Church's argument against the verification principle

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Definitions

- **VERIFICATION PRINCIPLE:** A non-analytic, non-contradictory sentence S is empirically meaningful iff S expresses a statement that is either directly or indirectly verifiable.

- **D1**: S is directly verifiable iff (a) S is an observation statement; or (b) S by itself, or in conjunction with one or more observation statements P, Q, R, . . . , logically entails an observation statement that is not entailed by P, Q, R, . . . alone.

- **D2:** S is indirectly verifiable iff (a) S by itself, or in conjunction with other premises P, Q, R, . . . , logically entails a directly verifiable statement D that is not entailed by P, Q, R, . . . alone; and (b) the other premises P, Q, R, ... , are all either analytic, directly verifiable, or can be shown independently to be indirectly verifiable.

Instrumental proofs:

We are going to prove that the negation of a directly verifiable statement is always indirectly verifiable. This will be useful later on. The proof has two steps.

Step One
We will show that where O is any observation statement and Q is any directly verifiable statement, O→Q is always meaningful.

1. Let O be an observation statement and Q be any directly verifiable statement
2. O→Q + O entails Q
3. O either by itself entails Q, or O doesn't by itself entail Q
4. If O does entail Q by itself, then O→Q is analytic, and so meaningful
5. If O doesn't by itself entail Q, then O→Q is directly verifiable (by D1), and so meaningful

Step Two
We will show that the negation of any directly verifiable statement is meaningful.

1. Let O and ¬O be observation statements, where ¬O is not entailed by directly verifiable statement Q
2. In Step One we proved that O→Q is either directly verifiable or analytic
3. ¬Q together with O→Q entails ¬O
4. By hypothesis, ¬O is not entailed by Q alone
5. So ¬O is not entailed by ¬O v Q alone (¬O v Q entails ¬O iff ¬O entails ¬O and Q entails ¬O)
6. But ¬O v Q ≡ O→Q
7. So ¬O is not entailed by O→Q alone
8. So ¬Q is indirectly verifiable (by D2)

Hence, if Q is directly verifiable, then ¬Q is indirectly verifiable.

Now, on to a reconstruction of Church's argument.

> It would seem, however, that the amended definition of verifiability is open to nearly the same objection as the original definition. For let O_1, O_2, O_3 be three "observation-statements" (or "experiential propositions") such that no one of the three taken alone entails any of the others. Then using these we may show of any statement S whatever that either it or its negation is verifiable, as follows. Let $\bar{O}_1$ and $\bar{S}$ be the negations of O_1 and S respectively. Then (under Ayer's definition) $\bar{O}_1O_2 \vee O_3\bar{S}$ is directly verifiable, because with O_1 it entails O_3. Moreover S and $\bar{O}_1O_2 \vee O_3\bar{S}$ together entail O_2. Therefore (under Ayer's definition) S is indirectly verifiable—unless it happens that $\bar{O}_1O_2 \vee O_3\bar{S}$ alone entails O_2, in which case $\bar{S}$ and O_3 together entail O_2, so that $\bar{S}$ is directly verifiable.

From: Alonzo Church (1949) 'Review of Ayer's *Language, Truth and Logic*'

Church's argument spelled out[1]

1. Let P, Q, R be observation sentences that are logically independent
2. Let S be any sentence you like ('Time is a vortex channelling the Absolute')
3. Let X be the sentence (¬P & Q) ∨ (R & ¬S)
4. X is directly verifiable, because X together with P entails R, where R is not entailed by P alone (cf. D1)

Now, clearly either X does not entail Q or X does entail Q. We will now show that S comes out as meaningful in either of there two conditions:

Condition one

1. Assume X does not entail Q
2. Note, X together with S do entail Q
3. X is directly verifiable (see above)
4. So, S is indirectly verifiable, according to Ayer's definition (D1)

Condition two

1. Assume X entails Q
2. This means that ¬P & Q entails Q, and that R & ¬S entails Q
3. But this means that R together with ¬S entail Q, while R alone does not entail Q
4. So, ¬S is directly verifiable (by D2)

   Church ends his argument here, but it seems that we need more to show that the verification principle is trivial. (Or don't we? Consider, should a verificationist be happy to accept that either S or ¬S is meaningful, but that we just don't know which?)

   To go beyond Church's disjunctive conclusion, we can add the following lines:

5. The negation of a directly verifiable statement is always indirectly verifiable (Step Two)
6. So, ¬¬S is indirectly verifiable
7. But ¬¬S ≡ S
8. So, S is indirectly verifiable

And so we have shown that for any sentence S, S is indirectly verifiable. (In the end, how different is this argument from Hempel's objection?)

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[1] I'm not going to copy Church's notation; I'll get rid of the subscripts, so that 'O_1, O_2, O_3' becomes 'P, Q, R'. And I'll use standard notation for negation, so that 'Ō' becomes '¬O'. The reconstruction here is based on Soames' (2003) presentation (pp. 289ff).