Practical 1

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The 1980s Munich rent data

The `rent` data come from a survey conducted in April 1993 by Infratest Sozialforschung, where a random sample of accommodation with new tenancy agreements or increases of rents within the last four years in Munich was selected, including single rooms, small apartments, flats and two-family houses. The data were analysed by Stasinopoulos, Rigby, and Fahrmeir (2000) and they are in the package `gamlss.data` (which is automatically loaded when `gamlss` is loaded). There are 1,969 observations on nine variables in the data set but, for the purpose of demonstrating GAMLSS, we will use only the following five variables:

```
library(gamlss.ggplots)
library(broom)
library(knitr)
library(gamlss.ggplots)
# remove two variables
da <- rent[, -c(4,5, 6, 8)]
da  |> head() |> kable(digits = c(2, 0, 0, 0, 0,0,0), format="pipe")
```

Table 1: Variables in Munich rent data

R	Fl	A	H	loc
693.3	50	1972	0	2
422.0	54	1972	0	2
736.6	70	1972	0	2
732.2	50	1972	0	2
1295.1	55	1893	0	2
1195.9	59	1893	0	2

```
library(gamlss.prepdata)
data_xyplot(da, response=R)
```

```
100 % of data are ploted,
that is, 1969 observations.
```

```
`geom_smooth()` using method = 'gam' and formula = 'y ~ s(x, bs = "cs")'
`geom_smooth()` using method = 'gam' and formula = 'y ~ s(x, bs = "cs")'
```

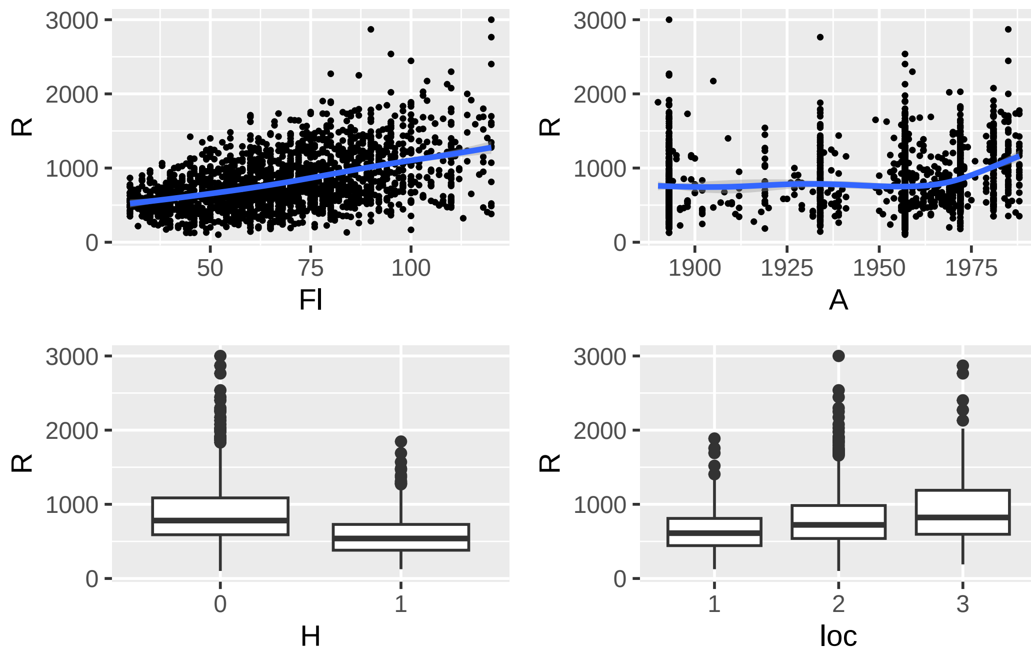

Figure 1: Plot of the rent `R` against explanatory variables `Fl`, `A`, `H` and `loc`.

Figure **??** shows plots of the rent, `R`, against each of the explanatory variables. Although these are bivariate exploratory plots and take no account of the interplay between the explanatory variables, they give an indication of the complexity of these data. The first two explanatory variables, `Fl` and `A`, are continuous. %the plots also show exploratory univariate

The plot of rent, `R`, against floor space, `Fl`, suggests a positive relationship, with increased variation for larger floor spaces, with the result that an assumption of homogeneity of variance would be violated here. There is also some indication of positive skewness in the distribution of rent, `R`. The peculiarity of the plot of `R` against year of construction, `A`, is due to the method of data collection. Many of the observations of `A` were collected on an interval scale and assigned the value of the interval midpoint, while for the rest the actual year of construction was recorded. The plot suggests that for flats up to 1960 the median rent is roughly constant but, for those constructed after that year, there is an increasing trend in the median rent. The two boxplots display how the rent varies according to the explanatory factors. The median rent increases if the flat has central heating, and increases as the location changes from below average to average and then to above average. There are no surprises in the plots here but again the problem of skewness is prominent, with asymmetrical boxes about the median and longer upper than lower whiskers.

In summary, any statistical model used for the analysis of the rent data should be able to deal with the following statistical problems:

- **Complexity of the relationship between rent and the explanatory variables**. The dependence of the median of the response variable rent on floor space and age of construction is nonlinear, and nonparametric smoothing functions may be needed. Median rent may also depend on linear or nonlinear interactions between the explanatory variables.
- **Non-homogeneity of variance of rent.** There is clear indication of non-homogeneity of the variance of rent. The variance of rent may depend on its mean and/or explanatory variables. A statistical model in which this dependence can be modelled explicitly, is needed.
- **Skewness in the distribution of rent.** There is clear indication of positive skewness in the distribution of rent which may depend on explanatory variables and this has to be accounted for within the statistical model.

The linear regression model

Linear regression is a simple but effective model, which served the statistical community well for most of the last century. With response variable Y, r covariates $x_1, \dots, x_r$ and sample size n, it is defined as

$$\begin{aligned} & Y_i = \beta_0 + \beta_1 x_{i1} + \dots + \beta_r x_{ir} + \epsilon_i \\ \text{where} \qquad & \epsilon_i \overset{\text{ind}}{\sim} \mathcal{N}(0, \sigma^2) \ , \qquad \text{for } i = 1, 2, \dots, n \end{aligned}$$

i.e. ϵ_i for $i = 1, 2, \dots, n$ are independently distributed each with a normal distribution with mean zero and variance σ^2. This specification is equivalent to

$$\begin{aligned} & Y_i \overset{\text{ind}}{\sim} \mathcal{N}(\mu_i, \sigma^2) \\ \text{where} \qquad & \mu_i = \beta_0 + \beta_1 x_{i1} + \dots + \beta_r x_{ir} \ , \qquad \text{for } i = 1, 2, \dots, n \ . \end{aligned}$$

We rewrite model (Equation **??**) in matrix form as:

$$\begin{aligned} \mathbf{Y} &\overset{\text{ind}}{\sim} \mathcal{N}(\mu, \sigma^2 \mathbf{I}) \\ \mu &= \mathbf{X}\beta \end{aligned}$$

where $\mathbf{Y} = (Y_1, \dots, Y_n)^\top$ is the response vector, $\mathbf{X}$ is the $n \times p$ design matrix ($p = r + 1$) containing the r covariate columns, plus a column of ones (if the constant is required), $\beta = (\beta_0, \dots, \beta_r)^\top$ is the coefficient vector, and $\mu = (\mu_1, \dots, \mu_n)^\top$ is the mean vector. Note that in order for the model to be fitted, both β and σ^2 have to be estimated from the data. The usual practice is to estimate β using the least squares estimator, obtained by minimizing the sum of squared differences between the observations y_i and the fitted means $\hat{\mu}_i = \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \dots + \hat{\beta}_r x_{ir}$, with respect to the $\hat{\beta}$'s . In matrix form this is written as In matrix form this is written as

$$\hat{\beta} = \text{argmin}_{\hat{\beta}} (\mathbf{y} - \mathbf{X}\hat{\beta})^\top (\mathbf{y} - \mathbf{X}\hat{\beta})$$

which has solution

$$\hat{\beta} = (\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{Y} \ .$$

It can be shown that $\hat{\beta}$ is also the maximum likelihood estimator (MLE) of β. Let

$$\hat{\mu} = \mathbf{X}\hat{\beta}$$

be the fitted values of the model and $\hat{\epsilon} = \mathbf{Y} - \hat{\mu}$ the standard residuals (i.e. fitted errors). Then the MLE for σ^2 is

$$\hat{\sigma}^2 = \frac{\hat{\epsilon}^\top \hat{\epsilon}}{n} \ ,$$

which is a biased estimator, i.e. $E\left(\hat{\sigma}^2\right) \neq \sigma^2$. An unbiased estimator of σ^2 is given by

$$s^2 = \frac{\hat{\epsilon}^\top \hat{\epsilon}}{n - p} \ .$$

Sometimes s^2 is referred as the REML (Restricted Maximum Likelihood) estimator of σ^2.

A linear regression model can be fitted in `R` using the function `lm()`. Here we compare the results from `lm()` to the ones obtained by `gamlss2()`. The notation

```
R ~ Fl+A+H+loc
```

refers to a formula in `R` for more information type `?formula`.

```
library(gamlss2)
r1 <-  gamlss2(R ~ Fl+A+H+loc, family=NO,  data=rent, trace=FALSE)
l1 <- lm(R ~ Fl+A+H+loc,data=rent)
coef(r1)
```

```
    mu.p.(Intercept)              mu.p.Fl               mu.p.A              mu.p.H1
        -2775.038803             8.839445             1.480755          -204.759562
           mu.p.loc2            mu.p.loc3 sigma.p.(Intercept)
          134.052349           209.581472             5.731647
```

```r
coef(l1)
```

```
 (Intercept)           Fl            A           H1         loc2         loc3
-2775.038803     8.839445     1.480755  -204.759562   134.052349   209.581472
```

The coefficient estimates for the μ parameter of the two fits are identical. Note the `gamlss2` produce an extra coefficient from the variance model which is a constant. Note that the two factors of the `rent` data, `H` and `loc`, are fitted as dummy variables as explained in more detail in later section Section.

The fitted objects `r1` and `l1` use the methods `fitted()` and `resid()` to obtain fitted values and residuals respectively. Note that the `gamlss2` object residuals are the normalized (randomized) quantile residuals as explained in the lecture and not the usual residuals $\hat{\epsilon}$ that might be expected.

The MLE of σ can be obtained from a `gamlss2` fitted object using the command `fitted(r1, type="parameter", what="sigma")[1]`. (Here `[1]` shows the first element of the fitted vector for σ) since it is constant for all observations. `summary()` will show the standard errors and t-tests of the estimated coefficients. The method used to calculate standard errors in the `summary()` function of a `gamlss2` model are the standard methods based on the second derivative of the likelihood function.

```r
head(fitted(r1, type="parameter"),5)
```

```
        mu    sigma
1 721.0349 308.4768
2 756.3927 308.4768
3 897.8238 308.4768
4 721.0349 308.4768
5 648.2525 308.4768
```

```r
summary(r1)
```

```
Call:
gamlss2(formula = R ~ Fl + A + H + loc, data = rent, family = NO,
    ... = pairlist(trace = FALSE))
```

```
---
Family: NO
Link function: mu = identity, sigma = log
*--------
Parameter: mu
---
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -2775.0388   526.8431  -5.267 1.54e-07 ***
Fl              8.8394     0.3386  26.108  < 2e-16 ***
A               1.4808     0.2673   5.540 3.43e-08 ***
H1           -204.7596    19.3784 -10.566  < 2e-16 ***
loc2          134.0523    25.1343   5.333 1.07e-07 ***
loc3          209.5815    27.1218   7.727 1.74e-14 ***
*--------
Parameter: sigma
---
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  5.73165    0.01594   359.7   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
*--------
n = 1969 df =  7 res.df =  1962
Deviance = 28159.0039 Null Dev. Red. = 2.8%
AIC = 28173.0039 elapsed =  0.02sec
```

The fitted model is given by

$$y \sim \mathcal{N}(\hat{\mu}, \hat{\sigma}^2)$$

where

$$\begin{aligned}\hat{\mu} &= -2775.03 + 8.83\,Fl + 1.48\,A - 204.75\texttt{if H=1)}+ \\ &\quad + 134.0\texttt{if loc=2)} \quad + 209.5\texttt{(if loc=3)} \\ \log(\hat{\sigma}) &= 5.73\end{aligned}$$

Note that σ is fitted on the log scale (indicated by the log link function, so its fitted value is computed from its intercept as

$$\hat{\sigma} = \exp(5.73).$$

R^2 is obtained from the `gamlss` fitted object as

```r
Rsq(r1)
```

```
[1] 0.3372029
```

One way of checking the adequacy of a model is to examine the residuals.

```r
resid(r1)
```

```
            1             2             3             4             5 
-0.0899092418 -1.0840125931 -0.5226449046  0.0361942620  2.0969084516 
            6             7             8             9            10 
 1.6607077479 -0.2065187784  0.5693444210  0.2407603294  0.0446620148 
           11            12            13            14            15 
 0.3641030052  0.9139607490 -0.4692763311  1.1024808206  0.8964343239 
           16            17            18            19            20 
 0.5835564320  1.5534836382 -1.5514809716 -0.5553857137 -0.7159297647 
           21            22            23            24            25 
-0.8066443509 -0.9346928958 -0.6951466569 -1.4917778935 -1.2457301831 
           26            27            28            29            30 
 5.5125980017  2.3320774819  0.1400533447 -0.1152675288 -0.5413738508 
           31            32            33            34            35 
-0.0370048380 -0.6537889171  0.3477591220 -0.2310190970 -0.0762082524 
           36            37            38            39            40 
-0.6637674253 -0.4314446520  1.7606828667 -1.2696228707 -0.5493092838 
           41            42            43            44            45 
 0.3665838597  0.9747630714  0.1724990684  0.3153591737  0.3270096182 
           46            47            48            49            50 
 0.1375959786  1.1130066017  0.1486137782 -0.0313948697 -0.6325332328 
           51            52            53            54            55 
-0.7621432109 -0.4559245915 -1.0308903021  0.8296521981 -0.3540782368 
           56            57            58            59            60 
 0.4368817473  0.8825766848 -0.2894284560  0.5153727450  0.6927406689 
           61            62            63            64            65 
 0.8757168422  1.2379248779 -0.3084346352 -1.1467473876  0.8416926129 
           66            67            68            69            70 
 0.0852139672  3.5557824589 -0.2909497871 -0.1365657508 -0.7241699259 
           71            72            73            74            75 
-0.2233455817 -1.6304283312 -0.2393262385  0.0056277042 -0.5721968538 
           76            77            78            79            80 
-0.6075621954 -0.8569795104  0.0245347574 -0.8467068254 -1.2018137122 
           81            82            83            84            85 
```

```
 0.2543301850 -0.2602232138  0.2103916942 -0.4923224620  0.2145091603
           86            87            88            89            90
-0.8519350355 -1.1781587624 -1.4968213438  0.3843606141 -1.3546250777
           91            92            93            94            95
-1.0860995251 -0.7135566059  0.4466131454  0.6382281782 -0.1916560624
           96            97            98            99           100
 0.6237448059 -0.5805106397  1.5633713616 -0.2974207109 -0.5046540333
          101           102           103           104           105
 0.2310165068  0.5448559906  1.0485027896 -0.4461839287 -0.3219090987
          106           107           108           109           110
-0.1987231568 -0.5753947995  0.0207423240 -0.9659691601  0.8086049725
          111           112           113           114           115
-0.3074502802 -0.3349095403 -1.0320753686  0.1759915139  0.1343748644
          116           117           118           119           120
 0.1514696258  0.0513090051  0.1420865943 -0.3186785767 -0.9320653322
          121           122           123           124           125
-0.4223906740  0.1770601883 -0.0316066980  0.0488225974  1.5421531872
          126           127           128           129           130
-1.1471039942  0.1227204479 -0.3804465919 -1.0733387298 -0.2345054996
          131           132           133           134           135
-0.4779093878 -0.2716517241 -0.0429995159 -0.3226244038 -1.6504919588
          136           137           138           139           140
-0.5341458488 -0.0296039645 -1.8518037219 -0.9489745321  1.2194623198
          141           142           143           144           145
-1.2216652403 -0.6824257363  0.0219673459 -1.1464298362  0.5383570631
          146           147           148           149           150
 0.3477964129  1.8655858878  0.4026112206 -0.1870604332 -0.6608711289
          151           152           153           154           155
-0.6726188113  0.2877794932 -0.7970299167 -0.5109035573 -0.5971998155
          156           157           158           159           160
-0.8995038511 -0.8139373743 -1.6112614874 -0.8555578348 -0.1539990497
          161           162           163           164           165
-0.2940420153 -0.6388006315 -0.3899654658 -0.1769073518 -0.7073651177
          166           167           168           169           170
-0.8451086483 -0.0018648580 -0.9363846998  1.1981366354 -0.1297368664
          171           172           173           174           175
-0.6114827832 -1.1989890426  0.3819685512  0.4447134132  0.8134320440
          176           177           178           179           180
-0.4877231004 -0.1793980702 -0.1601245444  1.0408050886  0.1403725702
          181           182           183           184           185
-0.2596963985 -0.2535371014  0.2868569565  0.2889570143 -0.6639934315
          186           187           188           189           190
 0.2268836299 -0.0379481445 -0.4927276074 -0.9232016386  1.4189337279
```

191	192	193	194	195
-0.7467322959	0.0334737219	0.2805941034	-0.8886188208	-0.8178301220
196	197	198	199	200
-1.4309334201	-0.3365600101	0.6390728295	-1.7908057068	0.0980122806
201	202	203	204	205
-0.8177641886	-0.1318782118	-1.3409410476	-1.6521476379	-0.1911450776
206	207	208	209	210
-0.5730779799	0.5694878377	1.9514345816	0.4243823727	0.6927980324
211	212	213	214	215
1.6950629436	2.5500958499	0.4075779231	-0.5168459990	-0.1938445114
216	217	218	219	220
-0.1848013646	-0.2848773539	-0.7051822180	-0.7908708029	-0.6058541522
221	222	223	224	225
-0.0697096148	-0.2059993678	0.4454839917	-0.1904458040	0.4335468106
226	227	228	229	230
0.5156941519	1.1797267485	0.5824712341	-0.1441150870	-1.2031024305
231	232	233	234	235
-1.6248576268	0.0931573892	0.0920257118	0.0372313845	-0.1216136459
236	237	238	239	240
2.2604888693	1.0622750620	0.4670039516	-0.3045324963	0.6157343867
241	242	243	244	245
0.1735537139	1.0203771277	-1.6858145127	-0.5769302985	-0.7913243031
246	247	248	249	250
-0.1555922890	-0.5692387398	1.3005698738	1.1094204792	-1.6076003257
251	252	253	254	255
-0.4455895843	-0.4698341643	-0.6996696295	-0.9452231459	-0.7505731568
256	257	258	259	260
0.7410854138	-1.4998120548	-0.7291362701	2.0250931235	0.0496949676
261	262	263	264	265
0.4599520707	0.7367962665	1.4358804418	-0.2063260184	-1.2819861776
266	267	268	269	270
-1.0251649263	-0.5764747047	-0.5144758211	0.0213830265	-0.7196776665
271	272	273	274	275
-0.6105564327	-0.9588528665	-1.9712577088	0.4916736741	0.3016575506
276	277	278	279	280
0.3081410213	-0.5939276936	1.1513721659	0.1441852172	0.3707298689
281	282	283	284	285
0.0382829610	-0.4380601322	-0.6476725378	-0.8317465817	-1.2328039579
286	287	288	289	290
0.5179117225	0.9124208853	0.3879729776	-0.4352920683	-0.5066102452
291	292	293	294	295
-0.4816213501	0.6128495651	-0.4848482051	1.6114409118	-1.6116464913
296	297	298	299	300

```
 0.5668896446  0.2406260831  0.9271930234  2.2275442107  1.9728806847
          301           302           303           304           305
-0.0497167450  0.1535371045  1.3338565013  1.2177933766 -0.2886088982
          306           307           308           309           310
 1.0360084361  0.1290454793 -0.6530487024  0.6186057230 -0.9813673374
          311           312           313           314           315
-1.5305047307  2.0048890893  0.9784482598  1.9147316180  0.3689281539
          316           317           318           319           320
 0.1711822997  1.4552300893 -0.3569184844 -0.3067214892  0.1111465663
          321           322           323           324           325
 0.1757435348  0.2470617118 -0.0725429353 -0.1506687564  1.3442589857
          326           327           328           329           330
 1.2604182414 -0.8767194255 -1.3572041544  0.8238682064 -0.4821027550
          331           332           333           334           335
-1.4854025492 -1.5862799519  0.6370995769  0.3031529494  0.1095079141
          336           337           338           339           340
 0.0509459419  1.1564164526 -1.0694889329 -0.3318212773  0.3231094068
          341           342           343           344           345
 1.9081541047  0.3998250967 -0.6917176178 -0.7044734119 -0.2690849264
          346           347           348           349           350
 1.0550727182 -0.6929955935 -0.5388078614 -0.5708001730 -0.3317239778
          351           352           353           354           355
-0.5930672787  0.9562454887 -0.1139297987  0.4959180493 -0.0801977506
          356           357           358           359           360
 0.5600754994  0.6530174589  0.1442135074  1.4157513209 -0.1546950017
          361           362           363           364           365
-0.3534043760  0.6239204440  0.0285055343 -0.4295296657  0.9840545223
          366           367           368           369           370
 0.8142197380  0.0744557391 -0.6232761830 -0.0328905687  1.0924727312
          371           372           373           374           375
 1.1264750938  1.7178034734  1.2903004504  0.4638205836 -0.0157568430
          376           377           378           379           380
 1.0456043840  0.1381396215 -0.1804300253  1.2160050692  0.0805746272
          381           382           383           384           385
 0.2573770712  0.5058632992 -1.2446737708 -0.8815994155 -0.7061170984
          386           387           388           389           390
-0.7212253850 -0.7258592506 -1.5165232082 -0.7454548152 -3.1561399525
          391           392           393           394           395
-2.5489629280 -2.0680712792 -2.1999919056  0.8268375758 -0.9457522948
          396           397           398           399           400
 1.8953620839  0.1397897032 -1.2146869594 -1.3831825693 -1.3074143041
          401           402           403           404           405
-0.7641296857 -0.4166538542  0.3843155508  0.0809489277 -0.0505469442
```

406	407	408	409	410
-0.7766827987	-0.1360690350	-0.5188639731	-0.5188639731	-0.7413658846
411	412	413	414	415
-0.5156222378	-0.6110983927	1.1720417158	0.1246690130	-1.7378303182
416	417	418	419	420
-1.1171827299	0.0423386540	1.3267348722	-0.7465392107	-0.2713322648
421	422	423	424	425
1.5081257090	1.0536827726	-0.3501117435	-0.7439031000	-0.6142336874
426	427	428	429	430
-0.3093466967	0.4848657651	0.1864731762	1.2070611081	2.2698863331
431	432	433	434	435
1.2818208931	0.9519423202	-0.3543836497	-0.1345939953	-0.2291968007
436	437	438	439	440
-0.0207532199	0.0699595030	0.9839407983	1.1501403380	0.3201374119
441	442	443	444	445
0.0735512229	-0.4454591709	0.6966624246	1.1102047902	1.5443197075
446	447	448	449	450
0.8698748909	-0.2186591118	-0.2813111604	0.1240550070	-0.5736853017
451	452	453	454	455
1.0615472568	1.2412852634	0.4576665972	0.0565480222	-0.9890911057
456	457	458	459	460
0.6016148373	0.6899323394	-1.0992444629	0.3933091929	0.6731657729
461	462	463	464	465
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attr(,"type")
[1] "quantile"
attr(,"class")
[1] "gamlss2.residuals" "numeric"
```

The important issue here is that the distributional assumption of normality is easily rejected by looking at the normal Q-Q plot (bottom right panel, Figure **??**). There is a systematic departure from a linear relationship between the observed (normalized quantile) residuals and their approximate expected values, indicating that the residuals are positively skewed. Note also that the plot of residuals against fitted values (top left panel, Figure **??**) is not randomly scattered about a horizontal line at 0, but fans out, indicating variance heterogeneity, in particular that the variance increases with the mean.

Given that the normal (or Gaussian) assumption is violated because of the positive skewness, we consider the generalized linear model next.