

Supplementary File 1

1 Data Generating Mechanisms

1.1 Single Covariate Setting

We assume a baseline risk factor $x_i \sim N(0, 1)$ and error term $e_i \sim N(0, 60^2)$ for all i . Outcomes are generated under each covariate-outcome relationship as follows:

- *Linear*:

$$y_i = 395 + 110x_i + 40\mathbb{I}(\text{arm}_i = 1) + e_i$$

- *Flattening-off*:

$$y_i = 700 - \exp(-x_i + 4) + 40 + 40\mathbb{I}(\text{arm}_i = 1) + e_i.$$

- *J-shaped*:

$$y_i = 200 + 100x_i + 60x_i^2 + 40\mathbb{I}(\text{arm}_i = 1) + e_i.$$

- *Two-tier*:

$$y_i = 180 + 470\mathbb{I}(x_i > 0) + 40\mathbb{I}(\text{arm}_i = 1) + e_i,$$

- *Full quadratic*

$$y_i = 100 + 104x_i^2 + 40\mathbb{I}(\text{arm}_i = 1) + e_i$$

- *Harmonic*:

$$y_i = 400 + 300 \cos(0.9\pi x_i + 4) + 40\mathbb{I}(\text{arm}_i = 1) + e_i.$$

where $\mathbb{I}(\cdot)$ denotes the indicator function.

1.2 Covariate-treatment interaction

We assume a baseline risk factor $x_i \sim N(0, 1)$ and error term $e_i \sim N(0, 60^2)$ for all i . Outcome under each interaction setting are generated as follows:

- *Small interaction*:

$$y_i = 395 + 80x_i + (20x_i + 40)\mathbb{I}(\text{arm}_i = 1) + e_i.$$

- *Large interaction:*

$$y_i = 500 + 20x_i + (90x_i - 65)\mathbb{I}(\text{arm}_i = 1) + e_i.$$

- *Different shapes:*

$$y_i = 450 + 100x_i + (100 + 100x_i - \exp(-(x_i - 3.5)))\mathbb{I}(\text{arm}_i = 1) + e_i.$$

- *Absent in one group:*

$$y_i = 13x_i^4\mathbb{I}(\text{arm}_i = 1) + e_i.$$

	Response	True Diff	Method	Sample Size	Missingness Mech	Tuning param	No. Failed reps
1	Two-tier	40.00	MI-PMM	50	MAR-X-30	none	40.00
2	Two-tier	40.00	MI-CART	50	MCAR30	10	40.00
3	Two-tier	40.00	MI-CART	50	MCAR30	20	40.00
4	Two-tier	40.00	MI-CART	50	MAR-Z-30	10	58.00
5	Two-tier	40.00	MI-CART	50	MAR-Z-30	20	58.00
6	Two-tier	40.00	MI-CART	50	MAR-X-30	5	39.00
7	Two-tier	40.00	MI-CART	50	MAR-X-30	10	53.00
8	Two-tier	40.00	MI-CART	50	MAR-X-30	20	53.00
9	Harmonic	0.00	MI-norm	50	MCAR30	none	45.00
10	Harmonic	0.00	MI-norm	50	MAR-Z-30	none	34.00
11	Harmonic	0.00	MI-norm	50	MAR-X-30	none	73.00
12	Harmonic	0.00	MI-PMM	50	MCAR30	none	46.00
13	Harmonic	0.00	MI-PMM	50	MAR-Z-30	none	44.00
14	Harmonic	0.00	MI-PMM	50	MAR-X-30	none	35.00
15	Harmonic	0.00	Complete Cases	50	MCAR30	none	33.00
16	Harmonic	0.00	Complete Cases	50	MAR-Z-30	none	43.00
17	Harmonic	0.00	Complete Cases	50	MAR-X-30	none	38.00
18	Harmonic	0.00	MI-RF	50	MCAR30	10	26.00
19	Harmonic	0.00	MI-RF	50	MCAR30	20	27.00
20	Harmonic	0.00	MI-RF	50	MAR-Z-30	20	33.00
21	Harmonic	0.00	MI-RF	50	MAR-X-30	5	29.00
22	Harmonic	0.00	MI-RF	50	MAR-X-30	10	29.00
23	Harmonic	0.00	MI-RF	50	MAR-X-30	20	31.00
24	Harmonic	0.00	MI-CART	50	MCAR30	5	27.00
25	Harmonic	0.00	MI-CART	50	MCAR30	10	32.00
26	Harmonic	0.00	MI-CART	50	MCAR30	20	32.00
27	Harmonic	0.00	MI-CART	50	MAR-Z-30	5	32.00
28	Harmonic	0.00	MI-CART	50	MAR-Z-30	10	46.00
29	Harmonic	0.00	MI-CART	50	MAR-Z-30	20	46.00
30	Harmonic	0.00	MI-CART	50	MAR-X-30	5	35.00
31	Harmonic	0.00	MI-CART	50	MAR-X-30	10	35.00
32	Harmonic	0.00	MI-CART	50	MAR-X-30	20	35.00
33	Harmonic	0.00	MI-SL	50	MAR-X-10	none	35.00
34	Harmonic	40.00	MI-norm	50	MCAR10	none	37.00
35	Harmonic	40.00	MI-norm	50	MCAR30	none	81.00
36	Harmonic	40.00	MI-norm	50	MAR-Z-10	none	44.00
37	Harmonic	40.00	MI-norm	50	MAR-Z-30	none	92.00
38	Harmonic	40.00	MI-norm	50	MAR-X-10	none	40.00
39	Harmonic	40.00	MI-norm	50	MAR-X-30	none	147.00
40	Harmonic	40.00	MI-PMM	50	MCAR10	none	34.00
41	Harmonic	40.00	MI-PMM	50	MCAR30	none	116.00
42	Harmonic	40.00	MI-PMM	50	MAR-Z-10	none	33.00
43	Harmonic	40.00	MI-PMM	50	MAR-Z-30	none	101.00
44	Harmonic	40.00	MI-PMM	50	MAR-X-10	none	35.00
45	Harmonic	40.00	MI-PMM	50	MAR-X-30	none	88.00
46	Harmonic	40.00	Complete Cases	50	MCAR10	none	28.00
47	Harmonic	40.00	Complete Cases	50	MCAR30	none	74.00
48	Harmonic	40.00	Complete Cases	50	MAR-Z-10	none	35.00
49	Harmonic	40.00	Complete Cases	50	MAR-Z-30	none	81.00
50	Harmonic	40.00	Complete Cases	50	MAR-X-10	none	27.00
51	Harmonic	40.00	Complete Cases	50	MAR-X-30	none	82.00
52	Harmonic	40.00	MI-RF	50	MCAR10	10	28.00
53	Harmonic	40.00	MI-RF	50	MCAR10	20	28.00
54	Harmonic	40.00	MI-RF	50	MCAR30	5	60.00
55	Harmonic	40.00	MI-RF	50	MCAR30	10	63.00
56	Harmonic	40.00	MI-RF	50	MCAR30	20	60.00
57	Harmonic	40.00	MI-RF	50	MAR-Z-10	5	26.00
58	Harmonic	40.00	MI-RF	50	MAR-Z-10	10	40.00
59	Harmonic	40.00	MI-RF	50	MAR-Z-30	5	79.00
60	Harmonic	40.00	MI-RF	50	MAR-Z-30	10	53.00
61	Harmonic	40.00	MI-RF	50	MAR-Z-30	20	61.00
62	Harmonic	40.00	MI-RF	50	MAR-X-10	5	28.00
63	Harmonic	40.00	MI-RF	50	MAR-X-10	20	29.00
64	Harmonic	40.00	MI-RF	50	MAR-X-30	5	75.00
65	Harmonic	40.00	MI-RF	50	MAR-X-30	10	63.00
66	Harmonic	40.00	MI-RF	50	MAR-X-30	20	67.00
67	Harmonic	40.00	MI-CART	50	MCAR10	10	35.00
68	Harmonic	40.00	MI-CART	50	MCAR10	20	35.00
69	Harmonic	40.00	MI-CART	50	MCAR30	5	69.00
70	Harmonic	40.00	MI-CART	50	MCAR30	10	67.00
71	Harmonic	40.00	MI-CART	50	MCAR30	20	67.00
72	Harmonic	40.00	MI-CART	50	MAR-Z-10	5	34.00
73	Harmonic	40.00	MI-CART	50	MAR-Z-10	10	31.00
74	Harmonic	40.00	MI-CART	50	MAR-Z-10	20	31.00
75	Harmonic	40.00	MI-CART	50	MAR-Z-30	5	75.00
76	Harmonic	40.00	MI-CART	50	MAR-Z-30	10	92.00
77	Harmonic	40.00	MI-CART	50	MAR-Z-30	20	92.00
78	Harmonic	40.00	MI-CART	50	MAR-X-10	5	36.00
79	Harmonic	40.00	MI-CART	50	MAR-X-10	10	33.00
80	Harmonic	40.00	MI-CART	50	MAR-X-10	20	33.00
81	Harmonic	40.00	MI-CART	50	MAR-X-30	5	91.00
82	Harmonic	40.00	MI-CART	50	MAR-X-30	10	75.00
83	Harmonic	40.00	MI-CART	50	MAR-X-30	20	75.00
84	Harmonic	40.00	MI-SL	50	MCAR10	none	30.00
85	Harmonic	40.00	MI-SL	50	MCAR30	none	42.00
86	Harmonic	40.00	MI-SL	50	MAR-Z-30	none	65.00
87	Harmonic	40.00	MI-SL	50	MAR-X-10	none	62.00

Table 1: Simulation settings where more than 25 repetitions (out of 5000) failed

Table 2: Time taken in seconds to run one repetition of a simulation under the single covariate setting. For brevity, we focus on the Linear and Full quadratic responses and the MCAR30 Missingness Mechanism.

	Method	Tuning	Sample size	Miss mech	True diff	Response	Time (s)
1	RF	5	50	mcar30	40	Linear	1.08
2	CART	5	50	mcar30	40	Linear	1.27
3	KNN	5	50	mcar30	40	Linear	0.03
4	PMM-default	none	50	mcar30	40	Linear	0.66
5	SuperLearner	none	50	mcar30	40	Linear	43.77
6	RF	5	100	mcar30	40	Linear	1.18
7	CART	5	100	mcar30	40	Linear	1.40
8	KNN	5	100	mcar30	40	Linear	0.03
9	PMM-default	none	100	mcar30	40	Linear	0.64
10	SuperLearner	none	100	mcar30	40	Linear	88.21
11	RF	5	200	mcar30	40	Linear	1.35
12	CART	5	200	mcar30	40	Linear	1.39
13	KNN	5	200	mcar30	40	Linear	0.04
14	PMM-default	none	200	mcar30	40	Linear	0.68
15	SuperLearner	none	200	mcar30	40	Linear	187.71
16	RF	5	500	mcar30	40	Linear	1.89
17	CART	5	500	mcar30	40	Linear	1.61
18	KNN	5	500	mcar30	40	Linear	0.06
19	PMM-default	none	500	mcar30	40	Linear	0.72
20	SuperLearner	none	500	mcar30	40	Linear	833.01
21	RF	5	50	mcar30	40	Full quadratic	1.05
22	CART	5	50	mcar30	40	Full quadratic	1.26
23	KNN	5	50	mcar30	40	Full quadratic	0.03
24	PMM-default	none	50	mcar30	40	Full quadratic	0.66
25	SuperLearner	none	50	mcar30	40	Full quadratic	50.95
26	RF	5	100	mcar30	40	Full quadratic	1.16
27	CART	5	100	mcar30	40	Full quadratic	1.39
28	KNN	5	100	mcar30	40	Full quadratic	0.03
29	PMM-default	none	100	mcar30	40	Full quadratic	0.62
30	SuperLearner	none	100	mcar30	40	Full quadratic	94.49
31	RF	5	200	mcar30	40	Full quadratic	1.22
32	CART	5	200	mcar30	40	Full quadratic	1.37
33	KNN	5	200	mcar30	40	Full quadratic	0.04
34	PMM-default	none	200	mcar30	40	Full quadratic	0.61
35	SuperLearner	none	200	mcar30	40	Full quadratic	203.60
36	RF	5	500	mcar30	40	Full quadratic	1.97
37	CART	5	500	mcar30	40	Full quadratic	1.76
38	KNN	5	500	mcar30	40	Full quadratic	0.07
39	PMM-default	none	500	mcar30	40	Full quadratic	0.99
40	SuperLearner	none	500	mcar30	40	Full quadratic	926.75