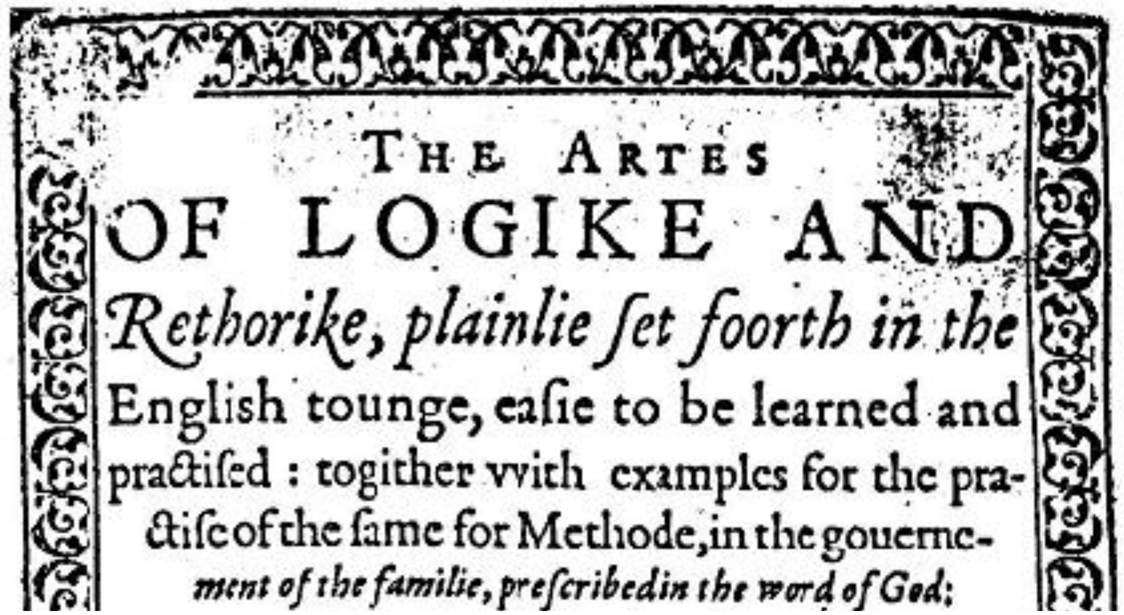

THE ARTES
OF LOGIKE AND
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English tounge, eaſie to be learned and
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ment of the familie, preſcribed in the word of God:

Philosophical Logic

LECTURE SIX | **MICHAELMAS 2017**

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1584

Last week

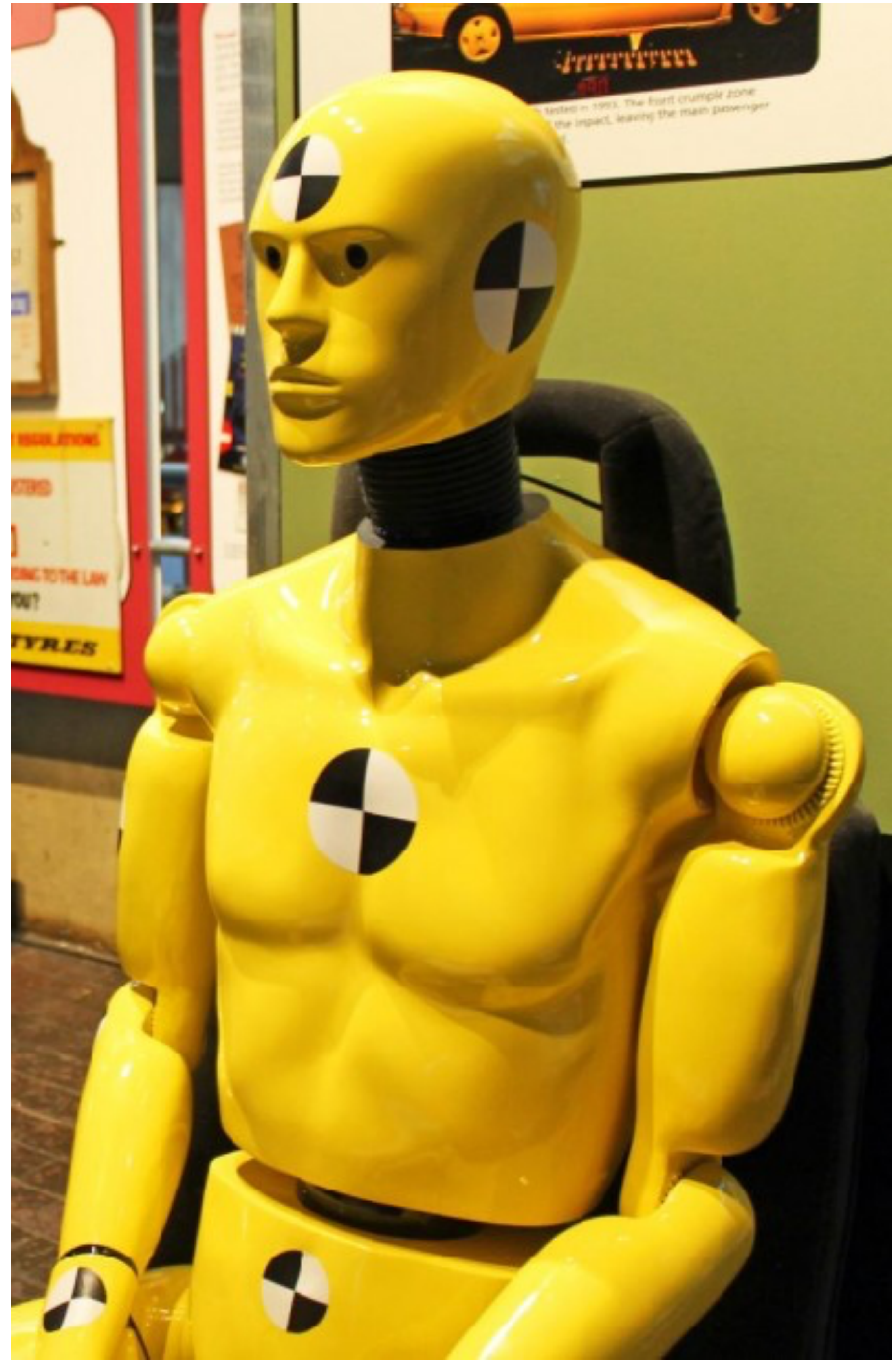
- ~~Lecture 1: **Necessity, Analyticity, and the A Priori**~~
- ~~Lecture 2: **Reference, Description, and Rigid Designation**~~
- ~~Lecture 3: **What Could ‘Meaning’ Mean?**~~
- ~~Lecture 4: **Natural Language**~~
- ~~Lecture 5: **Formal Translations**~~
- Lecture 6: **Conditionals**
- Lecture 7: **Deeper into ‘the’**
- Lecture 8: **Quantification and Existence**

Today

1. Ambiguities of scope
2. Conditionals: indicative and subjunctive
3. The Equivalence Thesis
4. Paradoxes of the material conditional

Ambiguities of scope

Reexamining
some case studies



Quantifier scope ambiguities

Sanna wants to steal some military secrets. A spy warns her: “There is a guard standing in front of every building”.

This of course didn’t worry Sanna at all. She very well knew that no guard can stand in front of more than one building. Sanna concluded: “What that spy tells me must be false!”

Quantifier scope ambiguities

“There is a guard standing in front of every building”

1. $\forall x\, Bx \rightarrow (\exists \mathbf{y}\, Gy \wedge Syx)$
   Existential quantifier takes narrow scope

2. $\exists \mathbf{y}\, \forall x\, Gy \wedge (Bx \rightarrow Syx)$
   Existential quantifier takes wide scope

(G for ‘…is a guard’, B for ‘…is a building’, S for ‘…stands in front of…’)

Quantifier scope ambiguities

"There is a guard standing in front of every building"

1. $\forall x\ Bx \rightarrow (\exists y\ \boxed{Gy \wedge Syx})$
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2. $\exists y\ \boxed{\forall x\ Gy \wedge (Bx \rightarrow Syx)}$
   Existential quantifier takes wide scope

(G for '…is a guard', B for '…is a building', S for '…stands in front of…')

Modal scope ambiguities

1. If I think, then I must exist
2. If I must exist, then I exist in all possible worlds
3. But I think!
4. Therefore, I exist in all possible worlds

*we use '☐' to represent the concept of necessity

Modal scope ambiguities

If I think, then I must exist

1. $\Box (A \rightarrow B)$
   Necessity operator takes wide scope

2. $A \rightarrow (\Box B)$
   Necessity operator takes narrow scope

(A ≡ I think, B ≡ I exist)

*we use '☐' to represent the concept of necessity

Modal scope ambiguities

If I think, then I must exist

1. $\Box\boxed{(A \rightarrow B)}$
   Necessity operator takes wide scope

2. $A \rightarrow (\Box\boxed{B})$
   Necessity operator takes narrow scope

   $(A \equiv$ I think, $B \equiv$ I exist$)$

*we use '☐' to represent the concept of necessity

Conditionals

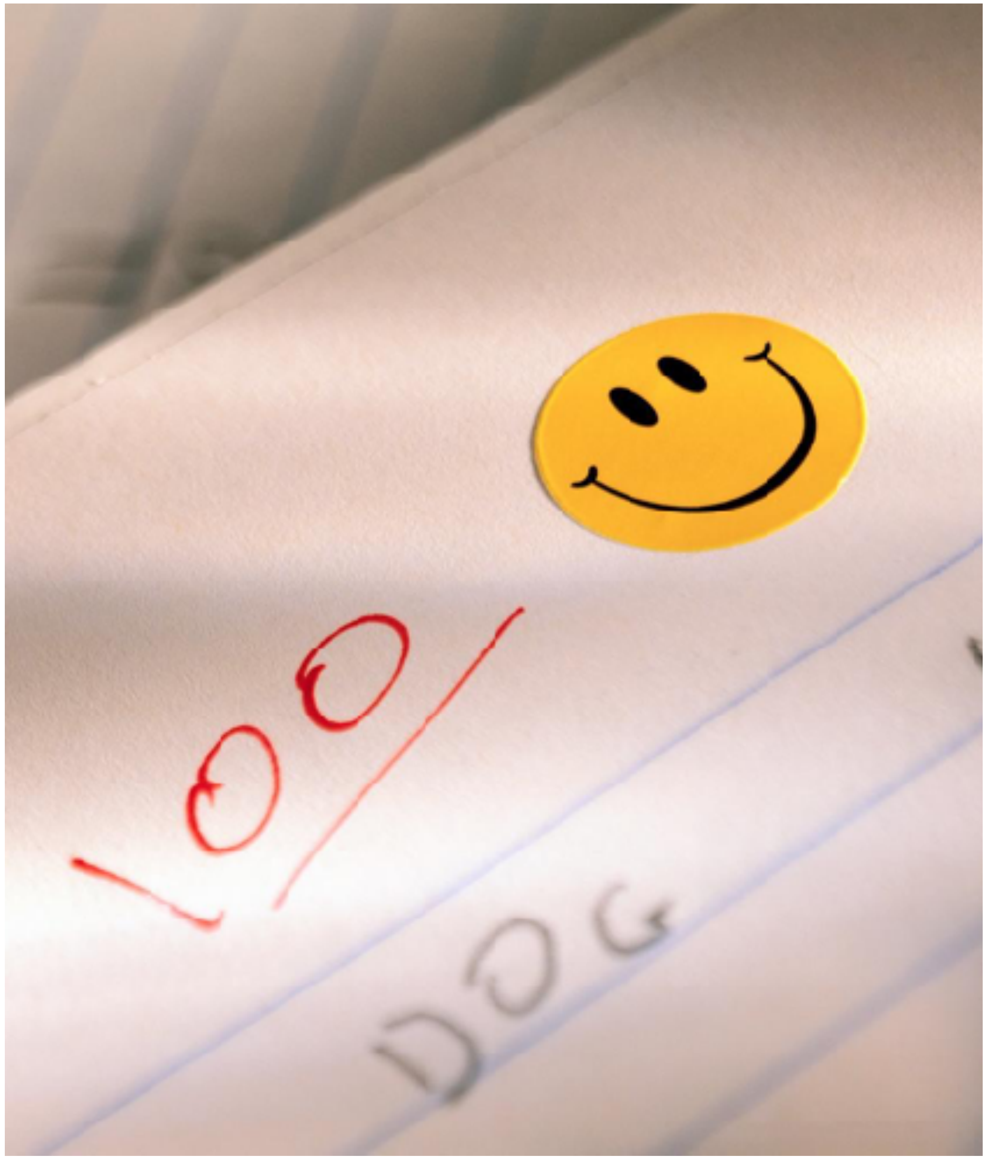
Conditional uses of ‘if’

Contrast:

1. ‘Paula wonders if her argument is valid’

2. ‘If Paula’s argument is valid, she gets full marks’

It is only in (2) that ‘if’ is used to construct a *conditional.* Roughly, it is an ‘if…, then…’ statement



The form of conditionals in English

If Paula's argument is valid, then she gets full marks

Paula's argument is valid	If…, then…	[Paula] gets full marks
Antecedent	***Connective***	***Consequent***

Compare with the material conditional

$A \rightarrow B$

$A \mid \rightarrow \mid B$

Indicative vs Subjunctive

1. If Oswald didn’t shoot Kennedy, someone else did

2. If Oswald hadn’t shot Kennedy, someone else would have

   Difference in mood: (1) *indicative*; (2) *subjunctive*

   Apparent difference in truth conditions: (1) seems clearly true, (2) doesn’t seem clearly true

   (2) is also called a *counterfactual* conditional

Indicative conditionals

- **‘If Paula’s argument is valid, then she gets full marks’**
- **‘If Paula’s argument is valid, then her argument is sound’**
- Indicative conditionals in English present an antecedent claim as a sufficient condition for a consequent claim
- Indicative conditionals in English appear to be truth-functional (the truth value of a complex statement is determined by the truth values of its component claims)

1. Socrates is a man.
2. All men are carrots.

So: Therefore, Socrates is a carrot.

The Equivalence Thesis (ET)

‘if.. then…’, and →

- **Equivalence Thesis:** the truth conditions for the indicative conditional (if…,then…) are the truth conditions for the material conditional (→)

- This implies that an indicative conditional is true unless its antecedent is true and its consequent is false

A	B	A→B	If A then B
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Logical form

- It also implies that an indicative conditional behaves just as the exclusive disjunction (or ∨)
- So the Equivalence Thesis takes the *logical form* of the indicative conditional to be that of A→B
- Logical form is what is left of a sentence once we abstract away from its subject matter, and leave its internal logical structure

A	¬A	B	¬A ∨ B	A→B	If A then B
T	F	T	T	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T

Argument for ET

1. The indicative conditional is truth-functional

2. If the indicative conditional is truth-functional, then the logical form of the indicative conditional is that of $A \rightarrow B$

3. Therefore, the logical form of the indicative conditional is that of $A \rightarrow B$

‘if.. then...’, and $\rightarrow$

A	B	$A \rightarrow B$	$A \vee B$	$A \wedge B$	
T	T	T	T	T	**If Paula’s argument is valid, then she gets full marks**
T	F	F	T	F	**If Paula’s argument is valid, then her argument is sound**
F	T	T	T	F	**If Trump doesn’t use Twitter, then Paris is in France**
F	F	T	F	F	**If pigs can fly, then the moon is made of cheese**

“If ‘if’ is truth-functional, this is the right truth function to assign to it: of the sixteen possible truth-functions of A and B, it is the only serious candidate…”

– Dorothy Edgington (SEP: ‘Conditionals’)

Paradoxes of the material conditional

Potential problem cases

Contraposition
If Corbyn has majority support, it isn't a large majority. Therefore, if Corbyn is supported by a large majority, he doesn't have majority support.
($A \to B \vdash \neg B \to \neg A$)

Antecedent Strengthening
If I strike this match then it will light. Therefore, if I pour water on this match and strike it, it will light.
($A \to C \vdash (A \wedge B) \to C$)

A	B	A→B
T	T	T
T	F	F
F	T	T
F	F	T

Serious problem cases

A	B	A→B
T	T	T
T	F	F
F	T	T
F	F	T

Ice is not warmer than water. Therefore, if ice is warmer than water, ice is not warmer than water ($\neg A \vdash A \rightarrow B$)

Ice floats on water. Therefore, if ice is denser than water, ice floats on water ($B \vdash A \rightarrow B$)

Argument against ET

1. If the indicative conditional is truth-functional, then the logical form of the indicative conditional is that of A→B

2. The logical form of the indicative conditional is not that of A→B

3. Therefore, the indicative conditional is not truth-functional

More than logical?

- The indicative conditional is used to express more than a logical connection between antecedent and consequent

- Take 'If Paula's argument is valid, then she gets full marks'

  We can only correctly say this, if the validity of her argument *causes* or *explains* Paula's getting the full marks. The statement is true only if that further connection holds

- What goes wrong in the previous inferences is that we do not take this further connection into account

Non-truth-functional interpretation

Edgington: “Non-truth-functional accounts agree that ‘If A, B’ is false when A is true and B is false; and they agree that the conditional is sometimes true for the other three combinations of truth-values for the components; but they deny that the conditional is always true in each of these three cases.”

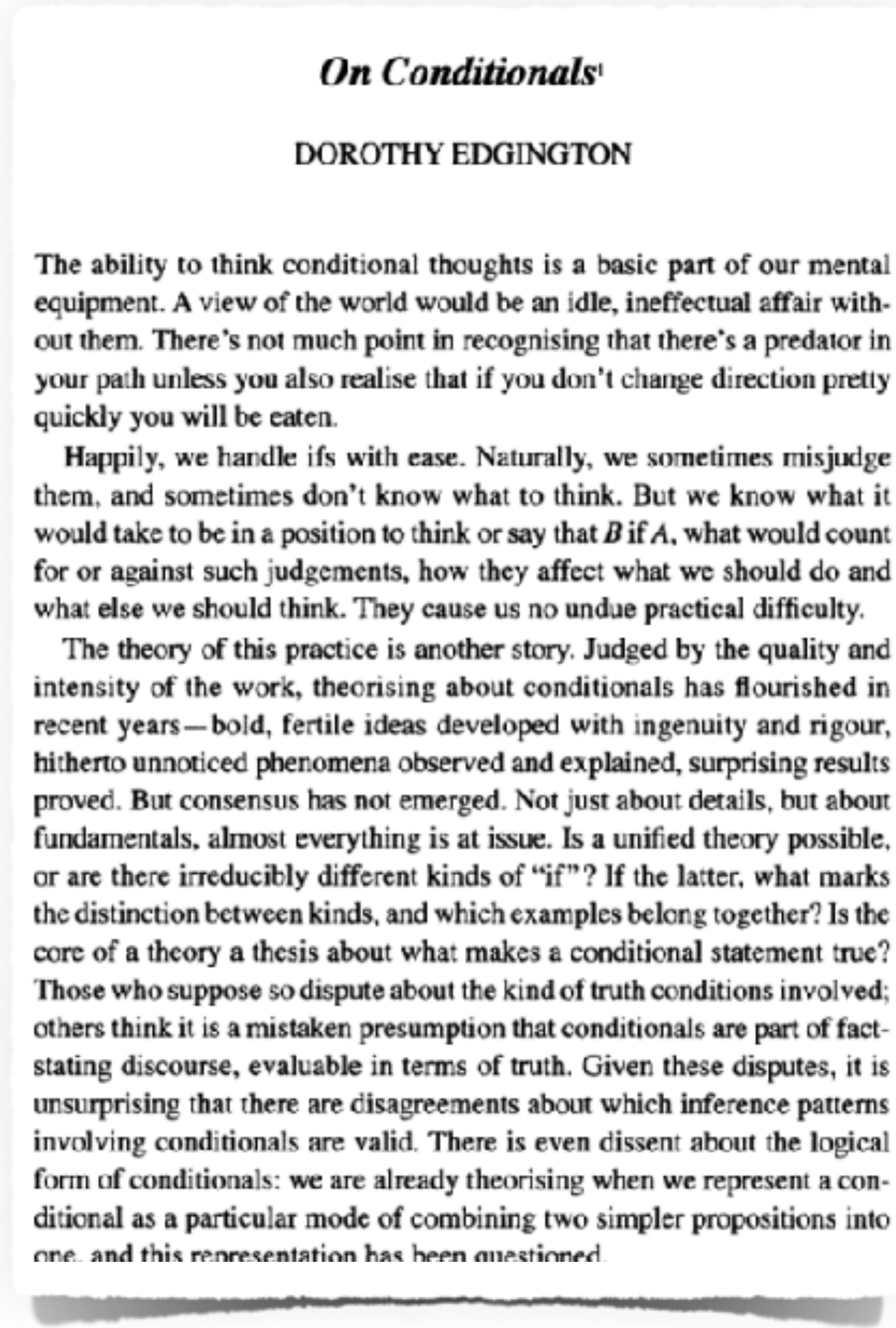
A	B	A→B	If A then B
T	T	T	T
T	F	F	F
F	T	T	T/F
F	F	T	T/F

Two directions

- Either we follow Edgington, and accept that ‘if…, then…’ constructions in English are not truth-functional

- In that case we have to find a non-truth-functional interpretation of the correctness of inferences like:

  If Paula’s argument is valid, then she gets full marks. But look, her argument is valid, and so she gets full marks.



On Conditionals[1]

DOROTHY EDGINGTON

The ability to think conditional thoughts is a basic part of our mental equipment. A view of the world would be an idle, ineffectual affair without them. There’s not much point in recognising that there’s a predator in your path unless you also realise that if you don’t change direction pretty quickly you will be eaten.

Happily, we handle ifs with ease. Naturally, we sometimes misjudge them, and sometimes don’t know what to think. But we know what it would take to be in a position to think or say that *B* if *A*, what would count for or against such judgements, how they affect what we should do and what else we should think. They cause us no undue practical difficulty.

The theory of this practice is another story. Judged by the quality and intensity of the work, theorising about conditionals has flourished in recent years—bold, fertile ideas developed with ingenuity and rigour, hitherto unnoticed phenomena observed and explained, surprising results proved. But consensus has not emerged. Not just about details, but about fundamentals, almost everything is at issue. Is a unified theory possible, or are there irreducibly different kinds of “if”? If the latter, what marks the distinction between kinds, and which examples belong together? Is the core of a theory a thesis about what makes a conditional statement true? Those who suppose so dispute about the kind of truth conditions involved; others think it is a mistaken presumption that conditionals are part of fact-stating discourse, evaluable in terms of truth. Given these disputes, it is unsurprising that there are disagreements about which inference patterns involving conditionals are valid. There is even dissent about the logical form of conditionals: we are already theorising when we represent a conditional as a particular mode of combining two simpler propositions into one, and this representation has been questioned.

Two directions

- On the other hand, we can also employ Grice's framework and say that *what is said* in 'if…, then…' statements is always truth-functional, but that this does leave room for further *implicatures*

- Suppose I believe that the match will be cancelled anyway, and I tell you that *if it rains, the match will be cancelled*

- What I say can indeed be asserted based on what I believe. However, by uttering just '*If it rains, the match will be cancelled'* I also imply (conventionally) that the condition of rain is somehow responsible if the match will be cancelled. So I have said something misleading

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