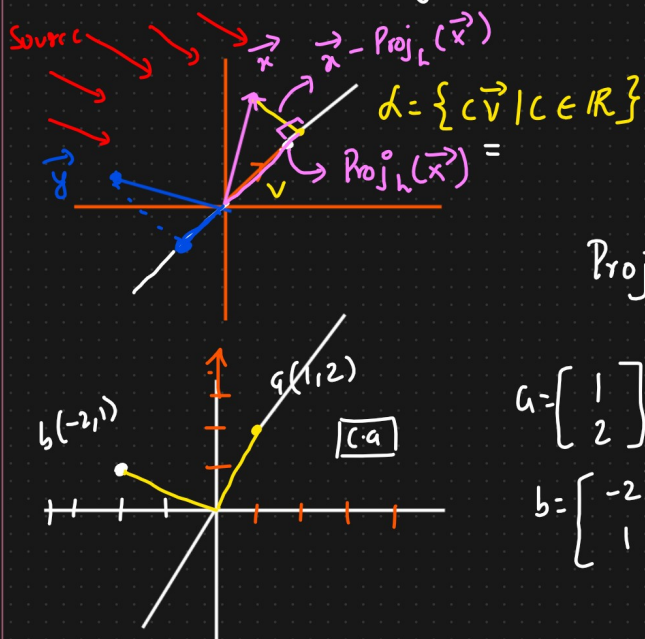


Introduction To Projections



$\text{Proj}_L(\vec{x}) \Rightarrow$ Project the \vec{x} on the line L .

$\text{Proj}_L(\vec{x}) \Rightarrow$ Some Vector in Line where $\vec{x} - \text{Proj}_L(\vec{x})$ is perpendicular to L

$$a = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$b = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$a \cdot b = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$a \cdot b = a_1 b_1 + a_2 b_2$$

$$a \cdot b = (1)(-2) + 2(1)$$

$$\boxed{a \cdot b = 0} \Rightarrow \text{Dot Product is always 0.}$$

$\text{Proj}_L(\vec{x}) \Rightarrow$ Some Vector in Line where $\vec{x} - \text{Proj}_L(\vec{x})$ is perpendicular to L

$$(\vec{x} - c\vec{v}) \cdot \vec{v} = 0$$

$$\vec{x} \cdot \vec{v} - c\vec{v} \cdot \vec{v} = 0$$

$$\vec{x} \cdot \vec{v} = c\vec{v} \cdot \vec{v}$$

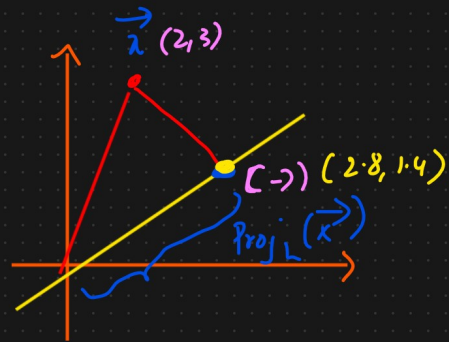
$$c = \frac{\vec{x} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}$$

$$\text{Proj}_L(\vec{x}) = c\vec{v}$$

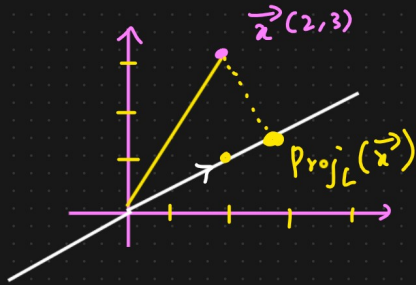
$$= \left(\frac{\vec{x} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \right) \cdot \vec{v}$$

$$\text{Proj}_b(a) = \left(\frac{a \cdot b}{b \cdot b} \right) \cdot b$$

$$\text{Proj}_L(\vec{x}) = \left(\frac{\vec{x} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \right) \cdot \vec{v}$$



$$L = \left\{ c \begin{bmatrix} 2 \\ 1 \end{bmatrix} \mid c \in \mathbb{R} \right\}, \quad \vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$



$$\text{Proj}_L(\vec{x}) = \left(\frac{\begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}}{\begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}} \right) \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= \frac{7}{5} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\text{Proj}_L(\vec{x}) = \begin{bmatrix} 14/5 \\ 7/5 \end{bmatrix} = \begin{bmatrix} 2.8 \\ 1.4 \end{bmatrix}$$