Measurement of Weak Coupling Constant blahblah

1 Introduction

The muon, a fundamental particle produced in the upper atmosphere as a secondary product of cosmic ray collisions, was originally discovered in 1936 []. It decays via the weak interaction with a mean decay lifetime of $2.2\mu s$, longer than every known particle other than the neutron []. With muons comprising 80% of cosmic ray flux at sea level, the muon is a good candidate for the study of the weak force [].

Our experiment consists of two main components: the muon lifetime measurement and the muon mass measurement. In section 2, *Background*, we introduce the theoretical basis for these measurement as well as that of muon creation and decay. We describe the experimental setup which consists of a system of three scintillators and photomultiplier tubes (PMTs) in *Setup*. Using this system, the cosmic ray muons passing through the scintillators and their decay products can be detected along with their enegy (*Procedure*). The muon lifetime and mass results are presented in *Results* and *Discussion* with the relevant statistical analysis of data, and compared to previous experimentally established values. Finally, we use the muon mass and lifetime values to calculate the weak force coupling constant, g_w.

2 Background

Muons (μ^-) and antimuons(μ^+) are the most numerous charged particles at sea level [**?**]. Most of them are produced at a height of about 15 km in decays of charged kaons and pions which are formed via the interaction of cosmic ray particles with the Earth's upper atmosphere [**?**]. The muon is produced in a weak decays as shown for example in Figure and at sea level forms 80% of cosmic ray flux.

2.1 Muon Decay

In free space, negatively charged muons decay weakly into an electron, muon neutrino, and electron antineutrino [**?**] (Figure):

$$\mu^- \rightarrow e^- \nu_\mu \overline{\nu_e} \tag{1}$$

with a corresponding antimatter process of

$$\mu^+ \rightarrow e^+ \overline{\nu_\mu} \nu_e \tag{2}$$

The muon lifetime is approximately 2.2μs [?], second only to the lifetime of the neutron. In matter, another decay is possible for μ^- via nucleus capture:

$$\mu^- p^+ \rightarrow n \nu_\mu \tag{3}$$

(**insert stuff about the muon capture lifetime here**)

The decay of the muon is desribed by an exponential function

$$N(t) = N_0 e^{-\Gamma_\mu t} \tag{4}$$

here Γ_μ is the decay rate, which gives the decay lifetime $\tau_\mu = 1/\Gamma_\mu$.

In this experiment, we measure the time between the start event, when a $\mu^-(\mu^+)$ comes to rest in a scintillator in the lab and the stop event, which signals the emission of $e^-(e^+)$ in the muon decay. The histogram of the recorded times is then fit to (4) to give the lifetime of the muon.

2.2 Effects of Relativistic Time Dilation

Even with velocities within a percent of the speed of light, the travel time of the muon from the point of creation in the atmosphere takes approximately 50μs - over 20 decay lifetimes - to reach the ground. According to Newtonian physics, the flux would be reduced by a factor of over 10^{10}. However, the flux of muons at sea level, where the lab is located, remains large at 10^{-2} cm$^-$2s$^-$1sr$^-$1, only reduced by a factor of 5 from the peak flux at 15km [**?**].

This effect is due to the relativistic time dilation predicted by Special Relativity. While in the frame of the laboratory, the time of flight of the muons is 50μs, the muon itself experiences a proper time reduced by a factor of γ: $t_\mu = \gamma t_{lab}$, where $\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}}$. Since the particles are travelling close to the speed of light, the relativistic correction becomes non-negligible. With muon speeds ranging from .994c to .998c, the proper time experienced by the muon is between 3.2 and 5.5μs, less than 2 lifetimes on average.

The time in flight is still on the same order, even greater than, the lifetime of muon decay which our experiment seeks to measure. Nevertheless, the time the muons experience in the atmosphere prior to stopping in the detector has no effect on the decay rate measurement. While we do sample fewer short decay times and slow moving muons, this fact simply decreases the amount of data without affecting the parameters of the exponential.

2.3 Muon Mass

The experimental setup which records the time between a muon stopping event and a released electron or positron event needs to be only slightly modified in order to measure the mass of the muon.

In order to measure muon mass, consider the μ^- decay (1) (the antimatter decay analysis is equivalent to the following). For a muon that is stopped in the scintillator, the center of mass frame is the same as the lab frame. Then, since the electron mass is only 0.5% of the muon mass and the neutrinos are essentially massless, to a good approximation we can assume that the rest energy of the muon is fully converted to the kinetic energy of the e^- and neutrinos. Then, measuring the energy distribution of the emitted electrons will provide information regarding the initial muon mass. Specifically, due to conservation of momentum, the magnitude of the electron momentum, p_e, must equal the sum of the neutrino momenta. Fixing p_e along the x-axis, the only possible scenario of the decay is pictured in Figure **??** where $0 \leq \theta < \pi/2$ and is measured from the negative x-axis.

The total momentum of the neutrinos is minimized when there is no y component, that is $\theta = 0$. In this case, $p_e = p_e^{max} = \frac{1}{2}p_{tot}$. Again neglecting electron mass, we have $E_e = p_e c$, so the energy of the electron is maximized when the momentum is maximized, and so

$$E_e^{max} = \frac{1}{2}E_\mu = \frac{1}{2}m_\mu \tag{5}$$

Then by measuring the electron energy spectrum, which is a β decay spectrum with a cutoff at $\frac{1}{2}m_\mu$, we can find the maximum electron energy and thus measure the muon mass.

2.3.1 Energy Calibration

Our intruments enable us to measure the height of the pulse from the electron that registers on the PMT. In order to convert the pulse height distribution to an energy distribution, we have to calibrate the pulse height voltage in terms of energy left in the detector.

To do so, we find the pulse height which corresponds to the minimum ionization energy (approximately $2\text{MeVg}^{-1}\text{cm}^2$) by finding the maximum in the peak height distribution of muons which pass through all three scintillators. Since the incoming muons have a relatively random distribution of momenta, the most common energy loss amount will be near a local extremum, which in this case is a minimum.

See Appendix for more details?

Assuming the detector response is linear, and measuring the scintillator density and thickness, we find a convesion ratio between pulse voltage and electron energy.

2.4 Weak Force Coupling Constant

The decay rate Γ_μ is proportional to the square of the amplitude of the decay diagram (Figure **??**, which depends on the product of the couplings at each vertex. In this case, the coupling at each of the two vertices is proportional to $\sqrt{G_F}$, the Fermi constant, so we have

$$\Gamma_\mu \propto G_F^2 \tag{6}$$

A more involved calculation gives that the lifetime of the muon is

$$\tau_\mu = \frac{192\pi^3\hbar^7}{G_F^2 m_\mu^5 c^4} \tag{7}$$

where c is the speed of light, $\hbar$ is Planck's constant, and m_μ is the rest mass of the muon.

Once we establish the value of m_μ, we can find the Fermi constant G_F and the

The weak decay of the muon is the clearest of all weak interaction phenomena in both its experimental and theoretical aspects. Thus, the muon decay is an effective means of studying the weak force, and specifically finding the weak coupling constant g_w.

$$\tag{8}$$