Homework 4

16-720A Computer Vision

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Q 1.1

We know

$$pFp' = 0$$

Given the problem constraint that both points in question lie at $(0,0)$ on their camera planes, we can write

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{1,1} & f_{1,2} & f_{1,3} \\ f_{2,1} & f_{2,2} & f_{2,3} \\ f_{3,1} & f_{3,2} & f_{3,3} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} f_{3,1} & f_{3,2} & f_{3,3} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$f_{3,3} = 0$$

Q 1.2

Given the formula for the coefficients of an epipolar line: $l = \mathcal{E} * p'$ and the formula for the essential matrix: $\mathcal{E} = [t_\times]R$, the knowledge that $t = \begin{bmatrix} x & 0 & 0 \end{bmatrix}$ and $R = I$, because there is only x-axis translation:

$$\mathcal{E} = t_\times R = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix} I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -x \\ 0 & x & 0 \end{bmatrix}$$

$$l = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -x \\ 0 & x & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ -xc \\ xb \end{bmatrix}$$

Because the resulting line coefficients have 0 for their X component, the line must be parallel to the x axis.

Q 1.3

$$R_{rel} = R_i$$

$$t_{rel} = t_i$$

$$\mathcal{E} = [t_{rel_\times}]R_{rel}$$

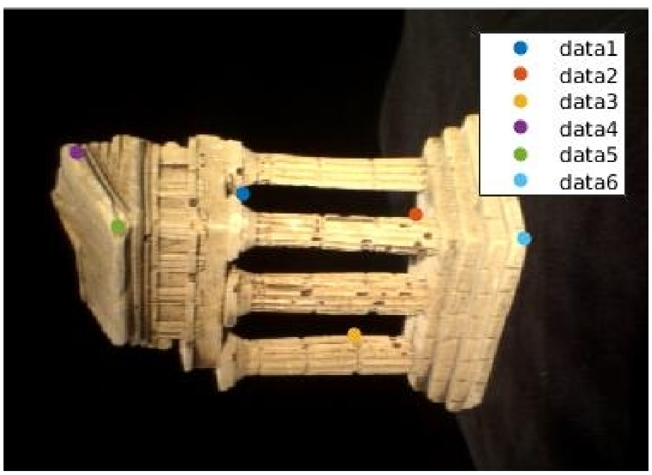
$$F = K^{-T}\mathcal{E}K^{-1}$$

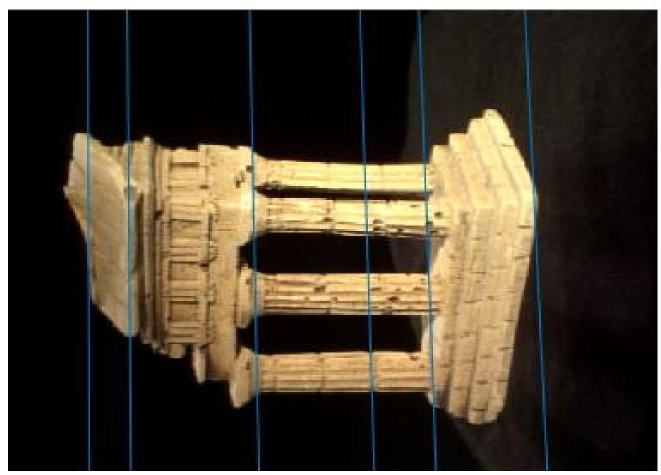
Q 1.4

The plane of the mirror is functionally equivalent to the image plane of a different camera. Because it has no real camera center but is being obeserved by the current camera, we can treat the mirror as having the same intrinsic matrix as the real camera. To find its center, define the translation from the real camera's center to its image plane, and the translation and rotation from the real camera image plane to the mirror. Composing these movements gives us the "center" of the mirror "camera". Because the mirror is a projecction plane and we have an R and t, we can define the essential and fundamental matrices as we normally would, therefore creating a skew symmetric fundamental matrix.

Q 2.1

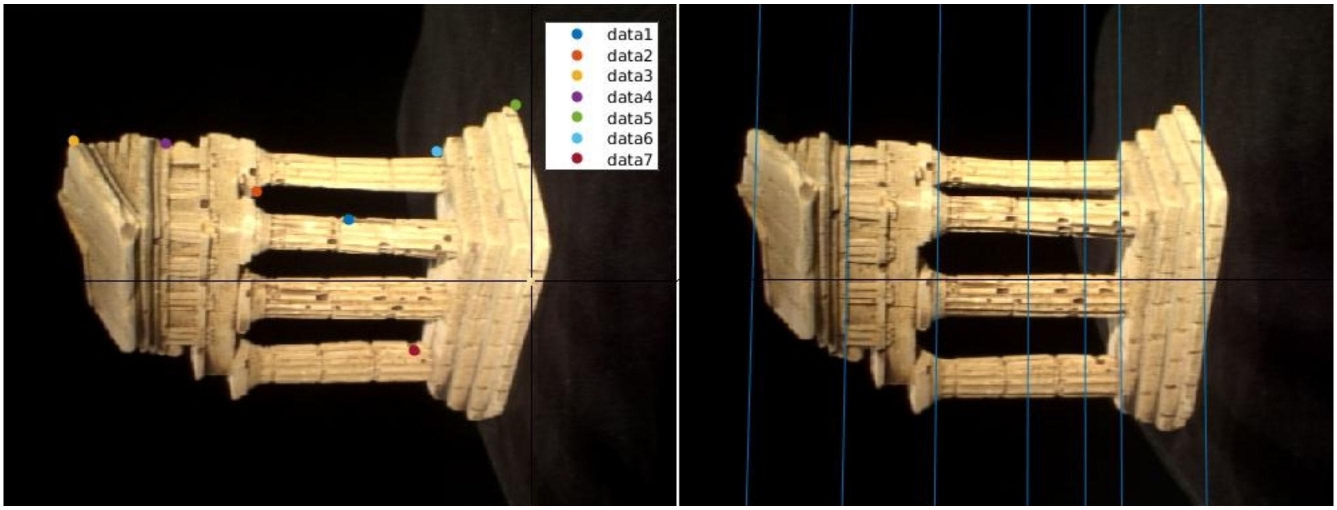
$$F = \begin{pmatrix} -1.4444E10^{-9} & -7.8627E10^{-8} & 0.0011327 \\ -1.1205E10^{-7} & 1.2618E10^{-9} & 4.1543E10^{-6} \\ -0.0010881 & 0.000015377 & -0.0046423 \end{pmatrix}$$





Q 2.2

$$F = \begin{pmatrix} 3.0708E10^{-8} & 3.5025E10^{-7} & 0.0016039 \\ -1.5483E10^{-7} & 9.2714E10^{-9} & 0.000054917 \\ -0.0016465 & -0.000030733 & -0.0091395 \end{pmatrix}$$



Q 3.1

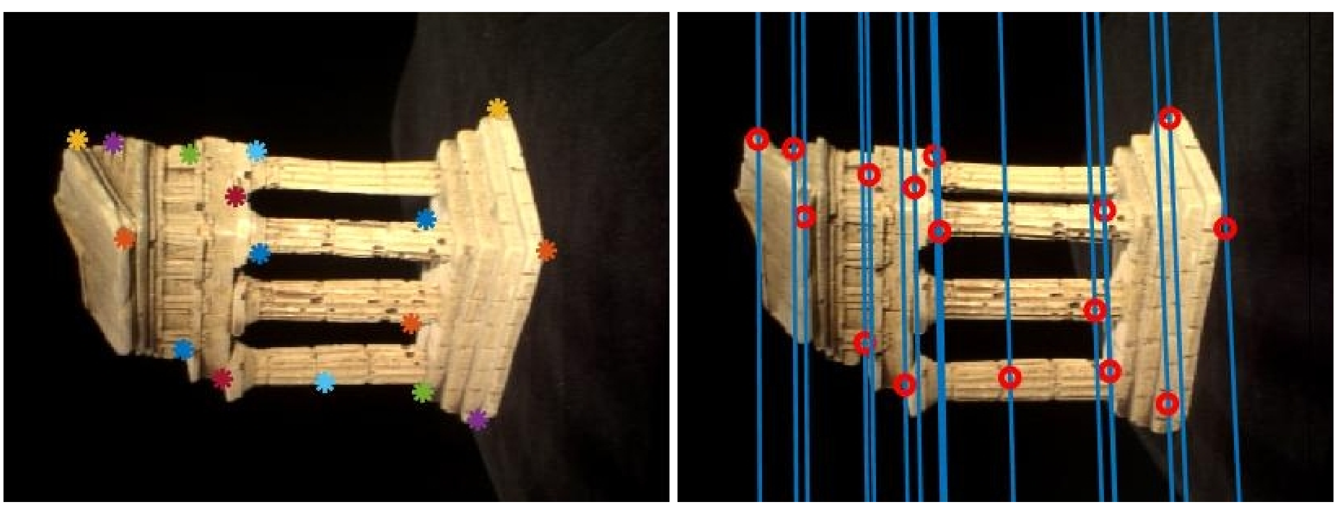
$$E = \begin{pmatrix} -0.0033389 & -0.18241 & 1.692 \\ -0.25994 & 0.002938 & -0.044874 \\ -1.6971 & -0.012332 & -0.00062738 \end{pmatrix}$$

Q 3.2

Given points $p_1 = [x_1 \ y_1]$ and $p_2 = [x_2 \ y_2]$ and camera matrices C_1 and C_2:

$$A = \begin{bmatrix} \begin{bmatrix}0 & 0 & 1\end{bmatrix} C_1 * x_1 - \begin{bmatrix}1 & 0 & 0\end{bmatrix} C_1 \\ \begin{bmatrix}0 & 0 & 1\end{bmatrix} C_1 * y_1 - \begin{bmatrix}0 & 1 & 0\end{bmatrix} C_1 \\ \begin{bmatrix}0 & 0 & 1\end{bmatrix} C_1 * x_2 - \begin{bmatrix}1 & 0 & 0\end{bmatrix} C_2 \\ \begin{bmatrix}0 & 0 & 1\end{bmatrix} C_1 * y_2 - \begin{bmatrix}0 & 1 & 0\end{bmatrix} C_2 \end{bmatrix} \tag{1}$$

Q 4.1



Q 4.2

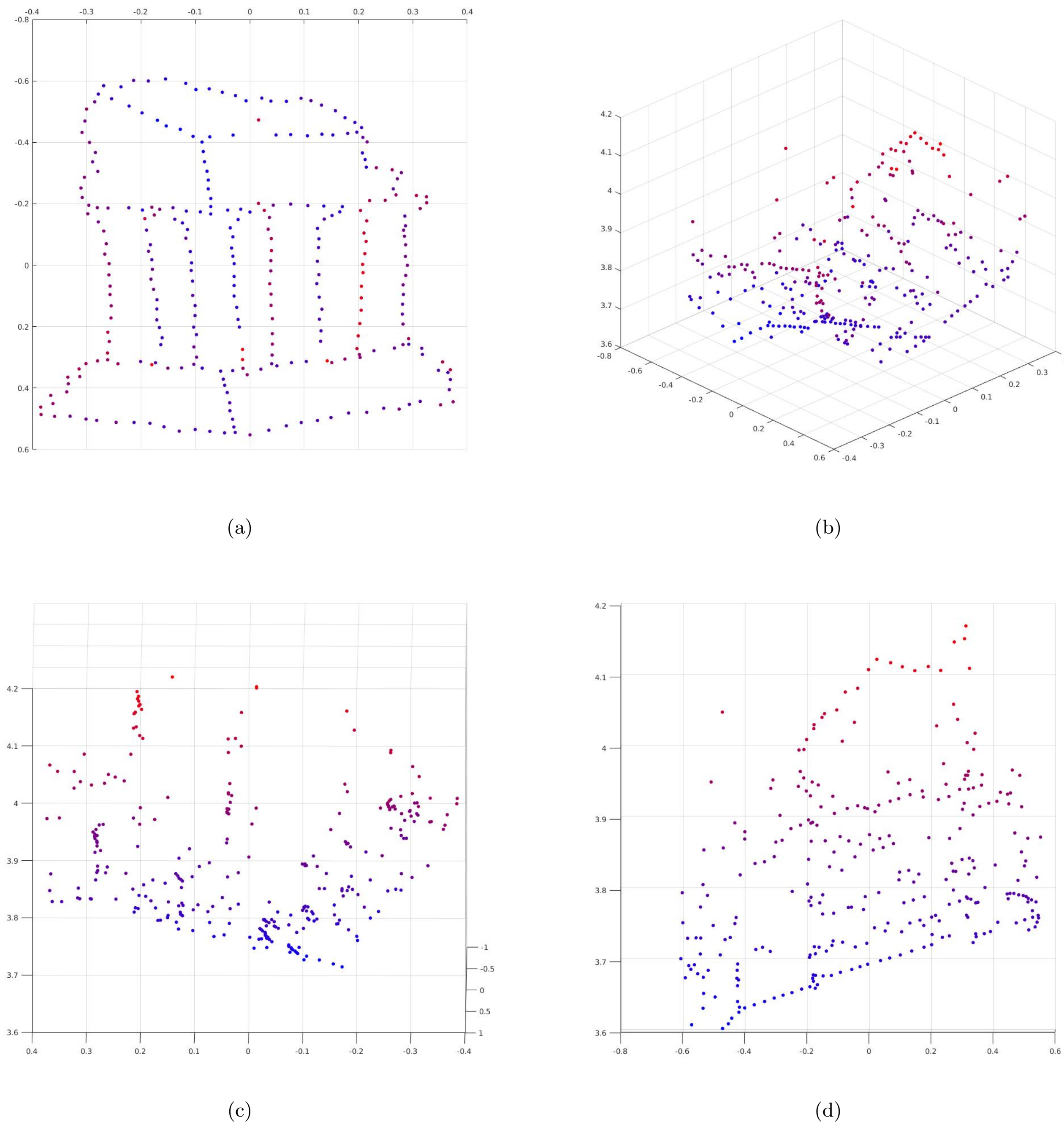


Figure 1: Blue dots are closer to the camera, red dots are further away.