Sidenote:

$g(n)$

f is $O(n)$ means..

$\exists\ c \in \mathbb{R}_{>0},\ n_0 \in \mathbb{N}$

such that

$\forall\ n \geq n_0,$

$$f(n) \leq c \cdot gn = c \cdot n$$

Claim: $c = 2$

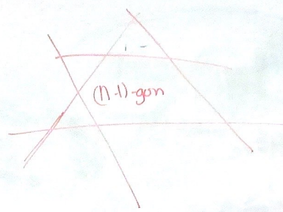
$n_0 = 1$

Arrangements: ~~R~~IC

INPUT: $\mathcal{L} = \{L_1, L_2, \ldots, L_n\}$, a set of lines

1) Solve the problem for $\mathcal{L} \setminus \{L_n\}$

2) Add L_n back in.

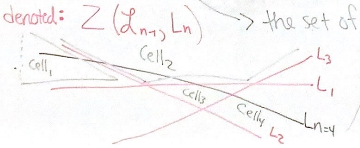


recursion: $T(n) = T(n-1) + \Theta(\text{dealing w/ } L_n)$ → We'll show this is over-counting. Complexity is $\Theta(n) \Rightarrow T(n) = \Theta(n^2)$

→ $\boxed{\text{If}}$ this is $\Theta(n^2)$, then $T(n)$ is $\Theta(n^3)$.

proportional to the "complexity of the zone" of L_n in $\mathcal{L} \setminus \{L_n\} =: \mathcal{L}_{n-1}$

denoted: $Z(\mathcal{L}_{n-1}, L_n)$ → the set of all cells intersecting L_n



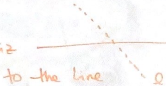
→ intersects n cells ($\mathcal{L}_{n-1}$ has $n-1$ lines)
each has the complexity $O(n-1) = O(n)$
$\therefore \boxed{\text{WC is } O(n^2)}$

Claim: $|Z(\mathcal{L}, \ell \notin \mathcal{L})| = O(|\mathcal{L}|)$

Proof: We proceed by induction on $|\mathcal{L}|$.

Base case: $|\mathcal{L}| = 1$.

Rotate so our line is horiz



Choose any $\ell \notin \mathcal{L}$ and $\parallel$ to the line

$\exists$ two 2-cells, each w/ complexity 1
No matter choice of ℓ, goes through both ✓
$\therefore |Z| = 2 \cdot 1 = 2 \leq 2$

I.A. Let $K \geq 1$. Assume that:
For all $(\mathcal{L}, \ell \notin \mathcal{L})$ sat GP w/ $|\mathcal{L}| = k$,
$|Z(\mathcal{L}, \ell)| \leq c \cdot K = 2K$

Note: When using induction with asymptotics you MUST state what n_0 and c are. OTW, you risk them changing w/ n.

Claim: $|Z(\mathcal{L}, \ell \notin \mathcal{L})| = O(|\mathcal{L}|)$

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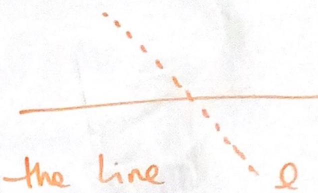


Choose any $\ell \notin \mathcal{L}$ and $\parallel$ to the line

$\exists$ two 2-cells, each w/ complexity 1.

No matter choice of ℓ, goes through both. ✓

$\therefore |Z| = 2 \cdot 1 = 2 \leq 2$

I.A. Let $K \geq 1$. Assume that: At most 1 LBS, 1 RBS,

For all $(\mathcal{L}, \ell \notin \mathcal{L})$ sat GP w/ $|\mathcal{L}| = K$,

$|Z(\mathcal{L}, \ell)| \leq c \cdot K = 2K$, at most K LBS, K RBS

Note: When using induction with asymptotics you MUST state what n_0 and c are. OTW, you risk them changing w/ n.

By I.H.

$$|Z(\mathcal{L} \setminus \{\ell_R\}, \ell)| \leq 2K. \quad (1)$$

Restricting to LBS, bound $\leq k$.

Now, let's look at

inductive case   max # LBS to add

$$|Z(\mathcal{L}, \ell)| \underset{LBS}{\leq} (k) + 1$$
$$= k+1$$

Similarly, # RBS, $\leq k+1$

$$\therefore |Z(\mathcal{L}, \ell)| \leq \max_{LBS} \# + \max_{RBS} \#$$
$$= (k+1) + (k+1)$$
$$= 2k+2 \quad \square$$

Inductive Step:

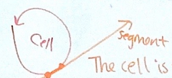
<u>WTS</u>: For all $(\mathcal{L}, \ell)$ sat GP with $|\mathcal{L}| = k+1$, $Z(\mathcal{L}, \ell) \leq 2 \cdot (k+1)$

<u>Sub proof</u>:

Let $(\mathcal{L}, \ell)$ sat GP w/ $|\mathcal{L}| = k+1$.

Let $\ell_R \in \mathcal{L}$ be the line s.t. $\ell_R \cap \ell$ is rightmost on ℓ.

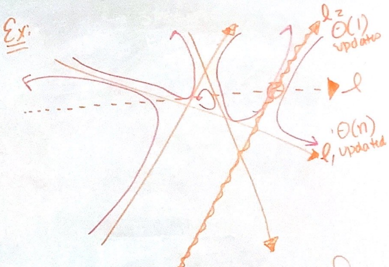
<u>Def'n</u>: right-bounding segment (LBS)
left-bounding segment (RBS)



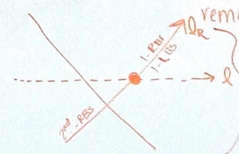
The cell is to the L of the segment, then seg is RB.

Note: Complexity $= |LBS| + |RBS|$

<u>Ex</u>:



the chosen one for removal.



Lemma: ℓ_R has at most 1 LBS.

Proof: by contradiction

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$P : (0,1) \to \mathbb{R}^2$ continuous