Question: $G=(V,E)$ (n verts, m edges)

If G is a planar graph, how many edges can it have (at most)?

- planar = a graph that can be embedded into $\mathbb{R}^2$.
- plane graph = an embedding of a (planar) graph into $\mathbb{R}^2$

def: $|G| \to \mathbb{R}^2$ is a homeo. onto its image

aka: G and $\mathrm{im}(G)$ is a bijection

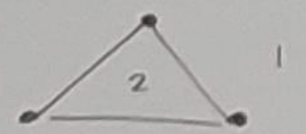
aka: no edges cross

Answer: $2n-3$

$3n$ $\boxed{3n-6}$ WHY?

$3n-5$

example:



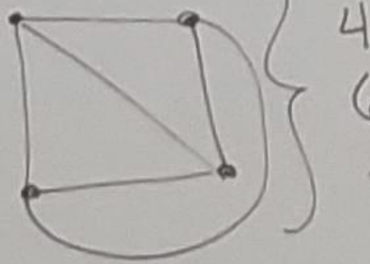
3 verts
3 edges
2 faces

$6-3=3$ ✓

$3\cdot 3 \geq 3$

but is it tight?

$3\cdot 3-5 = 4 \geq 3$



4 verts
6 edges
4 faces

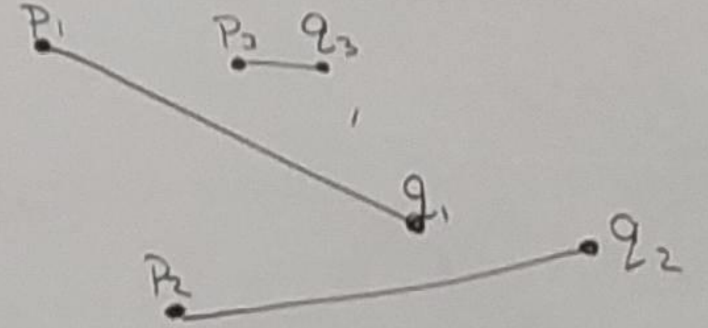
$2\cdot 4-3=8-3=5$ ✗

$3\cdot 4=12 \geq 6$

is it tight?

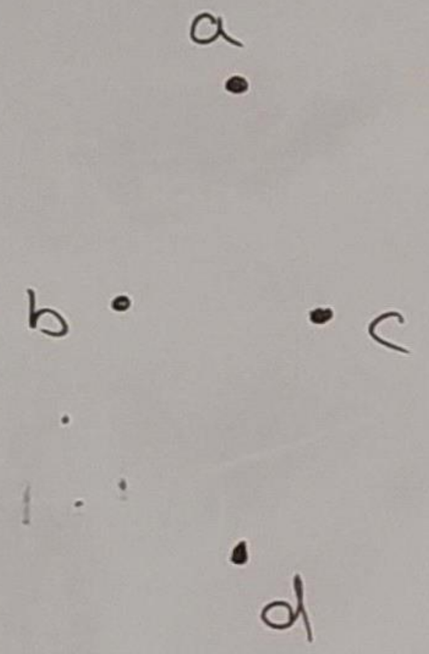
$3\cdot 4-5=7 \geq 3$

$\boxed{Q1}$ Give a trapezoidal map & search structure for:



$p_1 < p_2 < p_3 < q_3 < q_1 < q_2$

$\boxed{Q2}$ What is the Voronoi dgm for



$\|a-d\| > \|b-c\|$

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2 faces

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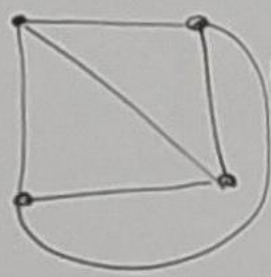
but is it tight?
$3 \cdot 3 - 5 = 4 \geq 3$

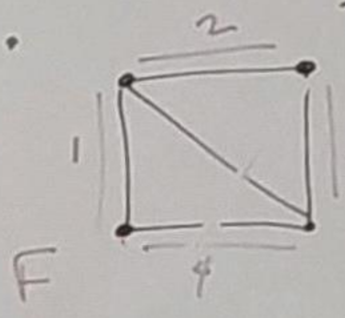


4 verts
6 edges
4 faces

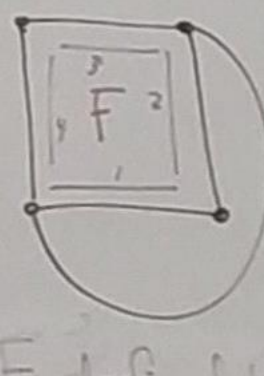
$2 \cdot 4 - 3 = 8 - 3 = 5$ ✗
$3 \cdot 4 = 12 \geq 6$
is it tight?
$3 \cdot 4 - 5 = 7 \geq 3$

Given a plane graph, we know:

$|V| - |E| + |F| = 2$ (inc the external face)



doesn't have maximal # faces



neither does this one.

∃ face F defined by 4 faces

Lemma:

Every face in a maximal plane graph has 3 edges defining it.

no edges can be added

- each face "sees" 3 edges
- each edge "sees" 2 faces

$$|E| = \frac{1}{2}\sum_{f\in F} \#\text{ edges for } f$$
$$= \frac{1}{2}\cdot(3|F|) = \frac{3}{2}|F|$$

$|V|-|E|+|F|=2$

$n - \frac{3}{2}|F| + |F| = 2$

$2n-4 = |F|(3-2)$

$|F| = 2n-4$ EVEN!!

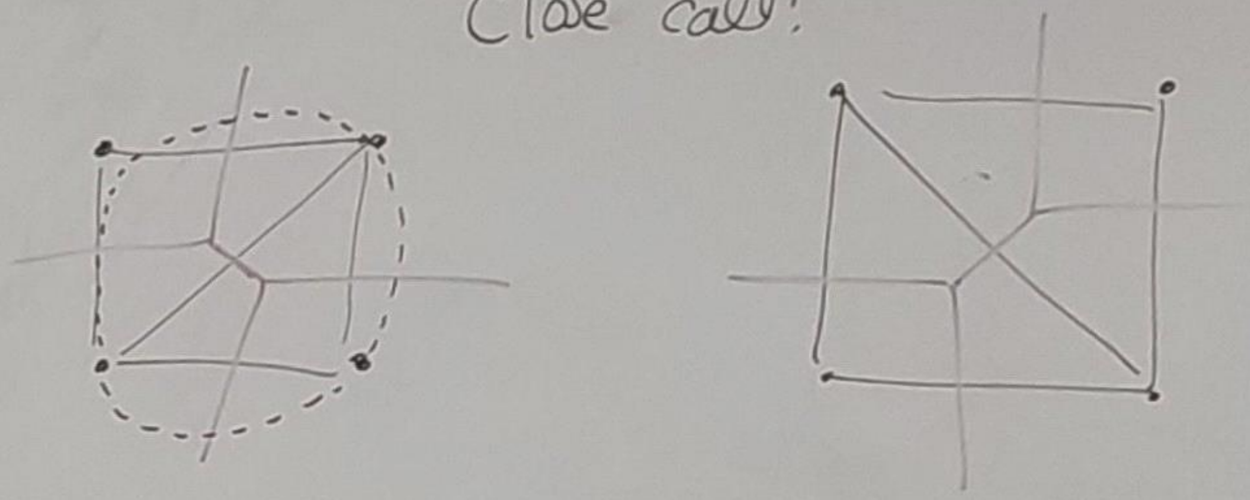
$|E| = \frac{3}{2}|F|$

$= 3n-6$

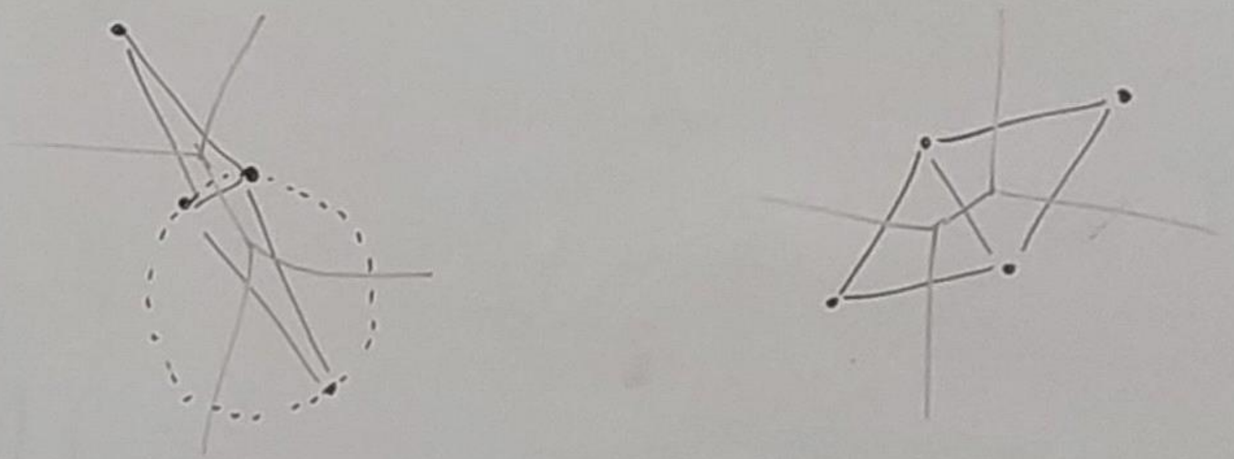
$\square$

Delaunay Triangulations

Close call:



dramatic:



Circum circle property:

the circum circle of 3 pts defining a Δ in DT has no other pts inside it.

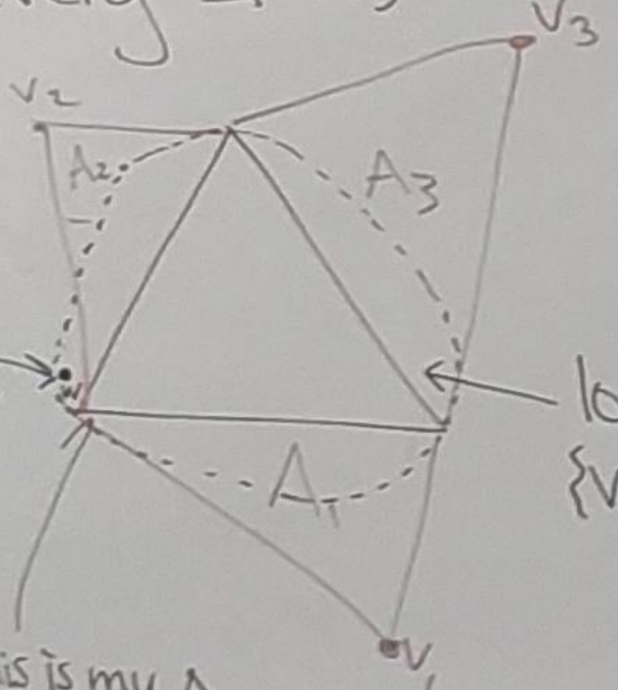
in ALL of my input point set

GLOBAL property

Local Delaunay Property:

Given a frugal triangulation T of CH(P)

we say that <u>T is locally Delaunay</u> iff ∀ every $\Delta \in T$,



could have another "distant" vertex in there

locally D. iff $\{V_1, V_2, V_3\} \cap \text{Circumcir}(\Delta) = \emptyset$

this is my Δ

has at most 3 adjacent Δs

Delaunay Thm

A triangulation is Delaunay

$\Updownarrow$

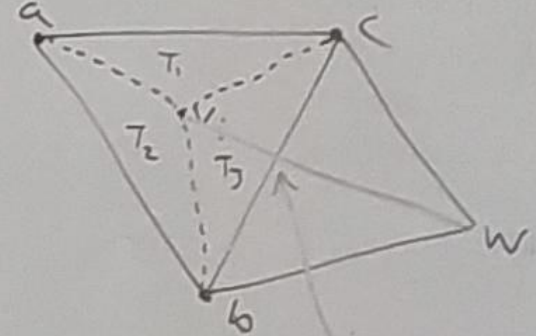
<u>it is locally Delaunay</u>

note: may say

- a triangle satisfies the local Delaunay property
- a triangulation is (locally) Delaunay

Algorithm:

- Randomly order verts, $\{v_1, v_2, \ldots, v_n\}$
- Create $\Delta\, v_1 v_2 v_3$
  (+ now $V_3 = \{v_1, v_2, v_3\}$ is triangulated
- For $i = 4 \ldots n$
  - → locate v_i in (a, b, c)
  - → add edges $v_i a$, $v_i b$, $v_i c$
  - → check edges ac, bc, ab
  (might have "cascade" on checks)



is $v_i w$ or bc the right edge locally?

In end:

} everything outside $st(v_i)$ is same as before.