Problem Set 1

16-642 Manipulation, Estimation, and Control

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1 Question 1

A 4

B 1

C 6

D 1

E ∞, because the trunk is being modeled as an infinitely variable curve. However, in real life the DOF of an elephant trunk is probably somewhere between 6-8, due to the limited number of major muscle groups in the trunk (the internet varies in the actual number).

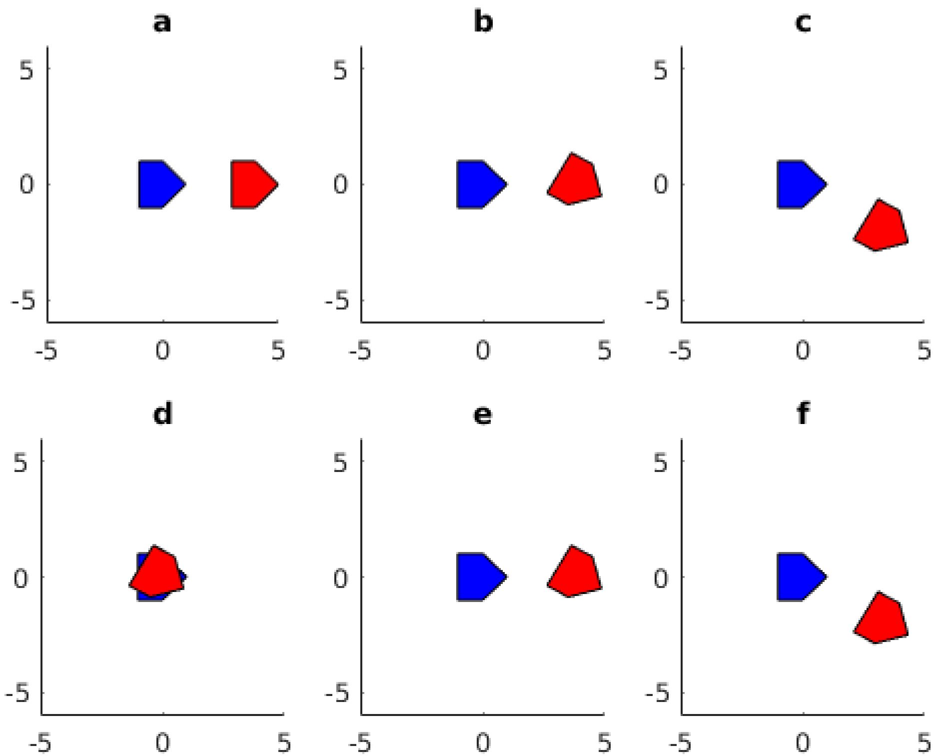
2 Question 2

$$H_0^1 = \begin{bmatrix} & \left[R_1^0\right]^{-1} & & \left[R_1^0\right]^{-1} d_1^0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3 Question 3

$$H_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & -1 & 5 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4 Question 4



5 Question 5

$$H_w^b = H_c^b H_s^c \left[H_t^w\right]^{-1}$$

6 Question 6

$$H_{Greenwhich}^{Pittsburgh} = H_{Center}^{Pittsburgh} H_{Greenwhich}^{Center}$$

$$H_{Center}^{Pittsburgh} = \left[H_{Pittsburgh}^{Center}\right]^{-1} \qquad H_P^C = H_{P_Z}^C H_{P_Y}^{P_Z} H_{P_X}^{P_Y}$$

$$H_{P_Z}^C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 6000 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_{P_Y}^{P_Z} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 80 & -\sin 80 & 0 \\ 0 & \sin 80 & \cos 80 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_{P_X}^{P_Y} = \begin{bmatrix} \cos 40.5 & 0 & -\sin 40.5 & 0 \\ 0 & 1 & 0 & 0 \\ \sin 40.5 & 0 & \cos 40.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_P^C = \begin{bmatrix} 0.76041 & 0 & 0.64945 & 0 \\ 0.63958 & 0.17365 & -0.74885 & 0 \\ -0.11278 & 0.98481 & 0.13204 & 6000 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_{Greenwhich}^{Center} = \begin{bmatrix} \cos -51 & 0 & -\sin -51 & 0 \\ 0 & 1 & 0 & 0 \\ \sin -51 & 0 & \cos -51 & 6000 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_{Greenwhich}^{Pittsburgh} = \begin{bmatrix} 0.76041 & 0 & 0.64945 & 0 \\ 0.63958 & 0.17365 & -0.74885 & 0 \\ -0.11278 & 0.98481 & 0.13204 & 6000 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \cos -51 & 0 & -\sin -51 & 0 \\ 0 & 1 & 0 & 0 \\ \sin -51 & 0 & \cos -51 & 6000 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.41993 & 0.67276 & 0.64691 & -452.46 \\ -0.80505 & 0.18266 & 0.7688 & -6215.4 \\ 0.47733 & -0.78771 & 0.43919 & 529.76 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7 Question 7

We know $p' = Rp$ and $p' = QPQ^*$, where $Q^* = [q_0, -q_1, -q_2, -q_3]^T$ and $P = [0, p_1, p_2, p_3]^T$
Therefore, $Rp = QPQ^*$ The Hamiltonian product of two quaternions is

$$(q_1, q_2, q_3, q_4)(p_1, p_2, p_3, p_4) = \begin{pmatrix} p_1\,q_1 - p_2\,q_2 - p_3\,q_3 - p_4\,q_4 \\ p_1\,q_2 + p_2\,q_1 + p_3\,q_4 - p_4\,q_3 \\ p_1\,q_3 + p_3\,q_1 - p_2\,q_4 + p_4\,q_2 \\ p_1\,q_4 + p_2\,q_3 - p_3\,q_2 + p_4\,q_1 \end{pmatrix}$$

For our specific P and Q, this becomes:

$$QP = \begin{pmatrix} -p_1\,q_1 - p_2\,q_2 - p_3\,q_3 \\ p_1\,q_0 - p_2\,q_3 + p_3\,q_2 \\ p_2\,q_0 + p_1\,q_3 - p_3\,q_1 \\ p_2\,q_1 - p_1\,q_2 + p_3\,q_0 \end{pmatrix}$$

This is also a quaternion, so:

$$QPQ^* =$$
$$\begin{pmatrix} q_1\,(p_1\,q_0 - p_2\,q_3 + p_3\,q_2) - q_0\,(p_1\,q_1 + p_2\,q_2 + p_3\,q_3) + q_2\,(p_2\,q_0 + p_1\,q_3 - p_3\,q_1) + q_3\,(p_2\,q_1 - p_1\,q_2 + p_3\,q_0) \\ q_0\,(p_1\,q_0 - p_2\,q_3 + p_3\,q_2) + q_2\,(p_2\,q_1 - p_1\,q_2 + p_3\,q_0) + q_1\,(p_1\,q_1 + p_2\,q_2 + p_3\,q_3) - q_3\,(p_2\,q_0 + p_1\,q_3 - p_3\,q_1) \\ q_0\,(p_2\,q_0 + p_1\,q_3 - p_3\,q_1) - q_1\,(p_2\,q_1 - p_1\,q_2 + p_3\,q_0) + q_2\,(p_1\,q_1 + p_2\,q_2 + p_3\,q_3) + q_3\,(p_1\,q_0 - p_2\,q_3 + p_3\,q_2) \\ q_0\,(p_2\,q_1 - p_1\,q_2 + p_3\,q_0) + q_1\,(p_2\,q_0 + p_1\,q_3 - p_3\,q_1) - q_2\,(p_1\,q_0 - p_2\,q_3 + p_3\,q_2) + q_3\,(p_1\,q_1 + p_2\,q_2 + p_3\,q_3) \end{pmatrix}$$

$$QPQ^* = \begin{pmatrix} 0 \\ p_1\,{q_0}^2 + 2\,p_3\,q_0\,q_2 - 2\,p_2\,q_0\,q_3 + p_1\,{q_1}^2 + 2\,p_2\,q_1\,q_2 + 2\,p_3\,q_1\,q_3 - p_1\,{q_2}^2 - p_1\,{q_3}^2 \\ p_2\,{q_0}^2 - 2\,p_3\,q_0\,q_1 + 2\,p_1\,q_0\,q_3 - p_2\,{q_1}^2 + 2\,p_1\,q_1\,q_2 + p_2\,{q_2}^2 + 2\,p_3\,q_2\,q_3 - p_2\,{q_3}^2 \\ p_3\,{q_0}^2 + 2\,p_2\,q_0\,q_1 - 2\,p_1\,q_0\,q_2 - p_3\,{q_1}^2 + 2\,p_1\,q_1\,q_3 - p_3\,{q_2}^2 + 2\,p_2\,q_2\,q_3 + p_3\,{q_3}^2 \end{pmatrix}$$

$$QPQ^* = \begin{pmatrix} 0 \\ \left({q_0}^2 + {q_1}^2 - {q_2}^2 - {q_3}^2\right)\,p_1 + (2\,q_1\,q_2 - 2\,q_0\,q_3)\,p_2 + (2\,q_0\,q_2 + 2\,q_1\,q_3)\,p_3 \\ (2\,q_0\,q_3 + 2\,q_1\,q_2)\,p_1 + \left({q_0}^2 - {q_1}^2 + {q_2}^2 - {q_3}^2\right)\,p_2 + (2\,q_2\,q_3 - 2\,q_0\,q_1)\,p_3 \\ (2\,q_1\,q_3 - 2\,q_0\,q_2)\,p_1 + (2\,q_0\,q_1 + 2\,q_2\,q_3)\,p_2 + \left({q_0}^2 - {q_1}^2 - {q_2}^2 + {q_3}^2\right)\,p_3 \end{pmatrix}$$

From this last form, we can see that the bottom three entries in the vector have a very similar form to a 3x3 matrix multiplied by $[p1, p2, p3]^T$. Discarding the 0 and pulling out p, we get

$$R = \begin{bmatrix} {q_0}^2 + {q_1}^2 - {q_2}^2 - {q_3}^2 & 2\,q_1\,q_2 - 2\,q_0\,q_3 & 2\,q_0\,q_2 + 2\,q_1\,q_3 \\ 2\,q_0\,q_3 + 2\,q_1\,q_2 & {q_0}^2 - {q_1}^2 + {q_2}^2 - {q_3}^2 & 2\,q_2\,q_3 - 2\,q_0\,q_1 \\ 2\,q_1\,q_3 - 2\,q_0\,q_2 & 2\,q_0\,q_1 + 2\,q_2\,q_3 & {q_0}^2 - {q_1}^2 - {q_2}^2 + {q_3}^2 \end{bmatrix}$$

8 Question 8

$$R_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This arrangement of linkages results in a transformaiton matrix similar to a XYX euler angle rotation.