Restart Schedules for Ensembles of Problem Instances

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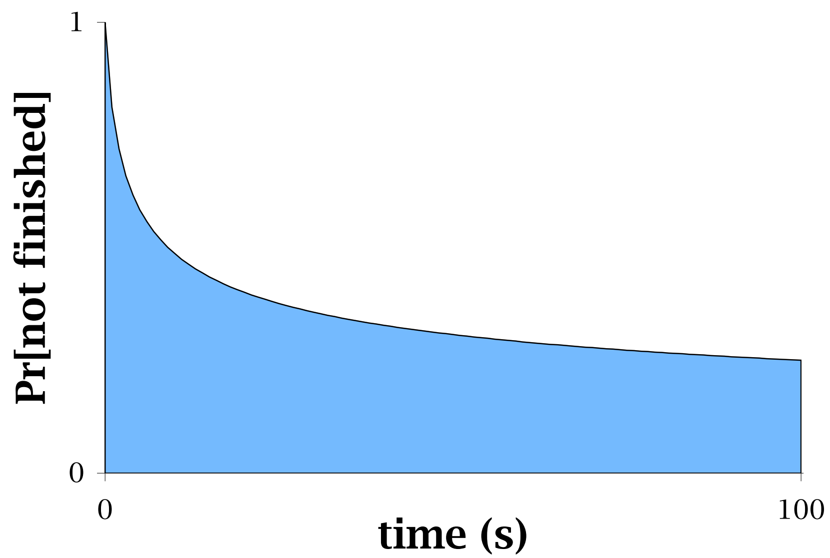
Restart schedules

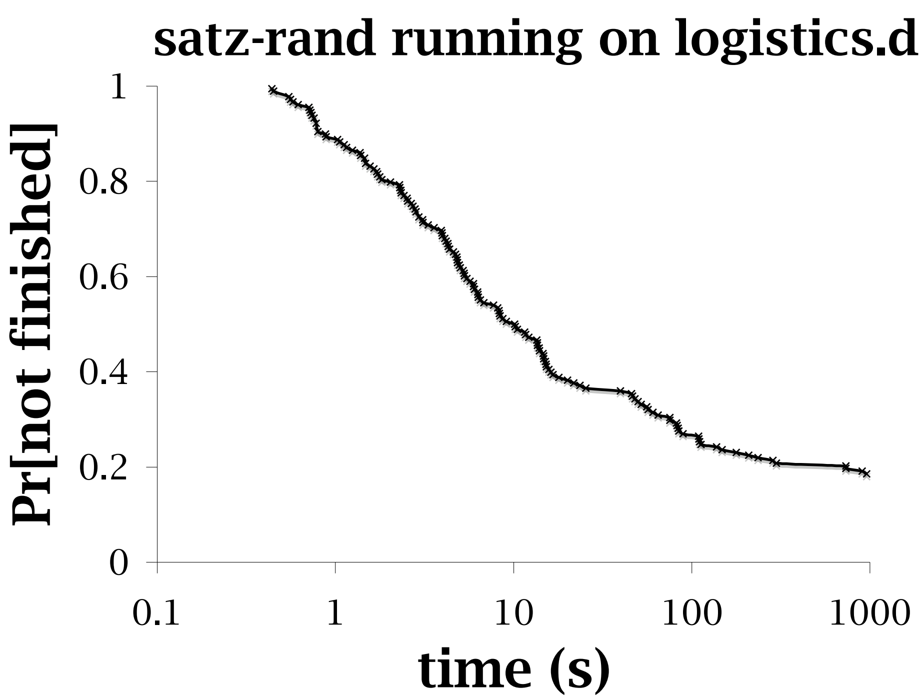
- A Las Vegas algorithm A always returns a correct yes/no answer, but running time depends on random seed. Behavior of A on instance x can be represented as a run length distribution (RLD):



- A *restart schedule* is a sequence $\langle t_1, t_2, \ldots \rangle$ of integers, meaning "run A for time t_1; if it doesn't return an answer then restart and run for time t_2, ..."

Restarts and heavy tails

- Algorithms based on chronological backtracking often exhibit heavy-tailed RLDs (Gomes et al. 1998)
- Restart schedules can improve performance by orders of magnitude

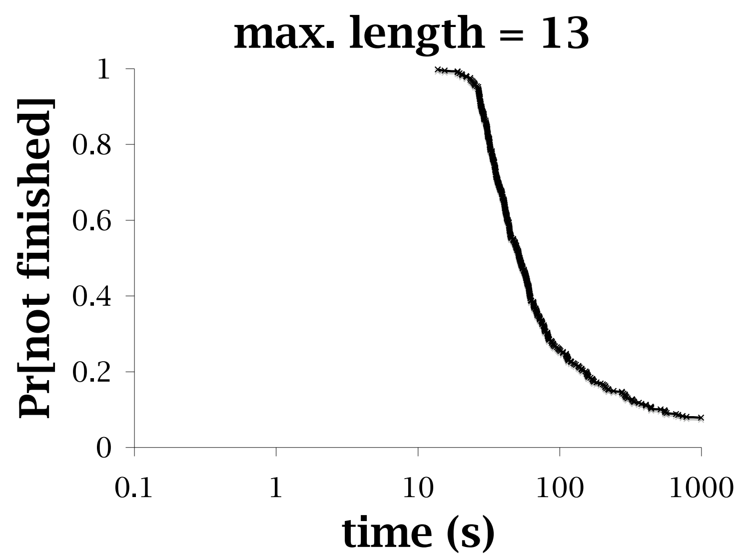


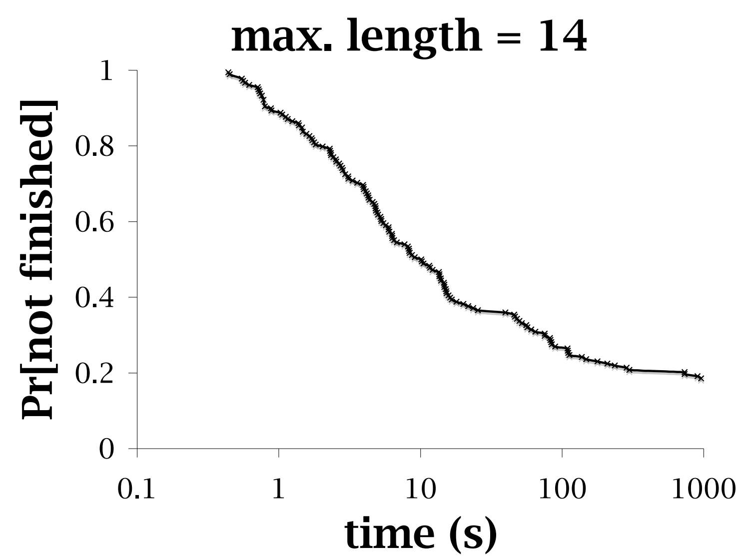
Previous work

- Luby et al. (1993) considered solving a single instance with unknown RLD, and gave a *universal restart schedule* with optimal competitive ratio
- Gomes et al. (1998) showed that restart schedules could improve performance of a then state-of-the-art SAT solver
- Kautz et al. (2002) use features to predict RLD
- Ruan et al. (2002) show how to compute optimal schedule when there are k distinct RLDs, but running time is exponential in k
- Gagliolo et al. (2007) used multi-armed bandit solver to select restart schedules online

RLDs vary across instances

- Here are RLDs for two SAT instances created by SatPlan when solving the logistics.d planning benchmark
- Restart helps in both cases
- Optimal schedule for average of two RLDs performs poorly





This talk

- **Goal:** efficiently construct a *single* restart schedule to use in solving a series of problem instances, each with a different RLD
- We consider three settings:
    - **Offline:** given a set of instances with known RLDs, compute an optimal restart schedule
    - **Learning-theoretic:** PAC-learn an optimal restart schedule from training instances
    - **Online:** you are fed an *arbitrary* sequence of instances, and must solve each instance before moving on to the next

The offline setting

- Given a set of RLDs, want to compute schedule that minimizes E[total CPU time]
- Assume CPU time for any instance capped at B
- We think this problem is NP-hard

Quasi-polynomial time approximation scheme

- Can get α^2 approximation to best schedule in time $O(n(\log_\alpha B)\ B^{\log_\alpha \log_\alpha B})$
- Uses shortest path formulation (generalization of algorithm from last talk)

Greedy approximation algorithm

- Algorithm: Greedily append run of length t to schedule, where t is chosen to maximize E[#(new instances solved)/t]

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<u>Instance x_1</u>	<u>Instance x_2</u>
$time(A,x_1) = 10$	$time(A,x_2) = 1$ w/prob. 1/100 ∞ w/prob. 99/100

$S = \langle \qquad \rangle$

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- Performance
  - Gives 4-approximation to optimal schedule (may do better)
  - **New variant** also returns optimal schedule if all instances have same RLD

The learning-theoretic setting

- Draw instances from some distribution. Each instance has its own RLD. Want to PAC-learn optimal restart schedule (with prob. $\geq 1-\delta$, schedule's expected cost is $\leq \epsilon$ worse than optimal)
- Two questions:
    - how many training instances?
    - how may runs per instance?

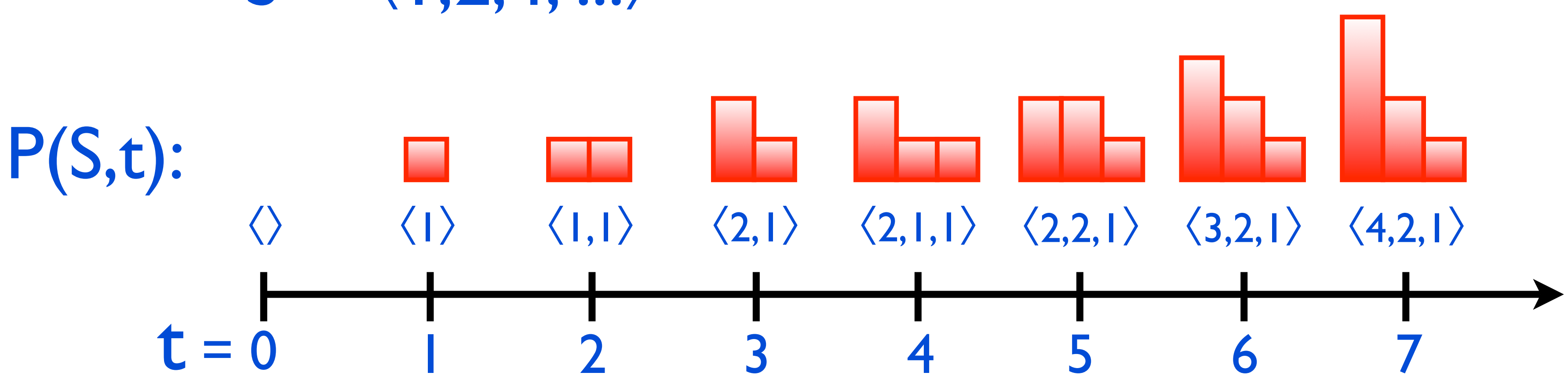
How many training instances?

- Need $O((B/\epsilon)^2 \mathrm{sqrt}(B) \log \delta^{-1})$ instances, assuming RLD of each instance is known exactly
- Proof uses shortest path formulation + Hoeffding bounds

How many runs per instance?

- A *profile* $\langle \tau_1, \tau_2, \ldots, \tau_k \rangle$ is a non-increasing list of integers
- State of schedule S at time t can be represented as a profile $P(S,t)$

$S = \langle 1,2,4, \ldots \rangle$



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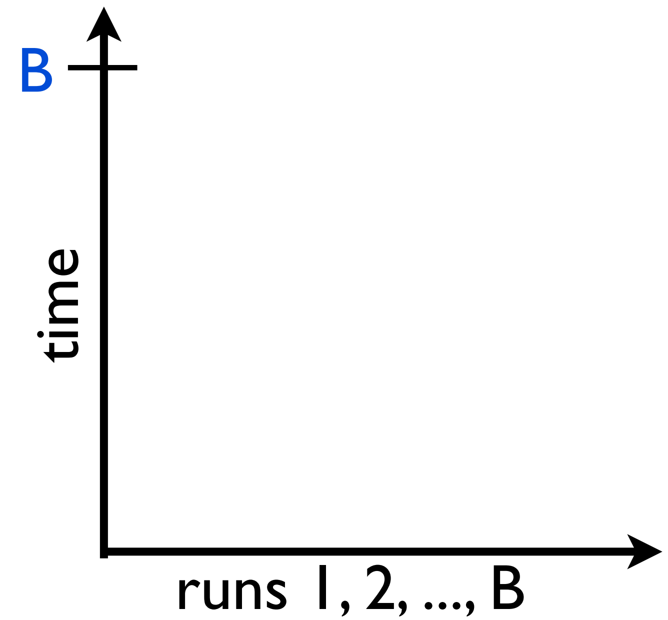

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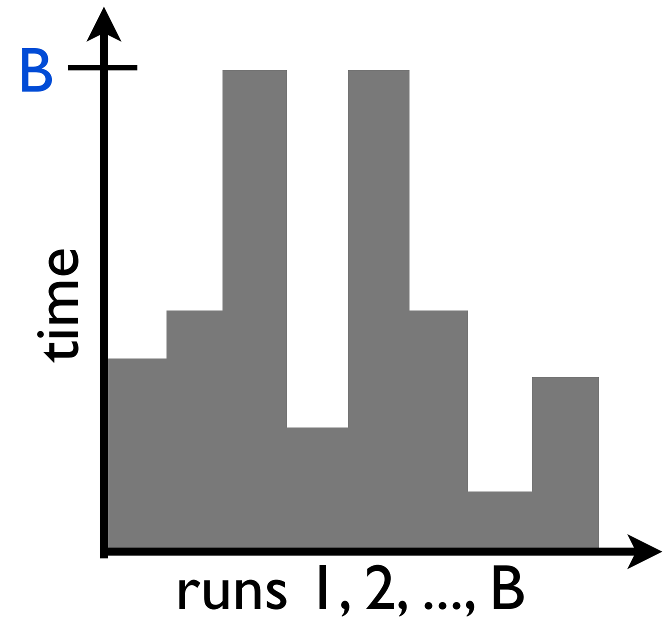

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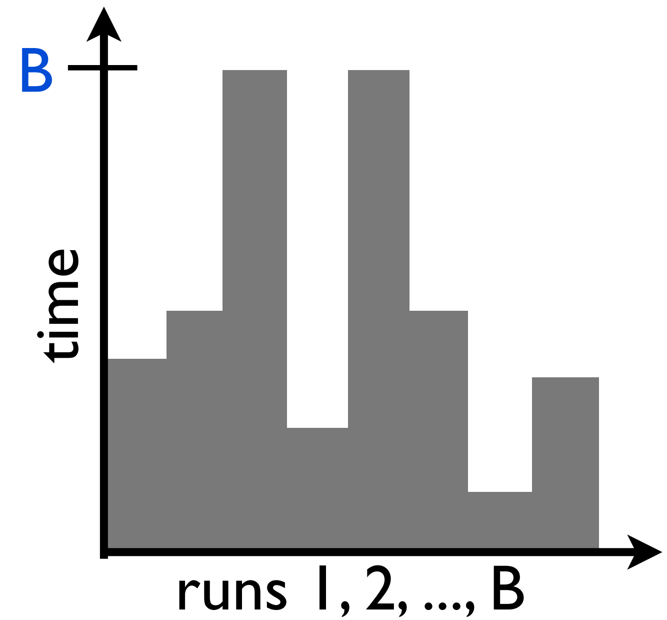

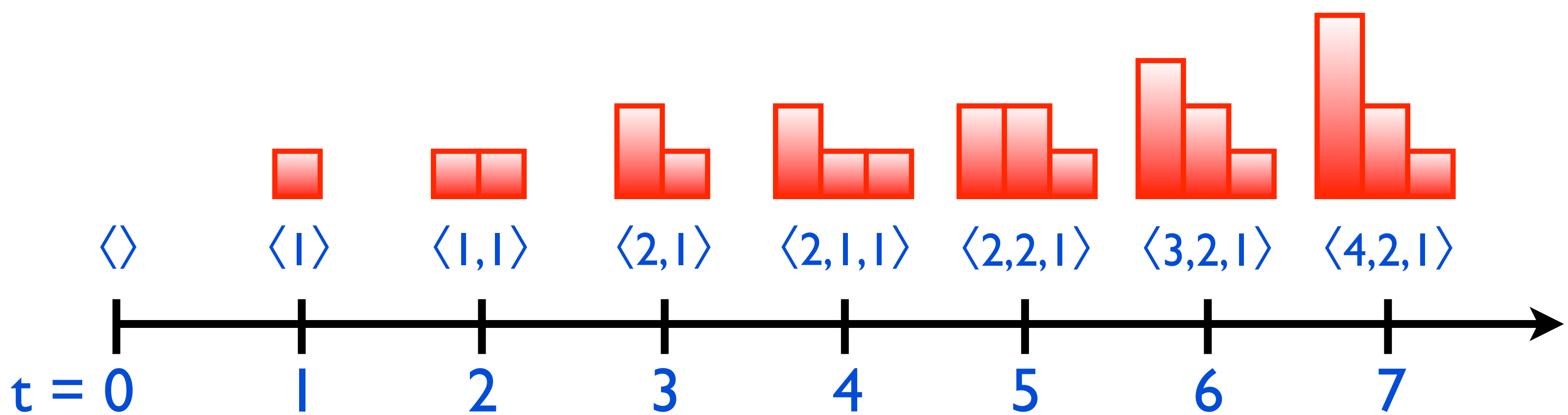

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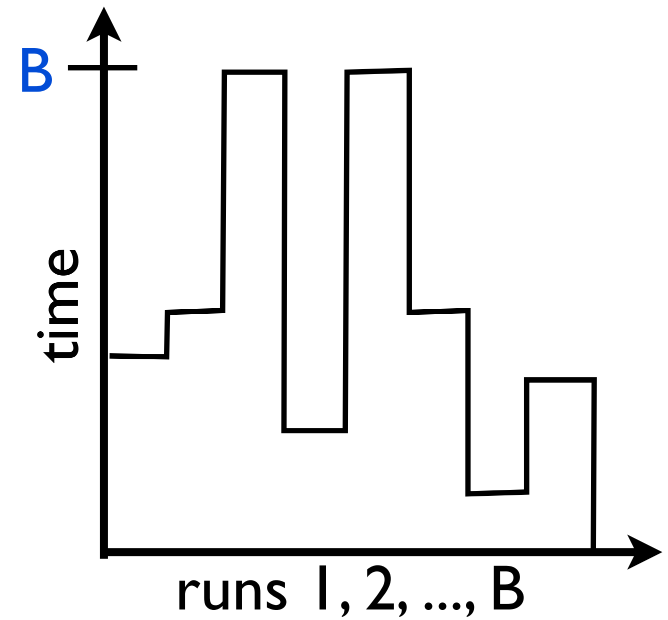

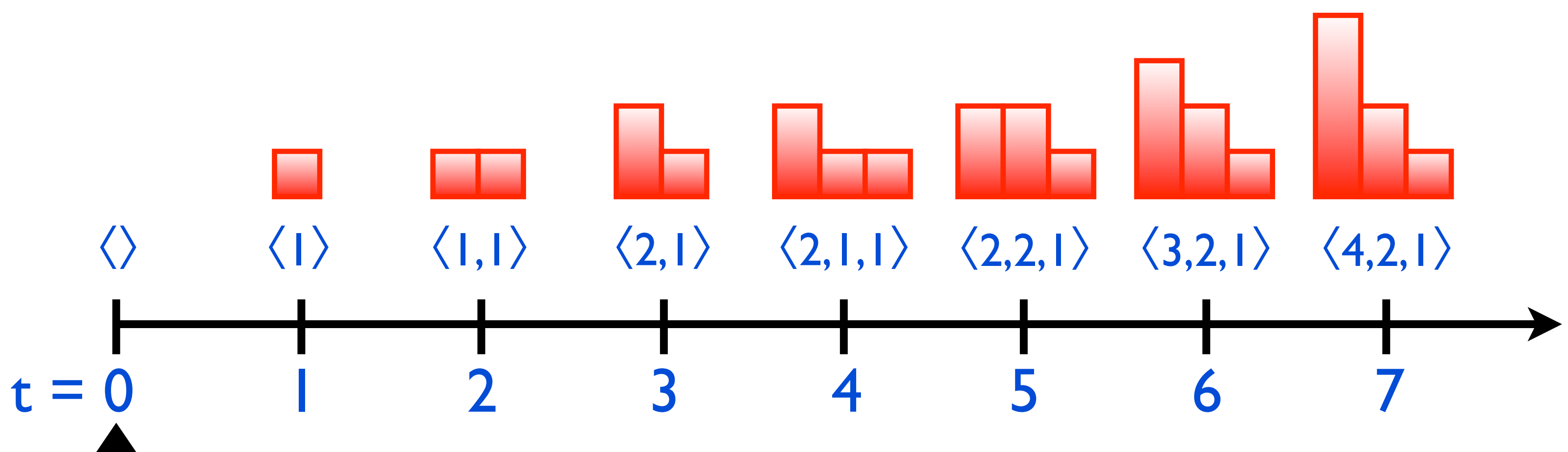

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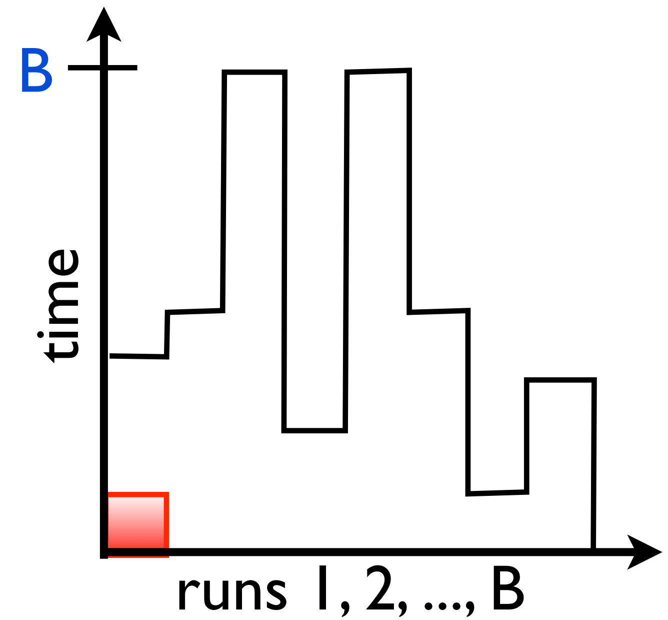

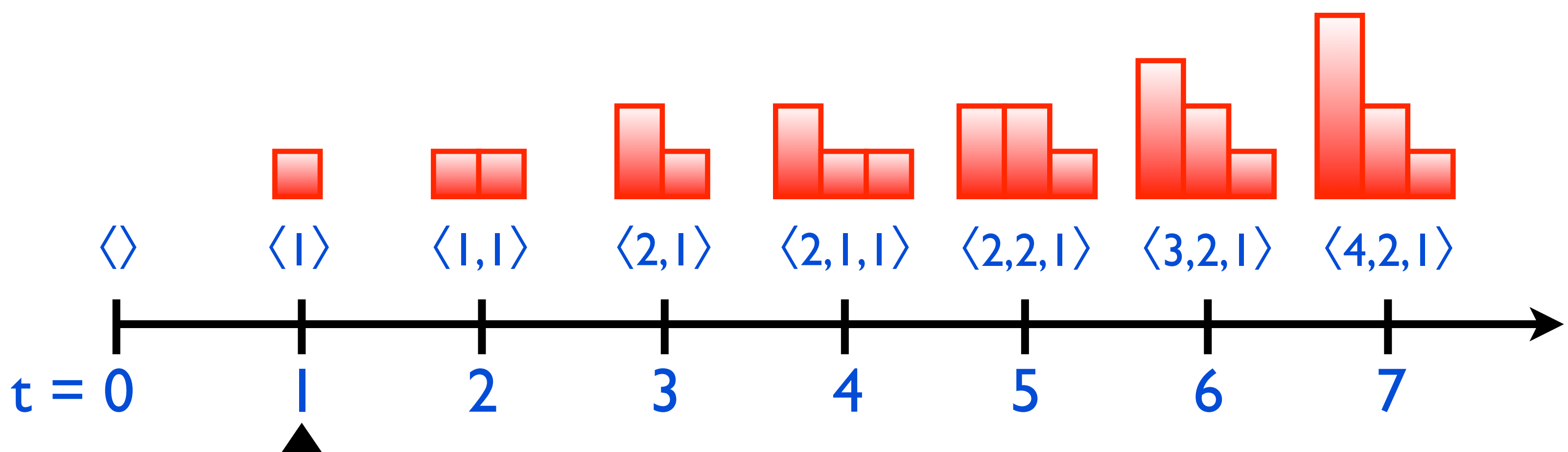

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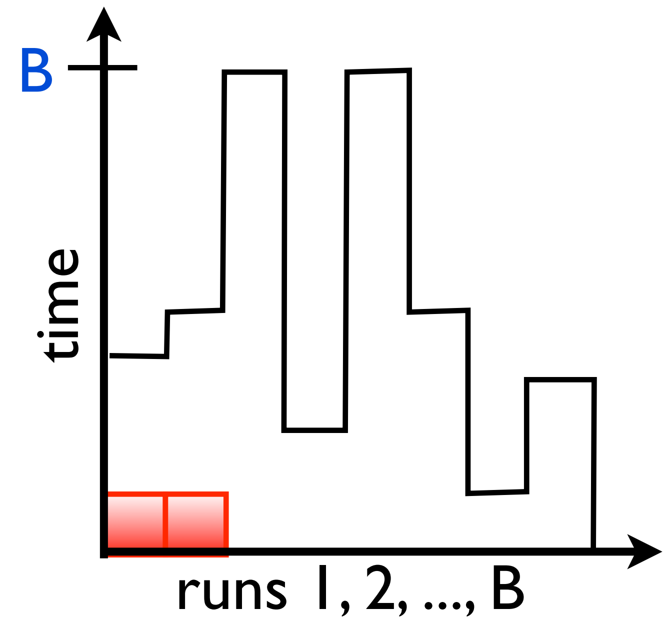

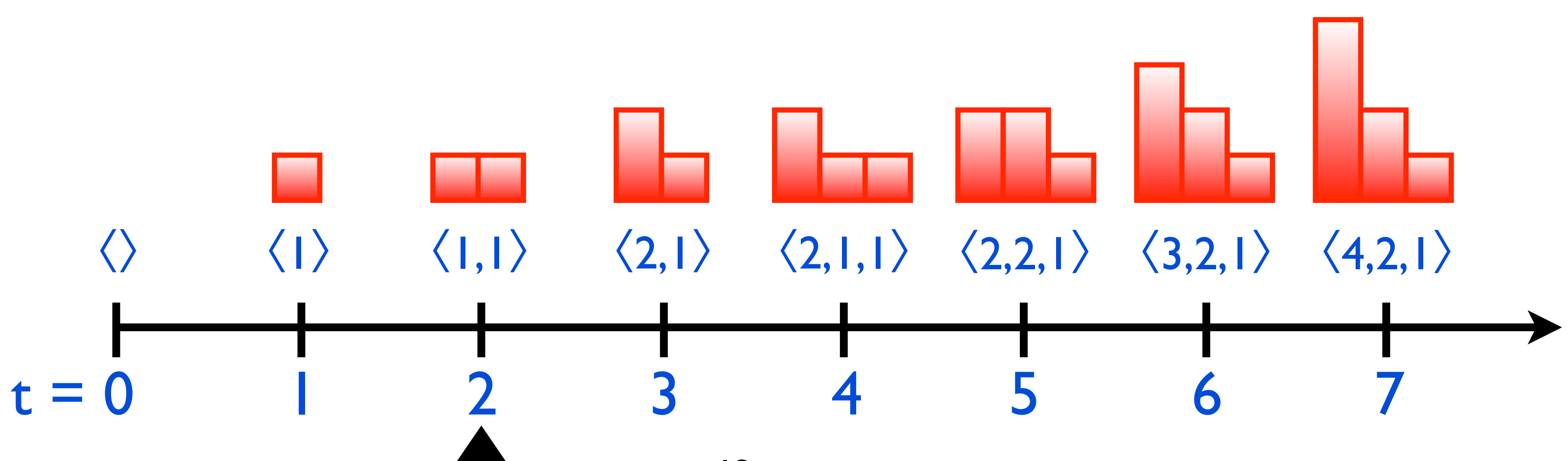

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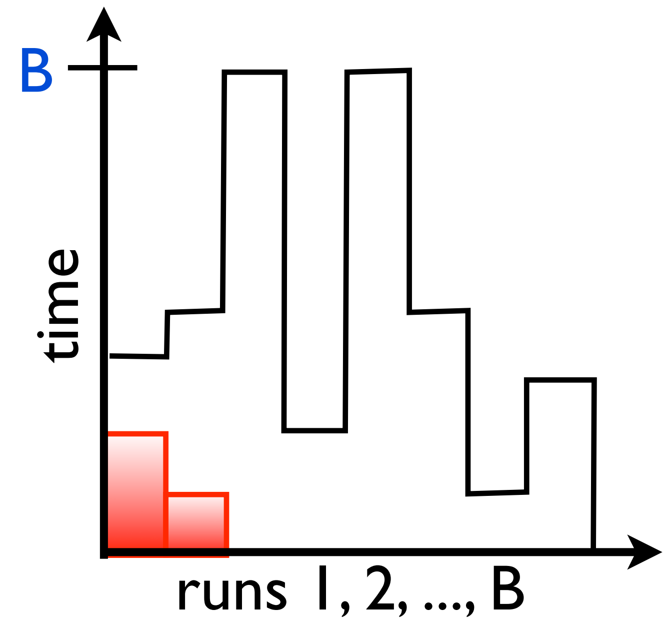

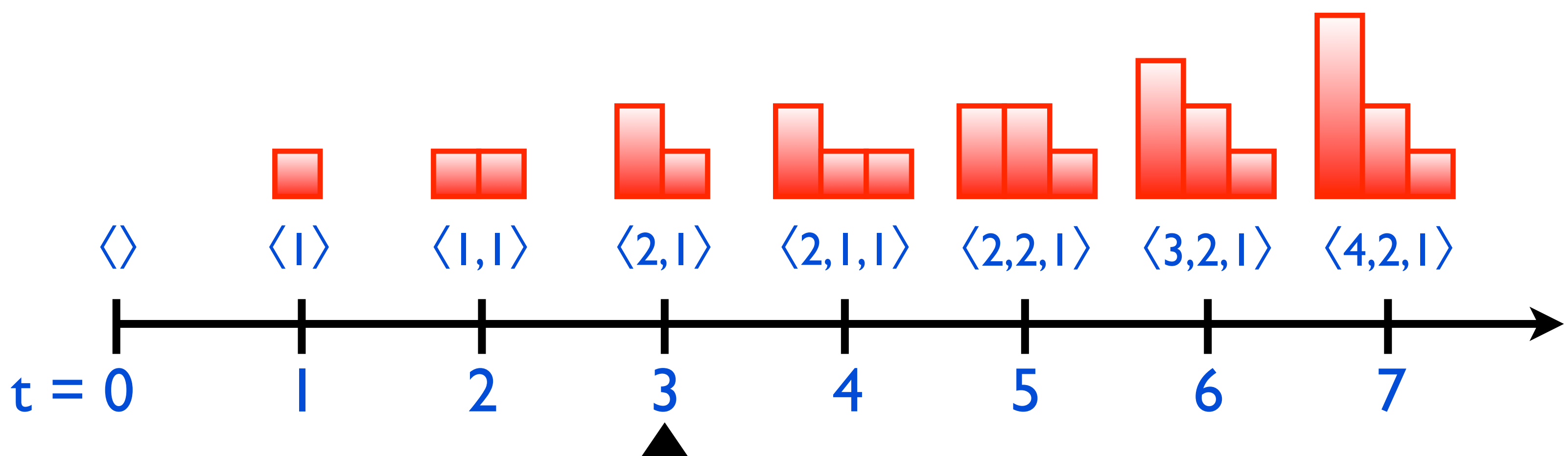

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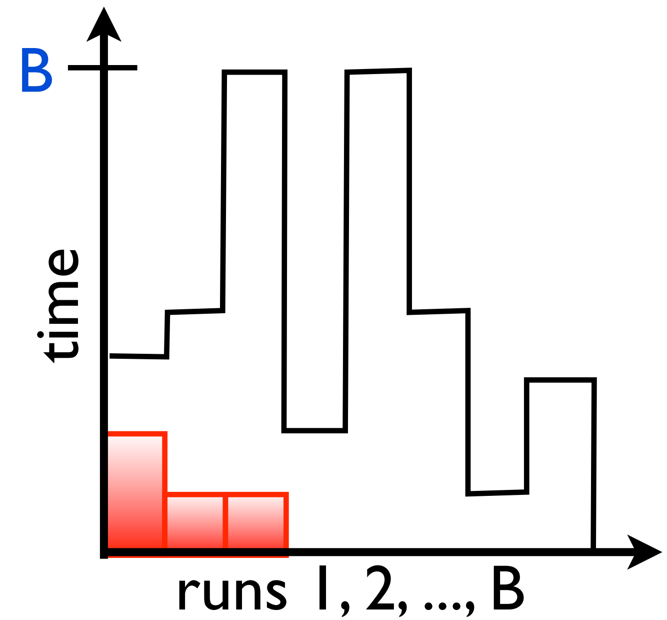

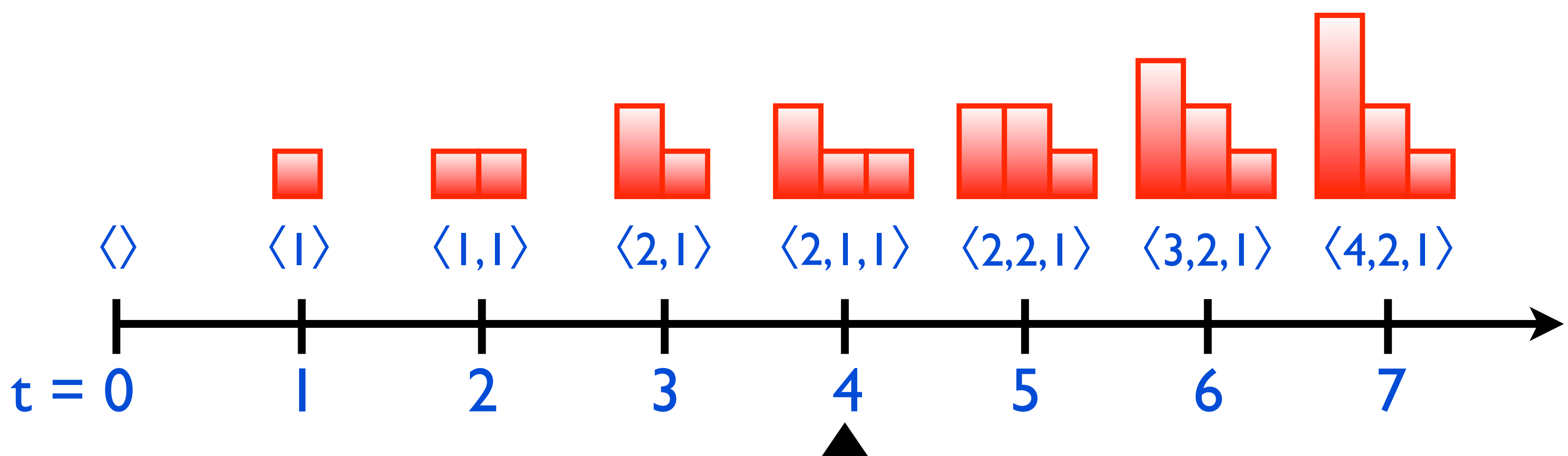

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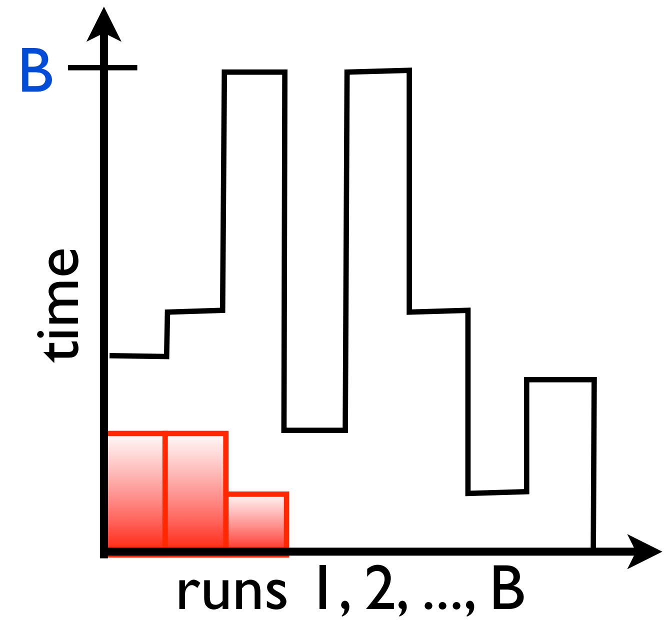

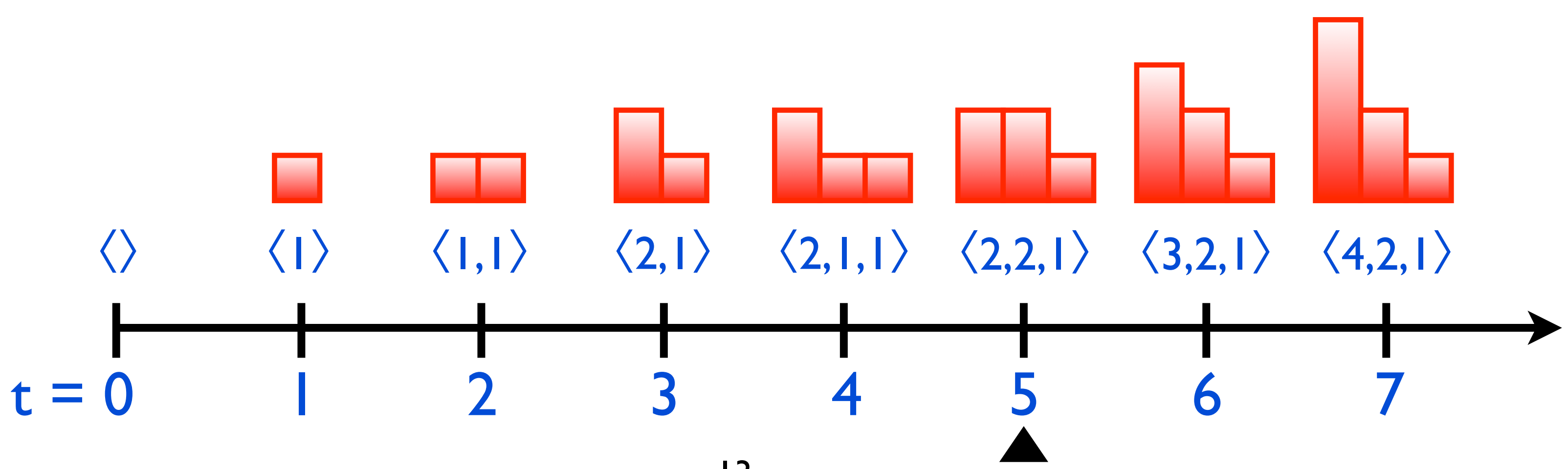

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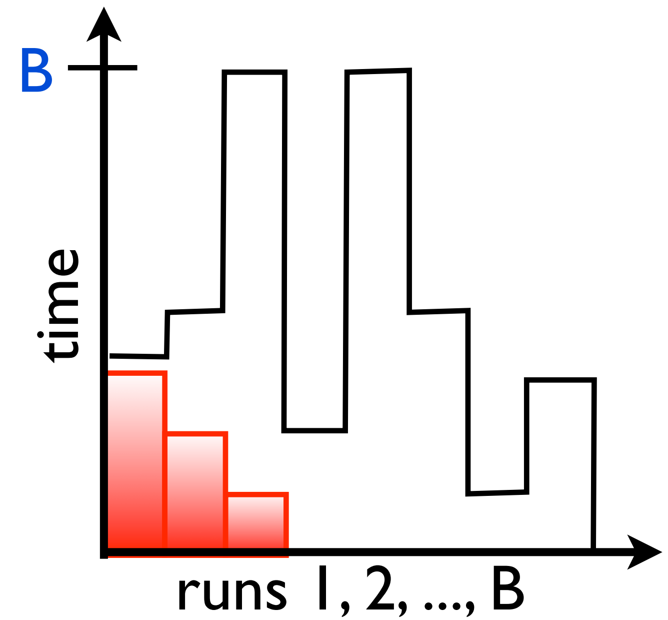

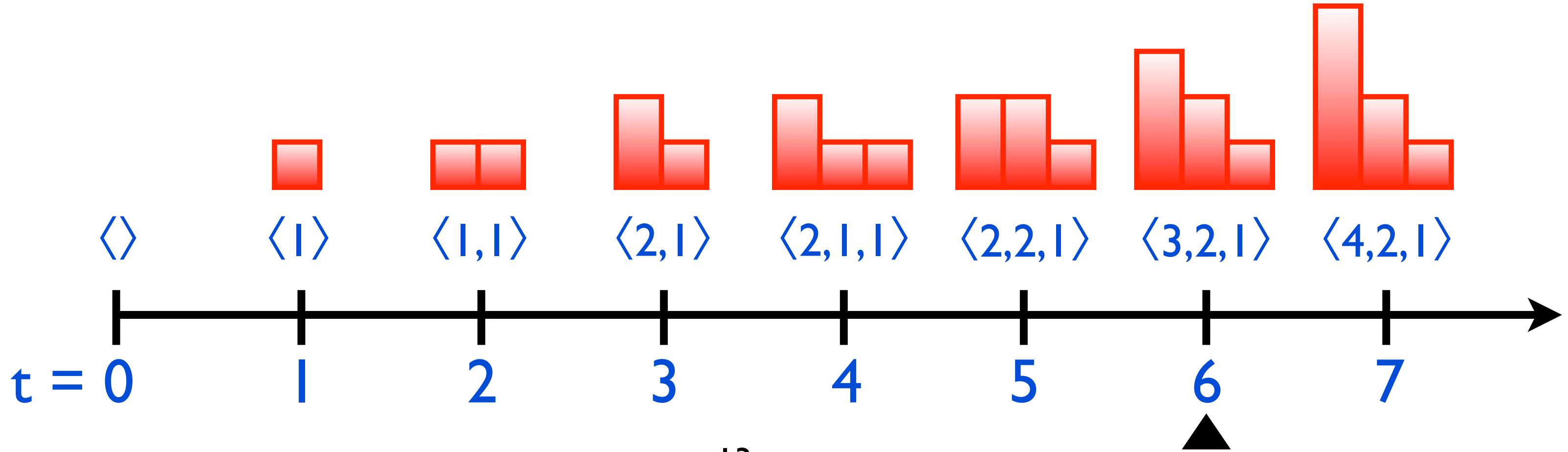

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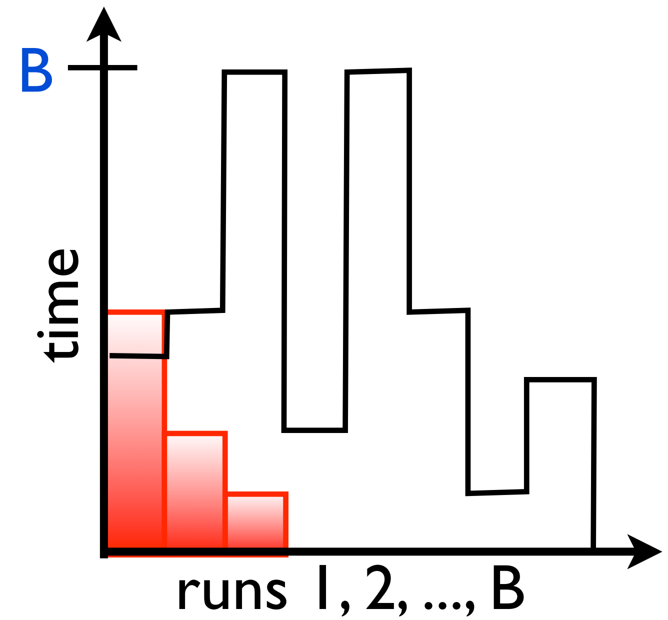

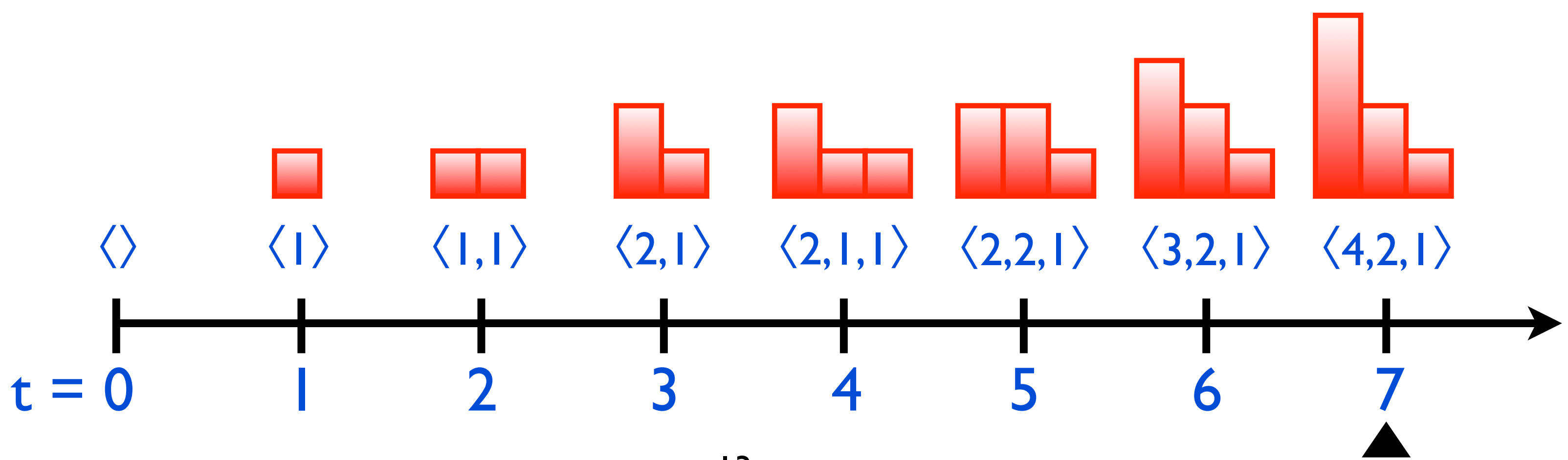

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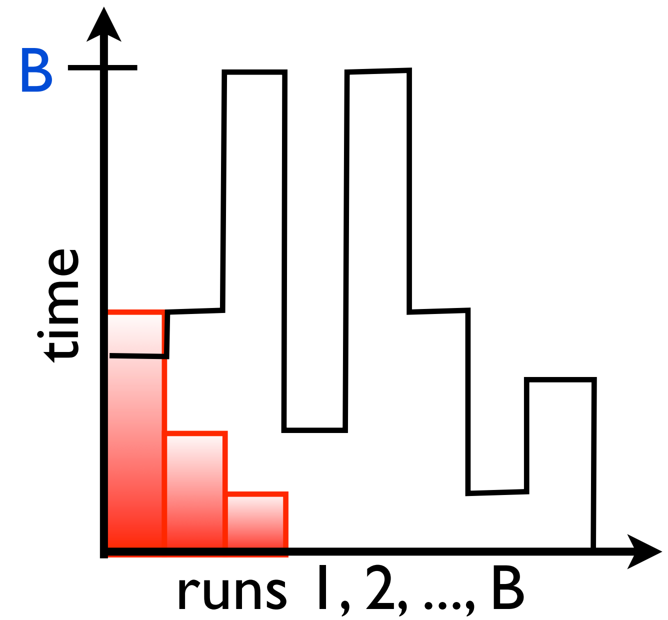

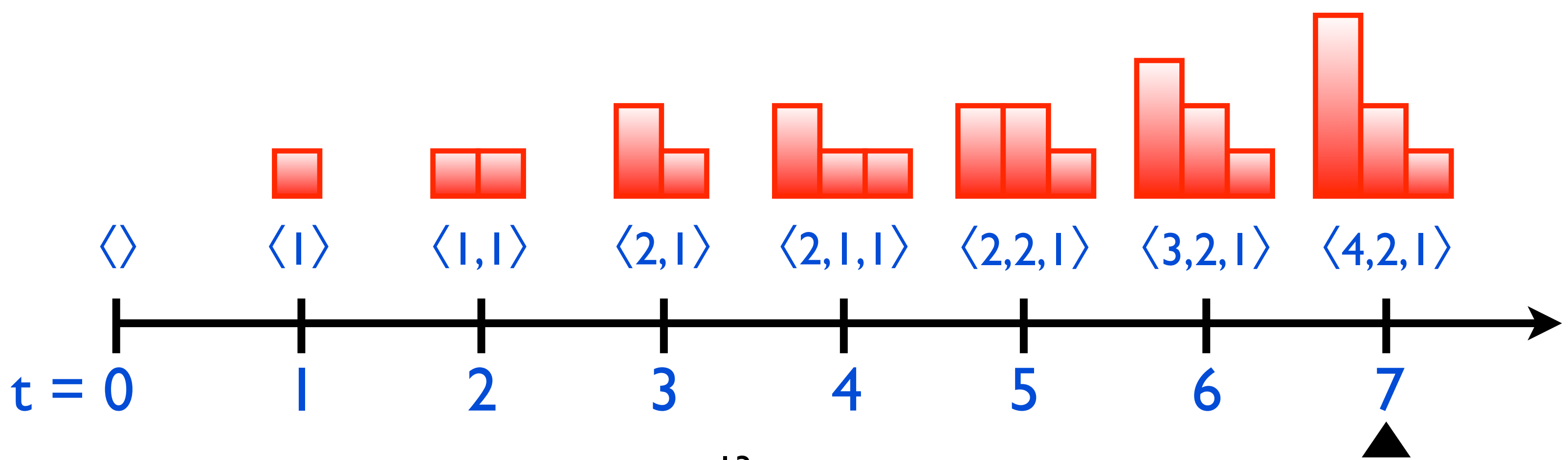

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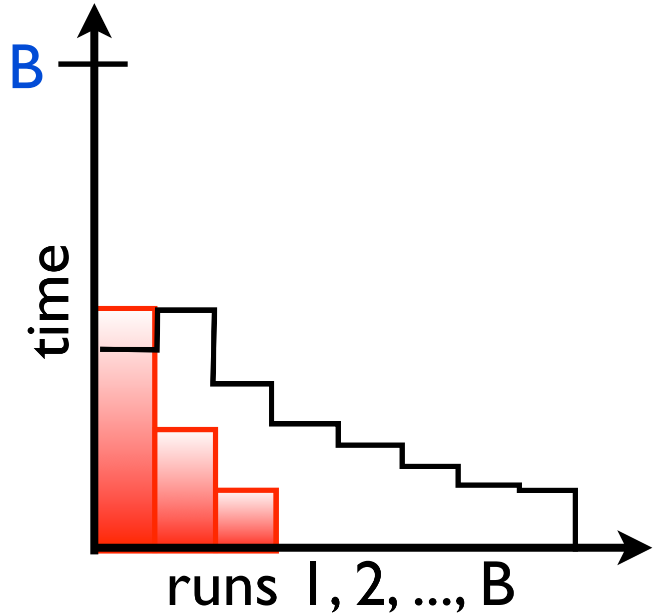
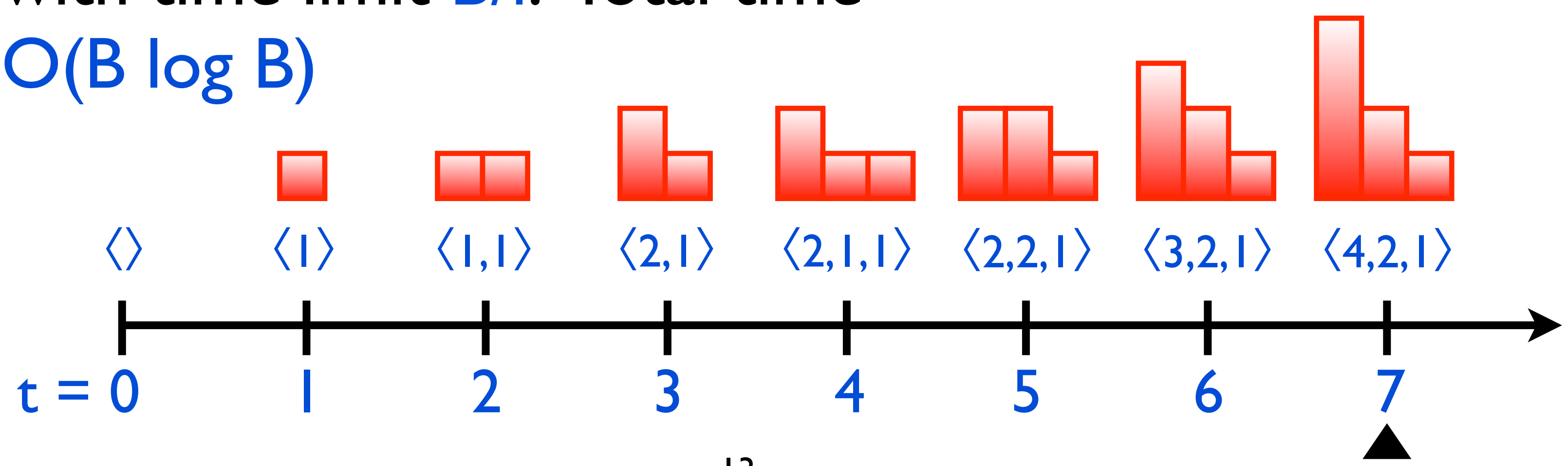
- Answer: after B runs, can get unbiased estimate of CPU time required by any schedule S
- If each run has time limit B, total CPU time $\leq B^2$
- Can actually perform i^{th} run with time limit B/i. Total time = O(B log B)





The online setting

- World secretly selects sequence of n instances/RLDs
- For i from 1 to n
  - You select schedule S_i to use to solve i^{th} instance
  - As feedback you observe how much time S_i takes
- regret = **E**[your total time] - $\min_{(\text{schedules } S)}$ (**E**[S's total time])

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- We give a schedule selection strategy whose worst-case regret is $o(n)$, assuming schedules come from a small pool.
- Uses unbiased estimation procedure + technique from Cesa-Bianchi et al. (2005)
- **Ongoing work:** online version of greedy approx. algorithm

Experimental evaluation

- Ran satz-rand on formulae generated by SatPlan when solving randomly-generated logistics planning benchmarks

Restart schedule	**Avg. CPU (s)**
Greedy schedule	21.9
Best geometric $\langle \alpha^0, \alpha^1, \alpha^2, \ldots \rangle$	23.9
Best uniform $\langle t, t, t, \ldots \rangle$	33.9
Luby's universal schedule	37.2
No restarts	74.1

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Restart schedule	**Avg. CPU (s)**
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Generalization:
multiple Las Vegas algorithms

- If we have multiple Las Vegas algorithms, can consider restart schedules of the form
  $\langle(a_1,t_1), (a_2,t_2), \ldots\rangle$
- Results for all three settings generalize
- With multiple algorithms, it is NP-hard to get a $4-\epsilon$ approximation for any $\epsilon > 0$ (so greedy 4-approx is optimal)