sides by the least common denominator, 6, to eliminate fractions.

$$\begin{aligned}
\frac{x+5}{2} - \frac{x}{3} &= 1 \\
6 \times \left(\frac{x+5}{2} - \frac{x}{3}\right) &= 6 \\
3(x+5) - 2x &= 6 \\
3x + 15 - 2x &= 6 \\
x + 15 &= 6 \\
x &= 9
\end{aligned}$$

To solve $600(1+i)^5 = 747.72$, first isolate the variable expression in brackets.

$$600(1+i)^5 = 747.72$$

$$(1+i)^5 = \frac{747.72}{600}$$ Divide both sides by 600.

$$(1+i)^5 = 1.2462$$

$$\sqrt[5]{(1+i)^5} = \sqrt[5]{1.2462}$$ Take the fifth root of both sides.

$$1 + i \doteq 1.0450$$ Simplify.

$$i \doteq 0.0450$$

1. Solve and check.

a) $7x - 5 = 3x - 17$ **b)** $3x - 7 = 5(x - 3)$

c) $\frac{x+1}{3} + \frac{x+5}{5} = 4$ **d)** $\frac{x+1}{2} - \frac{x-7}{6} = 3$

2. Solve. Round your answers to four decimal places, if necessary.

a) $\frac{850}{w-5} = 200$ **b)** $-6(p-3)^2 = -31.74$

c) $275.38 = 200(1+i)^{10}$ **d)** $\frac{2026.12}{(k+4)^3} = 5$

Solve Equations Involving Rational Expressions

To solve $\frac{200}{x} = 10$, isolate x.

$$\frac{200}{x} = 10$$

$$200 = 10x$$ Multiply both sides by x.

$$20 = x$$ Divide both sides by 10.

1. Solve.

a) $\frac{16}{x} = 8$ **b)** $\frac{a}{5} = 7$ **c)** $45 = \frac{135}{c}$

d) $\frac{12}{r} = \frac{4}{9}$ **e)** $\frac{k}{20} = \frac{3}{8}$ **f)** $\frac{36}{t} = \frac{2}{15}$

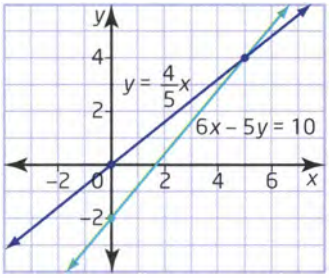
Solve Linear Systems of Equations

To determine the point of intersection of a linear system, graph the linear relations or solve the system algebraically.

To solve the linear system $y = \frac{4}{5}x$ and $6x - 5y = 10$ by graphing, find the point of intersection of the two lines. First, express the second equation in the form $y = mx + b$.

$$6x - 5y = 10$$
$$-5y = -6x + 10$$
$$y = \frac{6}{5}x - 2$$

Graph each line using the y-intercept and the slope. Then, identify the point of intersection. The point of intersection is (5, 4).



To solve a linear system algebraically, use either the substitution or the elimination method.
Substitution is suitable when one of the variables is easily isolated. The following system is best solved using the elimination method.

$$5x + 2y = 5$$
$$2x + 3y = 13$$

To make the coefficients of the y-terms the same, multiply the first equation by 3 and the second equation by 2. Then, subtract to eliminate y.

$$\begin{array}{llrl} 5x + 2y = 5 & ① \times 3 \rightarrow & 15x + 6y = & 15 \quad ③ \\ 2x + 3y = 13 & ② \times 2 \rightarrow & 4x + 6y = & 26 \quad ④ \\ \hline & & 11x \qquad = & -11 \quad ③ - ④ \\ & & x \qquad = & -1 \end{array}$$

Substitute $x = -1$ in ①.

$$5(-1) + 2y = 5$$
$$2y = 10$$
$$y = 5$$

The point of intersection is $(-1, 5)$.

1. Graph to find the point of intersection of each pair of lines.

a) $y = 3x - 5$
$y = 2x - 4$

b) $x + y = 1$
$y = \frac{2}{5}x - 6$

c) $x - 3y - 2 = 0$
$2x + y = 4$

2. Solve algebraically to find the point of intersection.

a) $x - 2y = 7$
$2x - 3y = 13$

b) $y = 2x - 7$
$3x + y = -17$

c) $4x - 7y = 20$
$x - 3y = 10$

d) $4x - 3y = 8$
$6x - 3y = 18$

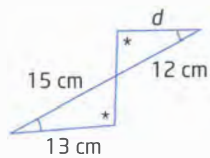
e) $4x + 3y = -2$
$4x + y = -6$

f) $5x - 2y = 20$
$2x + 5y = 8$

Use Similar Triangles

In similar triangles, the corresponding angles are equal and the lengths of corresponding sides are proportional.

The triangles shown are similar because all corresponding pairs of angles are equal. Write a proportion involving corresponding sides to solve for the length of side d.



$$\frac{d}{13} = \frac{12}{15}$$

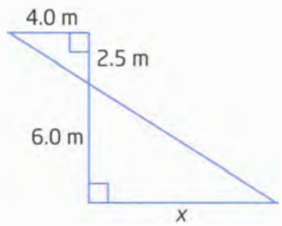
$$d = 13 \times \frac{12}{15}$$

$$d = 10.4$$

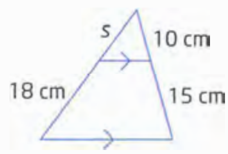
The length of side d is 10.4 cm.

1. Use similar triangles to determine the unknown length in each. If necessary, round answers to the nearest tenth.

a)



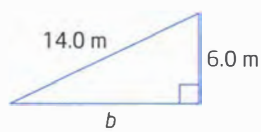
b)



Use the Pythagorean Theorem

The Pythagorean theorem states that, in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

To find the length, b, to the nearest tenth, write an equation using the Pythagorean theorem.



$$b^2 + 6^2 = 14^2$$

$$b^2 + 36 = 196$$

$$b^2 = 160$$

$$b = \sqrt{160}$$

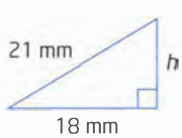
$$b \doteq 12.6$$

The length of side b is 12.6 m, to the nearest tenth of a metre.

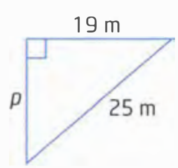
1. Determine the measure of the unknown side in each triangle. Round to the nearest tenth, if necessary.

a)

b)



c)



Work With Fractions

To evaluate or simplify expressions involving adding or subtracting fractions, it is best to determine the least common denominator (LCD), which is the same as determining the least common multiple (LCM) of the denominators.

To find the LCM of 24 and 30, write each number in factored form and determine the greatest common factor (GCF).
$24 = 2 \times 2 \times 2 \times 3$
$30 = 2 \times 3 \times 5$

The GCF is 2×3, or 6. Find the LCM by multiplying the GCF by the other factors of the original numbers.
$2 \times 3 \times 2 \times 2 \times 5 = 120$
The LCM is 120.

To find the LCM of $x^2 + x - 12$ and $x^2 - 8x + 15$, factor each expression and determine the GCF.
$x^2 + x - 12 = (x + 4)(x - 3)$
$x^2 - 8x + 15 = (x - 3)(x - 5)$
The GCF is $(x - 3)$.
The LCM is $(x - 3)(x + 4)(x - 5)$.

To add or subtract rational numbers in fraction form, find the LCM of the denominators, multiply the numerators accordingly to get equivalent fractions, and add or subtract the numerators.

To simplify $\frac{3x}{4} + \frac{2y}{10}$, first find the LCM of 4 and 10.

$$\frac{3x}{4} + \frac{2y}{10} = \frac{3x \times 5}{4 \times 5} + \frac{2y \times 2}{10 \times 2}$$

The LCM is $2 \times 2 \times 5 = 20$. Multiply to write equivalent fractions with denominator 20.

$$= \frac{15x}{20} + \frac{4y}{20}$$

Simplify.

$$= \frac{15x + 4y}{20}$$

1. Determine the LCM of each set.

a) 24, 40 **b)** $-10x^2, 35x, -55x^3$ **c)** $x^2 + 7x + 12, x^2 - 9$

2. Add or subtract.

a) $\frac{5}{9} + \frac{5}{6}$ **b)** $\frac{5}{24} - \frac{7}{60}$ **c)** $\frac{3a}{16} + \frac{5b}{36}$ **d)** $\frac{4x}{45} - \frac{k}{18}$

To simplify $\left(-\frac{14}{9}\right)\left(\frac{6}{7}\right)$, look for common factors to reduce the fractions.

Then, multiply numerators and multiply denominators.

$$\left(-\frac{14}{9}\right)\left(\frac{6}{7}\right) = \left(-\frac{\overset{2}{\cancel{14}}}{\underset{3}{\cancel{9}}}\right)\left(\frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{7}}}\right)$$

Divide by common factors.

$$= -\frac{4}{3}$$

Simplify.

To divide, multiply by the reciprocal of the second fraction.

3. Simplify.

a) $\left(\frac{8}{15}\right)^{\text{TM}}\left(\frac{3}{4}\right)$ **b)** $\left(\frac{5}{6}\right)\left(-\frac{3}{10}\right)$ **c)** $\frac{3}{8} \div \left(-\frac{9}{20}\right)$ **d)** $-\frac{11}{30} \div \frac{33}{9}$

Work With Polynomials

To expand $2(x - 5)(x + 4)$, expand the brackets. Then, multiply by the outside term.

$$
\begin{aligned}
2(x - 5)(x + 4) &= 2(x^2 + 4x - 5x - 20) \\
&= 2(x^2 - x - 20) && \text{Collect like terms.} \\
&= 2x^2 - 2x - 40 && \text{Multiply.}
\end{aligned}
$$

A perfect square trinomial has the general form $a^2 + 2ab + b^2$ and can be expressed in factored form as $(a + b)^2$.

To factor $4x^2 + 20x + 25$, check for the general form.

$4x^2 = (2x)^2$ $\quad 20x = 2(2x)(5)$ $\quad 25 = 5^2$

Therefore, $4x^2 + 20x + 25 = (2x + 5)^2$.

1. Expand and simplify.

a) $3(x + 2)(x + 5)$ **b)** $-2(x - 4)(x + 7)$ **c)** $(x - 6)^2$

2. Factor fully.

a) $x^2 + 3x - 10$ **b)** $x^2 + 14x + 49$ **c)** $9y^2 - 25$

d) $3a^2 + 48a + 192$ **e)** $25x^2 - 60x + 36$ **f)** $-p^2 - 20p - 100$

3. What value of k makes each quadratic expression a perfect square trinomial?

a) $x^2 + 16x + k$ **b)** $x^2 - 30x + k$ **c)** $x^2 + 40x + k$

d) $4x^2 + 12x + k$ **e)** $x^2 - 7x + k$ **f)** $9x^2 - 4x + k$

Zero and Negative Exponents

Any base, other than zero, raised to the exponent zero is equal to one.

$3^0 = 1$

A base raised to a negative exponent is equal to the reciprocal of the base raised to the positive value of the exponent.

1. Evaluate.

a) $10^3 \times 10^0$ **b)** 4^{-3} **c)** -7^0

d) 5^{-2} **e)** $(3^{-3})^2$ **f)** $6^4 \times 6^{-3} \times 6^2$

g) $2^3 \div 2^{-2}$ **h)** $2(3^4)^{-1}$ **i)** $\left(\frac{2}{3}\right)^{-2}$

2. Simplify. Write your answers using only positive exponents.

a) $3x^{-4}$ **b)** $(5y^{-2})^2$ **c)** $2(3x)^{-3}$

d) $\frac{4x^5y^6}{8x^2y^8}$ **e)** $\frac{(3a^{-4})2b^3}{6ab^{-3}}$ **f)** $\left(\frac{4m^3n^{-2}}{3m^{-4}n}\right)^{-3}$

Technology Appendix

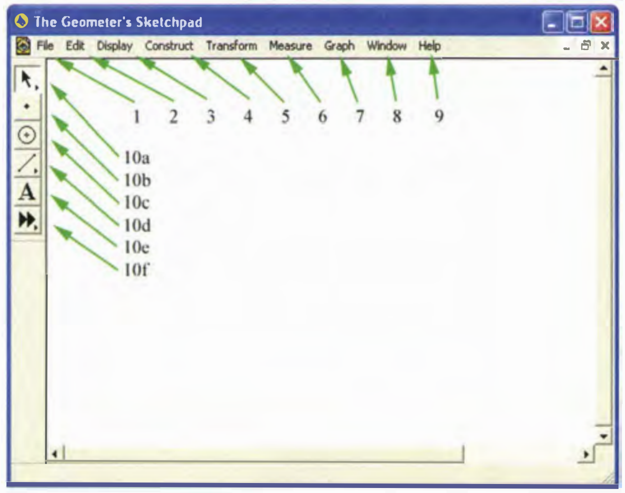
CONTENTS

The Geometer's Sketchpad®, Geometry Software Package

TI-83 Plus and TI-84 Plus Graphing Calculators

The Geometer's Sketchpad®, Geometry Software Package

Menu Bar

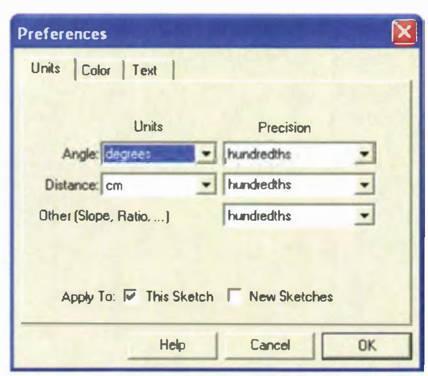


1 **File** menu—open/save/print sketches
2 **Edit** menu—undo/redo actions/ set preferences
3 **Display** menu—control appearance of objects in sketch
4 **Construct** menu—construct new geometric objects based on objects in sketch
5 **Transform** menu—apply geometric transformations to selected objects
6 **Measure** menu—make various measurements on objects in sketch
7 **Graph** menu—create axes and plot measurements and points
8 **Window** menu—manipulate windows
9 **Help** menu—access the help system, an excellent reference guide
10 **Toolbox**—access tools for creating, marking, and transforming points, circles, and straight objects (segments, lines, and rays); also includes text and information tools
10a **Selection Arrow Tool** (Arrow)—select and transform objects
10b **Point Tool** (Dot)—draw points
10c **Compass Tool** (Circle)—draw circles
10d **Straightedge Tool**—draw line segments, rays, and lines
10e **Text Tool** (Letter A)—label points and write text
10f **Custom Tool** (Double Arrow)—create or use special "custom" tools

Creating a Sketch

- From the **File** menu, choose **New Sketch** to start with a new work area.

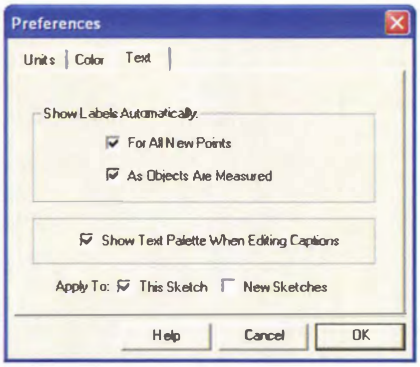
Setting Preferences



- From the **Edit** menu, choose **Preferences...**.
- Click the **Units** tab.
- Set the units and precision for angles, distances, and calculated values such as slopes and ratios.
- Click the **Text** tab.
- If you check the auto-label box **For All New Points**, then *The Geometer's Sketchpad®* will label points as you create them.
- If you check the auto-label box **As Objects Are Measured**, then *The Geometer's Sketchpad®* will label any measurements that you define.

You can also choose whether the auto-labelling functions will apply only to the current sketch, or also to any new sketches that you create.

Be sure to click **OK** to apply your preferences.



Selecting Points and Objects

- Choose the **Selection Arrow Tool**. The mouse cursor appears as an arrow.

To select a single point:
- Select the point by moving the cursor to the point and clicking it.

The selected point will now appear as a darker point, similar to a bull's-eye ⊙.

To select an object such as a line segment or a circle:
- Move the cursor to a point on the object until it becomes a horizontal arrow.
- Click the object. The object will change appearance to show it is selected.

To select a number of points or objects:
- Select each object in turn by moving the cursor to the object and clicking it.

To deselect a point or an object:
- Move the cursor over it, and then click the left mouse button.
- To deselect all selected objects, click in an open area of the workspace.

Hiding Points and Objects

Open a new sketch. Draw several objects, such as points and line segments.

To hide a point:
- Select the point.
- From the **Display** menu, choose **Hide Point**.

To hide an object:
- Select another point and a line segment.
- From the **Display** menu, choose **Hide Objects**.

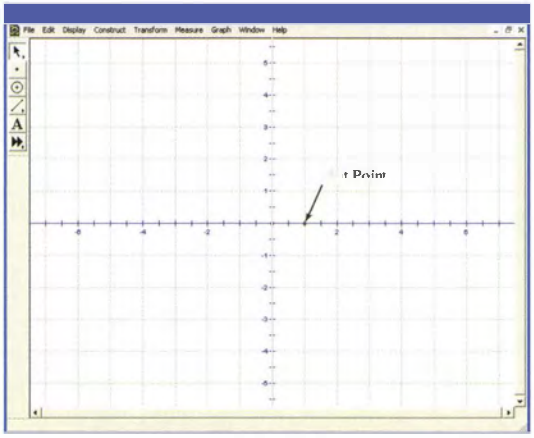
Shortcut: You can hide any selected objects by holding down the CTRL key and typing H.

You can make hidden objects reappear by choosing **Show All Hidden** from the **Display** menu.

Using a Coordinate System and Axes

- From the **Graph** menu, choose **Show Grid**.

The default coordinate system has an origin point in the centre of your screen and a unit point at (1, 0). Drag the origin to relocate the coordinate system and drag the unit point to change the scale.



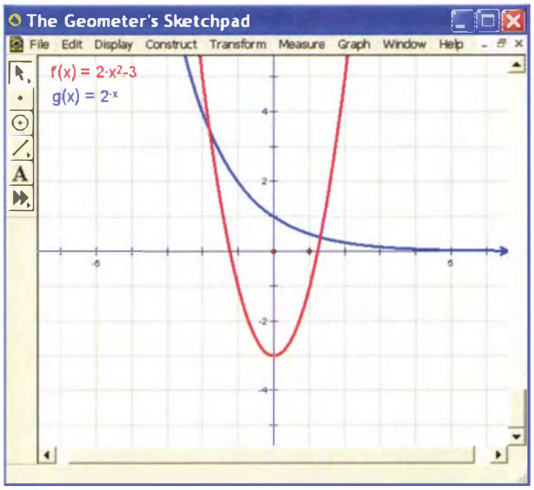
Graphing Relations

Consider the equations $y = 2x^2 - 3$ and $y = 2^{-x}$ as examples.
- From the **Graph** menu, select **Show Grid**.
- From the **Graph** menu, select **Plot New Function....**

The calculator interface will appear.

Enter the first equation: 2 * x ^ 2 − 3.
- Press **OK**. The graph of the first equation appears, along with the equation in function notation. You can move the equation next to the line.

Use the same procedure to graph the second equation.



Plotting Points

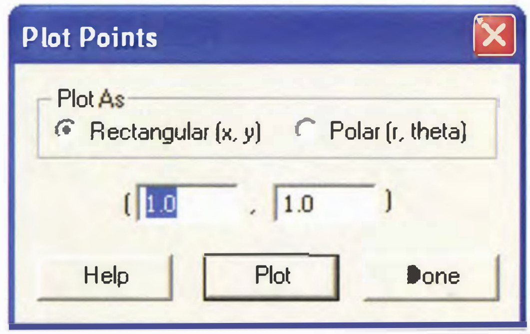
- From the **Graph** menu, choose **Show Grid**.
- If you want points plotted exactly at grid intersections, also choose **Snap Points**.
- Choose the **Point Tool**.

If you have enabled **Snap Points**, a point will "snap" to the nearest grid intersection as you move the cursor over the grid.

- Click the left mouse button to plot the point.

Alternatively, you can plot points by typing in the desired coordinates.

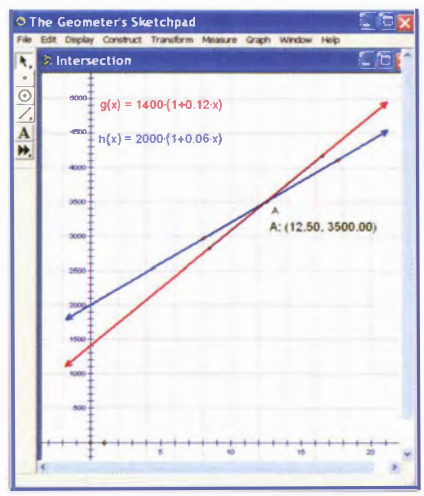
- From the **Graph** menu, choose **Plot Points...**. A dialogue box will appear. Type the desired x- and y-coordinates in the boxes. Then, press **Plot**.
- When you are finished plotting points, click **Done**.



Finding a Point of Intersection

Consider the equations $y = 1400(1 + 0.12x)$ and $y = 2000(1 + 0.06x)$ as examples.

- Turn on the grid, and then plot the graphs of the two equations.
- Use the **Point Tool** to plot two points on each line, such that the intersection lies between the points.
- Select one pair of points on a line. From the **Construct** menu, choose **Segment**. Use the same procedure to construct a segment on the other line.
- Select the two segments. From the **Construct** menu, choose **Intersection**. The point of intersection will appear.
- Select the point of intersection. From the **Measure** menu, choose **Coordinates**. The coordinates of the point of intersection will appear.



Using the Measure Menu

To measure the distance between two points:

- Ensure that nothing is selected.
- Select the two points.
- From the **Measure** menu, choose **Distance**.

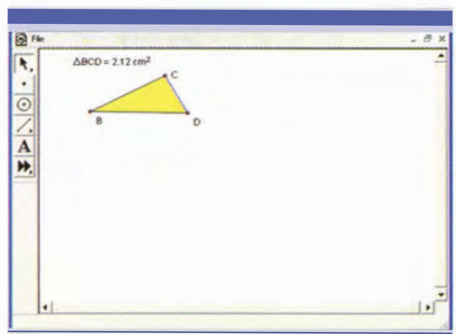
The Geometer's Sketchpad® will display the distance between the points using the units and accuracy selected in **Preferences...** under the **Edit** menu.

To measure the length of a line segment:
- Ensure that nothing is selected.
- Select the line segment (but not the endpoints).
- From the **Measure** menu, choose **Length**.

To measure an angle:
- Ensure that nothing is selected.
- Select the three points that define the angle so that the second point selected is the vertex of the angle.
- From the **Measure** menu, choose **Angle**.

To calculate the ratio of two lengths:
- Select the two lengths to be compared.
- From the **Measure** menu, choose **Ratio**.

To measure the area of a triangle or other closed figure:
- Use the **Straightedge Tool** to draw a triangle.
- Select the points on the vertices of the triangle.
- From the **Construct** menu, choose **Triangle Interior**.
- Select the triangle interior.
- From the **Measure** menu, choose **Area**.

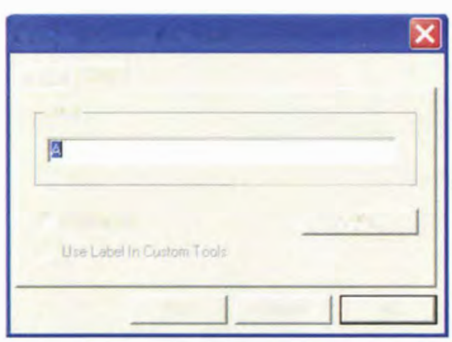


You can construct and measure the area of other closed figures in a similar manner.

To measure the slope of a line, ray, or line segment:
- Select the line, ray, or line segment.
- From the **Measure** menu, choose **Slope**.

Labelling a Vertex

To label a point on a line segment:
- Use the cursor to select the **Text Tool**.
- Move the cursor over the point on the line segment and click once.
- A letter label will appear.
- Move the cursor to the other point on the segment and click once.
- The next letter label in the alphabet will appear.
- If you double-click the point you can input the letter desired.
- Click **OK**.



Changing Labels of Measures

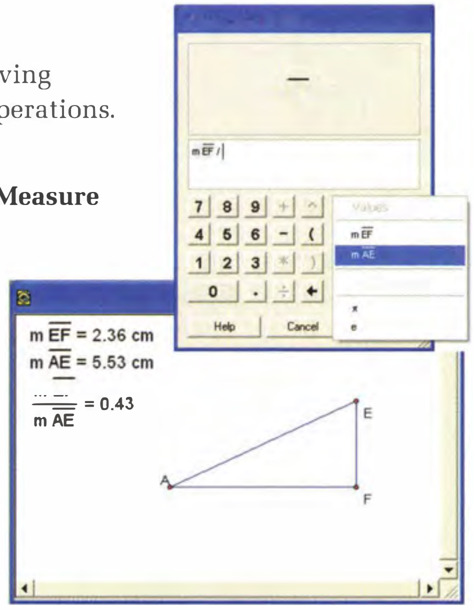
- Right-click the measure and choose **Label Measurement** (or **Label Distance Measurement** depending on the type of measure) from the drop-down menu.
- Type in the new label.
- Click **OK**.

Using the On-Screen Calculator

You can use the on-screen calculator to do calculations involving measurements, constants, functions, or other mathematical operations.

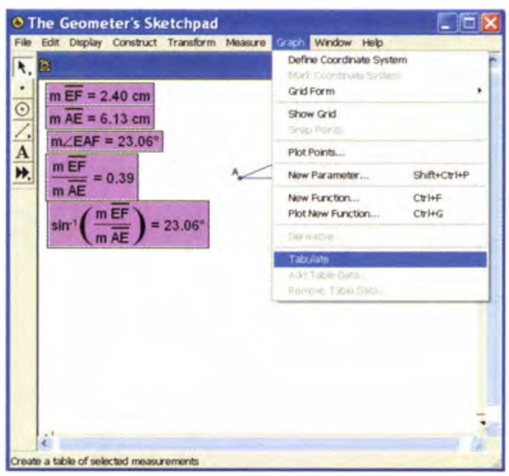
To calculate the sine ratio of two lengths:

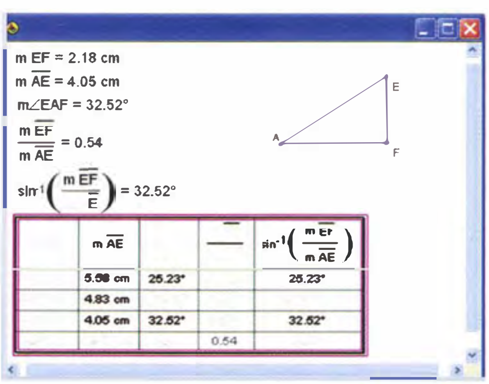
- Select the two measures. Then, choose **Calculate** from the **Measure** menu.
- Click the **Values** button and select the first measure.
- Press the (÷) key.
- Click the **Values** button and select the second measure.
- Click **OK** and the ratio will be calculated.



Using Tabulate to Construct a Data Table

- Click the measures in the order that they are to appear in the table. This will highlight them.
- Choose **Graph** and then **Tabulate**.
- The first row of the data table will be completed.
- Select a vertex and manipulate the object.
- Double-click the table to add a row of data.
  Note: The bottom row is always the active row.
  It changes as you manipulate the object. If you highlight only the table, you can choose the **Graph** menu and select **Add Table Data** to add a row to the table.

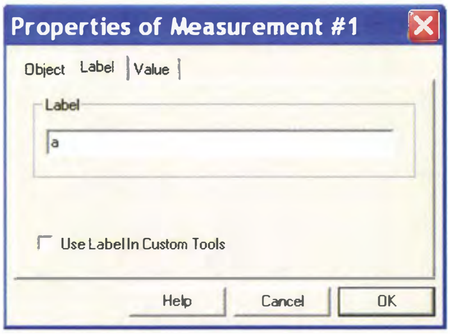


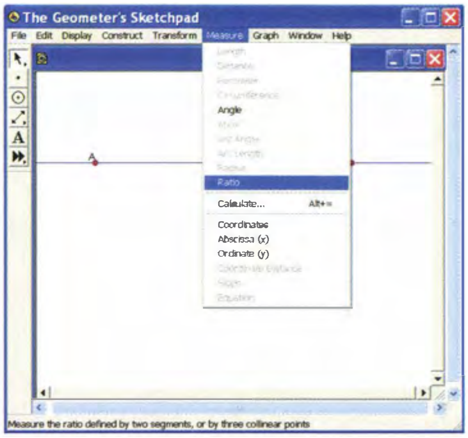


Constructing a Slider for a Function

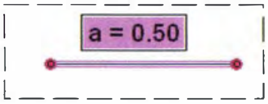
To construct a slider for a function of the form $y = a(x + h)^2 + k$:

- Construct a line through two points.
- Use the **Text Tool** to label the points A and B.
- Use the **Point Tool** to create another point on the line between A and B.
- Label the point C.
- Click A, B, and then C, in that order.
- From the **Measure** menu, choose **Ratio**.
- Click the white background to turn off the highlight on the ratio. Click the line and the point B to highlight them.
- Hide the line and point B by holding down the (Ctrl) key and pressing H. The points A and C and the ratio measure will remain.
- Construct a line segment from point A to point C.
- Select the **Text Tool** and double-click the ratio measure.
- Change its label to the letter *a*.

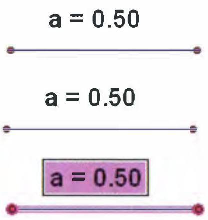




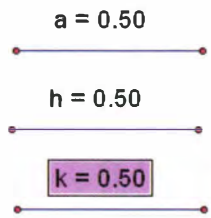
- Click the endpoints of the line segment to hide the letters A and C.
- Click the **Selection Arrow Tool** and click on the white background to turn off the highlighted ratio measure.
- Move the measure of *a* to the line segment.
- Select the right point and move it. **Note:** The value of *a* is positive when this point is on the right but negative when it is to the left of the other point.
- Manipulate the slider for the parameter *a* so that its value is 0.5.
- Select the measure and the slider for parameter *a* by holding the left mouse button until a rectangle is drawn around the objects as shown on the right.



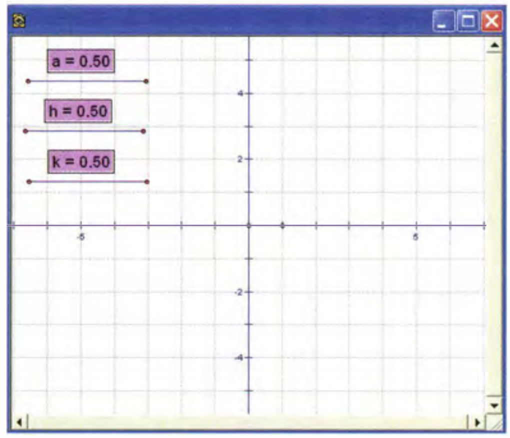
- From the **Edit** menu, choose **Copy**.
- From the **Edit** menu, choose **Paste** to insert a copy of the slider in the document.
- Move the copy below the original.
- From the **Edit** menu, choose **Paste** to insert another copy of the slider in the document.
- Move this copy below the last copy.



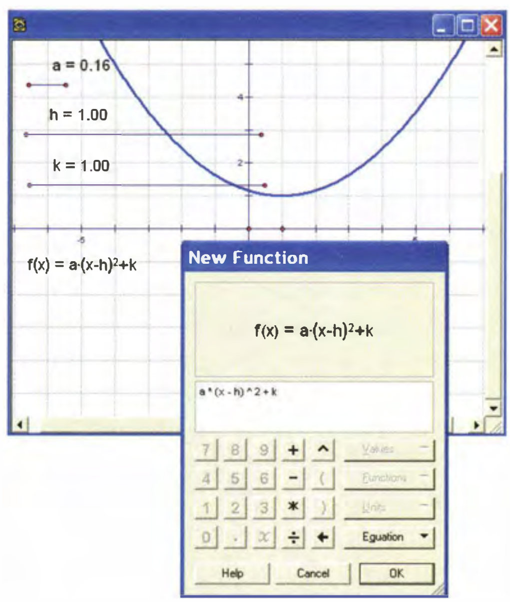
- Click the white background to turn off the highlight.
- Select the **Text Tool** and double-click the middle measure of *a*.
- Change its label to *h*.
- Click **OK**.
- Double-click the last measure.
- Change its label to *k*.
- Click **OK**.



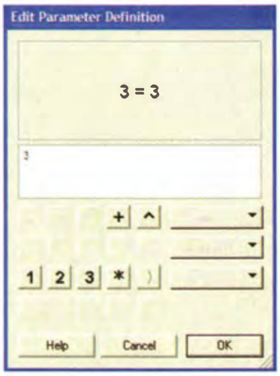
- Click the **Selection Arrow Tool**.
- Move the sliders and their values to the left side of the window.
- From the **Graph** menu, choose **Show Grid**.
- Click each of the measures *a*, *h*, and *k* (not the line segments).
- From the **Graph** menu, choose **Plot New Function**.



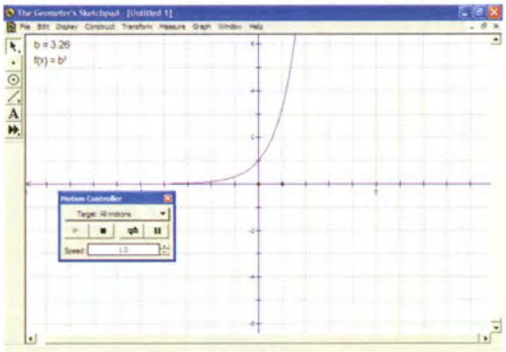
- Click (x) ^ 2 to begin the quadratic equation.
- Use the left arrow key and cursor to the beginning of the expression.
- Click the **Values** button and select the parameter *a*.
- Insert a multiplication symbol (*) between *a* and the opening parenthesis.
- Use the right arrow and cursor to the right of *x*.
- Insert a negative sign (–).
- Click the **Values** button and select the parameter *h*.
- Use the right arrow key and cursor to the far right of the expression.
- Insert an addition symbol (+).
- Click the **Values** button and select the parameter *k*.
- Click **OK**. The function will appear highlighted on your sketch.
- From the **Graph** menu, choose **Plot Function**. The graph of the quadratic function will appear.
- Move the right endpoints of the sliders and the values and the graph will change.



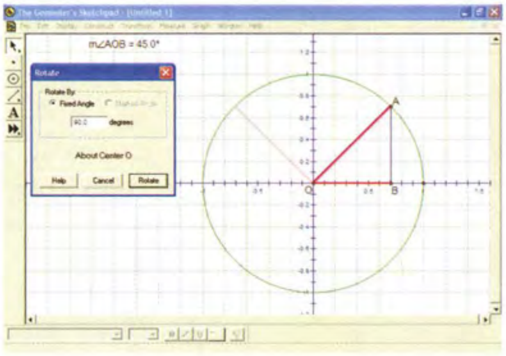
Changing the Value of a Parameter



- From the **Graph** menu, choose **New Parameter**. Set the name as b and its initial value to 3. Click **OK**.
- From the **Graph** menu, choose **Plot New Function**. Click on the parameter b, and then click on ^x and **OK**.
- Click on parameter b and press the + and − keys on the keyboard to increase or decrease the value of b in unitary increments.
- Right-click on parameter b and choose **Edit Parameter** to enter a specific value.
- Right-click on parameter b and choose **Animate Parameter**. Use the various buttons on the **Motion Controller** to see the effects of changing b continuously.



Rotating a Terminal Arm About the Origin



- From the **Graph** menu, choose **Show Grid**.
- Move the cursor to one of the increment numbers on either axis. Then, you can change the scale of both axes by dragging, so that you can create a large unit circle.
- Use the **Compass Tool** to draw a unit circle with centre (0, 0).
- Draw a line segment from the origin to the unit circle to create a terminal arm and then another one from the origin on the x-axis to create an initial arm that will form a right triangle when joined.
- To create the initial arm, select the x-axis and the point on the unit circle from the terminal arm. From the **Construct** menu, choose **Perpendicular Line**.
- Select the perpendicular line and the x-axis. From the **Construct** menu, choose **Intersection**. Select the origin and the point of intersection. From the **Construct** menu, choose **Segment**. You should now have an initial arm.
- Select the three points, making sure that the origin is your second choice. From the **Measure** menu, choose **Angle**. (Ensure that this is 45° by moving the point on the unit circle).
- Select the terminal arm. From the **Transform** menu, choose **Rotate**. Rotating the terminal arm by 90° is shown.

TI-83 Plus and TI-84 Plus Graphing Calculators

Keys

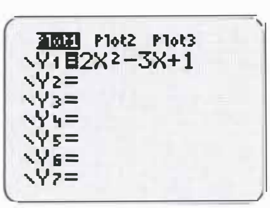
The keys on the TI-83 Plus and TI-84 Plus are colour-coded to help you find the various functions.

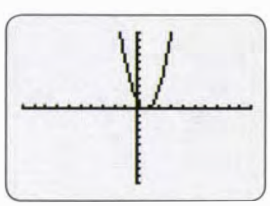
- The white keys include the number keys, decimal point, and negative sign. When entering negative values, use the white (–) key and not the grey – key.
- The grey keys on the right side are math operations.
- The grey keys across the top are used when graphing.
- The primary function of each key is printed on the key, in white.
- The secondary function of each key is printed in blue and is activated by pressing the 2nd key. For example, to find the square root of a number, press 2nd x^2 for [$\sqrt{\ }$].
- The alpha function of each key is printed in green and is activated by pressing the green ALPHA key.

Graphing Relations and Equations

Press Y=. Enter the equation.
To display the graph, press GRAPH.

For example, enter $y = 2x^2 - 3x + 1$ by pressing
Y= 2 X, T, θ, n x^2 – 3 X, T, θ, n + 1.
Press GRAPH.





Setting Window Variables

The WINDOW key defines the appearance of the graph.
The standard (default) window settings are shown.

To change the window settings:
- Press WINDOW. Enter the desired window settings.

In the example shown,
- the minimum x-value is -47
- the maximum x-value is 47
- the scale of the x-axis is 10
- the minimum y-value is -31
- the maximum y-value is 31
- the scale of the y-axis is 10
- the resolution is 1, so equations are graphed at each horizontal pixel

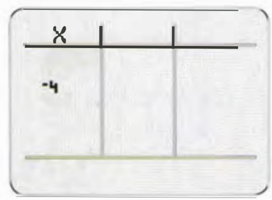
Note: The greater the resolution, the faster the graph plots because the horizontal pixels are omitted.

Setting Up a Table of Values

The standard (default) table settings are shown.
This feature allows you to specify the x-values of the table.

To change the **Table Set up** settings:
- Press 2nd WINDOW. Enter the desired values.

In the example shown,
- The starting x-value of the table is -5.
- The change in x-values is 0.5.



- Press 2nd GRAPH.

The table of values will appear as shown.

Tracing a Graph

- Enter a function using Y=.
- Press TRACE.
- Press ◄ and ► to move along the graph.

The x- and y-values are displayed at the bottom of the screen.

If you have more than one graph plotted, use the ▲ and ▼ keys to move the cursor to the graph you wish to trace.

You may want to turn off all Stat Plots before you trace a function:
- Press 2nd Y= for [STAT PLOT]. Select **4:PlotsOff**.
- Press ENTER.

Using Zoom

Use the (ZOOM) key to change the area of the graph that is displayed in the graphing window.

To set the size of the area you want to zoom in on:

- Press (ZOOM). Select **1:ZBox**. The graph screen will be displayed, and the cursor will be flashing.
- If you cannot see the cursor, use the (►), (◄), (▲), and (▼) keys to move the cursor until you see it.
- Move the cursor to an area on the edge of where you would like a closer view.
- Press (ENTER) to mark that point as a starting point.
- Press the (►), (◄), (▲), and (▼) keys, as needed, to move the sides of the box to enclose the area you want to look at.
- Press (ENTER) when you are finished. The area will now appear larger.

To zoom in on an area without identifying a boxed-in-area:

- Press (ZOOM). Select **2:Zoom In**.

To zoom out of an area:

- Press (ZOOM). Select **3:Zoom Out**.

To display the viewing area where the origin appears in the centre and the *x*- and *y*-axes intervals are equally spaced:

- Press (ZOOM). Select **4:ZDecimal**.

To display an equation with the minimum and maximum *y*-values constructed to match the scale of the *x*-axis and without changing the current minimum and maximum *x*-values, so that a square grid results:

- Press (ZOOM). Select **5:ZSquare**.

To reset the axes range on your calculator:

- Press (ZOOM). Select **6:ZStandard**.

To display all data points in a Stat Plot:

- Press (ZOOM). Select **9:ZoomStat**.

To display an equation with the minimum and maximum *y*-values constructed to best show the equation without changing the current minimum and maximum *x*-values:

- Press (ZOOM). Select **0:ZFit**.

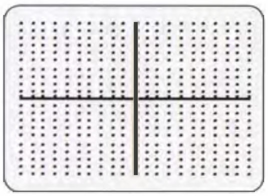
Setting the Format

To define a graph's appearance:

- Press (2nd) (ZOOM) for [FORMAT] to view the choices available.

The default settings, shown here, have some of the features on the left turned on.

To use **GridOn**:



- Pressing (2nd) (ZOOM) for [FORMAT].

Cursor down and right to **GridOn**.

- Press (ENTER).
- Press (GRAPH) to see the grid turned on.
- Press (2nd) (MODE) for [QUIT].

Entering Data Into Lists

To enter data:

- Press STAT. The cursor will highlight the **Edit** menu.
- Press 1 or ENTER to select **1:Edit...**.

This allows you to enter new data, or edit existing data, in lists **L1** to **L6**.

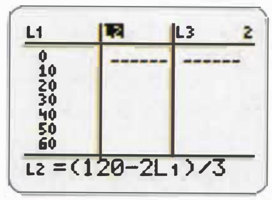
For example, press STAT, select **1:Edit...**, and then enter values in **L1**.

- Use the cursor keys to move around the editor screen.
- Complete each data entry by pressing ENTER.
- Press 2nd MODE for [QUIT] to exit the list editor when the data are entered.

Using List Operations

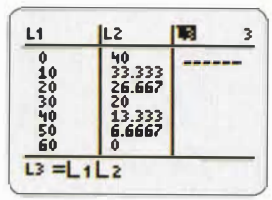
You can use a list in the list editor and apply the order of operations to produce another list. This is useful when tabulating data in a list affected by the same order of operations.

For example, for the area of a fenced-in yard:
Width = (120 − 2 × length) ÷ 3
Area = length × width



Use the list operations to compute the width values.

- Press STAT 1 to enter the list editor.
- Enter the given lengths into **L1**. For the example given, use 0, 10, 20, 30, 40, 50, 60.
- Press ▶ for **L2**. Cursor up to the title for **L2**.
- Press (120 − 2 2nd 1) ÷ 3 ENTER.

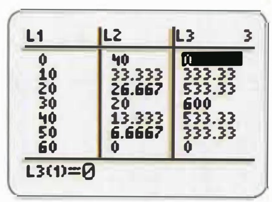


The width data are pasted into **L2**.

Use the list operations to compute the area values.

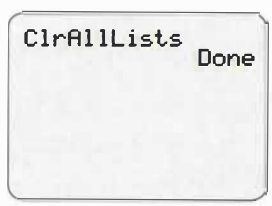
- Press ▶ for **L3**. Cursor up to the title for **L3**.
- Press 2nd 1 × 2nd 2 ENTER.

The area data are pasted into **L3**.



This process is useful when constructing the data for simple and compound interest problems.

Clearing Lists



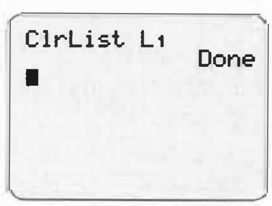
To clear all lists from the calculator without resetting the RAM:

- Press 2nd + for [MEM]. Select **4:ClrAllLists**.

This will paste the **ClrAllLists** command to the home screen of the calculator.

To clear all the lists:

- Press ENTER.

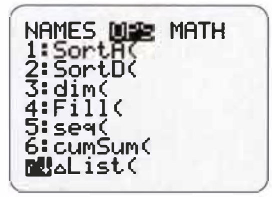


You may need to clear only a specific list before you enter data into it. For example, to clear list **L1**:

- Press STAT and select **4:ClrList**.
- Press 2nd 1 for [L1] and press ENTER.

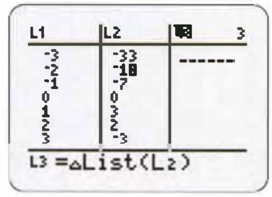
Calculating First and Second Differences

The calculator can compute the first and second differences of a list.



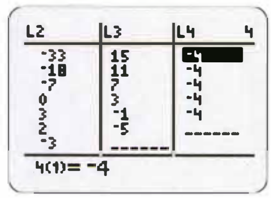
- Press STAT then select **1:Edit…**.
- Enter data into **L1**.
- Enter data into **L2**.

To find the first differences:



- Press ► and cursor over to **L3**.
- Press ▲ and cursor up to the title for **L3**.
- Press 2nd STAT for [LIST]. Cursor over to the **OPS** menu. Select **7:Δ List(** to paste the command in the title for **L3**.
- Press 2nd 2) ENTER to compute the first differences of **L2**.

To find the second differences:



- Press ► and cursor over to **L4**.
- Press ▲ and cursor up to the title for **L4**.
- Press 2nd STAT for [LIST]. Cursor over to the **OPS** menu. Select **7:Δ List(** to paste the command in the title for **L4**.
- Press 2nd 3) ENTER to compute the second differences of **L2** (first differences of **L3**).

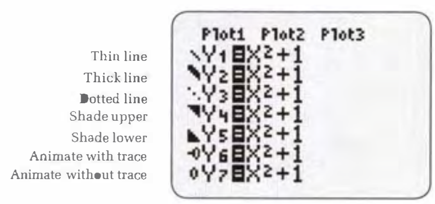
Turning Off All Plots

To turn off all the plots without resetting the RAM:

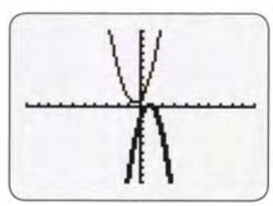
- Press 2nd Y= and select **4:PlotsOff**.
- Press ENTER.

Changing the Appearance of a Line

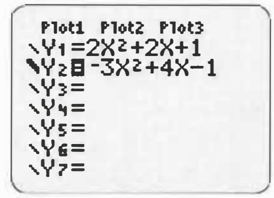
The default style is a thin solid line. The line style is displayed to the left of the equation.
There are seven options for the appearance of a line.



- Press Y= and clear any previously entered equations.
  Enter the relation $2x^2 + 2x + 1$ for **Y1**.
  Enter the relation $-3x^2 + 4x - 1$ for **Y2**.
- Press ◄ until the cursor is to the left of **Y2 =**.
- Press ENTER repeatedly until the thick solid line shows.
- Press GRAPH.

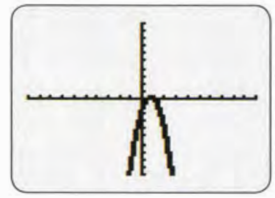


Turning Off an Equation



In the equation editor, move the cursor over the equal sign of the equation of the graph you do not want to display.

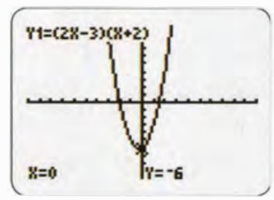
- Press (ENTER) to turn off the graph for that equation.



- Press (GRAPH).

Note: Any equation without a highlighted equal sign is turned off and will not be plotted.

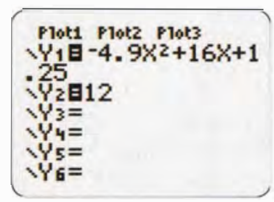
Finding a *y*-Intercept



To find the y-intercept of a function:
Enter $(2x \quad 3)(x + 2)$ for **Y1**.

- Press (GRAPH).
- Press (TRACE) (ENTER).

Finding an Intersection Point



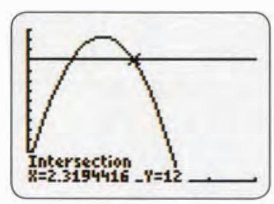
There must be at least two equations in the calculator's Equation Editor.

- Press (Y=) and enter the relations for **Y1** and **Y2**.

Be sure to use the appropriate window settings.

- Press (GRAPH).

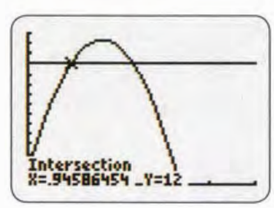
Note: An intersection point *must* be visible in order to find its coordinates. If an intersection point is not visible, adjust the window settings accordingly.



To find an intersection point:
(TRACE) lect **5:Intersect.**

- Press (ENTER) (ENTER) (ENTER).

The coordinates of the intersection point will appear at the bottom of the screen.



To find the other intersection point:

- Press (2nd) (TRACE) and select **5:Intersect.**
- Press and hold the (◄) or (►) key to move closer to the other intersection point.
- Press (ENTER) (ENTER) (ENTER).

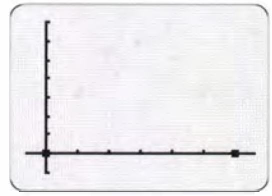
Creating a Scatter Plot

To create a scatter plot:

- Enter the two sets of data in lists **L1** and **L2**.
- Press (2nd) (Y=) for [STATPLOT].
- Press 1 (ENTER) to select **1:Plot1...**.
- Press (ENTER) to select **On**.
- Cursor down then press (ENTER) to select the top left graphing option, a scatter plot.
- Cursor down to **Xlist** and press (2nd) 1 for [L1].
- Cursor down to **Ylist** and press (2nd) 2 for [L2].

(ENTER).

- Press (2nd) (MODE) for [QUIT] to exit the Stat Plots editor after you have entered the data.



To display the scatter plot:

- Press (Y=) and use the (CLEAR) key to remove any graphed equations.
- Press (2nd) (MODE) for [QUIT].
- Press (ZOOM) select **9:ZoomStat** to display the scatter plot for the data in the .
- Press (WINDOW) to change **Xscl** and **Yscl** appropriately to place tick marks on both axes if they are not visible.

Turning On the Diagnostic Mode

To turn on the Diagnostic mode:

- Press (2nd) 0 for [CATALOG].
- Press (x^{-1}) for the Ds in the alphabetic list of commands.
- Scroll down to **DiagnosticOn**.

(ENTER)

(ENTER)

Note: If the diagnostic mode is turned on, you will see the values for r and r^2 for a linear regression and the value of R^2 for a quadratic regression.

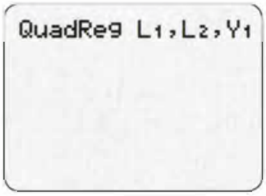
Turning Off the Diagnostic Mode

To turn off the Diagnostic mode:

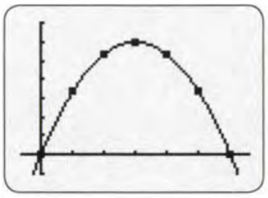
- Press (2nd) 0 for [CATALOG].
- Press (x^{-1}) for the Ds in the alphabetic list of commands.
- Scroll down to **DiagnosticOff**.
- Press (ENTER) to paste the command to the home screen of the calculator.
- Press (ENTER) to turn off the Diagnostic mode.

Finding a Curve or a Line of Best Fit

You can add a curve of best fit to data in the lists by using an appropriate model.



- With the scatter plot displayed, press STAT ▶ for the **CALC** menu.
- Select the model that would best represent the data displayed in the scatter plot:
  4:LinReg(ax+b) for a linear regression
  5:QuadReg for a quadratic regression
  0:ExpReg for an exponential regression
  C:SinReg for a sinusoidal regression
- Press 2nd 1 for **L1** and press , .
- Press 2nd 2 for **L2** and press , .
- Press VARS ▶ for the **Y-VARS** menu. Select **1:Function...**, and then select .
- Press ENTER to perform the regression analysis.
- Press GRAPH to view the model.

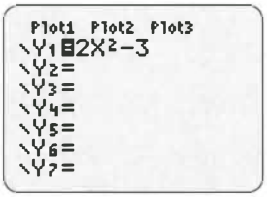


Finding a Zero or a Maximum/Minimum Value

There must be at least one equation in the Equation Editor.

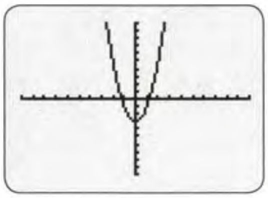
- Press Y= .

The example shows a parabola.



To calculate a zero (*x*-intercept), maximum, or minimum there *must* be one visible.

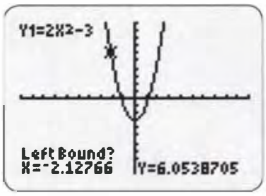
- Press GRAPH.



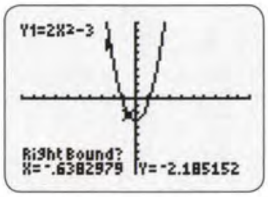
Note: If a zero (*x*-intercept), maximum, or minimum is not visible, adjust the window settings accordingly.

To find a zero:

- Press 2nd TRACE 2.
- Move to the left side of a zero (*x*-intercept) by pressing and holding the left cursor key.
- Press ENTER.



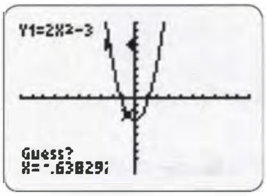
- Move to the right side of a zero (*x*-intercept) by pressing and holding the right cursor key.
- Press ENTER.

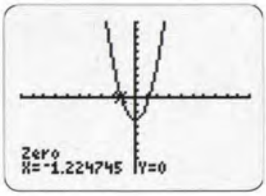


To find the value of a zero (*x*-intercept) using the calculator's guess feature:

- Press ENTER.

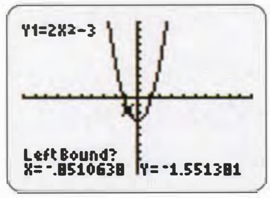
A zero (*x*-intercept) is shown at the bottom of the screen.

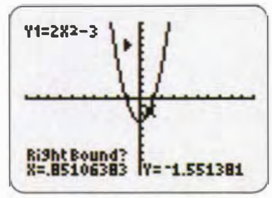




To find a minimum:

- Press 2nd TRACE 3.
- Move to the left side of a minimum by pressing and holding the left cursor key.
- Press ENTER.



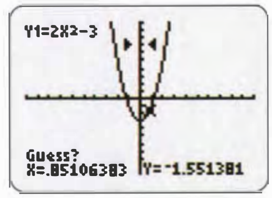


- Move to the right side of a minimum by pressing and holding the right cursor key.
- Press ENTER.

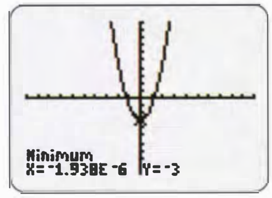
To find the value of a minimum using the calculator's guess feature:

- Press ENTER.

A minimum value is shown at the bottom of the screen.



Sometimes the values of x and/or y are not exact because of the method used by the calculator to determine the values. The value for x, shown, is −1.938E−6. This means the number is -1.938×10^{-6} in scientific notation. Moving the decimal six places to the left will give the number in standard form (−0.000 001 938). In this case, assume that the x-value of the minimum is 0 rather than −0.000 001 938.



To find a maximum:

- Press 2nd TRACE 4. Follow a similar procedure as above.

Finding the Sum of an Arithmetic or Geometric Series

Start by listing the first ten terms and storing them **L1**.

- Press 2nd STAT for [LIST] and cursor over to the **OPS** menu.

For an arithmetic series, such as one with $a = 2$, $d = 4$, to find S_{10}:

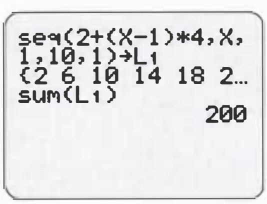
- Select **5:seq(** and enter $2 + (x - 1) \times 4$, x, 1, 10, 1).

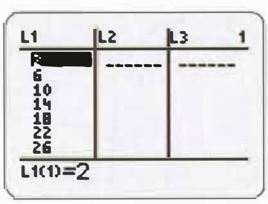
For a geometric series, such as one with $a = 2$, $r = -3$, to find S_{10}:

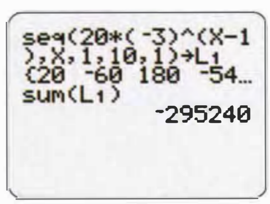
- Select **5:seq(** and enter $20 \times (-3)^{x-1}$, x, 1, 10, 1).
- Press STO▶, 2nd 1 for [L1], ENTER.

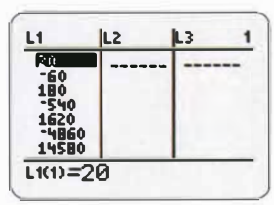
Now determine the sum.

- Press 2nd STAT [LIST] and cursor over to the **MATH** menu.
- Select **5:sum(** and enter **L1**).
- Press ENTER.



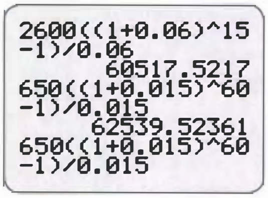


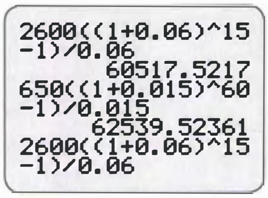




Repeating Calculations for Other Scenarios

- Press 2nd ENTER for [ENTRY] to recall the previous calculation.
  Use the cursor keys, the DEL key, and the [INS] command to modify the equation to fit each scenario.
- Press ENTER to perform the new calculation.
  In the example shown, press 2nd ENTER for [ENTRY] twice to see the first calculation again, and then alter the amount, 2600.





About the Finance Applications: The TVM Solver

The **TVM Solver** is used to work with annuities (for example, loans and investments with regular payments, and mortgages) and can also be used for non-annuities (for example, loans or investments with no regular payments). **TVM** stands for **T**ime **V**alue of **M**oney.

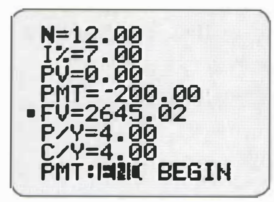
To open the TVM Solver:

- On the TI-83 Plus/TI-84 Plus, press APPS 1 and 1.

What the TVM Solver Variables Represent

When There Are Regular Payments (Ordinary Annuities and Mortgages)

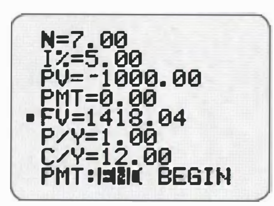
N	Number of Payments
I%	Annual Interest Rate
PV	Present Value
PMT	Payment
FV	Future Value
P/Y	Number of Payments/Year
C/Y	Number of Compounding Periods/Year
PMT: END BEGIN	Payments at End of Payment Interval



A savings annuity invested at 7%, compounded quarterly, with quarterly deposits of $200, for 3 years has a future value of $2645.02.

When There Are No Regular Payments

N	Number of Years
I%	Annual Interest Rate
PV	Present Value, or Principal
PMT	Always set **PMT=0.00**.
FV	Future Value, or Final Amount
P/Y	Always set **P/Y=1.00**.
C/Y	Number of Compounding Periods/Year
PMT: END BEGIN	END or BEGIN



$1000 invested at 5%, compounded monthly, for 7 years has a future value of $1418.04.

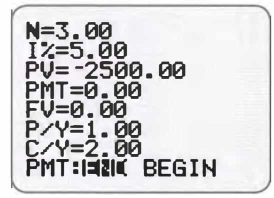
Investments and Loans (No Regular Payments)

Final Amount If you know the principal, or present value, interest rate, compounding frequency, and term of an investment or loan, you can determine its final amount.

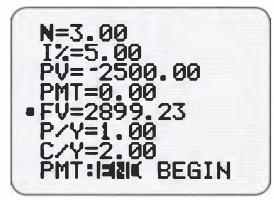
For example, to determine the final amount of a **$2500** investment earning 5% interest, **compounded semi-annually**, for **3 years**, follow these steps:

Open the **TVM Solver** and enter the values as shown:

To solve for **FV**, cursor to **FV=0.00** and press ALPHA ENTER.



Term is 3 years.
Annual interest rate is 5%.
Principal is $2500.
Final amount is unknown.
2 compounding periods/year



The final amount is $2899.23.

Present Value, or Principal If you know the final amount, interest rate, compounding frequency, and term of an investment or loan, you can determine its present value, or principal.

Open the **TVM Solver** and enter the known values.
To solve for **PV**, cursor to **PV=0.00** and press ALPHA ENTER.

Interest Rate To find the annual interest rate, enter the known values for **N**, **PV**, **FV**, and **C/Y**. Set **I%=0.00**, **PMT=0.00**, and **P/Y=1.00**. Then, cursor to **I%=0.00** and press ALPHA ENTER.

Term To find the term, in years, enter the known values for **I%**, **PV**, **FV**, and **C/Y**. Set **N=0.00**, **PMT=0.00**, and **P/Y=1.00**. Then, cursor to **N=0.00** and press ALPHA ENTER.

Important Points About the TVM Solver

- Set the number of decimal places to 2.
- A value must be entered for each variable.
- Money paid out (cash outflow), such as a loan payment, is negative.
- Money received (cash inflow), such as the final amount of an investment, is positive.
- To quit the **TVM Solver** and return to the Home Screen, press 2nd MODE.

Ordinary Annuities (Regular Payments)

Future Value Enter the known values for **N**, **I%**, **PMT**, **P/Y**, and **C/Y** and set **PV = 0.00**, **FV = 0.00**, and **PMT:END**. Then, cursor to **FV = 0.00** and press ALPHA ENTER.

Present Value Enter the know values for **N**, **I%**, **PMT**, **P/Y**, and **C/Y** and set **PV = 0.00**, **FV = 0.00**, and **PMT:END**. Then, cursor to **PV = 0.00** and press ALPHA ENTER.

Payment To find the payment given the present value or the future value, enter the known values for **N**, **I%**, **PV** or **FV**, **P/Y**, and **C/Y** and set **PV = 0.00** or **FV = 0.00**, and **PMT:END**. Then, cursor to **PMT = 0.00** and press ALPHA ENTER.