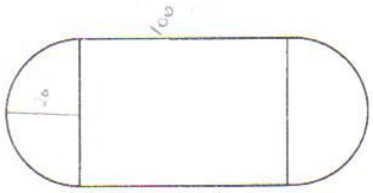
THINK: 3/7 APP: 11/14 Name: Uni Lee

67% **OPTIMIZATION: In Class Performance Task**

There are 3 parts to this performance task. Only complete ONE (1) question from each part. Please complete all your work on a separate piece of paper.

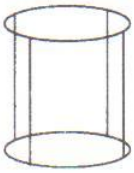
Part A: Area Question (Application) – Choose to complete 1 of the following 2 questions. [7 marks]

1) A 400m track has the shape of two semi-circles at the ends of a rectangle. The straight sections of the track must be at least 100m in length, and the radius of the semi-circles must be at least 20m. Find the dimensions of the tract that encloses the maximum area.



2) The perimeter of an isosceles triangle is 36cm. Find the lengths of the sides of the triangle of maximum area.

Part B: Volume Question (Application) – Choose to complete 1 of the following 2 questions. [7 marks]

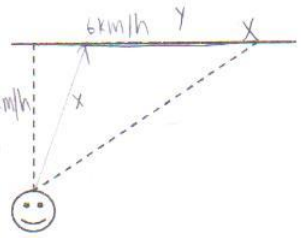


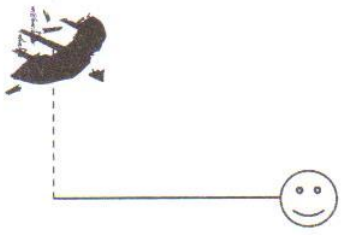
3) A piece of wire 180cm long is cut into 6 sections, 2 of one length and 4 of another length. Each of the two sections have the same length is bent into the form of a circle, and the two circles are then joined by the four remaining sections to make a frame for a model of a right cylinder. Find the lengths that will maximize the volume of the cylinder.

4) A fancy cylindrical drinking cup is to be made with two different materials. The bottom is to be made of crystal and the rest is to be made with silver. The volume of the cup is to be 100π cm^3. The radius and the height are to be no smaller than 2cm. If crystal costs 3 times more than silver, determine the dimensions that will produce a cup of least cost.

Part C: Thinking – Choose to complete 1 of the following 2 questions. [7 marks]

5) Sailor Sara is in a row boat 8km off a straight shoreline. She wants to reach a point on shore that is 10km from the point directly opposite her present position. If Sara can row at 4km/h and run at 6km/h, where should she land on shore to minimize her time?





6) A pair of scuba divers wishes to dive on a wreck that lies 0.4km off the shore from a point 0.8km from their present position. If they can walk carrying their gear at 5km/h and swim at 3km/h, what course should they follow to reach the wreck in minimum time?

question (Application)

x > 10cm
y > 40m

$A_r = xy$

$A_c = \pi(\frac{y}{2})^2$

$A_c = \frac{\pi y^2}{4}$

$A_{TL} = xy + \frac{\pi y^2}{4}$

$= \frac{400y - \pi y^2}{2} - \frac{\pi y^2}{4}$ ✓

$= \frac{800y - 2\pi y^2}{4} - \frac{\pi y^2}{4}$

$= \frac{800y - 3\pi y^2}{4}$

$P = 2\pi(\frac{y}{2}) + 2x$

$400 = \pi y + 2x$ ✓

~~$\frac{400}{2} = \frac{\pi y}{2} + x$~~

$\frac{400 - \pi y}{2} = x$

$A_{TL} = \frac{800y}{4} - \frac{3\pi y^2}{4}$

$A_{TL} + \frac{3\pi y^2}{4} = \frac{800y}{4}$

$4A_{TL} + 3\pi y^2 = 800y$

$0 = 3\pi y^2 - 800y$ X

$A_{TL}' = 6\pi y - 800$

$800 = 6\pi y$

$\frac{800}{6\pi} = y$ ✓ ok...

$\frac{400}{3\pi} m = y$

can't multiply everything by 4 because you've just multiplied the area by 4

$x = \frac{400 - \pi y}{2}$

$= \frac{400 - \pi(\frac{400}{3\pi})}{2}$

$= \frac{400 - \frac{400}{3}}{2}$

$= \frac{800}{3} \div 2$ ✓ ok...

$x = \frac{400}{3}$ m

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Therefore, the dimensions of the rectangle will be $\frac{400}{3\pi}$ m and $\frac{400}{3}$ m

check endpoints !!

Uni

...ne Question (Application)

$180\text{ cm} = 4\pi r + 4h$

$180 - 4\pi r = 4h$

$h = \frac{180 - 4\pi r}{4}$

$h = 45 - \pi r$

$V = \pi r^2 h$

$= \pi r^2 (45 - \pi r)$

$= 45\pi r^2 - \pi^2 r^3$

$= 45(3.14159)r^2 - (3.14159)^2 r^3$

$= 141.37155 r^2 - 9.869587 r^3$

you can keep "π" as "π" no decimals

$V' = 282.7431 r - 29.608761 r^2$

$= r(282.7431 - 29.608761 r)$

$r = 9.549305\text{ cm}$

$L = 60 + 60 + 4(15)$

$= 180$

Wire 1 (x2)

$C = 2\pi r$

$= 2\pi (9.549305)$

$= 60.00055\text{ cm}$

$C \doteq 60\text{ cm}$

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Wire 2 (x4)

$h = 45 - \pi r$

$= 45 - \pi (9.549305)$

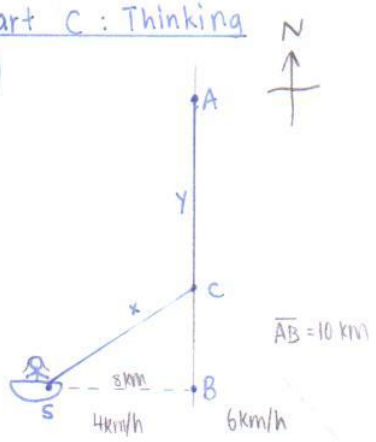
$= 14.999973357\text{ cm}$

$h \doteq 15\text{cm}$

Therefore, the length of the 2 type wire is 60 cm and 4 type is 15 cm

Part C : Thinking

5)



$8^2 + (10-y)^2 = x^2$

$x = \sqrt{64 + (100 - 20y + y^2)}$

$x = \sqrt{164 - 20y + y^2}$

$t = 4x + 6y$

$= 4(\sqrt{164 - 20y + y^2}) + 6y$ ← can't set $t = 0$!! need to take derivative then set to zero.

$[-6y]^2 = [4\sqrt{164 - 20y + y^2}]^2$

$36y^2 = 16(164 - 20y + y^2)$

$2.25y^2 = 164 - 20y + y^2$

$0 = -1.25y^2 - 20y + 164$

$= (y - 5.971)(y + 21.97)$

$y = 5.971$ $\quad$ $y = -21.97$ rejected. can't go backwards.

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Therefore, she should land about 5.971 km south of point A, along the shore

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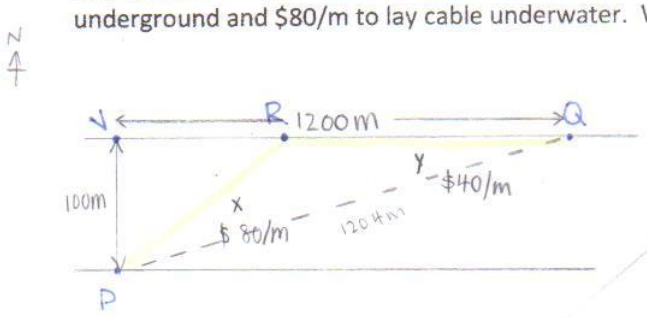
Name: Uni Lee

Optimization PT Part 2

A cable television company is laying cable in an area with underground utilities. Two subdivisions are located on opposite sides of Willow Creek, which is 100 m wide. The company has to connect points P and Q with cable, where Q is on the north bank 1200 m east of P. It costs \$40/m to lay cable underground and \$80/m to lay cable underwater. What is the least expensive way to lay the cable?

\$ 96320
58000



$Y = 1200 - \overline{VR}$

$Y + \overline{VR} = 1200$

$\overline{VR} = 1200 - Y$

$\overline{VR}^2 + \overline{VP}^2 = X^2$

$\overline{VR}^2 + 10000 = X^2$

$X = \sqrt{\overline{VR}^2 + 10000}$

$= \sqrt{(1200-y)^2 + 10000}$

$C = 80x + 40y$ ✓

$= 80(\sqrt{(1200-y)^2+10000}) + 40y$ ✓

$= 80\sqrt{1440000 - 2400y + y^2 + 10000} + 40y$

you've just see $C = 0$!!

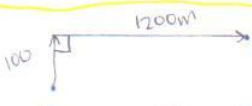
$-40y = 80\sqrt{y^2 - 2400y + 1450000}$ X

$1600y^2 = 6400(y^2 - 2400y + 1450000)$

$1600y^2 = 6400y^2 - 15360000y + 9280000000$

$0 = 4800y^2 - 15360000y + 9280000000$

1204m × \$80/m
= \$96320

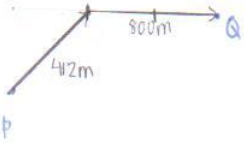


(directly across)

100m × \$80/m + 1200m × \$40/m

= \$8000 + 48000

= \$56000



412(80) + 800(40)

= \$64960

806(80) + 400(40)

= \$80480 X

$C' = 9600y - 15360000$

$0 = 9600y - 15360000$

$15360000 = 9600y$

$y = 4266.666...$ m X

← check end points

$y \doteq 4267$ m ← can't be max y distance is 1200m

There fore, it is least expensive if they lay 1200m underground away from Q, then diagonally underwater for the rest