

# Practical Dependent Types: Type-Safe Neural Networks

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# Preface

Slide available at <https://mstksg.github.io/talks/lambdaconf-2017/dependent-types/dependent-types.html>.

Exercises we will be doing available at <https://github.com/mstksg/talks/tree/master/lambdaconf-2017/dependent-types>.

Libraries required: (available on Hackage)

- ▶ *hmatrix*
- ▶ *singletons*
- ▶ *MonadRandom*

GHC 8.x assumed.

# The Big Question

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Dependent types are simply the extension of this question, pushing the power of types further.

# Artificial Neural Networks

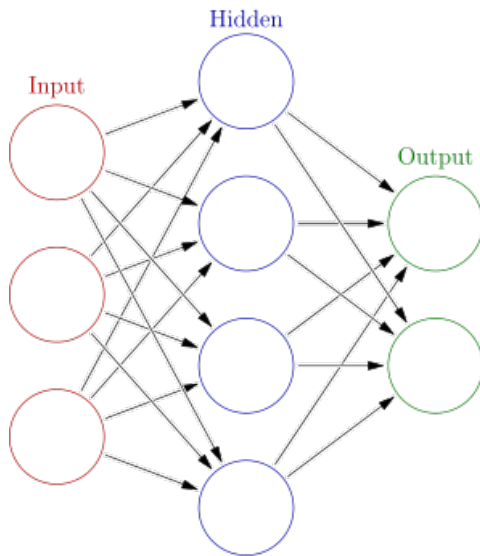


Figure 1: Feed-forward ANN architecture

## Parameterized functions

Each layer receives an input vector,  $\mathbf{x} : \mathbb{R}^n$ , and produces an output  $\mathbf{y} : \mathbb{R}^m$ .

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Each layer receives an input vector,  $\mathbf{x} : \mathbb{R}^n$ , and produces an output  $\mathbf{y} : \mathbb{R}^m$ .

They are parameterized by a weight matrix  $W : \mathbb{R}^{m \times n}$  (an  $m \times n$  matrix) and a bias vector  $\mathbf{b} : \mathbb{R}^m$ , and the result is:

$$\mathbf{y} = f(W\mathbf{x} + \mathbf{b})$$

Where  $f$  is some (differentiable) activation function. A neural network would take a vector through many layers.

# Networks in Haskell

```
data Weights = W { wBiases :: !(Vector Double)  --  $n$ 
                  , wNodes  :: !(Matrix Double)  --  $n \times m$ 
                  }                               -- " $m$  to  $n$ "
```

```
data Network :: Type where
    0      :: !Weights -> Network
    (:~)   :: !Weights -> !Network -> Network
infixr 5 :~
```



# Networks in Haskell

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data Weights = W { wBiases :: !(Vector Double)  -- n
                  , wNodes  :: !(Matrix Double)  -- n x m
                  }                               -- "m to n"
```

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infixr 5 :~
```

A network with one input layer, two hidden layers, and one output layer would be:

```
h1 :~ h2 :~ 0 o
```

## Running them

```
runLayer :: Weights -> Vector Double -> Vector Double
runLayer (W wB wN) v = wB + wN #> v
```

```
runNet :: Network -> Vector Double -> Vector Double
runNet (O w)      !v = logistic (runLayer w v)
runNet (w :~ n') !v = let v' = logistic (runLayer w v)
                      in  runNet n' v'
```

## Generating them

```
randomWeights :: MonadRandom m => Int -> Int -> m Weights
randomWeights i o = do
  seed1 :: Int <- getRandom
  seed2 :: Int <- getRandom
  let wB = randomVector seed1 Uniform o * 2 - 1
      wN = uniformSample seed2 o (replicate i (-1, 1))
  return $ W wB wN

randomNet :: MonadRandom m => Int -> [Int] -> Int -> m Netv
randomNet i [] o = 0 <$> randomWeights i o
randomNet i (h:hs) o = (:~) <$> randomWeights i h <*> randomNet i hs o
```

# Haskell Heart Attacks

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# Haskell Heart Attacks

- ▶ What if we mixed up the dimensions for `randomWeights`?
- ▶ What if the *user* mixed up the dimensions for `randomWeights`?
- ▶ What if layers in the network are incompatible?
- ▶ How does the user know what size vector a network expects?
- ▶ Is our `runLayer` and `runNet` implementation correct?



# Backprop

```
train :: Double           -- ^ learning rate
      -> Vector Double    -- ^ input vector
      -> Vector Double    -- ^ target vector
      -> Network          -- ^ network to train
      -> Network

train rate x0 target = fst . go x0
  where
```

## Backprop (Outer layer)

```
go :: Vector Double      -- ^ input vector
  -> Network              -- ^ network to train
  -> (Network, Vector Double)
-- handle the output layer
go !x (O w@(W wB wN))
  = let y      = runLayer w x
      o        = logistic y
      -- the gradient (how much y affects the error)
      -- (logistic' is the derivative of logistic)
      dEdy     = logistic' y * (o - target)
      -- new bias weights and node weights
      wB'      = wB - scale rate dEdy
      wN'      = wN - scale rate (dEdy `outer` x)
      w'       = W wB' wN'
      -- bundle of derivatives for next step
      dWs      = tr wN #> dEdy
  in (O w', dWs)
```

## Backprop (Inner layer)

```
-- handle the inner layers
go !x (w@(W wB wN) :~ n)
  = let y          = runLayer w x
      o          = logistic y
      -- get dWs', bundle of derivatives from rest
      (n', dWs') = go o n
      -- the gradient (how much y affects the error)
      dEdy      = logistic' y * dWs'
      -- new bias weights and node weights
      wB'      = wB - scale rate dEdy
      wN'      = wN - scale rate (dEdy `outer` x)
      w'       = W wB' wN'
      -- bundle of derivatives for next step
      dWs      = tr wN #> dEdy
  in (w' :~ n', dWs)
```

# Compiler, O Where Art Thou?

- ▶ Haskell is all about the compiler helping guide you write your code. But how much did the compiler help there?

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- ▶ Haskell is all about the compiler helping guide you write your code. But how much did the compiler help there?
- ▶ How can the “shape” of the matrices guide our programming?

# Haskell Red Flags

- ▶ How many ways can we write the function and have it still typecheck?

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- ▶ How many ways can we write the function and have it still typecheck?
- ▶ How many of our functions are partial?

## A Typed Alternative

```
data Weights i o = W { wBiases :: !(R o)
                       , wNodes  :: !(L o i)
                       }
```

An  $o \times i$  layer



## A Typed Alternative

From HMatrix:

```
R :: Nat -> Type
```

```
L :: Nat -> Nat -> Type
```

An  $R\ 3$  is a 3-vector, an  $L\ 4\ 3$  is a  $4 \times 3$  matrix.

## A Typed Alternative

From HMatrix:

```
R :: Nat -> Type
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```

An  $R\ 3$  is a 3-vector, an  $L\ 4\ 3$  is a  $4 \times 3$  matrix.

Operations are typed:

```
(+)   :: KnownNat n          => R n    -> R n    -> R n
(<#)  :: (KnownNat m, KnownNat n) => L m n -> R n    -> R m
```

(KnownNat n lets hmatrix use the n in the type)

Typed holes can guide our development, too!

# Data Kinds

With `-XDataKinds`, all values and types are lifted to types and kinds.

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In addition to the values `True`, `False`, and the type `Bool`, we also have the **type** `'True`, `'False`, and the **kind** `Bool`.

In addition to `:` and `[]` and the list type, we have `':` and `'[]` and the list kind.

# Data Kinds

```
ghci> :t True
```

```
Bool
```

```
ghci> :k 'True
```

```
Bool
```

```
ghci> :t [True, False]
```

```
[Bool]
```

```
ghci> :k '[ 'True, 'False ]
```

```
[Bool]
```

## A Typed Alternative

```
data Network :: Nat -> [Nat] -> Nat -> Type where
  0      :: !(Weights i o)
          -> Network i '[] o
  (:~) :: KnownNat h
          => !(Weights i h)
          -> !(Network h hs o)
          -> Network i (h ': hs) o
infixr 5 :~
```

## A Typed Alternative

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data Network :: Nat -> [Nat] -> Nat -> Type where
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infixr 5 :~

h1 :: W 10 8
h2 :: W 8 5
o  :: W 5 2

h1 :~ h2 :~ o :: Network 10 '[8, 5] 2

h2 :~ h1 :~ o -- type error
```

## Running

```
runLayer :: (KnownNat i, KnownNat o)
```

```
    => Weights i o
```

```
    -> R i
```

```
    -> R o
```

```
runLayer (W wB wN) v = wB + wN #> v
```

```
runNet :: (KnownNat i, KnownNat o)
```

```
    => Network i hs o
```

```
    -> R i
```

```
    -> R o
```

```
runNet (O w) !v = logistic (runLayer w v)
```

```
runNet (w :~ n') !v = let v' = logistic (runLayer w v)  
                      in runNet n' v'
```

Exactly the same! No loss in expressivity!



# Running

Much better! Matrices and vector lengths are guaranteed to line up!

## Generating

```
randomWeights :: (MonadRandom m, KnownNat i, KnownNat o)
               => m (Weights i o)
randomWeights = do
  s1 :: Int <- getRandom
  s2 :: Int <- getRandom
  let wB = randomVector s1 Uniform * 2 - 1
      wN = uniformSample s2 (-1) 1
  return $ W wB wN
```

No need for explicit arguments! User can demand `i` and `o`. No reliance on documentation and parameter orders.

## Generating

But, for generating nets, we have a problem:

```
randomNet :: forall m i hs o. (MonadRandom m, KnownNat i, P
    => m (Network i hs o)
randomNet = case hs of [] -> ??
```

## Pattern matching on types

The solution for pattern matching on types: singletons.

```
-- (not the actual impelentation)
```

```
data Sing :: Bool -> Type where
```

```
  SFalse :: Sing 'False
```

```
  STrue  :: Sing 'True
```

```
data Sing :: [k] -> Type where
```

```
  SNil  :: Sing '[]
```

```
  SCons :: Sing x -> Sing xs -> Sing (x ': xs)
```

```
data Sing :: Nat -> Type where
```

```
  SNat :: KnownNat n => Sing n
```

## Pattern matching on types

```
ghci> :t SFalse
```

```
Sing 'False
```

```
ghci> :t STrue `SCons` (SFalse `SCons` SNil)
```

```
Sing '[True, False]
```

```
ghci> :t SNat @1 `SCons` (SNat @2 `SCons` SNil)
```

```
Sing '[1, 2]
```

## Random networks

```
randomNet' :: forall m i hs o. (MonadRandom m, KnownNat i,  
    => Sing hs -> m (Network i hs o)  
randomNet' = \case  
    SNil          ->    0 <$> randomWeights  
    SNat `SCons` ss -> (:~) <$> randomWeights <*> randomNet
```

## Implicit passing

```
class SingI x where  
  sing :: Sing x
```

```
randomNet :: forall m i hs o. (MonadRandom m, KnownNat i, S  
    => m (Network i hs o)
```

```
randomNet = randomNet' sing
```

# Backprop

```
train :: forall i hs o. (KnownNat i, KnownNat o)
    => Double           -- ^ learning rate
    -> R i              -- ^ input vector
    -> R o              -- ^ target vector
    -> Network i hs o   -- ^ network to train
    -> Network i hs o
train rate x0 target = fst . go x0
```



# Backprop

```
go  :: forall j js. KnownNat j
    => R j                -- ^ input vector
    -> Network j js o    -- ^ network to train
    -> (Network j js o, R j)
-- handle the output layer
go !x (O w@(W wB wN))
    = let y      = runLayer w x
        o      = logistic y
        -- the gradient (how much y affects the error)
        -- (logistic' is the derivative of logistic)
        dEdy = logistic' y * (o - target)
        -- new bias weights and node weights
        wB'   = wB - konst rate * dEdy
        wN'   = wN - konst rate * (dEdy `outer` x)
        w'    = W wB' wN'
        -- bundle of derivatives for next step
        dWs  = tr wN #> dEdy
    in (O w', dWs)
```

# Backprop

```
-- handle the inner layers
go !x (w@(W wB wN) :~ n)
  = let y          = runLayer w x
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      -- get dWs', bundle of derivatives from rest
      (n', dWs') = go o n
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      w'         = W wB' wN'
      -- bundle of derivatives for next step
      dWs        = tr wN #> dEdy
  in  (w' :~ n', dWs)
```

Surprise! It's actually identical! No loss in expressivity. Typed holes can write our code for us in many cases. And shapes are all verified.

# Type-Driven Development

We wrote an untyped implementation, then realized what was wrong. Then we added types, and everything is great!

## Further reading

Blog series: <https://blog.jle.im/entries/series/+practical-dependent-types-in-haskell.html>