

Third order equations from pages 46 and 48, and derived 4th order equations, of  
[www.autodiff.org/Docs/euroad/17thEuroAdWorkshop-MuWang-HigherOrderReverseModeWithMPI.pdf](http://www.autodiff.org/Docs/euroad/17thEuroAdWorkshop-MuWang-HigherOrderReverseModeWithMPI.pdf)  
ZTA, July 2016

# 1 Equations

Given that

$$\frac{\hat{\partial} f_i}{\hat{\partial} v_c} = \frac{\partial f_{i+1}}{\partial v_c} + \left( \frac{\partial \phi_i}{\partial v_c} * \frac{\partial f_{i+1}}{\partial v_i} \right) \quad (1)$$

Then

$$\begin{aligned} \frac{\hat{\partial}}{\hat{\partial} v_b} \left[ \frac{\hat{\partial} f_i}{\hat{\partial} v_c} \right] &= \frac{\partial^2 f_{i+1}}{\partial v_b \partial v_c} + \left( \frac{\partial \phi_i}{\partial v_b} * \frac{\partial^2 f_{i+1}}{\partial v_i \partial v_c} \right) \\ &+ \left( \frac{\partial^2 \phi_i}{\partial v_b \partial v_c} * \frac{\partial f_{i+1}}{\partial v_i} \right) \\ &+ \left( \frac{\partial \phi_i}{\partial v_c} * \frac{\partial^2 f_{i+1}}{\partial v_b \partial v_i} \right) + \left( \frac{\partial \phi_i}{\partial v_c} * \frac{\partial \phi_i}{\partial v_b} * \frac{\partial^2 f_{i+1}}{\partial v_i \partial v_i} \right) \end{aligned} \quad (2)$$

It then follows that

$$\begin{aligned} \frac{\hat{\partial}}{\hat{\partial} v_a} \left[ \frac{\hat{\partial}}{\hat{\partial} v_b} \left( \frac{\hat{\partial} f_i}{\hat{\partial} v_c} \right) \right] &= \frac{\partial^3 f_{i+1}}{\partial v_a \partial v_b \partial v_c} + \left( \frac{\partial^3 \phi_i}{\partial v_a \partial v_b \partial v_c} * \frac{\partial f_{i+1}}{\partial v_i} \right) + \left( \frac{\partial^2 \phi_i}{\partial v_b \partial v_c} * \frac{\partial^2 f_{i+1}}{\partial v_a \partial v_i} \right) \\ &+ \left( \frac{\partial^2 \phi_i}{\partial v_a \partial v_c} * \frac{\partial^2 f_{i+1}}{\partial v_b \partial v_i} \right) + \left( \frac{\partial \phi_i}{\partial v_c} * \frac{\partial^3 f_{i+1}}{\partial v_a \partial v_b \partial v_i} \right) \\ &+ \left( \frac{\partial^2 \phi_i}{\partial v_a \partial v_b} * \frac{\partial^2 f_{i+1}}{\partial v_i \partial v_c} \right) + \left( \frac{\partial \phi_i}{\partial v_b} * \frac{\partial^3 f_{i+1}}{\partial v_a \partial v_i \partial v_c} \right) \\ &+ \left( \frac{\partial^2 \phi_i}{\partial v_a \partial v_b} * \frac{\partial \phi_i}{\partial v_c} * \frac{\partial^2 f_{i+1}}{\partial v_i \partial v_i} \right) + \left( \frac{\partial \phi_i}{\partial v_b} * \frac{\partial^2 \phi_i}{\partial v_a \partial v_c} * \frac{\partial^2 f_{i+1}}{\partial v_i \partial v_i} \right) \\ &+ \left( \frac{\partial \phi_i}{\partial v_b} * \frac{\partial \phi_i}{\partial v_c} * \frac{\partial^3 f_{i+1}}{\partial v_a \partial v_i \partial v_i} \right) \\ &+ \frac{\partial \phi_i}{\partial v_a} * \left[ \frac{\partial^3 f_{i+1}}{\partial v_i \partial v_b \partial v_c} + \left( \frac{\partial^3 \phi_i}{\partial v_i \partial v_b \partial v_c} * \frac{\partial f_{i+1}}{\partial v_i} \right) + \left( \frac{\partial^2 \phi_i}{\partial v_b \partial v_c} * \frac{\partial^2 f_{i+1}}{\partial v_i \partial v_i} \right) \right. \\ &+ \left( \frac{\partial^2 \phi_i}{\partial v_i \partial v_c} * \frac{\partial^2 f_{i+1}}{\partial v_b \partial v_i} \right) + \left( \frac{\partial \phi_i}{\partial v_c} * \frac{\partial^3 f_{i+1}}{\partial v_i \partial v_b \partial v_i} \right) \\ &+ \left( \frac{\partial^2 \phi_i}{\partial v_i \partial v_b} * \frac{\partial^2 f_{i+1}}{\partial v_i \partial v_c} \right) + \left( \frac{\partial \phi_i}{\partial v_b} * \frac{\partial^3 f_{i+1}}{\partial v_i \partial v_i \partial v_c} \right) \\ &+ \left( \frac{\partial^2 \phi_i}{\partial v_i \partial v_b} * \frac{\partial \phi_i}{\partial v_c} * \frac{\partial^2 f_{i+1}}{\partial v_i \partial v_i} \right) + \left( \frac{\partial \phi_i}{\partial v_b} * \frac{\partial^2 \phi_i}{\partial v_i \partial v_c} * \frac{\partial^2 f_{i+1}}{\partial v_i \partial v_i} \right) \\ &\left. + \left( \frac{\partial \phi_i}{\partial v_b} * \frac{\partial \phi_i}{\partial v_c} * \frac{\partial^3 f_{i+1}}{\partial v_i \partial v_i \partial v_i} \right) \right] \end{aligned} \quad (3)$$

[illegible]

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