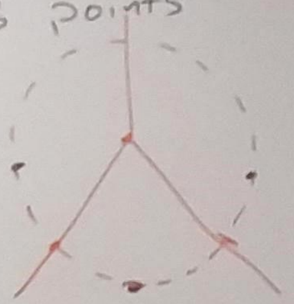
Voronoi Diagrams - Fortune's Algorithm

(post office problem)



3 points - find center of circle containg 3 points.

center (C_x, C_y)

radius r

$$(x - C_x)^2 + (y - C_y)^2 = r^2$$

plug in 3 points $(x + y)$

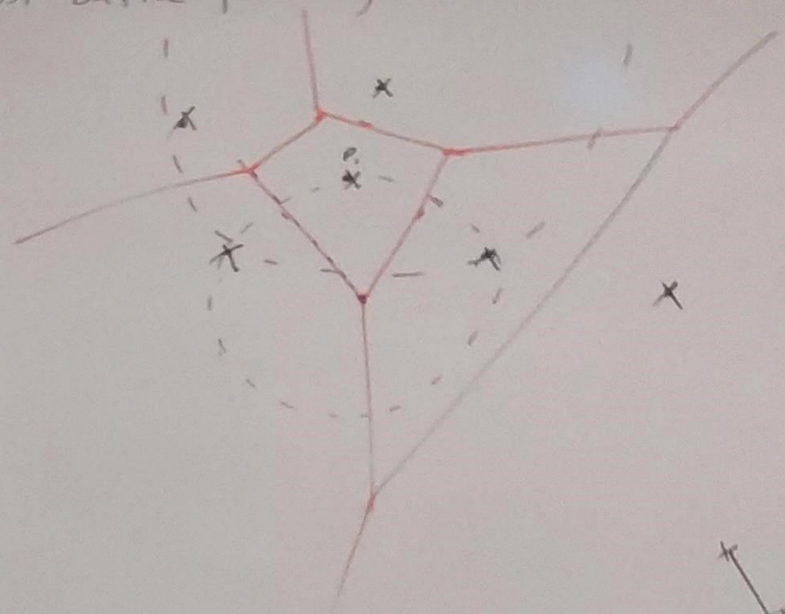
3 equations 3 unknowns

$\Rightarrow$ solve C_x C_y + r

Empty circle property -

If circle containg 3 sites does not contain any other sites

$\Rightarrow$ vertex of voronoi diagram.



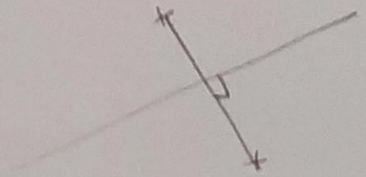
Problem:

Given P $\{p_1 \ldots p_n\}$

Build Voronoi Diagram.

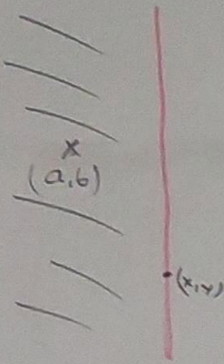
Voronoi diagram is a planar graph.

Stored in DCEL.



Small cases:

2 points - perpendicular bisector



$$(a-x)^2+(b-y)^2=(c-x)^2+(d-y)^2$$

$$\vdots$$

$$(a-c)x+(b-d)y=\frac{1}{2}(a^2+b^2)-\frac{1}{2}(d^2+c^2)$$

its a line!

Algo 1)

for each p_i
  for each p_j $(j \neq i)$
    find half-plane containing p_i - $h(p_i, p_j)$

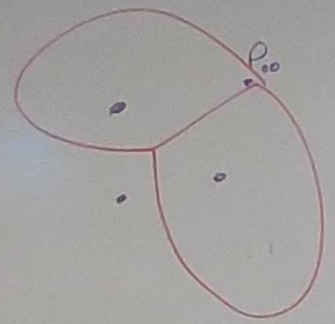
$$V(p_i)=\bigcap_{i \neq j} h(p_i p_j)$$

Runtime: $O(n^2)$ - intersection of n-half planes for each point.

Properties of Voronoi Diagram

- Each cell is convex
  Cell is intersection of half-planes.
- Assumption - no 4 points co-circular
  $\Rightarrow$
  each vertex on Voronoi graph has degree 3
- Sites with unbounded voronoi cells are on the convex hull of P.

Applications
- Giraffes
- Sports
- Trees
- Fires



- Nearest Neighbor Queries.
- Facility Locations
- Cellular network

Bound number of vertices (edges too)

planar Graph $\Rightarrow$ Euler's formula.

$$n_v - n_e + n_f = 2$$

connect unbounded edges to p_∞ - point at infinity

$$\Rightarrow (n_v+1) - n_e + n_f = 2$$

Degree at least 3.

Each vtex $\Rightarrow$ 3 edges but counted twice.

$$2n_e \geq 3(n_v+1)$$

$$n_e \geq \tfrac{3}{2}(n_v+1)$$

$$(n_v+1) - \tfrac{3}{2}(n_v+1) + n \geq 2$$

(one site per face.)

$$-\tfrac{1}{2}n_v - \tfrac{1}{2} + n \geq 2$$

$$-n_v - 1 + 2n \geq 4$$

$$\boxed{2n - 5 \geq n_v}$$

Similar argument

$$\boxed{3n - 6 \geq n_e}$$

Dual – (verts ↔ faces)

Each vertex has degree 3 in dual

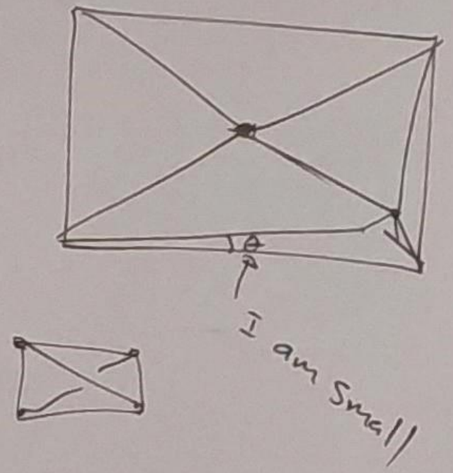
every face is a triangle

Get Delaunay triangulation

(Best triangulation)

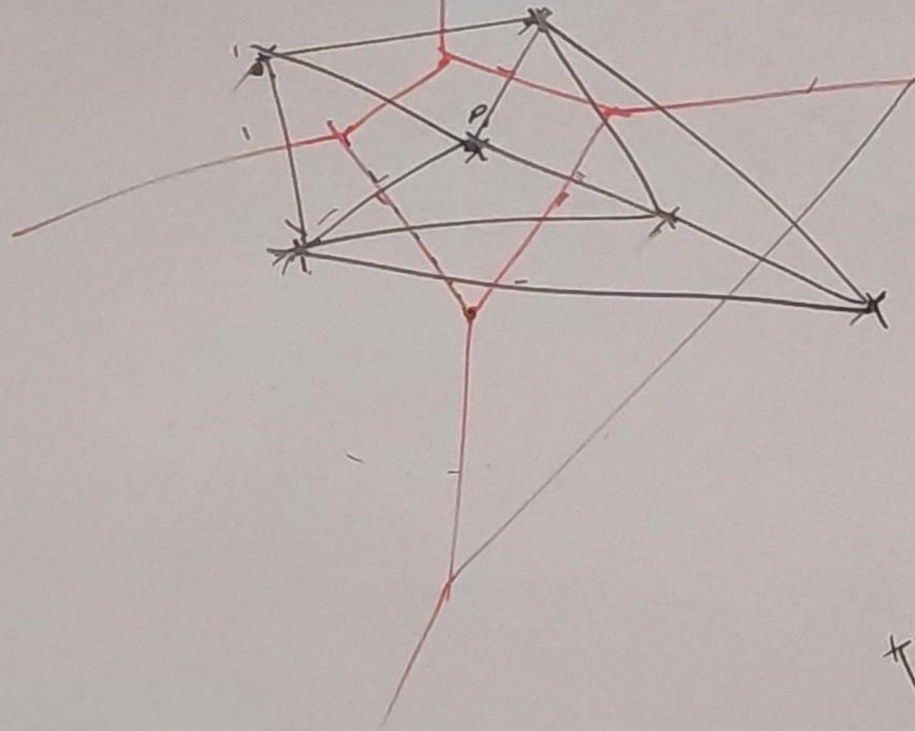
maximizes minimum angle

reduce Slivers



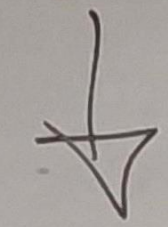
Voronoi Diagrams

(post office problem)

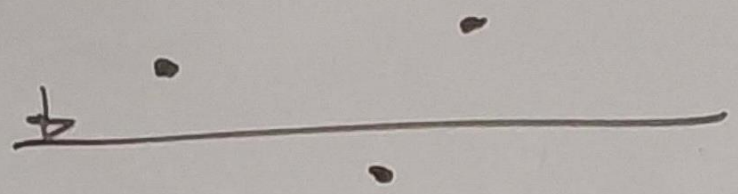


Fortune's Algo - $O(n \log n)$

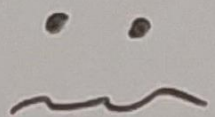
Sweep line



Problem



points below
affect what happens
above



Add another line beach line
above the sweep.