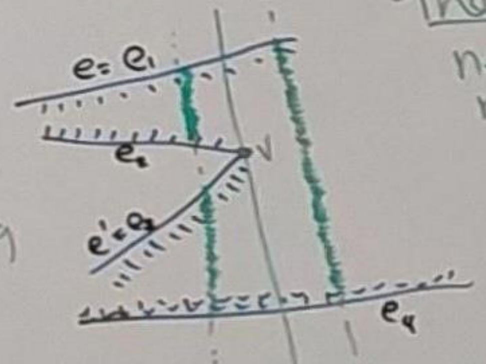
# Polygon Triangulation

Step 1: Subdivide into monotone polygons.
Step 2: Triangulate the monotone polygons

A plane sweep Events:

1) Merge




2 vertical segments combine as tracking $P \cap \ell$.

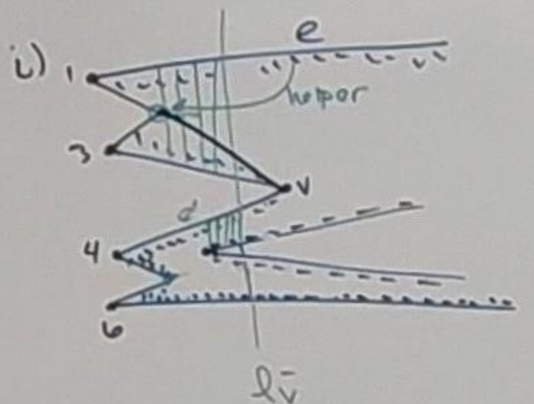
The event: $v_i$
next vertex to right of sweepline.

The sweepline DS:
Keeps track of edges intersecting $\ell$.
Note: only really need "upper edges $\frac{\text{outside poly}}{\text{poly inside}}$"

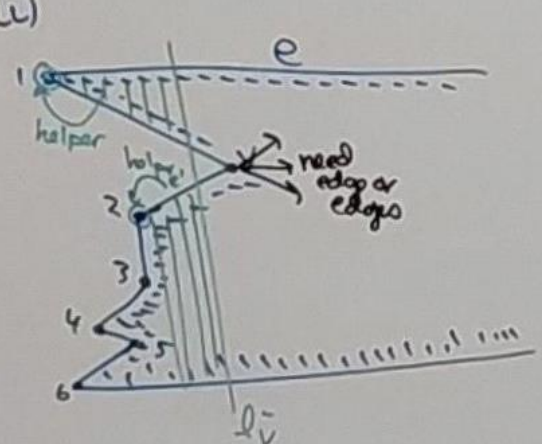
a. remove $e_3$ (and $e_2$) from the sweepline.
b. fixup$(v, e_1)$ and fixup$(v, e_3)$
c. set helper of e to v

Cases:

i)




ii)

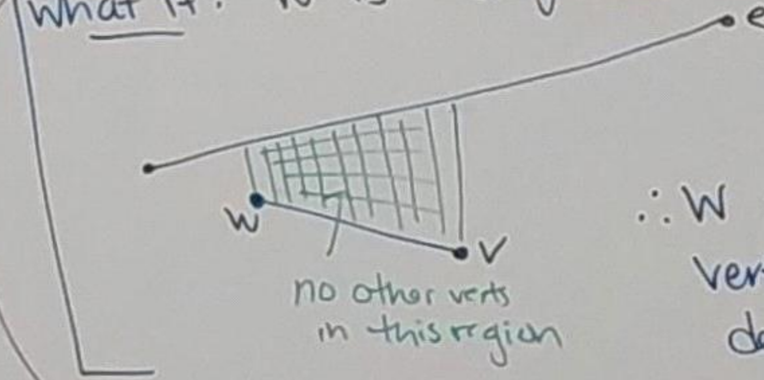



(more cases)

2) fix-up$(v, e)$: if helper(e) is a merge vertex, add diagonal between v and helper(e)

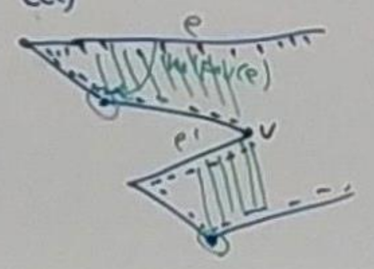
W = helper(v)

What if: w is adjacent to v?




∴ W can't be a Merge vertex. Proof: contradiction + def'n of merge vertex.

iii)




Cases: 1) w is adjacent
2) w is a merge vertex
3) w is a split vertex
4) w is none of the above

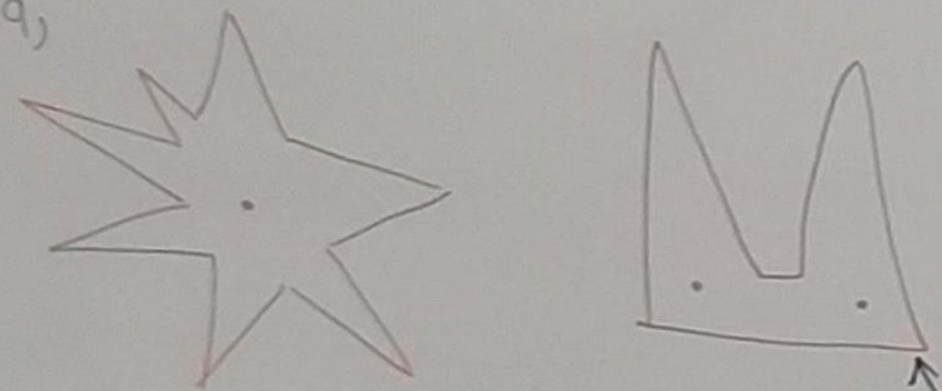
not adj.

# 2D Polygonal Guarding

Given: A polygon in $\mathbb{R}^2$
(think: aerial view of a museum)

Want: Find a minimal # of points (guards) that can "see" the whole polygon.

e.g.,

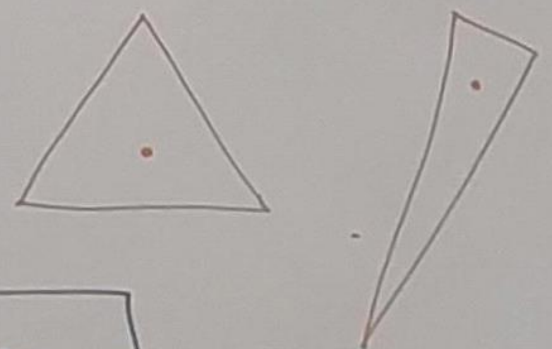


Rishi's Conjecture: If all verts are co-circular, then it is 1-guardable.

Exercise:
- → try for small $n$ = # of edges & make conjecture.
- → try to see if conjecture is tight.

$\boxed{n=3}$

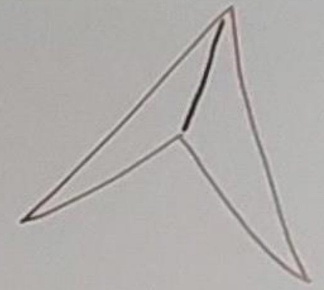
only one "combinatorial" polygon:



$\boxed{\text{Claim: } \lfloor \frac{n}{3} \rfloor \text{ suffices!}}$

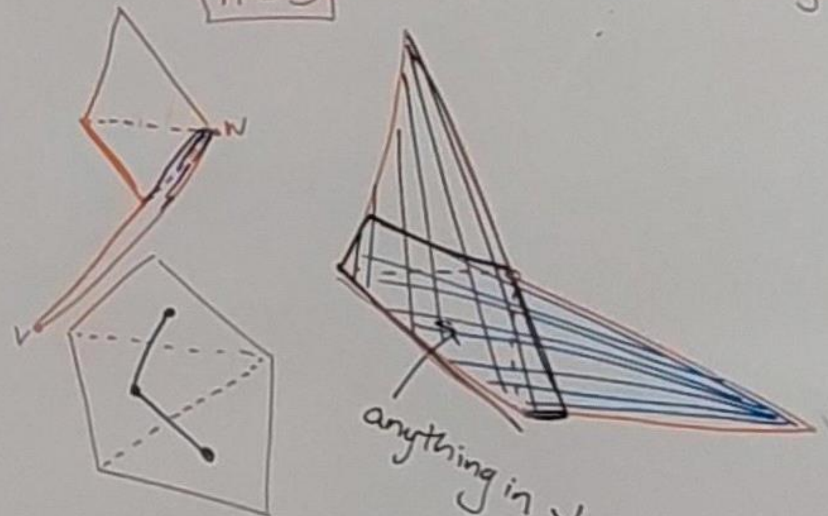
generalizing this shows the claim is tight.

$\boxed{n=4}$



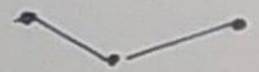
Frugal $\Delta$ulation of 4-gon has 2 triangles
n-gon n-2

$\boxed{n=5}$ " 5-gon " 3 triangles



anything in there will do

Dual graph:



By a counting argument:
1 $\Delta$ w/ 2 neighbors &
2 w/ 1 neighbor.

C = cellular partition of space X
(triangulation of polygon)
(verts/edges/faces of plane graph)

$C_0$ = the zero-dim cells (vertices)
$C_1$ = the 1-dim cells (edges)
$C_2$ = the 2-dim cells
⋮

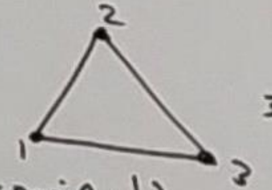
$\bigcup_{i=0}^{k} C_i$ ← the k-skeleton

The one-skeleton is a graph!

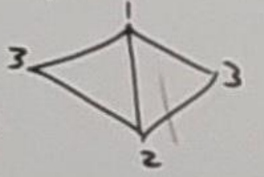
Claim: The 1-skeleton of a frugal triangulation of a polygon in $\mathbb{R}^2$ is 3-colorable.

Proof:
Use induction on # of edges.

Base: $n=3$



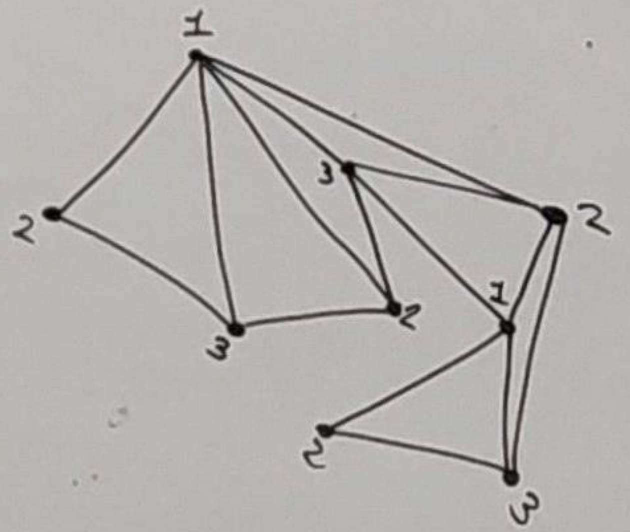

$n=4$




(only graph to consider!)

I.A. Let $\boxed{k \geq 4}$.
If G is the 1-skeleton of a frugal Δulation of an n-polygon, then G is 3-colorable.

Def: $G=(V,E)$ is k-colorable iff $\exists$ a labeling of V by k colors $\{c_1, \dots, c_k\}$ such that no adjacent verts have the same color.



Let $P$ be an $(k+1)$ polygon.
Fix some frugal $\Delta$ulation.
By prev. lemma, $\exists$ an ear.
"pluck it off" and we have
a frugal $\Delta$ulation of a $k$-polygon.
By I.A., it has a 3-coloring.
Choose one. Add the ear back in.
The vertex added has 2 neighbors. Choose the
3rd color. $\square$

Alg:

Find a 3-coloring.
One color has $\lfloor \frac{n}{3} \rfloor$ verts
(otw - counting issues). Use
the vertices labeled by
that color as the guards.