

Prestige
Centrality
(and some Linear Alg)

Def

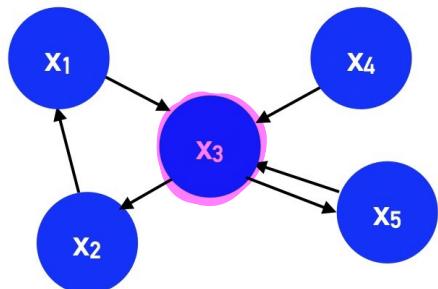
Prestige Centrality - a node is prestigious if prestigious point to it

Let $N_i = \{x_j \in V : d(x_i, x_j) = 1\}$ ← all verts
1 hop from x_i

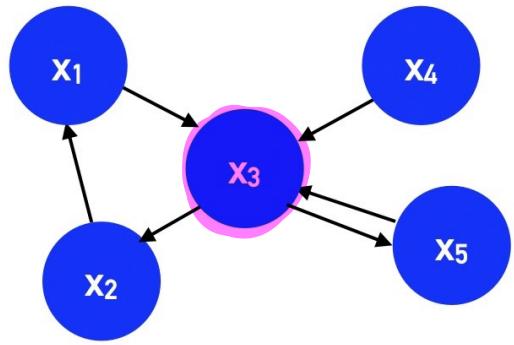
The prestige of x_i is

$$p(x_i) = \sum_{x_j \in N_i} p(x_j)$$

Example:



$$p(x_3) = p(x_1) + p(x_2) + p(x_4) + p(x_5)$$



$$\begin{aligned}
 p(x_1) &= \\
 p(x_2) &= \\
 p(x_3) &= p(x_1) + p(x_5) + p(x_4) \\
 p(x_4) &= \\
 p(x_5) &=
 \end{aligned}$$

Graph as Matrix

$$A = \left(\begin{array}{c} \\ \\ \\ \\ \end{array} \right)$$

$$P = \begin{pmatrix} p(x_1) \\ p(x_2) \\ \vdots \\ p(x_5) \end{pmatrix}$$

Linear Alg Review: Matrix Transpose

Transpose of Matrix A is denoted A^T

$$A_{ij}^T = A_{ji}$$

Example

$$A = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 2 & 4 \\ -4 & 1 & 2 \end{pmatrix}$$

$$A^T = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

Lin Alg Review: Mat-vec Prod

For $A \in \mathbb{R}^{m \times n}$, $v \in \mathbb{R}^n$ i^{th} elt of $A \cdot v$ is

$$(Av)_i = \sum_{j=1}^n A_{i;j} v_j$$

example

$$A \cdot v = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 2 & 4 \\ -4 & 1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} \quad \\ \quad \\ \quad \end{pmatrix} =$$

example 2: Compute AV for:

$$A = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 2 & 4 \\ -4 & 1 & 2 \end{pmatrix} \quad v = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & 0 \\ 0 & 2 & 4 \\ -4 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \quad \\ \quad \\ \quad \end{pmatrix} = \begin{pmatrix} \quad \\ \quad \\ \quad \end{pmatrix}$$

Lin Alg & Graphs

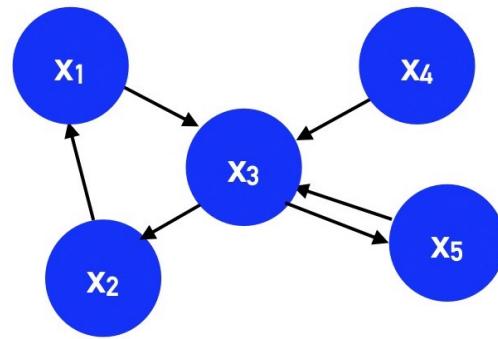
$$p(x_1) = p(x_2)$$

$$p(x_2) = p(x_3)$$

$$p(x_3) = p(x_1) + p(x_5) + p(x_4)$$

$$p(x_4) = 0$$

$$p(x_5) = p(x_3)$$

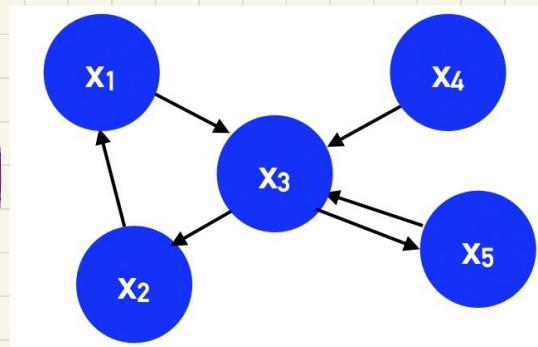


$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad p = \begin{pmatrix} p(x_1) \\ p(x_2) \\ \vdots \\ p(x_5) \end{pmatrix}$$

How to write $p(x_3)$ w/ mat-vec?

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$P = \begin{pmatrix} p(x_1) \\ p(x_2) \\ \vdots \\ p(x_5) \end{pmatrix}$$

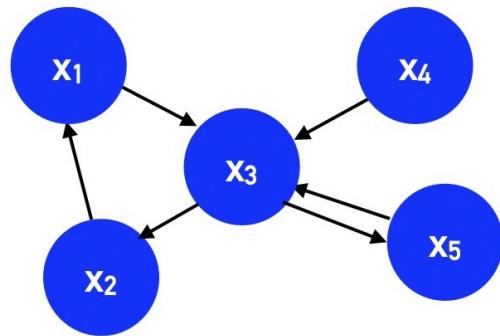


$$p(x_3) = p_1(x_1) + p(x_4) + p(x_5) = (1 \ 0 \ 0 \ 1 \ 1) \begin{pmatrix} p(x_1) \\ p(x_2) \\ \vdots \\ p(x_5) \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\rho = \begin{pmatrix} \rho(x_1) \\ \rho(x_2) \\ \vdots \\ \rho(x_5) \end{pmatrix}$$



$$\rho(x_2) = A_2^T \rho =$$

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad A^T = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad P = \begin{pmatrix} p(x_1) \\ p(x_2) \\ \vdots \\ p(x_5) \end{pmatrix}$$

Repeat for each $p(x_i)$

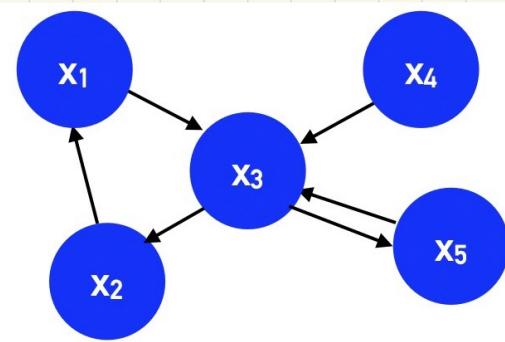
$$p(x_1) =$$

$$p(x_2) = A_2^T P$$

$$p(x_3) = A_3^T P$$

$$p(x_4) =$$

$$p(x_5) =$$



Power Iteration

How to solve $p = A^T p$ for p ?

Idea If we were given vector p' ,

how do we check if it is the prestige vector?

Formally
pick p_0 (how, TBP)

$$p_1 = A^T p_0$$

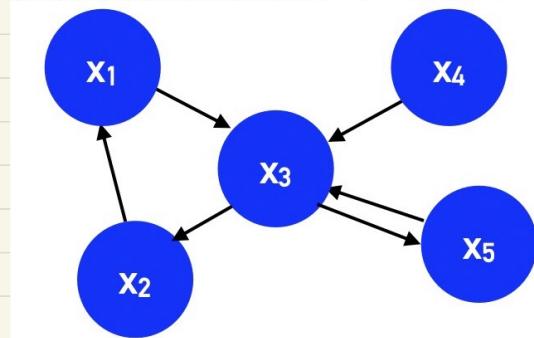
Formally
pick p_0 (how, TBP)

$$p_1 = A^T p_0$$

This process is called
power iteration
and p converges to the
dominant eigenvector
of A^T (for any $p_0 \neq 0$)

Example w/ $\rho_0 = \overline{1}$

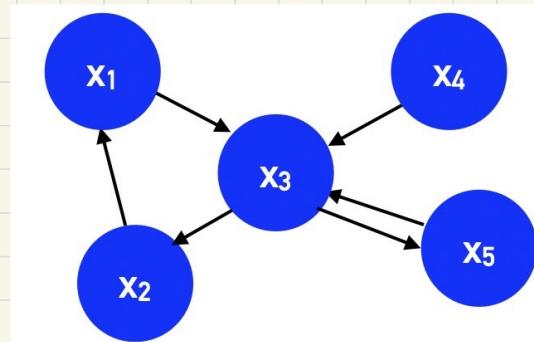
$$p_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \\ 1 \end{pmatrix}$$



Example w/ $\rho_0 = \overline{1}$

$$p_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \\ 1 \end{pmatrix}$$

$$p_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 3 \\ 1 \\ 3 \end{pmatrix}$$



Example w/ $\rho_0 = \frac{1}{5}$

$$p_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \\ 1 \end{pmatrix}$$

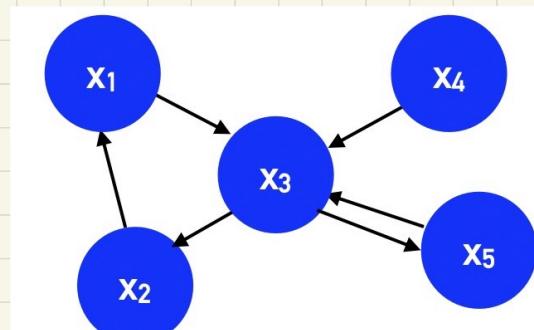
$$p_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 3 \\ 1 \\ 3 \end{pmatrix}$$

$$p_3 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 3 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 5 \\ 1 \\ 3 \end{pmatrix}$$

...

$$p_{10} = \begin{pmatrix} 33 \\ 49 \\ 71 \\ 23 \\ 49 \end{pmatrix} \quad p_{11} = \begin{pmatrix} 49 \\ 71 \\ 105 \\ 33 \\ 71 \end{pmatrix}$$

is this
converging?



Example w/ $\rho_0 = \overline{1}$

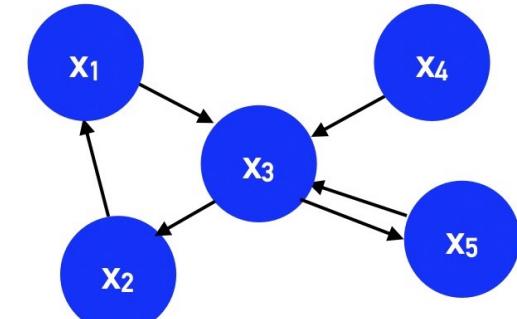
$$p_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \\ 1 \end{pmatrix}$$

$$p_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 3 \\ 1 \\ 3 \end{pmatrix}$$

$$p_3 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 3 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 5 \\ 1 \\ 3 \end{pmatrix}$$

is this
converging?

... $p_{10} = \begin{pmatrix} 33 \\ 49 \\ 71 \\ 23 \\ 49 \end{pmatrix}$ $p_{11} = \begin{pmatrix} 49 \\ 71 \\ 105 \\ 33 \\ 71 \end{pmatrix}$



$$\frac{1}{||p_{10}||} p_{10} = \begin{pmatrix} 0.15 \\ 0.22 \\ 0.32 \\ 0.10 \\ 0.22 \end{pmatrix}$$

$$\frac{1}{||p_{11}||} p_{11} = \begin{pmatrix} 0.15 \\ 0.22 \\ 0.32 \\ 0.10 \\ 0.22 \end{pmatrix}$$

Example w/ $\rho_0 = \overline{1}$

$$p_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \\ 1 \end{pmatrix}$$

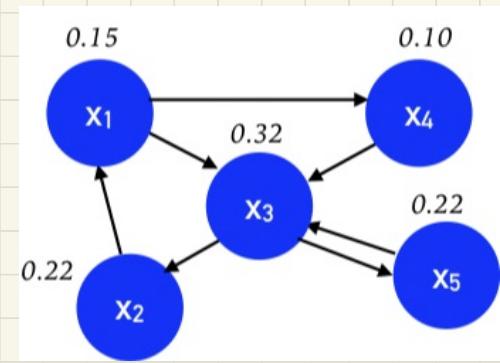
$$p_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 3 \\ 1 \\ 3 \end{pmatrix}$$

$$p_3 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 3 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 5 \\ 1 \\ 3 \end{pmatrix}$$

...

$$p_{10} = \begin{pmatrix} 33 \\ 49 \\ 71 \\ 23 \\ 49 \end{pmatrix} \quad p_{11} = \begin{pmatrix} 49 \\ 71 \\ 105 \\ 33 \\ 71 \end{pmatrix}$$

is this
converging?



$$\frac{1}{\|p_{10}\|} p_{10} = \begin{pmatrix} 0.15 \\ 0.22 \\ 0.32 \\ 0.10 \\ 0.22 \end{pmatrix}$$

$$\frac{1}{\|p_{11}\|} p_{11} = \begin{pmatrix} 0.15 \\ 0.22 \\ 0.32 \\ 0.10 \\ 0.22 \end{pmatrix}$$

PowerIteration(A, ϵ)

$k = 0$

$p_0 = 1 \in R^n$

Repeat:

$k = k + 1$

$p_k = A^T p_{k-1}$

$i = \text{argmax}_j\{p_k[j]\}$

$p_k = \frac{1}{p_k[i]} p_k$

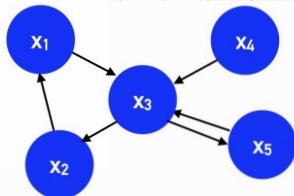
Until $||p_k - p_{k-1}|| \leq \epsilon$

$p = \frac{1}{||p_k||} p_k$

Return p

$$p_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$p_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \\ 1 \end{pmatrix} \rightarrow i = 3 \rightarrow p_1 = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 1 \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$$



PowerIteration(A, ϵ)

$k = 0$

$p_0 = 1 \in R^n$

Repeat:

$k = k + 1$

$p_k = A^T p_{k-1}$

$i = \text{argmax}_j \{p_k[j]\}$

$p_k = \frac{1}{p_k[i]} p_k$

Until $||p_k - p_{k-1}|| \leq \epsilon$

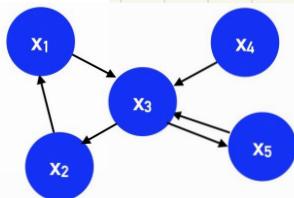
$p = \frac{1}{||p_k||} p_k$

Return p

$$p_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$p_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \\ 1 \end{pmatrix} \rightarrow i = 3 \rightarrow p_1 = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 1 \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$$

$$p_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ 1 \\ \frac{1}{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 1 \\ \frac{1}{3} \\ 1 \end{pmatrix} \rightarrow i = 2 \rightarrow p_2 = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 1 \\ \frac{1}{3} \\ 1 \end{pmatrix}$$



PowerIteration(A, e)

$k = 0$

$p_0 = 1 \in R^n$

Repeat:

$k = k + 1$

$p_k = A^T p_{k-1}$

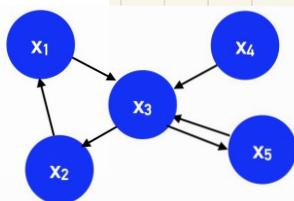
$i = \text{argmax}_j \{p_k[j]\}$

$p_k = \frac{1}{p_k[i]} p_k$

Until $||p_k - p_{k-1}|| \leq \epsilon$

$p = \frac{1}{||p_k||} p_k$

Return p



$$p_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$p_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \\ 1 \end{pmatrix} \rightarrow i = 3 \rightarrow p_1 = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 1 \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$$

$$p_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 1 \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 1 \\ \frac{1}{3} \\ 1 \end{pmatrix} \rightarrow i = 2 \rightarrow p_2 = \begin{pmatrix} \frac{1}{3} \\ 1 \\ \frac{1}{3} \\ \frac{1}{3} \\ 1 \end{pmatrix}$$

$$p_3 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{3} \\ 1 \\ \frac{1}{3} \\ \frac{1}{3} \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ \frac{5}{3} \\ \frac{1}{3} \\ 1 \end{pmatrix} \rightarrow i = 3 \rightarrow p_3 = \begin{pmatrix} \frac{3}{5} \\ \frac{3}{5} \\ 1 \\ \frac{1}{5} \\ \frac{3}{5} \end{pmatrix}$$

⋮

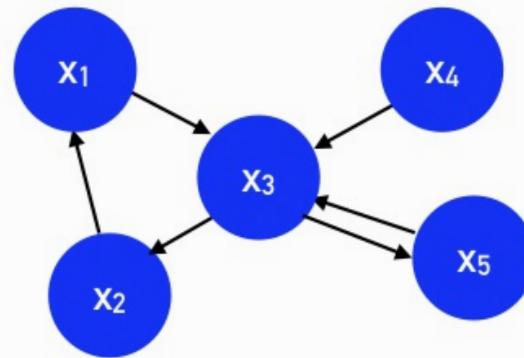
Prestige + Normalization

how likely am I to end up somewhere in a random walk

$$p = N^T p$$

$$A^T = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$N^T = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$$



$$p = \begin{pmatrix} p(x_1) \\ p(x_2) \\ p(x_3) \\ p(x_4) \\ p(x_5) \end{pmatrix}$$

$$\begin{pmatrix} p(x_1) \\ p(x_2) \\ p(x_3) \\ p(x_4) \\ p(x_5) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} p(x_1) \\ p(x_2) \\ p(x_3) \\ p(x_4) \\ p(x_5) \end{pmatrix} \rightarrow p(x_5) = \frac{1}{2}p(x_3)$$

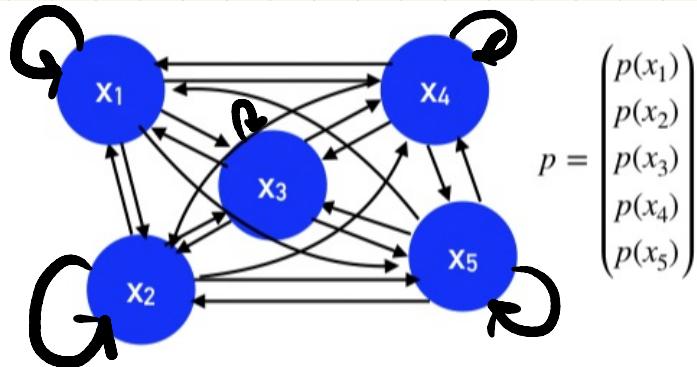
Prestige + Normalization + Random jumps

avoid getting stuck
in a sink

$$p = N_r p$$

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$N_r = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$$



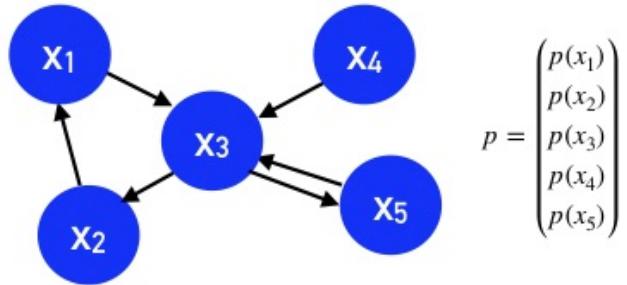
$$\begin{pmatrix} p(x_1) \\ p(x_2) \\ p(x_3) \\ p(x_4) \\ p(x_5) \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix} \begin{pmatrix} p(x_1) \\ p(x_2) \\ p(x_3) \\ p(x_4) \\ p(x_5) \end{pmatrix}$$

→ $p(x_3) = \frac{1}{5}p(x_1) + \frac{1}{5}p(x_2) + \frac{1}{5}p(x_3) + \frac{1}{5}p(x_4) + \frac{1}{5}p(x_5)$

Page Rank = Prestige + Normalization + Random jumps

$$p = (1 - \alpha)N^T p + \alpha N_r p$$

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$



$$p = \begin{pmatrix} p(x_1) \\ p(x_2) \\ p(x_3) \\ p(x_4) \\ p(x_5) \end{pmatrix}$$

$$\begin{pmatrix} p(x_1) \\ p(x_2) \\ p(x_3) \\ p(x_4) \\ p(x_5) \end{pmatrix} = (1 - \alpha) \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} p(x_1) \\ p(x_2) \\ p(x_3) \\ p(x_4) \\ p(x_5) \end{pmatrix} + \alpha \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix} \begin{pmatrix} p(x_1) \\ p(x_2) \\ p(x_3) \\ p(x_4) \\ p(x_5) \end{pmatrix}$$

Next Time

- Clustering Coeff
- Graphs w/ Python (NetworkX)