Applicative Regular Expressions w/ the Free Alternative

Justin Le (@mstk)
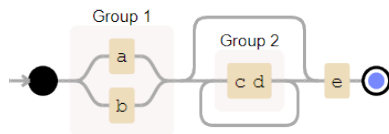
C ▫ mp ▫ se Conference 2019, June 24

Preface

Slide available at https://talks.jle.im.
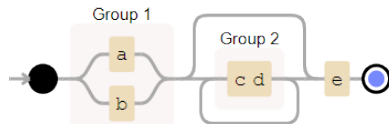
Regular Expressions

`(a|b)(cd)*e`

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Matches:

- ae
- acdcdcde
- bcde

Doesn't match:

- acdcd
- abcde
- bce

"Captures"

`(a|b)(cd)*e`
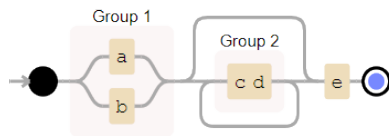
"Captures"

(a|b)(cd)*e

- ae →
- acdcdcde →
- bcde →
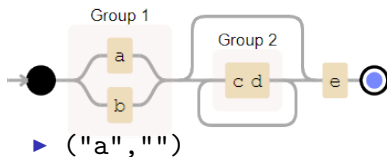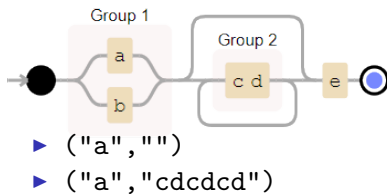
- ("a","")

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- ae →
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- ("a","")
- ("a","cdcdcd")

"Captures"

`(a|b)(cd)*e`

- ae →
- acdcdcde →
- bcde →

- ("a","")
- ("a","cdcdcd")
- ("b","cd")

Applicative Regular Expressions

"Type-indexed" regular expressions.

```haskell
type Regexp a
          -- ^ type of "result"
```
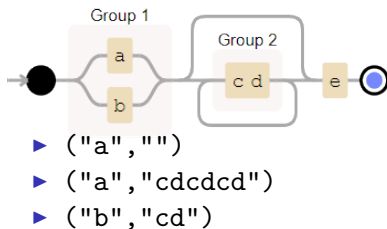
Applicative Regular Expressions

"Type-indexed" regular expressions.

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type Regexp a
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char   :: Char   -> RegExp Char
string :: String -> RegExp String
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type Regexp a
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char   :: Char   -> RegExp Char
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runRegexp :: RegExp a -> String -> Maybe a
```

Applicative Regular Expressions

```haskell
char   :: Char      -> RegExp Char
string :: String    -> RegExp String
(<|>)  :: RegExp a -> RegExp a -> RegExp a
many   :: RegExp a -> RegExp [a]

myRegexp :: RegExp (Char, [String])
myRegexp = (,) <$> (char 'a' <|> char 'b')
               <*> many (string "cd")
               <*  char 'e'
```

Applicative Regular Expressions

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char   :: Char      -> RegExp Char
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myRegexp :: RegExp (Char, [String])
myRegexp = (,) <$> (char 'a' <|> char 'b')
               <*> many (string "cd")
               <*  char 'e'

runRegexp myRegexp :: String -> Maybe (Char, [String])
```

Applicative Regular Expressions

```
runRegexp myRegexp "ae"
Just ('a', [])

runRegexp myRegexp "acdcdcde"
Just ('a', ["cd","cd","cd"])

runRegexp myRegexp "bcde"
Just ('b', ["cd"])

runRegexp myRegexp "acdcd"
Nothing
```

Applicative Regular Expressions

```
myRegexp2 :: RegExp (Bool, Int)
myRegexp2 = (,) <$> ((False <$ char 'a') <|> (True <$ char
               <*> fmap lengnth (many (string "cd"))
               <*  char 'e'
```

Applicative Regular Expressions

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myRegexp2 :: RegExp (Bool, Int)
myRegexp2 = (,) <$> ((False <$ char 'a') <|> (True <$ char
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runRegexp myRegexp2 "ae"
Just (False, 0)

runRegexp myRegexp2 "acdcdcde"
Just (False, 3)

runRegexp myRegexp2 "bcde"
Just (True, 1)
```

What's so Regular about Regexps?

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Regular Language Base Members

1. Empty language: Always fails to match
2. Empty string: Always succeeds, consumes nothing
3. Literal: Matches and consumes a given char

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Regular Language Operations

1. Concatenation: `RS`, sequence one after the other
2. Alternation: `R|S`, one or the other
3. Kleene Star: `R*`, the repetition of R

An Alternative Perspective

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```haskell
class Functor f => Applicative f where
    -- | Always succeed, consuming nothing
    pure  :: a -> f a
    -- | Concatenation
    (<*>) :: f (a -> b) -> f a -> f b
```

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1. Empty language: empty
2. Empty string: pure x

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4. Concatenation: <*>

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1. Empty language: empty
2. Empty string: pure x
3. Literal: ???
4. Concatenation: <*>
5. Alternation: <|>
6. Repetition: many

Functor combinator-style

- Define a primitive type

```
type Prim a
```

Functor combinator-style

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  ```
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- Add the structure you need

Functor combinator-style

- Define a primitive type

  ```
  type Prim a
  ```

- Add the structure you need
  - If this structure is from a typeclass, use the free structure of that typeclass

Easy as 1, 2, 3

```haskell
-- | Free Alternative, from 'free'
data Alt :: (Type -> Type) -> (Type -> Type)
          -- ^ take a Functor; ^ return a Functor
instance Functor     (Alt f)
instance Applicative (Alt f)
instance Alternative (Alt f)

liftAlt :: Prim a -> Alt Prim
```

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instance Functor     (Alt f)
instance Applicative (Alt f)
instance Alternative (Alt f)

liftAlt :: Prim a -> Alt Prim

data Prim a = Prim Char a
  deriving Functor

type RegExp = Alt Prim

char :: Char -> RegExp Char
char c = liftAlt (Prim c c)
```

Unlimited Power

```
empty :: RegExp a
pure  :: a -> RegExp a
char  :: Char -> RegExp Char
(<*>) :: RegExp (a -> b) -> RegExp a -> RegExp b
(<|>) :: RegExp a -> RegExp a -> RegExp a
many  :: RegExp a -> RegExp [a]
```

Unlimited Power

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char  :: Char -> RegExp Char
(<*>) :: RegExp (a -> b) -> RegExp a -> RegExp b
(<|>) :: RegExp a -> RegExp a -> RegExp a
many  :: RegExp a -> RegExp [a]

string :: String -> RegExp String
string = traverse char

digit :: RegExp Int
digit = asum [ intToDigit i <$ char i | i <- [0..9] ]
```

Parsing

Options:

1. *Interpret* into an `Alternative` instance, "offloading" the logic

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1. *Interpret* into an `Alternative` instance, "offloading" the logic
2. Direct pattern match on structure constructors (Haskell 101)

What is freeness?

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```haskell
class Semigroup m where
    (<>)   :: m -> m -> m
class Monoid m where
    mempty :: m

type FreeMonoid = [] :: Type          -> Type
                       -- ^ take a type; ^ return a type
instance Semigroup (FreeMonoid a)
instance Monoid    (FreeMonoid a)

injectFM :: a -> FreeMonoid a
runFM    :: Monoid m => (a -> m) -> (FreeMonoid a -> m)

runFM f
    -- ^ substitute injectFM for f
```

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injectFM :: a -> FreeMonoid a
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runFM f
    -- ^ substitute injectFM for f

(:[])    :: a -> [a]
foldMap :: Monoid m => (a -> m) -> ([a] -> m)
```

What is freeness?

```haskell
myMon :: FreeMonoid Int
myMon = injectFM 1 <> mempty <> injectF1 2 <> injectF1 3 <>
```

What is freeness?

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myMon :: FreeMonoid Int
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foldMap Sum myMon          -- mempty = 0, <> = +
Sum 1 + Sum 0 + Sum 2 + Sum 3 + Sum 4
Sum 10
```

What is freeness?

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myMon :: FreeMonoid Int
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foldMap Sum myMon        -- mempty = 0, <> = +
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Sum 10

foldMap Product myMon    -- mempty = 1, <> = *
Product 1 * Product 1 * Product 2 * Product 3 * Product 4
Product 24
```

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Sum 10

foldMap Product myMon      -- mempty = 1, <> = *
Product 1 * Product 1 * Product 2 * Product 3 * Product 4
Product 24

foldMap Max myMon          -- mempty = minBound, <> = max
Max 1 `max` Max minBound `max` Max 2 `max` Max 3 `max` Max
Max 4
```

What is freeness?

```haskell
type Alt a

liftAlt :: f a -> Alt f a
runAlt  :: Alternative g
        => (forall b. f a -> g a)
        -> (Alt f a -> g a)


runAlt f
    -- ^ substitute liftAlt for f
```

Goal

Find an Alternative where:

- Prim a = consumption

Goal

Find an `Alternative` where:

- `Prim a` = consumption
- `<*>` = sequencing consumption

Goal

Find an `Alternative` where:

- `Prim a` = consumption
- `<*>` = sequencing consumption
- `<|>` = backtracking

Hijacking StateT

`StateT [Char] Maybe`

- `Prim a` can be interpreted as *consumption* of state
- `<*>` sequences consumption of state
- `<|>` is backtracking of state

Hijacking StateT

```
processPrim :: Prim a -> StateT String Maybe a
processPrim (Prim c x) = do
    d:ds <- get              -- ˆ match on stream
    guard (c == d)           -- ˆ fail unless match
    put ds                   -- ˆ update stream
    return x                 -- ˆ return result value

matchPrefix :: Regexp a -> String -> Maybe a
matchPrefix re = evalStateT (runAlt processPrim re)
```

This works?

This works?

Yes!

```
matchPrefix myRegexp2 "ae"
Just (False, 0)

matchPrefix myRegexp2 "acdcdcde"
Just (False, 3)

matchPrefix myRegexp2 "bcde"
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```

What just happened?

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```haskell
data Prim a = Prim Char a
  deriving Functor

type RegExp = Alt Prim

matchPrefix :: RegExp a -> String -> Maybe a
matchPrefix re = evalStateT (runAlt processPrim re)
  where
    processPrim (Prim c x) = do
      d:ds <- get
      guard (c == d)
      put ds
      pure x
```

What just happened?

1. Offload `Alternative` functionality to `StateT`: `empty`, `<*>`,
   `pure`, `empty`, `many`.

What just happened?

1. Offload `Alternative` functionality to `StateT`: empty, `<*>`, pure, empty, many.
2. Provide `Prim`-processing functionality with `processPrim`: liftAlt.

What do we gain?

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   - `StateT String Maybe` is **not** a regular expression type.
     ```
     notARegexp :: StateT String Maybe ()
     notARegexp = put "hello"        -- no regular expression
     ```

What do we gain?

1. Interpretation-invariant structure
2. Actually meaningful types
   - `StateT String Maybe` is **not** a regular expression type.
     ```
     notARegexp :: StateT String Maybe ()
     notARegexp = put "hello"        -- no regular expression
     ```
   - `Alt Prim` **is** a regular expression type.

Direct matching

```haskell
newtype Alt f a = Alt { alternatives :: [AltF f a] }

data AltF f a = forall r. Ap (f r) (Alt f (r -> a))
              |                     Pure a
```

Direct matching

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data AltF f a = forall r. Ap (f r) (Alt f (r -> a))
              |            Pure a

-- | Chain of </>s
newtype Alt f a
    =             Choice (AltF f a) (Alt f a       )  -- ^ c
    |             Empty                               -- ^ n

-- | Chain of <*>s
data AltF f a
    = forall r. Ap     (f r     ) (Alt f (r -> a))  -- ^ c
    |           Pure a                              -- ^ n
```

Direct Matching

```haskell
matchAlts :: RegExp a -> String -> Maybe a
matchAlts (Alt res) xs = asum [ matchChain re xs | re <- re
```

Direct Matching

```haskell
matchAlts :: RegExp a -> String -> Maybe a
matchAlts (Alt res) xs = asum [ matchChain re xs | re <- re

matchChain :: AltF Prim a -> String -> Maybe a
matchChain (Ap (Prim c x) next) cs = _
matchChain (Pure x)             cs = _
```

One game of Type Tetris later...

```haskell
matchChain :: AltF Prim a -> String -> Maybe a
matchChain (Ap (Prim c x) next) cs = case cs of
    []   -> Nothing
    d:ds | c == d    -> matchAlts (($ x) <$> next) ds
         | otherwise -> Nothing
matchChain (Pure x)             _      = Just x
```

This works?

This works?

Yes!

```
matchChain myRegexp2 "ae"
Just (False, 0)

matchChain myRegexp2 "acdcdcde"
Just (False, 3)

matchChain myRegexp2 "bcde"
Just (True, 1)
```

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- First-class program rewriting, Haskell 101-style
- **Normalizing** representation
  - Equivalence in meaning = equivalence in structure

```haskell
-- | Not regularizing
data RegExp a = Empty
              | Pure a
              | Prim Char a
              | forall r. Seq (RegExp a) (RegExp (r -> a))
              | Union (RegExp a) (RegExp a)
              | Many (RegExp a)

-- a|(b|c) /= (a|b)|c
```

Free your mind

Is this you?

> *"My problem is modeled by some (commonly occurring) structure over some primitive base."*

► Use a "functor combinator"!

Free your mind

Is this you?

> "My problem is modeled by some (commonly occurring) structure over some primitive base."

- Use a "functor combinator"!
- If your structure comes from a typeclass, use a free structure!

Further Reading

- Blog post:
  https://blog.jle.im/entry/free-applicative-regexp.html
- Functor Combinatorpedia:
  https://blog.jle.im/entry/functor-combinatorpedia.html
- *free*: https://hackage.haskell.org/package/free
- *functor-combinators*:
  https://hackage.haskell.org/package/functor-combinators
- Slides: https://talks.jle.im/composeconf-2019/