28 November 2022

Reminders

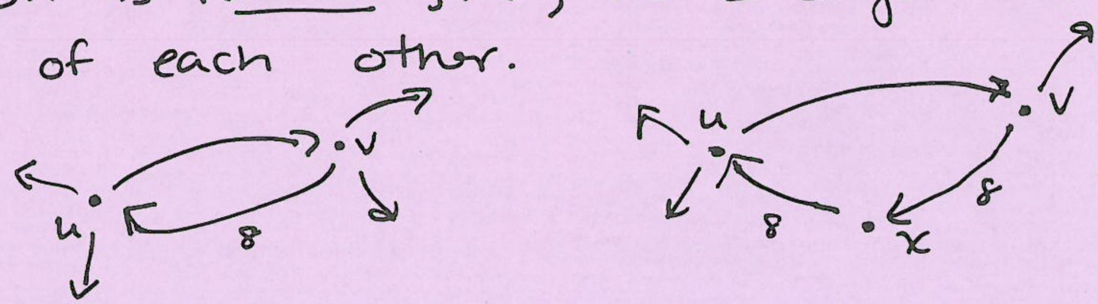
- WED: exam 2 — loop inv., dec. functions
- HW: LDOC (Fri, Dec. 9)
  - → to be posted
- $(n+1)$st HW: due @ FINAL
- PROJECT
- FINAL EXAM

BTF will send today
- HW 4
- personal email

<u>INPUT</u>: a directed graph $G = (V, E)$
a source $s \in V$
a sink/terminal $t \in V$ $s \neq t$
capacity fcn $c : E \to \mathbb{R}_{\geq 0}$

} "a flow network"

(assume: graph is <u>reduced</u>, i.e., no 2 edges are reverses of each other.
easy fix



)

<u>MAX FLOW PROBLEM</u>:
find a feasible flow that maximizes the value of the flow.

<u>MIN CUT PROBLEM</u>:
find a cut that minimizes the cost of the cut.

Given a flow network (G, s, t, c),

a <u>flow</u> on the network is:

a function $f: E \to \mathbb{R}_{\geq 0}$ such that $\forall v \in V \setminus \{s, t\}$, we have conservation of flow at v.

a flow is <u>feasible</u> iff $\forall e \in E$,

$$f(e) \leq c(e).$$

the value of the flow

$$|f| = \partial f(s) = \partial f(t)$$

abs. val of "flow in" - "flow out"

Residual Graphs:

$G_r = (V_r, E_r)$

$V_r = V$

$E_r = E \cup$ "$-E$"

$c_r : E_r \to \mathbb{R}_{\geq 0}$

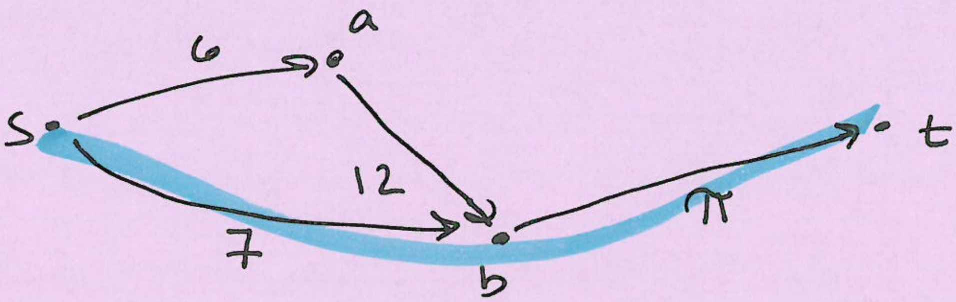
defined by: $c_r(e) = \begin{cases} c(e) - f(e), & \text{if } e \in E \\ -f(e), & \text{if } e \notin E \end{cases}$

Max Flow Algorithm

① Find the residual graph, which, initially is the graph w/ capacities given.

② Find any path from s to t.

e.g.,



and add flow along that path = min residual capacity of edges along that path. call this value v.

③ update residual graph

- for every edge e in the path:

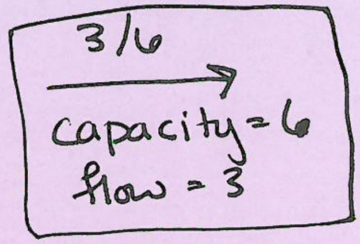
$$c_r(e) = c_r(e) - v$$

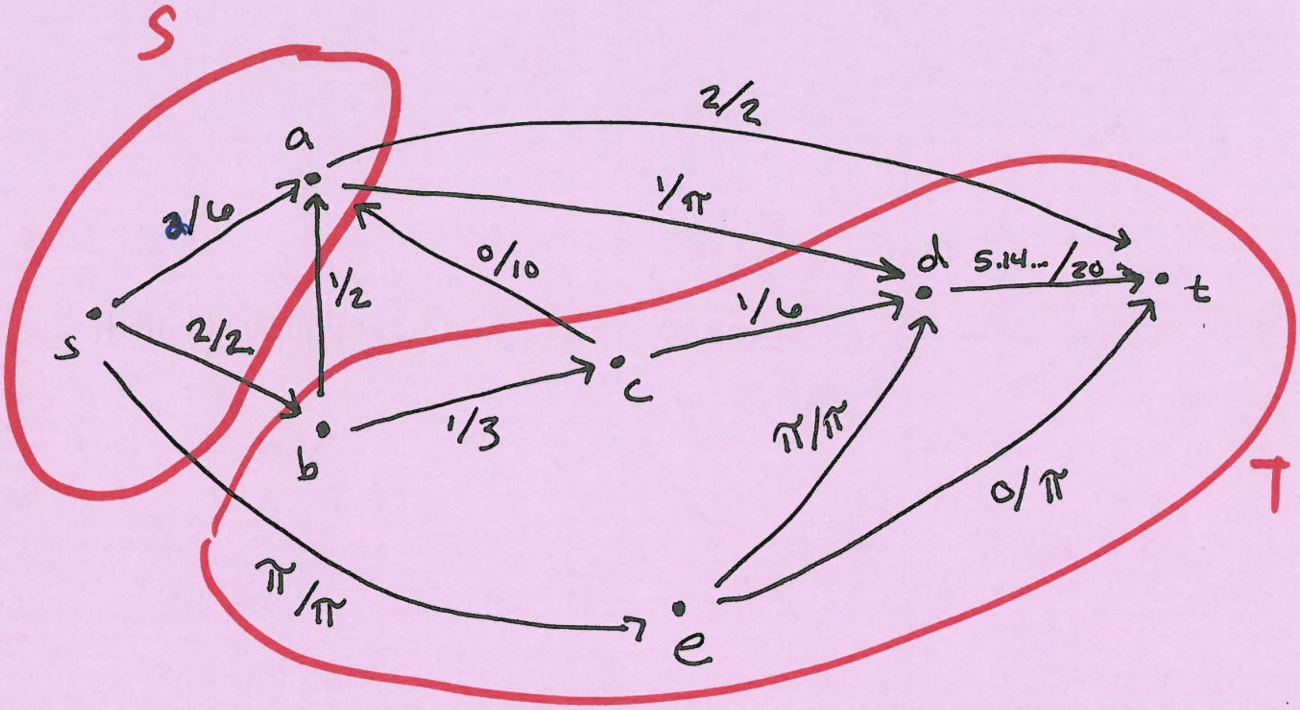
$$c_r(-e) = c_r(-e) + v$$

allows us to "undo"

④ Repeat.

Follow the algorithm to update the flow:



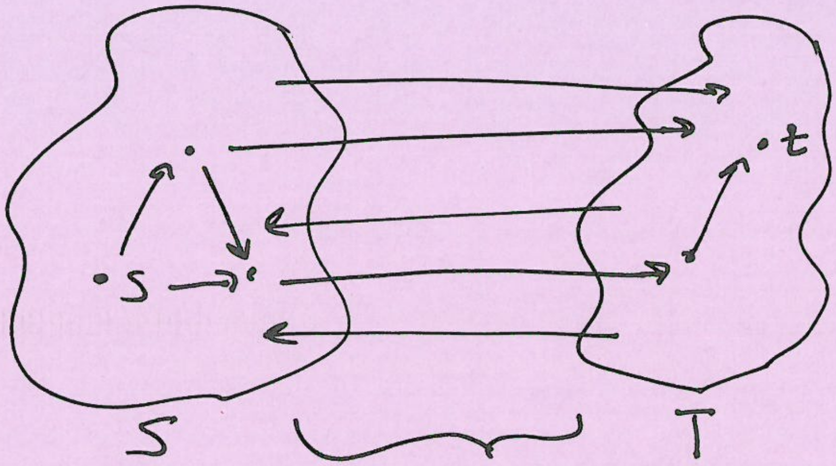


① What is max flow?

② What is min cut?

(s,t)-cut

partition of verts $V = S \sqcup T$
such that $s \in S$ and $t \in T$



we have to make this jump!

cost of a cut:

$$c(S,T) := \sum_{a \in S} \sum_{b \in T} c(a,b)$$