Problem Set 2

16-642 Manipulation, Estimation, and Control

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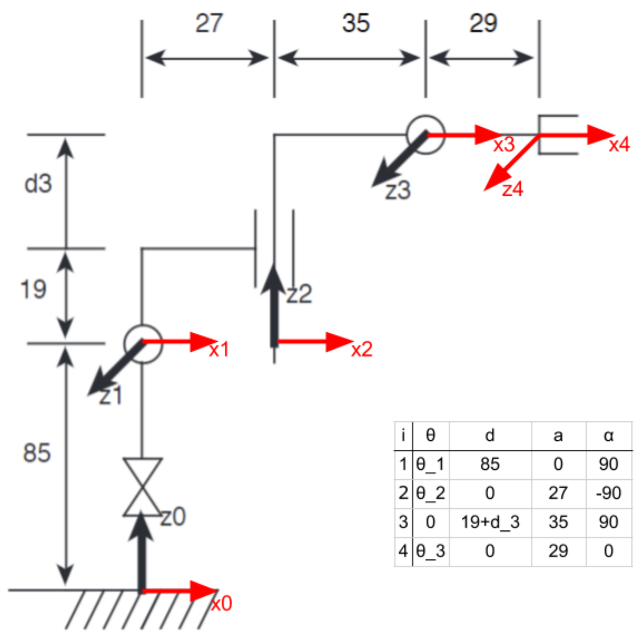
Question 1

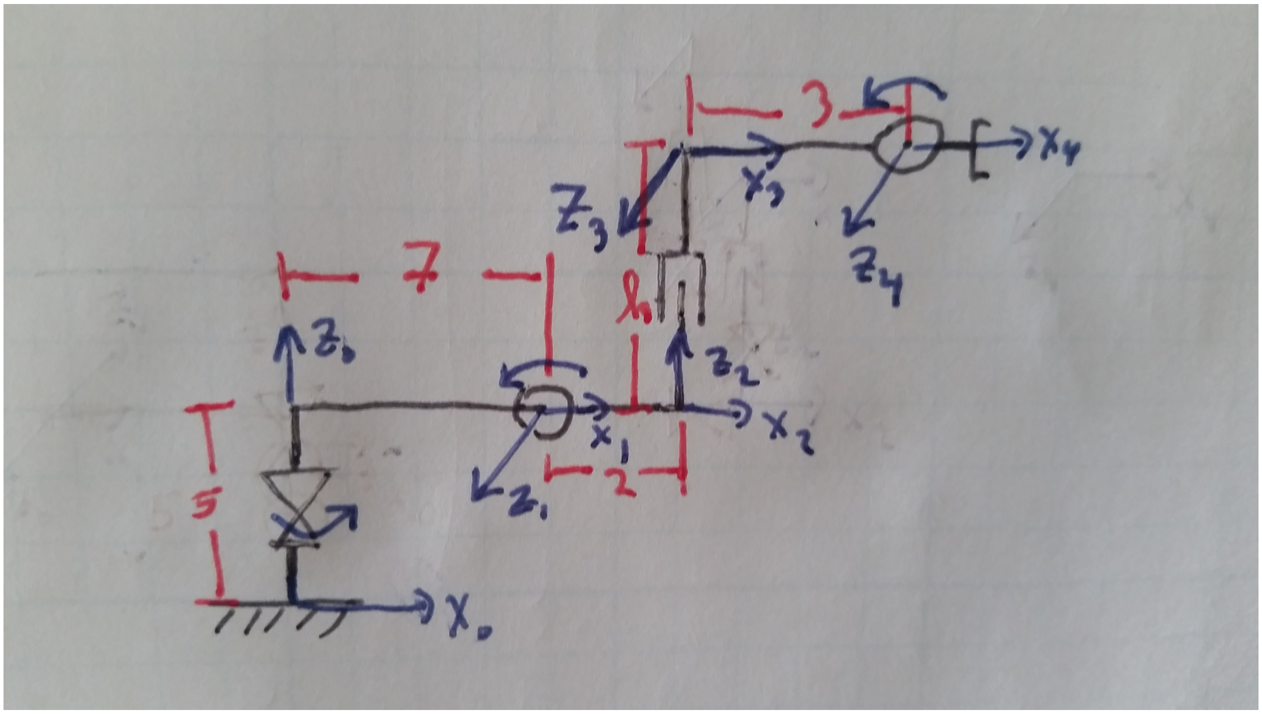


i	θ	d	a	α
1	θ_1	85	0	90
2	θ_2	0	27	-90
3	0	19+d_3	35	90
4	θ_3	0	29	0

All frames were chose to have their x axis pointing in the same direction because it made translations between them convenient. Frames 1,2,3 have their origins uniquely defined by DH convention. I chose the location of frame 0 to coincide with the ground point for convience of describing the translation from world frame origin to the end effector, and I chose the origin of frame 4 to be on the center point of the EE and with the same alignment as frame 3 for convenient translation.

*Worked with Georgia Crowther, Adam Driscoll, David Robinson, Jennifer Isaza, Joe Phaneuf

Question 2



Question 3

a

By inspection, can write the location of (x, y) from the ground frame as:

$$\begin{aligned} x =& cos(\theta_1)\big(9 + 5\cos\theta_3\big) \\ y =& sin(\theta_1)\big(9 + 5\cos\theta_3\big) \\ z =& 10 + d_2 + 5\sin\theta_3 \end{aligned}$$

Differentiating with respect to each joint space variable, we get:

$$J = \begin{bmatrix} -\sin\theta_1\big(9 + 5\cos\theta_3\big) & 0 & -5\sin\theta_3\cos theta_1 \\ \cos\theta_1\big(9 + 5\cos\theta_3\big) & 0 & -5\sin\theta_3\sin theta_1 \\ 0 & 1 & 5\cos\theta_3 \end{bmatrix}$$

b

Doing bad things to the DH convention we learned:

i	θ	d	a	α
1	θ_1	10	0	0
2	0	d_2	9	90
3	θ_3	0	5	0

From this, we can do DH stuff and get the following matrices:

$$H_1^0 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} H_2^1 = \begin{bmatrix} 1 & 0 & 0 & 9 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_3^2 = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 5\,\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 5\,\sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^0 = \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & 9\,\cos(\theta_1) \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & 9\,\sin(\theta_1) \\ 0 & 1 & 0 & d_2+10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = \begin{bmatrix} \cos(\theta_1)\,\cos(\theta_3) & -\cos(\theta_1)\,\sin(\theta_3) & \sin(\theta_1) & 9\,\cos(\theta_1)+5\,\cos(\theta_1)\,\cos(\theta_3) \\ \cos(\theta_3)\,\sin(\theta_1) & -\sin(\theta_1)\,\sin(\theta_3) & -\cos(\theta_1) & 9\,\sin(\theta_1)+5\,\cos(\theta_3)\,\sin(\theta_1) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & d_2+5\,\sin(\theta_3)+10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From these, we can generally define $R_{i-1}^0 = H_{i-1}^0(1:3, 1:3)$, and the various origin translations:

$$o_3^0 = \begin{bmatrix} 9\,\cos(\theta_1)+5\,\cos(\theta_1)\,\cos(\theta_3) \\ 9\,\sin(\theta_1)+5\,\cos(\theta_3)\,\sin(\theta_1) \\ d_2+5\,\sin(\theta_3)+10 \end{bmatrix} o_3^2 = \begin{bmatrix} 5\cos(\theta_3) \\ 5\sin(\theta_3) \\ 0 \end{bmatrix}$$

Given $J_v 1 = R_{i-1}^0\left(z \times o_n^{i-1}\right)$ for revolute joints and $J_v 1 = R_{i-1}^0 z$ for prismatic joints:

$$J_{v_1} = \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 9\,\cos(\theta_1)+5\,\cos(\theta_1)\,\cos(\theta_3) \\ 9\,\sin(\theta_1)+5\,\cos(\theta_3)\,\sin(\theta_1) \\ d_2+5\,\sin(\theta_3)+10 \end{bmatrix} = \begin{bmatrix} -\sin(2\,\theta_1)\,(5\,\cos(\theta_3)+9) \\ \cos(2\,\theta_1)\,(5\,\cos(\theta_3)+9) \\ 0 \end{bmatrix}$$

$$J_{v_2} = \begin{pmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) \\ \sin(\theta_1) & 0 & -\cos(\theta_1) \\ 0 & 1 & 0 \end{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_{v_3} = \begin{pmatrix} \cos(\theta_1)\,\cos(\theta_3) & -\cos(\theta_1)\,\sin(\theta_3) & \sin(\theta_1) \\ \cos(\theta_3)\,\sin(\theta_1) & -\sin(\theta_1)\,\sin(\theta_3) & -\cos(\theta_1) \\ \sin(\theta_3) & \cos(\theta_3) & 0 \end{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 5\,\cos(\theta_3) \\ 5\,\sin(\theta_3) \\ 0 \end{bmatrix} = \begin{bmatrix} -5\cos(\theta_1)\sin(\theta_3) \\ -5\sin(\theta_1)\sin(\theta_3) \\ 5\cos(\theta_3) \end{bmatrix}$$

c

There are two singular configurations: The configuration depicted, and the configuration depicted except with $\theta_3 = 180\circ$. These were found using the geometric observation that a singular configuration occurs whenever the movement of two or more joints produces the same end effector movement. In both listed cases, moving the prismatic joint or the θ_3 joint will cause the end effector to move in only the z_0 direction.

Question 4

a

We know the first two links of the manipulator must meet at $x = .75$, the halfway point between 0 and 1.5. From this, and the knowledge that both links have a length of 1, we can get $\theta_1 = \arccos .75 = 41.41\circ$ Because the second length is also 1, and the location of the third joint is required to be (1.5,0), we can write $\theta_2 = -2\theta_1 = -82.82\circ$, and $\theta_3 = \theta_1 = 41.41\circ$ because link 3 is required to be parallel to the x axis. The first two links have an invertible configuration, so another valid solution exists: $\theta_1 = -41.41\circ, \theta_2 = 82.82\circ, \theta_3 = -41.41\circ$.

b

By intuition, for there to be only a y axis compenent to the midpoint of link 3's velocity, it must be moving tangent to a circle whose center lies on the x axis. Therefore, a joint must lie on the x axis to move the third link at the desired velocity. Joint 1 and joint 3 both lie on the x axis, so both can create the required velocity.

Either $\dot{\theta}_1 = \frac{10}{2.5} = 4$ or $\dot{\theta}_3 = \frac{10}{.5} = 20$ will create the correct movement by only moving one joint. However, because the system is currently in a singular configuration, any linear combination of $\dot{\theta}_1$ and $\dot{\theta}_3$ such that

$$10 = 2.5\dot{\theta}_1 + .5\dot{\theta}_3$$

is valid will create the desired velocity.

Question 5

$$\mathcal{L}(q,\dot{q}) = K(q,\dot{q}) - P(q)$$

$$K_1 = \frac{\dot{q_1}^2\ (I_1 + 4\,m_1)}{2}$$

$$K_2 = \left(\frac{\dot{q_1}^2}{2} + \dot{q_1}\,\dot{q_2} + \frac{\dot{q_2}^2}{2}\right)\ I_2 + \left(9\,\dot{q_1}\,\dot{q_2} + 12\,\dot{q_1}^2\ \cos(q_2) + \frac{25\,\dot{q_1}^2}{2} + \frac{9\,\dot{q_2}^2}{2} + 12\,\dot{q_1}\,\dot{q_2}\ \cos(q_2)\right)\ m_2$$

$$K = K_1 + K_2$$

$$K = \left(\frac{I_1}{2} + \frac{I_2}{2} + 2\,m_1 + \frac{25\,m_2}{2} + 12\,m_2\ \cos(q_2)\right)\ \dot{q_1}^2 + (I_2 + 9\,m_2 + 12\,m_2\ \cos(q_2))\ \dot{q_1}\,\dot{q_2} + \left(\frac{I_2}{2} + \frac{9\,m_2}{2}\right)\ \dot{q_2}^2$$

$P_1 = 2gm_1 \sin(q_1)$ $\qquad$ $P_2 = gm_2\,(4\sin(q_1) + 3\sin(q_1+q_2))$

$P = P_1 + P_2 = gm_2\,(4\sin(q_1) + 3\ \sin(q_1+q_2)) + 2gm_1\sin(q_1)$

$$\begin{aligned}\mathcal{L} = &\left(\frac{I_1}{2} + \frac{I_2}{2} + 2\,m_1 + \frac{25\,m_2}{2} + 12\,m_2\ \cos(q_2)\right)\dot{q_1}^2 + \\ &\Big(I_2 + 9\,m_2 + 12\,m_2\ \cos(q_2)\Big)\dot{q_1}\dot{q_2} + \\ &\left(\frac{I_2}{2} + \frac{9\,m_2}{2}\right)\dot{q_2}^2 - gm_2\,(4\sin(q_1) + 3\ \sin(q_1+q_2)) - 2\,g\,m_1\ \sin(q_1)\end{aligned}$$

Question 6

$$\mathcal{L} = q_{1\text{dot}}^{\ \ 2} + 3\,q_{1\text{dot}}\,q_{2\text{dot}} + 2\,q_{2\text{dot}}^{\ \ 2} - 10\,q_2\,(t)$$

$$\frac{\partial\mathcal{L}}{\partial\dot{q}} = \begin{bmatrix}2 & 3\\3 & 4\end{bmatrix}\begin{bmatrix}\dot{q_1}\\\dot{q_2}\end{bmatrix} \qquad \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}} = \begin{bmatrix}2 & 3\\3 & 4\end{bmatrix}\begin{bmatrix}\ddot{q_1}\\\ddot{q_2}\end{bmatrix} \qquad \frac{\partial\mathcal{L}}{\partial q} = \begin{bmatrix}0\\-10\end{bmatrix}$$

$$\tau = \begin{bmatrix}\ddot{q_1}\\\ddot{q_2}\end{bmatrix} - \begin{bmatrix}0\\-10\end{bmatrix}$$