

1. **The abstract does not mention the scope of the results or any methodology to address or remove the identified deficiency. Therefore, it is primarily descriptive and lacks any indication of novelty or a proposed solution.** It highlights the existence of certain stabilizable trajectories that do not fit within the conventional VHC framework, but does not articulate a new method, theoretical extension, or design innovation to address this issue.
2. The sentence stating that “a typical criterion is the existence of stabilizing feedback which ensures that the planned motion can be executed on a physical system” appears to be incomplete in the context of underactuated mechanical systems. While such a criterion is indeed relevant, it is important to emphasize that, in underactuated systems, the mere existence of a stabilizing feedback law is not sufficient. For a motion plan to be truly feasible, the associated **zero dynamics** must also be stable. It is therefore recommended that the authors clarify this important aspect, particularly given the focus of the paper on systems with a single degree of under actuation.
3. Please clarify whether the “closed-loop system” mentioned in Definition 1 specifically refers to system (1).
4. It is recommended to move Figures 1 and 2 after the relevant discussion in the main text, rather than placing it beforehand. This adjustment would improve the logical flow of presentation and help readers interpret the figure with the necessary context.
5. In the sentence “According to [8, Chapter 6], the set Γ is controlled invariant if and only if, for every ...”, the reference to an entire chapter may be too vague for a claim of this specificity. Moreover, I was unable to locate a statement in Chapter 6 of [8] that explicitly supports the equivalence claimed by the authors. For better clarity and traceability, especially when referring to precise mathematical conditions such as controlled invariance, it is advisable to cite a specific theorem or lemma rather than an entire chapter. This would help readers and reviewers verify the claim more efficiently. **please point me to the exact location (e.g., theorem number or page) in [8] where your statement—specifically the “if and only if” characterization of controlled invariance—is formally stated.**
6. In the paragraph following “We define the new coordinates as...”, the trajectory $x^*(\tau)$ is referenced without an explicit definition of its functional form. For clarity and completeness, it is important to define it at once after call it—for example, if $x^*(\tau) = \sin(\tau)$, this should be explicitly stated.
7. In your construction of the transverse coordinates, you define $\tau := \text{atan2}(x, \dot{x})$ and claim that the transformation $(q, \dot{q}) \mapsto (\tau, \rho)$ is diffeomorphic within a tubular neighborhood of the orbit. However, since $\text{atan2}(x, \dot{x})$ is undefined when $\dot{x} = 0$, it appears that the transformation may become singular at such continues points. This raises the question: **Could you please clarify how you handle the singularity at $\dot{x} = 0$ for arbitrary x , and whether the transformation remains diffeomorphic in a strict mathematical sense throughout the entire neighborhood?** Furthermore, considering that the feedback control law (11) is expressed in terms of $\tau(q, \dot{q})$, **what happens to the control input $u(q, \dot{q})$ near such continuous singular points?** In particular, in Section D, you initialize the simulation with $\dot{x} = 0$, which corresponds to a singularity of the transformation.
8. The manuscript predominantly cites works published before 2016. It would benefit from including more recent literature to better reflect the current state of research in the field and to contextualize the contributions with respect to the latest developments. For instance, the paper *“Dynamic virtual holonomic constraints for stabilization of closed orbits in*

underactuated mechanical systems" could provide useful insights and a contemporary perspective closely related to the topic.

9. The formatting of the references is inconsistent. Please ensure that all citations are presented in a uniform style and fully conform to the journal's formatting requirements. Additionally, verify that the formatting and in-text referencing of figures, tables, and equations adhere to the journal's submission guidelines.