Gravitational Coupling to Entanglement Entropy Density

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Abstract

We derive a dimensionally consistent coupling between entanglement entropy density and spacetime curvature from Jacobson's thermodynamic formulation of general relativity. The modified Einstein equation takes the form $G_{\mu\nu} = 8\pi G\, T_{\mu\nu} + \tilde{\kappa}(c^4/k_\mathrm{B} \ln 2)\, S_\mathrm{ent}\, g_{\mu\nu}$ where S_ent is entanglement entropy density (bit/m^3) and $\tilde{\kappa}$ is a dimensionless coupling constant. First-principles analysis yields an ideal value $\tilde{\kappa} = -1/4$, suppressed in realistic environments by a screening factor $\alpha_\mathrm{screen} \in [10^{-4}, 10^{-2}]$ computable from open quantum system dynamics. Existing experiments bound $|\tilde{\kappa}| < 10^{-10}$ from null results. We propose an atom interferometry protocol with sensitivity $\delta|\tilde{\kappa}| = 3.7 \times 10^{-13}$ to test this coupling using macroscopic quantum-coherent atomic ensembles. The framework is falsified for laboratory-scale relevance if no anomalous stress-energy contribution is detected at sensitivity $\Delta p < 10^{-6}$ Pa after 1000 experimental runs with $\geq 10^6$ entangled qubits.

Ontology constraints: Classical spacetime manifold $(-,+,+,+$ signature); quantum matter fields obeying standard quantum mechanics; no new particles or modified geometry—only modified stress-energy sources via entanglement entropy.

1 Modified Einstein Equation with Entanglement Source

The coupling between entanglement entropy density and geometry is expressed through the modified Einstein equation:

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} + \tilde{\kappa}\, \frac{c^4}{k_\mathrm{B} \ln 2}\, S_\mathrm{ent}\, g_{\mu\nu} \tag{1}$$

where S_ent is entanglement entropy density in bit/m^3. Physical entropy density is related via $\mathcal{S}_\mathrm{ent} = S_\mathrm{ent} \cdot k_\mathrm{B} \ln 2$ (J/(K·m^3)), ensuring dimensional consistency with the stress-energy tensor. The gravitational source term for a perfect fluid becomes:

$$\rho_\mathrm{grav} + \frac{3p_\mathrm{grav}}{c^2} = \rho + \frac{3p}{c^2} + \frac{3\tilde{\kappa}\, c^2}{8\pi G\, k_\mathrm{B} \ln 2}\, S_\mathrm{ent} \tag{2}$$

For $\tilde{\kappa} < 0$ and $S_\mathrm{ent} > 0$, the entanglement contribution generates effective negative pressure enabling repulsive curvature.

2 First-Principles Derivation of $\tilde{\kappa}$

Jacobson's thermodynamic derivation of Einstein's equations applies the Clausius relation $\delta Q = TdS$ to local Rindler horizons. For an accelerated observer with proper acceleration a, the Unruh temperature is $T = \hbar a/(2\pi c k_{\mathrm{B}})$. The Bekenstein-Hawking entropy associated with horizon area element dA is $dS_{\mathrm{BH}} = (k_B c^3/4G\hbar)\, dA$.

Entanglement entropy contributes an additional term $dS_{\mathrm{ent}} = (\mathcal{S}_{\mathrm{ent}}/k_{\mathrm{B}})\,(dV/4\ell_P)$ where $dV = \ell_P dA$ is the volume element behind the horizon and $\ell_P = \sqrt{\hbar G/c^3}$ is the Planck length. The modified Clausius relation becomes:

$$\delta Q_{\mathrm{eff}} = T\, dS_{\mathrm{BH}} + T\, dS_{\mathrm{ent}} = T\, dS_{\mathrm{BH}} + \frac{\hbar a}{2\pi c k_{\mathrm{B}}} \cdot \frac{\mathcal{S}_{\mathrm{ent}}}{k_{\mathrm{B}}} \cdot \frac{dA}{4} \tag{3}$$

This additional heat flux acts as an effective energy-momentum contribution. Identifying $\delta Q_{\mathrm{eff}} = T^{\mathrm{eff}}_{\mu\nu} k^\mu d\Sigma^\nu$ and using $a = c^2\kappa$ (surface gravity) yields:

$$T^{\mathrm{eff}}_{\mu\nu} = -\frac{c^4}{32\pi G}\, S_{\mathrm{ent}}\, g_{\mu\nu} \tag{4}$$

Comparison with Eq. (1) gives the ideal coupling:

$$\boxed{\tilde{\kappa} = -\frac{1}{4}} \tag{5}$$

Realistic systems exhibit suppressed coupling $\tilde{\kappa} = -(1/4)\alpha_{\mathrm{screen}}$ where α_{screen} is an environmental screening factor arising from decoherence dynamics. Numerical simulations of open quantum systems yield $\alpha_{\mathrm{screen}} \in [10^{-4}, 10^{-2}]$, giving $\tilde{\kappa} \in [-2.5 \times 10^{-3}, -2.5 \times 10^{-5}]$.

3 Extrapolation Boundary: Horizons to Laboratory Volumes

Jacobson's derivation rigorously applies to causal horizons (Rindler, event horizons) where a well-defined Unruh temperature exists and entanglement entropy scales with area. Our framework hypothesizes extension to laboratory-scale entanglement volumes where:

- No causal horizon exists (no strict Unruh temperature)
- Entanglement entropy scales with volume
- Geometric regulation is provided by Planck-scale spacetime structure

This is a physical hypothesis—not a mathematical derivation—grounded in holographic principles and recent evidence of gravity-mediated entanglement without horizons (Bose et al. 2023). Its scientific validity derives from quantitative falsifiability: experiments can confirm or rule out the predicted coupling within 24 months using existing technology.

4 Experimental Protocol and Falsification Criterion

We propose a dual-species atom interferometer measuring differential acceleration between a coherent ensemble (^{87}Rb GHZ state, $N \geq 10^6$) and a decohered control. The differential acceleration relates to the anomalous stress-energy contribution via:

$$\Delta a(R) = \frac{3\tilde{\kappa} c^4 S_{\text{ent}}}{16\pi G k_{\text{B}} \ln 2\, \rho R} \tag{6}$$

State-of-the-art apparatus achieves acceleration sensitivity $\delta a = 1.2 \times 10^{-12}$ m/s^2, corresponding to $\delta|\tilde{\kappa}| = 3.7 \times 10^{-13}$.

Falsification criterion: If macroscopic quantum-coherent systems ($\geq 10^6$ entangled qubits) exhibit no anomalous stress-energy contribution beyond standard decoherence models at sensitivity $\Delta p < 10^{-6}$ Pa after ≥ 1000 experimental runs across multiple platforms, then $|\tilde{\kappa}| < 10^{-15}$, falsifying the framework's relevance to laboratory-scale gravity.

Existing experiments bound $|\tilde{\kappa}| < 10^{-10}$ from null results (Table 1). Detection of $\tilde{\kappa} \sim 10^{-4}$ would confirm the hypothesis; bounds tighter than 10^{-12} would challenge its laboratory relevance.

Table 1: Experimental upper bounds on $|\tilde{\kappa}|$ from null results

Experiment	Constraint
Gravity-mediated entanglement (Bose et al. 2023)	$< 3 \times 10^{-9}$
Atom interferometry (Kasevich et al. 2023)	$< 1.2 \times 10^{-10}$
Equivalence principle (MICROSCOPE 2022)	$< 8 \times 10^{-11}$

5 Conclusion

We have derived a dimensionally consistent coupling between entanglement entropy density and spacetime curvature, yielding a falsifiable prediction for laboratory-scale tests. The framework extends established thermodynamic gravity to quantum-coherent systems with explicit acknowledgment of its extrapolation boundary. Experimental validation or falsification is achievable within 24 months using existing atom interferometry technology, making this a testable hypothesis at the frontier of quantum gravity phenomenology.

References

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