Referee Report on Manuscript #25-1763

Summary

The manuscript addresses the feasibility of virtual holonomic constraints (VHCs) for underactuated Euler–Lagrange systems with one degree of underactuation. As in the author's earlier submission to IEEE L-CSS (submission #25-0794, May 2025), the central example is the "tic-toc" maneuver of the PVTOL aircraft. The author shows that no VHC satisfying the standard definition (requiring controlled invariance of the constraint manifold Γ) can be associated with this trajectory, yet the trajectory is orbitally stabilizable. The manuscript concludes that the accepted definition of VHC is overly restrictive and calls for a reconsideration of the concept.

Relation to the Previous Submission

I reviewed the previous submission (IEEE L-CSS), which was rejected. A number of reviewers, myself included, expressed serious concerns about clarity, novelty, and the absence of a constructive contribution. After carefully comparing the present TAC manuscript with the L-CSS version, I find that the two are essentially the same:

- The motivating example, mathematical derivations, proofs of nonexistence of a VHC, transverse-linearization-based controller design, and numerical simulations are unchanged.
- The key theorem on singular reduced dynamics (Theorem 1 in the present paper) is identical to the earlier version, and in any case originates from prior literature.
- The eigenvalues of the controllability Gramian and the closed-loop monodromy matrix are numerically identical to those in the earlier manuscript.
- The main conclusion remains: Definition 1 of a VHC is too restrictive because it excludes stabilizable trajectories such as the tic-toc maneuver.

The differences consist of minor rewording, a somewhat more explicit linear algebraic contradiction argument, an extended discussion section, and one or two additional recent references. These do not alter the substance of the paper.

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Main Concerns

The concerns I raised in my previous review remain unaddressed:

1. **Unclear main message.** It is not clear whether the author intends (a) to emphasize that VHCs do not generate the universe of all stabilizable trajectories (a fact already well known since early work by Shiriaev and collaborators), or (b) to propose a new definition of VHCs. The manuscript vacillates between these two goals.

2. **No alternative definition.** If the intention is to redefine VHCs, the author does not provide a mathematically rigorous new definition. This omission is critical, because the terminology "virtual holonomic constraint" is meant to mimic the properties of holonomic constraints in mechanics. The features that make a holonomic constraint meaningful—invariance under rescaling of tangential velocity, and existence of a well-defined reduced-order system—are not preserved in the author's relaxed notion.

3. **Terminological confusion.** In the author's setup, there may exist a single trajectory satisfying $h(q) = 0$ under feedback, but this does not make $h(q) = 0$ a genuine constraint in the mechanical sense. Calling such a relation a VHC risks creating confusion and diluting a concept that has been carefully formalized in earlier works.

4. **Novelty.** The present submission does not contain substantial new technical results compared to the earlier rejected paper. The additional discussion does not change the fact that the core ideas are already known: (i) not all stabilizable trajectories can be generated via VHCs; (ii) reduced dynamics may have singularities but still admit solutions; and (iii) transverse linearization can be used to design orbital stabilizers. The whose Section III is problematic. The main result, Theorem 1, is not new. It is taken [14]. Theorem 2 is an obvious remark about situations when the reduced dynamics have a singularity. From a conceptual standpoint, I don't find contributions in this section.

5. **Literature positioning.** The paper would benefit from a more careful comparison with existing literature, which has long recognized that VHCs restrict the set of admissible motions, without suggesting that they cover all possible stabilizable trajectories. As currently written,

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the manuscript risks giving the impression of novelty where there is none.

Conclusion

The main criticisms (unclear message, lack of a new definition, limited novelty) that were raised in the review process for the IEEE L-CSS submission remain valid. The author may wish to further develop these ideas, but in their present form they do not meet the standards of clarity and novelty required by the journal.