

HW 0 Due: 28 jan 2024

1. Learn the wonderful world of LaTeX! For drawings, you can use `tgif` (downloadable from the web), your favorite drawing tool, or TikZ. Reproduce the text and figure in the rectangular box below, as it appears, including the rectangular border.

This is an inline equation: $x + y = 3$.

This is a displayed equation:

$$x + \frac{y}{z - \sqrt{3}} = 2.$$

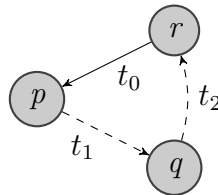
This is how you can define a piece-wise linear function:

$$f(x) = \begin{cases} 3x + 2 & \text{if } x < 0 \\ 7x + 2 & \text{if } x \geq 0 \text{ and } x < 10 \\ 5x + 22 & \text{otherwise.} \end{cases}$$

This is a matrix:

| | | | |
|---|---|---|---|
| 9 | 8 | 7 | 9 |
| 6 | 6 | 6 | |
| 3 | | 3 | 3 |

This is a graph with two types (solid and dashed) of labeled edges:



Files `Notation.tex` and `Figure.tex` should give you enough LaTeX hints to get you started.

Points will be subtracted if your font, font size, spacing, or alignment substantially differ from the one shown, but not if your figure is slightly different, since different tools may draw slightly different figures.

As for all other assignments, you must turn in: (1) a single source LaTeX file, plus one eps or pdf file for each figure you need to include (just one for this homework; of course, if you are using TikZ, you should have the TikZ figure directly in your source latex file, so no additional file is needed), and (2) a single pdf file obtained by running `pdflatex` on your source latex file.

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2. Prove that \mathbb{N} (natural numbers) and \mathbb{Z} (integer numbers) are equinumerous. 25
3. Prove that the relation R defined by $\forall m, n \in \mathbb{N}, (m, n) \in R \Leftrightarrow (m - n) \bmod 3 = 0$ is an equivalence relation, and describe its equivalence classes. 25
4. Show that $\sum_{i=1}^n i^2 = (2n + 1)(n + 1)n/6$. 25
5. Show that, for $n \geq 1$, $\sum_{i=1}^n \frac{1}{i^2} \leq 2 - \frac{1}{n}$. 25

6. Using a formal proof by induction, show that

$$\forall a \in \mathbb{R}, a \neq 1, \sum_{i=0}^n a^i = \frac{1 - a^{n+1}}{1 - a}.$$

Using the previous result, what is the sum of the first 30 powers of 2 (starting from 2^0)?

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