Importance Sampling in BRDF Evaluation

Advanced Monte Carlo Techniques for Realistic Rendering

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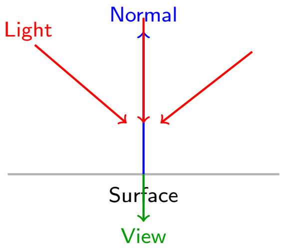
Exploratory Presentation on Advanced Graphics Topics

The Rendering Challenge (Why sampling?)

Goal: Compute outgoing light toward the camera.

Rendering equation (single bounce):

$$L_o(\hat{\mathbf{v}}) = \int_{\Omega^+} f_r(\hat{\mathbf{l}}, \hat{\mathbf{v}})\, L_i(\hat{\mathbf{l}})\, (\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})\, \mathrm{d}\hat{\mathbf{l}}$$



Why sample?

- Light can come from any direction on the hemisphere.
- Exact integration is rarely possible.
- Use **Monte Carlo**: estimate with random samples.

BRDF: the surface's reflection rule

BRDF $f_r(\hat{\mathbf{l}}, \hat{\mathbf{v}})$ says: given incoming direction $\hat{\mathbf{l}}$ and view direction $\hat{\mathbf{v}}$, how much light is reflected?

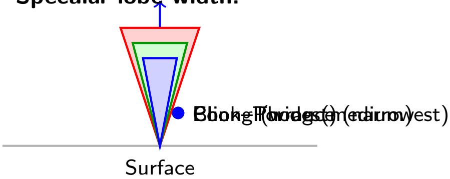
Three simple specular models (non–IS):

Phong: $f_r = \max(\hat{\mathbf{r}} \cdot \hat{\mathbf{v}}, 0)^n, \quad \hat{\mathbf{r}} = \text{reflect}(-\hat{\mathbf{l}}, \hat{\mathbf{n}})$

Blinn–Phong: $f_r = \max(\hat{\mathbf{n}} \cdot \hat{\mathbf{h}}, 0)^n, \quad \hat{\mathbf{h}} = \dfrac{\hat{\mathbf{l}} + \hat{\mathbf{v}}}{\|\hat{\mathbf{l}} + \hat{\mathbf{v}}\|}$

Cook–Torrance: $f_r \propto D(\theta)\, F(\theta)\, G(\theta)$

Specular lobe width:



Meaning:

- n = shininess (larger $n \Rightarrow$ tighter highlight).
- D = microfacet distribution, F = Fresnel, G = geometry (shadowing/masking).

Monte Carlo: estimating the integral

Integral:

$$I = \int_{\Omega^+} f(\omega)\,\mathrm{d}\omega \quad \approx \quad \frac{1}{N}\sum_{i=1}^{N} \frac{f(\omega_i)}{p(\omega_i)} \qquad (\omega_i \sim p)$$

Uniform

$$p(\omega) = \frac{1}{2\pi} \quad \text{(hemisphere)}$$

Simple but wastes samples where f is small.

Importance sampling

$$p(\omega) \propto f(\omega)$$

More samples where contribution is large $\Rightarrow$ less noise, faster.

Importance Sampling PDFs (simple, slide-ready)

1) Phong

$$\boxed{f_r^{\text{Phong}}(\theta_R) = \cos^n(\theta_R)}, \qquad \boxed{p_{\text{Phong}}(\theta_R) = \frac{n+1}{2\pi}\cos^n(\theta_R)}$$

Sampling: $\theta_R = \arccos\left(u_1^{1/(n+1)}\right), \ \phi = 2\pi u_2.$

2) Blinn–Phong

$$\boxed{f_r^{\text{Blinn-Phong}}(\theta_H) = \cos^n(\theta_H)}, \qquad \boxed{p_{\text{Blinn-Phong}}(\theta_H) = \frac{n+1}{2\pi}\cos^n(\theta_H)}$$

Sampling: $\theta_H = \arccos\left(u_1^{1/(n+1)}\right), \ \phi = 2\pi u_2.$

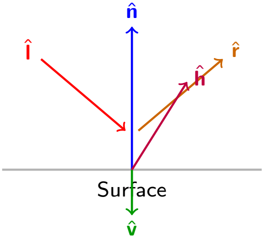
3) Cook–Torrance (microfacet)

$$\boxed{f_r^{\text{Cook-Torrance}}(\theta) \ \propto \ D(\theta)\, F(\theta)\, G(\theta)}, \qquad \boxed{p_{\text{Cook-Torrance}}(\theta) \ \propto \ D(\theta)\, \cos\theta}$$

Pick a distribution D (e.g., GGX or Beckmann) and normalize over the hemisphere.

Variables (quick guide)

- $\hat{\mathbf{n}}$ – surface normal (up direction at the point).
- $\hat{\mathbf{l}}$ – incoming light direction.
- $\hat{\mathbf{v}}$ – view (camera) direction.
- $\hat{\mathbf{r}}$ – perfect mirror reflection of $-\hat{\mathbf{l}}$ about $\hat{\mathbf{n}}$.
- $\hat{\mathbf{h}}$ – halfway direction between $\hat{\mathbf{l}}$ and $\hat{\mathbf{v}}$.
- θ_R – angle to the reflection direction $\hat{\mathbf{r}}$ (Phong).
- θ_H – angle to the halfway direction $\hat{\mathbf{h}}$ (Blinn–Phong).
- θ – angle from the normal (used in microfacet $D(\theta)$).
- n – shininess exponent (higher n = tighter highlight).
- D, F, G – microfacet distribution, Fresnel, geometry terms.
- u_1, u_2 – uniform random numbers in $[0, 1)$ for



Why importance sampling reduces noise

$$L_o \approx \frac{1}{N} \sum_{i=1}^{N} \frac{f_r(\hat{\mathbf{l}}_i, \hat{\mathbf{v}})\, L_i(\hat{\mathbf{l}}_i)\, (\hat{\mathbf{n}} \cdot \hat{\mathbf{l}}_i)}{p(\hat{\mathbf{l}}_i)}$$

If $p(\omega)$ is large where $f_r(\omega)$ is large, the ratio is stable $\Rightarrow$ lower variance and faster convergence.

Implementation hint

- Build a local frame aligned to the axis you sample around (reflection for Phong, normal for Blinn/Cook).
- Convert (θ, ϕ) to a direction in the local frame, then rotate to world.
- Always clamp dot products with $\max(\cdot, 0)$.

When to use which

Phong / Blinn–Phong

- Simple highlights, teaching, previews.
- IS PDF: $\frac{n+1}{2\pi}\cos^n(\cdot)$.

Cook–Torrance

- Modern PBR look (metals, roughness).
- IS PDF: $p(\theta) \propto D(\theta)\cos\theta$ (normalize).

Conclusion

- Importance sampling = sample more where the BRDF is strong.
- Use the simple PDFs shown for each model.
- Keep variables straight: θ_R (Phong), θ_H (Blinn), $D(\theta)$ for Cook–Torrance.