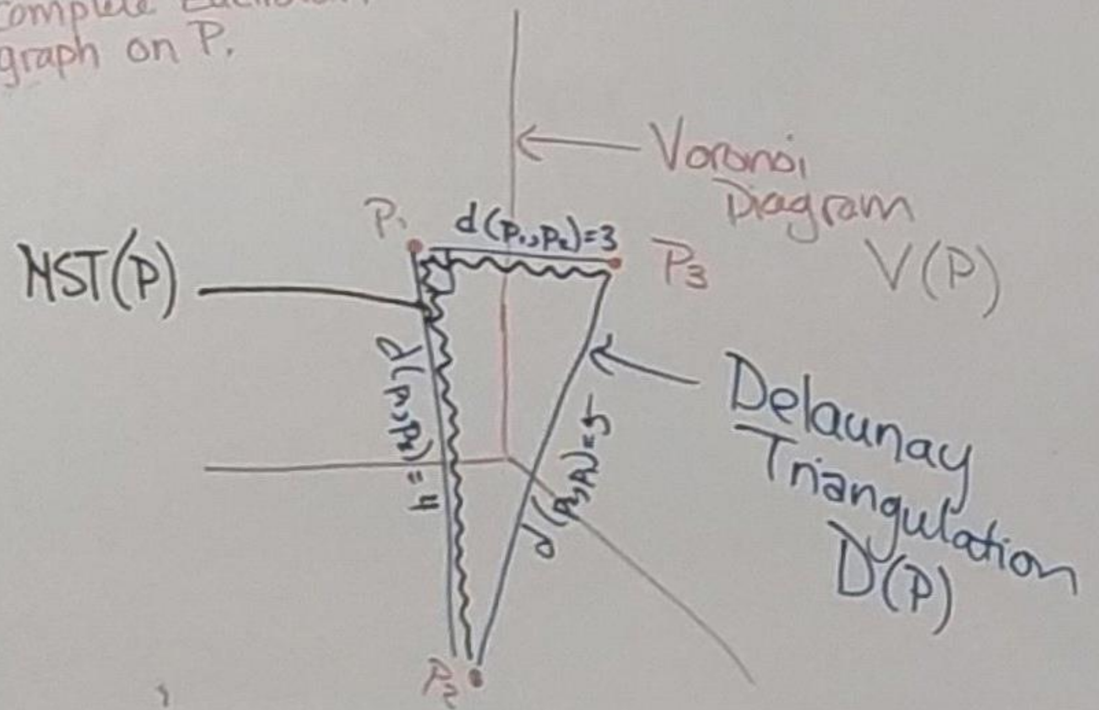
Assorted thoughts on DT

Let $P \subseteq \mathbb{R}^2$, $|P| < \infty$.

$\boxed{\text{Lemma}}$

① $\underbrace{MST(P)}_{\text{the MST on the complete Euclidean graph on } P}$ is a subgraph of $D(P)$.



naive approach to computing MST(P)

(1) compute complete graph $\rightarrow$ this graph has $\binom{n}{2}$ edges

$\Downarrow$ $\Theta(n^2)$ computation

(2) Use Kruskall

$\hookrightarrow$ a graph w/ k edges ($k \geq$ # verts)

Kruskall takes $\Theta(k \log k)$

↑ * remember this *

$\Rightarrow k = n^2$, finding MST(P) takes $\Theta(n^2 \log n)$ time

$\Theta(n^2) + \Theta(n^2 \log n) = \Theta(n^2 \log n)$ time

Alt. Approach (using $\boxed{\text{Lemma}}$) * you need to know this!!

(1) compute D(P): has $\Theta(n)$ edges, in $\Theta(n \log n)$ time

(2) compute MST of D(P) using Kruskall $\Theta(n \log n)$

$\Theta(n \log n) + \Theta(n \log n) = \Theta(n \log n)$

MST Excision Theorem:

If G, H are graphs $\boxed{\text{w/ same vertex set}}$ such that $H \subseteq G$ and $MST(G) \subseteq H$, then

$MST(G) = MST(H)$.

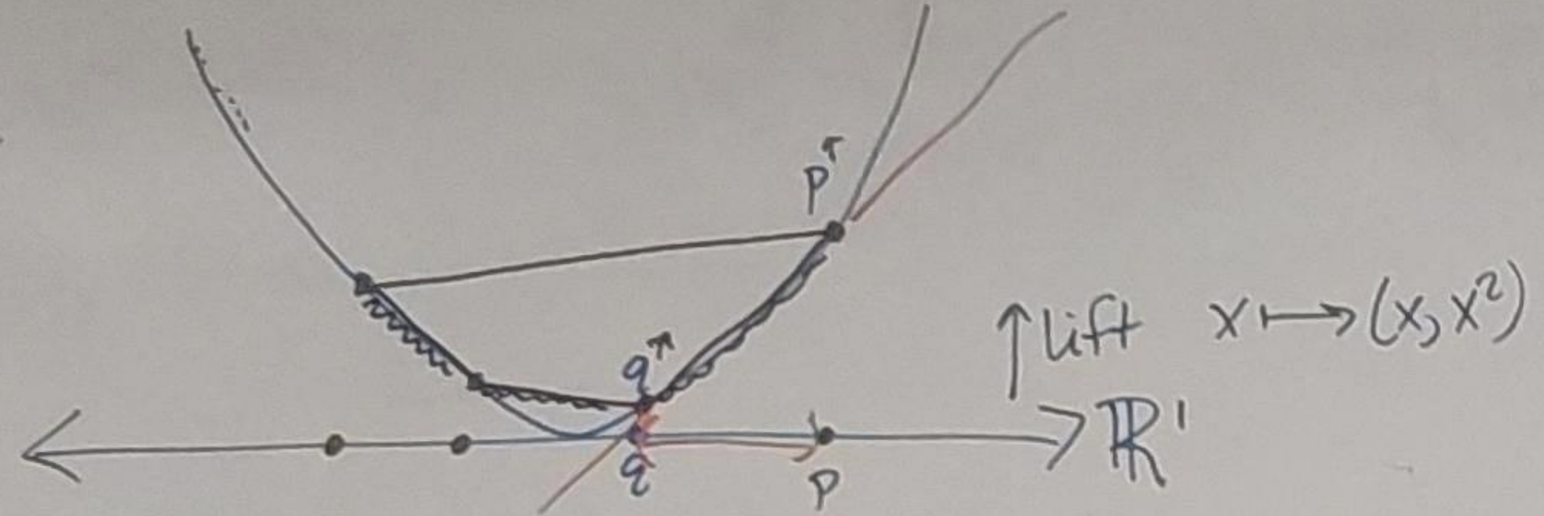
is this necessary?

In our setting:

$H = DT(P)$

$G =$ complete Euclidean graph on P.

$\boxed{\mathbb{R}^1}$



the lower hull of $P^\uparrow$ projects onto the DT(P).

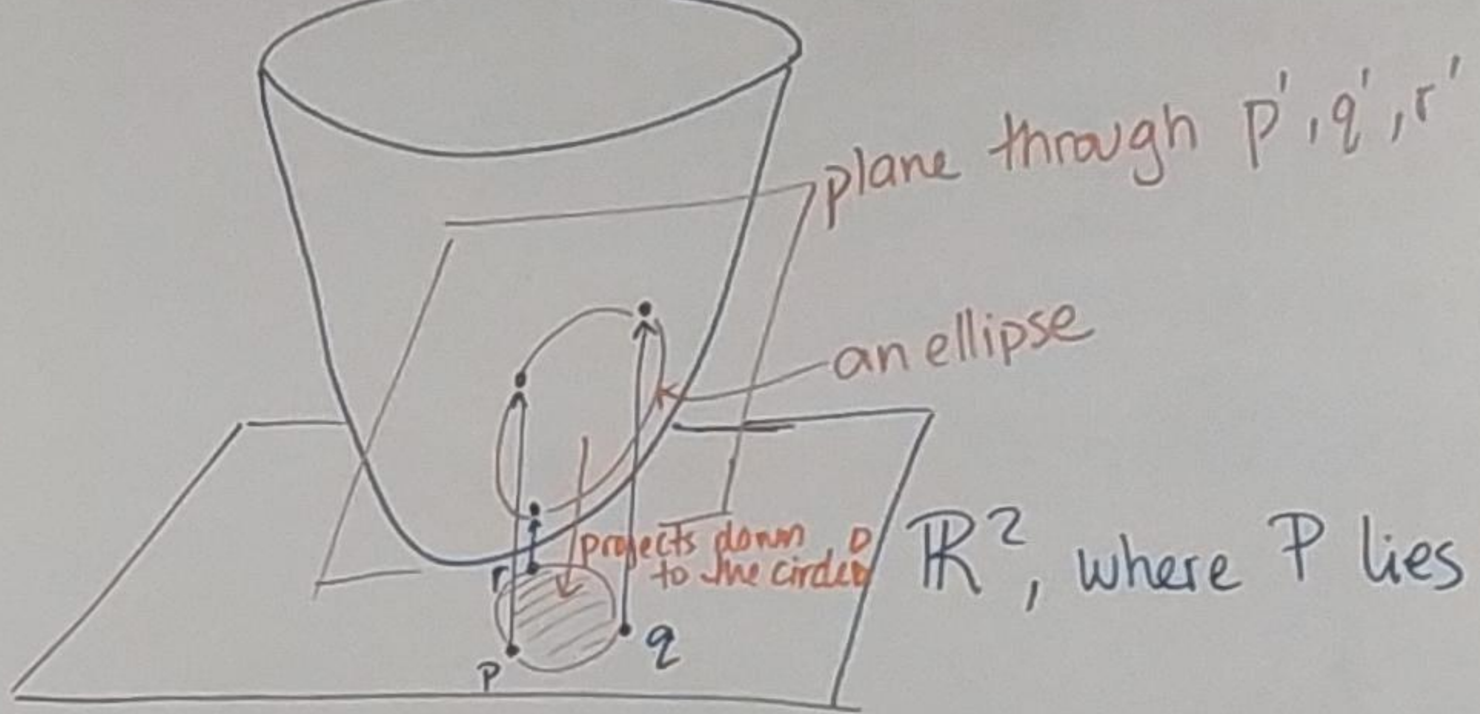
$\Downarrow$

$(q,p) \in DT(P) \iff$ the circle defined by $q + p$ (interval btwn them) is empty

$\Updownarrow$

$(q',p') \in LCH(P^\uparrow) \iff$ the line $q'p'$ is a support line / all of P' to one side of it

$\boxed{\mathbb{R}^2}$



1D		2D
1-d ball (an interval)	$\longleftrightarrow$	2d ball (inside of a circle)
line	$\longleftrightarrow$	plane

G = a positive weighted graph
$H \subseteq G$. Let $t \in \mathbb{R}$.
Then, we call H
a t-spanner of G iff
$\forall a,b \in vert(G)$,

$$\delta_G(a,b) \underset{\uparrow}{\leq} \delta_H(a,b) \leq t \cdot \delta_G(a,b)$$

easy since $H \subseteq G$

where

$$\delta_H(a,b) := \min_{\substack{\text{paths } p \\ \text{from } a \text{ to} \\ b \text{ in } H}} \text{length}(p)$$

Delaunay Stretch Factor:

G = the complete Euclidean graph on P

$H = DT(P)$, one skeleton of

$\boxed{\text{Thm}}$ Kiel & Gutwin, '92

$DT(P)$ is a 2.418-spanner of G, def. above

$\boxed{\text{Lemma}}$ If $t < 1.5846$,
then $D(P)$ is not always a t-spanner of G.

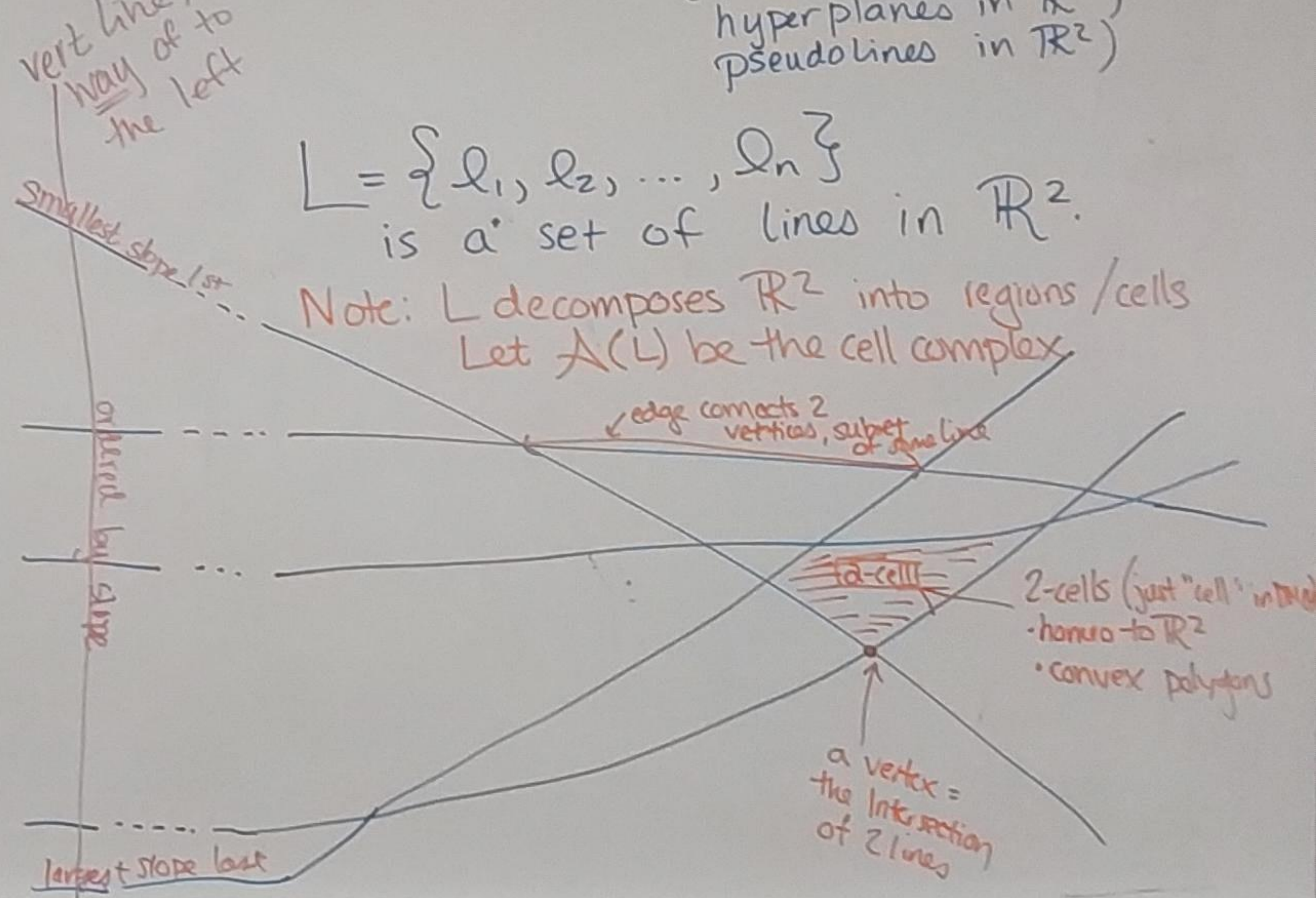
OpenQ: What is min t s.t.
$D(P)$ is a t-spanner of G?

Arrangements of Lines in $\mathbb{R}^2$

(planes in $\mathbb{R}^3$,
hyperplanes in $\mathbb{R}^d$,
pseudolines in $\mathbb{R}^2$)

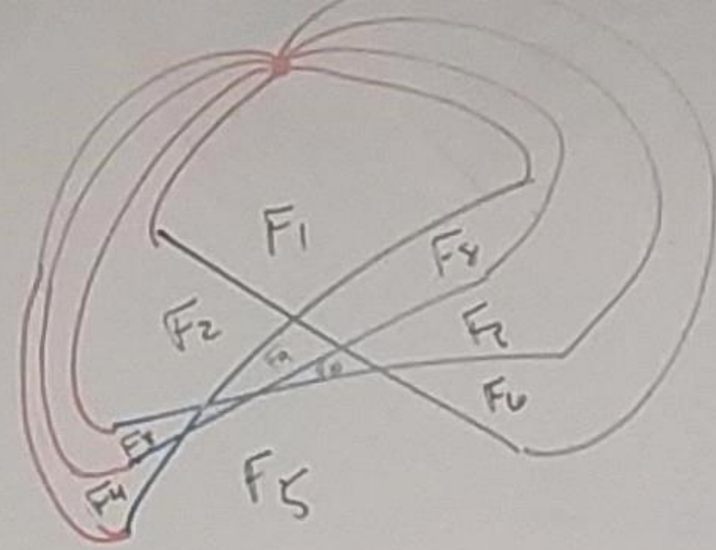
$L = \{\ell_1, \ell_2, \ldots, \ell_n\}$
is a set of lines in $\mathbb{R}^2$.

Note: L decomposes $\mathbb{R}^2$ into regions/cells
Let $\mathcal{A}(L)$ be the cell complex



L is in general position means (today)

- no 2 lines parallel
- no 3 lines meet at a single pt.



Lemma: L is a finite set of lines in $\mathbb{R}^2$

(i) $A(L)$ has $\underline{\binom{n}{2}}$ vertices
+ 1 at infinity

(ii) $A(L)$ has $\underline{n^2}$ edges

(iii) $A(L)$ has $\underline{\frac{n^2+n+2}{2}}$ 2-cells

Note: for us to call this a cell complex, we need to add a pt at infty.

(i) Every pair of lines intersect exactly once.
$\therefore \exists \binom{n}{2} = \frac{n(n-1)}{2} = \frac{1}{2}(n^2-n)$ vertices

(ii) Each line has $n-1$ verts $\Rightarrow$ n edges
$\exists$ n lines $\Rightarrow n^2$ edges

Can make another argument w/ $\binom{n}{2}$ vert of deg 4, and 1 (inf) w/ deg $2n$
Try it!

(iii) $F+V-E=2$

$F = 2 + (E=n^2) - (V=\frac{1}{2}(n^2-n)+1)$

$F = 2 + n^2 - \frac{1}{2}n^2 + \frac{1}{2}n - 1$

$= 2 + \frac{n^2+n}{2} - 1$

$= 1 + \frac{n^2+n}{2} = \frac{n^2+n+2}{2}$

$\square$