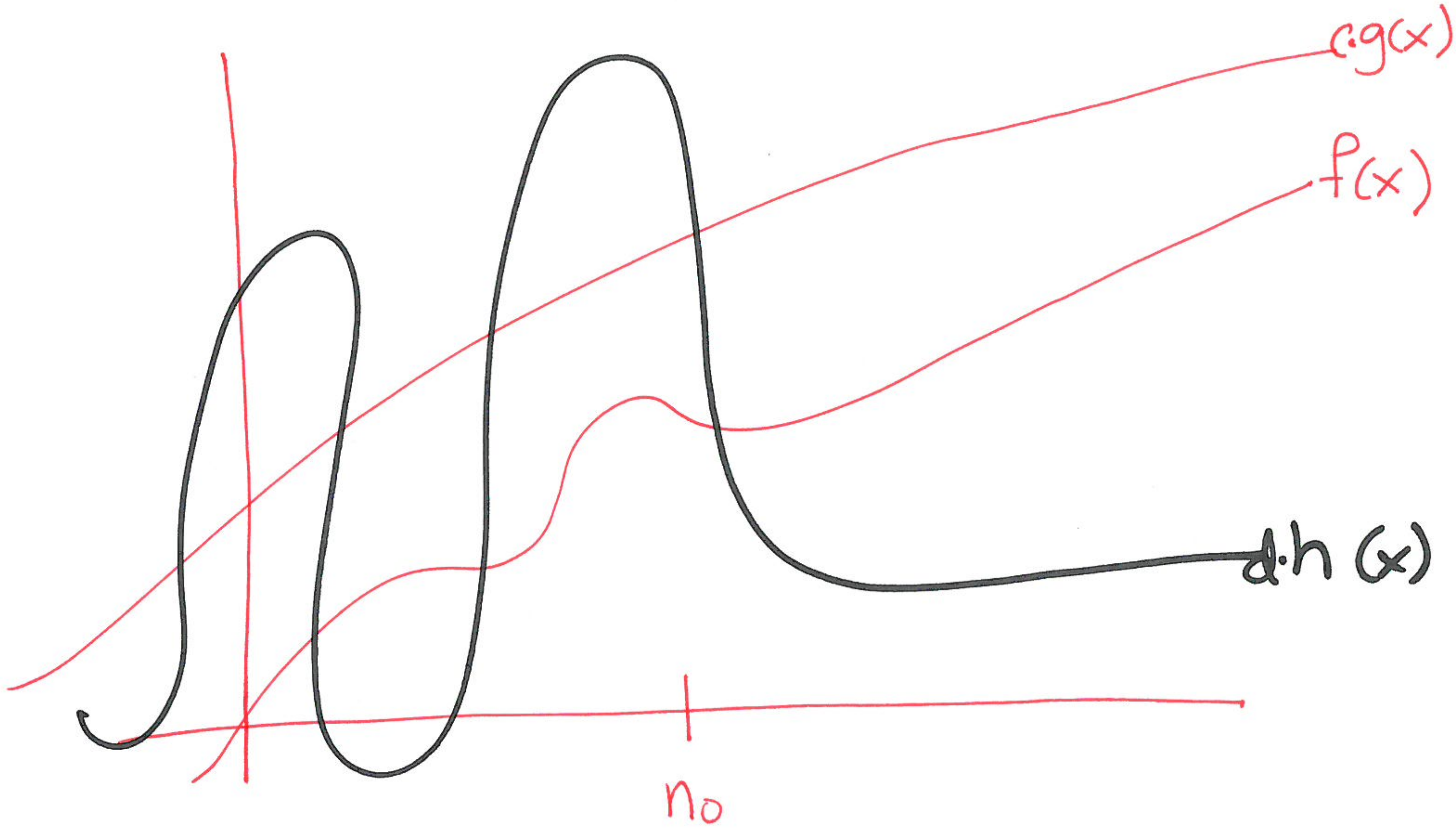
2 Sept 2022



<u>Big O</u>: $f(x)$ is $O(g(x))$ means that after some initial time n_0, $\exists c > 0$ s.t. $f(x) \leq c \cdot g(x)$ $\forall x \geq n_0$

"At worst, $f(x)$ "looks like" $g(x)$

is asymptotically upper-bounded by $g(x)$"

What is $f(x)$? It can represent:

→ the worst-case runtime given an input of size x.

AKA: <u>Any</u> input of size x will take at most $f(x)$ time.

e.g.)

$f_1(x)$ = worst-case search time in unsorted array of size x

$\Rightarrow$ time is at worst $\Theta(x)$.

(I have to check everything

$f_2(x)$ = worst-case runtime of searching in a sorted array of size x

$\Rightarrow$ time is at worst $\Theta(\log x)$

$f_3(x)$ = expected runtime of quicksort on an input array of size x

$\Rightarrow$ time is $\Theta(n \log n)$

But

$f_4(x)$ = worst-case RT of Quicksort $= \Theta(n^2)$

without the randomization we cannot say expected runtime is $\Theta(n \log n)$!

Recap: Quicksort!

- pick a random elt p
- create 2 arrays
  - $A_- \leftarrow$ all elts $\leq p$
  - $A_+ \leftarrow$ all elts $> p$
- sort A_- and A_+
- return A_- followed by elt p followed by A_+

Big Ω notation : lower-bounding our fcn

We say $f: \mathbb{N} \to \mathbb{R}$ is $O(h(x))$, where $g: \mathbb{N} \to \mathbb{R}$ when

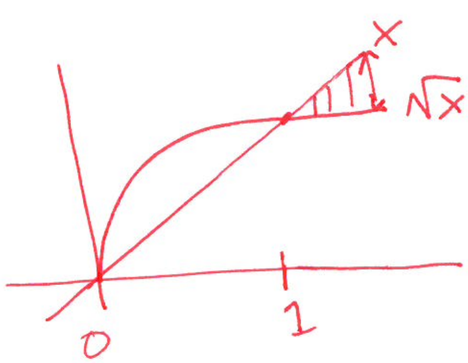
$\exists\, d > 0$, $d \in \mathbb{R}$ and $n_0 \in \mathbb{N}$ such that

$\forall\, n \geq n_0,\ n \in \mathbb{N}$

$$0 \leq d \cdot h(x) \leq f(x)$$

$f(x)$	$h(x)$	n_0	d
x	$\sqrt{x} = x^{\frac{1}{2}}$	1	1
x	x	1	1
x	$\sqrt{x} + \log x$		

↑ an exercise. start w/ graphing!



If $f(x) = O(g(x))$ and $f(x) = \Omega(g(x))$,
then we say $f(x) = \Theta(g(x))$.

tight asymtotic bound

f	g		for big O: n_0	for big O: c	for Ω: n_0	for Ω: d
$\frac{1}{2}x$	x		1	1	1	1/2
$x + x^2$	x^2		EXERCISE			
x^2	$x^2 + x$					

we can always go bigger!

note: we can always go smaller

to combine, pick the bigger n_0!

Alternate def'n for Θ:

f is $\Theta(g)$ iff

$\exists\, c \in \mathbb{R},\ c \geq 1$ and $n_0 \in \mathbb{N}$ such that $\forall\, n \geq n_0$,

$$0 \leq \frac{1}{c}\, g(x) \leq f(x) \leq c\, g(x).$$

- $f(x) = \Theta(g(x)) \iff g(x) = \Theta(f(x))$
- $f(x) = O(g(x)) \iff g(x) = \Omega(f(x))$

$\Theta(\log x) < \Theta(x^c)$

for c a const!