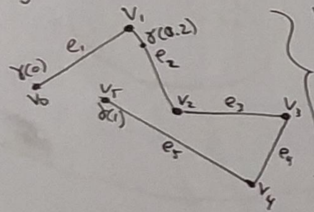
X = some "space", e.g., $\mathbb{R}^2$

path: $\gamma : [0,1] \longrightarrow X$ continuous

In $\mathbb{R}^d$, γ is a polygonal path if it is comprised of only straight line segments, connected at the endpoints



In this case, we can think of this as a sequence of verts

$[v_0, v_1, v_2, v_3, v_4, v_5]$

or a sequence of edges

$[e_1, e_2, e_3, e_4, e_5]$

$P : \underline{n} \longrightarrow \mathbb{R}^2$ } giving us the sequence of verts

$\underline{n} := \{1, 2, 3, \dots, n\}$

γ is closed if starts & ends at same place.

γ is simple if

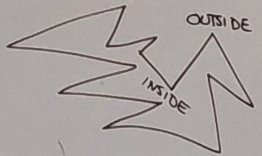
(a) edges only intersect if adjacent

(b) turning angles $\notin \{0, 180\}$ ← why bother? Consider one big edge.

↑ no backtracking

Theorem: JORDAN CURVE THEOREM

If γ is a simple closed curve in $\mathbb{R}^2$, then $\mathbb{R}^2 \setminus \gamma$ has 2 path-connected components, the "inside" and the "outside"



Manifolds

a d-manifold M is

→ (informal) looks locally like $\mathbb{R}^d$

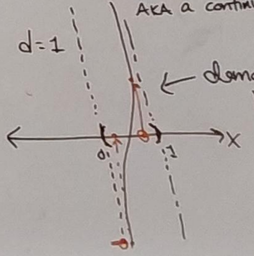
→ (formally)

$\forall x \in M, \exists U \ni x$ ← an open neighborhood containing x

such that:

$U \cong \mathbb{R}^d \cong \mathbb{B}_d$ ← open unit ball of dim d

↑ is homeomorphic to AKA a continuous bijection

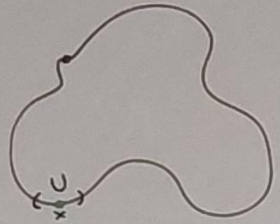


← demonstrates bijection between $(0,1)$ and $\mathbb{R}$

d=2

$\mathbb{B}_2 =$ 

a simple closed curve is a 1-fd



$\cong$ $\longleftrightarrow$ $\mathbb{B}_1$

S^2

looks like a little cap $\cong$

$\mathbb{B}_2$

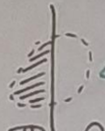
2-manifolds w/ boundary

X is a 2 mfd w/ boundary if each pt locally looks like

INTERIOR POINTS →

$\mathbb{B}_2$

or



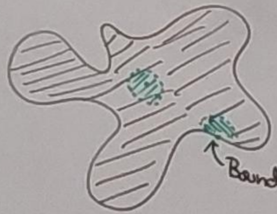
← The Boundary points

$\mathbb{B}_2 \cap H$

↑ closed half plane centered at zero

(This generalizes in dimension)

In $\mathbb{R}^2$:



M = manifold

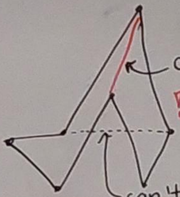
∂M = the boundary of M

$= \{x \in M \text{ s.t. } \exists\, U \ni x \text{ with } U \cong \mathbb{B}_d \cap H\}$

TRIANGULATIONS in $\mathbb{R}^2$

- M is a manifold w/ bdry in $\mathbb{R}^2$
  ∂M is a polygon
- A triangulation of M is a decomposition of M into a finite # of

pts	edges	triangles	[more general analogs, called simplices in $\mathbb{R}^d$ for $d>2$]
0-simplex	1-simplex	2-simplex	

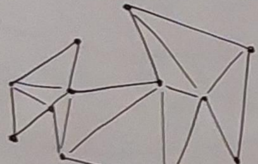


can use this edge

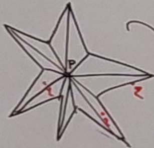
!idea! Find one, then break up my polygon into 2 smaller polygons.

can't use this edge

example:



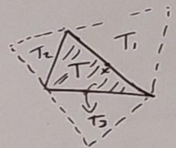
No added pts



add one pt

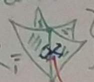
Star shaped shape S
$\exists p \in S$ such that
$\forall q \in S$, the line segment
$\overline{pq} \in S$.

T is a triangle in a 2-mfd (w/ bdry)

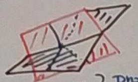


What is not a manifold?

2-complex in $\mathbb{R}^3$

$K=$  } book w/ 4 pages

no matter how small I make the neighborhood, it looks like 2 copies of $\mathbb{R}^2$ intersecting at a line

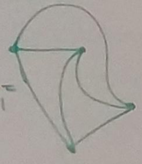


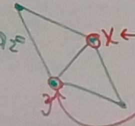
"X" $\underset{\uparrow \text{ cross}}{\times}$ I } product space

$\partial(K)=$ 

$\partial(\text{"X"}\times S^1) = 4$ copies of S^1

Graph

$G_1=$ 

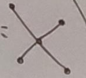
$G_2=$ 

x ← doesn't look like $\mathbb{R}^1$

y

Want: $\mathbb{R}^1$ $\longleftrightarrow$

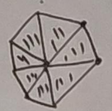
have: 

$\partial\Big(G_3 =$  $\Big) =$ 

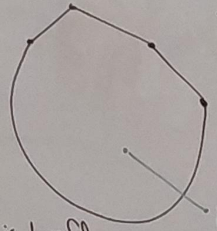
examples of stratified spaces

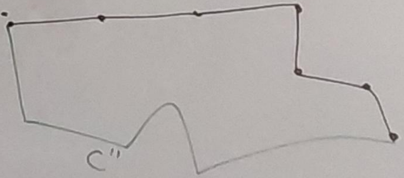
ISA manifold (w/ boundary)

$d=2$



$d=1$

$A=$ 

$B:$ 

A is 1-mfd

B is 1-mfd w/ bdry

A∪B is a 1-mfd w/ bdry

B∪C is 1-mfd

A∪B is 1-mfd w/ bdry

A∪D is not a mfd