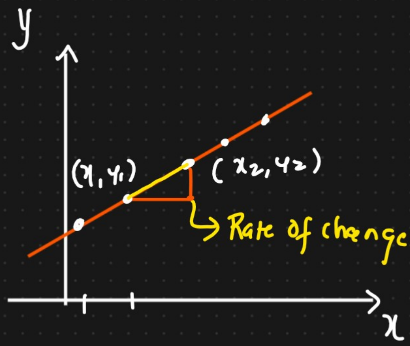
Slope → Derivative As a Concept

The **slope** of a line is a measure of how steep the line is, and it represents the rate of change of one variable with respect to another. In the context of a two-dimensional Cartesian coordinate system, the slope indicates the ratio of the vertical change (rise) to the horizontal change (run) between two points on a line.



$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{Rise}}{\text{Run}}$$

Where $y_2 - y_1$ is the vertical (rise)

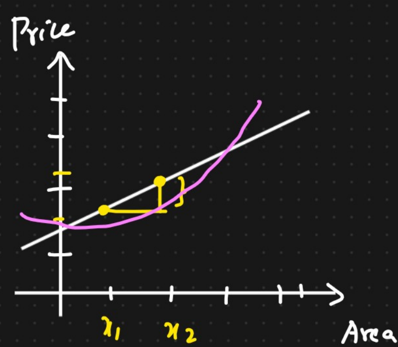
$x_2 - x_1$ is the horizontal (run)

Since the line is straight

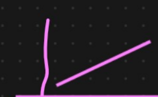
Rate of change = same = constant

Dataset

Area Price



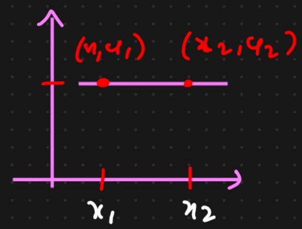
Interpretation of slope



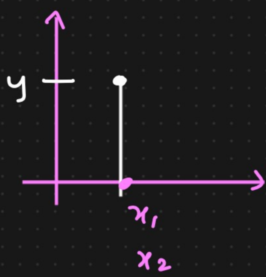
1. Positive slope : if Slope > 0 , the line rises as it moves from left to right, The larger the slope , the steeper the line.

2. Negative slope : if slope < 0 , the line falls as it moves from left to right , the more negative the slope, the steeper the line in the downward direction.

3. Zero slope : if Slope = 0 , the line is horizontal , meaning there is no vertical change as the line move from left to right.

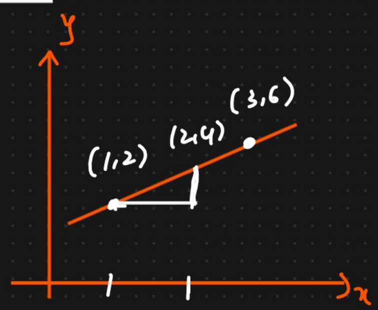


Slope = 0



④ <u>Undefined Slope</u> : If $x_2 = x_1$, the line is vertical, and the slope is undefined because you cannot divide by 0.

<u>Examples</u>

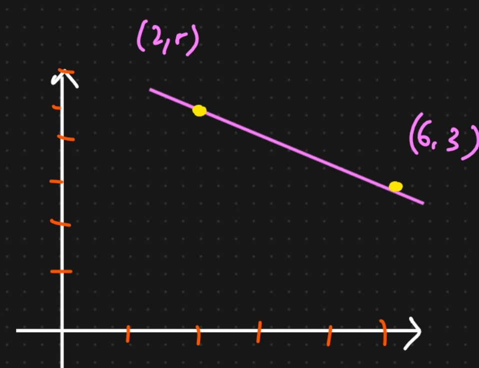


$$\text{Slope} = \frac{6-2}{3-1} = \frac{4}{2} = 2$$

This means the for every 1 unit you move horizontally from left to right the line moves <u>2 units vertically up</u>

2) <u>Negative slope</u>

(2,5) (6,3)

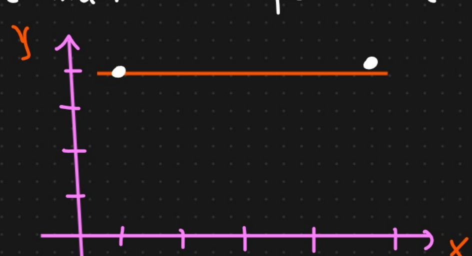


$$\underline{\text{Slope}} : \frac{3-5}{6-2} = \frac{-2}{4} = -\frac{1}{2}$$

This means that for every 2 units you move horizontally to the right, the line moves 1 unit vertically down.

3) <u>Zero slope</u> : Consider the point (1,4) (5,4)

X is not related to Y



$$\text{Slope} = \frac{4-4}{5-1} = \frac{0}{4} = 0$$

The means line is horizontal

4 Undefined Slope

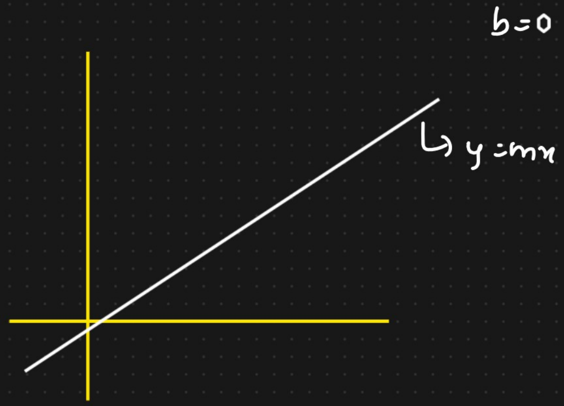
(3,2) (3,7)

$$\text{slope} = \frac{7-2}{3-3} = \frac{5}{0}$$

Slope in a Equation of a line

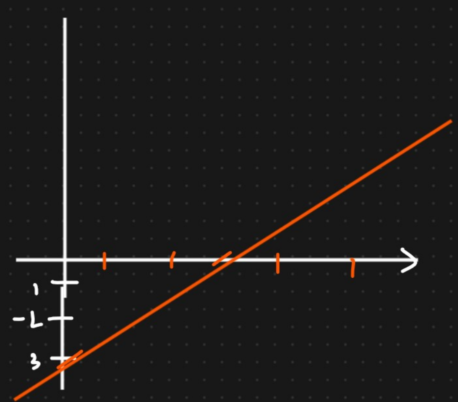
$$\boxed{y = mx + b}$$

- m is the slope of line
- b is the y intercept where the line crosses the y axis

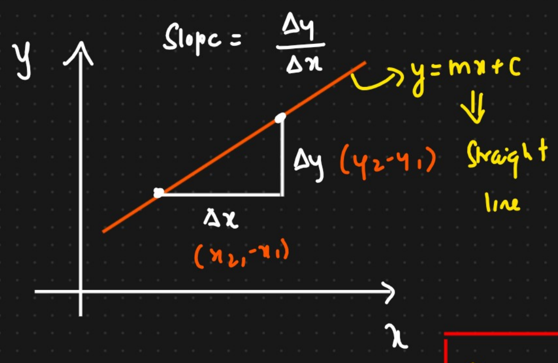


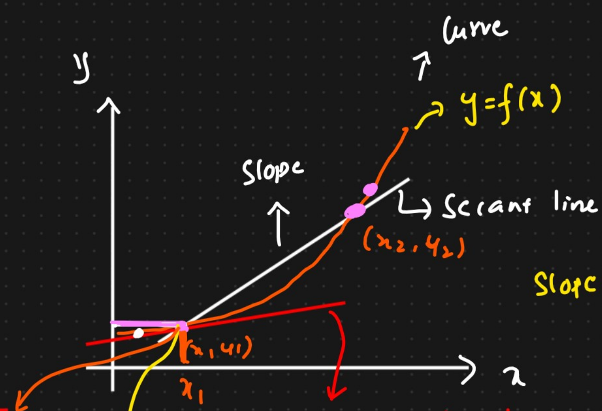
m=2 b=-3

$\Rightarrow \boxed{y = 2x - 3}$



Derivative



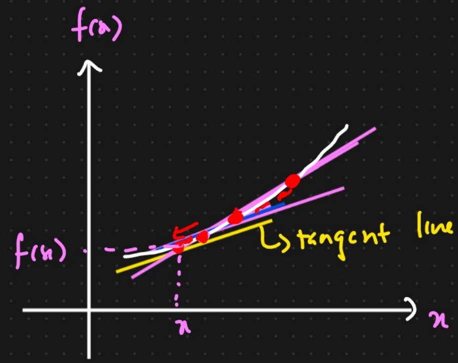


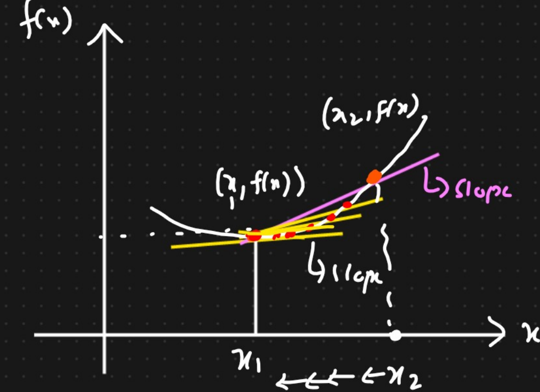
Slope = $\frac{\Delta y}{\Delta x}$

$$\boxed{\text{Slope} = \frac{\partial y}{\partial x}}$$

Slope of the tangent line

Instantaneous rate of change of x with respect to y

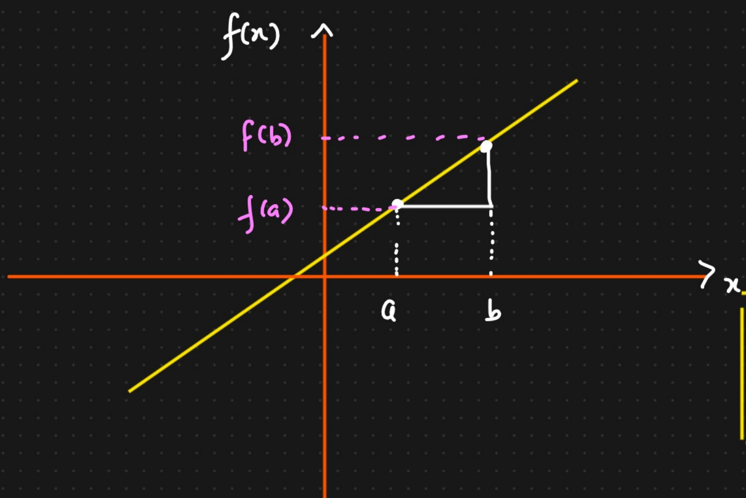




Limits

Mathematical Notation of Derivative With Limits

The derivative is a fundamental concept in calculus that represents the rate at which a function is changing at any given point. It is essentially the slope of the tangent line to the function's graph at that point. The derivative is used to understand how a function behaves as its input changes, and it is a key tool for analyzing the dynamics of systems in mathematics, physics, economics, engineering, and many other fields.

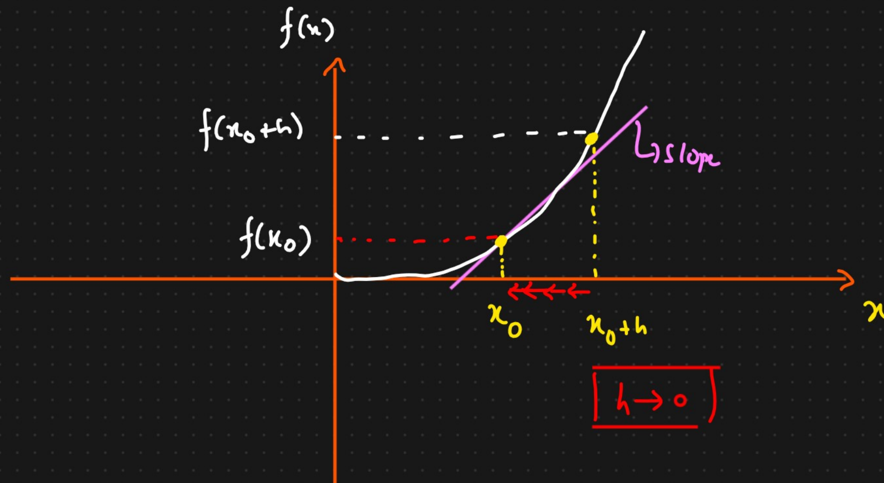


$f(x) = mx + c$ $\quad$ $m \rightarrow$ slope

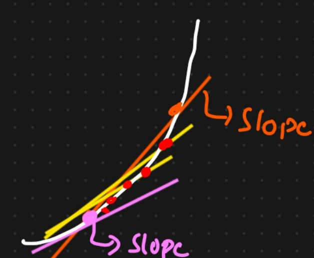
$c \rightarrow$ intercept

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\boxed{\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}}$$



$\boxed{h \rightarrow 0}$



Secant line slope is going to give the average rate of change.

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_0 + h) - f(x_0)}{(x_0 + h) - x_0} = \frac{\Delta y}{\Delta x}$$

$$\boxed{\text{Slope of Secant line} = \frac{f(x_0+h) - f(x_0)}{h}}$$

$\lim_{h \to 0} \rightarrow$

$$f'(x) = \boxed{\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}} \leftarrow \boxed{\text{Derivative of } f(x)}$$

$$f'(x) = \frac{\partial y}{\partial x} = \frac{\partial(f(x))}{\partial x} \text{ //.}$$