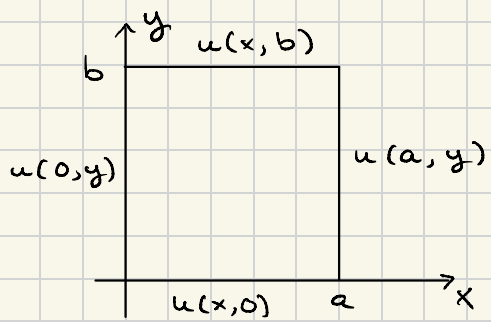
OBLIG TMA4121

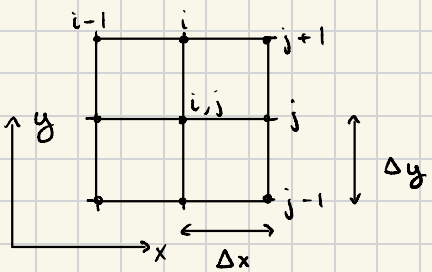
LAPLACES LIKNING

$$u_{xx} + u_{yy} = 0$$



$x \in [0, a]$ $y \in [0, b]$

Diskretisering av laplaces likning:



$$x_i = i \cdot \Delta x \qquad i = 0, 1, 2 \ldots$$

$$y_j = j \cdot \Delta y \qquad j = 0, 1, 2 \ldots$$

$$u_x \simeq \frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x}$$

$$u_{xx} \simeq \frac{u_{i+1,j} - 2u_{ij} + u_{i-1,j}}{(\Delta x)^2}$$

$$u_y \simeq \frac{u_{i,j+1} - u_{i,j-1}}{2\Delta y}$$

$$u_{yy} \simeq \frac{u_{i,j+1} - 2u_{ij} + u_{i,j+1}}{(\Delta y)^2}$$

$$0 = u_{xx} + u_{yy}$$

$$0 = \frac{u_{i-1,j} - 2u_{ij} + u_{i-1,j}}{(\Delta x)^2} + \frac{u_{i,j+1} - 2u_{ij} + u_{i,j-1}}{(\Delta y)^2}$$

$$0 = \Delta y^2 (u_{i-1,j} - 2u_{ij} + u_{i-1,j}) + \Delta x^2 (u_{i,j+1} - 2u_{ij} + u_{i,j-1})$$

$$u_{ij} = \frac{\Delta y^2 (u_{i+1,j} + u_{i-1,j}) + \Delta x^2 (u_{i,j+1} + u_{i,j-1})}{2(\Delta x^2 + \Delta y^2)}$$

Setter $\Delta x = \Delta y = 1$

$$u_{ij} = \frac{1}{4} (u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1})$$

Eksempelplott

$a = b = 3$

$u(x, 0) = np.sin(x+1)$

$u(0, y) = np.sin(y-1)$

$u(x, b) = 1$

$u(a, y) = 1$