Applicative Regular Expressions w/ the Free Alternative

Justin Le https://blog.jle.im

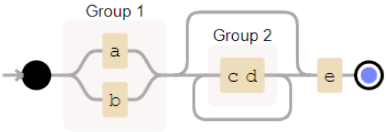
C∘mp∘se 2019, June 24

Preface

Slide available at https://talks.jle.im.

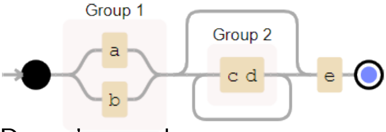
Regular Expressions

`(a|b)(cd)*e`



Regular Expressions

`(a|b)(cd)*e`



Matches:

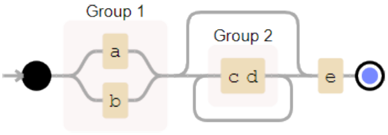
- ae
- acdcdcde
- bcde

Doesn't match:

- acdcd
- abcde
- bce

“Captures”

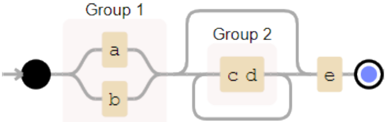
`(a|b)(cd)*e`



“Captures”

`(a|b)(cd)*e`

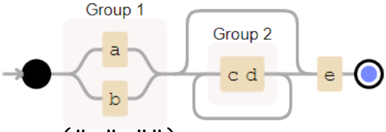
- ae →
- acdcdcde →
- bcde →



- `("a","")`

“Captures”

`(a|b)(cd)*e`

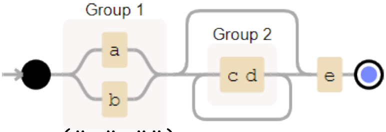


- ae →
- acdcdcde →
- bcde →

- `("a","")`
- `("a","cdcdcd")`

“Captures”

`(a|b)(cd)*e`



- ae →
- acdcdcde →
- bcde →

- `("a","")`
- `("a","cdcdcd")`
- `("b","cd")`

Applicative Regular Expressions

"Type-indexed" regular expressions.

```
type Regexp a
        -- ^ type of "result"
```

Applicative Regular Expressions

"Type-indexed" regular expressions.

```
type Regexp a
        -- ^ type of "result"
```

```
char   :: Char   -> RegExp Char
string :: String -> RegExp String
```

Applicative Regular Expressions

"Type-indexed" regular expressions.

```
type Regexp a
         -- ^ type of "result"

char   :: Char   -> RegExp Char
string :: String -> RegExp String

runRegexp :: RegExp a -> String -> Maybe a
```

Applicative Regular Expressions

```
char   :: Char      -> RegExp Char
string :: String    -> RegExp String
(<|>)  :: RegExp a -> RegExp a -> RegExp a
many   :: RegExp a -> RegExp [a]

myRegexp :: RegExp (Char, [String])
myRegexp = (,) <$> (char 'a' <|> char 'b')
               <*> many (string "cd")
               <*  char 'e'
```

Applicative Regular Expressions

```
char   :: Char     -> RegExp Char
string :: String   -> RegExp String
(<|>)  :: RegExp a -> RegExp a -> RegExp a
many   :: RegExp a -> RegExp [a]

myRegexp :: RegExp (Char, [String])
myRegexp = (,) <$> (char 'a' <|> char 'b')
               <*> many (string "cd")
               <*  char 'e'

runRegexp myRegexp :: String -> Maybe (Char, [String])
```

Applicative Regular Expressions

```
runRegexp myRegexp "ae"
Just ('a', [])

runRegexp myRegexp "acdcdcde"
Just ('a', ["cd","cd","cd"])

runRegexp myRegexp "bcde"
Just ('b', ["cd"])

runRegexp myRegexp "acdcd"
Nothing
```

Applicative Regular Expressions

```
myRegexp2 :: RegExp (Bool, Int)
myRegexp2 = (,) <$> ((False <$ char 'a') <|> (True <$ char
                <*> fmap lengnth (many (string "cd"))
                <*  char 'e'
```

Applicative Regular Expressions

```
myRegexp2 :: RegExp (Bool, Int)
myRegexp2 = (,) <$> ((False <$ char 'a') <|> (True <$ char
                <*> fmap lengnth (many (string "cd"))
                <*  char 'e'

runRegexp myRegexp2 "ae"
Just (False, 0)

runRegexp myRegexp2 "acdcdcde"
Just (False, 3)

runRegexp myRegexp2 "bcde"
Just (True, 1)
```

What's so Regular?

What's so Regular?

Regular Language Base Members

1. Empty set: Always fails to match
2. Empty string: Always succeeds, consumes nothing
3. Literal: Matches and consumes a given char

What's so Regular?

Regular Language Base Members

1. Empty set: Always fails to match
2. Empty string: Always succeeds, consumes nothing
3. Literal: Matches and consumes a given char

Regular Language Operations

1. Concatenation: `RS`, sequence one after the other
2. Alternation: `R|S`, one or the other
3. Kleene Star: `R*`, the repetition of `R`

An Alternative Perspective

An Alternative Perspective

```
class Functor f => Applicative f where
    -- | Always succeed, consuming nothing
    pure  :: a -> f a
    -- | Concatenation
    (<*>) :: f (a -> b) -> f a -> f b
```

An Alternative Perspective

```
class Functor f => Applicative f where
    -- | Always succeed, consuming nothing
    pure  :: a -> f a
    -- | Concatenation
    (<*>) :: f (a -> b) -> f a -> f b

class Applicative f => Alternative f where
    -- | Always fails to match
    empty :: f a
    -- | Alternation
    (<|>) :: f a -> f a -> f a
    -- | Reptition
    many  :: f a -> f [a]
```

An Alternative Perspective

1. Empty set: `empty`

An Alternative Perspective

1. Empty set: `empty`
2. Empty string: `pure x`

An Alternative Perspective

1. Empty set: `empty`
2. Empty string: `pure x`
3. Literal: ???

An Alternative Perspective

1. Empty set: `empty`
2. Empty string: `pure x`
3. Literal: ???
4. Concatenation: `<*>`

An Alternative Perspective

1. Empty set: `empty`
2. Empty string: `pure x`
3. Literal: ???
4. Concatenation: `<*>`
5. Alternation: `<|>`

An Alternative Perspective

1. Empty set: `empty`
2. Empty string: `pure x`
3. Literal: ???
4. Concatenation: `<*>`
5. Alternation: `<|>`
6. Repetition: `many`

Functor combinator-style

- Define a primitive type

```
type Prim a
```

Functor combinator-style

- Define a primitive type

  ```
  type Prim a
  ```

- Add the structure you need

Functor combinator-style

- Define a primitive type

```
type Prim a
```

- Add the structure you need
  - If this structure is from a typeclass, use the free structure of that typeclass

Easy as 1, 2, 3

```
data Prim a = Prim Char a
  deriving Functor

data Alt :: (Type -> Type) -> (Type -> Type)
          -- ^ take a Functor
                                  -- ^ return a Functor

type RegExp = Alt Prim

liftAlt :: Prim a -> Alt Prim

char :: Char -> RegExp Char
char c = liftAlt (Prim c c)
```

Unlimited Power

```
empty :: RegExp a
pure  :: a -> RegExp a
char  :: Char -> RegExp Char
(<*>) :: RegExp (a -> b) -> RegExp a -> RegExp b
(<|>) :: RegExp a -> RegExp a -> RegExp a
many  :: RegExp a -> RegExp [a]
```

Unlimited Power

```
empty :: RegExp a
pure  :: a -> RegExp a
char  :: Char -> RegExp Char
(<*>) :: RegExp (a -> b) -> RegExp a -> RegExp b
(<|>) :: RegExp a -> RegExp a -> RegExp a
many  :: RegExp a -> RegExp [a]

string :: String -> RegExp String
string = traverse char

digit :: RegExp Int
digit = asum [ intToDigit i <$ char i | i <- [0..9] ]
```

Parsing

Options:

1. *Interpret* into an `Alternative` instance, “offloading” the logic

Parsing

Options:

1. *Interpret* into an `Alternative` instance, "offloading" the logic
2. Direct pattern match on structure constructors (Haskell 101)

What is freeness?

What is freeness?

```haskell
type FreeMonoid = []

injectFM :: a -> FreeMonoid a
runFM    :: Monoid m => (a -> m) -> (FreeMonoid a -> m)
```

What is freeness?

```haskell
type FreeMonoid = []

injectFM :: a -> FreeMonoid a
runFM    :: Monoid m => (a -> m) -> (FreeMonoid a -> m)

(:[])    :: a -> [a]
foldMap :: Monoid m => (a -> m) -> ([a] -> m)
```

What is freeness?

```haskell
myMon :: FreeMonoid Int
myMon = [1] <> [2] <> [3] <> [4]
```

What is freeness?

```
myMon :: FreeMonoid Int
myMon = [1] <> [2] <> [3] <> [4]
```

```
foldMap Sum myMon
Sum 10
```

What is freeness?

```
myMon :: FreeMonoid Int
myMon = [1] <> [2] <> [3] <> [4]
```

```
foldMap Sum myMon
Sum 10
```

```
foldMap Product myMon
Product 24
```

What is freeness?

```
myMon :: FreeMonoid Int
myMon = [1] <> [2] <> [3] <> [4]
```

```
foldMap Sum myMon
Sum 10
```

```
foldMap Product myMon
Product 24
```

```
foldMap Max myMon
Max 4
```

What is freeness?

```
type Alt a

liftAlt :: f a -> Alt f a
runAlt  :: Alternative g
        => (forall b. f a -> g a)
        -> (Alt f a -> g a)
```

Hijacking StateT

`StateT [Char] Maybe`

- `Prim a` can be interpreted as *consumption*
- `<*>` sequences consumption
- `<|>` is backtracking

Hijacking StateT

```haskell
processPrim :: Prim a -> StateT String Maybe a
processPrim (Prim c x) = do
    d:ds <- get                 -- ^ match on stream
    guard (c == d)              -- ^ fail unless match
    put ds                      -- ^ update stream
    return x                    -- ^ return result value

matchPrefix :: Regexp a -> String -> Maybe a
matchPrefix re = evalStateT (runAlt processPrim re)
```

This works?

This works?

Yes!

```
matchPrefix myRegexp2 "ae"
Just (False, 0)

matchPrefix myRegexp2 "acdcdcde"
Just (False, 3)

matchPrefix myRegexp2 "bcde"
Just (True, 1)
```

What just happened?

What just happened?

```
data Prim a = Prim Char a
  deriving Functor

type RegExp = Alt Prim

matchPrefix :: RegExp a -> String -> Maybe a
matchPrefix re = evalStateT (runAlt processPrim re)
  where
    processPrim (Prim c x) = do
      d:ds <- get
      guard (c == d)
      put ds
      pure x
```

What just happened?

1. Offload `Alternative` functionality to `StateT`: `empty`, `<*>`, `pure`, `empty`, `many`.

What just happened?

1. Offload `Alternative` functionality to `StateT`: `empty`, `<*>`, `pure`, `empty`, `many`.
2. Provide `Prim`-processing functionality with `processPrim`: `liftAlt`.

What do we gain?

1. Interpretation-invariant structure

What do we gain?

1. Interpretation-invariant structure
2. Actually meaningful types

What do we gain

`StateT String Maybe` is **not** a regular expression type.

```
notARegexp :: StateT String Maybe ()
notARegexp = put "hello"        -- no regular expression
```

What do we gain

`StateT String Maybe` is **not** a regular expression type.

```
notARegexp :: StateT String Maybe ()
notARegexp = put "hello"         -- no regular expression
```

`Alt Prim` **is** a regular expression type

Direct matching

```
newtype Alt f a = Alt { alternatives :: [AltF f a] }

data AltF f a = forall r. Ap (f r) (Alt f (r -> a))
              |           Pure a
```

Direct matching

```haskell
newtype Alt f a = Alt { alternatives :: [AltF f a] }

data AltF f a = forall r. Ap (f r) (Alt f (r -> a))
              |           Pure a

-- | Chain of <|>s
newtype Alt f a
    =           Choice (AltF f a) (Alt f a      )  -- ^ c
    |           Empty                              -- ^ n

-- | Chain of <*>s
data AltF f a
    = forall r. Ap     (f r     ) (Alt f (r -> a))  -- ^ c
    |           Pure a                              -- ^ n
```

Direct Matching

```
matchAlts :: RegExp a -> String -> Maybe a
matchAlts (Alt res) xs = asum [ matchChain re xs | re <- re
```

Direct Matching

```
matchAlts :: RegExp a -> String -> Maybe a
matchAlts (Alt res) xs = asum [ matchChain re xs | re <- re

matchChain :: AltF Prim a -> String -> Maybe a
matchChain (Ap (Prim c x) next) cs = _
matchChain (Pure x)             cs = _
```

One game of Type Tetris later...

```
matchChain :: AltF Prim a -> String -> Maybe a
matchChain (Ap (Prim c x) next) cs = case cs of
    []  -> Nothing
    d:ds | c == d    -> matchAlts (($ x) <$> next) ds
         | otherwise -> Nothing
matchChain (Pure x)             _      = Just x
```

This works?

This works?

Yes!

```
matchChain myRegexp2 "ae"
Just (False, 0)

matchChain myRegexp2 "acdcdcde"
Just (False, 3)

matchChain myRegexp2 "bcde"
Just (True, 1)
```

What do we gain?

- First-class program rewriting, Haskell 101-style

What do we gain?

- First-class program rewriting, Haskell 101-style
- **Normalizing** representation

What do we gain?

- First-class program rewriting, Haskell 101-style
- **Normalizing** representation
  - Equivalence in meaning = equivalence in structure

What do we gain?

- First-class program rewriting, Haskell 101-style
- **Normalizing** representation
  - Equivalence in meaning = equivalence in structure

What do we gain?

- First-class program rewriting, Haskell 101-style
- **Normalizing** representation
  - Equivalence in meaning = equivalence in structure

```
-- | Not regularizing
data RegExp a = Empty
              | Pure a
              | Prim Char a
              | forall r. Seq (RegExp a) (RegExp (r -> a))
              | Union (RegExp a) (RegExp a)
              | Many (RegExp a)

-- a|(b|c) /= (a|b)|c
```

Free your mind

Is this you?

> *"My problem is modeled by some (commonly occurring) structure over some primitive base."*

- Use a "functor combinator"!

Free your mind

Is this you?

> *"My problem is modeled by some (commonly occurring) structure over some primitive base."*

- Use a "functor combinator"!
- If your structure comes from a typeclass, use a free structure!

Further Reading

- Blog post:
  https://blog.jle.im/entry/free-applicative-regexp.html
- Functor Combinatorpedia:
  https://blog.jle.im/entry/functor-combinatorpedia.html
- Slides:
  https://talks.jle.im/composeconf-2019/free-alternative.md