7 November 2022

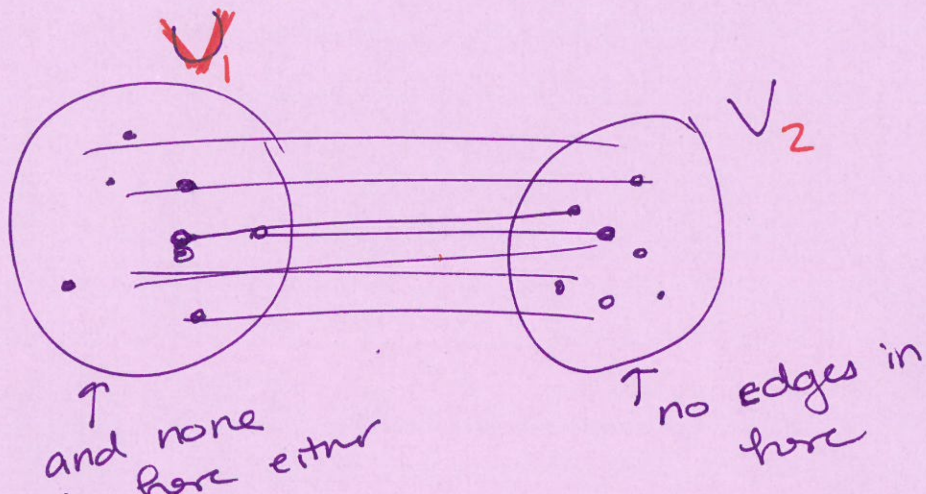
A graph $G=(V,E)$ is bi<u>partite</u> iff

$\exists\, V_1, V_2 \subseteq V$ such that $V = V_1 \sqcup V_2$

and $V_1 \neq \emptyset$ and $V_2 \neq \emptyset$

$\forall e=(u,v) \in E$, either $u \in V_1$ and $v \in V_2$

or $v \in V_1$ and $u \in V_2$



"every edge has to go between U and V"

Induction: (Weak form)

1. State what we're going to show.
2. Base case (say, $n=n_0$)
3. Let $k \geq n_0$, assume [stmt holds using $n=k$]
4. Prove stmt holds for "k+1" case.
   ☆☆ Start w/ k+1 + "tear it up" until you find the I.A (k-case) ☆☆
5. Conclude

Claim: $\forall a \in A$, $F(a)$ is true.

Proof: Let $a \in A$.

"generalizing from the general particular"

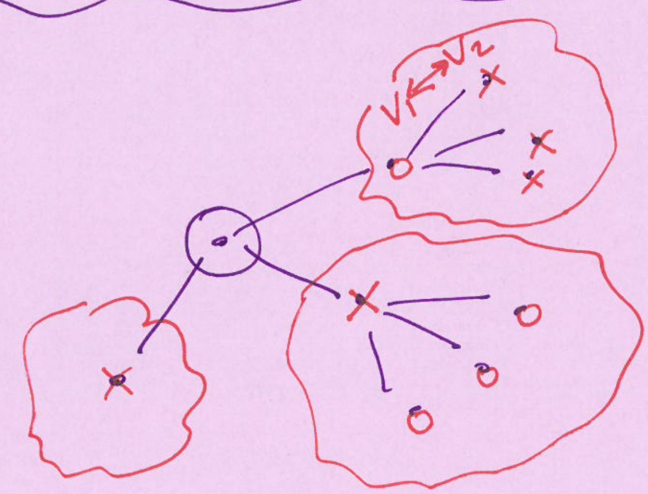
$\boxed{Q1}$ Prove that every tree is a bipartite graph.

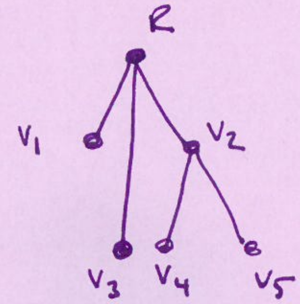
Suppose you have V_1 + V_2 + want to show they work in the def'n of bipartite. Then, you can

(a) Show $\forall e = (u,v) \in E$, e is between V_1 + V_2.

OR

(b) Show $\forall a, b \in V_i$, $(a,b) \notin E$.





Define tree, layer / depth

Q2 Come up w/ an (efficient) algorithm to determine if a graph is bipartite.

odd-length cycles are bad!