

10 October 2022

Let W_1, W_2 be words.

$$\text{edit}(W_1, W_2) = \begin{cases} 0, & \text{if } W_1 = W_2 \\ \text{length}(W_1), & \text{if } W_2 = \emptyset \\ \text{length}(W_2), & \text{if } W_1 = \emptyset \\ \min \begin{cases} \text{edit}(W_1^n, W_2^{m-1}) + 1 \\ \text{edit}(W_1^{n-1}, W_2^m) + 1 \\ \text{edit}(W_1^{n-1}, W_2^{m-1}) + \mathbb{1}_{W_1[n] \neq W_2[m]} \end{cases} \end{cases}$$

Let $n = \text{length}(W_1)$, $m = \text{length}(W_2)$

$W_i^j :=$ the substring from 1st through jth elt of W_i

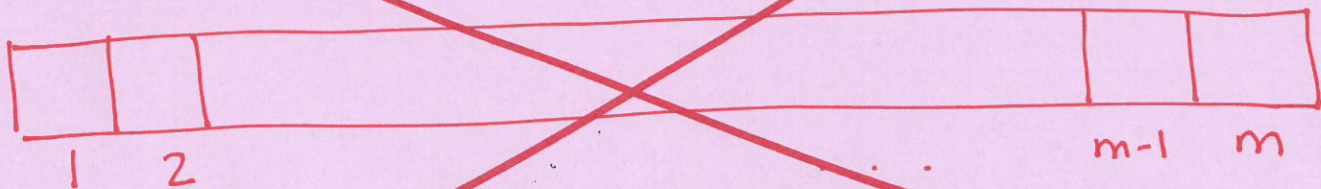
e.g., $W_1 = \text{FIX}$, $n = 3$

$W_2 = \text{FIND}$, $m = 4$

↓ how does this relate to

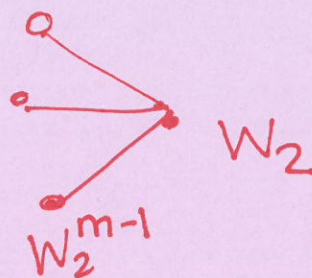
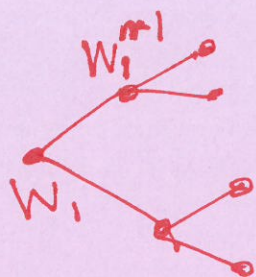
$\text{edit}(W_1^3 = \text{FIX}, W_2^3 = \text{FIN})$

m subproblems



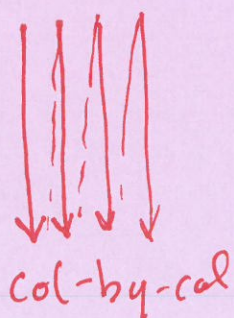
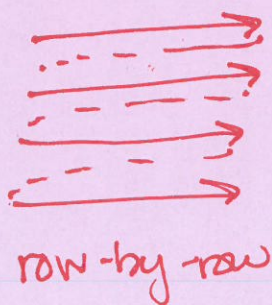
not enough storage
try 2D array next.

ABC
A A A B



equiv g : what is SP from w_1 to w_2 ?

Q1: What order can we evaluate these cells in?



		1	2	3	4	5	6	7	8
M		T O P O L O G Y							
1	T	0	1						
2	O								
3	P			0	1				
4	O			1	0				
5	G				2				
6	R								
7	A								
8	P								
9	H								
10	Y						5	5	

row-by-row
eval order

$$\text{edit}(TO, T) = \min \begin{cases} \text{edit}(TO, \emptyset) + 1 = 2 + 1 \\ \text{edit}(T, T) + 1 = 0 + 1 \\ \text{edit}(T, \emptyset) + 1_{O=T} = 1 + 1 \end{cases}$$

note: we're evaluating w/ the table above, not recursion.

compute 4,4:

$$\text{edit}(TOPO, TOPO) = 0 \quad (\text{base case!})$$

compute ^{col}row 4, ^{row}col 5:

want: $TOPO \rightsquigarrow TOPOG$

$$\text{edit}(TOPO, TOPOG) = \begin{cases} \text{edit}(TOPO, TOPO) + 1 \\ \text{edit}(TOP, TOPOG) + 1 \\ \text{edit}(TOP, TOPO) + 1_{O=T} = 1 + 1 = 2 \end{cases}$$

sol'n r/c

(4/4) $0 + 1 = 1$
 $TOPO \rightsquigarrow TOPOG$
 (see r 5, col 3) $2 + 1 = 3$
 $TOPO \rightsquigarrow TOP \rightsquigarrow TOPOG$
 see r 4, col 3.
 $TOPO \rightsquigarrow TOP \rightsquigarrow TOPO \rightsquigarrow TOPOG$

3

row 10, col 8

depends on

$$M[10, 7] = 6$$

$$M[9, 8] = 5$$

$$M[9, 7] = 5$$

$$M[10, 8] = \min \begin{cases} M[10, 7] + 1 = 6 + 1 \\ M[9, 8] + 1 = 5 + 1 \\ M[9, 7] + \underline{1}_{y \neq y} = 5 + 0 \end{cases}$$