CENG 384 - Signals and Systems for Computer Engineers
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Homework 4

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1. (a)

$$\dot{y}(t) = x(t) + \dot{x}(t) - \int x(t)dt - 2y(t) - \int y(t)$$
$$\ddot{y}(t) + 2\dot{y}(t) + y(t) = \ddot{x}(t) + \dot{x}(t) - x(t)$$

   (b)

$$x(t) = e^{j\omega t} \quad y(t) = H(j\omega)e^{j\omega t}$$
$$H(j\omega)e^{j\omega t}\left(j^2\omega^2 + 2j\omega + 1\right) = e^{j\omega t}\left(j^2\omega^2 + j\omega - 1\right)$$
$$H(j\omega) = \frac{1 - \omega^2 + 2j\omega}{-1 - \omega^2 + j\omega}$$

   (c)

   (d)

2. (a)

$$\dot{h}(t) = \delta(t+1) - \delta(t-1)$$
$$h(t) = u(t+1) - u(t-1)$$

   (b)

$$\begin{aligned} H(j\omega) &= \int_{-\infty}^{\infty} \left[u(t+1) - u(t-1)\right] e^{-j\omega t} dt \\ &= \int_{-1}^{1} e^{-j\omega t} dt \\ &= -\frac{e^{-j\omega t}}{j\omega}\Big|_{-1}^{1} \\ &= \frac{e^{j\omega} - e^{-j\omega}}{j\omega} \\ &= \frac{(\cos\omega + j\sin\omega) - (\cos\omega - j\sin\omega)}{j\omega} \\ &= \frac{2\sin\omega}{\omega} \end{aligned}$$

3. (a)

$$\begin{aligned} h[n] &= h_1[n] * h_2[n] \\ &= \sum_{0}^{\infty} \frac{1}{2^n} \\ &= \left(2 - \frac{1}{2^n}\right) u[n] \end{aligned}$$

(b)

$$
\begin{aligned}
x[n] &= \frac{1}{\sqrt{2}}(\sin\frac{\pi}{3}n + \cos\frac{\pi}{3}n) \\
X(e^{j\omega}) &= \frac{1}{\sqrt{2}}\left(\frac{\pi}{j}\sum_{-\infty}^{\infty}\left(\delta(\omega-\frac{\pi}{3}-2k\omega)-\delta(\omega+\frac{\pi}{3}-2k\omega)\right)+\pi\sum_{-\infty}^{\infty}\left(\delta(\omega-\frac{\pi}{3}-2k\omega)+\delta(\omega+\frac{\pi}{3}-2k\omega)\right)\right) \\
&= \frac{3}{2}(1+j)\sqrt{\pi}(\delta(\pi-3\omega)-j\delta(\pi+3\omega))
\end{aligned}
$$

(c)

$$
\begin{aligned}
Y(e^{j\omega}) &= H(e^{j\omega})X(e^{j\omega}) \\
&= \left(2-\frac{1}{2^n}\right)u[n]\frac{3}{2}(1+j)\sqrt{\pi}(\delta(\pi-3\omega)-j\delta(\pi+3\omega))
\end{aligned}
$$

4. (a)
   (b)
   (c)