21 Sept. 2022

$$\sum_{i=0}^{n} \binom{n}{i} = 2^n$$

} # of elts. in the power set of set of size n

fix $i$. The $i$th summand counts # of subsets of size $i$.

for each of the $n$ elements, we have 2 choices $\Rightarrow \underbrace{2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2}_{n \text{ times}} = 2^n$ sets

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$$\sum_{i=0}^{n} i \binom{n}{i} = n 2^{n-1}$$

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$$\sum_{i=0}^{n} i = \frac{n(n-1)}{2}$$

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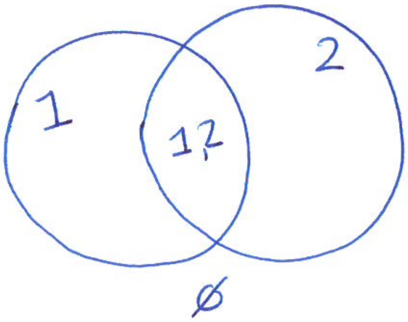
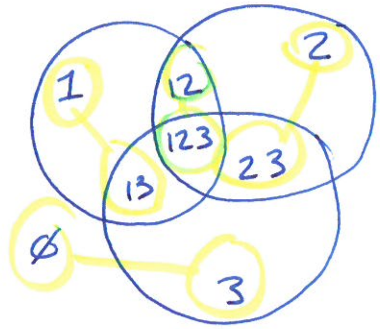
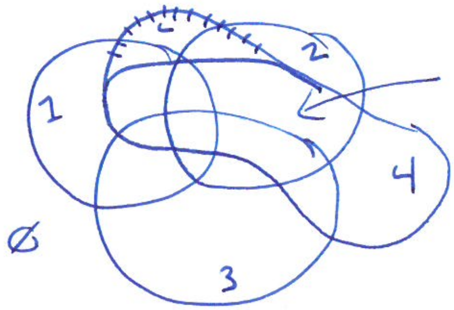
Q: Can we come up w/ a recursive solution to the problem of finding all subsets of a set of size n? (i.e., find the power set)

- try small examples
- come up w/ solution
- time / space complexity?
- correctness?




pretend I draw this nicely

note: $\underline{3} = \{1, 2, 3\}$

$\underline{n} = \{1, 2, \ldots, n\}$

$\underline{n} = 1{:}n$

$\mathcal{P}(\underline{3})$

$\mathcal{P}(\underline{2})$ subproblem 1 | $\{3\} + \mathcal{P}(\underline{2})$ subproblem 2

$\mathcal{P}(\underline{1})$ | $\{2\} + \mathcal{P}(\underline{1})$ | $\{3\} + \mathcal{P}(\underline{1})$ | $\{3, 2\} + \mathcal{P}(\underline{1})$

$\boxed{\emptyset \quad \{1\} \quad \{2\} \quad \{1,2\} \quad \{3\} \quad \{1,3\} \quad \{3,2\} \quad \{3,2,1\}}$

The elements of $\mathcal{P}(\underline{3})$

$\mathcal{P}(\underline{n})$

if $n = 0$
| return $\emptyset$
elseif
return $\mathcal{P}(\underline{n-1}) \cup \underbrace{\{n\} + \mathcal{P}(n-1)}_{\text{add } n \text{ to each elt. of } \mathcal{P}(n-1).}$

<u>Runtime Recursion:</u> $T(n) = 2T(n-1) + \Theta(2^{n-1})$ assuming ptrs to arrays!

Q: How many elts in $\mathcal{P}(n)$? $2^n$
$\Rightarrow$ my time complexity is at <u>least</u> $\Omega(2^n)$

$\mathcal{P}(A, B)$ = the powerset of A ~~including~~ ~~all~~ plus B added to each set

1st call: $\mathcal{P}(A, \emptyset)$

$\mathcal{P}(A, B)$
if $|A| = 0$
| return B
end if
return $\mathcal{P}(A \setminus A[0], B) \cup \mathcal{P}(A \setminus A[0], B \cup A[0])$

runtime:
$T(n) = 2T(n-1) + \Theta(1)$
$= \Theta(n\, 2^{n-1})$

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In Recursion, How do we know we will end?

$b = 0$
while $b < 2^n$
| $b \leftarrow b+1$
end while

$k = 20$
while $k > 0$
| $k \leftarrow k-2$
end while

$r = \pi$
while $r > 0$
| $r \leftarrow r/2$
end while

$k = 20$
while $k \neq 0$
| $k \leftarrow k-2$
end while

Well-ordered set: A set such that <u>every</u> subset has a smallest elt.

<u>example</u>: $\mathbb{N}$, $\mathbb{Z}_+$

<u>non-example</u>: $\mathbb{R}$
$(0,1)$
$[0,1] \subseteq \mathbb{R}$