

① Catching Polygons

② Snow Blowing

$O(n)$

after  
- each cut, run  
LCS

-  $O(n) \cdot O(n^2)$   
 $= O(n^3)$

▷ Can be done  
in  $O(n^2)$

Non-geometric version:

Longest square subsequence problem:

- Given a sequence  $S$  of length  $n$ ,  
compute a longest subsequence in the  
form of  $X^2 = XX$ , where  $X$  is a subsequence  
of  $S$ .

- EX.  $S = \text{ACTGTATCTGTCT}$   
Solution:  $\text{ACTGT} \cdot \text{ACTGT}$

Geometric version:

- Given a polygonal chain  
 $P$  with  $n$  vertices (2D or 3D)

Compute 2 subsequence

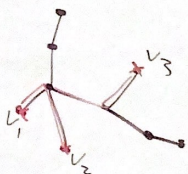
$XY$  s.t.  $X, Y$  are  
subchains of  $P$ ,  $d_{\text{F, translation or rotation}}(X, Y) \leq \epsilon$

AMS-MRC - Trees

Graph Theory

Given degree sequence

$$D = \{4, 3, 2, 2, 1, 1, 1, 1, 1\}$$



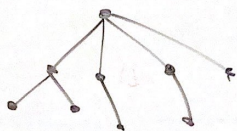
$$r=3$$

$$rsw(v_1, v_2, v_3) = 4$$

$$k=2$$

Q: which trees max. the number of vertices  $(u, v)$  with  $d(u, v) \leq k$

A: Greedy trees



$\perp$ -Stienne weine distance

Q: Give degree sequence which tree maximizes the number of  $k$ -tuples with  $s.w \leq t$

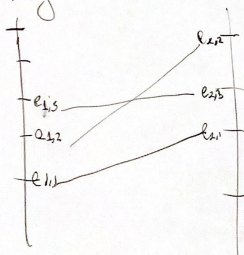
① Catching Polygons

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③ subchains

④ greedy trees

⑤ parametric graphs



## Parametric MST

Input:  $G_1 = (V, E, \varepsilon_1: E \rightarrow \mathbb{R})$ ,  $p: E \rightarrow \{0, 1\}$   
 $G_2 = (V, E, \varepsilon_2: E \rightarrow \mathbb{R})$

$$I_\lambda(G_1, G_2) := (V, E, \lambda \varepsilon_1 + (1-\lambda) \varepsilon_2)$$

Output: Sequence of intervals over  $\mathbb{R}([0, 1])$ :

$$(-\infty, t_0), (t_0, t_1), (t_1, t_2), (t_2, t_3), \dots, (t_k, \infty)$$

s.t. interval contain all MST, for  $I_\lambda(G_1, G_2)$

- Shortest paths - direct case

- Min Cut Max flow

activation of function  
is 1 if edge present

Solved in

$$O(m \log n)$$

$$O(p n^{2/3} \log^{O(1)} n)$$

where  $p$  is # of trees,  
or interval.

Q1: Which faster

Also known

$$p \text{ is } O(m^2 n)$$

$$p \text{ is } \Omega(m \log n)$$

Q3: Multiple parameters  
i.e. 3 graphs