ICRA8



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OVERVIEW OF TALK

MOTIVATION

DESIGN FOR PERSONALIZED MEDICINE

ALGORITHM

RESULTS FROM SIMULATIONS

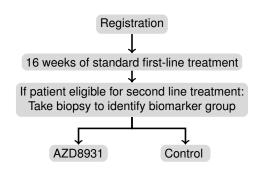
DISCUSSION



MOTIVATION

FOCUS4 (2004-present, UK) aims to find biomarker-targeted treatments for colon cancer.

Kaplan (2015)



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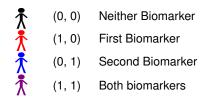
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MOTIVATION

PERSONALIZED CLINICAL TRIALS

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GIVEN THE BIOMARKER STATUS OF A PATIENT, HOW DO WE DECIDE WHETHER TO GIVE THEM THE TREATMENT OR THE CONTROL?





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- ► Sequential.
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 - ... a random allocation can be unlucky.





Myopic Approaches:

MINIMIZATION (POCOCK AND SIMON, 1975) Commonly used in clinical trials ATKINSON (1982) Optimal design based approach



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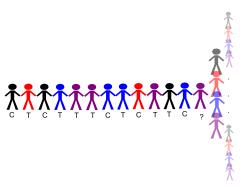


Sequential Design with Covariates



Sequential Design with Covariates





Nonmyopic Approaches:

HUAN AND MARZOUK (2016) Dynamic Programming CHENG AND BERRY (2007) Clinical trials for dose-finding

Sequential Design with Covariates



Sequential Design with Covariates





Pseudo-nonmyopic Approach

Approximation to the computationally expensive non-myopic approach

HOW DOES IT COMPARE TO THE MYOPIC APPROACH?

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$$t_{i,j} = \begin{cases} 1 & \text{if patient } i \text{ receives new treatment } j, \quad j \in \{1,...,t\} \\ 0 & \text{otherwise} \end{cases}$$

$$t_{i,1} = ... = t_{i,j} = 0 \text{ implies patient } i \text{ receives standard treatment } (j = 0)$$

THE MODEL

We assume a logistic model for the response:

$$y_i = \mathsf{Bernoulli}(\pi_i), \mathsf{where logit}\left(\frac{\pi_i}{1-\pi_i}\right) = \mathbf{x}_i^{\top} \boldsymbol{\beta}.$$

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▶ Model matrix \boldsymbol{X} has ith row $\boldsymbol{x}_i = \begin{bmatrix} 1 & b_{i,1} \dots b_{i,k} & t_{i,1} \dots t_{i,t} & b_1 t_1 \dots b_k t_t \end{bmatrix}$

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There are $R = 2^k \times t$ hypotheses to consider:

$$H0: \boldsymbol{c}_r^{\top} \boldsymbol{\beta} \geq \tau_r$$

 $H1: \boldsymbol{c}_r^{\top} \boldsymbol{\beta} < \tau_r$

- ▶ c_r indicate subsets of parameters, for $r \in \{1, ..., R\}$.
- $\blacktriangleright \tau_r$ are thresholds for effective treatment difference.

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- ► Four hypotheses:

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with vectors \mathbf{c}_r given by:

$$\begin{array}{ll} & \boldsymbol{c}_1^\top = (0,0,0,1,0,0) \\ & \boldsymbol{c}_2^\top = (0,0,0,1,1,0) \\ & \boldsymbol{c}_3^\top = (0,0,0,1,0,1) \\ & \boldsymbol{c}_4^\top = (0,0,0,1,1,1) \\ \end{array}$$

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$$\operatorname{Var}\left(\boldsymbol{c}_{r}^{\top}\hat{\boldsymbol{\beta}}\right) = \boldsymbol{c}_{r}^{\top}(\boldsymbol{X}^{\top}\boldsymbol{W}\boldsymbol{X})^{-1}\boldsymbol{c}_{r},$$

where **W** is a diagonal matrix with *i*th entry given by $\hat{\pi}_i (1 - \hat{\pi}_i)$.

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Lee and Wason (2019) propose a **weighted** *L***-optimal objective function**, which seeks to minimize the variance of a linear combination of parameters in β , weighted by their probabilities of success:

$$\Psi_L(\boldsymbol{X}) = \sum_{r=1}^R w_r \boldsymbol{c}_r^\top (\boldsymbol{X}^\top \boldsymbol{W} \boldsymbol{X})^{-1} \boldsymbol{c}_r.$$

MOTIVATION





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- 3. Fit the model to obtain the **initial parameter estimates** $\hat{\beta}_0$.
- 4. Calculate initial weights $w_{r_0} = P(\boldsymbol{c}_r^{\top} \hat{\beta}_0 < \tau_r)$ for all r.

MYOPIC DECISION FOR *i*TH PATIENT



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ITERATE

MYOPIC DECISION FOR ITH PATIENT



ITERATE

For $i = n_0 + 1, ..., n$, given biomarker profile of patient i:

1. Calculate the **weighted** *L***-optimal criterion** for $t \in \{0, 1\}$:

$$\Psi_{L}(\mathbf{X}_{i,t}) = \sum_{r=1}^{R} \mathbf{w}_{r_{i-1}} \mathbf{c}_{r}^{\top} (\mathbf{X}_{i,t}^{\top} \mathbf{W}_{i-1} \mathbf{X}_{i,t})^{-1} \mathbf{c}_{r},$$

where $\mathbf{X}_{i,j}$ is the design matrix where patient i receives treatment t.

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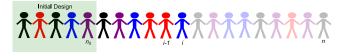
2. Assign the treatment t which minimizes $\Psi_L(\mathbf{X}_{i,t})$



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 - b. Assuming patient i gets treatment $t \in \{0,1\}$, select treatments $t_{i+1}, ..., t_n$ to minimize the weighted L-optimality criterion. Denote the resulting design matrix by $\mathbf{X}_{i,t}^t$.
 - c. Compute the objective function at the end of the trial:

$$\Psi_L(\mathbf{X}_{i,t}^m) = \sum_{r=1}^R w_{r_{i-1}} \mathbf{c}_r^\top (\mathbf{X}_{i,t}^{m\top} \mathbf{W}_{i-1} \mathbf{X}_{i,t}^m)^{-1} \mathbf{c}_r.$$



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2. Compute the average objective function for $t \in \{0, 1\}$:

$$\overline{\Psi}_{L}(t_{i}=t)=\frac{1}{M}\sum_{m=1}^{M}\Psi_{L}\left(\boldsymbol{X}_{i,t}^{m}\right).$$



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$$\overline{\Psi}_{L}(t_{i}=t)=\frac{1}{M}\sum_{m=1}^{M}\Psi_{L}\left(\boldsymbol{X}_{i,t}^{m}\right).$$

Assign the treatment t which minimizes $\overline{\Psi}_{l}(t)$.



ITERATE

For $i = n_0 + 1, ..., n$, given biomarker profile of patient i:

3. Observe **response** y_i .

UPDATE TO INCLUDE ITH PATIENT



ITERATE

- **3.** Observe **response** y_i .
- **4.** Refit model and update **parameter estimates** $\hat{\beta}_i$.
- **5.** Update **weights** w_{r_i} for all r.

RESULTS FROM SIMULATIONS

A simple case with two biomarkers and one treatment:

$$y_i = \mathsf{Bernoulli}(\pi_i), \mathsf{where logit}\left(rac{\pi_i}{1-\pi_i}
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- ▶ Design matrix has *i*th row given by $\mathbf{x}_i = \begin{bmatrix} 1 & b_{i,1} & b_{i,2} & t_i & b_{i,1}t_i & b_{i,2}t_i \end{bmatrix}$
- ▶Linear combinations of interest given by these contrast vectors:

▶ Biomarkers generated independently. For $i \in \{1, ..., 100\}$,

$$b_{i,1} \sim \text{Bernoulli}(0.5)$$

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True distributions assumed to be known for the pseudononmyopic approach.

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B_{1,2} / ○ Dernoum(o.7

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- ightharpoonup Design constructed with myopic and pseudononmyopic (M=100) algorithms.

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- ▶ Threshold for effective treatment difference $\tau_r = -1$ for all r.
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- ► Simulation repeated 100 times.

Results: Parameter Estimates

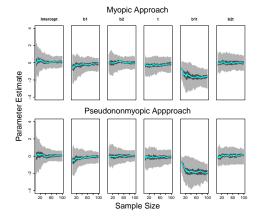


Figure 1: Distribution of $\hat{\beta}$ vs sample size, $\beta = (0, 0, 0, 0, -2, 0)$ median $\blacksquare 40 - 60\% \blacksquare 10 - 90\%$

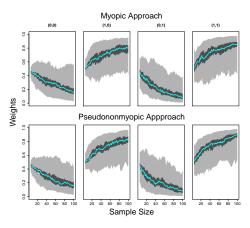


Figure 2: Distribution of w_r vs sample size median $\blacksquare 40 - 60\% \blacksquare 10 - 90\%$

SIMULATIONS

Results: L-Optimality



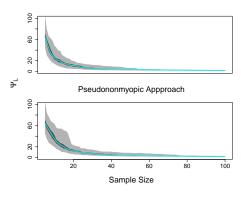


Figure 3: Distribution of Ψ_L vs sample size median $\blacksquare 40 - 60\% = 10 - 90\%$

DISCUSSION

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Also true for examples with:

- ▶ linear models
- ▶ D-, D_A- and A- optimality
- ▶ time-varying biomarkers

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WHAT ABOUT FOR (MULTIPLE) CONTINUOUS TREATMENT(S)?

Thank you



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Slides and R code provided here: https://github.com/mst1g15/biasedcoin

References

Atkinson, A. C. (1982) Optimum biased coin designs for sequential clinical trials with prognostic factors. *Biometrika* **69**, 61-67.

Atkinson, A. C. (1999) Optimum biased-coin designs for sequential treatment allocation with covariate information. *Statistics in Medicine* **18**, 1741-1752.

Cheng, Y. and Berry, D. A. (2007) Optimal adaptive randomized designs for clinical trials. *Biometrika* **94**, 673-687.

Huan, X and Marzouk, Y. M. (2016) Sequential Bayesian optimal experimental design via approximate dynamic programming. *SIAM/ASA Journal on Uncertainty Quantification*, 1-34.

Kaplan, R. (2015) The FOCUS4 design for biomarker stratified trials. *Chinese Clinical Oncology*, **4**(3).

Lee, K.M and Wason, J. (2019) Design of experiments for a confirmatory trial of precision medicine. *Journal of Statistical Planning and Inference* **199**, 179-187.

Pocock, S.J. and Simon, R. (1975) Sequential treatment assignment with balancing for prognostic factors in the controlled clinical trial. *Biometrics* **31**, 103-115.