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A hybrid fuzzy regression-fuzzy cognitive map algorithm for forecasting and optimization of housing market fluctuations

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ABSTRACT

This paper presents a hybrid algorithm based on fuzzy linear regression (FLR) and fuzzy cognitive map (FCM) to deal with the problem of forecasting and optimization of housing market fluctuations. Due to the uncertainty and severe noise associated with the housing market, the application of crisp data for forecasting and optimization purposes is insufficient. Hence, in order to enable the decision-makers to make decisions with respect to imprecise/fuzzy data, FLR is used in the proposed hybrid algorithm. The best-fitted FLR model is then selected with respect to two indicators including Index of Confidence (IC) and Mean Absolute Percentage Error (MAPE). To achieve this objective, analysis of variance (ANOVA) for a randomized complete block design (RCBD) is employed. The primary objective of this study is to utilize imprecise/fuzzy data in order to improve the analysis of housing price fluctuations, in accordance with the factors obtained through the best-fitted FLR model. The secondary objective of this study is the exhibition of the resulted values in a schematic way via FCM. Hybridization of FLR and FCM provides a decision support system (DSS) for utilization of historical data to predict housing market fluctuation in the future and identify the influence of the other parameters. The proposed hybrid FLR-FCM algorithm enables the decision-makers to utilize imprecise and ambiguous data and represent the resulted values of the model more clearly. This is the first study that utilizes a hybrid intelligent approach for housing price and market forecasting and optimization.

1. Introduction

Over the past three decades, a considerable attention in literature has paid to modeling, forecasting, and explaining long-run equilibrium of house prices. In the long run, employment and population growth as well as levels and growth of real incomes, real construction costs, and real interest rates affect the house prices. Housing markets fluctuates similar to other typical markets, i.e. the prices fall when the supply exceeds the demand and vice versa (McCue & Belsky, 2007). In situations where the supplier's production level is higher than the real demand, due to inventory carrying issues, the supplier has to reduce its output and lower its asking price. As a consequence, the quantities of supplied product fall and so the demand rise. Nevertheless, in some instances, the system may encounter different behaviors when the supply and demand quantities are not equal.

Housing supply and demand often are not in equilibrium in large run. Since, in real-world problems, there is a long time-lag between the times in which the land is purchased with the intention of building homes and the building process is completed. Therefore, when the building process is completed, the level of housing market demand may be far less or higher than its expected value. McCue and Belsky (2007) discussed on a number of factors that disturb the equilibrium between the supply and demand quantities in housing markets (Table 1).

Maisel (1963) introduced a theoretical framework based on feedback with lags for modeling housing market. DiPasquale and Wheaton (1996) employed a stock-flow model to describe the housing market and developed their model for the Singapore housing market further. Borgersen, Sommervoll, and Wennemo (2006) proposed a model with macroeconomic and demographic variables. Edelstein and Tsang (2007) presented a dynamic approach including demand and supply of housing in terms of econometric methods. Sæther (2008) classified housing market variables into three classes including endogenous, exogenous, and excluded. He classified housing prices, construction costs, housing demand/supply, labor, etc. into endogenous class, price elasticity, adjustment times, labor productivity, etc. into exogenous class, and debt, financial sector, housing expenditures, national economy, etc. into excluded class.

The profitability rate of housing market can be evaluated in comparison with different classes of variables. Endogenous variables including land price, material price, wages, cost expectations, and tolls and exogenous variables including cash flow, stock/gold

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Nomenclature

FLR	Fuzzy Linear Regression
CLR	Conventional Linear Regression
FCM	Fuzzy Cognitive Map
CM	Cognitive Map
ANOVA	Analysis of Variance
RCBD	Randomized Complete Block Design
MAPE	Mean Absolute Percentage Error
IC	Index of Confidence
DSS	Decision Support System

market, general inflation, profitability rate of other sectors, deposit profits of banks, and oscillations in currency market affect the profitability of housing market in Iran. In general, analyzing the supply and demand of housing market is a sophisticated problem and so, there is not a comprehensive to deal with this problem. Shakoorifar and Kaveh (2001) designed a dynamic system with respect to housing parameters categorized as supply system, demand system, and dealing system. Construction is an economic procedure which needs sufficient profitability of the investment. The governments usually support suppliers and affect housing market to control it (supply system). On the other hand, balanced conditions in population structure of a society leads to demanding for houses. Marriage, immigration, and death rate are the factors which directly affect the housing market. Hence, population factors are the most significant controllers (demand system). Dealing system lies between supply and demand systems. Table 2 represents the mentioned housing parameters and their relevant effective factors (Shakoorifar & Kaveh, 2001).

Table 1
The reasons of the supply and demand fluctuations in housing markets.[a]

Demand becomes lower than supply	– Job growth decreases dramatically or turns negative – Housing affordability decreases due to increasing in real rates – Confidence in future house prices decrease which makes buyers to wait on the sidelines or exit markets – Lenders constrain access to mortgage credit – Returns on other investments become relatively more attractive
Supply becomes higher than demand	– House building increases with respect to the quantity of housing demand – Second owners or speculators exit the market and place their unoccupied houses on the market for sale – Job losses force distressed sales by owners unable to afford their mortgage payments or lenders that foreclose on defaulted loans – Interest rate increases or expiring teaser rates on adjustable mortgage cause mortgage payment shocks that force distressed sales or defaults

[a] McCue and Belsky (2007).

Table 2
The housing parameters and their related effective factors.[a]

Supply system	– Available stock – Governmental policies (Loans, Credit, etc.) – Investment in other markets – Construction of new houses – Under construction and completed housing projects – Completion rate of houses – Houses ready for sell – Selling rate of completed houses – Selling price of houses – Number of sold houses in each period – The policy of supply corporation – Attractions of other markets and investments
Demand system	– New households – Total number of households – Inexistency rate of households – Immigrant households – General conditions of society – Potential applicants – Saver household – Household income and expenditure – Households which receive loans – House prices
Dealing system	– Effective applicants – The number of vacancies – Maximum price which can be paid by applicants – Minimum price which a supplier offers – Unsold houses – Delayed effective applicants – Number of transactions – House price (The most important variable)

[a] Shakoorifar and Kaveh (2001).

Aggregation of the three systems presented in Table 2 forms a real system in which, suppliers and applicants construct two sides of system and the dealing system connects their components. However, the resulted model based on the three mentioned systems (Shakoorifar & Kaveh, 2001) seems to be incomplete. Since, it merely proposes a general framework of components and relationships of the supply and demand system. Hence, in order to investigate on this model more precisely, some of the defined components have to be eliminated and provide a simplified model for housing market to be analyzed.

In many Asian countries like Iran, discrepancies between demand and availability in the housing market bring on problems of inefficiency and dissatisfaction. To solve this problem, housing suppliers have been increasingly searching for ways to reengineer the process and satisfy the customers' demand more appropriately. Based on this motivation, due to the uncertainty/incompleteness of the house demand data in the third world countries such as Iran, this paper proposes a hybrid FLR-FCM algorithm to evaluate the fluctuations of housing markets in Iran. Several test problems have been used to show the superiority of the proposed hybrid algorithm and its applicability in real-world problems.

The remainder of this paper is organized as follows. Section 2 represents the proposed methodology for the hybridization of FLR and FCM. Section 3 describes the experimentation procedure. Section 4 presents the results and analysis and concluding remarks of this study are presented in Section 5.

2. Method: the hybrid FLR-FCM algorithm

The proposed hybrid FLR-FCM algorithm to deal with fluctuations existing in housing markets is presented in this section. Hybridization of LFR and FCM will result in a comprehensive perception of housing market fluctuations with respect to its complexity and uncertainty. In summary, the steps of the proposed algorithm can be drawn as follows:

Step 1: Definition of input variables (endogenous, exogenous, and excluded);
Step 2: Representation of variables in a schematic diagram as nodes and the relations between each pair of variables;

Step 3: Collection of reliable data in order to sufficiently evaluate and optimize the system;
Step 4: Application, calculation, evaluation, and defuzzification of appropriate, well-known and mostly-used FLR models for forecasting of housing market fluctuations;
Step 5: Application of FCM by representation of fuzzy weights for relations, application of subjective utility functions for weights definition, and replacement of the defined weights;
Step 6: Selection of the best-fitted FLR model with respect to their related IC and MAPE indicators via ANOVA-RCBD;
Step 7: Hybridization of the developed FLR and FCM models for forecasting and optimization of housing market fluctuations.

Fig. 1 illustrates the proposed hybrid algorithm based on hybridization of FLR and FCM for forecasting and optimization of housing market fluctuations. The incoming sections describe each of the methods used in the hybrid algorithm in detail.

2.1. Fuzzy linear regression

Regression analysis is a well-known tool recommended to be utilized for identifying the functional relationship between independent and dependent variables. This would be usual when we have crisp variables; however, in real situations, regression variables may be given as non-numerical entities such as linguistic variables (Bargiela, Pedrycz, & Nakashima, 2007).

Real-world problems usually encounter uncertainty/incompleteness. Uncertainty would be due to (1) impreciseness, (2) incompleteness, (3) vagueness, (4) judgments, (5) ambiguousness associated with the data, (6) not knowledge-based data, and/or (7) stochastic nature of an event's results. Except the last one, all first six uncertainties are based on fuzzy logic. Therefore, it would be better to use fuzzy modeling approach under uncertainty. Fuzzy data have several applications in enormous fields such as psychometry, reliability, marketing, quality control, ballistics, ergonomy, image recognition, artificial intelligence, etc. (D'Urso & Gastaldi, 2000). There are many cases in which, we have to use approximate description of observations or appropriate intervals.

Conventional linear regression (CLR) models are applicable in all cases of FLR, when we have to make decisions with imprecise and/or incomplete data (Hong & Hwang, 2004). Several fuzzy regression approaches have been proposed in literature. Those models which have been used in this paper will be explained more to define the required relationships. FLR was introduced by Tanaka, Uejima, and Asia (1982) to construct a fuzzy linear relationship by $Y = A_0X_0 + A_1X_1 + \cdots + A_kX_k$, where regression coefficients A_j ($j = 0,\ldots,K$) are supposed to be symmetric triangular fuzzy numbers with center α_j, having membership function equal to one, and spreads c_j, $c_j \geqslant 0$.

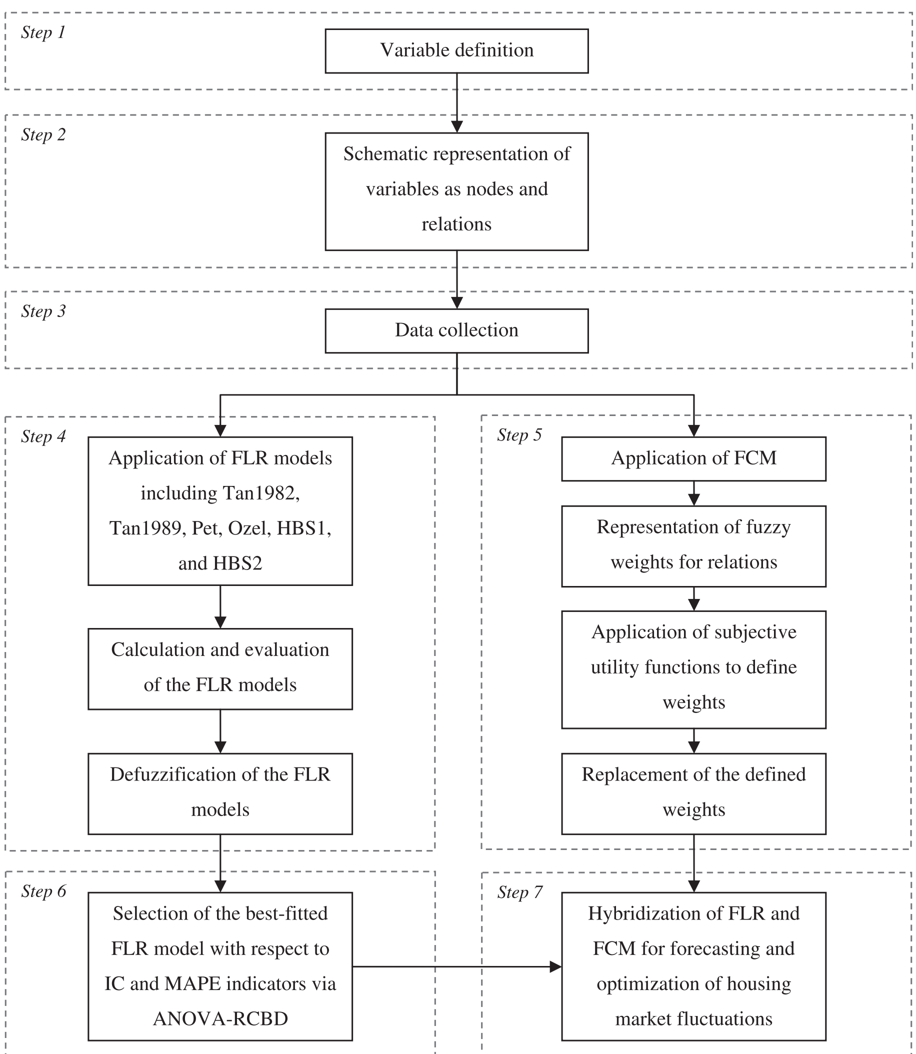

Fig. 1. Schematic representation of the proposed hybrid FLR-FCM algorithm.

2.1.1. Notation

The following notations help us to discuss more about different FLR models. Two different classes of notations have been considered in which in the first class, x is crisp and y is fuzzy and in the second class, both x and y are fuzzy.

2.1.1.1. If x is crisp and y is fuzzy

k	Number of independent variable values
n	Number of input data sets of values $(y_i, x_{i0}, x_{i1}, \ldots, x_{ik})$, $n \geqslant k+1$
i	Index for input data sets, $(i = 1,2,\ldots,n)$
j	Index for independent variables, $(j = 1,2,\ldots,k)$
x_{ij}	A crisp number
y_i	A symmetric triangular fuzzy number
$\bar{y}_i$	Symmetric triangular fuzzy number center
e_i	half-width, $(e_i \geqslant 0)$

2.1.1.2. If both x and y is fuzzy

k	Number of independent variables
n	Number of input data sets, $(y_i, x_{i0}, x_{i1}, \ldots, x_{ik})$, $n \geqslant k+1$
i	Index for input data sets, $(i = 1,2,\ldots,n)$
j	Index for independent variables, $(j = 1,2,\ldots,k)$
x_{ij}	A symmetric triangular fuzzy number
$\bar{x}_{ij}$	Symmetric triangular fuzzy number center
f_{ij}	Half-width, $(f_{ij} \geqslant 0)$
y_i	A symmetric triangular fuzzy number
$\bar{y}_i$	Symmetric triangular fuzzy number center
e_i	Half-width, $(e_i \geqslant 0)$

2.1.2. FR models

Tanaka et al. (1982) introduced the following linear programming (LP) formulation for estimation of $A_j (j = 1,2,\ldots,k)$ as follows:

$$\begin{aligned} \min\ & \sum_{i=1}^{n}\sum_{j=0}^{k} c_j x_{ij} \\ \text{s.t.}\ & \sum_{j=0}^{k}(\alpha_j + (1-H)\times c_j)\times x_{ij} \geqslant \bar{y}_i + (1-H)\times \bar{e}_i \quad i=1,\ldots,n \\ & \sum_{j=0}^{k}(\alpha_j - (1-H)\times c_j)\times x_{ij} \leqslant \bar{y}_i - (1-H)\times \bar{e}_i \quad i=1,\ldots,n \\ & \alpha_j = free,\quad c_j \geqslant 0,\quad j=0,\ldots,k \end{aligned} \tag{1}$$

Note that in Model (1), c_j is supposed to be non-negative, because the fuzziness in the estimated intervals usually increases for larger values of independent variables x_j (Hojati et al., 2005). However, there are some criticisms on the FR model presented by Tanaka et al. (1982). For instance, the results are x_j-scale dependent and many c_js might be equal to zero (Jozsef et al., 1992). To deal with this problem and replacement for sum of spreads of FR model's coefficients, the sum of spreads of the estimated intervals can be used as an objective function. Another comment on the FR model of Tanaka et al. (1982) is that each H-certain estimated interval is required to involve the corresponding H-certain observed interval.

Peters (1994) proposed another FR formulation which is a little complex in comparison with the other methods. Assume that y_{iU}, $\bar{y}_i$ and y_{iL} are the upper, center, and lower values of the ith observed interval, respectively. Also, let $\hat{y}_{iU}$ and $\hat{y}_{iL}$ be the upper and lower values of the ith estimated interval. This model permits $\hat{y}_{iL}$ to be greater than y_{iL} and smaller than y_{iU}, and similarly $\hat{y}_{iU}$ to be smaller than y_{iU} and greater than y_{iL}. This objective function is balanced against the total spreads of estimated intervals. By changing the objective function of the FR model of Tanaka, Hayashi, and Watada (1989) into a constraint, Peters (1994) formulation can be drawn as follows:

$$\begin{aligned} \min\ & \bar{\lambda} \\ \text{s.t.}\ & \sum_{j=0}^{k}(\alpha_j + c_j)\times x_{ij} \geqslant \bar{y}_i - (1-\lambda_i)\times \bar{e}_i \quad i=1,\ldots,n, \\ & \sum_{j=0}^{k}(\alpha_j - c_j)\times x_{ij} \leqslant \bar{y}_i + (1-\lambda_i)\times \bar{e}_i \quad i=1,\ldots,n, \\ & \bar{\lambda} = (\lambda_1 + \lambda_1 + \cdots \lambda_1)/n, \\ & \sum_{i=1}^{n}\sum_{j=0}^{k} c_j x_{ij} \leqslant P_0 \times (1-\bar{\lambda}), \\ & 0 \leqslant \lambda_i \leqslant 1,\quad i=1,\ldots,n,\quad \bar{\lambda} \geqslant 0, \\ & \alpha_j = \text{free},\quad c_j \geqslant 0,\quad j=0,\ldots,k \end{aligned} \tag{2}$$

It is obvious that determining a proper value for P_0 is difficult and the result is sensitive to this parameter (Peters, 1994; Hojati et al., 2005). In addition, this formulation considered with only one membership function to which the result belongs to the set of good solution λ for all constraints. Therefore, a new FR formulation is proposed by Peters (1994) as follows:

$$\begin{aligned} \max\ & \lambda \\ \text{s.t.}\ & \sum_{i=1}^{n}\sum_{j=0}^{k} c_j x_{ij} \leqslant P_0 \times (1-\lambda) \\ & \sum_{j=0}^{k}(\alpha_j + c_j)\times x_{ij} \geqslant \bar{y}_i - (1-\lambda)\times \bar{e}_i \\ & i=1,\ldots,n, \\ & \sum_{j=0}^{k}(\alpha_j - c_j)\times x_{ij} \geqslant \bar{y}_i + (1-\lambda)\times \bar{e}_i \\ & i=1,\ldots,n, \\ & 0 \leqslant \lambda \leqslant 1,\quad c_j \geqslant 0 \end{aligned} \tag{3}$$

We have to guess a good value for P_0, since the solution is sensitive to this value. Ozelkan and Duckstein (2000) introduced a similar model to Peters (1994) which did not require the estimation intervals to be divided to the observed intervals. The formulation of Ozelkan and Duckstein (2000) can be written as follows:

$$\begin{aligned} \min\ & \sum_{i=1}^{n}(d_{iU} + d_{iL}) \\ \text{s.t.}\ & \sum_{j=0}^{k}(\alpha_j + (1-H)\times c_j)\times x_{ij} \geqslant \bar{y}_i + (1-H) \\ & \times \bar{e}_i - d_{iU} \quad i=1,\ldots,n, \\ & \sum_{j=0}^{k}(\alpha_j - (1-H)\times c_j)\times x_{ij} \leqslant \bar{y}_i - (1-H) \\ & \times \bar{e}_i + d_{iL} \quad i=1,\ldots,n, \\ & \sum_{i=1}^{n}\sum_{j=0}^{k} c_j x_{ij} \leqslant \upsilon, \\ & d_{iL}, d_{iU} \geqslant 0,\quad i=1,\ldots,n, \\ & \alpha_j = \text{free},\quad c_j \geqslant 0,\quad j=0,\ldots,k, \end{aligned} \tag{4}$$

where υ is a parameter which should be diversified over all possible amounts of total spreads of estimated intervals, and d_{iU} and d_{iL} $(i = 1,\ldots,n)$ are upper and lower shift variables. Hojati et al. (2005) introduced a simple goal programming-like method to select the FR coefficients in order to minimize the total deviation of the estimated upper values of H-certain and the related observed intervals, and the estimated deviation of lower values of H-certain and the related observed intervals. This can be obtained via the following formulation:

$$\begin{aligned}
\min\ & \sum_{i=1}^{n}\left(d_{iU}^{+}+d_{iU}^{-}+d_{iL}^{+}+d_{iL}^{-}\right)\\
\text{s.t.}\ & \sum_{j=0}^{k}(\alpha_j+(1-H)\times c_j)\times x_{ij}-d_{iU}^{-}\geqslant \bar{y}_i+(1-H)\\
&\quad\times \bar{e}_i-d_{iU}^{+}\quad i=1,\dots,n,\\
& \sum_{j=0}^{k}(\alpha_j-(1-H)\times c_j)\times x_{ij}-d_{iL}^{-}\leqslant \bar{y}_i-(1-H)\\
&\quad\times \bar{e}_i+d_{iL}^{+}\quad i=1,\dots,n,\\
& \sum_{i=1}^{n}\sum_{j=0}^{k}c_jx_{ij}\leqslant \upsilon,\\
& d_{iU}^{+},d_{iU}^{-},d_{iL}^{+},d_{iL}^{-}\geqslant 0,\quad i=1,\dots,n,\\
& \alpha_j=\text{free},\quad c_j\geqslant 0,\quad j=0,\dots,k,
\end{aligned}\tag{5}$$

Note that for each index $i = 1,\dots,N$, at least one of d_{iU}^{+} or d_{iU}^{-} and d_{iL}^{+} or d_{iL}^{-} should be positive. Consequently, $|d_{iU}^{+}-d_{iU}^{-}|$ is the distance between upper value of H-certain estimation interval and the upper value of the H-certain observed interval, and $|d_{iL}^{+}-d_{iL}^{-}|$ is the distance between lower value of H-certain estimated interval and the lower value of the H-certain observed interval. The objective is to minimize the sum of these two intervals (Hojati et al., 2005).

In another FR formulation, Hojati et al. (2005) selected the FR coefficients so that the total difference between the upper values of estimated and observed intervals and the total difference between the lower values of estimated and observed intervals are minimized at both upper and lower values of each of the independent variable (except x_0). For simplicity, the following formulation is considered in which it is assumed that there is only one independent variable in addition to (x_0):

$$\begin{aligned}
\min\ & \sum_{i=1}^{n}\left(d_{ilU}^{+}+d_{ilU}^{-}+d_{ilL}^{+}+d_{ilL}^{-}+d_{irU}^{+}+d_{irU}^{-}+d_{irL}^{+}+d_{irL}^{-}\right)\\
\text{s.t.}\ & \sum_{j=0}^{l}(\alpha_j+(1-H)\times c_j)\times(\bar{x}_{ij}-(1-H)\times f_{ij})-d_{ilU}^{-}\\
&\quad=\bar{y}_i+(1-H)\times\bar{e}_i-d_{ilU}^{+},\quad i=1,\dots,n,\\
& \sum_{j=0}^{l}(\alpha_j+(1-H)\times c_j)\times(\bar{x}_{ij}+(1-H)\times f_{ij})-d_{irU}^{-}\\
&\quad=\bar{y}_i-(1-H)\times\bar{e}_i-d_{irU}^{+},\quad i=1,\dots,n,\\
& \sum_{j=0}^{l}(\alpha_j-(1-H)\times c_j)\times(\bar{x}_{ij}-(1-H)\times f_{ij})-d_{ilL}^{-}\\
&\quad=\bar{y}_i+(1-H)\times\bar{e}_i-d_{ilL}^{+},\quad i=1,\dots,n,\\
& \sum_{j=0}^{l}(\alpha_j-(1-H)\times c_j)\times(\bar{x}_{ij}+(1-H)\times f_{ij})-d_{irL}^{-}\\
&\quad=\bar{y}_i-(1-H)\times\bar{e}_i-d_{irL}^{-},\quad i=1,\dots,n,\\
& \sum_{i=1}^{n}\sum_{j=0}^{k}c_jx_{ij}\leqslant\upsilon,\\
& d_{ilU}^{+},d_{ilU}^{-},d_{ilL}^{+},d_{ilL}^{-},d_{irU}^{+},d_{irU}^{-},d_{irL}^{+},d_{irL}^{-}\geqslant 0,\quad i=1,\dots,n,\\
& \alpha_j=\text{free},\quad c_j\geqslant 0,\quad j=0,\dots,k,
\end{aligned}\tag{6}$$

where l and r represent the lower and upper values of the intervals of the independent variable, respectively. Moreover, U and L refer to the upper and lower values of the observed and estimated intervals, respectively (Hojati et al., 2005). Because the stated six FLR models are used in this study, there is no need to review other available FLR methods in this section. Körner and Näther (1998) studied on the problems when only fuzzy data were available for well-justified statistical models. Cheng and Lee (1999) represented the fuzzy adaptive network application in fuzzy regression analysis. Yen et al. (1999) extended the results of a FLR model that used symmetric triangular coefficient to one with non-symmetric ones. D'Urso and Gastaldi (2000) introduced a least-square approach to FLR analysis. Chen (2001) focused on non-fuzzy input and fuzzy output data type and proposed approaches to handle the outlier problem. Lee and Chen (2001) concentrated on the fuzzy regression model with fuzzy input and output data, since in modeling a fuzzy system with fuzzy linear functions, the vagueness of fuzzy output data may be caused by the vagueness of the input data. IP, Kwong, and Wong (2003) described the fuzzy regression concept and its application in modeling transfer moulding for microchip encapsulation, because fuzzy regression is a well-known method to deal with the problems with a high degree of fuzziness. An extended fuzzy regression model using regularization method was proposed by Hong and Hwang (2004). The parametric representation of fuzzy numbers and application to fuzzy calculus by Stefanini, Sorini, and Guerra (2006), an omission approach for detecting outliers in fuzzy regression models by Hung and Yang (2006), and multiple regression with fuzzy data by Bargiela et al. (2007) are some other researches from fuzzy point of view.

2.1.3. Defuzzification method

There may be situations where output of a fuzzy process needs to be a single scalar as oppose to a fuzzy set. Defuzzification is conversion of a fuzzy quantity to a precise quantity. Three methods of Defuzzification which are popular for Defuzzifying fuzzy quantities are summarized below. Assume that $\widetilde{A}$ is a fuzzy quantity, its support is $x \in X$ universe, and its crisp quantity is x^*. The defuzzification method used in this study is center of area. This procedure is the most common defuzzification method which is expressed by $x^* = \int \mu_{\widetilde{A}(x)} x d_x / \int \mu_{\widetilde{A}(x)} d_x$ (Fig. 2).

2.2. Fuzzy cognitive map

A typical cognitive map (CM) is a qualitative model which demonstrates how a given system operates. CM is based on the defined variables (either measurable physical quantities or complex aggregate ideas) in the problem and the causal relationships between them. Measurable physical quantities may include amount of precipitation or percent vegetation cover, and complex aggregate and abstract ideas may be political forces or aesthetics. Using the CM, the decision-maker can define the important variables affecting the system under study and then draw causal relationships among them and their relative strength with values between −1 and 1. The directions of the causal relationships are indicated with arrowheads. CMs are applicable tools in terms of modeling complex relationships between variables. They consist of directed graphs, or digraphs (i.e. having directed links, e.g. line or connection between variables, e.g. points or nodes). Thus, they have their historical origins in graph theory introduced by Euler in 1736 (Biggs et al., 1976).

FCM was firstly proposed by Kosko (1986) so that he modified the binary CMs by applying fuzzy causal functions with real numbers in [−1,1] to the connections (Fig. 3). He also computed the outcome of FCMs, or the FCM inference, and modeled the effect of different policy options using a neural network computational method (Kosko, 1987). FCMs, as an extension of CMs, enable the decision-makers to represent the concepts linguistically (and not necessarily precise) via associated fuzzy sets. To define the

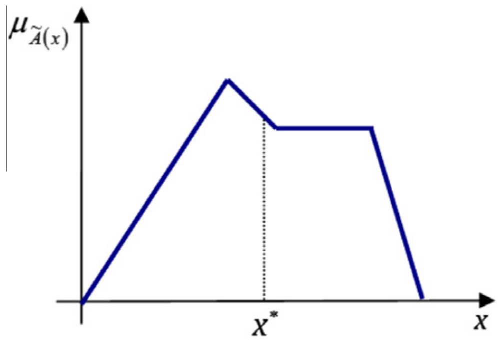

Fig. 2. The center of area defuzzification method.

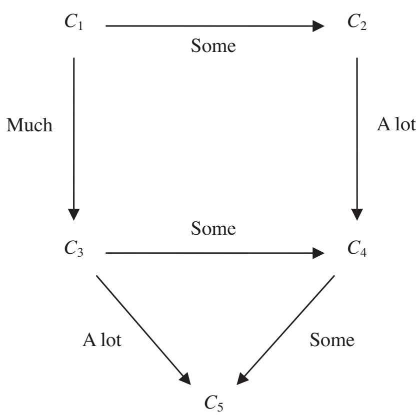

Fig. 3. An example of FCM (Kosko, 1986).

relationship degrees among concepts, FCMs utilize a number between [0,1] and [−1,1], or fuzzy linguistic terms such as "often", "always", "some", "a lot", etc.

As shown in Fig. 5, three paths connect C_1 to C_5. Therefore, there are three indirect effects of C_1 on C_5 as follows:

$$p_1(C_1, C_2, C_4, C_5) \to I_1(C_1, C_5) = \min\{e_{12}, e_{24}, e_{45}\} = \text{some}, \tag{7}$$
$$p_2(C_1, C_5, C_5) \to I_2(C_1, C_5) = \min\{e_{13}, e_{35}\} = \text{much}, \tag{8}$$
$$p_1(C_1, C_3, C_4, C_5) \to I_3(C_1, C_5) = \min\{e_{13}, e_{34}, e_{45}\} = \text{some}, \tag{9}$$

Thus, the total effect of C_1 on C_5 is

$$TE(C_1, C_5) = \max_{i=1,\ldots,3}\{I_i(C_1, C_5)\} = \text{much} \tag{10}$$

As Rodriguez-Repiso, Setchi, and Salmeron (2007) mentioned, FCM consists of nodes and links between them so that they indicate the most relevant factors of a decisional environment and the relationship between those factors, respectively. As Chunmei (2007) discussed, FCM approaches to representation of knowledge by emphasizing on the connections (as basic units for storing knowledge) and the structure (for representing the significance of system).

Over the two last decades, FCMs have attracted wide varieties of researchers in terms of representing knowledge and artificial intelligence in engineering applications, e.g. such as decision-making in complex war games (Klein and Cooper, 1982), strategic planning (Diffenbach, 1982; Ramaprasad & Poon, 1985), information retrieval (Johnson & Briggs, 1994), distributed decision process modeling (Zhang, Wang, & King, 1994). fault detection (Ndouse & Okuda, 1996; Pelaez & Bowles, 1995), modeling the supervision of distributed systems (Stylios, Georgopoulos, & Groumpos, 1997), geographical information systems (Liu & Satur, 1999), web data mining (Hong & Han, 2002; Lee, Kim, Chung, & Kwon, 2002), medical diagnosis (Georgopoulos, Malandraki, & Stylios, 2002), sup-

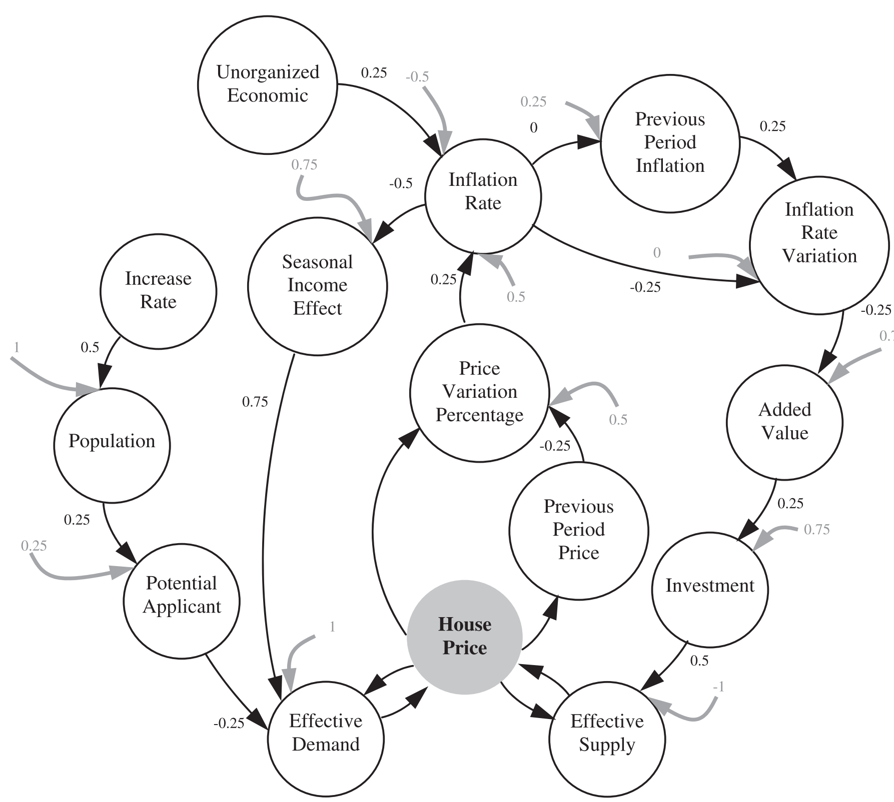

Fig. 4. The FCM structure for period 15.

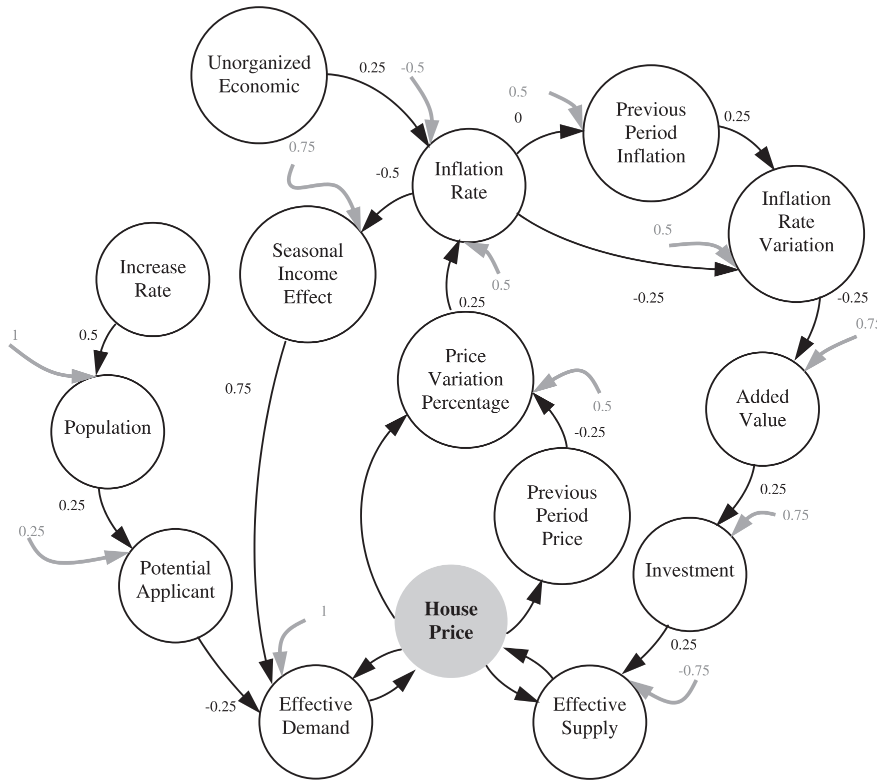

Fig. 5. The FCM structure for period 16.

porting urban design (Xirogiannis, Stefanou, & Glykas, 2004), analysis of critical success factors (CSFs) in IT projects (Chunmei, 2007) etc.

Despite the extensive utilization of FCMs in different fields of knowledge, contribution of FCM to the housing market problem is so minimal. Thus, we have applied FCM in the proposed hybrid algorithm of this study in order to linguistically model the relationships among variables associated with the housing market fluctuations from a FCM point of view. The formula used in this study for calculating the values of FCM is as follows:

$$C_i^{t+1} = f\left(\sum_{j=1}^{n} W_{ji} C_j^t\right), \tag{11}$$

where C_i^{t+1} is the value of C_i in iteration $t+1$, C_j^t is the value of interconnected concept of C_i in iteration t, W_{ji} is the calculated weigh of the relation between concept C_j to C_i, and f is a threshold function, which can be defined in various formulations. If lb is the value of the lower boundary of the nodes, either -1 or 0, the first proposed function and the one used in this study would be:

$$f(x) = \max\{lb, \min(1, x)\} \tag{12}$$

Hereafter, assume that a continuous function with the value interval of $[-1, 1]$ is considered; then

$$f(x) = \tanh(x) \tag{13}$$

On the other hand, if the interval is constrained to [0, 1], the function will be

$$f(x) = \frac{1}{1+e^{-x}} \tag{14}$$

Thus, the formula (11) can be rewritten as

$$S^{t+1} = f(S^t W) \tag{15}$$

where S is the row vector with the values of all concepts, the superscript t refers to the iteration number, and W is the influence matrix, including all relation weights.

2.3. Analysis of variance for a randomized complete block design

ANOVA-RCBD is applied in this study to test of the equality of results obtained by each of the six used FLR models, i.e. Tan1982, Tan1989, Pet, Ozel, HBS1, and HBS2. The experiment is designed such that variability arising from extraneous sources can be systematically controlled. The test of hypothesis can be defined as:

$$\begin{aligned} H_0: &\quad \mu_1 = \mu_2 = \cdots = \mu_6 \\ H_1: &\quad \mu_i \neq \mu_j, \quad \forall i,j = 1,\ldots,6, \quad i \neq j \end{aligned} \tag{16}$$

where μ_1 to μ_6 are the MAPE/IC values obtained by Tanaka et al. (1982), Tanaka et al. (1989), Peters (1994), Ozelkan et al. (2000), and Hojati et al. (2005) (two models) FLR models, respectively. Thus, we are dealing with 6 treatments. Moreover, there are 3 blocks related to each observation including the defuzzified value of the forecasted fuzzy triangular number ($\widehat{Y}$), the MAPE, and the IC values. If the obtained F_0 value is less than $F_{\alpha,a-1,\,(a-1)(b-1)}$ value resulted from the ANOVA-RCBD at significance level of $\alpha = 0.05$, the null hypothesis (i.e. the equality of mean efficiencies obtained by the six mentioned models) will be rejected. Note that a represents the number of treatments which is equal to 6, b represents the number of blocks and is equal to 3, and N represents the total number of observations from all treatments. The test procedure is summarized in Table 3. where y_{ij} denotes each observation, $y_{i.}$ is the total of all observation taken under treatment i, $y_{.j}$ is the total of all observation in block j, and $y_{..}$ is the grand total of all observation. Similarly, $\bar{y}_{i.}$ is the average of the observation taken under treatment i, $\bar{y}_{.j}$ is the average of the observation in block j, and $\bar{y}_{..}$ is the grand average of all observations.

Table 3
ANOVA-RCBD table for comparing the results of the six FLR models.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Between treatments	$SS_{\text{Treatments}} = \frac{1}{b}\sum_{i=1}^{a} y_{i.}^2 - \frac{y^2}{N}$	$a-1$	$MS_{\text{Treatments}} = \frac{SS_{\text{Treatments}}}{a-1}$	$F_0 = \frac{MS_{\text{Treatment}}}{MS_{\text{Error}}}$
Between blocks	$SS_{Blocks} = \frac{1}{a}\sum_{j=1}^{b} y_{j}^2 - \frac{y^2}{N}$	$b-1$	$MS_{Blocks} = \frac{SS_{Blocks}}{b-1}$	
Error (within treatment)	$SS_{\text{Error}} = SS_T - SS_{\text{Tr}} - SS_{Blocks}$	$(a-1)(b-1)$	$MS_{\text{Error}} = \frac{SS_{\text{Error}}}{(a-1)(b-1)}$	
Total	$SS_{\text{Total}} = \sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}^2 - \frac{y^2}{N}$	$N-1$		

3. Experiment: the algorithm implementation

3.1. Data collection

Economic evaluations and analysis need reliable data in order to have efficient results. These data can be collected from annual reports of reliable institutions. In this study, the data for 16 periods related to housing market in Iran (in certain areas) has been collected and used. The independent variables to test the proposed hybrid algorithm include unorganized economic, inflation rate, inflation rate variation, inflation related to the previous period, seasonal income effect, increase rate, population, percentage of price variation, prices related to the previous period, added values, potential applicants, investment, effective demand, and effective supply. These data are gathered from Iran's Central Bank (www.cbi.ir) and presented in Appendix A.

3.2. Application of fuzzy linear regression

Six well-known FLR models, i.e. Models (1) up to (6), are employed for forecasting the fluctuations of housing market in Iran based on a 16 period data set (Appendix A). The data of the first 14 periods are used for FLR calculations and the data of the two remaining periods are used for selection of the best-fitted FLR model. Comparison of these FLR models is based on two indicators including IC and MAPE. For this purpose, ANOVA-RCBD is applied as a useful tool for representing whether the performance of the FLR models are significantly different.

3.3. Application of fuzzy cognitive map

FCM is used in this study for representing a casual diagram for the housing market in Iran, which allows the prediction of interactions and complicated consequences. Triangular fuzzy numbers are used for initial generation of FCMs. In other words, the predicted values by the best-fitted FLR are used as initial weight of relationships in FCM. Then, these weighs are converted into defined numbers in the range of $[-1,1]$ in order to finalize an applicable FCM.

4. Results and analysis

Tables 4 and 5 present the historical values of Y and the defuzzified value of the forecasted fuzzy triangular number $(\widehat{Y})$ related to each of the six FLR models for periods 15 and 16, respectively, with respect to the following relations including:

1. Unorganized economic-inflation rate,
2. Inflation rate-seasonal income,
3. Inflation rate-previous period inflation,
4. Increase rate-population,
5. Population-potential applicants,
6. Potential applicants-seasonal income-effective applicants,
7. Percentage of price variation,
8. Inflation rate-previous period inflation-inflation rate variation,
9. Inflation rate variation-added value,
10. Added value-investment,
11. Investment-effective supply,
12. Previous period price-percentage of price variation.

Tables 6 and 7 respectively represent the MAPE values related to periods 15 and 16 (test periods). Similarly, Tables 8 and 9 represent the IC values related to the test periods 15 and 16. Rough-cut comparison of the results presented in Tables 4–9 show that the Tan1982 FLR model is identified as the preferred model for relation No. 1 (i.e. added value-investment). Similarly, it can be concluded that Tan1989 is the preferred model for relations 2 and 4, Pet is the preferred model for relations 1, 7, and 12, HBS1 is the preferred model for relations 5, 8, and 11, and HSB2 is preferred for relations 3, 6, and 9 and Ozel is not preferred in any relation. Therefore, it can be concluded that there is no FLR model that is comprehensively superior to the other models in all cases, and depending on the nature of the problem, FLR models may reveal different performances. Nevertheless, in order to perform more precise and scientific comparison between the six FLR models, ANOVA-RCBD is applied in this study. The FLR models are coded in LINGO software package. For briefness, two samples of the FLR coding schemes related to relations 1 (i.e.

Table 4
The actual and forecasted values of Y by each FLR model for different relations (Period 15).

Relation No.	$\widehat{Y}$						Y
	Tan1982	Tan1989	Pet	Ozel	HBS1	HBS2	
1	6.07	5.75	5.03	36.68	6.48	6.44	5.03
2	395.69	309.04	213.59	529.38	233.25	233.25	354.88
3	8.64	8.35	10.23	8.56	10.58	7.30	6.73
4	28110244.4	27699161.1	25070665.6	30368618.4	27627657.3	26456186.6	27730660.0
5	1257804.87	1194036.02	1194022.54	1304247.89	1194022.66	1139521.62	1194022.63
6	174200.56	173458.93	141503.80	258591.45	177323.14	201718.83	220991.59
7	7.10	6.17	4.76	8.02	7.08	6.73	5.03
8	3.26	1.70	1.05	6.30	1.70	2.47	1.70
9	658.97	658.97	617.00	750.63	666.95	669.70	675.10
10	763.16	760.15	722.79	794.90	760.10	747.86	775.50
11	478.60	475.00	381.06	580.57	497.43	441.57	494.70
12	5.64	3.60	− 1.00	4.45	3.55	5.27	2.50

Table 5
The actual and forecasted values of Y by each FLR model for different relations (Period 16).

Relation No.	$\widehat{Y}$						Y
	Tan1982	Tan1989	Pet	Ozel	HBS1	HBS2	
1	6.07	5.75	5.03	36.68	6.48	6.44	5.03
2	395.69	309.04	213.59	529.38	233.25	233.25	354.88
3	8.64	8.35	10.23	8.56	10.58	7.30	6.73
4	28257665.60	27870585.75	25070770	30546541.68	27799082	26469419.96	27830660
5	1261677.05	1198341.80	1198328.32	1308556.29	1198328.44	1139860.00	1198328.42
6	185322.10	173794.73	136212.58	222575.40	165178.99	177714.80	228417.03
7	8.62	6.17	5.18	8.74	6.71	6.73	5.03
8	2.35	−0.44	0.05	4.14	− 0.45	0.04	1.70
9	658.97	658.97	617.00	750.63	666.95	669.70	686.2
10	768.82	765.48	723.99	808.82	767.89	748.38	786
11	494.11	493.00	386.00	598.57	520.60	444.30	510
12	5.38	3.44	−1.35	4.28	3.38	5.25	4.5

Table 6
The MAPE values related to each FLR model for different relations (Period 15).

Relation No.	Tan1982	Tan1989	Pet	Ozel	HBS1	HBS2
1	0.207097416	0.143638171	4.97018E−07	6.291640027	0.287276342	0.280498907
2	0.115001159	0.12918408	0.398139365	0.491726276	0.342735573	0.342735573
3	0.283090702	0.240948825	0.520059450	0.271654864	0.572169428	0.08469539
4	0.013688257	0.00113589	0.095922505	0.095127865	0.003714395	0.045958999
5	0.053417946	1.12142E−05	7.63607E−08	0.092314215	2.16840E−08	0.045644873
6	0.211732155	0.215088077	0.359686922	0.170141595	0.197602300	0.08721038
7	0.411045909	0.225646123	0.05270552	0.593464877	0.407006262	0.337972167
8	0.919715391	0.002618292	0.380572598	2.705881530	2.61229E−16	0.451600495
9	0.023897007	0.023897007	0.086055543	0.111877464	0.012072222	0.00800511
10	0.01591236	0.019793033	0.067968500	0.025010315	0.019854699	0.035636582
11	0.032529229	0.039821724	0.229714715	0.173583370	0.005516366	0.107390161
12	1.256256000	0.441655734	1.395920825	0.778879734	0.419065080	1.107434876

Table 7
The MAPE values related to each FLR model for different relations (Period 16).

Relation No.	Tan1982	Tan1989	Pet	Ozel	HBS1	HBS2
1	0.207097416	0.143638171	4.97018E−07	6.291640027	0.287276342	0.280498907
2	0.115001159	0.129184076	0.398139365	0.491726276	0.342735573	0.342735573
3	0.283090702	0.240948825	0.520059450	0.271654864	0.572169428	0.08469539
4	0.015342992	0.001434596	0.099167249	0.097585960	0.00113465	0.048911526
5	0.052864165	1.11689E−05	8.02588E−08	0.091984688	1.74336E−08	0.048791651
6	0.188667782	0.239134106	0.403667135	0.02557441	0.276853466	0.221972185
7	0.713620700	0.225646123	0.02838001	0.736614499	0.334338827	0.337972167
8	0.38516971	1.260322870	0.968053548	1.434194240	1.266666667	0.973573925
9	0.039686490	0.039686490	0.100839547	0.093891687	0.028052983	0.02405166
10	0.02186103	0.026105352	0.078894735	0.029030770	0.023046425	0.047868938
11	0.03115725	0.033332949	0.243137307	0.173669992	0.020794114	0.128830765
12	0.196302222	0.236322607	1.300113587	0.04897594	0.249918361	0.167664093

Table 8
The IC values related to each FLR model for different relations (Period 15).

Relation No.	Tan1982	Tan1989	Pet	Ozel	HBS1	HBS2
1	0.902244209	0.969774797	1	0.556907284	0.844697906	0.752887052
2	0.311143720	0.74613150	0.031290200	0.403798971	0.617007764	0.617007764
3	0.835997906	0.889106043	0.570038029	0.831437387	0.049709094	0.97416282
4	0.987433622	1	3.43653E−07	0.505834745	0.999597248	0.824100618
5	0.994883153	0.999999996	1	0.500658950	1	0.826360080
6	0.590223970	0.507941482	0.238071125	0.733471484	0.242449621	0.90695602
7	0.776488426	0.9	0.93166213	0.397157493	0.316389640	0.640676870
8	0.727408056	0.999998837	0.539507069	0.070999474	1	0.972900618
9	0.987458252	0.987458252	0.042745309	0.167373089	0.984051271	0.99643398
10	0.99691836	0.991521835	0.035740722	0.989457879	0.987327644	0.878547864
11	0.999032090	0.995619694	0.077474804	0.832482057	0.99991530	0.900864659
12	0.526045418	0.747067737	0.84217425	0.226777227	0.450911245	0.724074175

unorganized economic-inflation rate) and 6 (i.e. potential applicants-seasonal income-effective applicants) are presented in Appendix B.

Table 10 presents the ANOVA-RCBD results for comparing the performance of the FLR models with respect to different relations. Note that for all relations, the degree of freedom for sources of variation including between treatments, between blocks, and within treatments (error) are $a - 1 = 3$, $b - 1 = 2$, and $(a - 1)(b - 1) = 10$, respectively. Thus, at significance level of $\alpha = 0.05$, we have $F_{0.05,5,10} = 3.33$.

Table 9
The IC values related to each FLR model for different relations (Period 16).

Relation No.	Tan1982	Tan1989	Pet	Ozel	HBS1	HBS2
1	0.902244209	0.969774797	1	0.556907284	0.844697906	0.752887052
2	0.311143720	0.74613150	0.031290200	0.403798971	0.617007764	0.617007764
3	0.835997906	0.889106043	0.570038029	0.831437387	0.049709094	0.97416282
4	0.985689410	0.999851581	3.19226E−07	0.491280617	0.99996213	0.804184281
5	0.994967031	0.999999996	1	0.500647138	1	0.805261861
6	0.544788202	0.419266784	0.151013430	0.69962404	0.255363485	0.704906929
7	0.704541117	0.900000000	0.97917541	0.435702905	0.406834657	0.640676870
8	0.96552344	0.900596346	0.538703710	0.969690103	0.899761784	0.911216687
9	0.965072862	0.965072862	0.030516321	0.208682718	0.917079094	0.96770044
10	0.99126189	0.982802878	0.026081347	0.985467733	0.982545369	0.796030055
11	0.99909858	0.996734465	0.065886976	0.823668876	0.998722397	0.855929898
12	0.865074770	0.760992124	0.655008258	0.95814880	0.416106089	0.972478134

Table 10
The results of ANOVA-RCBD for different relations.

Relation No.	$MS_{\text{Treatment}}$	MS_{Error}	F_0	$F_{0.05,5,10}$
1	84.12375546	96.10932285	0.875292354	3.33
2	396503.5692	396394.4403	1.000275304	3.33
3	264.0033882	263.2793401	1.002750113	3.33
4	7.86145E+13	7.86145E+13	1.000000028	3.33
5	1.35447E+11	1.35447E+11	1.000000133	3.33
6	79272442521	79272440603	1.000000024	3.33
7	113.7353748	113.4838110	1.002216737	3.33
8	224.2387850	225.8116178	0.993034757	3.33
9	189799.3307	189627.5579	1.000905843	3.33
10	71669.60150	71621.36076	1.000673552	3.33
11	325762.0151	325668.3924	1.000287479	3.33
12	324.0089913	327.0319159	0.990756485	3.33

The FCM structure is developed using the forecasted fuzzy numbers (by the preferred FLR model) for periods 15 and 16, respectively. The fuzzy numbers include variable coefficients and the constant values of FLRs. However, the relations, which relate to the house price, have no values; thus, an influence matrix should be developed in order to determine the relations' values. Figs. 4 and 5 illustrate the FCMs for periods 15 and 16, respectively in which, the triangular fuzzy numbers are replaced by values varies between −1 and +1. These values are −1, −0.75, −0.5, −0.25, 0, 0.25, 0.5, 0.75, and 1.

In order to determine the values for house price relations, one approach is to form the weight matrix, which has to be filled with the weights of the network. The influence matrix of the house price fluctuations includes 9 types of weights $(-1, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 1)$. The '0'-value represents the weight between two

Table 11
Results of Scenario 1.

Iteration	0	1	2	3	4	5	6	7
Unorganized economic	0	0	0	0	0	0	0	0
Inflation rate	0	0.25	−0.06	0	0	0	0	0
Previous period inflation	0	0	0	0	0	0	0	0
Inflation rate variation	0	0	−0.06	0.02	0	0	0	0
Seasonal income effect	0	0	−0.13	0.03	0	0	0	0
Price variation percentage	1	−.025	0	0	0	0	0	0
Added value	0	0	0	0.02	0	0	0	0
Increase rate	0	0	0	0	0	0	0	0
Population	0	0	0	0	0	0	0	0
Potential applicant	0	0	0	0	0	0	0	0
Previous period price	0	0	0	0	0	0	0	0
Investment	0	0	0	0	0	0	0	0
House price	1	1	1	1	1	1	1	1
Effective demand	1	1	1	0.91	0.93	0.93	0.93	0.93
Effective supply	1	1	1	1	1	1	1	1

Table 12
Results of Scenario 2.

Iteration	0	1	2	3	4	5	6	7
Unorganized economic	1	0	0	0	0	0	0	0
Inflation rate	1	0.5	−0.06	0	0	0	0	0
Previous period inflation	1	0	0	0	0	0	0	0
Inflation rate variation	1	0	−0.13	0.02	0	0	0	0
Seasonal income effect	1	−0.5	−0.25	0.03	0	0	0	0
Price variation percentage	1	−0.25	0	0	0	0	0	0
Added value	1	−0.25	0	0.03	0	0	0	0
Increase rate	1	0	0	0	0	0	0	0
Population	1	0.5	0	0	0	0	0	0
Potential applicant	1	0.25	0.13	0	0	0	0	0
Previous period price	1	0	0	0	0	0	0	0
Investment	1	0.25	−0.06	0	0	0	0	0
House price	1	1	1	1	1	1	1	1
Effective demand	1	1.5	1.06	0.84	0.86	0.86	0.86	0.86
Effective supply	1	1.25	1.31	1.30	1.30	1.30	1.30	1.30

Table 13
Results of Scenario 3.

Iteration	0	1	2	3	4	5	6	7
Unorganized economic	1	0	0	0	0	0	0	0
Inflation rate	1	0.5	−0.06	0	0	0	0	0
Previous period inflation	1	0	0	0	0	0	0	0
Inflation rate variation	1	0	−0.13	0.02	0	0	0	0
Seasonal income effect	1	−0.5	−0.25	0.03	0	0	0	0
Price variation percentage	1	−0.25	0	0	0	0	0	0
Added value	1	−0.25	0	0.03	0	0	0	0
Increase rate	1	0	0	0	0	0	0	0
Population	1	0.5	0	0	0	0	0	0
Potential applicant	1	0.25	0.13	0	0	0	0	0
Previous period price	1	0	0	0	0	0	0	0
Investment	1	0.25	−0.06	0	0	0	0	0
House price	0	0	0	0	0	0	0	0
Effective demand	1	1.5	1.06	0.84	0.86	0.86	0.86	0.86
Effective supply	1	1.25	1.31	1.30	1.30	1.30	1.30	1.30

Table 14
Unique features of the proposed hybrid FLR-FCM algorithm *versus other methods.*

Method	Features						
	Fuzzy data	Crisp data	Numeral fluctuations	Forecasting	Cognitive modeling	Visualization	Flexible structure
The hybrid FLR-FCM	✓	✓	✓	✓	✓	✓	✓
FLR	✓	✓		✓			
FCM	✓		✓		✓	✓	
CLR		✓		✓			
System Dynamics		✓	✓	✓		✓	✓

concepts that have no direct causal link. Since the proposed FCM takes weight values between −1 and 1, the weights will be linearly scaled in this interval. Using this information, generation of the influence matrix of Figs. 4 and 5 is almost simple.

As shown in Figs. 4 and 5, some nodes affect some others with positive influence while negative impacts of some nodes enforce the application of negative weights on edges. The house price, effective demand, and effective supply are assumed to be influenced positively by themselves. Thus, in the first scenario, some nodes would be activated to show the converged solution. These nodes are price variation percentage, previous period price, house price, effective demand, and effective supply. The results of running successive iterations are presented in Table 11. In this scenario, running seven iterations resulted in converged vector matrix.

Tables 12 and 13 demonstrate the final vector matrices of two other scenarios (scenarios 2 and 3), which are shown in iteration 0. It is possible to use several different initial vector matrices to obtain appropriate results. For instance, in Table 12, all of the defined nodes of FCM structure play a positive role in iterations, while in Table 13 the effect of house price is not considered.

5. Conclusion

The primary objective of this study was to utilize imprecise/fuzzy data in order to improve the analysis of housing price fluctuations, in accordance with the factors obtained through the best-fitted FLR model. The secondary objective of this study was the exhibition of the resulted values in a schematic way via FCM. Hybridization of FLR and FCM provides a decision support system (DSS) for utilization of historical data to predict housing market fluctuation in the future and to identify the influence of the other parameters. In this study, a set of non-crisp data have been used for analyzing housing price fluctuations problem. The best-fitted FLR among six models for the pre-defined relations of the housing market parameters was identified. These models have been selected among all available methods to confirm a comparison among old and most recent approaches. However, analysis of variance showed that there is no significant difference in resulted values obtained by each of the FLR models. After fuzzy calculations of FLRs, the forecasted values have been used to form a diagram with fuzzy relations. The hybridization between FLR and FCM was as simple as replacement of fuzzy values with predefined values between −1 and +1. These substitutions occur by using the subjective utility functions, which can be different for decision makers. They can be simply defined and used, since we only want to show the relative weights of each relation according to the influences of nodes on one another. The hybrid algorithm of this study leads to more effective analytical perception of available data.

In summary, this study presented a hybrid algorithm based on fuzzy linear regression (FLR) and fuzzy cognitive map (FCM) to deal with the problem of forecasting and optimization of housing market fluctuations. Due to the uncertainty and severe noise associated with the housing market, the application of crisp data for forecasting and optimization purposes is insufficient. Moreover, the proposed hybrid FLR-FCM algorithm enables the decision-makers to utilize imprecise and ambiguous data and represent the resulted values of the model more clearly. This is the first study that utilizes a hybrid intelligent approach for housing price and market forecasting and optimization. Table 14 compares the hybrid FLR-FCM algorithm with other general approaches in some features. In addition to handling both fuzzy and crisp data in proposed methodology, the proposed hybrid algorithm is able to evaluate fluctuated behaviors in different situations, forecast future behavior of a system, utilize cognitive models, visualize results, and implement required changes as needed. In this study, application of these two approaches is proposed for forecasting and optimization of the housing market fluctuations and its effective factors.

Acknowledgment

The authors are grateful for the valuable comments and suggestion from the respected reviewers. Their valuable comments and suggestions have enhanced the strength and significance of our pa-

Table A.1
The raw data from annual reports of the Iran's Central Bank[a].

Periods	Unorganized economic	Inflation rate	Seasonal income effect	Previous period inflation	Inflation rate variation	Increase rate	Population	Price variation percentage	Added value	Previous period price	Potential applicant	Investment	Effective demand	Effective supply
1	24	6.2	200.5	9.7	3.5	825000	25270560	7.3	635.48	100	1088096.01	745.2	183908.69	419.2
2	24	6.2	200.5	9.7	3.5	850000	25495560	7	664.6	107.03	1097784.03	762	205403.79	464
3	24	6.2	200.5	9.7	3.5	875000	25570560	7.3	693.72	114.81	1101013.37	778.8	226946.86	508.8
4	24	6.2	200.5	9.7	3.5	900000	25645560	7.3	706.2	123.19	1104242.71	786	236183.42	528
5	28	7.3	233.25	6.2	−1.1	925000	25888590	5.8	703.05	132.19	1114707.06	779.7	226007.33	506.7
6	28	7.3	233.25	6.2	−1.1	950000	26138590	6.7	695.7	139.85	1125471.52	765	202131.95	457
7	28	7.3	233.25	6.2	−1.1	975000	26221923	4.8	688.35	149.22	1129059.67	750.3	178369.29	407.3
8	28	7.3	233.25	6.2	−1.1	1000000	26305257	5.3	685.2	156.38	1132674.83	744	168245.24	386
9	27	6.73	233.25	7.3	0.58	1025000	26575290	4.5	674.25	164.67	1144274.88	739.8	169919.86	389.3
10	27	6.73	233.25	7.3	0.58	1050000	26850290	4.1	648.7	172.08	1156115.79	730	173693.69	397
11	27	6.73	233.25	7.3	0.58	1075000	26941957	4.4	623.15	179.14	1160062.76	720.2	177348.87	404.7
12	27	6.73	233.25	7.3	0.58	1100000	27033623	5.2	612.2	187.02	1164009.73	716	178902.18	408
OSR[b]	25	5.03	354.88	6.73	1.7	1125000	27330660	5	623.3	196.75	1176799.49	726.5	186414.67	423.3

[a] www.cbi.ir.
[b] On Side Range.

per. The authors would like to acknowledge the financial support of University of Tehran for this research under grant number8106013/1/05.

Appendix A

Table A.1.

Appendix B

The LINGO codes for FLR for relations 1 and 6 are respectively presented as follows:

B.1. Unorganized economic-inflation rate

B.1.1. Tanaka 1982

```
!Unorganized Economic-Inflation Rate;
Model:
Sets:
Period/1..14/:Ybar,e;
Coefficient/1..2/:C,Alpha;
Link (Period,Coefficient):X;
End Sets
Data:
  X = 1 24
  1 24
  1 24
  1 24
  1 28
  1 28
  1 28
  1 28
  1 27
  1 27
  1 27
  1 27
  1 25
  1 25;
Ybar = 6.2
  6.2
  6.2
  6.2
  7.3
  7.3
  7.3
  7.3
  6.73
  6.73
  6.73
  6.73
  5.03
  5.03;
e = 2.27;
H = 0;
End Data
Min=@Sum (Coefficient (j):C (j));
@For (Period (i): (Ybar (i)+(1-H)*e (i))<=(@Sum (Coefficient (j):
  (Alpha (j)+((1-H)*C (j)))*X (i,j))));
@For (Period (i): (Ybar (i)-(1-H)*e (i))>=(@Sum (Coefficient (j):
  (Alpha (j)-((1-H)*C (j)))*X (i,j))));
@For (Coefficient (j): @Free (Alpha (j)));
End
```

B.1.2. Tanaka 1989

```
!Unorganized Economic-Inflation Rate;
Model:
Sets:
Period/1..14/:Ybar,e;
Coefficient/1..2/:C,Alpha;
Link (Period,Coefficient):X;
End Sets
Data:
X = 1 24
  1 24
  1 24
  1 24
  1 28
  1 28
  1 28
  1 28
  1 27
  1 27
  1 27
  1 27
  1 25
  1 25;
Ybar = 6.2
  6.2
  6.2
  6.2
  7.3
  7.3
  7.3
  7.3
  6.73
  6.73
  6.73
  6.73
  5.03
  5.03;
e = 2.27;
H = 0;
End Data
Min = @Sum (Link (i,j):C (j)*X (i,j));
@For (Period (i): (Ybar (i)+(1-H)*e (i))<=(@Sum (Coefficient (j):
  (Alpha (j)+((1-H)*C (j)))*X (i,j))));
@For (Period (i): (Ybar (i)-(1-H)*e (i))>=(@Sum (Coefficient (j):
  (Alpha (j)- ((1-H)*C (j)))*X (i,j))));
@For (Coefficient (j):@Free (Alpha (j)));
End
```

B.1.3. Pet

```
!Unorganized Economic-Inflation Rate;
Model:
Sets:
Period/1..14/:Ybar,e,Landa;
Coefficient/1..2/:C,Alpha;
Link (Period,Coefficient):X;
End Sets
Data:
X = 1 24
  1 24
  1 24
  1 24
  1 28
  1 28
  1 28
  1 28
  1 27
  1 27
  1 27
  1 27
  1 25
  1 25;
Ybar = 6.2
  6.2
  6.2
  6.2
  7.3
  7.3
  7.3
  7.3
  6.73
  6.73
  6.73
  6.73
  5.03
  5.03;
e = 2.27;
n = 7;
P0 = 1000;
End Data
Max = @Sum (Period (i):Landa (i))/n;
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)+C (j))*X (i,j))>
  =Ybar (i)- (1-Landa (i))*e (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)-C (j))*X (i,j))<
  =Ybar (i)+ (1-Landa (i))*e (i));
@Sum (Link (i,j): (C (j)*X (i,j)))<=P0*(1-(@Sum (Period (i):
Landa (i))/n));
@For (Period (i):Landa (i)>=0);
@For (Period (i):Landa (i)<=1);
@For (Coefficient (j):@Free (Alpha (j)));
End
```

B.1.4. Ozel

```
!Unorganized Economic-Inflation Rate;
Model:
Sets:
Period/1..14/:Ybar,e,dU,dL;
Coefficient/1..2/:C,Alpha;
Link (Period,Coefficient):X;
End Sets
Data:
X = 1 24
  1 24
  1 24
  1 24
  1 28
  1 28
  1 28
  1 28
  1 27
  1 27
  1 27
```

```
  1 27
  1 25
  1 25;
Ybar = 6.2
  6.2
  6.2
  6.2
  7.3
  7.3
  7.3
  7.3
  6.73
  6.73
  6.73
  6.73
  5.03
  5.03;
e = 2.27;
v = 1000;
End Data
Min = @Sum (Period (i):dU (i)+dL (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)+(1-H)*C (j))*
  X (i,j))>= Ybar (i)+(1-H)*e (i)-dU (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)-(1-H)*C (j))*
  X (i,j))>= Ybar (i)-(1-H)*e (i)+dL (i));
@Sum (Link (i,j): (C (j)*X (i,j)))<=v;
@For (Coefficient (j):@Free (Alpha (j)));
End
```

B.1.5. HBS1

```
!Unorganized Economic-Inflation Rate;
Model:
Sets:
Period/1..14/:Ybar,e,dpU,dpL,dmU,dmL;
Coefficient/1..2/:C,Alpha;
Link (Period,Coefficient):X;
End Sets
Data:
X = 1 24
  1 24
  1 24
  1 24
  1 28
  1 28
  1 28
  1 28
  1 27
  1 27
  1 27
  1 27
  1 25
  1 25;
Ybar = 6.2
  6.2
  6.2
  6.2
  7.3
  7.3
  7.3
  7.3
  6.73
  6.73
  6.73
  6.73
  5.03
  5.03;
e = 2.27;
H = 0;
End Data
Min = @Sum (Period (i):dpU (i)+dpL (i)+dmU (i)+dmL (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)+(1-H)*C (j))*
  X (i,j)+dpU (i)- dmU (i))=Ybar (i)+(1-H)*e (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)-(1-H)*C (j))*
  X (i,j)+dpL (i)- dmL (i))=Ybar (i)-(1-H)*e (i));
@For (Coefficient (j):@Free (Alpha (j)));
End
```

B.1.6. HBS2

```
!Unorganized Economic-Inflation Rate;
Model:
Sets:
Period/1..14/:Ybar,e,dplU,dplL,dmlU,dmlL,dprU,dprL,dmrU,
  dmrL;
Coefficient/1..2/:C,Alpha;
Link (Period,Coefficient):Xbar,f;
End Sets
Data:
Xbar = 1 24
  1 24
  1 24
  1 24
  1 28
  1 28
  1 28
  1 28
  1 27
  1 27
  1 27
  1 27
  1 25
  1 25;
f = 0 4
  0 4
  0 4
  0 4
  0 4
  0 4
  0 4
  0 4
  0 4
  0 4
  0 4
  0 4
  0 4
  0 4;
Ybar = 6.2
  6.2
  6.2
  6.2
  7.3
  7.3
```

(continued on next page)

```
    7.3
    7.3
    6.73
    6.73
    6.73
    6.73
    5.03
    5.03;
e = 2.27;
H = 0;
End Data
Min = @Sum (Period (i):dplU (i)+dplL (i)+dmlU (i)+dmlL (i)
    +dprU (i)+dprL (i)+dmrU (i)+dmrL (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)+(1-H)*C (j))*
    (Xbar (i,j)-(1- H)*f (i,j))+dplU (i)-dmlU (i))=Ybar (i)+(1-H)*e
    (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)+(1-H)*C (j))*
    (Xbar (i,j)+(1- H)*f (i,j))+dprU (i)-dmrU (i))=Ybar (i)+
    (1-H)*e (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)-(1-H)*C
    (j))*(Xbar (i,j)-(1- H)*f (i,j))+dplL (i)-dmlL (i))=Ybar (i)-(1-
    H)*e (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)-(1-H)*C (j))*
    (Xbar (i,j)+(1- H)*f (i,j))+dprL (i)-dmrL (i))=Ybar (i)-(1-H)*e
    (i));
@For (Coefficient (j):@Free (Alpha (j)));
End
```

B.2. Potential applicants-seasonal income effect-effective applicants

B.2.1. Tanaka 1982

```
!Potential Applicant & Seasonal Income Effect-Effective
    Applicant;
Model:
Sets:
Period/1..14/:Ybar,e;
Coefficient/1..3/:C,Alpha;
Link (Period,Coefficient):X;
End Sets
Data:
X = 1 1088096.01 200.5
    1 1097784.03 200.5
    1 1101013.37 200.5
    1 1104242.71 200.5
    1 1114707.06 233.25
    1 1125471.52 233.25
    1 1129059.67 233.25
    1 1132674.83 233.25
    1 1144274.88 233.25
    1 1156115.79 233.25
    1 1160062.76 233.25
    1 1164009.73 233.25
    1 1176799.49 354.88
    1 1189716.85 354.88;
Ybar = 183908.69
    205403.79
    226946.86
    236183.42
    226007.33
    202131.95
    178369.29
    168245.24
    169919.86
    173693.69
    177348.87
    178902.18
    186414.67
    203702.86;
e = 60171.79;
H = 0;
End Data
Min = @Sum (Coefficient (j):C (j));
@For (Period (i): (Ybar (i)+(1-H)*e (i))<=(@Sum (Coefficient (j):
    (Alpha (j)+((1-H)*C (j)))*X (i,j))));
@For (Period (i): (Ybar (i)-(1-H)*e (i))>=(@Sum (Coefficient (j):
    (Alpha (j)- ((1-H)*C (j)))*X (i,j))));
@For (Coefficient (j):@Free (Alpha (j)));
End
```

B.2.2. Tanaka 1989

```
!Potential Applicant & Seasonal Income Effect-Effective
    Applicant;
Model:
Sets:
Period/1..14/:Ybar,e;
Coefficient/1..3/:C,Alpha;
Link (Period,Coefficient):X;
End Sets
Data:
X = 1 1088096.01 200.5
    1 1097784.03 200.5
    1 1101013.37 200.5
    1 1104242.71 200.5
    1 1114707.06 233.25
    1 1125471.52 233.25
    1 1129059.67 233.25
    1 1132674.83 233.25
    1 1144274.88 233.25
    1 1156115.79 233.25
    1 1160062.76 233.25
    1 1164009.73 233.25
    1 1176799.49 354.88
    1 1189716.85 354.88;
Ybar = 183908.69
    205403.79
    226946.86
    236183.42
    226007.33
    202131.95
    178369.29
    168245.24
    169919.86
    173693.69
    177348.87
    178902.18
    186414.67
    203702.86;
e = 60171.79;
H = 0;
End Data
Min = @Sum (Link (i,j):C (j)*X (i,j));
@For (Period (i): (Ybar (i)+(1-H)*e (i))<=(@Sum (Coefficient (j):
```

```
   (Alpha (j)+((1-H)*C (j)))*X (i,j))));
@For (Period (i): (Ybar (i)-(1-H)*e (i))>=(@Sum (Coefficient (j):
   (Alpha (j)- ((1-H)*C (j)))*X (i,j))));
@For (Coefficient (j):@Free (Alpha (j)));
End
```

B.2.3. Pet

```
!Potential Applicant & Seasonal Income Effect-Effective
   Applicant;
Model:
Sets:
Period/1..14/:Ybar,e,Landa;
Coefficient/1..3/:C,Alpha;
Link (Period,Coefficient):X;
End Sets
Data:
X = 1 1088096.01 200.5
   1 1097784.03 200.5
   1 1101013.37 200.5
   1 1104242.71 200.5
   1 1114707.06 233.25
   1 1125471.52 233.25
   1 1129059.67 233.25
   1 1132674.83 233.25
   1 1144274.88 233.25
   1 1156115.79 233.25
   1 1160062.76 233.25
   1 1164009.73 233.25
   1 1176799.49 354.88
   1 1189716.85 354.88;
Ybar = 183908.69
   205403.79
   226946.86
   236183.42
   226007.33
   202131.95
   178369.29
   168245.24
   169919.86
   173693.69
   177348.87
   178902.18
   186414.67
   203702.86;
e = 60171.79;
n = 7;
P0 = 1000;
End Data
Max = @Sum (Period (i):Landa (i))/n;
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)+C (j))*X (i,j))>
   =Ybar (i)- (1-Landa (i))*e (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)-C (j))*X (i,j))<
   = Ybar (i)+(1-Landa (i))*e (i));
@Sum (Link (i,j): (C (j)*X (i,j)))<=P0*(1-(@Sum (Period (i):
   Landa (i))/n));
@For (Period (i):Landa (i)>=0);
@For (Period (i):Landa (i)<=1);
@For (Coefficient (j):@Free (Alpha (j)));
End
```

B.2.4. Ozel

```
!Potential Applicant & Seasonal Income Effect-Effective
   Applicant;
Model:
Sets:
Period/1..14/:Ybar,e,dU,dL;
Coefficient/1..3/:C,Alpha;
Link (Period,Coefficient):X;
End Sets
Data:
X = 1 1088096.01 200.5
   1 1097784.03 200.5
   1 1101013.37 200.5
   1 1104242.71 200.5
   1 1114707.06 233.25
   1 1125471.52 233.25
   1 1129059.67 233.25
   1 1132674.83 233.25
   1 1144274.88 233.25
   1 1156115.79 233.25
   1 1160062.76 233.25
   1 1164009.73 233.25
   1 1176799.49 354.88
   1 1189716.85 354.88;
Ybar = 183908.69
   205403.79
   226946.86
   236183.42
   226007.33
   202131.95
   178369.29
   168245.24
   169919.86
   173693.69
   177348.87
   178902.18
   186414.67
   203702.86;
e = 60171.79;
H = 0;
v = 1000;
End Data
Min = @Sum (Period (i):dU (i)+dL (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)+(1-H)*C (j))*
   X (i,j))>= Ybar (i)+(1-H)*e (i)-dU (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)-(1-H)*C (j))*
   X (i,j))>= Ybar (i)-(1-H)*e (i)+dL (i));
@Sum (Link (i,j): (C (j)*X (i,j)))<=v;
@For (Coefficient (j):@Free (Alpha (j)));
End
```

B.2.5. HBS1

```
!Potential Applicant & Seasonal Income Effect-Effective
   Applicant;
Model:
Sets:
Period/1..14/:Ybar,e,dpU,dpL,dmU,dmL;
Coefficient/1..3/:C,Alpha;
Link (Period,Coefficient):X;
```

(*continued on next page*)

```
End Sets
Data:
X = 1 1088096.01 200.5
   1 1097784.03 200.5
   1 1101013.37 200.5
   1 1104242.71 200.5
   1 1114707.06 233.25
   1 1125471.52 233.25
   1 1129059.67 233.25
   1 1132674.83 233.25
   1 1144274.88 233.25
   1 1156115.79 233.25
   1 1160062.76 233.25
   1 1164009.73 233.25
   1 1176799.49 354.88
   1 1189716.85 354.88;
Ybar = 183908.69
   205403.79
   226946.86
   236183.42
   226007.33
   202131.95
   178369.29
   168245.24
   169919.86
   173693.69
   177348.87
   178902.18
   186414.67
   203702.86;
e = 60171.79;
H = 0;
End Data
Min = @Sum (Period (i):dpU (i)+dpL (i)+dmU (i)+dmL (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)+(1-H)*C (j))*
   X (i,j)+dpU (i)- dmU (i))=Ybar (i)+(1-H)*e (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)-(1-H)*C (j))*
   X (i,j)+dpL (i)- dmL (i))=Ybar (i)-(1-H)*e (i));
@For (Coefficient (j):@Free (Alpha (j)));
End
```

B.2.6. HBS2

```
!Potential Applicant & Seasonal Income Effect-Effective
   Applicant;
Model:
Sets:
Period/1..14/:Ybar,e,dplU,dplL,dmlU,dmlL,dprU,dprL,dmrU,
   dmrL;
Coefficient/1..3/:C,Alpha;
Link (Period,Coefficient):Xbar,f;
End Sets
Data:
Xbar = 1 1088096.01 200.5
   1 1097784.03 200.5
   1 1101013.37 200.5
   1 1104242.71 200.5
   1 1114707.06 233.25
   1 1125471.52 233.25
   1 1129059.67 233.25
   1 1132674.83 233.25
   1 1144274.88 233.25
   1 1156115.79 233.25
   1 1160062.76 233.25
   1 1164009.73 233.25
   1 1176799.49 354.88
   1 1189716.85 354.88;
f = 0 110232.41 154.38
   0 110232.41 154.38
   0 110232.41 154.38
   0 110232.41 154.38
   0 110232.41 154.38
   0 110232.41 154.38
   0 110232.41 154.38
   0 110232.41 154.38
   0 110232.41 154.38
   0 110232.41 154.38
   0 110232.41 154.38
   0 110232.41 154.38
   0 110232.41 154.38
   0 110232.41 154.38;
Ybar = 183908.69
   205403.79
   226946.86
   236183.42
   226007.33
   202131.95
   178369.29
   168245.24
   169919.86
   173693.69
   177348.87
   178902.18
   186414.67
   203702.86;
e = 60171.79;
H = 0;
End Data
Min = @Sum (Period (i):dplU (i)+dplL (i)+dmlU (i)+dmlL (i)
   +dprU (i)+dprL (i) +dmrU (i)+dmrL (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)+(1-H)*C (j))*
   (Xbar (i,j)-(1- H)*f (i,j))+dplU (i)-dmlU (i))=Ybar (i)+
   (1-H)*e (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)+(1-H)*C (j))*
   (Xbar (i,j)+(1- H)*f (i,j))+dprU (i)-dmrU (i))=Ybar (i)+
   (1-H)*e (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)-(1-H)*C (j))*
   (Xbar (i,j)-(1- H)*f (i,j))+dplL (i)-dmlL (i))=Ybar (i)-(1-H)*e
   (i));
@For (Period (i):@Sum (Coefficient (j): (Alpha (j)-(1-H)*C (j))*
   (Xbar (i,j)+(1- H)*f (i,j))+dprL (i)-dmrL (i))=Ybar (i)-(1-H)*e
   (i));
@For (Coefficient (j):@Free (Alpha (j)));
End
```

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