Uni Lee

Name: Uni

KU: 14.5 / 25 TH: 4 / 10 COMM: 3 / 3 APPS: 14.5 / ~~17~~ 15 Date: May 1 2015

68%

UNIT 4b QUEST: Exponential and Logarithmic Functions

Instructions:

1. Read the instructions carefully.
2. Attempt all problems and don't spend all your time on one problem. Other problems will feel left out ☹
3. Show all work in the space provided. Three (3) marks will be given to the overall mathematical form on this test.
4. ENJOY ☺

PART A: KNOWLEDGE AND UNDERSTANDING

1. Find the **simplified** form of $\frac{dy}{dx}$ for each of the following and complete the chart. Show any work you do in the space provided. [15]

y	$\frac{dy}{dx}$
$y = e^{\pi}$	$\frac{dy}{dx} = 0$ ✓
$y = x^e$	$\frac{dy}{dx} = e\cdot\cancel{x}^e$ ✗ (−1)
$y = \ln(7^e \cdot e^{-7})$	$\frac{dy}{dx} = -\frac{7e^7}{7^e e^8}$ ✗ (−1)
$y = (e^x)^x$	$\frac{dy}{dx} = 2xe^x$ ✓ ✓
$y = \frac{x^7}{\ln 3}$	$\frac{dy}{dx} = \frac{(\ln 3)(7x^6)}{(\ln 3)^2}$ ✗ (−1)
$y = 2^{-5x}$	$\frac{dy}{dx} = 2^{-5x}(-5)(\ln 2)$ ✗ (−1)
$y = \log_5 x$	$\frac{dy}{dx} = \frac{\ln 5(\frac{1}{x})}{(\ln 5)^2}$ ✗ (−2)
$y = \ln\sqrt{1-x}$	$\frac{dy}{dx} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2}(1-x)^{-\frac{1}{2}} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{1-x}}$; $\frac{dy}{dx} = -\frac{1}{2(1-x)}$ ✓ ✓ (−1)
$y = xe^x$ $\ln y = x^2 \ln e$	$\frac{dy}{dx} = (xe^x)(2x)$ ✗ (−2)

$\frac{d}{dx}\ln y = \frac{d}{dx}x^2 \ln e$

$\frac{1}{y} = 2x(1)$

/6

2. Solve for x. Express answers as EXACT answers only. No decimals! [10]

a) $e^{2x} = 12$

$2x \ln e = \ln 12$

$x = \frac{\ln 12}{2}$

b) $\ln(e^{x^2}) = 16$

$\sqrt{x^2} = \sqrt{16}$

$x = 4$ OR $\boxed{x = -4}$

½.

8.5/10.

c) $\ln\left(\frac{x+1}{x}\right) = 1$

$\frac{x+1}{x} = e^1$

$1 + \frac{1}{x} = e$

$\frac{1}{x} = e - 1$

$1 = x(e-1)$

$\frac{1}{e-1} = x$

d) $e^x - 30e^{-x} = 1$

$(e^x)^2 - 30 = e^x$

$(e^x)^2 - e^x - 30 = \emptyset$

$(e^x+5)(e^x-6) =$

$e^x = -5$ $e^x = 6$

−1

continue to solve for x!! (i.e. ln Both sides)

PART B: APPLICATIONS

3. Given the function $y = \ln\left(\frac{x^2}{x^2+1}\right)$, find its derivative using 2 different ways. Express answer in simplified form. As well, give a brief explanation to each of your approach. [5]

Quotient Rule

$y = \ln\left(\frac{x^2}{x^2+1}\right)$

$y' = \frac{1}{\frac{x^2}{x^2+1}} \cdot \left(\frac{(x^2+1)(2x) - (x^2)(2x)}{(x^2+1)^2}\right)$

$= \frac{x^2+1}{x^2} \cdot \frac{\cancel{2x^3} + 2x - \cancel{2x^3}}{(x^2+1)^2}$

$= \frac{(x^2+1)2x}{x^2(x^2+1)^2}$

$= \frac{2x}{x^2(x^2+1)}$

$= \frac{2}{x(x^2+1)}$

log laws

$y = \ln\left(\frac{x^2}{x^2+1}\right)$

$= \ln x^2 - \ln(x^2+1)$

$= 2\ln x - \ln(x^2+1)$

$y' = 2\frac{1}{x} - \frac{1}{x^2+1}(2x)$

$= \frac{2}{x} - \frac{2x}{x^2+1}$

$= \frac{2(x^2+1) - 2x^2}{x(x^2+1)}$

$= \frac{\cancel{2x^2} + 2 - \cancel{2x^2}}{x(x^2+1)}$

$= \frac{2}{x(x^2+1)}$

5/5

8.5+5\

4. A fruit fly population is given by the equation $P(t) = \frac{240}{1+11e^{-0.4t}}$ where t is in days. [3]

a) What is the initial fruit fly population?

$P(\emptyset) = \frac{240}{1+11e^{-0.4(0)}}$

$= \frac{240}{1+11e^{0}}$

$= \frac{240}{2}$ 12

$= 120$ flies.

$\boxed{\text{there fore, initial population is 120 flies}}$

1/2

b) Find the instantaneous growth rate at $t = 5$ days. Round your answer to 2 decimal places.

$P(t) = \frac{240}{1+11e^{-0.4t}}$

$P'(t) = \frac{(1+11e^{-0.4t})(\emptyset) - (240)(\frac{d}{dx} 1+11e^{-0.4t})}{(1+11e^{-0.4t})^2}$

$= \frac{-240(11e^{-0.4t})(-0.4)}{(1+11e^{-0.4t})^2}$

$P(5) = \frac{-240(11e^{-0.4(5)})(-0.4)}{(1+11e^{-0.4(5)})^2}$

$= \frac{142.914591\ldots}{6.193568537\ldots}$

$= 23.0746766\ldots$

$\doteq 23.07$ flies/day

$\boxed{\text{Therefore, the instantaneous growth rate is about 23.07 flies/day}}$

2.5/3

5. The spread of a rumor in a certain school is modeled by the equation $P(t) = \frac{300}{1+2^{4-t}}$, where $P(t)$ is tht total number of students who have heard the rumor t days after the rumor first started to spread. [5]

a) Estimate the initial number of students who first heard the rumor.

$P(0) = \frac{300}{1+2^{4-0}}$

$= \frac{300}{1+2^4}$

$= 17.647\ldots$

$\doteq 17$

cant have 0.647 of a student.

round down because that student only heard 64.7%

$\boxed{\therefore \text{ about 17 students heard}}$

2/3

b) How fast is the rumor spreading after 4 days?

$P(t) = \frac{300}{1+2^{4-t}}$

$P'(t) = \frac{(1+2^{4-t})(\emptyset) - (300)(\frac{d}{dx} 1+2^{4-t})}{(1+2^{4-t})^2}$ $\ln 2$

$= \frac{-300(2^{4-t})(-1)}{(1+2^{4-t})^2}$ $\ln 2$

X

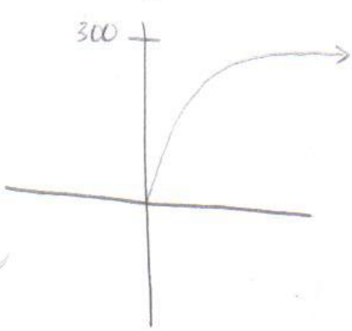
$P'(4) = \frac{300}{4}$ $\ln 2$

$= 75$ students/day

ok

$\boxed{\text{Therefore, the speed of spreading after 4 days is 75 students/day}}$

c) When will the rumor spread at jts maximum [redacted] What is that rate?

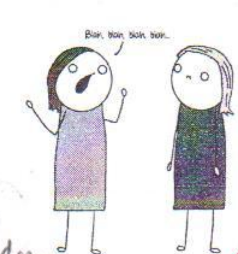


approaches 300.

reaches max students at ∞ days.

about 299 students on day 10

rate almost ∅ students/day



4.5

6. Kim visited the campus of Western University while sick with the Norwalk virus. The number T of days it took for the flu virus to infect x people is given by the equation: [4]

$$T = -0.93 \ln\left(\frac{7000 - x}{6999x}\right)$$

a) How many days did it take for 6000 people to become infected. Round answer to 1 decimal place.

$$T = -0.93 \ln\left(\frac{7000-x}{6999x}\right)$$

$$= -0.93 \ln\left(\frac{7000-6000}{6999(6000)}\right)$$

$$\doteq 9.9 \text{ days}$$

therefore, it took about 9.9 days

4/4

b) After 2 weeks, how many people were infected?

2weeks = 14 days

$$T = -0.93 \ln\left(\frac{7000-x}{6999(x)}\right)$$

$$14 = -0.93 \ln\left(\frac{7000-x}{6999x}\right)$$

$$-\frac{14}{0.93} = \ln\left(\frac{7000-x}{6999x}\right)$$

$$e^{\frac{-14}{0.93}} = \frac{7000-x}{6999x}$$

$$= \frac{7000}{6999x} - \frac{\cancel{x}}{6999\cancel{x}}$$

$$= \frac{7000}{6999x} - \frac{1}{6999}$$

$$e^{\frac{-14}{0.93}} + \frac{1}{6999} = \frac{7000}{6999x}$$

$$\frac{6999e^{\frac{-14}{0.93}}}{6999} + \frac{1}{6999} = \frac{7000}{6999x}$$

$$\frac{1+6999e^{\frac{-14}{0.93}}}{6999} = \frac{7000}{6999x}$$

$$\cancel{6999}x(1+6999e^{\frac{-14}{0.93}}) = 7000(\cancel{6999})$$

$$\frac{7000}{1+6999e^{\left(\frac{-14}{0.93}\right)}} = x$$

$$x = 6985.8261\ldots$$

$$\doteq 6985$$

person #6986 isn't fully infected yet.

∴ After two weeks, 6985 people were infected

PART C: THINKING

7. Find the 1000th derivative of $f(x) = e^{-3x}$. Explain briefly how you arrived at your answer. [3]

$f(x) = e^{-3x}$

first der $f'(x) = e^{-3x}(-3)$ one set of (-3)

2nd der $f''(x) = -3e^{-3x}(-3)$ two sets of (-3)

2/3

third der $f'''(x) = (-3)(-3)e^{-3x}(-3)$ three sets of (-3)

$\vdots$

$$\boxed{f^{1000}(x) = 1000(-3)e^{-3x} = -3000e^{-3x}}$$ one thousand sets of (-3)

$(-3)^{1000}$

X

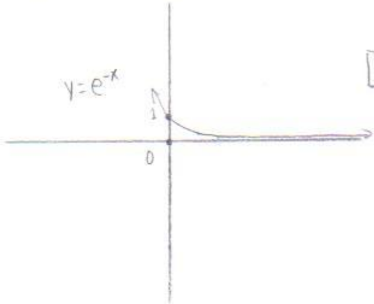
4+2\

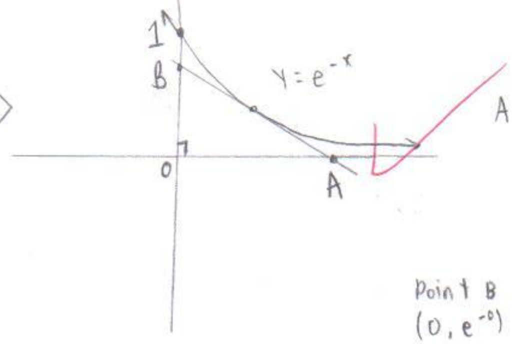
∴ the 1000th derivative of $f(x) = e^{-3x}$ is $f^{1000}(x) = -3000e^{-3x}$

X

Uni Lee

8. The tangent line to the curve $y = e^{-x}$ at the point (a, e^{-a}) where $a > 0$, intersects the x-axis at the point A and the y-axis at the point B. Find the expression for the area of ΔAOB. Express your answer as EXACT answer (no decimals!) [7]





$A = \frac{bh}{2}$

$= \frac{\overline{AO} \cdot \overline{BO}}{2}$

Point B
$(0, e^{-0})$
$= (0, 1)$

Point at $x = 1$
$(1, e^{-1})$
$= (1, 0.3678...)$

$y = e^{-x}$

$y' = e^{-x}(-1)$

$= -e^{-x}$

∴ passes throug (a, e^{-a})

$e^{-a} = (-a)(\ln e(a)) + b$?

$e^{-a} = -(a^2)(1) + b$

$e^{-a} + a^2 = b$ X

equation

$y = [(-x)$

☺ The end ☺

/2