

Find eigen value and eigen vector for following matrices

Q)

$$\begin{bmatrix} 85 & -28 & -28 \\ -10 & -11 & -11 \\ -46 & -2 & -2 \end{bmatrix}$$

A)

$$\text{Let } A = \begin{bmatrix} 85 & -28 & -28 \\ -10 & -11 & -11 \\ -46 & -2 & -2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{bmatrix} 85 - \lambda & -28 & -28 \\ -10 & -11 - \lambda & -11 \\ -46 & -2 & -2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 85 - \lambda & -28 & -28 \\ -10 & -11 - \lambda & -11 \\ -46 & -2 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 85 - \lambda & -28 & -28 \\ -10 & -11 - \lambda & -11 \\ -46 & -2 & -2 - \lambda \end{vmatrix} = 0$$

$$(85 - \lambda)((-11 - \lambda) - (-2 - \lambda)) - (-28)((-10)(-2 - \lambda) - (-11)(-46)) + (-28)[(-10)(-2) - (-11 - \lambda)(-46)]$$

$$(85-x)(-9) + 28(10x - 486) - 28(-46x - 126)$$

$$\Rightarrow x^3 - 72x^2 - 2673x = 0$$

$$\Rightarrow (\lambda)(\lambda+27)(\lambda-99) = 0$$

$$\Rightarrow \lambda = 0, 99, -27$$

(axel: $\lambda=0$

$$(A - \lambda I)x = 0$$

$$\begin{pmatrix} 85 & -28 & -28 \\ -10 & -11 & -11 \\ -46 & -2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$R_1 \rightarrow R_1 - 14R_3$$

$$\begin{pmatrix} 729 & 0 & 0 \\ -10 & -11 & -11 \\ -46 & -2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\Rightarrow 729x_1 = 0$$

$$-10x_1 - 11x_2 - 11x_3 = 0$$

$$-46x_1 - 2x_2 - 2x_3 = 0$$

$$\therefore \text{Eigen vector} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

Case 2: $\lambda = 99$

$$(A - \lambda I) X = 0$$

$$\left(\begin{array}{ccc} 85 - 99 & -28 & -28 \\ -10 & -11 - 99 & -4 \\ -46 & -2 & -2 - 99 \end{array} \right) \left(\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right) = 0$$

$$\left(\begin{array}{ccc} -14 & -28 & -28 \\ -10 & -110 & -11 \\ -46 & -2 & -101 \end{array} \right) \left(\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right) = 0$$

$$-14x_1 - 28x_2 - 28x_3 = 0$$

$$-10x_1 - 110x_2 - 11x_3 = 0$$

$$-46x_1 - 2x_2 - 101x_3 = 0$$

$$\frac{x_1}{-14x_1 - 28x_2 - 28x_3} = \frac{-x_2}{-10x_1 - 110x_2 - 11x_3} = \frac{x_3}{-46x_1 - 2x_2 - 101x_3}$$

$$\frac{x_1}{-14(-1) - (-28)(-10)} = \frac{-x_2}{-10(-1) - (-28)(-10)} = \frac{x_3}{-46(-1) - (-2)(-10)}$$

$$\frac{x_1}{-2772} = \frac{x_2}{126} = \frac{x_3}{1260}$$

$$\frac{x_1}{-22} = \frac{x_2}{1} = \frac{x_3}{10}$$

$$\therefore \text{Eigenvector} = \begin{bmatrix} -22 \\ 1 \\ 10 \end{bmatrix}$$

Case 3: $\lambda = -22$

$$(A - \lambda I) X = 0$$

$$\left(\begin{array}{ccc} 85 - (-22) & -28 & -28 \\ -10 & -11 - (-22) & -11 \\ -46 & -2 & -2 - (-22) \end{array} \right) \left(\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right) = 0$$

$$\begin{pmatrix} 112 & -28 & -28 \\ -10 & 16 & -4 \\ -46 & -2 & 25 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$112x_1 - 28x_2 - 28x_3 = 0$$

$$-10x_1 + 16x_2 - 4x_3 = 0$$

$$-46x_1 - 2x_2 + 25x_3 = 0$$

$$\frac{x_1}{(-28)(-1) - (-28)(16)} = \frac{-x_2}{(112)(-1) - (-28)(-10)} = \frac{x_3}{(4)(6) - (-28)(-1)}$$

$$\frac{x_1}{308 + 448} = \frac{x_2}{1232 + 280} = \frac{x_3}{1792 - 280}$$

$$\frac{x_1}{756} = \frac{x_2}{1512} = \frac{x_3}{1512}$$

$$\therefore x_1 = \frac{x_2}{2} = \frac{x_3}{2}$$

$$\therefore \text{Eigen vector} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

2) $\begin{bmatrix} 6 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 7 \end{bmatrix}$

$$\text{Let } A = \begin{bmatrix} 6 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 7 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\Rightarrow \left| \begin{bmatrix} 6 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 7 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 6-\lambda & 2 & -2 \\ 2 & 5-\lambda & 0 \\ -2 & 0 & 7-\lambda \end{bmatrix} \right| = 0$$

$$\Rightarrow (6-\lambda)(5-\lambda)(2-\lambda) - 2(2\lambda - \lambda^2)$$

$$-2(0 - (-2)(5-\lambda))$$

$$\Rightarrow (6-\lambda)(\lambda^2 - 12\lambda + 35) - 2(14-2\lambda)$$

$$-2(10-2\lambda)$$

$$\Rightarrow \lambda^3 - 18\lambda^2 + 99\lambda - 142 = 0$$

$$\Rightarrow (\lambda-3)(\lambda-6)(\lambda-9) = 0$$

$$\Rightarrow \lambda = 3, 6, 9$$

Case 1: $\lambda = 3$

$$(A - \lambda I)x = 0$$

$$\begin{pmatrix} 6-3 & 2 & -2 \\ 2 & 5-3 & 0 \\ -2 & 0 & 7-3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} 3 & 2 & -2 \\ 2 & 2 & 0 \\ -2 & 0 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\Rightarrow R_1 \rightarrow R_1 - R_3$$

$$\begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 0 \\ -2 & 0 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{aligned}
 & x_1 + 0x_2 - 2x_3 = 0 \\
 & 2x_1 + 2x_2 + 0x_3 = 0 \\
 & -2x_1 + 0x_2 + 4x_3 = 0 \\
 \Rightarrow \frac{x_1}{0 - (-2)x_2} &= \frac{-x_2}{0 - (-2)x_2} = \frac{x_3}{2 - 0} \\
 \Rightarrow \frac{x_1}{2} &= \frac{x_2}{-2} = \frac{x_3}{1} \\
 \therefore \text{Eigen vector} &= \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}
 \end{aligned}$$

Lage 2: $\lambda = 6$

$$(A - \lambda I)X = 0$$

$$\begin{pmatrix} 6-6 & 2 & -2 \\ 2 & 5-6 & 0 \\ -2 & 0 & 7-6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & 2 & -2 \\ 2 & -1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$0x_1 + 2x_2 - 2x_3 = 0$$

$$2x_1 - 2x_2 + 0x_3 = 0$$

$$-2x_1 + 0x_2 + x_3 = 0$$

$$\frac{x_1}{0-(-1)(-2)} = \frac{-x_2}{0-(-2)(2)} = \frac{x_3}{0-4}$$

$$\frac{x_1}{-2} = \frac{x_2}{-4} = \frac{x_3}{-4}$$

$$\Rightarrow \frac{x_1}{2} = \frac{x_2}{2} = \frac{x_3}{2}$$

\therefore Eigen vector = $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

$$\text{Case 3: } \lambda = 9$$

$$(A - \lambda I) X = 0$$

$$\begin{pmatrix} 6-9 & 2 & -2 \\ 2 & 5-9 & 0 \\ -2 & 0 & 7-9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} -3 & 2 & -2 \\ 2 & -4 & 0 \\ -2 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$-3x_1 + 2x_2 - 2x_3 = 0$$

$$2x_1 - 4x_2 + 0x_3 = 0$$

$$-2x_1 + 0x_2 - 2x_3 = 0$$

$$\frac{x_1}{0 - (-4)(-2)} = \frac{-x_2}{0 - (-2)(2)} = \frac{x_3}{(+3)(-4) - (2)(4)}$$

$$\frac{x_1}{2} = \frac{x_2}{(-1)} = \frac{x_3}{-2}$$

∴ Eigen vector = $\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

3)

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 6 & 4 & 2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\Rightarrow \left(\begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 6 & 4 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) = 0$$

$$\Rightarrow \begin{bmatrix} 1-\lambda & 0 & 0 \\ 2 & 4-\lambda & 0 \\ 6 & 4 & 2-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (1-\lambda)(4-\lambda)(2-\lambda) = 0$$

$$\Rightarrow \lambda = 1, 2, 4$$

(Case 1: $\lambda = 1$)

$$(A - \lambda I) X = 0$$

$$\begin{pmatrix} 1-1 & 0 & 0 \\ 2 & 4-1 & 0 \\ 6 & 4 & 2-1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 2 & 3 & 0 \\ 6 & 4 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$2x_1 + 3x_2 + 0x_3 = 0$$

$$6x_1 + 4x_2 + x_3 = 0$$

$$\frac{x_1}{3-0} = \frac{x_2}{2-0} = \frac{x_3}{8-18}$$

$$\frac{x_1}{3} = \frac{x_2}{2} = \frac{x_3}{-10}$$

\therefore eigen vector = $\begin{pmatrix} 3 \\ -2 \\ -10 \end{pmatrix}$

Case 2: $\lambda = 2$

$$(A - \lambda I)X = 0$$

$$\begin{pmatrix} -2 & 0 & 0 \\ 2 & 4-2 & 0 \\ 6 & 4 & 2-2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} -1 & 0 & 0 \\ 2 & 2 & 0 \\ 6 & 4 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$-x_1 + 0x_2 + 0x_3 = 0$$

$$2x_1 + 2x_2 + 0x_3 = 0$$

$$6x_1 + 4x_2 + 0x_3 = 0$$

$$\Rightarrow \frac{x_1}{0} = \frac{x_2}{0} = \frac{x_3}{-2}$$

\therefore Eigen vector = $\begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$

(case 3: $\lambda = 4$)

$$(A - \lambda I) X = 0$$

$$\begin{pmatrix} 1-4 & 0 & 0 \\ 2 & 4-4 & 0 \\ 6 & 4 & 2-4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} -3 & 0 & 0 \\ 2 & 0 & 0 \\ 6 & 4 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$-3x_1 + 0x_2 + 0x_3 = 0$$

$$2x_1 + 0x_2 + 0x_3 = 0$$

$$6x_1 + 4x_2 - 2x_3 = 0$$

$$\frac{x_1}{0} = \frac{-x_2}{-4} = \frac{x_3}{8}$$

$$\Rightarrow \frac{x_1}{0} = \frac{x_2}{4} = \frac{x_3}{2}$$

$$\therefore \text{Eigen vector} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$4) \begin{bmatrix} 0.5 & 0.2 & 0.1 \\ 0 & 1 & 1.5 \\ 0 & 0 & 3.5 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{pmatrix} 0.5 & 0.2 & 0.1 \\ 0 & 1 & 1.5 \\ 0 & 0 & 3.5 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right| = 0$$

$$\left| \begin{pmatrix} 0.5 - \lambda & 0.2 & 0.1 \\ 0 & 1 - \lambda & 1.5 \\ 0 & 0 & 3.5 - \lambda \end{pmatrix} \right| = 0$$

expand along G₁

$$(0.5 - \lambda)(- \lambda(3.5 - \lambda)) = 0$$

$$\Rightarrow \lambda = 0.5, 1, 3.5$$

(one): $\lambda = 0.5$

$$\cdot (A - \lambda I) X = 0$$

$$\left(\begin{pmatrix} 0.5 - 0.5 & 0.2 & 0.1 \\ 0 & 1 - 0.5 & 1.5 \\ 0 & 0 & 3.5 - 0.5 \end{pmatrix} \middle\| \begin{pmatrix} ? \\ ? \\ ? \end{pmatrix} \right) \xrightarrow{\text{Row operations}} \left(\begin{pmatrix} 0 & 0.2 & 0.1 \\ 0 & 0.5 & 1.5 \\ 0 & 0 & 3 \end{pmatrix} \middle\| \begin{pmatrix} ? \\ ? \\ ? \end{pmatrix} \right) \xrightarrow{\text{Row operations}} \left(\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \middle\| \begin{pmatrix} ? \\ ? \\ ? \end{pmatrix} \right)$$

$$0x_1 + 0.2x_2 + 0.1x_3 = 0$$

$$0x_1 + 0.5x_2 + 1.5x_3 = 0$$

$$0x_1 + 0x_2 + 3x_3 = 0$$

$$\frac{x_1}{(0.2)(1.5) - (0.5)(0.1)} = \frac{-x_2}{0} = \frac{x_3}{0}$$

$$\frac{x_1}{0.25} = \frac{x_2}{0} = \frac{x_3}{0}$$

\therefore Eigen value = $\begin{bmatrix} 0.25 \\ 0 \\ 0 \end{bmatrix}$

(case 2: $\lambda = 1$)

$$(A - \lambda I)x = 0$$

$$\begin{pmatrix} 0.5 - 1 & 0.2 & 0.1 \\ 0 & -1 & 1.5 \\ 0 & 0 & 3.5 - 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} -0.5 & 0.2 & 0.1 \\ 0 & 0 & 1.5 \\ 0 & 0 & 2.5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$-0.5x_1 + 0.2x_2 + 0.1x_3 = 0$$

$$0x_1 + 0x_2 + 1.5x_3 = 0$$

$$0x_1 + 0x_2 + 2.5x_3 = 0$$

$$\frac{x_1}{(0.2)(1.5) - 0} = \frac{-x_2}{(-0.5)(1.5)} = \frac{x_3}{0}$$

$$\frac{x_1}{2} = \frac{x_2}{5} = \frac{x_3}{0}$$

\therefore Eigen vector $\begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}$

Case 3: $\lambda = 3.5$

$$\left(\begin{array}{ccc} 0.5 - 3.5 & 0.2 & 0.1 \\ 0 & 1 - 3.5 & 1.5 \\ 0 & 0 & 3.5 - 3.5 \end{array} \right) \left(\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right) = 0$$

$$\left(\begin{array}{ccc} -3 & 0.2 & 0.1 \\ 0 & -2.5 & 1.5 \\ 0 & 0 & 0 \end{array} \right) \left(\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right) = 0$$

$$-3x_1 + 0.2x_2 + 0.1x_3 = 0$$

$$0x_1 - 2.5x_2 + 1.5x_3 = 0$$

$$0x_1 + 0x_2 + 0x_3 = 0$$

$$\frac{x_1}{(0.2)(1.5) - (-2.5)(0.1)} = \frac{-x_2}{(-3)(1.5) - 0} = \frac{x_3}{1.5 / (-2.5) - 0}$$

$$\frac{x_1}{11} = \frac{x_2}{90} = \frac{x_3}{150}$$

∴ Eigen vector = $\begin{pmatrix} 11 \\ 90 \\ 150 \end{pmatrix}$

5) $\begin{bmatrix} 7 & -2 & 2 \\ -2 & 1 & 4 \\ -2 & 4 & 1 \end{bmatrix}$

Let $A = \begin{bmatrix} 7 & -2 & 2 \\ -2 & 1 & 4 \\ -2 & 4 & 1 \end{bmatrix}$

$$(A - \lambda I) = 0$$

$$\Rightarrow \begin{bmatrix} 7-\lambda & -2 & 2 \\ -2 & 1-\lambda & 4 \\ -2 & 4 & 1-\lambda \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 7-\lambda & -2 & 2 \\ -2 & 1-\lambda & 4 \\ -2 & 4 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (7-\lambda)((1-\lambda)(1-\lambda) - (4)(4)) - (-2)((-2)(1-\lambda) - (4)(-2)) + 2((-2)(4) - (-2)(1-\lambda))$$

$$\Rightarrow (7-\lambda)(\lambda^2 - 2\lambda - 15) + 2(2\lambda + 6)$$

$$+ 2(-2\lambda - 6)$$

$$\Rightarrow (7-\lambda)(\lambda-5)(\lambda+3) = 0$$

$$\Rightarrow \lambda = -3, 5, 7$$

$$\text{Case 1: } \lambda = -3$$

$$(A - \lambda I)x = 0$$

$$\begin{pmatrix} 7 - (-3) & -2 & 2 \\ -2 & 1 - (-3) & 4 \\ -2 & 4 & 1 - (-3) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$10x_1 - 2x_2 + 2x_3 = 0$$

$$-2x_1 + 10x_2 + 10x_3 = 0$$

$$-2x_1 + 10x_2 + 10x_3 = 0$$

$$\frac{x_1}{-2(4) - (4)(2)} = \frac{-x_2}{10(4) - (2)(-2)} = \frac{x_3}{(6)(4) - (-2)(-2)}$$

$$\frac{x_1}{4} = \frac{x_2}{11} = \frac{x_3}{9}$$

$\therefore \text{Eigen vector} = \begin{pmatrix} 4 \\ 11 \\ -9 \end{pmatrix}$

$$\text{Case 2: } \lambda = 5$$

$$(A - \lambda I)x = 0$$

$$\begin{pmatrix} 7 - 5 & -2 & 2 \\ -2 & 1 - 5 & 4 \\ -2 & 4 & 1 - 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$2x_1 - 2x_2 + 2x_3 = 0$$

$$-2x_1 - 4x_2 + 4x_3 = 0$$

$$-2x_1 + 4x_2 - 4x_3 = 0$$

$$\frac{x_1}{(-2)(4) - (2)(-2)} = \frac{-x_2}{(2)(4) - (2)(-2)} = \frac{x_3}{(2)(-2) - (-2)(4)}$$

$$\Rightarrow \frac{x_1}{0} = \frac{x_2}{1} = \frac{x_3}{1}$$

$$\therefore \text{Eigen vectors} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

(Case 3: $\lambda = 2$)

$$(A - \lambda I) X = 0$$

$$\begin{pmatrix} 2-2 & -2 & 0 \\ -2 & 1-2 & 4 \\ -2 & 4 & 1-2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$0x_1 - 2x_2 + 2x_3 = 0$$

$$-2x_1 - 6x_2 + 6x_3 = 0$$

$$-2x_1 + 4x_2 - 6x_3 = 0$$

$$\frac{x_1}{(-2)(4) - (2)(-6)} = \frac{-x_2}{0 - (2)(-2)} = \frac{x_3}{(0)(-2) - (-2)(-2)}$$

$$\frac{x_1}{-4} = \frac{x_2}{1} = \frac{x_3}{1}$$

$$\therefore \text{Eigen Vector} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$