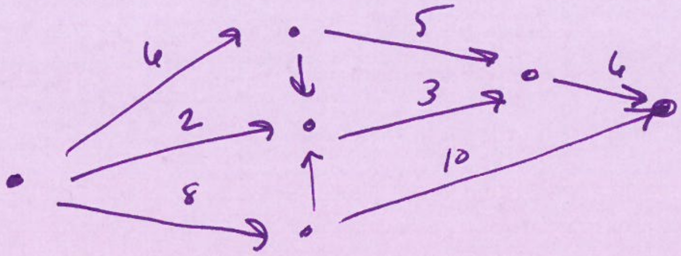
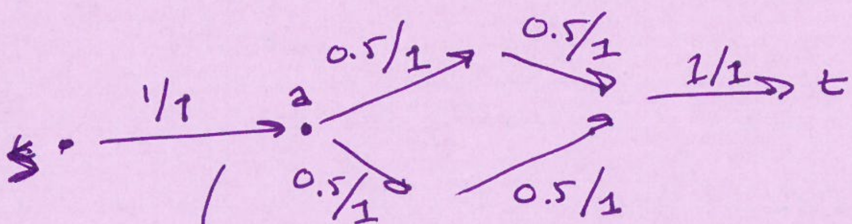
5 December 2022

Max flow



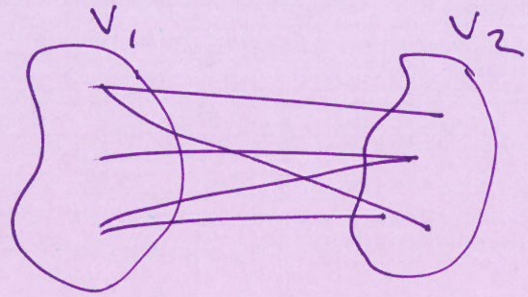
Lemma: If (G, c) is a flow network such that $c(v) \in \mathbb{Z}$ $\forall v \in V_G$, then the value of the max flow is an integer and there exists an integral flow (ie., one where there is integer flow on each edge)



at a, it "split", but OK to "push" it down b/c the capacities are integers.

Bipartite Graph Matching

- $G=(V,E)$ is a bipartite graph iff $V = V_1 \sqcup V_2$ such that $E \subset V_1 \times V_2$



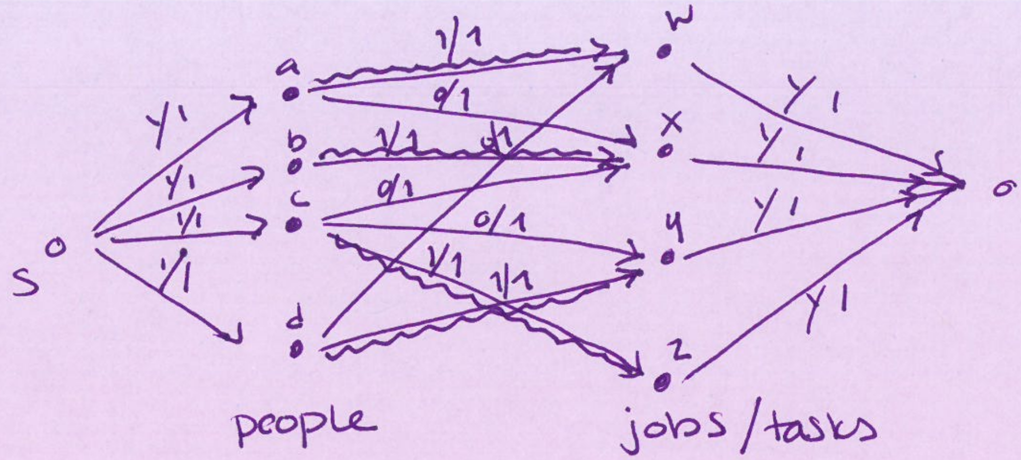
Recap:

- to test if bipartite:

  find $\boxed{\text{spanning tree}}$ for each $\boxed{\text{connected comp.}}$ of G.

  ~~Use spanning forest~~

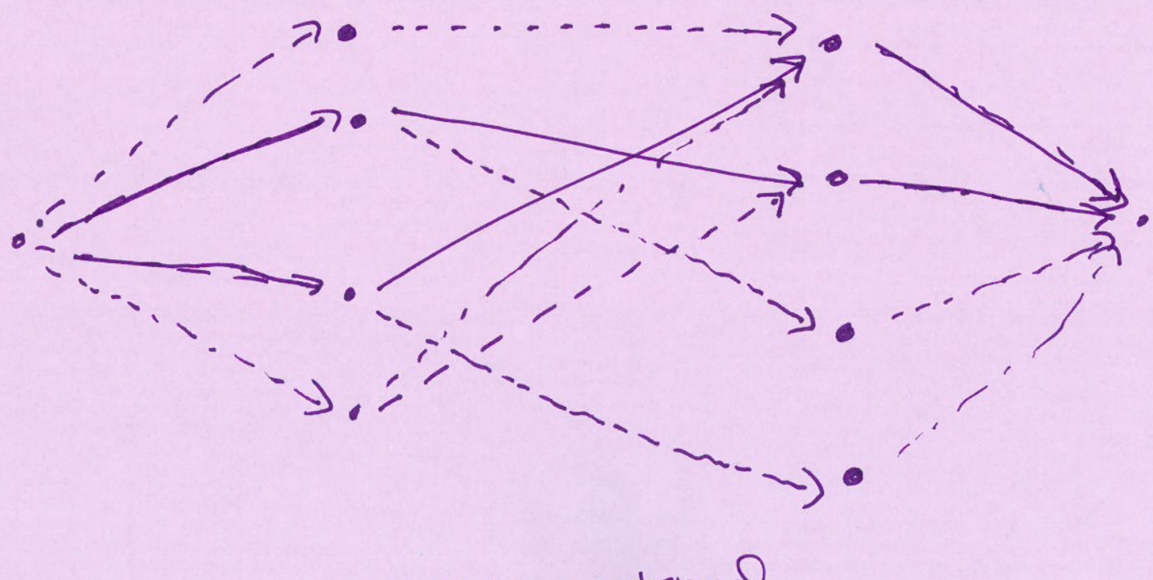
  The set of these spanning trees is called a $\boxed{\text{spanning forest.}}$ Use this forest to color all verts (each layer a diff "color" than above/below). Then, check all edges have 2 diff. color verts.



<u>Problem</u>: Find a maximum (= most # edges) matching in $G = (V, E)$, a bipartite graph.

A <u>matching</u> $M \subseteq E$ such that each vertex in V appears at most once in M.

Given a matching, can I find a flow?



Solid = matched
dashed = not in matching
maximal (not maximum) matching

Reductions

problem A (given)

① transform the problem.

③ interpret result

problem B ② solve here

*note: not every prob. B can "go back" to prob. A. But, any $A \rightsquigarrow B$ can "be interpreted backwards"

To solve prob A, I could:

① transform to prob B + solve
② transform to prob C + solve
③ think + find a clever sol'n
⋮

"fastest way" is minimum over all these

Assuming the reduction is "negligible", solving prob. A is at most as hard as solving prob B. "$A \leq B$"