CENG 384 - Signals and Systems for Computer Engineers
Spring 2022
Homework 1

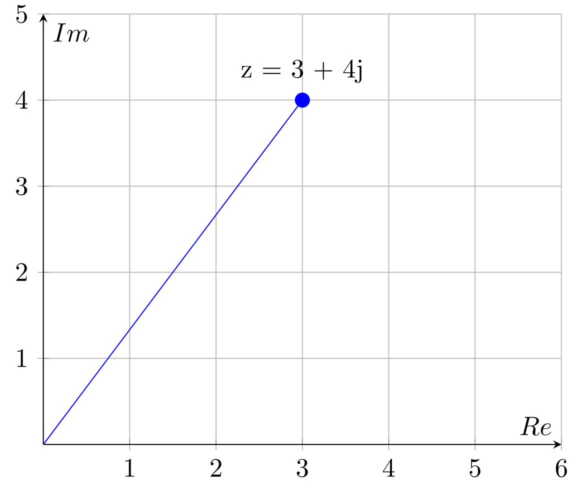
Sezgin, Mustafa
e2380863@ceng.metu.edu.tr

March 31, 2022

---

1. (a) i.

$$
\begin{aligned}
2z - 9 &= 4j - \bar{z} \\
2x + j2y - 9 &= 4j - (x - jy) \\
(3x - 9) + j(y - 4) &= 0 \\
x &= 3, \ y = 4 \\
|z|^2 &= 3^2 + 4^2 = 25
\end{aligned}
$$



ii.

(b)

$$
\begin{aligned}
z &= \sqrt[3]{-27j} = 3j \\
r &= |z| = 3, \ \theta = \frac{\pi}{2} \\
z &= 3e^{j\pi/2}
\end{aligned}
$$

(c)

$$
\begin{aligned}
z &= \frac{(1+j)(\sqrt{3}-j)}{\sqrt{3}+j} \cdot \frac{\sqrt{3}-j}{\sqrt{3}-j} \\
&= \frac{(1+j)(2-2\sqrt{3}j)}{4} \\
&= \left(\frac{1+\sqrt{3}}{2}\right) + j\left(\frac{1-\sqrt{3}}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
|z| &= \sqrt{\left(\frac{1+\sqrt{3}}{2}\right)^2 + \left(\frac{1-\sqrt{3}}{2}\right)^2} \\
&= \sqrt{\left(\frac{4+2\sqrt{3}}{4}\right) + \left(\frac{4-2\sqrt{3}}{4}\right)} \\
&= \sqrt{2}
\end{aligned}
$$

$$\theta = \arctan\left(\frac{1-\sqrt{3}}{1+\sqrt{3}}\right)$$
$$= \frac{-\pi}{12}$$

(d)

$$\begin{aligned} z &= -(1+j)^8 e^{j\pi/2} \\ &= -(2j)^4 e^{j\pi/2} \\ &= -(-4)^2 e^{j\pi/2} \\ &= -16e^{j\pi/2} \end{aligned}$$

2. (a)

$$\begin{aligned} E_x &= \sum_{n=-\infty}^{\infty} |nu[n]|^2 \\ &= \sum_{n=0}^{\infty} n^2 \\ &= \infty \end{aligned}$$

E_x is not finite; thus, $x[n]$ is not an energy signal.

$$\begin{aligned} P_x &= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-\infty}^{\infty} |nu[n]|^2 \\ &= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{\infty} n^2 \\ &= \lim_{N\to\infty} \frac{N(N+1)}{6} \\ &= \infty \end{aligned}$$

P_x is not finite; thus, $x[n]$ is not a power signal.

(b)

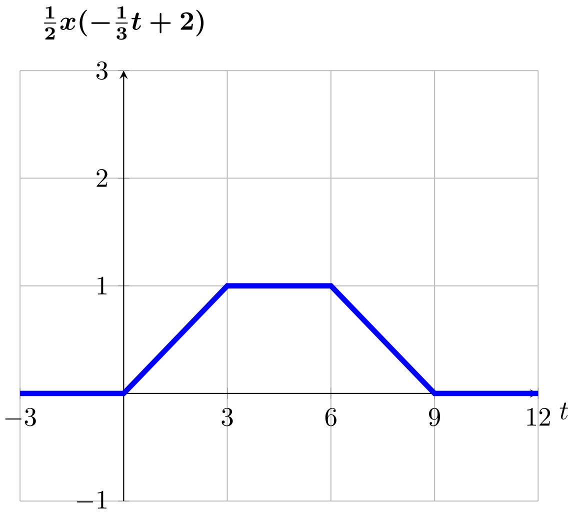
$$\begin{aligned} E_x &= \int_{t=-\infty}^{\infty} |e^{-2t}u(t)|^2 dt \\ &= \int_{t=0}^{\infty} e^{-4t} dt \\ &= \frac{e^{-4t}}{4}\bigg|_0^{\infty} \\ &= \frac{1}{4} \end{aligned}$$

$E_x < \infty$; thus, $x(t)$ is an energy signal.

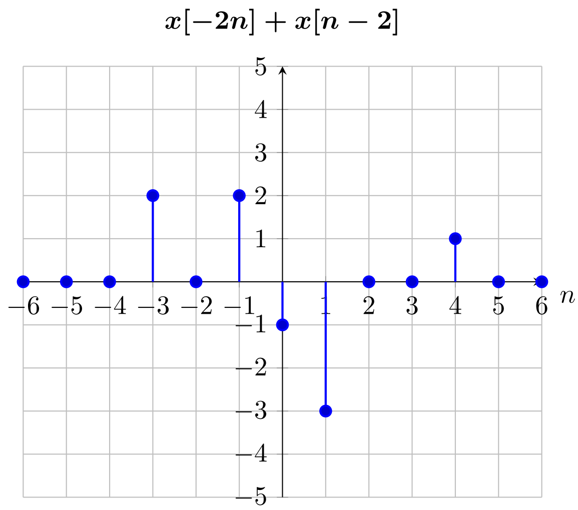
$$\begin{aligned} P_x &= \lim_{T\to\infty} \frac{1/4}{2T} \\ &= 0 \end{aligned}$$

P_x is not a nonzero value; thus, $x(t)$ is not a power signal.

3. We apply time reverse, time scale by $\frac{1}{3}$, time shift to right by 6, respectively, and multiply the result by $\frac{1}{2}$.



4. (a) The graph is the following.



(b) $x[-2n]+x[n-2]=2\delta[n+3]+2\delta[n+1]-\delta[n]-3\delta[n-1]+\delta[n-4]$

5. (a)

$$x(t)=\frac{e^{j3t}}{-j}=je^{j3t}=je^{j3t}e^{j3T}=x(t+T)$$

$$\begin{aligned} e^{j3T}&=\cos 3T+j\sin 3T=1\\ 3T&=2k\pi,\ k=0,\pm1,\pm2,\dots \end{aligned}$$

$x(t)$ is a periodic signal with $T_0=\frac{2\pi}{3}$.

(b)

$$\begin{aligned} x[n]&=\frac{1}{2}\sin\left[\frac{7\pi}{8}n\right]+4\cos\left[\frac{3\pi}{4}n-\frac{\pi}{2}\right]\\ x[n+N]&=\frac{1}{2}\sin\left[\frac{7\pi}{8}n+\frac{7\pi}{8}N\right]+4\cos\left[\frac{3\pi}{4}n+\frac{3\pi}{4}N-\frac{\pi}{2}\right] \end{aligned}$$

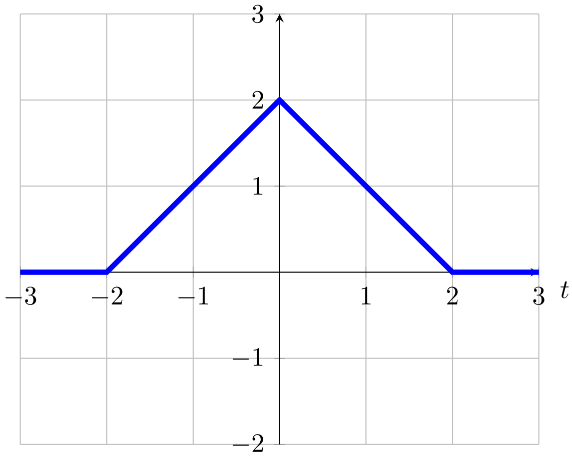
$$\begin{aligned} \frac{7\pi}{8}N=2k\pi &\implies N=\frac{16}{7}k,\ k=0,\pm1,\pm2,\dots\\ \frac{3\pi}{4}N=2l\pi &\implies N=\frac{8}{3}l,\ l=0,\pm1,\pm2,\dots \end{aligned}$$

$x[t]$ is a periodic signal with $N_0=16$.

6. (a) The signal is not even because $x(t)\neq x(-t)$ when, for example, $t=1$: $x(1)=2\neq 0=x(-1)$.
The signal is not odd because $x(t)\neq -x(-t)$ when, for example, $t=1$: $x(1)=2\neq 0=-x(-1)$.

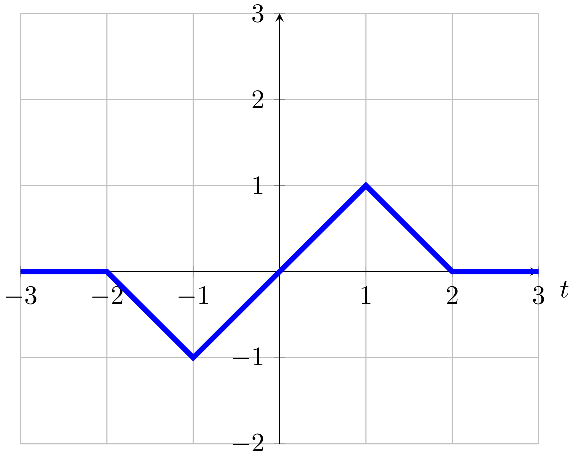
(b) $Even\{x(t)\}=\frac{x(t)+x(-t)}{2}$.

$Even\{x(t)\}$



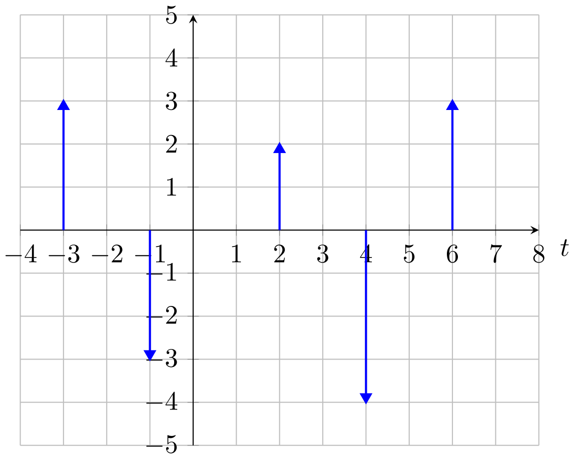
$Odd\{x(t)\} = \frac{x(t)-x(-t)}{2}$.

$Odd\{x(t)\}$



7. (a) $x(t) = 3u(t+3) - 3u(t+1) + 2u(t-2) - 4u(t-4) + 3u(t-6)$
   (b) $\frac{dx(t)}{dt} = 3\delta(t+3) - 3\delta(t+1) + 2\delta(t-2) - 4\delta(t-4) + 3\delta(t-6)$

$x'(t)$



8. (a) i. $y[n]$ has memory, e.g. $y[0] = x[-2]$ when $n = 0$.
   ii. $y[n]$ is stable because the output signal is bounded when the input signal is bounded.
   iii. $y[n]$ is not causal because the output signal depends on the future values of the input signal too, e.g. $y[3] = x[4]$ when $n = 3$.
   iv. $y[n]$ is linear. Suppose two inputs x_1 and x_2 generates the following outputs.

$$y_1[n] = x_1[2n-2]$$
$$y_2[n] = x_2[2n-2]$$

Superposition of the two inputs generates the same superposition in the output.

$$y[n] = ay_1[n] + by_1[n] = ax_1[2n-2] + bx_2[2n-2] = x[2n-2]$$

v. $y[n]$ is invertible because we can obtain the input from the output, that is, $x[n] = y[\frac{n+2}{2}]$.

vi. $y[n]$ is time-invariant because a time shift at the input generates the same amount of time shift at the output.

$$x[2(n-n_0)-2] \longrightarrow y[n-n_0] = x[2(n-n_0)-2]$$

(b) i. $y(t)$ has memory, e.g. $y(2) = 2x(0)$ when $t = 2$.

ii. $y(t)$ is not stable because a bounded input, e.g. $x(t) = u(t)$, may generate an unbounded output, $y(t) = tu(\frac{t}{2}-1)$, as $t \to \infty$.

iii. $y(t)$ is not causal because the output signal depends on the future values of the input signal too, e.g. $y(-4) = -4x(-3)$ when $t = -4$.

iv. $y(t)$ is linear. Suppose two inputs x_1 and x_2 generates the following outputs.

$$y_1(t) = tx_1\left(\frac{t}{2}-1\right)$$
$$y_2(t) = tx_2\left(\frac{t}{2}-1\right)$$

Superposition of the two inputs generates the same superposition in the output.

$$y(t) = ay_1(t) + by_2(t) = atx_1\left(\frac{t}{2}-1\right) + btx_2\left(\frac{t}{2}-1\right) = t\left[ax_1\left(\frac{t}{2}-1\right) + bx_2\left(\frac{t}{2}-1\right)\right] = tx\left(\frac{t}{2}-1\right)$$

v. $y(t)$ is invertible when $t \neq 0$ because we can obtain the input from the output, that is, $x(t) = \frac{y(2t+2}{2t+2}$.

vi. $y(t)$ is not time-invariant because a time shift at the input does not generate the same amount of time shift at the output.

$$x\left(\frac{t-t_0}{2}-1\right) \longrightarrow y(t-t_0) = tx\left(\frac{t-t_0}{2}-1\right) \neq y(t-t_0) = (t-t_0)x\left(\frac{t-t_0}{2}-1\right)$$