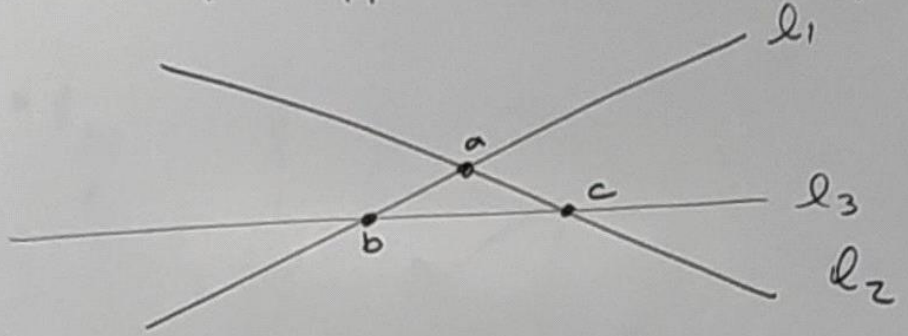
Quiz #4 !

① What is the upper + lower envelope of:



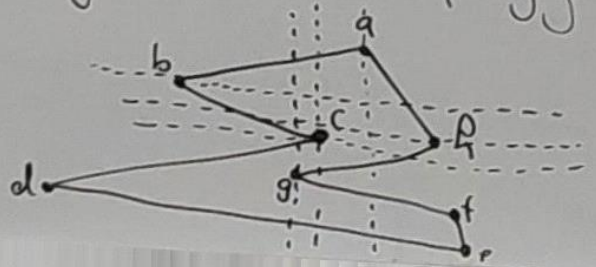
② Give an example (equations or graphs) of linear programs that

a) have a feasible, finite solution (state what it is)

b) are infeasible

c) are unbounded

③ triangulate this polygon in 2 ways:



Trapezoid Maps, a RIC.

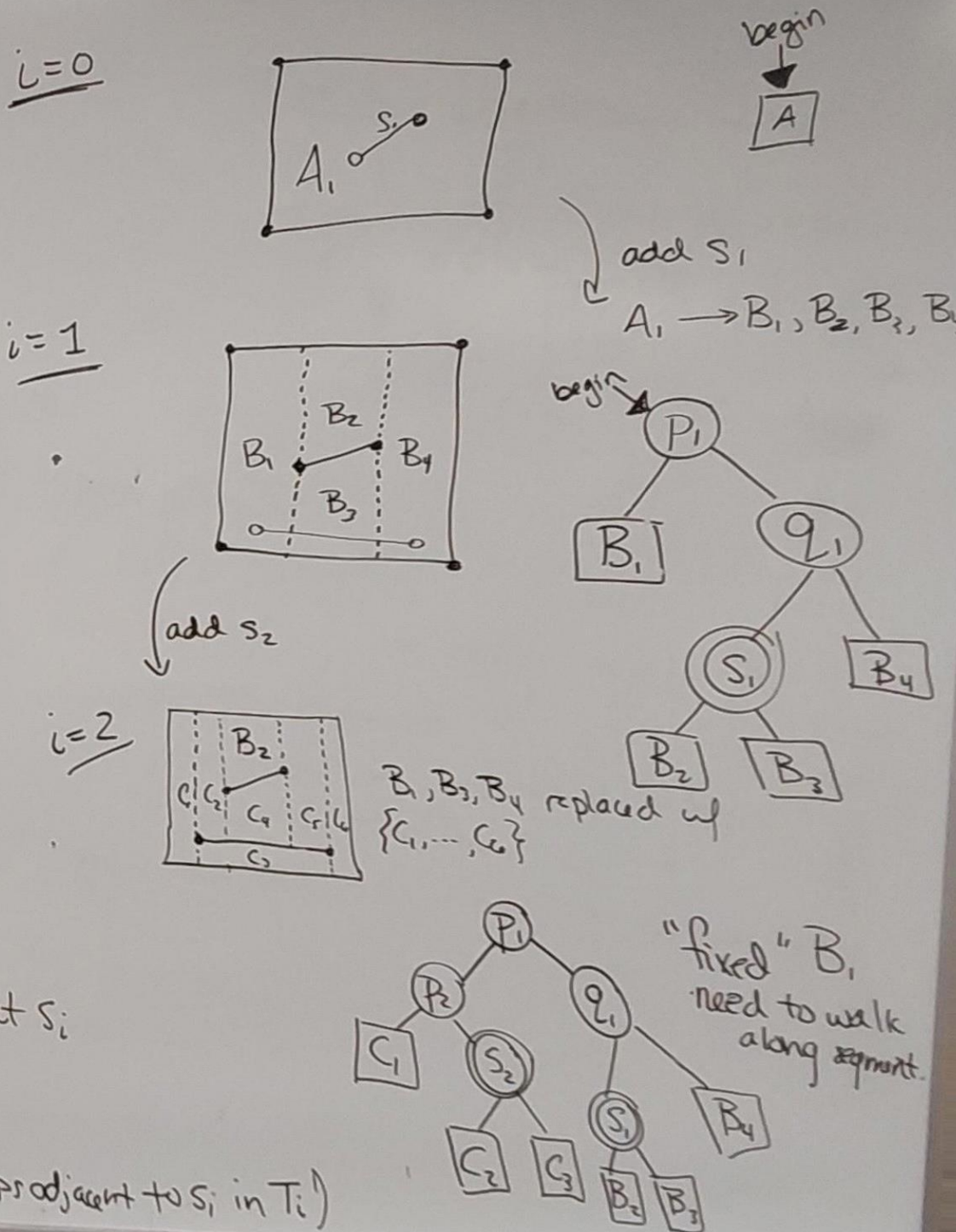
1. Randomize $S = \{s_1, s_2, \ldots, s_n\}$
2. Start w/ base case: create a bounding box
   → 1 trapezoid, 4 vertices
3. Iteratively add next segment s_i
   a) find left endpoint
   b) fix the trap. map for that cell
   *c) fix the search DS for that cell
   d) walk along segment to next trap
   (repeat (c)+(d) until endpoint reached)

Notation: $S_i := \{s_1, s_2, \ldots, s_i\}$, s_i begins at p_i, ends at s_i

$T_i :=$ the trap. map for S_i

$D_i :=$ the DS. for searching in T_i

$k_i :=$ # of traps in T_{i-1} that intersect $s_i = \Theta$(traps adjacent to s_i in T_i)



Cont. of $i=2$

"fix B_3"

Was:

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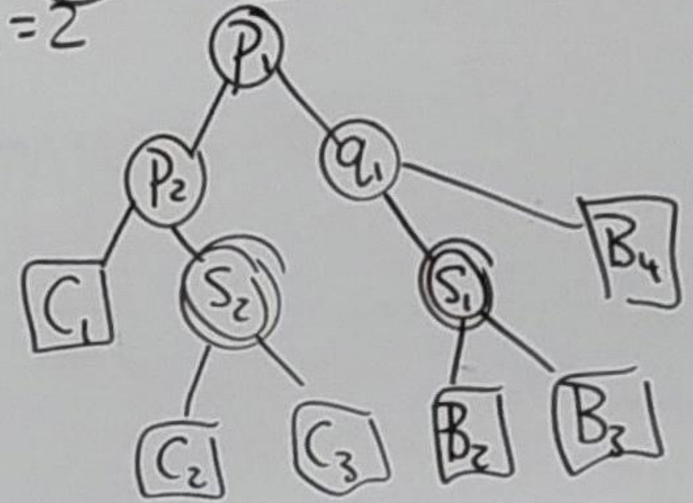
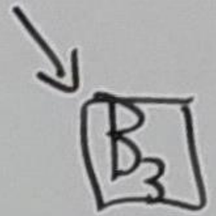
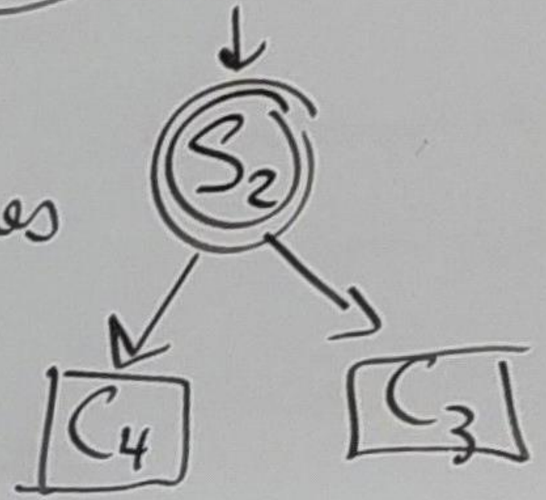
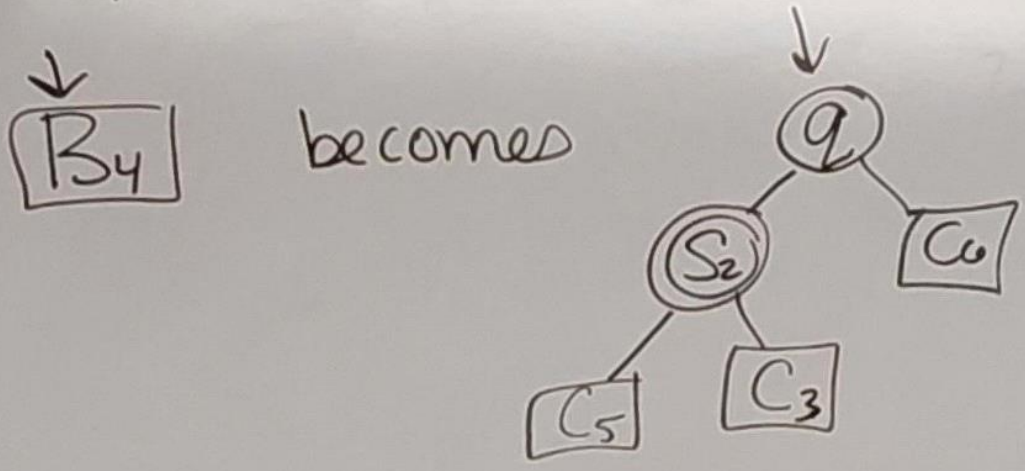




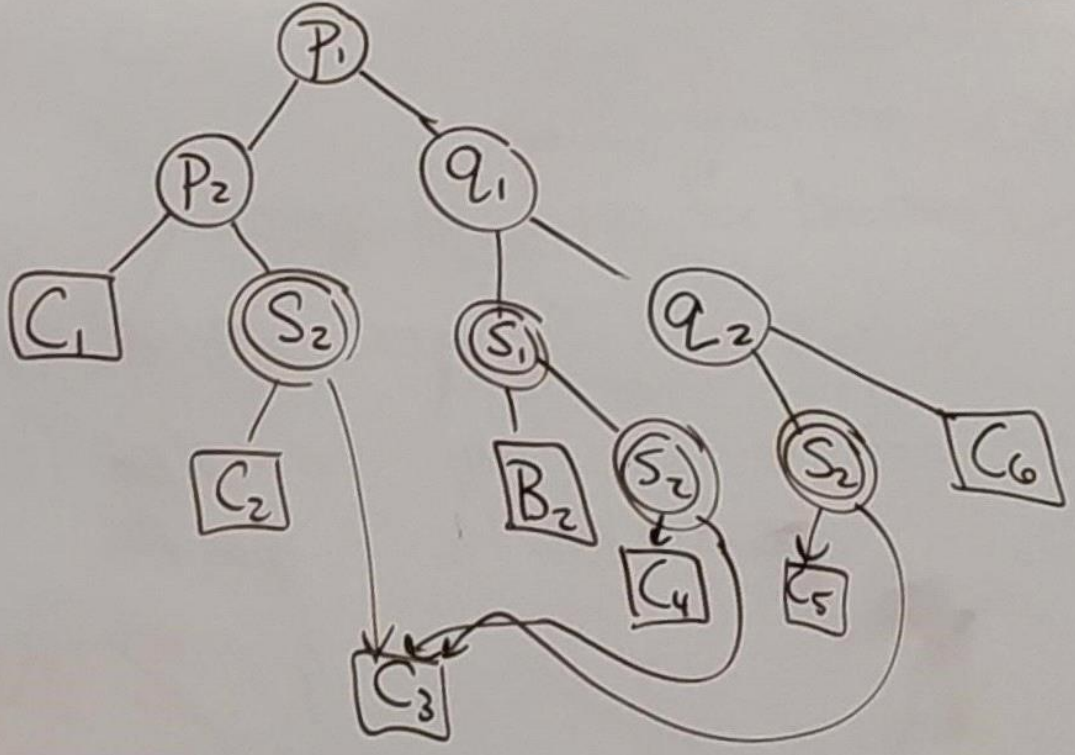
becomes



"fix B_4"

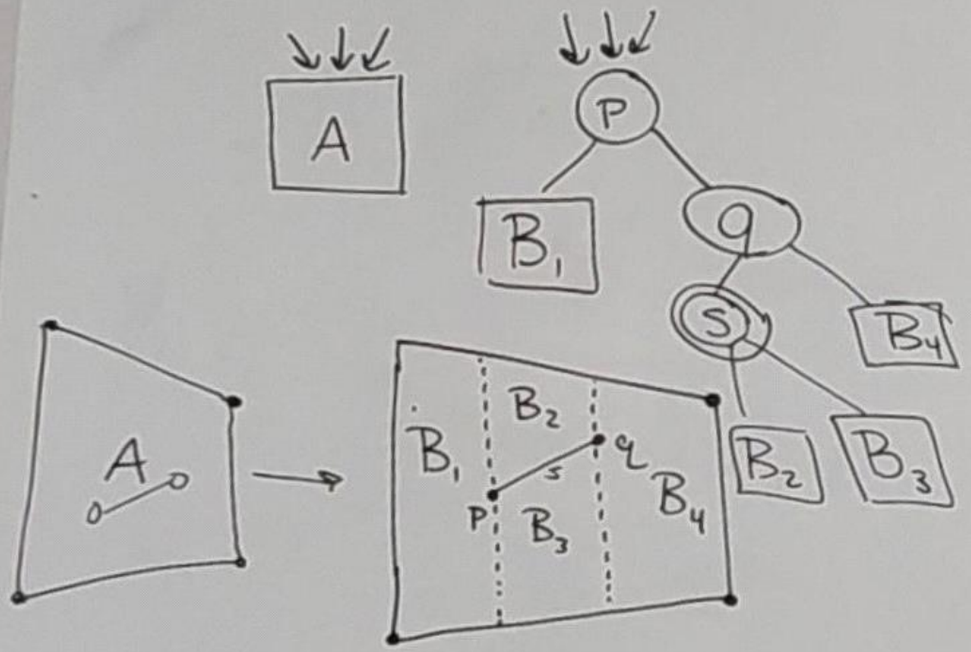


Result after all fixes for adding S_2:



Updating the search DS: ⟶ a DAG

Case 1: both endpoints are in the same trap.



→ leaves ⟺ traps
↳ nothing points out

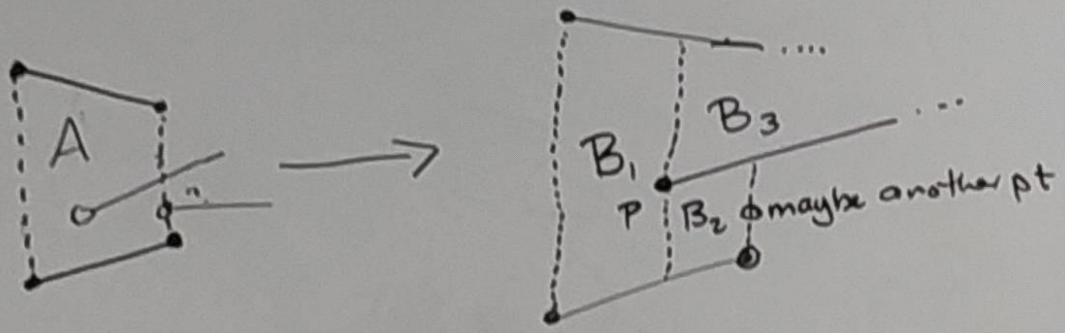
→ internal nodes are of 2 types
(i) left/right of an endpoint
(ii) above/below a segment
(only do this if in the same vertical space!)

Observation:

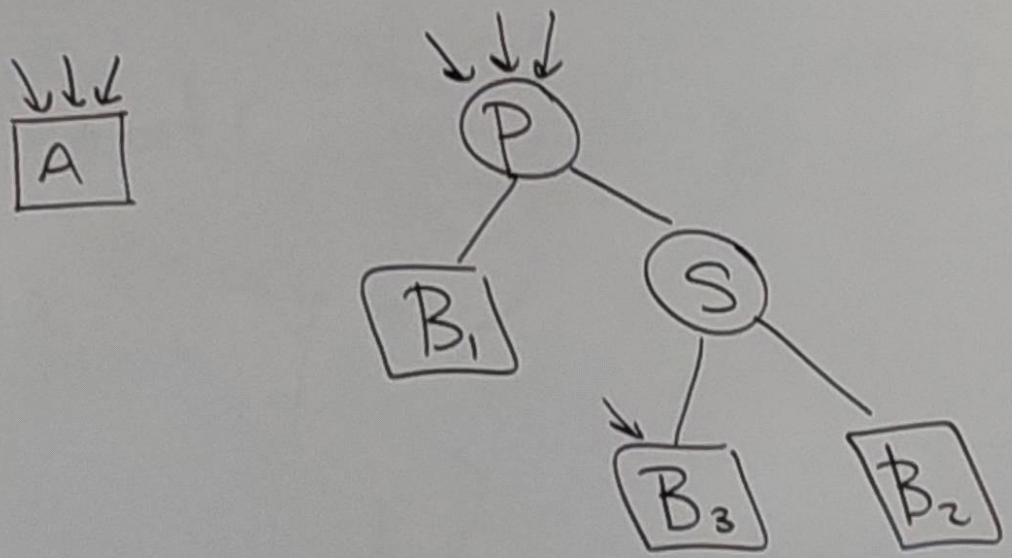
Time to add S_i is $\Theta(k_i)$.
"remove k_i traps, then add k_i+3 traps"
plus, updating the DS.
worst: $k_i = \Theta(n)$
"best": $k_i = \Theta(1)$
$\mathbb{E}(k_i) = \Theta(1)$ ⟵ super!

note: cap. letters rep. trapezoids, □ is leaf, ○ is L/R internal node, ◎ above/below internal node

Case 2: left endpoint in the current trap (but not the right one)



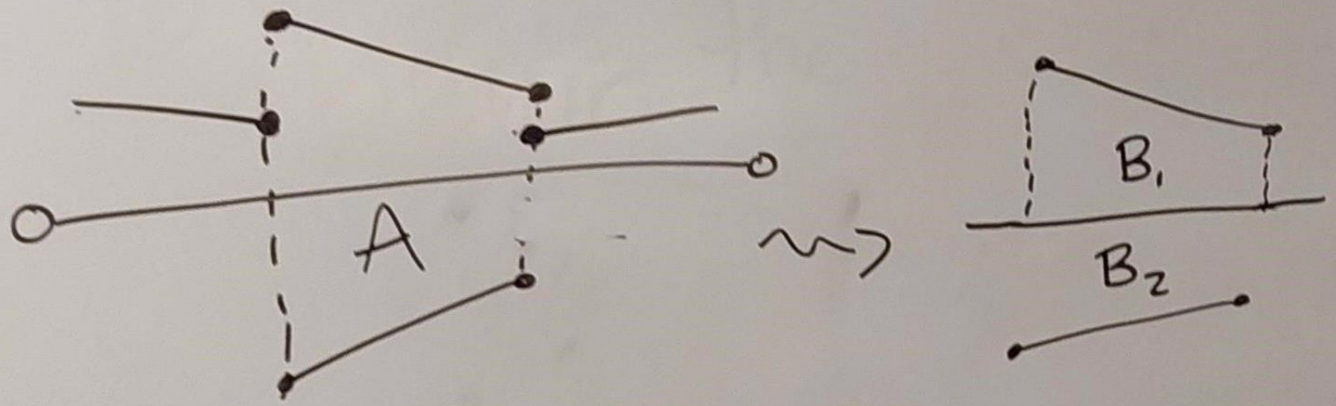
(if right vert. wall came from below. Symmetric case if from above)

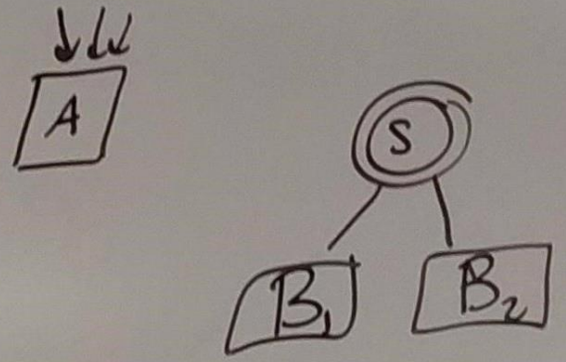


Case 4: trap contains right endpoint, but not left endpt.

Symmetric to case 2.

Case 3: "through trap"





note: left/right walls defined by segment endpoints. We know (or can check) if they are above/below the cur. segment, So 4 total outcomes. All symmetric.