MAT-2002

Digital assignment-1

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15BCE0517
C2 SLOT

Find eigon value and eign vector for follawing matrices

1) $\begin{bmatrix} 85 & -28 & -28 \\ -10 & -11 & -11 \\ -46 & -2 & -2 \end{bmatrix}$

A) let $A = \begin{bmatrix} 85 & -28 & -28 \\ -10 & -11 & -11 \\ -46 & -2 & -2 \end{bmatrix}$

$|A - \lambda I| = 0$

$\Rightarrow \left| \begin{bmatrix} 85 & -28 & -28 \\ -10 & -11 & -11 \\ -46 & -2 & -2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$

$\Rightarrow \left| \begin{bmatrix} 85 & -28 & -28 \\ -10 & -11 & -11 \\ -46 & -2 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$

$\Rightarrow \begin{vmatrix} 85-\lambda & -28 & -28 \\ -10 & -11-\lambda & -11 \\ -46 & -2 & -2-\lambda \end{vmatrix} = 0$

$(85-\lambda)\left(\left(-11-\lambda\right) - \left(-2-\lambda\right)\right) - (-28)\left((-10)(-2-\lambda)\right.$

$\left. - (-11)(-46)\right) + (-28)\left[(-10)(-2) - (-11-\lambda)(-46)\right]$

$(85-\lambda)(-9) + 28(10\lambda - 486) - 28(-46\lambda - 486)$

$\Rightarrow \lambda^3 - 72\lambda^2 - 2673\lambda = 0$

$\Rightarrow (\lambda)(\lambda + 27)(\lambda - 99) = 0$

$\Rightarrow \lambda = 0, 99, -27$

Case 1: $\lambda = 0$

$(A - \lambda I)X = 0$

$$\begin{bmatrix} 85 & -28 & -28 \\ -10 & -11 & -11 \\ -46 & -2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_1 = R_1 - 14R_3$

$$\begin{bmatrix} 729 & 0 & 0 \\ -10 & -11 & -11 \\ -46 & -2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\Rightarrow 729x_1 = 0$

$-10x_1 - 11x_2 - 11x_3 = 0$

$-46x_1 - 2x_2 - 2x_3 = 0$

$\therefore$ Eigen vector $= \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

Case 2: $\lambda = 99$

$(A - \lambda I)X = 0$

$$\begin{pmatrix} 85-99 & -28 & -28 \\ -10 & -11-99 & -4 \\ -46 & -2 & -2-99 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} -14 & -28 & -28 \\ -10 & -110 & -11 \\ -46 & -2 & -101 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$-14x_1 \; -28x_2 \; -28x_3 = 0$

$-10x_1 \; -110x_2 \; -11x_3 = 0$

$-46x_1 \; -2x_2 \; -101x_3 = 0$

$$\frac{x_1}{(-28)(-11)-(-28)(-110)} = \frac{-x_2}{(-14)(-11)-(-28)(-10)} = \frac{x_3}{(-14)(-110)-(-28)(-10)}$$

$$\frac{x_1}{-2772} = \frac{x_2}{126} = \frac{x_3}{1260}$$

$$\frac{x_1}{-22} = \frac{x_2}{1} = \frac{x_3}{10}$$

$\therefore$ Eigen vector $= \begin{bmatrix} -22 \\ 1 \\ 10 \end{bmatrix}$

Case 3: $\lambda = -27$

$(A - \lambda I)X = 0$

$$\begin{pmatrix} 85-(-27) & -28 & -28 \\ -10 & -11-(-27) & -11 \\ -46 & -2 & -2-(-27) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} 112 & -28 & -28 \\ -10 & 16 & -11 \\ -46 & -2 & 25 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$112x_1 - 28x_2 - 28x_3 = 0$$
$$-10x_1 + 16x_2 - 11x_3 = 0$$
$$-46x_1 - 2x_2 + 25x_3 = 0$$

$$\frac{x_1}{(-28)(-11) - (-28)(16)} = \frac{-x_2}{(112)(-11) - (-28)(-10)} = \frac{x_3}{(112)(16) - (-28)(-10)}$$

$$\frac{x_1}{308 + 448} = \frac{x_2}{1232 + 280} = \frac{x_3}{1792 - 280}$$

$$\frac{x_1}{756} = \frac{x_2}{1512} = \frac{x_3}{1512}$$

$$\frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{2}$$

$\therefore$ Eigen vector $= \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

2] $\begin{bmatrix} 6 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 7 \end{bmatrix}$

Let $A = \begin{bmatrix} 6 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 7 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\Rightarrow \left| \begin{bmatrix} 6 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 7 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 6-\lambda & 2 & -2 \\ 2 & 5-\lambda & 0 \\ -2 & 0 & 7-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (6-\lambda)\left((5-\lambda)(7-\lambda)-0\right) - 2\left(2(7-\lambda)-0\right)$

$-2\left(0-(-2)(5-\lambda)\right)$

$\Rightarrow \quad (6-\lambda)\left(\lambda^2-12\lambda+35\right) - 2(14-2\lambda)$

$-2(10-2\lambda)$

$\Rightarrow \quad \lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0$

$\Rightarrow (\lambda-3)(\lambda-6)(\lambda-9)=0$

$\Rightarrow \lambda = 3, 6, 9$

Case 1: $\lambda = 3$

$(A-\lambda I)X = 0$

$$\begin{pmatrix} 6-3 & 2 & -2 \\ 2 & 5-3 & 0 \\ -2 & 0 & 7-3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} 3 & 2 & -2 \\ 2 & 2 & 0 \\ -2 & 0 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$\Rightarrow \quad R_1 \to R_1 - R_3$

$$\begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 0 \\ -2 & 0 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$\Rightarrow x$

$$x_1 + 0x_2 - 2x_3 = 0$$
$$2x_1 + 2x_2 + 0x_3 = 0$$
$$-2x_1 + 0x_2 + 1x_3 = 0$$

$$\Rightarrow \frac{x_1}{0-(-2)(2)} = \frac{-x_2}{0-(-2)(2)} = \frac{x_3}{2-0}$$

$$\Rightarrow \frac{x_1}{2} = \frac{x_2}{-2} = \frac{x_3}{1}$$

$\therefore$ Eigen vector $= \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

Case 2: $\lambda = 6$

$$(A - \lambda I)X = 0$$

$$\begin{pmatrix} 6-6 & 2 & -2 \\ 2 & 5-6 & 0 \\ -2 & 0 & 7-6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & 2 & -2 \\ 2 & -1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$0x_1 + 2x_2 - 2x_3 = 0$
$2x_1 - x_2 + 0x_3 = 0$
$-2x_1 + 0 + x_3 = 0$

$$\frac{x_1}{0-(-1)(-2)} = \frac{-x_2}{0-(-2)(2)} = \frac{x_3}{0-4}$$

$$\frac{x_1}{-2} = \frac{x_2}{-4} = \frac{x_3}{-4}$$

$$\Rightarrow \frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{2}$$

$$\therefore \text{Eigen vector} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

Case 3: $\lambda = 9$

$$(A - \lambda I)X = 0$$

$$\begin{pmatrix} 6-9 & 2 & -2 \\ 2 & 5-9 & 0 \\ -2 & 0 & 7-9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} -3 & 2 & -2 \\ 2 & -4 & 0 \\ -2 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$-3x_1 + 2x_2 - 2x_3 = 0$
$2x_1 - 4x_2 + 0x_3 = 0$
$-2x_1 + 0x_2 - 2x_3 = 0$

$$\frac{x_1}{0-(-4)(-2)} = \frac{-x_2}{0-(-2)(2)} = \frac{x_3}{(-3)(-4)-(2)(4)}$$

$$\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{-2}$$

$\therefore$ Eigen vector $= \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

3)

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 6 & 4 & 2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\Rightarrow \left| \begin{pmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 6 & 4 & 2 \end{pmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & 0 \\ 2 & 4-\lambda & 0 \\ 6 & 4 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)(4-\lambda)(2-\lambda) = 0$$

$$\Rightarrow \lambda = 1, 2, 4$$

Case 1: $\lambda = 1$

$$(A - \lambda I) X = 0$$

$$\begin{pmatrix} 1-1 & 0 & 0 \\ 2 & 4-1 & 0 \\ 6 & 4 & 2-1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 2 & 3 & 0 \\ 6 & 4 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$2x_1 + 3x_2 + 0x_3 = 0$$
$$6x_1 + 4x_2 + x_3 = 0$$

$$\frac{x_1}{3-0} = \frac{-x_2}{2-0} = \frac{x_3}{8-18}$$

$$\frac{x_1}{3} = \frac{x_2}{-2} = \frac{x_3}{-10}$$

$\therefore$ eigen vector $= \begin{bmatrix} 3 \\ -2 \\ -10 \end{bmatrix}$

Case 2: $\lambda = 2$

$$(A - \lambda I)X = 0$$

$$\begin{pmatrix} -2 & 0 & 0 \\ 2 & 4-2 & 0 \\ 6 & 4 & 2-2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} -1 & 0 & 0 \\ 2 & 2 & 0 \\ 6 & 4 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$-x_1 + 0x_2 + 0x_3 = 0$$
$$2x_1 + 2x_2 + 0x_3 = 0$$
$$6x_1 + 4x_2 + 0x_3 = 0$$

$$\Rightarrow \frac{x_1}{0} = \frac{x_2}{0} = \frac{x_3}{-2}$$

$\therefore$ Eigen vector $= \begin{bmatrix} 0 \\ 0 \\ \cancel{-2} \end{bmatrix}$

<u>Case 3</u>: $\lambda = 4$

$(A - \lambda I)X = 0$

$$\begin{pmatrix} 1-4 & 0 & 0 \\ 2 & 4-4 & 0 \\ 6 & 4 & 2-4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} -3 & 0 & 0 \\ 2 & 0 & 0 \\ 6 & 4 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$-3x_1 + 0x_2 + 0x_3 = 0$$
$$2x_1 + 0x_2 + 0x_3 = 0$$
$$6x_1 + 4x_2 - 2x_3 = 0$$

$$\frac{x_1}{0} = \frac{-x_2}{-4} = \frac{x_3}{8}$$

$$\Rightarrow \frac{x_1}{0} = \frac{x_2}{1} = \frac{x_3}{2}$$

$\therefore$ Eigen vectors $= \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

4)
$$\begin{bmatrix} 0.5 & 0.2 & 0.1 \\ 0 & 1 & 1.5 \\ 0 & 0 & 3.5 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{pmatrix} 0.5 & 0.2 & 0.1 \\ 0 & 1 & 1.5 \\ 0 & 0 & 3.5 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} 0.5-\lambda & 0.2 & 0.1 \\ 0 & 1-\lambda & 1.5 \\ 0 & 0 & 3.5-\lambda \end{vmatrix} = 0$$

Expand along C_1

$$(0.5-\lambda)(1-\lambda)(3.5-\lambda) = 0$$

$$\Rightarrow \lambda = 0.5, 1, 3.5$$

Case 1: $\lambda = 0.5$

$\cdot (A - \lambda I)x = 0$

$$\begin{pmatrix} 0.5-0.5 & 0.2 & 0.1 \\ 0 & 1-0.5 & 1.5 \\ 0 & 0 & 3.5-0.5 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$0x_1 + 0.2x_2 + 0.1x_3 = 0$
$0x_1 + 0.5x_2 + 1.5x_3 = 0$
$0x_1 + 0x_2 + 3x_3 = 0$

$$\frac{x_1}{(0.2)(1.5) - (0.5)(0.1)} = \frac{-x_2}{0} = \frac{x_3}{0}$$

$$\frac{x_1}{0.25} = \frac{x_2}{0} = \frac{x_3}{0}$$

$\therefore$ Eigen value = $\begin{bmatrix} 0.25 \\ 0 \\ 0 \end{bmatrix}$

Case 2: $\lambda = 1$

$(A - \lambda I)x = 0$

$$\begin{pmatrix} 0.5-1 & 0.2 & 0.1 \\ 0 & 1-1 & 1.5 \\ 0 & 0 & 3.5-1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} -0.5 & 0.2 & 0.1 \\ 0 & 0 & 1.5 \\ 0 & 0 & 2.5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$-0.5x_1 + 0.2x_2 + 0.1x_3 = 0$
$0x_1 + 0x_2 + 1.5x_3 = 0$
$0x_1 + 0x_2 + 2.5x_3 = 0$

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$$\frac{x_1}{(0.2)(1.5)-0} = \frac{-x_2}{(-0.5)(1.5)} = \frac{x_3}{0}$$

$$\frac{x_1}{2} = \frac{x_2}{5} = \frac{x_3}{0}$$

$\therefore$ Eigen vector $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

<u>Case 3:</u> $\lambda = 3.5$

$$\begin{pmatrix} 0.5-3.5 & 0.2 & 0.1 \\ 0 & 1-3.5 & 1.5 \\ 0 & 0 & 3.5-3.5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} -3 & 0.2 & 0.1 \\ 0 & -2.5 & 1.5 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$-3x_1 + 0.2x_2 + 0.1x_3 = 0$$

$$0x_1 - 2.5x_2 + 1.5x_3 = 0$$

$$0x_1 + 0x_2 + 0x_3 = 0$$

$$\frac{x_1}{(0.2)(1.5) - (-2.5)(0.1)} = \frac{-x_2}{(-3)(1.5)-0} = \frac{x_3}{(-3)(-2.5)-0}$$

$$\frac{x_1}{11} = \frac{x_2}{90} = \frac{x_3}{150}$$

$\therefore$ Eigen vector $= \begin{bmatrix} 11 \\ 90 \\ 150 \end{bmatrix}$

5) $\begin{bmatrix} 7 & -2 & 2 \\ -2 & 1 & 4 \\ -2 & 4 & 1 \end{bmatrix}$

Let $A = \begin{bmatrix} 7 & -2 & 2 \\ -2 & 1 & 4 \\ -2 & 4 & 1 \end{bmatrix}$

$|A - \lambda I| = 0$

$\Rightarrow \left| \begin{pmatrix} 7 & -2 & 2 \\ -2 & 1 & 4 \\ -2 & 4 & 1 \end{pmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$

$\Rightarrow \begin{vmatrix} 7-\lambda & -2 & 2 \\ -2 & 1-\lambda & 4 \\ -2 & 4 & 1-\lambda \end{vmatrix} = 0$

$\Rightarrow (7-\lambda)\left((1-\lambda)(1-\lambda) - (4)(4)\right) - (-2)\left((-2)(1-\lambda) - (4)(-2)\right) + 2\left((-2)(4) - (-2)(1-\lambda)\right)$

$\Rightarrow (7-\lambda)(\lambda^2 - 2\lambda - 15) + 2(2\lambda + 6) + 2(-2\lambda - 6)$

$\Rightarrow (7-\lambda)(\lambda - 5)(\lambda + 3) = 0$

$\Rightarrow \lambda = -3, 5, 7$

Case 1: $\lambda = -3$

$$(A - \lambda I)X = 0$$

$$\begin{pmatrix} 7-(-3) & -2 & 2 \\ -2 & 1-(-3) & 4 \\ -2 & 4 & (-1-3) \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$10x_1 - 2x_2 + 2x_3 = 0$$
$$-2x_1 + 4x_2 + 4x_3 = 0$$
$$-2x_1 + 4x_2 + 4x_3 = 0$$

$$\frac{x_1}{(-2)(4) - (4)(2)} = \frac{-x_2}{(10)4 - (2)(-2)} = \frac{x_3}{(10)(4) - (-2)(-2)}$$

$$\frac{x_1}{4} = \frac{x_2}{11} = \frac{x_3}{9}$$

$\therefore$ Eigen vector $= \begin{pmatrix} 4 \\ 11 \\ -9 \end{pmatrix}$

Case 2: $\lambda = 5$

$$(A - \lambda I)X = 0$$

$$\begin{pmatrix} 7-5 & -2 & 2 \\ -2 & 1-5 & 4 \\ -2 & 4 & 1-5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$2x_1 - 2x_2 + 2x_3 = 0$$
$$-2x_1 - 4x_2 + 4x_3 = 0$$
$$-2x_1 + 4x_2 - 4x_3 = 0$$

$$\frac{x_1}{(-2)(4) - (2)(-4)} = \frac{-x_2}{(2)(4) - (2)(-2)} = \frac{x_3}{(2)(-4) - (-2)(-2)}$$

$$\Rightarrow \quad \frac{x_1}{0} = \frac{x_2}{1} = \frac{x_3}{1}$$

$\therefore$ Eigen vectors $= \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

<u>Case 3:</u> $\lambda = 7$

$$(A - \lambda I) X = 0$$

$$\begin{pmatrix} 7-7 & -2 & 0 \\ -2 & 1-7 & 4 \\ -2 & 4 & 1-7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$0x_1 - 2x_2 + 2x_3 = 0$$
$$-2x_1 - 6x_2 + 4x_3 = 0$$
$$-2x_1 + 4x_2 - 6x_3 = 0$$

$$\frac{x_1}{(-2)(4) - (2)(-6)} = \frac{-x_2}{0 - (2)(-2)} = \frac{x_3}{(0)(-6) - (-2)(-2)}$$

$$\frac{x_1}{-1} = \frac{x_2}{1} = \frac{x_3}{1}$$

$\therefore$ Eigen Vector $= \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$