

Power Law method:

x corresponds to a discretization of a solution on a grid. Suppose the grid is two dimensions:

$$\|x\| = \sqrt{\sum_{i=0}^N \sum_{j=0}^M x_{ij}^2 / (NM)} \quad (1)$$

$$Ax = \lambda x \quad (2)$$

$$\text{given } x^{(0)}, \|x^{(0)}\| = 1 \quad (3)$$

for $k = 0, 1, 2, 3, \dots$,

$$\lambda^{(k)} = \|Ax^{(k)}\| \quad (4)$$

$$x^{(k+1)} = Ax^{(k)} / \lambda^{(k)} \quad (5)$$

Let $q^{(n)}$ be some “base state” at time t^n .

The nonlinear operator, $N(q)$, advances q from t^n to t^{n+1} :

$$q^{(n+1)} \equiv N(q^{(n)}) \quad (6)$$

Define the following, approximately linear, operator:

$$L(x) \equiv \frac{1}{\epsilon}(N(q^{(n)} + \epsilon x) - q^{(n+1)}) \approx \quad (7)$$

$$\frac{1}{\epsilon}(N(q^{(n)}) + \epsilon \frac{\partial N}{\partial q} \cdot x - N(q^{(n)})) \rightarrow \quad (8)$$

$$\frac{\partial N}{\partial q} \cdot x \text{ as } \epsilon \rightarrow 0 \quad (9)$$

Applying the perturbation (just on the initial step of the evaluation of $N()$)
For reference, see Jemison, Sussman, Arienti, JCP (7 equation model, see Saurel and Remi Abgrall)

Conservation of mass (materials $m = 1, \dots, M$):

$$\frac{\partial \rho_m F_m}{\partial t} + \nabla \cdot (\rho_m F_m u) = \epsilon(\delta(\rho_m)) F_m \quad (10)$$

Conservation of energy (materials $m = 1, \dots, M$):

$$\frac{\partial \rho_m E_m F_m}{\partial t} + \nabla \cdot ((p u + \rho_m u E_m) F_m) = \epsilon(\delta(\rho_m E_m)) F_m \quad (11)$$

Conservation of momentum:

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (p I + \rho u u^T) = \epsilon(\delta(\rho u)) - \sum_{m=1}^M \delta(\phi_m) \nabla H(\phi_m) \quad (12)$$

ϕ_m corresponds to the signed distance function for material m .

ϕ_m is positive if $\vec{x} \in \Omega_m$ and negative otherwise.

$H(\phi) = 1$ if $\phi > 0$ and $H(\phi) = 0$ otherwise.

Outline of the code modifications:

- read in the checkpoint of the base state $q^{(n)}$ and save it.
- compute $q^{n+1} = N(q^n)$ and save it. (no regridding!)
- initialize $x^{(0)}$
- carry out the power law algorithm.