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KU: 13 / 16 TH: 16 / ~~10~~ 12 COMM: 5+3 / 6+3 APPS: 13 /13 Date: March 11 2015

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UNIT 2 TEST: Derivatives
Beware of Careless DERIVERS

Instructions:
1. Read the instructions carefully.
2. Attempt all problems and don't spend all your time on one problem. Other problems will feel left out ☹
3. Show all work in the space provided. Three (3) marks will be given to the overall mathematical form on this test.
4. ENJOY ☺

PART A: KNOWLEDGE AND UNDERSTANDING

Multiple Choice – Circle the BEST answer. [6]

1. If $y = -\frac{1}{5x}$, then $\frac{dy}{dx} =$

   a. $\frac{1}{25x^2}$ b. $-\frac{1}{5}$ c. $\frac{5}{x^2}$ d. $-\frac{1}{5x^2}$ e. $\frac{1}{5x^2}$

2. If $f(x) = \sqrt{x}$, then $f'(a)$ equals:

   a. $\lim_{h\to\infty} \frac{\sqrt{a+h}-\sqrt{a}}{h}$ b. $\lim_{h\to\infty} \frac{\sqrt{a+h}-a}{h}$ c. $\lim_{h\to 0} \frac{\sqrt{a+h}-\sqrt{h}}{h}$ d. $\lim_{h\to 0} \frac{\sqrt{a+h}-\sqrt{a}}{h}$ e. $\lim_{h\to 0} \frac{\sqrt{a+h}-\sqrt{a}}{a}$

3. Simplify $\frac{d(3x-7)^2}{d(3x-7)}$

   a. 3 b. 9 c. $3x-7$ d. $2(3x-7)$ e. $6(3x-7)$

4. Find $\frac{dy}{dx}$ if $y = (4x+5)^3$

   a. $(4x+5)^2$ b. $3(4x+5)^2$ c. $12(4x+5)^2$ d. $12(4x+5)^3$ e. none of these

5. If $h(x) = (9x-1)(2x+9)$, find $h'(x)$

   a. $18x-9$ b. 36 c. $36x$ d. $36x+79$ e. $36x-79$

6. Find the equation of the tangent of $y = x^2+1$ at the point where $x = 3$.

   a. $y = 2x$ b. $y = 2x+1$ c. $y = 6x+1$ d. $y = 6x+8$ e. $y = 6x-8$

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Complete solutions must be shown for full marks.

7. Find $\frac{dy}{dx}$ for each of the following and leave **UNSIMPLIFIED**.

[2]

a) $y = \frac{-2x^6+x^4+1}{x^5}$

$= (-2x^6+x^4+1)(x^{-5})$

$\frac{dy}{dx} = (-2x^6+x^4+1)(-5x^{-6}) + (x^{-5})(-12x^5+4x^3)$

2/2.

[3]

b) $y = (x^8)\sqrt{7x^2+1}$

$= (x^8)(7x^2+1)^{\frac{1}{2}}$

$\frac{dy}{dx} = (x^8)(\frac{1}{2})(7x^2+1)^{-\frac{1}{2}}(14x) + (7x^2+1)^{\frac{1}{2}}(8x^7)$

3/3

8. Find $\frac{dy}{dx}$ and leave answer in **SIMPLIFIED FULLY FACTORED** form.

[5]

$$y = \left(\frac{x^2+4}{3x-2}\right)^5$$

$y = \left(\frac{x^2+4}{3x-2}\right)^5$

$\frac{dy}{dx} = 5\left(\frac{x^2+4}{3x-2}\right)^4 \cdot \frac{d}{dx}\left(\frac{x^2+4}{3x-2}\right)$

$= 5\left(\frac{x^2+4}{3x-2}\right)^4 \cdot \left(\frac{d}{dx}(x^2+4)(3x-2)^{-1}\right)$ what happened?

$= 5\left(\frac{x^2+4}{3x-2}\right)^4 \cdot \left[(x^2+4)(-1)(3x-2)^{-2}(3) + (3x-2)^{-1}(2x)\right]$

$= 5\left(\frac{x^2+4}{3x-2}\right)^4 \cdot \left[\frac{(x^2+4)(-1)}{(3x-2)^2} + \frac{2x}{(3x-2)} \cdot \frac{3x-2}{3x-2}\right]$

ok $= 5\left(\frac{x^2+4}{3x-2}\right)^4 \cdot \left[\frac{-(x^2+4)}{(3x-2)^2} + \frac{2x(3x-2)}{(3x-2)^2}\right]$

$= 5\left(\frac{x^2+4}{3x-2}\right)^4 \cdot \frac{-x^2-4+6x^2-4x}{(3x-2)^2}$

ok

$= \frac{5(x^2+4)^4}{(3x-2)^4} \cdot \frac{5x^2-4x-4}{(3x-2)^2}$

ok $\boxed{= \frac{(5)(x^2+4)^4(5x^2-4x-4)}{(3x-2)^6}}$ X -1

4/5

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PART B: APPLICATIONS

9. Find the slope of the tangent to the graph of $y = \frac{5x-6}{\sqrt{x+1}}$ at $x = 0$. [3]

$y = (5x-6)(x+1)^{-\frac{1}{2}}$

$y' = (5x-6)(-\frac{1}{2})(x+1)^{-\frac{3}{2}} + (x+1)^{-\frac{1}{2}}(5)$

$m_{tan} = (5(0)-6)(-\frac{1}{2})(0+1)^{-\frac{3}{2}} + (0+1)^{-\frac{1}{2}}(5)$

$= (-6)(-\frac{1}{2})(1)^{-\frac{3}{2}} + (1)^{-\frac{1}{2}}(5)$

$= (3)(1) + (1)(5)$

$= 3+5$

$= 8$

Therefore, the slope at $x = 0$ is 8

10. After t hours, a side of beef that has been put in a freezer has a temperature (°C) given by: [3]

$T(t) = 15 - 3t + \frac{4}{t+1}$ where $0 \le t \le 5$.

Find the rate of change of temperature at the one hour mark.

$T(t) = 15 - 3t + 4(t+1)^{-1}$

$T'(t) = -3 + (-4)(t+1)^{-2}$

$= -3 - 4(t+1)^{-2}$

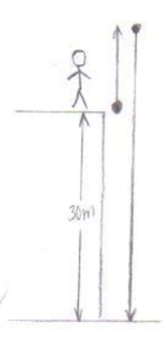
$T'(1) = -3 - 4(1+1)^{-2}$

$= -3 - 4(0.25)$

$= -4$ °c / hour

Therefore, the rate of change at the one hour mark is -4°c / hour

11. A pebble is thrown vertically upwards from the top of a cliff of height 30m. The pebble's height h (in meters) in relation to the top of the cliff t seconds later is $h(t) = -5t^2 + 25t$. Find the pebble's velocity on impact at the base of the cliff. [3]



Let down be (-)

$h(t) = -5t^2 + 25t$

$-30 = -5t^2 + 25t$

$= -5t^2 + 25t + 30$

$= 5(t+1)(t-6)$

$\cancel{t = -1s}$ $\quad t = 6s$

hits ground at 6s

$h'(t) = -10t + 25$

$h'(6) = -10(6) + 25$

$= -60 + 25$

$= -35$ m/s

$= 35$ m/s [down]

Therefore, the pebble's final velocity is 35 m/s [down]

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12. The cost of producing x fruit tarts is $C(x) = -0.0005x^2 + 1.5x + 300$.

a) Determine the marginal cost of producing 200 fruit tarts. [2]

$C'(x) = -0.001x + 1.5$

$C'(200) = -0.001(200) + 1.5$

$\$1.3$ /bar

∴ it ~~will~~ cost \$1.30 ~~per~~ for next bar ~~when~~ after you produce 200 bars

b) Determine the actual cost of producing the 201st fruit tart. [2]

$C(200) = 580$

$C(201) = 581.2995$

$581.2995 - 580$

$= \$1.2995$

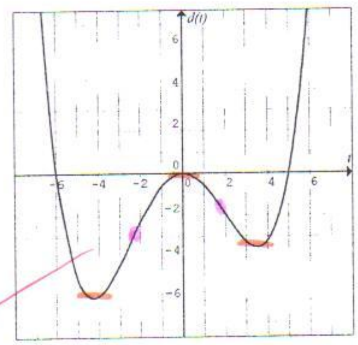
∴ the actual cost of producing the 201st bar is \$1.2995

4/4.

PART C: COMMUNICATION

13. Given the following position-time graph, [2] [2] [2]

i) Explain how can we use it to help sketch its corresponding velocity-time graph.

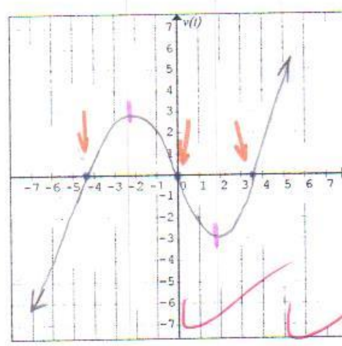


- where tangent is Ø on the d-t graph, that point will be zero on the v-t graph. where x value is, y-value will be zero.
- where the curve on d-t graph changes (where the middle of the curve), it will be where it changes direction on v-t graph
- Depending on how the rate of change on the d-t graph, we graph the slope of the curve on the v-t graph

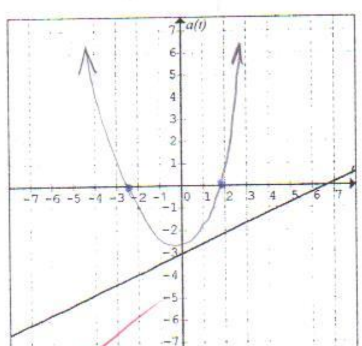
when will v(t) be positive/negative? -1

5/6

ii) Sketch its corresponding velocity-time graph



iii) Give two reasons explaining why the given acceleration-time graph is not the correct.



- We follow same rules as 13 i) because you are just finding derivative of graph above, then graphing it.
- v-t is a cubic function. a-t is always supposed to be one degree less to its suppose to be a quadratic
- x-int on a-t graph's supposed to be where v-t graph has tangent of zero.

/4+5

PART D: THINKING

14. Choose to do ONE of the following two questions ONLY. Circle the one that you want to be marked. [6]

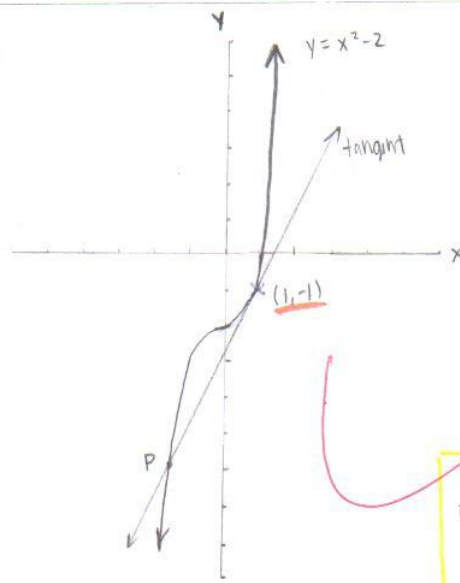
a) The tangent to the curve $y = x^3 - 2$ at the point $(1, -1)$ meets the curve at a second point P.
   i) Find the coordinates of point P.
   ii) Clearly show algebraically why there are no other points of intersection.
   Include a sketch

= OR =

b) The function of $f'(x)$ is given below. If $f(x)$ is a continuous function and it has a point at $(10, -31)$. **Accurately graph** the function $f(x)$ on the grid given. Show all your work.

$$f'(x) = \begin{cases} -2x - 2, & \text{for } x \le 1 \\ -4, & \text{for } x > 1 \end{cases}$$

a)



$y' = 3x^2 \rightarrow 3(1)^2 = 3$

$y = mx + b$

$-1 = 3(1) + b$

$-1 = 3 + b$

$b = -4$

equation of tan line

$y = 3x - 4$

Find POI of

$y = 3x - 4$ ①

$y = x^3 - 2$ ②

sub ① into ②

$3x - 4 = x^3 - 2$

$0 = x^3 - 3x + 2$

$= (x+2)(x^2 - 2x + 1)$

$= (x+2)(x-1)(x-1)$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -3 & 2 \\ - & \downarrow & 2 & -4 & 2 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$\therefore (x+2)$ is a factor

$x = -2$ $\quad$ $x = 1$ $\quad$ $x = 1$

sub $x = -2$ into: $y = x^3 - 2$

$= (-2)^3 - 2$

$= -8 - 2$

$= -10 \rightarrow (-2, -10)$

point where tangent touches cubic function, not intersect. $(1, -1)$

Therefore, point P is $(-2, -10)$, where it intersects the cubic function

$f'(x) = \begin{cases} -2x - 2, & x \le 1 \\ -4, & x > 1 \end{cases}$

$f(x) = \begin{cases} -x^2 - 2x + m, & x \le 1 \\ -4x + n, & x > 1 \end{cases}$

sub $(10, -31)$

$-4x + n = y$

$-4(10) + n = -31$

$-40 + n = -31$

$n = +9$

continuous means connects at $x = 1$ from both pieces

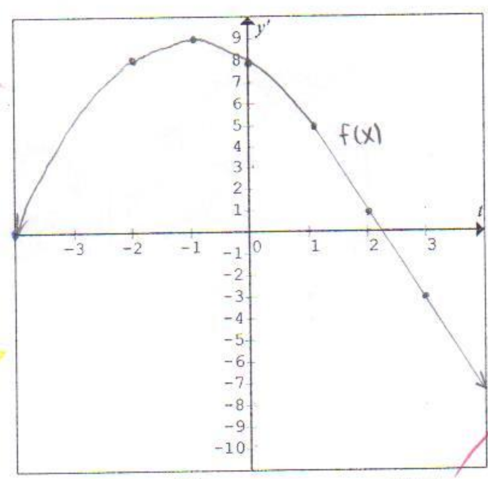
sub $x = 1$ into $f(x)$

$-x^2 - 2x + m = -4x + 9$

$-(1)^2 - 2(1) + m = -4(1) + 9$

$-1 - 2 + m = 5$

$m = 8$



$\therefore f(x) = \begin{cases} -x^2 - 2x + 8 & x \le 1 \\ -4x + 9 & , x > 1 \end{cases}$

x-int

$0 = -x^2 - 2x + 8$

$= (x+4)(x-2)$

$x = -4$ $\quad$ $x = 2$

yint

$y = -0^2 - 2(0) + 8$

$= 8$

$\frac{-4+2}{2}$

$= -1$

vertex

$(-1, -(-1)^2 - 2(-1) + 8)$

$(-1, 9)$

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15. Derive the proof of the PRODUCT RULE using first principles given $F(x) = f(x) \times g(x)$ [4]

Let $F(x)$ be replaced by $p(x)$

$$p(x) = f(x) \cdot g(x)$$

$$p'(x) = \lim_{h \to 0} \left[\frac{p(x+h) - p(x)}{h} \right]$$

$$= \lim_{h \to 0} \left[\frac{f(x+h)g(x+h) - f(x)g(x)}{h} \right]$$

$$= \lim_{h \to 0} \left[\frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h} \right]$$

$$= \lim_{h \to 0} \left[f(x+h) \frac{g(x+h) - g(x)}{h} + g(x) \frac{f(x+h) - f(x)}{h} \right]$$

$$= \lim_{h \to 0} f(x+h) \cdot \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} + \lim_{h \to 0} g(x) \cdot \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\boxed{= f(x) \cdot g'(x) + g(x) \cdot f'(x)}$$

4/4

☺ The end ☺

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