# CENG 384 - Signals and Systems for Computer Engineers
## Spring 2022
## Homework 2

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1. (a)

$$\dot{y}(t) = x(t) - 2\dot{x}(t) + 3y(t) - 2\int y(t)dt$$
$$\ddot{y}(t) = \dot{x}(t) - 2\ddot{x}(t) + 3\dot{y}(t) - 2y(t)$$
$$\ddot{y}(t) - 3\dot{y}(t) + 2y(t) = -2\ddot{x}(t) + \dot{x}(t)$$

(b) Using Laplace transform, we have the following.

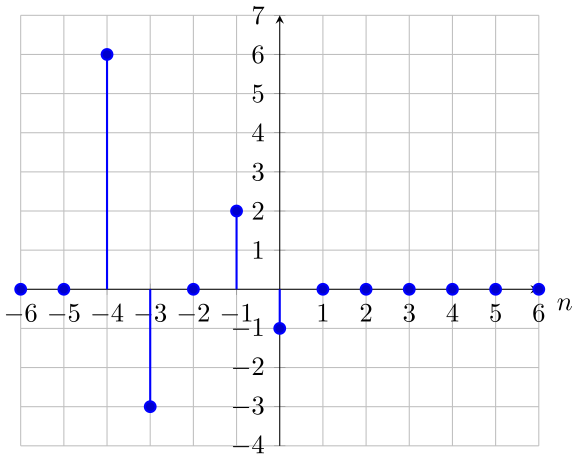
$$(s^2 - 3s + 2)Y(s) = (-2s^2 + s)X(s)$$
$$Y(s) = X(s)\frac{-2s^2 + s}{s^2 - 3s + 2}$$

$$x(t) = (e^{-t} + e^{-2t})u(t) \implies X(s) = \frac{1}{s+1} + \frac{1}{s+2}$$
$$Y(s) = \frac{-4s^3 - 4s^2 + 3s}{(s-2)(s-1)(s+1)(s+2)}$$
$$Y(s) = \frac{-7/2}{s-2} + \frac{5/6}{s-1} + \frac{-1/2}{s+1} + \frac{-5/6}{s+2}$$
$$y(t) = -\frac{7}{2}e^{2t} + \frac{5}{6}e^{t} - \frac{1}{2}e^{-t} - \frac{5}{6}e^{-2t}$$

2. (a)

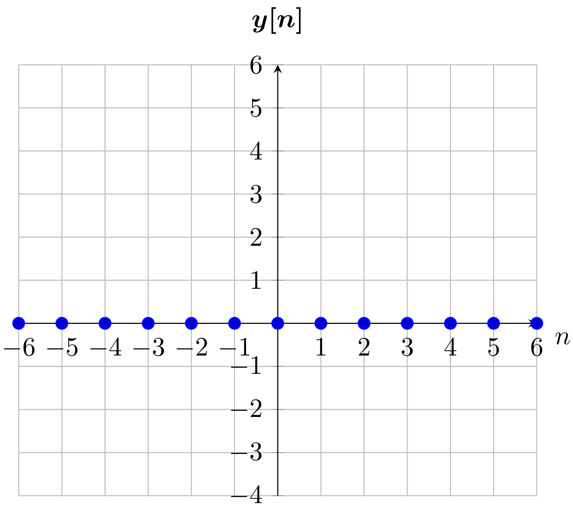
$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= (\delta[n-1] + 3\delta[n+2]) * (2\delta[n+2] - \delta[n+1]) \\ &= \delta[n-1] * 2\delta[n+2] - \delta[n-1] * \delta[n+1] + 3\delta[n+2] * 2\delta[n+2] - 3\delta[n+2] * \delta[n+1] \\ &= 2\delta[n+1] - \delta[n] + 6\delta[n+4] - 3\delta[n+3] \end{aligned}$$

$\boldsymbol{y[n]}$



(b)

$$
\begin{aligned}
y[n] &= x[n] * h[n] \\
&= (u[n+1] - u[n-2]) * (u[n-4] - u[n-6]) \\
&= u[n+1] * u[n-4] - u[n+1] * u[n-6] - u[n-2] * u[n-4] + u[n-2] * u[n-6] \\
&= (n+1-4+1) - (n+1-6+1) - (n-2-4+1) + (n-2-6+1) \\
&= 0
\end{aligned}
$$




3. (a)

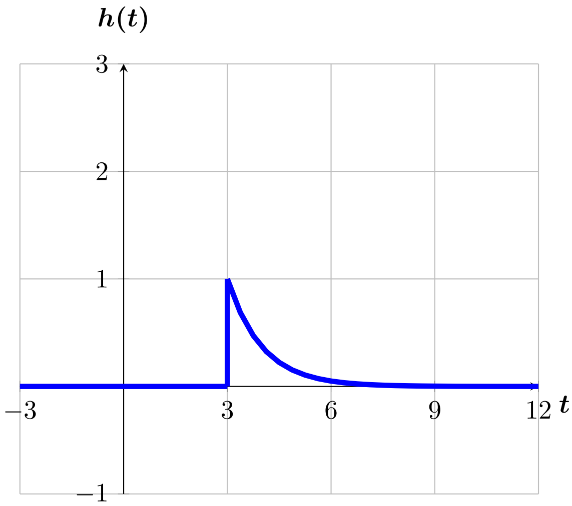
$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \int_{-\infty}^{\infty} e^{-\tau}u(\tau)e^{-\frac{1}{2}(t-\tau)}u(t-\tau)d\tau \\
&= \int_{0}^{t} e^{-\tau}e^{-\frac{1}{2}(t-\tau)}d\tau \\
&= e^{-\frac{t}{2}} \int_{0}^{t} e^{-\frac{\tau}{2}}d\tau \\
&= \left. -2e^{-\frac{t}{2}}e^{-\frac{\tau}{2}} \right|_{\tau=0}^{t} \\
&= -2(e^{-t} - e^{-\frac{t}{2}})
\end{aligned}
$$

(b)

$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= [e^{-3t}u(t) * u(t)] - [e^{-3t}u(t) * u(t-4)] \\
&= \int_{0}^{t} e^{-3\tau}d\tau - \int_{0}^{t-4} e^{-3\tau}d\tau \\
&= -\frac{1}{3}(e^{-3t} - 1) + \frac{1}{3}(e^{-3(t-4)} - 1) \\
&= e^{-3t}\left(\frac{e^{12} - 1}{3}\right)
\end{aligned}
$$

4. (a)

$$
\begin{aligned}
h(t) &= \int_{-\infty}^{t} e^{-(t-\tau)}x(\tau-3)d\tau \\
&= e^{3-t}u(t-3)
\end{aligned}
$$




(b) The input is $x(t) = u(t+2) - u(t-1)$, which implies $x(\tau-3) = u(\tau-1) + u(\tau-4)$. So, the response of the system is as follows.

$$
\begin{aligned}
y(t) &= \int_{-\infty}^{t} e^{-(t-\tau)} u(\tau-1) d\tau - \int_{-\infty}^{t} e^{-(t-\tau)} u(\tau-4) d\tau \\
&= e^{\tau-t} u(t-1)\Big|_{\tau=-\infty}^{t} - e^{\tau-t} u(t-4)\Big|_{\tau=-\infty}^{t} \\
&= u(t-1) + u(t-4)
\end{aligned}
$$

5. (a)

(b)

(c)