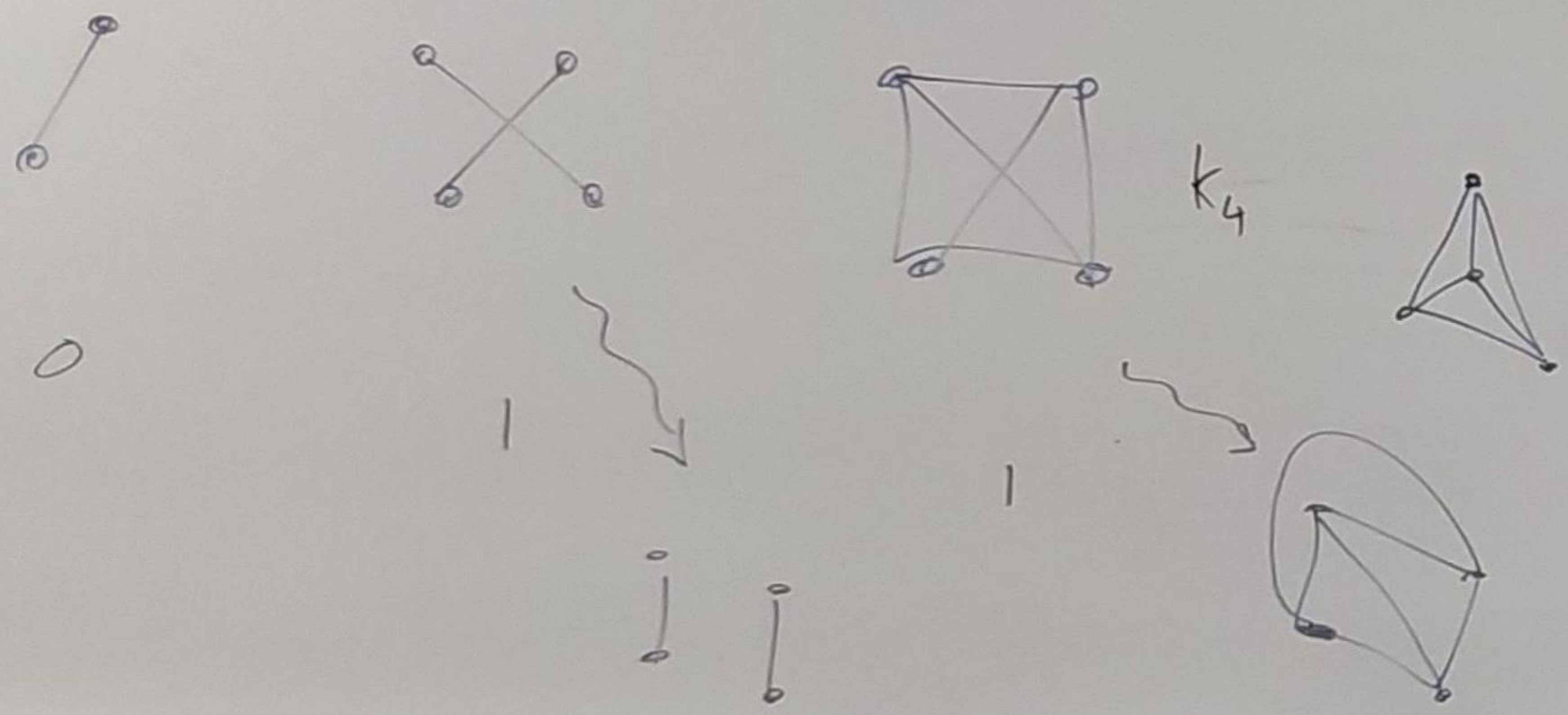
Crossing #s of graphs



0

1

1

K_4

Q1) Given $G=(V,E)$,

what is

$$cr(G) = \min_{V \hookrightarrow \mathbb{R}^2} \{\#\text{ of crossings in } G\}$$

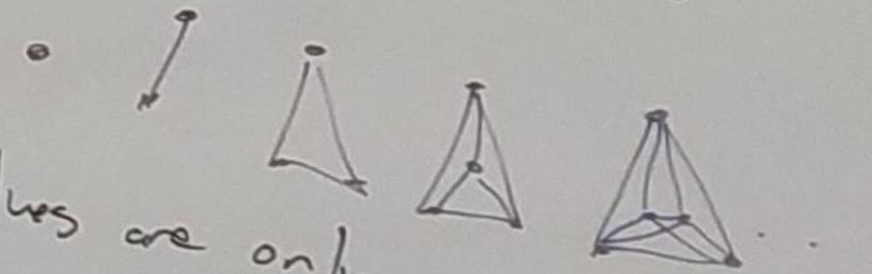
When are $cr(G)$ and $\overline{cr}(G)$ different?

Variations...

1. restrict edges to being straight lines (rectilinear)

   $\overline{cr}(G)$

1.5. focus on $\overline{cr}(K_n)$



Values are only known up to around $n=27$

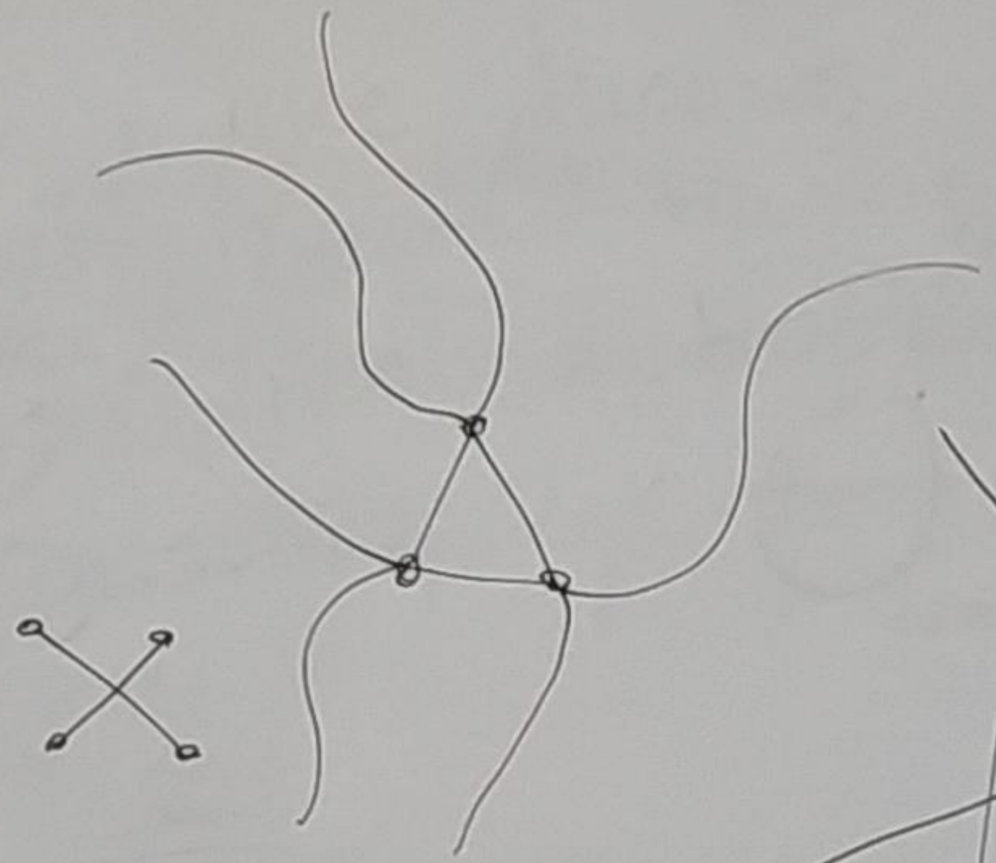
convex hull

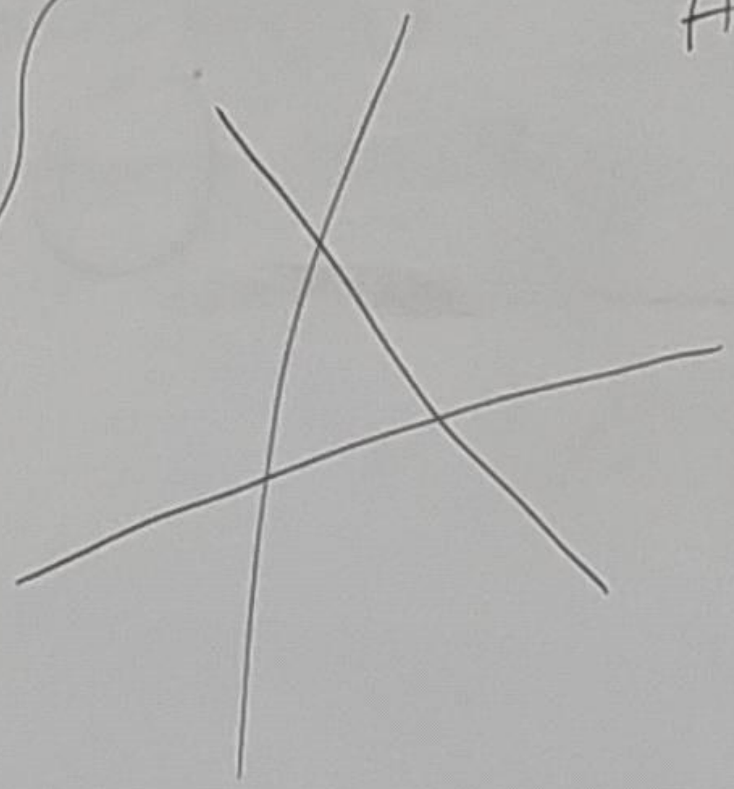
~ why are they triangular? ✓

~ why does K_n have a copy of K_{n-3} in the middle?

2. pseudo linear $\tilde{cr}(G)$

- edges can curve, but either
  - cross every other edge exactly once
  - can be extended to cross everyone else exactly once.





3. is every pseudoliner graph/arrangement stretchable?

A: NO.

(yes if you have 8 or fewer pseudolines)

↑ Grünbaum's conjecture

Observe.

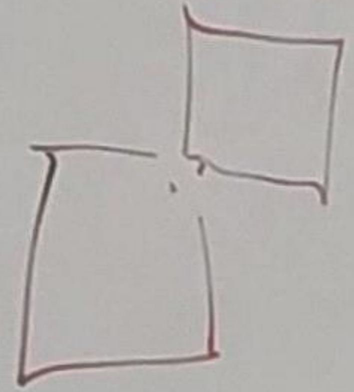
if G is stretchable,

$\overline{cr}(G) = \tilde{cr}(G)$

The Snowblower Problem

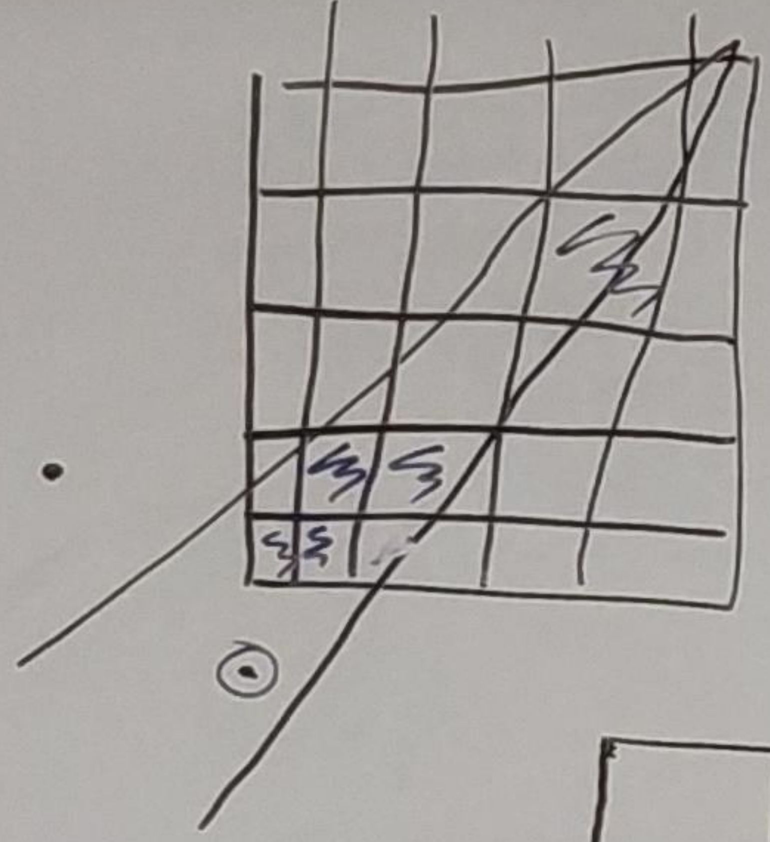
※snow-movers-dist

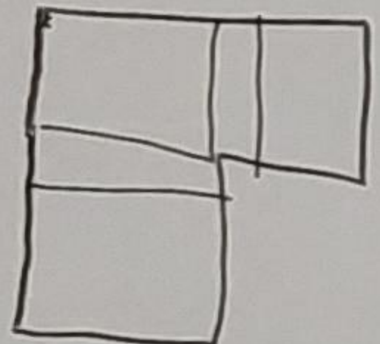
(2006) Joe Mitchell et. al



depth = 1

Snowblower
→ Max depth = D
→ Throwing Snow
1) "Default" any direction
2) "adjustable" → L,R, Forward
3) "fixed" → R





Questions

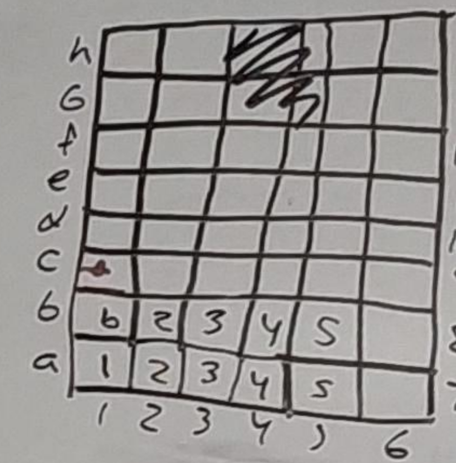
1. Can we show their structures are connected?

2. If these their Voronoi structures aren't connected, how might we fix their results?

The Snowblower Problem

#snow-moves-dist

(2006) Joe Mitchell et. al



Q: Is the Discrete Voronoi Dgm always 4-colorable?

Def'n: a graph G is k-colorable iff

$\exists f : V_G \to \{1, 2, \ldots, k\}$

such that

⊛ Edges are multi-colored

i.e., $\forall (v,w) \in E_G,\ f(v) \neq f(w)$

To get our graph,

vertices = Voronoi regions

edges = represent adjacent regions.