In [1]:
```
#Done by SANJAY MALLASAMUDRAM SANTHANAM ; USC ID:3124715393
#part 1: PCA
import pandas as pd
import numpy as np
import math
from operator import truediv,add
import copy
```

In [2]:
```
#read data
data=np.loadtxt('C:/Users/Lenovo/Desktop/data.txt',delimiter="\t")
```

In [3]:
```
#Check data
np.shape(data)
```

Out[3]:
```
(6000, 3)
```

In [4]:
```
#calculate mean
u=np.mean(data,axis=0)
```

In [5]:
```
#Value of reduced dimension as given in question
k=2
```

In [6]:
```
#Center data around mean
data=data-u
```

In [7]:
```
#Compute covariance matrix
n=np.shape(data)[1]
N=np.shape(data)[0]
E=np.zeros(shape=(n,n))
for i in range(0,N):
    E+=np.reshape(data[i],(3,1))*data[i]
E/=N
```

In [8]:
```
#Find eigenvalue and eigenvector
V,U=np.linalg.eig(E)
```

In [9]:
```
#eigenvector
U
```

Out[9]:
```
array([[ 0.86667137, -0.4962773 , -0.0508879 ],
       [-0.23276482, -0.4924792 ,  0.83862076],
       [ 0.44124968,  0.71496368,  0.54233352]])
```

In [10]:
```
#take only necesary eigenvectors(first two as k=2) and leave rest of the eigenvectors.
U=U[:,0:k]
```

In [11]:
```
print("Direction:")
U
```

```
Direction:
```

Out[11]:
```
array([[ 0.86667137, -0.4962773 ],
       [-0.23276482, -0.4924792 ],
       [ 0.44124968,  0.71496368]])
```

In [12]:
```python
#Correesponding eigenvalues
print("Corresponding eigenvalues:")
V[0:k]
```

```
Corresponding eigenvalues:
```

Out[12]:
```
array([101.60286375,  19.89589866])
```

In [13]:
```python
z=U.T.dot(np.transpose(data))
```

In [14]:
```python
print("New data points:")
z.T
```

```
New data points:
```

Out[14]:
```
array([[ 10.87667009,   7.37396173],
       [-12.68609992,  -4.24879151],
       [  0.43255106,   0.26700852],
       ...,
       [ -2.92254009,   2.41914881],
       [ 11.18317124,   4.20349275],
       [ 14.2299014 ,   5.64409544]])
```

In [15]:
```python
#library function
from sklearn.decomposition import IncrementalPCA
```

In [16]:
```python
data=np.loadtxt('C:/Users/Lenovo/Desktop/data.txt',delimiter="\t")
```

In [17]:
```python
#use library function to find transformed data
pca = IncrementalPCA(n_components=k)
#fit data
pca.fit(data)
#map data from 3D to 2D
Z=pca.transform(data)
```

In [18]:
```python
print("New data points from library function:")
Z
```

```
New data points from library function:
```

Out[18]:
```
array([[ 10.87702383,   7.36575086],
       [-12.68565055,  -4.2656881 ],
       [  0.43247706,   0.26959768],
       ...,
       [ -2.92251403,   2.41954659],
       [ 11.18319958,   4.20450064],
       [ 14.23036055,   5.63145537]])
```

In [19]:
```python
"""
It can be seen that the coordinates of the new points matches exactly with the one calculate
dd by the Library function.
The runtime of the program can be decreased if we use matrix multiplication instead of using
for loop for summation while
calculating Covariance matrix
"""
```

Out[19]:
```
'\nIt can be seen that the coordinates of the new points matches exactly with the one calcul
atedd by the Library function.\nThe runtime of the program can be decreased if we use matrix
multiplication instead of using for loop for summation while \ncalculating Covariance matrix
\n'
```

In [31]:
```python
#Done by SANJAY MALLASAMUDRAM SANTHANAM ; USC ID:3124715393
#part 2: Fastmap
import pandas as pd
import numpy as np
import math
from operator import truediv,add
import copy
```

In [32]:
```python
#read data
data=np.loadtxt('C:/Users/Lenovo/Desktop/fastmap_data.txt',delimiter="\t")
```

In [33]:
```python
#Read list of words from txt file
wordlist=[]
with open('C:/Users/Lenovo/Desktop/fastmap_wordlist.txt') as single:
    single=single.readlines()
    wordlist=(single)
```

In [34]:
```python
#clean data by removing newline character at the end and separate all words
for i in range(0,len(wordlist)):
    wordlist[i]=wordlist[i].split('\n')[0]
```

In [35]:
```python
wordlist
```

Out[35]:
```
['acting',
 'activist',
 'compute',
 'coward',
 'forward',
 'interaction',
 'activity',
 'odor',
 'order',
 'international']
```

In [36]:
```python
#n= no of words=10
n=len(wordlist)
#D represents the distance matrix which contains all initial distances between all pairs. It is symmetric
D=np.zeros((n,n))

#for each pair, update in 2 cells of the matrix.
for t in data:
    D[int(t[0])-1][int(t[1])-1]=t[2]
    D[int(t[1])-1][int(t[0])-1]=t[2]
```

In [37]:
```python
D
```

Out[37]:
```
array([[ 0.,  4.,  7.,  6.,  7.,  7.,  4.,  6.,  6., 10.],
       [ 4.,  0.,  7.,  7.,  8.,  9.,  2.,  8.,  8., 11.],
       [ 7.,  7.,  0.,  5.,  6., 10.,  6.,  6.,  6., 12.],
       [ 6.,  7.,  5.,  0.,  2., 10.,  7.,  4.,  5., 12.],
       [ 7.,  8.,  6.,  2.,  0., 10.,  8.,  5.,  4., 11.],
       [ 7.,  9., 10., 10., 10.,  0.,  9., 10.,  9.,  4.],
       [ 4.,  2.,  6.,  7.,  8.,  9.,  0.,  8.,  8., 11.],
       [ 6.,  8.,  6.,  4.,  5., 10.,  8.,  0.,  2., 12.],
       [ 6.,  8.,  6.,  5.,  4.,  9.,  8.,  2.,  0., 11.],
       [10., 11., 12., 12., 11.,  4., 11., 12., 11.,  0.]])
```

In [38]:

```
#k-dimension of transformed data = 2 as given in question
k=2
#p is array of all 'k' dimensional transformed data points .Initially it is assigned zero
p=np.zeros((n,k))
```

In [39]:
```
#iterate once for each dimension of the transformed data
for it in range(0,k):
    #farthest pair calculation.Initially Oa is set to 0 and Ob is chosen randomnly.Distance
 values are compared using
    #square values because 1)it saves time spent in calculating square root value, which is
 actual distance 2) Comparing
    #square distances instead of actual distance values  wont change the result.
    Oa=0
    #set Ob randomnly
    Ob=np.random.randint(low=0 ,high=n-1)
    #distance btwn 2 points is initial distance minus distance between all lower coordinate
s.
    max_dis=D[Oa][Ob]**2
    #p is a n*k matrix which contains coordinates of all points and all dimensions. For iter
ation it, we take into considerations
    # iterations 0 to iterations it-1 only.
    for j in range(0,it):
        max_dis-=(p[Oa][j]-p[Ob][j])**2
    #Use heuristic method to find max distance between pairs.The iteration process is run 35
times so that the chances of
    #choosing a sub optimal solution is reduced.
    #In each iteration, Ob and Oa is updated once. i.e. given some  (Oa,Ob), first a new val
ue of Oa say Oa' is found.
    #Now given this (Oa',Ob), a new value for Ob is found say Ob'. So both pair values are u
pdated in 1 iteration itself.
    #On running 35 iterations, 70 times the coordinates are tested.
    #Loop through all n points to find max dist btwn a fixed coordinate and the new coordina
te.
    for _ in range(0,35):
        #Fix Ob and find farthest point pt from Ob and assign it as Oa
        for pt in range(0,n):
            #Initial Old distance
            t=D[Ob][pt]**2
            #New Distance
            for j in range(0,it):
                t-=(p[Ob][j]-p[pt][j])**2
            if(t>max_dis):
                Oa=pt
                max_dis=t
            #print("Ob:",Ob," pt:",pt," Dist:",t)
        #Fix Oa and find farthest point pt from Oa and assign it as Ob
        for pt in range(0,n):
            t=D[Oa][pt]**2
            for j in range(0,it):
                t-=(p[Oa][j]-p[pt][j])**2
            if(t>max_dis):
                Ob=pt
                max_dis=t
            #print("Oa:",Oa," pt:",pt," Dist:",t)
    #after determining the farthest pait, now iterate through every point and find their coo
rdinates.
    print("\nFarthest pair:",Oa,Ob,"\n","Max distance between farthest pair:",math.pow(max_d
is,0.5),"\n")
    for i in range(0,n):
        #Distance calculations
        x1=D[Oa][i]**2
        x2=D[Oa][Ob]**2
        x3=D[i][Ob]**2
        for j in range(0,it):
            x1-=(p[Oa][j]-p[i][j])**2
            x2-=(p[Oa][j]-p[Ob][j])**2
            x3-=(p[Ob][j]-p[i][j])**2
```

```
        p[i][it]=(x1+x2-x3)/(2*np.sqrt(x2))
```

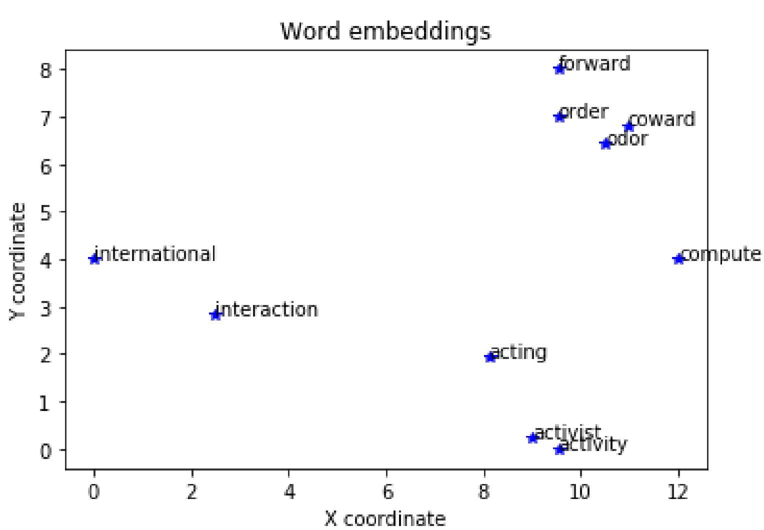
```
Farthest pair: 9 2
 Max distance between farthest pair: 12.0


Farthest pair: 6 4
 Max distance between farthest pair: 8.0
```

In [40]:
```
#Note: Distance can be calculated from either one of Oa or Ob. Both gives different but symm
etric results.
print("Points in ",k," dimensions are:")
p
```

```
Points in  2  dimensions are:
```

Out[40]:
```
array([[ 8.125     ,  1.9375    ],
       [ 9.        ,  0.25      ],
       [12.        ,  4.        ],
       [10.95833333,  6.8125    ],
       [ 9.54166667,  8.        ],
       [ 2.5       ,  2.8125    ],
       [ 9.54166667,  0.        ],
       [10.5       ,  6.4375    ],
       [ 9.54166667,  7.        ],
       [ 0.        ,  4.        ]])
```

In [41]:
```
#plot graph.
import matplotlib.pyplot as plt
figure,axis = plt.subplots()
#title
plt.title("Word embeddings")
#label of x axis
plt.xlabel("X coordinate")
#label of y axis
plt.ylabel("Y coordinate")
#plot each point
for pt in range(0,n):
    axis.plot(p[pt][0],p[pt][1],'b*')
    #show words along with point
    axis.annotate(wordlist[pt],(p[pt][0],p[pt][1]))
print(plt.show())
```




```
None
```

In [ ]:

```

```

In [ ]:

```

```

INF 552 MACHINE LEARNING FOR DATA SCIENCE

HOMEWORK 3

Team member:

SANJAY MALLASAMUDRAM SANTHANAM – 3124715393

Part 1:

Language used:

Python 3, jupyter notebook

Data structures used:

Numpy array to store data

Code-level optimization:

In farthest pair calculation in Fastmap, distance values are compared using square values because

1) saves time spent in calculating square root value, which is actual distance

2) Comparing square distances instead of actual distance values will not change the result.

3) Sometimes distance values turn out to be negative and taking square root of it will raise error.

In PCA, IncrementalPCA method is used as library function instead of PCA because IncrementalPCA is designed for large data sets. The problem question contains 6000 samples so Incremental PCA is used.

Challenges:

In fastmap, sometimes calculating distance between two points can give negative result also, so to circumvent this problem, distance values aren't reduced to square root, instead the square value is taken as such. (This occurs due to distortion of distances). Also if the 'k' i.e. number of dimensions is huge (say 10), then the farthest pair sometimes results in the same point itself(like (0,0) being returned as farthest pair) again because lot of distortion occurs. So a fine balance ahs to be maintained, the number of dimensions shouldn't go too low(as it cant embed properly) and it shouldn't go too high(as distortion occurs). In this problem, k=2 seems ideal. If k is fixed something above 5, then too much distortion happens and the data points cant be properly embedded.

For PCA, theoretically there are many solutions, but the method we follow to obtain the solution is deterministic i.e. gives only one result. If **w** is one PCA component then –**w** is also theoretically correct.When using PCA library function , the 1^{st} coordinate of the result of the library function exhibited this property (i.e. if my calculations gave -1,2 then the library function gave 1,2) .But using IncrementalPCA gave the exact result as the implementation. (i.e. if my calculations gave -1,2 then the

Incremental PCA library function gave -1,2 correctly). This occurs because the PCA library function might use different method to calculate the principal components.

Applications:

PCA is used where dimentionality reduction is needed like Face recognition, image compression and also it is used for noise reduction. PCA is used in astronomy to find the environmental differences in star populations of early type galaxies

Fastmap is used in data mining, indexing and visualizing traditional and multimedia datasets. This embedding is later used for further analysis like finding nearest neighbor. Fastmap is also used to find shortest distance between two points in a graph