Prove: $\forall n \geq 0$, $\boxed{\sum_{i=0}^{n} i = \frac{n(n+1)}{2}}$ P(n)

Proof: We proceed by induction. ← abbreviated claim

The base case is $n=0$.

$\sum_{i=0}^{0} i := 0$, by def'n of ~~an empty sum~~ (~~the sum [illegible]~~)

Also, $\frac{n(n+1)}{2} = \frac{0(1)}{2} = 0$

$\therefore \sum_{i=0}^{0} i = \frac{0(1)}{2}$.

1st proof!

Let $k \geq 0$, and assume that

$\sum_{i=0}^{k} i = \frac{k(k+1)}{2}$. P(k)

Let $n = k+1$. We want to show (wts)

START $\sum_{i=0}^{k+1} i = \frac{(k+1)(k+2)}{2}$ END. Claim

2nd sub-proof!

START $\sum_{i=0}^{k+1} i = \sum_{i=0}^{k} i + \sum_{i=k+1}^{k+1} i$ by expanding the sum

$= \frac{k(k+1)}{2} + \sum_{i=k+1}^{k+1} i$ by our I.A.

expand sum + mult by $\frac{2}{2}$

$= \frac{k(k+1)}{2} + \frac{(k+1)2}{2}$

$= \frac{k(k+1) + (k+1)2}{2} = \frac{(k+1)(k+2)}{2}$ END

$\therefore$ We have shown...

Scratch work (not a proof)

$$\sum_{i=0}^{k+1} i = \frac{(k+1)(k+2)}{2}$$

$$\sum_{i=0}^{k} i + (k+1) = \frac{k^2+k+2k+2}{2}$$

$$\frac{k(k+1)}{2} + (k+1) = \frac{k^2+k+2k+2}{2}$$

$$\frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{k^2+\cancel{k}+3k+2}{2}$$

$$\frac{k^2+k+2k+2}{2}$$

Make a U-shaped argument from scratch work!

Expanding Sums

$$\sum_{i=x}^{x+1} i^2 = (x)^2 + (x+1)^2$$

$$\sum_{i=x}^{x} i = x$$

$$\sum_{i=k+1}^{k+1} i = k+1 = \frac{2(k+1)}{2}$$