10 Problems of Dynamic Programming

AMMAR FATHIN SABILI

1. Tribonacci

$$F(1) = F(2) = F(3) = 1$$

$$F(n) = F(n-1) + F(n-2) + F(n-3)$$

Given n, find $F(n)$ in modulo $1{,}000{,}000{,}007$.

Constraints:

$1 \leq n \leq 2{,}000{,}000$

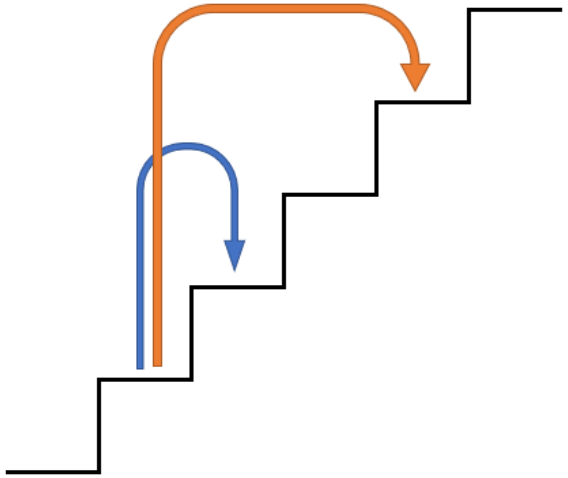
$1, 1, 1, 3, 5, 9, 17, 31, 57, 105, 193, 355, \cdots$

2. Ladders

There is a ladder with n steps.

You can climb either **1 step** or **3 steps** (by jumping) at once.

How many ways you can climb all n steps?



Constraints:

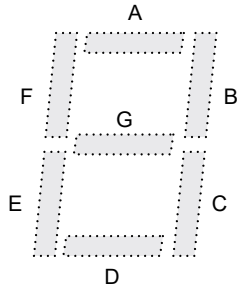
$1 \leq n \leq 2{,}000{,}000$

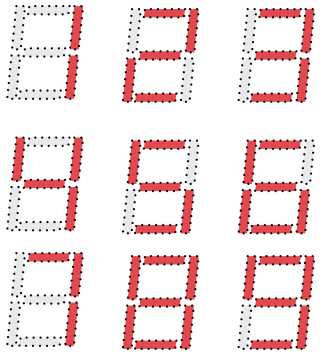
3. Sum of the Digits

You have a 7-segment-display displaying digits 1 to 9 (not 0)

How many string it can display such that:

Sum of the digits is exactly n?





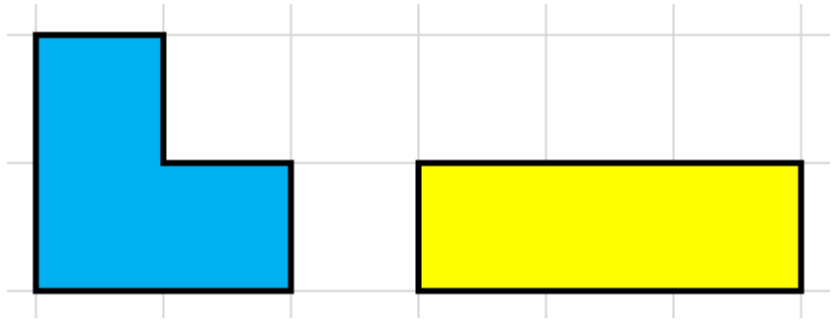
Constraints:

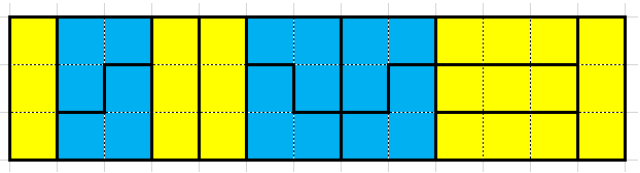
$1 \leq n \leq 2{,}000{,}000$

4. Tiling

How many ways you can fully tile these two kind of pieces to **a $3 \times n$ board**?

(Also, each column should have the same colour!)





Constraints:

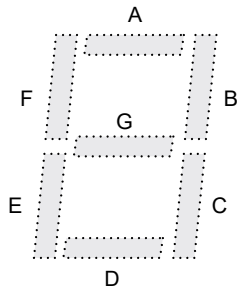
$1 \leq n \leq 2{,}000{,}000$

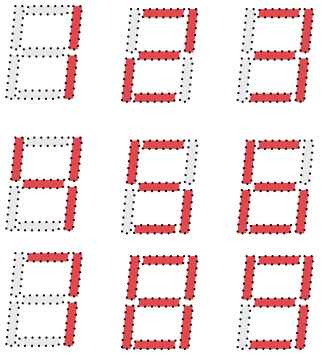
5. Sum of the Segments

You have a 7-segment-display displaying digits 1 to 9 (not 0)

How many string it can display such that:

Sum of the segments is exactly n?





Constraints:

$1 \leq n \leq 2{,}000{,}000$

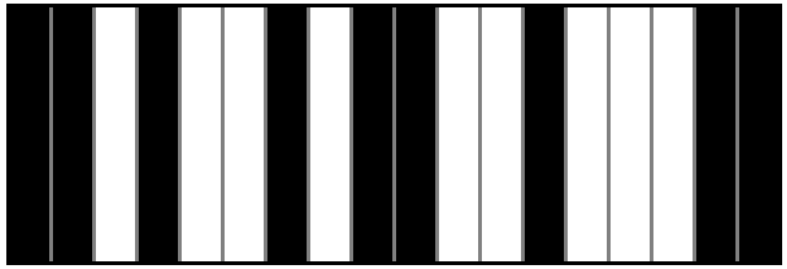
6. Barcode

A barcode of length n is a series of n bars coloured either black or white.

A barcode is valid if it

doesn't have 3 consecutive black or 4 consecutive white bars.

How many valid barcodes are there for length n?



Constraints:

$1 \leq n \leq 2{,}000{,}000$

7. Fastest Way to Finish

There are n cells, cell i has a number A_i written on it.

You are from cell 1 and want to go to cell n.

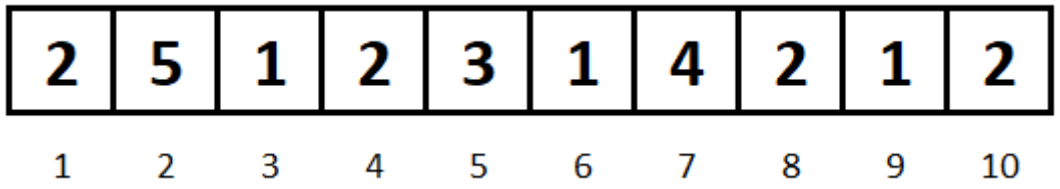
In one step, you can go from cell x to either cell $x + 1$ or $x + A_i$.

What will be the minimum steps?

Constraints:

$1 \leq n \leq 2{,}000{,}000$

$1 \leq A_i \leq 2{,}000{,}000$



8. Fastest Refreshing Way to Finish

There are n cells, cell i has a number A_i written on it.

You are from cell 1 and want to go to cell n, **visiting at least k different cells**.

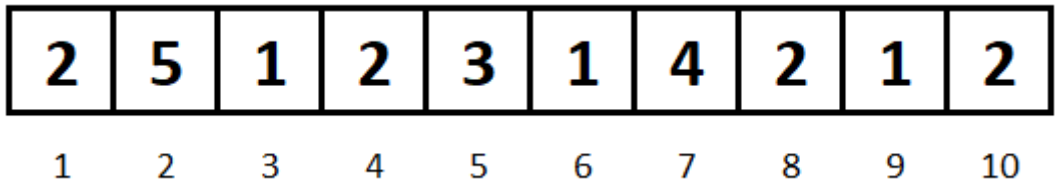
In one step, you can go from cell x to either cell $x + 1$ or $x + A_i$.

What will be the minimum steps?

Constraints:

$1 \leq k \leq n \leq 2{,}000$

$1 \leq A_i \leq 2{,}000$



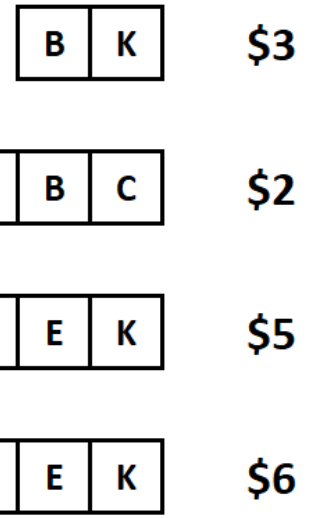
9. Vitamins

You want to buy vitamins **A, B, C, D, E, K**.

The shop sells in n bundles e.g. (A+B) for $3, (B+E+K) for $7, etc.

You don't mind having multiple similar vitamins as long as it's complete.

What is the minimum price you should pay?



Constraints:

$1 \leq n \leq 2{,}000$

$1 \leq price \leq 2{,}000{,}000$

10. Cutting Chocolate

Initially, there is a piece of chocolate with n cells. Cell i has a toughness of A_i.

You want to cut this chocolate into n smaller pieces.

One cutting of a piece is to pick which cells to be separated.

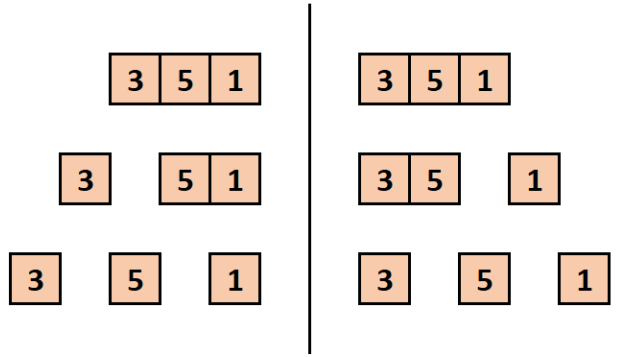
In one cutting, you need the energy equals to the sum of toughness of the piece.

What is the minimum energy to be taken?

Constraints:

$1 \leq n \leq 200$

$1 \leq A_i \leq 2{,}000{,}000$



Extra. Prefix Sums

Given a matrix of $n \times n$ cells each having an integer.

Given q queries, find the sum of some submatrix.

Constraints:

$1 \leq n \leq 2{,}000$

$1 \leq q \leq 200{,}000$

1	5	7	8
3	2	2	4
8	9	3	5
7	8	3	1