Practical Dependent Types: Type-Safe Neural Networks

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Preface

Slide available at
https://mstksg.github.io/talks/kievfprog/dependent-types.html.

All code available at
https://github.com/mstksg/talks/tree/master/kievfprog.

Libraries required: (available on Hackage) *hmatrix*, *singletons*, *MonadRandom*. GHC 8.x assumed.

The Big Question

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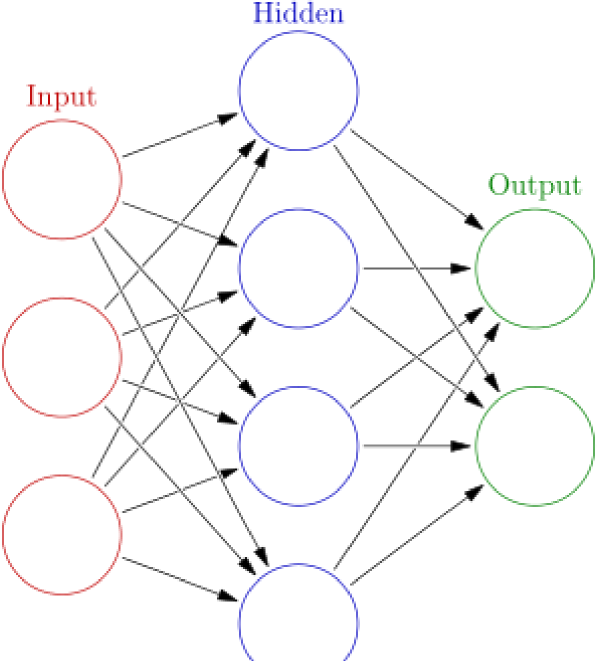
Dependent types are simply the extension of this question, pushing the power of types further.

Artificial Neural Networks



Parameterized functions

Each layer receives an input vector, $\mathbf{x} : \mathbb{R}^n$, and produces an output $\mathbf{y} : \mathbb{R}^m$.

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They are parameterized by a weight matrix $W : \mathbb{R}^{m \times n}$ (an $m \times n$ matrix) and a bias vector $\mathbf{b} : \mathbb{R}^m$, and the result is: (for some activation function `f`)

$$\mathbf{y} = f(W\mathbf{x} + \mathbf{b})$$

A neural network would take a vector through many layers.

Networks in Haskell

```haskell
data Weights = W { wBiases :: !(Vector Double)  -- n
                 , wNodes  :: !(Matrix Double)  -- n x m
                 }                              -- "m to n
```

```haskell
data Network :: Type where
    O    :: !Weights -> Network
    (:~) :: !Weights -> !Network -> Network
infixr 5 :~
```

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A network with one input layer, two hidden layers, and one output layer would be:

```haskell
h1 :~ h2 :~ O o
```

Running them

```haskell
runLayer :: Weights -> Vector Double -> Vector Double
runLayer (W wB wN) v = wB + wN #> v

runNet :: Network -> Vector Double -> Vector Double
runNet (O w)      !v = logistic (runLayer w v)
runNet (w :~ n') !v = let v' = logistic (runLayer w v)
                      in  runNet n' v'
```

Generating them

```
randomWeights :: MonadRandom m => Int -> Int -> m Weights
randomWeights i o = do
    seed1 :: Int <- getRandom
    seed2 :: Int <- getRandom
    let wB = randomVector  seed1 Uniform o * 2 - 1
        wN = uniformSample seed2 o (replicate i (-1, 1))
    return $ W wB wN

randomNet :: MonadRandom m => Int -> [Int] -> Int -> m Netw
randomNet i []     o =     O <$> randomWeights i o
randomNet i (h:hs) o = (:~) <$> randomWeights i h <*> rando
```

Haskell Heart Attacks

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Haskell Heart Attacks

- What if we mixed up the dimensions for `randomWeights`?
- What if the *user* mixed up the dimensions for `randomWeights`?
- What if layers in the network are incompatible?
- How does the user know what size vector a network expects?
- Is our `runLayer` and `runNet` implementation correct?

Backprop

```
train :: Double                  -- ^ learning rate
      -> Vector Double           -- ^ input vector
      -> Vector Double           -- ^ target vector
      -> Network                 -- ^ network to train
      -> Network
train rate x0 target = fst . go x0
  where
```

Backprop (Outer layer)

```
go :: Vector Double      -- ^ input vector
   -> Network            -- ^ network to train
   -> (Network, Vector Double)
-- handle the output layer
go !x (O w@(W wB wN))
    = let y    = runLayer w x
          o    = logistic y
          -- the gradient (how much y affects the error)
          --   (logistic' is the derivative of logistic)
          dEdy = logistic' y * (o - target)
          -- new bias weights and node weights
          wB'  = wB - scale rate dEdy
          wN'  = wN - scale rate (dEdy `outer` x)
          w'   = W wB' wN'
          -- bundle of derivatives for next step
          dWs  = tr wN #> dEdy
      in  (O w', dWs)
```

Backprop (Inner layer)

```
-- handle the inner layers
go !x (w@(W wB wN) :~ n)
    = let y           = runLayer w x
          o           = logistic y
          -- get dWs', bundle of derivatives from rest
          (n', dWs') = go o n
          -- the gradient (how much y affects the error
          dEdy        = logistic' y * dWs'
          -- new bias weights and node weights
          wB'  = wB - scale rate dEdy
          wN'  = wN - scale rate (dEdy `outer` x)
          w'   = W wB' wN'
          -- bundle of derivatives for next step
          dWs  = tr wN #> dEdy
      in  (w' :~ n', dWs)
```

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- Haskell is all about the compiler helping guide you write your code. But how much did the compiler help there?
- How can the “shape” of the matrices guide our programming?
- We basically rely on naming conventions to make sure we write our code correctly.

Haskell Red Flags

- How many ways can we write the function and have it still typecheck?

Haskell Red Flags

- How many ways can we write the function and have it still typecheck?
- How many of our functions are partial?

A Typed Alternative

```haskell
data Weights i o = W { wBiases :: !(R o)
                     , wNodes  :: !(L o i)
                     }
```

An `o x i` layer

A Typed Alternative

From HMatrix:

```
R :: Nat -> Type
L :: Nat -> Nat -> Type
```

An `R 3` is a 3-vector, an `L 4 3` is a 4 x 3 matrix.

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Operations are typed:

```
(+)  :: KnownNat n                    => R n   -> R n -> R n
(<#) :: (KnownNat m, KnownNat n) => L m n -> R n -> R m
```

`KnownNat n` lets hmatrix use the `n` in the type. Typed holes can guide our development, too!

Data Kinds

With `-XDataKinds`, all values and types are lifted to types and kinds.

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In addition to the values `True`, `False`, and the type `Bool`, we also have the **type** `'True`, `'False`, and the **kind** `Bool`.

In addition to `:` and `[]` and the list type, we have `':` and `'[]` and the list kind.

Data Kinds

```
ghci> :t True
Bool
ghci> :k 'True
Bool
ghci> :t [True, False]
[Bool]
ghci> :k '[ 'True, 'False ]
[Bool]
```

A Typed Alternative

```haskell
data Network :: Nat -> [Nat] -> Nat -> Type where
    O    :: !(Weights i o)
         -> Network i '[] o
    (:~) :: KnownNat h
         => !(Weights i h)
         -> !(Network h hs o)
         -> Network i (h ': hs) o
infixr 5 :~
```

A Typed Alternative

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data Network :: Nat -> [Nat] -> Nat -> Type where
    O     :: !(Weights i o)
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infixr 5 :~

h1 :: Weight 10 8
h2 :: Weight 8  5
o  :: Weight 5  2

            O o :: Network 5  '[]     2
      h2 :~ O o :: Network 8  '[5]    2
h1 :~ h2 :~ O o :: Network 10 '[8, 5] 2
h2 :~ h1 :~ O o -- type error
```

Running

```haskell
runLayer :: (KnownNat i, KnownNat o)
         => Weights i o
         -> R i
         -> R o
runLayer (W wB wN) v = wB + wN #> v

runNet :: (KnownNat i, KnownNat o)
       => Network i hs o
       -> R i
       -> R o
runNet (O w)      !v = logistic (runLayer w v)
runNet (w :~ n') !v = let v' = logistic (runLayer w v)
                      in  runNet n' v'
```

Exactly the same! No loss in expressivity!

Running

Much better! Matrices and vector lengths are guaranteed to line up!

Also, note that the interface for `runNet` is better stated in its type. No need to reply on documentation.

```
runNet
    :: (KnownNat i, KnownNat o)
    => Network i hs o -> R i -> R o
```

The user knows that they have to pass in an `R i`, and knows to expect an `R o`.

Generating

```haskell
randomWeights :: (MonadRandom m, KnownNat i, KnownNat o)
              => m (Weights i o)
randomWeights = do
    s1 :: Int <- getRandom
    s2 :: Int <- getRandom
    let wB = randomVector  s1 Uniform * 2 - 1
        wN = uniformSample s2 (-1) 1
    return $ W wB wN
```

No need for explicit arguments! User can demand `i` and `o`. No reliance on documentation and parameter orders.

Generating

But, for generating nets, we have a problem:

```
randomNet :: forall m i hs o. (MonadRandom m, KnownNat i, K
          => m (Network i hs o)
randomNet = case hs of [] -> ??
```

Pattern matching on types

The solution for pattern matching on types: singletons.

```haskell
-- (not the actual impelentation)

data Sing :: Bool -> Type where
    SFalse :: Sing 'False
    STrue  :: Sing 'True

data Sing :: [k] -> Type where
    SNil  :: Sing '[]
    SCons :: Sing x -> Sing xs -> Sing (x ': xs)

data Sing :: Nat -> Type where
    SNat :: KnownNat n => Sing n
```

Pattern matching on types

```
ghci> :t SFalse
Sing 'False
ghci> :t STrue `SCons` (SFalse `SCons` SNil)
Sing '[True, False]
ghci> :t SNat @1 `SCons` (SNat @2 `SCons` SNil)
Sing '[1, 2]
```

Random networks

```
randomNet' :: forall m i hs o. (MonadRandom m, KnownNat i,
           => Sing hs -> m (Network i hs o)
randomNet' = \case
    SNil            ->    O <$> randomWeights
    SNat `SCons` ss -> (:~) <$> randomWeights <*> randomNet
```

Implicit passing

Explicitly passing singletons can be ugly.

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```
class SingI x where
    sing :: Sing x
```

We can now recover the expressivity of the original function, and gain demand-driven shapes.

Implicit passing

```
randomNet :: forall i hs o m. (MonadRandom m, KnownNat i, S
          => m (Network i hs o)
randomNet = randomNet' sing
```

Now the shape can be inferred from the functions that use the `Network`.

Implicit passing

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randomNet :: forall i hs o m. (MonadRandom m, KnownNat i, S
          => m (Network i hs o)
randomNet = randomNet' sing
```

Now the shape can be inferred from the functions that use the `Network`.

We can also demand them explicitly:

```
randomNet @1 @'[8,5] @2
```

Backprop

```haskell
train :: forall i hs o. (KnownNat i, KnownNat o)
      => Double           -- ^ learning rate
      -> R i              -- ^ input vector
      -> R o              -- ^ target vector
      -> Network i hs o   -- ^ network to train
      -> Network i hs o
train rate x0 target = fst . go x0
```

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      -> Network i hs o   -- ^ network to train
      -> Network i hs o
train rate x0 target = fst . go x0
```

Ready for this?

Backprop

```haskell
go  :: forall j js. KnownNat j
    => R j                  -- ^ input vector
    -> Network j js o       -- ^ network to train
    -> (Network j js o, R j)
-- handle the output layer
go !x (O w@(W wB wN))
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    -- handle the inner layers
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Surprise! It's actually *identical*! No loss in expressivity.

Also, typed holes can help you write your code in a lot of places.

Backprop

```haskell
    -- handle the inner layers
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Type-Driven Development

The overall guiding principle is:

1. Write an untyped implementation.
2. Realize where things can go wrong:
   - Partial functions?
   - Many, many ways to implement a function incorrectly with the current types?
   - Unclear or documentation-reliant API?
3. Gradually add types in selective places to handle these.

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I recommend not going the other way (use perfect type safety before figuring out where you actually really need them). We call that "hasochism".

Further reading

- Blog series: https://blog.jle.im/entries/series/+practical-dependent-types-in-haskell.html
- Extra resources:
  - https://www.youtube.com/watch?v=rhWMhTjQzsU
  - http://www.well-typed.com/blog/2015/11/implementing-a-minimal-version-of-haskell-servant/
  - https://www.schoolofhaskell.com/user/konn/prove-your-haskell-for-great-safety
  - http://jozefg.bitbucket.org/posts/2014-08-25-dep-types-part-1.html