Sequences and Series IB Problems Worksheet Name Maksim Stepnov

1.. Find the sum of the arithmetic series 17 + 27 + 37 +...+ 417.

$417 = 17 + (n-1)10$ $\quad 410 = 10n$ $\quad n = 41$ $\quad S_{41} = 41\left(\frac{17+417}{2}\right) = 8897$

$417 = 17 + 10n - 10$

2. Gwendolyn added the multiples of 3, from 3 to 3750 and found that $3 + 6 + 9 + \ldots + 3750 = s$. Calculate s.

$3750 = 3 + (n-1)3$ $\quad n = 1250$

$3750 = 3 + 3n - 3$ $\quad S_{1250} = 1250\left(\frac{3+3750}{2}\right) = 2345625$

$3750 = 3n$

3. The second term of an arithmetic sequence is 7. The sum of the first four terms of the arithmetic sequence is 12. Find the first term, a, and the common difference, d, of the sequence.

$7 = a + d$ $\quad 12 = a + (a+d) + (a+2d) + (a+3d)$ $\quad 12 = 28 - 4d + 6d$ $\quad a = 7 - (-8) = 15$

$a = 7 - d$ $\quad 12 = 4a + 6d$ $\quad -16 = 2d$

$d = -8$

4. Find the sum of the positive terms of the arithmetic sequence 85, 78, 71, ...

$a_n = 85 + (n-1)(-7)$ $\quad \frac{85}{7} > n-1$ $\quad a_{12} = 85 + (12-1)\cdot -7 =$

$85 + (n-1)(-7) > 0$ $\quad = 85 - 77 = 8$ $\quad S_{12} = 12\left(\frac{85+8}{2}\right) = 558$

$85 > -7(n-1)$ $\quad 13.14 > n$

5. Consider the infinite geometric series $1 + \left(\frac{2x}{3}\right) + \left(\frac{2x}{3}\right)^2 + \left(\frac{2x}{3}\right)^3 + \ldots\ldots$

(a) For what values of x does the series converge? $-1 < \frac{2x}{3} < 1$ $\quad -\frac{3}{2} < x < \frac{3}{2}$

(b) Find the sum of the series if $x = 1.2$. $\quad -1 < \frac{2x}{3}$ $\quad -\frac{3}{2} < x$

$\frac{2\cdot 1.2}{3} = 0.8$ $\quad \left(\frac{2\cdot 1.2}{3}\right)^2 = 0.64$ $\quad r = 0.8$ $\quad S_\infty = \frac{1}{1-0.8} = 5$

6. A geometric sequence has all positive terms. The sum of the first two terms is 15 and the sum to infinity is 27. Find the value of

(a) the common ratio; $\quad r = \frac{3}{2}$ $\quad a + ar = 15$ $\quad 27 = \frac{a}{1-r}$ $\quad 27(1-r) = a$ $\quad -27r^2 + 27 = 15$

$a(1+r) = 15$ $\quad 27 - 27r = \frac{15}{1+r}$ $\quad -27r^2 = 12$

(b) the first term. $\quad a = \frac{15}{1+r}$ $\quad (27 - 27r)(1+r) = 15$ $\quad r^2 = \sqrt{\frac{12}{27}}$

$a = \frac{15}{1+2/3} = \frac{15}{5/3} = \frac{45}{5} = 9$ $\quad r = \sqrt{\frac{4}{9}} = \boxed{\frac{3}{2}}$

7. The first four terms of an arithmetic sequence are 2, $a - b$, $2a + b + 7$, and $a - 3b$, where a and b are constants. Find a and b.

$a - b - 2 = d$ $\quad \begin{cases} a - b - 2 = a + 2b + 7 \\ a - b - 2 = -a - 4b - 7 \end{cases}$ $\quad -3b - 9 = 0$ $\quad 2a + 3b + 5 = 0$

$a + 2b + 7 = d$ $\quad b = -3$ $\quad 2a - 4 = 0$

$-a - 4b - 7 = d$ $\quad 2a = 4$ $\quad a = 2$

8. An infinite geometric series is given by $\sum_{k=1}^{\infty} 2(4-3x)^k$. Find the values of x for which the series has a finite sum.

$r = 4 - 3x$ $\quad a = 2(4-3x)$

$-1 < 4 - 3x < 1$

$1 < x < \frac{5}{3}$

$4 - 3x < 1$ $\quad$ ~~4-3x<1~~

$-3x < -3$ $\quad 4 - 3x > -1$

$x > 1$ $\quad$ ~~-3x>-3~~ $-3x > -5$

$x < \frac{5}{3}$

9. The *Acme* insurance company sells two savings plans, Plan A and Plan B.

For Plan A, an investor starts with an initial deposit of \$1000 and increases this by \$80 each month, so that in the second month, the deposit is \$1080, the next month it is \$1160 and so on. For Plan B, the investor again starts with \$1000 and each month deposits 6% more than the previous month.

(a) Write down the amount of money invested under Plan B in the second and third months.

$M_1 = 1000 \quad M_2 = 1000 \cdot 1.06 = \underline{1060} \quad M_3 = 1000 \cdot (1.06)^2 = \underline{1124}$

(2)

Give your answers to parts (b) and (c) correct to the nearest dollar.

(b) Find the amount of the 12th deposit for each Plan.

$a_1 = 1000 \quad d = 80$

$a_{12} = 1000 + (12-1) \cdot 80 = \underline{1880}$

$a_{12} = 1000 \cdot (1.06)^{11} = \underline{1898}$

(4)

(c) Find the total amount of money invested during the first 12 months

(i) under Plan A; $S_{12} = \frac{12}{2}(2 \cdot 1000 + 11 \cdot 80) = \underline{17280}$

(2)

(ii) under Plan B.

$S_{12} = 1000 \cdot \frac{(1.06)^{12} - 1}{0.06} = 16870$

(2)

(Total 10 marks)

10. (a) Consider the geometric sequence −3, 6, −12, 24, ….

(i) Write down the common ratio. $r = \frac{6}{-3} = \underline{-2}$

(ii) Find the 15th term. $a_{15} = -3(-2)^{14} = \underline{-49152}$

(3)

Consider the sequence $x - 3$, $x + 1$, $2x + 8$, ….

(b) When $x = 5$, the sequence is geometric.

(i) Write down the first three terms. $\underline{2;\ 6;\ 18}$

(ii) Find the common ratio. $r = \frac{6}{2} = \underline{3}$

(2)

(c) Find the other value of x for which the sequence is geometric.

$\frac{x+1}{x-3} = \frac{2x+8}{x+1}$

$(x+1)^2 = (x-3)(2x+8)$

$x^2 + 2x + 1 = 2x^2 + 2x - 24$

$x^2 - 25 = 0$

$\cancel{x = \pm 5}$

$\underline{x = -5}$

(4)

(d) For this value of x, find

(i) the common ratio; $r = \frac{-4}{-8} = \underline{\frac{1}{2}}$

(ii) the sum of the infinite sequence.

$S_\infty = \frac{-8}{1 - 1/2} = -8 \cdot 2 = \underline{-16}$

(3)

(Total 12 marks)