14 Nov 2022

- Exam 2: Wed, (30 Nov) dec. fcn + partial correctness (loop inv.)
- Final: during finals week, cumulative
- Project Presentations
  → by Wed, I'll email each group
- HW: Fri, Dec. 9th and n+1st due at FINAL.

Proof Writing: * Ott's 10 tips for mathematical writing

- present tense (not future): you're walking me through the proof with you.
- no weird line breaks. Use paragraphs.
- polished writing: don't start lines w/ math symbols and spell out # ≤ 10.
- define terms before you use them
- n and N are 2 diff symbols.

Flow Networks

INPUT: Directed Graph $G = (V, E)$

source s

sink/terminal t

capacity fcn $c : E \to \mathbb{R}_{\geq 0}$

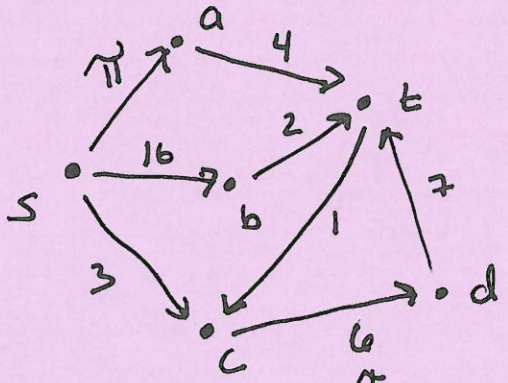
Physical:

- Plumbing "network"
- road networks
- streams/rivers

Abstract/Physical:

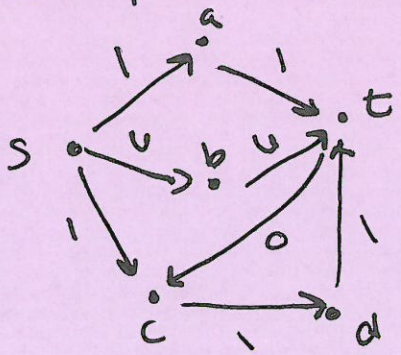
- Computer Networks
  Vertices = routers/relays
- social networks

Example:



capacity fcn $c : E \to \mathbb{R}_{\geq 0}$

Example flows on this graph:



A flow on a directed graph $G=(V,E)$ (with source s and sink t) is a function on the edges $f: E \to \mathbb{R}_{\geq 0}$ such that

$\forall v \in V \setminus \{s,t\}$, we have conservation of flow

"flow in = flow out"

$$\sum_{u \in V} f((u,v)) = \sum_{w \in V} f((v,w))$$

"in-flow = out-flow"

note: if $(a,b) \notin E$, then define $f(a,b)=0$

Q: come up w/ 2 flows on our example graph

Def'n: The value of a flow is:

$$|f| = \sum_{w \in V} f((s,w)) - \sum_{w \in V} f((w,s))$$

"flow out of source" — "flow into source"

Q: for two flows you made
1. What is the value of flow?
2. What is "flow out" - "flow in" at the sink?
3. Is there a relationship between the 2?

Def'n: A flow f is feasible if $\forall e \in E$,

$$f(e) \leq c(e).$$

Q: What feasible flows can you make on your network?