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ESTIMATION OF UNKNOWN WALL HEAT FLUX IN TURBULENT CIRCULAR PIPE FLOW

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ABSTRACT

An inverse heat convection problem is solved for the estimation of a nonuniform wall heat flux in a thermally developing, hydrodynamically developed turbulent flow in a circular pipe based on temperature measurements obtained at several different locations in the stream. The direct problem of turbulent forced convection is solved with a finite difference method with appropriate algebraic turbulence modelling. The unknown wall heat flux is represented by a one-dimensional finite element interpolation. Nodal values at several chosen points are determined as unknown parameters by the Levenberg-Marquardt algorithm. The effects of sensor number and position are examined. © 2000 Elsevier Science Ltd

Introduction

A wide variety of inverse heat conduction problems have been solved in the last two decades for the estimation of initial or boundary conditions, physical properties, geometric parameters, or heat source intensities. Beck et al. [1], Hensel [2], and Özisik and Orlande [3], among others, present some of the methods developed for the solution of such problems. Despite many potential applications, inverse convection problems have only received some attention recently. Moutsoglou [4] apparently was the first to address an inverse convection problem that has utilized a sequential function specification algorithm for the estimation of the asymmetric heat flux in mixed convection in a vertical channel. The same author [5] has also applied the whole domain regularization technique in a similar forced convection problem. Raghunath [6] has determined the inlet temperature profile in steady state laminar forced convection by using the quasi-Newton conjugate gradient method, which is a special case of the conjugate gradient method. Huang and Özisik [7] have applied the conjugate gradient method with an adjoint equation for the estimation of steady state wall heat flux in hydrodynamically developed laminar flow in a parallel plate duct. The same method has been applied by Bokar and Özisik [8] to estimate the time dependence of inlet temperature in similar flow conditions. Liu and Özisik [9] have used the Levenberg-Marquardt algorithm for the minimization procedure for estimation of the thermal conductivity and specific heat of laminar flow through a circular duct by utilizing transient temperature readings at a single downstream location. Machado and Orlande [10] have used the conjugate gradient method with an adjoint equation to estimate the timewise and spacewise variation of the wall heat flux in a parallel plate channel. An inverse problem for estimating the heat flux to a power-law non-Newtonian fluid in a parallel plate channel flow was solved by Machado and Orlande [11] by using the same method. Few work have been done on inverse problems in turbulent flow despite its obvious technological relevance [12].

In this work, we solve an inverse heat convection problem to estimate spatially nonuniform wall heat flux in a thermally developing, hydrodynamically developed turbulent flow in a circular pipe based on temperature measurements obtained at several different positions in the stream. We present firstly the mathematical formulation of the direct problem of turbulent forced convection in a circular pipe. For the solution of the inverse problem, we use the Levenberg-Marquardt algorithm for minimization to estimate the unknown spatially nonuniform wall heat flux, which is represented by an interpolation function with unknown coefficients to be determined. The effects of sensor number and position, as well as magnitude of measurement errors, are examined by using simulated experimental data.

Mathematical Formulation of the Direct Problem

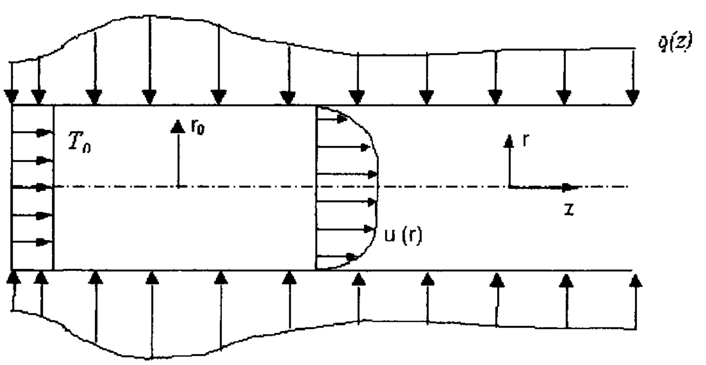

FIG. 1
Schematic Representation of the Physical Problem

We consider a thermally developing, hydrodynamically developed turbulent flow through a circular pipe of a Newtonian fluid with constant properties. Fluid enters the circular pipe with a given uniform temperature T_0. The circular pipe is subjected to an axisymmetric and longitudinally nonuniform wall heat flux. A schematic representation of the physical problem is given in Fig. 1.

Assuming axisymmetry of the problem and neglecting axial conduction, the governing equation for the steady state temperature field, $T(r, z)$, is written as

$$u(r)\frac{\partial T}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\Big(r(\alpha + \epsilon_H)\frac{\partial T}{\partial r}\Big), \quad \text{in} \quad 0 \le r \le r_o, \quad 0 \le z \le z_{max}, \tag{1}$$

with the boundary conditions,

$$T(r,0) = T_0, \quad \text{in} \quad z = 0, \quad \text{for} \quad 0 \le r \le r_o, \tag{2}$$

$$-k\frac{\partial T}{\partial r}|_{r=r_o} = q(z), \quad \text{in} \quad r = r_o, \quad \text{for} \quad 0 \le z \le z_{max}, \tag{3}$$

where r and z are the radial and longitudinal coordinates, $q(z)$ the unknown wall heat flux, T_0 the uniform inlet temperature, $u(r)$ the hydrodynamically developed velocity profile, α the thermal diffusivity, k the thermal conductivity, and ϵ_H the turbulent thermal diffusivity. The fully developed velocity profile of turbulent flow of a Newtonian fluid in a circular pipe is obtained from the following expression for the dimensionless velocity in wall parameters,

$$u^+ = 2.5 \ln\Big[y^+ \frac{1.5(1 + r/r_o)}{1 + 2(r/r_o)^2}\Big] + 5.5, \tag{4}$$

where, $u^+ = u/u_\tau$, $y^+ = yu_\tau/\nu$, $u_\tau = \sqrt{\tau_w/\nu}$.

Equation (4) is in quite good agreement with experiment [13] and is obtained from an empirical equation proposed by Reichardt [14] for the kinematic turbulent viscosity,

$$\epsilon_M = \frac{\nu\kappa y^+}{6}\Big(1 + \frac{r}{r_o}\Big)\Big[1 + 2\Big(\frac{r}{r_o}\Big)^2\Big]. \tag{5}$$

The turbulent thermal diffusity is obtained by using the concept of the turbulent Prandtl number, $Pr_t = \epsilon_M/\epsilon_H$. The turbulent Prandtl number Pr_t is given by the following expression, which fits well available experimental data [13],

$$Pr_t = \frac{1}{\frac{1}{2Pr_\infty} + (CPe_t)^2\sqrt{\frac{1}{Pr_\infty}} - (CPe_t)^2[1 - \exp\big(-\frac{1}{CPe_t\sqrt{Pr_\infty}}\big)]}, \tag{6}$$

where, $Pe_t = \epsilon_M Pr/\nu$, $Pr_\infty = 0.86$, and $C = 0.2$.

The direct problem given by the equations (1-6) can be solved to obtain the temperature field of the fluid inside the circular pipe when the wall heat flux $q(z)$ and the inlet temperature T_0 are known. The direct problem defined by Eqs. (1-6) is solved by using an implicit finite difference method. As the governing partial differential equation (1) is a linear parabolic equation, the problem is solved by a spatially marching procedure with inversion of a tri-diagonal matrix at each space marching step. No iteration is needed in the solution of the direct problem.

Solution of the Inverse Problem

In the inverse problem considered in this work, we are looking for the unknown spatially nonuniform wall heat flux $q(z)$ from temperature measurements taken at several interior points in the flow field. The unknown wall heat flux is represented as a linear combination of N_P linearly independent functions defined in the domain $[0, z_{max}]$ with coefficients as parameters to be determined. Although any consistent set of base functions can be used, we choose to adopt a finite element interpolation to represent the unknown wall heat flux. The domain $[0, z_{max}]$ is divided into a set of subdomains, $[z_i, z_{i+1}]$, i=1,..,$I-1$. In each subdomain, $[z_i, z_{i+1}]$, we have

$$q_e(z) = \sum_{j=1}^{n} q_j^e \psi_j^e, \tag{7}$$

where the coefficients q_j^e are taken to be the values of $q(z)$ at the preselected nodes in the element $\bar{\Omega}_e$. The interpolation or shape functions ψ_j^e, are given in standard textbooks on finite element method. As we may have more experimental data than unknowns, the inverse problem is solved as a finite dimensional optimization problem. Consider the norm given by the summation of the square of the residues between the calculated temperatures, $T(r_m, z_m)$, and the measured temperatures, $Z_m(r_m, z_m)$, at the points (r_m, z_m), $m = 1, 2, ..., M$, with M being the total number of sensors,

$$R(\vec{q}) = \sum_{m=1}^{M} \left[T(r_m, z_m) - Z_m(r_m, z_m)\right]^2, \tag{8}$$

or

$$R(\vec{q}) = \vec{F}^T \vec{F} \quad \text{with} \quad F_m = T(r_m, z_m) - Z_m(r_m, z_m), \quad m = 1, 2, ..., M. \tag{9}$$

As the inverse problem is solved as an optimization problem it is seeked to minimize the norm R,

$$\frac{\partial R}{\partial q_j} = \frac{\partial}{\partial q_j}(\vec{F}^T \vec{F}) = 0, \quad j = 1, 2, ..., N_P. \tag{10}$$

Making a Taylor's expansion,

$$F(q^{k+1}) = F(q^k + \Delta q^k) = F(q) + \sum_{n=1}^{NP} \frac{\partial F(q)}{\partial q_n} \Delta q_n + O(\Delta q_n^2) \tag{11}$$

where N_P is the number of unknown parameters. Keeping up only to the first order terms in Eq. (11) and plugging the resulting expression into Eq. (10), we obtain the following equation,

$$J^T J \Delta q^n = -J^T \vec{F}, \tag{12}$$

where the elements of the Jacobian matrix are

$$J_{mn} = \frac{\partial T_m}{\partial q_n}, \quad m = 1, 2, ..., M \quad \text{and} \quad n = 1, ..., N_P. \tag{13}$$

Summing up with a damping factor λ to improve the convergence behaviour we have the Levenberg-Marquardt Method,

$$(J^T J + \lambda D)\Delta\vec{q} = -J^T \vec{F}, \tag{14}$$

where D represents the diagonal matrix. Equation (14) is then written in a form convenient to be used in an iterative procedure,

$$\Delta q^k = -(J^{kT} J^k + \lambda^k D^k)^{-1} J^{kT} \vec{F}^k, \tag{15}$$

where k is the iteration index. A new estimation of the parameters, $\vec{q}^{k+1}$, is calculated by

$$\vec{q}^{k+1} = \vec{q}^k + \Delta\vec{q}^k. \tag{16}$$

It should be noticed that the problem given by Eq. (14) is different from that given by Eq. (12). Nevertheless, it is seeked to reduce the value of the dumping factor through the iterations so that when convergence is achieved, the solution obtained be the same as that for original problem. The iterative procedure is initialized with an initial estimation of the parameters, $\vec{q}^0$, and new estimates, $\vec{q}^{k+1}$ are sequentially obtained using Eq. (16) with $\Delta\vec{q}^k$ given by Eq. (15) until the convergence criterion

$$|\frac{\Delta q_n^k}{q_n^k}| < \epsilon, \quad n = 1, 2, .., N_P,$$

is satisfied, where ϵ is a small real number, such as 10^{-5}. The elements of the Jacobian matrix as well as the right hand term of Eq. (14) are calculated by using the solution of the direct problem defined by Eqs. (1-6), as described in the previous section.

Results and Discussion

The inverse analysis presented in the previous section is applied in testing cases to estimate unknown wall heat flux in turbulent circular pipe flow. As real experimental data were not available, we generated simulated temperature data, $Z_m(r_m, z_m)$, $m = 1, 2, ..., M$, adding random errors to computed exact temperatures, $T_m(r_m, z_m)$,

$$Z_m = T_m + \sigma e_m, \quad m = 1, 2, ..., M,$$

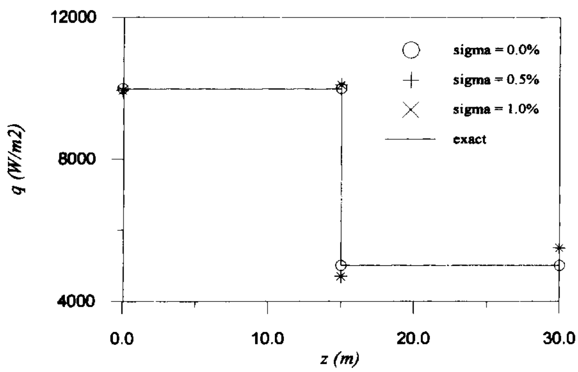

FIG. 2
Testing Case 1, with 60 measurement points.

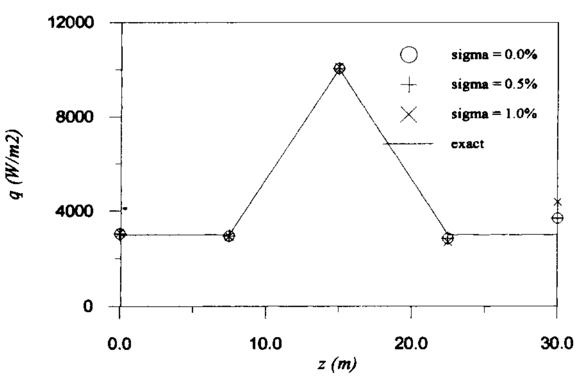

FIG. 3
Testing Case 2, with 60 measurement points.

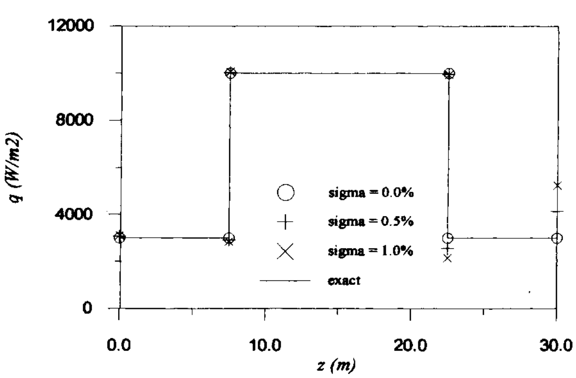

FIG. 4
Testing Case 3, with 60 measurement points.

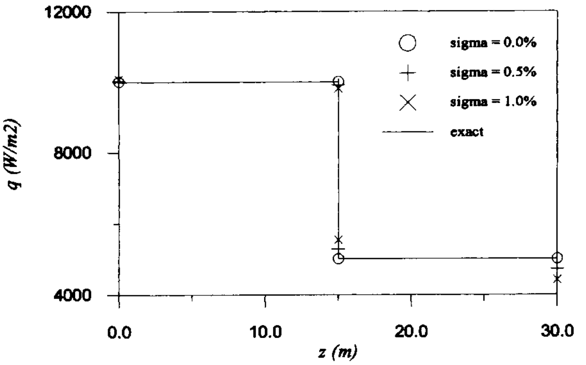

FIG. 5
Testing Case 1, with 80 measurement points.

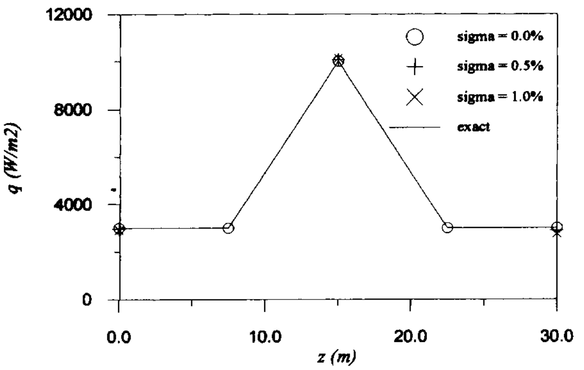

FIG. 6
Testing Case 2, with 80 measurement points.

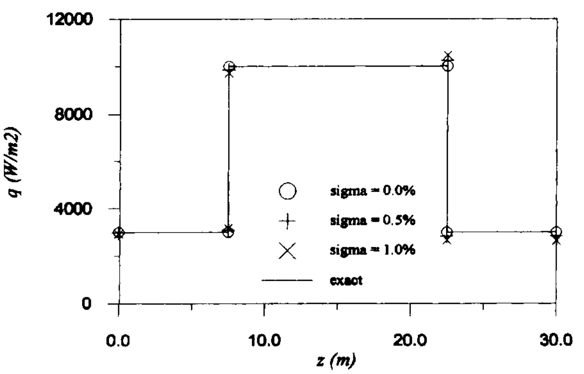

FIG. 7
Testing Case 3, with 80 measurement points.

where σ is the standard deviation of measurement errors which is assumed to be the same for all measurements and e_m is a normally distributed random error. For normally distributed error, there is a 99% probability of the value of e_m lying in the range $-2.576 < e_m < 2.576$..

For the cases considered here, the fluid properties are taken as ρ=985.46 kg/m^3, c_p=4184.3 J/kg^{o}C, k=0.651 W/mK, $\mu = 4.71 \times 10^{-5}$ kg/(m s). The mean velocity is taken as 2 m/s with a uniform inlet temperature of 60°C. The circular pipe considered has an inner diameter of 0.05 m and a heated length of 30m.

In Figures 2 to 4, 60 measurement points are distributed with uniform longitudinal spacings along the heated length, and increasing distance from the pipe wall. Figures 1 to 3 show comparison of estimated wall heat flux with σ=0.0%, 0.5%, and 1.0% for three different wall heat flux distribution. In Figures 5 to 7, 80 measurement points are used with the same kind of wall heat flux distribution. In all cases, estimation errors increased along the heated length with highest error always at the outlet of the pipe. Better estimations are obtained with 80 sensors than with 60 sensors. The nonuniform distribution of estimation error is due to downstream propagation of information. Wall heat flux at the inlet of the pipe influences all downstream measurement temperature, while wall heat flux at the outlet only influences the last measurement reading through diffusion. More uniform error distribution could be obtained with a nonuniform distribution of measurement points with more points near the outlet of the pipe.

Conclusion

An inverse analysis is presented for the estimation of unknown wall heat flux in turbulent circular pipe flow. The direct problem of turbulent forced convection is solved by using an implicit finite difference method with appropriate turbulence modelling. By representing the unknown wall heat flux by finite element interpolation, the Levenberg-Marquardt method is applied to obtain unknown coefficients in interpolation representation. The proposed method is tested against several cases. Numerical results are encouraging. Better estimation is obtained in the entrance section of the pipe than in downstream section.

Acknowledgement

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Nomenclature

C constant in Eq. (6)

C_P specific heat

D	diagonal matrix
e	normally distributed random error
F	difference vector between calculated and measured temperatures
J	Jacobian matrix
k	thermal conductivity
M	number of temperature sensors
N_P	number of estimated parameters
Pe_t	turbulent Peclet number
Pr_t	turbulent Prandtl number
q	wall heat flux
R	squared residue
r	radial coordinate
T	temperature
u	velocity
y	distance from the wall
Z	measured temperatures
z	axial coordinate

Greek symbols

α	thermal diffusivity
ϵ	convergence criterion
ϵ_H	turbulent thermal diffusivity
ϵ_M	kinematic turbulent viscosity
λ	damping factor
μ	viscosity
ν	kinematic viscosity
ρ	density
σ	standard deviation
τ_w	skin friction
ψ	finite element shape function

Subscripts

0	entrance condition
k	iteration index
m	temperature sensors

max pipe outlet

o pipe wall

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