Derivatives Pricing workbook with C++ code and QuantLib

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Abstract

These are my notes on derivatives pricing. This is the material I wish I had access to when originally applying for jobs as a junior quantitative analyst. I have written everything I learned. I have broadly divided it into four parts. Part I covers the basics of financial products : discount factor curve, forwards, interest rate swaps and cross-currency swaps, OIS, basis swaps, CDS contracts. Part II is dedicated to FX options, how FX options are risk managed, the volatility surface and the instruments that are used to define it. Part III covers various FX option spreads. Part IV covers exotic derivatives trading, starting with the most basic products and slowly increasing the complexity up to advanced volatility and multi-asset products. I wish to explain derivatives pricing and trading from first principles, to build a gut, intuitive feel for things. This material will be mostly useful to an aspiring quantitative analyst.

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1.1. Foreign Exchange Spot

1.1.1 Introduction

The foreign exchange(FX) market is an international marketplace for trading currencies. Currencies are denoted with a three letter code and currency pairs are written CCY1/CCY2 where CCY1 is the base currency, CCY2 is the terms currency. For example, if USD/JPY = 100, it costs 100 yens to buy 1 dollar. If the dollar-yen goes higher, dollar is getting relatively stronger against the yen, since it will cost more yens to buy the dollar. If the FX rate goes lower, dollar is getting relatively weaker against the yen, because one dollar will now buy fewer yens.

All major currencies are traded 24 hours a day, in markets around the world from 5 PM EST Sunday to 4 PM EST Friday. Each day of forex trading starts with the opening of the Australasia market followed by Europe and then North America.

1.1.2 Exchange rate quotation terms

- The major currency pairs can be quoted in either *European* or *American* terms.
- Those that quote in number of US dollars per one unit of another currency is American. An example of this is EUR/USD which is quoted as the number of USD per one Euro.
- A pair quoted as number of units of a specific currency per one USD is quoted in European terms. When rates are spoken, the base currency comes first. It is imperative you remember these conventions.

Nobody on the trading floor calls USD/JPY "you-ess-dee-jay-pee-why". Major currency pairs have names that are well established and widely used. Standardized language is common in financial markets. It enables quick and accurate communication, but it exposes those who are not experienced market participants. For this reason, using the correct market terms is important.

American Terms	European Terms	Other Major Cross-rates
EUR/USD - "Euro-Dollar"	USD/JPY - "Dollar-Yen"	EUR/NOK - "Euro-Nockie"
AUD/USD - "Aussie-Dollar"	USD/CAD - "Dollar-Cad"	EUR-SEK - "Euro-Stockie"
NZD/USD - "Kiwi-Dollar"	USD/CHF - "Dollar-Swissie"	USD/MEX - "Dollar-Mex"
GBP/USD - "Sterling (Cable)	USD/SEK - "Dollar-Stockie"	"USD/ZAR" - "Dollar-Rand"
	USD/NOK - "Dollar-Nockie"	GBP/JPY - "Sterling-Yen"
		EUR/GBP - "Euro-Sterling"

The most commonly quoted FX rate is the **spot rate**, often just called spot. For example, if EUR/USD spot rate is 1.3105, EUR $1,000,000$ would be exchanged for USD $1,310,500$. Within a spot transaction, the two cash flows actually hit the bank(*settle*) on the **spot date**, which is usually two clear business after the transaction is agreed(called *T+2* settlement). However, in some currency pairs, for example, USD/CAD and USD/TRY(Turkish Lira), the spot date is only one day after the transaction date.

Do not let the terminology confuse you, a "dollar-yen" is quoted as Yens per Dollar.

$$\frac{Numerator}{Denominator} = \frac{\text{Terms currency}}{\text{Base currency}}$$

Another set of commonly traded FX contracts are **outright forwards**. Within a forward transaction, the cash flows settle on some future date other than the spot date. When rates are quotes on forwards, the **tenor** or **maturity** of the contract must also be specified. For example, if the EUR/USD 1Y (one year) forward FX rate is 1.3245, by transacting in this contract in EUR10m (ten million euros) notional, each EUR will be exchanged for 1.3245 USD(i.e., EUR10m will be exchanged for USD13.245m in one year's time). In a given currency pair, the spot rate and the forward rates are linked by the interest rates in each currency. By a no-arbitrage argument, delivery to the forward maturity must be equivalent to spot trading and putting the cash balances in each currency in a risk-free money-market account(MMA) until maturity.

Differences between the spot rate and a forward rate are called **swap points** or **forward points**. For example, if the EUR/USD spot is 1.3105 and the EUR/USD 1y forward is 1.3245, the EUR/USD 1Y swap points are 0.0140. In the market, swap points are quoted as a number of **pips**. Pips are the smallest increment in the FX rate usually quoted for a particular currency pair. In EUR/USD, where FX rates are quoted to four decimal places, a pip is 0.0001. In USD/JPY, where FX rates are usually only quoted to two decimal places, a pip is 0.01. In the above example, an FX swaps trader would say that EUR/USD 1Y swap points are at 140("one-fourty").

Pips(sometimes called "points") are also used to describe the magnitude of FX moves, for example, EUR/USD has jumped fourty pips higher, if the EUR/USD spot rate moves from 1.3105 to 1.3145. Another term used to describe spot moves is **figure**, meaning one hundred pips, for example, USD/JPY has dropped a figure if the USD/JPY spot rate moves from 101.20 to 100.20.

FX swap contracts contain two FX deals in opposite directions(one a buy, the other a sell). Most often one deal is a spot trade and the other deal is a forward trade to a specific maturity. The two trades are called the **legs** of the transactions and the notionals on

the two legs of the FX swap are often equal in base currency, for example buy EUR10m EUR/USD spot against sell EUR10m EUR/USD 1yr forward. FX swaps are quoted in swap point terms - the difference in FX rate between the two legs.

1.1.3 Long and Short positions

A trader takes up a new FX position by buying USD10M USD/CAD spot at a rate of 0.9780. This means buying USD10m and simultaneously selling CAD9.78m. This position is described as **long ten dollar-cad**, meaning USD10m has been bought and an equivalent amount of CAD has been sold. If USD10m USD/JPY has been sold at 101, the position is described as **short 10 dollar-yen**. Note that the long/short refers to the base currency(CCY1) position. The concept of selling something you don't initially own is a strange one in the real world but it quickly becomes normal in financial markets where trading positions can flip often between long(a net bought position) and short(a net sold position).

USD/CAD spot jumps to 0.9900 after it was bought at 0.9780. The trader is a hero! Time to sell USD/CAD spot and lock in a profit. Selling USD10m USD/CAD spot at 0.9900 results in selling USD10m against buying CAD9.9m. The initial bought USD10m and new sold USD10m cancel out, leaving no net USD position, but the initial sold CAD9.78m and the new bought CAD9.9m leave a CAD120,000 profit.

A long position in a financial instrument makes money if the price of the instrument *rises* and loses if the price of the instrument *falls*. Mathematically, the intra-day P&L from a long spot position is:

$$P\&L_{CCY2} = Notional_{CCY1} \cdot (S_T - S_0)$$

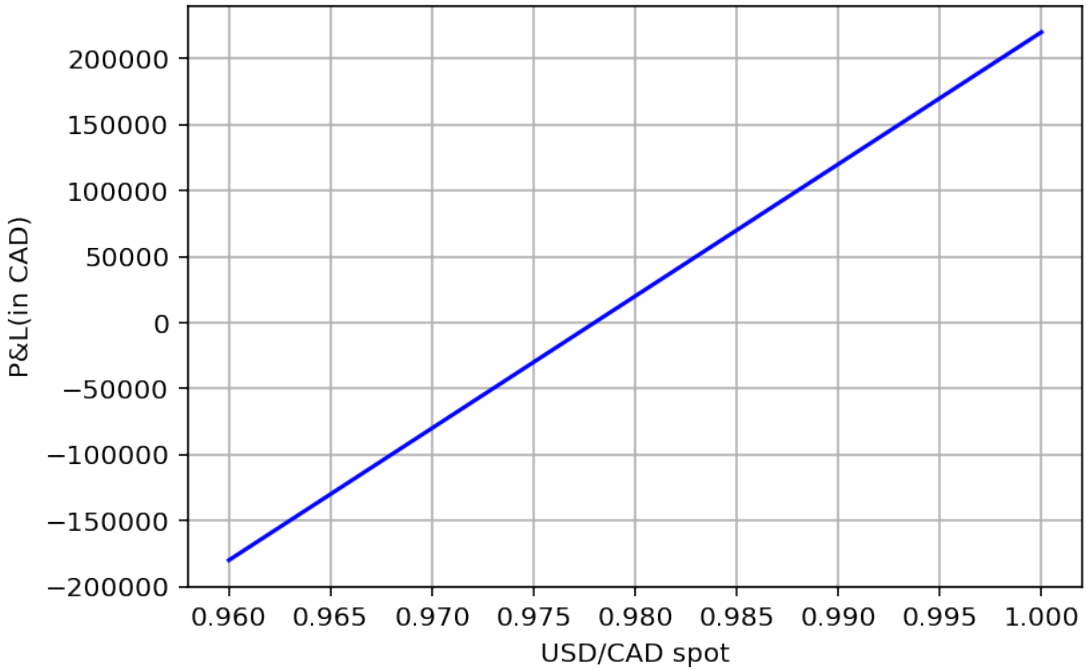
where S_0 is the initial spot rate and S_T is the new spot rate. The below code snippet shows the P&L from a long USD10m USD/CAD spot position.

```
import numpy as np
import matplotlib.pyplot as plt

notional = 10000000
S_0 = 0.9780
S_t = np.arange(0.9600,1.000,0.0001)
pnl = notional*(S_t - S_0)

plt.plot(S_t,pnl,'b-')
plt.xlabel('USD/CAD spot', fontsize=10)
plt.ylabel('P&L(in CAD)', fontsize=10)
plt.grid(True)
plt.show()
```

A short position in a financial instrument *makes money* if the price of the instrument falls and loses money if the price of the instrument *rises*. The intraday P&L from a short position is also:



$$P\&L_{CCY2} = Notional_{CCY1} \cdot (S_T - S_0)$$

However, the notional will be negative to denote a short position. The below code snippet shows the P&L from a short USD10m USD/CAD spot position.

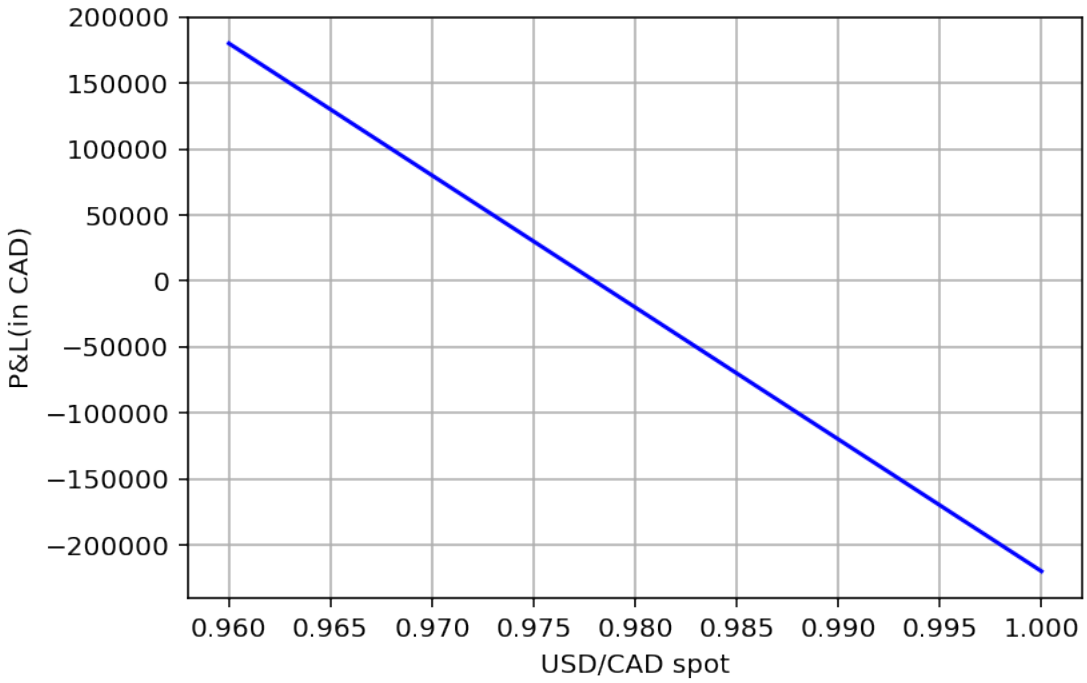
1.1.4 Bids and Offers

The international foreign exchange market is enormous, with trillions of dollars's worth of deals transacted each day. The most important international center for FX is London, followed by New York. In Asia, Tokyo, Hong Kong and Singapore are roughly equally important.

The USD is by far the most frequently traded currency. The dollar is also the global reserve currency. EUR/USD is the most traded currency pair, followed by USD/JPY and then GBP/USD.

FX traders draw a distinction between the **major currency pairs**: the most commonly traded currency pairs against the USD, and **cross currency pairs**. For example, EUR/USD and AUD/USD are majors while EUR/AUD is a cross. FX rates in cross pairs are primarily determined by the trading activity in the majors. The FX market is highly efficient, so if the EUR/USD spot is trading at 1.2000 and AUD/USD spot is trading at 0.8000, EUR/AUD spot will certainly be trading at $1.2/0.8 = 1.5000$.

The below table is a mocked-up screen-grab of a market data tool showing live spot rates in major G10 currency pairs. In practice these rates change(*tick*) many times a second.



Pair	Bid	Offer	Day Low	Day High
EUR/USD	1.3651	1.3652	1.3647	1.3688
GBP/USD	1.6898	1.6899	1.6856	1.6913
USD/CHF	0.8933	0.8934	0.8923	0.8950
AUD/USD	0.9237	0.9238	0.9221	0.9274
NZD/USD	0.8567	0.8568	0.8556	0.8587

Bid is the rate at which the dealer is willing to buy. **Offer** (also called *Ask*): is the rate at which the dealer is willing to sell. If a client wants to buy, the contract must be bought at a dealer's offer, and if the client wants to sell, the contract must be sold at the dealer's bid. The difference between the bid and the offer is called the **spread**.

G10 currency pairs are (mostly) freely floating with no restrictions on their trading. The G10 FX markets are tradable 24 hours a day between Wellington Open (9 a.m. Wellington, New Zealand time) on Monday through to New York Close (5 p.m. New York time) on Friday.

In G10 pairs, the market convention for quoting a currency pair can be deduced from this ordering : $EUR > GBP > AUD > NZD > USD > CAD > CHF > NOK > SEK > JPY$. For example, the CAD against the GBP FX rate is quoted in the market as GBP/CAD.

Emerging market(EM) countries often have mechanisms in place to control currency flows. For example, some EM currencies have limited spot open hours and some peg their currency at a fixed level or maintain it within a trading band by buying and selling spot or by restricting transactions. When trading in an emerging market currency, it is vital to

learn exactly how the FX market functions in that country. EM majors are quoted as the number of EM currency units needed to buy one USD(i.e. USD/CCY).

In currency pairs with restrictions on spot transactions, **Non-Deliverable Forward (NDF)** contracts are often traded. NDFs settle into a single cash payment (usually in USD) at maturity rather than the two cash flows in a regular FX settlement. The **fix**, a reference FX rate published at a certain time every business day in the appropriate country is used to determine the settlement payment.

Example. If a dealer quotes the Cable at 1.5460/70, he will buy the GBP at 1.5460 dollars and the sell the GBP at 1.5470 dollars. This means *you* can buy the GBP at 1.5470(offer) and sell it at 1.5460(bid).

Example. Decide where the client will deal.

Client wants to:	Market quote	Client deals at:
Buy 5 GBP versus USD	1.5471/73	1.5473
Sell 10 USD versus JPY	125.06/12	125.06
Sell 07 NOK versus USD	7.5946/78	7.5978
Buy 1 USD versus CAD	1.5626/32	1.5632
Sell 5 EUR versus SEK	9.1268/9.1318	9.1268

Exercise. A corporation obtains the following quotes from a competition bank. Decide whom and at which rate the client will deal at.

Client wants to:	Market quote	Competition quote	Client deals at:
Buy 5 GBP versus USD	1.5471/73	1.5472/75	
Sell 10 USD versus JPY	125.06/12	125.01/05	
Sell 07 NOK versus USD	7.5946/78	7.5950/80	
Buy 1 USD versus CAD	1.5626/32	1.5620/25	
Sell 5 EUR versus SEK	9.1268/9.1318	9.1260/9.1300	

When traders pay offers they say "mine!" (i.e. they're buying it). This is sometimes accompanied with a raised index finger. When traders give bids they say "yours!" (i.e., they're selling it), sometimes accompanies with an index finder pointing down.

If bids in the market are getting "given" or "hit", this is a sign the market is moving lower. If offers in the market are getting "paid" or "lifted", this is sign that the market is moving higher. These terms can be confusing until they are used day-to-day, at which point they quickly become second nature.

1.1.5 Cross rates

EUR/JPY is a classic, widely traded cross-rate whose price movements reflect the market's view on the Euro and the yen against various currencies.

The market quotes for dollar-yen and dollar-swissie are :

Pair	Bid	Offer
USD/JPY	111.58	111.61
USD/CHF	1.0195	1.0199

Suppose that you want the CHF/JPY cross; that is, the number of yens for 1 CHF. In this cross, the Swiss Franc is the base and the yen is the terms currency. Now, work with what you know. Since, CHF is the base, you know that 1) the currency being traded is the CHF, so 2) the bid and offer of the cross are for CHF.

To get the bid side of the cross, buy the base and sell the terms currency.
To get the ask side of the cross, sell the base and buy the terms currency.

Given a currency quote of USD/JPY=100.00, we could write 1 dollar = 100 yens. The ratio of 1 dollar/1 yen = 100.

$$\frac{USD}{JPY} = 100$$

The cross CHF/JPY can be expressed as the division of the two ratios,

$$\frac{CHF}{JPY} = \frac{USD/JPY}{USD/CHF} \tag{1}$$

$$\left(\frac{CHF}{JPY}\right)_{bid} = \frac{111.61}{1.0195}$$

$$\left(\frac{CHF}{JPY}\right)_{offer} = \frac{111.58}{1.0199}$$

```
#include <iostream>
#include <string>

using namespace std;

int main()
{
        const double usd_jpy_bid = 111.58; const double usd_jpy_offer = 111.61;
        const double usd_chf_bid = 1.0195; const double usd_chf_offer = 1.0199;

        double chf_jpy_bid = usd_jpy_offer / usd_chf_bid;
        double chf_jpy_offer = usd_jpy_bid / usd_chf_offer;

        cout << "Pair\tBid\tOffer";
        cout << "\nCHF/JPY\t"<<chf_jpy_bid<<"\t"<<chf_jpy_offer;

        return 0;
}
```

```
Pair    Bid     Offer
CHF/JPY 109.475 109.403
```

Shortcut rule: If the two currencies are quoted in the same terms, divide the base currency into the terms currency.

The market quotes for the cable and dollar-singie are :

Pair	Bid	Offer
GBP/USD	1.2917	1.2922
USD/SGD	1.3617	1.3625

Suppose you are interested in GBP/SGD cross. The base of the new cross is GBP. We know that,

$$\frac{GBP}{SGD} = \frac{GBP}{USD} \times \frac{USD}{SGD} \tag{2}$$

```
#include <iostream>
#include <string>

using namespace std;

int main()
{
        double gbd_usd_bid = 1.2917;
        double gbp_usd_offer = 1.2922;
        double usd_sgd_bid = 1.3617;
        double usd_sgd_offer = 1.3625;

        double gbp_sgd_bid = gbd_usd_bid * usd_sgd_bid;
        double gbp_sgd_offer = gbp_usd_offer * usd_sgd_offer;

        cout << "Pair\tBid\tOffer\n";
        cout << "GBP/SGD\t" << gbp_sgd_bid << "\t" << gbp_sgd_offer;

        return 0;
}
```

```
Pair    Bid     Offer
GBP/SGD 1.75891 1.76062
```

Shortcut rule : If the two currencies are quoted on different terms, multiply one by the other.

Example. Calculate the cross rates for the following currencies.

1. EUR/SEK
USD/EUR = 0.9772 SEK/USD = 9.3622
$EUR/SEK = 0.9722 \times 9.3622 = 9.1487$

2. EUR/GBP
USD/EUR = 0.9772 GBP/USD = 1.5465
$GBP/EUR = 0.9722 \times 1.5465 = 1.51123$
$EUR/GBP = 1/1.51123 = 0.66171$

3. AUD/NZD
AUD/USD = 0.5535 NZD/USD = 0.4841
$AUD/NZD = 0.5535/0.4841 = 1.1433$

Example. Determine the bid-offer rates.

1. What is the EUR/NOK cross?

Pair	Bid	Offer
EUR/USD	0.9785	0.9789
USD/NOK	7.5853	7.5865

Shortcut rule. Although it is essential to be able to derive the cross rates, there are times when you cannot take time to think - you have to react immediately. These rules may help you:

1. Currencies on the same terms.
   (a) Bid terms on Offer base = Bid cross
   (b) Offer terms on Bid base = Offer cross
2. Currencies on the different terms.
   (a) Bid base times Bid terms = Bid cross
   (b) Offer base tiems Offer terms = Offer cross

Shortcut rule : When multiplying rates, perform parallel multiplication. When dividing rates, perform cross-division.

EUR/NOK bid = $0.9785 \times 7.5853 = 7.4222$.
EUR/BOK offer = $0.9789 \times 7.5865 = 7.4264$.

2. What is the CAD/JPY rate?

Pair	Bid	Offer
USD/CAD	1.5675	1.5685
USD/JPY	125.11	125.17

CAD/JPY bid = $125.11/1.5685 = 79.7641$
CAD/JPY offer = $125.17/1.5675 = 79.8532$

1.1.6 Cash-date, Tom date and Spot date

The deal date is the day, the terms of a FX contract are agreed upon. The value date(settlement date) is the day, the FX contract is executed. By market convention, foreign exchange spot trades two clear business days after the trade date(T+2). For example, you enter a contract to purchase EUR1m EUR/USD spot at 1.1170. Assume that the deal date is 01st January, 2018 - a Monday. Two clear business days from today, 1 million Euros will be delivered in exchange for a payment of 1.117 million Dollars. You will receive a credit of EUR1m in your Euro account and incur a debit USD1.117m in your Dollar account on 03rd Jan. 03rd Jan is the settlement date.

The market convention for settlement is as follows:

Settlement	Settlement/Value Date	Definition
Cash	January 01	Deal Date
Value "Tomorrow"	January 02	One clear business day after deal date
Spot	January 03	Two clear business days after deal date
Forward outright	January 04 or later	Three clear business days or more after deal date, always longer than spot

1.1.7 Market terminologies

Squaring off your positions.
Squaring off your position is, to take an offsetting position, so that you are left with a profit or loss in your own domestic currency.

Mark-to-market(MTM).
Mark-to-market reflects the hypothetical exercise of squaring off all your trades. It reflects the profit or loss that will be realised from closing out your positions.

1.1.8 FX Spot Trading

Based on macroeconomic factors, traders will often take directional views on a currency, buying it if they think it will strengthen and selling it if they think it will weaken.

Example 1. Assume, an FX trader does the following trades during the day. Find the total net values of all USD cashflows and do the same for JPY. Find out the average price for which you have taken this position. Find out the MTM value of the net position using the USD/JPY closing price of 99.10/15.

TradeId	Counter-party	USD	Bid-Offer	Deal price	JPY	Brokerage	Intra-day position	Average rate
1	Interbank	5	98.90-98.95			0	USD5M	
2	Interbank	5	99.00-99.05			0	USD10M	
3	Client	-2	98.98-99.03			0		
4	Interbank	5	99.05-99.10			0		
5	Broker	-3	99.13-99.17			1 pip		

Solution. For every trade, the deal price, JPY amount, intra-day position and average rate can calculated as below.

TradeId	Counter-party	USD	Bid-Offer	Deal price	JPY	Brokerage	Intra-day position	Average rate
1	Interbank	5	98.90-98.95	98.95	-494.75	0	USD5M	
2	Interbank	5	99.00-99.05	99.05	-495.25	0	USD10M	99.00
3	Client	-2	98.98-99.03	99.03	198.06	0	USD8M	98.9925
4	Interbank	5	99.05-99.10	99.10	-495.5	0	USD13M	99.0384
5	Broker	-3	99.13-99.17	99.12	297.36	1 pip	USD10M	99.008

As long as the dealer buys at a bid lower than the average rate or sells at an ask above the average rate, he is profitable.

For a 1 pip movement in USD/JPY spot, the trader gains 10,000 yens. The value of 1 pip is 10,000 yens. This is called the sensitivity of the portfolio.

The dealer is long ten million dollars, short 990.08 million yens. The MTM value of his long ten dollar-yen net position = (99.10-99.008) = 9.2 pips times 10,000 yens = 92,000 yens.

1.1.9 What happens to the spot trade at maturity?

Assume you buy USD1m USD/JPY spot at 100. This trade will settle on the spot date, which is two clear business days away from the trade date. Let's understand what

happens to the trade on the settlement date. On the settlement date, your Dollar bank account is credited USD1m. Your Yen account will be debited JPY100m. Receiving 1 million dollars sounds great! But, what if, you don't have 100 million yens to pay to the counterparty? In such case, you will have to take an interest-bearing yen loan from another bank or another unit within your investment bank. Similarly, the USD balance can put in a interest-earning deposit account.

On trade settlement, therefore, the spot trade will mature and will not contribute to your MTM, daily P&L and cash-flows of the FX Spot portfolio. The trade will appear as an expired trade. Your buy USD/sell JPY position has simply moved from FX spot books to the cash books. You can keep the USD deposit-JPY loan going for a few days and then do a spot, tom or a cash trade to square off your cash positions.

1.2. FX Forwards

FX forwards are contracts to trade currencies at a pre-determined rate at some future date(past the spot date).

1.2.1 Calculating the forward rate

The rate of exchange for any date other than the spot is a function of the *spot* and the *relative interest rates* in each currency.

The classic no-arbitrage argument goes like this. Assume, you are a dollar-based investor who has $1,000,000 dollars to invest in a Sterling-based stock for one year. Your investment parameters do not permit you to be exposed to the exchange rate risk, so you must set the rate at which you will re-convert the Sterling into Dollars at the time you enter into the investment. There is 3% interest rate differential between the Sterling and the Dollar market, since you can earn 2% if you invest in the Eurodollar market for the year or 5% if you invest in Euro Sterling. In an arbitrage-free market, **the forward rate will eliminate this 3% interest rate differential between the dollar and the Sterling**.

To determine what forward rate would eliminate the benefit of being invested in Sterling, we follow the below steps -

1. determine how much you would earn if you invested in dollars
2. buy one million dollars worth of GBP spot against the dollar
3. determine how many pounds you will have at the end of the investment period.
4. determine what forward exchange rate will convert pound return into a dollar amount equal to what you would have earned had you invested in dollars.

- Maturity value of 1 million dollar deposited invested at the rate of 2 percent

$$\$1,000,000\left(1+0.05\times\frac{365}{360}\right)=1,020,277$$

- Buy one million dollars worth of GBP spot against the dollar. The spot rate is currently 1.55.

$$\$1,000,000/1.55=645,161 \text{ pounds}$$

- Maturity value of your pound deposit at the rate of 5 percent

$$645,161 \times \left(1 + 0.05 \times \frac{365}{365}\right) = 677,419 \text{ pounds}$$

- The rate that equates these two cash flows is:

$$\frac{1,020,277 \text{ USD}}{677,419 \text{ GBP}} = 1.5061$$

$$F = S_0 \times \frac{\text{Maturity value of the variable currency}}{\text{Maturity value of the fixed currency}} = S_0 \times \frac{\left(1 + r_v \times \frac{Act}{360}\right)}{\left(1 + r_f \times \frac{Act}{360}\right)}$$

Be sure to use the exact days(Actual) over the correct day-count convention(360-day year for most currencies, except Sterling which uses a 365-day year).

By buying the pound at 1.55 and selling it forward at 1.5061, the benefit of 3% interest rate differential is completely eliminated. At a forward rate of 1.5061, you be totally indifferent as to which currency you invest in. This rate is also the equilibrium rate between the spot and the forward markets; at 1.5061, you would be not better off if you bought the currency forward or if you bought the currency spot today and invested it for the year before you needed it.

It is important to note that the forward rate reflects the current interest rate, and it assumes that you invest at that rate. If the interest rate differential changes, you will experience a gain or loss. For example, assume you do a forward in Canadian dollars with a 1% interest rate differential priced into a contract. The instant after you do a forward, the Candian rate drops 10 basis points. Now you will have fewer CAD at the end of one year than you *should* have, so you will experience a loss vis-a-vis what you would have earned had you remained in dollars. *If you do not earn the interest rate differential implied in the forward rate, you will experience a gain or a loss.*

Rather than using the aforementioned cash flow analysis technique, it is more common to think of the forward rate in terms of how much it differs from the spot rate. Normally, you will know the spot rate and the forward points (discussed momentarily), and given these two pieces of information you can derive the forward rate. The difference between the forward rate and the spot is referred to as *swap points* or forward points.

	Spot	swap points	Forward rate
GBP:	1.5500	-0.0439	1.5061
JPY:	122.50	-0.2287	122.2713
AUD:	0.5575	-0.0048	0.5527

The forward rate neutralizes the interest rate differential, making you indifferent as to whether you buy a currency spot or forward.

1.2.2 How do you calculate swap points?

The swap points can be calculated as:

$$\text{Swap points} = S_0 \times (r_2 - r_1) \times \left(\frac{Days}{360}\right) \times 10000 \tag{3}$$

where r_2 is the secondary currency interest rate, r_1 is the dominant currency interest rate.

Example. What are the one year swap points for EUR/USD? The "Euro-Dollar" pair is currently trading at 1.0110. The one year Euro interest rate is 2.97 percent. The one year Dollar LIBOR is 1.52 percent. Solution.

$$\text{Swap points} = 1.0110 \times (0.0152 - 0.0297) \times \left(\frac{365}{360}\right) \times 10000 = -148.6$$

1.2.3 Pay and earn forward points

Since the information you normally have is spot and forward points not the forward rate, you must decide whether (i) pay or earn the forward points (ii) whether to add or subtract the points to get the forward rate. Let's see how this works in the following examples.

Remember, the forward rate neutralizes the interest rate differential, making your indifferent as to whether you buy the currency spot or forward.

- If you benefit from the differential by having an interest bearing deposit in the higher interest rate currency from the period between today and the forward date, you will pay for it in forward points.
- If the differential costs you, you will earn the forward points.

Let's say the client wants to sell spot, buy GBP forward against the dollar(buy spot, sell USD forward). We will assume, you have dollar deposit from today to the forward date. If the dollar rates are lower than the pound interest rates, then you will not *earn* the differential during that period. Therefore, you will earn the forward points by paying less USD per GBP in the future. You will buy the pound at a lower, more advantageous rate (that is pay fewer dollars per pound) in the future, so you subtract the points to get the rate. Cable is trading at 1.5500. The forward points are 439. So, the forward rate is $1.5500 - 0.0439 = 1.5061$.

Next, suppose you'd like to sell Aussie forward against the dollar. If the AUD rates are higher than the USD rates, you will earn the differential, so you will pay forward points. You shall receive less US dollars per Australian Dollar in the future, so you subtract the points. Aussie-Dollar is trading at 0.5575 and the forward points are 48, so the forward rate is $0.5575 - 0.0048 = 0.5527$.

Suppose, you want to buy the yen forward against the dollar. If the dollar interest rates are higher then the yen interest rate, you will earn the interest rate differential, so you must pay the forward points. You will receive fewer yens per dollar in the future. This means you must subtract the points to get the forward rate. Dollar-yen pair is trading at 122.50 and the points are 22.87, so the forward rate is $122.50 - 0.2287 = 122.2713$.

Intuition. If you hold the physical and receive cash-flows, it must be cheaper to buy it forward. If you hold the physical and pay cash-flows, then it must be more expensive to buy it forward.

1.2.4 Premium vs. Discount points

Points are the cost of carrying a currency.

Premium points. Low yield currencies trade at a forward premium. Swap points are quoted low-high to maintain buy-low maxim and are added to the spot rate.

Examples: USD/CAD, USD/SEK, USD/NOK, USD/ZAR

- The bid will always be quoted lower than the offer.
- For example, 1 year USD/CAD is quoted as 215/220.
- These points will be added to the spot rate.

$$1.5690 + 0.0215 = 1.5905$$

.

- As the interest rate differential widens, points will move to the right(i.e. become more positive).

Discount points. High-yield currencies trade at a forward discount. Swap points are quoted high-low to maintain high-low maxim and are subtracted from the spot rate.

Examples: USD/JPY, EUR/USD, GBP/USD, USD/CHF

- The bid will always be quoted higher than the offer.
- 1 year EUR/USD is quoted 149/148.
- These points will be subtracted from the spot rate.

$$1.0310 - 0.0149 = 0.9981$$

- As the interest rate differential widens, the points will move to the left(become more negative).

Example. You, the client wishes to sell CAD forward one year against the USD.

Euro USD rates = 5.0%

Euro CAD rates = 7.0%

USD/CAD spot = 1.45

The client will earn the interest rate differential of 2%, so he should pay the forward points. He will pay more Canadian dollars/US dollar in the future. So, the swap points must be added to the spot rate.

Forward points = $(0.07 - 0.05) \times \frac{360}{360} \times 1.45 = 0.0294$.

Forward rate to deal = $1.4500 + 0.0294 = 1.4794$

1.2.5 Forward date conventions

Trading date conventions are the same as they are in the Eurocurrency market, and they dictate what the exact straight dates are. Dealing for proper value dates is imperative. Not accounting for interest rate differentials, even for a few days, can be an unnecessary expense. These conventions are :

1. *Date-to-date*: The market trades date-to-date, which means the appropriate date for each of the forward periods is the date corresponding to the spot date being traded on a given day. If the spot value date were July 6th, the forward date for each of the regular forward periods would be the sixth of the appropriate month unless the sixth were a holiday or a weekend. For example, the one month forward date would be August 6th, the two month forward date would be September 6th and so on.
2. *Holidays/weekends*: If the forward date is a holiday or a weekend in either of the centers concerned, the value date moves to the next business day. Since January 7th is a Sunday, the six month forward date would be July 8th.
3. *End-to-end*:If the spot date is the last business day of the month, the forward value date for each month must be the last business day of the appropriate month. Assuming spot were January 31, 2018, the last business day of the month, the forward value dates would be February 28th, April 30th, all the last days of the month.

When market-makers deal with clients, they will do the forward for any number of days the client wishes; that is they will do odd-dated forwards. They will also do odd-dates with other dealers with whom they deal regularly, but it is more common to trade straight dates. Straight dates are limited to spot against one month, two months, three months, six months, nine months and twelve months.

1.2.6 Calculating Odd dates

Suppose the spot date is 01st January, 2018 and the client wants to buy EUR/sell USD for value date July 17, 2018, which is 197 calendar days. We take the average forward points between 6 months and 7 months and then adjust the points by linear interpolation for the number of days in the odd date. The 6m forward points are 83.2 and 7m forward points are 95.5.

```
import numpy as np
from datetime import date

x = (date(2018,7,17) - date(2018,1,1)).days + 1
x1 = (date(2018,6,30) - date(2018,1,1)).days + 1
x2 = (date(2018,7,31) - date(2018,1,1)).days + 1
y1 = 83.2
y2 = 95.5

y = y1 + ((y2-y1)/(x2-x1))*(x-x1)
print(y)
```

```
89.94516129032259
```

1.2.7 Trade ideas

Trade ideas are formed by a series of analytical techniques. They are presented here merely as a list.

Many trade ideas are based on simple fundamentals

1. Interest rate exhibit patterns better than currencies, allowing one to generate ideas about interest rates and their probable future direction. Trade ideas are more useful in interest rates also because their movements aren't as volatile as spot.
2. Forming trade ideas is more about taking a medium term view.

Trade ideas are often influenced by the carry or points per day available by remaining in a high-yielding currency.

Exercise.

	EURO	AUD	NZD	JPY	GBP	CAD
Spot	0.9957	0.5585	0.4982	124.64	1.56865	1.5357
1mo	13.9/13.8	16.2/16	19/18.7	17.3/17.2	33.4/33.3	18.8/19.2
2mo	27.3/27.1	32.4/32.1	38.5/38.1	31.8/31.5	67.65/67.35	37.5/37.8
3mo	39.1/38.9	46.8/46.4	54.9/54.4	45.3/44.9	98.5/98.2	55/56
6mo						

1.3. FX Swaps

2. Constructing the DF Curve, bond pricing

3. Interest rate swaps and cross currency swaps

4. OIS and basis swaps

5. Introduction to QuantLib in C++

6. FX Options Risk/Reward characteristics

7. Vanilla FX Options greeks

8. Volatility spreads

9. Bull and bear spreads

10. Option arbitrage

11. Position Analysis

12. The binomial tree method

13. The Black Scholes framework and pricing fundamentals

14. FX derivatives risk management

15. Vanilla FX derivatives miscellaneous

16. ATM Curve construction

17. Volatility smile market instruments

18. Probability density functions

19. Vanilla FX options trading exposures

20. Vanilla FX derivatives trading topics

21. ATM volatility and correlation

22. FX derivatives market analysis

23. Exotic FX derivatives pricing

24. FX derivatives pricing models

25. Exotic FX derivatives product classification

26. European digital options

27. European barrier options

28. Touch options

29. American barrier options

30. Exotic FX derivatives trading topics

Figure 1: The Universe

REFERENCES

[1] D. Adams. *The Hitchhiker's Guide to the Galaxy*. San Val, 1995.