

Prove:  $\forall n \geq 0$ ,

$$\sum_{i=0}^n i = \frac{n(n+1)}{2} \quad P(n)$$

Proof: We proceed by induction.

The base case is  $n=0$ .

← abbreviated claim

$$\sum_{i=0}^0 i := 0, \text{ by def'n of an empty sum (no summands)}$$

$$\text{Also, } \frac{n(n+1)}{2} = \frac{0(1)}{2} = 0$$

$$\therefore \sum_{i=0}^0 i = \frac{0(1)}{2}$$

1st proof!

Let  $k \geq 0$ , and assume that

$$\sum_{i=0}^k i = \frac{k(k+1)}{2} \quad P(k)$$

Let  $n = k+1$ . We want to show (wts)

$$\text{START } \sum_{i=0}^{k+1} i = \frac{(k+1)(k+2)}{2} \text{ END}$$

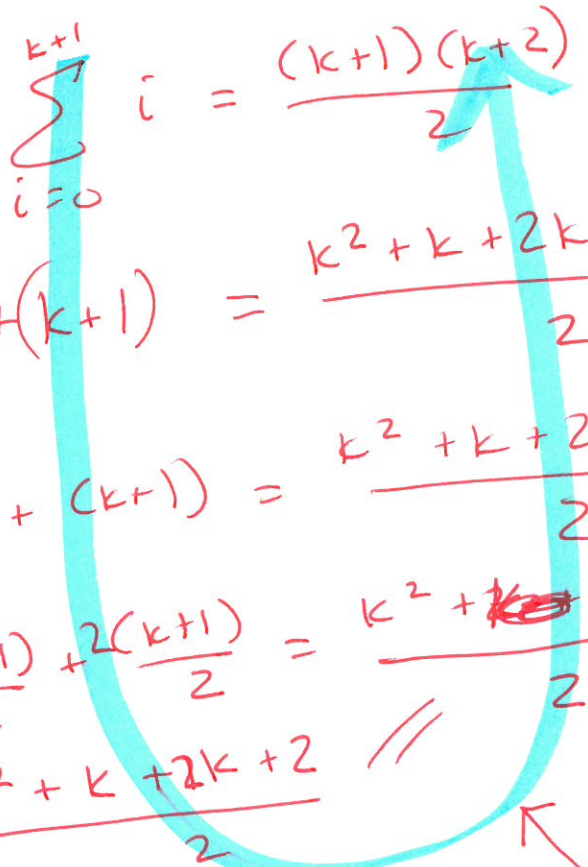
Claim

2nd sub-proof!

$$\begin{aligned} \text{START } \sum_{i=0}^{k+1} i &= \sum_{i=0}^k i + \sum_{i=k+1}^{k+1} i && \text{by expanding the sum} \\ &= \frac{k(k+1)}{2} + \sum_{i=k+1}^{k+1} i && \text{by our I. A.} \\ &= \frac{k(k+1)}{2} + (k+1) \text{ (2)} && \text{expand sum + mult by } \frac{2}{2} \\ &= \frac{k(k+1) + (k+1) \text{ (2)}}{2} = \frac{(k+1)(k+2)}{2} \text{ (2) END} \end{aligned}$$

∴ We have shown...

Scratch work (not a proof)

$$\begin{aligned}\sum_{i=0}^{k+1} i &= \frac{(k+1)(k+2)}{2} \\ \sum_{i=0}^k i + (k+1) &= \frac{k^2 + k + 2k + 2}{2} \\ \frac{k(k+1)}{2} + (k+1) &= \frac{k^2 + k + 2k + 2}{2} \\ \frac{k(k+1)}{2} + \frac{2(k+1)}{2} &= \frac{k^2 + \cancel{k} + 3k + 2}{2} \\ \frac{k^2 + k + 2k + 2}{2} & \quad // \end{aligned}$$


Make a U-shaped argument from scratch work!

Expanding Sums

$$\sum_{i=x}^{x+1} i^2 = (x)^2 + (x+1)^2$$

$$\sum_{i=x}^x i = x$$

$$\sum_{i=k+1}^{k+1} i = k+1 = \frac{2(k+1)}{2}$$