RESOURCE LETTER

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This is one of a series of Resource Letters on different topics intended to guide college physicists, astronomers, and other scientists to some of the literature and other teaching aids that may help improve course contents in specified fields. No Resource Letter is meant to be exhaustive and complete; in time there may be more than one letter on some of the main subjects of interest. Comments on these materials as well as suggestions for future topics will be welcomed. Please send such communications to Professor Roger H. Stuewer, Editor, AAPT Resource Letters, School of Physics and Astronomy, 116 Church Street SE, University of Minnesota, Minneapolis, MN 55455. Reprints: When ordering request Resource Letter SH-1. Enclose 55 cents per copy (not in stamps) together with a stamped and self-addressed envelope and send to: Executive Office, American Association of Physics Teachers, Graduate Physics Building, SUNY at Stony Brook, Stony Brook, NY 11794.

Resource letter SH-1: superfluid helium

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The resource letter covers the general subject of superfluid helium and treats ³He and ³He-⁴He mixtures as well as ⁴He. No effort has been made to include the fascinating experiments on either solid helium or the equally fascinating work on adsorbed helium where the helium coverage is below that necessary for superfluidity. An earlier resource letter by C. T. Lane [Am. J. Phys. 35, 367 (1967)] may be consulted for additional comments on some of the cited earlier manuscripts, but the present work is self-contained and may be used independently. Many high-quality research reports have not been cited here. Rather, the author has tried in most cases to include works particularly readable or relevant. There is a relatively heavy emphasis on experimental references. The primary reason is that these works tend to be more generally readable. No doubt some works that might have been included, have not, and for this the author takes responsibility with apology. Articles selected for incorporation in a reprint volume (to be published separately by the American Association of Physics Teachers) are marked with an asterisk(*). Following each referenced work the general level of difficulty is indicated by E, I, or A for elementary, intermediate, or advanced.

I. INTRODUCTION

Discovered in the atmosphere of the sun in 1868, helium is one of the most remarkable elements known to mankind. Of all of the elements, it is the only one that cannot be solidified by merely reducing the temperature. This fact alone makes it a unique substance in nature. It remains a liquid even at absolute zero although it can be readily solidified by the application of pressure. The first liquefaction of helium was accomplished by Kamerlingh Onnes in 1908 at Leiden in The Netherlands and it was a number of years before workers began to realize that the liquid was, in fact, something really unusual. The interruption of dramatic progress in the field was caused in large measure by the discovery in 1911, also by Onnes, of superconductivity in metals. The beginning of the recognition that there was a lot more to be learned about helium probably stems from the observations of Onnes and Boks in 1924 that the density of helium attains a maximum near 2.3 K. Equally interesting was the early work of Dana and Onnes that showed the unexpected result that the specific heat of helium attains very large values near 2.2 K. The results were so surprising, in fact, that they were not published and not until 1930 was the observation confirmed elsewhere and published. Keesom noted that the temperature dependence of the specific heat resembled the Greek letter lambda and introduced the notations He I for the fluid at higher temperatures and He II for the liquid at lower temperatures.

Subsequent work showed the heat conduction to be quite anomalous and, finally, serious attention was focused on what was by 1936 an entirely perplexing situation. At this point, however, only a few of the truly spectacular properties of the fluid had yet been uncovered, but the pace of progress took a dramatic turn.

II. GENERAL PROPERTIES OF ⁴He AND EARLY HISTORY

In the span of forty odd years since the beginnings of a substantial understanding of superfluidity in helium, there have been a number of painstaking and often ingenious steps. As we know it today, helium in its liquid state undergoes a transition to a fluid with unique properties as the temperature is lowered below 2.17 K (T_1) . Below this temperature the fluid can be considered to be made up of two independent fluids that interpenetrate without (in many situations) apparent interaction. In this model, called the two-fluid model, one considers the density to be given by the sum of a normal component density and a superfluid component density. Briefly, the normal component of density ρ_n possesses viscosity and entropy while the superfluid component of density $\rho_{\rm c}$ is inviscid and has no entropy. For a wide range of situations, this simple picture provides a valuable aid for the understanding of experimental phenomena. As we shall see later, one can place a more microscopic and more elegant basis behind the two-fluid model. The experiments that led to the development of this model were, for the most part, all conducted in a remarkably short period of time and published almost exclusively in one famous volume of Nature. We begin with a very readable 'quick survey" of the early work.

- II.1. "Superfluidity," E. M. Lifshitz, Sci. Am. 198 (6), 30 (1958). Very readable introduction to the basic properties of superfluid ⁴He including its high thermal transport, specific heat at T_{λ} , and the concept of second sound. The mobile film is mentioned as is the famous Andronikashvili experiment to measure the density of the normal component. Written well before the observation of superfluidity in ³He. (E)
- II.2. "Further Experiments with Liquid Helium," H. K. Onnes and J. D. A. Boks, Leiden Comm. 170b (1924). A detailed study of the density of helium that clearly shows a maximum as a function of temperature at approximately 2.3 K. The results confirmed the much earlier results [H. K. Onnes, Leiden Comm. 119 (1911)] that such a maximum in the density was present. Given that helium is a "simple" element these and the previous measurements were regarded as startling. (E)
- II.3. "Two Different Liquid States of Helium," W. H. Keesom and M. Wolfke, Leiden Comm. 190b (1927). Suggestion that some sort of a liquid to liquid transition takes place in liquid helium at a specific temperature and pressure. The authors refer to the liquid at higher temperatures as He I and that at lower temperatures as He II. This terminology remains in use today. (E)
- II.4. "Specific Heat of Liquid Helium," W. H. Keesom and K. Clusius, Leiden Comm. 219e (1930). Observation of a spectacular peak in the specific heat of liquid helium at 2.19 K. These very careful measurements confirmed clearly Keesom's earlier conclusion that something extraordinary takes place in helium and showed that heat capacity measurements offer a particularly graphic signature of the transition. (E)
- II.5. "On the Heat Conductivity of Liquid Helium," W. H. Keesom and A. P. Keesom, Physica 3, 359 (1936). Observation that the thermal conductivity of liquid helium at 1.4 and 1.75 K was 200 times larger than that of copper at room temperature. The authors pointed out that He II was therefore the best thermal conductor known and termed it supra-heat-conducting. (E)
- II.6. "Viscosity of Liquid Helium Below the λ Point," P. Kapitza, Nature 141, 74 (1938). Observed the viscosity to decrease by more than a factor of 10^3 upon cooling through the lambda transition. His observation that the liquid helium viscosity was 10^4 times smaller than hydrogen gas (the substance previously observed to have the smallest viscosity) lead him to suggest liquid helium below T_λ might be called a "superfluid." In his apparatus the conclusion that helium had a tiny viscosity was deduced by observing the rate of flow of the liquid through two optical flats pressed together. Kapitza proposed the erroneous suggestion that turbulent flow might explain the large heat flow seen by others in tubes. (E)
- II.7. "Flow of Liquid Helium II," J. F. Allen and A. D. Misener, Na-

- ture 141, 75 (1938). Study of the rate of flow of ⁴He through two capillaries of different size. Measured an upper limit to the viscosity consistent with Kapitza's. Suggested that the helium "slips" over the surface of the tube (i.e., doesn't interact with it). Concluded that turbulent flow cannot account for the high thermal conductivity seen in other experiments. (E)
- *II.8. "New Phenomena Connected with Heat Flow in Helium II," J. F. Allen and H. Jones, Nature 141, 243 (1938). An outgrowth of studies on the thermal conductivity of helium in tubes. This paper describes the first observations of the fountain effect in He II in which a mechanical pressure difference between two reservoirs connected by a superleak is supported by a difference in temperature between the reservoirs. (E)
- II.9. "Transfer of Helium II on Glass," J. G. Daunt and K. Mendelssohn, Nature 141, 911 (1938) (supplement). Systematic study of the flow of helium out of a beaker by means of the moving film. Showed that vapor condensation was not responsible by the use of wicks to increase the surface area available to the flow. Observed drops on the bottom of the outside of the container. (E)
- II.10. "Experiments with Liquid Helium II," A. K. Kikoin and B. G. Lasarew, Nature 141, 912 (1938) (supplement). Study of the temperature of the upper end of a glass tube immersed in liquid helium. Supports the idea of a film of helium atoms. Discusses evaporation of the film. The work provides a rough estimate of the thickness of the film. High thermal conductivity in the thin film seems to rule out the convection idea of Kapitza. (E)
- *II.11. "The λ Phenomenon of Liquid Helium and the Bose-Einstein Degeneracy," F. London, Nature 141, 643 (1938). Suggestion that the superfluid transition in ⁴He is connected with the idea of Bose-Einstein condensation. This idea was the genesis for our present picture of the superfluid transition, although a true microscopic picture is not yet at hand. Part of the difficulty concerns the presence of interactions among the atoms. London was aware even then that this would present a difficulty. (E)
- II.12. "Transport Phenomena in Helium II," L. Tisza, Nature 141, 913 (1938) (supplement). The beginning of the two-fluid model of He II. Although not yet fully developed, this paper clearly discussed how the "condensate" atoms differ from the "excited-state" atoms. The various viscosity experiments are hence understood and so, too, the fountain effect experiments of Allen and Jones. Predicted the inverse of the fountain effect called the mechanocaloric effect. (E)
- *II.13. "Surface Transport in Liquid Helium II," J. G. Daunt and K. Mendelssohn, Nature 143, 719 (1939). Experimental observation of the mechanocaloric effect. The flow of helium results in the creation of a temperature difference. The experiments confirmed Tisza's predictions. (E)
- *II.14. "The Study of Heat Transfer in Helium II," P. L. Kapitza, J. Phys. (USSR) 4, 181 (1941). A remarkably thorough and ingenious study of the flow of heat in tubes filled with superfluid helium. Several experiments, including the famous "Kapitza Spider" are described in great clarity. As a result of this work Kapitza concluded that the huge values of thermal conductivity observed for He II were the result of the flow of fluid down essentially the full bore of the channel. To explain how the helium lost at one end could be replaced, Kapitza supports his earlier suggestion of a return flow of helium in a surface layer. Although incorrect in detail, this suggestion of counterflow in a channel was perceptive. (E)
- II.15. "The Problem of Liquid Helium—Some Recent Aspects," J. G. Daunt and R. S. Smith, Rev. Mod. Phys. 26, 172 (1954). An extensive and quite complete review of work on superfluid ⁴He up until mid-1953. No mention of the solid phase or ³He was intended and none is given. The text is very readable. (I)

III. TOWARD A MICROSCOPIC UNDERSTANDING OF ⁴He

The seminal contribution to our more microscopic understanding of the nature of liquid helium stems from the

work of Landau in the 1940s. In this work it was assumed that at absolute zero superfluid helium can be considered to be an inert, totally noninteracting fluid and all of the thermal properties encountered at higher temperatures are due to elementary excitations or quasiparticles called phonons and rotons that populate the inert fluid. The phonons are excitations of long wavelength; the rotons are more localized. Thus in these terms, the transport of heat in the fluid is merely the flow of these excitations. Given the density of these excitations as a funtion of temperature, one can compute ρ_s and ρ_n (at least far from T_{λ}) and hence make a connection to the two-fluid model. Thus one can consider the excitation model of Landau as extending significantly and providing a microscopic basis for the two fluid model of Tisza. A number of classic early experiments confirmed several of the predictions of Landau.

- III.1. "Superfluidity and 'Quasi-Particles,' " F. Reif, Sci. Am. 203 (5), 138 (1960). Well-written description (and overview) of the elementary excitation picture of superfluid helium from the point of view of understanding the basic experiments. The term quasiparticle is discussed and given intuitive meaning. (E)
- III.2. "The Theory of Superfluidity of Helium II," L. D. Landau, J. Phys. (USSR) 5, 71 (1941). A classic work in which the details of the two-fluid model are enunciated in substantial detail. Liquid helium is considered to consist of a superfluid component and a normal component. The normal component consists of excitations called phonons and rotons. Given the temperature dependence of the densities of these excitations, the specific heat and ρ_n are calculated. A moment of inertia technique for measuring ρ_n is suggested, the second sound is predicted, the viscosity paradox and fountain effect are explained and heat flow in capillaries is explained. The paper is our present understanding in many regards. The one error, the shape of the dispersion curve, was corrected later by Landau [J. Phys. (USSR) 11, 91 (1947)]. (E)
- *III.3. "A Direct Observation of Two Kinds of Motion in Helium II," E. Andronikashvili, J. Phys. (USSR) 10, 201 (1946). A classic experimental confirmation of the essential validity of the two-fluid model of He II. In the experiment the period of torsional oscillation of a stack of closely spaced aluminum plates immersed in He II was measured. A decrease in temperature results in a decrease in the amount of normal fluid present and hence a decrease in the moment of inertia of the apparatus as less helium "sticks" to the plates. A measurement of the temperature dependence of ρ_n results. (E)
- III.4. "Radiation of Sound in Helium II," E. Lifshitz, J. Phys. (USSR) 8, 110 (1944). Suggestion that second sound in He II can be most efficiently radiated by means of a source whose temperature is made to vary. This idea led to the first observation of second sound. (E)
- *III.5. "Determination of the Velocity of Propagation of the Second Sound in Helium II," V. Peshkov, J. Phys. (USSR) 10, 389 (1946). Landau's theory of superfluidity predicted that not only would ordinary sound propagate in He II but also a new wave mode. This new wave mode was a temperature wave and termed the second sound. Peshkov clearly describes the thermal generation of the second sound and its detection with a thermometer. Both standing and traveling waves were measured. The temperature dependent values of the velocity were analyzed to yield ρ_n/ρ values in agreement with those of Andronikashvili. (E)
- III.6. "Second Sound in Liquid Helium II," C. T. Lane, H. A. Fairbank, and W. M. Fairbank, Phys. Rev. 71, 600 (1947). Measurements of the velocity of second sound as a function of temperature. A novel technique, which employed a microphone in the ⁴He vapor above the He II bath, was used. Second sound resonances in the superfluid resulted in pressure waves in the vapor. (E)
- III.7. "On the Theory of Superfluidity of Helium II," L. D. Landau, J. Phys. (USSR) 11, 91 (1947). Observation that the measured velocity of second sound demands a change in the elementary excitation spectrum initially proposed. Rather than a spectrum with

- two branches, it is now proposed that there is only a single branch. As later measured by neutron scattering experiments the single branch spectrum is essentially correct. (I)
- III.8. "The Theory of Liquid Helium," L. Tisza, Phys. Rev. 72, 838 (1947). Detailed phenomenological description of the two-fluid model of He II. The paper carefully describes the assumptions inherent in the model. A discussion of second sound is presented. (E)
- III.9. "On the Theory of Superfluidity," N. Bogoliubov, J. Phys. (USSR) 11, 23 (1947). A classic theoretical paper in which a gas of Bose particles of zero spin is considered for the case of weak interactions. The excitation spectrum is obtained. (E)
- III.10. "Application of Quantum Mechanics to Liquid Helium," R. P. Feynman, in Progress in Low Temperature Physics (North-Holland, Amsterdam, 1955), Vol. 1, p. 17. Extremely readable and intuitive discussion of the fundamental reasons why the assumptions of Landau are correct. Helium in rotation and the quantization of circulation are also discussed. The article is one of the best "seat of the pants" discussions to be found in the literature of He II. (See also article V.9.) (E)
- III.11. "Atomic Theory of the Two Fluid Model of Liquid Helium," R. P. Feynman, Phys. Rev. 94, 262 (1954). Fundamental discussion of the form of the wave function appropriate to liquid helium. By use of the variational principle an expression for the elementary excitation spectrum is obtained. A discussion of the connection to the two-fluid model is presented. (I)
- III. 12. "Energy Spectrum of the Excitations in Liquid Helium," R. P. Feynman and M. Cohen, Phys. Rev. 102, 1189 (1956). Presentation of an improved wave function for liquid helium. The improvement in this wave function over Feynman's previous one is the inclusion of a term to represent a localized excitation. The physical description of this new wave function gives us our present picture of a roton. (A).
- III.13. "Inelastic Scattering of Thermal Neutrons from Liquid Helium," R. A. Cowley and A. D. B. Woods, Can. J. Phys. 49, 177 (1971). Detailed discussion of the elementary excitation spectrum in He II as obtained by inelastic neutron scattering. The paper essentially summarizes the work at the Chalk River Lab and is nearly a complete treatment of the excitation spectrum. (I)
- III. 14. "Bose-Einstein Condensation and Liquid Helium," O. Penrose and L. Onsager, Phys. Rev. 104, 576 (1956). Theoretical discussion of Bose-Einstein condensation in the case of interacting particles. A first principle argument suggests that helium II represents a Bose-Einstein condensed state with 8% of the atoms in the condensed phase at absolute zero. The work was a pivotal theoretical development in the understanding of superfluidity. (A)

The question of what fraction of the atoms in helium are actually a part of the Bose-Einstein condensate has been asked repeatedly since the original suggestion of London. Current theoretical predictions hover near 10% at absolute zero, which is quite close to the 8% originally suggested by Penrose and Onsager. The experimental situation is quite complicated. It is clear from a number of accounts that as helium is cooled through T_{λ} , there is a loss in spatial order as might be expected of a system that picks up an increase in momentum order. Whether this is actually due to the formation of the Bose-Einstein condensate remains to be seen.

The spatial order present in an atomic or molecular system gives rise to a nontrivial diffraction pattern. That constituent of the diffraction pattern directly related to the real-space spatial order is given the name the liquid structure factor. An increase of structure in the liquid structure factor is the result of an increase in the spatial order in the system under study. Hence, a study of the diffraction pattern allows one to gain a measure of the spatial order in the system under examination.

III.15. "Effect of the λ Transition on the Atomic Distribution in Liquid Helium by Neutron Diffraction," D. G. Henshaw, Phys. Rev. 119, 9 (1960). Measurements of the spatial order in liquid helium above and below the lambda transition revealed a lessening of the spatial order upon cooling through the transition. This behavior has been confirmed in detail by subsequent work with both x rays and neutrons. (I)

The degree of spatial order in liquid helium provides a powerful testing ground for theoretical advances as they are made. A detailed microscopic theory of the ⁴He superfluid state still does not exist in spite of decades of progress. The problem is made difficult by the interactions among the atoms. Within the past 15 years or so numerical calculations have resulted in an improvement in agreement between calculations and experiment both from the point of view of the equilibrium energy per atom in the system and the spatial structure present.

- III.16. "Ground State of Liquid 4He," W. L. McMillan, Phys. Rev. 138, A442 (1965). Monte Carlo calculation of the ground state of 4He. The work marks the beginning of the modern era of numerical calculations on the nature of liquid helium. (I)
- III.17. "Energy and Structure of the Ground State of Liquid 'He," C. C. Chang and C. E. Campbell, Phys. Rev. B 15, 4238 (1977). Most extensive use of the variational technique to compute the ground-state properties of 'He liquid. The calculation is conducted as a function of density for the case of two different potentials. (A)
- III.18. "Properties of Liquid and Solid ⁴He," P. A. Whitlock, D. M. Ceperley, G. V. Chester, and M. H. Kalos, Phys. Rev. B 19, 5598 (1979). An extensive Monte Carlo computation of the properties of ⁴He at absolute zero in both the liquid and solid phases. Within the context of the potential chosen (Lennard–Jones) the calculation is essentially exact and hence the results serve as a test for other theoretical developments. (A)
- III. 19. "Neutron-Diffraction Study of the Static Structure Factor and Pair Correlations in Liquid ⁴He," E. C. Svensson, W. F. Sears, A. D. B. Woods, and P. Martel, Phys. Rev. B 21, 3638 (1980). A thorough investigation of the structure in liquid helium at saturated vapor pressure that shows clearly the evolution as a function of temperature. The scattering data is transformed to reveal directly the spatial order in the helium. A detailed comparison is made to relevant theory at the lowest experimental temperature (1 K). (E)

IV. IONS IN HELIUM

Ions placed in He II constitute an effective technique to probe the superfluid since they interact with the excitations that are present. Thus measurements of the mobility offer an effective measure of the density and scattering cross section of those excitations. The actual structure of the positive and negative ions was an early question that has been answered by picturing the negative ion to be an electron distributed inside a bubble of substantial size due to its large zero-point motion. The positive ion, on the other hand, is considered to be a snowball of sorts. The high pressures caused by electrostriction near the positive ion result in formation of a localized region of solid helium.

- *IV.1. "Ions in Liquid Helium," K. R. Atkins, Phys. Rev. 116, 1339 (1959). Proposal that the positive helium ion is, in fact, surrounded by a "snowball" of helium atoms frozen to it by the strong effects of electrostriction. The account of how this comes about is clear and quite readable. (I)
- IV.2. "Experimental Behavior of Ionic Structures in Liquid Helium II," G. Careri, U. Fasoli, and F. S. Gaeta, Nuovo Cimento 15, 774 (1960). Concise but readable account of the experiments that led to the accepted physical picture of the positive ion as a "snow-

- ball" and the negative ion as a negative charge in a large bubble (created by zero-point motion).
- IV.3. "Study of Superfluidity in Liquid He by Ion Motion," F. Reif and L. Meyer, Phys. Rev. 119, 1164 (1960). A study of the mobility of positive and negative ions in He II by the use of a time-of-flight method, which is described in detail. From the temperature dependence of the mobility it can be deduced that the ion mobility is limited by scattering from rotons. (E)
- IV.4. "Heat Flush and Mobility of Electric Charges in Liquid Helium," G. Careri, F. Scaramuzzi, and J. O. Thomson, Nuovo Cimento 13, 186 (1959). Experiments in which ions in helium were shown to interact with the elementary excitations (normal component) of the fluid and be dragged along with the normal fluid flow. The experiments constitute a clear confirmation of earlier conclusions by others that ³He impurities are carried by the flow of ρ_n .

V. 4He IN ROTATION, VORTICES, AND OUANTIZATION

The two-fluid model had spectacular early success in explaining and predicting a number of phenomena. One assumption built into the model was that the superfluid must be curl-free everywhere. Thus the free surface of helium II in a rotating bucket would have a meniscus whose shape is determined by the amount of normal fluid present and hence the shape should be temperature dependent. In a classic experiment Osborne showed that this was in fact not the case. Rather, the free surface of rotating He II formed a meniscus with a temperature-independent shape exactly as predicted by classical mechanics for a classical fluid. This observation is consistent with the suggestion that tornadolike structures called vortices must be present in rotating He II. These vortices were further suggested by Onsager and Feynman to have only quantized circulation states. That is, the integral of the velocity along any closed path that encircles the vortex core, must be an integer multiple of h/m. Subsequent work by Vinen and Rayfield and Reif established that this was the case.

- *V.1. "The Rotation of Liquid Helium II," D. V. Osborne, Proc. Phys. Soc. London 63, 909 (1950). Startling observation that the meniscus between the vapor and rotating free surface of He II was parabolic in shape and in agreement with predictions for a classical fluid. The results obtained were independent of temperature and in disagreement with the assumption of the two-fluid model that curl $v_x = 0$ and directly led to the idea that vortices were present in rotating superfluid helium. (E)
- *V.2. "The Detection of Single Quanta of Circulation in Liquid Helium II," W. F. Vinen, Proc. R. Soc. London A 260, 218 (1961). Detailed experimental observations that the circulation ($\oint v \cdot d I$) around a closed path in He II is quantized at the value h/m. In these experiments the precession of the plane of vibration of a vibrating wire is used to determine the circulation and, although a variety of values are seen, there is stability for the value h/m. [See also, W. F. Vinen, Nature 181, 1524 (1958).] (E)
- V.3 "Observation of Quantization of Circulation in Superfluid 4He," P. W. Karn, D. R. Starks, and W. Zimmermann, Jr., Phys. Rev. B 21, 1797 (1980). Measurements of the circulation in rotating 4He are reported using the vibrating wire technique originally introduced by Vinen. The results display in a beautiful and picturesque way the quantization of circulation in 4He. Discrete quantum steps are clearly observed.
- V.4. "Quantized Vortex Rings in Superfluid Helium," F. Reif, Sci. Am. 211 (6), 116 (1964). Elementary account of the experiments that resulted in the identification of charged vortex rings in superfluid helium. The quantization of circulation is discussed. (E)
- *V.5. "Quantized Vortex Rings in Superfluid Helium," G. W. Rayfield and F. Reif, Phys. Rev. 136, A1194 (1964). Extensive discussion

205

- of the creation, identification, and properties of charged, quantized vortex rings in superfluid helium. It is shown that at low temperatures the motion of negative ions causes vortex rings to be created and that these rings have circulation h/m. Various scattering experiments involving the rings are also described. [See also G. W. Rayfield and F. Reif, Phys. Rev. Lett. 11, 305 (1963).]
- V.6. "Size of Quantized Vortex Rings in Liquid Helium II," G. Gamota and T. M. Sanders, Jr., Phys. Rev. A 4, 1092 (1971). Detailed account of measurements of the size of quantized vortex rings. The measurements were conducted by a study of the transmission of charged vortex rings through rectangular mesh patterns. A geometric interpretation allowed a measurement of the ring diameter. The results for the quantum of circulation confirm Rayfield and Reif's result
- V.7. "Observations on Single Vortex Lines in Rotating Superfluid Helium," R. E. Packard and T. M. Sanders, Jr., Phys. Rev. A 6, 799 (1972). Detailed discussion of the detection of individual rectilinear vortex lines in He II. The technique used involves the trapping of charge on the vortex lines followed by a measurement of the total amount of charge trapped. The total charge trapped is found to be quantized and a function of the rotation speed of the apparatus.
- *V.8. "Observation of Stationary Vortex Arrays in Rotating Superfluid Helium," E. J. Yarmchuk, M. J. V. Gordon, and R. E. Packard, Phys. Rev. Lett. 43, 214 (1979). Experimental observation of stationary vortex line patterns in rotating He II. The vortices are contained in a rotating bucket and observed in top view by means of an imaging process that involves decorating the vortices with ions and then sweeping the charge to a phosphor screen. As the rotation rate is increased, a hierarchy of patterns involving first one, then two, etc., vortices is observed in agreement with theoretical predictions.

Superfluid coupling between two reservoirs of He II connected by a weak link has been sought for in ⁴He since the observation of the Josephson effect in superconductivity. The requirements for such tunneling may include a link between the two superfluid reservoirs that is of dimensions comparable to the coherence length in the superfluid system under study. In superconductivity this dimension is of the order of microns and hence such experiments are relatively easy. In helium, on the other hand, the relevant dimension is of the order of an Angstrom. In spite of this formidable potential requirement, a number of workers have attempted to do the experiments. In the early work an orifice of several microns provided the link between the two reservoirs. Stimulated by a sound source near the orifice, apparently quantized level differences between the two coupled reservoirs were seen. Presently, experiments of this nature are not considered as evidence for the Josephson effect sought for, due primarily to the observation of acoustical resonances in several experiments. [See, for example, P. Leiderer and F. Pobell, Phys. Rev. A 7, 1130 (1973).1

V.9. "Considerations on the Flow of Superfluid Helium," P. W. Anderson, Rev. Mod. Phys. 38, 298 (1966). A detailed discussion of the dynamics of superfluid flow in which the fundamental basis is the existence of a macroscopic wave function and its associated phase. An elegant development and understanding of macroscopic interference phenomena is presented based on the Josephson equation and phase slippage. Although mathematical ideas are crucial, there is a genuine appeal to physical visualization and analogy. (I)

VI. ⁴He FILMS AND CONSTRICTED GEOMETRIES

All materials present in the vapor phase above a solid surface will be in equilibrium with like atoms adsorbed onto the solid. A closed container, which has helium gas in it, will also have a coating of helium on the surfaces exposed to the gas. What is remarkable in the case of helium is that for temperatures not too near the transition temperature and for thicknesses above a few atomic layers the film is highly mobile. This is understandable in the context of the two-fluid model since, while the normal component is clamped to the walls by its viscosity, the superfluid component is free to move, feeling neither the walls nor the normal component through which it passes without interaction. Perhaps even more remarkable than the fact that the films are mobile is the fact that wave motions can be supported on the surface of the film, even if the film is only a few layers thick. These wave motions are known as third sound and are analogous to long-wavelength (tidal) waves on the ocean.

It is customary to use the terms saturated and unsaturated when referring to these films. A saturated film is a film on a surface in a container that also contains a free surface on bulk liquid. In the unsaturated case the pressure in the container is lower than saturated vapor pressure and no bulk liquid is present. At a given temperature the unsaturated film thickness is a function of the pressure.

- VI.1. "On the 'Film' Phenomenon of Liquid Helium II," B. V. Rollin and F. Simon, Physica 6, 219 (1939). A belated publication of strong evidence that a mobile helium film coats the walls of containers containing helium. The idea that such a film exists had been proposed earlier [Physica 3, 226 (1936)]. Shortly afterward these authors concluded that the film was, in fact, mobile but circumstances interrupted the publication of the evidence. (E)
- *VI.2. "The Transfer Effect in Liquid He II: I. The Transfer Phenomena; II. Properties of the Transfer Film," J. G. Daunt and K. Mendelssohn, Proc. R. Soc. London A 170, 423, 439 (1939). Classic experimental observations that directly showed the presence and effects of a mobile superfluid film. The experiments are remarkably clear and established many of the fundamental properties of the film. (E)
- VI.3. "Some Experiments on Flow in the Unsaturated Helium II Film," E. Long and L. Meyer, Phys. Rev. 85, 1030 (1952). Observation that an unsaturated film of He II also demonstrated superfluidity. The work was notable in that it demonstrated that superflow started at a temperature that was directly related to the apparent film thickness at least for some of the measurements. The workers were not convinced of all of their observations. (E)
- VI.4. "Transfer of the Unsaturated Helium II Film," D. F. Brewer and K. Mendelssohn, Proc. R. Soc. London A 260, 1 (1961). Belated report of detailed studies of the onset of superfluidity in unsaturated He II films. Clearly observed was the suppression in temperature of the onset of superflow as the film thickness was reduced. The work concluded that the onset was independent of the nature of the substrate. (E)
- *VI.5. "Third and Fourth Sound in Liquid Helium II," K. R. Atkins, Phys. Rev. 113, 962 (1959). Prediction of third and fourth sound as possible propagating wave modes in He II. In both cases the normal component remains fixed while the superfluid component oscillates. Third sound is a wave that propagates on a thin superfluid film; fourth sound involves the oscillation of the superfluid component in a porous medium in which the normal component remains fixed due to its viscosity. (I)
- VI.6. "Third Sound in Liquid Helium Films," C. W. F. Everitt, K. R. Atkins, and A. Denenstein, Phys. Rev. 136, A1494 (1964). The detailed description of the experiments that resulted in the first experimental observation of third sound. Attenuation of the third sound was observed which was far in excess of expectations. [See also, Phys. Rev. Lett. 8, 161 (1962).] (E)
- VI.7. "Third Sound," K. R. Atkins and I. Rudnick, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland, Amsterdam, 1970), Vol. 6, p. 37. A very readable review of the early work on third sound in superfluid helium films. The theory

fundamental to third sound is reviewed and the experimental techniques and basic experimental properties are discussed. Although now out of date in many respects, the article constitutes a good introduction to the subject. (I)

- *VI.8. "Fourth Sound in He II," I. Rudnick and K. A. Shapiro, Phys. Rev. Lett. 9, 191 (1962). First experimental observation of the propagation of fourth sound. Mylar transducers acted as sources and detectors of the pressure wave in the superfluid component and measurements of the velocity as a function of temperature were carried out. (E) [See also Phys. Rev. 137, A1383 (1965).]
- VI.9. "Superfluid Fifth Sound," I. Rudnick, J. Maynard, G. Williams, and S. Putterman, Phys. Rev. B 20, 1934 (1979). Many propagating modes in helium have been given the name "sound." In this article a new mode, termed fifth sound, is proposed. Fifth sound is a propagating thermal wave that can be present in a superleak if there is a pressure-released boundary condition. The relation of fifth sound to other sound modes is discussed. (I)

There has also been a great deal of work done on the free surface of liquid helium. This includes surface tension, interfacial tension in mixtures, and surface waves. The following review represents an excellent survey of the subject.

VI.10. "The Free Surface of Liquid Helium," D. O. Edwards and W. F. Saam, Progress in Low Temperature Physics, edited by D. F. Brewer (North-Holland, Amsterdam, 1978), Vol. VIIA, p. 283. A detailed review of the free surface of liquid ⁴He and ³He-⁴He mixtures. Both experimental and theoretical aspects are discussed. (I).

VII. PERSISTENT FLOW IN 4He

Perhaps the most striking experimental manifestation of superfluidity in helium is the observation of persistent flow. In such experiments the fluid is set into motion and it can remain in apparently the same flow state for immeasurably long times. The earliest observation of persistent flow was probably the already mentioned work of Vinen in which long-lived circulation states were observed in the flow of helium around a wire in bulk helium. Persistent flow has also been observed both in bulk fluid contained in a porous material as well as in helium films. In these two systems a number of spectacular properties have been observed. For example, Reppy and his colleagues showed that the angular momentum of the circulating superfluid was temperature dependent. That it must be so is easy to see. For a given stable circulation state characterized by a quantum number, the angular momentum is directly proportional to the value of the temperature-dependent density of the superfluid component. In the case of a superfluid film Telschow and Hallock showed that the velocity of the persistent flow state for He II flowing around a ring was unchanged by changes of the film thickness (hence amount of superfluid present) as large as a factor of two.

- *VII.1. "Persistent Currents in Superfluid Helium," J. D. Reppy and D. Depatie, Phys. Rev. Lett. 12, 187 (1964). Observation of macroscopic persistent currents as a function of temperature. The experiments successfully tested the two-fluid model prediction that the angular momentum of rotating superfluid helium should be a function of temperature. (E)
- VII.2. "Flow of Superfluid Helium in a Porous Medium," J. B. Mehl and W. Zimmermann, Jr., Phys. Rev. 167, 214 (1968). A detailed report of measurements of the flow of superfluid helium in a sphere filled with fine powder. Use of a gyroscopic technique results in the conclusion that stable persistent flow can exist in such a medium. Warming and cooling the apparatus shows that the angular momentum is a function of the temperature as would be expected on the basis of the two-fluid model. Also described in detail are the results of torsional oscillation experiments in the same apparatus. [Phys. Rev. Lett. 14, 815 1965).] (I)
- VII.3. "Temperature Dependence of the Superfluid Healing Length,"

- R. P. Henkel, E. N. Smith, and J. D. Reppy, Phys. Rev. Lett. 23, 1276 (1969). Experimental evidence from a gyroscopic technique that shows that the addition of mass to a circulating persistent film current causes a proportional increase in the angular momentum of the flowing film. (I)
- VII.4. "Intrinsic Critical Velocities in Superfluid Helium," J. S. Langer and J. D. Reppy, in Progress in Low Temperature Physics, edited by C. J. Gorter (North-Holland, Amsterdam, 1970), Vol. 6, p. 1. A review of the general notion that the flow of superfluid helium is a condition of metastability. The general rule for superfluid deceleration is presented in an appealing and highly readable way. (I)
- VII.5. "Superfluidity of Thin 'He Films," M. H. W. Chan, A. W. Yanof, and J. D. Reppy, Phys. Rev. Lett. 32, 1347 (1974). Consider a helium film in persistent flow at low temperature. As the temperature is raised beyond a certain value, T_c , there is observed a dramatic lessening of the angular momentum associated with the persistent flow. These gyroscopic measurements illustrate how T_c depends on film thickness. (I)
- *VII.6. "Stability of Persistent Currents in Unsaturated Superfluid ⁴He Films," K. L. Telschow and R. B. Hallock, Phys. Rev. Lett. 37, 1484 (1976). (E) Observation of persistent currents and the stability of circulation in unsaturated films. Persistent currents are generated without rotation by the use of heaters. The stability of the flow is observed to be an extremely sharp function of film thickness. [See also, D. T. Ekholm and R. B. Hallock, Phys. Rev. Lett. 42, 449 (1979).]

VIII. SUPERFLUID ⁴He IN TWO DIMENSIONS

The nature of superfluidity in two dimensions has been a subject of discussion ever since the early experiments on helium films. Numerous experiments, both in films and in porous restricted geometries, have been carried out. Many such experiments have sought to characterize the behavior of the superfluid density as a function of temperature and establish when superflow ceased as the temperature was raised. In recent years there has been a substantial rebirth of interest in this general area due to the theoretical developments of Kosterlitz and Thouless. Based on this theoretical work, Nelson and Kosterlitz showed that ρ_s should, under static conditions, exhibit a finite universal jump at a characteristic temperature rather than the smooth behavior observed in three dimensions. Well before these predictions Rudnick and co-workers had observed an abrupt cessation of superfluid properties of a film with a finite value of ρ_s . The elegant subsequent experiments of Bishop and Reppy showed clearly the abrupt nature of the jump in ρ_s as well as the enhanced dissipation in the superfluid that accompanies the jump.

- VIII.1. "Ordering, Metastability and Phase Transitions in Two-Dimensional Systems," J. M. Kosterlitz and D. J. Thouless, J. Phys. C 6, 1181 (1973). A pivotal contribution in the field of two-dimensional physics that has stimulated a great deal of activity. In this work the authors show that the adoption of a new definition of long-range order allows superfluidity to exist theoretically in two dimensions and predicts a transition temperature. The work also shows that the value of the superfluid mass per unit area must be finite at the transition temperature.
- *VIII.2. "Universal Jump in the Superfluidity Density of Two-Dimensional Superfluids," D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. 39, 1201 (1977). Prediction that under static conditions the superfluid density in ⁴He films will undergo a universal jump at a critical temperature. In particular, it is predicted that at the transition the relation $m^2 k_B T / \hbar^2 \sigma_s = \pi/2$ will hold. (Here σ_s is the superfluid density per unit area.) (A)
- VIII.3. "Superflow in Helium Films: Third-Sound Measurements." R.

- S. Kagiwada, J. C. Fraser, I. Rudnick, and D. Bergman, Phys. Rev. Lett. 22, 338 (1969). Observations on the velocity of third sound as a function of temperature and film thickness. The experiments led to the conclusion that ρ_s/ρ is finite and large just before the disappearance of the third sound signal with decreasing film thickness. It was concluded, incorrectly, that ρ_s/ρ is finite after superflow stops. A rejected conclusion, that ρ_s/ρ discontinuously jumps to zero, is closer to the truth. In retrospect the experiments constitute the first test of the Kosterlitz-Nelson-Thouless predictions for He II. (I)
- VIII.4. "Superfluid Fraction in Thin Helium Films," M. Chester and L. C. Yang, Phys. Rev. Lett. 31, 1377 (1973). Quartz microbalance measurements of the superfluid content of a helium film as a function of the total coverge of the film. The superfluid fraction was observed to drop sharply to zero at a temperature-dependent total helium coverage. Later work has shown this observation to be consistent with the predictions of Kosterlitz and Nelson concerning the two-dimensional phase transition.
- VIII.5. "Critical Surface Density of the Superfluid Component in ⁴He Films," I. Rudnick, Phys. Rev. Lett. 40, 1454 (1978). A comment that shows that the early work on third sound in thin helium films is in agreement with the two-dimensional calculations of the value of σ_s at the transition temperature. (E)
- *VIII.6. "Study of the Superfluid Transition in Two-Dimensional ⁴He Films," D. J. Bishop and J. D. Reppy, Phys. Rev. Lett. 40, 1727 (1978). Elegant measurement of the superfluid mass and dissipation in the vicinity of the superfluid transition by means of a sensitive Andronikashvili technique. As the temperature is raised the superfluid mass is observed to drop dramatically but continuously to zero. The magnitude of the temperature dependent jump in σ_s is found to be in good agreement with the predictions of theory. [See also D. J. Bishop and J. D. Reppy, Phys. Rev. B 22, 5171 (1980).] (E)
- VIII.7. "Clustering and Percolation Transitions in Helium and Other Thin Films," J. G. Dash, Phys. Rev. B 15, 3136 (1977). Suggestion that ⁴He films grow in thickness by forming clusters or droplets until a critical thickness is reached. The role of surface imperfections in the nucleation of the droplets is stressed. Stresses that superflow onset is a percolation transition. The suggestions are controversial but have been stimulating. (E)

IX. VICINITY OF T_{λ}

A substantial effort has gone into the investigation of the region of the phase diagram in immediate proximity to the superfluid transition. The early work of Kellers, Buckingham, and Fairbank at temperatures within a few μK of the transition was in many ways well ahead of its time. A major reason for the substantial interest in the close proximity to T_{λ} has been the recognition that the case of liquid helium constitutes an extremely good system for the study of the critical properties of the transition. Thus liquid helium has been a testing ground for critical phenomena the flavor of which is illustrated by the Ahler's articles cited below.

- IX.1. "The Nature of the λ -Transition in Liquid Helium," M. J. Buckningham and W. M. Fairbank, in Progress in Low Temperature Physics (North-Holland, Amsterdam, 1961), Vol. 3, p. 80. Detailed description of the first high-precision measurements of the heat capacity of ⁴He near T_{λ} . Temperatures within a few microkelvin of the transition were attained and the logarithmic nature of the specific heat over four to five decades in reduced temperature was revealed. In many respects the measurements were well ahead of their time. (I)
- IX.2. "Heat Capacity near the Superfluid Transition in ⁴He at Saturated Vapor Pressure," G. Ahlers, Phys. Rev. A 3, 696 (1971). Precision measurements of the heat capacity of ⁴He in the vicinity of the lambda transition. The article illustrates in detail the care necessary for accurate measurements near the superfluid transi-

- tion. (I)
- IX.3. "Thermodynamics and Experimental Tests of Static Scaling and Universality Near the Superfluid Transition in "He Under Pressure," G. Ahlers, Phys. Rev. A 8, 530 (1973). Precision measurements of the specific heat of "He under pressure are reported in the vicinity of the superfluid transition. A detailed comparison with the theories of critical phenomena is made. (A)
- IX.4. "Thermal Expansion Coefficient, Scaling, and Universality near the Superfluid Transition of 'He under Pressure," K. H. Mueller, G. Ahlers, and F. Pobell, Phys. Rev. B 14, 2096 (1976). A detailed discussion of the experimental technique and measurements of the isobaric thermal expansion coefficient for 'He in the vicinity of the lambda point. The work includes a detailed comparison between the experimental results and relevant critical point theory. (I)
- IX.5. "Velocity and Attenuation of First Sound near the λ Point of Helium," M. Barmatz and I. Rudnick, Phys. Rev. 170, 224 (1968). A detailed report of sound resonance measurements in the vicinity of the lambda transition. Measurements of both the velocity and attenuation are reported to within a few microkelvin of the transition. (I)
- IX.6. "Light Scattering from First and Second Sound near the λ Transition in Liquid He," G. Winterling, F. S. Holmes, and T. J. Greytak, Phys. Rev. Lett. 30, 427 (1973). Brillouin scattering is routinely used to measure the spectrum of density fluctuations in liquids. In this work peaks in the scattered intensity are clearly observed both for first sound as well as for second sound, which is much harder to see. The changes in the second sound data as T_{λ} is approached are dramatic. (I)

X. ³He-⁴He MIXTURES

The addition of the lighter isotope ³He to ⁴He causes the superfluid transition to be suppressed to lower temperatures. The effect becomes more complicated, however, below 0.9 K. Below that temperature in the presence of gravity a mixture of ³He and ⁴He undergoes a phase separation whereby the lighter ³He-rich phase floats on top of the more dense ⁴He-rich phase. The lower ⁴He-rich phase remains a superfluid while the upper phase does not. The first observation of phase separation is a classic experiment.

- *X.1. "Phase Separation in ³He-⁴He Solutions," G. K. Walters and W. M. Fairbank, Phys. Rev. 103, 262 (1956). Discovery of phase separation in ³He-⁴He mixtures. Nuclear magnetic resonance techniques were used to unambiguously show that in the presence of a gravitational field a ³He-rich phase floats on top of a ³He-poor phase. (E)
- X.2. "Sound Measurements in Liquid and Solid ³He, ⁴He, and ³He— ⁴He Mixtures," J. H. Vignos and H. A. Fairbank, Phys. Rev. 147, 185 (1966). Measurement of the longitudinal velocity of sound are reported from 1 to 4.2 K and 1-150 atm. A time of flight technique was used and the effect of added ³He concentration on the phase diagram was carefully documented. (E)
- *X.3. "Phase Separation and the Superfluid Transition in Liquid ³He⁴He Mixtures," E. H. Graf, D. M. Lee, and J. D. Reppy, Phys. Rev. Lett. 19, 417 (1967). Precision measurement of the ³He-⁴He phase separation curve in the vicinity of its maximum. The measurements established that the λ line intersected the phase separation curve at its maximum and also showed that the phase separation curve failed to have a horizontal tangent (i.e., there was a cusp at the highest position of the curve). (E)
- X.4. "Number Density and Phase Diagram of Dilute 3 He- 4 He Mixtures at Low Temperatures," D. O. Edwards, E. M. Ifft, and R. E. Sarwinski, Phys. Rev. 177, 380 (1969). Detailed measurements on the solubility of 3 He in 4 He at low concentrations. In this work the T=0 K limiting solubility of 3 He in 4 He was measured. Also measured was the volume occupied by a 3 He atom immersed in 4 He. (I)
- X.5. "Dilute Solutions of ³He in ⁴He at Low Temperatures," J. C. Wheatley, Am. J. Phys. 36, 181 (1968). A readable tutorial review

- of the properties of dilute solutions of ³He in ⁴He. The treatment assumes no prior knowledge of ³He-⁴He mixtures. An extensive discussion of the operating principles of a dilution refrigerator is given including a careful thermodynamic analysis. (I)
- X.6. "Condensation of ³He Atoms onto Quantized Vortex Lines in Superfluid Helium," R. M. Ostermeier, E. J. Yarmchuk, and W. I. Glaberson, Phys. Rev. Lett. 35, 957 (1975). It is interesting to ask where the ³He resides in a ³He-⁴He mixture system with vortices present. Here, experiments on ion mobility in which the ions move along the cores of the vortex lines are used to show that there is a concentration dependent critical temperature below which the ³He atoms dress the vortex cores. (E)

XI. SUPERFLUIDITY IN 3He

In many ways the discovery of and early measurements in superfluid ³He paralleled the early development of ⁴He. In the case of ³He, however, the pace was much faster since no development such as the discovery of superconductivity diverted the investigators. The early work was completely dominated by those few laboratories that were in a position to realize the ultralow temperatures necessary for the measurements. The first suggestion that something of interest was available for study came during rather careful studies of the pressure-temperature coordinates of the melting curve in ³He. The melting curve is simply the line in the phase diagram that separates the solid phase from the liquid phase. Osheroff, Richardson, and Lee at Cornell University used a compressional cooling cell to make continuous measurements of the pressure versus temperature. The resulting chart recording trace was not smooth; rather, two anomalies labeled A and B by Osheroff et al. were observed both on cooling and on warming the cell. These features were taken to be evidence for a magnetic phase transition in the solid that was expected in the same temperature range. The tentative identification of a solid transition was quickly shown to be incorrect by Osheroff and his colleagues and the two transitions were clearly shown to be associated with what appeared to be two separate new fluid phases. These observations at Cornell University kindled a revolution in the field and a fever of activity took place reminiscent of the 1936-38 years in superfluid ⁴He. A key difference in the case of ³He, however, was the previous existence of the theory of superconductivity. This coupled with the early insight of Leggett allowed a rapid convergence of theoretical guidance. In fact, the microscopic theory of superfluid ³He is in a much better position than the theory of ⁴He for which no microscopic theory of similar power

An important part of our understanding of ³He is based on the description of the normal phase due to Landau and referred to simply as Fermi liquid theory. Although not directly on the subject of superfluid ³He we mention several important works in the normal liquid.

- XI.1. "The Theory of a Fermi Liquid," L. D. Landau, JETP (USSR) 30, 1058 (1956). A perturbation theory of the normal Fermi liquid is presented. The effective mass is calculated as is the compressibility and the magnetic susceptibility. (A)
- XI.2. "Oscillation in a Fermi Liquid," L. D. Landau, JETP (USSR) 32, 59 (1957). The propagation and absorption of waves in a Fermi liquid is presented. The Fermi liquid parameters are defined and related to the effective mass. Spin waves are also considered. The paper is a classic. (A)
- *XI.3. "Propagation of Zero Sound in Liquid ³He at Low Temperatures," W. R. Abel, A. C. Anderson, and J. C. Wheatley, Phys. Rev. Lett. 17, 74 (1966). Measurement of the velocity and attenuation of sound in normal ³He as a function of temperature at

- 15 and 45 MHz. A transition from ordinary sound to "collisionless" or zero sound is observed as the temperature is reduced. The experiments constitute a striking verification of Landau's Fermi liquid theory. [See also related work by B. E. Keen, P. W. Mathews, and J. Wilks, Proc. R. Soc. London A 284, 125 (1965).] (E)
- XI.4. "Measurements of the Melting Curve of Pure ³He Below the Minimum," J. L. Baum, D. F. Brewer, J. G. Daunt, and D. O. Edwards, Phys. Rev. Lett. 3, 127 (1959). First detailed measurements of the shape of the phase boundary in the P-T plane between liquid and solid ³He at low temperatures (0.1 < T < 0.5 K). The experiments confirmed the earlier experimental conclusions of Walters and Fairbank and the theory of Pomeranchuk. The existence of a minimum in the melting curve led to the important development known as Pomeranchuk refrigeration in which a sample of ³He liquid-solid mixture cools under the application of pressure. In such a device the superfluid phases of ³He were discovered in 1972. (E)

The general subject of superfluid ³He is more complicated and formidable in appearance than that of superfluid ⁴He. Fortunately, there are now several excellent review works that both discuss the theory in detail and provide appropriate connection to the rapidly growing body of experimental results. Particularly readable are the works discussed later in this section (Leggett, XI.23; Wheatley, XI.24; and Richardson and Lee in the book edited by Benneman and Ketterson, XIII.17).

We next cite a number of experimental reports and begin with an elementary but very well written article that provides general orientation.

- XI.5. "Superfluid Helium 3," N. David Mermin and David M. Lee, Sci. Am. 235 (6), 56 (1976). Clear description of the superfluid phases of ³He including both experimental observations and the essential ideas behind the theory. The discussion provides an intuitive feel for the quantum ideas that are involved and a comparison that clearly showed the essential difference between the cases of ³He and ⁴He. (E)
- *XI.6. "Evidence for a New Phase of Solid ³He," D. D. Osheroff, R. C. Richardson, and D. M. Lee, Phys. Rev. Lett. 28, 885 (1972). First observation of the A and B phases of superfluid ³He. The experiments were made in a Pomeranchuk cell with solid present and anomalies in the P-T phase line were interpreted as being due to new solid phases. Later work showed this interpretation to be incorrect and that the phase transitions were associated with the liquid. (E)
- *XI.7. "New Magnetic Phenomena in Liquid ³He Below 3 mK," D. D. Osheroff, W. J. Gully, R. C. Richardson, and D. M. Lee, Phys. Rev. Lett. 29, 920 (1972). Nuclear magnetic resonance measurements below 2.7 mK in ³He. The results showed a resonance frequency associated with the liquid phase that was temperature dependent. Below the B feature the NMR signal showed a drop in amplitude. The experiments were important in that they clearly showed that the A and B features were associated with the liquid ³He and not the solid. The frequency shifts in the A phase were to be crucial to the theoretical understanding. (I)
- *XI.8. "Observation of a Second-Order Phase Transition and its Associated P-T Phase Diagram in Liquid ³He," R. A. Webb, T. J. Greytak, R. T. Johnson, and J. C. Wheatley, Phys. Rev. Lett. 30, 210 (1973). First specific heat measurements across the normal fluid-a superfluid transition. The experiments were significant in that they not only showed the classic signature associated with the second-order phase transition but they were carried out at pressure for which no solid was in the experimental cell.
- XI.9. "Evidence for Superfluidity in the Newly Formed Phases of ³He," T. A. Alvesalo, Yu D. Anufriyev, H. K. Collan, O. V. Lounasmaa, and P. Wennerström, Phys. Rev. Lett. 30, 962 (1973). Measurements on the damping of a vibrating wire in ³He. The experiments were the first to show that the A and B phases of ³He had anomalous flow properties. (I)

- XI.10. "Viscosity Measurements in Superfluid 3He-B from 2 to 29 Bar," C. N. Archie, T. A. Alvesalo, J. D. Reppy, and R. C. Richardson, J. Low Temp. Phys. 42, 295 (1981). Measurements made simultanesously with vibrating wire and torsional viscometers are described in detail. The importance of mean-free-path effects is dramatically illustrated. The pressure dependence of the normal fluid viscosity is confirmed to be substantially greater than observed in the early work of Wheatley. (I)
- XI.11. "Propagation of Fourth Sound in Superfluid ³He," H. Kojima, D. N. Paulson, and J. C. Wheatley, Phys. Rev. Lett. 32, 141 (1974). Report of the propagation of pressure waves through a superleak in both the A and B phases of ³He. Measurements of ρ_s/ρ are presented for both phases showing no anomalous features at the A-B transition. The experiments clearly showed both phases to be superfluid phases. (I)
- XI.12. "Observation of Second Sound in ³He-A₁," L. R. Corruccini and D. D. Osheroff, Phys. Rev. Lett. 45, 2029 (1980). A novel experiment that provides strong experimental evidence that in ³He-A₁ only one spin species has undergone pairing. In the experiment second sound was generated acoustically and detected both with a capacitive microphone as well as with NMR. The work suggests the paired spins are aligned "up" relative to an applied magneitc field. (1)
- *XI.13. "Determination of the ³He Superfluid-Density Tensor for the A and B Phases," J. E. Berthold, R. W. Gianetta, E. N. Smith, and J. D. Reppy, Phys. Rev. Lett. 37, 1138 (1976). Measurements of the density of superfluid ³He by use of a torsional oscillator technique. The superfluid component ρ_s was observed to be dependent on the orientation of the applied magnetic field. A jump in the value of ρ_s at the A-B transition was observed. The anisotropic nature of ³He was firmly established by these studies. (E)
- XI.14. "Static Nuclear Magnetism in Extraordinary Liquid ³He," D. N. Paulson, R. T. Johnson, and J. C. Wheatley, Phys. Rev. Lett. 31, 746 (1973). Observation that the static magnetization in ³He-A is essentially independent of temperature while in ³He-B it decreases with a decrease in temperature. (I)
- XI.15. "Pulsed NMR Frequency Shifts in Superfluid ³He," D. D. Osheroff and L. R. Corruccini, Phys. Lett. 51A, 447 (1975). Measurements of the NMR frequency of ³He A and B are reported for which the nuclear magnetization is tipped far from equilibrium. The reduced frequency shift is observed to obey a simple universal curve as a function of the tipping angle and is independent of both temperature and external field. (I)
- XI.16. "Tip-Angle-Dependent Magnetic Relaxation in Superfluid ³He," R. A. Webb, Phys. Rev. Lett. 40, 883 (1978). A careful study of the relaxation of the longitudinal magnetization in ³He as a function of the initial rotation angle of the magnetization. The experiments in both ³He A and ³He B were carried out in several magnetic fields and demonstrated that the time evolution of the recovery showed a strong dependence on the initial rotation angle; there was observed a marked transition from exponential to nonexponential behavior. (A)
- XI.17. "Experimental Studies of Solitons in Superfluid ³He-A," C. M. Gould, T. J. Bartolac, and H. M. Bozler, J. Low Temp. Phys. 39, 291 (1980). Experiments are described in which magnetic textures are created and observed to move in superfluid ³He-A. Detection of the textures is by means of NMR and the observed properties allow the conclusion that solitons consistent with the model developed by Maki exist and move in the superfluid. (I)
- *XI.18. "Attenuation of Zero Sound and the Low-Temperature Transitions in Liquid ³He," D. T. Lawson, W. J. Gully, S. Goldstein, R. C. Richardson, and D. M. Lee, Phys. Rev. Lett. 30, 541 (1973). Sound attenuation measurements in ³He. Conducted in a magnetic field, the measurements revealed the A transition attenuation peak to split into two peaks as a function of the applied magnetic field. This appears to be one of the first clear observations of the A₁ phase. (Previous evidence for it was present in unpublished strain gauge measurements due to Osheroff et al.) (E)

- XI.19. "Field Splitting of the New Sound Attenuation Peak in ³He-B," O. Avenel, E. Varoquaux, and H. Ebisawa, Phys. Rev. Lett. 45, 1952 (1980). Observation of the J=2 multiplet splitting with magnetic field in the propagation of zero sound in ³He-B. The work provides a firm basis for the theoretical interpretation of the new sound mode as well as references to earlier work that first identified the mode. [See also, V. E. Koch and P. Wölfle, Phys. Rev. Lett. 46, 486 (1981).] (I)
- XI.20. "Specific Heat of Normal and Superfluid ³He on the Melting Curve," W. P. Halperin, C. N. Archie, F. B. Rasmussen, T. A. Alvesalo, and R. C. Richardson, Phys. Rev. B 13, 2124 (1976). Detailed results of specific heat measurements in ³He including the A₁ phase. (E)
- XI.21. "Pressure Dependence of the Specific Heat Jump at the Superfluid Transition and the Effective Mass of ³He," T. A. Alvesalo, T. Haavasoja, M. T. Manninen, and A. T. Soinne, Phys. Rev. Lett. 44, 1076 (1980). Measurements of the specific heat of ³He from 1-10 mK are reported as a function of pressure. A determination of the ³He effective mass is made as is the value of the heat capacity jump at the superfluid transition. The results that are of high accuracy suggest that the Fermi liquid parameters should be lowered by 30% and if confirmed elsewhere this will require the reinterpretation of a number of other experimental results. (E)
- XI.22. "Observations of a Critical Current in ³He B," J. P. Eisenstein, G. W. Swift, and R. E. Packard, Phys. Rev. Lett. 43, 1676 (1979). A study is made of the temperature dependence of the maximum rate of flow of ³He B through a channel of irregular geometry. The temperature dependence is observed to be $[1 T/T_c]^{3/2}$ in agreement with predictions based on Ginzburg-Landau theory. The work is the first such critical superfluid current measurement in ³He B and is patterned after the wide body of literature in ⁴He. (E)
- XI.23. "A Theoretical Description of the New Phases of Liquid ³He,"

 A. J. Leggett, Rev. Mod. Phys. 47, 331 (1975). A review of the theoretical understanding of the superfluid phases of ³He. Landau-Fermi liquid theory is reviewed as is the weak coupling BCS theory applied to superfluids. The Anderson-Brinkmann-Morel and Balian-Werthamer theories identified with superfluid ³He-A and ³He-B are discussed in detail. (A)
- XI.24. "Experimental Properties of Superfluid ³He," J. C. Wheatley, Rev. Mod. Phys. 47, 415 (1975). A review of the experimental properties of superfluid ³He as of late 1974. Thermodynamic and magnetic properties are discussed in detail. The emphasis is experimental but appropriate connection to the theoretical development and the implications of the experimental results are freely discussed. (I)
- XI.25. "Further Experimental Properties of Superfluid ³He," J. C. Wheatley, Progress in Low-Temperature Physics, edited by D. F. Brewer (North-Holland, Amsterdam, 1978), Vol. VIIA, p. 1. A further review of the properties of superfluid ³He with special emphasis on orbital effects. Topics include ultrasonic propagation, critical velocity effects, and some discussion of ion mobility. Also discussed are a number of spin dynamics effects including nonlinear ringing, T₁ effects, wall pinning, and linewidths. Some discussion of thermodynamic data is also given. (A)
- XI.26. "Spin and Orbital Dynamics of Superfluid ³He," W. F. Brinkman and M. C. Cross, in *Progress in Low-Temperature Physics*, edited by D. F. Brewer (North-Holland, Amsterdam, 1978), Vol. VIIA, p. 105. A detailed review of the theoretical understanding of the dynamics of superfluid ³He. Superflow energetics is discussed in detail as are singularities and textures. Spin and orbital dynamics are treated in full. (A)
- XI.27. "Sound Propagation and Kinetic Coefficients in Superfluid ³He," P. Wölfle, in *Progress in Low-Temperature Physics*, edited by D. F. Brewer (North-Holland, Amersterdam, 1978), Vol. VIIA, p. 191. A review of sound propagation and transport phenomena in superfluid ³He. The review begins with a discussion of the normal Fermi liquid. It goes on to discuss sound propagation and the kinetic coefficients of superfluid ³He in detail. (A)

Another excellent review is that by Richardson and Lee in XIII.16.

XII. REVIEW ARTICLES

Given the great breadth of the subject of superfluidity there are no current reviews of the entire subject. There are many excellent reviews that are topical. A number of these have been included among the cited manuscripts in the earlier sections. Topical reviews are also to be found in some of the books listed in Sec. XII. Particularly good are the topical reviews to be found in the recent two volume set edited by Bennemann and Ketterson (XIII.16 and XIII.17).

XIII. BOOKS

Here there are a number of excellent works. The last really complete books are those of Wilks and Keller (which complement each other well) which became available in the late 1960s. More recent books tend to be more topical or collections of very complete topical review articles (e.g., the two volume set edited by Bennemann and Ketterson). (See also Sec. XIV.)

- XIII.1. The Quest for Absolute Zero, K. Mendelssohn (Oxford University, London, 1966). A highly readable account of the history of developments leading to ever lower temperatures. Topics covered include various technical developments beginning with the liquefaction of oxygen, superfluidity, and superconductivity. The treatment is historical and places the development of superfluidity into a wider context. An excellent book. (E)
- XIII.2. Superfluids (Vol. II, Macroscopic Theory of Superfluid Helium), F. London (Wiley, New York, 1954) (now available as a Dover paperback). Historically significant, this second volume of London's famous two-volume work on superconductivity and superfluidity provides a review of the known properties of superfluid helium up until the early 1950s. The book places emphasis on the theoretical interpretation of the available experimental results. ³He and ³He-⁴He mixtures are discussed although the data available at the early date are few. (I)
- XIII.3. Liquid Helium, K. R. Atkins (Cambridge University, Cambridge, 1959). A survey of work in the field up until the late 1950s. Relevant experimental work is treated as is theory where the physical significance rather than the mathematical formalism is emphasized. Topics treated include the full spectrum of work on ⁴He but also included is ³He-⁴He mixtures as well as liquid ³He. Although outdated seriously in a number of areas the book is highly readable. (I)
- XIII.4. Superfluid Physics, C. T. Lane (McGraw-Hill, New York, 1962). A classic textbook on superfluid helium. While the work is obviously not at all up to date, it contains a detailed treatment of work in superfluid helium up until about 1960 over a broad front. (I)
- XIII.5. Men of Physics: L. D. Landau (Pergamon, New York, 1965).

 Contains an introduction both to superfluid ⁴He and Fermi liquids. The introduction is followed by a bibliography of works by Landau and a collection of reprints including his works on both superfluidity in ⁴He and Fermi liquid theory. (I)
- XIII.6. Experimental Superfluidity, R. J. Donnelly (University of Chicago, Chicago, 1967). A general introduction to superfluid ⁴He at an intermediate level. Vorticity and ions are given special emphasis. The book concludes with tables of useful numerical information pertinent to helium. (I)
- XIII.7. The Properties of Liquid and Solid Helium, J. Wilks (Clarendon, Oxford, 1967). A readable and quite detailed account of the state of knowledge of the quantum liquids as of the mid-1960s. Numerous experimental results are discussed and placed in context. Coverage includes the solid phase. (I)
- XIII.8. Helium-3 and Helium-4, W. E. Keller (Plenum, New York,

- 1969).
- A readable and detailed account of the liquid phases of helium. Coverage includes a chapter on gaseous helium and a substantial coverage of critical velocity and flow effects. (I)
- XIII.9. An Introduction to Liquid Helium, J. Wilks (Clarendon, Oxford, 1970). An excellent introduction to the subject of liquid helium including normal fluid ³He. The text contains relevant developments through the mid-1960s and provides the reader with a thorough grounding in the field in a highly digestable manner. (E)
- XIII.10. Helium 4, Z. M. Galasiewicz (Pergamon, New York, 1971). ISBN 08-015816-1. The book contains a substantial introduction by the author to the subject to liquid helium. The main part of the book consists of a selection of reprints of mostly early works in the field of superfluidity. The works were chosen for their importance but most are easily readable. An excellent "reader" for study. (I)
- XIII.11. Superfluid Hydrodynamics, S. J. Putterman (North-Holland, American Elsevier, Amsterdam, New York, 1974). A detailed discussion of liquid helium from the point of view of hydrodynamics—both classical and quantum. Many classical hydrodynamic effects are discussed in detail. The reader will encounter occasional speculative remarks and errors. (A)
- XIII.12. Superfluidity and Superconductivity, D. R. Tilley and J. Tilley (Wiley, New York, 1974). A text that stresses the similarities between superfluidity and superconductivity and describes the fundamental properties of both systems. The chapter on vorticity is particularly well done. Given its limited scope, the book does not provide a broad spectrum introduction to the full realm of superfluidity. (A)
- XIII.13. Superfluid Helium, edited by J. F. Allen (Academic, New York, 1966). LOC 66-25087. Proceedings of a symposium on superfluid helium held in 1965. Topics treated include vorticity, ions, He II flow properties and films with the principal emphasis on flowing helium in constricted geometries. (I)
- XIII. 14. Liquid and Solid Helium, edited by C. G. Kuper, S. G. Lipson, and M. Revzen (Wiley, New York, 1975). ISBN 7065-1459-9. Proceedings of a conference on superfluidity. A broad spectrum of topics is presented by means of topical reviews and shorter reports on specific experimental work. Special emphasis was placed on superfluid ³He as the conference took place in the period of explosive developments in the subject. Solid helium is also treated. (I)
- XIII.15. The Helium Liquids, J. G. M. Armitage and I. E. Farquhar (Academic, London, 1975). ISBN 0-12-062550-4. Proceedings of the Scottish Universities Summer School in Physics (1974). The volume contains a set of tutorial lectures that deal with both theory and experiment in ³He and ⁴He as well as ³He-⁴He mixtures. The first chapter is a quite readable introduction to superfluidity by G. V. Chester. (A)
- XIII.16. The Physics of Liquid and Solid Helium, edited by K. H. Bennemann and J. B. Ketterson (Wiley, New York, 1976), Part I. ISBN 0-471-06600-1. The first volume of a two-volume series, which contains self-contained review articles. Topics covered include: the phenomenological theory of superfluid 4 He, experiments near T_{λ} in 4 He and 3 He- 4 He mixtures, vortices and ions, light scattering from helium, microscopic calculations and solid helium. (A)
- XIII.17. The Physics of Liquid and Solid Helium, K. H. Bennemann and J. B. Ketterson (Wiley, New York, 1978), Part II. ISBN 0-471-06600-1. Collection of self-contained review articles. Topics covered are Landau-Fermi liquid theory, ³He-⁴He solutions at low temperature, theory of superfluid ³He, superfluid ³He (experiment and theory summary), monolayer helium, helium films thicker than one layer, and neutron scattering. (A)
- XIII.18. Quantum Liquids, edited by J. Ruvalds and T. Regge (North-Holland, Amsterdam, 1978). Proceedings of the "Ettore Majorana" summer school on quantum liquids. Review and tutorial articles are included, which cover critical phenomena and T_{λ} , light scattering and neutron scattering from helium, and several discussions of ³He. (A)

211

- XIII.19. Progress in Low-Temperature Physics, edited by C. J. Gorter (and recently D. F. Brewer) (a series) (North-Holland, Amsterdam, 1955—). A series of books each containing a selection of review articles (typically) on subjects of current interest in low-temperature physics. A wide range of subjects is treated and only a fraction of the articles have to do with superfluid helium. Nonetheless, some exceptional articles are to be found here. (A)
- XIII.20. Proceedings of the International Conferences on Low-Temperature Physics. An international conference on low-temperature physics has been held triannually since 1972 and biannually for many years before then. The proceedings represent the state of the art in many fields of low-temperature physics including superfluid helium and techniques. The volumes have various publishers and editors and are available in most physics libraries. (A)

XIV. TEXTBOOKS

Quite readable and often detailed discussions of superfluidity in the case of ⁴He or Fermi liquids are available in a number of textbooks. Some of these books are written so as to cover a broad range of topics, others are specific to quantum liquids.

- XIV.1. Heat and Thermodynamics (5th ed.), Mark W. Zemansky (McGraw-Hill, New York, 1968). A very clear discussion of classical thermodynamics. Chapter 15 contains a survey of many of the basic properties of superfluid ⁴He including the various sounds, helium films, and persistent flow. (E)
- XIV.2. States of Matter, David L. Goodstein (Prentice-Hall, Engelwood Cliffs, NJ, 1975). A unified text in which thermodynamics and statistical mechanics are freely used to describe solids, liquids, and gases. Chapter 5 contains a good discussion of the fundamental properties of superfluid ⁴He and a clear discussion of the Feynman theory. (I)
- XIV.3. Fluid Mechanics, L. D. Landau and E. M. Lifshitz (Pergamon, New York, 1959). A classic textbook in fluid dynamics that is also used as a standard reference. Chapter 16 deals specifically with superfluid helium and includes the two-fluid model, hydrodynamics, and the propagation of sound. Included are several problems with solutions. (I)
- XIV.4. Statistical Physics, L. D. Landau and E. M. Lifshitz (Addison-Wesley, Reading, MA, 1958). A classic text in statistical physics with sections devoted to Bose-type quantum liquids and superfluidity, and Fermi quantum liquids. The treatment is clear but very brief. (I)
- XIV.5. Statistical Physics, E. M. Lifshitz and L. P. Pitaevskii (Pergamon, New York, 1980), Part 2. ISBN 0-08-023072-5. A companion volume to Landau and Lifshitz's original Statistical Physics. An excellent work at the graduate level, this book contains an extensive discussion of Fermi liquid theory and superfluidity in ⁴He. There is also a full discussion of Green's function and diagrammatic techniques. Hydrodynamic fluctuations, superconductivity, and electromagnetic phenomena are also treated. (I)
- XIV.6. Statistical Mechanics, R. P. Feynman (Benjamin, Reading, MA, 1972). A collection of edited lecture notes on the general topic of statistical mechanics. In spite of the format the book is quite readable with a chapter devoted to superfluidity. The basic properties of the superfluid are reviewed and Feynman's wave function is described in detail. An intuitive discussion of the origin of the idea of vortex lines is presented. (I)
- XIV.7. An Introduction to the Theory of Superfluidity, I. M. Khalatnikov (Benjamin, New York, 1965). A classic work that details the

- theory of superfluidity and includes elementary excitations, hydrodynamics, kinetic phenomena with some discussion of impurities. Contains a valuable reference list to Soviet work on helium through the early 1960s. (A)
- XIV.8. Quantum Theory of Many-Particle Systems, A. L. Fetter and J. D. Walecka (McGraw-Hill, New York, 1971). ISBN 07-020653-8. An excellent graduate-level text on nonrelativistic many-particle physics with an emphasis on second quantization and field theory. The book includes a review of the statistical mechanics of Fermi and Bose systems, an extensive treatment of diagrammatic techniques and in Chapter 14 a discussion of superfluidity including a review of the experimental facts, the Landau quasiparticle theory and a treatment of the interacting Bose gas. (A)
- XIV.9. The Theory of Quantum Fluids, D. Pines and P. Nozières (Benjamin, New York, 1966). An extensive treatment of the Fermi liquid with emphasis in the first half of the book to the neutral case. (A)
- XIV.10. The Theory of Interacting Fermi Systems, P. Nozières (Benjamin, New York, 1964). A discussion of infinite Fermi systems at zero temperature. The treatment is general in tone with the result that it may be applied to ³He. The Landau quasiparticle is discussed as are response theory, perturbation methods, and interactions. (A)
- XIV.11. The Many-Body Problem in Quantum Mechanics, N. H. March, W. H. Young, and S. Sampanthar (Cambridge University, Cambridge, 1967). A text on many-body theory including second quantization and perturbation theory. Separate chapters treat Fermi fluids and many-boson systems. Included is a treatment of the Feynman and Bogoliubov theories. (A)
- XIV.12. Theory of Quantum Fluids, E. Feenberg (Academic, New York, 1969). A microscopic description of ⁴He and normal ³He. There is heavy emphasis on the case of ⁴He, particularly variational techniques. (A)

XV. FILMS

There are few films on superfluid helium but the few there are, are rather uniformly good.

- XV.1. Liquid Helium II, The Superfluid (by A. Leitner), Michigan State University, Instructional Media Center, East Lansing, Mich. 48823. (39 min, 16 mm, black and white, sound), LC F1A64-81 (1963). This remains an excellent introduction to superfluid helium with comments on the two-fluid model and clear visual demonstrations of the fountain effect, second sound, viscosity vanishing, and mobil superfluid films. (I)
- XV.2. The Unusual Properties of Liquid Helium (by I. Rudnick) available directly from Isadore Rudnick, Department of Physics, UCLA, Los Angeles, CA 90024 (17 min, 16 mm, color, sound). This film won first prize in the category of Technical and Scientific Films at the 21st Annual San Francisco International Film Festival 1977. A number of the properties of the superfluid are shown in remarkable clarity including the fountain effect, vanishing viscosity, etc. The film is a professional production. A teacher's manual accompanies the film. (I)
- XV.3. Superfluid Helium (by J. F. Allen), currently in production is the fifth edition with information on availability (expected 1982) available from J. F. Allen, Dept. of Physics, St. Andrews University, North Haugh, St. Andrews KY169SS, Scotland (4th ed., 30 min, 16 mm, color, sound). This is a well-done film with a number of demonstrations not ordinarily considered. Included are demonstrations of the fountain effect, superleaks, and the mobil film. Unusual demonstrations include inertial flow oscillations.