26 Sept. 2022

2n
4 coins (lined up), 2 players (Alice / Bob)

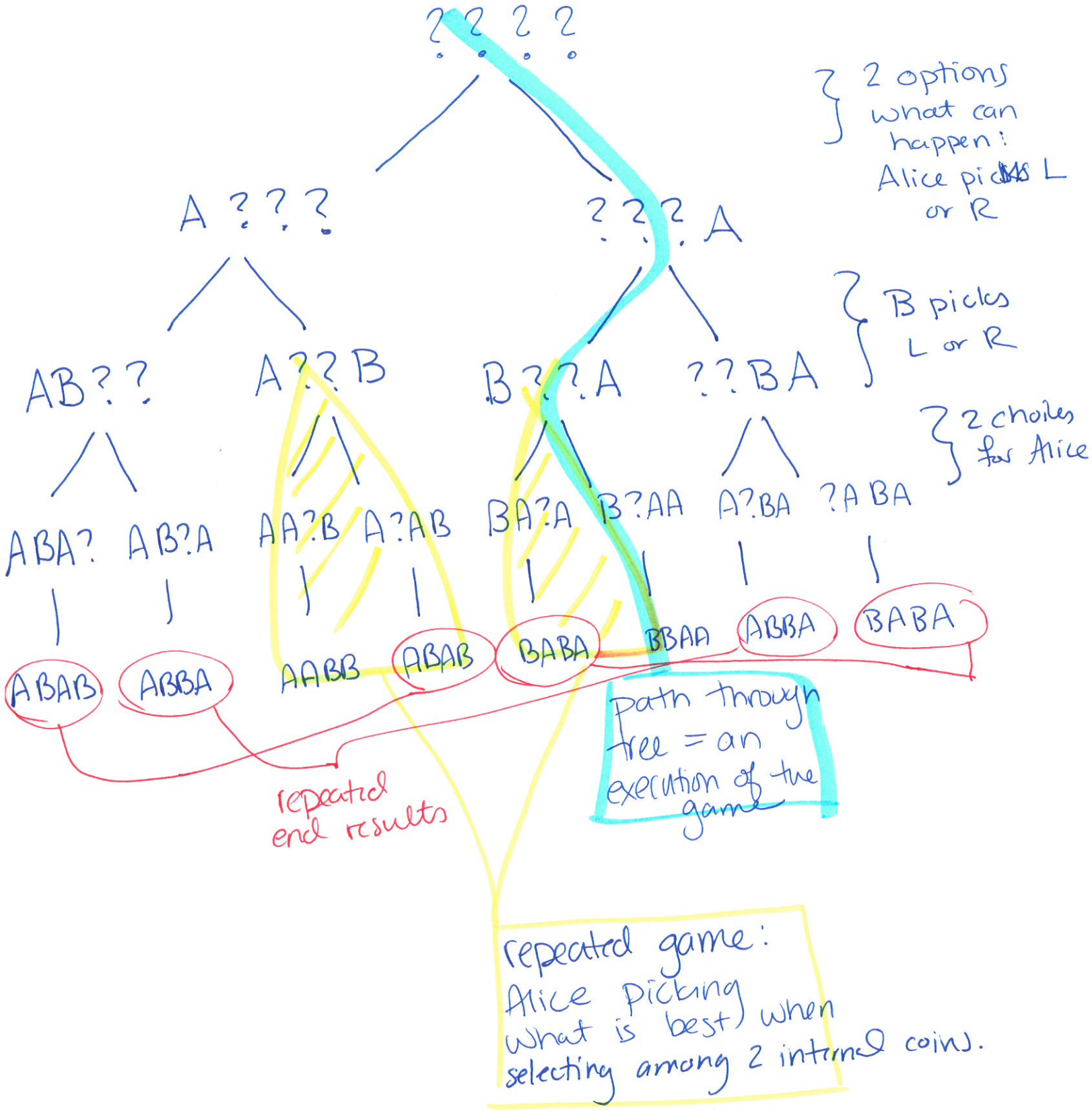
→ Alice picks one on end of line of coins
→ Bob picks one, on end of line of coins
→ repeat until all coins taken
→ winner: the one of more $ at the end

Questions:

1. Is there a best strategy for Alice (or Bob)?
2. If so, does "greedy strategy" exist/work?
3. What is the expected winning for Alice?
4. can you draw out a "game tree"?
5. Is it possible to pick 2n coins + an arrangement of them so that Alice always wins (if she plays optimally)?
6. If all selections are random, what is the probability that Alice wins?

not always best strategy. ex:

5¢ 10¢ 25¢ 10¢
B A B A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A???
???A
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B??A
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ABA?
AB?A
AA?B
A?AB
BA?A
B?AA
A?BA
?ABA
ABAB
ABBA
AABB
ABAB
BABA
BBAA
ABBA
BABA
2 options what can happen: Alice picks L or R
B picks L or R
2 choices for Alice
path through tree = an execution of the game
repeated end results
repeated game: Alice picking what is best when selecting among 2 interned coins.

Decrementing Functions

- can be used to prove that an algorithm terminates. Specifically, if a loop or recursion terminates.

- A function from the state space to a well-ordered set (WOS)
  - → state space: a <u>state</u> is a snapshot in the execution of an algorithm. Includes status/value of all variables, stack traces, ~~etc~~. # of times a loop executed, etc
    think: what you can "see" using a breakpoint + maybe some extra variables
  - → WOS: a set such that every subset has a minimum element.
    our favorite WOS is $\mathbb{N} = \{0, 1, 2, \ldots\}$ ← some sources start N at 1, not 0.
    $\mathbb{Z}_+ = \{1, 2, 3, \ldots\}$
  - → this function must strictly decrement (i.e., not stay the same value) each time we reach a new call to the fun (in recursion) or top of loop (for a loop)

e.g., $D : S \to \mathbb{N}$, $D(S) := n - i$, i = the iteration through loop, n = size of input