Applicative Regular Expressions w/ the Free Alternative

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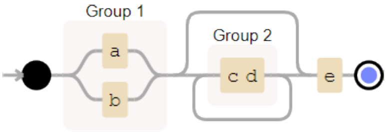
Preface

Slide available at https://talks.jle.im.

Regular Expressions

`(a|b)(cd)*e`



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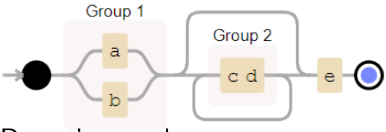


Matches:

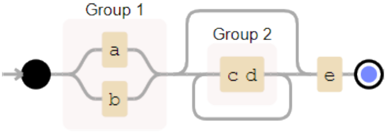
- ae
- acdcdcde
- bcde

Doesn't match:

- acdcd
- abcde
- bce

“Captures”

`(a|b)(cd)*e`



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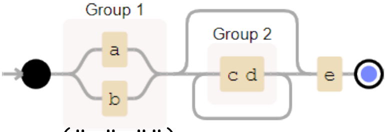

- ae →
- acdcdcde →
- bcde →

- `("a","")`

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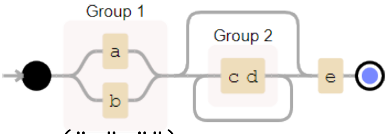

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- `("a","cdcdcd")`

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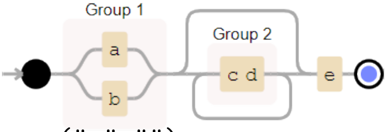

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- `("a","")`
- `("a","cdcdcd")`
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type Regexp a
          -- ^ type of "result"
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char   :: Char   -> RegExp Char
string :: String -> RegExp String
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char   :: Char   -> RegExp Char
string :: String -> RegExp String

runRegexp :: RegExp a -> String -> Maybe a
```

Applicative Regular Expressions

```
char   :: Char      -> RegExp Char
string :: String    -> RegExp String
(<|>)  :: RegExp a -> RegExp a -> RegExp a
many   :: RegExp a -> RegExp [a]

myRegexp :: RegExp (Char, [String])
myRegexp = (,) <$> (char 'a' <|> char 'b')
               <*> many (string "cd")
               <*  char 'e'
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Applicative Regular Expressions

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runRegexp myRegexp :: String -> Maybe (Char, [String])
```

Applicative Regular Expressions

```
runRegexp myRegexp "ae"
Just ('a', [])

runRegexp myRegexp "acdcdcde"
Just ('a', ["cd","cd","cd"])

runRegexp myRegexp "bcde"
Just ('b', ["cd"])

runRegexp myRegexp "acdcd"
Nothing
```

Applicative Regular Expressions

```
myRegexp2 :: RegExp (Bool, Int)
myRegexp2 = (,) <$> ((False <$ char 'a') <|> (True <$ char
                <*> fmap lengnth (many (string "cd"))
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runRegexp myRegexp2 "ae"
Just (False, 0)

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Just (True, 1)
```

What's so Regular about Regexps?

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Regular Language Base Members

1. Empty language: Always fails to match
2. Empty string: Always succeeds, consumes nothing
3. Literal: Matches and consumes a given char

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Regular Language Operations

1. Concatenation: `RS`, sequence one after the other
2. Alternation: `R|S`, one or the other
3. Kleene Star: `R*`, the repetition of `R`

An Alternative Perspective

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```
class Functor f => Applicative f where
    -- | Always succeed, consuming nothing
    pure  :: a -> f a
    -- | Concatenation
    (<*>) :: f (a -> b) -> f a -> f b
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Components of a Regular Language

1. Empty language: `empty`

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2. Empty string: `pure x`

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Components of a Regular Language

1. Empty language: `empty`
2. Empty string: `pure x`
3. Literal: ???
4. Concatenation: `<*>`
5. Alternation: `<|>`
6. Repetition: `many`

Functor combinator-style

- Define a primitive type

```
type Prim a
```

Functor combinator-style

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- Add the structure you need

Functor combinator-style

- Define a primitive type

  ```
  type Prim a
  ```

- Add the structure you need
  - If this structure is from a typeclass, use the free structure of that typeclass

Easy as 1, 2, 3

```haskell
-- | Free Alternative, from 'free'
data Alt :: (Type -> Type) -> (Type -> Type)
          -- ^ take a Functor; ^ return a Functor
instance Functor     (Alt f)
instance Applicative (Alt f)
instance Alternative (Alt f)

liftAlt :: Prim a -> Alt Prim
```

Easy as 1, 2, 3

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data Alt :: (Type -> Type) -> (Type -> Type)
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instance Alternative (Alt f)

liftAlt :: Prim a -> Alt Prim

data Prim a = Prim Char a
  deriving Functor

type RegExp = Alt Prim

char :: Char -> RegExp Char
char c = liftAlt (Prim c c)
```

Unlimited Power

```
empty :: RegExp a
pure  :: a -> RegExp a
char  :: Char -> RegExp Char
(<*>) :: RegExp (a -> b) -> RegExp a -> RegExp b
(<|>) :: RegExp a -> RegExp a -> RegExp a
many  :: RegExp a -> RegExp [a]
```

Unlimited Power

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empty :: RegExp a
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(<*>) :: RegExp (a -> b) -> RegExp a -> RegExp b
(<|>) :: RegExp a -> RegExp a -> RegExp a
many  :: RegExp a -> RegExp [a]

string :: String -> RegExp String
string = traverse char

digit :: RegExp Int
digit = asum [ intToDigit i <$ char i | i <- [0..9] ]
```

Parsing

Options

1. *Interpret* into an `Alternative` instance, “offloading” the logic

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1. *Interpret* into an `Alternative` instance, "offloading" the logic
   - Analogy: Process a list using `foldMap`
2. Direct pattern match on structure constructors (Haskell 101)
   - Analogy: Process a list using pattern matching and recursion

What is freeness?

What is freeness?

```
class Semigroup m where
    (<>)   :: m -> m -> m
class Monoid m where
    mempty :: m

type FreeMonoid = [] :: Type        -> Type
                    -- ^ take a type; ^ return a type
instance Semigroup (FreeMonoid a)
instance Monoid    (FreeMonoid a)

injectFM :: a -> FreeMonoid a
runFM    :: Monoid m => (a -> m) -> (FreeMonoid a -> m)

runFM f
  -- ^ substitute injectFM for f
```

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injectFM :: a -> FreeMonoid a
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runFM f
  -- ^ substitute injectFM for f

(:[])   :: a -> [a]
foldMap :: Monoid m => (a -> m) -> ([a] -> m)
```

What is freeness?

```
myMon :: FreeMonoid Int
myMon = injectFM 1 <> mempty <> injectF1 2 <> injectF1 3 <>
```

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foldMap Sum myMon       -- mempty = 0, <> = +
Sum 1 + Sum 0 + Sum 2 + Sum 3 + Sum 4
Sum 10
```

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Sum 10

foldMap Product myMon   -- mempty = 1, <> = *
Product 1 * Product 1 * Product 2 * Product 3 * Product 4
Product 24
```

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Product 1 * Product 1 * Product 2 * Product 3 * Product 4
Product 24

foldMap Max myMon        -- mempty = minBound, <> = max
Max 1 `max` Max minBound `max` Max 2 `max` Max 3 `max` Max
Max 4
```

What is freeness?

```haskell
type Alt a

liftAlt :: f a -> Alt f a
runAlt  :: Alternative g
        => (forall b. f a -> g a)
        -> (Alt f a -> g a)


runAlt f
    -- ^ substitute liftAlt for f
```

Goal

Find an `Alternative` where:

- `Prim a` = consumption

Goal

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Goal

Find an `Alternative` where:

- `Prim a` = consumption
- `<*>` = sequencing consumption
- `<|>` = backtracking

Hijacking StateT

`StateT [Char] Maybe`

- `Prim a` can be interpreted as *consumption* of state
- `<*>` sequences consumption of state
- `<|>` is backtracking of state

Hijacking StateT

```haskell
processPrim :: Prim a -> StateT String Maybe a
processPrim (Prim c x) = do
    d:ds <- get                  -- ^ match on stream
    guard (c == d)               -- ^ fail unless match
    put ds                       -- ^ update stream
    return x                     -- ^ return result value

matchPrefix :: Regexp a -> String -> Maybe a
matchPrefix re = evalStateT (runAlt processPrim re)
```

This works?

This works?

Yes!

```
matchPrefix myRegexp2 "ae"
Just (False, 0)

matchPrefix myRegexp2 "acdcdcde"
Just (False, 3)

matchPrefix myRegexp2 "bcde"
Just (True, 1)
```

What just happened?

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```haskell
data Prim a = Prim Char a
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type RegExp = Alt Prim

matchPrefix :: RegExp a -> String -> Maybe a
matchPrefix re = evalStateT (runAlt processPrim re)
  where
    processPrim (Prim c x) = do
      d:ds <- get
      guard (c == d)
      put ds
      pure x
```

What just happened?

1. Offload `Alternative` functionality to `StateT`: `empty`, `<*>`, `pure`, `empty`, `many`.

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1. Offload `Alternative` functionality to `StateT`: `empty`, `<*>`, `pure`, `empty`, `many`.
2. Provide `Prim`-processing functionality with `processPrim`: `liftAlt`.

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   - `StateT String Maybe` is **not** a regular expression type.

```
notARegexp :: StateT String Maybe ()
notARegexp = put "hello"        -- no regular expressio
```

What do we gain?

1. Interpretation-invariant structure
2. Actually meaningful types
   - `StateT String Maybe` is **not** a regular expression type.

     ```
     notARegexp :: StateT String Maybe ()
     notARegexp = put "hello"          -- no regular expressio
     ```
   - `Alt Prim` **is** a regular expression type.

Direct matching

```
newtype Alt f a = Alt { alternatives :: [AltF f a] }

data AltF f a = forall r. Ap (f r) (Alt f (r -> a))
              |           Pure a
```

Direct matching

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newtype Alt f a = Alt { alternatives :: [AltF f a] }

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-- | Chain of <|>s
newtype Alt f a
    =          Choice (AltF f a) (Alt f a      )  -- ^ c
    |          Empty                              -- ^ n

-- | Chain of <*>s
data AltF f a
    = forall r. Ap     (f r     ) (Alt f (r -> a))  -- ^ c
    |          Pure a                               -- ^ n
```

Direct Matching

```
matchAlts :: RegExp a -> String -> Maybe a
matchAlts (Alt res) xs = asum [ matchChain re xs | re <- re
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Direct Matching

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matchAlts :: RegExp a -> String -> Maybe a
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matchChain :: AltF Prim a -> String -> Maybe a
matchChain (Ap (Prim c x) next) cs = _
matchChain (Pure x)             cs = _
```

One game of Type Tetris later. . .

```
matchChain :: AltF Prim a -> String -> Maybe a
matchChain (Ap (Prim c x) next) cs = case cs of
    []  -> Nothing
    d:ds | c == d    -> matchAlts (($ x) <$> next) ds
         | otherwise -> Nothing
matchChain (Pure x)             _        = Just x
```

This works?

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matchChain myRegexp2 "ae"
Just (False, 0)

matchChain myRegexp2 "acdcdcde"
Just (False, 3)

matchChain myRegexp2 "bcde"
Just (True, 1)
```

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- **Normalizing** representation
  - Equivalence in meaning = equivalence in structure

```haskell
-- | Not regularizing
data RegExp a = Empty
              | Pure a
              | Prim Char a
              | forall r. Seq (RegExp a) (RegExp (r -> a))
              | Union (RegExp a) (RegExp a)
              | Many (RegExp a)

-- a|(b|c) /= (a|b)|c
```

Free your mind

Is this you?

> *"My problem is modeled by some (commonly occurring) structure over some primitive base."*

- Use a "functor combinator"!

Free your mind

Is this you?

> *"My problem is modeled by some (commonly occurring) structure over some primitive base."*

- Use a "functor combinator"!
- If your structure comes from a typeclass, use a free structure!

Further Reading

- Blog post: https://blog.jle.im/entry/free-applicative-regexp.html
- Functor Combinatorpedia: https://blog.jle.im/entry/functor-combinatorpedia.html
- *free*: https://hackage.haskell.org/package/free
- *functor-combinators*: https://hackage.haskell.org/package/functor-combinators
- Slides: https://talks.jle.im/composeconf-2019/